LEPTONIC AND HADRONIC SYMMETRIES

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ABSTRACT

Two models for a gauge theory of the weak and electromagnetic interactions of leptons and hadrons are considered. The first model is based on the group $[SU(3)]^4 = SU(3) \times SU(3) \times SU(3) \times SU(3)$, the fundamental representation of which is spanned by left and right-handed lepton and quark triplets. The lepton triplet involves the ν_e and $\bar{\nu}_\mu$ combined to a 4 component field, the e^- and the μ^+ . A spontaneous symmetry breakdown is caused by the appearance of tadpoles. It is proved that the tadpoles responsible for the lepton and quark masses, by themselves, do not break the symmetry in a manner compatible with the observed universality properties of weak interactions. Many additional scalar fields are needed and they are classified into specific $[SU(3)]^4$ multiplets. The model can account for the suppression of $|\Delta S| = 1$ neutral current induced semileptonic decays ($K \rightarrow \pi \nu \bar{\nu}$, $K^0 \rightarrow \mu^+ \mu^-$, etc...) A quartet model based on the symmetry $U(4) \times U(4)$ is discussed. The model is simpler, but has to confront the absence of "charmed" hadrons in the low ($m < 2-3$ GeV) mass region.

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1. INTRODUCTION

The close similarity between weak and electromagnetic currents raises the interesting possibility of constructing a common gauge theory^{[1],[2]} of the Higgs type^[3] for the weak and electromagnetic interactions. It has been recently noted^[4] that such a gauge theory should be renormalizable. It could therefore provide the natural quantum field theoretic framework for weak interactions. On general grounds however, one is not given of the symmetry upon which the gauge theory is to be erected. For the weak and electromagnetic interactions encountered in nature, a minimal symmetry group is^{[5],[6],[1]} $SU(2)_+ \times U(1)_Y$ (here $SU(2)_+$ is a "left-handed isospin" group and $U(1)_Y$ a hypercharge-like group). The disadvantages of choosing this minimal group as the basis of a gauge theory are that A) it treats (ν_e, e^-) and $(\nu_\mu, \bar{\mu})$ as independent doublets and therefore sheds no light on the e- μ problem and B) it leads to unacceptably high rates for semileptonic transitions like $K^0 \rightarrow \mu^+ \mu^-$, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, etc...that involve a neutral $|\Delta S| = 1$ hadronic current.

To overcome the difficulties A) and B) one has to search for a more adequate group on which to base the gauge theory. Two kinds of models have emerged: those with an underlying triplet structure^[7] and those with a quartet structure^[8]. The corresponding groups are products of $SU(3)$ and $SU(4)$ factors respectively. In this paper we are going to give a more detailed discussion of the precise groups involved, of the gauge bosons and the spin zero fields for each of these models. Our conclusion is that for the spontaneously broken symmetry $SU(3) \times SU(3) \times SU(3)$, the triplet model has no obvious faults as far as agreement with experiment is concerned. If the so obtained theory can be faulted, it is primarily on account of the rather strong proliferation of spin zero fields to which it leads.

The quartet $U(4) \times U(4)$ model is much more economical, it achieves its goals more elegantly, but it requires the existence of four rather than three building blocks for hadrons. Unless the fourth quark is very heavy (which

would be somewhat contrary to the lepton-quark symmetry that underlies the model) the quartet model is hard to reconcile with the observed hadron mass spectrum.

2. THE TRIPLET MODEL

The basic hadronic building blocks (the quarks) appear to span a triplet. It is then only natural to seek a basic triplet structure also in the leptonic world. The electron and the muon are described by a Dirac field each. The two neutrinos ν_e and ν_μ are described by "half" a Dirac field (i.e. a Weyl field) each. One can then try to unite the lefthanded electronic neutrino ν_e and the righthanded muonic antineutrino $\bar{\nu}_\mu^C$ into a unique Dirac field.^[10] The leptons are then ν, e^-, μ^+ where $\nu_+ = \frac{1+\gamma_5}{2} \nu = \nu_e$, $\nu_- = \frac{1-\gamma_5}{2} \nu = \bar{\nu}_\mu^C$. We introduce a three componet lepton triplet ℓ

$$\ell = \begin{pmatrix} \nu \\ e^- \\ \mu^+ \end{pmatrix} \quad (2.1)$$

The left and right handed parts of ℓ each form a triplet

$$\ell_+ = \frac{1+\gamma_5}{2} \ell \quad \ell_- = \frac{1-\gamma_5}{2} \ell \quad (2.2)$$

The basic group is then $SU(3)_+^{(\ell)} \times SU(3)_-^{(\ell)}$ where $SU(3)_\pm^{(\ell)}$ are the $SU(3)$ groups acting on $\ell_\pm$. There are sixteen spin 1 gauge bosons $W_{\pm\mu i}^{(\ell)}$ and $W_{-\mu i}^{(\ell)}$ spanning the $(8,1) + (1,8)$ representation of $SU(3)_+^{(\ell)} \times SU(3)_-^{(\ell)}$. Along with these "originally" massless gauge bosons, we introduce following Higgs^[3] a set of spin zero mesons. Some of these have a non-vanishing vacuum expectation value and will confer mass to the gauge bosons and leptons to which they couple. What multiplets of $SU(3)_+^{(\ell)} \times SU(3)_-^{(\ell)}$ do these scalar mesons

belong to? As they have to give mass to the leptons they must* have a part that transforms like $(3,3) + (\bar{3},3)$. Let us now show that these cannot be the only scalar mesons.

Call the 16 gauge fields $W_{+\lambda i}^{(\ell)}$ and $W_{-\lambda i}^{(\ell)}$ ($\lambda=0,1,2,3$ Lorentz index; $i=1,\dots,8$ SU(3) index). Electron-muon universality requires, that following the diagonalization of the gauge boson mass matrix, among the "physical" fields one should find the following combinations of gauge fields.

$$\begin{aligned} W_{+\lambda 1}^{(\ell)} + W_{-\lambda 4}^{(\ell)} \\ W_{+\lambda 2}^{(\ell)} - W_{-\lambda 5}^{(\ell)} \end{aligned} \quad (2.3)$$

For this to happen, there must exist at least one neutral scalar field ϕ of nonvanishing vacuum expectation value ("tadpole") that couples to $(W_{+1}^{(\ell)} + iW_{+\lambda 2}^{(\ell)})(W_{-\lambda 4}^{(\ell)} + iW_{-\lambda 5}^{(\ell)})$ and to $(W_{+1}^{(\ell)} - iW_{+\lambda 2}^{(\ell)})(W_{-\lambda 4}^{(\ell)} - iW_{-\lambda 5}^{(\ell)})$ and thus provides the mixing leading to the "physical" fields (2.3).

Define a leptonic hypercharge

$$Y^{(\ell)} = \frac{1}{\sqrt{3}} (F_{8+}^{(\ell)} + F_{8-}^{(\ell)}) \quad (2.4)$$

where $F_{8\pm}^{(\ell)}$ are the eighth generators of $SU(3)_{\pm}^{(\ell)}$. It is then clear that $W_{+\lambda 1}^{(\ell)} + iW_{+\lambda 2}^{(\ell)}$ and $W_{-\lambda 4}^{(\ell)} - iW_{-\lambda 5}^{(\ell)}$ have $Y^{(\ell)} = 0$ and $Y^{(\ell)} = -1$ respectively.*

* We assume a Yukawa type $SU(3)_+ \times SU(3)_-$ invariant trilinear coupling of the scalar mesons to leptons.

+ i.e. $[Y^{(\ell)}, W_{+\lambda 1}^{(\ell)} + iW_{+\lambda 2}^{(\ell)}] = 0$, $[Y^{(\ell)}, W_{-\lambda 4}^{(\ell)} - iW_{-\lambda 5}^{(\ell)}] = -(W_{-\lambda 4}^{(\ell)} - iW_{-\lambda 5}^{(\ell)})$

Therefore the tadpole ϕ that mixes them must have $Y^{(\ell)} = +1$. Furthermore all tadpoles must be electrically neutral. All electrically neutral members of the $(3,3) + (\bar{3},3)$ representation, however, have $Y^{(\ell)} = 0$ (as they can be viewed as $\nu\bar{\nu}$, $e\bar{e}$ or $\mu\bar{\mu}$ composites). Thus if the only available tadpoles would belong to $(3,3) + (\bar{3},3)$ then μ decay could not proceed in the standard V-A way. The next simple representation from which one would think of selecting the tadpoles is $(8,1) + (1,8)$. Such tadpoles cannot mix W_+ with W_- mesons and again could not lead to the desired combinations (2.3). The simplest representation that both can couple to W_+W_- (and thus mix them) and contains $Y^{(\ell)} = 1$ electrically neutral mesons is $(8,8)$. One is thus led to introduce in addition to the $(3,3) + (\bar{3},3)$ spin zero mesons that account for the lepton masses, also an $(8,8)$ set of spin zero mesons to achieve the mixing between the gauge fields that leads to the "physical" fields (2.3). Some of the tadpoles from this $(8,8)$ multiplet can give some of the "undesired" (e.g. doubly charged) gauge bosons a very large mass and suppress the weak interactions that they mediate.

One could now go into details of how these $(8,8)$ mesons are to be used. But we are not yet done. Indeed at this stage we have not yet included the hadrons into the model. One might think that this is straightforward, but it is not. The reason is that the leptonic gauge mesons $W_{\pm}^{(\ell)}$ cannot all couple to hadrons. It is convenient - though obviously not necessary - to think of the hadrons in the context of the quark model. There are no doubly charged ($|\Delta Q| = 2$) currents in the quark model. Yet as the leptonic charge assignments follow a different pattern, there are $|\Delta Q| = 2$ currents formed with the leptonic triplet (2.1). Correspondingly some of the gauge bosons ($W_{+6} \pm iW_{+7}$ and $W_{-6} \pm iW_{-7}$) will carry two units of charge. These bosons obviously cannot couple to quarks. The way to include the hadrons then is to consider the quark triplet

$$q = \begin{pmatrix} \bar{q} \\ \chi \end{pmatrix} \quad (2.5)$$

and the corresponding left- and right-handed triplets

$$q_+ = \frac{1+\gamma_5}{2} q, \quad q_- = \frac{1-\gamma_5}{2} q. \quad (2.6)$$

We then introduce two new $SU(3)$ groups $SU(3)_{\pm}^{(h)}$ that act on q_+ and q_- respectively and leave the leptons unaffected. The symmetry structure is thereby extended* to $[SU(3)]^4 \equiv SU(3)_{\pm}^{(l)} \times SU(3)_{\pm}^{(h)} \times SU(3)_{\pm}^{(h)} \times SU(3)_{\pm}^{(h)}$. The triplets ℓ_+ , ℓ_- , q_+ and q_- span the representations $(3,1,1,1)$, $(1,3,1,1)$, $(1,1,3,1)$ and $(1,1,1,3)$ of this semisimple group. The leptonic gauge bosons span $(8,1,1,1) + (1,8,1,1)$. It is now natural to introduce "hadronic" gauge bosons $W_{\pm\lambda}^{(h)}$ spanning the representation $(1,1,8,1) + (1,1,1,8)$. So far the hadrons only couple to $W_{\pm}^{(h)}$ and the leptons only to $W_{\pm}^{(l)}$. To allow for the possibility of β -decay we have to introduce a mixing between $W_{\pm}^{(h)}$ and $W_{\pm}^{(l)}$. Moreover we have to account for the universal character of weak interactions i.e. the near (up to a $\cos \theta$ factor) equality of the vector coupling constants in O^{14} - β -decay and μ -decay. To achieve all this, we have to further extend the family of scalar fields from which the tadpoles are to be extracted. Possible candidates for the tadpole containing representation are, $R_1 = (8,8,1,1) + (8,1,8,1) + (8,1,1,8) + (1,8,8,1) + (1,8,1,8) + (1,1,8,8)$, $R_2 = (8,8,1,1) + (8,1,8,1) + (1,8,8,1)$. In the representation R_2 we allowed for the possibility that the right-handed quarks q_- never participate in

*The $[SU(3)]^4$ group introduced here is not to be confused with the purely hadronic $[SU(3)]^4$ group considered^[11] in the early days of the quark model.

weak interactions. (This would even further increase the asymmetrical way in which leptons and hadrons participate in this model of weak and electromagnetic interactions.) In addition to these fields we of course need scalar fields in the representations $(3,3,1,1) + (\bar{3},3,1,1)$ and $(1,1,3,3) + (1,1,\bar{3},3)$ to get lepton- and quark-mass terms. In short, the model is "flooded" with scalar fields.

We now come to the problem of the $|\Delta S| = 1$ neutral current. Suppose $W_{\mu\text{bad}}$ is a neutral "physical" boson that emerges from the grand diagonalization of the gauge boson (mass)² matrix. Let $W_{\mu\text{bad}}$ couple to a current

$$j_{\mu\text{bad}} = j_{\mu\text{bad}}^{(l)} + j_{\mu\text{bad}}^{(h)}$$

in the form $\mathcal{L}_{\text{Ibad}} = g_{\text{bad}} j_{\mu\text{bad}} W_{\mu\text{bad}}^{\mu}$. Here $j_{\text{bad}}^{(h)}$ is a neutral hadronic current that has a $|\Delta S| = 1$ part and $j_{\text{bad}}^{(l)}$ the corresponding leptonic current. Assume that we can arrange the tadpole strengths so that there exist also another "physical" boson $W'_{\mu\text{bad}}$ which couples to $j'_{\mu\text{bad}} = j_{\mu\text{bad}}^{(l)} - j_{\mu\text{bad}}^{(h)}$, in the form $\mathcal{L}'_{\text{Ibad}} = g'_{\text{bad}} j'_{\mu\text{bad}} W'^{\mu}_{\mu\text{bad}}$. If the parameters can be so chosen that $g'^2_{\text{bad}}/m'^2_{W'_{\text{bad}}} = g^2_{\text{bad}}/m^2_{W_{\text{bad}}}$ then the $\mathcal{L}_{\text{Ibad}}$ and $\mathcal{L}'_{\text{Ibad}}$ induced contributions will cancel in the effective contact term and the undesired neutral $|\Delta S| = 1$ decays will be suppressed. In view of the large spin zero meson representations required by the triplet model, sufficient freedom is available to achieve such a cancellation. From the point of view of suppressing these undesired decays, the flood of scalar fields is thus a blessing in disguise. With a tighter supply of scalar fields such intricate cancellations might be unachievable.

This mechanism of settling the neutral $|\Delta S| = 1$ current problem is essentially a generalization to $[SU(3)]^4$ of the mechanism discussed by Schechter and Ueda^[12] for the minimal $SU(2) \times U(1)$ scheme. Within this minimal scheme there is no

place for W'_{bad} and they are forced to enlarge the minimal group to $SU(2) \times U(1) \times U(1)$. On the contrary in the $[SU(3)]^4$ scheme the cancellation is possible without further enlarging the symmetry and its occurrence provides a welcome constraint on the model.

To sum up, we have a model based on the group $[SU(3)]^4$ that accommodates the renormalizable universal weak and electromagnetic interactions of leptons and hadrons. It involves 32 gauge bosons and many spin zero fields. The leptons and hadrons both exhibit an underlying triplet structure. The undesired semi-leptonic neutral $|\Delta S| = 1$ transitions can be suppressed. Thus the model does not disagree in any obvious way with the known experimental facts. Elegance is not one of its distinctive features.

3. THE QUARTET MODEL

Unlike the triplet model, the quartet model is, on the one hand, very elegant, but on the other hand hard to reconcile with the known hadron spectrum. The basis of the scheme is the extension to four leptons ($\nu_e, \nu_\mu, e^-, \mu^-$) of the old Gamba, Marshak, Okubo^[13] idea of a lepton-hadron symmetry. Instead of the lepton triplet of Section 2, one starts now from a lepton quartet.^[8]

$$L = \begin{pmatrix} \nu_\mu \\ \nu_e \\ e^- \\ \mu^- \end{pmatrix} \quad (3.1)$$

(Note that now $\nu_{\mu_L}, \nu_{e_L}, e^-$ and μ^- are all assigned lepton number + 1).

Define a quark quartet

$$q = \begin{pmatrix} \Theta' \\ \Theta \\ \mathcal{N} \\ \lambda \end{pmatrix} \quad (3.2)$$

with $\Theta, \mathcal{N}, \lambda$ the usual three quarks and Θ' is a "charmed" fourth quark carrying the same electric charge as Θ . Now define a "rotated" quark quartet

$$\tilde{q} = \begin{pmatrix} \Theta' \cos \theta_1 - \Theta \sin \theta_1 \\ \Theta \cos \theta_1 + \Theta' \sin \theta_1 \\ \mathcal{N} \cos \theta_2 + \lambda \sin \theta_2 \\ \lambda \cos \theta_2 - \mathcal{N} \sin \theta_2 \end{pmatrix} \quad (3.3)$$

and require the weak interactions to be invariant under the substitution

$$L \longleftrightarrow \tilde{q} \quad (3.4)$$

One can visualize this idea in the form of an extended Nagoya^[14] type model by viewing the quark quartet as a composite of leptons and of a baryonically and leptonically charged Bose field B of electric charge $+2/3$.

$$\tilde{q} = L B \quad (3.5)$$

If only integrally charged particles are to appear, then a four quartet extension of the three triplet model^[15] is readily available. One considers instead of B four Bosons B_1, B_2, B_3, B_4 carrying respectively electrical charge $+1, +1, 0, 0$. The four quartets are then

$$\tilde{q}_i = \begin{pmatrix} \nu_\mu B_i \\ \nu_e B_i \\ e^- B_i \\ \mu^- B_i \end{pmatrix} \quad i = 1, \dots, 4 \quad (3.6)$$

Omitting all $\bar{Q}'$ quarks and all quarks that contain a B_4 one obtains the usual three triplets.

In the quartet model the most general charged "V-A"-type hadronic current w_μ that obeys the nonlinear universality constraint^[6]

$$[[W, W^\dagger], W] = 8W \quad (3.7)$$

(where $W = \int w_0 d^3x$) is found to be*

$$w_\mu = \bar{\psi}_{Y_\mu}(1+\gamma_5)((\cos\theta)n + (\sin\theta)\lambda) + \bar{\psi}'_{Y_\mu}(1+\gamma_5)((\cos\theta)\lambda - (\sin\theta)\lambda') \quad (3.8)$$

The corresponding neutral weak charge $W_0 = \frac{1}{8} [W, W^\dagger]$ contains no $|\Delta S| = 1$ part and thus the problem of neutral $|\Delta S| = 1$ transitions is automatically solved in the quartet model^[9].

The natural starting symmetry here is $U(4)_+ \times U(4)_-$ (or maybe $SU(4) \times SU(4) \times U(1)$ so as not to couple a gauge field to lepton number).** There are 32 (31) gauge bosons for $U(4) \times U(4)$ ($SU(4) \times SU(4) \times U(1)$). Because both the leptons and the quarks have charge assignments of the type $q \ q \ q-1 \ q-1$, the same gauge bosons couple to both leptons and quarks (and therefore hadrons). Unlike the triplet model, no extra doubling of the gauge boson set is required. Spin zero fields, some of which lead to tadpoles must appear. The spin zero fields must span a $(4, \bar{4}) + (\bar{4}, 4)$ representation and possibly some higher representations of $U(4)_+ \times U(4)_-$.

As can be seen, the model works naturally and leads to all the required couplings and cancellations more or less automatically. However it makes it hard to understand why hadrons fall only in $SU(3)$ rather than larger $SU(4)$ multiplets. One could always claim that the $\bar{Q}'$ quark is much heavier than the

other three. This would violate however, the spirit of the lepton-hadron analogy (3.4). One might get around this difficulty by requiring that the hadronic counterparts of the leptons be not the quarks but rather the anti-quarks. So instead of (3.4) one has

$$\begin{aligned} \nu_e &\leftrightarrow \bar{N} \cos\theta_2 + \bar{\lambda} \sin\theta_2 \\ \nu_\mu &\leftrightarrow \bar{\lambda} \cos\theta_2 - \bar{N} \sin\theta_2 \\ e^- &\leftrightarrow \bar{Q} \cos\theta_1 + \bar{Q}' \sin\theta_1 \\ \mu^- &\leftrightarrow \bar{Q}' \cos\theta_1 - \bar{Q} \sin\theta_1 \end{aligned} \quad (3.9)$$

Then, for small θ_1 , $\bar{Q}'$ becomes essentially the counterpart of the muon. The muon is heavier than the other leptons and this might be the reason for $\bar{Q}'$ being heavier than the other quarks. In the case (3.9), left-handed leptons have to correspond to right-handed antiquarks. The gauge mesons that couple to left-(right-) handed leptons will therefore couple to right-(left-) handed antiquarks. One can consider Nagoya-type^[14] realizations of this case as well, the $B(B_i)$'s carrying now electric charge + 1/3 (+1, +1, 0, 0).

In view of its elegance and of its archetypal appeal* the quartet scheme should be kept in mind for the time being.

4. THE MUON PUZZLE

Trying to understand why the muon exists and why it has a mass of ~ 105.6 MeV was one of the motivations for the higher symmetry schemes considered here. In both the triplet and quartet models the muon was an essential component of the lepton multiplet and as such acquired a raison d'être. In either case a parallel (in the triplet model a rather peculiar one at that) between hadron

* This form of the current can also be derived directly from the lepton-quark analogy (3.4) and the known form of the weak lepton current. Then the angle θ in Eq. (3.8) is related to θ_1 and θ_2 of Eq. (3.3) in the form $\theta = \theta_2 - \theta_1$.

** I wish to thank Dr. S. Weinberg for a useful comment on this point. My thanks are also due to Dr. J. L. Rosner for a valuable discussion of the material of this section.

* We quote from Jung's book ref. 15. "...whereas the central Christian symbolism is a Trinity, the formula of the unconscious mind is a quaternity... Contrary to the dogma there are not three but four aspects. It could be easily inferred that the fourth represents the devil."

and lepton states forced upon us the muon. In both the triplet model and the quartet model based on (3.4), the muon appears to be the leptonic counterpart of the λ -quark. It is therefore gratifying^[13] that the μ -e mass difference (~ 105 MeV) has the same sign and order of magnitude as the λ - Λ (quark) mass difference (~ 140 MeV) as inferred from the hadronic mass spectrum*. Unfortunately at the present stage nothing more quantitative can be said about the ratio $(m_\mu - m_e)/(m_\lambda - m_\Lambda)$. The important point is that both models have a potential (as evidenced by the qualitative argument just offered) to predict this ratio. In the same vein, in both models $m_e - m_\nu$ and $m_\lambda - m_\Lambda$ are expected to be related and indeed they are of the same order of magnitude.

* Not all hadronic mass splittings (e.g. not $m_\Sigma - m_\Lambda$) can be explained by just "counting" strange quarks. It may be possible that $m_\lambda - m_\Lambda$ is really equal to $m_\mu - m_e$ but that the mechanism that causes the Σ - Λ mass splitting also causes an extra mass splitting that happens to be proportional to the number of strange quarks but does not originate in an excess mass of the strange quark. For instance the mass-splittings of the baryon octet can be viewed as arising from a mass excess of the strange quark of exactly $m_\mu - m_e$ plus an extra term that transforms like the eighth component of an octet and splits m_Σ from m_Λ , while having a D/F ratio of $\sim -2/3$.

5. CONCLUSIONS

Weak and electromagnetic interactions involve both hadrons and leptons. If these interactions have an underlying symmetry structure, then this should reflect itself both on hadrons and leptons. The abundance of hadrons in nature has facilitated the task of establishing a basic hadronic symmetry ($SU(3)$). By contrast, the scarcity of leptons has made it rather difficult to pin down a natural leptonic symmetry. The path followed in this paper was to search for a common symmetry structure for leptons and hadrons. As in the past^{[7][8]} triplet and quartet structures were explored. The new results obtained in this paper concern to a large extent the triplet model. We found the group $SU(3)_+^{(2)} \times SU(3)_-^{(2)} \times SU(3)_+^{(h)} \times SU(3)_-^{(h)}$ necessary once both hadrons and leptons were included. Thirty-two gauge fields and large tadpole furnishing spin zero multiplets had to be introduced. The net result was a model that accommodated the known features of weak and electromagnetic interactions. Foremost among these features in determining the spin-zero field proliferation was the universality exhibited by weak and electromagnetic interactions. We stress that the model does not deliver this universality in an obvious way but by exploiting all the freedom it can find. To this extent the model has - at present - a certain artificiality to it.

The model also has a rather limited lepton-hadron analogy. True, the basic leptonic building block is a triplet and so is the basic hadronic building block. But here the analogy seems to end. The charge assignments of the leptonic and hadronic triplets are different and could hardly be more different. Therefore not all the particles with which the leptonic triplet interacts can even in principle couple to hadrons. This is the reason for much of the complexity of the triplet model. If the charge operator is always to be an $SU(3)$ generator then an arbitrary linear combination of F_3 and F_8 is the most general form for it. Therefore, there are at most two independent candidates for the title of charge operator. One can, maybe, view this lepton-hadron dissimilarity as nature's way of making use of both of them, one for the leptons the other for the hadrons.*

* While this work was in progress Dr. D. Gordon kindly gave me the preprint of a paper by S. Weinberg (to be published in The Physical Review) which also discusses the triplet model. It may be useful to spell out here the similarities and differences between Weinberg's work and the present paper. Weinberg discusses leptons on the basis of the smaller group $SU(3)_l^{(L)} \times SU(3)_l^{(R)}$. He, of course, also has gauge fields in the representation $(8,1) + (1,8)$ but makes the interesting additional assumption that the left and right handed gauge fields couple with equal strengths to leptons. This leads him to a value $M_W = 74.6$ GeV for the charged weak intermediate boson mass. He also realizes the insufficiency of the $(3,3) + (\bar{3},\bar{3})$ spin zero multiplet and correspondingly introduces a "superstrong symmetry breaking". He does not identify the $SU(3) \times SU(3)$ multiplet (which as we have seen must be at least $(8,8)$) to which the extra scalar fields belong. He does not include the hadrons (and the corresponding extra hadronic $SU(3) \times SU(3)$ group and the new gauge and spin zero fields) into the model. He suggests a possible mechanism for explaining the electron-muon mass ratio as an electromagnetic effect. Finally, we would like to mention one more point. Weinberg makes the assumption that the left and right-handed gauge bosons couple with equal strength to leptons is in order to reduce the

By contrast the quartet model exhibits a beautiful lepton-hadron analogy. It makes up its mind as to which charge operator it is to employ and then sticks to it for both leptons and hadrons. The trouble with this model is "only" that, other than maybe the suppression of decays like $K^0 \rightarrow \mu^+ \mu^-$, $K \rightarrow \pi \nu \bar{\nu}$, etc...(which, if less naturally, can still be accommodated also by the triplet model), there seems to be no evidence favoring a basic hadronic quartet structure.

Both the triplet and quartet models contain the "minimal" $SU(2) \times U(1)$ symmetry of weak and electromagnetic interaction. They offer the two simplest embeddings of this minimal symmetry, that are a) able to remove the more obvious difficulties that arise in the context of a gauge theory and that b) have the potential of solving the μ -e puzzle.

number of arbitrary parameters in the model. He justifies this assumption in terms of parity conservation. It is interesting that a similar reduction in the number of arbitrary couplings would result if the symmetry were increased from $[SU(3)]^4$ to $SU(6)_L^{(L)} \times SU(6)_R^{(R)}$ where $SU(6)_L^{(L)}$ shuffles the six (three left- and three right-handed) leptons while $SU(6)_R^{(R)}$ shuffles the six (three left- and three right-handed) quarks. One could also go to $SU(12)^{(h)}$ if he allows shuffling of quarks and leptons or to $SU(12)_L^{(L)} \times SU(12)_R^{(R)}$ by shuffling the twelve components of the six two-component leptons (quarks) among themselves, or finally, to the maximal $SU(24)$ that shuffles "everything."

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