DAEDALUS:
A CODE TO GENERATE A LINEAR
PROGRAMMING SIMULATION
OF A NUCLEAR POWER ECONOMY

W. I. Neef

JANUARY 1968

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OF A NUCLEAR POWER ECONOMY

by

W. I. Neef

Advanced Concepts and Analysis
Mathematics Department

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D A E D A L U S:
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INTRODUCTION

DAEDALUS is a code designed to incorporate the detailed characteristics of individual nuclear reactor types into the projected future of an expanding power industry. These reactor characteristics are then used in a linear programming format to determine the combination of reactors producing the optimum value of some overall objective function, which might be the minimum cost of power over a period of time or the minimum total amount of raw material and uranium that is needed. The reactor characteristics that concern DAEDALUS are the power level, the operating history--that is, the plant factor as a function of time--, and the impact of the reactor on various materials requirements, especially uranium, plutonium, thorium, and U^{233} .

The discussion that follows will consider the output from the DAEDALUS code and the input and calculations that are required to generate this output. The basic philosophy of DAEDALUS and linear programming is to simulate the history of the nuclear reactors as a series of linear equations, with the reactors and certain other items being the variables in these equations. The values of these variables are the number of reactors that are built. The coefficients in the equations are the effect that the building of one reactor has on the property being represented by the equation. These properties are: the total power requirement that is needed in a period, the total uranium that is used in a period, and so on. Each coefficient, then, is the total power that is generated per reactor in a period which, when multiplied by the number of reactors, is the total power that is generated by all reactors of one type during the period. Likewise, the amount of uranium used in a period by a reactor type is multiplied by the number of such reactors to find the total uranium requirements of all reactors of that type in that period. The sum of all these products for all reactor types then gives the total uranium required in a given period from all reactors.

The output from the DAEDALUS⁽¹⁾ consists of: (1) some preliminary initializing; (2) a list of row names that are generated to give labels to each of the equations in the matrix; (3) a list of matrix elements that are not associated with the reactor variables, being the coefficients that tie the various reactor characteristics together into the general equations used in the simulation; (4) the detailed listing of the data read into the DAEDALUS code to describe the characteristics of each individual type of reactor; these are the data that will be translated into coefficients on each of the rows defined for each reactor variable. These variables will be defined as the calculations proceed. Also included in the listing of the data read into the DAEDALUS code are the details used in calculating the cost coefficient for each reactor variable that is to be generated; (5) the right-hand side (or B-vector) of the linear programming problem which is equal to the total energy, capacity retirement, and certain other constraints of the problem that must be satisfied before an optimal solution is reached.

The next output, EQLIST,⁽²⁾ is not, strictly speaking, an output from DAEDALUS, but rather an output of the linear programming code. For convenience in operation, the DAEDALUS code, which was written independently, has been incorporated as a subroutine of the linear programming code for solution. The EQLIST output is the result of the calculations by DAEDALUS. A detailed discussion of these equations will follow a discussion of the output of DAEDALUS proper. The equations in the EQLIST will be discussed in an order which is much more logical for presentation than that actually followed in the DAEDALUS calculations. Mathematically, the actual order of the equations in the EQLIST is completely immaterial to the solution of the linear programming problem.

(1) "Appendix to DAEDALUS: A Code to Generate a Linear Programming Simulation of a Nuclear Power Economy, BNWL-796, APPENDIX, pp. 1-13, January 1968.

(2) *ibid.* p. 14.

INITIALIZATION

Two things are performed by the DAEDALUS code in preparing the tables of numbers to be used as the calculations proceed and in performing the housekeeping necessary to allow efficient operation of the code. First, it prorates the capital costs of reactors operating beyond the time period being simulated. As will be explained below, the cost of building a reactor near the end of the problem will be charged at the time a reactor is built, even if this is before the end of the period being simulated. And since part of this cost should not be represented as a cost of power during the period, the capital cost of such reactors is prorated on the basis of an accumulative discounted depreciation schedule. Second, once a data deck is set up, it may be easier to prepare additional data cards, rather than modify the original ones, when constructing a problem similar to a problem already run. The reason for this is that it is usually easier to modify a previously run problem by adding data cards requiring the unneeded data to be deleted than to remove and modify cards in the original program decks.

A. The Accumulative Discounted Depreciation Schedule (See Appendix, p. 1)

Let us assume that a reactor, once built, will operate for exactly 30 years before being shut down. Let us further assume that this reactor will be built exactly 10 years before the end of the time period being considered in the problem. Now, we do not wish to charge the entire capital cost of the reactor to the energy which will be produced during the first 10 years of the reactor lifetime. Only a part of the capital cost should be charged off to this energy. The rest of the capital cost should be charged off to the energy which is produced after the end of the problem. This prorating is done in DAEDALUS by considering the entire capital cost of the plan as an equivalent annual cost determined by a sinking fund calculation. The annual payments at the time they are made are then present-worthed back to the day the reactor is built. Thus, for this assumed reactor the capital cost will be spread out to 30 equal payments that will repay the capital cost at a 7 percent discount rate by the end of 30 years.

Only the first 10 of these annual payments will be considered in the problem as part of the capital cost of this reactor. The present worth of these 10 payments at the time the reactor is built will be added together to give the "total" capital cost of the reactor--the capital cost associated with the operation of the reactor during the last 10 years of the problem, which are the same as the first 10 years of the reactor lifetime. The output from this calculation is shown on the DAEDALUS printout under the heading "Accumulative Discounted Depreciation Schedule." (See Appendix, page 1.) The number printed under this heading, 1.4570-01, means that 0.14570 of the total capital cost will be charged off for a reactor which is built during the last period of the problem. Each period is two years long. So if a reactor costs \$100,000,000 a year, the present worth in the last period of the problem of the first two annual payments will be \$14,570,000. If a reactor is built four years before the end of the problem, so that only the payments for the first four years will be considered, this \$100,000,000 will be shown as a cost coefficient of \$27,296,000. This represents the present worth of the first four annual payments at the time the reactor was built. Likewise, our reactor which operates for 10 years will have the sum of the first 10 of the annual payments or the first five biennial present-worth sums indicated. In other words, this \$100,000,000 reactor which operates only for 10 years will have a total capital cost of \$56,601,000. If the reactor operates for the full 30 years before the end of the problem, then the total fraction of 1 or \$100,000,000 will be shown as the capital cost of the reactor.

B. The Delete Modification (See Appendix, p. 1).

If a problem being run is similar to one previously set up, but requires some of the data from the first problem to be deleted, the delete activities can be used. Certain rows of the problem may not be pertinent. For instance, the time period of the second problem might be only through 2020, whereas the time period of the first problem might be through 2040. Consequently, the rows and the column vectors associated with descriptions, material balance, energy requirements, etc. for the years beyond 2020 must be deleted. Rather than completely revamp the input deck, it is simpler to specify the data which is to be deleted. Then, in the course of

the LP loading routines and in generating the matrix, data will be generated to go in these rows beyond 2020 but will then be ignored because of the deletion. This will occur at the expense of a slight amount of machine time, but at a great saving of human error in setting up input decks. In the normal course of operation, these delete data are not needed. However, the capability has been built into DAEDALUS because it is available in the linear programming codes. We did not wish DAEDALUS to be any more restricted than the linear programming codes.

ROW ID LIST (See Appendix, p. 1)

Immediately following the indication of the delete information is the list of row names generated for the matrix. Many of the row names are standard for all problems of this type and can be set up by pattern information on the input data. The pattern for most of the ROW ID to be generated is given by two-digit codes indicating the years and by four-character codes indicating types of variables. Only the beginning year, the ending year, and the difference are given in addition to the four-character code. A special character of either plus, minus, or blank is also given as a type indicator for the row. The rows which are generated automatically are very similar to those generated from separate input cards. The ones which are generated automatically are the energy and capacity requirements, various material balances, and special reactor characteristics. The rows which have to be generated individually are the restraints for the coal plants, the annual and cumulative high-priced uranium requirements, and reactor introduction restraints. For example, the first row printed out on the special ROW ID cards is +70COLL. The 70 indicates that the row will be the restraint for 1970, the COLL is the four-character code indicated for the coal-fired power plants. The COL indicates coal-fired power plants and the second L of COLL indicates that this is in the low-priced region of the country; that is, where the lowest price of coal power can be obtained. The plus sign in front of the name indicates that a positive slack vector will be generated for this row. This means that row 70COLL is going to be a less-than inequality; in other words, the sum of the real variables on this row must be less than or equal to the total number

shown on the right-hand side. The real total plus the slack total then will equal the required total on the right-hand side. If the sign were negative, this would be a greater-than inequality, because the total value of real terms of the left side of the equation would have to be equal to or greater than the number of the right-hand side, and the slack would be taken up by a negative coefficient of the slack variable on that particular row. Other rows generated include the low- and medium-priced coal plant capacity for each of the years from 1970 through 1990. The numbers from 76SHT1 through 86SMS1 are the reactor introduction restraints. They are rows which will limit the total amount of a new kind of reactor that can be built during the first few years after its introduction. The details of the row names and the significance of each of these rows or each of these equations will be discussed when the equations themselves come up for discussion later in this report. The purpose of setting up the ROW ID's is simply to allow enough space in the matrix for each equation to be generated.

Following ROW ID is a message indicating that the LP system selected its own initial basis. It is possible in the linear programming to specify the starting set of vectors for the solution of the problem, and it has been our experience in running these problems that the LP system reached a solution more rapidly if a starting point for the iterating solution is a basis picked by the PRESLV routine rather than a basis that we might specify. The message "the LP system selected as its own initial basis" means that the PRESLV routine was used to start up this particular problem.

THE GENERATION OF THE MATRIX

A. Matrix Generation by Subroutine TOMUCH

The next message printed out says, "At least part of the matrix was generated by TOMUCH." This indicates that some of the matrix elements for the linear programming problem were generated from a pattern specified by the subroutine called TOMUCH. Some column vectors are similar from one equation to the other and can be generated by use of a pattern. Each individual matrix element must have associated with it a column name that is the name of a vector, a row name that is the name of an equation,

and a value that is the size of the coefficient. These three things can be specified for certain vectors as part of the input matrix. These vectors are generated automatically by TOMUCH. The detailed structure of these vectors is unimportant at this point of the discussion. Here it suffices to say that the vectors are generated correctly. The actual values of the coefficients and the names of the vectors will be indicated in detail as each row is discussed below. The details about how these vectors are generated will not be discussed at this point.

B. The Nonstandard Matrix Elements

Some matrix elements, in addition to those associated with reactor vectors, do not fit the overall pattern used in subroutine TOMUCH. These matrix elements are printed out after the word "matrix" on the output. There are none of these nonstandard elements in this problem. Had there been some, they would have been printed at this point in the output, one matrix element per line of output. Once these elements are written, DAEDALUS generates the part of the matrix associated with the operation of each type of reactor to be considered.

REACTOR VECTOR INPUT DATA

A reactor vector describes how the problem will be affected by building a reactor in a given year and operating the reactor at a predetermined load factor history for its entire lifetime. In this particular model we have two predetermined histories. The first is a heavy-load history in which the reactor operates at a plant factor of 85 percent for 15 years and then at progressively lower factors for the last 15 years. The second is a light-load history in which a reactor operates at a plant factor of 85 percent for only 5 years before operating at progressively lower factors. Each history requires a separate set of input data.

The input data needed to generate a reactor vector is printed on subsequent pages of output. The first page⁽¹⁾ contains fuel fabrication and reprocessing costs as functions of time. These data are obtained from the FUELCO code which derives them from total industry size and plant size considerations. The data on the next few pages come from the PACTOLUS

(1) See Appendix, p. 2.

code and describe the history of a reactor for its lifetime. The data consist of: (1) a general description of the reactor, (2) the total annual cost and throughput, (3) the total throughputs of each individual isotope fabricated each year before processing losses, (4) the total (by year) of each isotope processed after losses, and (5) the details of the cost coefficient calculations. This last item is not input data from PACTOLUS but is presented as output of the cost calculations for each reactor, and appears immediately following the printout describing the input data for each reactor. Because of that, it will be discussed when the individual reactor data are presented. The detailed calculations of the other elements of the reactor vector will be left until the discussion of the equation list (EQLIST).

A. General Description of the Reactor

The first page of output⁽¹⁾ presents a general description of the reactor and the fuel cycle. The top of the page contains a heading which gives a brief name of the reactor and possibly the type of fuel cycle the reactor will have. To the right of the heading, the four-digit code LWAA is repeated twice. In order for a matrix to be properly constructed, these two code designations must be identical. The last code LWAA is taken from the data card in the DAEDALUS input deck that indicates the reactor history we wish to consider with this name. The first code LWAA is taken from the input tape prepared by PACTOLUS. When these two code designations are the same, it is known that the intended tape has been properly read. Some of the other items on that page indicate a general description of the reactor, including the startup date for the reactor. This startup date is the one that corresponds to the uranium price, the fuel fabrication cost, and the reprocessing cost schedules for a reactor starting in that year. This date is not necessarily the date that a new reactor is introduced into the problem, nor is it necessarily the date that this reactor will be introduced into the problem. The 1970 date for the reactor LWAA indicates that this reactor history is based on an assumed startup of 1970. The vector of the actual reactor built, as will be indicated below, may be 1970,

(1) See Appendix, p. 3.

1972, or even 1990. The reactor power in thermal megawatts times the electrical conversion efficiency (3077 times 32.5 percent) gives the total electrical power level of this reactor--1000.0 megawatts. Various plant charge data are also indicated.

The nuclear liability cost of \$332,000 per year, and the plant base operating cost of \$1,003,000 per year are added together for the operating and insurance cost. And the combined State and local taxes can be included as part of an annual charge.

The total plant construction cost of \$133,000,000 is used as a basis for the capital cost calculation and the purchase of the first core is considered as part of the fuel cycle cost. Uranium cascade price schedule parameters are those prevailing for 1970, but these numbers will change if the price of uranium varies during the time the reactor operates. The fixed process times for prefabrication, fabrication, shipping, postreactor cooling, and so forth, are received from the Fuel Recycle Task Force and are used for calculating the length of time the fuel must be purchased before a core is loaded, and the length of time after the fuel is discharged before it is available from this separation process. Shipping charges are included in the reprocessing and fabrication costs. Reprocessing charges constitute the base throughput and operating costs for the reprocessing cost description. The total fabrication, shipping, purchase, and loss charges for the first core purchase are included with the total amount of fuel which must be purchased and are not considered as capital cost. This is of no particular importance for most reactors because the total cost, whenever it is spent, is actually capitalized and it is this total cost which serves as basis of the reactor cost coefficient. Only for reactors where the problem ends before the end of the reactor lifetime will there be an error caused by not considering as capital cost the fabrication charge and the purchase price of the first core. In any case, when the problem ends, there will be a treatment of the core cost such that the entire cost of purchasing the first core will be charged off to the operation of the reactor during the short period of time; however, the credit for the fuel later in the fuel cycle will not be credited to this reactor. Thus, a certain error will be introduced.

The isotope prices at the bottom of the page printed out are those prevailing for the price schedule indicated immediately above them. As these price schedules change, so will the isotope prices used in calculating the total fuel-cycle and operating reactor cost to be generated by the report code MERCURY. These data, however, are not used per se in the DAEDALUS program to formulate the linear programming matrix.

B. The Total Annual Cost and Throughput

The second page⁽¹⁾ of the printout of the input data from PACTOLUS indicates, year by year, several overall costs for the reactor. The first line is for the period two years before the indicated startup date with the plant factor 0. This is because certain fuel fabrication and separative work requirements may occur that long before the reactor operates. The two years after the reactor shuts down are included so as to account for the possible chemical processing cost and separative duty credit that occur after the reactor shuts down. The plant factor column indicates, year by year, the average plant factor for this particular reactor. The third and fourth columns contain the shipping costs for each year. The next column is the variable operating cost, which can be added to the fixed operating cost from the previous page so as to give the total operating costs. The last two columns give the annual taxes.

The next page⁽²⁾ shows various annual throughputs. The first two columns after the year list the amount of fuel fabricated and reprocessed each year. The next three columns indicate the amount of U_3O_8 which must be purchased for the fuel fabricated, the separative work needed to convert the U_3O_8 to reactor fuel composition, and the cascade tailings produced in the process. The last three columns show the effective crediting of these three items by fuel reprocessed after irradiation. These are the amount of natural uranium which is the equivalent of the lesser enriched material returned from the reprocessing stream and the separative work savings which can be obtained from using that material. The last column indicates the amount of cascade tails which are generated and which might be used for each kind of reactor during each year.

(1) See Page 4 of the Appendix.

(2) See Page 5 of the Appendix.

C. The Total Isotopes Fabricated by Year

The next two pages of the computer printout⁽¹⁾ indicate for each year of the reactor from 1968 through 2000, the total amount of isotopes in the fuels that are fabricated. They are: Th^{232} , Pa^{233} , U^{233} , U^{234} , U^{235} , U^{236} , U^{238} , Pu^{239} , Pu^{240} , Pu^{241} , Pu^{242} , Np^{237} , and a special poison column if reactor poisons are to be considered as a separate isotope. By dividing the total U^{235} by the sum of the U^{235} and U^{238} , one can obtain the total initial enrichment of the fuel being charged to the reactor.

D. Isotopes Reprocessed per Year

The next two pages of the printout⁽²⁾ indicate for the same isotopes the total amount of material that is recovered from the reprocessing stream each year after reprocessing losses have been accounted for. And again, by dividing U^{235} kilograms by the total of U^{235} and U^{238} , one can obtain the discharge enrichment of the fuel.

E. The Details of the Cost Coefficient for a Reactor Vector

So far all of the data that has been discussed has been the input data from PACTOLUS; no calculations have been done on these data in the DAEDALUS code other than to rearrange their format slightly and print them out. From now on, however, we will be considering not the input data to DAEDALUS, but rather the details of the calculation of the cost coefficient: the output from DAEDALUS.

The total costs which are incurred for each separate portion of the reactor operation are shown on the next page⁽³⁾ of the computer printout. The data received on an annual basis are first consolidated into two-year periods. The capital cost is translated by a sinking fund calculation into an annual cost. This annual cost is then added two years at a time and the present worth is calculated as of the year the reactor is built for each of these annual payments. These payments are shown in the capital cost column in units of millions of dollars. The data from the total annual cost and throughput pages are added together, converted to the present worth

(1) See Pages 6 and 7 of the Appendix.

(2) See Pages 8 and 9 of the Appendix.

(3) See Page 10 of the Appendix.

as of the year the reactor is built, and then printed out for each pair of years in the next five columns. The bottom of each column gives a total for the entire column. The cost of thorium, of which there is none in the LWAA reactor, is taken from the isotope-fabricated column for thorium, converted to thorium oxide, then multiplied by a price (\$5 per pound). The present worths are then added together in pairs to give the total cost of thorium oxide for each period. The last column represents a total across the page of each of the other columns. Finally, the total at the bottom of the total column, the number 283.9, represents the present worth, as of the year the reactor is built, of the total cost of the reactor and its operation. This number, if the reactor is to be built some year other than 1970, the base year, will then be discounted to 1970. For this particular reactor, which is built in 1970, a second discounting is not necessary. On page 11 of the Appendix, the cost coefficients are summarized and are discounted to 1970. One line is produced for each vector that is generated from the PACTOLUS history. The name of the vector and the name of a cost row is printed once for each cost row where the reactor coefficient is to be presented. The capital cost, the operating, separative work, thorium, fabrication, chemical processing, and total cost are also shown. The data are the same as the total line across the bottom of the previous page. However, if these data must be discounted to 1970 the coefficients reflect this. The part of the calculation starting with the reading in of the general reactor description and ending with the calculation of all the cost coefficients is presented for each reactor that is read in and for each reactor vector that is generated from this data. When the last reactor has been completed, all of the vectors have been calculated and the matrix is complete. However, only the left-hand side of each equation in the model has been calculated. In the next step each of these left-hand sides must be equated to a number on the right-hand side of each equation.

F. B Vectors

The B vector, or right-hand side of the equation, must contain numbers for each equation in the model. Each of these numbers represents a sum which is the total of all the terms on the left-hand side of an equation.

This number may be zero. In the particular model we are using, there are 178 rows, of which 89 have nonzero elements on the right-hand side. These indicate the capacity that must not be exceeded in the low- and medium-priced regions by the coal plants, the total capacity and energy which must be produced in the entire country during each year, the amounts of low- and premium-priced uranium that must be purchased, and the restrictions on the total number of reactors which can be constructed during the time when a reactor is being introduced. The significance of each of these numbers will be discussed below when the EQLIST is considered. When the first right-hand side is complete, the problem may continue with the addition of a second right-hand side, or the NEXT B vector. The two vectors might be required, for instance, when one wishes to change the desired growth curve. Again, the same reactor descriptions and matrix may be used if the total amount of uranium available at a low price is to be considered unlimited. In this case, the limiting quantities would have to be revised to some large number. Each line of the printout constitutes an element of the B vector. The line contains the row name and the size of the nonzero element. At the end of the last right-hand side is a card which says ENDATA. When the machine encounters this card, it knows that the complete matrix and right-hand side has been read in, and that the problem is ready to continue on to the solution of the problem generated.

LOADED STATISTICS AND DUMP⁽¹⁾

The next page of the printout⁽²⁾ contains the loader statistics for the problem which has just been completely generated. This page indicates that for the structural matrix generated, there were 178 equations or rows, 260 columns or vectors. These columns contain 2949 nonzero elements or coefficients in the matrix. This represents a matrix which is 6.372 percent dense. In other words, 2949 divided by the product of 178 and 260 gives 6.372 percent. The coefficients in this matrix ranged from 0.00005

(1) The other reactor histories prepared by DAEDALUS for the linear program are similar in format to those listed in Pages 2-11 of the Appendix and explained above.

(2) See Page 12 of the Appendix.

to 215. There is one cost row loaded. For the cost row named DSCNTD there were 171 nonzero elements, which represents a 65.769 percent density. The coefficients on this cost row range from 0.001 to 0.389. There was one right-hand side or B vector loaded. The right-hand side, number *B1, contained 89 nonzero elements, or a 50.000 percent density. The coefficients on this right-hand side ranged from 40 to 1027. As an initial point to start the solution, the problem contained 89 initial artificial vectors, that is, artificial variables that were nonzero-valued in order to balance the equation. There were 22 of these variables which have negative values and represent infeasibilities. Once these loader statistics have been computed and printed out, the linear programming code used a routine called COST to add additional slack prices to the cost row. Then, the entire matrix as generated was dumped onto the tape and a card containing information to identify the particular dump was punched. If the problem were to be reloaded later, the computer would not repeat all of the DAEDALUS program before proceeding with the solution, but would read this data card produced by the dump and reload all of the information which has been processed and converted to machine language form. A dump at this point and the punching out of that card will save the necessity for repeating the linear-programming-matrix-generation part of the problem.

THE EQLIST OR DETAILED DISCUSSION OF THE LINEAR PROGRAMMING MODEL

The remaining pages of the printed output constitute a list of the equations in the model. They are the model generated by DAEDALUS. This EQLIST contains all of the equations and all of the vectors which are generated by the subroutines and by the main program DAEDALUS. The EQLIST is printed out by the LP codes, and will be discussed below in a logical order rather than in the actual order shown in the Appendix. Some of the equations represent capacity and energy requirements for each period during which these requirements must be satisfied. There are also certain material balances, particularly U^{233} , Pu^{239} , and Pu^{241} , which must be satisfied. Uranium requirements must be calculated year by year in order to determine the total amount of money that must be spent

on uranium purchases. There are other restraints on the fossil capacity and the reactor introduction rates that must be considered as separate equations. There is also a cost row: an objective function whose value is to be minimized.

A. The Capacity and Energy Requirements

These requirements are represented by Equations 2 (70MWYH) through 23 (90MW H). (See Pages 12-25 of the Appendix.)

1. Capacity

Equations 13 through 23 represent the capacity which must be in service during each 2-year period considered in the problem. The period considered in this problem extends from 1970 through 1990. In the early years of the problem, starting with Equation 13, each equation represents the total capacity which must be constructed after 1970, to satisfy the expanding energy requirements of our country. The first term in each of these equations, as well as in all the other types to be discussed later, is the slack variable. For purposes of presentation, this first term will be discussed after the other terms of the equation have been considered.

The total capacity (70MW H) must be equal to 34.783779 for the first right-hand side, represented by the coefficient of the term (*B1).⁽¹⁾ This means that 34,784 megawatts or 34.784 gigawatts, of electrical generating capacity must be built during the 1970-71 period to satisfy the increased energy demands during that period. This capacity represents the minimum amount which must be constructed; if an amount greater than this is built, the total will be greater than 34.784. We will see in a moment how this is accounted for. The individual plants which are built in 1970 are DOLH70, DOLM70, DOLL70, COLH70, COLM70, COLL70, LWAA70, LWAC70, LWZA70, and LWZC70. The coefficients of each of these variables represents the total number of gigawatts per plant of the type represented by the variable constructed. In other words, the value of the variable, COLH70, in the final solution represents the number of 1000 megawatt coal plants constructed in the high-fuel-cost region of the country in 1970. The

(1) See Page 20 of the Appendix.

coefficient of 1.00 of this variable means that each of these plants is exactly 1000 megawatts. So the product of the number of plants and the coefficient, 1 gigawatt per plant, gives a total that is the number of gigawatts built of this type of plant. The same is true of the other coal plants, The light-water reactors, represented by the vector LWAA70, for instance, are actually 1,000.02 megawatt plants. Each of these plants will have 1.00002 gigawatts of capacity represented by its construction. The product of the number of plants and the 1.00002 gigawatts per plant represents the total capacity of each of the reactors represented by the LWAA70 vector. The sum of all these products, the 1 times COLH70, 1 times COLM70, 1 times COLL70, 1.00002 times LWAA70, and so forth, represent the total generating capacity built in the year 1970. The sum of these terms must be greater than 34.784, the value on the right-hand side of the equations shown in the EQLIST as the coefficient of a variable *B1. Now, if the sum of these products is greater than 34.784, then to make an equality we must have another variable which is the amount by which the sum of these products exceeds 34.784. This variable is the variable, 70MW H, the first variable shown in Equation 13, and it is the slack variable. There is no restraint in the LP formulation of the problem which requires exactly 34.784 gigawatts of capacity to be constructed during the 1970 period. If 35 or 36 gigawatts are constructed, then the value of the variable 70MW H, the slack variable, will be equal to 0.216 or 1.216; that is, the amount by which the total exceeds 34.784. We must remember, however, that the problem is a cost minimization problem and if excess reactors are constructed during a year, we must pay the price of building these excess reactors. The total cost to be minimized means that all the reactor capacity and, as we will see later, the energy curves must be satisfied as exactly as possible. In other words, as few excess reactors as feasible must be constructed. In actual practice it turns out that the total number of reactors operational during any given period is within about 1 percent of the total requirements for each period. In other words, less than 1 percent excess capacity is in service at any point in time.

The next equation, Equation 14, represents the total capacity which must be in service in 1972. This equation has the name 72MW H.

Something new has been added for this equation. We see that the total capacity, the coefficient of the variable *B1, is 72.305799. This total is composed of the reactors that are built in the 1972 period plus the reactors that have already been constructed in the 1970 period and are still in service during the 1972 period. In other words, we have the total of all reactors that are brand new and of all the reactors that are two years old. The coefficient of COLH70 is 1. The product of this coefficient and the value of the variable COLH70 is the total number of gigawatts available in 1972 from coal plants built in the high-priced region during the 1970 period. Likewise, the product of the coefficient 1 and the variable COLH72 represents the total number of gigawatts available in 1972 from coal plants built in the high-priced region of the country during the 1972 period. The sum of these two products represents the total number of gigawatts available from coal plants in 1972. Similar reasoning applies to the COLM variables and the COLL variables. The LWAA70 and LWAA72 variables times their respective coefficients represent the total number of light-water reactors having this particular load factor operational in the 1972 period. Adding all the coal and nuclear plants together, the sum of these terms must equal or exceeds 72.306. Again, the amount by which the sum of these products exceed 72.306 is the value of the first variable shown in equation 72 MW H. (See Page 20 of the Appendix.)

Equation 15 represents the same type of restraint for the year 1974. We have terms in this equation for reactors built in 1974, for reactors two years old (those built in 1972) and for reactors four years old (those built in 1970). This same type of designation applies to the equations representing 1976 through 1990.

If the problem extends past the year 2000, the row representing the total capacity in operation in the year 2000, which would have the name 00MW H, would reflect the assumption that the plants built in 1970 had been shut down. The only terms in this equation would be those representing plants built in 1972 through 2000, namely, plants that are less than 30 years old and still supply capacity. In 2002, the terms for the plants built in 1972 would also be dropped. This process would continue--dropping off

plants which are 30 years old, adding plants which are brand new--until we get to the last year of the problem.

2. The Energy Requirements for Each Period of the Problem

The equations representing the energy requirements for the problem are Equations 2 through 12.

Equation 2 is the total amount of energy that must be produced in the 1970 period. This equation has the name 70MWYH. If we take the 1000 megawatt coal plant built in the high-priced region of the country during 1970, operate it at an 85 percent plant factor (as befits a brand new plant) for two years, then we will produce 1.7 gigawatt years of energy from that plant in the two-year period. This 1.7 is the coefficient of the variable COLH70. It represents the energy produced per plant of this type. If we multiply that by the number of plants, we get a product which is the total energy supplied by the plants built in 1970 in the high-priced coal region of the country. Similarly, adding up the COLM70 and COLL70 products we get the total energy produced in the 1970 period from fossil plants. The 1000.02 megawatt reactor operated at 85 percent capacity, namely, reactor LWAA70, produces 1.70004 gigawatt years of energy per reactor in the period. This coefficient times the number of reactors gives the total energy produced by this type of reactor. The sum of all these products gives the total energy produced in the 1970 period.

Equation 3 gives the total energy that must be supplied within the scope of the problem in the 1972 period, and the total of the products of each variable and its coefficient must equal or exceed 118.721458 gigawatt years, the coefficient of the variable *B1. Again, we have the 1.7 gigawatt year per period for each of the coal plants times the number of each of these coal plants representing the total energy that is produced by all the coal plants. The energy supplied by the nuclear plants may be slightly different in Equation 4, which describes the energy required in the 1974 period, from that shown in the previous equations. This difference is due to different load factor histories for each type of reactor considered. For instance, the reactor LWAA70 operates at 85 percent capacity during the fifth and sixth year of its lifetime during the 1974 period, and produces

1.70004 gigawatt years of energy per reactor. The reactor LWZA70, however, operates at 85 percent capacity only during the fifth year of its life and at 84 percent capacity during the sixth year of its life. Since both of these years are included in the 1974 period, this reactor produces only 1.69004 gigawatt years of energy per plant. Thus, the difference in the coefficients of the vectors LWAA70 and LWZA70 is entirely due to the slightly different plant factor in the sixth year. Summing all the products of these coefficients (the energy per reactor) and the number of reactors gives us the total energy produced in the 1974 period. This energy must exceed the 187.214610 gigawatt years represented as the coefficient of *B1 in Equation 3. Again, the value of the variable 74MWYH represents the total by which the energy produced exceeds the requirement.

The effect of the reducing plant factor is even greater in Equation 5, which represents the energy that must be produced during the 1976 period. We see, for instance, that reactor LWAA70 during the seventh and eighth year of its life, represented by a coefficient of 1.70004, produces considerably more energy than that from the reactor LWZA70, represented by a coefficient of 1.59004. This reactor operates at 81 percent capacity during the seventh year of its life and at 78 percent capacity during the eighth year of its life. Again, the total of all these gigawatt years per reactor times the number of reactors must exceed 267.123270 gigawatt years of energy, the requirement shown as the coefficient of *B1 in Equation 5.

B. Material Balances

The availability of certain isotopes may restrain the linear programming problem. We must produce fissile plutonium in some reactor before we require it as inventory for some other reactor. The same thing is true of U^{233} . In this formulation, we do not consider that isotopes such as U^{235} and U^{238} are to be produced or to be used in some reactor before they are needed in another. We consider that the amount of U^{235} and U^{238} is unlimited; however, if we exceed the amount available at a lower price, as we will discuss in Section C below, we must pay the penalty of higher prices for our uranium. But this is not an absolutely limiting factor in the problem. However, the total amount of U^{233} , Pu^{239} , and Pu^{241} must

limit the problem. We must satisfy, during each biennium, a material balance which says the total amount of material available at the beginning of a period plus the net amount of material produced during the period must be equal to or exceed the amount of material available at the end of the period. In other words, we can never require material before we have produced it.

1. U²³³ Balances

Equations 24 through 35 are the balances on the U²³³ in this problem. Equation 24, which has the name 68 U33, represents the material balance for U²³³ in the 1968 period. This material balance is trivial. It is equal to U3368, which is the stockpile at the end of the 1968 period and which must exceed a slack variable 68 U33. It is obvious that both of these variables must have a value of zero.

Equation 25, which has the name 70 U33, represents the U²³³ stockpile at the end of the 1970 period. It contains three terms: U3368 (the previous stockpile, which will be zero), U3370 (the stockpile in the 1970 period and equal to zero because there are no production terms), and the slack variable 70 U33. The same trivial equations are carried through Equation 26 in 1972. It is only in 1974 that we see some real operations going on with U²³³. Equation 27, which has the name 74 U33, determines the stockpile of U²³³ at the end of the 1974 period. This equation includes terms both for the 1972 stockpile (U3372) and for the total production of U²³³ in the 1974 biennium. The total production consists both of -1.29704 kilograms of U²³³ "produced" in the 1974 biennium times the number of reactors of this type HTAE76 and of -1.79704 kilograms of U²³³ times the number of reactors of the type HTZE76. These "productions" with a negative coefficient actually represent requirements--in this case they represent the requirements for start-up cores. The sum of these production number and the initial zero stockpile gives us the value of the variable U3374 which is the stockpile at the end of the 1974 period.

This variable is now carried forward to Equation 28, the calculation of the U²³³ material balance in the 1976 period. This equation has the name, 76 U33. We see the U3374 variable, the stockpile from the previous

period, as an additive term. We add to this 0.03484 kilograms per reactor times the number of HTAE76 reactors, the product of -1.2974 and the number of HTAE78 reactors, the product of 0.3484 kilograms per reactor and the number of HTZE76 reactors, and -1.2974 kilograms per reactor and the number of HTZE78 reactors to give the value of the variable U3376, the stockpile of U^{233} available at the end of the 1976 biennium. This same initial stockpile plus net production gives us the final stockpile for each of the periods from 1978 on in Equations 29 through 35.

2. Fissile Plutonium Balances

Equations 36, 68FSPU, through 47, 90FSPU, show the balance of fissile plutonium for each period of the problem. Because these are material balances and equalities, the slack variables 68 FSPU, etc. are zero.

This "material," fissile plutonium, is handled slightly differently than U^{233} because the total amount of Pu^{239} at any point in time will be allowed to become negative if there is a corresponding positive amount of Pu^{241} for use as a substitute. In other words, the requirement is not that an individual plutonium isotope be available but that the total amount of fissile plutonium available be positive.

Equation 36, 68FSPU, contains terms for the final stockpile and for the use of plutonium in 1969 by reactors which will be started in 1970. This coefficient, 1.57937 kilograms of fissile plutonium per reactor type LWAC70, which is built in 1970, represents the total quantity of fissile plutonium required to supply enrichment for the initial core of the reactor built in 1970. Add this to the 1.57937 times the number of reactors LWZC70 and we have the total amount of plutonium used in the 1968 period. This total use of plutonium, if either of these reactors is built, will mean that the total quantity of plutonium produced will be negative. In other words, there is a net use. If there is no initial stockpile, as these equations imply, then the total "stockpile" at the end of the 1968 period will be less than zero if either of these reactors is built. Therefore none will be built. The coefficients in these equations represent the total Pu^{239} and Pu^{241} . By referring back to the Isotopes Fabricated table in the input, one can determine that the coefficient for Reactor LWAC70, -1.57937,

is the sum of 1331.7 kilograms Pu^{239} and 247.6 kilograms Pu^{241} fabricated in 1969.

Equation 37 represents the fissile plutonium balance in 1970. It has the name 70FSPU. The real production or plutonium use is represented by the coefficients of the reactor vectors for those reactors built either in 1970 or 1972. For the reactor LWAC70, we require 1.44951 kilograms of fissile plutonium per reactor built. The product of this coefficient and the number of reactors built is, as before, the total quantity of fissile plutonium used in the 1970 period for the construction or operation of this type of reactor. The sum of all these reactor coefficients and reactor variables represents the total production during the 1970 period. Reactor LWAA70 produces 0.13039 tonnes of fissile plutonium per reactor. By adding the total usage and production to the initial stockpile we will arrive at the final stockpile. Note that we only add 0.98989 of the initial stockpile, the variable FSPU68. This is due to the decay of Pu^{241} in the stockpile. We assume that 10 percent of the stockpile is Pu^{241} and decays with a 13.1 year half-life. Thus about 10 percent of the Pu^{241} decays during the period, reducing the total stockpile by about 1 percent.

C. Uranium Requirements

Each reactor considered in the problem requires a certain amount of uranium for each year that it operates. Each reactor will return some uranium to the process stream after it has been reprocessed after irradiation. Included in the information we want from the model is the total amount of uranium purchased during each period in the problem. We also wish to know the total amount of uranium used through each year of the problem, in order to determine the price of uranium during a period. We wish to know also how much high-priced uranium must be purchased during each period, and we wish to know the cumulative amount of premium-priced uranium that we must purchase during the course of the problem.

1. U_3O_8 Requirements for Each Period

Equations 48 through 59 calculate the amount of U_3O_8 which must be purchased during each period of the problem. The first, Equation 48,

which has the name 68U3O8, describes the total amount of U_3O_8 which must be purchased during the 1968 biennium. The coefficient 0.48751 of the variable LWAA70 indicates that 487.51 metric tons of U_3O_8 must be purchased during the 1968 biennium for each 1000 MW reactor of type LWAA that will be built during the 1970 biennium. Likewise, the 0.48751 coefficient of the vector LWZA70 indicates that 487.51 metric tons of uranium must be purchased for each reactor of type LWZA that is built during the 1970 biennium. The coefficients 0.09818 of the reactor variables LWAC70 and LWZC70 indicate that 98.18 tons of uranium must be purchased to provide inventories for each of the plutonium recycle reactors of the types LWAC and LWZC constructed in the 1970 biennium. As in all these equations, the product of the coefficient and the value of the variable gives the total uranium requirements during this biennium for the reactor indicated. The sum of these four products gives the total uranium requirements during the 1968 biennium. This total requirement will be represented as the value of the variable U3O868 which has the coefficient -1.00000. Thus the sum of the individual products yields the total requirements for each year. The origins of the coefficients of the reactor variables can be seen by turning to the U_3O_8 Purchased and the U_3O_8 Credited columns in the PACTOLUS printout. (See page 5 of the Appendix.)

There is one other term in Equation 48, namely, 1.0 (NTAL70). Some reactors use cascade tails to supply a portion of their fuel. It is conceivable that, at some point in the future, there may not be enough tails available to charge the reactor. In that case, the material needed will come from natural uranium. The amount is the value of the variable NTAL. It represents a deficiency in the stockpile which is added to the requirements for natural uranium. How this deficiency is calculated will be discussed later.

Equation 49 solves for the uranium requirements in the 1970 biennium. In this equation, more reactors are added than were present in the previous equations. We have the requirements in 1970 for the reactors that were started up in 1970. These requirements represent replacements for the first core. We also require first core uranium for the

reactors that will be started up in the 1972 biennium. Thus for the reactor LWAA70 that required 487.51 tons of uranium for the first core in the previous biennium we will require 367.70 tons of uranium as replacement during the 1970 biennium. The sum of the product 367.70 and the number of reactors of type LWAA70 gives us the total uranium which is required by reactor LWAA. The sum of such products for all reactors gives the total uranium requirement in 1970. This total will be the value of the variable $U3O870$ which has the coefficient of -1.00000 in this equation. There is no accumulation of uranium at this point in the model. This equation is simply intended to total the uranium requirement needed in a given period and not the cumulative amount used through the period.

Equation 50, $72U3O8$, is similar to the other equations, and the sum of the products of the coefficients and the values of the corresponding variables is the total U_3O_8 requirement in 1972. The 347.51 tons of uranium needed for the reactor LWAA70 represents replacements for the second core, less the credit from the first core that has returned from the reprocessing plant. Thus, although nearly 500 tons of uranium will be required to supply the new fuel to be loaded into this reactor during this biennium, we have the equivalent of over 100 tons being returned as a credit against the new uranium that must be purchased. It is only the net 347.51 tons that is shown as required additional uranium to support this reactor. These equations continue to tally the total uranium requirement during each biennium of the problem through Equation 59, $90U3O8$, which gives the uranium requirements that will be needed in the 1990 biennium. In this equation, some of the coefficients are negative. This represents the fact that a net amount of uranium will be returned to the process stream, particularly from the low enrichment reactors that require material at an enrichment less than natural, and from reactors that are being shut down and are returning their entire inventory as a credit.

2. The Cumulative U_3O_8 Requirements

Equations 105, $70CUNN$, through Equation 115, $90CUUN$, give the cumulative amount of uranium that must be purchased from the beginning of the problem through the year represented by the equation. In Equation

105, the total uranium for 1968 and 1970 are added together to give the cumulative uranium requirement through 1970. The variable CUUN70 is the cumulative amount of uranium needed through 1970. The variables U3O868 and U3O876 represent the uranium requirements in each of these bienniums.

In Equation 106, 72CUUN, we have the cumulative uranium requirement through 1970, CUUN70, added to the uranium requirement in 1972, U3O872, to get the total requirement through 1972, CUUN72. Equations 107 through 115 continue in this same pattern of adding the cumulative requirements through the previous period to the requirements in the latest period to give the total requirements through the latest period. These equations do not apply any restraint to the amount of uranium which is used in any given year; they only tally the amount that is actually used through the time indicated so that we can determine what the price of uranium will be in a period.

3. The Premium-Priced U_3O_8 Needed in a Period

Equation 72, 70NUP1, through Equation 82, 90NUP1, show the amount of uranium which must be purchased at the first premium price. If uranium is available at the lower price, that is, if the total amount of uranium which is used at a given point of time, is less than the amount of low-priced uranium available, then this low-price will be paid for all the uranium. However, if the amount of uranium which is used through a given point in time is greater than the amount available at a low price, then a premium must be paid for the additional uranium purchased. It is these equations that determine year by year how much of this premium-priced uranium must be purchased. The actual number of dollars spent to purchase this high-priced uranium will be discussed below when we consider the cost row. (See page 34 below.)

In Equation 72, we wish to calculate the amount of first-premium uranium that must be purchased during the 1970 period. The amount of uranium that is available at the base price is the 272,160 tons indicated as the coefficient on the right-hand side of the equation--the term 272.16 (*B1).

Let us suppose first, that the total amount of uranium needed through 1970 is 300,000 tons. This 300,000 tons will be the value of the variable CUUN70. If we subtract the 272,000 tons available at the lower price by bringing the right-hand side number over to the left side of the equation, we have a deficiency of 27,840 tons. This 27,840 tons will be the value of the variable NUP170, which, when multiplied by its coefficient of -1, will balance the equation, forcing the value of the slack variable 70NUP1 to be zero. Thus, the -27,840 resulting from the product of the negative coefficient and the positive value of the variable NUP170, and the positive product represented by the 300,000 tons of uranium CUUN70 times its positive coefficient gives a total of 272,160, and will balance the equation. The 27.84, which will be the value of the variable NUP170 represents the amount of this premium-priced uranium which must be purchased during the 1970 period. Let us suppose, however, that only 200,000 tons of uranium will be needed through 1970. This means that the variable CUUN70 will have the value of 200,000. If we then subtract the 272,160 tons on the right-hand side from this 200,000 tons, we have a total of -72,160 tons. In order to balance the equation, the product of a variable and its coefficient must be positive. In other words, we are looking for a variable times its coefficient which will be +72.16. But variables must have either zero or positive values. They cannot have a negative value. If we multiply a positive value of the variable NUP170 times its negative coefficient we will get a negative product--but we want a positive number. If we multiply a positive value of the variable 70NUP1 by its positive coefficient +1, we will get the positive product we are looking for to balance the equation. Thus, NUP170 will be zero and 70NUP1 will be equal to 72.16. This means that we have purchased no premium-priced uranium during the 1970 period; moreover, the value of the slack variable, 70NUP1, indicates that we still have 72,160 tons of low-priced uranium available to us before we must start purchasing high-priced uranium.

Equation 73, or 72NUP1, is very similar to the Equation 72, or 70NUP1. If we take the total amount of uranium, CUUN72, and subtract the available low-priced uranium, 272.16, we have a difference that represents the total amount of premium-priced uranium that we have to purchase through the 1972 period. From this difference, we have to further subtract

the amount of premium-priced uranium we have purchased previously, in order to determine how much uranium we have to purchase in the 1972 period. Let us suppose now that the total uranium required through 1970 was 200,000 tons and that the total required through 1972 is 300,000 tons. If we take the 300,000 tons, the value of the variable CUUN72, and subtract 272.16 from it, we find that there is a difference of 27.84 tons of uranium. Because the uranium in 1970 was less than 272.160, the previous equation will force the value of the variable NUP170 to be zero. Then the positive difference of 27.84 tons will require a negative product to balance the equation. This means that the variable 72NUP1, which will produce a positive product, must be zero, and the variable NUP172, which produces a negative product, will have to be equal to the 27,840 tons of U_3O_8 that will be needed during the 1972 period. In this equation, the variable NUP170 represents the total amount of premium-priced uranium which is purchased before the 1972 period.

In Equation 74, 74 NUP1, we wish to determine the amount of premium-priced uranium that will be purchased in the 1974 biennium. As we will see on the next page, the variable CUP172 in this equation represents the total amount of high-priced uranium that we had to purchase before the 1974 biennium. This equation is in other respects similar to the previous two. To determine the amount of high-priced uranium that is needed through 1974, we calculate the cumulative amount of uranium needed through 1974, the value of the variable CUUN74, and subtract the amount of low-priced uranium, 272.16, from that amount. Then we subtract the variable CUP172, the total premium-priced uranium purchased in earlier periods, from this difference to determine the amount of premium-priced uranium that must be purchased during the 1974 period. If the total amount of uranium here is less than 272,160, then the variable CUP172 and the variable NUP174 will be zero and the value of the variable 74NUP1, the slack variable in this equation, will represent the amount of low-priced uranium that we still have available to us in 1974. The equations for the first-premium uranium continue in this manner in each period through 1990, as represented in Equation 82.

It is possible that we will consume the uranium available at the first-premium price. Consequently, additional uranium must be purchased and for this we will have to pay a still higher price. The amount of material that we have to pay the second premium for is determined by Equations 83 through 93.

In Equation 83, we determine the amount of uranium that must be purchased at the second-premium price during the 1970 biennium. We calculate as before the total amount of uranium needed through 1970. Then we subtract from that total the 453.6 on the right-hand side, and the result will represent the kilotons of uranium that we have available below the second premium. In other words, the difference between 453.6 and 272.16 represents 180,440 tons of material that we have to pay the second premium for. Above 453,600 tons we have to pay a third premium. The logic of these equations is identical to the logic of the previous equations describing the first premium. From the total amount of uranium required we subtract the amount we can purchase at a lower price than the premium we are considering. The difference represents, if it is positive, the amount of material we have to pay the second-premium price for, or if the difference is negative, the amount of material we still have available before we have to pay the premium price.

Starting with Equation 94, 70NUP3, we determine the amount of material for which we have to pay the third-premium price. In addition to the material at the first premium price, there is 226,800 tons available at the second premium price. This is the difference between 680.4, the coefficient on the right-hand side of these equations, and 453.6, the coefficient on the right-hand side in the equations for the second premium. Again, we determine the total amount of uranium needed through 1970 and subtract from it 680,400 tons, the total amount of material we have available to use below the third premium price. The remainder represents the total amount of third-premium material that we will have to buy through the year 1970. As before, either the variable NUP170 or the slack variable 70NUP3 will have a value. The real variable will indicate the amount of third-premium uranium we have to buy, and the slack variable, if it is not

zero, will represent the amount of low-priced material that we have before we must pay the third premium.

4. Cumulative Premium-Priced U_3O_8 Equations

When we were determining in Equation 74 the amount of high-priced uranium that we had to purchase we saw the need for a number which represented the total amount of premium-priced uranium that we had purchased through the previous period. These values are calculated for the first premium in Equations 116 through 125. Equation 116 totals the amount of first-premium uranium that we must purchase through the year 1972. This is the sum of the variables NUP170 and NUP172, the total amount of uranium that we must purchase in 1970 and 1972. The sum of these two terms equals the value of the variable CUP172, the total we purchase through 1972. Equation 117, 74CUP1, is the sum of CUP172, the material we purchase through 1972, and NUP174, the material we purchase in 1974. The sum of these two terms is the value of the variable CUP174, the cumulative first-premium material that we have to purchase through 1974. These equations continue to accumulate in this manner, adding the total of the previous purchase to the latest to get the total cumulative material that we need through each period.

Equations 126 through 135 perform the same functions for the material at the second premium price. Likewise, Equations 136 through 145 give the total amount of \$100 uranium that we have to purchase through each period from 1972 to 1990.

5. Cascade Tails Requirements

Equations 60, 68TAIL through 71, 90TAIL, tally the total change in the supply of cascade tails that results from the construction and operation of reactors. Reactors LWAA and LWZA, for example, produce 324.73 metric tons of tails material in the 1968 period. This material comes from production of enriched material for first cores. However, reactor LWAC and LWZC actually require 3.18 metric tons of tails for their make-up. This requirement arises because the material postulated for the core of these reactors is at an enrichment very slightly below that of natural

uranium. The sum of the products of these coefficients and their reactor vectors is the total change in the tails stockpile during the 1968 period. If the total is positive, that is, tails are produced, the variable TAIL68, which represents the stockpile at the end of the 1968 period, will equal this total. But if the total is negative, the variable, NTAL70, will have a positive value equal to the magnitude of the total. NTAL70 represents, not a stockpile, but a requirement, which is assumed to be filled by the purchase of natural uranium. If NTAL68 has a value, it will add to the total natural uranium requirements. (See Equation 68, the 1968 U_3O_8 requirements.) The tails balances in the remaining periods are similar to the fissile plutonium balances discussed above. The stockpile at the end of a period is added to the production in the next period to determine the stockpile at the end of the next period.

D. Other Restraints

There are two other types of restraints that we have to include in our general description of this expanding energy economy. First of all, we must restrict the amount of fossil capacity that is built each year in each of the price regions of the country. In other words, we cannot build plants using low-priced coal in a part in the country where we cannot get low-priced coal. Also, we wish to arbitrarily restrain the rate at which a new reactor type can be introduced into the economy.

1. Limitations on Fossil Plant Capacity

Equations 146, 70COLL, through 156, 90COLL, represent the total amount of capacity that we need in the low-cost region of the country--the region where we can get low-priced coal. In other words, there is a maximum demand in 1970, as shown in Equation 146, for 10.20682 new plants in the region of the country where we can obtain coal at a low cost. Consequently, the number of fossil plants that we build in this part of the country must be equal to or less than 10.20682 thousand megawatt plants. If all of these are built as coal plants, the value of the variable COLL70 will be 10.20682; if part of the plants built in this part of the country are nuclear, the value of the variable COLL70 will be the difference between 10.20682 and the number of nuclear plants built in this part of the country.

The value of the slack variable 70COLL will then be the total of the nuclear plants of all types which are built in this part of the country. So the total of the coal plants that are built, COLL70, and the coal plants which are not built, 70COLL, is the total power requirement in the 1970 period in the low-priced region of the country. In 1972, the total amount of new coal capacity must be less than 12.65291 gigawatts. If less than 12.65291 coal plants are built during the 1972 period, then the value of the slack variable, 72COLL, will reflect this.

In the medium-priced region of the country we can build coal plants that can supply capacity to the extent required only where the medium price prevails for coal. This fact is represented for each period in Equations 157 through 167. These equations are similar to those described for the low-priced region of the country, except that the numbers on the right-hand side are slightly different so as to account for the different demands in the different parts of the country. Thus, in 1970 the total number of coal plants built in the medium-priced region of the country must be less than 7.11792 gigawatts. In 1972, Equation 158 states that the total number of coal plants built in 1972 must be equal to or less than 8.16048 gigawatts. If coal plants are not built to this extent, the difference will be represented as the value of the slack variable 72COLM. The same limitations apply in each biennium in a pattern similar to that for the low-priced coal for each equation through Equation 167 that represents the new capacity in the medium-priced region of the country in the year 1990.

2. Reactor Introduction Restraints

When a new type of reactor is developed and introduced into this energy economy as a lower-cost power producer than other reactors in the system, it will tend to be chosen (by the model) for construction in very large numbers as soon as it is introduced. The realities of the marketplace, however, do not permit this. Consequently, we have included in the model an arbitrary limitation on the rate at which a new reactor type can be introduced into the economy. This rate is such that during the first two years that a reactor can be built, not over 4000 megawatts of the new reactor type can be constructed. During the second two years, not over

8000 megawatts (4000 more than were introduced in the first period) can be constructed. If, for example, only 2000 were built in the first two-year period, then only 1000 will be built during the second two-year period. During the third two-year period, the limit is 8000 more than were produced in the second period, and in the fourth period 16,000 more than were constructed in the third period. This means that as many as 32,000 megawatts can be built during the fourth two-year period. From that time on, we assume there will be a large enough and viable enough industry to support this reactor type and no further restraints will be necessary.

Separate equations are written to represent these introduction restraints for each type of reactor. These equations begin with Equation 168, 76SHT1, and extend through Equation 179, 86SMS1. Because all the equations are similar for each type of reactor, let us consider only the equations which apply to one kind of reactor, the HTGR. These restraints are written in Equation 168, 76SHT1, through 171, 82SHT1. Equation 168 limits the number of HTGR's that can be built during the 1976 biennium. This is the first time these reactors are said to be available for introduction into the problem. We wish to build not more than 4000 megawatts of these reactors during the biennium. The number of reactors that are built is equal to the sum of the vectors HTAE76 and HTZE76, the heavy- and light-load factor histories of the HTGR. Thus the total number of reactors built with a heavy-load history, plus the total number of reactors built with the light-load history, must be equal to or less than four, the number on the right-hand side: the coefficient of the B vector (*B1). If less than four reactors are built, the slack variable, 76SHT1, will have a value and will equal the number of reactors that we could have built but did not. This number may vary anywhere from zero to four. If we built four reactors, the slack variable will be equal to zero; if we built no reactors, the slack variable will be equal to four. Equation 169, 78SHT1, shows the number of HTGR's that can be built during the 1978 period. We do not want this number to be more than four plus the number of HTGR's that were built during the previous period. If we built four during the previous period, we want not more than eight built during this period. If we did not build any during the previous period, we do not want more than four built during this period.

If we built four, then the total of the variables HTAE76 and HTZE76 will be equal to the four from the previous equation. This four becomes a constant which we can, in effect, add to the right-hand side, which already contains a four; thus the remaining equation has one times the variable HTAE78, plus one times the variable HTZE78, being equal to or less than eight on the right-hand side--the four we transferred, plus the coefficient of the variable (*B1). Thus if four reactors are built during the 1978 period, we can build anywhere from zero to eight of these reactors during the 1982 period. If we build none, then the slack variable 78SHT1 will have the value of eight. If we build the eight that we are allowed, then the slack variable will be equal to zero. If we build six, the slack variable will be equal to two.

Let us now consider the other situation where we did not build any HTGR reactors in the previous period. In other words, the vectors HTAE76 and HTZE76 will be equal to zero; thus we have the slack variable, plus HTAE78 and HTZE78, being required to add up to four. If four reactors are built during this period, then the variables HTAE78 and HTZE78 will add up to four, and the slack variable will be equal to zero. If no reactors are built during this period, the slack variable will be equal to four, and the real variables will be equal to zero.

Equation 170, 80SHT1, applies this same type of restraint to the 1980 biennium. If four reactors were built in 1976, and eight reactors were built in 1978, we wish to allow as many as 16 reactors to be built during the 1980 biennium. In this case, HTAE76 and HTZE76 will be equal to four and HTAE78 and HTZE78 will be equal to eight. This total of 12 can be transferred to the right-hand side to give us a total constant of 16, which must be the sum of the slack variable 80SHT1 and the reactor construction variables during this period, HTAE80 and HTZE80. Thus, if 12 reactors are already built and we do not wish to build more, then the variable 80SHT1 will equal 16 and the real variables will equal zero. If the full 16 reactors are built, then the slack variable will be equal to zero and the total of the two variables HTAE80 and HTZE80 will be equal to 16. Again, if no reactors are built before the 1980 period, then all the reactor construction variables will be equal to zero for 1976 and 1978, with the slack variable for 1980,

plus the construction variables for 1980, being equal to four. Thus not over four reactors can be built during the 1980 period. Equation 171, 82SHT1, is the final restraint on the introduction of this type of reactor. Again, depending upon how many reactors have been built previously, the number that can be built during this period will be restrained to as few as 4 and to as many as 32. If the full $4 + 8 + 16$ reactors have been built, then 32 reactors can be built during this period. If no reactors have been built previously, then only four reactors can be built during this period. This is the last period in which we restrain the number of HTGR reactors that can be built. During the 1984 biennium, the number of reactors that can be built is limited only to the new reactors needed in the system. This may be as few as 1 or as many as 100 or more reactors. If it turns out that only 1 or 2 HTGR's were built during 1982 then perhaps 102 HTGR's could be built in the 1984 biennium. However, if this unrealistic result occurred, we would have to go back and apply the introduction restraints either over a later set of periods or over more periods. Perhaps we would apply the restraint from 1980 to 1990. In actual practice, if a reactor is advantageous, it will be built up to the limit during all four periods and this will mean that 32 reactors are built during the 1982 period and that probably 70 or 80 might be built during the 1984 period. Once a reactor industry becomes large enough that 32 reactors are being built in a two-year period, then it is not too terribly unrealistic to think that three times that might be built during the next two-year period. In actual practice, reactors are built within the model to the extent of 4, 8, 16, 32, and then about 50 or 80. Following that, the reactors increase at a rate that is normal for the increasing demands of the entire economy. Other restraints from Equation 172 through 179 limit other types of reactors: LMFBR, which can be constructed only after 1980, and a molten salt breeder, after 1980. These complete the list of equations in the model.

E. The Cost Row

In the formulation of this model, the cost row, called DSCNTD, totals the present worth in 1970 of the entire cost of producing power for the period 1970 through 1990. We will discuss in some detail the coefficients of the cost row DSCNTD.

1. The Objective Function, DSCNTD (See Equation 1, page 14 of the Appendix.)

The purpose of the cost row is to total all the costs associated with all the activities that we perform in the solution of this problem. These activities are the purchase of uranium during each period, the purchase of high-price uranium during the period when we require high-price uranium, and the construction and operation of each of the several types of fossil and nuclear power plants. The sum total of the cost of all these activities is the total cost of power, the minimum value of which we wish to find. The first group of coefficients in the cost row DSCNTD are the base cost of buying uranium during each year. These vectors, from U3O868 through U3O890, represent the number of kilotons of U_3O_8 which is purchased during the indicated biennium. The base price is \$15.98 per kilogram, or 15.98 million dollars per kiloton. This is the coefficient of the variable U3O870. We are using a discounting value of 7 percent. During 1972 the price of uranium is still \$15.98. However, this must be discounted to 1970 prices by dividing by the square of 1.07 to make prices comparable.

The coefficients, represent the base price of \$15.98 per kilogram of U_3O_8 that must be paid for all uranium that is purchased, regardless of when it is purchased. However, after a while, the price of uranium might be expected to rise. So we have to calculate how much the premium will increase on the uranium that must be purchased after the rise in price. These premiums are the coefficients of the variables NUP170 through NUP390. Consider the vectors U3O870 and NUP170. If we must buy first-premium uranium in 1970, then we will pay \$15.983, the base price for all uranium we purchase in 1970. This is the coefficient of the variable U3O870. Also, we pay \$3.858 per kilogram of premium uranium that we have to buy in 1970. The sum of these two, or 19.841 dollars per kilogram, represents the total price of the first-premium uranium that we must purchase in 1970. Thus the sum of the base price and the premium price for the amount of uranium that we must purchase at the high price represents the total cost of uranium in each period. Suppose, as another example, that we have to purchase third-premium uranium in the year 1980. Our base premium price is the coefficient of the vector U3O880, the uranium

that we have to buy in the year 1980. This is 8.125 dollars per kilogram. Then we have to pay the first premium, 1.961 dollars, the coefficient of NUP180. We must also pay the second premium, the coefficient of the variable NUP280, 2.521 dollars, and finally we must pay the third premium, 2.802 dollars, the coefficient of the variable NUP380. The total of these coefficients is 15.309 dollars per kilogram of uranium. This represents the 1970 present worth of the total price per kilogram uranium that we must purchase in the year 1980. The sum of all these variables times their respective coefficients gives us the total amount of money that we have to spend for uranium in this problem.

The next few variables in the cost row give the total cost of constructing and operating the coal plants within this model. The first variable DOLH70 represents the number of coal plants that are built in the high-cost region of the country in the 1970 biennium and operate with the light-load-factor history. The cost coefficient of 357.25811 for this variable represents the total present worth in 1970 of the cost of constructing one of these plants and operating it over a 30-year period with the predetermined load-factor history. The capital cost of this plant is 105.9 million dollars. The difference between that and the cost coefficient, or 251.35811, represents slightly over 251 million dollars, the present worth in 1970 of the cost of operating the coal plant for the 20 years of the problem. This coefficient times the number of such plants built gives the total cost of building this type of plant in this period and of operating it over its predetermined life history. The coefficient 298.41855 of the variable DOLH72 represents the total present worth in 1970 of building this plant in 1972 and of operating it over the remaining 18 years of the problem. The capital cost of this plant is 105.9 million dollars discounted to 1970; the difference represents the cost of operating this plant over its history. This same procedure applies through all of the DOLH, DOLM, DOLL, COLH, COLM, and COLL variables all the way through the plant COLL90. The coefficient of 10.26927 for this variable represents the present worth in 1970 of building the reactor in 1990 and of operating it in 1990 only. Plants that are built in this problem will operate beyond the end of the problem. Consequently, the total cost of these plants over their entire lifetime will not be considered as part of the

problem. For the plant which is built in 1970, the total capital cost is divided into 30 equal annual payments at a 7 percent interest rate, which will exactly pay for the plant at the end of 30 years. Each annual payment is then converted to its present worth in the year the plant is built. The sum total of these present worths will, of course, be the total capital costs. Since the plant built in 1990 will only operate for the first two years of its life, only the present worth in 1990 of these first two payments will be included as part of the capital cost of the plant. Then, this cost, together with the operating cost of this plant for the first two years of its life only, is discounted back to 1970, so that the number 10.26927, the coefficient of the variable COLL90, represents the 1970 present worth of the partial capital cost of the plant built in 1990 and the operation of that plant in the 1990-91 biennium. Likewise, for the plants built in 1988, the coefficient 22.07063 represents the present worth in 1970 of operating this plant from 1988 through 1991, as well as the present worth of the capital cost payments made over the first four years of this plant's life.

After we have established the total cost of operating the coal plants, we must then consider the total cost of building and operating the nuclear plants. The same procedure used for calculating the capital and operating cost of the coal plants is used for calculating the capital and operating cost of the nuclear plants. However, there is the additional item of uranium purchases in the nuclear plants--purchases that are not considered as part of the cost coefficients of the reactor variable itself. These uranium costs are calculated separately, as we discussed above. The coefficient of the first reactor variable, LWAA70 is 283.87222. This number represents the 1970 present worth of the capital cost of this plant, which is 133 million dollars, and the present worth in 1970 of all the fabrication, reprocessing, separative work, insurance, and other costs of this plant for the 20 years of the problem. The details of the calculation of this coefficient 283.87222 were discussed previously in the output of the DAEDALUS code. The same type of coefficients apply to all vectors; the coefficient printed on the cost row is the present worth in 1970 of all the dollars spent within the span of the problem for each reactor that is constructed. The product of each coefficient and the value of the corresponding variable gives the total number

of dollars spent for that particular reactor being built in that period. The sum of all these, together with the sum of the cost of operating and building coal plants and the total cost of uranium, gives the total cost of power during the period of the problem. It is this grand total which must be minimized by selecting the proper values for these variables.

In future work, we intend to include other cost rows which have costs from 1970 through 1980, from 1990 through 2010, and so forth, to see whether optimizing on the shorter time periods produces reactor mixes or paths that are different from the solution that is an optimum for the total power cost from 1970 through 1990. However, these additional optimizations have not been performed at the time of this writing.

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