ENRICO FERMI ATOMIC POWER PLANT

NUCLEAR TEST SERIES

OSCILLATOR TESTS IN THE ENRICO FERMI REACTOR

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FOREWORD

This is one of a series of reports on the low-power (up to 1 Mwt) and high-power (up to 200 Mwt) nuclear testing of the Enrico Fermi fast-breeder reactor. The Nuclear Test Program is planned, directed, and evaluated by Atomic Power Development Associates, Inc. (APDA). The tests are conducted by Power Reactor Development Company (PRDC). The reactor proper is owned and operated by PRDC. The steam generators and electrical generating facilities are owned by The Detroit Edison Company (DECo).

Many individuals have contributed to the nuclear testing of the Fermi reactor. Listed below are those, exclusive of the authors, who made significant contributions to phases of the work described in this report.

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SUMMARY

The requirement of two oscillator rods was anticipated for making oscillator tests in the Enrico Fermi reactor - one rod for low power and the other for higher power. Prior to the tests, the oscillator rods were installed in the core and their worths were measured. The gross worth and amplitude of both rods agreed reasonably well with the predicted values. The differences were explainable by the uncertainties in the predictions. Measurement of the wave shapes revealed sinusoidal characteristics to within the accuracy of the measurements.

The zero-power transfer function was measured by means of oscillator tests, using four measurement techniques. The most satisfactory of these from the standpoint of frequency capability, completeness of information and ease of making measurements, was the method which employed the synchronous transfer function analyzer. However, the zero-power transfer function measured by this technique had a consistent amplitude discrepancy with the predicted zero-power transfer function, the difference being about 7 per cent at low frequencies (0.005 to 0.01 cps).

The reactor transfer function was measured at powers up to 111 Mwt with the reactor in both manual and automatic control. The reactor stability was investigated, using the results of the oscillator tests, and it was stable under both modes of control. A simple amplitude criterion was used for the cases with the reactor in manual control and the Nyquist criterion was used for the cases with the reactor in automatic control.

The feedback transfer function was extracted from the reactor transfer function data obtained at 67 Mwt and 111 Mwt. The feedback contained a substantial component with a long time constant (several seconds), which was not anticipated. The transfer function of the automatic control system was also extracted and analyzed. It fitted a simple model which represented the physical system reasonably well and the time constants agreed with the predicted values to within the probable experimental uncertainty.

I. PURPOSE OF TESTS

The purpose of the oscillator tests was twofold: (1) to evaluate the reactivity oscillation equipment and the transfer function analysis equipment, and (2) to evaluate the stability and the feedback mechanisms of the reactor. Oscillator tests require the use of unique equipment. Although certain aspects of the performance of this equipment can be evaluated with bench tests, the interaction between this equipment and the neutronics of the reactor can be evaluated experimentally only by in-reactor testing.

The first purpose of the test required studies of the gross reactivity perturbation caused by the presence of an oscillator rod in the core and the amplitude of the reactivity perturbation caused by the rotation of the rod. The second purpose required measurement of the reactor transfer function by determining the reactor response to a sinusoidal reactivity perturbation. To determine reactor stability and reactor feedback, it was necessary to measure the transfer function at various power levels and with both modes of reactor control, manual and automatic.

II. GENERAL DESCRIPTION OF THE ENRICO FERMI REACTOR

The Enrico Fermi reactor and its associated structures are shown in perspective in Fig. 1. The reactor is contained in a stainless steel vessel sealed at the top by a rotating shield plug which supports the control mechanisms, the holddown mechanism and the fuel-handling mechanism. The reactor vessel is surrounded by borated and plain graphite neutron shielding material which is contained inside the primary shield tank. The reactor is the fast-breeder type, cooled by sodium, and operated at essentially atmospheric pressure. The maximum reactor power with the first core loading (Core A) is 200 Mwt.

The core and blanket, located in the lower reactor vessel, consist of square (cross section) subassemblies containing the fuel pins and blanket rods arranged to approximate a cylinder about 80 in. in diam and 70 in. high. The core, contained in the central portion of the core subassemblies, approximates a right cylinder 31 in. in diam and 31 in. high; it is axially and radially surrounded by breeder blankets. The fuel in Core A consists of zirconium-clad pins containing U-10 w/o molybdenum alloy with the uranium enriched to 25.6 w/o U-235. Each fuel subassembly in the core contains 140 fuel pins having a total mass of approximately 4.75 kg of U-235. The blanket is depleted U-3 w/o molybdenum alloy.

The reactor cross section, shown in Fig. 2, indicates the placement of individual components within the lower reactor vessel. There is a total of 149 central lattice positions that are occupied by core and inner radial blanket subassemblies, the neutron source, and the ten operating control- and safety-rod channels. These positions are supplied with sodium coolant flowing upward from a high-pressure plenum which is connected to the discharge lines of the three primary sodium pumps. The coolant flows upward through the individual core and inner radial blanket subassemblies into a large upper plenum and from there by gravity to the three intermediate heat exchangers and then to the suction side of the primary pumps. Sodium also is used in the secondary cooling system.

The lattice positions surrounding the inner radial blanket comprise the outer blanket area; when filled with outer radial blanket subassemblies, they form an annular region having a top and bottom at the same elevation as the top and bottom of the inner radial blanket. Surrounding the outer radial blanket are lattice positions for stainless steel subassemblies which provide thermal and neutron shielding. The outer radial blanket and shielding lattice positions are supplied with sodium coolant from the low-pressure plenum. The sodium from the low-pressure plenum, after cooling the blanket and

FIG. 1 PERSPECTIVE VIEW OF REACTOR

FIG. 2 REACTOR CROSS SECTION

thermal shield subassemblies, is mixed in the upper plenum with the sodium from the core before flowing to the intermediate heat exchangers.

The reactor is controlled by two operating control rods and seven safety rods. Provisions have been included in the design for an eighth safety rod. The rods are of the poison type, containing boron carbide (B_4C) in which the boron is enriched in boron-10 (B-10). One operating control rod is for regulating and the other is for shimming; the maximum reactivity insertion rates of these rods are approximately one cent per sec and one cent per min, respectively. Both rods have approximate reactivity worths of 46 cents. The seven safety rods, which are spaced around the center of the core, provide shutdown reactivity. During operation of the reactor, the safety rods are held above the axial blanket section of the core so that they can be rapidly inserted into the core if a scram becomes necessary. Each of the safety rods has a reactivity worth in excess of \$1.00. The movements of the control rods and safety rods are actuated from the top. During fuel reloading, they are detached from their drive extensions to allow them to remain in the core.

The neutron detectors (fission chambers and ion chambers) for normal reactor operation at-power, are located in six neutron-counter tubes embedded in the graphite neutron shield surrounding the reactor vessel (Fig. 3). Eleven channels of nuclear instrumentation are distributed throughout the six neutron-counter tubes in a manner which will cover the full power range during reactor operation. The detectors located in these channels are designed to operate at temperatures up to 500 F. Additional information concerning the reactor design may be found in Reference 1.

During certain of the tests described in this report, two of the channels contained BF_3 proportional counters, with RIDL 49-51 scalers attached. The channels designated 3 and 4 were located in the most sensitive neutron-counter tube and were used for period measurements.

Although there were two oscillator rods available for the tests (one was designed for use during low-power tests and the other for use during high-power tests), they were not used simultaneously. The oscillator rod in use was placed in one safety-rod position because the flux gradient in this region is sufficient to provide adequate reactivity perturbation when the rod is rotated.

FIG. 3 SCHEMATIC PLAN VIEW OF NEUTRON DETECTOR LOCATIONS

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III. EXPERIMENTAL APPARATUS AND EQUIPMENT

A. INTRODUCTION

Two general categories of equipment were unique to these tests. The first contains the reactivity oscillator mechanism which includes the oscillator rod, the rod extension, and the rod drive mechanism. The second contains the transfer function analysis equipment. Because four methods were used for transfer function analysis, the second category includes all of the equipment used in the four methods. This equipment is described below, in context relevant to the methods used.

B. REACTIVITY OSCILLATOR MECHANISM

The oscillator mechanism, shown in Fig. 4, consisted of three units of equipment: a rotating oscillator-rod assembly, a drive extension assembly, and a variable speed drive mechanism. The oscillator-rod assembly contained a rotating B_4C poison section which was installed in one of the eight safety-rod positions in the core (See Fig. 2). The poison cartridge was mounted eccentrically to the axis of rotation. The variable speed drive mechanism was located above the reactor on top of the rotating shield plug and was used to drive and control the speed of the poison rotor section. A drive shaft, mounted in and housed by the drive extension assembly, provided the connection between the variable speed drive mechanism and the poison rotor section in the reactor.

Rotation of the poison section in the reactor resulted in a sinusoidal perturbation of the reactivity of known amplitude and phase angle. The reactivity perturbation caused a flux perturbation whose amplitude and phase angle are measured relative to the reactivity perturbation. These data defined the response of the reactor power to sinusoidal variations of reactivity at various power levels and frequencies and were used to plot the response curves or transfer functions of the reactor. From an analysis of transfer functions, the dynamic behavior of the reactor under various operating conditions can be predicted.

The three major units of equipment comprising the oscillator mechanisms are described in more detail below:

1. Variable Speed Drive Mechanism

The variable speed drive mechanism is an electromechanical device which was designed to drive the oscillator rotor shaft poison section assembly over a variable speed range of 0.001 to 10 cps. The drive con-

FIG. 4 OSCILLATOR MECHANISM

sists of an electronically controlled d-c motor driving a variable speed transmission. The transmission drives an external output shaft. One end of the output shaft is coupled to the oscillator drive extension by means of a timing belt. The other end of the output shaft is coupled to the transfer function analysis equipment which is described below. Overall speed control is accomplished by tachometer feedback through a servo loop and provides ± 2 per cent regulation of the set speed. Visual indication of oscillator speed is provided by a panel meter which registers to within ± 5 per cent. Backlash between the drive mechanism and the oscillator-rod drive shaft was designed to be less than 15 min of arc. Output torque is 140 in-lb, minimum, at 10 cps.

The variable speed drive mechanism was designed for operation in clean, dry air or inert gas at an ambient temperature of 150 F, and it was designed to run dry, or with dry lubricants, insofar as is possible. Where hydrocarbon lubricants were required, high-integrity seals and full-capacity drip pans were provided.

All oscillator-rod speed control and regulating equipment was supplied in a self-contained package which utilized chassis and panel construction for mounting on a control console panel. The control equipment was designed for operation in 90 F ambient air.

2. Drive Extension Assembly

The drive extension assembly consists of an oscillator-rod drive shaft and housing with appropriate bearings and seals to provide a penetration through the rotating shield plug for the drive shaft. The drive shaft connects the variable speed drive mechanism with the oscillator-rod poison rotor shaft (See Fig. 4). The upper portion of the drive shaft is mounted in standard ball and needle bearings. The lower portion, which operates in sodium, is guided radially by three journal-type bearings. The lower end of the drive shaft terminates in an asymmetric two-pronged drive fork which engages a drive collar on the poison rotor shaft in a unique orientation.

Penetration of the oscillator-rod drive extension through the holddown column mounting plate was achieved by means of a seal thimble similar in design to those used for the control-rod drive extensions. A stainless steel bellows seal permits 11 in. of vertical movement of the drive extension. This movement is required to raise the drive extension clear of the fuel subassemblies during refueling operations. O-ring-type seals were used between the seal thimble and the mounting plate. Packing-type seals were used as backup for the stainless steel bellows. The seal arrangement prevents any contaminated portion of the drive extension from being drawn up into the region above the biological shield plug. Boron stainless steel was used extensively in the design of the drive extension assembly to maintain the integrity of the rotating plug shield. During oper-

ation, the drive extension housing engages the oscillator-rod lower bearing carrier, maintaining it stationary while the drive shaft and poison rotor shaft revolve.

3. Oscillator-Rod Assembly

Two oscillator-rod assemblies were available: a low-power oscillator rod, which had a nominal peak-to-peak amplitude of 6 cents, and a high-power oscillator rod, which had a nominal peak-to-peak amplitude of 2 cents. Two rods were available since it was found after the first rod was built that it did not have sufficient cooling capacity to permit its use at powers greater than 20 Mwt without incurring intolerable thermal distortions. Since the rods differ in mechanical design, separate descriptions of the rods are given below.

a. Low-Power Oscillator-Rod Assembly

The low-power oscillator-rod assembly consists of a stationary housing tube, lower bearing carrier, and the poison rotor shaft assembly. This latter assembly consists in turn of a poison cartridge, a poison rotor shaft (which has an eccentric center section machined to accept and locate the poison cartridge), and a handling head-drive collar. A cross section of the poison section of the low-powered oscillator rod is shown in Fig. 5. Assembled, the oscillator-rod assembly is similar in size and configuration to a safety rod and it is placed in and removed from the reactor by the offset fuel-handling mechanism. The lower end of the lower bearing carrier is built to the dimensions of a safety-rod dash ram to permit it to seat and locate in the dash pot. At the upper end, the lower bearing carrier tube has a radial bearing of Colmonoy which locates the oscillator-rod assembly in the lower guide tube. The oscillator-rod assembly is held down in the guide tube by the drive extension which is driven down against the lower bearing carrier. A compression spring in the dash ram section provides for relatively constant loading of the drive extension against the oscillator rod and it also accommodates differential expansion.

The poison rotor shaft is supported on a conical Kantanium radial thrust bearing at the lower end and by a journal-type radial bearing at the upper end. A retainer collar secures the rotor shaft in the lower bearing carrier. At the top of the rotor shaft is the drive collar and sub-assembly-type handling head. The drive fork at the lower end of the oscillator-rod drive shaft mates with the drive collar of the rotor shaft to drive the rotor shaft during oscillator operation.

The central portion of the rotor shaft is eccentric with respect to the axis of rotation. It is machined to accept the poison cartridge and is so located as to provide the required physical movement of the poison.

CROSS SECTION OF HIGH-POWER OSCILLATOR ROD

CROSS SECTION OF LOW-POWER OSCILLATOR ROD

FIG. 5 OSCILLATOR-ROD CROSS SECTIONS

Radial movement of the poison cartridge with respect to the core center during rotation produces the desired reactivity change.

The poison cartridge is a stainless steel container of 1-in. OD and 7/16-in. ID and it is approximately 31-in. long. It is filled with slugs of boron carbide (B_4C) having an enrichment of 92.0 a/o in B-10. The total amount of B-10 in the form of B_4C used is 265 g. It was predicted that this rod would have a total negative reactivity worth of approximately 70 cents and would provide a peak-to-peak reactivity change of about 6 cents when rotated. The rotor shaft poison cartridge assembly was dynamically balanced during fabrication.

Ports were provided at the lower end of the lower bearing carrier to admit sodium for cooling. The sodium flows upward through and around the lower bearing carrier, through and around the poison cartridge and around the rotor shaft. Holes were provided at the top of the lower bearing carrier for the sodium to exit from the oscillator-rod assembly.

b. High-Power Oscillator-Rod Assembly

The high-power oscillator-rod assembly consists of a stationary housing tube with lower bearing carrier and a poison rotor shaft assembly. The poison assembly, located eccentric to the center of rotation, consists of seven poison tubes of 0.250-in. diam, each of which contains stainless-steel-clad, helium-bonded B_4C pins. The nominal diameter of the B_4C pins is 0.1935 in. and their height is 30.000 in. In addition to the poison section, the rod contains a steel counterbalance tube (for dynamic stability) and two spring tubes (to position the poison section and counterbalance tube). The oscillator rod is mounted in a stationary housing containing bearings which permit rotation of the rod. The housing is mounted in a safety-rod guide tube. The poison section contains 110.25 g of B-10 in the form of B_4C enriched to 74.69 a/o in B-10. A cross section of the poison section is shown in Fig. 5.

The mechanical construction of the high-power oscillator-rod assembly is very similar to that of the low-power oscillator-rod assembly. However, the lightweight construction of this rod (17.7 lb) makes it subject to lifting by the coolant flow; hence it was necessary to provide a bearing arrangement with a spring-loading feature to eliminate vertical play of the oscillator rod in its housing. This feature fixes the rod in the vertical direction while allowing thermal expansion to take place downward from the handling head. The housing retaining nut acts as a stop to limit the upward travel when the drive extension is retracted.

The rod is maintained in radial position by radial bearings located near the upper and lower ends of the rod housing. These bearings,

of necessity, have clearances which allow the rod to move sideways until contact is made.

It was predicted that this rod would have a total negative reactivity worth of approximately 28 cents and would provide a peak-to-peak reactivity change of about 2 cents when rotated.

C. TRANSFER FUNCTION ANALYSIS EQUIPMENT

The transfer function of the reactor at low power levels was determined by rotating each of the above two oscillator-rod assemblies within the reactor at various frequencies. The amplitude and phase of the response of the reactor power relative to this sinusoidal reactivity input was measured as a function of frequency, and the ratio of the power output to reactivity input was defined as the reactor transfer function. Four methods were used to measure the transfer function:

- (1) Synchronous
- (2) Null-balance
- (3) Sanborn
- (4) Keithley

The synchronous method was considered to be the primary method because it has a wider frequency range and better reliability than any of the other methods. The other methods were used either in conjunction with or alternate to the synchronous method to provide a substantiation of the results obtained with the synchronous method and also to provide alternate methods in case of failure of the analysis equipment used with the synchronous method. Specially constructed transfer function analyzers were used in the first two methods which are described in the following. In the last two methods, the pertinent information was recorded in a unique manner on available recorders; these are also described in the following.

1. Synchronous Transfer Function Analyzer

The oscillator rod, rotating at a set frequency, provided a sinusoidal reactivity perturbation in the reactor which in turn caused a power perturbation which was detected by an ion chamber. This signal was used as the input signal to the synchronous transfer function analyzer.

A block diagram of this analyzer is shown in Fig. 6. The ion chamber current in this instrument, which is proportional to reactor power, is converted to a voltage signal of approximately 1 volt by an amplifier and feedback circuit. The steady-state d-c component is suppressed by an adjustable d-c power supply leaving only that portion of the signal resulting from the power oscillations. The signal is then modulated by a solid-state chopper at a frequency of 210 cps and fed to an a-c amplifier. Its output

FIG. 6 BLOCK DIAGRAM, SYNCHRONOUS METHOD TRANSFER FUNCTION ANALYZER

is fed to a synchro-resolver which is mechanically coupled to the oscillator rod. The resolver multiplies the modulated signal by $\sin \omega t$ and $\cos \omega t$; ωt being the time-dependent attitude of the oscillator rod. The resolver outputs are the in-phase ($B/2 \cos \theta$) and quadrature ($B/2 \sin \theta$) components of the reactor power oscillations. These two outputs of the resolver are separately demodulated by a solid-state chopper and then detected by amplifier-integrators which provide digital voltmeter readouts. The amplitude, B , and phase angle, θ , are calculated from the two readouts. The amplitude is the square root of the sum of the squares of the two components. The phase angle is the angle whose tangent is the quadrature component divided by the in-phase component. Further discussion of the theory of this technique is given in Appendix A.

The in-phase and quadrature readouts are based on a precise 1-volt output at the preamplifier. Since it is not practical to adjust the feedback circuit with the accuracy necessary to accomplish this, the output of the preamplifier is connected to an amplifier-integrator which provides the mean level of reactor power. This digital readout is used as a multiplying factor to correct the in-phase and quadrature readouts for changes in reactor power.

A single digital voltmeter, mounted on the control panel, is switched by the operator to permit reading the values obtained for the in-phase, quadrature, or the reactor power mean level. A timer and cycle counter are used to accurately determine the frequency and integration period used in the experiment.

A rod-position selsyn transmitter, coupled to the oscillator rod, is provided as a means of rapidly checking oscillator-rod position during operation.

2. Null-Balance Transfer Function Analyzer

Equipment was also available to measure the transfer function by means of a manual null-balance method. In this method, the reactor output signal is mixed with the input of a resolver which is driven by the oscillator rod. A null balance is obtained by manual adjustments of a second resolver and a voltage divider located remotely from the reactor. At the null-balance point, the shaft position of the second resolver and the position of the voltage divider are proportional to the phase and amplitude of the reactor transfer function, respectively. A block diagram of this analyzer, the null-balance transfer function analyzer, is shown in Fig. 7.

The details of the operation of this analyzer are as follows: The oscillator rod causes the flux perturbation which is detected by the ion chamber. The ion chamber signal is then amplified and modulated. This signal is applied to a resolver rotated by the oscillator rod and to one terminal of

FIG. 7 BLOCK DIAGRAM, NULL-BALANCE METHOD TRANSFER FUNCTION ANALYZER

a nulling transformer. The outputs of the resolver (in-phase and quadrature components of the ion chamber signal) are fed into a second (adjustable) resolver. The output of the second resolver is applied to a precision resistor voltage divider, which is also manually adjusted, and then fed into the second terminal of the primary of the nulling transformer. The secondary voltage from the nulling transformer is then demodulated and passed through a band-pass filter which removes the direct current component and a large part of the harmonics of the signal, but allows the fundamental frequency to pass. This nulling signal is then fed into a recorder.

The operating procedure for determining the transfer function by means of the null-balance method is as follows:

- (1) Set adjustable resolver to some phase angle determined by calculation.
- (2) Adjust voltage divider several times on either side of apparent null and observe recordings.
- (3) From recordings, determine voltage divider setting for apparent null. Then adjust resolver several times on either side of its apparent null and observe recordings.

The voltage divider and resolver settings determined from the recordings will be proportional to the amplitude and phase angle of the transfer function, respectively.

3. Sanborn Method

Two signals which could be used to determine the reactor transfer function were recorded on a two-channel Sanborn recorder (Model No. 150). The two signals recorded were signals proportional to reactor power and to oscillator-rod position.

The power signal was the signal from the same neutron detector that was attached to the synchronous transfer function analyzer, amplified by its head amplifier. Thus, the input signal for the Sanborn method was the signal appearing at the output of the head amplifier of the synchronous transfer function analyzer.

The oscillator-rod position signal was simulated with a sine potentiometer mounted on the drive shaft of the oscillator-rod drive mechanism. This signal is proportional to reactivity perturbation in the reactor. Because 60-cycle noise was found to be present on the reactivity signal, both signals (power and reactivity) were fed through low-pass filters with a 5 cps cut-off before being fed to the Sanborn recorder. The power signal had no noise but it was filtered to introduce the same attenuation and phase

lag into both signals. A block diagram of the equipment used in this method is shown in Fig. 8. The strip-chart records of the two pertinent signals could be analyzed to obtain relative values for the transfer function amplitude and absolute values for the transfer function phase angle.

4. Keithley Method

In this method, the signal from a neutron detector during normal operation was amplified by a Keithley micro-microammeter, (Model No. 415) and it was recorded on a Leeds and Northrup recorder. During the oscillator tests, a large amount of the signal was suppressed and the oscillatory component was amplified to produce a recorded signal with a large amplitude on the strip chart. This recorded signal was analyzed to obtain a relative measurement of the transfer function amplitude. A block diagram of the equipment used in this method is given in Fig. 9. No information on phase angle can be obtained by this method.

FIG. 8 BLOCK DIAGRAM, SANBORN METHOD EQUIPMENT

FIG. 9 BLOCK DIAGRAM, KEITHLEY METHOD EQUIPMENT

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IV. EXPERIMENTAL PROCEDURE AND PLANT CONDITIONS

A. INTRODUCTION

The experimental procedures and plant conditions related to the oscillator tests can be separated into three categories: (1) low-power rod worth and wave-shape measurements (2) high-power rod worth and wave-shape measurements, and (3) oscillator tests.

B. LOW-POWER WORTH AND WAVE-SHAPE MEASUREMENTS

The gross worth of the low-power oscillator rod was determined by making critical control-rod measurements with and without the oscillator rod in place in location P03-P00* (See Fig. 10) in the core. The predicted worth of the low-power oscillator rod was -223 inhours (-70¢) with a ± 10 per cent uncertainty. Therefore, to obtain critical rod position measurements both without and with the oscillator rod in place, but with the same fuel loading, it was necessary to add fuel to the reactor before inserting the rod, until the excess reactivity was close to the maximum allowable limit for the Fermi reactor, 293.5 inhours (92¢) at 517 F.

The amount of fuel addition required to achieve this excess reactivity condition was determined from previous reactivity measurements. To gain an additional margin of operational flexibility, the critical rod measurements after the addition of fuel (but before the oscillator rod was installed) were made at a primary system isothermal temperature of approximately 525 F. Following this measurement, the reactor was shut down, the primary system temperature was reduced to 517 F, and the oscillator rod was installed in core lattice position P03-P00, using the following procedure.

- (1) Core subassemblies were removed from the four adjacent lattice positions P04-P00, P03-N01, P02-P00 and P03-P01 and placed in storage positions at the edge of the blanket (See Fig. 2).
- (2) The oscillator rod was removed from the transfer rotor container and installed in position P03-P00.

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* The coordinate system used to locate subassemblies in the core lattice is shown in Figure 10. The first position number given is the X-coordinate and the second is the Y-coordinate. "P" represents positive values and "N" negative values. The core center is designated P00-P00.

NO6	NO5	NO4	NO3	NO2	NO1	POO	PO1	PO2	PO3	PO4	PO5	PO6
SR = Safety Rod						NS = Neutron Source						
CR = Control Rod						CS = Core Shim Subassembly						
OR = Oscillator Rod												

FIG. 10 CORE LOADING NO. 109

- (3) The four core subassemblies were returned from storage to positions P03-P01, P02-P00, P03-N01 and P04-P00.

Throughout the fuel-handling operations, the subcritical count-rate was monitored on the two source range safety system channels. Counts were taken both before and after each of the oscillator-rod and fuel assembly transfers. After the fuel handling was completed, the reactor was made critical again and critical-rod measurements were made with the oscillator rod in place and rotated to the azimuth position, where it was known that the rod would have its minimum negative reactivity worth. The primary system temperature was held at approximately 517 F for this measurement and the reactor power was kept below 1 Mwt. The core loading for these measurements (No. 109) is shown in Fig. 10.

Following the measurement of gross rod worth, the reactivity wave shape during rod rotation was determined. The reactor was first made critical with the shim-control rod fully withdrawn, and the oscillator rod was rotated by hand in a clockwise direction in 90-deg increments. At each angular position of the oscillator rod, the reactivity state of the reactor was determined by means of positive period measurements and by regulating critical-rod measurements. For the positive period measurements, signals from a Reuter Stokes high-sensitivity BF_3 ion chamber located in NCT-3, and two Anton BF_3 ion chambers, one each located in NCT-3 and NCT-4, were monitored.

Following these measurements, the reactor was reloaded to allow criticality to be attained with the shim rod at its fully inserted position. Wave-shape measurements were then made by rotating the oscillator rod in 30-deg increments in the clockwise direction, again utilizing both critical regulating-rod and positive period measurements. Following this, critical-rod and positive period measurements were also obtained at 30-deg increments with the shim rod fully inserted when the oscillator rod was rotated in the counterclockwise direction. The wave shape was obtained for different shim-rod positions to determine the effect of the shadowing of the oscillator rod by the shim rod. The wave shape was obtained for both directions of rotation because it was anticipated that the looseness in the mechanical coupling between the oscillator rod and the rod extension might cause the phase angle of the wave shape to be dependent upon the direction of rotation.

C. HIGH-POWER ROD WORTH AND WAVE-SHAPE MEASUREMENTS

The procedure for determining the gross worth of the low-power oscillator rod has been discussed in the previous section. A similar procedure was used to determine the gross worth of the high-power oscillator rod. However, the reference case selected for this determination was that with the low-power rod present. Thus, only the incremental worth between

the two rods was measured. The gross worth of the high-power rod was determined by adding this measured increment to the measured gross worth of the low-power rod.

The worth of the high-power rod was estimated to be -90 inhours $(-28.2\text{¢}) \pm 10$ per cent. Since the high-power rod replaced the low-power rod which had a predicted worth of -223 inhours, the reactivity change due to the oscillator-rod interchange was predicted to be an increase of 133 inhours.

Before making the rod interchange, a base reactivity measurement was made with the low-power oscillator rod at its azimuth position of minimum negative worth and with the primary system temperature at approximately 425 F. This measurement indicated that a reactivity override of approximately 112 inhours remained in the operating control rods; since the predicted reactivity increase (133 inhours) was greater than the override available, it was necessary to raise the primary system temperature to approximately 455 F before making the oscillator-rod interchange. When this was done, the reactor was shut down and the oscillator rods were interchanged using the following procedure.

- (1) Core subassemblies were removed from the four adjacent core lattice positions P04-P00, P03-N01, P03-P00, and P03-P01 and placed in storage positions (see Fig. 2).
- (2) The low-power oscillator rod was removed from position P03-P00 and placed in the transfer rotor container.
- (3) The high-power oscillator rod was removed from the transfer rotor and installed in position P03-P00.
- (4) The four core subassemblies were returned from storage to positions P03-P01, P02-P00, P03-N01 and P04-P00.

Throughout the fuel-handling operations, the subcritical count-rate was monitored on the two source range safety system channels. Counts were taken both before the fuel-handling operation was started and after each oscillator-rod and fuel subassembly transfer.

After the above fuel handling was completed, the reactor was made critical again and critical rod measurements were made with the high-power oscillator rod in place and rotated to the azimuth position where it was known the rod would have its minimum negative reactivity worth. The primary system temperature was held at approximately 455 F for this measurement and the power was kept below 1 Mwt. The core loading for these measurements (No. 128) is shown in Fig. 11.

NO6	NO5	NO4	NO3	NO2	NO1	POO	PO1	PO2	PO3	PO4	PO5	PO6
SR =	Safety Rod							OR =	Oscillator Rod			
CR =	Control Rod							NS =	Neutron Source			

FIG. 11 CORE LOADING NO. 128

Following the measurement of gross rod worth, the reactivity wave shape during rod rotation was determined. The reactor was first made critical at about 5 kwT with the shim rod at its fully inserted position and the oscillator rod was rotated by hand in a clockwise direction for two complete revolutions. At 30-deg intervals during the rotation, the reactivity state of the reactor was determined by critical regulating-rod measurements. Following these measurements, the reactor was reloaded into the configuration shown in Fig. 12 (Loading No. 129 and 131) and the measurements were repeated. The basic difference between Loadings 128 and 129 was that the core subassembly located in position P06-P00 in Loading 128 was in position N06-P00 for Loading 129. Thus, whereas there were three core subassemblies in place between the oscillator rod and the core-blanket interface in Loading 128, there were only two core subassemblies between the oscillator rod and the core-blanket interface in Loading 129.

The effect of the number of subassemblies between the oscillator rod and the core-blanket interface was investigated because it was found (as explained below) that, with three subassemblies, the peak-to-peak amplitude of the rod was substantially less than predicted. The experiment was performed with two subassemblies to determine if this change would increase the amplitude.

D. ZERO-POWER OSCILLATOR TESTS

With the core in the configuration shown in Fig. 12, (Core Loadings No. 131 and 129), the reactor was made critical and brought to a power level of 2 MwT or less. The high-power oscillator rod was rotated in a clockwise direction at low power at a number of frequencies in the range from 0.005 to 5 cps, and the corresponding power perturbation was monitored by a Reuter-Stokes, B-10 lined, uncompensated ion chamber (No. F-793) which was being used as a spare power-range detector. It was located in NCT-6 and the bottom of the detector was 8.5 in. above the bottom of the tube. At this position, the ion chamber had a sensitivity of 2.67×10^{-13} amps per watt. The signal from this detector was used with the synchronous transfer function analyzer, and alternately with the null-balance transfer function analyzer, to obtain the reactor transfer function data.

On one run, signals from the ion chamber and from the sine potentiometer of the ion chamber simulator (mechanically coupled to the oscillator rod) were also recorded on a Sanborn recorder strip chart, employing the Sanborn method for transfer function analysis. In another test, a trace of the power oscillation was obtained on the recorder attached to the Keithley micro-microammeter, employing the Keithley method for transfer function analysis. The detector attached to this latter instrument was a Reuter-Stokes, B-10 lined, compensated ion chamber (No. F-774), which was being used as a spare intermediate-range detector. It was located in NCT-5 and

FIG. 12 CORE LOADING NO. 129 AND 131

SR = Safety Rod
CR = Control Rod

OR = Oscillator Rod
NS = Neutron Source

the bottom of the detector was 8.5 in. above the bottom of the tube. At this position, the ion chamber had a sensitivity of 3.35×10^{-13} amps per watt.

The results of these tests were then analyzed by the four methods outlined in Section III-C to obtain the reactor transfer function, $\frac{(\delta P/P)}{(\delta k/k)}$.

Since the tests were run with the reactor at a nominal power level of 2 Mwt or less, feedback was negligible and the measured transfer function was the zero-power transfer function.

E. HIGH-POWER OSCILLATOR TESTS

In a similar manner, the transfer function was measured for reactor powers greater than 2 Mwt with two basic flow rates and two modes of control. Nominal powers of 6, 13.3, 20, 30, 50, 67, and 100 Mwt were selected. On some of the tests only two primary coolant loops were used; on other tests all three primary coolant loops were used. Details including actual powers and total primary coolant flow rates are given in Table I. The normal mode of control was the manual control of the reactor by the reactor operator. Under this mode, no effort was made by the operator to null-out with the regulating rod the reactivity perturbation caused by the oscillator rod.

At all powers, except the lowest power, tests were also performed with the reactor under automatic control. Under this mode, the control system did attempt to null-out the oscillator-rod reactivity perturbation with appropriate regulating-rod motion. The tests under manual control were performed to determine reactor stability and feedback. The tests under automatic control were performed to determine the effects of the automatic control system on stability and the feedback transfer function of the control system.

The instrumentation and methods that were used in the zero-power oscillator tests to measure the transfer function were also used in the high-power oscillator tests. The automatic control system was unique to the high-power oscillator tests and a description of the system is required to clarify its action.

The reactor can be controlled by an automatic control system designed to control the reactor power and temperature and to regulate the generation of steam and electric power. The control system can be divided into a feedwater and steam control system and a reactor control system. The reactor control system consists of three major channels: (1) a temperature error or demand power rate-of-change channel (2) a neutron flux or actual power rate-of-change channel, and (3) a regulating-rod velocity demand channel. A block diagram of the reactor control system is shown in Fig. 13. The actions of the three channels are described as follows.

TABLE I - SUMMARY OF HIGH-POWER OSCILLATOR TESTS

<u>Nominal power, Mwt</u>	<u>Actual power,* Mwt</u>	<u>Primary coolant loops</u>	<u>Total primary † coolant flow rate, 10⁶ lb/hr</u>	<u>Mode of control ‡</u>	<u>Date</u>
6.6	6.6	1, 3	6.2	M	1-10-66
13.3	13.3	1, 3	6.0	M	1-12-66
			5.6	M	1-13-66
13.3	12.7	1, 3	6.2	A	1-21-66
20	20.1	1, 2, 3	(Not avail.)	M	1-28-66
20	19.4	1, 2, 3	(Not avail.)	A	1-31-66
30	30	1, 2, 3	8.9	M	3-29-66
30	30	1, 2, 3	8.9	A	3-29-66
50	46	1, 2, 3	8.9	M	3-30-66
50	46	1, 2, 3	8.9	A	3-30-66
67	66.7	1, 2	5.9	M	4- 1-66
67	67	1, 2	6.0	A	4- 4-66
100	111	1, 2, 3	11.5	M	7-11-66
100	111	1, 2, 3	11.5	A	7-11-66

* As measured with a Keithley micro-microammeter attached to an ion chamber

† These values are meter values. The actual flow was about 10 per cent less than the meter flow.

‡ M = manual, A = automatic

1. Temperature Error Channel

The control signal produced by this channel is obtained from two signals: (1) ST, the temperature signal received from a bridge circuit connected to a temperature resistance element which measures the reactor sodium outlet temperature, and (2) SA, the actual set point for this temperature. The actual temperature signal is compared to the actual set-point by a summing-type operational amplifier, and the algebraic sum is the temperature error signal, SA-ST. The temperature error signal then enters a limiter that limits the maximum value of the error signal. The outlet from the limiter is then sent through a lag unit. This is an R-C delay network with a fixed time constant of about 10 sec. The output from the lag unit is the damped-demand power rate-of-change signal, ΔT .

2. Neutron Flux Channel

The highest flux signal from two neutron detectors, NE-106 and NE-107, is selected by auctioneering unless one of the signals is known

FIG. 13 BLOCK DIAGRAM — AUTOMATIC CONTROL SYSTEM

to be faulty, in which case the other signal is selected manually. This signal, n , which is proportional to reactor power, is differentiated by a rate-type operational amplifier, giving an actual power rate-of-change signal, dn/dt . The differentiating circuit has a time constant of about 10 sec.

3. Regulating-Rod Velocity Demand Channel

The damped-demand power rate-of-change signal, ΔT , and the actual power rate-of-change signal, dn/dt , are compared by a summing-type operational amplifier. Its output is the power rate-of-change error signal, $\Delta T - \frac{dn}{dt}$. This signal is then divided by the neutron flux by a servodivider, to maintain a constant-loop gain at any power level. The output of the divider is the velocity demand signal that is used to determine the speed and direction at which the regulating rod is to be moved. If the signal is positive, the rod will be driven out of the reactor.

To insure that a balance point exists on the servodivider for all power levels, a limiter is used. The limiter prevents the apparent neutron flux signal, as seen by the servodivider, from dropping below 15 per cent of the normal 200 Mwt value. The velocity demand signal from the control system is one input to a double input magnetic amplifier. The other input is the actual rod velocity that is transmitted from a tachometer on the rod drive mechanism. The two inputs are subtracted and the error signal is a command signal to the silicon rectifiers that furnish a variable voltage to the regulating-rod motor. The time constant of this portion of the system is a double-pole time constant of about 0.4 sec.

V. EXPERIMENTAL RESULTS AND ANALYSIS

A. LOW-POWER ROD WORTH AND WAVE SHAPE

1. Rod Worth - Experimental Results and Analysis

The experimental data obtained in the measurements of the gross worth of the low-power oscillator rod are given in Table II. These data were used in conjunction with the regulating-and shim-rod calibration curves shown in Figs. 14 and 15 to obtain an average rod worth from all measurements of -206.9 inhours (-64.9¢) for the low-power oscillator rod. This value is about 7 per cent less in absolute value than the predicted value of -70¢ (the data were corrected for temperature drift and power drift between measurements).

TABLE II - DATA OF THE MEASUREMENT OF GROSS WORTH
OF THE LOW-POWER OSCILLATOR ROD

Oscillator-rod azimuth position, deg	Critical reg.-rod position, in. withdrawn	Critical shim-rod position, in. withdrawn	System isothermal temp, F	Power, kw	Power drift, %/5 min
Not installed	0.93	0	526.2	4.1	0
Not installed	0.94	0	526.1	4.0	+0.3
Not installed	0.96	0	526.2	9.8	0
210	7.22	20	519.4	3.9	-0.6
210	7.23	20	519.4	4.1	+1.3
210	7.23	20	519.1	9.7	+0.9

2. Wave Shape - Experimental Results and Analysis

The raw critical-rod and period data taken during the low-power oscillator-rod wave-shape measurements were converted to reactivity values using the regulating-rod calibration curves shown in Fig. 14 and the period-reactivity curve shown in Fig. 16. The data were adjusted to an isothermal reactor temperature of 518 F, using a temperature coefficient value of -0.82 inhours per F. In addition, a power drift correction as given in Reference 2 was applied to the critical-rod measurements.

FIG. 14 REGULATING ROD CALIBRATION CURVES

FIG. 15 SHIM-ROD CALIBRATION CURVES

FIG. 16 PERIOD-REACTIVITY CURVE

The corrected reactivity data for each of the four reactivity measurements (three positive period and one critical rod measurement made at each angular position for each direction of rod rotation and shim-rod position) are presented in Tables III, IV, and V and the average reactivity values after adjustment were plotted in Figures 17-22. Before plotting, the data were adjusted by the average value of all of the data points of the cycle, resulting in data points that varied above and below a null-reactivity level. Each data point plotted was assigned an uncertainty of ± 0.5 inhour, as indicated by the error flags in the figures.

The corrected data for each low-power oscillator-rod rotation were Fourier-analyzed and it was found that the data could be fitted well by the following expressions:

Shim rod fully withdrawn, clockwise rotation,

$$R(\theta) = 8.05 \sin(\theta - 121); \text{ positive period data} \quad (1)$$

$$R(\theta) = 7.90 \sin(\theta - 120); \text{ critical-rod data} \quad (2)$$

Shim rod fully inserted, clockwise rotation,

$$R(\theta) = 7.74 \sin(\theta - 128); \text{ positive period data} \quad (3)$$

$$R(\theta) = 7.80 \sin(\theta - 127); \text{ critical-rod data} \quad (4)$$

Shim rod fully inserted, counterclockwise rotation,

$$R(\theta) = 7.73 \sin(\theta - 122); \text{ positive period data} \quad (5)$$

$$R(\theta) = 7.69 \sin(\theta - 121); \text{ critical-rod data} \quad (6)$$

where,

R = reactivity relative to a null position, inhours
 θ = angular position as denoted by the azimuth dial
on the oscillator rod, deg

The fitted sine-wave curves obtained from the above equation are also shown in Figs. 17-22.

The standard deviations between the experimental data and the fitted curves in Figs. 17-22 are 0.9, 0.7, 0.3, 0.5, 0.2, and 0.8 inhours, respectively. This indicates that the experimental results can be reasonably well fitted by a sine wave. With one exception, the data for the case with the shim rod fully withdrawn were taken only at rotational increments of 90 deg. Because of the small amount of data available, the fit cannot be expected to be very good. For the other sets of data, the fits agree with the experimental results well enough such that the standard deviation has about the same magnitude as the uncertainty in the experimental data.

TABLE III — CRITICAL-ROD AND POSITIVE PERIOD DATA OF THE WAVE-SHAPE MEASUREMENTS FOR THE LOW-POWER OSCILLATOR ROD WITH THE SHIM ROD FULLY WITHDRAWN AND WITH CLOCKWISE ROTATION OF THE ROD

Oscillator-rod position azimuth, deg	Isothermal temp, F	Reg. rod position, in.	Power drift, %/5 min	Reactor Period			Corrected Reactivity		Adjusted Reactivity	
				Channel 3, sec	Channel 4, sec	Keithley, sec	Avg. of period measurements, inhours	Critical-rod measurements, inhours	Avg. of period measurements, inhours	Critical-rod measurements, inhours
214	519.2	11.45	-	64.5	64.0	61.6	40.3	-	8.0	-
214	519.4	7.21 (critical)	-0.7	-	-	-	-	40.7	-	8.0
304	519.5	11.45	-	85.0	86.0	85.1	32.9	-	0.6	-
304	519.6	7.96 (critical)	-1.2	-	-	-	-	33.2	-	-0.5
34	519.7	11.45	-	129.0	130.0	127.3	24.2	-	-8.1	-
34	519.8	8.81 (critical)	-1.9	-	-	-	-	24.9	-	-7.8
124	519.9	11.45	-	89.6	89.3	88.6	32.0	-	-0.3	-
124	520.0	8.07 (critical)	0.0	-	-	-	-	32.1	-	+0.6
218	520.1	11.45	-	63.0	64.0	62.7	41.1	-	8.8	-
218	520.5	7.21 (critical)	-3.1	-	-	-	-	40.8	-	8.1

TABLE IV - CRITICAL-ROD AND POSITIVE PERIOD DATA OF THE WAVE-SHAPE MEASUREMENTS FOR THE LOW-POWER OSCILLATOR ROD WITH THE SHIM ROD FULLY INSERTED AND WITH CLOCKWISE ROTATION OF THE ROD

Oscillator-rod position azimuth, deg	Isothermal temp, F	Reg. rod position, in.	Power drift, %/5 min	Reactor Period			Corrected Reactivity		Adjusted Reactivity	
				Channel 3, sec	Channel 4, sec	Keithley, sec	Avg. of period measurements, in hours	Critical-rod measurements, in hours	Avg. of period measurements, in hours	Critical-rod measurements, in hours
214	518.1	10.60	-	61.0	61.0	61.0	40.5	-	7.3	-
214	518.1	6.36	+0.36	-	-	-	-	41.5	-	7.1
244	518.1	10.60	-	60.8	61.0	60.5	40.5	-	7.3	-
244	518.1	6.41	-1.08	-	-	-	-	41.1	-	6.7
274	518.1	10.60	-	68.2	68.0	66.9	37.5	-	4.3	-
274	518.1	6.63	-2.07	-	-	-	-	39.2	-	4.8
304	518.0	10.60	-	78.0	77.0	77.3	33.8	-	0.6	-
304	518.0	6.99	-1.90	-	-	-	-	35.2	-	0.8
334	518.0	10.60	-	91.3	92.0	90.0	29.9	-	-3.3	-
334	517.9	7.34	-3.10	-	-	-	-	31.4	-	-3.0
4	517.9	10.60	-	105.8	105.0	103.5	26.3	-	-6.9	-
4	517.9	7.62	-2.70	-	-	-	-	28.8	-	-5.6
34	518.0	10.60	-	109.3	110.5	110.0	25.8	-	-7.4	-
34	518.1	7.76	-3.50	-	-	-	-	27.5	-	-6.9
64	518.4	10.60	-	108.0	109.2	109.3	26.3	-	-6.9	-
64	518.6	7.78	+0.20	-	-	-	-	26.3	-	-8.1
94	518.6	10.61	-	97.5	99.0	97.2	28.7	-	-4.5	-
94	518.3	7.49	-1.30	-	-	-	-	30.4	-	-4.0
124	518.4	10.60	-	81.0	82.5	82.2	32.7	-	-0.5	-
124	518.6	7.11	-2.50	-	-	-	-	33.6	-	-0.8
154	518.6	10.60	-	70.0	71.2	70.4	36.8	-	3.6	-
154	518.6	6.81	+1.65	-	-	-	-	36.8	-	2.4
134	518.6	10.60	-	63.8	64.5	62.9	39.6	-	6.4	-
134	518.7	6.47	-2.64	-	-	-	-	40.8	-	6.4
230	518.7	10.60	-	63.0	62.2	61.4	40.3	-	7.1	-
230	518.7	6.38	-2.80	-	-	-	-	41.8	-	7.4

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FIG. 17 LOW-POWER OSCILLATOR-ROD WAVE SHAPE WITH SHIM ROD FULLY WITHDRAWN - CLOCKWISE ROTATION - PERIOD MEASUREMENT DATA

FIG. 18 LOW-POWER OSCILLATOR-ROD WAVE SHAPE WITH SHIM ROD FULLY WITHDRAWN - CLOCKWISE ROTATION - CRITICAL ROD MEASUREMENT DATA

FIG. 19 LOW-POWER OSCILLATOR ROD WAVE SHAPE WITH SHIM ROD FULLY INSERTED - CLOCKWISE ROTATION - PERIOD MEASUREMENT DATA

FIG. 20 LOW-POWER OSCILLATOR-ROD WAVE SHAPE WITH SHIM ROD FULLY INSERTED - CLOCKWISE ROTATION - CRITICAL ROD MEASUREMENT DATA

FIG. 21 LOW-POWER OSCILLATOR-ROD WAVE SHAPE WITH SHIM ROD FULLY INSERTED - COUNTERCLOCKWISE ROTATION - PERIOD MEASUREMENT DATA

FIG. 22 LOW-POWER OSCILLATOR-ROD WAVE SHAPE WITH SHIM ROD FULLY INSERTED - COUNTERCLOCKWISE ROTATION - CRITICAL ROD MEASUREMENT DATA

The standard deviation for the fit to the critical-rod data is rather large for the case with the shim rod fully inserted and with counterclockwise oscillator-rod rotation (Fig. 22), because one data point appears to be incorrect. If it were omitted, the fit would not change appreciably but the standard deviation would be reduced from 0.8 to 0.5 inhour.

From Core A critical experiment studies made for the Fermi Reactor on ZPR-III, it was predicted that the low-power oscillator rod would have a peak-to-peak* worth of 19.2 inhours with the adjacent shim rod fully inserted, and 20.8 inhours with the shim rod at its normal operating or fully withdrawn position. It was also predicted that there would be a phase difference of 8 deg between results obtained from clockwise vs. counterclockwise rotation.³

From the Fourier analyses, Eqs. (1)-(6), the measured peak-to-peak worth was 16.0 inhours (5.0¢) with the shim rod fully withdrawn, and 15.5 inhours (4.85¢) with the shim rod fully inserted. (The result for the shim rod fully inserted was obtained by averaging the amplitudes for the two directions of rotation.) Furthermore, the phase difference between the two directions of rotation was found experimentally to be 6 to 8 deg. The experimental results are, therefore, in satisfactory agreement with the predicted values.

It was anticipated that the low-power oscillator rod would have a peak-to-peak worth slightly less than predicted since it had been found in earlier testing that the radial flux profile in the reactor is slightly flatter than it was in the critical assembly. The difference in oscillator worth for the two cases with the shim rod in its extreme withdrawal positions is slightly less than that found in the critical experiment. This can be explained by the flatter flux profile in the reactor.

B. HIGH-POWER ROD WORTH AND WAVE SHAPE

1. Rod Worth - Experimental Results and Analysis

The experimental data obtained in the measurements of the gross worth of the high-power oscillator rod relative to the low-power oscillator rod are given in Table VI. These data were used in conjunction with the regulating-rod calibration curve shown in Fig. 14 and with the appropriate temperature and power drift corrections, to obtain a measured reactivity of 105.3 inhours when the high-power rod is substituted for the low-power rod. Since the measured worth of the low-power rod was -206.9 inhours,

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* This usage of peak-to-peak worth is consistent with industry usage; more appropriately it should be considered to be the trough-to-crest worth.

the worth of the high-power rod is $-206.9 + 105.3$ or -101.6 inhours (-31.75%). The magnitude of the measured worth is about 12.9 per cent greater than the predicted worth of -90 inhours (28.2%).

TABLE VI - DATA OF THE MEASUREMENT OF GROSS WORTH OF THE HIGH-POWER OSCILLATOR ROD RELATIVE TO THE LOW-POWER OSCILLATOR ROD

Meas. no.	Oscillator-rod azimuth position for minimum negative worth, deg	Critical reg.-rod position, in. withdrawn	Critical shim-rod position, in. withdrawn	System isothermal temp, F	Power, kw	Power drift, %/5 min.
1	51.0	12.59	0	425.89	4.80	0
2	51.0	12.58	0	426.23	4.83	-1.1
1	202.5	3.77	0	453.98	4.87	+0.6
2	202.5	3.83	0	454.51	4.92	+0.9

The worth of the high-power oscillator rod was predicted from the low-power oscillator rod worth measurement made in the critical experiment, taking into account only the difference in B-10 loading. The differences in born enrichment (72.8 vs 91w/o), and the differences in other material constituents (steel, sodium, B-11, and carbon) were not considered specifically but were at first estimated to introduce a 10 per cent uncertainty in the predicted worth. Investigation indicated that the reduction in self-shielding of the boron due to the lower enrichment used in the high-power rod alone would account for two-thirds of the observed discrepancy. Therefore, the substantial differences in composition between the high-power and low-power rods can readily account for 12 per cent of the remaining difference. (The high-power rod has 4.5 kg less steel, 0.6 liters more sodium, and lesser amounts of B-11 and carbon.)

2. Wave Shape - Experimental Results and Analysis

The critical-rod raw data for wave-shape measurements, taken during the clockwise rotation of the high-power oscillator rod with the shim rod fully inserted, were converted to reactivity values using the regulating calibration curve shown in Fig. 14. The data were adjusted to a common isothermal temperature of 455 F, using a temperature coefficient value of -0.82 inhours per F, and a power drift correction was applied as given in Reference 2. The corrected reactivity data for each angular position with each core loading investigated are given in Tables VII and VIII.

TABLE VII - CRITICAL-ROD DATA OF WAVE-SHAPE
MEASUREMENTS FOR THE HIGH-POWER
OSCILLATOR ROD WITH CORE LOADING NO. 128

Oscillator- rod azimuth position, deg	Critical regulating- rod position, withdrawn in.	Power drift, %/5 min.	Isothermal temp, F	Corrected reactivity, inhours	Adjusted reactivity, inhours
52.5	4.20	+0.70	453.79	33.27	1.85
142.5	3.84	0	453.85	29.70	-1.72
322.6	4.09	+0.20	453.90	32.14	0.72
352.5	4.21	+0.37	454.08	33.17	1.75
22.5	4.25	-0.56	454.25	33.54	2.12
55.5	4.25	+0.56	454.53	33.18	1.76
82.5	4.15	-0.20	454.52	32.27	0.85
112.5	4.02	-0.73	454.30	31.22	-0.20
142.5	3.89	0	454.20	29.92	-1.50
172.5	3.79	-0.73	454.05	29.13	-2.29
202.5	3.77	+0.60	453.99	28.82	-2.60
232.5	3.79	-0.75	453.99	29.18	-2.24
262.5	3.89	+0.20	454.02	30.04	-1.38
292.5	4.03	0	454.04	31.46	0.04
322.5	4.16	-0.75	454.05	32.83	1.41
352.5	4.29	+1.1	454.20	33.79	2.37
22.5	4.31	-0.75	454.30	34.12	2.70
52.5	4.30	-0.56	454.39	33.93	2.51
82.5	4.23	+0.75	454.47	33.00	1.58
112.5	4.10	0	454.44	31.82	0.40
172.5	3.85	-1.1	454.43	29.46	-1.96
202.5	3.83	+0.91	454.51	28.95	-2.47
232.5	3.85	0	454.60	29.19	-2.23
262.5	3.94	0	454.67	30.03	-1.39
292.5	4.06	-1.5	454.72	31.37	-0.05

TABLE VIII - CRITICAL-ROD DATA OF WAVE-SHAPE
MEASUREMENTS FOR THE HIGH-POWER
OSCILLATOR ROD WITH CORE LOADING NO. 129

Oscillator- rod azimuth position, deg	Critical regulating- rod position, withdrawn in.	Power drift, %/5 min.	Isothermal temp, F	Corrected reactivity, inhours	Adjusted reactivity, inhours
202.5	2.34	+4.29	464.29	15.91	-2.75
232.5	2.30	-1.27	464.40	16.12	-2.54
262.5	2.42	-0.54	464.41	17.10	-1.56
292.5	2.58	-1.27	464.45	18.60	-0.06
322.5	2.78	0	464.62	20.11	1.45
352.5	2.89	-0.74	464.87	20.99	2.33
23.5	2.95	0	465.03	21.31	2.65
52.5	2.91	-1.48	465.10	21.07	2.41
82.5	2.84	0	465.25	20.13	1.47
113.0	2.68	+0.36	465.28	18.63	-0.03
142.5	2.53	0	465.30	17.30	-1.36
172.5	2.42	-0.55	465.35	16.33	-2.33
202.5	2.40	0	465.53	15.95	-2.71
232.5	2.43	-0.20	465.60	16.18	-2.48
262.5	2.55	0	465.70	17.16	-1.50
292.5	2.71	-0.20	465.76	18.57	-0.09
322.5	2.88	0	465.83	20.02	1.36
352.5	3.00	0	465.89	21.05	2.39
22.5	3.06	+0.55	465.98	21.45	2.79
52.5	3.02	+0.36	466.08	21.03	2.37
82.5	2.94	0	466.20	20.26	1.60
112.5	2.77	-0.72	466.29	18.74	0.08
142.5	2.64	0	466.40	17.39	-1.27
172.5	2.53	-0.88	466.48	16.45	-2.21

The corrected data have been plotted on Figs. 23 and 24. They have been Fourier-analyzed and in each case the data for the high-power oscillator rod could be fitted to within the experimental accuracy by the following expressions:

Shim rod fully inserted - clockwise rotation,

$$\begin{aligned} &\text{Loading No. 128} \\ &R(\theta) = 2.50 \sin(\theta - 112) \end{aligned} \tag{7}$$

$$\begin{aligned} &\text{Loading No. 129} \\ &R(\theta) = 2.77 \sin(\theta - 111) \end{aligned} \tag{8}$$

where,

R = the reactivity (in inhours) relative to a null position
 θ = the angular position (in deg) as denoted by the azimuth dial on the oscillator rod

The fitted sine-wave curves obtained from the above equations are shown in Figs. 23 and 24. Each data point was assigned an uncertainty of ± 0.5 inhours, as indicated by the flags.

The standard deviation between the experimental data and the fitted curves was 0.3 inhours for the data taken with core loading 128 and 0.1 inhours for the data taken with core loading 129. Since the standard deviations of the fits were less than the uncertainty of the measurements, the use of higher harmonics to describe the wave shape was not justified.

It was anticipated (from physics calculations) that the peak-to-peak worth of the high-power oscillator rod could be estimated from the measured peak-to-peak worth of the low-power oscillator rod by scaling according to the difference in B-10 loading. Since the two rods contained 110.25 g and 254 g of B-10, respectively, and the low-power rod had a measured peak-to-peak worth of 15.5 inhours with the shim rod fully inserted, the predicted peak-to-peak worth of the high-power rod was 6.7 ± 1.3 inhours.

From the curves fitted by Fourier analyses it is apparent that the measured peak-to-peak worth of the high-power oscillator rod is 5.00 inhours (1.57¢) for core loading 128 and 5.54 inhours (1.74¢) for core loading 129. Core loading No. 109 (Fig. 10) was used for the measurement of the peak-to-peak worth of the low-power oscillator rod. The loading contained a core-shim subassembly at core lattice position P01-P06. The subassembly had only a minimal effect on the flux gradient at the oscillator rod and on the rod worth. Thus, this loading can be compared with loading No. 129 (Fig. 12) for prediction of the high-power oscillator-rod worth. Loading No. 128 (Fig. 11) was expected to result in a smaller flux gradient at the oscillator-rod position and therefore the oscillator rod in this loading

FIG. 23 HIGH-POWER OSCILLATOR-ROD WAVE SHAPE WITH CORE LOADING NO. 128

FIG. 24 HIGH-POWER OSCILLATOR-ROD WAVE SHAPE WITH CORE LOADING NO. 129

should have had a smaller peak-to-peak worth. This was found to be true, with the reactivity amplitude being 10 per cent less than in core loading No. 129.

The discrepancy between the predicted nominal value (6.7 in-hours) and the value measured with core loading No. 129 (5.54 in-hours) is believed to be due to three effects: (1) the prediction was based on the low-power rod loading which had a B-10 enrichment of 92 per cent compared to the high-power rod which had an enrichment of 75 per cent; the decrease in enrichment could be expected to decrease the peak-to-peak worth since additional B-11, which has a positive reactivity effect, was added; (2) the difference in the B-10 content of the two rods would be expected to cause a slight decrease in self-shielding in the high-power rod, which would tend to increase the peak-to-peak worth; (3) the high-power rod had a slightly smaller eccentricity for the boron loading. This would also tend to decrease the peak-to-peak worth. These factors led to the assignment of a ± 20 per cent uncertainty in the predicted value. The measured value was slightly larger than the lower limit of prediction.

C. ZERO-POWER OSCILLATOR TESTS

1. Experimental Results

The zero-power transfer function was measured with the synchronous transfer function analyzer on December 20, 28, 29 in 1965 and on January 4 and 6 in 1966. A total of 16 passes was made through the frequency spectrum of 0.005 to 5.0 cps, in a power range between 340 kw and 2.0 Mwt, as measured on the Keithley micro-microammeter. This power range was chosen as the zero-power range because it had no detectable feedback reactivity. The core loading during these measurements is shown in Fig. 25. The data obtained with this method are: (1) a measurement proportional to the in-phase (or real) component, (2) a measurement proportional to the quadrature (or imaginary) component of the transfer function as a function of frequency and, (3) a measurement proportional to the integral of the power level over the measurement time period.

The unnormalized or relative amplitude, $|G|$, and the phase angle, θ , of the transfer function were obtained at each frequency for each pass through the spectrum by using the following relationships:

$$|G| = \frac{\sqrt{I^2 + Q^2}}{IP} \cdot (AGS) \cdot (IR) \quad (9)$$

$$\theta = \tan^{-1} \frac{I}{Q} \quad (10)$$

SR = Safety Rod
CR = Control Rod

OR = Oscillator Rod
NS = Neutron Source

FIG. 25 CORE LOADING FOR OSCILLATOR TESTS

where,

|G = transfer function amplitude
 θ = transfer function phase angle
I = in-phase measurement
Q = quadrature measurement
IP = integrated power measurement
AGS = amplifier gain setting
IR = integration rate

In the above, AGS and IR are instrument settings made at the time of measurement.

The synchronous transfer function analyzer was calibrated with an overall gain such that when the nominal values given on the instrument were used, the amplitude was in units of per cent, i. e., the ratio of the peak-to-peak power perturbation to the mean power level.

The ion chamber was compensated for gamma rays on at least one pass through the frequency spectrum. Also, on at least one pass through the frequency spectrum, a less sensitive ion chamber was used (the lower sensitivity was accomplished by using a chamber of the same type located more remotely from the core). The results within the experimental uncertainty were no different for these two situations than those for the nominal situation.

The data for each pass through the frequency spectrum were normalized in amplitude and phase angle. The amplitude normalization was accomplished by adjusting the amplitude values so that the average value over the range from 2 to 5 cps was $1.000 \text{ } \mu\text{V}^{-1}$. From theoretical considerations (see Appendix B) it was expected that the amplitude would be constant over that frequency range. Similarly, the phase angles were adjusted by an additive constant to normalize the measured phase angle to -1.0 deg at 5 cps, which is the theoretical value. It was anticipated that such normalization would be necessary since, from the quantities measured, only power perturbation or relative amplitude can be determined and since there is no way of aligning the synchro-resolver of the synchronous transfer function analyzer precisely with respect to the angular orientation of the poison section of the oscillator rod.

The normalized results for all 16 runs were averaged at each frequency to obtain an average measured set of data for the zero-power transfer function. The results are given in Table IX and Figs. 26 and 27.

A calibration measurement was made periodically to determine that the measurements that were made were those of the zero-power transfer function and that they did not contain any frequency-dependent transfer function of the analyzer. A sine potentiometer was set up with batteries so

TABLE IX - DATA OF THE MEASUREMENT OF THE ZERO-
POWER TRANSFER FUNCTION USING THE
SYNCHRONOUS TRANSFER FUNCTION ANALYZER

<u>Frequency,</u> <u>cps</u>	<u>Normalized</u> <u>amplitude,</u> <u>δ^{-1}</u>	<u>Normalized</u> <u>phase-angle,</u> <u>deg</u>
5.0	1.000	- 1.0
4.0	0.999	- 1.2
3.0	0.999	- 1.5
2.0	0.999	- 2.3
1.0	1.022	- 4.5
0.7	1.033	- 6.0
0.5	1.059	- 7.8
0.4	1.079	- 9.1
0.3	1.112	- 11.0
0.2	1.169	- 13.7
0.1	1.315	- 20.1
0.07	1.420	- 24.4
0.05	1.582	- 28.1
0.04	1.703	- 29.5
0.03	1.929	- 33.8
0.02	2.275	- 36.3
0.01	3.210	- 43.5
0.007	3.821	- 47.5
0.005	4.634	- 52.7

that it provided an a-c signal superimposed on a d-c signal, where the amplitude of the a-c signal is a few per cent of the d-c signal and is independent of frequency. The sine potentiometer was mounted on the oscillator-rod extension so that it rotated with the rod, and the signal from the sine potentiometer was used as an ion chamber signal simulant supplied to the synchronous transfer function analyzer. With the reactor shut down, the oscillator rod was operated and the transfer function of the synchronous transfer function analyzer was measured by making the appropriate measurements over the frequency range of 0.005 to 5.0 cps.

In one calibration measurement, the sine potentiometer was set so that it was about 45 deg out of phase with respect to the synchro-resolver. Lack of a precision read-out prevented a setting with an accuracy of less than several deg. The amplitude and phase angle of the sine potentiometer signal were measured and compared with the input signal. The results of this calibration measurement are given in Table X and Figs. 28 and 29. The measured amplitude was compared to the predicted amplitude (Fig. 28) and was found to have a constant (with frequency) deficiency of about one per cent,

FIG. 26 ZERO-POWER TRANSFER FUNCTION AMPLITUDE, SYNCHRONOUS METHOD

FIG. 27 ZERO-POWER TRANSFER FUNCTION PHASE ANGLE, SYNCHRONOUS METHOD

TABLE X - DATA OF A CALIBRATION MEASUREMENT ON THE
SYNCHRONOUS TRANSFER FUNCTION ANALYZER

<u>Frequency,</u> <u>cps</u>	<u>Ratio of experimental</u> <u>amplitude to predicted</u> <u>amplitude</u>	<u>Phase</u> <u>shift,</u> <u>deg</u>
5	0.992	43.4
2	0.987	43.4
1	0.989	43.5
0.5	0.987	43.5
0.2	0.986	43.9
0.1	0.988	43.8
0.05	0.989	43.9
0.02	0.987	43.7
0.01	0.991	44.1
0.007	0.986	44.4

Amplifier gain setting = 5%
Integration range = 50 sec

which can be attributed to lack of precision in setting the a-c signal at the predicted value. It was also found that there is a scatter of the amplitude data of about 0.5 per cent above and below the constant level.

The measured phase angle had a small frequency-dependent transfer function, with the difference between the low-frequency phase angle and the high-frequency phase angle being one deg, with an average value of about 44 deg. The measured phase angle was not considered to have a consistent error since the precision of the measurement was greater than the precision to which the sine potentiometer could be set with respect to a desired value. The phase-angle data obtained with this instrument were not corrected for this small phase-angle transfer function since the correction is on the order of the precision to which the phase angle could be measured. From the results obtained, it was concluded that the synchronous transfer function analyzer does not have a frequency-dependent transfer function.

The Keithley method was used on many of the same tests that were conducted with the synchronous transfer function analyzer, described above. Data were taken on December 28 and 29 in 1965 and January 4 in 1966. The Keithley method provided a strip-chart recording from which the unnormalized amplitude only could be obtained manually. A portion of a typical chart is shown in Fig. 30. The data for all runs at each frequency were averaged to obtain an averaged, unnormalized amplitude. Since the unnormalized amplitudes were expressed in terms of the ratio of the power

FIG. 28 SYNCHRONOUS TRANSFER FUNCTION ANALYZER, TRANSFER FUNCTION AMPLITUDE

FIG. 29 SYNCHRONOUS TRANSFER FUNCTION ANALYZER, TRANSFER FUNCTION PHASE ANGLE

FIG. 30 TYPICAL KEITHLEY CHART RECORDING

perturbation, ΔP , to the d-c power level, P_0 , it was justifiable to average the results of a large number of tests. The average values of the amplitude are given in Table XI.

TABLE XI - DATA OF THE MEASUREMENT OF THE
ZERO-POWER TRANSFER FUNCTION
BY THE KEITHLEY METHOD

<u>Frequency,</u> <u>cps</u>	<u>Relative</u> <u>amplitude</u>
0.7	1.55
0.5	1.65
0.4	1.68
0.3	1.75
0.2	1.87
0.1	2.07
0.07	2.27
0.05	2.49
0.04	2.73
0.03	3.08
0.02	3.64
0.01	5.05
0.007	5.92
0.005	7.15

The null-balance analyzer was used in a test on September 3, 1965, conducted at 600 kw. The experimental data obtained were the direct readings of the unnormalized transfer function amplitude and phase angle as a function of frequency. The data obtained are given in Table XII.

The Sanborn method was used in a test on August 23, 1965, conducted at 300 kw. The experimental data obtained was in the form of a strip chart that was reduced manually to obtain the amplitude and phase angle as a function of frequency. A portion of a typical chart is shown in Fig. 31. The results are given in Table XIII. The data in this table have been corrected at high frequencies for slight differences which existed in the noise filters on the two recording channels.

2. Analysis

a. Synchronous Method

The zero-power transfer function obtained by the synchronous method was compared with the theoretical zero-power transfer function ob-

TABLE XII - DATA OF THE MEASUREMENT OF THE
ZERO-POWER TRANSFER FUNCTION
USING THE NULL-BALANCE ANALYZER

<u>Frequency,</u> <u>cps</u>	<u>Relative</u> <u>amplitude</u>	<u>Relative</u> <u>phase angle,</u> <u>deg</u>
5.00	4.08	- 13.0
4.00	4.35	- 11.0
3.00	4.35	- 12.0
2.00	4.30	- 12.5
1.00	4.35	- 14.0
0.7	4.45	- 16.0
0.5	4.57	- 18.8
0.4	4.65	- 20.0
0.3	4.75	- 22.0
0.2	4.90	- 25.0
0.1	5.45	- 31.0
0.07	6.00	- 35.0
0.05	6.55	- 38.1

TABLE XIII - DATA OF THE MEASUREMENT OF THE
ZERO-POWER TRANSFER FUNCTION
BY THE SANBORN METHOD

<u>Frequency,</u> <u>cps</u>	<u>Relative</u> <u>amplitude</u>	<u>Relative</u> <u>phase angle,</u> <u>deg</u>
5.0	2.38	- 29.5
4.0	2.24	- 32.0
3.0	2.24	(Not avail.)
2.0	2.20	- 25.6
1.0	2.60	- 27.0
0.7	2.30	- 22.0
0.5	2.30	- 19.8
0.4	2.41	- 21.1
0.3	2.50	- 22.2
0.2	2.58	- 28.4
0.1	3.04	- 32.7
0.07	3.36	- 36.7
0.05	3.51	- 41.5
0.04	3.85	- 42.7
0.03	4.36	- 46.1
0.02	5.13	- 49.5
0.01	7.25	- 56.3
0.007	8.55	- 67.0
0.005	9.25	- 73.7

FIG. 31 TYPICAL SANBORN CHART RECORDING

tained from a point kinetics model. The experimental results are shown on Figs. 26 and 27. The ratio of the experimental amplitude to the theoretical amplitude is given as a function of frequencies (Fig. 32) and the difference between the experimental and theoretical phase angles is given (Fig. 33) as a function of frequency. Also shown on these graphs are simple functional fits to the data. From these it can be seen that the amplitude ratio is not constant with frequency, as would be expected if there were agreement between experiment and theory. Also, the phase-angle difference is not constant. Rather, the amplitude ratio and the phase-angle difference increase with decreasing frequency. The cause for this apparent discrepancy is discussed below.

The standard deviation of the data at each frequency was calculated to determine the statistical significance of the data obtained. The standard deviation on a single observation of the amplitude was less than one per cent when the frequency was greater than or equal to 0.2 cps; between one and two per cent when the frequency was between 0.02 and 0.2 cps; and two to four per cent when the frequency was below 0.02 cps. Since it was the practice to take three times as much data at each frequency below 0.02 cps as at higher frequencies, the standard deviation can be reduced accordingly.

The standard deviation of all of the amplitude data at each frequency was about 0.5 per cent. Similarly, the phase-angle data was analyzed; for the body of data analyzed, the standard deviation of the phase angle at each frequency was less than 0.3 deg. This confirmed the belief that the measured average amplitude and measured average phase angle at each frequency can be treated with confidence and that the observed differences between them and the respective theoretical values are consistent and outside the domain of statistical scatter.

b. Keithley Method

The data obtained by the Keithley method were analyzed in a similar manner. However, the amount of data was limited. Only a recording of the power level during the experiment was available. From this, the amplitude of the zero-power transfer function was obtained. Because the chart of the power level was recorded by a Leeds and Northrup recorder with a long time constant, the power trace was not reliable for frequencies above 0.5 cps. The data available was the unnormalized amplitude expressed as the ratio of peak-to-peak power perturbation to d-c power level, given in per cent.

The ratio of this unnormalized, experimental amplitude to the theoretical amplitude was determined at each frequency. The results are given in Fig. 34. Also, a simple functional fit to the data is included on this figure. These results show the same general characteristics found

FIG. 32 ZERO-POWER TRANSFER FUNCTION — RATIO OF EXPERIMENTAL AMPLITUDE TO THEORETICAL AMPLITUDE, SYNCHRONOUS METHOD

FIG. 33 ZERO-POWER TRANSFER FUNCTION — DIFFERENCE BETWEEN EXPERIMENTAL PHASE ANGLE AND THEORETICAL PHASE ANGLE, SYNCHRONOUS METHOD.

FIG. 34 ZERO-POWER TRANSFER FUNCTION — RATIO OF EXPERIMENTAL AMPLITUDE TO THEORETICAL AMPLITUDE, KEITHLEY METHOD

by the synchronous method and the amount of change in the ratio in the decade from 0.05 to 0.5 cps is about the same. The most marked difference is that at very low frequencies the Keithley method indicates that the ratio is constant with frequency while the synchronous method indicates that it varies with frequency. There is not sufficient data to support either position unequivocally. The data of the Keithley method was analyzed statistically and, for the data used in the analysis (9 separate runs or passes through the frequency spectrum), the standard deviation was one per cent at all frequencies.

c. Null-Balance Method

The third method was the one in which the null-balance analyzer was used. Only one pass was made through the frequency spectrum with this analyzer. The results are given on Figs. 35 and 36. Before the phase-angle differences were taken, the phase-angle data were normalized by adjusting all of the data appropriately to get agreement between experiment and theory at 2.0 cps. The phase-angle data at higher frequencies appeared to be rather erratic and would provide a dubious normalization. Consequently, the highest frequency was selected at which it appeared there would be reasonable agreement between experiment and theory after an additive correction. From the results shown in Fig. 36, it appears that the normalization would be good over a larger frequency range if the normalization adjustment were increased by one deg. However, this is a minor problem since relative differences are the main items of interest. It was expected that it would be necessary to normalize the data because the resolver in the analyzer was not aligned with the oscillator-rod azimuth position. Consequently it provides only relative values of the phase angle. Likewise, the amplitude measurement is only a relative measurement, since the reading taken at each frequency was a setting of a ten-turn potentiometer. However, it was not necessary to normalize the amplitude data for the comparison that was made. A simple functional fit to the data is also shown on Fig. 35.

It is not apparent that the amplitude ratio and the phase-angle differences have the same characteristics found by the other methods. Rather, it appears that the amplitude ratio is constant, indicating agreement between experiment and theory. However, the scatter in the data is quite large and the frequency range is quite limited. With the exception of one low-frequency point (0.05 cps) the phase-angle data shows the same general characteristics of the phase angle found with the synchronous method. The method of determining the transfer function with the null-balance analyzer suffers from the handicap of a frequency limitation. Because of the limited capability of the Krohn-Hite band pass filter it is not possible to measure the transfer function with this analyzer at frequencies below 0.02 cps. However, the operators of the equipment could not detect the null point reliably at frequencies below 0.05 cps. Since much of the interesting infor-

FIG. 35 ZERO-POWER TRANSFER FUNCTION - RATIO OF EXPERIMENTAL AMPLITUDE TO THEORETICAL AMPLITUDE, NULL - BALANCE METHOD

FIG. 36 ZERO-POWER TRANSFER FUNCTION — DIFFERENCE BETWEEN EXPERIMENTAL PHASE ANGLE AND THEORETICAL PHASE ANGLE, NULL-BALANCE METHOD

mation on reactor transfer functions lies below 0.05 cps, the null-balance analyzer is of dubious value for the measurement of the reactor transfer function.

d. Sanborn Method

The fourth method used to determine the zero-power transfer function was the Sanborn method, discussed in Section III-C-3. The results were put in a form similar to that for the other methods. The amplitude ratio and the data of phase-angle difference are shown as a function of frequency on Figs. 37 and 38. Before the phase-angle differences were taken, the phase-angle data were normalized by adjusting all of the data appropriately to get agreement between experiment and theory at 0.5 cps. This frequency was selected because, although the absolute value of the phase angle should vary inversely with frequency, it was observed that the experimental data followed this rule only up to 0.05 cps, after which it varied directly with frequency. Therefore the minimum point was selected for the normalization point.

It was expected that it would be necessary to normalize the data because the signal which is proportional to reactivity perturbation is obtained from a sine potentiometer mounted on the shaft of the oscillator-rod extension. There was no precise azimuthal alignment between the signal from the sine potentiometer and the poison section of the oscillator rod. Consequently, the measured phase angle is only a relative phase angle.

Also shown in Figs. 37 and 38 are simple functional fits to the data. The amplitude ratio is the same characteristic function of frequency that was found with the synchronous transfer function analyzer and by the Keithley method. The phase-angle difference is constant with frequency up to 0.5 cps, indicating agreement between experiment and theory. However, the scatter of the data is quite large, with the standard deviation of all of the data below 0.5 cps being about 2.5 deg. This is larger than the range of the functional fit to the data taken with the synchronous transfer function analyzer. Lack of sufficient data prevents the determination of the precision of the data at any given frequency. Because of this and the large scatter, the phase-angle data obtained by the Sanborn method at frequencies below 0.5 cps probably cannot be trusted as much as that obtained with the synchronous transfer function analyzer. Above 0.5 cps, the phase-angle difference for the Sanborn method shows a sharp drop, which is characteristic of a filter roll-off.

Although the input signals were filtered to remove the strong 60-cycle signal present, the amplifier characteristics for the channel used for recording reactor power were different from those for the channel used for recording oscillator-rod azimuthal position or reactivity perturbation. Consequently, it was necessary to use different filters. Although the filters

FIG 37 ZERO-POWER TRANSFER FUNCTION — RATIO OF EXPERIMENTAL AMPLITUDE TO THEORETICAL AMPLITUDE, SANBORN METHOD

FIG. 38 ZERO POWER TRANSFER FUNCTION — DIFFERENCE BETWEEN EXPERIMENTAL PHASE ANGLE AND THEORETICAL PHASE ANGLE, SANBORN METHOD

were designed to have the same frequency characteristics, inspection of the results indicated that they probably were different. However, some of the roll-off could be attributed to the amplifiers. The two amplifiers used had different high-frequency limitations. It was necessary to use different amplifiers for the two channels because the signals for the two channels differed by several orders of magnitude in their strengths.

3. Conclusions on Results

Three of the four methods used are in agreement that at low frequencies the measured amplitude is greater than the theoretical amplitude. Because there is such a meager amount of data for the fourth method and because it does not extend very far in the low-frequency domain, no conclusion can be drawn with respect to that method. Two of the three methods used for the measurement of the phase angle indicate that the absolute value of the measured phase angle is less than the absolute value of the theoretical phase angle. The third method shows agreement between measurement and theory.

Several reasons have been advanced for these differences. The most obvious is that the reactivity worth of the rod might be a function of frequency, i. e. at high frequencies centrifugal forces might cause bowing of the rod. The designers of the rod examined the design and concluded that: (1) the design was adequate to prevent mechanical movements which would cause such effects, (2) the amount of distortion necessary to produce the effects observed would be much larger than the amount that could occur without causing the oscillator rod to bind in its channel, and (3) any such distortions would increase the amplitude at high frequencies and cause an apparent decrease in the low-frequency amplitude after normalization, which is contrary to the observed effects.

Several neutronics effects were studied. One was the change required in the relative number of delayed neutrons produced by Uranium-235 and Uranium-238 and the relative number of delayed neutrons produced by the different precursor groups. In both instances, the measured zero-power transfer function could be obtained by an appropriate set of data. However, the data necessary to accomplish this differ from the nominal values by more than the uncertainties listed in the literature. Consequently, this approach was considered to be only remotely appropriate for an explanation of the observed differences. Another approach was a study of the spatial effects on reactor kinetics. The spatially-dependent transfer function was calculated for the Enrico Fermi reactor by the methods of Reference 4. It was found that for a detector located in the graphite shield and an oscillator rod located in the core, the measured transfer function should show only negligible differences from the transfer function calculated from a point kinetics model. Consequently, it appears

that the observed differences between the measured and theoretical zero-power transfer functions cannot be attributed to mechanical effects. Neither can they be attributed to the models used in the calculations or the basic delayed neutron data when the uncertainties given in the literature are considered to be correct.

An analysis was made of the differences in the transfer functions obtained from the linear and nonlinear forms of the neutron kinetics equations. For the amount of reactivity that was oscillated, the effect on the transfer function due to this cause is negligible and not the cause of the observed difference between the measured and theoretical transfer functions.

Two other possible reasons were discarded from experimental work. One was the possibility of the appearance at the detector of short half-life gamma rays produced in the blanket and shield. Such gamma rays would influence an uncompensated ion chamber as pseudo-delayed neutrons and cause apparent errors in the transfer function measured. This was discarded when it was found that the results obtained were not dependent upon gamma compensation. The other possible reason was a spatially-dependent effect on the transfer function. Not only was this shown analytically not to be a cause, but it was also shown experimentally that the results obtained were not dependent upon the position of the ion chamber in the shield, and hence not a cause of the observed difference between the measured and theoretical transfer function.

4. Conclusions on Methods

From the results obtained on the measurement of the zero-power transfer function, the best method for making measurements appears to be the one which uses the synchronous transfer function analyzer. This method measures the transfer function with good precision over the frequency range from 0.005 to 5.0 cps and it permits the obtaining of both the amplitude and the phase angle.

The other methods have one or more deficiencies. The method using the null-balance analyzer can be used only at frequencies above 0.05 cps. Also, the method is very subjective; the results obtained depend strongly on the skill and experience of the operator.

The Keithley method is relatively simple. However, it provides information on the amplitude only in the range from 0.005 to 0.5 cps and it does not provide information on the phase angle. To some extent, the results of this method are dependent upon the skill of the operator in discounting the recorded noise when he is measuring the amplitude.

The Sanborn method provides information on the amplitude and phase angle, but, as it was shown above, the phase-angle data is of dubious

value at frequencies above 0.5 cps. Also, the amplitude measurements made by this method probably are not as precise as those made with the synchronous transfer function analyzer, because of the difficulties in resolution associated with the reading of a small-width strip chart.

5. The Dynamic Amplitude of the Oscillator Rod

The dynamic amplitude of the high-power oscillator rod was determined with two of the methods used to measure the zero-power transfer function. With the synchronous method and the Keithley method, the measured amplitude is the amplitude of the power perturbation, expressed as a percentage of the nominal power level. At the high frequencies used for normalization, the measured transfer function amplitude, expressed as $\Delta P/P_0$, in units of per cent, can be compared with the theoretical value, expressed in units of $\$^{-1}$, and the amount of reactivity that was oscillated can be extracted in units of cents.

Following this procedure, it was found that for the synchronous method the average value of the power perturbation was constant with respect to frequency over the frequency range of 2 to 5 cps and had a value of 1.57 per cent. Since the zero-power transfer function is $1.00 \$^{-1}$ for this frequency range, the measured dynamic, high-frequency, high-power oscillator-rod amplitude is 1.57 cents.

The highest frequency at which Keithley measurements are reliable is about 0.5 cps. At that frequency, the measured power perturbation was 1.65 per cent and the theoretical value of the transfer function at that frequency is $1.06 \$^{-1}$. Thus, by this method, the measured dynamic, high-frequency rod amplitude is 1.56 cents.

The above amplitudes must be compared with the static value of 1.74 cents for the same core loading. The specific cause for the discrepancy has not been completely determined. It could be caused by some systematic experimental error or it could be caused by a systematic theoretical error. For example, the static amplitude was measured by methods which are traceable to period measurements.

The period measurements were converted to reactivity measurements using the inhour equation, to obtain reactivity values in inhours. The inhour equation is not very sensitive to the values used for the delayed neutron characteristics. The reactivity values in inhours were converted into values in cents, using a conversion factor which is very sensitive to the values of the delayed neutron characteristics. It was found that a set of delayed neutron characteristics that would give agreement between static and dynamic amplitudes at high frequency would also give agreement between the experimental and theoretical zero-power transfer function amplitudes at low frequencies.

However, such a set lies outside the range of the nominal values when reported uncertainties are applied.

D. HIGH-POWER OSCILLATOR TESTS

1. Experimental Results

The high-power oscillator tests were conducted as previously outlined. The purpose of these tests was to measure the reactor transfer function at various power levels and under two modes of control, to establish the stability of the reactor and to learn as much as possible about the feedback characteristics of the reactor. The basic method used to measure the reactor transfer function was the synchronous method, using the synchronous transfer function analyzer, since the experiments at zero powers showed that the synchronous method is the only method that supplies all of the desired information over the desired frequency range.

At each power level, the frequency spectrum from 0.005 to 5.0 cps was scanned, with three passes being made through the frequency spectrum. Three data points were taken on each pass at each frequency in the range from 0.005 cps to 0.01 cps, to add to the body of data. Exceptions to this procedure were made at power levels of 6.6, 30, and 46 Mwt because only a limited frequency range was of interest. These three power levels were not in the original schedule but they were included in the experimental program to provide additional information on reactor stability requested by the reactor operator (PRDC) during the experimental program.

The experimental results for a particular power level and mode of control were combined at each frequency to obtain an average amplitude and average phase angle. These averages were normalized to an amplitude of $1.000\text{ }^{\circ}\text{ }^{-1}$ and a phase angle of -1.0 deg at 5 cps.

Data for the normalized measured transfer functions are shown in Appendix C, Tables XXIII and XXIV and Figs. 46 through 71. These data are pertinent to the analyses of stability and feedback which are discussed in Section V-D-2 which follows.

2. Analysis

a. Stability Analysis - Manual Control

The results obtained with the reactor in manual control were analyzed by a method developed by H. A. Bethe⁵ to determine reactor stability. The method basically consists of an examination of the ratio of the at-power reactor transfer function amplitude to the zero-power transfer function amplitude. If this ratio is less than unity, the reactor is stable; if it is greater than unity, there is evidence of a possible resonance in the

amplitude and the reactor is conditionally unstable. In the latter case it becomes necessary to examine the phase-angle difference between the at-power reactor transfer function and the zero-power transfer function to determine if a certain assumption is satisfied for instability. The details of the derivation of this stability criterion are presented in Appendix D.

The ratio of the amplitudes, defined as γ , the stability figure of merit, has been determined for each frequency and at each power level for which the reactor transfer function was measured. The results are given in Table XIV. With the exception of the run at 46 Mwt, all values of γ are less than or equal to 1.000 to within the experimental error of 0.5 per cent assigned to the measurement of the transfer function amplitude.

TABLE XIV - THE STABILITY FIGURE OF MERIT -
MANUAL CONTROL

Frequency, cps	Power, Mwt						
	6.6	13.3	20.1	30	46	66.7	111
5	1.000	1.002	1.002	1.000	1.000	0.995	1.000
4	1.000	0.999	0.999	*	*	0.997	0.997
3	1.000	0.999	0.998	*	*	1.002	0.997
2	1.003	1.001	1.004	*	*	1.005	1.003
1	1.000	0.994	0.991	*	*	0.991	0.984
0.7	1.006	1.000	0.998	*	*	0.991	0.974
0.5	1.000	0.994	0.995	0.987	1.019	0.974	0.957
0.4	1.000	0.994	0.992	0.983	1.015	0.965	0.947
0.3	1.005	0.987	1.004	0.975	1.008	0.954	0.939
0.2	0.994	0.986	0.991	0.972	1.028	0.936	0.923
0.1	0.990	0.988	0.989	0.924	1.011	0.906	0.897
0.07	0.997	0.984	0.982	0.948	1.041	0.892	0.891
0.05	0.985	0.983	0.977	0.942	1.032	0.858	0.872
0.04	1.008	0.980	0.969	0.934	1.047	0.864	0.871
0.03	0.997	0.967	0.973	0.922	1.038	0.845	0.850
0.02	0.970	0.932	0.936	*	*	0.803	0.818
0.01	0.958	0.935	0.928	*	*	0.740	0.726
0.007	*	0.916	0.897	*	*	0.689	0.714
0.005	*	0.941	0.909	*	*	0.647	0.684

* Data not available

The lack of resonances indicates that the reactor can be classified as stable over the frequency range from 0.005 to 5 cps and at all powers through 111 Mwt when operated under manual control. The exceptional data, that for the 46 Mwt run, can only be considered to be in error due to malfunctioning of the synchronous transfer function analyzer. Analysis of the raw data indicates that, at high frequencies, an error of 3 to 4 per cent in the amplitude was occurring. Such an error at that frequency would cause the normalization to be in error and cause consequent errors in all of the values of the amplitude and of γ at lower frequencies. Such an error is sufficient to explain the inconsistency of the data for the 46 Mwt run.

b. Feedback Analysis - Manual Control

The closed-loop transfer function of a linear system can be defined in terms of the open-loop transfer function and the feedback. From the first principles of linear systems with feedback it is possible to write the expression

$$G_p(\omega, P) = \frac{G_o(\omega)}{1 - G_o(\omega) H(\omega, P)} \quad (11)$$

where,

$G_o(\omega)$ = open-loop transfer function as a function of frequency ω

$G_p(\omega, P)$ = closed-loop transfer function as a function of frequency, ω , and gain level, P

$H(\omega, P)$ = feedback as a function of frequency, ω , and gain level, P

This expression can be solved for the feedback,

$$H(\omega, P) = \left[\frac{1}{G_o(\omega)} - \frac{1}{G_p(\omega, P)} \right] \quad (12)$$

In reactor analysis, the open-loop transfer function is usually designated as the zero-power transfer function and the closed-loop transfer function is usually designated as the at-power transfer function and the gain level, P , is the reactor power. In a reactor there exists a transfer function due to the fission product decay which must be taken into account. The theory of this transfer function and its effect on the feedback transfer function are developed in Appendix E. The at-power transfer functions measured at 67 Mwt and 111 Mwt, along with the measured zero-power transfer, were used to extract the raw feedback reactivity transfer function. Then the fission

product transfer function was taken into account to obtain a corrected feedback reactivity transfer function. The results are given in Table XV and Figs. 39 and 40.

TABLE XV - FEEDBACK - MANUAL CONTROL

Frequency, cps	67 Mwt		111 Mwt	
	Amplitude, \$	Phase angle, deg	Amplitude, \$	Phase angle, deg
1.0	0.015	130.5	0.039	117.4
6.7	0.023	116.8	0.053	123.8
0.5	0.041	133.9	0.066	137.0
0.4	0.054	136.9	0.071	143.8
0.3	0.063	142.8	0.077	148.5
0.2	0.077	151.9	0.084	161.1
0.1	0.097	161.3	0.101	165.7
0.07	0.110	161.9	0.108	165.3
0.05	0.126	168.8	0.113	168.7
0.04	0.115	168.9	0.107	170.2
0.03	0.123	169.1	0.113	170.8
0.02	0.134	173.7	0.123	173.4
0.01	0.142	173.5	0.142	179.3
0.007	0.150	177.4	0.124	183.4
0.005	0.152	178.7	0.124	184.0

These transfer functions were fit to simple models to gain some insight into the kinetic mechanisms existing in the reactor. The first characteristic which can be identified is some evidence of a loop in the reactivity transfer function for the 111-Mwt conditions. However, there are not sufficient data to permit complete identification and analysis of this loop. Also, there is no evidence of the loop on the 67-Mwt data. Such a loop was expected and can be attributed to the signals which are transported and attenuated through the primary coolant loop with a time lag of about 45 sec and which cause coolant inlet temperature oscillations. The time-lag term in the feedback transfer function would cause the loop.

The data above the very low frequency portion of the spectrum were fitted by simple first-order time-constant models. From inspection it is obvious that more than one such mechanism must be acting, since a single first-order time-constant mechanism would produce a semi-circular transfer function, rather than the elliptical functions that were found. Also, from inspection, it appears that a sum of first-order time-constant or single-pole models is more appropriate than higher-order models because

FIG. 39 FEEDBACK TRANSFER FUNCTION - 67 Mw

FIG. 40 FEEDBACK TRANSFER FUNCTION - 111 Mw

there is no strong evidence that the measured transfer functions fall in the first quadrant, which is required for higher-order models.

Each of the transfer functions was fitted by a least-squares technique⁶ with a transfer function model, using the sum of three first-order time-constant models. The time constants of two of these models were assigned values thought to be appropriate for the feedback mechanisms anticipated. These time constants were calculated using the methods of Reference 7. The third time constant was assigned parametric values and the value giving the best overall fit was considered to be the most proper value.

Early attempts with a function containing fewer first-order time-constant models showed that an appreciable amount of feedback was due to a long time-constant model, which was not anticipated in the theoretical development of the feedback mechanisms. For this reason, the third time constant was allowed to be a parameter.

Following the methods of Reference 7, the first-order time constant for the fuel and the coolant expansion effects was found to be 0.4 sec for the 67-Mwt conditions and 0.3 sec for the 111-Mwt conditions. These were the time constants assigned to the first term for these two cases.

The core expansion is known to depend on the expansion of the wrapper can at the spacer pads. This expansion follows the coolant temperature with a time constant of 0.5 sec.⁸ Therefore the second term assigned to this mechanism was assumed to have a first-order time constant equal to that of the first term plus 0.5 sec. The results of this fitting procedure are given in Table XVI. Although a 4-sec time constant is given for the third term in Table XVIII, the fit for the 4-sec value was only slightly better than that for values of 2 sec and 6 sec. Consequently, it is not possible to specify the value for the third-term time constant with accuracy. Rather, it is most appropriate to categorize it merely as long.

The amount of feedback ascribed to each mechanism is strongly dependent upon the value of the time constant for the third mechanism. For example, the amount of feedback in the 111-Mwt data which is ascribed to the first mechanism varies from 75 per cent when the third mechanism time constant is 2 sec, to 69 per cent when the time constant is 6 sec. Likewise, the power coefficient associated with the second mechanism becomes positive when the third mechanism time constant is 3 sec or less. This is evidence that the fitting process is not very exact and little information about feedback can be obtained from the fitting of experimental data with such simple but somewhat meaningful forms. An attempt at fitting the data was made with the method of Reference 6, where the time constant and the gain of each term are determined by the fit. This fitting technique did not realize the physical limitation that time

constants must be positive. For this reason the results using this technique are not reported.

TABLE XVI - FITTED TRANSFER FUNCTIONS

Power = 67 Mwt		
Term	Time constant, sec	Gain, per cent
1	0.4	46
2	0.9	20
3	4	34

Power = 111 Mwt		
Term	Time constant, sec	Gain, per cent
1	0.3	71
2	0.8	6
3	4	23

Although the amount of feedback and the value of the time constant for the third mechanism cannot be determined exactly, there is evidence of the presence of an appreciably large amount of feedback with a time constant of several sec. Since no such mechanism was anticipated when feedback predictions were made, the possible sources of such feedback were investigated after the tests and a mechanism was postulated. The only process which might cause such a feedback is the expansion of the hold-down mechanism. It has been estimated that it has a time constant of several sec. However, the amount of feedback associated with the expansion of the holddown mechanism was predicted to be negligible.¹ This shows that there is a lack of understanding of some of the feedback reactivity effects which occur in the Enrico Fermi reactor. This is understandable when considering that the core, the support structure, and the holddown mechanism form a very large elastic structure for which a dynamic analysis is very difficult.

The available feedback data can also be analyzed for zero-frequency characteristics. The data in Figs. 65 and 66 can be extrapolated to zero frequency to obtain feedbacks of -0.15% for the 67-Mwt data and about -0.13% for the 111-Mwt data. These power levels are reported on the basis of a nuclear calibration. On a heat balance calibration, these two power levels are 59.7 Mwt and 99.0 Mwt, respectively. The consequent power coefficients are $-0.25\%/Mwt$ and $-0.13\%/Mwt$. However, the total flow in the 14-in. primary sodium pipes which feed the core was, for these two cases respectively, 5.1×10^6 lb per hr and 10.0×10^6 lb per hr, using the readings on the flow meters. Thus, from this measurement, the power coefficient appears to vary inversely as the primary coolant flow rate. Ac-

cordingly, then, the power coefficient for the nominal Core A coolant flow rate of 8.8×10^6 lb per hr would be -0.15¢/Mwt . This is roughly 10 per cent less than the value found by precision power coefficient methods⁹ and can be attributed to the problems associated with extracting feedback data from transfer function measurements and extrapolating feedback to zero frequency.

c. Stability Analysis - Automatic Control

The data obtained from the oscillator tests with the reactor in automatic control were analyzed for reactor stability, using the same method used for data taken with the reactor in manual control. The transfer function amplitude ratio, γ , was calculated for each frequency at each power level for which data were available. The results are given in Table XVII. From this table it can be seen that the ratio, γ , has very low values at low frequencies. However, in and near the decade from 0.05 to 0.5 cps, the amplitude ratio exceeds unity, and has values as high as 1.26. This is an indication of resonances in the amplitude. Such resonances are evident in the data shown in Fig. 41. The resonances are in the frequency range of 0.07 to 0.2 cps.

From Figs. 61, 63, 65, 67, 69 and 71 it can be seen that the phase angle of the at-power transfer function approaches the phase angle of the zero-power transfer function at the high-frequency end of this range. (The data for 12.7 Mwt shows an intersection of these phase angles). The phase-angle portion of the amplitude criterion is not satisfied exactly but the difference between the phase angles is small enough to cause concern about stability. However, the phase angles have a large difference at the low end of the frequency range over which the resonances exist. Consequently, it was considered only doubtful that an instability did exist. To determine whether or not these resonances were true resonances and signs of instability, a Nyquist analysis was performed.

For each power level and at each frequency for which data were available, the amplitude and phase angle of the quantity G_0H was calculated, where H is the total feedback present. The results are given in Table XIX. The plots of G_0H for 67 Mwt and 111 Mwt are given in Fig. 42. The phase angle at the frequency at which the amplitude is unity is defined as the phase margin. When the phase margin becomes zero the reactor is unstable and a phase margin that increases with reactor power is a sign of a stable reactor. The phase margin at each power level is given in Table XVIII. From the data in this table it can be seen that the reactor is stable under automatic control with increasing power since the phase margin is increasing with power above 19.4 Mwt.

TABLE XVII — STABILITY ANALYSIS DATA - AUTOMATIC CONTROL

Frequency, cps	γ , THE STABILITY FIGURE OF MERIT						AMPLITUDE OF G_0H						PHASE ANGLE OF G_0H , DEGREES					
			Power, Mwt						Power, Mwt						Power, Mwt			
	12.7	19.4	30	46	67	111	12.7	19.4	30	46	67	111	12.7	19.4	30	46	67	111
5	-	1.002	1.000	1.000	1.000	1.000	-	-	-	-	-	-	-	-	-	-	-	-
1	0.984	0.989	-	-	0.985	0.985	0.016	0.013	-	-	0.019	-	-	-	-	-	-	-
0.7	0.977	0.995	-	-	0.994	-	0.025	0.016	-	-	0.020	-	-	-	-	-	-	-
0.5	0.980	0.990	1.007	1.035	0.997	0.982	0.020	0.030	0.025	0.041	0.021	0.042	-	-	-	-	-	-
0.4	0.987	0.994	1.013	1.055	1.003	0.994	0.054	0.032	0.027	0.052	0.011	0.039	-	-	-	1.8	-	-
0.3	0.993	1.013	1.051	1.103	1.041	1.035	0.023	0.042	0.058	0.106	0.056	0.067	-	-	31.2	26.6	-	58.8
0.2	0.993	1.092	1.176	1.193	1.140	1.118	0.011	0.109	0.176	0.222	0.182	0.216	-	36.9	29.2	38.6	42.8	54.7
0.1	1.103	1.261	1.256	1.074	1.002	0.987	0.126	0.415	0.711	0.640	0.756	0.675	38.7	49.2	52.3	66.7	67.6	71.4
0.07	1.149	1.214	0.969	0.880	0.801	0.832	0.284	0.777	1.100	1.011	1.191	1.032	55.4	53.4	58.6	68.8	68.9	72.7
0.05	1.078	0.960	0.704	0.672	0.594	0.634	0.505	1.129	1.617	1.467	1.681	1.509	67.0	58.2	60.5	71.3	72.7	74.6
0.04	0.968	0.731	0.556	0.511	0.477	0.528	0.625	1.481	2.028	1.981	2.137	1.888	74.9	63.6	63.8	73.9	74.6	75.4
0.03	0.807	0.555	0.384	0.348	0.344	0.390	0.826	1.926	2.854	2.852	2.998	2.584	84.6	67.6	70.6	98.0	74.9	77.6
0.02	0.673	0.401	-	-	0.226	0.263	1.041	2.548	-	-	4.288	3.742	92.6	75.2	-	-	86.6	84.1
0.01	0.435	0.250	-	-	0.173	-	1.533	3.588	-	-	5.170	-	130.5	107.2	-	-	93.5	-
0.007	0.440	0.223	-	-	0.077	-	1.509	3.955	-	-	9.469	-	128.5	115.3	-	-	93.0	-
0.005	0.642	0.180	-	-	0.145	-	0.963	5.120	-	-	6.427	-	119.1	111.2	-	-	92.9	-

FIG. 41 STABILITY FIGURE OF MERIT — AUTOMATIC CONTROL

FIG. 42 NYQUIST DIAGRAMS—67 Mwt AND 111 Mwt—AUTOMATIC CONTROL

TABLE XVIII - AUTOMATIC CONTROL PHASE MARGIN

<u>Power,</u> <u>Mwt</u>	<u>Phase margin,</u> <u>deg</u>
12.7	90
19.4	56
30	57
46	68
67	68
111	72

d. Feedback Analysis - Automatic Control

There are two forms of feedback when the reactor is operated in the automatic mode. One is through the feedback transfer function $X(\omega)$ and its contribution to the total feedback, PX , where P is the nominal power at which the reactor is operated. The other is through the automatic control system. Since the automatic control system was designed and built so that above 30 Mwt the signal to the regulating rod is proportional to $dn/dt/n$ (where n is the neutron signal), the signal to the regulating rod should be independent of power and dependent only on frequency. The feedback of the automatic control system can be represented as H_A . Therefore the at-power transfer functions at power P under manual and automatic control can be written respectively as:

$$G_M = \frac{G_o}{1 - P G_o X} \quad (13)$$

$$G_A = \frac{G_o}{1 - G_o(PX + H_A)} \quad (14)$$

The difference between these two expressions produces the expression

$$H_A = \frac{1}{G_M} - \frac{1}{G_A} \quad (15)$$

where,

- G_o = zero-power transfer function
- G_M = at-power transfer function under manual control
- G_A = at-power transfer function under automatic control
- X = reactivity feedback transfer function
- H_A = automatic control system transfer function
- P = reactor power

The data from the tests at 67 and 111 Mwt were treated in this manner, neglecting small changes in flow between the tests made for manual and automatic control, to determine the automatic control system feedback transfer function. The data are given in Table XIX and Figs. 43 and 44. These data were fitted to the ratio of two polynomials in $i\omega$ according to the method of Reference 6. It was found that the fit to a second-order polynomial is quite good and the fit can be approximated well by the model

$$H_A = \frac{G}{(1 + i \omega \tau_1)(1 + i \omega \tau_2)^2} \quad (16)$$

where,

$$\begin{aligned} G &= \text{transfer function gain} \\ \tau_1, \tau_2 &= \text{time constants} \\ \omega &= \text{frequency} \end{aligned}$$

This is the model which is valid for the automatic control system. The first term in the denominator is characteristic of the differentiating circuit and the second term is characteristic of the servomechanism. The values found by this experimental approach are given in Table XX, along with the expected values. The expected values are based on results obtained in the Enrico Fermi Pre-Operational Test Program.

The lack of precise agreement between the results for the two runs demonstrates the uncertainty in the values obtained by fitting the data to the model that was chosen. It also explains some of the reasons for the discrepancy between the fitted values and the expected values. The measured time constants can be considered to be in agreement with the expected values, to within the experimental uncertainties. The difference between the measured and expected gains can be attributed to a suspected change in the amplification of one component between the time at which the expected values were determined and the time at which the oscillator tests were conducted.

Since the reactor was found to be stable under automatic control, it was not necessary to adjust the settings of the automatic control system to obtain better agreement between the fitted values and the expected values or to obtain better reactor control.

TABLE XIX - AUTOMATIC CONTROL SYSTEM TRANSFER FUNCTION

Frequency, cps	67 Mwt		111 Mwt	
	Amplitude, \$	Phase angle, deg	Amplitude, \$	Phase angle, deg
0.5	0.025	-21.7	0.0283	-23.3
0.4	0.020	-46.3	0.0466	-8.0
0.3	0.079	2.3	0.0893	9.7
0.2	0.178	31.1	0.205	44.2
0.1	0.553	77.8	0.492	79.8
0.07	0.804	85.7	0.692	88.5
0.05	1.020	94.1	0.913	96.0
0.04	1.198	99.8	1.052	100.7
0.03	1.496	104.5	1.288	106.9
0.02	1.508	123.6	1.581	117.6

TABLE XX - AUTOMATIC CONTROL SYSTEM PARAMETERS

	Fitted Values		Expected values
	67 Mwt	111 Mwt	
G, \$	2.3	2.75	3.96
τ_1 , sec	7	10	11
τ_2 , sec	0.4	0.6	0.356

FIG. 43 TRANSFER FUNCTION AMPLITUDE - AUTOMATIC CONTROL SYSTEM

FIG. 44 TRANSFER FUNCTION PHASE ANGLE - AUTOMATIC CONTROL SYSTEM

VI. CONCLUSIONS

Each of many aspects of the testing program was necessary to fulfill the purposes of the oscillator tests. The conclusions on each aspect are presented in the following.

The measured gross worths and the peak-to-peak amplitudes of the oscillator rods were in reasonable agreement with predicted values. The differences were explainable. The wave shapes of the rods were sinusoidal, as predicted.

The most satisfactory technique for measuring the zero-power transfer function, from the standpoint of frequency capability, completeness of information, and ease of making measurements was that which employed the synchronous transfer function analyzer. However, the zero-power transfer function measured by this technique had a consistent amplitude discrepancy with the predicted zero-power transfer function. The difference was about 7 per cent at low frequencies (0.005 to 0.01 cps). Analysis of the theory and the experiment revealed no single factor which could explain the discrepancy.

For all conditions of reactor transfer function measurement, the reactor was stable at all times; it would be expected to be stable at higher powers (beyond 111 Mwt) based on extrapolation of the results obtained. The feedback transfer function and the automatic control system feedback were in reasonable agreement with prediction.

Prediction of reactor stability from the results was relatively easy; however, it was more difficult to extract and interpret feedback. Only at high powers could feedback be extracted, since the at-power and zero-power transfer functions were too close at lower powers to permit extraction with satisfactory accuracy. Even at high powers the probable errors were sufficiently large to introduce a large uncertainty into the analysis. Since the automatic control system had a strong feedback transfer function and a simple model was anticipated for it, the analysis for this aspect could be made with more certainty and less difficulty than could the analysis of the normal reactor feedback.

APPENDIX A: THEORY OF THE SYNCHRONOUS TRANSFER FUNCTION ANALYZER

The transfer function of a linear dynamic system is defined as the ratio of the Laplace transform of the output or response of the system to the Laplace transform of an input or stimulus to the system. Although the input to the system can be an arbitrary function, it will be considered here to be sinusoidal with amplitude, A , i.e.

$$A \sin \omega t$$

and the system will be considered to be linear so that the response of the system to the above input will be sinusoidal with amplitude, B , and with a phase lag, θ , i.e.

$$B \sin (\omega t - \theta)$$

The Laplace transform of the input is

$$\frac{A \omega}{\omega^2 + s^2}$$

and the Laplace transform of the output is

$$\frac{B(\omega \cos \theta + s \sin \theta)}{\omega^2 + s^2}$$

Thus, by definition, the transfer function, G , of the system is

$$G(s) = \frac{B}{A} \left(\cos \theta - \frac{s}{\omega} \sin \theta \right) \quad (17)$$

Going from the s domain to the ω domain by the relation $s = i\omega$,

$$G(\omega) = \frac{B}{A} (\cos \theta - i \sin \theta) \quad (18)$$

The amplitude is B/A , which is the ratio of the amplitudes of the output and input sine waves. The phase angle is $-\theta$, which is the phase angle of the output sine wave with respect to the input sine wave.

The transfer function can be obtained analytically in another way which is more amenable to use with electronic devices. If the function e^{-st} is expressed in the frequency domain as

$$e^{-i \omega t}$$

and then as

$$\cos \omega t - i \sin \omega t,$$

the Laplace transforms can be taken by integration directly. The Laplace integrals of the output and input signals become:

Output:

$$\begin{aligned} & B \int_0^{\infty} (\sin \omega t \cos \theta - \cos \omega t \sin \theta) \cos \omega t \, dt \\ & - i B \int_0^{\infty} (\sin \omega t \cos \theta - \cos \omega t \sin \theta) \sin \omega t \, dt \end{aligned}$$

Input:

$$A \int_0^{\infty} \sin \omega t \cos \omega t \, dt - i A \int_0^{\infty} \sin^2 \omega t \, dt$$

The above expressions can be reduced by noting that since the ratio of the integrals will be taken, the integration can be performed over a complete cycle, or any number of complete cycles to obtain equivalent results, and that,

$$\int_0^{2\pi} \sin \omega t \cos \omega t \, dt = 0 \quad (19)$$

Thus the integrals become:

Output:

$$-B \sin \theta \int_0^{2\pi} \cos^2 \omega t \, dt - i B \cos \theta \int_0^{2\pi} \sin^2 \omega t \, dt$$

Input:

$$- i A \int_0^{\omega t = 2\pi} \sin^2 \omega t dt$$

Since the above two definite integrals have the value of π/ω , it can be seen that integration of the output and input functions given above over a full number of cycles produces the following results:

Output:

$$- \frac{B \pi N}{\omega} i (\cos \theta - i \sin \theta)$$

Input:

$$- \frac{A \pi N}{\omega}$$

The ratio of the integrals is then

$$\frac{B}{A} (\cos \theta - i \sin \theta)$$

which is the transfer function of the system.

The real part of the transfer function thus is proportional to the integral of the product of the output function and $\sin \omega t$, which is the input function, and the imaginary part of the transfer function is proportional to the integral of the product of the output function and $\cos \omega t$, which can be related to the input function.

This analysis describes the method used in the synchronous transfer function analyzer. Briefly, the output signal is multiplied by synchro-resolvers by $\sin \omega t$ and $\cos \omega t$ and the products are integrated electronically over a full number of cycles. The integrals, called the in-phase and quadrature components, are proportional to the real and imaginary components of the transfer function.

APPENDIX B: THEORY OF THE ZERO-POWER TRANSFER FUNCTION

The commonly used neutron kinetic equations for the point reactor model are

$$\frac{dn}{dt} = \frac{\delta k - \beta}{\ell} n + \sum_{i=1}^6 \lambda_i C_i \quad (20)$$

and

$$\frac{dC_i}{dt} = \frac{\beta_i}{\ell} n - \lambda_i C_i ; i = 1, 6 \quad (21)$$

where

n = neutron density

C_i = precursor concentration for the i^{th} group of delayed neutrons

λ_i = decay constant for the i^{th} group of delayed neutrons

k = reactivity

β = total delayed neutron fraction

β_i = delayed neutron fraction of the i^{th} group of delayed neutrons

ℓ = prompt neutron lifetime

Let n_o and C_{io} be the equilibrium neutron density and delayed neutron emmitters concentration. Define

$$n_1 + n_o = n \quad (22)$$

$$C_{i1} + C_{io} = C_i \quad (23)$$

where

n_1 = change in neutron density

C_{i1} = change in precursor concentration

At steady state as $C_i = C_{io}$ and Eq. 21 reduces

$$\frac{\beta_i n_o}{\ell} - \lambda_i C_{io} = 0 \quad (24)$$

or

$$\lambda_i C_{io} = \frac{\beta_i n_o}{\ell} \quad (25)$$

The time-dependent variables can now be transformed into the s-plane by Laplace Transform method, hence

$$s \bar{n}_1 = \frac{\overline{n_1 \delta k}}{\ell} + \frac{n_o \overline{\delta k}}{\ell} - \frac{\beta}{\ell} \left(\bar{n}_1 + \frac{n_o}{s} \right) + \sum_{i=1}^6 \lambda_i \left(\bar{C}_{il} + \frac{C_{io}}{s} \right) \quad (26)$$

and

$$s \bar{C}_{il} = \frac{\beta_i}{\ell} \left(\bar{n}_1 + \frac{n_o}{s} \right) - \lambda_i \left(\bar{C}_{il} + \frac{\bar{C}_{io}}{s} \right) \quad (27)$$

The product $\overline{n_1 \delta k}$ represents Laplace Transform of the respective time-varying quantities and not the product of the transforms of each quantity.

Equation (27) can further be simplified by substituting Eq. (25) in Eq. (27), hence

$$s \bar{C}_{il} = \frac{\beta_i}{\ell} \bar{n}_1 + \lambda_i \bar{C}_{il} \quad (28)$$

and therefore

$$\bar{C}_{il} = \frac{\beta_i \bar{n}_1}{\ell} \cdot \frac{1}{s + \lambda_i} \quad (29)$$

Collecting terms for $\bar{n}_1$ and n_o in Eq. (26) yields

$$s \bar{n}_1 = \frac{\overline{\delta k n_1} - \beta \bar{n}_1}{\ell} + \sum_{i=1}^6 \frac{\overline{C_{i1}}}{\ell} + \frac{\overline{\delta k}}{\ell} n_o - \frac{\beta/\ell}{s} n_o + \sum_{i=1}^6 n_o \frac{\beta_i}{\ell} \quad (30)$$

By definition

$$\beta = \sum_{i=1}^6 \beta_i \quad (31)$$

and therefore

$$\frac{\beta n_o}{\ell} = \sum_{i=1}^6 \beta_i \frac{n_o}{\ell} \quad (32)$$

Also $\overline{n_1 \delta k}$ is much less than $n_o \overline{\delta k}$ because of small perturbations assumed; therefore

$$s \bar{n}_1 = \frac{\overline{\delta k} n_o}{\ell} + \frac{\bar{n}_1}{\ell} \left[\sum_{i=1}^6 \lambda_i \overline{C_{i1}} \ell / \bar{n}_1 - \beta \right] \quad (33)$$

Substituting the value of $\overline{C_{i1}}$ from Eq. (29) in Eq. (33) gives,

$$\begin{aligned} s \bar{n}_1 &= \frac{\overline{\delta k} n_o}{\ell} + \frac{\bar{n}_1}{\ell} \left[\sum_{i=1}^6 \frac{\lambda_i \ell}{\bar{n}_1} \cdot \frac{\beta_i n_1}{\ell} \cdot \frac{1}{s + \lambda_i} - \sum_{i=1}^6 \beta_i \right] \\ &= \frac{\overline{\delta k} n_o}{\ell} + \frac{\bar{n}_1}{\ell} \left[\sum_{i=1}^6 \frac{\beta_i \lambda_i}{s + \lambda_i} - \sum_{i=1}^6 \beta_i \right] \\ &= \frac{\overline{\delta k} n_o}{\ell} - \frac{n_1}{\ell} \left[\sum_{i=1}^6 \frac{s \beta_i}{s + \lambda_i} \right] \end{aligned} \quad (34)$$

Rearranging Eq. (33) for n_1 and δk , the Eq. becomes

$$\bar{n}_1 = \left[s + \frac{1}{\ell} \sum_{i=1}^6 \frac{s \beta_i}{s + \lambda_i} \right] \cdot \frac{\bar{\delta k} n_o}{\ell}$$

Hence

$$\begin{aligned} \frac{\bar{n}_1/n_o}{\delta k} &= \frac{1}{s \ell} \left[\frac{1}{1 + 1/\ell \sum_{i=1}^6 \frac{\beta_i}{s + \lambda_i}} \right] \\ &= \frac{1}{s \ell + \sum_{i=1}^6 \frac{s \beta_i}{s + \lambda_i}} \end{aligned} \quad (35)$$

For a fast reactor, prompt neutron lifetime is very small. Hence the term

$s \ell$ is small as compared to $\sum_{i=1}^6 \frac{s \beta_i}{s + \lambda_i}$ for all experimental frequencies.

Hence the transfer function of a fast reactor is denoted by

$$\frac{\bar{n}_1/n_o}{\delta k} = \frac{1}{\sum_{i=1}^6 \frac{s \beta_i}{s + \lambda_i}} \quad (36)$$

The appropriate prompt and delayed neutron data for the Enrico Fermi reactor are given in Table XXI. For application in the frequency domain, the variable s is replaced by $i\omega$. The zero-power transfer function, calculated with Eq. (36) and the data in Table XXI, is given in Table XXII.

TABLE XXI - PROMPT AND DELAYED NEUTRON CHARACTERISTICS

<u>Group</u>	<u>Decay constant, sec⁻¹</u>	<u>Relative abundance</u>
1	0.0127	0.0317
2	0.032	0.1918
3	0.120	0.1858
4	0.323	0.3943
5	1.400	0.1571
6	3.940	0.0393

effective delayed neutron fraction = 0.00662

prompt neutron lifetime = 1.37×10^{-7} sec

TABLE XXII - ZERO-POWER TRANSFER FUNCTION

<u>Frequency, cps</u>	<u>Amplitude, \$⁻¹</u>	<u>Phase angle, deg</u>
5	1.000	-1.0
4	1.000	-1.2
3	1.001	-1.7
2	1.003	-2.5
1	1.018	-4.6
0.7	1.030	-6.2
0.5	1.048	-7.8
0.4	1.063	-9.1
0.3	1.092	-10.9
0.2	1.140	-13.8
0.1	1.270	-19.8
0.07	1.370	-23.8
0.05	1.500	-28.5
0.04	1.62	-30.8
0.03	1.82	-34.0
0.02	2.17	-38.2
0.01	2.92	-45.3
0.007	3.50	-49.8
0.005	4.18	-55.0

APPENDIX C: REACTOR TRANSFER FUNCTION DATA FOR HIGH-POWER LEVELS

This appendix contains data relative to the transfer functions measured at high-power levels. The reduced data for the transfer functions measured under manual control are given in Table XXIII. The data for the transfer functions measured under automatic control are given in Table XXIV. These data are also shown graphically in Figs. 46 through 71.

TABLE XXIII - REACTOR TRANSFER FUNCTIONS - MANUAL CONTROL

Frequency, cps	AMPLITUDE, $\$/^{-1}$								PHASE ANGLE, deg.							
	Zero	6.6	13.3	Power, Mwt		46	66.7	111	Zero	6.6	13.3	Power, Mwt		46	66.7	111
				20.1	30							20.1	30			
5	1.000	1.000	1.002	1.002	1.000	1.000	0.995	1.000	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0
4	1.000	1.000	0.999	0.999	*	*	0.997	0.997	-1.2	-1.1	-1.0	-0.8	*	*	-1.0	-1.3
3	1.000	1.000	0.999	0.998	*	*	1.002	0.997	-1.5	-1.6	-1.5	-1.5	*	*	-1.8	-1.3
2	1.000	1.003	1.001	1.004	*	*	1.005	1.003	-2.3	-2.0	-2.6	-2.4	*	*	-2.6	-1.6
1	1.002	1.022	1.016	1.013	*	*	1.013	1.006	-4.5	-4.2	-4.4	-4.1	*	*	-3.8	-2.4
0.7	1.033	1.041	1.033	1.031	*	*	1.024	1.006	-6.0	-5.7	-6.0	-5.4	*	*	-4.7	-3.3
0.5	1.060	1.060	1.054	1.055	1.046	1.080	1.032	1.013	-7.8	-6.7	-7.7	-7.5	-4.8	-4.2	-5.9	-4.9
0.4	1.079	1.079	1.073	1.070	1.061	1.095	1.041	1.022	-9.1	-8.8	-8.7	-8.7	-5.8	-5.5	-6.6	-6.2
0.3	1.113	1.114	1.099	1.117	1.085	1.122	1.061	1.044	-11.0	-10.6	-10.4	-10.5	-7.5	-7.0	-8.1	-7.8
0.2	1.167	1.161	1.151	1.157	1.134	1.200	1.094	1.079	-13.7	-13.8	-13.0	-13.2	-10.1	-14.7	-10.4	-10.8
0.1	1.315	1.304	1.299	1.301	1.215	1.330	1.192	1.180	-20.1	-20.3	-19.1	-19.4	-15.4	-17.4	-15.7	-16.0
0.07	1.422	1.418	1.399	1.397	1.348	1.480	1.266	1.265	-24.3	-23.7	-22.7	-20.9	-20.4	-16.6	-18.8	-19.3
0.05	1.580	1.557	1.553	1.543	1.488	1.630	1.358	1.379	-28.1	-27.3	-26.6	-24.6	-22.7	-19.4	-21.6	-22.2
0.04	1.711	1.725	1.676	1.658	1.598	1.792	1.471	1.483	-30.5	-29.6	-28.6	-29.1	-26.0	-27.3	-23.5	-24.1
0.03	1.927	1.924	1.863	1.875	1.777	2.000	1.630	1.640	-33.8	-32.8	-31.3	-31.0	-28.3	-28.5	-25.4	-26.5
0.02	2.260	2.209	2.111	2.116	*	*	1.826	1.861	-37.0	-35.8	-34.5	-33.9	*	*	-27.2	-27.9
0.01	3.207	3.060	2.998	2.976	*	*	2.375	2.33	-43.5	-41.5	-38.6	-39.8	*	*	-27.8	-29.5
0.007	3.920	*	3.593	3.518	*	*	2.632	2.73	-46.7	*	-43.1	-41.8	*	*	-29.1	-33.1
0.005	4.66	*	4.386	4.237	*	*	3.002	3.17	-53.5	*	-46.9	-44.6	*	*	-30.1	-34.8

*Data not available

TABLE XXIV - REACTOR TRANSFER FUNCTIONS - AUTOMATIC CONTROL

Frequency, cps	AMPLITUDE, $\text{\$}^{-1}$							PHASE ANGLE, deg.						
	Power, Mwt							Power, Mwt						
	Zero	12.7	19.4	30	46	67	111	Zero	12.7	19.4	30	46	67	111
5	1.000	*	*	1.000	1.000	1.000	1.000	-1.0	*	*	-1.0	-1.0	-1.0	-1.0
1	1.022	1.006	1.011	*	*	1.012	1.007	-4.5	-4.4	-4.1	*	*	-3.6	-2.3
0.7	1.033	1.011	1.028	*	*	1.027	-	-6.0	-6.8	-5.4	*	*	-4.9	-
0.5	1.060	1.039	1.050	1.067	1.097	1.056	1.040	-7.8	-7.8	-6.2	-6.4	-7.5	-6.6	-5.7
0.4	1.079	1.065	1.073	1.093	1.138	1.082	1.072	-9.1	-12.1	-7.3	-7.7	-9.0	-9.7	-6.9
0.3	1.113	1.100	1.128	1.170	1.228	1.158	1.151	-11.0	-12.1	-8.7	-9.2	-8.0	-8.6	-7.6
0.2	1.167	1.160	1.275	1.372	1.392	1.333	1.307	-13.7	-14.2	-9.6	-7.9	-4.2	-5.6	-2.3
0.1	1.315	1.453	1.656	1.651	1.412	1.318	1.298	-20.1	-15.1	+3.2	+24.8	+18.4	+24.2	+19.1
0.07	1.422	1.634	1.727	1.378	1.252	1.138	1.181	-24.3	-8.7	+25.0	+41.2	+31.8	+38.5	+30.6
0.05	1.580	1.703	1.517	1.112	1.062	0.940	1.004	-28.1	+2.0	+39.0	+53.7	+41.0	+44.6	+39.5
0.04	1.711	1.657	1.250	0.952	0.875	0.813	0.900	-30.5	+5.3	+45.1	+57.6	+46.2	+47.7	+43.5
0.03	1.927	1.559	1.070	0.740	0.672	0.664	0.752	-33.8	+7.9	+47.7	+60.2	+46.2	+51.9	+46.2
0.02	2.260	1.531	0.908	*	*	0.514	0.599	-37.0	+7.8	+44.9	*	*	+40.2	+43.6
0.01	3.207	1.388	0.802	*	*	0.557	-	-43.5	-13.2	+15.5	*	*	+6.4	*
0.007	3.920	1.724	0.876	*	*	0.393	-	-46.7	-15.6	+6.3	*	*	+10.4	*
0.005	4.66	2.573	0.838	*	*	0.672	-	-53.5	-23.7	+5.6	*	*	+2.4	*

*Data not available

FIG. 45 TOTAL POWER-TO-FISSION POWER TRANSFER FUNCTION

FIG. 46 REACTOR TRANSFER FUNCTION AMPLITUDE - MANUAL CONTROL - 6.6 Mw

FIG. 47 REACTOR TRANSFER FUNCTION PHASE ANGLE - MANUAL CONTROL - 6.6 Mw

FIG. 48 REACTOR TRANSFER FUNCTION AMPLITUDE - MANUAL CONTROL - 13.3 Mwt

FIG. 49 REACTOR TRANSFER FUNCTION PHASE ANGLE - MANUAL CONTROL - 13.3 Mwt

FIG. 50 REACTOR TRANSFER FUNCTION AMPLITUDE - MANUAL CONTROL - 20.1 Mw

FIG. 51 REACTOR TRANSFER FUNCTION PHASE ANGLE - MANUAL CONTROL - 20.1 Mw

FIG. 52 REACTOR TRANSFER FUNCTION AMPLITUDE - MANUAL CONTROL - 30 Mwt

FIG. 53 REACTOR TRANSFER FUNCTION PHASE ANGLE - MANUAL CONTROL - 30 Mwt

FIG. 54 REACTOR TRANSFER FUNCTION AMPLITUDE - MANUAL CONTROL - 46 Mw

FIG. 55 REACTOR TRANSFER FUNCTION PHASE ANGLE - MANUAL CONTROL - 46 Mw

FIG. 56 REACTOR TRANSFER FUNCTION AMPLITUDE - MANUAL CONTROL - 66.7 Mw

FIG. 57 REACTOR TRANSFER FUNCTION PHASE ANGLE - MANUAL CONTROL - 66.7 Mw

FIG. 58 REACTOR TRANSFER FUNCTION AMPLITUDE - MANUAL CONTROL - 111 Mwt

FIG. 59 REACTOR TRANSFER FUNCTION PHASE ANGLE - MANUAL CONTROL - 111 Mwt

FIG. 60 REACTOR TRANSFER FUNCTION AMPLITUDE - AUTOMATIC CONTROL - 12.7 Mw

FIG. 61 REACTOR TRANSFER FUNCTION PHASE ANGLE - AUTOMATIC CONTROL - 12.7 Mw

FIG. 62 REACTOR TRANSFER FUNCTION AMPLITUDE - AUTOMATIC CONTROL - 19.4 Mw

FIG. 63 REACTOR TRANSFER FUNCTION PHASE ANGLE - AUTOMATIC CONTROL - 19.4 Mw

FIG. 64 REACTOR TRANSFER FUNCTION AMPLITUDE - AUTOMATIC CONTROL - 30 Mwt

FIG. 65 REACTOR TRANSFER FUNCTION PHASE ANGLE - AUTOMATIC CONTROL - 30 Mwt

FIG. 66 REACTOR TRANSFER FUNCTION AMPLITUDE - AUTOMATIC CONTROL - 46 Mw

FIG. 67 REACTOR TRANSFER FUNCTION PHASE ANGLE - AUTOMATIC CONTROL - 46 Mw

FIG. 68 REACTOR TRANSFER FUNCTION AMPLITUDE - AUTOMATIC CONTROL - 67 Mwt

FIG. 69 REACTOR TRANSFER FUNCTION PHASE ANGLE - AUTOMATIC CONTROL - 67 Mwt

FIG. 70 REACTOR TRANSFER FUNCTION AMPLITUDE - AUTOMATIC CONTROL - 111 Mw

FIG. 71 REACTOR TRANSFER FUNCTION PHASE ANGLE - AUTOMATIC CONTROL - 111 Mw

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APPENDIX D: FISSION PRODUCT DECAY HEAT

Since the decay heat of a reactor occurs as the fission products decay, the total heat, which is the sum of the fission heat and the decay heat, is related to the fission heat by a transfer function which is defined as $T(s)$. Thus,

$$T(s) = \frac{P_t(s)}{P_f(s)} = \frac{P_f(s) + P_d(s)}{P_f(s)} = \frac{P_d(s)}{P_f(s)} + 1 = 1 + D(s) \quad (37)$$

where,

$P_f(s)$ = fission heat Laplace transform

$P_D(s)$ = decay heat Laplace transform

$P_t(s)$ = total heat Laplace transform

$D(s)$ = decay heat to fission heat transfer function

Evaluation of $D(s)$ has been made¹⁰ by noting that the decay power following a burst can be expressed as

$$P_d(t) = 0.0307e^{-1.085t} \text{sec}^{-1}; 0 \leq t \leq 1.2 \text{ sec} \quad (38)$$

$$P_d(t) = 0.01039 t^{-1.2} \text{sec}^{-1}; t \geq 1.2 \text{ sec} \quad (39)$$

By integration of these expressions over the complete time domain after a burst it can be shown that the total decay heat is 7.05 per cent of the fission heat or 6.59 per cent of the total heat. The sum of the Laplace transforms of the above expressions gives the transfer function $D(s)$, i.e.

$$D(s) = \int_0^{\infty} P_d(t) e^{-st} dt \quad (40)$$

Numerically, the transfer function is:

$$\begin{aligned}
 D(s) = & \frac{0.0307}{1.085 + s} \left[1 - e^{-1.2(1.085 + s)} \right] \\
 & + 0.01039 \left\{ -5.821s^{0.2} \right. \\
 & + 5(1.2)^{-0.2} e^{-1.25} \left[1 + \frac{1.2}{0.8} s + \frac{(1.2)^2}{(0.8)(1.8)} s^2 + \dots \right. \\
 & \left. \left. \dots + \frac{(1.2)^{n+1}}{(0.8)(1.8) \dots (0.8 + n)} s^{n+1} + \dots \right] \right\}
 \end{aligned} \tag{41}$$

This has been evaluated and the transfer function is given in Fig. 45.

In making measurements, the neutron signal is measured and it is related to the fission heat. However, the total heat is the driving mechanism for the feedback mechanisms. Therefore the reactor transfer function must be written

$$G_p(s) = \frac{G_o}{1 - P_f T(s) X(s) G_o(s)} \tag{42}$$

where,

G_o = zero-power transfer function

G_p = at-power transfer function

P_f = nominal fission power

This demonstrates the application of the decay heat transfer function to oscillator tests and indicates where it must be taken into account to extract the feedback transfer function from reactor transfer function data.

APPENDIX E: STABILITY CRITERIA

A power reactor is a complicated system and it often is necessary to use nonlinear differential equations to describe the neutron kinetics and the feedback mechanisms in such a reactor. Such descriptions are difficult to handle analytically and for interpretation one must often resort to the describing function technique. However, if one uses small signal theory, the nonlinear differential equations reduce to linear differential equations and the reactor behavior can be described by transfer functions.

It is necessary to use appropriate stability criteria to analyze a system represented by transfer functions for stability. A number of criteria have been developed by various analysts; two of these were found especially useful in the analysis of the Enrico Fermi reactor. They are developed below. The first is suitable for determining if a reactor is unconditionally or conditionally stable and the second can be used to determine the degree of instability for a potentially unstable reactor.

The at-power transfer function for a reactor with feedback is usually expressed as

$$G_p(\omega) = \frac{G_o(\omega)}{1 - P G_o(\omega) X(\omega)} \quad (43)$$

where

$G_o(\omega)$ = zero-power transfer function

$X(\omega)$ = feedback transfer function at frequency ω

P = reactor power

If it is assumed that the feedback transfer function is independent of power, an open-ended resonance will occur at frequency ω_r and power P_r when

$$1 - P_r G_o(\omega_r) X(\omega_r) = 0 \quad (44)$$

Implicit in this statement are the conditions that at a resonance the phase angle of the quantity $G_o(\omega_r) X(\omega_r)$ should be a multiple of π , where the multiplier may be zero, or that phase angles of $G_p(\omega_r)$ and $G_o(\omega_r)$ should be the same. From the above statement, at a resonance

$$P_r = \frac{1}{G_o(\omega_r) X(\omega_r)} \quad (45)$$

or

$$G_o(\omega_r) X(\omega_r) = \frac{1}{P_r} \quad (46)$$

At some lower power, P_1 , at the resonant frequency

$$G_p(\omega_r, P_1) = \frac{G_o(\omega_r)}{1 - P_1 G_o(\omega_r) X(\omega_r)} \quad (47)$$

which can be written as

$$G_p(\omega_r, P_1) = \frac{G_o(\omega_r)}{1 - \frac{P_1}{P_r}} \quad (48)$$

or

$$\frac{G_p(\omega_r, P_1)}{G_o(\omega_r)} = \frac{1}{1 - \frac{P_1}{P_r}} \quad (49)$$

For this statement to be true, G_p and G_o must have the same phase angle at the resonant frequency, ω_r . Thus, it is possible then to write Eq. (49) as

$$\frac{|G_p(\omega_r, P_1)|}{|G_o(\omega_r)|} = \frac{1}{1 - \frac{P_1}{P_r}} \quad (50)$$

If the ratio of the transfer function amplitudes is defined as

$$\gamma(\omega, P_1) = \frac{|G_p(\omega, P_1)|}{|G_o(\omega)|} \quad (51)$$

Eq. (50) can be written as

$$\gamma(\omega, P_1) = \frac{1}{1 - \frac{P_1}{P_r}} \quad (52)$$

or

$$\frac{P_r}{P_1} = \frac{\gamma(\omega, P_1)}{\gamma(\omega, P_1) - 1} \quad (53)$$

From an inspection of the frequency-dependent values of $\gamma(\omega, P_1)$ at a power level, it is possible to predict the presence of a resonance, within the bounds imposed by the initial assumptions. If $\gamma(\omega, P_1)$ does not exceed unity anywhere in the frequency spectrum, no resonance exists and the reactor is unconditionally stable. If $\gamma(\omega, P_1)$ does exceed unity, a resonance may exist and the power at which it will occur can be determined from the above equation. It is recommended that this criterion be applied to transfer function data. It is simple to apply and provides strong evidence for a stable reactor. If the presence of a resonance is detected, methods other than a simple amplitude analysis must be used.

For a resonance to exist, the phase-angle condition specified above must also exist. Often the difference between the phase angle of at-power transfer function and that for the zero-power transfer function may be small. Consequently, when experimental error is taken into account, the presence of a true resonance may depend upon the reliability of the phase angle measurements. The above criterion places a condition on the phase angle for instability but does not provide a measure, based on the phase angle, for the approach to an instability. For a detailed analysis of a potential instability, it is recommended that the Nyquist Criterion be used. Basically, this criterion involves an analysis of the complex quantity, $G_o H$, a frequency-dependent quantity, where H is the product of the power level, P , and the feedback transfer function, X . The reactor is deemed to be unstable if this quantity encircles the point (1.0, 0i) when it is plotted on an Argand plot. If it does not encircle this point, a measure of the reactor stability at power, P , is the phase margin, where the phase margin at power P is defined as the phase angle of the quantity $G_o H$ when the amplitude is unity. If the phase margin increases as the power level is increased, the stability of the reactor is increasing.

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