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MASTER

COMPUTATION OF LOSSES DUE TO MULTIPLE SCATTERING
OF CHARGED PARTICLES IN THIN FOILS

J. B. Ball

OAK RIDGE NATIONAL LABORATORY

operated by

UNION CARBIDE CORPORATION

for the

U.S. ATOMIC ENERGY COMMISSION

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ELECTRONUCLEAR DIVISION
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COMPUTATION OF LOSSES DUE TO MULTIPLE SCATTERING OF CHARGED PARTICLES IN THIN FOILS

J. B. Ball

ABSTRACT

The losses of charged particles from a detector aperture, due to multiple scattering of the particles in thin layers of material preceding the detector, are calculated with the aid of a high-speed digital computer. The root-mean-square multiple-scattering angle is computed after the method of Moliere, and graphs are presented for protons and alpha particles scattering in nitrogen, aluminum, argon, copper, silver, and gold, and for deuterons and tritons scattering in aluminum and copper. The range of particle energy covered is from 5 to 100 Mev. A second program is used to construct a map from which losses, as a function of the root-mean-square multiple-scattering angle and the scattering configuration, may be directly read. The FORTRAN listings for both programs are given.

INTRODUCTION

In the study of nuclear reactions induced by or producing fast charged particles, it is frequently necessary to pass the charged particles through thin layers of material before or during the detection process. In passing through such foils the particle may undergo many small-angle Coulomb deflections. Under certain conditions this multiple scattering may result in enough spreading of the particle trajectories to affect the experimental results. The careful design of an experiment often requires an estimate to be made of the effects of multiple scattering.

Since these small-angle Coulomb deflections are randomly oriented, the first-order distribution of the scattering should follow a normal, or Gaussian, distribution:

$$P(\theta) d\theta = (2\theta/\theta_{av}^2) \exp(-\theta^2/\theta_{av}^2) d\theta, \quad (1)$$

where $P(\theta)$ is the probability for scattering into an angular increment $d\theta$ about a spatial angle θ measured with respect to the axis of the unscattered particles, and θ_{av} is the root-mean-square angle of the multiple scattering. The distribution is normalized to unity over all space.

Since, in general, both the area of the scattering foil and of the detector must be considered of finite size, each point on the surface of the scatterer will give rise to a two-dimensional Gaussian distribution of multiply scattered particles. The sum of these two-dimensional Gaussians is then incident on the plane containing the detector aperture.

The calculation of multiple-scattering losses thus requires two distinct steps: (1) the computation of the root-mean-square multiple-scattering angle, which characterizes the multiple-scattering distribution, and (2) the calculation of the fraction of all these distributions, arising from the scatterer surface,

falling outside the detector aperture. While the first of these is relatively easy to compute, the second requires a long and tedious numerical integration over the two-dimensional Gaussians. Dickinson and Dodder¹ have presented a graphical method of performing this integration by constructing a two-dimensional Gaussian graph and counting appropriate area increments. This reduces the required time of solution to about half an hour. They have also calculated the losses for a few representative cases, but the interpolation or extrapolation of these values to match a given experimental situation generally involves sufficient uncertainty to make a specific graphical integration necessary.

In this work, advantage of the fast computing capabilities of the IBM 7090 is made in computing enough cases to allow the mapping of a multiple-scattering-loss surface, so that the loss from any given experimental situation may be read directly. The computer is also used to calculate the root-mean-square multiple-scattering angle θ_{av} as a function of foil thickness for a number of different particles, at energies between 5 and 100 Mev, scattering from nitrogen, argon, aluminum, copper, silver, and gold.

CALCULATION OF THE ROOT-MEAN-SQUARE MULTIPLE-SCATTERING ANGLE

The first analytical treatment of multiple scattering was given by Williams.² He considered the scattering as following the Gaussian distribution given by Eq. (1). The root-mean-square multiple-scattering angle needed to establish the width of the distribution was then calculated from the relationship

$$\theta_{av}^2 = \chi_c^2 \ln (\theta_1/\theta_2), \quad (2)$$

where χ_c is an angular unit, and the logarithm term is a measure of the mean number of scatterings. In the logarithm term, θ_1 is the angle beyond which a particle traversing the foil suffers, on the average, only one collision, and θ_2 is the angle below which no Coulomb scattering occurs due to the shielding of the nuclear charge by the orbital electrons. Williams has solved for θ_2 in the classical case, valid when $\gamma \gg 1$, and with the Born approximation, valid for $\gamma \ll 1$. Here $\gamma = Zz/137\beta$, where z is the charge on the particle, Z is the nuclear charge of the scatterer, and $\beta = v/c$ for the particle. Neither of these methods is valid near $\gamma = 1$, and the results of the two methods do not extrapolate through this region. Since the condition $\gamma = 1$ occurs, for example, for protons of 22 Mev on copper, or alphas of 28 Mev on aluminum, this singularity in the simple theory occurs in the region of interest for this paper.

¹W. C. Dickinson and D. C. Dodder, *Rev. Sci. Instr.* **24**, 428 (1953).

²E. J. Williams, *Revs. Modern Phys.* **17**, 217 (1945); *Proc. Roy. Soc. (London)* **A169**, 531 (1939).

A more complete theory of multiple scattering has been developed by Moliere.³ An expression for the minimum scattering angle was developed quantum mechanically that is valid for all values of γ . In Moliere's theory, the root-mean-square multiple-scattering angle θ_{av} is calculated from the expression

$$\theta_{av}^2 = B\chi_c^2, \quad (3)$$

where χ_c is the same angular unit used in Eq. (2).

Moliere also treats the multiple-scattering distribution in more detail. He represents the probability distribution with the series

$$P(\theta) d\theta = \left(\frac{\theta}{\theta_{av}^2} \right) \left[2 \exp(-\theta_{av}^2) + \frac{1}{B} f^1(\theta) + \frac{1}{B^2} f^2(\theta) + \dots \right] d\theta. \quad (4)$$

The first term in the series is just the Gaussian distribution of Eq. (1). The second term in the series represents the contribution to the distribution from large-angle single scatterings. The third term represents the contribution from large-angle double scatterings, etc.

The mean number Ω_b of scatterings in the Moliere theory is given by the equation

$$\log \Omega_b = 5.215 + \log [t\gamma^2 Z^{-2/3}/A(1.13 + 3.76\gamma^2)], \quad (5)$$

where t is the scatterer thickness in mg/cm^2 , and Z and A are the atomic number and atomic weight, in atomic mass units, of the scatterer respectively. Instead of using Ω_b to determine the width of the Gaussian, Moliere defines the quantity B which is slightly greater than Ω_b and allows the first term of Eq. (4) to yield a more accurate approximation to the total distribution. The quantity B is related to Ω_b by the equation

$$e^B/B = \Omega_b/1.167. \quad (6)$$

In this work the spatial distribution of the multiple scattering will be approximated by the first term of Eq. (4), that is, Eq. (1), and the root-mean-square scattering angle θ_{av} will be computed from Eq. (3), with B defined in Eq. (6) and χ_c^2 given by the equation

$$\chi_c^2 = 4\pi N t z^2 Z^2 e^4 / (p v)^2, \quad (7)$$

where N is the number of atoms per milligram of scatterer, t is the scatterer thickness in mg/cm^2 , z is the charge of the scattered particle, Z is the atomic number of the scatterer material, $p = mv/\sqrt{1 - \beta^2}$ for the particle, and e is the electron charge.

The approximation of using only the first term in Eq. (4) to represent the probability distribution may introduce errors up to 10% (ref 1). Nigam *et al.*⁴ have pointed out that certain assumptions made by Moliere have introduced errors in the expression for the minimum scattering angle that affect the higher-order terms of the distribution. This difference does not seem to be important for this paper, where only

³G. Moliere, *Z. Naturforsch.* 3a, 78 (1948) (also available as AEC-tr-2119 and AERE Lib/Trans 626).

⁴B. P. Nigam, M. K. Sundaresan, and Ta-You Wu, *Phys. Rev.* 115, 491 (1959).

the Gaussian term is considered. The values of θ_{av} calculated for protons and alpha particles scattering in nitrogen, aluminum, argon, copper, silver, and gold over an energy range 5 to 100 Mev are shown in Figs. 1-12. The program used to compute θ_{av} is described in more detail in Appendix I; a FORTRAN listing is also given.

In the simple theory of Williams,² the root-mean-square scattering angle θ_{av} depends only on the charge and not on the mass of the particle. This is not true in the case of the Moliere treatment, and the values of θ_{av} for protons, deuterons, and tritons deviate somewhat from each other. The values of θ_{av} calculated for deuterons and tritons in aluminum and copper are shown in Figs. 13-16. The deviations become smaller as the scatterer Z is increased. It is interesting to note that the spacing of the energies in these figures is approximately logarithmic. This makes it convenient to use a logarithmic scale for interpolating between energies.

CALCULATION OF THE MULTIPLE-SCATTERING LOSSES

The geometry considered in these calculations is shown in Fig. 17. Almost all the experimental configurations encountered can be treated in this geometry. The notation used is that of Dickinson and Dodder.¹

Since the angles entering into the computation are never very large, it is convenient to make the approximation $\tan \theta \approx \theta$. The value of the root-mean-square displacement then becomes

$$\rho_{av} = \theta_{av} D, \quad (8)$$

and the distribution of Eq. (1) becomes

$$P(\rho) d\rho = 2\rho/\rho_{av}^2 \exp(-\rho^2/\rho_{av}^2) d\rho. \quad (9)$$

Since the unit area in the plane containing ρ is given by

$$dA = 2\pi\rho d\rho, \quad (10)$$

the probability per unit area on the detector surface will be

$$P(\rho) = (\pi\rho_{av}^2)^{-1} \exp(-\rho^2/\rho_{av}^2). \quad (11)$$

By using Eq. (11) to compute the probability for scattering into a given area of the detector surface from a given area of the scatterer surface and taking appropriate sums over both surfaces, the loss of particles may be computed for any combination of R_1/ρ_{av} . A map of the multiple-scattering losses computed in this fashion is shown in Fig. 18. The terms R_1 and R_2 are the detector and scatterer radii in either order, since the geometry of the scattering is symmetrical about the midplane. The multiple-scattering loss computation is discussed in more detail in Appendix III; the FORTRAN listing for the program is also given.

Estimation of the multiple-scattering losses with Fig. 18 requires the following steps. The value of θ_{av} is estimated from one of the graphs in the preceding section. The value of ρ_{av} is computed from Eq. (8). The values of R_1/ρ_{av} and R_2/ρ_{av} are then computed, and the loss corresponding to these values is read from Fig. 18.

Fig. 1. Root-Mean-Square Multiple-Scattering Angle, as a Function of Incident Particle Energy and Scatterer Thickness, for Protons on Nitrogen.

Fig. 2. Root-Mean-Square Multiple-Scattering Angle, as a Function of Incident Particle Energy and Scatterer Thickness, for Protons on Aluminum.

Fig. 3. Root-Mean-Square Multiple-Scattering Angle, as a Function of Incident Particle Energy and Scatterer Thickness, for Protons on Argon.

Fig. 4. Root-Mean-Square Multiple-Scattering Angle, as a Function of Incident Particle Energy and Scatterer Thickness, for Protons on Copper.

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Fig. 5. Root-Mean-Square Multiple-Scattering Angle, as a Function of Incident Particle Energy and Scatterer Thickness, for Protons on Silver.

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Fig. 6. Root-Mean-Square Multiple-Scattering Angle, as a Function of Incident Particle Energy and Scatterer Thickness, for Protons on Gold.

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Fig. 7. Root-Mean-Square Multiple-Scattering Angle, as a Function of Incident Particle Energy and Scatterer Thickness, for Alpha Particles on Nitrogen.

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Fig. 8. Root-Mean-Square Multiple-Scattering Angle, as a Function of Incident Particle Energy and Scatterer Thickness, for Alpha Particles on Aluminum.

Fig. 9. Root-Mean-Square Multiple-Scattering Angle, as a Function of Incident Particle Energy and Scatterer Thickness, for Alpha Particles on Argon.

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Fig. 10. Root-Mean-Square Multiple-Scattering Angle, as a Function of Incident Particle Energy and Scatterer Thickness, for Alpha Particles on Copper.

Fig. 11. Root-Mean-Square Multiple-Scattering Angle, as a Function of Incident Particle Energy and Scatterer Thickness, for Alpha Particles on Silver.

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Fig. 12. Root-Mean-Square Multiple-Scattering Angle, as a Function of Incident Particle Energy and Scatterer Thickness, for Alpha Particles on Gold.

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Fig. 13. Root-Mean-Square Multiple-Scattering Angle, as a Function of Incident Particle Energy and Scatterer Thickness, for Deuterons on Aluminum.

Fig. 14. Root-Mean-Square Multiple-Scattering Angle, as a Function of Incident Particle Energy and Scatterer Thickness, for Deuterons on Copper.

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Fig. 15. Root-Mean-Square Multiple-Scattering Angle, as a Function of Incident Particle Energy and Scatterer Thickness, for Tritons on Aluminum.

Fig. 16. Root-Mean-Square Multiple-Scattering Angle, as a Function of Incident Particle Energy and Scatterer Thickness, for Tritons on Copper.

Fig. 17. Scattering Configuration Considered in This Work.

Fig. 18. Multiple-Scattering-Loss Map. Loss contours are in percent.

Appendix I

ROOT-MEAN-SQUARE MULTIPLE-SCATTERING ANGLE PROGRAM

By making the appropriate substitutions in Eq. (7), the expression for χ_c^2 becomes

$$\chi_c^2 = (1.810 \times 10^{-10}) t z^2 Z^2 (1 - \beta^2) / A \mu^2 \beta^4, \quad (12)$$

and the expression for B , from Eqs. (5) and (6), becomes

$$e^B/B = (1.45 + 10^5) t \gamma^2 / (1 + 3.33 \gamma^2) A Z^{2/3}. \quad (13)$$

In the above equations, t is the thickness of the scatterer in mg/cm^2 , Z is the nuclear charge of the scatterer, A is the atomic weight of the scatterer in atomic mass units, z is the charge of the particle, μ is the reduced mass of the particle, $\beta = v/c$ of the particle, and $\gamma = zZ/137\beta$.

Since in many cases the particle will lose a significant amount of energy in traversing the scatterer, the program calculates χ_c^2 and B for the β corresponding to the average energy of the particle traversing the foil. The program computes the final energy of the particle after passing through the scattering foil, the quantity required to calculate the average β , by calling a dE/dx subroutine. The FORTRAN listing of this dE/dx subroutine is given in Appendix II.

The program calculates the quantity B by setting $B = 4.4$, computing the value of e^B/B , and comparing this value with the right side of Eq. (13). If the value is less than the right side, B is incremented in steps of 0.1 until the left side is equal to the right side. The value of B thus determined is accurate to within 1 or 2%, which is more than sufficient for the purpose of this work. If the value of e^B/B , when $B = 4.4$, is found to be greater than the right side of Eq. (13), the particle has not undergone sufficient collisions to make the Moliere theory of multiple scattering applicable, and the program will print out the message `B LESS THAN 4.5 - THEORY NOT VALID`.

A typical output sheet from the program is shown in Fig. 19. The top two lines of the heading identify the particular case being computed. With the initial conditions as given by the second line, that is, the charge and mass of the particle and of the scatterer and the mean ionization potential of the scatterer, the root-mean-square multiple-scattering angle is computed for as many combinations of initial energy and scatterer thicknesses as desired. The program outputs a list of the initial energy, the scatterer thickness, the final energy as computed by the dE/dx subroutine, the value of B from Eq. (13), using the average value of β , the value of χ_c^2 from Eq. (12), using the average value of β , and the value of θ_{av} from the square root of the product of B and χ_c^2 . This last column is labeled `MOLIERE RMSSA`. The program also computes, for the sake of comparison, the value of θ_{av} from Eq. (9) of Dickinson and Dodder,¹ which employs a nonrelativistic range-energy equation due to Bethe, and this column is labeled `BETHE RMSSA`. The values in the column `MOLIERE RMSSA` were used for Figs. 1-16.

The input to the program consists of four types of cards:

The Title Card. - This card contains any desired alphanumeric information to identify the case. The contents of this card are printed out in the first line of the heading for each page. For the output sheet shown in Fig. 19, the title card contained the words `PROTONS ON COPPER`.

RMS MULTIPLE SCATTERING ANGLE FOR PROTONS ON COPPER

PART Z# 1. PART A# 1. SCAT Z#29. SCAT A# 64. SCAT I#371.0

INITIAL ENERGY	SCATTERER THICKNESS	FINAL ENERGY	MOLIERE B	MOLIERE CHI2	MOLIERE RMSSA	BETHE RMSSA
70.00	0.5	69.997	B IS LESS THAN 4.5 - THEORY NOT VALID			
70.00	1.0	69.994	5.2	1.17E-07	0.000778	0.000771
70.00	2.0	69.988	6.0	2.33E-07	0.001183	0.001172
70.00	3.0	69.982	6.5	3.50E-07	0.001508	0.001494
70.00	5.0	69.969	7.1	5.83E-07	0.002034	0.002015
70.00	10.0	69.939	7.9	1.17E-06	0.003035	0.003007
70.00	20.0	69.878	8.7	2.33E-06	0.004507	0.004465
70.00	30.0	69.816	9.1	3.50E-06	0.005647	0.005594
70.00	50.0	69.694	9.7	5.85E-06	0.007533	0.007463
70.00	70.0	69.571	10.1	8.21E-06	0.009103	0.009018
70.00	100.0	69.387	10.5	1.18E-05	0.011108	0.011004
70.00	200.0	68.769	11.2	2.37E-05	0.016294	0.016139
70.00	300.0	68.148	11.7	3.59E-05	0.020486	0.020289
70.00	500.0	66.893	12.3	6.08E-05	0.027357	0.027088
70.00	700.0	65.621	12.6	8.67E-05	0.033058	0.032726
70.00	1000.0	63.675	13.0	1.27E-04	0.040700	0.040293

Fig. 19. Output Listing from the Root-Mean-Square Multiple-Scattering-Angle Program.

The Primary Data Card. — This card inputs the particle charge, the particle mass, the scatterer charge, the scatterer mass, and the scatterer mean excitation potential (in units of ev) in five successive fields of eight columns each. The decimal point is assumed to be at the right edge of each field unless it is specifically entered. One of these cards is required for each title card.

The Secondary Data Cards. — Data are read from the first nine fields of eight columns each. The first field contains the initial energy of the particle, and the remaining eight fields contain the desired thicknesses of the absorber. The calculation will be performed for every entry of absorber thickness. As many of these cards as needed can follow the title and primary card. The program will ignore blank fields corresponding to no scatterer, but each of these secondary cards must have the particle energy in the first field.

The Control Card. — Normally the program will print 25 lines of information from the cases on the secondary data cards and then begin a new page, repeating the heading of the previous page. If it is desired to change the information on the title and the primary data card, a control card containing a 0 in the first eight-column field must be inserted before the new title card. In addition, the control card allows a certain flexibility in the output of the information from the secondary cards. By placing a control card containing a -1 in the first field before a secondary data card, the information from the secondary data card will be printed on a new page. This allows groups of secondary data cards to produce separate pages of output without inserting a new title and primary card.

The values of the mean ionization potential used in this work are given in Table 1. The FORTRAN listing for the program is shown in Table 2.

Table 1. Mean Ionization Potentials Used to Compute Energy Losses

Scatterer Material	I (ev)	Source
Nitrogen	80.5	^a
Aluminum	166	^b
Argon	207	^a
Copper	371	^b
Silver	585	^b
Gold	1037	^b

^aW. A. Aron, B. G. Hoffman, and F. C. Williams, AECU-663, UCRL-121 (1949).

^bH. Bichsel, R. F. Mozley, and W. A. Aron, *Phys. Rev.* 105, 1788 (1957).

Table 2. FORTRAN Listing for the Root-Mean-Square Multiple-Scattering-Angle Program

```

C      PROGRAM FOR COMPUTING RMS MULTIPLE SCATTERING ANGLE          15880108
C      SECOND REVISION      1-MAR-1962              J.B.BALL        15880109
      DIMENSION TITLE(10),SCAT(8),DUMMY1(10),DUMMY2(40),DUMMY3(40)  15880111
11     READINPUTTAPE10,41,TITLE,PZ,PA,SZ,SA,SI                     15880113
12     WRITEOUTPUTTAPE9,43,TITLE,PZ,PA,SZ,SA,SI                    15880115
      L#0                                                            15880117
13     READINPUTTAPE10,42,EI,(SCAT(I),I#1,8)                       15880119
      IF(EI)12,11,14                                                15880121
14     REDPA#PA*SA/(PA+SA)                                          15880123
      SZTTP#SZ**0.6666667                                           15880124
      DC24I#1,8                                                     15880125
      IF(SCAT(I))24,24,15                                           15880202
15     ST#SCAT(I)                                                  15880204
      N#1                                                            15880206
      CALL SDODX(PZ,PA,EI,SZ,SA,SI,ST,N,NLIM,DUMMY1,              15880208
      IDUMMY2,DUMMY3,EF)                                           15880210
      IF(EF)24,24,16                                                15880212
16     N#(12.0*EI/EF)-11.0                                         15880214
      IF(N-2)18,17,17                                              15880215
17     CALL SDODX(PZ,PA,EI,SZ,SA,SI,ST,N,NLIM,DUMMY1,              15880216
      IDUMMY2,DUMMY3,EF)                                           15880218
      IF(EF)24,24,18                                                15880220
18     EA#(EI+EF)/2.0                                              15880222
      BETA2#1.0-(931.1/((EA/PA)+931.1))*2                          15880224
      ALPHA2#((SZ*PZ/137.0)**2)/BETA2                             15880302
      L#L+1                                                         15880304
      IF(L-25)20,19,19                                             15880306
19     WRITEOUTPUTTAPE9,43,TITLE,PZ,PA,SZ,SA,SI                  15880308
      L#0                                                            15880310
20     RHS#(1.243E5)*ST*ALPHA2/(SA*SZTTP*(1.0+3.33*ALPHA2))      15880312
      B#4.4                                                         15880314
      CLHS#(EXPF(B))/B                                              15880316
      IF(CLHS-RHS)22,21,21                                         15880318
21     WRITEOUTPUTTAPE9,44,EI,ST,EF                                15880320
      GCT024                                                         15880322
22     B#B+0.1                                                     15880324
      CLHS#(EXPF(B))/B                                              15880402
      IF(CLHS-RHS)22,23,23                                         15880404
23     CHI2#(1.81E-1C)*ST*(1.0-BETA2)*((PZ*SZ/(REDPA*BETA2))**2)/SA  15880406
      RMSSA#SQRTF(B*CHI2)                                           15880408
      BRMSSA#SQRTF((B*SZ*LOGF(EI/EF))/                             15880410
      1(7360.0*PA*LOGF(297.0*SQRTF(BETA2/SZ))))                  15880411
      WRITEOUTPUTTAPE9,45,EI,ST,EF,B,CHI2,RMSSA,BRMSSA           15880413
24     CONTINUE                                                    15880415
      GCT013                                                         15880417
41     FCRMAT(10A6/5F8.0)                                          15880420
42     FCRMAT(9F8.0)                                               15880422
43     FORMAT(35H1RMS MULTIPLE SCATTERING ANGLE FOR ,10A6/1H0,    15880502
      13X7HPART Z#F3.0,4X7HPART A#F3.0,4X7HSCAT Z#F3.0,          15880504
      24X7HSCAT A#F4.0,4X7HSCAT I#F5.1//9H0 INITIAL,            15880506
      33X9HSCATTERER,3X5HFINAL,4X7HMCLIERE,3X7HMOLIERE,          15880508
      45X7HMCLIERE,6X5HBETHE/8H ENERGY,4X9HTHICKNESS,           15880510
      53X6HENERGY,6X1HB,7X4HCHI2,8X5HRMSSA,7X5HRMSSA)           15880512
44     FCRMAT(1H0,F8.2,3XF7.1,3XF8.3,                             15880514
      16X37HB IS LESS THAN 4.5 - THEORY NOT VALID)              15880516
45     FORMAT(1H0,F8.2,3XF7.1,3XF8.3,4XF5.1,4X1PE8.2,4X0PF8.6,4XF8.6)  15880518
      END(1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)

```

Appendix II

THE dE/dx SUBROUTINE

This is a general-purpose subroutine which employs a form of the dE/dx expression similar to that used by Aron, Hoffman, and Williams,⁵

$$-\frac{dE}{dx} = \frac{4\pi e^4 z^2}{mv^2} NZ \left[\ln \frac{2mv^2}{I} - \ln(1 - \beta^2) - \beta^2 \right], \quad (14)$$

where e is the electron charge, m is the electron mass, v is the velocity of the charged particle, z is the charge of the particle, $\beta = v/c$ for the charged particle, I is the mean excitation potential of the stopping material, and NZ is the number of electrons per unit volume of the stopping material.

With appropriate substitutions, the formula is reexpressed for this work as

$$-\frac{dE}{dx} = (3.071 \times 10^{-4}) \frac{z^2 Z}{\beta^2 A} \left[\ln \left(\frac{1.022 \times 10^6}{1 - \beta^2} \right) \frac{\beta^2}{I} - \beta^2 \right], \quad (15)$$

where the only new definitions are Z and A , the atomic number and atomic weight, in atomic mass units, of the stopping material. The value of the mean excitation potential I is in electron volts, and dE/dx has the units of Mev/mg cm⁻². Note that this form of the energy-loss equation does not contain the shell correction terms necessary at very low energies or the density factor necessary for very high energies.⁶

The program computes, by performing a numerical integration, the energy of a particle after the particle traverses an absorber. The absorber is divided into a number of increments. At each increment the program uses the initial particle energy to calculate a value of dE/dx and to estimate the particle energy after the absorber increment. This estimated energy is then used to obtain the average energy of the particle in traversing the increment. The dE/dx corresponding to this average energy is then used to compute the final particle energy after the absorber increment. The final energy is then used as the initial energy for the next increment. The entire process is repeated for each increment of the absorber.

The FORTRAN listing for the program is given in Table 3. Information is inputted and outputted from the subroutine by means of the argument list: PZ, PM, EINIT, ABSZ, ABSA, ABSI, ABSTH, N, NLIM, E, DEDX, RTABS, EFINAL. The first eight quantities are input required by the subroutine and are, in order, the charge of the particle, the mass of the particle in atomic mass units, the initial energy of the particle in Mev, the atomic number of the absorber material, the atomic weight of the absorber material in atomic mass units, the mean ionization potential of the absorber in ev, and the number of increments desired for the numerical integration. The last five quantities in the list are output from the subroutine. If the thickness of the absorber exceeds the range of the particle, the subroutine will return control to the calling program as soon as the particle energy becomes negative. In this case the

⁵W. A. Aron, B. G. Hoffman, and F. C. Williams, AECU-663, UCRL-121 (1949).

⁶R. M. Sternheimer, *Phys. Rev.* **115**, 137 (1959).

value of NLIM is the number of the absorber increment causing the energy to go negative. The next three quantities in the list are arrays. The array E contains the initial particle energy before the particle traverses each absorber increment, the array DEDX contains the value of dE/dx corresponding to each entry in the E array, and the array RTABS contains the total absorber traversed to reach the corresponding energy in the E array. The final argument in the list, EFINAL, is the particle energy after the particle traverses the entire absorber.

In the root-mean-square multiple-scattering program only the final particle energy is of interest, and no use is made of the array information. The value of N is set by the program and depends on the thickness of the absorber.

The root-mean-square multiple-scattering angle program chooses N in the following way: The subroutine is called with N set equal to 1. The final energy calculated is returned to the program, and the value of N for the numerical integration is then obtained from the expression

$$N = 12 \left(\frac{E_{\text{FINAL}}}{E_{\text{INIT}}} \right) - 11. \quad (16)$$

The value of N is restricted by the program to be a maximum of 40. This provides sufficient accuracy over the energy range of interest in this program.

For use in other programs, the size of the E, DEDX, and RTABS arrays may be easily enlarged by changing the dimension statement.

Table 3. FORTRAN Listing for the dE/dx Subroutine

```
C      SLROUTINE FOR COMPUTING ENERGY LOSS OF CHARGED PARTICLES IN  
C      TRAVERSING AN ABSORBER J. B. BALL 21-FEB-1962  
SLROUTINE SDEDX(PZ,PM,EINIT,ABSZ,ABSA,ABSI,ABSTH,N,NLIM,E,DEDX,  
I RTABS,EFINAL)  
DIMENSION E(40),DEDX(40),RTABS(40)  
FLTGN#N  
DABSTH#ABSTH/FLTGN  
ARG1#(3.071E-4)*(PZ**2)*ABSZ/ABSA  
ARG2#(1.022E6)/ABSI  
EPART#EINIT  
DO 103 I#1,N  
E(I)#EPART  
SQBETA#1.0-(931.1/((E(I)/PM)+231.1))**2  
DEDX(I) #(ARG1/SQBETA)*(LOGF(ARG2*SQBETA/(1.0-SQBETA))-SQBETA)  
FLTGI#I  
RTABS(I) # (FLTGI-1.0)*DABSTH  
NLIM#I  
IF(ABSTH)100,104,100  
100 EF1#EPART-DEDX(I)*DABSTH  
IF(EF1)101,101,102  
101 EPART#EF1  
GO TO 104  
102 EA#(EPART+EF1)/2.0  
SQBETA#1.0-(931.1/((EA/PM)+231.1))**2  
ADEDX#(ARG1/SQBETA)*(LOGF(ARG2*SQBETA/(1.0-SQBETA))-SQBETA)  
EPART#EPART-ADEX#DABSTH  
IF(EPART)104,104,103  
103 CONTINUE  
104 EFINAL#EPART  
RETURN  
END(1,1,0,0,0,0,0,0,0,0,0,0,0,0)
```

Appendix III

PROGRAM FOR COMPUTING MULTIPLE-SCATTERING LOSSES

The problem of performing the numerical integration can be easily seen from Fig. 17. The vector r_1 defines a point on the scatterer surface which has a maximum radius of R_1 . The vector r_2 defines a point on the detector surface which has a maximum radius of R_2 . The probability per unit area of multiple scattering from the point r_1 to the point r_2 is given by Eq. (11) and the vector difference ρ . It is necessary to calculate the probability at each value of r_2 from all points throughout the range of r_1 and Φ . The total fraction of particles passing through the scattering foil and scattering inside the detector aperture will then be the sum of all the probabilities properly normalized for the areas. For this computation, the scatterer and detector areas are divided into small-area elements, and the value of ρ is calculated for the distance between the centers of these elements.

In the first program these area increments were all chosen to be the same for the respective apertures. Although this made the normalization easier, it was found to be rather cumbersome, since a large number of radial divisions were required to obtain a significant number of divisions near the center of the apertures. In the program used to construct Fig. 18, the radii of the respective apertures were divided into a number of equal lengths and the areas computed for each area segment.

The scatterer radius R_1 is divided into N parts. The average radius for the n th annulus on the scatterer surface will then be

$$r_1(n) = (n - 1/2)R_1/N . \quad (17)$$

Similarly, the detector radius is divided into M parts, and the average radius for the m th annulus on the detector surface will be

$$r_2(m) = (m - 1/2)R_2/M , \quad (18)$$

and the outer diameter of each detector annulus is

$$R_2'(m) = mR_2/M . \quad (19)$$

Since there is symmetry about $\Phi = 180^\circ$, Φ being the angle between the two radius vectors, Φ need only be incremented from 0 to π and the resulting sum doubled. The angle Φ is divided into $2I$ parts, and the i th angle between radius vectors is

$$\Phi(i) = (i - 1/2)\pi/I . \quad (20)$$

This particular method of incrementing Φ is chosen so that the symmetry about $\Phi = \pi$ is readily separable. The value of the root-mean-square displacement ρ is a function of the two average radius vectors and the angle between them and is solved for by the law of cosines:

$$\rho^2 = r_1^2(n) + r_2^2(m) - 2r_1(n)r_2(m) \cos \Phi(i) . \quad (21)$$

The total probability for scattering into the detector aperture is

$$\Pi = 2 \sum_{m=1}^M \sum_{n=1}^N \sum_{i=1}^I P(\rho) \cdot A \cdot F , \quad (22)$$

where $P(\rho)$ is the probability per unit area for multiple scattering with a root-mean-square displacement of ρ as given by Eq. (11), A is the area of the detector segment, and F is the fractional area of the scatterer surface represented by the scatterer segment. The quantity F normalizes the incident particle flux to unity.

The area of each annulus on the scatterer surface defined by a given $r_1(n)$ is

$$a_1(n) = 2\pi r_1(n)R_1/N. \quad (23)$$

Since the total area of the scatterer is πR_1^2 , the fractional contribution from each annulus will be

$$f_1(n) = 2r_1(n)/R_1N. \quad (24)$$

Each annulus is divided into $2I$ parts, so that the fractional area of each scatterer is

$$F = r_1(n)/R_1NI. \quad (25)$$

Similarly, the area of each detector annulus is

$$a_2(m) = 2\pi r_2(m)R_2/M. \quad (26)$$

This is also divided into $2I$ parts, each of which will receive an identical flux of scattered particles due to the symmetry about the central axis. However, since the total contribution over the entire annulus simply requires multiplying by the number of parts of the annulus, in this calculation the entire annulus can be taken as the area segment of the detector surface:

$$A = 2\pi r_2(m)R_2/M. \quad (27)$$

Substituting Eqs. (11), (25), and (27) into Eq. (22), the probability for scattering into the detector aperture becomes

$$\Pi = \frac{4R_2}{\rho_{av}^2 R_1 M N I} \sum_{m=1}^M r_2(m) \sum_{n=1}^N r_1(n) \sum_{i=1}^I \exp\left(\frac{-\rho^2}{\rho_{av}^2}\right). \quad (28)$$

With the aid of Eqs. (17) and (18) this simplifies to

$$\Pi = \frac{4R_2^2}{\rho_{av}^2 M^2 N^2 I} \sum_{m=1}^M (m - 1/2) \sum_{n=1}^N (n - 1/2) \sum_{i=1}^I \exp\left(\frac{-\rho^2}{\rho_{av}^2}\right). \quad (29)$$

This program utilizes the form of the equation given by (28), because it is possible to extract more information from this form. The partial sum at each value of m will correspond to the contribution inside a detector area of radius $R_2'(m)$ given by Eq. (19). Thus, for any given value of the scatterer radius R_1 , the program will compute the multiple-scattering loss for detector apertures corresponding to each division of R_2 .

An output listing from the program is shown in Fig. 20. The heading outputs the following quantities: CASE, an identification number; RMSSA, the root-mean-square multiple-scattering angle characterizing the distribution; SRMAX, the radius of the scatterer denoted above as R_1 ; DRMAX, the radius of the detector denoted above as R_2 ; D , the distance between the foils measured in the same units as R_1 and R_2 ; MI, the number of angular increments from $\Phi = 0$ to π ; MN, the number of divisions of the scatterer radius; MM, the number of divisions of the detector radius; RHOAV, the value of ρ_{av}

PROGRAM FOR NUMERICAL INTEGRATION WITH CONSTANT RADIAL INCREMENT

CASE#108 RMSSA#0.0100 SRMAX# 1.0 DRMAX# 4.0 D# 100.00
 MI# 10 MN# 20 MM# 80 RHOAV# 1.000 ASRMAX# 1.00 RTP#1.0001250

INDEX M	DR(M)	PROB(M)	TPROB(M)	TLOSS(M)
1	0.050	0.15804	0.15804	36.78245
2	0.100	0.47275	0.63080	36.92032
3	0.150	0.78334	1.41414	37.14935
4	0.200	1.08713	2.50126	37.46838
5	0.250	1.38150	3.88276	37.87583
6	0.300	1.66397	5.54673	38.36966
7	0.350	1.93221	7.47894	38.94744
8	0.400	2.18406	9.66299	39.60628
9	0.450	2.41756	12.08055	40.34296
10	0.500	2.63099	14.71154	41.15384
11	0.550	2.82288	17.53441	42.03500
12	0.600	2.99202	20.52643	42.98214
13	0.650	3.13748	23.66391	43.99074
14	0.700	3.25865	26.92256	45.05600
15	0.750	3.35517	30.27773	46.17292
16	0.800	3.42701	33.70475	47.33633
17	0.850	3.47442	37.17917	48.54094
18	0.900	3.49793	40.67711	49.78135
19	0.950	3.49834	44.17544	51.05214
20	1.000	3.47668	47.65212	52.34788
21	1.050	3.43424	51.08636	48.91364
22	1.100	3.37247	54.45883	45.54117
23	1.150	3.29303	57.75186	42.24814
24	1.200	3.19768	60.94953	39.05047
25	1.250	3.08830	64.03783	35.96217
26	1.300	2.96685	67.00468	32.99532

Fig. 20. Output Listing from the Multiple-Scattering-Loss Program.

from Eq. (21); ASRMAX, the dimensionless measurement of the scatterer aperture R_1/ρ_{av} ; RTP, the total fraction of particles within the detector radius DRMAX.

For a sufficiently large value of DRMAX, the value of RTP becomes a measure of the accuracy of the numerical integration, since the distribution over all space should normalize to unity.

The output also lists four quantities as a function of the index m of the partial sum. The column headed DR(m) is the detector radius corresponding to the value of m and is given in the dimensionless units R_2'/ρ_{av} . Each value of DR(m) can be considered as a separate R_2/ρ_{av} . The column headed PROB(m) represents the scattering into the detector annulus defined by the value of m . The column headed TPROB(m) represents the probability for scattering into all annuli of the detector surface up to and including the m th, that is, the probability for scattering inside a detector radius of the corresponding DR(m). The column headed TLOSS(m) is the probability for losing particles by scattering outside the detector radius given by DR(m). For all values of DR(m) greater than the scatterer radius, the value of TLOSS(m) is simply

$$\text{TLOSS}(m) = 1 - \text{TPROB}(m). \quad (30)$$

When the detector radius is smaller than the scatterer radius, the probability for losing particles is corrected for the fraction of the scatterer not subtended by the detector in the case of no scattering. For these cases,

$$\text{TLOSS}(m) = 1 - \text{TPROB}(m)[R_1/R_2'(m)]^2. \quad (31)$$

All probabilities are printed out as percent.

The results of the computation shown in Fig. 20 are plotted graphically in Fig. 21. This figure can be compared with Fig. 2 of ref 1.

The probability of losing particles is always a maximum when $R_1 = R_2$. The probability of losing particles as a function of R_1/ρ_{av} when $R_1 = R_2$ is shown in Fig. 22. The map in Fig. 18 was constructed with the aid of Fig. 22 and many different figures of the type shown in Fig. 21.

The FORTRAN listing of the numerical integration program is given in Table 4. The input to the program consists of one card per case. The card is divided into eight fields of eight columns each. The fields are in the order: CASE, RMSSA, SRMAX, DRMAX, D, MI, MN, MM, where these quantities have been defined in the output description. The values of CASE, MI, MN, and MM must be integers. For all the remaining fields the decimal point is assumed to be at the right edge of the field unless it is specifically entered. The quantity RMSSA is in units of radians.

Experience has shown that MI = 20 is a sufficient number of angular divisions for the area covered in Fig. 18. The radial divisions MM and MN may vary from 20 to 100, depending on the aperture size. An approximate running time for the program may be estimated by the relationship

$$\text{running time} = 2(\text{MI})(\text{MN})(\text{MM}) \text{ msec.}$$

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Fig. 21. Multiple-Scattering Losses as a Function of R_2/ρ_{av} , when $R_1/\rho_{av} = 1.0$.

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Fig. 22. Multiple-Scattering Losses as a Function of R/ρ_{av} , when $R = R_1 = R_2$.

Table 4. FORTRAN Listing for the Multiple-Scattering-Loss Program

```

C      PROGRAM FOR NUMERICAL INTEGRATION WITH CONSTANT RADIAL INCREMENT 15880106
C      PART TWO OF MULTIPLE SCATTERING COMPUTATION J.B.BALL 15880107
      DIMENSION INDEX(300), ASR(300), DR(300), PROB(300), TPROB(300), TLOSS(300) 15880108
      ID) 15880108
11 READ INPUT TAPE 10, 41, NCASE, RMSSA, SRMAX, DRMAX, D, MI, MN, MM 15880109
41 FORMAT(18, 4F8.0, 3I8) 15880110
      RHOAV#RMSSA*D 15880111
      ASRMAX#SRMAX/RHOAV 15880112
      A2#1.0/(RHOAV**2) 15880114
      FLTGM#MI 15880115
      FLTGMN#MN 15880116
      FLTGM#MM 15880117
      PI#3.141593 15880118
      SRI#SRMAX/FLTGMN 15880119
      DO20N#1, MN 15880120
      FLTGN#N 15880121
20 ASR(N)#(FLTGN-0.5)*SRI 15880122
      DRI#DRMAX/FLTGMN 15880123
      DELTA#PI/FLTGM 15880124
      SNORM#1.0*A2*DRI*SRI/(FLTGM*(SRMAX**2)) 15880125
      RTPROB#0.0 15880126
      DO23M#1, MM 15880127
      FLTGM#M 15880128
      BSUM#0.0 15880129
      INDEX(M)#M 15880130
      DR(M)#(FLTGM*DRI)/RHOAV 15880131
      ADR#(FLTGM-0.5)*DRI 15880132
      DO22N#1, MN 15880133
      ASUM#0.0 15880134
      DO21I#1, MI 15880135
      FLTGI#I 15880136
      THETA#(FLTGI-0.5)*DELTA 15880137
      RHO2#(ASR(N)**2)+(ADR**2)-2.0*ASR(N)*ADR*COSF(THETA) 15880138
      APROR#EXP(-A2*RHO2) 15880139
21 ASUM#ASUM+APROB 15880140
      BPROB#ASR(N)*ASUM 15880141
22 BSUM#BSUM+BPROB 15880142
      PROB(M)#SNORM*ADR*BSUM 15880143
      RTPROB#RTPROB+PROB(M) 15880144
      TPROB(M)#RTPROB 15880145
      IF(DR(M)-ASRMAX)24, 25, 25 15880146
24 TLOSS(M)#1.0-TPROB(M)*((ASRMAX/DR(M))**2) 15880147
      GOT023 15880148
25 TLOSS(M)#1.0-TPROB(M) 15880149
23 CONTINUE 15880150
      WRITE OUTPUT TAPE 9, 42, NCASE, RMSSA, SRMAX, DRMAX, D, 15880151
      IMI, MN, MM, RHOAV, ASRMAX, RTPROB, (INDEX(M), DR(M), 15880152
      IPROB(M), TPROB(M), TLOSS(M), M#1, MM) 15880153
42 FORMAT(67H1 PROGRAM FOR NUMERICAL INTEGRATION WITH CONSTANT RADIAL 15880303
      IL INCREMENT/ 78H10 CASE#13, 5X6HRMSSA#F6.4, 4X6HSRMAX#F5.1, 15880304
      15X6HDRMAX#F5.1, 5X2HD#F7.2/6HD MI#13, 2X3HMN#13, 15880305
      12X3HMM#13, 4X6HRHOAV#F6.3, 4X7HASRMAX#F6.2, 4X4HRT#F9.7// 15880306
      11CHD INDEX M, 8X5HDR(M), 8X7HPROB(M), 9X8HTPROB(M), 7X8HTLOSS(M)/ 15880307
      1(IH0, 4X13, 8X0PF7.3, 7X2PF9.5, 6X2PF10.5, 7X2PF10.5)) 15880308
      GOT011 15880309
      END 15880310

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