MASTER

SPIN DEPENDENT INELASTIC ELECTRON-NUCLEON SCATTERING*

C. E. Carlson and Wu-Ki Tung

The Enrico Fermi Institute and the Department of Physics

The University of Chicago, Chicago, Illinois 60637

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It is shown that the simultaneous measurement of the longitudinal and transverse polarization effects in deep inelastic lepton hadron scattering offers a practical way of determining the two spin dependent structure functions. These functions furnish interesting information on the spin dependence of scattering amplitudes at high energies as well as provide the natural testing ground for theoretical models based on light cone singularities, partons, etc.

The interesting results from the deep inelastic electron-nucleon scattering¹ over a wide kinematic region have stimulated much theoretical speculation on the origin of the observed behavior of the structure functions W_1 and W_2 . In so far as most of these ideas are designed to explain the observed phenomena, further experimental studies are desirable and indeed necessary in order to test the predictions of the various theories and distinguish the good ones from the bad ones. The spin dependent part of the inelastic lepton-hadron scattering provides a most natural testing ground for this purpose². This process is sufficiently simple to allow for definitive predictions from most of the models and also is accessible to experimental study in the near future at SLAC and NAL.

Furthermore, the spin dependence of scattering amplitudes at high energies

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is an interesting problem in general, and so far has not been well understood. Measurements on polarization effects in inelastic lepton-nucleon scattering can yield detailed information on the forward helicity amplitudes for virtual Compton scattering off nucleon targets. Thus, these experiments should provide additional information on the controversial question about s-channel helicity conservation³ and other conjectures concerning spin dependence at high energies.

We shall show that a practical way to gain complete information on the spin dependent structure functions in deep inelastic e-N and μ -N scattering is to measure the cross section with the target polarization vector both parallel and perpendicular to the incoming lepton helicity. Also, we enumerate the important predictions from various models which can be directly confronted with such experiments. In particular, simple and direct consequences of the far reaching hypothesis of scale invariance of leading light cone singularities⁴ can be subjected to experimental test.

Measurement of the Structure Functions: The kinematics of this process is well known. Fig. 1 defines our designation of momentum and polarization variables for each of the particles. Thus we consider a lepton ($e^\pm$ or $\mu^\pm$) with energy E and helicity λ incident on a nucleon target with polarization vector $\hat{n}$. The outgoing lepton with energy E' and scattering angle θ is measured. The final state hadrons (with total 4-momentum p') and the polarization of the outgoing lepton are not detected. Since the asymmetry with $\hat{n}$ perpendicular to the scattering plane is predicted to be zero by time reversal invariance⁵ and this is indeed found to hold, within errors, by experiment⁶, we shall confine our attention to the case where $\hat{n}$ is a vector in the scattering plane. Also, we shall set the lepton mass equal to zero throughout this paper. If we choose the z-axis in the Lab. frame to be along

the direction of the recoil hadrons p' (or q), the kinematic configuration will be as illustrated in Fig. 2. The difference in the differential cross sections with fixed lepton helicity λ but target polarization vectors $\hat{n}$ and $-\hat{n}$ is

$$\frac{d\sigma_{\Delta}}{dE'd\Omega} - \frac{d\sigma_{-\Delta}}{dE'd\Omega} = \frac{4\alpha^2}{Mq^2} \frac{E'}{E} \times \left[\cos \varphi \frac{E+E'}{\sqrt{q^2+v^2}} (W_{1\frac{1}{2},1\frac{1}{2}} - W_{1-\frac{1}{2},1-\frac{1}{2}}) + \sin \left\{ \cos \frac{\theta}{2} \cdot \sqrt{\frac{2EE'}{q^2+v^2}} W_{1\frac{1}{2},0-\frac{1}{2}} \right\} \right], \quad (1)$$

where $W_{m\sigma,n\tau}$ are the absorptive parts of the helicity amplitudes for forward virtual photon hadron scattering,

$$\gamma(q, n) + N(p, \tau) \rightarrow \gamma(q, m) + N(p, \sigma) \quad (2)$$

These helicity amplitudes can be related to an appropriate set of invariant structure functions by the following formulas:

$$W_{m\sigma,n\tau} = \frac{1}{4\pi} \int d^4x e^{-iqx} \epsilon_{\mu}^{(m)*}(q) \langle p\sigma | [j^{\mu}(x), j^{\nu}(0)] | p\tau \rangle \epsilon_{\nu}^{(n)}(q) \\ = \epsilon_{\mu}^{(m)*}(q) \bar{u}_{\sigma}(p) W^{\mu\nu}(q, p) u_{\tau}(p) \epsilon_{\nu}^{(n)}(q), \quad (3)$$

$$W^{\mu\nu} = (-g^{\mu\nu} + q^{\mu}q^{\nu}/q^2) W_1 \\ + (1/M^2) [p^{\mu} - q^{\mu}(q \cdot p)/q^2] [p^{\nu} - q^{\nu}(q \cdot p)/q^2] W_2 \\ + (1/M) [\gamma^{\mu} \not{q} \gamma^{\nu} - \gamma^{\nu} \not{q} \gamma^{\mu}] G_1 \\ + (1/M^2) [q^2 [\gamma^{\mu}, \gamma^{\nu}] - 2 \not{q} (q^{\mu} \gamma^{\nu} - \gamma^{\mu} q^{\nu})] G_2, \quad (4)$$

where M is the nucleon mass, $u_{\sigma}(p)$ its wave function and $\epsilon_{\mu}^{(m)}$, $m=1,0,-1$, are the helicity polarization vectors for the virtual photon. We obtain then the relations,

$$\frac{1}{2}(W_{1\frac{1}{2},1\frac{1}{2}} + W_{1-\frac{1}{2},1-\frac{1}{2}}) = W_1 \\ W_{0\frac{1}{2},0\frac{1}{2}} = (\frac{v^2}{q^2} + 1) W_2 - W_1 \\ \frac{1}{2}(W_{1\frac{1}{2},1\frac{1}{2}} - W_{1-\frac{1}{2},1-\frac{1}{2}}) = -2[(v/M) G_1 + (q^2/M^2) G_2] \\ W_{1\frac{1}{2},0-\frac{1}{2}} = 2(2q^2/M^2)^{1/2} [-G_1 + (v/M) G_2]. \quad (5)$$

The first two lines are proportional to the familiar σ_T and σ_S , respectively.

It is customary to consider the special case of (1) where the polarization

axis of the target is parallel (and anti-parallel) to the incident lepton momentum⁷. The difference in cross sections then takes the familiar form, in terms of the invariant structure functions,

$$\frac{d\sigma_{\uparrow\uparrow}}{dE'd\Omega} - \frac{d\sigma_{\downarrow\downarrow}}{dE'd\Omega} = \frac{16\alpha^2}{Mq^2} \frac{E'}{E} \left[\frac{E+E'\cos\theta}{M} G_1 + \frac{q^2}{M^2} G_2 \right] \quad (6)$$

(longitudinal polarization).

In principle, detailed measurements of the left-hand side of this equation should enable one to determine the two structure functions very much in the way W_1 and W_2 are determined from the spin averaged measurements. In practice, this is tedious and hard to achieve because of (i) increasing difficulty of measurement arising from the use of the polarized target and (ii) decreasing statistics due to taking the difference of two cross sections. In addition, we shall show in the next section that it is very likely that G_2 will not contribute noticeably to this difference in the deep inelastic region. This implies that the measurement will yield direct information on G_1 , leaving G_2 completely undetermined.

This situation suggests that one should utilize the full power of the polarization measurements by varying the target polarization axis. In particular, a different combination of the structure functions is measured if the target polarization vector is placed perpendicular to the momentum (and helicity) of the incident lepton. A simple calculation using Eqns. (1)-(5) yields,

$$\frac{d\sigma_{\rightarrow}}{dE'd\Omega} - \frac{d\sigma_{\leftarrow}}{dE'd\Omega} = \frac{16\alpha^2}{Mq^2} \frac{E'^2}{ME} \sin\theta \left[G_1 - \frac{2E}{M} G_2 \right] \quad (7)$$

(transverse polarization).

In this expression, we shall see, G_2 is no longer suppressed relative to G_1 . A simultaneous measurement of the longitudinal and transverse polarizations will thus allow the direct determination of both G_1 and G_2 . This is very important because the behavior of G_2 is especially useful in distinguishing various theoretical models.

Phenomenological Expectations: The most provocative feature of the results from the spin averaged deep inelastic e-N scattering experiments is the scaling behavior^{1,8} found to hold over a considerable range of the observed kinematic region: that is,

$$\nu W_2 \rightarrow F_2(\omega), \quad (8a)$$

with some indication of

$$W_1 \rightarrow F_1(\omega), \quad (8b)$$

where the limit is taken with $\nu \rightarrow \infty$, $q^2 \rightarrow \infty$ and $\omega = 2M\nu/q^2$ fixed. These results, together with certain rigorous inequalities satisfied by the structure functions place restrictions on the polarization measurements discussed in the previous section. These inequalities follow from positivity conditions on the amplitude $W_{\mu\nu}$ due to unitarity⁹. Aside from the familiar relations $\sigma_T \propto W_1 \geq 0$ and $\sigma_S \propto [(\nu^2/q^2 + 1)W_2 - W_1] \geq 0$, we shall need,

$$W_1 \geq \frac{1}{2} |W_{1\frac{1}{2}, 1\frac{1}{2}} - W_{1-\frac{1}{2}, 1-\frac{1}{2}}| \quad (9a)$$

$$\text{and} \quad W_{1\frac{1}{2}, 1\frac{1}{2}} \cdot W_{0\frac{1}{2}, 0\frac{1}{2}} \geq (W_{1\frac{1}{2}, 0-\frac{1}{2}})^2 \quad (9b)$$

Then from Eqns. (5), (8), and (9) we can conclude that

$$\begin{aligned} G_1 &\rightarrow O(\nu^{-1}) \\ G_2 &\rightarrow O(\nu^{-3/2}), \end{aligned} \quad (10)$$

where $O(\nu^{-\alpha})$ means "of the order of $\nu^{-\alpha}$ or less". This means the contribution from G_1 to the longitudinal cross section, Eqn. (6), can be finite while the contribution from G_2 must vanish in the deep inelastic region. In contrast, the contribution from G_2 to the transverse cross section, Eqn. (7), is likely to be of higher order than that from G_1 if Eqn. (10) is correct. In fact, even if G_2 falls off as ν^{-2} , as suggested by some models, it still contributes at least equally with G_1 .

Another feature of the observed results is the smallness¹ of σ_S which is usually taken as an indication that perhaps

$$\sigma_s \propto \left(\frac{\nu^2}{q^2} + 1 \right) W_2 - W_1 \rightarrow 0 \quad (11)$$

in the scaling limit. If so, Eqn. (9b) implies

$$W_{\frac{1}{2}, 0 - \frac{1}{2}} \rightarrow 0. \quad (12)$$

This, in turn, implies through the last of Eqns. (5) that in the scaling region

$$G_1 \approx (\nu/M) G_2. \quad (13)$$

Theoretical Implications: The measurement of both G_1 and G_2 , or equivalently $(W_{\frac{1}{2}, \frac{1}{2}} - W_{\frac{1}{2}, 1 - \frac{1}{2}})$ and $W_{\frac{1}{2}, 0 - \frac{1}{2}}$ yields immediate information on the spin dependence of scattering amplitudes at high energies. Thus, if $W_{\frac{1}{2}, 0 - \frac{1}{2}}$ is found to be small, one would have further evidence for s-channel helicity conservation³ (only in the forward direction, though). The same result can also be interpreted as the dominance of the scattering mechanism by a single set of intrinsic quantum numbers (i. e., those of the vacuum) in the t-channel¹⁰. Similarly, the size of $(W_{\frac{1}{2}, \frac{1}{2}} - W_{\frac{1}{2}, 1 - \frac{1}{2}})$ found in these experiments will indicate whether the diagonal scattering matrix elements become spin independent as some intuitive arguments would suggest¹¹.

Of intense interest, of course, will be the possible scaling behavior of these new structure functions, in analogy to that found for the spin averaged structure functions. Here one will have the first opportunity to test the various models which have been constructed to accommodate the well known results for W_1 and νW_2 . The most ambitious and theoretically far reaching idea among these is probably that of scale invariance applied to the leading singularity on the light cone for current commutators⁴. The widely accepted rule of assigning dimension "J+2"¹² to operators which appear in the light cone expansion implies the following scaling behavior for the new structure functions:

$$(\nu/M) G_1(\nu, q^2) \rightarrow F_3(\omega)$$

$$\text{and} \quad (\nu/M)^2 G_2(\nu, q^2) \rightarrow F_4(\omega). \quad (14)$$

We very briefly indicate the derivation of this result. One writes the part

of the current commutator antisymmetric in (μ, ν) as

$$[j^\mu(x), j^\nu(0)]^{(A)} = \epsilon^{\mu\nu\lambda\sigma} \partial_\lambda \theta_\sigma(x, 0) + \epsilon^{\mu\nu\lambda\sigma} \partial_\lambda \partial^\tau \theta_{\sigma\tau}(x, 0) + \dots \quad (15)$$

where $\theta_\sigma(x, 0)$ and $\theta_{\sigma\tau}(x, 0)$ are axial vector and pseudotensor bilocal operators.

The leading singularities of the light cone expansions of these operators are assumed⁴ to consist of a smooth operator of dimension $J+2$ (in this case, 3 and 4, respectively) and a singular c-number coefficient free from dimension carrying parameters. Conservation of dimension then determines the character of the singular functions. This implies

$$\theta_\sigma(x, 0) \sim \delta(x^2) \theta_\sigma(x, 0) \quad (16a)$$

$$\text{and} \quad \theta_{\sigma\tau}(x, 0) \sim \theta(x^2) \theta_{\sigma\tau}(x, 0). \quad (16b)$$

These results, when substituted into Eqns. (3) and (4), imply Eqn. (14).

For comparison, a naive application of Bjorken's original "derivation" of scaling⁸ yields the same behavior for G_1 but

$$(v/M)^{3/2} G_2(v, q^2) \rightarrow F_4'(\omega). \quad (17)$$

Notice that both scaling behaviors (14) and (17) are consistent with the inequalities (10) which follow from unitarity and the observed behavior of W_1 and W_2 . However, if in addition the longitudinal cross section indeed vanishes, the inequality (9b) excludes a behavior like (17).

Another extensively studied idea is the parton model emphasizing the point-like structure of the constituents of the nucleon¹³. A very naive application of this model assuming quarks as the basic constituent² yields again the same scaling behavior for G_1 but predicts

$$G_2 = 0. \quad (18)$$

More sophisticated parton models, however, can result in behaviors for G_1 and G_2 both agreeing with (14). Models in which the helicity flip amplitude vanishes¹⁵ would predict, in addition to (14),

$$F_3(\omega) = F_4(\omega) \quad (19)$$

as a consequence of Eqn. (13).

Thus we see that many interesting ideas can be tested by measuring the spin dependent structure functions in the deep inelastic region. In particular, if the observed scaling behavior of these functions does not follow Eqn. (14), the credibility and subsequent predictive power of the light cone commutator approach would be severely impaired.

We have not discussed more detailed predictions from the various models, such as those involving sum rules¹⁵ and superconvergence relations^{15,16}. Many relations of this kind can be derived in most of the models mentioned previously (usually with more detailed assumptions). In spite of their theoretically appealing features, a conclusive quantitative comparison with data would require detailed measurements covering the entire kinematic region. Going into such detail both in theory and experiment is beyond the scope of this paper.

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Fig. 1

Fig. 2