EFFECT OF FUEL INJECTION ON A
CYCLIC β -LIMITED RADIATION
DOMINATED FUSION REACTOR

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I. INTRODUCTION

Thermal stability of a deuterium-tritium fusion plasma near its ignition temperature depends on the existence of an energy loss mechanism which is larger than, and increases with temperature faster than, the energy gain due to slowing down of the product alpha particles. Conditions for the thermal stability of a Tokamak reactor have been studied by several authors, including Ohta, et al,¹ Kolesnichenko,² and Horton and Kammash.³

It is clear that steady-state reactor operation requires rapid particle diffusion for both temperature stability and removal of fusion products. Since there is no certainty that a satisfactory diffusion mechanism will exist in a fusing Tokamak plasma, it seems prudent to investigate cyclic operation. We consider a limiting case by assuming that the present neoclassical theory will describe particle diffusion and thermal conductivity in a plasma confined by a Tokamak device of the large size and high magnetic field suitable for a reactor. In this case, radiation would be the dominant energy loss from

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the plasma. Radiation losses alone, however, provide neither the temperature dependence nor the magnitude necessary for thermal stability at low temperatures.

Since Tokamak experimental evidence so far indicates diffusion and heat conductivity similar to that predicted by neoclassical theory for the ions, we are motivated to investigate post-ignition operation of a reactor with particle and energy confinement times long compared to fuel burning time.

The opposite limiting case is of the type considered by Mills^{4,5} and Rose.⁶ They assume the existence of a mechanism that would give a sufficiently short confinement time with the necessarily strong inverse temperature dependence so that thermally stable ignited plasma conditions may be found.

Horton and Kammash³ consider reactor start-up and operation for three different types of possible dominant particle loss mechanisms. Two of their cases, one governed by the drift wave instability and the other by Bohm's turbulent diffusion, are shown to exhibit thermal stability under wide ranges of reactor parameters. Their third case, with neoclassical diffusion only, can not produce steady-state operation for any of the reactor parameters considered.

If radiation (without wall reflection) is the dominant energy loss mechanism after ignition and if one load of 50% D-50% T fuel is allowed to fusion without replenishment, then for fuel densities large enough to make fusion power density economically attractive ($\geq 10^{19}/\text{m}^3$), the ion temperature rises to about 100 keV or more before fuel burnout and

synchrotron radiation cause T_i to decrease. The result is that an ignited plasma with no new fuel input experiences a rapid temperature rise, followed by a slightly slower decrease to quenching. The peak temperature and power density are determined by the density and composition of the initial fuel charge and the magnetic field.

Associated with the temperature peak is a peak in the ratio of particle pressure to magnetic field pressure, β . This sharp peaking in temperature and β is an undesirable mode of operation for several reasons. There is reason to believe that the peak β in a Tokamak reactor will be limited by equilibrium considerations.⁷ In addition, if the reactor could operate at a more nearly constant β then the magnetic field, achieved with great effort and expense, would be more efficiently used. Also, if the time during which the fusion rate is large could be increased, then the average power density would be higher, an important economic consideration. A more uniform radiation and neutronic wall loading would also result from a flattening of the burning rate curve.

In this paper we show that by introducing more fuel at or after the T_i and β peaks, it is possible to extend the burn time and operate near a limit on maximum β and/or at a nearly constant power production density. We calculate the dynamics of a thermally unstable reactor and by using a model for introducing fuel via the injection either of an energetic neutral beam or of cold fuel liquid or pellets, we explore the attainable range of operating parameters for such a reactor.

II. MODEL

The plasma is assumed to operate in the neoclassical collisionless diffusion regime. We take this to mean that no significant particle diffusion or thermal conductivity heat loss occurs during times less than about 500 seconds due to the high magnetic field (> 60 kG), high temperature and large reactor radius (5 m). This slow particle loss necessitates cyclic reactor operation to avoid exceeding a reasonable β limit and destroying confinement.

In this regime of no significant particle or conductive energy loss, the dominant energy loss mechanism is radiation from electrons. If we use Trubnikov's expression for synchrotron radiation energy loss as in Rose's paper,⁶ and assume zero wall reflectivity, then synchrotron radiation is the dominant energy loss mechanism in a pure D-T plasma above about 15 keV.

Charged products are assumed to thermalize as quickly as they are produced. They, along with energetic injected particles, if any, provide the only energy source to the plasma in this model. All fuel and product ions are assumed to thermalize to the same temperature due to the rapid energy exchange between particles of nearly equal mass. Electrons and ions are assumed to have the same initial temperature just after ignition.

With these assumptions, we may write the time-dependent differential equations for the density of each species of particle and for the temperature of ions and electrons.

Let S_D and S_T be the source terms for deuterons and tritons, respectively, introduced by either energetic neutral or frozen pellet

injection. The particle density equations are then

$$\begin{aligned}
 \dot{n}_e &= S_D + S_T, \\
 \dot{n}_H &= n_D n_{He^3} \langle \sigma v \rangle_{DHe^3} + \frac{1}{2} n_D^2 \langle \sigma v \rangle_{DD}, \\
 \dot{n}_D &= S_D - n_D n_T \langle \sigma v \rangle_{DT} - 2n_D^2 \langle \sigma v \rangle_{DD} - n_D n_{He^3} \langle \sigma v \rangle_{DHe^3}, \\
 \dot{n}_T &= S_T + \frac{1}{2} n_D^2 \langle \sigma v \rangle_{DD} - n_D n_T \langle \sigma v \rangle_{DT} - n_T^2 \langle \sigma v \rangle_{TT}, \\
 \dot{n}_{He^3} &= \frac{1}{2} n_D^2 \langle \sigma v \rangle_{DD} - n_D n_{He^3} \langle \sigma v \rangle_{DHe^3}, \\
 \dot{n}_\alpha &= n_D n_T \langle \sigma v \rangle_{DT} + n_D n_{He^3} \langle \sigma v \rangle_{DHe^3} + \frac{1}{2} n_T^2 \langle \sigma v \rangle_{TT}. \quad (\text{Eq. 1})
 \end{aligned}$$

Values for the reaction parameters ($\langle \sigma v \rangle$'s) are interpolated from tables published by Greene.⁷ The D-D, D-He³ and T-T reactions are included for the sake of completeness and because high ion temperatures (> 100 keV) are reached. However, for the cases we will consider here, the temperature does not rise so high that D-T reactions fail to dominate.

The temperature equations are obtained from the energy density equations. Energy input to the plasma is from the slowing down of the seven energy species of charged reaction products and, in the case of neutral beam injection, of the two species of energetic injected particles. The nine fast ion, energy species with their initial energies U_{oi} and rates of production P_i are shown in Table I. Since the alpha particles produced in the T-T reaction is one of three products, its energy is an average based on the assumption of equal probabilities for all possible combinations of product momenta. The fast ion energy is

Fast Ion	Initial Energy = U_{oi} (keV)	Production Rate = P_i
H	14670	$n_D n_{He^3} \langle \sigma v \rangle_{DHe^3}$
H	3030	$\frac{1}{2} n_D^2 \langle \sigma v \rangle_{DD}$
T	1010	$\frac{1}{2} n_D^2 \langle \sigma v \rangle_{DD}$
He ³	820	$\frac{1}{2} n_D^2 \langle \sigma v \rangle_{DD}$
α	3670	$n_D n_{He^3} \langle \sigma v \rangle_{DHe^3}$
α	3520	$n_D n_T \langle \sigma v \rangle_{DT}$
α	1890	$\frac{1}{2} n_T^2 \langle \sigma v \rangle_{TT}$
D	E_0	S_D
T	E_0	S_T

TABLE I. PRODUCTION RATES AND INITIAL ENERGIES OF THE FAST IONS.

distributed between electrons and thermal ions as in Rose's paper,⁶ in a manner dependent on electron temperature, the fast ions charge z_i , mass m_i , and energy u_{oi} . The only energy loss from the thermal ions is to the electrons. The ion energy density equation is therefore

$$\begin{aligned} \frac{d}{dt} \left(\frac{3}{2} \sum_{\substack{\text{thermal} \\ \text{ions}}} n_i T_i \right) = K_1 n_e \frac{(T_e - T_i)}{T_e^{3/2}} \sum_{\substack{\text{thermal} \\ \text{ions}}} \frac{z_i^2 n_i}{m_i} \\ + \sum_{\substack{\text{fast} \\ \text{ions}}} U_{oi} (1 - f_i) p_i \quad (\text{keV/sec m}^3) . \end{aligned} \quad (\text{Eq. 2})$$

where $K_1 = 3 \times 10^{-18}$ and f_i is the fraction of the fast ion energy which is delivered to the electrons. This fraction is given by

$$f_i = \frac{2}{3} \int_0^{\ln[(\gamma_i + 1)\gamma_i]} [(1 + \gamma_i)e^{-x} - \gamma_i]^{2/3} dx \quad (\text{Eq. 3})$$

where

$$\gamma_i = \frac{57}{n_e} \left(\frac{m_i T_e}{U_{oi}} \right)^{3/2} \sum_{\substack{\text{thermal} \\ \text{ions}}} \frac{z_j^2 n_j}{m_j} .$$

Rearranging the ion energy density equation gives the ion temperature equation:

$$\begin{aligned} \frac{dT_i}{dt} = \frac{1}{\sum_{\substack{\text{thermal} \\ \text{ions}}} n_i} \left[-T_i \sum_{\substack{\text{thermal} \\ \text{ions}}} \dot{n}_i + \frac{2}{3} K_1 \frac{(T_e - T_i)}{T_e^{3/2}} \sum_{\substack{\text{thermal} \\ \text{ions}}} \frac{z_i^2 n_i}{m_i} \right. \\ \left. + \frac{2}{3} \sum_{\substack{\text{fast} \\ \text{ions}}} U_{oi} (1 - f_i) P_i \right] . \end{aligned} \quad (\text{Eq. 4})$$

The electrons gain energy from both thermal and fast ions and lose energy through bremsstrahlung and synchrotron radiation. The electron energy density equation is:

$$\frac{d}{dt} \left(\frac{3}{2} n_e T_e \right) = K_1 \frac{(T_i - T_e)}{T_e^{3/2}} \sum_{\substack{\text{thermal} \\ \text{ions}}} \frac{z_i^2 n_i}{m_i} + \sum_{\substack{\text{fast} \\ \text{ions}}} U_{oi} f_i P_i \quad (\text{Eq. 5})$$

$$- \frac{3}{2} K_2 n_e \frac{B^{5/2} T_e^{11/4}}{\sqrt{n_e}} (1-R) - \frac{3}{2} K_3 n_e T_e^{1/2} \sum_{\substack{\text{thermal} \\ \text{ions}}} z_i^2 n_i \quad (\text{keV/sec m}^3)$$

where $K_2 = 10^5 / \sqrt{L}$ and $K_3 = 2 \times 10^{-21}$. B is the average magnetic field in the plasma. L is a characteristic plasma length which we arbitrarily take to be 5 meters. R is the wall reflectivity for synchrotron radiation and is taken to be zero unless otherwise indicated.

Noting that $\dot{n}_e = S_D + S_T$, we rearrange this equation to obtain the electron temperature equation:

$$\begin{aligned} \frac{dT_e}{dt} = & - \frac{T_e}{n_e} (S_D + S_T) + \frac{2}{3} K_1 \frac{(T_i - T_e)}{T_e^{3/2}} \sum_{\substack{\text{thermal} \\ \text{ions}}} \frac{z_i^2 n_i}{m_i} + \frac{2}{3} \sum_{\substack{\text{fast} \\ \text{ions}}} U_{oi} f_i P_i \\ & - K_2 B^{5/2} T_e^{11/4} (1-R) / \sqrt{n_e} - K_3 T_e^{1/2} \sum_{\substack{\text{thermal} \\ \text{ions}}} z_i^2 n_i \quad (\text{Eq. 6}) \end{aligned}$$

To compare the case of no fuel injection with various injection strategies, we introduce two important parameters which are functions of burn-time.

In analogy to a parameter of the same name for steady-state devices, we define

$$Q \equiv \frac{\int_0^{T_B} (P_{N+B} + P_{rad}) dt}{E_{ig} + \int_{t_0}^{T_B} \frac{P_{inj}}{\epsilon_{inj}} dt}, \quad (\text{Eq. 7})$$

where T_B = burn-time; i.e., duration of burn from ignition to quench;

t_0 = time of start of injection;

P_{N+B} = thermal power density output, including energy carried by fusion product neutrons plus energy generated in the blanket due to neutron reactions;

P_{rad} = thermal power density radiated from the plasma in the form of bremsstrahlung and synchrotron radiation;

E_{ig} = electrical energy density required to create, confine, and heat the plasma until it ignites.

P_{inj} = injected beam's electrical power per unit plasma volume;

ϵ_{inj} = injection efficiency.

Q is a measure of how much thermal energy is gained for each unit of input electrical energy in a cycle. It indicates how efficiently the reactor uses the energy required to keep it going. Q will obviously increase until quench in the case with no injection. In the other cases, Q will increase until injection starts, then decrease. Optimization of $Q(T_B)$ will involve a maximization of peak Q combined with a minimization of $\left| \frac{dQ}{dT_B} \right|$ after injection starts.

The other significant parameter is the electrical energy gain per unit of plasma volume. We define this quantity:

$$\Delta E = \epsilon \int_0^{T_B} (P_{N+B} + P_{rad}) dt - \int_{t_0}^{T_B} \frac{P_{inj}}{\epsilon_{inj}} dt - E_{ig} = (\epsilon - \frac{1}{Q}) \int_0^{T_B} (P_{N+B} + P_{rad}) dt \quad (\text{Eq. 8})$$

where ϵ is the efficiency of conversion from thermal to electrical power.

The net electrical power production density during the burn is given by

$\frac{\Delta E}{T_B}$. The overall net power production density is $\frac{\Delta E}{T_B + T_P} = \eta \frac{\Delta E}{T_B}$, where η is the reactor duty factor equal to $\frac{T_B}{T_B + T_P}$ and T_P is the pumping, re-fueling and heating time between burns. T_P is expected to be of the order of 10 minutes mainly because of the current penetration time in a large tokamak. It is essential that ΔE and $\Delta E/T_B$ be as large as possible for an economically sized reactor. However, maximizing $\Delta E/T_B$ may not maximize η and therefore both parameters must be considered separately.

We assume a thermal efficiency of $\epsilon = 40\%$ for all calculations. The energy needed for ignition (E_{ig}) is estimated to be a factor of 5 times the energy contained in the plasma just before ignition. The results are quite insensitive to this estimate since the other terms are much larger after a few seconds of burn. For example, taking E_{ig} as 20 times the plasma kinetic energy at ignition typically changes ΔE by less than 2% after injection starts.

The value for ϵ_{inj} is taken to be either 80% or 20%. These values roughly bracket predictions ranging from pessimistic to cautiously optimistic, based on present and probable future technology.⁸ The 20% efficiency is characteristic of D_2^+ or D_3^+ ion source systems while the 80% would be for D^- systems.

In calculating P_{N+B} we assume an average of 4.8 MeV released in blanket reactions for each fusion neutron. This quantity is then given by

$$P_{N+B} = 1.6 \times 10^{-16} \times \left[7.03 \times \frac{1}{2} n_D^2 \langle \sigma v \rangle_{DD} + 18.67 n_D n_T \langle \sigma v \rangle_{DT} + 18.63 \times \frac{1}{2} n_T^2 \langle \sigma v \rangle_{TT} \right] \text{ (kW/m}^3\text{)} \quad (\text{Eq. 9})$$

Since all charged particles are assumed to be retained in the plasma, fractional burn-up (f_B) which is the ratio of burned fuel to input fuel may be calculated for each cycle according to:

$$f_B = \frac{[n_{ef} - (n_{Df} + n_{Tf})]}{n_{ef}} \quad (\text{Eq. 10})$$

where the subscript f denotes final values of the labelled quantities; i.e., values at time of quench.

III. THE FREE BURNING REACTOR (NO FUEL INJECTION DURING BURN)

The six density and two temperature equations were solved simultaneously on a computer using the Runge-Kutta method. Results for T_i , T_e , β and all the densities were obtained as functions of time starting with initial $T_i = T_e = 6 \text{ keV}$ and ending at quench.

Peak values for β , T_i , T_e and radiation wall loading, along with values of final ΔE , burn duration T_B , and total net electrical power $\frac{\Delta E}{T_B}$ are listed in Table II for various on-axis magnetic fields, initial fuel densities and wall reflectivities. Wall loadings are calculated for a ratio of plasma radius to wall radius equal to .8.

Greater power densities are obtained with greater values of B or initial density. But higher fuel density incurs a penalty of higher

B (kG)	Initial n_e $=2n_{D0}=2n_{T0}$ ($10^{19}/m^3$)	Peak β	Peak T_i (keV)	Peak T_e (keV)	Peak Radiation Wall Loading (MW/m ²)	Final ΔE (MJ/m ³)	T_B (sec)	$\frac{\Delta E}{T_B}$ (MW/m ³)	R
60	4	.07	75	37	.24	17	171	.10	0
	6	.13	98	46	.49	29	135	.21	
	8	.21	118	54	.78	40	118	.34	
	9	.25	127	57	.94	46	110	.42	
80	6	.06	71	36	.56	25	110	.23	
	10	.12	100	47	1.3	48	84	.57	
	12	.17	112	51	1.8	60	76	.79	
	15	.23	128	58	2.6	77	67	1.1	
100	9	.05	73	37	1.2				
	10	.06	78	39	1.5	44	71	.62	
	15	.12	101	47	3.0	72	57	1.3	
	20	.19	120	55	4.8	102	49	2.1	
	25	.27	138	62	6.8	131	42	3.1	
	30	.36	156	69	8.8	161	38	4.2	
60	6	.13	98	46	.49	29	135	.21	
		.14	109	50	.46	30	150	.20	
		.16	126	57	.42	31	167	.19	
		.19	148	66	.38	32	189	.17	
		.21	166	74	.34	33	206	.16	

TABLE II. RESULTS FOR REACTOR OPERATION WITH NO FUEL INJECTION.

peak β and higher B pushes the limits of technological and economic feasibility. Allowing higher wall reflectivity leads to lower total power density, but higher peak β because temperatures climb higher. Figure 1 shows the increase in peak value of the crucial parameter β with increasing fuel density, decreasing magnetic field and increasing wall reflectivity. Changing the fuel composition away from a 50%D-50%T mix will have qualitatively the same effect as reducing the fuel density while enhancing the radiation losses. The details of this variation are not explored here.

IV. BURNING CONTROL

If the range of values for β (5%-25%) found in the free burn case can be assumed acceptable, then we may use feedback-controlled injection of fuel during the burn to maintain the reaction near a chosen value of β for a much longer burn-time. The result will be shown to maintain or increase power density averaged over burn-time for a modest investment in recirculating power. The power density averaged over total reactor operation time benefits even more because the interval between burns needed for pumping and ignition is likely to require many minutes.

To investigate the effects of fuel injection we choose two basically different strategies of feedback control, and a variant of each, to determine beam current and energy. Each strategy is applied to a reference reactor with 60 kG for the magnetic field in the plasma and initial $n_e = 2n_D = 2n_T = 6 \times 10^{19} \text{ m}^{-3}$. This gives the moderate value of 13.2% for peak β with no injection. Fractional burn-up with no injection is 68%. Table III summarizes the reference reactor's parameters.

Average B in the plasma = 60 kG.

Initial $n_e = 6 \times 10^{19} \text{ m}^{-3}$.

Initial fuel composition is 50%D and 50%T.

Plasma radius = 5 m.

Ratio of plasma radius to wall radius = .8.

Wall reflectivity = 0.

Injection efficiency = 80%.

Thermal to electric energy conversion efficiency = 40%.

TABLE III. REFERENCE REACTOR PARAMETERS AND ASSUMPTIONS

Both methods prevent β from exceeding a maximum which we may choose. Method I keeps the fuel density and ion temperature nearly constant until a chosen maximum β is attained and thereafter keeps β constant. Method II keeps the fusion power output constant. Both methods start injection immediately after peak ion temperature is reached.

In Method I the beam currents S_D and S_T are then adjusted to keep n_D and n_T constant according to the following equations: From $\dot{n}_D = 0$,

$$S_D = n_D n_T \langle \sigma v \rangle_{DT} + 2 n_D^2 \langle \sigma v \rangle_{DD} + n_D n_{He3} \langle \sigma v \rangle_{He3}$$

and from $\dot{n}_T = 0$,

$$S_T = n_D n_T \langle \sigma v \rangle_{DT} + n_T^2 \langle \sigma v \rangle_{TT} - \frac{1}{2} n_D^2 \langle \sigma v \rangle_{DD} \quad (\text{Eq. 11})$$

The beam energy E_0 is adjusted to keep T_i nearly constant until β rises to a selected value called $\beta_{\max}$. E_0 is calculated from:

$$\begin{aligned} \dot{T}_i = 0 = \frac{1}{\sum_{\text{thermal ions}} n_i} & \left[-T_i \sum_{\text{thermal ions}} \dot{n}_i + \frac{2}{3} K_1 n_e \frac{(T_e - T_i)}{T_e^{3/2}} \sum_{\text{thermal ions}} \frac{z_i^2 n_i}{m_i} \right. \\ & \left. + \frac{2}{3} \sum_{\text{fast ions}} U_{oi} (1-f_i) P_i \right] \quad (\text{Eq. 12}) \end{aligned}$$

and $U_{OD} = U_{OT} = E_0$. This gives

$$E_0 = \frac{\frac{3}{2} T_i \sum_{\text{thermal ions}} \dot{n}_i + K_1 n_e \frac{(T_i - T_e)}{T_e^{3/2}} \sum_{\text{thermal ions}} \frac{z_i^2 n_i}{m_i} - \sum_{\text{fast ions } i \neq D, T} U_{oi} (1-f_i) P_i}{S (1-f_D) + S (1-f_T)} \quad (\text{Eq. 13})$$

When β exceeds the chosen $\beta_{\max}$, E_0 is reduced by a factor of .8 every second until β is again below $\beta_{\max}$. This strategy keeps β oscillating very near $\beta_{\max}$ until the end of the burn because, although the total density is increasing, T_e and T_i are allowed to decrease. If the pre-injection β peak is higher than the chosen $\beta_{\max}$, injection is delayed until $\beta < \beta_{\max}$. Several values for $\beta_{\max}$ were tried, and the best results for ΔE and Q are obtained from choosing $\beta_{\max}$ slightly higher than the peak β for no injection, namely, $\beta_{\max} = 15\%$.

A variant of this strategy, Method IA, is to wait for T_i to decrease to some chosen value before starting injection. One might hope thereby to spend more time operating closer to the maximum for the reaction parameter^{3,5} $\langle\sigma v\rangle_{DT}/(T_i+T_e)$ or $\langle\sigma v\rangle_{DT}/(T_i+T_e)^2$.

The other injection strategy, Method II, keeps the D-T fusion reaction rate constant to produce a constant power output. Injection is again started at the natural T_i peak. Beam energy, E_0 , is chosen to be some constant value high enough so that the beam will penetrate into the plasma. Beam current is equally divided between deuterium and tritium, and its magnitude is calculated to keep the fusion reaction rate constant. Since

$$RR = \text{Reaction Rate} = n_D n_T \langle\sigma v\rangle_{DT} ,$$

we have

$$\frac{d}{dt} RR = 0 = \langle\sigma v\rangle_{DT} (\dot{n}_D n_T + n_D \dot{n}_T) + n_D n_T \frac{d\langle\sigma v\rangle_{DT}}{dt}$$

or

$$\frac{\dot{n}_T}{n_T} + \frac{\dot{n}_D}{n_D} = - \frac{d(\ell n \langle \sigma v \rangle_{DT})}{dT_i} \frac{dT_i}{dt} \quad (\text{Eq. 14})$$

Using the equations for n_T and n_D and letting $S_D = S_T = S$, we finally obtain:

$$\begin{aligned} S = \frac{1}{(n_D + n_T)} & \left[n_D (n_D n_T \langle \sigma v \rangle_{DT}) - \frac{1}{2} n_D^2 \langle \sigma v \rangle_{DD} + n_T^2 \langle \sigma v \rangle_{TT} \right. \\ & + n_T (n_D n_T \langle \sigma v \rangle_{DT}) + 2 n_D^2 \langle \sigma v \rangle_{DD} + n_D n_{\text{He}^3} \langle \sigma v \rangle_{D\text{He}^3} \\ & \left. + n_D n_T T_i \frac{d(\ell n \langle \sigma v \rangle_{DT})}{dT_i} \right] \quad (\text{Eq. 15}) \end{aligned}$$

Delaying injection until T_i is less than 40 keV starts a variant strategy labelled Method IIA. In an attempt to continue the burn for a longer time, we have switched strategies to Method IA at the time when T_i approaches zero and S would start to sharply increase. This hybrid strategy gives the longest burning time attained. Switching strategies was also tried for the other methods, but the required beam energy became excessively high very quickly. The four variants of injection strategy are summarized in Table IV.

V. RESULTS

Figures 2-9 show the time evolution in the reference reactor of nine important quantities for an example of each injection strategy. These quantities are ΔE , thermal power density from neutrons and blanket reactions, neutron wall loading, Q , β , T_i , n_e/n_{eo} , n_D/n_{eo} , and radiation wall loading. Time starts with ignition and goes to quench. The Method I example uses $\beta_{\text{max}} = 15\%$, and the Method II example uses beam energy $E_0 = 300$ keV. On each graph the case with no injection is included for comparison.

METHOD I

Injection started when T_i peaks.

Beam current and energy separately programmed to:

- a) hold T_i , n_D and n_T constant until β equals a chosen $\beta_{\max}$;
- b) then hold β , n_D and n_T constant.

Graphed results are for $\beta_{\max} = .15$.

METHOD IA

Injection delayed until T_i decays below 40 keV.

Beam current and energy separately programmed to keep T_i , n_D and n_T constant.

METHOD II

Injection started when T_i peaks.

Beam energy is constant and beam current programmed to hold fusion power constant.

Zero beam energy corresponds to pellet injection.

Graphed results are for beam energy = 300 keV.

METHOD IIA

Injection delayed until T_i decays below 40 keV.

Method II followed until 225 sec after ignition.

Switch to Method IA for remainder of burn.

Graphed results are for Method II beam energy = 300 keV:

TABLE IV. SUMMARY OF INJECTION STRATEGIES

Net electrical energy gain ΔE is shown plotted against time in Fig. 2. The greatest energy gain is obtained with Method II during the shortest burn duration. The constant slope implies constant power density. The deviation from constant slope in Methods I, IA, and IIA result from beam energies increasing to keep T_i constant as the burn proceeds. Maxima occur for Methods IA and IIA, meaning that after the time when the maximum occurs, more beam power is needed to maintain constant T_i than is obtained from fusion.

If an injection efficiency of 20% rather than 80% is assumed, then the maxima for Methods IA and IIA occur sooner. Then also the Method I curve acquires a maximum of 54 MJ/m^3 at 200 sec, although quench may be delayed until 350 sec, at which time ΔE would decrease to zero.

The time development of thermal power density from neutrons and blanket reactions and approximate neutron wall loading is shown in Fig. 3. Neutron wall loading is approximate because the calculation is based on one neutron and 22.4 MeV per fusion event which is not true for the small fraction of non-DT reactions occurring at the higher temperatures. Note that the peak in thermal power density and neutron wall loading occurs before injection starts.

Method II maintains power density constant as it was designed to do. Method IA keeps both fuel temperature and density constant and therefore maintains constant power. Method IIA is a serial combination of II and IA, and thus also keeps thermal power constant. Method I is designed primarily to maintain a constant β and is not directly tied to power density, which declines with time.

Figure 4 shows Q as it changes with time. Without injection the only power cost is that needed to contain and ignite the plasma. Therefore, Q can only increase for that case. After injection starts, Q decreases sharply for each strategy. Methods I and IA start injection with low beam energy, but it quickly increases while beam current decreases much less quickly to give a net decreasing Q as the burn continues. Because Method II uses a constant beam energy, Q does not decrease as fast Method IIA has the highest Q value for the longest time. The Q values for $\epsilon_{inj} = 20\%$ would be one-fourth of the corresponding values for $\epsilon_{inj} = 80\%$.

In Fig. 5 β is plotted against time. Method I allows β to rise to β_{max} and then keeps β constant by allowing T_i and T_e to decline while maintaining constant fuel density and letting n_e and n_α increase. Method II has no deliberately built-in control for β , but the strategy followed seems to limit β automatically. Methods IA and IIA let T_i and T_e decay so that β decreases until injection starts. Then total density simply builds up while T_i and T_e remain nearly constant, thus increasing β until quench. Method IIA gives two β peaks of about equal magnitude. Method IA was run with β_{max} set at 25%, but the second peak could be reduced by setting β_{max} equal to a lower value, say 15%, so that β would level off at 500 sec.

Figure 6 shows the behavior of T_i with time. The steep initial rise in temperature is responsible for the early β peak of Fig. 5. Methods I and II merely extend the T_i decay, while Methods IA and IIA maintain a nearly constant T_i until quench. Quench occurs when heating by charged fusion products and injected high energy particles is no

longer sufficient to maintain T_i above an ignition temperature determined by an increasing fuel plus ash ion population. The ignition temperature increases because radiation losses rise due to an increasing electron population.

The increase in electron density with time is shown in Fig. 7. The slope of these curves is the fuel injection rate which is fairly constant except for Method I. The injection rates are nearly constant for Methods II, IA, and IIA because they are tied to keeping the fusion power density constant, and the only fuel particle loss in this model is by fusion.

Requirements for neutral beam energy, current and injected power are summarized in Table V for each injection strategy that we investigated.

For Method I, S_D and S_T are greatest during the early part of injection, with a maximum value for each of about 4.10×10^{17} particles per sec per cubic meter of plasma volume. This corresponds to 66 milliamperes/ m^3 of D or T ions. After β reaches β_{max} , the temperature declines and reaction rates decrease. As a result, S_D and S_T decrease and E_0 increases as the feedback system compensates for reduced α heating. The smallest values for E_0 are about 300-400 keV, and the largest are about 4-5 MeV. Total injected power varies widely from .035 to .03 MW/ m^3 .

Method II uses a neutral beam with constant energy and varying current. This is probably a more practical strategy than Method I which varies both. For the 300 keV beam, S ranges between 58 and 80 milliamperes/ m^3 , corresponding to .035-.05 MW/ m^3 . A higher energy

Method	Beam Energy E_0 (MeV)	Beam Current (ma/m ³) $S_D + S_T$ for Method I 2S for Method II	Injected Power (MW/m ³)
I (Constant β and fuel density) $\beta_{\max} = .15$	.28 - 4.9	58 - 131	.035 - .30
I (Constant T_i and fuel density) ⁱ Injection delayed until $T_i \leq 40$ keV	.45 - 5.0	39 - 113	.019 - .20
II (Constant Fusion Power)	0 .30 .65 1.0	112 - 154 117 - 160 113 - 170 110 - 194	--- .035 - .050 .073 - .10 .11 - .19
II (Constant Fusion Power) Injection delayed until $T_i \leq 40$ keV Method II initially with $E_0 = 300$ keV followed by Meth. I	.30 - 5.3	29 - 54	.08 - .20

TABLE V. SUMMARY OF NEUTRAL BEAM REQUIREMENTS FOR THE VARIOUS
STRATEGIES OF INJECTION CONTROL.

beam would presumably be used only if lower energy particles would not sufficiently penetrate the plasma.

Deuteron density as a function of time is shown in Fig. 8. Triton density is almost the same. In Methods I, IA, and the latter part of IIA, fuel injection is programmed to keep fuel density constant. In Method II, fuel is burned at the fastest rate and the programming of high constant thermal power thus requires fuel to be replaced at the fastest rate.

For Method I about 88% of the fuel entering the reactor is consumed by fusion. For Method II about 78% is burned. This compares with 68% fractional burnup in the no injection case.

The change in radiation wall loading with time shown in Fig. 9 is the combined result of the behavior in time of n_e and T_e . Electron temperature follows the same qualitative variation with time as ion temperature (see Fig. 6), but with lower values. Thus, during the main part of the burn, T_e is nearly constant for Methods IA and IIA. Constant T_e with steadily increasing n_e (cf. Fig. 7) leads to steadily increasing radiation losses for these strategies. In Methods I and II, T_e is decaying while n_e is increasing so that radiation loss is more nearly constant.

The quantities of most importance for economic considerations are ΔE , which is the net electrical power density, and Q , which is the ratio of thermal energy produced per unit of input electrical power density. A summary of these quantities as achieved in our reference reactor is listed in Table VI. The various strategies of burn control may be compared with each other as well as with the case with no injection.

Burn Control Method	Duration of Burn, T_B , min.	Net Electrical Power Density Produced During Burn, $\Delta E/T_B$, MW/m ³	Q	Net Electrical Power Density Produced During Whole Cycle, $\eta \Delta E/T_3$, MW/m ³			
				$T_P = 5$ min.		$T_P = 20$ min.	
				Duty Factor, η	$\eta \Delta E/T_B$	Duty Factor, η	$\eta \Delta E/T_B$
I (Constant β and fuel density) $\beta_{\max} = .15$	5	.36	9	55%	.20	23%	.083
	4	.45	16	44	.20	17	.075
	3	.49	20	38	.18	13	.064
IA (Constant T_i and fuel density). Inj. delayed until $T_i \leq 40$ keV	7	.14	6	58	.082	26	.036
	5	.18	9	50	.090	20	.036
	3	.23	17	38	.086	13	.030
II (Fusion Power Constant) $E_0 = 0$ (pellet inj.) $E_0 = 300$ keV $E_0 = 650$ keV $E_0 = 1000$ keV	4.5	.53	-	47	.25	18	.097
	4.5	.51	28	47	.24	18	.092
	4.2	.46	15	46	.22	17	.083
	4.3	.46	10	46	.21	18	.081
IIA (Fusion Power Constant) Injection delayed until $T_i \leq 40$ keV. Meth. II initially with $E_0 = 300$ keV, followed by Method I	10	.16	8	67	.10	33	.052
		.19	15	58	.11	26	.049
	7	.23	37	44	.10	17	.039
	4	.25	44	38	.095	13	.033
No Injection During Burn	2	.23	-	29	.064	9	.02

TABLE VI. SUMMARY OF NET ELECTRICAL POWER DENSITY AND Q VALUES WITH NO NEUTRAL INJECTION AND WITH THE VARIOUS STRATEGIES OF INJECTION CONTROL.

The duration of burn, T_B , has a maximum possible value determined by the particle and energy dynamics and the β limit. A burn time less than this maximum may be chosen by stopping injection, after which T_i will fall below ignition in tens of seconds. This quench time is not necessarily small enough to neglect, but may be included in T_P for our purposes here. The net electrical power density produced over a whole cycle will depend upon T_P because T_P and T_B together determine the duty factor η . T_P would be difficult to estimate, but is probably bracketed by 5 to 20 minutes which are the values used in Table VI. The best choice of burn duration for control by Methods, I, IA, and IIA are determined by T_P .

Table VI shows clearly that the lengthening of burn duration made possible by fuel injection substantially increases the output power density, even taking into account the power cost of injection and conversion efficiencies. This increase in burn time is accomplished while at the same time observing the limitation of maximum β and with reasonable wall loadings.

VI. CONCLUSION

We have explored the dynamics of a fusion reactor operating in a purely neoclassical manner. Particle losses are negligible in such a device and thus it must operate in a cyclic manner to avoid exceeding the maximum equilibrium β . The thermal runaway that occurs after ignition places additional restrictions on the range of parameters available for operation. The reactor must be controlled so that the peak β at any time during the cycle is less than this critical β . To accomplish this while assuring a reasonable output power requires

careful control of the particle and energy input rates from neutral beams. We have found that beams between .02 and .3 MW/m³ are required to operate the reactor in a controlled manner after ignition. At this power level the average reactor output can be maintained at an economically interesting level without exceeding the equilibrium β limitation. The start up of a reactor using neutral beam heating has been shown⁹ to require about the same range of beam powers, between 0.09 and 0.25 MW/m³.

A second consequence of the thermal runaway is the rather short burning time of an uncontrolled reactor relative to a recycle time. For reasonable values of β we have found that the burning time with no fuel injection during the burn is less than two minutes. However, the power produced is concentrated in a small part of this burning period so that the duty cycle of such a reactor is quite low. Both the burning time and the power duty cycle can be improved by the judicious use of neutral injection to fuel and heat the plasma. The burning time can be extended to about 700 sec with a moderate expenditure of power if the injection efficiency is high. Extending the burning time beyond this point results in uneconomical operation because of excessive beam power requirements.

The general picture of a neoclassical reactor operation which emerges from our calculations involves pulsed operation at lower densities ($n \sim 6 \times 10^{19} \text{ m}^{-3}$) and higher temperatures ($T_1 \sim 50\text{-}100 \text{ keV}$) than have hitherto been envisioned. These values are for our reference reactor with $B = 60 \text{ KG}$ in the plasma and β limited to about 15%.

Of course, we are considering a limiting case in which particle and heat conductivity losses are totally negligible as predicted by

neoclassical theory for large devices. An equally limiting case is to postulate a short, typically one second, particle confinement time with the appropriate temperature dependence to stabilize the thermal runaway. This model used by Mills^{4,5} and Rose⁶ results in a steady state, high density, low temperature reactor. However, the problem of economic operation of such a device is very severe. A one second confinement time essentially transforms a closed toroidal system into one analogous to an open mirror system with all of the associated problems of low Q and large recirculating power unless a zero energy fueling scheme can be invented. In reality the situation will probably be between these extremes with a long, but not infinite, particle containment time. This will improve the picture of reactor operation given in this paper, but the main features of thermal runaway and the necessity for fueling and energy input control will probably persist.

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FIGURE CAPTIONS

- Figure 1a. The peak β for a free burning reactor synchrotron radiation absorbing walls as a function of the toroidal magnetic field and initial density.
- Figure 1b. The increase in peak β as the wall reflectivity is raised.
- Figure 2. The net electrical energy gain and burning time for different injection strategies is shown. In each case the burn is terminated when the ion temperature falls below ignition.
- Figure 3. The thermal power density including neutrons and blanket reactions for the different injection strategies. This power density can be related to an approximate reactor wall loading which is given on the right hand scale.
- Figure 4. The reactor Q for the various injection strategies is shown as a function of time. The penalty of maintaining the reactor burn through injection is evident.
- Figure 5. The variation in reactor β with and without injection. Clearly, low β limits imposed by stability would seriously limit the burning time.
- Figure 6. The range of ion temperatures covered in the controlled and free burning reactor.
- Figure 7. The increase of electron density caused by the injection of the neutral beams.
- Figure 8. The time dependence of the deuterium and tritium density is shown during a burn. Control Methods I, IA, and IIA have significantly lower levels of unburned fuel at quench than Method II.

FIGURE CAPTIONS (Cont.)

Figure 9 The variation of radiation wall loading for the different control strategies is shown.

ORNL-DWG 73-3564

Figure 1

Figure 2

Figure 3

ORNL-DWG 73-3567

Figure 4

ORNL-DWG 73-3568

Figure 5

ORNL-DWG 73-3571

Figure 8

ORNL-DWG 73-3572

Figure 9

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