AN EVALUATION OF THE NEUTRON KINETICS MODEL USED IN THE DESIGN OF THE FORT ST. VRAIN NUCLEAR GENERATING STATION

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SUMMARY

The average neutron number density will increase when positive reactivity is added to the reactor. In addition, the spatial shape of the flux will change, giving rise to the so-called space-time problem. The time-dependent flux shape gives rise to local fission densities which are larger than those that would be predicted on the basis of only the time-dependent average flux.

For small reactors, the lack of an accurate treatment of the space-time problem may not lead to large errors in estimates of peak fission densities. For large reactors, however, and even in some cases for small reactors, the space-time effect is very important in determining core temperatures, and it must be taken into account in hazards analyses.

A "point" kinetics model has in the past been used extensively to study the transient behavior of small reactors. The time-dependent flux shape cannot be determined with this model since the reactor is treated as a "point". However it is possible to use a point kinetics model in such a way as to yield conservative estimates of peak core and system temperatures, that is, temperature estimates which are higher than those which could be obtained. This method is known as the Adiabatic Approximation. The degree of conservatism is very difficult to determine, however, and this is the chief disadvantage of the method. This difficulty and the attending consequences of an overly conservative and costly protective system are the chief motivations for continuing analytical and experimental work in space-time reactor kinetics.

A discussion of this application of the point kinetics formulation to the analysis of a rod withdrawal accident in the Fort St. Vrain reactor (or the PSC Plant for Public Service of Colorado) is presented below. A familiarity with reactor kinetics calculations is assumed.


THE FORT ST. VRAIN REACTOR

The Fort St. Vrain (PSC) reactor is a 330 MW(e) HTGR scheduled for completion in 1971. It is graphite moderated and helium cooled, and other core constituents are limited to uranium and thorium. The core is about 15 ft. in height and has an equivalent diameter of about 20 ft. It is a "small" reactor as far as graphite reactors are concerned; sustained xenon oscillations are not possible, for example. About 20% of the core volume is occupied by coolant passages, 20% by the thorium-uranium-graphite fuel material, and 10% by the structural core graphite.

The fuel material is a compact mixture of graphite coated fissile particles, graphite coated fertile particles, and a graphite matrix or binder. The fissile particle kernel is ThC$_2$-UC$_2$ with Th/U = 4.25. The uranium is fully enriched. The fertile kernel is ThC$_2$. The particle kernels are typically 200 to 400 $\mu$m in diameter and the graphite coatings are typically 100 $\mu$m thick.

The core is divided into a central refueling region or patch, and 36 surrounding patches arranged in a hexagonal pattern. The fuel lifetime is 6 years, and one-sixth of the core (or about 6 patches) is refueled each year. A significant fraction of the total fuel exposure of 100,000 MWD/tonne comes from fission in the U$^{233}$ which is "manufactured" by neutron capture in the Th$^{232}$.

The uranium and thorium concentrations are both zoned radially to flatten the power distribution - 3 or 4 radial fuel zones appear sufficient. The core is also zoned axially to reduce the peak fuel temperature. Two axial zones appear adequate.

Control is achieved with 74 control rods operated as 37 "rod-pairs". There is one rod pair in each refueling region or patch. The worth of an average control rod is about $0.005 \Delta\rho$. The worst reactivity hazard involves the unplanned withdrawal of a control rod.

BASIC INPUT TO A POINT KINETICS CALCULATION

The basic equations of the point kinetics formalism are approximately:

$$\frac{dn}{dt} = \frac{\delta k_0 - \beta - \delta k_{PB}}{\lambda} n + \sum_{i=1}^{n} \lambda_i C_i$$
\[
\frac{dC_1}{dt} = \frac{b_i n}{\ell} - \lambda_i C_i
\]

where:

\( \delta k_\varnothing \) = perturbing or driving reactivity
\( \beta \) = the delayed neutron fraction
\( C \) = total population of delayed neutron precursors
\( n \) = total neutron population
\( \ell \) = prompt neutron lifetime
\( \lambda \) = delayed neutron decay constant
\( \delta k_{PB} \) = feedback reactivity

= (temperature coefficient) x (change in temperature)

i \rightarrow delayed neutron group.

In a rod withdrawal accident analysis, \( \delta k_\varnothing \) would be the control rod worth. Feedback reactivity \( \delta k_{PB} \), would be dependent on both the temperature coefficients and the temperature changes that take place during the transient. Obviously, an important component of the digital program which solves the kinetics equation is the fluid flow and heat transfer model of the core within whose framework system temperatures are computed.

Since the point kinetics equations represent only the total neutron and precursor populations, and exhibit none of the spatial effects which exist during a transient, their use in a hazards analysis must be with a core. Each physical piece of the model must be considered separately and treated in such a way as to give a conservative answer. In the above equations, there are 5 pieces of input required. These are:

1. The initial or driving reactivity perturbation.
2. The neutron lifetime.
3. The delayed neutron fraction.
4. The temperature coefficients.
5. The heat transfer model.

A rod withdrawal accident will be used as a vehicle to discuss each of these items in more detail.

**Initial Perturbing Reactivity** - The worth of a single rod is determined with two-dimensional diffusion theory (x-y) calculations. The difference in
eigenvalue \( k_{\text{eff}} \) between a "rod-in" calculation and a "rod-out" calculation is assumed to be a good estimate of the rod worth. This will be true provided the rod worth is small, of the order of .02 to .03 \( \Delta k \) or smaller.

The maximum rod worth is believed to be less than .015 \( \Delta k \) at any time in life under any reasonable operating rod program. The qualification "reasonable" is used to exclude rod configurations such as all the rods being fully inserted in half of the reactor and fully withdrawn in the other half. Reasonable configurations which give rise to a power distribution in which the relative power density in the hottest refueling region at full power is less than the design power density of 1.83. They also include symmetric rod arrangements which might be used under some low power conditions which could give rise to relative region power densities greater than the design value. Non-symmetric configurations can occur during the approach to operating temperature. Since the rods must be moved singly, and not in groups of 3 or 6, non-symmetric rod distributions, and therefore some flux tilting, will occur at source power levels.

The rod worth under all normal operating rod configurations is believed to be less than .011 \( \Delta k \).

**Neutron Lifetime** - The neutron lifetime in kinetics calculations is assumed to be equal to its value in the initial critical core, \( \sim 1.8 \times 10^{-4} \) sec. The withdrawal of a rod and/or the operation of the reactor will lead to a longer neutron lifetime. From a hazards point of view, short neutron lifetimes are conservative, although for credible reactivity transients the safety margins are essentially independent of the possible variations in the value of the neutron lifetime.

**Delayed Neutron Fractions** - The delayed neutron fraction is governed by the relative abundances of \( ^{235}U \) and \( ^{233}U \). In the equilibrium core, the effective delayed neutron fraction is least, about .0046, and the use of this is conservative in an accident analysis.

**Temperature Coefficients** - The Doppler and moderator coefficients are least negative at the end of an equilibrium cycle. At this time the core is unrodded and the \( ^{233}U \) concentration is a maximum. These are the coefficients used in accident analyses.
Even though the moderator coefficient at the end of an equilibrium cycle is positive, the coefficient which is effective in a nuclear transient is never less negative than the isothermal coefficient, which is about $-1.5 \times 10^{-5}/^\circ C$. Initially in a transient, the fuel heats up and the temperature coefficient feedback is proportional to the Doppler coefficient which is about $-3.0 \times 10^{-5}/^\circ C$. As heat flows from the fuel to the moderator, the net effective coefficient gets less negative and approaches the isothermal value.

Heat Transfer Model - The heat transfer model in General Atomic kinetics codes treats the average fuel element very well. Feedback coefficients are based on volume-averaged temperatures deduced from this 2-dimensional \((r-z)\) model.

In summary the input (and components) of the point kinetics model are each chosen to be either accurate or conservative. The final result of the calculation is also conservative, as described below.

CORE POWER VS. TIME: POINT KINETICS VS. SPACE-TIME KINETICS

The estimate of the core power level versus time obtained with the above input to a point kinetics calculation would be conservative when compared with an exact 3-dimensional kinetics code. The arguments supporting this statement are given below.

The perturbing reactivity in a rod withdrawal accident is not computed with perturbation theory as was done by Yaskinsky and Henry when they found such large discrepancies between one-dimensional and point kinetics calculations. Instead, a conservative estimate of the actual rod worth is made, which then yields an initial period in the point kinetics calculation at least as short as the actual period.

The negative feedback reactivity in the point model will be less than that in the 3-dimensional model for two reasons. First the (conservative) temperature coefficient which is used will be less negative than is the actual case. And second, for reactivity perturbations in high worth locations in the core (that is, in rings 1 or 2 or 3 in PSC) the worth of the feedback in the actual case is greater than the worth of the feedback.
in the point kinetics model. This is simply because the feedback in the point model is based on an average core temperature, the changes in which have only an average worth.

In PSC, the worst rod-withdrawal accident does involve rods in high worth regions.

The heat transfer and fluid flow model in General Atomic's kinetics codes yields an excellent estimate of the time dependent temperatures in an average fuel element. Again, a volume weighting of the temperature changes (as assumed in the point model) will be conservative provided the actual reactivity perturbation occurs in a high-worth region.

Hence, with the above methods, and providing the rod withdrawal accident involves a high-worth rod, that is, a rod in a neutronically important region of the core, the plot of core power versus time for the point kinetics will be qualitatively as shown below when compared with a 3-dimensional space time result:

![Graph showing core power versus time for point kinetics and 3-dimensional space-time result.](image)

The core power level is overestimated in the point kinetics calculation, a conservative situation from the hazards point of view.

This approach to reactor kinetics is essentially equivalent to the Adiabatic Approximation described by Henry in ref.(2). He also showed that the Adiabatic Approximation gave conservative estimates of the transient power level.
MAXIMUM TEMPERATURES: POINT KINETICS VS. SPACE-TIME KINETICS

Additional features of the Adiabatic Approximation are employed in computing the highest temperatures in the reactor at any time. The basic assumption is that the highest (spatial) power peaking factor experienced at any time during the transient exists during the whole transient. This highest peaking factor might occur in the initial or the asymptotic power distributions, and it might be in the fuel region from which the rod is being withdrawn or in some other region. This is clearly a conservative approach. This limiting peaking factor together with the time-dependent core power lead directly to conservative estimates of peak core temperatures and peak outlet gas temperatures.

For the slow transients which are possible in PSC, there is no overshoot to the time-dependent power level which would not be conservatively estimated in point kinetics calculations. The overshoot is a function of the feedback, and if the feedback in the point model is weaker than in the space-time model, the overshoot will be larger in the former than the latter. In fact, in a three-dimensional model, feedback from the high-power-density region will preferentially reduce the local reactivity in that region. This will flatten the power distribution and tend to reduce peak temperatures. This effect is neglected completely in the point kinetics model. It will be a stronger effect in a rod withdrawal accident from low or source power levels.

CONCLUSION

The modified Adiabatic Approximation used with the point kinetics model in the analysis of PSC rod withdrawal accidents gives a conservative estimate of peak temperature occurring during the transient. Important in this regard are:

1. The rod withdrawal accident must involve rods located in regions of high worth, i.e., greater than average worth.
2. In computing peak temperatures, care must be taken to use the highest obtainable power density for a local region, whether this occurs initially or asymptotically, in the perturbed region or in some other region.

Both of these conditions are fulfilled in PSC accident analyses.