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MAGNETIC FIELD EFFECTS ON THE HEAT TRANSFER
OF POTENTIAL FUSION REACTOR COOLANTS

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MAGNETIC FIELD EFFECTS ON THE HEAT TRANSFER
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ABSTRACT

In several conceptual designs of fusion reactor containment vessels, it is proposed to use liquid lithium for the three-fold function of neutron moderator, tritium breeder and heat transfer fluid. Since the lithium must flow through a region of intense magnetic fields, it is of interest to ascertain the magnitude of the effect of the magnetic field on the heat transfer to the lithium.

A previous study revealed that the magnetic field can have a very powerful and detrimental effect on lithium pumping power requirements as well as on the structural design of the blanket. This present study shows that the effects of the magnetic field on the heat transfer to liquid metals such as lithium can be severe in the regimes of interest to fusion reactors if very thin walls cannot be employed. The heat transfer capabilities of lithium are compared to those of other potential heat transfer fluids such as flibe (lithium fluoride - beryllium fluoride mixture), and high pressure helium gas. It is shown that at high Reynolds numbers, these other fluids can have higher heat transfer coefficients than liquid lithium in magnetically-laminarized flow regimes. Consequently, the MHD effect on liquid metal heat transfer capabilities is one more important factor which must be taken into consideration in the choice of a suitable coolant for a fusion reactor.

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I. Introduction

In most conceptual designs of fusion reactors, the high energy neutrons released in the nuclear reactions are moderated (i.e. slowed down) by coolants and other materials in the plasma containment vessel. This containment vessel, usually referred to as the "blanket", may be required to perform several functions including:

1. Slowing down all neutrons before they reach the magnet coils.
2. Absorbing the plasma radiation and charged particle energy impinging on the first wall.
3. Transferring the neutron and plasma energy to the working fluid of an electric power generation system.
4. Breeding new tritium to replace that used in a D-T reactor.
5. Withstanding the compressive loads due to the near vacuum in the plasma region.

In one advanced design concept¹, the first four functions have been separated from the severe structural requirements of function number 5. In this approach the principal load bearing functions are removed from the hot inner walls to a relatively cool wall in the region between the blanket and the magnet coils. This permits the use of much thinner hot blanket walls and brings the structural and heat transfer design of the first wall facing the plasma within the realm of technical feasibility. This blanket design philosophy is discussed in more detail in Reference 2.

The first four blanket functions can all be performed by one unique coolant, lithium, which is the coolant employed in many conceptual designs including Reference 1. Since the liquid lithium must flow through a region of intense magnetic fields, it is of interest to ascertain the magnitude of the MHD (magnetohydrodynamic) effects on the heat transfer to the lithium. A previous study³ revealed that the magnetic field can have a very powerful and detrimental effect on the lithium pumping power requirements, and in turn on the structural design of coolant flow passages in the blanket.

In this study we wish to evaluate the effects of the magnetic fields on the heat transfer to the lithium.

Other blanket conceptual designs have been proposed which employ more than one fluid to accomplish the first four functions of the blanket. In one concept⁴, potassium is used as the heat transfer fluid. The potassium liquid is boiled and vaporized as it flows through the blanket and then used directly as the working fluid in a potassium Rankine topping cycle. The lithium is still used for functions 1, 2 and 4 in this concept. In other blanket concepts^{5,6} high pressure gaseous helium is used as the heat transfer fluid.

It has also been proposed to use flibe (a lithium fluoride-beryllium fluoride mixture) as a blanket coolant. This liquid apparently comes close to performing all of functions 1 through 4, although it is more difficult to achieve the necessary tritium breeding ratios and perhaps more difficult to work with from a chemical corrosion point of view, than lithium. One of the objectives of this study is to compare the heat transfer performance of flibe and helium to that of lithium in the presence of strong magnetic fields.

In Section II of this report, the basic magnetic effects on electrically conducting fluids are discussed. The specific flow regimes of interest to this study are then defined. In Section III, an analytical solution to the magnetic-laminar flow between parallel plates is described. Several important limiting cases for strong magnetic field environments are obtained, and these results are compared to the nonmagnetic cases. The parallel plate results are then extended to the round pipe case in Section IV. Estimation techniques for evaluating the difference between the wall and bulk fluid temperatures in round pipes are described. Finally, in Section V, a comparison is made of the heat transfer capabilities of several possible fusion reactor coolants.

II. General Effects of Magnetic Fields

A. Flow Regimes for Fusion Reactor Coolants

In order to characterize the flow regimes in the presence of magnetic effects, it is necessary to specify several dimensionless magnetic numbers in addition to the usual Reynolds number, Re , and Prandtl number, Pr . The magnetic Reynolds number, is defined as $R_m \equiv \sigma \bar{u} D_h$, where σ is the electrical conductivity of the fluid, $\bar{u}$ is the mean fluid velocity, μ is the magnetic permeability of the fluid, and D_h is the channel hydraulic diameter. It is assumed, with justification, that R_m is less than unity in this study; this implies that the effect of induced magnetic fields is negligible.

The Hartmann number is a measure of the ratio of magnetic forces to viscous forces. There are three useful Hartmann numbers:

$$H_{\perp} \equiv a B_{\perp} \sqrt{\frac{\sigma}{\eta}}$$

$$H_{\parallel} \equiv a B_{\parallel} \sqrt{\frac{\sigma}{\eta}}$$

$$H_r \equiv a B_r \sqrt{\frac{\sigma}{\eta}}$$

where a is the channel half height or pipe radius in the $B_{\perp}$ direction, and η is the fluid viscosity. These are the most important magnetic parameters in this study. $H_{\perp}$ is a function of $B_{\perp}$, the transverse magnetic field component perpendicular to the flow velocity; $H_{\parallel}$ is a function of $B_{\parallel}$, the magnetic field component parallel to the flow velocity, and H_r is related to the radial field component, B_r , in an axisymmetric situation.

Finally, we define a wall conductance ratio, $C \equiv \frac{\sigma_w t_w}{\sigma a}$, where σ_w is the wall material electrical conductivity, and t_w is the

effective wall thickness. While the parameter C plays a dominant role in determining the magnitude of the magnetic pressure drops in liquid metal flows³, it will be shown to have only a negligible effect on the heat transfer in the flow regimes of interest here.

In Figure 1 is shown a sketch of the typical boundaries of the magnetic flow regimes expected inside the fusion reactor magnetic field region for both lithium and flibe. It can be seen that the maximum likely value of H_L for flibe is about two orders of magnitude lower than that for lithium. This is due both to the lower electrical conductivity and higher viscosity of flibe. Pertinent property data and other key parameters are summarized in Table 1 for these and several other possible fusion reaction coolants, as well as for niobium, a possible wall material.

Also shown on Figure 1 are lines of constant magnetic interaction parameter, I , defined as follows in this study:

$$I \equiv \frac{4 [H(a)]^2}{Re_D}$$

This is effectively the ratio of magnetic forces acting on the fluid to fluid inertial forces. High interaction parameter usually means that there are strong magnetic effects on the velocity profile, fluid turbulence, free convection and consequently on the temperature profile as well.

It can be seen from Figure 1 that the possible transverse magnetic interaction parameters for lithium coolant flows is over 100 while for flibe it is less than 0.1; consequently, the MHD effects in fusion reactor blankets can be very important for lithium, while they are likely to be very weak effects for flibe.

For helium and potassium vapor, there are negligible MHD effects at the temperatures of interest (up to 1200°K). However, since the potassium coolant enters the reactor blanket as a liquid, there may be some important MHD effects on the heat transfer and, in particular, on the boiling heat transfer to a two-phase potassium flow. The nature of the MHD effects in modifying boiling and vaporization of liquid metals is not yet fully understood. This is an area requiring further research.

B. Turbulence Suppression by Magnetic Fields

The Reynolds numbers for the flow regimes of primary interest in the study of liquid lithium lie between 10^4 and 10^6 (based on the hydraulic diameter, D_h). Therefore, for smooth-walled flow channels, the fully-developed flow would normally be turbulent. However, the presence of magnetic fields acts so as to damp the turbulent eddies and tends to laminarize the flow of electrically conducting fluids.

Experimental results indicate that the flow can be fully laminarized in strong transverse magnetic fields (i.e. a B field perpendicular to the flow velocity) if there is sufficient entrance length and if the transverse Hartmann number is sufficiently large. For circular pipes the requirement for full laminarization is given approximately by:

$$H_1 \approx \frac{Re}{500}$$

This has been verified experimentally^{7,8,9} up to $H_1 \approx 10^3$, as shown in Figure 2. It should be noted that the constant is closer to 400 for the data points at smaller $H_1 < 100$.

For strong parallel magnetic fields (i.e. a B field parallel to

the flow velocity), the experimental data^{10,11} are less complete as shown in Figure 2. However, extrapolations indicate that almost-full laminarization occurs when

$$H_{||} \approx \frac{Re}{60}$$

There is some experimental evidence to indicate that the flow is never fully laminarized by a $B_{||}$ field¹². The effect of this residual turbulence on the heat transfer has not yet been established; it will be neglected in this study.

The flow lengths required for turbulence suppression in a parallel magnetic field are estimated to be of the order³:

$$\frac{L_{TS}}{D_h} \approx 4 \times 10^8 \frac{\mu_0 \rho}{I_{||}} ; \text{ (MKS units)}$$

For fusion reactor applications, essentially full turbulence suppression is expected to occur within a very short length after entry into the magnetic field (on the order of a few hydraulic diameters at the most) due to the very high magnetic fields anticipated. The turbulence suppression lengths are expected to be even shorter in strong transverse magnetic fields.

The strong suppression of turbulence by magnetic fields is the most important magnetic effect on liquid lithium insofar as the heat transfer capabilities are concerned. However, for flibe, the amount of turbulence suppression is expected to be very small due to the much weaker magnetic interaction; this is discussed in more detail in Appendix A.

C. Useful Limiting Cases for Liquid Lithium Flows

The entrance geometry, magnetic field profile and entrance flow conditions determine whether there will be sufficient entrance length available for the flow to become laminarized and fully-developed in three separate senses:

1. Full turbulence suppression
2. Fully-developed velocity profile
3. Fully-developed temperature profile

The necessary conditions for fully-developed magnetic flows are as yet only partially established; some of the available data are described in Reference 3. Using preliminary estimates of the inlet development lengths, the following limiting cases for the velocity profile appear to be of practical importance for fusion reactor applications using liquid lithium:

- Limiting Case I. Fully-developed magnetic-laminar velocity profile (also called Hartmann profile)
- Limiting Case II. Uniform velocity profile (i.e. slug flow)
- Limiting Case III. Fully-developed mean turbulent velocity profile, but without turbulence (i.e. approximately a one-seventh power law profile)
- Limiting Case IV. Fully-developed laminar parabolic velocity profile

In all of these Limiting Cases, we will assume full turbulence suppression. This is consistent with the large magnetic interaction parameters expected for lithium flows in a fusion reactor blanket.

Limiting Case I arises when the liquid metal crosses a region of strong transverse magnetic field on entry into the fusion reactor. For this situation, the fully-developed Hartmann velocity profile can develop in a very few pipe diameters. In addition, the velocity profile can respond to changes in the local H_z rather rapidly, if I_z is large. We will consider only simplified cases where H_z is constant inside the blanket.

At very high H_L , the Hartmann flow approaches a nearly uniform velocity profile in the plane containing u and B_L . As Limiting Case II we shall consider the perfectly uniform case, (i.e. the slug flow limit).

In the case of an open reactor geometry, such as a mirror machine, it may be possible to bring the fluid piping in along field lines. In this case the turbulence can be almost fully suppressed by the strong parallel magnetic field. However, the mean turbulent velocity profile (approximately a one-seventh power law profile) may remain almost unchanged for hundreds of diameters if there is no transverse or radial component of the magnetic field. As Limiting Case III we shall consider this possibility.

Finally, if there are sufficiently long inlet pipes oriented parallel to field lines, and a sufficiently low flow Reynolds number, the mean turbulent velocity profile will eventually develop into the pure parabolic laminar profile (except possibly for the small residual turbulence near the pipe center line mentioned previously). This limiting behavior is chosen as Case IV.

Clearly, these four Limiting Cases are idealizations and do not represent all the possible magnetic field variations and various oblique orientations in an actual fusion reactor. However, they will provide an insight into the magnitude of the magnetic effects on liquid metal flows in fusion reactor blankets. In particular, they will permit some useful quantitative estimates of heat transfer rates and wall temperature excesses to be made for preliminary design purposes.

The temperature profiles for the various Limiting Cases, should develop at approximately the same rate as for ordinary laminar flow, once the turbulence is suppressed. The thermal entrance lengths for

ordinary laminar flows are of the order

$$\frac{L_{th}}{D} \approx 0.05 Re_D Pr$$

for a parabolic velocity profile¹³; this is also a reasonable first estimate for the other possible velocity profiles. For liquid lithium, the L_{th}/D is expected to range between about 10 and 10^3 for Reynolds numbers from 10^4 to 10^6 . Consequently, only at the lower end of the Reynolds number range is the temperature profile likely to be fully-developed.

In this study the temperature profiles will be assumed to be fully-developed. This gives a good first estimate of the range of Nusselt numbers likely to be encountered in magnetic laminar flows. However, further work is certainly warranted to obtain more exact calculational techniques for developing temperature profiles.

Other possible effects on the lithium heat transfer are free convection and axial heat diffusion; these are expected to be negligible in the flow regimes of interest (see Appendix B).

III. Magnetic Effects on Flow Between Parallel Plates

A. Background

The case of fully-developed flow between infinite parallel plates in the presence of a transverse magnetic field has been studied by many analysts since the original works of Hartmann and Lazaras^{14,15}. (This particular example of magnetic-laminar flow is often referred to as Hartmann flow.) Hartmann analysed primarily the effects of the magnetic field on the velocity profile and the resultant increase in the viscous pressure drop. The recent work in this area of magnetic pressure drops is reviewed and extended in Reference 3.

Insofar as the heat transfer aspects of parallel plate flow are concerned, Siegel¹⁶ appears to have been among the first to publish in the U.S.A. while Regirer published an early work on the subject in the U.S.S.R.¹⁷. Siegel's analysis of the constant wall heat flux case with electrically insulating walls in the presence of an uniform transverse magnetic field was extended to the case of arbitrary wall electrical conductance by Alpher¹⁸. Yen¹⁹ further extended the results to the case of unequal wall conductances and also included viscous dissipation. This latter case was also apparently solved independently by Jagadeesan²⁰. In Appendix C of this report, a further extension is made to allow for unequal wall heat fluxes and for internal heat generation such as produced by neutron moderation.

The principal value of the parallel plate solutions lie in their relatively simple analytical form which permits us to analyse and compare the contributions of the wall heat flux, viscous dissipation, joule dissipation, and internal heat sources on a quantitative basis. As yet, no such simple solutions to the magnetic heat transfer problem exist for other more practical flow channel geometries such as round pipes or rectangular channels. However, the results of this parallel plate study will help us to identify the dominant mechanisms affecting the heat transfer and to predict the maximum wall temperatures in other geometries as well.

B. Fully-Developed Flow in Transverse Magnetic Fields

In this section we examine Limiting Case I, fully-developed Hartmann flow, for the infinite, parallel plate geometry. The full set of differential equations for this parallel plate heat transfer problem are given in Appendix C. These reduce to the usual continuity equation, the modified axial momentum equation including the magnetic body force for the steady state case:

$$\frac{dp}{dx} = j_y B_z + n \frac{d^2 u}{dz^2}$$

the Ohm's Law:

$$j_y = \sigma (E_y - u B_z)$$

and the steady state energy equation:

$$\rho u c \frac{\partial T}{\partial x} = k \frac{\partial^2 T}{\partial z^2} + n \left(\frac{\partial u}{\partial z} \right)^2 + \frac{j_y^2}{\sigma} + S$$

The geometry for this problem is shown in Figure 3a. In this section, $B_{\perp} \equiv B_z$ is the transverse component of the magnetic field. The assumption of low magnetic Reynolds number has been made so that $B_{\perp}$ is the only magnetic field component considered. E_y and j_y are the local electric field and local current density, respectively, which are purely in the y direction for the parallel plate geometry. S is the internal heat source term representing the effect of neutron moderation by the fluid.

The momentum and continuity equations and Ohm's Law are solved first to yield the classical fully-developed, magnetic-laminar or Hartmann velocity profile:

$$U = \frac{\left(1 - \frac{\cosh H_L Z}{\cosh H_L} \right)}{\left(1 - \frac{\tanh H_L}{H_L} \right)} = C_{ml} \left(1 - \frac{\cosh H_L Z}{\cosh H_L} \right)$$

where $U = u/\bar{u}$, $\bar{u}$ is the mean flow velocity and $Z \equiv \frac{z}{a}$.

This velocity profile reduces to the ordinary laminar parabolic profile as $H_1 \rightarrow 0$.

The energy equation can be conveniently nondimensionalized as follows:

$$\text{URePrA} = -\frac{d^2G}{dz^2} + \text{Pr}\left(\frac{dU}{dz}\right)^2 + \text{Pr}H_1^2Jy^2 + \hat{S}$$

where A is the nondimensional axial temperature gradient, and where

$$G \equiv \frac{c(T_{wi} - T)}{u^2}$$

note that G is a function only of Z for these fully-developed flows. Refer to the List of Symbols for the other definitions.

The complete solution for the temperature profile is given in Appendix C. The asymptotic solution for large $H_1 \gg 10$ for the case of constant wall heat flux, q_w , to both walls is given by:

$$G = G_Q + G_S + G_V + G_J$$

where the individual contributions are:

$$\text{Wall heat flux: } G_Q = + \hat{q}_w \frac{(1 - z^2)}{2}$$

$$\text{Internal sources: } G_S = + \hat{S} \frac{(1 - z^2)}{2H_1}$$

$$\text{Viscous dissipation: } G_V = \text{Pr} \frac{H_1}{4} (1 - z^2)$$

$$\begin{aligned} \text{Joule dissipation: } G_J &= + \text{Pr} \frac{H_1}{4} (1 - z^2) (2K^2 - 1) \\ &= \text{Pr} \frac{H_1}{4} (1 - z^2) \left[\frac{2}{(c+1)^2} - 1 \right] \end{aligned}$$

The effective loading parameter, K , commonly used in MHD problems, is related to the wall conductance ratio by $C = \frac{1}{K} - 1$.

The temperature profile is seen to be basically parabolic for large H_1 . The q_w contribution to G is essentially the same result one obtains for slug flow. However, in the true slug flow limit, both the uniform heat source contribution, G_s , and the viscous contribution G_v , would be identically zero, whereas in this Hartmann flow case they are finite. This is due to the fact that while the velocity profile is very flat like slug flow across most of the channel, there is a thin boundary layer with a steep velocity gradient near the wall over a distance δ_H of order $\frac{a}{H_1}$. Due to this boundary layer, there is a heat source and a viscous contribution in the Hartmann flow case.

One result of great interest to this study is the difference between the wall temperature on the fluid side, T_{w1} , and the bulk fluid temperature, T_B , defined as follows:

$$T_B = \frac{\int_0^a \rho u c T dz}{\int_0^a \rho u c dz}$$

In nondimensional terms:

$$G_B \equiv \frac{T_{w1} - T_B}{\left(\frac{U^2}{c}\right)} = \int_0^1 G U dz$$

The general results for all values of H_1 are given in Appendix C; for large $H_1 \gg 10$ the contributions to G_B are:

$$\text{Wall heat flux: } G_{BQ} = + \frac{\hat{q}_w}{3}$$

$$\text{Internal sources: } G_{BS} = + \frac{\hat{S}}{3H_1}$$

$$\text{Viscous dissipation: } G_{BV} = + \frac{PrH_1}{6}$$

$$\text{Joule dissipation: } G_{BJ} = + \frac{PrH_1}{6} (2K^2 - 1) = + \frac{PrH_1}{6} \left[\frac{2}{(C+1)^2} - 1 \right]$$

where the total

$$G_B = G_{BQ} + G_{BS} + G_{BV} + G_{BJ}$$

These results are plotted in terms of the general nondimensional variables in Figure 4 for a wall conductance ratio, $C = 0$. Each of these contributions will be examined in turn in the following sections.

C. Contribution of the Wall Heat Flux

We shall first examine the effects of the $\hat{q}_w$ contribution to G_B for various possible magnetic field orientations. In particular, we are interested in the values of the Nusselt number,

$$Nu \equiv \frac{h D_h}{k} = \frac{4 \hat{q}_w}{G_{BQ}}$$

for the four Limiting Cases of fully-magnetically-laminarized flows. (For the parallel plate geometry, $D_h = 4a$.)

For the case of a strong transverse field, B_z , such that $H_1 \gg 10$, the Nusselt number for these Hartmann flows approaches 12. This is the same as the result for slug flow. Consequently, the parallel plate flow with fully-developed Hartmann velocity profile, (Limiting Case I

of Section II C), does indeed behave essentially like slug flow, (Limiting Case II), at high H_1 , as far as the Nusselt number is concerned.

Up to now we have considered only the case where B_1 is oriented across the gap in the plates (i.e. along the z axis in Figure 3a). It is also possible to have a component of B_1 in the direction of the y axis (Figure 3b). In a true parallel plate geometry this will cause a uniform current density to flow in the z direction provided that a current return path exists through the walls or through some external circuit (e.g. external or internal connecting structure). The resultant $\bar{j} \times \bar{B}$ body force will be uniform across the flow passage, causing an MHD pressure drop. However, the most important magnetic effect of a B_y on the heat transfer is turbulence suppression. The magnetically laminarized flow will have an initial velocity profile and a temperature profile the same as the mean profiles of the incoming flow. Given sufficient flow length, the profiles will approach those of ordinary laminar flow. Consequently, the Nusselt number will be given by that of Limiting Case III or IV, or by some value in between.

A strong parallel magnetic field, (Figure 3c), can also lead to laminar-magnetic flows represented by Limiting Cases III and IV (defined in Section II C). For Limiting Case III, it is assumed that the turbulence is completely suppressed during entry along the magnetic field lines, leaving a mean turbulent velocity profile. Approximating this velocity profile by a one-seventh power law, the equations can be integrated to find G and G_B . The final result is a Nusselt number of 10.8.

For Limiting Case IV, it is assumed that the flow length is sufficiently long for the velocity profile to become the ordinary laminar, parabolic profile. Integration of the equations yields the classical result for the Nusselt number of 8.23, as expected.

The range of possible Nusselt numbers for all four of these magnetic laminar cases with full turbulence suppression can be seen to be quite small, namely from about 8.2 to about 12.

D. Comparison with Ordinary Turbulent Lithium Flows

It is of great interest to compare the Nusselt numbers for the various magnetic-laminar flows to those for ordinary turbulent flows between parallel plates. For liquid metals such as lithium, the Nusselt numbers have been shown to range between about 8 and 12 for all four magnetic-laminar flows with the various possible velocity profiles. It is interesting to note that the fully-developed Hartmann flows actually have higher Nusselt numbers (approaching 12 for large H_1) than ordinary nonmagnetic laminar flow where $Nu = 8.23$, signifying a slightly better heat transfer coefficient for the Hartmann flow.

However, the reverse is true when we compare Hartmann flow to ordinary turbulent flow of liquid metals. For turbulent flow Nusselt numbers we will use the empirical correlation of experimental results on narrow concentric annuli given in Reference 21:

$$Nu_{D_h} = 10.5 + 0.036 (Re_{D_h} Pr)^{.8}$$

Values of Nu representative of high temperature lithium are listed in Table 2 for three values of Reynolds numbers from 10^4 to 10^6 . The Nusselt numbers range from about 15 at $Re = 10^4$ to almost 100 for $Re = 10^6$.

It can be seen that the magnetic field reduces the Nusselt number and hence the heat transfer coefficient in all cases in comparison to the ordinary turbulent flow. At a Reynolds number of 10^6 , the reduction is about an order of magnitude. We can tentatively conclude from these results that the detrimental effect of the magnetic

fields in fusion reactors will probably result in no more than an order of magnitude reduction in the heat transfer coefficients for liquid metal coolants such as lithium.

E. Comparison of Various Contributions

It is useful to compare the magnitudes of the contributions to G_B in the regions corresponding to fusion reactor conditions. The various contributions to the wall temperature excess will then be estimated by assuming a typical lithium flow of about 1 m/sec in a 1 cm half-height. For the n^{th} contribution to the wall temperature excess, we have:

$$(T_{wi} - T_B)_n \approx 2.4 \times 10^{-4} G_{Bn} \text{ (}^\circ\text{K)}$$

Typical values of the nondimensional wall heat flux, $\hat{q}_w$, for first walls of fusion reactors are usually much greater than 10^5 (corresponding to $q_w \lesssim 15 \frac{\text{W}}{\text{cm}^2}$ for $\bar{u} \approx 1 \text{ m/s}$ and $a = .01 \text{ meters}$). This results in values of G_{BQ} greater than 3×10^4 (Figure 4). Thus the typical contribution to $(T_{wi} - T_B)$ due to the first wall heat flux is much greater than 7°K .

Typical nondimensional values of the internal heat generation, $\hat{S}$, in first wall lithium coolant flows are about 10^5 (corresponding to an internal power density due to neutron moderation of about 25 watts/cm^3). This results in $G_{BS} \approx 3 \times 10^2$, and the resulting wall temperature excess, $(T_{wi} - T_B)$, due to internal heat generation can be seen to be much less than 1°K for $H_L \gg 10$. This is because uniform internal heating tends to raise the temperature of the fluid uniformly. Only in the regions near the walls where the velocity is low is the temperature raised slightly more than elsewhere. As a result we can normally neglect the effect of neutron heating of the lithium on the wall temperature excess as a first approximation if $H_L \gg 10$.

For the limiting case where $H_L \rightarrow 0$ (and $H_v \rightarrow 0$) and the velocity profile tends to the laminar parabolic profile, internal heat generation will make a somewhat larger contribution to the wall temperature excess.

The solution for the limiting case yields $G_{BS} \approx .09 \hat{S}$. For the numerical values given above, $(T_{wi} - T_B) \approx 2 \times 10^{-5} \hat{S}$ ($^{\circ}K$). We see that for $\hat{S} \approx 10^5$, the wall temperature excess will be only about $2^{\circ}K$. Consequently, for velocity profiles other than the very flat Hartmann profiles, internal heat generation in the liquid coolant still does not contribute very much to $(T_{wi} - T_B)$.

From Figure 4, we see that the viscous dissipation and joule dissipation contributions to G_B are only significant at extremely high H_L . Even at $H_L \approx 10^5$, G_{BV} and G_{BJ} are only about 3×10^2 , and the contributions to $(T_{wi} - T_B)$ from the viscous and joule dissipation are much less than $1^{\circ}K$. The small effect of finite wall conductance on $(T_{wi} - T_B)$ is discussed in Appendix D.

In summary, it can be seen that the dominant contribution to the difference between T_{wi} and T_B is normally due to the wall heat flux for the fusion reactor applications, particularly near the first wall. Consequently, the Nusselt number is a good measure of the overall heat transfer effectiveness of the flowing fluid and of the overall wall temperature excess, $(T_{wi} - T_B)$.

F. Steady State Fusion Reactor Examples

In order to put these results into perspective, we shall evaluate the wall temperature excess for two steady state fusion reactors. As Example #1 we shall evaluate $(T_{wi} - T_B)$ for energy fluxes typical of those expected in D-T mirror-type reactors. We assume a plane parallel flow passage with a large enough B_L to make $H_L \gg 10$. One wall of this flow passage is assumed to face an uniform slab of D-T plasma, while the second wall is assumed to be insulated on the back face.

A nominal neutronic wall-loading, q_{NL} (i.e. total neutron energy flux passing through the surface of the first wall facing the plasma) of about 1000 watts/cm^2 is used. Neutronics calculations indicate that this should result in a neutronic power density in a niobium first wall, PD_{n1} , of about 250 watts/cm^3 due to neutron moderation. In addition, we shall assume an alpha particle energy flux to the first wall, $q_{\alpha 1}$, equal to 25 watts/cm^2 and a plasma radiation flux q_{r1} , of about

60 watts/cm². We can now estimate the first wall temperature excess above the bulk fluid temperature, $(T_{wi} - T_B)$, using the results of the previous sections. In particular, we shall assume that the contributions to $(T_{wi} - T_B)$ due to viscous and joule dissipation and neutronic energy deposition in the lithium itself are all negligible compared to the contribution due to the wall heat flux.

The first wall heat flux is given by:

$$q_{w1} = q_{n1} + q_{a1} + q_{r1}$$

where

$$q_{n1} = (PD)_{n1} / t_{w1}$$

is the neutronic contribution to the heat flux due to neutron moderation in the first wall, and t_{w1} is the first wall thickness.

The results for liquid lithium are shown in Figure 5 and Table 3 (Example #1) as a function of the wall thickness. The lithium properties listed in Table 1 have been used in these calculations. First we shall examine the symmetric heating case where both walls receive the same heat flux, assumed to be the maximum heat flux, q_{w1} . For these high H_L cases, the flow approaches slug flow, and the wall temperature excess for the symmetric heating case can be seen to range from 48° to 172°K for wall thicknesses from 0.03 cm to 1.0 cm respectively. Consequently, there is an incentive to use as thin walls as practical.

For comparison purposes, three cases of ordinary nonmagnetic turbulent flows are shown on Figure 5 for three values of Reynolds numbers. For the highest $Re_{Dh} = 10^6$, the wall temperature excess becomes very small and is almost an order of magnitude lower than the values for magnetically-laminarized lithium.

These results are all for a channel half-height, a , of 1 cm. If a is decreased, the wall temperature excess is decreased according to the relation for symmetric heat fluxes:

$$(T_{w1} - T_B) \cong \frac{4a q_{w1}}{k Nu} \cong \frac{a q_{w1}}{3k} \quad (\text{for } Nu \cong 12)$$

This is the same as the result for ordinary nonmagnetic heat transfer, as expected.

Next we consider the effects of the asymmetric (i.e. unequal) wall heating. In the fusion reactor, the alpha flux and radiation flux are only felt by the first wall facing the plasma. Furthermore, the neutron flux is attenuated by both the first wall and the lithium coolant before reaching the second wall. Consequently, the neutronic energy deposition in the second wall is very much lower than in the first wall. The resultant heat flux to the fluid from the second wall, q_{w2} , estimated assuming that all the neutronic energy deposited in that wall flows into the coolant, is given approximately by the following relation assuming $t_{w2} = t_{w1}$:

$$q_{w2} \cong \left\{ 1 - \left[\frac{q_{n1} + (PD)_{n(Li)}(2a)}{q_{NWL}} \right] \right\} q_{w1}$$

where $(PD)_{n(Li)}$ is the neutronic power density due to neutron energy deposition in the lithium coolant between the first and second walls, estimated as about 25 watts/cm³ for these examples.

The values of q_{w2} for the asymmetric case are shown in Table 3. For Example #1, for a wall thickness of 0.1cm, the heat flux from the second wall is only about 23 watts/cm² compared to 110 watts/cm² through the first wall. This asymmetry in the heat flux has a very strong

adverse affect on the first wall temperature excess. The wall temperature excess for the high H_1 (slug flow) limit is now given by the modified equation:

$$(T_{wi} - T_B)_{\text{first wall}} = \frac{2a q_{w1}}{3k} - \frac{a q_{w2}}{3k} \quad (\text{for } Nu \cong 12)$$

In the Limiting Case where q_{w2} approaches zero, the first wall temperature excess is twice as large as for the symmetric case (with $q_w = q_{w1}$ flowing through both walls).

The values of $(T_{wi} - T_B)$ for the asymmetric case for Example #1 can be seen to be about 45°K higher than the symmetric case for wall thicknesses up to 0.3 cm; for $t_w = 1$ cm, the asymmetric case is 82° higher. Consequently, one must take account of the unequal wall heat fluxes in estimating the maximum first-wall temperatures.

One additional important effect must be considered in estimating the maximum first-wall temperature; this is the temperature drop through the wall itself as given by:

$$(T_{wo} - T_{wi}) = \frac{q_{w1} t_w}{k_w}$$

where T_{wo} is the outer surface temperature (plasma-side) and T_{wi} is the inner surface temperature (fluid-side). These values are given for the first and second walls in Table 3. For first-wall thicknesses greater than about 3 mm, this temperature difference for Example #1 exceeds about 80°K. Consequently, one pays this additional severe wall temperature penalty for thick first-walls.

The preceding values of wall temperature excess are not only representative of mirror type reactors, but also represent the case of a toroidal reactor with good diversion of the alpha particle flux away from the first wall. We next consider for Example #2 the worst case of a toroidal reactor with little or no diversion of the alpha particle flux from the first wall. Assuming an alpha energy flux of 250 watts/cm^2 , ten times the previous example, we obtain the results shown in the bottom half of Table 3. All other energy fluxes have been assumed to remain the same as in the first example.

It can be seen that the inner-wall-surface temperature excess ($T_{wi} - T_B$), is increased to over 300°K for the asymmetric case with thin walls of about 1mm thickness. In addition, the temperature drop across the first wall material is now over 50°K for walls as thin as 1mm. The importance of using as thin walls as possible and employing good alpha particle diverters in toroidal machines is evident.

IV. Magnetic Effects on Flows in Round Pipes

A. General Comments

We would expect the general trends for the MHD effects in lithium flows in round pipes to be similar to those we have found for parallel plate flows in the preceding section. In particular, dimensional arguments can be used to verify that the effects of viscous and joule dissipation and internal heat sources on ($T_{wi} - T_B$) are also negligible in round pipes in the regimes of interest where the wall heat fluxes are large. We also expect the same four Limiting Cases of Section II C to be as useful for round pipe flows as for parallel plate flows.

The values of Nusselt number for the important Limiting Case I of fully-developed Hartmann flow in a round pipe in a transverse magnetic field can only be found exactly by evaluating rather complex expressions involving many terms of an infinite series. The round pipe

problem is complicated theoretically because the velocity profile tends toward the Hartmann profile in planes close to B_L ($\theta \rightarrow 0^\circ$) while it tends toward a more rounded profile in planes nearly perpendicular to B_L ($\theta \rightarrow 90^\circ$).⁸ (θ is defined in Figure 6.)

As a result, it is usually necessary to resort to approximations in solving round pipe MHD flow problems. Some of the approximate solutions and magnetic pressure drop estimates are reviewed in Reference 3. For the heat transfer problem, Gardner²² has used a simplified velocity profile to obtain approximate values of the Nusselt number around the tube periphery for constant wall heat flux. His analytical results for the local Nusselt number are shown in Figure 7. The high H_L Limiting Case can be approximated by:

$$Nu(\theta) = 7.1 + 1.6 \cos 2\theta \quad (H_L \rightarrow \infty)$$

The resultant Nusselt number averaged around the periphery of the tube as calculated by Gardner is shown in Figure 8. The experimental data of Gardner and Lykoudis²³ tend to corroborate the order of magnitude of these calculated results at the higher Hartmann numbers. For example, from these experiments, the average value of Nusselt number for $Re = 1.06 \times 10^5$ and $H_L \cong 370$ approached about 6, and for $Re = 3.87 \times 10^5$ and $H_L \cong 1230$, the average Nusselt number approached about 8. For lower Reynolds numbers, the strong free convection effects in mercury complicate the interpretation of the data.

Most of the other heat transfer experiments available for round pipes in transverse magnetic fields and in parallel magnetic fields have been run in the transition regime where the turbulence is only partially suppressed (e.g. References 24 and 25). This is a complex MHD flow regime where our understanding of the scaling laws is incomplete. This regime of partial turbulence suppression is discussed briefly in Appendix A in connection with the magnetic effects on flibe. We shall discuss only the fully-magnetically-laminarized Limiting Cases for lithium flows.

B. Useful Limiting Cases

In fusion reactor applications, we expect to be operating in the fully-laminarized (i.e. fully turbulence suppressed) flow regimes when using liquid lithium, as discussed in Section II. Hence, we can use the limited heat transfer data available for round pipes in magnetic fields to estimate the Nusselt numbers for the first of the fully-laminarized Limiting Cases of Section II C. In particular, for fully-developed Hartmann flows (Limiting Case I) at high H_L , we can estimate the average Nusselt number as about 7.1, based on Gardner's analysis of the uniform heat flux case²² and the experimental data of Gardner and Lykoudis²³.

For a uniform velocity profile (i.e. the slug flow Limiting Case II), the well known result for round tubes is a Nusselt number of 8.0. The fully-laminarized flow with parabolic velocity profile (Limiting Case IV) has the classical laminar value of Nusselt number of 4.36. The value of Nusselt number for Limiting Case III (mean turbulent velocity, profile but full turbulence suppression by the magnetic field) will be between 8.0 and the classical laminar value of 4.36. Based on the low Reynolds number and low Prandtl number Limiting Cases of Figures 9 and 8 from Reference 13, this Nusselt number is probably about 6.2.

In summary, the magnetically-laminarized flows in round tubes should have average Nusselt numbers ranging between 4.36 and 8.0 for uniform heat flux both axially and around the tube periphery. The effect of other magnetic field orientations is discussed in Appendix E, where it is shown that all cases where turbulence is fully suppressed have Nusselt numbers within the above range.

The effects of a nonuniform wall heat flux for nonmagnetic flows are discussed in Reference 26. Using this theory, it can be shown that highly asymmetric wall heating increases the wall temperature

excess of the tube side facing the plasma by about a factor of 2, as in the parallel plate case. (See Appendix F.)

C. Comparison with Ordinary Turbulent Lithium Flows

For ordinary turbulent flow of liquid metals in round tubes, many different empirical equations for the Nusselt number exist. Some of the differences between these data correlation equations are due to the strong effect of small amounts of impurities on the experimental heat transfer data in liquid metals. We shall use one of the more recent correlations of Skupinski¹³:

$$Nu = 4.82 + 0.0185 (Pr Re)^{0.827}$$

This equation correlates the data quite well, and in addition, matches the recommended values of Rust²⁷ based on various turbulence models.

Values of Nusselt number calculated from this equation are compared to the magnetic-laminar Nusselt numbers for uniform wall heat flux in Table 4. For high temperature lithium, the ordinary turbulent Nusselt numbers can be seen to range from about 6 to 63 for Reynolds numbers from 10^4 to 10^6 . As in the case of parallel plate flow, we find that the magnetic effects will reduce the Nusselt number and hence increase the wall-to-bulk temperature difference, $(T_{wi} - T_B)$, by no more than about an order of magnitude.

For higher Prandtl number fluids, the Nusselt numbers for constant wall heat flux for ordinary turbulent flows have been calculated using the familiar empirical equation:

$$Nu = .023 Re^{0.8} Pr^{0.4} \quad (Pr > 0.5)$$

These are shown in Table 4 for comparison purposes. They will be discussed in more detail in Section V.

It can be seen that the lithium results for round pipe flows in Table 4 are quite comparable to the results for parallel plate flows in Table 2. Consequently, the general trends deduced from the numerical examples in Section III, should also apply to the round pipe flows and to other shaped channels as well.

In order to estimate the upper limit to the wall temperature excess, $(T_{w1} - T_B)$, for round pipes, the following approximate procedure is suggested. Select the minimum Nusselt number for the flow and magnetic field situation from Table 4. For example, for Hartmann flow we would use $Nu_0 \approx 5.5$. For magnetic-laminar flow with a parabolic velocity profile we would use $Nu_0 = 4.36$. Then calculate $(T_{w1} - T_B)$ for a constant wall heat flux equal to the maximum value of wall heat flux on the plasma side. Finally, for the first wall situation with the usual highly asymmetric heat flux distribution, multiply the above $(T_{w1} - T_B)$ by a factor of about 2.

This estimation procedure is admittedly only a first approximation. However, this procedure is useful in making preliminary design estimates for liquid metal coolants and for comparing various alternative designs. Clearly, more refined computer calculation procedures will have to be developed before detailed blanket designs can be undertaken.

V. Comparison of Several Potential Coolants

The comparison of the heat transfer capabilities of various fusion reactor coolants is most meaningful in terms of $Nu \times k$ rather than Nu alone since:

$$Nu \times k = hD = \frac{q_w D}{(T_{w1} - T_B)_q}$$

Thus, the higher $Nu \times k$, the smaller the wall temperature excess (for a specified wall heat flux and pipe diameter).

In Figure 9, the parameter $Nu \times k$ is plotted as a function of Reynolds number for liquid lithium, flibe, helium and potassium vapor. For the case of lithium the ordinary turbulent flow curve is based on the empirical relation of Skripinshii¹³. In addition, two fully magnetically-laminarized cases are shown for $Nu = 4.36$ and 8.0 .

From Figure 9, it can be seen that the heat transfer capability of lithium is decreased substantially by the magnetic turbulence suppression above a Reynolds number of roughly 10^4 . The resultant magnetically-laminarized lithium flow has lower heat transfer capability than flibe at the higher Reynolds numbers. (The flibe data from Reference 31 agrees well with the Reference 6 data for $1200^\circ K$ and is probably better than Reference 28 for $1000^\circ K$.) We also see that helium is superior to lithium, as far as the heat transfer is concerned, above Reynolds numbers of about 10^6 .

Potassium vapor has very much lower heat transfer coefficients than the other possible coolants. However, the proposed use of potassium in Reference 4 envisions boiling liquid potassium inside the coolant tubes in the blanket. Consequently, the heat transfer process involves a two-phase potassium flow, where the latent heat of vaporization accounts for most of the energy absorbed. As mentioned earlier, the MHD effects on boiling heat transfer to two-phase liquid metal flows are not yet well defined enough to include this curve in Figure 9.

VI. Summary and Conclusions

It has been shown that magnetic fields can have a strong effect on the heat transfer to liquid metal flows. The most important magnetic effect, insofar as heat transfer is concerned, is turbulence suppression. A second magnetic effect of lesser importance is the modification of the velocity and temperature profiles. For example, the magnetic turbulence suppression can reduce the average Nusselt number of liquid metal coolants from about 63 to about 6 or 7 at a Reynolds number of 10^6 in smooth round pipes. Subsequent changes in velocity profile can reduce the Nusselt number to 4.36 at most.

The wall thickness can strongly affect the wall temperatures required to achieve a desired bulk fluid temperature, particularly for the first wall facing the plasma of a fusion reactor. For example, for a parallel plate geometry exposed to the energy flux typical of a mirror-type fusion reactor, a wall thickness of 1 mm yields first wall temperatures at the outer (plasma side) surface 120°K above the bulk fluid temperature. If the first wall thickness is increased to 3 mm, the outer wall temperature excess climbs to 210°K, while for 1 cm thick walls it soars to 814°K.

The situation can be far worse for toroidal-type fusion reactors if the alpha particles cannot be efficiently diverted from the first wall. For the worst case of no alpha particle diversion, the outer wall surface temperature would be 388°K above the bulk fluid temperature for a wall thickness of only 1 mm. (For 1 cm thick walls, this temperature difference would be an impossible 1418°K).

We conclude that thin first walls are very likely to be required in steady state fusion reactors. This requirement makes the blanket design approach of Werner^{1,2} imperative; specifically this involves moving the vacuum support structure to the outside of the blanket, thus eliminating the large vacuum loads from the first wall.

A comparison of several proposed fusion reactor coolants reveals several interesting points. First of all, the heat transfer capabilities of lithium and flibe are quite comparable when no magnetic fields are present. However, in the presence of strong enough magnetic fields to suppress the turbulence in the lithium flow, flibe becomes clearly superior to lithium at Reynolds numbers above about 10^5 . Even high pressure helium becomes comparable in heat transfer capability to magnetically-laminarized lithium flows at Reynolds numbers of about 10^6 .

We conclude that magnetic fields can reduce the heat transfer capabilities of lithium by factors from 2 to 10 in the range of

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TABLE 2

NUSSELT NUMBERS FOR FULLY-DEVELOPED FLOWS OF
LIQUID METALS BETWEEN PARALLEL PLATES*

LIMITING CASES (see Sec. II C)	FLOW TYPE	VELOCITY PROFILE	Pr	$Re_{D_h}^{**}$	$Nu_{D_h}^{**}$
	Ordinary-laminar	Parabolic	—	<2300	8.23
IV	Magnetic-laminar	Parabolic			8.23
III	Magnetic-laminar	One-seventh power law (Turbulence suppressed)		$<Re_t$	10.8
I	Magnetic-laminar	Hartmann		$<Re_t$	12.0
II	Ordinary-laminar	Uniform (slug flow)		<2300	12.0
	Ordinary turbulent	Usual turbulent	.017+	10^4	12.7
			.017	10^5	24.4
			.017	10^6	97.7

* Constant wall heat flux; equal flux on both walls.

** For parallel plates, $D_h = 4a$.

† Typical Pr for high temperature liquid lithium.

Table 1 - Property Data

	LITHIUM	FLIBE (Reference 28)	FLIBE (Reference 6)	HELIUM	NIOBIUM SOLID
Temperature, T (°K)	~1100°	~1000°	~1200°	~1200°	~1100°
Pressure, p (atm)	—	—	—	60	—
Density, ρ (Kg/m ³)	4.5×10^2	2.0×10^3	1.84×10^3	2.5×10^3	8×10^3
Viscosity, η ($\frac{\text{Newton-s}}{\text{m}^2}$)	2.5×10^{-4}	1×10^{-2}	3×10^{-3}	5×10^{-5}	—
Kinematic Viscosity, ν ($\frac{\text{m}^2}{\text{s}}$)	5.5×10^{-7}	5×10^{-6}	1.6×10^{-6}	2.0×10^{-8}	—
Specific Heat, c_p (joules/Kg °K)	4.2×10^3	2.7×10^3	2.4×10^3	5.2×10^3	—
Thermal Conductivity, k (watts/m °K)	65	3.9	1.0	0.4	60
Electrical Conductivity, σ (mhos/m)	2×10^6	1×10^2	3.94×10^2	~0	2×10^6
Prandtl No., $Pr \equiv \frac{\eta c_p}{k}$	.017	7.0	7.2	0.65	—
Typical Reynolds No., $Re \equiv \frac{\rho \bar{u} D_h}{\eta}$ (for $D_h = .02\text{m}$, $\bar{u} = 1 \text{ m/s}$)	3.6×10^4	4.0×10^3	1.2×10^4	1×10^6	—
Typical Hartmann No., $H_L \equiv a B_L \sqrt{\frac{\sigma}{\eta}}$ (for $a = .01\text{m}$, $B_L = 1.0 \text{ Tesla}$)	0.9×10^3	1.0	3.6	~0	—
Typical Hartmann No., $H_H \equiv a B_H \sqrt{\frac{\sigma}{\eta}}$ (for $a = .01\text{m}$, $B_H = 10 \text{ Tesla}$)	0.9×10^4	10	36	~0	—

TABLE 4

NUSSELT NUMBERS FOR FULLY-DEVELOPED
FLows IN SMOOTH ROUND PIPES

LIMITING CASES	FLOW TYPE	VELOCITY PROFILE	Pr	Re _D	Nu _D
	Ordinary-laminar	Parabolic	—	<2300	4.36
IV	Magnetic-laminar	Parabolic			
III	Magnetic-laminar	One-seventh power law (Turbulence suppressed)		<Re _t	~6.2
I	Magnetic-laminar	Hartmann		<Re _t	<div style="display: flex; align-items: center;"> <div style="font-size: 3em; margin-right: 5px;">{</div> <div> ~5.5 min. ~7.1 avg. ~8.7 max. </div> </div>
II	Ordinary-laminar	Uniform (Slug flow)		<2300	8.0
	Ordinary turbulent (Liquid metal)	Usual turbulent	.017	10 ⁴	6.1
				10 ⁵	13.5
				10 ⁶	63.1
	Ordinary turbulent (Typical gas)	Usual turbulent	0.7	10 ⁴	32
				10 ⁵	200
				10 ⁶	1250
	Ordinary turbulent (Flibe)	Usual turbulent	7.0	10 ⁴	80
				10 ⁵	502
				10 ⁶	3150

TABLE 3

TYPICAL WALL TEMPERATURES FOR A PARALLEL PLATE FLOW CHANNEL

Example Number	$t_{w1}=t_{w2}$ (cm)	q_{n1} (watts/cm ²)	q_{a1} (watts/cm ²)	q_{r1} (watts/cm ²)	q_{w1} (watts/cm ²)	Symmetric Case (q_{w1} to both walls)		q_{w2} (watts/cm ²)	Asymmetric Case First wall temperatures		
						$(T_{w1}-T_B)$ °K	$(T_{w2}-T_{w1})$ °K		$(T_{w1}-T_B)_1$ °K	$(T_{w2}-T_{w1})_1$ °K	$(T_{w2}-T_B)_1$ °K
1	0.03	7.5	25	60	92.5	48	4.6	7.1	91	4.6	96
	0.1	25.	25	60	110.	57	18.	23.	102	18.	120
	0.3	75.	25	60	160.	82	80.	66.	130	80.	210
	1.0	250.	25	60	335.	172	560.	175.	254	560.	814
2	0.03	7.5	250	60	317.5	163	16.	7.1	322	16.	338
	0.1	25.	250	60	335.	172	56.	23.	332	56.	388
	0.3	75.	250	60	385.	198	193.	66	362	193.	555
	1.0	250.	250	60	560.	287	934.	175.	484	934.	1418

Common Values for all Cases: Lithium flows for $H_1 \gg 10$ (Slug Flow Limit)

Parallel-plate half-height, $a = 1$ cm.

Figure 1 - Typical Flow Regimes for Two Potential Fusion Reactor Coolants

Figure 2 - Transition Reynolds Numbers for Flows of Conducting Fluids in Circular Pipes in Parallel and Transverse Magnetic Fields

Figure 3 - Parallel Plate Flows Channel Geometry Showing Three Possible Magnetic Field Orientations

Figure 4 - Nondimensionalized Contributions to the Wall Temperature Excess in a Parallel Plate Flow Channel

Figure 5 - First-Wall Excess Temperature Requirements for Possible Fusion Reactor Conditions for Lithium Coolant

Figure 6 - Round Pipe Flow Geometry with a Transverse Magnetic Field Component Shown

Figure 7 - Local Hartmann Numbers as a Function of the Angular Coordinate Measured from the Direction of the Magnetic Field (Ref. 22)

Figure 8 - Average Nusselt Numbers as a Function of the Hartmann Number. Dashed line indicates suggested interpolation between the high and low Hartmann number solutions. (Ref. 22)

Figure 9 - Comparison of the Heat Transfer Capabilities of Several Potential Coolants in Smooth Round Pipes

LIST OF SYMBOLS

- A Dimensionless axial temperature gradient, $\frac{ac}{u^2} \frac{dT}{dx}$
- a Channel half-height (or pipe radius) in the transverse (B_z) magnetic field direction (meters)
- B Magnetic field strength (tesla)
- C Wall conductance ratio, $\frac{\sigma_w t_w}{u a}$
- c Liquid specific heat capacity, $\left(\frac{\text{joules}}{\text{kg}^\circ\text{K}}\right)$
- D Pipe diameter (meters)
- D_h Hydraulic diameter (meters)
- G Dimensionless temperature difference, $\frac{c (T_{wi} - T)}{u^2}$
- G_B Dimensionless wall temperature excess, $\frac{c (T_{wi} - T_B)}{u^2}$
- H Hartmann number, $a B \sqrt{\frac{\sigma}{\eta}}$
- I Magnetic interaction parameter, $\frac{4H^2}{Re}$
- j_y Electric current density (amps/m²)
- Jy Dimensionless current density, $\frac{j_y}{\sigma u B_z}$
- K Load parameter, $\frac{1}{1 + C}$
- k Thermal conductivity $\left(\frac{\text{watt}}{\text{m}^\circ\text{K}}\right)$
- L Length of flow channel (meters)
- Nu Nusselt number, $\frac{h D_h}{k}$
- Pe Peclet number, $Re Pr$
- Pr Prandtl number, $\frac{\eta c}{k}$
- q Energy flux (watts/m²)

- $\hat{q}$ Dimensionless energy flux, $\frac{c a q}{k \bar{u}^2}$
- Re Reynolds number, $\frac{\rho \bar{u} D_h}{\eta}$
- R_m Magnetic Reynolds number, $\sigma \bar{u} \mu D_h$
- S Internal (heat generation) source strength $\left(\frac{\text{watts}}{\text{m}^3}\right)$
- $\hat{S}$ Dimensionless heat source strength, $\frac{c a^2 S}{k \bar{u}^2}$
- T Temperature ($^{\circ}\text{K}$)
- T_B Bulk fluid temperature, ($^{\circ}\text{K}$)
- T_{w1} Inner wall surface (fluid side) temperature, ($^{\circ}\text{K}$)
- t_w Wall thickness, (meters)
- U Dimensionless fluid velocity, $u/\bar{u}$
- u Local fluid velocity, (meters/sec)
- $\bar{u}$ Average fluid velocity, (meters/sec)
- x Direction of fluid velocity
- Z Dimensionless coordinate, z/a
- η Dynamic viscosity (Newton - secs/ m^2)
- θ Angle in polar coordinates
- μ Magnetic permeability, $4\pi \times 10^{-6}$ (henrys/m $\equiv$ kg - m/coul 2)
- ν Kinematic viscosity (meter 2 /sec 2), η/ρ
- ρ Density (kg/m 3)
- σ Electrical conductivity (ohm $^{-1}$ m $^{-1}$)

Subscripts

- B Bulk fluid property
- J Joule dissipation contribution to $(T_{wi} - T)$
- JB Joule dissipation contribution to $(T_{wi} - T_B)$
- Q Wall heat flux contribution to $(T_{wi} - T)$
- QB Wall heat flux contribution to $(T_{wi} - T_B)$
- S Internal heat generation contribution to $(T_{wi} - T)$
- SB Internal heat generation contribution to $(T_{wi} - T_B)$
- V Viscous dissipation contribution to $(T_{wi} - T)$
- VB Viscous dissipation contribution to $(T_{wi} - T_B)$
- w wall
- wi inner wall surface (fluid coolant side)
- wo outer wall surface (plasma side)
- w1 first wall facing the plasma
- w2 second wall
- ⊥ Transverse component, perpendicular to the fluid velocity
- || Parallel component, parallel to the fluid velocity.

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APPENDIX A

Effect of Magnetic Fields on Flibe

Flibe is a slightly electrically-conducting fluid. For a pipe diameter of 2 cm, some typical Hartmann numbers for flibe are approximately as follows:

B (tesla)	H (Reference 28)	H (Reference 6)
0.1	0.1	0.36
0.5	0.5	1.8
1.0	1.0	3.6
5.0	5.0	18.0
10.0	10.0	36.0

The two sets of values of H are for two different sets of property data given in the two references cited (see Table 1).

Since B_1 is unlikely to exceed 1.0 tesla inside the fusion reactor blanket, we see that $H_1 \leq 4$ for flibe. This small value of H_1 will result in a small amount of turbulence suppression and some change in the velocity profile. The net effect on the Nusselt number can be estimated using Blum's correlation equation²⁴ for $I_1 < 0.1$:

$$\frac{Nu_m - Nu_0}{Nu_0} \approx - \frac{H_1^2}{6 Re_0} = - 1.5 I_1$$

This equation agrees with both his rectangular channel data for flows of a 15% aqueous solution of KOH and the data of Gardner and Lykoudis for mercury flows²³ in the regime where the turbulence is just beginning to be suppressed. It appears to be reasonably accurate as long as $Re > 1.5 Re_{crit} \approx 750 H_1$.

The maximum amount of turbulence suppression for flibe would occur at

a low $Re \approx 10^4$ and a maximum H_L of about 4. The empirical relation of Blum predicts a maximum decrease in Nu of about 1% due to transverse magnetic fields. Consequently, it seems reasonable to neglect the effects of transverse magnetic fields on the heat transfer characteristics of filbe.

For the case of parallel magnetic fields, the maximum likely H_{II} for filbe is about 40 for B_{II} about 10 tesla. Kovner, et. al.²⁵ have measured the effect of B_{II} on the Nusselt number in liquid gallium in round tubes at $H_{II} = 190$ and 275 over a range of Reynolds numbers from about 8×10^3 to about 10^5 ($Pr = .033$). They give an empirical correlation equation for this partially-turbulence-suppressed regime:

$$Nu_m - Nu_0 = .005 Pe \left[\frac{1}{1 + 1890 \left(\frac{2 H_L}{Re} \right)^{1.7}} - 1 \right]$$

where they used $Nu_0 = 6.5 + .005 Pe^{0.8}$ for the $B = 0$ value. These equations fit the gallium data quite well.

For the limiting case of low magnetic interaction parameter, $I_{II} < 1$, (i.e. the high Reynolds number region of Kovner's data), the following simpler empirical equation has been found to fit the data reasonably well:

$$\frac{Nu_m - Nu_0}{Nu_0} \approx \left(0.15 \frac{H_{II}^2}{Re_0} \right) + 0.04 I_{II}$$

If this result can be assumed to have general validity, it indicates that a parallel magnetic field is much less effective in suppression of turbulence than a transverse magnetic field, at least for weak magnetic interaction. This is in accord with other studies (e.g. Reference 12).

Using this simplified empirical equation, we can estimate the effect of parallel magnetic fields on flibe heat transfer. For a low $Re \approx 10^4$ and a maximum $H_{||} \approx 40$, this relation predicts a maximum decrease in the Nusselt number of about 2.3% due to parallel magnetic fields. Consequently, for the purpose of this study, it seems reasonable to neglect the effects of parallel magnetic fields, as well as transverse magnetic fields, on flibe heat transfer.

APPENDIX B

Some Additional Effects on Lithium Heat Transfer

In liquid metal heat transfer experiments using mercury²³, natural convection effects were found to have an important influence on the forced convection and on the temperature profiles for values of Reynolds number, $\frac{\rho u D}{\mu}$, up to as high as 3×10^5 in a tube of 0.56 cm diameter. In general, free convection effects tend to scale as $\frac{Gr Pr}{Re^m} \left(\frac{D}{L}\right)^n$, where the exponents m and n appear to depend on many factors. The Grashof number, Gr, is defined as

$$Gr = \frac{D^3 \rho g (T_w - T_B)}{\nu^2}$$

where ρ is the volume coefficient of thermal expansion, g is the acceleration of gravity, $(T_w - T_B)$ is the difference between the wall and bulk fluid temperatures and ν is the kinematic viscosity.

Since lithium has a kinematic viscosity about a factor of six higher than mercury, the Grashof number for lithium is about a factor of thirty-six lower than for mercury, other things being equal. Based on the criterion used in Reference 23, we can tentatively conclude that free convection effects can probably be neglected in these lithium flow applications for $Re > 10^4$. There may even be additional significant magnetic damping effects on the free convection in the liquid metals, as suggested by the data of Reference 25.

With regards to axial diffusion of heat in liquid metal coolants, Reference 29 concludes that this effect has no significant influence on the heat transfer for Peclet numbers greater than 35. (The Peclet number is defined as $Re \times Pr$). Since we expect to operate at $Pe > 170$ with lithium, axial heat diffusion in the fluid should be negligible.

Finally, the variation of the fluid properties should be negligible for liquid lithium due to the small temperature differences normally encountered in liquid metals.

APPENDIX C

Fully-Developed, Magnetic-Laminar Flow Between Parallel Plates

The simplified set of equations governing the steady, fully-developed incompressible flow of an electrically conducting fluid between parallel plates in the presence of a transverse magnetic field (Figure 3a) is:

$$\text{Continuity:} \quad \rho u A = \text{constant} \quad (1)$$

$$\text{Momentum:} \quad + \frac{dp}{dx} = j_y B_z + \eta \frac{d^2 u}{dz^2} \quad (2)$$

$$\frac{dp}{dz} = -j_y B_x \quad (3)$$

$$\text{Ohm's Law:} \quad j_y = \sigma (E_y - u B_z) \quad (4)$$

$$j_x = j_z = 0 \quad (5)$$

$$\text{Maxwell's Eqs:} \quad \frac{dB_x}{dz} - \frac{dB_z}{dx} = \mu j_y \quad (6)$$

$$\nabla \times \vec{B} = 0 \quad (7)$$

$$\nabla \times \vec{E} = 0 \quad (8)$$

$$\nabla \cdot \vec{E} = 0 \quad (\text{in the bulk of the fluid}) \quad (9)$$

$$\text{Energy:} \quad \rho u c \frac{dT}{dx} = k \frac{d^2 T}{dz^2} + \eta \left(\frac{du}{dz} \right)^2 + \frac{j_y^2}{\sigma} + S \quad (10)$$

A further simplification made in this study is that the induced magnetic field, B_x , is always very much less than the applied magnetic field, B_z (i.e. low magnetic Reynolds number). This assumption is not necessary to obtain a solution of the equations (see e.g. Refs. 19 and 20), but it is justified in the cases of interest here. Equations (3) and (6) thus no longer enter the problem.

These simplifications together with equations (7) and (8) yield:

$$u = u(z) \text{ only}$$

$$\frac{dp}{dx}, B_z, \text{ and } E_y \text{ are constants}$$

$$j_y = j_y(z) \text{ only}$$

For constant uniform wall heat flux, q_w , and uniform internal heat sources, S , $\frac{dT}{dx}$, is also a constant. Equation (9) only indicates that there is no excess change in the bulk of the fluid.

The solution for the velocity profile is decoupled from the energy equation and involves only the solution of equations (1), (2) and (4). The momentum equation can be integrated directly using Ohm's Law and the continuity equation to yield the classical Hartmann velocity profile (e.g. see Ref. 14).

$$U(z) = C_m \left(1 - \frac{\cosh H_\perp z}{\cosh H_\perp} \right)$$

where

$$C_m \equiv \frac{H_1}{H_1 - \tanh H_1}$$

and

$$J_y(Z) = K - U(Z)$$

where

$$U \equiv \frac{u}{u}$$

$$Z \equiv \frac{z}{a}$$

$$K \equiv \frac{E_y}{\sigma B_z}$$

$$H_1 \equiv a B_z \sqrt{\frac{\sigma}{\eta}}$$

$$J_y \equiv \frac{j_y}{\sigma u B_z}$$

Using these relations for U and J_y , the energy equation can now be integrated to yield the fully-developed Hartmann temperature profile Using

$$G \equiv \frac{c(T_w - T)}{u^2},$$

$$G = G_Q + G_S + G_V + G_J$$

where

$$G_Q = C_m \hat{q}_w \left[\frac{(1 - Z^2)}{2} - \frac{1 - \frac{\cosh H_1 Z}{\cosh H_1}}{H_1^2} \right]$$

$$G_S = G_Q \times \frac{\hat{S}}{\hat{q}_w} = \hat{S} \frac{(1-z^2)}{2}$$

$$G_V = C_{m1}^2 \frac{Pr}{4} \left\{ - \left[1 - \frac{\cosh^2 H_1 Z}{\cosh^2 H_1} \right] - \left[\frac{H_1^2 (1-z^2)}{\cosh^2 H_1} \right] + 2 C_{m1} \left[\frac{\tanh H_1}{H_1} - \frac{2}{\cosh^2 H_1} \right] \right. \\ \left. \left[\frac{(1-z^2)}{2} H_1^2 - \left(1 - \frac{\cosh H_1 Z}{\cosh H_1} \right) \right] \right\}$$

$$G_J = C_{m1}^2 Pr \left\{ - C_{m2}^2 \frac{(1-z^2)}{2} - \frac{1}{4} \left[1 - \frac{\cosh^2 H_1 Z}{\cosh^2 H_1} \right] - \left[\frac{2 C_{m2}}{H_1} \left(1 - \frac{\cosh H_1 Z}{\cosh H_1} \right) \right] \right. \\ \left. + \left[\frac{H_1^2 (1-z^2)}{4 \cosh^2 H_1} \right] + C_{m1} \left[\frac{H_1 \tanh H_1}{2} - \tanh^2 H_1 + \frac{H_1^2 (1-K)^2}{C_{m1}^2} + \frac{H^2}{2 \cosh^2 H_1} \right] \right. \\ \left. \times \left[\frac{(1-z^2)}{2} - \frac{\left(1 - \frac{\cosh H_1 Z}{\cosh H_1} \right)}{H_1^2} \right] \right\}$$

where

$$\hat{S} \equiv \frac{S}{k \left(\frac{u^2}{ca^2} \right)}$$

$$\hat{q}_w \equiv \frac{q_w}{k \left(\frac{u^2}{ca^2} \right)}$$

and

$$C_{m2} \equiv H_1 (K-1) \cosh H_1 - K \sinh H_1$$

The nondimensional axial temperature gradient,

$$A \equiv \frac{ca}{u^2} \frac{dT}{dx}$$

is given by:

$$A \operatorname{Re} Pr = \hat{q}_w + \hat{S} + A_V + A_J$$

where

$$A_V = Pr \frac{C_{m1}^2}{2} \left[H_1 \tanh H_1 - \frac{H_1^2}{\cosh^2 H_1} \right]$$

and

$$A_J = Pr \frac{C_{m1}^2}{2} \left[H_1 \tanh H_1 + \frac{2H_1^2 (1-K)^2}{C_{m1}^2} - 2 \tanh^2 H_1 + \frac{H_1^2}{\cosh^2 H_1} \right]$$

The nondimensionalized difference between the wall temperature on the inner or fluid side, T_{w1} , and the bulk fluid temperature, T_B , is given by:

$$G_B = \int_0^1 G U dZ$$

where

$$G_B \equiv \frac{c (T_{w1} - T_B)}{u^2}$$

and

$$T_B \equiv \frac{\int_0^a \rho u c T dz}{\int_0^a \rho u c dz}$$

The results of this integration are:

$$G_B = G_{BQ} + G_{BS} + G_{BV} + G_{BJ}$$

where each of the contributions is given by ($H \equiv H_1$):

$$G_{BQ} = c_{m1}^2 \hat{q}_w \left\{ \frac{1}{3} - \frac{\tanh H}{2H} + \frac{1}{2} \left[\frac{\tanh H}{H} \left(1 + \frac{2}{H^2} \right) - \frac{2}{H^2} \right] - \frac{1}{H^2} + \frac{2 \tanh H}{H^3} - \frac{\tanh H}{2H^3} + \frac{1}{2H^2 \cosh^2 H} \right\}$$

$$G_{BS} = G_{BQ} \frac{\hat{S}}{\hat{q}_w} - \frac{c_{m1} \hat{S}}{2} \left\{ \frac{2}{3} - \frac{2}{H^2} - \frac{2 \tanh H}{H^3} \right\}$$

The expressions for G_{BV} and G_{BJ} are rather long and will be omitted, because they are of little importance to the problems considered in this report.

For large values of H , they approach the following values:

$$\left. \begin{aligned} G_{BV} &\Rightarrow - \frac{\text{Pr } H_1}{6} \left[1 + \frac{9}{2H_1} \right] \\ G_{BJ} &\Rightarrow \frac{H_1}{6} (1 - 2K^2) \end{aligned} \right\} (H_1 \gg 10)$$

APPENDIX D

Effect of Finite Wall Conductance

As the wall conductance ratio, C , is increased from zero (perfectly electrically insulating walls) to infinity (perfectly electrically conducting walls), the joule dissipation term, G_{BJ} , varies in magnitude and changes sign at a value to $C = (\sqrt{2} - 1) = 0.414$. It is at first surprising to note that the joule contribution to G_B has the same magnitude for $C = 0$ and $C = \infty$, even though the total joule dissipation for $C = \infty$, ($K = 0$) is roughly H_1^2 times as large as it is for $C = 0$, ($K = 1.0$). This result can be interpreted by referring to the sketch in Figure D-1 which shows the variation of the joule dissipation per unit volume across the channel. The joule heating contribution to G_B is determined not by the magnitude of the joule dissipation, but rather by its distribution throughout the channel. For $C = 0$ and $C = \infty$, the shape of the distribution is approximately the same, although the regions of high and low joule dissipation are reversed. This causes the reversal in the sign of the G_{BJ} contribution.

It should be noted that the joule dissipation in electrically conducting walls varies approximately as follows for $C > 0$ ($K < 1.0$)

$$\left(\frac{j^2}{\sigma}\right)_w \approx \sigma_w \bar{u}^2 B_1^2 \frac{1}{(1+C)^2}$$

For typical values of $C \ll 1$, and for $\bar{u} \approx 1$ meter/sec, $B_1 \approx 1$ Tesla, and $\sigma_w \approx 2 \times 10^6$ mhos/m, the joule dissipation in the walls is only about 2 watts/cm³. In the fusion reactor examples considered, this internal heat generation can be seen to be negligible compared to the typical neutron energy deposition in the walls (about 250 watts/cm³).

Figure D-1 - Comparison of the Relative Joule Dissipation for Perfectly Insulating Walls ($C = 0$) and Perfectly Conducting Walls ($C = \infty$) in a Parallel Plate Channel Flow of an Electrically Conducting Liquid

APPENDIX E

Effect of Magnetic Field Orientation on Round Pipe Flows

Several possible B field orientations (or combinations of B field components) are possible, for round pipe flows. A B_z orientation along the z axis (Figure E-1a) implies a maximum Nusselt number of about 8.7 at the $\theta = 0$ point where the neutron flux is a maximum (assuming the neutron flux is in the negative z direction as shown). Hence this is not the worst case from a heat transfer point of view. The worst case of Hartmann flow is where B_z is oriented along the y direction, (Figure E-1b); then the Nusselt number is a minimum of about 5.5 at the $\theta = 0$ point and the wall temperature excess will be a maximum.

The worst case from a heat transfer viewpoint of any of the magnetically-laminarized flows is that flow with a laminar parabolic velocity profile (which can occur for a strong $B_{||}$ along the x axis parallel to the flow velocity as shown in Figure E-1c). For this case, the Nusselt number takes on its lowest value of about 4.36 and the wall temperature excess is a maximum at the $\theta = 0$ point.

A fourth possibility occurs when the pipes essentially follow the field lines, but the field strength changes. (This pipe path along the field lines is desirable to minimize magnetic pressure drops³.) However, the changing $B_{||}$ field strength along the x direction creates a radial component of B (Figure E-1d) as pointed out by Fuller³⁰. For the asymmetric case where

$$K_x = \frac{dB_{||}}{dx} = \text{constant}$$

the radial field strength is given by:

$$B_r = K_x \frac{r}{2}$$

Figure E-1 - Various Possible Magnetic Field Components and Orientations for Round Pipes

This magnetic field causes an e.m.f. which generates azimuthal current loops which close completely in the fluid. These currents create an electromagnetic body force, $j_\theta B_r$, which can suppress turbulence, change the velocity and temperature profiles and cause a magnetic pressure drop.

For high radial Hartmann numbers, $H_R \equiv R_0 B_R \sqrt{\frac{\sigma}{\eta}}$, the velocity profile becomes very flat approaching the uniform, slug flow limit. The resulting temperature profiles for this magnetic laminar flow in round pipes approaches a parabolic shape for $H_R \gg 10$ for the case where the uniform wall heat fluxes are much greater than the energy addition due to joule dissipation³⁰. The Nusselt number for this high H_R limiting case approaches 8.0. This is exactly what we would obtain for the ordinary laminar slug flow in round pipes with no magnetic fields. This is due to the fact that the radial B field flattens the velocity profile in all planes. As discussed earlier, a transverse field, $B_\perp$, only tends to flatten the velocity profiles in round pipe flows in the planes close to the $B_\perp$ direction, leading to an average Nusselt number of about 7.1 rather than 8.0 in the limiting case of high $H_\perp$.

APPENDIX F

Effect of Nonuniform Wall Heat Flux on Round Pipe Flows

For a circumferential variation of the heat flux given by the following Fourier series:

$$q_w(\theta) = q_{w(AVG)} + \sum_{n=1}^{\infty} \left[b_n \cos(n\theta) \right]$$

Reynolds²⁶ obtained the following variation in the wall temperature excess for laminar flow:

$$(T_{wf}(\theta) - T_B) = \frac{a}{k} \left[\frac{2}{Nu_0} q_{w(AVG)} + \sum_{n=1}^{\infty} \frac{b_n}{n} \cos(n\theta) \right]$$

where Nu_0 is the classical value of 4.36 for a parabolic velocity profile.

In a fusion reactor, the first-wall tubes will actually see a variable radiation flux and alpha particle flux on the plasma side. However, on the back side, the heat flux is almost constant at a lower value than the front side; this heat flux is due only to the neutronic energy deposition in the back wall area.

We can consider two simple Limiting Cases in order to estimate the increase in the front (first) wall temperature due to the heat flux asymmetries:

- A. The total heat flux is given by a simple cosine which goes to zero at $\theta = \pi$.

$$q_w = q_{w(AVG)} \cos \theta$$

- B. The total heat flux is uniform on the plasma side and zero on the back side.

For Limiting Case A, we find

$$\frac{(T_{wi} - T_B)_{\max}}{(T_{wi} - T_B)_{\text{uniform}} \text{ at } q_{\max}} = 1.6$$

For Limiting Case B, we find

$$\frac{(T_{wi} - T_B)_{\max}}{(T_{wi} - T_B)_{\text{uniform}} \text{ at } q_{\max}} \approx 2.2$$

For the parallel plate case, this latter case was found to be exactly 2.0. Thus, the asymmetric wall heating on round pipes also results in about a factor of 2 increase in the first-wall temperature excess, $T_{wi} - T_B$.