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AND DEVELOPMENT PROGRAM

A PROGRAM OF TWO-PHASE FLOW INVESTIGATION

EIGHTH QUARTERLY REPORT JANUARY - MARCH, 1965

G. BIJWAARD
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GENERAL ELECTRIC

SAN JOSE, CALIFORNIA

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A PROGRAM OF TWO-PHASE FLOW INVESTIGATION
EIGHTH QUARTERLY REPORT JANUARY - MARCH, 1965

by

G. Bijwaard
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NOMENCLATURE
MLT Θ System of Units
With H defined by $H = ML^2/T^2$

- a = Dimensionless amplitude in Eq. 4
- A = Area [L^2]
- C_k = Velocity of kinematic wave [L/T]
- C_o = Distribution parameter defined by Eq. 9 [-]
- D = Diameter of the duct [L]
- g = Acceleration due to gravity [L/T^2]
- h_{fg} = Latent heat of vaporization [H/M]
- $\langle j \rangle$ = Volumetric flux density of the mixture [L^3/TL^2]
- $\langle j_g \rangle$ = Volumetric flux density of the vapor [L^3/TL^2]
- L_b = Boiling length [L]
- q = Heat flux density [H/TL^2]
- t = Time [T]
- t^* = Dimensionless time (Defined by Eq. 14)
- v_{fi} = Inlet liquid velocity [L/T]
- v'_{fi}^* = Dimensionless velocity perturbation defined by Eq. 41.
- $\bar{V}_{gj}$ = Drift velocity of the vapor [L/T]
- z = Axial distance along duct
- z^* = Dimensionless distance defined by Eq. 13
- α = Vapor volumetric concentration [-]
- Γ = Rate of mass formation per unit volume [M/TL^3]
- Γ^* = Dimensionless rate of mass formation defined by Eq. 15

- ϵ = Dimensionless power input amplitude (See Eq. 12)
- ϵ_f = Dimensionless amplitude of inlet flow oscillation
- ρ = Mass density $[M/L^3]$
- $\Delta\rho$ = $(\rho_f - \rho_g)$ $[M/L^3]$
- σ = Surface tension $[ML/T^2L]$
- τ = Time constant defined by Eq. 37 $[T]$
- τ^* = Dimensionless time constant defined by Eq. 34
- ω = Angular frequency of power input $[T^{-1}]$
- ω^* = Dimensionless angular frequency defined by Eq. 16
- Ω = Angular frequency of inlet flow oscillation $[T^{-1}]$
- Ω^* = Dimensionless angular frequency defined by Eq. 40.

Subscripts:

- DIA = Evaluated along a tube diameter
- f = liquid
- g = vapor

INTRODUCTION

The United States and the European Atomic Energy Community (Euratom), on May 29, and June 18, 1958, signed an agreement which provides a basis for cooperation in programs for the advancement of the peaceful applications of atomic energy. This agreement, in part, provides for the establishment of a Joint U. S. -Euratom research and development program which is aimed at reactors to be constructed in Europe under the Joint Program.

The work described in this report represents the Joint U. S. -Euratom effort which is in keeping with the spirit of cooperation in contributing to the common good by the sharing of scientific and technical information and minimizing the duplication of effort by the limited pool of technical talent available in western Europe and the United States.

SUMMARY

First Contract Year

During the first contract year of this effort, March 1963 to March 1964, the experimental equipment was designed, constructed and installed. In addition, the shakedown tests and preliminary experiments were completed to check out the various instrumentation systems and operating procedures. This work is summarized in [2] and given in greater detail in [1] and [4]. To avoid redundancy, only the most important characteristics of the metal and glass test sections are repeated here.

The results of the analytical investigations which have been carried out during the first contract year are summarized in the next section which describes the completed objectives.

Eighth Quarterly Period

During the eighth quarter of this program the oscillatory heat input tests have been completed at a reduced pressure of 0.12 and additional tests were carried out at a reduced pressure of 0.22 with very low inlet subcooling. The latter was required by the need for first comparing the results of the analysis of the void propagation equation with the data for low inlet subcooling. The complete test results of the dynamic void fraction measurements taken to-date are summarized in this report in addition to sampled data on the dynamic pressure drop.

The void propagation equation given in [3] was applied to the range of data taken in this investigation, and the computer solutions of the developed equations are compared with the data taken at oscillatory heat input and low inlet subcooling. Good agreement is shown between predicted and measured values of: 1) the average void fraction, 2) the amplitude of its periodic variation and 3) the phase angle, i. e., velocity of propagation, of the void oscillation. The loss of accuracy in predicting these values, which is introduced by not considering the effect of flow regime, was also demonstrated.

A computer solution and an approximate but simple, closed form analytical solution of the void propagation equation are presented for the case when the oscillatory power input to the liquid is of the form $(1 + \epsilon \sin \omega t)$. The solutions predict both the rate of propagation and the wave form of the void disturbance (as it propagates along the heated duct) caused by power oscillations.

An analytical solution of the void propagation equation was derived for the case when the inlet flow oscillates as $(1 + \epsilon_f \sin \Omega t)$. The solution predicts both the rate of propagation and the wave form of the void disturbance (as it propagates along the heated duct) caused by inlet flow oscillations.

A high speed motion picture has been prepared which shows the process of nucleation and the resulting vapor bubble behavior in the liquid film adjacent to the heated wall during the annular flow regime.

Second Contract Year

The experimental objectives for the second contract year, March 1964 to March 1965, which are listed in the next section were completed. The number of tests which have been carried out slightly exceeded those which have been initially planned both in the number of tests as well as in the range of parameters which have been covered by these experiments.

The analytical objectives listed in the following section have been completed.

- 1) An equation which can be used either for predicting the void fraction or for analyzing and interpreting experimental data was derived [6]. The analysis takes into account both the effect of non-uniform flow and concentration profiles as well as the effect of the local relative velocity.
- 2) The void propagation equation has been generalized in [3] to predict the void response to variations of 1) power density, 2) system pressure, 3) pressure drop, 4) energy storage in the liquid and in the vapor, 5) compressibilities of the liquid and of the vapor, 6) friction factor, 7) flow regime and 8) gravitational force field.

This void propagation equation, which is formulated in terms of the kinematic waves, predicts both the rate of propagation as well as the wave form of void disturbances which are caused by the various perturbations.

- 3) A computer and an approximate but simple, closed form analytical solution of the void propagation equation was derived [10] for the case of oscillating power input to the liquid and low inlet subcooling.

The values predicted by the analysis have been compared to the experimental data obtained in this research program. The predicted 1) average void fraction, 2) rate of propagation and 3) the wave form of the void variations were found to be in good agreement with the experimental data.

- 4) An analytical solution of the void propagation equation was derived for the case of oscillating inlet flow.
- 5) In analyzing the dynamic characteristics of the two phase mixture (item four in the analytical objectives for the second contract year) it was found necessary to predict the void fraction in subcooled boiling. Although an analysis of the behavior of a two-phase mixture in subcooled boiling was not one of the objectives of the present program, such an analysis had to be performed to render completeness to the investigation of the stability of the two-phase mixture. The results are reported in [9] which presents an equation for predicting the void fraction in subcooled boiling.

The analysis of the dynamic characteristics and of the stability of the two-phase mixture will continue as part of the final contract period.

STATEMENT OF PROBLEM

The design, cost, efficiency and reliability of high power systems which are characterized by the presence of two phases, depend to a large extent upon the understanding of the two-phase flow and boiling processes. Experiments and practice have shown that in two-phase flow systems steady-state oscillations of flow, of vapor content, of heat transfer coefficient and of local pressure can occur even when the independent parameters are kept constant. Under some conditions, not yet understood, the amplitudes of these oscillations may increase with time and lead to unstable operation. An understanding of the conditions leading to thermal-hydraulic oscillations and instabilities and the ability to predict them are therefore essential for an optimum design and reliable operation of engineering systems characterized by high power densities and change of phase.

The overall purpose of the proposed experimental and analytical research program is two-fold. First, the purpose is to gain an understanding of the various phenomena which lead to thermal-hydraulic oscillations in forced convection two-phase flow systems with heat addition and second, the purpose is to obtain equations and criteria which can be used for predicting the inceptions of these oscillations and instabilities as function of input parameters and system characteristics.

The experimental investigation is being carried out with Refrigerant-22* as the working fluid. This fluid was selected to meet the requirement of obtaining simultaneously recorded data on the heat transfer coefficient, pressure drop, vapor content and the visual identification of the two-phase flow regimes in a vertical forced convection boiling system over a reduced pressure range from 0.1 to 0.7. The use of Refrigerant-22 thus facilitates

* CHClF_2

and reduces the cost of obtaining such data over a wide pressure range. It also provides data which can be used to test the effect of thermodynamic and transport properties of the working fluid on the inception of thermal-hydraulic oscillations and instabilities. The experimental investigation will therefore complement other two-phase flow and heat transfer studies which are being carried out with water as the working medium under the sponsorship of the USAEC Division of Reactor Development.

The purpose of the analytical investigations is to determine the mechanisms which lead to thermal-hydraulic oscillations, stable or divergent, and to derive equations and criteria which may be used to predict the inceptions of these oscillations. The derived equations are to be of general form so that they may be used for predicting and for interpreting experimental results obtained with various working fluids.

OBJECTIVES COMPLETED IN FIRST CONTRACT YEAR

Experimental

1. Completed modification of the experimental loop installed at the Advanced Technology Laboratories of the General Electric Company to permit operation over the defined ranges of pressure, flow, inlet enthalpy, and either steady-state or oscillating, heat input.
2. Completed electrically heated metal test section instrumented for the measurement of local wall temperature, local static pressure, local void fraction, inlet and discharge fluid conditions. Provide and install appropriate instrumentation conversion and recording equipment.
3. Completed shakedown testing of the loop and test section(s).
4. Evaluated feasibility of employing a direct electrically heated (coated) glass test section identical to, and to be used in addition to, the metal

test section. Constructed and operated this test section after approval.

Analytical

1. All available formulations and analyses concerned with the transient behavior of two-phase flow systems have been examined and found to be basically incomplete if not incorrect.

In all analyses but one [10] the set of equations which describes the system is incomplete because the void diffusion equation, or better void propagation equation, is not taken into account.

All analyses are incorrectly formulated because none of them is formulated in terms of the center of mass of the mixture. It is well known from mechanics that the dynamic behavior of a multi-particle or component system gives meaningful solution only by considering the dynamic response of the center of mass of the system. This was not done in two-phase flow system.

2. In view of the conclusions above the set of equations which describe the two-phase flow system was derived in terms of the center of mass. Furthermore, the void propagation equation was formulated in terms of kinematic waves; it gives the void response to variations of pressure, of volumetric flux density of the mixture, of power density and of flow regime.
3. Since the ability to predict the void response depends upon the ability to predict the steady-state void fraction $\langle \alpha \rangle$, a general expression for either predicting $\langle \alpha \rangle$ or for analyzing and interpreting experimental data was derived. The analysis takes into account both the effect of non-uniform flow and concentration profiles and the effect of the local relative velocity. Good agreement with experimental data for adiabatic systems was shown. [11]
4. Since it was predicted analytically (from the void propagation equation) and confirmed experimentally [12] that the transient void response

depends on the flow regime an analysis of the thermal effects on the stability of a liquid film was performed.

Two problems dealing with the destabilizing effects of heat addition on the two-phase flow and flow patterns were analyzed. First, the destabilizing effect of the vapor thrust on a liquid film flowing over a heated surface was considered and relations which predict the destruction of such a film under these conditions were derived. The second problem considered deals with the stability of dry patches in a liquid film flowing over a heated surface. Criteria which predict the stability of such patches were derived. [2]

SECOND CONTRACT YEAR OBJECTIVES

Experimental

1. Carry out experiments at reduced pressures less than 0.25 and at steady-state heat input in the metal and glass test sections at two or more different mass flow rates ranging between about 50 lbs/hr and 400 lbs/hr. Steady-state power input data at each mass flow rate(s) will then be gathered at several heat input levels and degrees of subcooling below the instability thresholds. A total of 40-50 tests in both test sections are contemplated, with the photographic and visual identification of the flow mechanism to be carried out with the glass test section.
2. Develop minimum cost methods of steady-state data reduction including the use of digital computers to evaluate the experimental mass flow rate, heat flux, local heat transfer coefficient, local pressure drop and local void fraction.
3. Oscillatory power input tests in the metal test section will be carried out at several frequencies between .01 and 10 cps and at least two different amplitudes up to about $\pm 30\%$ of the average input. A minimum of four different operating conditions, i. e., combination of flow rate,

heat addition subcooling and/or reduced pressure, will be selected from the results of the steady-state experiments. The exact conditions and number of runs selected will be dictated by the previous results obtained.

4. Develop minimum cost methods of oscillatory data reduction including the use of digital computers and, possibly, tape recording equipment to evaluate the experimental parameters determined under item 3 above.
5. Replace the present facility condenser for high pressure operation by the type of unit found most stable during the above experimental program.

Analytical

1. If the heretofore proposed correlation for predicting the two-phase pressure drop, void distribution and the heat transfer coefficient to a two-phase mixture prove to be inadequate for predicting the steady-state data obtained from the experimental program, an attempt will be made to obtain improved correlations.
2. The transient void response predicted, by the void propagation equation, will be compared to the experimental data for various power oscillations.
3. The transient void response to flow and pressure oscillations, predicted by the void propagation equation, will be compared to experimental data available in the literature.
4. The void propagation equation will be used, together with the conservation equations, to analyze the stability of the two-phase mixture.

EXPERIMENTAL INVESTIGATION

1. Test Section Characteristics

The metal test section consists of a vertical circular stainless steel tube of 0.40" inside diameter, 0.020" wall thickness and 60.6" in heated length. The tube is heated directly by a power supply that can furnish constant or oscillating heat input. There are six pressure taps, six wall temperature measurement locations and six void fraction measurement stations along the length of this test section.

The glass test section consists of four vertical circular tubes aligned in series and heated by an internal electrically conducting and optically transparent coating. The internal diameter and total heated length of these sections is the same as those pertinent to the metal test section. The power supplies allow separate control of the heating current to each section. This test section is only employed for visual and photographic observations and both test sections are described in detail in [1] and [2].

2. Photography in Glass Test Section

The editing and the titling of the high speed (5000-7000 fps) motion pictures of the nucleation and presence of bubbles in the liquid film pertinent to the annular flow mechanism, referred to in [3], have been completed. The motion pictures show the generation of numerous small bubbles in the liquid film, the effect of different heat fluxes on the bubble density, the effect of different ways of lighting the test section, and the agglomeration and disappearance of bubbles into the vapor core.

3. Oscillating Heat Input in Metal Test Section

The power supply described in [2] yields a $P = P_{av} (1 + a \sin \omega t)^2$ power curve rather than $P = P_{av} [1 + (\sin \omega t) (\sin \omega t)]$ as was mentioned in [3]. Frequencies between 0.01 and 10 cps and amplitudes between 5% and 50%

of the average power level can be employed within the regulation tolerances given in [2]. The value of "a" for 25% and 40% of the average power amplitudes are 0.118 and 0.18 respectively. Only the data for the tests carried out at a reduced pressure of 0.12 will be discussed in this report. Table I however, gives a summary of all the tests which have been completed to-date. The results of the tests carried out at a reduced pressure of 0.22 are discussed in [3].

TABLE IA
Summary of Test Conditions Reduced To Date ($P_r = 0.22$)

Run No.	Inlet Sub. Cooling °F	G lbs/hr. ft ²	Exit Quality %	Average Heat Flux Btu/hr. ft ²	Amplitude of Power Oscil- lation %	Freq. Range cps
Preliminary Mapping Tests.						
SO62III3	12	172,000	10	3,280	25	.25-1.0
SO62III3B	12	172,000	10	3,280	25	.25-1.0
SO63III4	12	97,500	13	2,460	25 & 40	.1-1.0
SO64III5	18	266,000	35	21,200	25	.05-.75
SO67III8	22.4	362,000	30	17,800	25	.05-.75
SO68III9	21.2	230,000	30	11,200	25	.05-5.0
SO69III10	20.7	100,000	28	4,600	40	.05-6.0
SO70III11	21.3	347,400	12	9,000	25	.05-.75
SO73III12	22.2	95,500	12	3,460	40	.05-7.5
SO75III13	6.1	230,000	11	4,000	40	.05-5.0
SO76III14	6.1	495,000	28	19,000	25	.1-5.0
SO77III15	21.9	364,000	32	18,700	12.5	.05-7.5
SO91III27	2.7	113,200	11	1,770	40	.1-10.
SO92III28	1.1	337,900	11	4,820	40	.05-10.
SO93III29	2.2	111,400	30	4,430	25	.05-10.
SO94III30	1.5	341,000	30	13,300	25	.05-10.
SO95III31	1.2	222,900	11	3,300	40	.05-10.
SO96III32	1.6	223,000	29	8,440	25	.05-10.
SO97III33	0.4	511,300	11	6,920	40	.05-10.
SO98III34	1.5	506,200	31	20,200	25	.05-10.

TABLE IB
Summary of Test Conditions Reduced To Date ($P_r = .12$)

Run No.	Inlet Sub- Cooling °F	G lbs/hr. ft ²	Exit Quality %	Average Heat Flux Btu/hr. ft ²	Amplitude Power Oscil- lation %	Freq. Range cps
SO79III16	11.0	116,900	32	5,900	40	.05-5.
SO80III17	12.3	229,400	32	11,500	25	.1-5.
SO82III18	12.0	348,700	29	16,200	25	.2-5.
SO83III19	12.6	121,900	10	2,500	40	.05-2.
SO84III20	12.1	349,800	10	6,700	25	.1-5.
SO85III21	5.2	117,200	30	5,200	40	.1-5.
SO86III22	4.7	344,700	11	5,800	40	.05-5.
SO87III23	6.7	350,500	28	15,000	25	.1-5.
SO88III24	6.4	119,800	11	2,200	40	.05-10.
SO89III25	2.9	712,400	11	11,500	25	.05-10.
SO90III26	0.8	527,100	10	7,170	25	.05-10.

3.1 Experimental Method

During all the oscillatory tests the two-phase mixture was passed into the separator above the liquid level in the separator. From there the vapor traveled to the shell-tube condenser and the liquid, through the subcooler and control valve, to the pump inlet. The power was oscillated at a fixed amplitude in such a run and the frequency was varied after a sufficient time interval to insure quasi-steady-state conditions. All wall temperatures, the flow rate, power input, local pressure differences and the local void fraction were continuously recorded (see [2]). The local heat transfer coefficient and quality were then calculated employing the average power level. The oscillation in the heat transfer coefficient can later be calculated from the recorded data, but in view of the lesser importance of this effect, compared to the local void fraction and pressure oscillations, this evaluation has not yet been carried out.

The frequency response of the pressure transducers [2] does not cause an attenuation or phase shift of the local pressure measurement. The amplitude and phase shift of the pressure oscillations were thus taken directly from the chart readings. The frequency response of the metal test section wall, however, is such that the actual energy input oscillation into the fluid is significantly changed relative to the measured power oscillation above roughly 0.2 cps (see [4]). The response of the outside tube wall temperature, and the local effect of the presence of the temperature sensor itself, were calculated (See Appendix A of [3]) and compared to the measured data as shown in Figure 1. The attenuation and phase shift shown in Figure 1 are relative to the power input amplitude and phase angle respectively. The amplitude of the measured wall temperature oscillation was compared to its amplitude in the low frequency range, where this amplitude did not change with frequency. Note that the data compares with the calculated wall temperature within experimental error. The data points at a heat transfer coefficient of 150 Btu/hr ft² °F were obtained with single phase operation. Figure 1 thus gives sufficient confidence in the calculated inside wall temperature (not shown in

Figure 1. Comparison of Calculated and Measured Outside Tube Wall Temperature Attenuation and Phase Angles in the Metal Test Section.

Figure 1 to avoid confusion) attenuation and phase shift and, therefore, similar confidence in the calculation of the actual oscillation of energy addition to the fluid from the power oscillation at the higher frequencies. This calculation is used to serve as a reference in the void oscillation data and can later be employed to indirectly evaluate the possible heat storage limits in the liquid phase.

As pointed out in [3] the average void fraction was calculated by multiplying the measured chordal diameter voids by the ratio of the chordal to the average void fraction. This ratio was initially determined by the experimental traverses taken at both constant and oscillating heat input and plotted as a function of quality. Figure 2 is a plot of this ratio from the high pressure data and Figure 3 was prepared from the traverses at low pressure. At high pressure the maximum difference in this ratio between the constant and oscillating heat input, at the same quality, was 8% of the local void fraction while at low pressure this difference was only 4%.

No changes were made in the data reduction program, i. e., the maximum and minimum void fraction were calculated at each station and each frequency as pointed out in [3]. At higher frequencies, where the peak-to-peak void oscillation is less than about 3% in voids, the accuracy of this measurement is very poor (i. e., about $\pm 1 \frac{1}{2}\%$ out of 3%). It is for this reason that the power oscillation amplitude was kept large enough to clearly define the void amplitude variation from the low to the high frequencies. (See Table IB).

3.2 Experimental Results

3.20 Loop Behavior

As discussed under loop design in [4] minimum feedback of test section behavior on the inlet flow rate, system pressure and inlet subcooling was desired. In the tests completed to date, the maximum pressure oscillation

Figure 2. Measured Ratio of Chordal (Diameter) Void Fraction to Average Void Fraction for Constant and Oscillating Heat Input Experiments at Reduced Pressure of 0.22 Respectively.

Figure 3. Measured Ratio of Chordal (Diameter) Void Fraction to Average Void Fraction for Constant and Oscillating Heat Input Experiments at Reduced Pressure of 0.12 Respectively.

measured between the test section exit and the separator was $\pm .2$ psi at .05 cps and $\pm .05$ psi at 1-2 cps. While these oscillations are smaller than those anticipated due to the vapor 'storage' effect in the separator, the high frequency oscillations are more likely due to dynamic effects that are not affected by the separator. The maximum flow oscillations never exceeded 1-2%, since 25-30 psi of inlet pressure drop was taken across the throttle valve. The maximum inlet temperature oscillations were about $\pm 0.5^{\circ}\text{F}$.

Once the optimum condenser coolant rates, by-pass rates and compressor speed were selected, the slow drift in system pressure during a run was easily held to less than ± 0.5 psi. It can, therefore, be concluded that the measured test section dynamics are in reality those of the test section and are not the result of any significant coupling with loop behavior.

3.21 Heat Transfer Coefficients

The heat transfer coefficients as determined by the average power input and wall temperature were within experimental error of the values listed in [5] at constant heat input. The maximum calculated wall temperature variation during the oscillating heat input tests was $\pm 1/2^{\circ}\text{F}$ and $\pm 1^{\circ}\text{F}$ at the low and high heat flux values based on the heat transfer coefficients measured at constant heat input. At these respective conditions the maximum measured wall temperature oscillations, at frequencies where negligible wall attenuation takes place, are $\pm 5/8^{\circ}\text{F}$ and $\pm 1.3^{\circ}\text{F}$ respectively. While these values imply wall-to-saturation temperature differences changes of about $\pm 18\%$, their absolute variation is small relative to the effects of the void and test section pressure drop oscillations.

As the appearance of the temperature recorder traces, and also of the pressure, power, and void traces for that matter, did not change significantly enough, during these 'low' pressure test series, no samples of recorder traces will be shown here. Samples taken were virtually the same as those reported in [3] for the test runs at 0.22 reduced pressure.

3.22 Average Void Fraction

As indicated previously, the time-average void fraction at the respective vertical tube positions showed a maximum increase of approximately 4% of the void fraction when the measurements at oscillating versus constant heat input were compared. For the same local conditions the void fraction for oscillating heat input was in all cases higher than the corresponding value at constant heat input. The increase in average voids was negligible at the inlet but of significant value at the higher void measurement stations and at the outlet.

This increase in average void fraction is caused by the presence of increased voids near the wall of the test section, during both the time of maximum and minimum local heat input in the upper half of the test section. By taking traverse void reading and evaluating the average void fraction from these traverses, the values were approximately 4% larger than the average void fraction obtained for the same local conditions at constant heat input. (See [5] and [6]). This increase is about half of the increase reported for oscillating runs at a reduced pressure of 0.22 (see [3]).

3.23 Oscillation of the Void Fraction

Figures 4 to 11 show plots of the measured amplitudes and phase angles of the oscillations in the average void fraction at each axial location. Test conditions representative of the range of conditions investigated at a reduced pressure of 0.12 (see Table I-B) are shown in these plots of maximum void oscillation and phase angle versus the power input frequency.

Both the attenuation and phase lag of the energy addition to the fluid relative to the power input oscillation are also shown in Figures 4 to 11 for proper data evaluation.

Relative to the energy input oscillation to the fluid, the change of the phase lag of the void oscillation is essentially linear with respect to the frequency. Figures 4, 6, 8 and 10 show this linearity rather clearly. This

Run 82 III 18

Figure 4. Phase Lag of Void Fractions Relative to Test Section Power Input. Qualities at each Void Station Assume Thermodynamic Equilibrium at the Average Power Level.

Figure 5. Twice the Amplitude of the Average Void Fraction Oscillation. Zero Frequency Oscillations are Those Calculated Assuming Thermodynamic Equilibrium. Attenuation of Test Section Wall is Taken Relative to Zero Attenuation at the Low Frequency Void Oscillation.

Figure 6. Phase Lag of Void Fraction Relative to Test Section Power Input. Qualities at each Void Station Assume Thermodynamic Equilibrium at the Average Power Level.

Figure 7. Twice the Amplitude of the Average Void Fraction Oscillation. Zero Frequency Oscillations are Those Calculated Assuming Thermodynamic Equilibrium. Attenuation of Test Section Wall is Taken Relative to Zero Attenuation at the Low Frequency Void Oscillation.

Figure 8. Phase Lag of Void Fractions Relative to Test Section Power Input. Qualities at each Void Station Assume Thermodynamic Equilibrium at the Average Power Level.

Figure 9. Twice the Amplitude of the Average Void Fraction Oscillation. Zero Frequency Oscillations are Those Calculated Assuming Thermodynamic Equilibrium. Attenuation of Test Section Wall is Taken Relative to Zero Attenuation at the Low Frequency Void Oscillation.

Figure 10. Phase Lag of Void Fraction Relative to Test Section Power Input. Qualities at Each Void Station Assume Thermodynamic Equilibrium at the Average Power Level.

Figure 11. Twice the Amplitude of the Average Void Fraction Oscillation. Zero Frequency Oscillations are those Calculated Assuming Thermodynamic Equilibrium. Attenuation of Test Section Wall is Taken Relative to Zero Attenuation at the Low Frequency Void Oscillation.

implies that the void propagation time between the inlet and outlet is relatively constant.

Figures 5, 7, 9 and 11 show the void oscillation for the corresponding test runs. As can be seen decreases with increasing frequency as was expected. Except for the lower stations, at low subcooling and low frequencies, the void oscillations at higher frequency were less than the void oscillations calculated at zero frequency from the constant heat input data.

At low heat flux, see Figure 7, the decrease in the oscillation of the void fraction with increasing frequency was considerably greater than the decrease in the heat input oscillation to the fluid. This may in large part be due to the decreased role that nucleation plays at lower heat flux values. A greater part of the heat input oscillation would then go directly to sensible heat storage in the liquid phase.

A comparison of Figures 5 and 7 also shows that, while the void oscillation increases with increasing quality in Figure 7, these oscillations at the higher flow and heat flux pertinent to the data in Figure 5 increase with decreasing quality. This preserves the qualitative behavior calculated at these two conditions at zero frequency. Note that the void oscillation calculated at zero frequency is that void change determined at constant heat input using the minimum and maximum heat flux values respectively present during the corresponding oscillating heat input tests. It is interesting to note in this connection that the void oscillation always decreases, with increasing frequency and constant quality, below the zero frequency values except at low qualities for tests having significant inlet subcooling.

The oscillation of the average void fraction is also plotted versus time in Figures 12 to 15 for the same runs as above for frequencies of .05 cps and .75 cps. As is to be expected the variation in voids are greatest at low frequencies. At low frequencies the oscillation in the voids are also

Figure 12. Plot of Measured Average Void Fraction at Indicated Power Oscillation Frequency from Data Given in Figures 4-11 (See Table Ib for Test Parameters).

Figure 13. Plot of Measured Average Void Fraction at Indicated Power Oscillation Frequency from Data Given in Figures 4-11 (See Table Ib for Test Parameters).

Figure 14. Plot of Measured Average Void Fraction at Indicated Power Oscillation Frequency from Data Given in Figures 4-11 (See Table Ib for Test Parameters).

Figure 15. Plot of Measured Average Void Fraction at Indicated Power Oscillation Frequency from Data Given in Figures 4-11 (See Table Ib for Test Parameters).

almost in phase with the power oscillation. Runs SO82III18 and SO87III23 were identical for as far as flow rate and heat flux are concerned. The inlet subcooling was halved, however, in run SO87III23, and the effect of this is shown in Figure 12. A more detailed discussion and comparison of the void oscillations with the analysis is given in Section 4 of this report.

3.24 Calculation of the Oscillation in the Test Section Pressure Drop

The instantaneous pressure drop was calculated for a few test conditions and compared to the measured values. Only the pressure drop at the minimum and maximum energy input levels were calculated for run 83III19, at 0.05 cps, and for run 89III25, at 0.05 cps and 0.75 cps. The instantaneous measured void fractions were used to calculate the hydraulic head component (curves A in Figures 16, 17 and 18) and the method of Martinelli-Nelson [7] was employed to obtain the friction and momentum effect. Curve B represents the calculated pressure drop including friction effects. It was assumed that all the energy input to the fluid goes to vapor generation. The black arrow indicates the point of maximum power input to the fluid.

In Figure 16 it is shown that the pressure drop oscillations due to momentum and friction are negligible, i. e., curve B almost coincides with curve A. Agreement is excellent with the measured data. In run 89III19 (.05 cps) friction and momentum effects were of importance and caused an additional pressure drop of more than 1 psi. Furthermore, curve A and B are in the order of 180° out of phase. Again, curve B correlates satisfactorily with the measured pressure drop oscillations. Agreement is poor at a higher frequency (.75 cps), for otherwise the same operating conditions due to dynamic resonance effects. (See Figure 18.) These conclusions are consistent with observations made during the period in which tests at a reduced pressure of 0.22 were conducted. These observations were recorded and discussed in [3] in greater detail.

Figure 16. Comparison of Measured and Calculated Pressure Drop Across Test Section. (See Table 1b for Test Parameters.)

A - Calculated ΔP due to hydrostatic head

B - Calculated ΔP including momentum and friction effects

Figure 17. Comparison of Measured and Calculated Pressure Drop Across Test Section. (See Table Ib for Test Parameters.)
 A - Calculated ΔP due to hydrostatic head
 B - Calculated ΔP including momentum and friction effects

Figure 18. Comparison of Measured and Calculated Pressure Drop Across Test Section. (See Table Ib for Test Parameters.)

A - Calculated ΔP due to hydrostatic head

B - Calculated ΔP including momentum and friction effects

ANALYSIS

1. The Void Propagation Equation

A void propagation equation was formulated in [3] in terms of kinematic waves. The equation predicts the transient response of the vapor volumetric concentration to variations of: 1) power density; 2) pressure; 3) pressure drop; 4) energy storage in the liquid and in the vapor; 5) compressibilities of the liquid and of the vapor; 6) friction factor; 7) flow regime and 8) gravitational force field.

The void propagation equation, when expressed in terms of values averaged over the cross-sectional area of the duct, is given by:

$$\frac{\partial \langle \alpha \rangle}{\partial t} + \left[C_o \langle j \rangle + \bar{V}_{gj} \right] \frac{\partial \langle \alpha \rangle}{\partial z} = \left[1 - C_o \langle \alpha \rangle \frac{\Delta \rho}{\rho_f} \right] \frac{\langle \Gamma_g \rangle}{\rho_g} \quad (1)$$

where the velocity of kinematic waves is given by

$$C_k = C_o \langle j \rangle + \bar{V}_{gj} \quad (2)$$

whereas the other symbols are defined in the Nomenclature.

The general expressions for the vapor source term Γ_g and for the volumetric flux density of the mixture are given in [3]. In the present experiments the effects of compressibility of the liquid and of the vapor were negligible as were the effects of system pressure oscillations and of subcooling. Consequently, for the present experiments the expressions for j and Γ_g reduce to

$$j = v_{fi} + \int_0^z \frac{\Delta \rho}{\rho_f} \Gamma_g dz \quad (3)$$

and

$$\Gamma_g = \frac{4q}{h_{fg} D} (1 + a^2 \sin^2 \omega t + 2a \sin \omega t) \quad (4)$$

where q , is the average power density input to the liquid and $1 + a^2 \sin^2 \omega t + 2a \sin \omega t$ is the form of the oscillating component of the power density to the liquid.

Substituting Eq. 4 into Eq. 3 we obtain for the volumetric flux

$$j = v_{fi} + \frac{\Delta \rho}{\rho_f} \frac{4 q z}{\rho_g h_{fg} D} (1 + a^2 \sin^2 \omega t + 2a \sin \omega t) \quad (5)$$

whereas from Eq. 5 and Eq. 2 we obtain the following expression for the velocity of the kinematic wave

$$C_k = C_o \left[v_{fi} + \frac{\Delta \rho}{\rho_f} \frac{4 q z}{\rho_g h_{fg} D} (1 + a^2 \sin^2 \omega t + 2a \sin \omega t) \right] + \bar{V}_{gj} \quad (6)$$

Substituting Eq. 5 into Eq. 1 gives the void propagation equation appropriate to the experimental conditions discussed in this report:

$$\begin{aligned} \frac{\partial \langle \alpha \rangle}{\partial t} + \left[C_o v_{fi} + \bar{V}_{gj} + C_o \frac{\Delta \rho}{\rho_f} \frac{4 q z}{\rho_g h_{fg} D} (1 + a^2 \sin^2 \omega t + 2a \sin \omega t) \right] \frac{\partial \langle \alpha \rangle}{\partial z} = \\ = \left[1 - C_o \frac{\Delta \rho}{\rho_f} \langle \alpha \rangle \right] \frac{4 q}{\rho_g h_{fg} D} (1 + a^2 \sin^2 \omega t + 2a \sin \omega t) \end{aligned} \quad (7)$$

As discussed in [3] Eq. 7 shows that the transient response of the vapor volumetric concentration depends upon the flow regimes. This statement follows from the fact that both the distribution parameter C_o , and the weighted mean drift velocity $\bar{V}_{gj}$ which appear in Eq. 7 and Eq. 6 depend upon the flow regime and change with a change of flow pattern (See below). The dependence of the transient response of a two phase mixture upon the flow regimes is not surprising if one recalls that, even in single phase flow, the transient response of the laminar flow regime is different from the transient response of the turbulent flow regime. A brief review of the manner in which C_o and $\bar{V}_{gj}$ are determined is given below.

The tabulated and correlated experimental average void fractions determined at constant power input in the course of this investigation are given in [5] and [6]. These data as well as those reported by other investigators for fluids at various pressures were correlated by the following equation derived in [8]

$$\frac{\langle j_g \rangle}{\langle \alpha \rangle} = C_o \langle j \rangle + \frac{\langle \alpha \bar{V}_{gj} \rangle}{\langle \alpha \rangle} \quad (8)$$

where the distribution parameter C_o is given by

$$C_o = \frac{\int_A j \alpha \, dA}{\left[\int_A j \, dA \right] \left[\frac{1}{A} \int_A \alpha \, dA \right]} \quad (9)$$

whereas the other symbols are defined in the nomenclature. The various characteristics of this equation are discussed in more detail in [5, 6, 8]. Briefly, the distribution parameter C_o , takes into account the effects of radial profiles of the flow and of the concentration. The weighted mean drift velocity $\bar{V}_{gj}$, given by the last term on the right hand side of Eq. 8, accounts for the relative velocity between the phases. It was shown in [6] and [8] that the values of both C_o , and of $\bar{V}_{gj}$ change with a change of flow regime. The values and expressions for C_o as well as for $\bar{V}_{gj}$ valid for the various flow regimes are given in [5, 6, 8].

Equation 8 suggests a plot of the data in the vapor weighted mean velocity $\langle j_g \rangle / \langle \alpha \rangle$, versus mixture mean flux density $\langle j \rangle$, plane. Such a plane is shown in Figure , together with some of the experimental results reported in [5, 6]. The boundaries between the flow regimes that are indicated in this figure were determined, independently, from still photographs taken during identical runs in the glass test section [5, 6]. The reader will note that, in agreement with the analysis, the change of slope, i. e., of C_o , and of the intercept, i. e., of $\bar{V}_{gj}$, occurs in the region where a change of flow was visually observed.

Figure 19. The Volumetric Concentration of Refrigerant-22 at Constant Heat Addition on the $\langle j_g \rangle / \langle \alpha \rangle - \langle j \rangle$ Plane. Reduced Pressure = .22.

An analysis and correlation of all the steady-state experimental data obtained in the course of this investigation as well as of the available data reported by other investigators for various fluids, subcoolings, geometries and system pressures are given in [9]. It is shown there that these results exhibit characteristics similar to those shown in Figure 19. We note that the analysis reported in [9] is applicable to liquids at saturation temperature as well as to subcooled liquids.

2. Comparison with Experimental Results

In order to compare the predicted variation of the average void fraction at the various axial locations with the experimental results three data runs involving minimum inlet subcooling were selected. The significant parameters for these runs are given in Table 2.1. In each case the requirement that no visible bubble exist in the inlet sight glass in the test section was satisfied.

TABLE 2.1

Test Conditions

Reduced Pressure = 0.22 Sat. Temp. = 80°F

Run No.	Inlet Subcooling °F	G lbs/hr ft ²	Exit Quality %	Avg. Ht. Flux Btu/hr ft ²	Amplitude of Pwr. Oscil. %	Freq. Range CPS
SO92III28	1.1	340,000	11	4820	40	.05-10
SO95III31	1.2	220,000	11	3270	40	.05-10
SO96III32	1.6	220,000	29	8400	25	.05-10

The power supply delivers a sinusoidal voltage variation, added to the average input voltage, to the test section. The resulting shape of the power input oscillation to the test section thus follows the form (Average Power) $(1 + a^2 \sin^2 \omega t + 2a \sin \omega t)$. At frequencies of about 0.1 cps or less there is a negligible attenuation or phase angle shift of the test section input power by the test section wall for the test parameters investigated. As previously pointed out, above this frequency, Figure 1 indicates the attenuation and phase shift of the outside wall temperature due to the wall effect. A similar plot

of the attenuation and phase shift of the inside wall temperature without the local wall sensor effect, was employed to define the amplitude and phase angle of the energy input to the fluid relative to the energy input to the test section wall. That is, it was assumed that the change in the local saturation temperature or the local heat transfer coefficient was small enough during the higher frequency oscillations, so that the variation of the inside wall temperature is a direct measure of the energy input to the fluid. The validity of this assumption is in turn supported by the agreement shown in Figure 1.

At 0.1 cps and $h = 2200 \text{ Btu/hr ft}^{\circ}\text{F}$ for a 40% power input oscillation the value of the coefficient in the above power input term is $a = 0.36$. Whereas for the same conditions but at a frequency of 0.5 cps the value is $a = 0.22$. The calculated phase angle shift between the test section input and fluid power input is 108° , i. e., 0.6 sec. in Figure 21.

The variation in the experimental void fraction at the various axial tube locations is shown as a function of time by the data points in Figures 20 and 21 for run 92III28, in Figure 22 for run 95III31 and in Figure 23 for run 96III32. The plotted points are those taken from the corresponding data traces. The phase angle between the measured power input to the test section and the experimental void fraction at each axial location is shown in Figures 20 to 23 by the equivalent time between these plotted parameters. Note that a maximum frequency of 0.5 cps is shown in Figure 21. At higher frequencies, the measured void oscillation at the top station becomes too small for accurate data comparison.

In order to test the theoretical results Eq. 7 was solved on a computer to predict the void response for the flow regimes shown in Figure 19, i. e., for the bubbly-churn and for the annular flow regimes. The values of the distribution parameter C_o , and of the drift velocity $\bar{V}_{gj}$, which were used in Eq. 7 were appropriate to these two flow regimes. Thus, as shown in Figure 19, for the bubbly-churn regime these values are

$$\begin{aligned} C_o &= 1.15 \\ \bar{V}_{gj} &= 1.18 \left[\frac{\sigma g \Delta \rho}{\rho_f^2} \right]^{1/4} = 0.3 \text{ ft/sec} \end{aligned} \quad (10)$$

RUN 92 III 28

0.10 CPS

Figure 20. Comparison of Calculated and Measured Volumetric Concentration for Oscillating Heat Input Where $\omega t = 0$ at Zero Time in Eq. 7. Reduced Pressure = .22; Average $q = 4820$ Btu/hr ft² (See Table Ib).

RUN 92 III 28
0.5 CPS
(SEE FIG.20 FOR SYMBOLS)

4844
GEAP-4484

Figure 21. Comparison of Calculated and Measured Volumetric Concentration for Oscillating Heat Input Where $\omega t = 0$ at Zero Time in Eq. 7. Reduced Pressure = .22, Average $q = 4820$ Btu/hr ft² (See Table Ib).

RUN 95 III 31
0.10 CPS
(SEE FIG. 20 FOR SYMBOLS)

4844
GEAP-484

Figure 22. Comparison of Calculated and Measured Volumetric Concentration for Oscillating Heat Input Where $\omega t = 0$ at Zero Time in Eq. 7. Reduced Pressure = .22, Average $q = 4820$ Btu/hr ft² (See Table Ib).

4844

Figure 23. Comparison of Calculated and Measured Volumetric Concentration for Oscillating Heat Input Where $\omega t = 0$ at Zero Time in Eq. 7. Reduced Pressure = .22, Average $q = 8400 \text{ Btu/hr ft}^2$ (See Table Ib). This figure shows the degree of approximation which is obtained when the effect of flow regime is not taken into account.

whereas for the annular flow regime $C_o = 1.05$ and $\bar{V}_{gj} = 1.0$ ft/sec. The predicted void responses were then compared to the experimental data obtained in these two flow regimes and shown in Figures 20, 21 and 22.

For run 95III31, shown in Figure 22, the volumetric flux density of the mixture $\langle j \rangle$ was low so that the entire section was in the bubbly-churn flow regime. The values predicted by Eq. 7 for this regime are also shown in this figure. For run 92III28, shown in Figures 20 and 21, the exit volumetric flux density was such that a transition from the bubbly-churn flow to the annular flow regime occurred at approximately station no. 4 in the test section. Consequently, Eq. 7 was programmed to change the respective values of C_o and of $\bar{V}_{gj}$ from 1.15 and 0.3 ft/sec, pertinent to the churn flow, to 1.05 and 1.0 ft/sec pertinent to the annular flow regime. Note, again, that this flow regime transition is taken from the velocity-flux plot, of which Figure 19 is an example. This plot is supported by both the void measurements and the photography of the flow regimes. The predicted values for the two frequencies of 0.10 cps and of 0.50 cps are shown in Figures 20 and 21 respectively. It can be seen in Figure 20 that the predicted wave propagation as well as the wave form are in close agreement with the experimental data.

The agreement between predicted values of $\langle \alpha \rangle$ and experimental data appears satisfactory. The maximum deviation, either in terms of the mean values of $\langle \alpha \rangle$ or in terms of its amplitude is about 5%, with an average deviation of about 2% to 3% in voids. The latter is close to the confidence limit of the experimental void measurement. Note also that the data runs selected were those involving minimum inlet subcooling since the void propagation equation, when expressed in the form of Eq. 7, does not take the inlet subcooling into account. The inlet subcooling present in the data shown in Figures 20 to 23 involves roughly 5% of the total energy input per unit mass of fluid. A comparison of predicted results with experimental data obtained when the inlet subcooling is appreciable will be presented in a subsequent publication.

The preceding results tend to indicate that for every flow regime, rather close agreement exists between values predicted by Eq. 7 and experimental data. However, a definite and accurate method for predicting the boundaries of the various flow regimes is not yet available. It is therefore of practical interest to inquire what kind of agreement exists between predicted values and experimental data when no attention is paid to the flow regime. This was done in [6] for the steady-state values of $\langle \alpha \rangle$.

It was shown in [6] that, for the entire range covered by the experimental conditions of [5] and [6], a single straight line with the slope of $C_o = 1.13$ and $\bar{V}_{gj}$ given by Eq. 12 can be used to approximate the experimental data. It can be seen from Figure 19 that a straight line with $C_o = 1.13$ has a slope that is lower than the slope of the experimental data in the churn turbulent region and higher than that of the data in the annular regime. Consequently, for given $\langle j \rangle$ and $\langle j_g \rangle$, the steady-state values of $\langle \alpha \rangle$ predicted by this approximation are somewhat high in the churn regime and low in the annular regime.

It can be expected that similar results will be obtained in transients if the values of $C_o = 1.13$ and of $\bar{V}_{gj}$ given by Eq. 12 are inserted in Eq. 7 to predict the void response regardless of the flow regime. This is indeed the case as shown in Figure 23. It is seen there that at stations no. 2 and no. 3 where, according to the steady-state data, the flow was in the churn regime, the predicted values are higher than the experimental data. Further downstream, where according to the steady-state data, the flow is in the annular flow regime, the predicted values are lower than the experimental data. We can conclude therefore, as we did in [6] in connection with the steady-state values of $\langle \alpha \rangle$, that the simplification achieved by not paying any attention to the flow regime is achieved by sacrificing the accuracy with which the transient void fraction can be predicted if the flow regimes are taken into consideration.

Figures 20 to 23 show also the power input to the fluid as well as the

power input to the test section. The power input to the fluid was calculated by the method discussed previously. At the low frequency depicted in Figures 20, 22 and 23 a negligible power amplitude attenuation takes place and the phase shift is too small, as indicated by the arrows on the abscissa, to warrant a separately drawn power curve. At 0.5 cps, however, the considerable attenuation and phase shift of the power input to the fluid, relative to that into the wall, is indicated in Figure 21. In the solution of Eq. 7 the heat input is that into the fluid so that, at the higher frequency, the reduced amplitude of the power input to the fluid curve shown in Figure 21 is employed to establish the predicted void oscillation. The phase angles of the voids predicted by Eq. 7 are all shown relative to the curve that defines the power input to the fluid. The satisfactory agreement in the relative phase angles between the calculated and experimental results in Figures 20 to 23 also supports the method of calculating the phase angle difference between the power input to the test section and that into the fluid.

The phase angle between the void oscillation in the bottom and top axial void locations describes the void propagation time between these locations. A correspondence between this calculated phase angle and the measured values is thus as important as the agreement in the void values themselves. Table 2.2 compares these measured and calculated phase angles expressed in terms of propagation time.

TABLE 2.2

Propagation Time Between Void Station 1 and 6 (4 ft)

Run No.	Frequency (cps)	Calculated Time (sec)	Measured Time (sec)
92III28	0.1	0.62	0.6
92III28	0.5	0.45	0.53
95III31	0.1	0.80	0.75
96III32	0.15	0.70	0.75

It appears from the preceding results that the values predicted by the analysis are in qualitative and quantitative agreement with the experimental data.

3. The Velocity of Propagation and Wave Form of Void Disturbances in Boiling, Forced Convection Systems

3.1 Introduction

A void propagation was formulated in [3, 10] in terms of kinematic waves. The equation predicts the transient response of the vapor volumetric concentration to variations of 1) power density, 2) pressure, 3) pressure drop, 4) energy storage in the liquid and in the vapor, 5) compressibilities of the liquid and of the vapor, 6) friction factor, 7) flow regime and 8) gravitational free field.

It was shown in [3, 10] that the void disturbances in boiling systems are propagated by the velocity of kinematic waves given by

$$C_k = C_o \langle j \rangle + \bar{V}_{gj} \quad 11$$

The general expressions for the average volumetric flux density $\langle j \rangle$, the distribution parameter C_o , and for the weighted mean drift velocity of the vapor $\bar{V}_{gj}$, are given and discussed in more detail in [3, 10].

It was shown in the preceding section that both the rate of propagation as well as the wave form of the void disturbance predicted by the void propagation equation are in good agreement with experimental data obtained in this investigation. In these experiments the effects of the compressibility of the liquid and of the vapor were negligible as were the effects of system pressure oscillations and of subcooling. Consequently, these effects were also neglected in the computer solution of the void propagation equation. Furthermore, since in the experiments the oscillating power input term to the liquid was of the form of $(1 + a^2 \sin^2 \omega t + 2a \sin \omega t)$ the same form was used in the analysis in order to compare the predicted results with the experimental data.

It is of interest to reactor kinetics, however, to predict the void response when the oscillatory power input term is of the form $(1 + \epsilon \sin \omega t)$. It is of

further interest to predict the void response to inlet flow oscillations of the form $(1 + \epsilon_f \sin \Omega t)$. It will therefore be the purpose of this section to derive closed form analytical solutions of the void propagation equation giving the void response to these two perturbations. Both solutions are presented in dimensionless form so that they can be applied to various systems of practical interest.

3.2 The Dimensionless Form of the Void Propagation Equation and the General Form of the Solution

When the oscillatory power input term to the liquid is of the form $(1 + \epsilon \sin \omega t)$, and when we neglect 1) the effects of the compressibility of the liquid and of the vapor, 2) the effects of pressure variations and 3) the effects of energy storage in the two phases, i. e., when we neglect the effects of superheating and of subcooling, the void propagation equation reduces to:

$$\begin{aligned} \frac{\partial \langle \alpha \rangle}{\partial t} + [C_o v_{fi} + \bar{V}_{gj} + C_o \frac{\Delta \rho}{\rho_f} \frac{q}{\rho_g h_{fg}} \frac{4z}{D} (1 + \epsilon \sin \omega t)] \frac{\partial \langle \alpha \rangle}{\partial z} \\ = [1 - C_o \frac{\Delta \rho}{\rho_f} \langle \alpha \rangle] \frac{q}{\rho_g h_{fg}} \frac{4}{D} (1 + \epsilon \sin \omega t) \end{aligned} \quad 12$$

We define now the dimensionless length by

$$z^* = \frac{z}{L_b} \quad 13$$

the dimensionless time by

$$t^* = \frac{C_o v_{fi} + \bar{V}_{gj}}{L_b} t \quad 14$$

the dimensionless vapor source term

$$\Gamma^* = C_o \frac{\Delta \rho}{\rho_f} \frac{q}{\rho_g h_{fg} (C_o v_{fi} + \bar{V}_{gj})} \frac{4 L_b}{D} \quad 15$$

and the dimensionless angular frequency

$$\omega^* = \frac{\omega L_b}{C_o v_{fi} + \bar{V}_{gj}} \quad 16$$

Inserting these dimensionless parameters in Eq. 12 we obtain the dimensionless form of the void propagation equation, thus

$$\begin{aligned} \frac{\partial \langle \alpha \rangle}{\partial t^*} + [1 + \Gamma^* z^* (1 + \epsilon \sin \omega^* t^*)] \frac{\partial \langle \alpha \rangle}{\partial z^*} = \\ = [1 - C_o \frac{\Delta \rho}{\rho_f} \langle \alpha \rangle] \frac{\rho_f}{C_o \Delta \rho} \Gamma^* (1 + \epsilon \sin \omega^* t^*) \end{aligned} \quad 17$$

This is a first order linear, partial differential equation whose solution can be obtained by means of characteristics (see for example [12, 13]). The general solution of Eq. 17 is of the form:

$$u_2 = f(u_1) \quad 18$$

where $u_1[\langle \alpha \rangle, z^*, t^*] = c_1$ and $u_2[\langle \alpha \rangle, z, t] = c_2$ are solutions of any two independent differential equations which imply the relationships:

$$\frac{dt^*}{dz^*} = \frac{1}{1 + \Gamma^* z^* (1 + \epsilon \sin \omega^* t^*)} = \frac{d\langle \alpha \rangle}{[1 - C_o \frac{\Delta \rho}{\rho_f} \langle \alpha \rangle] \frac{\rho_f}{C_o \Delta \rho} \Gamma^* (1 + \epsilon \sin \omega^* t^*)} \quad 19$$

In what follows we shall obtain three solutions of Eq. 17, 1) a solution when the power input is steady, 2) an exact solution by means of a computer and 3) an approximate but simple, closed form analytical solution. A comparison of the simple analytical solution with the exact computer solution will permit then an evaluation of the accuracy of the former.

3.3 Steady Power Input

For a steady power input we set $\epsilon = 0$ in Eq. 17, and the set given by Eq. 19 reduces then to

$$dt^* = \frac{dz^*}{1 + \Gamma^* z^*} = \frac{d\langle\alpha\rangle}{[1 - C_o \frac{\Delta\rho}{\rho_f} \langle\alpha\rangle] \frac{\rho_f}{C_o \Delta\rho} \Gamma^*} \quad 20$$

We obtain the first solution u_1 , by considering the first and second term in Eq. 20 thus, upon integration:

$$u_1 = t^* - \frac{1}{\Gamma^*} \ln (1 + \Gamma^* z^*) \quad 21$$

From the first and third term of Eq. 20 we obtain the second solution u_2 , thus upon integration

$$u_2 = t^* + \frac{1}{\Gamma^*} \ln (1 - C_o \frac{\Delta\rho}{\rho_f} \langle\alpha\rangle) \quad 22$$

The general solution is given by Eq. 18, which, in view of Eq. 21 and Eq. 22 can be expressed as:

$$t^* + \frac{1}{\Gamma^*} \ln (1 - C_o \frac{\Delta\rho}{\rho_f} \langle\alpha\rangle) = f [t^* - \frac{1}{\Gamma^*} \ln (1 + \Gamma^* z^*)] \quad 23$$

The form of the function $f(u_1)$ in Eq. 18, i. e., in Eq. 23 is determined from the boundary conditions. Taking as the boundary condition: $\langle\alpha\rangle = 0$ when boiling starts, i. e., when $z^* = 0$, we obtain from Eq. 23 the following expression for $f(u_1)$

$$f(u_1) = t^* - \frac{1}{\Gamma^*} \ln (1 + \Gamma^* z^*) \quad 24$$

whence Eq. 23 becomes

$$1 - C_o \frac{\Delta\rho}{\rho_f} \langle\alpha\rangle = \frac{1}{1 + \Gamma^* z^*} \quad 25$$

or solving for $\langle\alpha\rangle$ we obtain

$$C_o \frac{\Delta\rho}{\rho_f} \langle\alpha\rangle = \frac{\Gamma^* z^*}{1 + \Gamma^* z^*} \quad 26$$

Introducing now in Eq. 26 the expressions for z^* and Γ^* , i. e., Eq. 13 and Eq. 15, we obtain the equation which predicts the vapor volumetric concentration in a uniformly heated circular duct when the liquid is at saturation temperature, i. e., in absence of subcooling, thus:

$$\langle \alpha \rangle = \frac{\frac{q}{\rho_g h_{fg}} \frac{4z}{D}}{C_o v_{fi} + \bar{V}_{gj} + C_o \frac{\Delta \rho}{\rho_f} \frac{q}{\rho_g h_{fg}} \frac{4z}{D}} \quad 27$$

which is of the same form as the expression derived previously in [6, 9] using a different approach.

Figure 24 shows the values of $\langle \alpha \rangle$ predicted by Eq. 27 for Refrigerant in the churn-turbulent flow regime. For this regime the value of the distribution parameter is $C_o = 1.2$, whereas the weighted mean drift velocity of the vapor $\bar{V}_{gj}$ is given by Eq. 10. Additional comparisons with experimental data for water and other fluids as well as for subcooled boiling are presented in [6, 9].

3.4 Response to Oscillatory Power Input

The general solution of Eq. 17 is given by Eq. 18 where the two solutions u_2 and u_1 are obtained from Eq. 19.

We obtain the u_2 solution considering the first and the third term of Eq. 19, thus upon integration

$$u_2 = \Gamma^* t^* - \frac{\Gamma^* \epsilon}{\omega^*} \cos \omega^* t^* + \ln \left[1 - C_o \frac{\Delta \rho}{\rho_f} \langle \alpha \rangle \right] \quad 28$$

The solution u_1 is obtained by considering the first and the second term of Eq. 19, i. e., by solving the following differential equation:

$$\frac{dz^*}{dt^*} - \Gamma^* (1 + \epsilon \sin \omega^* t^*) z^* = 1 \quad 29$$

Figure 24. Comparison of the Void Fraction for Constant Power Input Predicted by Eq. 27 with Experimental Data in the Churn-Turbulent Regime.

whose solution is given by:

$$z^* e^{-\Gamma^* t^* + \frac{\Gamma^* t^*}{\omega^*} \cos \omega^* t^*} - \int e^{-\Gamma^* t^* + \frac{\Gamma^* \epsilon}{\omega^*} \cos \omega^* t^*} dt^* = \text{const.} \quad 30$$

The integral in Eq. 30 can be evaluated by successive approximation to yield the u_1 solution of Eq. 17. However, the advantage of having an analytical solution is then lost because of the complexity of the resulting general solution given by Eq. 18.

It is advantageous, therefore, to obtain an approximate but simple, closed form solution which, as it will be seen in what follows, is in satisfactory agreement with the exact solution of Eq. 17 obtained by means of a computer. The simplification is obtained by letting $\epsilon = 0$ in the second term in Eq. 19. We consider, therefore, the following set:

$$dt^* = \frac{dz^*}{1 + \Gamma^* z^*} = \frac{dt^*}{\left[1 - C_o \frac{\Delta \rho}{\rho_f} \langle \alpha \rangle\right] \frac{\rho_f}{C_o \Delta \rho} \Gamma^* (1 + \epsilon \sin \omega^* t^*)} \quad 31$$

From the first and the second term we obtain the u_1 solution which is given by Eq. 21. The u_2 solution is again obtained from the first and third term; it is given by Eq. 28. The general solution then follows by inserting Eq. 21 and Eq. 28 into Eq. 18, thus

$$\Gamma^* t^* - \frac{\Gamma^* \epsilon}{\omega^*} \cos \omega^* t^* + \ln \left[1 - C_o \frac{\Delta \rho}{\rho} \langle \alpha \rangle \right] = f \left[t^* - \frac{1}{\Gamma^*} \ln (1 + \Gamma^* z^*) \right] \quad 32$$

Evaluating the function $f(u_1)$ from the boundary conditions: $\langle \alpha \rangle = 0$ at $z = 0$ we obtain from Eq. 32 the following expression for this function

$$f(u_1) = \Gamma^* \left[t^* - \frac{1}{\Gamma^*} \ln (1 + \Gamma^* z^*) \right] - \frac{\Gamma^* \epsilon}{\omega^*} \cos \left[t^* - \frac{1}{\Gamma^*} \ln (1 + \Gamma^* z^*) \right] \omega^* \quad 33$$

We introduce now the dimensionless time constant τ^* given by:

$$\tau^* = \frac{1}{\Gamma^*} \ln (1 + \Gamma^* z^*) \quad 34$$

which in view of Eq. 13 and Eq. 15 can be expressed also as

$$\tau^* = \frac{\rho_f}{C_o \Delta \rho} \frac{\rho_g h_{fg} (C_o v_{fi} + \bar{V}_{gj})}{q} \frac{D}{4 L_b} \ln \left[1 + \frac{C_o \Delta \rho}{\rho_f} \frac{q}{\rho_g h_{fg} (C_o v_{fi} + \bar{V}_{gj})} \frac{4z}{D} \right]$$

35

or as

$$\tau^* = \frac{C_o v_{fi} + \bar{V}_{gj}}{L_b} \tau \quad 36$$

where the time constant τ is defined by

$$\tau = \frac{\rho_f}{C_o \Delta \rho} \frac{\rho_g h_{fg}}{q} \frac{D}{4} \ln \left[1 + \frac{C_o \Delta \rho}{\rho_t} \frac{q}{\rho_g h_{fg} (C_o v_{fi} + \bar{V}_{gj})} \frac{4z}{D} \right] \quad 37$$

Substituting Eq. 34 and Eq. 33 into Eq. 32 we obtain after some rearrangement the following expression for the void fraction $\langle \alpha \rangle$, thus

$$C_o \frac{\Delta \rho}{\rho_f} \langle \alpha \rangle = 1 - \frac{e^{\frac{\Gamma^* \epsilon}{\omega^*}} [\cos \omega^* t^* - \cos \omega^* (t^* - \tau^*)]}{1 + \Gamma^* z^*} \quad 38$$

This equation predicts $\langle \alpha \rangle$, at various positions in the heated duct and at various times as function the oscillatory power input to the liquid. We note that if we set $\epsilon = 0$ in Eq. 38 we obtain again the steady state solution given by Eq. 25.

The values predicted by Eq. 38 are plotted as dashed lines in Figures 25-28. The values of the dimensionless angular frequency ω^* , of the dimensionless vapor source Γ^* and of the dimensionless power amplitude ϵ which have been used in preparing these figures cover a wide range of operating conditions.

Figure 25. Comparison of the Oscillatory Void Fraction Predicted by Eq. 38, with the Computer Solution of Eq. 18.

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Figure 26. Comparison of the Oscillatory Void Fraction Predicted by Eq. 38, with the Computer Solution of Eq. 18.

Figure 27. Comparison of the Oscillatory Void Fraction Predicted by Eq. 38, with the Computer Solution of Eq. 18.

Figure 28. Comparison of the Oscillatory Void Fraction Predicted by Eq. 38, with the Computer Solution of Eq. 18.

Since Eq. 38 is subject to the simplification introduced in Eq. 17, i. e., in the second term of Eq. 31, it is of interest to compare the values predicted by Eq. 38 with the exact solution of Eq. 17, i. e., of Eq. 19. For this purpose Eq. 17 was solved on a computer; the resulting solutions are plotted as full lines in Figures 25-28. It can be seen from these figures that the values predicted by the approximate but simple, closed form solution, i. e., by Eq. 38, are in relatively satisfactory agreement with the exact, computer solution. This is especially evident at high values of ω^* irrespective of the magnitude of Γ^* . (See Figures 25 and 26). However, at a low value of ω^* and at a high value of Γ^* the amplitude predicted by Eq. 28 overestimates that predicted by the computer (see Figure 28).

In the table below we compare the propagation time predicted by the approximate solution, i. e., by Eq. 38 with the experimental data obtained in [3].

Void Propagation Time Between Stations No. 1 and No. 6 (4 ft.)

Run No.	Frequency Cps	Calculated Time (sec)	Measured Time (sec)
92III28	0.1	0.6	0.6
92III28	0.5	0.5	0.53
96III32	0.15	0.55	0.75

It appears from this table and from Figures 25-28 that the approximate but simple, closed form solution given by Eq. 38 can be used for estimating the time of propagation and the wave form of void disturbances when the oscillatory power input to the liquid is given by $(1 + \epsilon \sin \omega t)$.

3.5 Response to Oscillatory Inlet Flow

For an oscillatory inlet flow of the form $(1 + \epsilon_f \sin \Omega t)$, where ϵ_f is the dimensionless amplitude, the void propagation equation for steady power input becomes

$$\frac{\partial \langle \alpha \rangle}{\partial t} + \left[C_o v_{fi} (1 + \epsilon_f \sin \Omega t) + \bar{V}_{gj} + C_o \frac{\Delta \rho}{\rho_f} \frac{q}{\rho_g h_{fg}} \frac{4z}{D} \right] \frac{\partial \langle \alpha \rangle}{\partial z} =$$

39

$$= \left[1 - C_o \frac{\Delta \rho}{\rho_f} \langle \alpha \rangle \right] \frac{q}{\rho_g h_{fg}} \frac{4}{D}$$

This equation is subject to the same assumptions enumerated in connection with Eq. 12.

We introduce now the dimensionless angular frequency

$$\Omega^* = \frac{\Omega L_b}{C_o v_{fi} + \bar{V}_{gj}} \quad 40$$

and the dimensionless velocity perturbation

$$v'_{fi}{}^* = \frac{C_o v_{fi} \epsilon_f}{C_o v_{fi} + \bar{V}_{gj}} \quad 41$$

Using these two groups together with the dimensionless length, time and vapor source given by Eq. 13, 14, and 15, we can render Eq. 39 dimensionless, thus

$$\frac{\partial \langle \alpha \rangle}{\partial t^*} + \left[1 + \Gamma^* z^* + v'_{fi}{}^* \sin \Omega^* t^* \right] \frac{\partial \langle \alpha \rangle}{\partial z^*} =$$

42

$$\left[1 - C_o \frac{\Delta \rho}{\rho_f} \langle \alpha \rangle \right] \frac{\rho_f}{C_o \Delta \rho} \Gamma^*$$

The solution of Eq. 42 was derived and will be reported in a following publication.

4. CONCLUSIONS

1. Experimental data on the transient response of the vapor volumetric concentration in a boiling fluid flowing upward in a vertical circular tube are presented.
2. The experiments were performed by oscillating the power input to the metal test section while maintaining a constant discharge pressure and a constant inlet liquid velocity.
3. The amplitude of the power oscillations was varied between 5% and 40% of the average power at frequencies between 0.01 and 10 cycles per second.
4. The experimental results are compared to the values predicted by the void propagation equation which is formulated in terms of kinematic waves. Good agreement is shown between predicted and measured values.
5. The results presented in this report confirm the results predicted by the analysis, in particular that:
 - a) The rate of propagation in the voids as well as the wave form of the void disturbance as it propagates along the duct, can be predicted by the void propagation equation derived in 1, 2.
 - b) The void response can be predicted if the steady state voids can be predicted.
 - c) The void response depends upon the flow regime.
 - d) The void response, i. e., the propagation velocity and the wave form can be predicted rather accurately if the effects of flow regime are taken into account.

The loss of accuracy which is introduced by not considering the effect of flow regime is demonstrated.
6. a) The oscillation in the test section pressure drop is predicted satisfactorily using the measured void fraction to calculate the momentum and hydrostatic head and employing the Martinelli-Nelson method to obtain the friction pressure drop.

- b) This pressure drop oscillation is less than the measured value at the frequency where a maximum in this pressure drop amplitude takes place; that is, it does not correctly predict the dynamic pressure drop at a resonance frequency.
7. A computer solution and an approximate but simple, closed form analytical solution of the void propagation equation are presented for the case when the oscillatory power input to the liquid is of the form $(1 + \epsilon \sin \omega t)$. The solutions predict the rate of propagation and the wave form of the void disturbance (as it propagates along the heated duct) caused by power oscillations.
8. An analytical solution of the void propagation equation is presented for the case when the inlet flow oscillates as $(1 + \epsilon_f \sin \Omega t)$. The solution predicts the rate of propagation and the wave form of the void disturbance (as it propagates along the heated duct) caused by inlet flow oscillations.

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