HIGH-FIELD BENDING MAGNETS FOR ISABELLE

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Abstract

Circular bending magnets with a cosine current distribution, a 3 in. aperture, and going up to 40 kG are studied. A magnet is proposed that fulfills the suggested tolerances for ISABELLE. Topics discussed include approximating the cosine current distribution using a ribbon winding, choice of the iron shield parameters, dependence of the multipole terms on the field and the iron shield parameters, the sextupole correction winding, and tolerances for random errors in the coil locations.

I. Introduction

The bending magnets in ISABELLE will go up to magnetic fields of the order of 40 kG. The presence of an iron shield in the magnet is desirable since it reduces by a considerable amount the current required to achieve the desired field, and it also reduces the leakage field by a large factor. However, as the iron saturates at the high-field end, nonlinear terms are introduced into the median plane magnetic field. There are severe tolerances in the magnitude of these nonlinear terms. This proposed tolerance on the nonlinear terms can be written as

\[ \frac{1}{B} \frac{d^2 B}{dx^2} < 1 \times 10^{-3}/\text{in.}^2 \]

If one writes the median plane magnetic field as

\[ B = B_0(1 + b_2x^2 + b_4x^4 + \ldots) \]

then the above tolerance implies that

\[ b_2 < 0.5 \times 10^{-3}/\text{in.}^2 \]

and also that \( b_4 < 0.1 \times 10^{-2}/\text{in.}^4 \) if one desires that \( (1/B) \frac{d^2 B}{dx^2} \leq 1 \times 10^{-3} \) for \( x \leq 0.75 \text{ in.} \) at 40 kG.

The above tolerances stem from the consideration that in view of the very long beam lifetimes one will want to make use of in ISABELLE, one should be cautious about the amount of nonlinearity one allows in the magnetic field. A possible guide that was used in setting the above tolerances is that the nonlinearities present in ISABELLE should not greatly exceed the nonlinearities present in the CERN ISR. The nonlinearity in the magnetic field corresponding to the above tolerances can cause a variation with momentum of the order of Δν = ±0.85 across a horizontal aperture of ±0.75 in., and in this sense the nonlinearity is not small.

One additional requirement which seems worthwhile to satisfy is that the nonlinearity present in the magnetic field should be much smaller than the above tolerance on B'/B for some region near the center of the magnet, and should approach the tolerance value near the outer edge of the good field aperture. Thus if the above tolerance were to turn out to be somewhat in error, one would still be certain of having some aperture with a good field.

The properties of iron magnets, including the effects due to finite permeability of the iron, can be studied through the use of magnet computer programs. The magnet programs available at Brookhaven include GRACY, LINDA, TRIM, and MAGFLD. The first three programs are mesh-iteration programs that allow for the finite permeability of the iron.

II. The Cosine Magnet

The results that are presented in this study are chiefly for the cosine magnet which is shown in Fig. 1. The magnet shown there has a physical aperture inside the main current coils of 3 in. The main current coils have a cosine current distribution which is approximated by four blocks of current within a quadrant, where within each block the current density is constant and proportional to the cosine of the azimuthal angle, and the blocks are 5/8 in. thick. The actual numbers of blocks, and the geometry of the blocks in the ISABELLE bending

magnets, may be different from that shown in Fig. 1. However, the main theoretical results presented should also be valid for the ISABELLE magnets.

In addition to the main current coils, the magnet in Fig. 1 has one correction coil which is used to correct the sextupole term, \( b_2 \), in the magnetic field. This \( b_2 \) coil is excited independently from the main coil, and is shown as simply six rectangular blocks of uniform current density which are spaced at 60° intervals around the magnet, and are located just within the main coil. The \( b_2 \) coil used in ISABELLE magnets may look somewhat different physically but will follow the same general principles which are described below.

An underlying principle in the study of the cosine magnet\(^5\) is that only one independently excited correction coil should be required, which is the \( b_2 \) coil. The iron shield is chosen so that no \( b_4 \) correction coil is needed. This largely determines the space between the main current coil and the iron, which is shown as 5/8 in., and the thickness of the iron shield which is shown as 4.5 in.

### Approximating the Cosine Current Distribution

An exact cosine current distribution may be shown to produce an exactly uniform magnetic field in the entire region inside the current distribution if the iron shield has infinite permeability. There are limitations in approximating the cosine distribution using a finite number of current-carrying conductors. One way of approximating the cosine current distribution is the method used by Sampson\(^6\) and co-workers which uses a 5/8-in.-thick ribbon which is wound in blocks within which the current density is uniform, as in shown in Fig. 1.

The use of rectangular blocks of current immediately limits the obtainable region of uniform field. It is possible to arrange the blocks as shown in Fig. 1, or somewhat differently as suggested by Beth\(^7\), both of which arrangements can be

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shown to make all the harmonics in the magnetic field up to roughly the 4N harmonic vanish, where N is the number of blocks in a quadrant. However, even if one uses a large number of blocks, only a certain fraction of the physical aperture will satisfy the tolerance on the magnetic field of $B''/B < 1 \times 10^{-3}$/in.$^2$. This is shown in Fig. 2, where the percentage of the physical aperture having a good field is plotted against the number of current blocks in a quadrant, for the case where the current coil has a radius of $R_c = 1.5$ in. and the current block thickness $H_c = 5/8$ in., and the iron has infinite permeability.

Another limitation in the ribbon method of approximating the cosine current distribution is due to the finite number of ribbons that one can use due to the nonzero thickness of the ribbons. The net result of these two effects leads to a roughly 63% good field region obtained by using about six blocks in a quadrant. Future studies and the use of other conductor geometries will undoubtedly improve on this result.

III. Computer Study Results for the Magnetic Field Shape

A computer study$^{2,5,8}$ was done using the GRACY magnet program to determine the geometry of a cosine bending magnet which satisfies the suggested tolerances while reaching a field of 40 kG. A goal of this study was to obtain a magnet that needs only one correction coil to correct the sextupole term, $b_2$, in the magnetic field, and which requires no $b_4$ correction coil. $b_4$ is made small by the choice of the magnet parameters, such as the iron shield thickness or the inner radius of the iron shield.

Choice of the Inner Radius of the Main Current Coil

The choice of the inner radius, $R_c$, of the main current coil is primarily determined by the region of good field that is needed at the low field levels existing during injection. It was stated in Section II that, with the ribbon method presently being considered, one may expect about 63% of the region within the main coil to have a good field shape within the suggested tolerances. Thus for $R_c = 1.5$ in., one would get a good field aperture of $\pm 0.95$ in.

Choice of the Iron Shield Parameters

A goal of this study was to obtain a magnet that required only one correction coil for the sextupole term, \( b_2 \). This means that the magnet parameters must be chosen so that the \( b_4 \) term is small enough not to require correction. This can be done by proper choice of the parameters of the iron shield, which are the inner radius of the iron shield, \( R_{Fe} \), and the thickness of the iron shield, \( t \).

If one has already chosen the inner radius, \( R_c \), of the main current coil, and the thickness of the current coil, then the choice of \( R_{Fe} \) can be considered as choosing the distance, \( d \), between the outside of the current coil and the iron shield. One would expect that as one increases this distance, \( d \), the saturation effects will go down, and the \( b_2 \) and \( b_4 \) due to saturation effects will also decrease.

In Fig. 3, \( b_2 \) and \( b_4 \) are plotted versus the central magnetic field, \( B \), for various choices of \( d \) from \( d = 1/8 \) in. to \( d = 5/8 \) in., or of \( R_{Fe} = 2.25 \) in. to \( R_{Fe} = 2.75 \) in. The inner radius of the current coil has been assumed to be \( R_c = 1.5 \) in., the thickness of the coil is \( 5/8 \) in., the thickness of the iron shield is \( t = 4.5 \) in., and the current coil has four blocks per quadrant as shown in Fig. 1.

We assume a tolerance on \( b_4 \) of \( b_4 < 0.1 \times 10^{-3} \) /in.\(^2\), which corresponds to \( B''/B < 1 \times 10^{-3} \) /in.\(^2\) for \( x < 0.75 \) in. One notices that the \( b_4 \) vs \( B \) curve has the interesting, and not easily understood, property of reaching a peak in the 35 to 45 kG region. If one requires that this peak in the \( b_4 \) curve be smaller than \( b_4 = 0.1 \times 10^{-3} \) /in.\(^2\), then this leads to a choice of \( d = 5/8 \) in. or \( R_{Fe} = 2.75 \) in.

The thickness, \( t \), of the iron shield also affects \( b_2 \) and \( b_4 \). One would expect that increasing \( t \) would decrease the saturation effects and decrease \( b_2 \) and \( b_4 \). In Fig. 4, \( b_2 \) and \( b_4 \) are plotted versus \( B \) for various choices of the thickness of the iron shield from \( t = 2.5 \) in. to \( t = 4.5 \) in. The coil to iron distance is \( 5/8 \) in. or \( R_{Fe} = 2.75 \) in. Figure 4 shows that the iron shield thickness of \( t = 3.5 \) in. is close to the critical thickness that causes \( b_2 \) to vanish. However, \( b_4 \) is somewhat above the tolerance at 40 kG and is increasing rapidly. On the other hand, the thickness of \( t = 4.5 \) in. has a \( b_4 \) which is below tolerance at 40 kG and is not increasing rapidly, so that this magnet may be satisfactory at 50 kG. The thickness of \( t = 4.5 \) in. also gives a lower leakage field. The leakage field just outside the iron shield on the median plane is 20 G for \( t = 4.5 \) in. and 180 G for \( t = 3.5 \) in.
Figure 6

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$R_c = 1.5$

$d = 5/8$

$b_2 / 10^{-3}$ (in.~²)

$b_4 / 10^{-4}$ (in.~⁴)

$B$ (kG)

$t = 4.5$

$t = 3.5$

$t = 2.5$

-0.4

-0.3

-0.2

-0.1

0

20 40 60

B (kG)
It would appear that the choice of iron shield parameters of \( t = 4.5 \) in., \( d = 0.625 \) in., \( R_e = 2.75 \) in. would be satisfactory. Table I lists some properties of this magnet and of two other magnets with slightly different parameters. The gradient and \( B''/B \) as a function of \( r \) are shown in Fig. 5 for the case \( t = 4.5 \) in., \( d = 0.625 \) in., \( R_e = 2.75 \) in., and with five current blocks per quadrant. In Fig. 5, the sextupole term in the magnetic field has been cancelled out using the sextupole correction coil.

**TABLE I.** A summary of some of the properties of three magnets. The magnets are distinguished by \( d \), the distance between the main coil and the iron shield, and \( t \), the thickness of the iron shield. The results are for 40-kG excitation. SE is the stored energy, NI is the required ampere turns, and \( B_{\text{leak}} \) is the leakage field just outside the iron shield on the median plane. The inner radius of the main current coil is \( R_c = 1.5 \) in. and the current block thickness is \( H_c = 5/8 \) in.

<table>
<thead>
<tr>
<th>( d ) (in.)</th>
<th>( t ) (in.)</th>
<th>( b_2/10^-3 ) (in.(^2))</th>
<th>( b_4/10^-4 ) (in.(^4))</th>
<th>NI (kA)</th>
<th>SE (kJ/m)</th>
<th>( b_6/10^-3 ) (in.(^6))</th>
<th>( b_8/10^-3 ) (in.(^8))</th>
<th>( B_{\text{leak}} ) (kG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.375</td>
<td>4.5</td>
<td>2.8</td>
<td>-0.18</td>
<td>386.0</td>
<td>52.3</td>
<td>-0.023</td>
<td>0.000</td>
<td>0.050</td>
</tr>
<tr>
<td>0.625</td>
<td>4.5</td>
<td>0.88</td>
<td>-0.086</td>
<td>405.0</td>
<td>54.5</td>
<td>0.003</td>
<td>0.000</td>
<td>0.020</td>
</tr>
<tr>
<td>0.625</td>
<td>3.5</td>
<td>0.27</td>
<td>-0.125</td>
<td>407.0</td>
<td>54.6</td>
<td>0.002</td>
<td>0.000</td>
<td>0.180</td>
</tr>
</tbody>
</table>

**Sextupole Correction Coil**

A simple approximation to the \( \cos 3\theta \) current distribution is to have coils with uniform current density at \( 60^\circ \) intervals with alternating current directions. This is shown in Fig. 1 where the sextupole coil consists of six rectangular blocks placed just within the main current coil. This simple approximation to a \( \cos 3\theta \) distribution will contain a \( \cos 9\theta \) harmonic which would generate a \( b_8 \) term in the median plane field. The tolerance on \( b_8 \) is assumed to be about \( b_8 < 0.018 \times 10^-3 \) in.\(^8\), which results from assuming that \( B''/B < 1 \times 10^-3 \) in.\(^2\) at \( x = 1 \) in.

The \( b_8 \) introduced by the sextupole coil can be controlled by properly choosing the azimuthal extent of the current block. There is an azimuthal extent for which \( b_8 \) will vanish, as is shown in Fig. 6 where \( b_8 \) is plotted versus the azimuthal extent, \( w \), of the current block, for a case where the current radius \( R_c = 1 \) in. These results can be scaled to other values of \( R_c \). \( b_8 \) vanishes
$R_c = 1.5$

N BLOCK $= 5$

d $= 5/8$

t $= 4.5$

Figure 5
Figure 6

\[ R_c = 1.0 \]
\[ d = 1/8 \]
\[ H_c = 5/8 \]
for \( w = 0.885 \) in. for the case where the blocks are tangent to a circle of radius 1.137 in. and are 5/8 in. thick.

This method of eliminating \( b_8 \) is similar to that suggested by Beth, where one considers each current block as made of two current-carrying wires whose separation is chosen to eliminate \( b_8 \).

The sextupole coils were placed further in than the main current coil since the region of acceptable field is about 0.95 in., and the sextupole coils do not then reduce the region of good field. The exact radial location of the sextupole coil does not appear to be critical.

In order to cancel out a \( b_2 \) of about \( b_2 = 8 \times 10^{-3} \) in.\(^2\), which is about the worst case that is likely to arise, the sextupole coil will require a small current which is about 0.0016 of the total current in the main coil.

IV. Tolerances on Coil Locations

Errors in the positions of current-carrying coils will produce perturbing magnetic fields in the median plane. These perturbing fields can move the central orbit, change the \( \nu \)-value and, if the perturbing fields are nonlinear, they can excite harmful nonlinear resonances.

For the purposes of this study, the current distribution will be assumed to be made up of four or five current blocks per quadrant, and error fields will be considered which are due to a displacement of these current blocks as a whole. The current blocks are made up of many ribbons or conductors, but one may expect that the more severe tolerances will be found when the current block moves as a whole.

The displacement of a single block of current will produce a perturbing field in the median plane which can be broken down into field multipoles. All multipoles will be generated, and the perturbing field will have a radial component as well as a vertical component in the median plane. If one writes each component in the perturbing field in the median plane as

\[
\Delta B = B_0 (\delta_0 + b_1 r + b_2 r^2 + b_3 r^3 + \ldots )
\]  

(4.1)

then the rms $b_\ell$ generated in each component of the magnetic field is given by

$$b_{\ell, \text{rms}} = \sqrt{\frac{2}{N_b}} \frac{\ell + 1}{R^{\ell + 1}} c$$

(4.2)

where $N_b$ is the total number of current blocks in the magnet. $R$ is the average radius of the circular current distribution. $c$ is the rms current block displacement. This formula is a rough formula which is obtained for a current distribution which is very thin, with no iron shield present. It is valid for the lower order multipoles for which $\ell \ll N_b$. The presence of iron does not change this result greatly and in general reduces the size of the multipoles.

The considerations given below lead to a tolerance on the coil locations of about

$$c < 2.5 \times 10^{-3} \text{ in.}$$

Displacement of the Central Orbit

An rms error in the position of current blocks will produce an rms dipole field, $b_o$, in the radial component of the magnetic field which causes a vertical displacement of the central orbit and which often gives rise to one of the more severe tolerances.

Assuming that the central orbit displacement gives a tolerance on the error dipole field in each magnet of

$$b_{o, \text{rms}} < 0.5 \times 10^{-3}$$

then one finds a tolerance from Eq. (4.2) of

$$c < 2.5 \times 10^{-3} \text{ in.}$$

where we assumed $N_b = 20$, $R = 1.8$ in.

Random Nonlinear Magnetic Fields

In previous sections we considered nonlinear terms in the magnetic field which are due to saturation of the iron, or due to the current distribution not

being exactly the required cosine distribution. These nonlinear terms, 
$b_2, b_4, \ldots$ are expected to be consistent or the same from magnet to magnet. 
For this consistent $b_2, b_4$ we assumed the tolerances

$$b_2 < 0.5 \times 10^{-3}/\text{in.}^2$$
$$b_4 < 0.1 \times 10^{-3}/\text{in.}^4$$

The tolerance on $b_4$ is $0.08 \times 10^{-3}/\text{in.}^4$ at low fields as the aperture required 
there is $x = \pm 1$ in.

Random errors in the location of the current blocks will introduce random 
b$_2$, b$_4$ which are different from magnet to magnet. These random b$_2$, b$_4$ can be 
estimated from Eq. (4.2) using $\epsilon = 2.5 \times 10^{-3}$ in. rms error in the coil positions, 
and one finds

$$b_{2,\text{rms}} = 0.41 \times 10^{-3}/\text{in.}^2$$
$$b_{4,\text{rms}} = 0.13 \times 10^{-3}/\text{in.}^4$$

For $\epsilon = 2.5 \times 10^{-3}$ in., the random $b_2, b_4$ are roughly equal to the suggested 
tolerances for the consistent $b_2, b_4$. However, the azimuthal harmonics due to 
the random $b_4$ will be smaller than $b_4$ by the factor $\sqrt{M}$, where $M$ is the number of 
magnets in the ring, which factor is about 15. Thus the azimuthal harmonics due 
to the random $b_4$ will be smaller than the harmonics due to the consistent $b_4$ by 
a factor of about 15 for an rms error in the coil positions of $\epsilon = 2.5 \times 10^{-3}$ in.

In spite of the smaller harmonics, the effects of the random $b_4$ may still 
be worrisome if the $\nu$-value of the accelerator is too close to some nonlinear 
resonance. The random $b_4$, unlike the consistent $b_4$, will give rise to essentially 
all the lower harmonics and can excite nonlinear resonances close to the $\nu$-value 
of the accelerator. This happens, for example, when one wants to use a slow 
extraction scheme based on using a one-third resonance. If some nonlinear 
resonance driven by the random $b_4$ causes difficulties, then one can reduce the 
particular harmonics which are causing trouble by exciting seступoles and octupoles around the ring.

Random Gradients

Random errors in the location of the blocks will also introduce random \( b_1 \) terms which are different from magnet to magnet. The random \( b_1 \) can be estimated from Eq. (4.2) using \( \varepsilon = 2.5 \times 10^{-3} \) in., and one finds

\[
b_{1, \text{rms}} = 0.49 \times 10^{-3} / \text{in}.
\]

The random \( b_1 \) can introduce a small shift in the \( n \)-value, and narrow stopbands at half-integer \( n \)-values of the order of \( \Delta n = 0.01 \) for \( n = 17 \). The presence of long straight sections may increase this result for \( \Delta n \) by a factor of 2 or 3.

V. Additional Correction Coils

The goal of this study was to design a bending magnet that required only a separately excited sextupole coil to control the sextupole term, \( b_2 \), in the median plane magnetic field. It would seem desirable nevertheless to have additional correction coils to control \( b_4, b_6, \) and \( b_8 \). It may also be desirable to have coils to control \( b_1 \) and \( b_2 \), which would ordinarily be absent because of the symmetry. The question of the number of additional correction coils and their geometry and power supplies needs to be studied further.