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THE USE OF SHAKER OPTIMIZED PERIODIC TRANSIENTS
IN MATCHING FIELD SHOCK SPECTRA

D. O. Smallwood

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SANDIA LABORATORIES

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ABSTRACT

Present methods of matching field shock spectra in the laboratory are often inadequate. Classical pulses produced on conventional shock machines often result in poor synthesis of field shock spectra at either the high-frequency or low-frequency end of the spectrum or both. The match of shock spectra is usually very limited in frequency range.

A technique has been developed and is described in this report which utilizes decaying periodics produced on vibration exciters to match shock spectra over a very wide frequency range. These periodics, consisting of decaying cosines superimposed on a cosine bell, are well suited to reproduction on both electrodynamic and electrohydraulic exciters. The acceleration, velocity, and displacement characteristics of these transients meet limitations imposed by the physical capabilities of vibration exciters.

The method of synthesizing shock spectra consists of adding several cosine transients having frequencies, decay rates, and amplitudes necessary to produce a composite shock spectrum closely matching the field shock spectrum. The mechanics of determining frequencies, decay rates, and amplitudes is simple since the spectra for individual decaying cosine components add in the frequency domain.

Digital techniques can be used to synthesize the transients and to provide properly conditioned input signals for accurate real time control and reproduction of these transients on vibration exciters.

Key Words: Shock Spectra, Transient Vibration

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1

TABLE OF CONTENTS

	<u>Page</u>
Nomenclature	5
Introduction	7
Vibration Exciter Limitations	10
Reproduction of Classical Pulses	10
Decaying Sinusoids to Match Shock Spectra	13
Shaker Optimized Transient Theory	17
Appendix A. Procedures for Matching a Shock Spectrum with a SHOC Pulse	27
Appendix B. Transient Waveform Reproduction on Electrodynamic Shakers	29

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LIST OF ILLUSTRATIONS

<u>Figure</u>		<u>Page</u>
1	Standard Boxcar Rail Hump Time History	8
2	Standard Boxcar Rail Hump Shock Spectra	8
3	Flight Shock Time History	9
4	Flight Shock Shock Spectrum	9
5	Half-Sine Acceleration, Velocity, and Displacement Characteristics	11
6	Modified Half-Sine Acceleration, Velocity, and Displacement Characteristics	12
7	Using a Half-Sine to Match the ATMX Rail Hump Shock Spectrum	13
8	Decaying Sinusoid Acceleration, Velocity, and Displacement Characteristics	15
9	Normalized Shock Spectra for a Single Frequency Decaying Sinusoid	15
10	Matched Shock Spectra Using Summed Decayed Sinusoids	16
11	Acceleration, Velocity, and Displacement Relationships for ATMX Car Shock Spectra Match Using Decaying Sinusoids	16
12	A SHOC Pulse	17
13	Normalized Shock Spectra for SHOC Pulses	20
14	Acceleration, Velocity, and Displacement of a SHOC Pulse	20
15	Using the SHOC Pulse to Match the ATMX Rail Hump Shock Spectrum	21
16	Time History of the SHOC Pulse Used to Match the ATMX Rail Hump Shock Spectrum	21
17	The Use of a SHOC Pulse to Match a Smooth Shock Spectrum	22
18	Test Setup for SHOC Test	22
19	Shock Spectra of a Launch Event and Simulation	23
20	Desired Time History of the SHOC Pulse Used to Simulate a Launch Event	24
21	Actual Time History of a SHOC Pulse Used to Simulate a Launch Event	24
22	Time History of a SHOC Pulse Whose Components Alternate in Sign	25
23	Shock Spectrum of the SHOC Pulse Shown in Figure 22	26
A1	Fitting Shock Spectra Using SHOC Pulse	28
B1	Electrodynamic Shaker Pulse Reproduction System	30

Nomenclature

$\ddot{x}(t)$	acceleration at any time
A	peak value of acceleration in the time domain
$\dot{x}(t)$	velocity at any time
$x(t)$	displacement at any time
τ	duration of a shock pulse or transient (sec)
ζ	decay rate
ω	circular frequency (rad/sec)
ω_n	circular natural frequency (rad/sec)
t	time
i	$\sqrt{-1}$ or an integer subscript
B	a constant

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THE USE OF SHAKER OPTIMIZED PERIODIC TRANSIENTS IN MATCHING FIELD SHOCK SPECTRA

Introduction

The work described in this report was initiated primarily for the purpose of developing a technique for producing transient vibrations on low-frequency electrohydraulic exciters. However, the resulting technique is one which is suitable for use on both electrohydraulic and electrodynamic exciters. The technique utilizes summed decaying periodics which have been optimized for exciter operation to match shock spectra over wide frequency ranges.

The normal procedure for matching (enveloping) shock spectra of field transients or an ensemble of field transients is to use shock spectra of "classical" pulses (half-sine, terminal peak sawtooth, haversine, etc.). These classical pulses are then produced on conventional shock machines. This generally results in a good match of field shock spectra over a very limited frequency range and severe overtests and/or undertests over the rest of the frequency range of interest.

Two distinctly different physical events will illustrate the problem. The first is a standard boxcar rail hump. The time history of a typical event (taken from Figure 36 of Reference 1) is shown in Figure 1. The shock spectrum* of this event is shown as Curve c on Figure 2 (taken from Figure 37 of Reference 1). The average and maximum¹ composite shock spectra for many similar events are shown as Curves a and b, Figure 2. Two half-sine pulses are included as Curves d and e. As can be seen, a low-level, long-duration half-sine (Curve e) closely matches the composite spectrum below 50 Hz but is a serious undertest at the higher frequencies. The higher-level, short-duration half-sine (Curve d) will cover the higher frequencies but overtest the low frequencies. Clearly, the ratio of high-frequency energy to low-frequency energy is larger for the field data than a half-sine.

The second event was the launch shock recorded on a missile structure during flight.** The time history and the shock spectrum of the event are shown as Figures 3 and 4. The oscillatory nature of the time history is even more evident than in the first case. As before, a classical pulse (in this case a 0.93-ms, 37-g, half-sine) will overtest the low frequencies or undertest the high frequencies (a 2.9-g, 5-ms half-sine). Again, it appears that an oscillatory type laboratory transient would be a more realistic test.

These are only two examples of many field records which indicate oscillatory transient vibrations which are poorly simulated by classical waveforms.

* All shock spectra reported in this report are for a 3 percent damped single degree of freedom system unless stated otherwise.

** From unpublished data.

Figure 1. Standard Boxcar Rail Hump Time History
(taken from Figure 36 of Reference 1)

- a - Maximum standard boxcar composite
- b - Average standard boxcar composite
- c - A single standard boxcar event time history (taken from Figure 37 of Reference 1)
- d - 200-g, -6-msec half-sine
- e - 43-g, 20-ms half-sine

Figure 2. Standard Boxcar Rail Hump Shock Spectra

Figure 3. Flight Shock Time History

Figure 4. Flight Shock Shock Spectrum

Vibration exciters (electrodynamic and electrohydraulic) provide a versatile means of producing transient vibrations or shock pulses as long as the acceleration, velocity, and displacement requirements meet the physical limitations of the exciters. 'Because of these limitations, vibration exciters are poorly suited to reproduction of classical pulses as will be explained in the next section. However, they are well suited for the reproduction of oscillatory type transients.

Vibration Exciter Limitations

The types of transient vibration or shock pulses which can be accurately reproduced on both electrodynamic and electrohydraulic exciters are very much dependent on the physical limitations of the exciters. These limitations are listed in Table I and are briefly discussed here.

The initial and final acceleration and velocity of a transient must be zero for both electrodynamic and electrohydraulic exciters. As with any type of testing machine, maximum attainable values of acceleration and velocity are limited. Acceleration is actually limited by the force capabilities of the exciter.

Flexures in electrodynamic exciters generate restoring forces which return the exciter table to its originating position (defined as zero), and limitation 3 holds. This is not a requirement for electrohydraulic systems; however, by imposing this limitation one can take advantage of both the forward and return portion of the stroke to generate the required transient. This effectively results in doubling the displacement capacity of the exciter for generating transients. This concept will be discussed in more detail later. Thus for the purposes of this report limitation 3 will also be considered a limitation for an electrohydraulic exciter.

TABLE I
Exciter Limitations

Limitation No.	Initial	Final	Maximum	Electrodynamic	Electrohydraulic
1	$\ddot{x}(0) = 0$	$\ddot{x}(\tau) = 0$	Limited	105000 lb	37,500 lb
2	$\dot{x}(0) = 0$	$\dot{x}(\tau) = 0$	Limited	200 in./sec	220 in./sec
3	$x(0) = 0$	$x(\tau) = 0$ (Electrodynamic)	Limited	$\pm 1/2$ in.	8 in.

Reproduction of Classical Pulses

Consider the half-sine shock pulse shown in Figure 5 where acceleration, velocity, and displacement are defined by the following relationships:

$$\ddot{x}(t) = A \sin \pi t / \tau \quad (1)$$

Figure 5. Half-Sine Acceleration, Velocity, and Displacement Characteristics

$$\dot{x}(t) = \frac{A\tau}{\pi} (1 - \cos \pi t/\tau) \quad (2)$$

$$x(t) = A\tau^2/\pi^2 (\pi t/\tau - \sin \pi t/\tau) \quad (3)$$

One can see that the limitations of zero final velocity and zero final displacement have not been met. Classical pulses (haversine, parabolic cusp, triangle, square wave, etc.) all result in residual velocities and displacements and therefore cannot be accurately reproduced on vibration exciters without modifying the pulses.

A modification will be considered which is particularly well suited to reproduction on both electrodynamic and electrohydraulic exciters. Figure 6 shows the acceleration, velocity, and displacement relationships of this pulse.

An initial, relatively low-level, long-duration pulse defined by

$$\ddot{x} = -nA \sin \pi t/\tau' \quad (0 \leq t \leq \tau')$$

is used to attain one-half of the required velocity change. The half-sine defined by

$$\dot{x}(t) = A \sin \pi(t - \tau')/\tau \quad (\tau' \leq t \leq \tau + \tau') \quad (4)$$

is used to attain the required velocity change. A second, relatively low-level, long-duration pulse

$$\ddot{x}(t) = -nA \sin \pi(t - \tau' - \tau/\tau') \quad (\tau + \tau' \leq t \leq \tau + 2\tau') \quad (5)$$

is used to bring the final velocity to zero. Notice that acceleration, velocity, and displacement are all zero at $t = 0$. Velocity passes through zero at $\tau' + \tau/2$; displacement therefore changes directions at $\tau' + \tau/2$; and the final acceleration, velocity, and displacement are all zero. Because of

Figure 6. Modified Half-Sine Acceleration, Velocity, and Displacement Characteristics

these characteristics this type of pulse is well suited to reproduction on electrodynamic exciters. The requirement of returning to the starting position does not apply to electrohydraulic exciters. However, by examining the characteristics of the modified half-sine one can see that, in requiring the specimen to return to its starting position, the forward portion of the stroke is used to attain the necessary initial velocity and to perform one-half the required pulse. The return portion of the stroke is used to perform the second half of the required pulse and to attain a zero final velocity. This in effect doubles the displacement capacity of the exciter. For example, rather than being limited to 8 inches of displacement in generating a pulse one can actually take advantage of 16 inches of displacement. This results in being capable of producing pulses having higher maximum levels of acceleration and velocity than if one is restricted to 8 inches of stroke in a single direction.

Figure 7 illustrates the effect on the shock spectrum of modifying the half-sine. The curve represented by the solid line is the envelope of an ensemble of field pulses obtained from switching/humping ATMX rail cars at 11-mph switching speeds.¹ The dashed line represents the spectrum of a half-sine used as a test pulse. The dotted line represents the spectrum of the modified half-sine. Two changes are noted. First, the levels at the low frequencies have been reduced as the low-frequency energy has been removed from the pulse. Second, an increase in the level is noted at the frequency of the low-level, long-duration half-sines used for velocity control. At higher frequencies the spectra are nearly the same.

Figure 7. Using a Half-Sine to Match the ATMX Rail Hump Shock Spectrum

Although the modified half-sine results in a poor means of matching shock spectra over wide frequency ranges, the reader should keep in mind its general acceleration, velocity, and displacement characteristics. These characteristics will manifest themselves later in a transient which can be used to match shock spectra over wide frequency ranges.

Decaying Sinusoids to Match Shock Spectra

The first attempt to develop a method for matching shock spectra over wide frequency ranges was to use decaying sinusoids. This provides an excellent means of matching shock spectra but results in velocities and displacements which do not meet the limitations of electrodynamic and electrohydraulic exciters. However, a brief discussion of the use of decaying sinusoids is included at this point since the technique provided a considerable contribution in the evolution of developing a more suitable transient for reproduction on vibration exciters.

Consider, therefore, a single frequency transient whose acceleration, velocity, and displacement characteristics are described in the time domain by the following relationships:

$$\ddot{x}(t) = Ae^{-\zeta\omega t} \sin \omega t, \quad (6)$$

$$\dot{x}(t) = \frac{-A e^{-\zeta \omega t}}{\omega(\zeta^2 + 1)} (\zeta \sin \omega t + \cos \omega t) + \frac{A}{\omega(\zeta^2 + 1)} \quad (7)$$

and

$$x(t) = \frac{A e^{-\zeta \omega t}}{2(\zeta^2 + 1)^2} (\zeta^2 \sin \omega t + 2\zeta \cos \omega t - \sin \omega t) + \frac{At}{\omega(\zeta^2 + 1)} - \frac{2\zeta A}{\omega^2(\zeta^2 + 1)^2} \quad (8)$$

These relationships are graphically illustrated in Figure 8. The reader can immediately observe that the final velocity and displacement are nonzero and do not meet the shaker limitations previously described. Disregarding these disadvantages at this point, consider the normalized shock spectra of a single frequency decaying sinusoid shown in Figure 9 for varying decay rates. Several things are interesting to note:

- 1) Shock spectra levels of greater than two can be obtained. (Classical pulses result in maximum values of shock spectra of two or less.)
- 2) Maximum shock spectra levels decrease with increasing decay rates.
- 3) Shock spectra for high decay rates ($\zeta = 0.5$) approach the shape of that obtained when using a half-sine.

By adding several decaying sinusoids of various frequencies, amplitudes, and decay rates one can obtain a shock spectrum which is the composite of the shock spectra of the individual single frequency decaying sinusoids. Thus by judiciously choosing frequencies, amplitudes, and decay rates of the individual decayed sinusoids one can obtain an accurate match of a field shock spectrum. The resulting transient is described by the following relationship,

$$\ddot{x}(t) = \sum_{i=1}^n A_i e^{-\zeta_i \omega_i t} \sin \omega_i t \quad (9)$$

Figure 10 illustrates an example of matching a field shock spectra using decayed sinusoids. The curve represented by the solid line is the same switching/humping envelope as shown in Figure 7. The dashed line represents a match obtained using five decaying sinusoids. A match to 1.0 Hz resulted in a pulse duration (τ) of 359 ms. The residual velocity and displacement were 185 inches/second and 62 inches, respectively. Shortening the pulse duration (i.e., truncation of the pulse) to 150 ms resulted in a match of shock spectra to 2.2 Hz with the residual velocity and displacement of 210 inches/second and 22 inches, respectively. Decreasing the duration of the pulse to 75 ms resulted in considerable degradation of the shock spectra and a match to approximately 5.5 Hz.

Figure 8. Decaying Sinusoid Acceleration, Velocity, and Displacement Characteristics

Figure 9. Normalized Shock Spectra for a Single Frequency Decaying Sinusoid

Figure 10. Matched Shock Spectra Using Summed Decayed Sinusoids

The residual velocity and displacement were approximately 170 inches/second and 8 inches, respectively. The acceleration, velocity, and displacement relationships for matching shock spectra to 2.2 Hz are shown in Figure 11. Decaying sinusoids provide a means of matching shock spectra over wide frequency ranges but residual velocities and displacement are nonzero.

Figure 11. Acceleration, Velocity, and Displacement Relationships for ATMX Car Shock Spectra Match Using Decaying Sinusoids

Shaker Optimized Transient Theory

One can see from previous examples that decaying periodics offer an excellent means of matching shock spectra over wide frequency ranges. The problem is to develop a periodic transient such that residual velocities and displacements are zero.

Consider the periodic function shown in Figure 12a which is defined by

$$\begin{aligned}\ddot{y}(t) &= Ae^{\zeta\omega t} \cos \omega t & (-\tau/2 \leq t \leq 0) \\ \ddot{y}(t) &= Ae^{-\zeta\omega t} \cos \omega t & (0 \leq t \leq \tau/2)\end{aligned}\tag{10}$$

where τ is of such length that acceleration, $\ddot{y}$, has sufficiently decayed to zero at $-\tau/2$ and $\tau/2$. Integrating the function from $t = -\tau/2$ to $t = \tau/2$ using $\dot{y}(-\tau/2) = 0$ results in a positive nonzero velocity at $\tau/2$.

Figure 12. A SHOC Pulse

For acceleration inputs, one way to assure that the velocity boundary conditions are met is to calculate the area under the acceleration input function (this is the residual velocity; it is also the zero frequency term of the Fourier spectrum), and superimpose a pulse with a residual velocity equal and opposite to the calculated value. Ideally, the superimposed pulse should change the input pulse as little as possible while reducing the zero frequency term of the Fourier spectrum of the input to zero. A good pulse for this purpose is the cosine bell described in Equation (11).

$$s(t) = \begin{cases} B \cos^2 \pi t / \tau & (-\tau/2 < t < \tau/2) \\ 0 & (\text{elsewhere}) \end{cases} \quad (11)$$

The Fourier spectrum, $S(\omega)$, is given by

$$S(\omega) = \frac{-iB}{2\omega} \left(1 - e^{-i\omega\tau} \right) \left(\frac{p^2}{p^2 - \omega^2} \right)$$

where

$$p = 2\pi / \tau$$

$$i = \sqrt{-1} \quad .$$

Thus

$$S(0) = B\tau/2 \quad ,$$

$$S(p) = S(0)/2 \quad ,$$

and for $\omega \gg p$

$$S(\omega) \propto \frac{1}{\omega^3} \quad .$$

Thus two independent variables (B , τ) are available for reducing the residual velocity to zero. One of the variables can be used to control the residual displacement. If the reciprocal of the pulse duration, $1/\tau$, is small compared to the lowest resonant frequency of the system, the effect on the response of the structure will be small.

If the cosine bell is superimposed on the previous function, the result is shown in Figure 12c and is defined by

$$\begin{aligned} \ddot{x}(t) &= Ae^{\zeta\omega t} \cos \omega t - B \cos^2 \pi t / \tau & (-\tau/2 \leq t \leq 0) \\ \ddot{x}(t) &= Ae^{-\zeta\omega t} \cos \omega t - B \cos^2 \pi t / \tau & (0 \leq t \leq \tau/2) \end{aligned} \quad (12)$$

The function described by Equation (12) is even (i.e., $\ddot{x}(-t) = \ddot{x}(t)$); its integral velocity is odd (i.e., $\dot{x}(-t) = -\dot{x}(t)$); and the second integral displacement is even. Keeping in mind these relationships one can adequately describe the function using the right side or

$$\ddot{x}(t) = Ae^{-\zeta\omega t} \cos \omega t - B \cos^2 \pi t/\tau, \quad 0 \leq t \leq \tau/2.$$

By integrating this function and evaluating the necessary constants, the value of B can be obtained such that the zero velocity boundary conditions are satisfied. The required B is given by,

$$B = \frac{4A\zeta}{\tau\omega(\zeta^2 + 1)} \quad (13)$$

Substituting into the previous equation

$$\ddot{x}(t) = Ae^{-\zeta\omega t} \cos(\omega t) - \frac{4A\zeta}{\tau\omega(\zeta^2 + 1)} \cos^2(\pi t/\tau), \quad (14)$$

$$\dot{x}(t) = \frac{Ae^{-\zeta\omega t}}{\omega(\zeta^2 + 1)} \left[-\zeta \cos(\omega t) + \sin(\omega t) \right] + \quad (15)$$

$$\frac{2A\zeta}{\omega(\zeta^2 + 1)} \left[\frac{1}{2} - \frac{t}{\tau} - \frac{1}{2\pi} \sin\left(\frac{2\pi t}{\tau}\right) \right],$$

and

$$x(t) = \frac{-A\zeta}{\omega(1 + \zeta^2)} \left[\frac{1}{4\tau} (2t - \tau)^2 - \frac{\tau}{2} \cos^2(\pi t/\tau) \right] + \quad (16)$$

$$\frac{Ae^{-\zeta\omega t}}{\omega^2(1 + \zeta^2)^2} \left[(\zeta^2 - 1) \cos(\omega t) - 2\zeta \sin(\omega t) \right], \quad 0 \leq t \leq \tau/2.$$

Figure 12 shows a typical resulting time history. The magnitude of the cosine bell has been exaggerated to illustrate the concept. The normalized shock spectra of a single-frequency SHOC (SHaker Optimized Cosine) for several decay rates is shown in Figure 13. Note that the general character (and hence the advantages) of the spectrum of the decaying sinusoids is preserved. If the pulse duration (τ) is decreased, the required displacement will decrease and because the low-frequency energy has been reduced the shock spectrum will start to roll off at a frequency approximately equal to $2/\tau$. Thus a compromise is frequently required between the lowest frequency at which the shock spectrum can be matched and the displacement available. It has also been found that because of the symmetry about $t = 0$, the pulses will add in the frequency domain (i.e., shock spectrum) as well as in the time domain. This simplifies the construction of complex spectrum spectra when the pulses are summed.

Figure 13. Normalized Shock Spectra for SHOC Pulses

An example of a typical acceleration, velocity, and displacement time history is shown in Figure 14. The time has been shifted by a factor of $\tau/2$ for convenience. Note that the desired boundary conditions have been met (i.e., the initial and final acceleration, velocity, and displacement are all zero).

Figure 14. Acceleration, Velocity, and Displacement of a SHOC Pulse

Examples of Matching Shock Spectra Using the SHOC Pulse

The first example will be for the ATMX rail hump (see Figure 7). The shock spectrum was matched using four frequencies. The resulting shock spectrum is shown in Figure 15. (See Appendix A for a detailed description of the matching procedure.) As can be seen the spectra match well above 3.5 Hz. The low frequency roll-off is a function of τ . If τ is increased, the spectra can be matched at a lower frequency; however, the required displacements will increase. The pulse duration (τ) was picked to limit the displacement at less than 8 inches (a physical limit of Sandia's low-frequency system). The acceleration, velocity, and displacement time histories are shown in Figure 16.

Figure 15. Using the SHOC Pulse to Match the ATMX Rail Hump Shock Spectrum

Figure 16. Time History of the SHOC Pulse Used to Match the ATMX Rail Hump Shock Spectrum

The second example is included to illustrate that relatively smooth spectra can also be enveloped using the SHOC technique. The envelope was developed to represent a portion of a flight environment of a missile system. As can be seen from Figure 17, three superimposed SHOC pulses envelope the required spectrum. The acceleration, velocity, and displacement limits are well within the capabilities of an electrodynamic system.

The third example illustrates a case where the pulse was duplicated in the laboratory. The simulation was of a transient launch vibration of a missile structure. The test setup is shown in Figure 18. As can be seen, the testing was performed on a slip table driven by an MB C-220 electrodynamic shaker. The pulse was reproduced using the LeBrun, Favour technique (see Appendix B). A Xerox Sigma V computer was used to perform the required calculations and to generate the input pulse. The shock spectrum envelope of the field event is shown in Figure 19 as the desired curve. Plus and minus 30 percent lines are included as an acceptable error band. The predicted shock spectrum, shown on Figure 19, is the spectrum of an ideal SHOC pulse with six components (the

Figure 17. The Use of a SHOC Pulse to Match a Smooth Shock Spectrum

Figure 18. Test Setup for SHOC Test

Figure 19. Shock Spectra of a Launch Event and Simulation

components are listed on Figure 19). The time history of the ideal SHOC pulse and the actual pulse measured at the input accelerometer are shown in Figures 20 and 21, respectively. The shock spectrum of the actual pulse is also included on Figure 19. As can be seen, the differences between the predicted and desired spectra are small. In other tests, cases have been found where the reproduced time history differed significantly from the desired pulse due to noise, system nonlinearities, etc. However, the shock spectra usually differed by less than 30 percent. This is not unreasonable as the shock spectrum is a tolerant measurement of the characteristics of a time history.

The acceleration peaks of a SHOC pulse are predominantly positive. For some transient testing a pulse may be desired which has approximately equal numbers of positive and negative peaks of comparable amplitude. This can be accomplished by alternating the signs of the components of the SHOC pulse. For example, if the signs of the components of the previous example are alternated the resulting time history is shown in Figure 22. Of course, the shock spectrum will be changed (see Figure 23). Different amplitudes, frequencies, and damping coefficients could still be picked to match a given spectrum. However, the procedures become slightly more tedious if the signs are allowed to change. In some cases a spectrum can be matched with a smaller required displacement by alternating the signs of the frequency components.

Figure 20. Desired Time History of the SHOC Pulse
Used to Simulate a Launch Event

Figure 21. Actual Time History of a SHOC Pulse
Used to Simulate a Launch Event

Figure 22. Time History of a SHOC Pulse Whose Components Alternate in Sign

Figure 23. Shock Spectrum of the SHOC Pulse Shown in Figure 22

Conclusions

It has been shown that the proposed SHOC pulse can be used to match a great variety of shock spectra. The pulse has characteristics lending itself toward reproduction on existing shaker systems. The technique has been used successfully on electrodynamic shaker systems.

APPENDIX A

PROCEDURES FOR MATCHING A SHOCK SPECTRUM WITH A SHOC PULSE

As was mentioned earlier in the text, the shock spectra of the components of the SHOC pulse can be added to produce the spectrum of the composite. The procedure for matching a complex spectrum is therefore a relatively simple procedure. Three items make this possible. First, the spectrum at frequencies much lower than the frequency of a SHOC component will be small. This implies that the high-frequency components will have little effect on the spectrum at the lower frequencies. Second, the principal effect on the spectrum of a SHOC component will be near the frequency of the component. The shock spectrum at this point can be many times the peak amplitude of the component. And third, at frequencies much higher than the frequency of a SHOC component the spectrum will be equal to the peak amplitude of the component. Since the peak amplitudes of all the components occur at the same time ($t = 0$), the peak amplitude of the composite will be equal to the sum of the peak amplitudes of the components. And the shock spectrum of the composite of high frequencies will be equal to this peak amplitude.

An example will illustrate the procedure. The spectrum to be matched in this example is the shock spectrum shown in Figure 15 of the text. Figure A1 is a form which can be used as an aid. The lowest frequencies are matched first as the higher frequencies to be matched later will have little influence on the values picked for the lower frequencies. In this example, a low-frequency, highly damped (4.0 Hz, $\zeta = 1.0$) component was chosen to match the low-frequency portion of the shock spectrum. The amplitude was determined by dividing the desired level by the amplification factor for this component. The amplification factors can be determined from the normalized shock spectrum (see Figure 13). Next a 13-Hz component was added to match the spectrum in this area. The damping coefficient ($\zeta = 0.1$) was chosen to fit the spectrum around 13 Hz. Again, the normalized shock-spectra curves (Figure 13) were used. The first component will contribute ≈ 9.2 g to the shock spectrum at 13 Hz. This is accounted for in the columns labeled c_1 and e_1 (Figure A1). A third component is added at 32 Hz to match the area around this frequency. Columns c_1 and e_1 now account for 12.1 g contributed by the two lower frequencies. And, finally, a fourth component is added at 118 Hz. It can be seen that the spectrum will approach 23.9 g (the peak amplitude of the composite) at still higher frequencies. The pulse duration of 1.0 was chosen to give a 2-Hz roll-off frequency. The sample rate was chosen at approximately 10 times the frequency of the highest component. The sampling theorem requires a sample rate of twice the highest frequency in the data. The SHOC components contain energy at frequencies above the fundamental frequency. It is felt that 10 times the highest fundamental is an adequate margin.

Figure A1

Fitting Shock Spectra Using SHOC Pulse

Sample Rate (samples/sec) = 1000

Pulse Duration (τ) (sec) = 1.0

i	Frequency (Hz)	ζ	$a_i = e_i/b_i$ Ampl. SHOC (g)	Amplifaction Factor b_i	$c_i = \sum_{j=1}^{i-1} a_j$ (g)	$d_i =$ Desired Level (g)	$e_i = d_i - c_i$ (g)
1	4.0	1.0	9.2	1.2	0	11.	11.
2	13.0	0.1	2.9	5.4	9.2	25.	15.8
3	32.0	0.2	2.3	3.5	12.1	20.	7.9
4	118	0.03	9.7	9.5	14.4 23.9	100	92

NOTE: Roll-off frequency = $2/\tau$

To start with zero velocity $\tau = \frac{1}{f} \left[\frac{1}{180} (\tan^{-1} \zeta) - n \right]$ where n is the number of zero crossings.

The shock spectrum is then computed and compared with the desired spectrum. The resulting displacements and velocities are also computed for inspection. Among other deviations it was found that the displacements were too large (therefore τ was shortened), and the level at 4 Hz was too low (therefore the amplitude of the 4-Hz component was increased). The process was then repeated to develop a refined pulse. Three iterations were required to produce the results shown in Figure 15.

One additional item should be mentioned. It is sometimes desirable to truncate the SHOC pulse before the acceleration has decayed to zero (this is done to shorten τ , which will reduce the required displacements). Within limits this can be done if a zero crossing of the velocity function is chosen (step changes in the acceleration are mechanically consistent, but step changes in the velocity are not consistent). The equation at the bottom of Figure A1 defines the velocity zero crossings for a single component.

APPENDIX B

TRANSIENT WAVEFORM REPRODUCTION ON ELECTRODYNAMIC SHAKERS

The method used to reproduce the SHOC pulses is basically a technique developed by LeBrun and Favour.² (The technique was implemented at Sandia by Normal F. Hunter.) The principal difference is the waveform used for the calibration pulse. LeBrun and Favour used an exponentially decaying pulse. An exponentially decaying pulse plus a fast sine sweep is used in reproducing the SHOC pulses. A block diagram illustrating the technique is shown in Figure B1. A calibration pulse is used as the input to the power amplifier, and the response of the system is observed. The discrete Fourier transform (DFT) is calculated for the input and the response. The ratio is then formed to determine the frequency transfer function, $H(\omega)$, of the system. The frequency transfer function is then stored for later use. The desired pulse is then defined (a recorded time history or analytic function) and the Fourier spectrum, $D(\omega)$, is calculated. A deconvolution is performed by dividing the Fourier spectrum of the desired pulse by the frequency transfer function, $H(\omega)$, to determine the frequency spectrum of the input, $O(\omega)$. An inverse DFT is then calculated to determine required time history of the input.

The technique has been used and is known to work well for reasonably linear systems. Cases have been found where nonlinear behavior can result in poor pulse reproduction. Efforts are being made at the present time to improve the technique for these cases.

Figure B1. Electrodynamic Shaker Pulse Reproduction System

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