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AIR POLLUTION, TURBULENCE AND DIFFUSION
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Edited by H. W. Church and R. E. Luna
Atmospheric Fluid Dynamics Division 5644
Sandia Laboratories
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ABSTRACT

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~~University Park, Pa.~~
Las Cruces, N. Mex.

Key words: Meteorology, environment, atmosphere

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SYMPOSIUM AIMS AND GOALS

Robert E. Luna
Sandia Laboratories
Albuquerque, New Mexico

At the first symposium of this series held in Albuquerque in December, 1967, we at Sandia had the hard job (it seemed at the time) of making all of the physical arrangements for the meeting, and Henry Rachele of WSMR performed this task. We looked forward to this day when we would have the easy job. Things, of course, are not always what they seem, and this job is not an exception.

It is difficult to state a realistic set of aims and goals for a gathering such as this, because of the disparities in the training, background and interests of those who have come together here. I have, however, a precedent; Henry, in his aims and goals statement of four years past*, resorted to the dictionary** for "official" definitions of the key words in that symposium's title. While I lacked the originality to search out a new approach, I would not inflict upon you the definitions of those same words. Instead, I looked up the word symposium in order to glean the facts and requirements of our current undertaking. I discovered at once (and I am sure many of you practiced symposium-goers are aware of this) that we would have little trouble in achieving consonance with the first definition listed which is "a drinking party especially one following a banquet and providing music, singing and conversation." I would wager that most of us are more practiced at symposiums than our travel vouchers and expense accounts would attest.

The second definition, "a meeting at which several speakers deliver short addresses on related topics or various aspects of the same topic", is somewhat closer to what the organizers (some of the organizers) had in mind in planning this meeting - yet there is something stated quite clearly in the first definition but only implied in the second which is the primary goal of this symposium. Conversation. People speaking to one another and exchanging information: not just "delivering short addresses."

We are interested, of course, in being the medium of communication among people working in the same general field, such as those in boundary layer dynamics or numerical prediction methods or air resources management. We are somewhat more interested in acting as the vehicle in bringing ideas from one field where they may not be new or even novel into an area where that idea may be exactly what is required to initiate a solution to a vexing problem. We have attempted to carry out this general goal by accepting a wide variety of papers from people both within and outside of the establishment group usually seen at this sort of meeting. To what degree this policy is successful is yet to be seen, but we hope that attendance at this meeting will provide someone the germ of an idea for a new or improved approach to a problem.

* Sandia Laboratories Report, SC-M-68-191, May 1968.

** Webster's Third New International Dictionary, G. & C. Merriam Co., Springfield, Massachusetts, 1967.

To put this general ideal of fostering communication into a more specific framework, I would offer the following questions which might be answered or at least fully discussed in the course of this symposium (in particular, we offer the informal session Tuesday evening for the purposes of unstructured discussion).

1. If the solution of air pollution problems is largely a political (enforcement) problem, is there any justification for diffusion modeling and boundary layer studies in the air pollution fields?
2. Is there any hope that the data and theory for flows (atmospheric or wind tunnel) over a vast flat plane, in near-neutral and steady conditions can ever be applied to conditions in the real world?
3. Does the direct solution of the Navier-Stokes equations, which is made possible by the current generation of computers, provide realistic solutions to real atmospheric flows of interest in air pollution studies?
4. Is the spatial and temporal density of measurements required to understand the state of the atmosphere within a city within the financial or physical capability of any organization interested in the data?
5. Is there information in the detailed measurements of atmospheric turbulent variables such as, coherence, quadrature spectra, covariance, etc., which is useful in the theory or practice of transport and diffusion prediction?
6. Are present sensor systems for measurements of atmospheric variables adequate for turbulence and diffusion studies on the microscale? mesoscale? Global scale?

While these questions may appear in some small way to be overstating the issues, I believe that they provide a starting point to discussions which may pinpoint what the current state-of-the-art is and help decide what the future state-of-the-art should be.

The Role of the Meteorologist in "Air Resource Management"

Victor H. Sussman, Director
Bureau of Air Quality and
Noise Control
Commonwealth of Pennsylvania

In April of this year I presented a talk on the principles of air resource management at a meeting of the American Meteorological Society in Raleigh, North Carolina. Shortly thereafter I received a call asking if I would come to this meeting and discuss the role of the meteorologist in air resource management—especially as it relates to the development of implementation plans under the Clean Air Act Amendments of 1970. This, as many of you know, is a critical period in the timetable set in the Clean Air Act. It is also a critical period for "air resource management"—in that it is being applied, really for the first time, and the results of these evaluations are being used to make decisions which will have lasting and significant social and economic impacts in this country.

The science of the air resource management system was legislatively mandated by Congress—and I am sure that there are many in EPA who can prove that its infallibility is prescribed by the Constitution and/or the Bill of Rights. Variations on a theme by Muskie (if I may refer to them as such) have been published in the Federal Register by EPA as regulations and standards for the preparation of State implementation plans. The States are now trying to follow this scheme to prepare the plans for submission to EPA by the end of next month.

To get some information on the meteorologist's role in the preparation of implementation plans, I discussed the Federal Register regulations and the plans being developed with meteorologists on my staff, people involved in diffusion modeling for other State agencies and some Feds. As a result of this investigation, I have come to the conclusion that the role of the meteorologist in developing implementation plans is similar to that of a priest administering the final rites. After pragmatic decisions based on the facts of life have been made, he gives you absolution, moral support and assurances that everything will work out well. By this I mean that for the most part implementation plans are being developed in the following manner—emission standards and regulations are developed based upon realistic enforcement techniques and what can reasonably be achieved ("the best available technology"). The meteorologist then judiciously selects the proper diffusion equation and constants to demonstrate that when the chosen emission standards are enforced, the Federally promulgated ambient air quality standards will be achieved. If this implies that in the area of air resource management, the meteorologist's role is somewhat quasi-scientific and philosophical—then it is consistent with the entire Federal scheme.

During this rush towards the preparation of implementation plans, I have observed that meteorologists in the field are divided into two broad categories—the ones who are actively involved in the mumbo-jumbo of developing emission standards from ambient standards, and the few others who will not be associated with this witchcraft. Of those who are involved there are also two broad categories. The first, ("the informed") who realize that their role is just so much window dressing, and are playing the game with a full realization of the sham of the whole business. Even though this group knows what's going on, I have detected strong cynicism and extreme discomfort with respect to the condition they now find themselves in. In the other category are placed those meteorologists who sincerely believe that at the present level of the state of the art and knowledge, we are performing meaningful evaluations which will lead to meaningful regulations and "cleaner air." They point out that everything has been done to make the meteorologist's life easy in this system—for example, standards have been set as annual means to make modeling simple. But it is just for such reasons that it can only be concluded that this group is comfortable in the system because they don't know what's going on. The other group—the small group which will not "play the game" of diffusion modeling with insufficient information and within a system that begets and compounds errors—this other group is composed of an elite of starving consultants.

I guess that it is now up to me to defend my position that: 1) There are built-in errors and subjective decisions in various elements of the air resource management system; 2) That the system is designed to compound these errors— 3) Inputs from meteorologists in this system are primarily for "window dressing."

An extemporaneous discussion of the air resource management system was presented. This discussion was based upon the following publication—"New Priorities in Air Pollution Control," by V. H. Sussman, Journal of the Air Pollution Control Association, Volume 21, Number 4, pages 201 to 203, April, 1971.

The discussion included an analysis of the errors involved in:

- 1) The designation of air quality control regions—it was pointed out that these are not true "air basins" and are not susceptible to accurate modeling techniques.
- 2) The issue of "air quality criteria"—it was pointed out that over-simplified conclusions were drawn from insufficient information.
- 3) The establishment of "ambient air quality standards" from the criteria—it was pointed out that erroneous interpretations of the criteria documents led to the establishment of standards of questionable validity. Also, the public is interested in the "cleanest air possible" and ambient standards that have been established are approaching background levels.
- 4) The development of emission standards, using emission inventory and diffusion models of questionable accuracy—it was pointed out that meteorologists are attempting to relate "Q's" and "X's" that are not necessarily the same substances. For example, the "particulate matter" which is emitted from a power plant stack {Q} is not necessarily the same material as the "particulate matter" {X} as measured by a high volume sampler.
- 5) Measuring air quality to determine progress towards obtaining ambient air quality standards—the limitations of ambient air sampling techniques was discussed and the lack of standardized methods was emphasized.

The August 14, 1971 Federal Register contains guidelines for the preparation of implementation plans. Meteorologists are now directly involved in Section 420.16—emergency episodes, Appendix A—Air Quality Estimation and Appendix H—Air Quality Data Summary. It is really only with respect to the emergency alert procedures that I believe that meteorologists are having objective meaningful input. That is because (at least in my experience) the meteorologists have been directly involved in a program operation—not an academic exercise.

That is, when atmospheric science becomes involved in direct operations instead of carrying on an insulated love affair with computers, data manipulations and self-serving experiments—it can do a job. Too many self-styled atmospheric scientists—who have no concept of the practical problems involved—have hopped onto the environmental bandwagon. They are using up resources (and over-dignifying this politically developed "science") in trying to justify regulatory standards on the basis of diffusion modeling. These resources can be put to better use in areas where we need real help from atmospheric science:

- 1) Criteria for the location of air monitoring stations.
- 2) Criteria for land resource management techniques.
- 3) Forecasting air pollution alerts.
- 4) Some common sense input to establish more effective substance and less meaningless form to our efforts.

Written discussion received after the symposium:

Comments on "The Role of the Meteorologist in 'Air Resource Management'," by Victor Sussman (received 18 January 1972):

Mr. Sussman's oft repeated tirade would appear to have us deny any knowledge of the causal chain between air quality and the sources of pollution. Any viable and lasting scheme for air pollution control on a national scale must involve consideration of cost effectiveness. So long as pollution continues to be released into the atmosphere, its economic control having regard to the demands of public health and welfare must necessarily utilize the available knowledge of this causal chain and hence meteorological dispersion modeling.

R. A. McCormick, NOAA
Director
Division of Meteorology

Author's reply (received 3 February 1972):

The thrust of Mr. McCormick's "oft repeated" diatribe is that—even though air resource management is a crooked wheel, it is the only wheel in town and therefore we have to gamble against it. My point is that there is a place for diffusion modeling but that its application in the present air resource management concept, with our existing knowledge, is not only premature, but is leading to costly, invalid decisions.

I completely agree that we have to involve cost effectiveness considerations in air pollution control programs—and that is why I am concerned about how the mandated air resource management concept is being used. The system itself could be considered as something like a computer (i.e., a data massaging machine). The system is logical but, as with any system, its meaningfulness depends almost entirely upon the quality of its input information. The computer people often talk about "GIGO" (GARBAGE IN, GARBAGE OUT). A form of "GIGO" exists as the air resource management system is presently being implemented—but what is happening, is, GARBAGE IN, GOSPEL OUT.

Mr. McCormick has misinterpreted my position, in his defense of the meteorologist's input into air pollution control programs. To clear this up once and for all ("and I would like to make this very clear" in language that I am sure conforms to Federal standards) I categorically state that I do not now, nor have I ever advocated the overthrow of meteorological diffusion modeling systems. I recognize the "causal chain," although at present I feel it is more of a "casual" chain. Diffusion modeling has its place—but, in view of the quality of information we now have (emission inventory, ambient air quality, control technology) they should be employed judiciously with respect to the development of emission standards.

COOLING TOWER PLUME RISE AND CONDENSATION

Steven R. Hanna
Air Resources Atmospheric Turbulence and Diffusion Laboratory
National Oceanic and Atmospheric Administration
Oak Ridge, Tennessee

1. INTRODUCTION

Recent regulations discourage the use of river or lake water for direct cooling in industrial processes. The trend is towards the increased use of evaporative cooling towers, in which hot water is cooled by the evaporation of part of the water into a passing air stream. The vertical flow of air through the tower can be driven by natural buoyancy forces or by fans. In order to estimate the effects on the environment of heat and moisture discharges from proposed cooling towers, it is necessary to develop theoretical or empirical models of these physical processes and verify them with observations. It is clear from the few environmental impact statements that we have seen (references 1 and 2 are good examples) that there currently are no standard methods for estimating plume rise and cloud formation from cooling towers. Furthermore, observations of cooling tower plumes are very limited. In this paper, methods of estimating cooling tower plume rise and the possibility of condensation will be outlined.

Our ultimate goal is to reduce the environmental impact of plumes from cooling towers as much as possible. The environmental impact can be described by several criteria, including increases in local and regional temperature, cloudiness, fog, and rainfall. Local surface effects depend strongly on plume rise. Regional temperature and moisture increases depend on the non-linear interaction between the heat and moisture in the plume and in the environment. As suggested by Hanna and Swisher⁽³⁾, the regional problem can be analyzed using mesoscale or synoptic-scale computer models of the atmosphere. The Gaussian plume model outlined by Slade⁽⁴⁾ may not be practical for estimating dispersion on regional scales (10 km or greater).

2. PLUME RISE

In this section problems involving total plume rise and the effects of tower configuration are discussed. In order to provide a comprehensive list of useful equations in the space available, lengthy derivations of many of the formulas will be omitted. Complete derivations are given in previous reports by Hanna⁽⁵⁾,⁽⁶⁾ and Briggs⁽⁷⁾.

2.1 Basic Plume Rise Formulas

According to Briggs⁽⁷⁾, the final rise H of dry plumes dominated by buoyancy in a stable atmosphere can be approximated by the equations:

$$H = 5.0 F_o^{1/4} s^{-3/8} \quad (\text{Calm}) \quad (1)$$

$$H = 2.9 (F_o / U s)^{1/3} \quad (\text{Windy}), \quad (2)$$

where U is the wind speed, s is the stability parameter $(g/T_p)(\partial\theta_e/\partial z)$, and F_o is the initial sensible heat flux

$$F_o = (g/T_p) w_o R_o^2 (T_{po} - T_{eo}).$$

The factor π is omitted from all flux formulations. The parameters g , T , θ , w , and R are the acceleration of gravity, the temperature, the potential temperature, the vertical speed of the plume, and the plume radius, respectively. The subscripts p, e, o, s represent plume, environment, initial, and saturation variables, respectively. In these equations, plume rise depends only on the initial energy flux from the tower and the wind speed and stability of the environment. Equations (1) and (2) have been verified by observations of plume rise from conventional smoke stacks over many orders of magnitude of initial heat flux F_o .

Unlike most conventional smoke stack plumes, the cooling tower plume carries more latent heat than sensible heat. For the range of typical cooling tower parameters listed in Table 1, about 90% of the energy leaving the tower is in the form of latent heat. If this latent heat is not released to the atmosphere through condensation, then plume rise can be calculated using the sensible heat flux F_o in equations (1) or (2). In order to account for the difference in molecular weight between air and water vapor, the virtual temperature T_v should be used rather than actual temperature T . The resulting plume rise calculations differ by less than 10%.

TABLE 1

Buoyancy due to molecular weight differences, and to potential latent heat release, divided by buoyancy due to initial temperature differences. Typical cooling tower plumes and environment initial temperature and environment relative humidities 60% are considered. The plume is initially saturated.

T_{po}	T_{eo}	$\frac{.61(m_{po} - m_{eo})T_{po}}{(T_{po} - T_{eo})}$		$\frac{L(m_{po} - m_{eo})}{c_p(T_{po} - T_{eo})}$	
		100%RH	60%RH	100%RH	60%RH
305°K	275°K	.15	.16	2.32	2.48
315	285	.26	.29	3.91	4.24
305	285	.19	.22	2.88	3.37
315	295	.31	.39	4.63	5.68
305	295	.24	.36	3.60	5.43

If all the latent heat is released and all the resulting liquid water drops from the plume, then the total heat flux F_o is the sum of the sensible heat flux and the latent heat flux:

$$F_c = w_o R_o^2 g \left[\frac{T_{po} - T_{eo}}{T_{po}} + \frac{L}{c_p T_{po}} (m_{po} - m_{eo}) \right] \quad (3)$$

where L is the latent heat of condensation, c_p is the specific heat of air at constant pressure, and m is the water vapor mixing ratio. The release of latent heat thus can increase the heat flux F_o several times and can theoretically increase plume rise by 50 to 80% for the cases considered in Table 1. If only a fraction of the initial latent heat flux is released, then the second term in equation (3) must be multiplied by this fraction in order to more accurately estimate plume rise. Techniques for calculating the fraction of latent heat released are suggested in Section 3.

The plume rise calculated using a numerical cloud growth mode (5) was compared with the rise predicted using equations (1) and (2) for a variety of initial and environment conditions. For dry plumes, agreement was $\pm 10\%$ and for cases in which condensation occurred, agreement was $\pm 50\%$. Consequently, it is believed that the simple equations (1) and (2) can be used to estimate plume rise within an error of $\pm 50\%$, even when a cloud forms in the plume.

2.2 Influence of Initial Radius on Plume Rise

If the energy flux, F_o , volume flux, $V_o = v_o R_o^2$, and environmental conditions are constant, then the question arises of whether plume rise can be increased by varying the cooling tower radius. Equations (1) and (2) are derived by assuming point sources of heat but are well proven by observations. If the full set of governing equations for plume rise are integrated, then factors such as initial source radius can be shown to have minor effects.

Since the relative rate at which environment air is entrained into the plume is inversely proportional to plume radius, it can be argued that the plume from a wide source will lose its buoyancy more slowly than a similar plume from a narrow source. On the basis of laboratory simulations in a calm environment, Brown and Sneek(8) state that increases in source radius can increase plume rise by several percent. However, integrations of the equations of motion(6) show that Brown and Sneek's conclusion is valid only for values of the dimensionless parameters $s R_o/b_o$ less than unity and $w_o^2 s/2 b_o^2$ less than two, where $b_o = F_o/v_o$. This usually occurs for small, buoyancy dominated energy sources (F_o less than about 10 megawatts). Since Brown and Sneek's model source is characterized by $s R_o/b_o$ less than .1, their observations validate the theory. However, for the energy fluxes typical of large hyperbolic cooling towers (100 or more megawatts per unit), it is predicted that increases in tower radius will decrease plume rise slightly. A good general rule of thumb is that when the ratio of expected plume rise to source radius drops below about ten, then increases in source radius (at constant volume and heat flux) will not increase total plume rise.

2.3 Downwash of Plume

Even if it were theoretically possible to increase plume rise by increasing the size of the tower opening, the resulting low efflux speed, w_0 , could result in the downwash of the plume behind the tower during windy conditions. Briggs⁽⁷⁾ suggests that downwash of the plume from a narrow, straight-sided stack will occur if the ratio w_0/U is less than about 1.5, for initial stack Froude numbers ($Fr_0 = w_0^2/2 R_0 b_0$) greater than about five. Overcamp and Hault⁽⁸⁾ on the basis of laboratory simulations and theoretical work, suggest that the critical value of w_0/U decreases to .5 at Fr_0 equal to .25 and decreases further to .2 at Fr_0 equal to .01. The Froude number Fr_0 ranges from about .5 to 1.5 for the typical data in Table 1, and for efflux speed and radius equal to about 5 m/sec and 30 m, respectively. Since the Froude number for most large cooling towers is on the order of unity, this theory predicts that downwash is likely to occur if the wind speed U is greater than the efflux speed w_0 . However, downwash is not observed, for example, at the large Keystone hyperbolic cooling towers, where the environment wind speed, U , often exceeds the efflux speed, 5 m/sec. The inhibition of downwash is probably partly due to the tower's hyperbolic shape, which displaces the low pressure zone downwards from the lip of the tower.

Downwash has been observed at banks of mechanical draft towers, where the bulky shapes of the towers present more of an obstacle to the airflow. The probability of downwash can be decreased by separating the individual towers, increasing the efflux speed, and encouraging flaired geometrical shapes. If the possibility of downwash can be eliminated, then problems associated with rain-out and fogging at the ground near the towers can be reduced.

2.4 Multiple Sources

The plume rise theory outlined above applies only to single, isolated, sources. At some large power plants, the problem exists of calculating the plume rise from a line of about ten mechanical draft cooling towers. The total rise of all the plumes is probably greater than the rise calculated using the heat flux from a single tower but less than the rise calculated using the combined flux from all the towers. Based on an unpublished analysis of TVA data, Briggs suggests that the total plume rise is influenced by the spacing of the towers and the angle between the wind direction and the line of the towers. He defines a "spacing factor," S , as the ratio of the crosswind component of the spacing of the towers to the plume rise expected from a single tower. The closer the towers, the more the individual plumes will interact. The rough rule that he suggests is that for S less than .1, the plume rise in windy conditions from two, three, and four towers will be 20%, 30%, and 40%, respectively, greater than the rise calculated for a single tower. For S greater than .25, interaction of the plumes can be ignored. Unfortunately, there are currently no published data on plume rise either from more than four multiple sources or from any banks of cooling towers to check these suggestions. There is a need for basic observations of cooling tower plumes.

On the basis of the discussions with respect to the problems of downwash and plume rise from multiple sources, it is clear that the use of a single hyperbolic tower will result in greater plume rise than the use of banks of mechanical draft towers, for a given heat flux. A single hyperbolic tower is not as susceptible to downwash as a bank of smaller towers, and its single plume rises higher than their combined plumes. Also, large hyperbolic towers have the added benefit of being taller than mechanical draft towers.

3. CONDENSATION IN THE PLUME

Qualitatively, condensation occurs in a plume if the flux of water from the cooling tower is sufficient to saturate the initial volume flux plus the flux of air entrained into the plume as it rises. It is important to know the characteristics of the entrained air since the flux of entrained air in the plume is usually at least an order of magnitude greater than the initial volume flux, by the time the plume nears its level of maximum rise.

The initial flux of water Q_0 (in mass per unit time) is defined by the equation:

$$Q_0 = \rho_0 V_0 (m_{p0} + r_{p0}) \quad (4)$$

where ρ_0 and r_{p0} are the initial air density and liquid water mixing ratio (gm water per gm of air). It is assumed that vertical speed, concentration of water vapor, and other variables are constant across a given plume cross-section. The flux of water Q_0 at level z that would saturate the plume is given by the relation:

$$Q_0(z) = \pi(z) \left[V_0 m_{p0}(z) + \left(V(z) - V_0 \right) \left(m_{pa}(z) - \bar{m}_0(z) \right) \right] \quad (5)$$

where $\bar{m}_0$ is the average mixing ratio of the air entrained into the plume, defined by

$$\bar{m}_0(z) = \left[\int_0^z m_0(\eta) \left(dV(\eta) / d\eta \right) d\eta \right] / \left[V(z) - V_0 \right] \quad (6)$$

The term $(\bar{m}_{ps} - \bar{m}_e)$ is sometimes called the "saturation deficit." Saturation is more likely to be achieved in an environment where the saturation deficit is small; e.g. a cold environment or a warm humid environment. Because the water vapor mixing ratio generally decreases with height, the average mixing ratio $\bar{m}_e$ in the plume at level z is generally greater than the mixing ratio σ_e in the local environment. It is then possible for Q_g to drop below zero at some height as T_p approaches T_e , even if the plume initially contains no excess water. In this case, condensation will occur at the level of natural cloud formation.

The ratio $(Q_0 - Q_g(z))/Q_0$ is defined as the fraction, A , of the initial water flux Q_0 that is condensed at any level. If there is an initial liquid water flux, $\rho_0 \sigma_{po} V_0$, the fraction, B , of this flux remaining as liquid water in the plume at level z can be estimated from the equation

$$B = -AQ_0 / \rho_0 V_0 \sigma_{po} \quad (Q_g > Q_0) . \quad (7)$$

When B increases to unity, all the initial liquid water has evaporated.

3.1 Level of First Condensation

The fraction A of the initial water flux that is condensed is thus equal to the ratio

$$A = 1 - \frac{\rho(z)}{\rho_0} \frac{V(z)}{V_0} \frac{\left[\bar{m}_{ps}(z) - \left(1 - \frac{V_0}{V(z)} \right) \bar{m}_e(z) \right]}{(\bar{m}_{po} + \sigma_{po})} . \quad (8)$$

It is seen that A is a function of the initial plume mixing ratio, the vertical variation of plume and environment mixing ratio, and the dimensionless ratios $\rho(z)/\rho_0$ and $V(z)/V_0$. For heights less than two or three kilometers, the ratio $\rho(z)/\rho_0$ can be approximated by the simple expression $\exp(-gz/R_d\bar{T})$, where R_d is the gas constant for dry air. The saturation mixing ratio $\bar{m}_{ps}$ is a function of height and temperature and can be approximated by the analytical expression:

$$\bar{m}_{ps} = \bar{m}_{es0} \exp \left[\frac{-gz}{R_d\bar{T}} + \frac{L}{R_v\bar{T}^2} (T_0 - T_{e0}) + \frac{L}{R_v\bar{T}^2} (T_p - T_e) \right] , \quad (9)$$

where R_v is the gas constant for water vapor. The temperature difference $(T_0 - T_{e0})$ is known from the vertical distribution of environment temperature and the temperature difference $(T_p - T_e)$ is obtained from the definition

$$(T_p - T_e) = (T_{po} - T_{e0}) \frac{F(z)}{F_0} \frac{V_0}{V(z)} . \quad (10)$$

The ratios $F(z)/F_0$ and $V(z)/V_0$ can be calculated using Briggs' (7) theory of plume rise. Detailed derivations of the following analytical expressions can be found in reference 6:

$$\frac{V(z)}{V_0} = \left[1 + .5 \frac{z}{L_0} \left(\frac{U}{w_0} \right)^{1/2} \right]^2 \quad \text{Windy} \quad (11)$$

$$\frac{V(z)}{V_0} = \left[1 + .2 \left(\frac{15}{8Fr_0} \right)^{1/5} \frac{z}{R_0} \right]^{5/3} \quad \text{Calm} \quad (12)$$

$$\frac{F(z)}{F_0} = 1 - \frac{z}{b_0} \left(\frac{w_0}{U} \right)^{1/2} \frac{R_0}{1.5} \left\{ \left[1 + .5 \frac{z}{R_0} \left(\frac{U}{w_0} \right)^{1/2} \right]^3 - 1 \right\} \quad \text{Windy} \quad (13)$$

$$\frac{F(z)}{F_0} = 1 - \frac{15}{8} \frac{w_0}{b_0} \left(\frac{15}{8Fr_0} \right)^{-2/5} \left\{ \left[1 + .2 \left(\frac{15}{8Fr_0} \right)^{1/5} \frac{z}{R_0} \right]^{8/3} - 1 \right\} \quad \text{Calm} \quad (14)$$

The ratios of volume fluxes $V(z)/V_0$ are plotted as a function of z/R_0 for various values of U/w_0 and Fr_0 in Figure 1.

Using the above formulas, it is possible to calculate the fraction A of the initial water flux that is condensed at any level z , based on the initial plume conditions and the vertical variation of environment temperature and wind speed. As Morton(9) concluded from his analysis of moist plume rise during calm conditions, plume rise is highly sensitive to slight variations in environmental mixing ratio. There is a thin line between no cloud development and explosive cloud development.

Figure 1. The ratio of the volume flux V to initial volume flux V_0 , as a function of the ratio of height z to initial radius R_0 , for windy conditions at various values of U/w_0 , and calm conditions at various values of $Fr_0 = w_0^2/2R_0b_0$.

As an example of the application of these techniques, consider a plume in a well-mixed atmosphere at great heights, such that potential temperature and mixing ratio are constant with height ($\theta_e = \theta_{e0}$ and $m_e = m_{e0}$) and plume temperature equals environment temperature. Under these conditions, the plume saturation mixing ratio m_{ps} can be approximated by $m_{eos} \exp(-4.5 gz/R_d \bar{T})$. Furthermore, assume that the scale height of the atmosphere, $R_d \bar{T}/g$, is 8000 m, and the initial liquid water content, σ_{p0} , is zero. Then equation (9) becomes:

$$A(z) = 1 - e^{\frac{-z}{8000m}} \left[1 + .5 \frac{z}{R_0} \left(\frac{U}{w_0} \right)^{1/2} \right]^2 \frac{\left[e^{\frac{-z}{1800m}} - RH_0 \left(1 - 1/(1 + .5 \frac{z}{R_0} \left(\frac{U}{w_0} \right)^{1/2})^2 \right) \right]}{m_{p0}/m_{eos}} \quad (15)$$

where RH_0 equals m_{e0}/m_{eos} . In a specific case, for example, $R_0 = 30$ m, $U/w_0 = 4.0$, $m_{p0}/m_{eos} = 3.2$, and $RH_0 = .8$. The first level of condensation ($A(z)=0$) is at 370 m, and the level at which a quantity of water equal to the initial water flux condenses ($A(z)=1$) is at 410m. This is the height at which an initially dry plume would begin to condense.

After working a few examples, one finds that the above condensation criterion is sensitive to slight variations in the variables U/w_0 , RH_0 , and m_{p0}/m_{eos} . However, we know from many years of experience that the analogous problem of the prediction of the formation of cloud layers is also very difficult.

3.2 Iterative Technique for Estimating Plume Rise

It was mentioned in Section 2 that the plume rise equations (1) and (2) are adequate for calculating cooling tower plume rise, provided that the initial latent heat flux is accounted for when calculating the total initial energy flux P_0 . Now that we have developed methods for estimating the fraction of the initial water flux that is condensed, it is possible to estimate the latent heat flux at the height, H , of final plume rise. The first estimate of plume rise, H_1 , is calculated assuming that no vapor has condensed. Then the fraction A_1 of the initial water flux condensed at level H_1 is calculated. The value, A_1 , is then multiplied by the total possible latent heat flux and the new heat flux is used to calculate a new estimate of plume rise H_2 . This procedure is repeated until plume rise, H , and fraction of initial water condensed, $A(H)$, approach constants.

4. ADDITIONAL REMARKS

It is seen that cooling tower plume rise can be easily calculated for either uncondensed or completely condensed plumes. Due to the sensitivity of the condensation processes to slight changes in, for example, environment mixing ratio, cooling tower plume rise in borderline cases can not be calculated with so much confidence. It is because of this sensitivity that observation programs of cooling tower plumes will require much attention to detail.

In writing environmental impact statements, it is necessary to determine the diffusion of water vapor or drops to the ground from the elevated cooling tower plume. The usual assumption is that water vapor diffuses in the same manner as an inert gas. Consequently the Gaussian dispersion equation (4) may possibly be used, knowing the approximate wind speed and stability class. It is necessary to conduct a comprehensive observation program to determine the limitations of the simple Gaussian equation. If a thin elevated inversion layer is between the plume axis and the ground, dispersion of moisture to the ground may be inhibited. Nonlinear interactions between plume and environment heat and moisture may also prove to be important.

The techniques discussed in this paper provide a starting point for the analysis of the environmental impact of cooling tower plumes. Observations of the rise, condensation, and dispersion of cooling tower plumes are practically non-existent. Much more research needs to be done with respect to cooling tower behavior.

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THE USE OF CLIMATOLOGICAL PREDICTION MODELS FOR INDUSTRIAL AIR RESOURCES MANAGEMENT

Wilfrid Bach,* Barry Root** and Anders Daniels**
University of Hawaii, Honolulu, Hawaii

INTRODUCTION

Air resource management in the U.S. has now reached the point where each of the states is attempting to complete implementation plans which should state in rather specific terms how they plan to meet the ambient air quality standards that have been set for their areas. One tool that could be used to good advantage by both the agencies writing these plans and the industries that will be legally required to comply with the regulations they set forth is the climatological prediction model. In this paper ground level isopleths of sulfur dioxide concentration in the vicinity of an oil refinery in Hawaii have been calculated and compared using both the standard 22.5° sector model and a 1° sector model for both arithmetic and harmonic mean wind values.

THE 22.5° SECTOR MODEL

The basic form of this diffusion model for a continuously emitting point source has been set forth by Martin:⁽¹⁾

$$(1) \quad \chi = \frac{360 F}{\theta 100} \frac{Q}{2 \pi \sigma_z \bar{u} x} \exp \left[-1/2 \left(\frac{H}{\sigma_z} \right)^2 \right]$$

where χ is the ground level pollutant concentration at a given downwind distance, x .

Q and H are source parameters representing the source strength and effective stack height, respectively.

σ_z is the standard deviation in the vertical direction (a function of atmospheric stability and downwind distance).

F and $\bar{u}$ are the frequency and arithmetic mean wind speed values for each sector of width, θ .

To this should be added a term, $\exp \left[-\frac{(\ln 2) x}{T_{1/2} \bar{u}} \right]$ which accounts for the decay of sulfur dioxide with downwind distance where $T_{1/2}$ represents the 'half life' of 4 hours. This form of the equation assumes perfect reflection of the pollutants at the ground. To allow for less than perfect reflection another term $(1 + \alpha)/2$, should be included where $\alpha(\theta)$ is the reflection coefficient ($\alpha = 1$ for $\bar{u}(\theta) > 7$ m/sec, $\alpha = \bar{u}(\theta)/7$ for $\bar{u}(\theta) < 7$ m/sec).⁽²⁾

Thus the equation for the model as used in this paper becomes:

$$(2) \quad \chi = \frac{360 F}{\theta 100} \frac{Q}{(2\pi)^{3/2} \sigma_z \bar{u} x} (1 + \alpha) \exp \left[-1/2 \left(\frac{H}{\sigma_z} \right)^2 \right] \exp \left[-\frac{(\ln 2) x}{T_{1/2} \bar{u}} \right]$$

Since climatological wind data is most often available in a format consisting of just the 16 cardinal compass directions the sector width, θ , becomes 22.5°. By computing pollution concentrations at various distances from the source within each of these 16 wind sectors a predicted pollution pattern such as the one shown in Figure 1A might result. The discontinuity at the boundaries between sectors results from the assumption inherent in the use of this model that whenever the wind blows into a specific angular sector each of the wind directions within the sector occur with equal frequency and that a single arithmetic mean wind speed, $\bar{u}$, can be used to describe the magnitude of the wind throughout the sector.

It is relatively simple to eliminate this discontinuity merely by interpolation linearly from the middle of one sector to the middle of the next, etc., until a somewhat smoother set of isolines is produced (Figure 1B). But this is a rather crude way to smooth out the errors in the assumption. A better way to eliminate these errors would be to eliminate the assumption.

* Department of Geography

** Department of Meteorology

THE 1° SECTOR MODEL

Since winds blow with different frequencies and wind speeds within any given sector it would be a much better approach to perform diffusion calculations for each individual 1° sector.

In this case it would be possible to use the Gaussian Diffusion Equation for one wind direction:

$$(3) \quad \chi = \frac{f}{100} \frac{Q}{2\pi \bar{u} \sigma_y \sigma_z} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] (1+\alpha) \exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right] \exp\left[-\frac{(1+\alpha)x}{1/2\bar{u}}\right]$$

where f and $\bar{u}$ are the frequency and mean wind speed (arithmetic or harmonic) for each 1° sector. In this case a Gaussian distribution with standard deviation σ_y can be included in the climatological calculations.

The geometry for this model is shown in Figure 2. A receptor at R^0 and a given downwind distance, x_0 , will receive pollutants from the source when the wind blows into any of the $R^0 \pm \beta^0$ sectors from $R^0 - 11^0$ to $R^0 + 11^0$. For winds outside this sector we neglect the very minute pollution contribution. The total pollutant concentration at the receptor will thus be the sum of 22 separate contributions.

This model obviously depends upon our being able to define mean wind speed and frequencies for each 1° sector. One of the best ways to interpolate wind data that has been given only for the 16 cardinal directions to data for every one degree is to apply harmonic analysis to the data.

HARMONIC ANALYSIS OF STANDARD WIND DATA

The harmonic analysis used in this paper was carried out as follows:⁽³⁾

1. A set of degrees was assigned to each of the 16 cardinal wind directions. Since 360° cannot be evenly divided into 16 sectors the major wind sectors were assigned 23 degrees and the minor ones 22 degrees. Thus the frequency of occurrence of the wind becomes:

$$(4) \quad F_M(J) = \sum_{\theta=\theta_L}^{\theta_L+22} f(\theta)$$

$$(5) \quad F_N(J) = \sum_{\theta=\theta_L}^{\theta_L+21} f(\theta)$$

where θ_L is the leftmost degree in the J-th sector, $F_M(J)$ is the total frequency of occurrence of winds from a major compass point-direction (e.g. NE) and $F_N(J)$ that for a minor compass point direction (e.g. NNE) and $f(\theta)$ is the occurrence for wind from θ degrees.

2. It was assumed that the actual wind statistics (speed or occurrence) can be written as a sum of harmonics with some amplitudes, a :

$$(6) \quad f(\theta) = a_1 + \sum_{i=1}^8 a_{(i+1)} \cos(i\theta) + \sum_{i=1}^7 a_{(i+7)} \sin(i\theta)$$

3. Summing all the $f(\theta)$ for the 8 major and the 8 minor compass sectors yields a set of 16 equations with 16 unknowns ($a_1 \dots a_{16}$).
4. A computer program solving this system of equations has been developed. The machine time required for the complete operation is about 8 seconds.

THE WIND DATA

The industrial operation used as the source for these computations is located at Barber's Point, Oahu, Hawaii, immediately adjacent to a naval air station where hourly wind data has been recorded continuously since 1950. A magnetic computer tape of this data was obtained from the National Weather Records Center at Asheville, N.C. Data for the month of January was extracted from this tape and fed into the program for harmonic analysis. January data was selected because onshore winds are most frequent during that month and the location of the refinery is such that only onshore winds will create any significant pollution concentrations over the inland area. Barber's Point itself is a relatively level coastal plain and effects of topography on the diffusion calculations are negligible.

The wind data from the tape was subdivided into 3-hourly periods and sorted for 20 speed classes (1, 2, 3, ... $\geq$ 20 knots) and for 16 compass points. Harmonic analysis was employed to obtain the frequency for each wind speed class and each degree of wind direction.

For this the following parameters were calculated:

For the 22.5° sector model:

$$(7) \quad F(j) = \sum_{i=1}^{20} FJ(i,j)$$

where i is the number of wind speed classes, and j is the number of wind direction sectors (j = 1 to 16), and FJ(i,j) is the frequency of winds from the j-th direction at speed i.

Similarly,

$$(8) \quad \bar{u}(j) = \sum_{i=1}^{20} FJ(i,j) \cdot u(j)/F(j)$$

and the reflection coefficient α is also expressed as a function of this arithmetic mean wind speed ($\alpha(\bar{u}(j))$). The stability class for each sector was determined using sector arithmetic mean wind.

For the 1° sector model $\bar{u}$ has, for comparative purposes, been defined in two ways:

In the first instance the arithmetic mean wind data has been used:

$$(9) \quad \bar{u}_A(\theta) = \sum_{i=1}^{20} f(i,\theta)u(i,\theta) / \sum_{i=1}^{20} f(i,\theta)$$

or, if

$$(10) \quad f_A(\theta) = \sum_{i=1}^{20} f(i,\theta) / \sum_{\theta=1}^{360} f_A(\theta)$$

where $\bar{u}_A(\theta)$ is the arithmetic mean wind speed for the θ -th degree. $f_A(\theta)$ is the fraction of the time the wind blows from the θ -th degree and the reflection coefficient α uses $\bar{u}_A(\theta)$. The stability class for each degree was here based on the arithmetic mean wind for the degree.

In the second instance the harmonic mean wind data has been used:

$$(11) \quad \bar{u}_H(\theta) = \sum_{i=1}^{20} f(i,\theta) / \sum_{i=1}^{20} [f(i,\theta) / u(i,\theta)]$$

and

$$(12) \quad f_H(\theta) = \sum_{i=1}^{20} \{ f(i,\theta) [1 + \alpha(u(i,\theta))] / u(i,\theta) \} \bar{u}_H(\theta) / \sum_{\theta=1}^{360} f_H(\theta)$$

Fig.1A and B: 22.5 Sector Model with Linear Interpolation

Fig.2: Geometry of a 1° Sector Model

Fig.3A&B SO₂ CONCENTRATIONS AS CALCULATED
USING DIFFERENT METHODS

Fig.4 PREDICTED SO_2 - CONCENTRATION ($\mu\text{g}/\text{m}^3$) AROUND AN OIL REFINERY

Fig.5 PREDICTED SO_2 - CONCENTRATION ($\mu\text{g}/\text{m}^3$) AROUND AN OIL REFINERY

where $\bar{u}_H(\theta)$ is the harmonic mean wind speed and $f_H(\theta)$ is the frequency of winds from θ -th degree multiplied by the reflection coefficient α . In this case the stability class for each degree was based on the harmonic mean wind.

The harmonic mean wind has been used here because the diffusion equations clearly show that the concentration, χ , is a function of the harmonic rather than the arithmetic wind speed (see also Hilst).⁽⁴⁾

CONCENTRATION COMPARISONS

Using the University of Hawaii's IBM 360 computer 3-hour climatological pollution concentrations for radial distances of 1 to 10 km were calculated for both of the models using only arithmetic mean winds for the 22.5° sector model and both arithmetic and harmonic mean winds for the 1° sector model. Comparisons of results are shown for the 1 km radial distance for the 3-hourly time periods from 0300 to 0500 LST and time 1200 to 1400 LST in Figures 3A and B.

Using the 1° sector model values of SO₂ concentration exceeding the set 3-hour air quality standard of 400 $\mu\text{g}/\text{m}^3$ were found (see Figure 3A). Values of this magnitude would not have been revealed using only the 22.5° sector model.

DAILY PATTERNS OF SULFUR DIOXIDE POLLUTANTS AROUND AN OIL REFINERY

Using the 1° sector model with harmonic mean winds and source parameters of $Q_{\text{AVG}} = 194 \text{ g/sec}$ and $H=43\text{m}$ ⁽⁵⁾ isolines of ground level sulfur dioxide concentrations are shown for the area in the vicinity of the Barber's Point oil refinery for each of the three-hourly time periods on an average January day (Figure 4). The three-hourly allowable standard for SO₂ set by the Hawaii State Department of Health is 400 $\mu\text{g}/\text{m}^3$. It can be seen that during the two 3-hour periods, namely between 3 to 5 and 6 to 8 a.m., the air quality standard for SO₂ is exceeded in a small downwind area of approximately 1-2 km away from the source. The average daily pattern is shown in Figure 5. The 24-hourly air quality standard of 80 $\mu\text{g}/\text{m}^3$ is exceeded up to 5 km away from the source over the shoreline, and about 2 km inland close to some settlements.

USEFULNESS OF THE MODEL AS A MANAGEMENT TOOL

It is sometimes necessary in refinery operations to 'flare' excess sulfur and if such activities must be conducted it is vital that they be done at a time when diffusion of pollutants is likely to be the greatest. From the maps presented here it can be seen that climatic factors combine to make the hours from 1200 - 1700 the best time of the day to conduct such operations.

Other managerial uses for the model should be obvious. Any time physical parameters such as stack heights or control devices are altered or production is increased or decreased a climatological model can, indeed should, be used to predict what the consequences in terms of ground level pollutant concentrations will be.⁽⁶⁾

ACKNOWLEDGEMENT

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A NUMERICAL STUDY OF CONVECTIVE TRANSPORT OF A GASEOUS AIR POLLUTANT IN THE VICINITY OF TALL BUILDINGS

D. Djurić and J. C. Thomas
Texas A&M University
College Station, Texas

INTRODUCTION

The numerical models, applied to transport and diffusion processes, are considerably simplified due to a) the need to limit the many parameters found in the model and in the governing equations, and b) the desire to study simple geometry, where causes and effects could be distinguished easily. The tendency is apparent to reproduce numerically the analytical cases, the latter ones being geometrically quite simple. With the advent of larger and faster computers it is becoming feasible to solve the transport and diffusion equations under more complicated conditions than previously attained. The flow around buildings of various shapes is geometrically complicated, but can be simulated in a three-dimensional model. Some possibilities of such computations are treated in this paper.

The experience is drawn from previous numerical models of convective flow that have been used by Brundage *et al.* (1), Deardorff (2), Harlow and Welch (3), and Lilly (4). Most of these models were applied in cloud dynamics, but they can be used for a study of diffusion processes, where larger eddies are treated explicitly. It is these large eddies that transport most of the diffusing material. Therefore, new information can be expected by a direct study of eddies larger than the double grid size.

THE NUMERICAL MODEL

Thomas (12) has studied small-scale wind patterns in downtown Houston, Texas, by micro-meteorological observations and wind-tunnel experiments. He also used a numerical model which simulated the air flow in a part of the city with several blocks of tall buildings. These blocks were geometrically similar to the actual buildings in Houston. A perspective view of the area in consideration is shown in Figure 1. This region covers a portion of four blocks in the central business district of downtown Houston. The blocks in the figure are numbered for later use in the paper. This area was simulated by a three-dimensional set of grid points in which the flow was computed. The mathematical model used was devised for studying the flow patterns around buildings covering a four square block area. This simulated region (Figure 2) has the same gross geometric features as a portion of the downtown region shown in Figure 1. The numbered buildings in Figure 2 correspond to similar numbered blocks found in Figure 1. The numerical model consisted of 14×14 grid points in the x-y plane and eight grid points in the vertical. The spacing between grid points in the model was 20 m, i.e., $\Delta x = \Delta y = \Delta z = 20$ m. The model is oriented in such a way that the ambient wind blows along the x-axis. Therefore, periodic boundary conditions in x were used, except for pressure, which had a slight general decrease in the x-direction. This decrease in pressure provided the driving force for the wind in the model. The walls of the buildings, the lateral boundaries, the top, and the bottom of the model were rigid with vanishing normal component of flow.

The equations used in this model follow those of Ogura and Phillips (10) in which the anelastic approximation is utilized. The equations are

$$\frac{\partial \vec{V}}{\partial t} = \vec{A} - \nabla p + \vec{K} \vec{T} + \frac{1}{\rho_0} \vec{\tau} = \vec{S}, \quad (1)$$

$$\vec{\tau} = (\rho_0 \vec{V}) = 0, \quad (2)$$

$$\frac{\partial T}{\partial t} = \Delta T + \frac{K_H}{\rho_0} \nabla \cdot (\rho_0 \vec{V} T), \quad \text{and} \quad (3)$$

$$\frac{\partial Q}{\partial t} = \Delta Q + \frac{K_H}{\rho_0} \nabla \cdot (\rho_0 \vec{V} Q), \quad \text{where} \quad (4)$$

$$\vec{A} = \frac{1}{\rho_0} \nabla \cdot (\rho_0 \vec{V} \vec{V}),$$

Figure 1. Reproductive rates of common loach, *Pisces* and *A. n. n.*

 

[illegible]

 

where the variables are $\vec{V}$, the 3-dimensional wind vector, p , the potential of the pressure force ($p = C \rho^{-1}$), and T , buoyancy ($T = g \theta' / \theta$). The Coriolis term is not included in (1) because of the small scale of motion under consideration. By use of (1) and (2) we can derive the balance equation, whereby pressure is evaluated at each time step. This equation is represented by

$$\nabla \cdot (\rho_0 \vec{V}) = F, \text{ where } F = \vec{V} \cdot (\rho_0 \vec{A}) + \frac{1}{f^2} (T \rho_0). \quad (7)$$

In these equations, the basic density, ρ_0 , is constant in the horizontal planes and varies only with height. The model that was used in this study previously had been used for deep convection, hence, the need for (1). The density variation with height was not neglected because of the increased versatility of the model, allowing for mountain effects when applied to the macro-scale motion. The advective terms in (1), (3), and (4) are of first order in variables u , v , w , T , and Q . The mathematical concept of cyclic boundary conditions for first-order terms, used with rigid boundaries in the v - and w -direction, requires

$$\begin{aligned} u(0) &= u(E), \\ Q(0) &= Q(E), \\ T(0) &= T(E), \\ v &= 0 \text{ at } z = 0, H, \text{ and} \\ v &= 0 \text{ at } y = 0, D. \end{aligned}$$

Also, all normal components of velocity are equal to zero on all rigid internal boundaries.

The diffusion terms represent second-order terms in variables u , v , w , T , and Q . All five variables, or their normal derivatives (viz., derivatives across the boundaries) must be prescribed on all boundaries. The diffusion terms, when, allow one to prescribe boundary conditions for flow along the boundaries. Boundary conditions also must be used in regard to the diffusion terms in addition to the boundary conditions used for the advective terms. These boundary conditions prescribe frictionless flow ($\partial u / \partial n = 0$) on the walls of the model, except at the lower boundary where a square law for friction is assumed in the form (5), where V_{10} is the wind at the first level of interior grid points ($z = 10$ m). On the sides and tops of buildings the stress was linear, which gave less friction than the square law (5). The particular form (6) was convenient from the programming standpoint.

In order to satisfy the consistency with the equation of motion, the derivative boundary conditions are applied to the balance equation. The boundary conditions in this case become

$$\begin{aligned} p(0) &= p(E) + 1.2, \text{ where } 1.2 \text{ units of } p \text{ correspond to } 0.01 \text{ mb}, \\ \frac{\partial p}{\partial y} &= 0 \text{ at } y = 0, D, \text{ and} \\ \frac{\partial p}{\partial z} &= T \text{ at } z = 0, H. \end{aligned} \quad (8)$$

The fact that $\partial p / \partial n \neq 0$ expresses the presence of a general pressure force which moves the air in the model. This driving force is counteracted by friction at the lower boundary and on the walls of the buildings. The coefficient c in (9) is chosen as

$$c = 0.16. \quad (9)$$

This value is larger than the values given by Sutton (11), but it is still small enough not to influence the development of flow in the interior of the model. This value of c has been chosen after some experimentation to provide an average speed in the model of about 3 m sec^{-1} , working under the influence of a pressure drop of about 0.01 mb per 100 m across the model, as described by the boundary condition (8). The values in (8) and (9) are greater than the typical values in nature, and were used for practical reasons because they allowed the quasi steady-state flow to be attained sooner after the beginning of the computation. More realistic values of c and $\partial p / \partial n$ are not essential in developed turbulent flow, since then the flow is greatly governed by the inertial forces in the eddies; the surface stress is of secondary importance.

The boundary conditions for T and Q in the diffusion terms do not allow diffusion across the solid boundaries; at these boundaries

$$\frac{\partial T}{\partial n} = \frac{\partial Q}{\partial n} = 0,$$

where n is the normal direction. In order to influence the internal distribution of T and Q , the values T and Q are prescribed on the lower boundary of the model.

In order to satisfy consistency between the equations in the finite difference form, it was suitable to use the alternating arrangement of grid points with various variables as shown in Figure 3.

Figure 3. Diagrammatic representation of alternating spacing of grid points with various variables.

- LEGEND**
- POINTS WITH u , AV
 - ⊕ POINTS WITH v , AV
 - △ POINTS WITH u , AH , T , AT , Q , AQ
 - * POINTS WITH p , F

In finite-difference, the advection terms are used in the form

$$\frac{\partial u}{\partial x} \approx \delta_x \bar{u}^x, \quad \frac{\partial v}{\partial y} \approx \delta_y \bar{v}^y, \quad \frac{\partial w}{\partial z} \approx \delta_z \bar{w}^z, \quad \text{etc.}$$

where the bars and deltas have the usual meaning of the Shuman's notation. The time extrapolation used in the model was of the form

$$\phi^{t+\Delta t} = \phi^t + \Delta t \left[1.8 \left(\frac{\partial \phi}{\partial t} \right)^t - 0.8 \left(\frac{\partial \phi}{\partial t} \right)^{t-\Delta t} \right],$$

where $\phi = u, v, w, T$, or Q . This formulation was presented by Mesinger⁽⁷⁾. Computational stability exists for both the diffusion and advection terms when

$$\Delta t \leq \frac{\Delta x^2}{9K} \quad \text{and} \quad \Delta t \leq \frac{0.4 \Delta x}{U_{\max}}$$

The time step was chosen "automatically" to satisfy both above conditions in all grid points. The time step varied between 1.5 sec and 2.0 sec. In any two time steps subsequent to each other, however, the changes in Δt were quite small, no more than 5%. As in results of other researchers with similar numerical schemes, this scheme showed no appreciable error which could be attributed to non-linear numerical instability.

The balance equation (7) was solved by a method of relaxation by columns as described by Beardorff⁽¹²⁾.

Initially, there was no motion in the region. The flow was developed in response to a pressure drop within the model (0.01 mb). Surface heating was simulated in the middle streets within the model. A continuous source representing an air-pollution tracer was introduced in the model to show the actual and average dispersion in the turbulent flow. This tracer was set at 99 arbitrary units in the streets, and it can be thought of as maximum carbon monoxide value in parts per million from vehicles in the downtown area (Ramsey⁽⁹⁾, Jaffe⁽¹⁰⁾). These values, then, represent a high traffic congestion in the middle streets of the model.

Initially, buoyancy (7) increased with height allowing for stable atmospheric conditions.

RESULTS

The numerical model simulated fluid motion and pollutant diffusion for the region of downtown Houston (Figure 1) during 16 minutes of actual time; this was accomplished in 1000 computational time steps. Outprints of buoyancy (also referred to as "temperature") were made every 200 time steps up to 1000 time steps. In addition, outprints also were made for an atmospheric tracer, Q , and a mean Q which was the original tracer averaged over each 200 time steps up to 800 time steps. Outprints were in the x - y plane in all eight vertical levels.

During the computation the vertical buoyancy distribution was examined every 200 time steps. An unstable region was found within the lower 20 m and between 140 and 160 m. Heating was simulated in the numerical model along the middle streets in the x - and y -direction and along the side streets perpendicular to the main flow. The side streets along the lateral boundaries were not heated. The thermal instability in the lower 20 m was a result of the heating from the paved streets. The model

continued to get warmer as the lower boundary was heated. Slightly stable stratification existed throughout the numerical experiment. Initially, however, stability increased at a greater rate between levels 2 and 7 than it did during the remainder of the experiment. The increase in stability during the remainder of the experiment was attributed to the small amount of heating from the building tops.

Wind speed profiles from the model showed an increase in speed from 0.0 m sec^{-1} at the bottom boundary to about 3.0 m sec^{-1} at the top boundary. The wind speed increased with height more in lower layers than in the upper ones. An average wind speed within the model was 2.0 m sec^{-1} .

Figure 4 shows the vertical Q distribution for 200, 400, 600, and 800 time steps. An almost constant concentration of Q with height is seen by 400 time steps (19 minutes simulated time), thereby showing the saturation of the region with the tracer. Increased mixing due to heating (unstable thermal regions) and convection around buildings (forced ascent and descent) cause the tracer to be well distributed vertically by 400 time steps. In the interior of the model a portion of the tracer was lost at the building tops due to the employed interior boundary condition.

The average tracer values were compared at each level for 200, 400, 600, and 800 time steps to determine general concentrations in the model. A low concentration of the average tracer occurred toward the top lee side of each building. A high concentration was found near the low three-quarters of the lee side of each building. The high concentration in these areas was caused by the updraft on the back portion of each building. The entire updraft, however, did not reach the top of the buildings. A downdraft was found near the upper portion of the lee side which caused a smaller tracer concentration to occur at the top of each building. A high concentration, which was caused by upward vertical motion, also was found near the upper one-third of the fronts of the buildings. The lower two-thirds of front faces were in a predominantly downward motion and thus a lower concentration was formed there. The increased concentration of pollutant in the upper one-third of the buildings is similar to the findings by Schnelle, Ziegler and Krenkel⁽¹⁰⁾ who have given vertical concentrations of carbon monoxide in a downtown region of Nashville, Tenn. It was found that under certain stable atmospheric conditions, maximum concentrations can occur near roof top levels (110 ft). This concentration distribution was also noted in the numerical model and was related to building locations, wind speed, and wind direction. The maximum of Q close to the building tops, of course, does not occur in the free air at a distance (several tens of meters) from the buildings.

In lower levels the tracer showed the influence of the lower boundary. A high concentration was found in the areas of the middle streets with low values near the lateral boundaries of the model. This was due to the assumed source of Q which was simulated in the middle streets of the model and on the streets perpendicular to the main flow (Figure 5).

Tracer concentrations are directly related to the wind patterns that occur around tall buildings. The variation of the wind among the buildings cause the concentration of gaseous pollutants to be increased or decreased at various locations and times around the buildings. If one considers the automobile as the principal polluter in the downtown region, then updrafts that occur near the earth's surface will tend to draw high pollutant concentrations upward. These updrafts can increase pollutant concentrations in upper levels.

In determining pollutant concentrations in the downtown region attention must be given to wind patterns that are caused by the buildings as well as atmospheric stability conditions. Knowledge of the diffusion coefficient K is also of great importance. In the numerical work this coefficient was taken as $10 \text{ m}^2 \text{ sec}^{-1}$.

CONCLUSIONS

The numerical simulation of turbulent flow offers a possibility to study the detailed distribution of pollutants in a three-dimensional space. The geometry of the model can be arbitrarily shaped, for simulation of flow around buildings. Here it has been shown that such a computation can be performed, using the experience from the computational models of convection with primitive equations.

The results of computation show several typical patterns of distribution of pollutant around buildings as observed by Schnelle, Ziegler, and Krenkel⁽¹⁰⁾. These patterns can be related to the typical distribution of stationary and transient eddies around the buildings. Much of the detail has not yet been computed since the computer capacity and time were limited.

Figure 4. Horizontally averaged vertical distribution of Q for 200, 400, 600, and 800 time steps.

Figure 5. Q field at level 1.

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APPENDIX A - LIST OF SYMBOLS

x	horizontal coordinate	$\vec{v}_{10}$	two-dimensional velocity vector parallel to boundaries and 10 m away from them
y	horizontal coordinate		
z	vertical coordinate	t	time
$\hat{i}$	unit vector in x-direction	Δt	time increment, time step
$\hat{j}$	unit vector in y-direction	Δx	horizontal grid increment
$\hat{k}$	unit vector in z-direction	Δy	horizontal grid increment
u	wind speed component in x-direction	Δz	vertical grid increment
v	wind speed component in y-direction	K	eddy exchange coefficient
w	wind speed component in z-direction	K_H	eddy exchange coefficient for heat
$\vec{v}$	three-dimensional velocity vector, $\hat{i}u + \hat{j}v + \hat{k}w$	K_M	eddy exchange coefficient for momentum
		E	total length of model in x-direction

D	total width of model in y-direction	$\bar{p}$	average pressure in y and z
H	total height of model in z-direction	θ	reference - potential temperature
c	drag coefficient	θ	potential temperature
C_p	reference-specific heat at constant pressure	θ'	potential temperature deviation, $\theta - \theta$
g	acceleration of gravity	π_0	reference - atmospheric pressure variable
V_{max}	maximum speed	π	dimensionless pressure, Ekmer function, $(p/p_0)^{1/\kappa}$
R	universal gas constant for dry air	π'	dimensionless pressure deviation, $\pi - \pi_0$
K	constant, R/C_p	A	inertial terms, $-\frac{1}{\rho_0} \nabla \cdot (\rho_0 \vec{V} \vec{V})$
ρ_0	reference - atmospheric density	AT	advective terms in the thermal equation
Q	concentration of a tracer	AQ	advective terms in the tracer (Q) equation
T_0	reference - atmospheric absolute temperature	S	stress tensor, $\rho_0 \vec{V} \vec{V}$
T	absolute temperature or buoyancy given by $g(\theta - \theta)/\theta$	$\vec{S}$	stress vector for skin friction
T'	temperature deviation, $T - T_0$	n	normal direction
p	a pressure variable given by $C_p \theta (\pi - \pi_0)$	$\delta_1 \psi$	the finite-difference representation of the derivative of ψ with respect to x_i , where i may be x, y, or z

THE NUMERICAL CALCULATION OF THREE-
DIMENSIONAL FLOWS OF AIR AND PARTICULATES
ABOUT STRUCTURES

Robert S. Hotchkiss
University of California
Los Alamos Scientific Laboratory
Los Alamos, New Mexico 87544

I. INTRODUCTION

The development of computer programs capable of solving the time dependent, three-dimensional equations governing fluid motions greatly distorted by structures, offers fluid dynamicists, meteorologists, and air pollution researchers tools to study highly complex fluid and particulate flows. In this paper such a numerical method, an extension of a previously published technique,⁽¹⁾ is described. The method employs finite differences to solve the full nonlinear Navier-Stokes equations for incompressible flows with buoyancy. In addition heat and particulate transport equations are solved. The full details are given in Ref. (2).

Two particulate transport models are described; one involves the solution of a transient concentration equation for pure convection and diffusion. In the second model a dynamic equation that includes Stokes drag, gravity, and diffusion is solved for each particle. In both cases particulate settling is allowed to occur.

Results of calculations are displayed as three-dimensional perspective views drawn by standard computer hardware. Newly developed techniques for computerizing these perspective views are described. Two adjacent perspective views are used to enable the viewer to see in stereo.

To establish the validity of the particulate transport calculations, results are presented for a comparison of the numerical calculation of a symmetric plume to the standard analytical model.

When strong flow distortions are present, simple analytical models are not applicable. In such cases a technique such as that described here must be employed. As an example, the formation of flow separation and particulate entrainment behind structures is studied.

II. THE METHOD

A. FLUID TRANSPORT

The basic method, as previously mentioned, is designed to predict the behavior of three-dimensional, time-dependent flows for viscous incompressible fluids. This is accomplished by solving the full nonlinear equations that specify such flows, namely the Navier-Stokes equations,

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = g_x - \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right), \quad (1a)$$

$$\frac{\partial v}{\partial t} + \frac{\partial vu}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} = g_y - \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right), \quad (1b)$$

$$\frac{\partial w}{\partial t} + \frac{\partial wu}{\partial x} + \frac{\partial wv}{\partial y} + \frac{\partial w^2}{\partial z} = g_z - \frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right), \quad (1c)$$

and the continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (2)$$

where u , v , and w are the fluid velocity components in the x , y , and z directions, respectively, t is time, p is the pressure to density ratio; the components of g are the body accelerations from gravity in the x , y , and z directions, respectively, and ν is the kinematic viscosity, or the eddy viscosity for turbulent flows. At present, the effect of turbulence is resolved only through a constant eddy viscosity.

The solutions of Eqs. (1) and (2) are accomplished by applying a finite difference approximation to them. A volume of space is selected to be the region to which Eqs. (1) and (2) apply. This main calculational region is then divided into many subregions called cells, all cells having dimensions δx , δy and δz . Each cell is labeled with an index (i, j, k) , which establishes the location of the cell in the mesh. For example, $[(i\delta x - 3\delta x/2), (j\delta y - 3\delta y/2), (k\delta z - 3\delta z/2)]$ gives the $[x, y, z]$ coordinate of the center of the (i, j, k) th cell. Pressures are evaluated at the center of the cell, whereas values of velocity are recorded at the centers of the cell faces as shown in Fig. (1). With the variables spatially defined in this manner, Eqs. (1) and (2) can be rewritten in finite difference form as given in Ref. (2). The time dependent solution of the resulting finite difference equations is then obtained by advancing all field variables through time in short steps of duration δt . At each time step, the evaluation of the field variables is accomplished in two phases. The first phase utilizes the known values of all field variables at a previous time, by performing a purely explicit calculation to find approximate values of the velocity components. This phase, however, does not ensure the requirement of incompressibility, namely that the divergence of the velocity field is zero. The second phase does ensure this requirement by iteratively adjusting the pressure field (which in turn is made to adjust the velocity field) in a way that conserves mass in every cell of the mesh. In other words, some amount of fluid is either pushed out or sucked into each cell with the pressure adjustment, thus, preserving continuity. After this is accomplished, the time is advanced by some amount (δt) to a new time, and the above steps are repeated.

The solution of the equations depends upon the various constraints or conditions imposed at the boundaries. The method is capable of using five different kinds of boundary conditions. They are: inflow, outflow and periodic boundaries, along with rigid walls that are either free-slip (no-shear at the wall) or no-slip (zero velocity at the wall).

In the computer program each computational cell is flagged as one of the following: input cell, output cell, fluid cell, rigid free-slip cell or rigid no-slip cell. The latter two types of cells can occur anywhere in the mesh and hence an obstacle or block type structure is easily formed by flagging any particular combination of cells as rigid cells with the appropriate slip condition. In this way the flow field can be made to conform to any of a great variety of boundary conditions.

Fig. (2a) shows a perspective view of a block type structure enclosed by the calculational region. Fluid enters the computing region uniformly from the left side, passes over and around the structure, and flows out the right side. The short lines in the picture are velocity vectors of fluid elements in a horizontal plane near the lower portion of the structure. The velocity vectors show the average relative magnitude and direction of fluid elements. Notice that directly behind the structure, a region of slowly moving fluid is apparent. Fig. (2b) shows that a recirculation region, or cavity, exists in the lee of the structure. This figure is a plane view of the magnified velocity vectors in Fig. (2a) as seen from above. The double vortex structure is revealed by the directions of the vectors near the structure.

Fig. (2c) is a vertical plane of expanded velocity vectors taken through the center of the top eddy in Fig. (2b), showing the recirculation taking place in the vertical direction. Fig. (2d) shows the vertical plane of velocity vectors directly behind the structure that an observer would see while viewing the flow field from a downstream position. Fig. (2e) shows the effect that recirculation has on particle transport in the vicinity of the building. Notice how particles, emitted from a smoke stack near the top of the building, are entrained into the cavity region.

The problem results, depicted in Figs. (2a) through (2e), demonstrate how complicated flow fields in three dimensions can be computed, even when relatively few cells are used to resolve the flow field structure. The computational region in Fig. (2a) is composed of 2112 real cells: 16 wide, 12 deep and 11 high; however, the computer must store information for 3276 cells: 18 wide, 14 deep, and 13 high. The additional two cells in each direction are needed to set boundary conditions around the entire mesh. The building

is 2 cells wide by 6 cells deep by 6 cells high. The computation time for this problem was approximately three seconds per time cycle on the CDC 7600 computer; the total time to reach the steady state patterns shown in the figures was approximately 300 seconds.

In addition to solving the Navier-Stokes equations, the time dependent heat equation is solved in order to incorporate features of thermal buoyancy in the method. Through the use of the Boussinesq approximation, a buoyancy term equal to $\beta g_z (T_0 - T)$ is added to the right hand side of Eq. (1c) where β is the coefficient of volumetric expansion and T_0 is a constant reference temperature.

B. PARTICLE TRANSPORT

In addition to solving the fluid and heat transport equations, the computer program includes the simultaneous solution of a particulate transport equation. With this feature, distributions of particulate matter can be predicted in a great variety of flow fields.

The basic idea of particle transport involves a technique described by Sklarew⁽³⁾ to approximate the solution of the concentration equation,

$$\frac{\partial c}{\partial t} + \nabla \cdot \vec{u}c = \nabla \cdot (\lambda \nabla c) \quad , \quad (3)$$

where c represents a scalar concentration and λ is a diffusion coefficient. Eq. (3) predicts the transient behavior of a distribution, c , that is convected with velocity $\vec{u}$, and diffused with coefficient λ . Eq. (3) can be rewritten as

$$\frac{\partial c}{\partial t} + \nabla \cdot [(\vec{u} - \lambda \frac{\nabla c}{c})c] = 0 \quad . \quad (4)$$

This equation is the same as Eq. (3), but indicates that Eq. (3) can equivalently be interpreted as a pure convection process with velocity

$$\vec{u}' = \vec{u} - \lambda \frac{\nabla c}{c} \quad . \quad (5)$$

For the calculation, marker particles can be placed in the fluid region in such a way that the number of particles in a cell is proportional to c in the cell. When these particles are moved with the $\vec{u}'$, given by Eq. (5), the resulting distribution of particles approximates the solution of Eq. (3).

The basic Sklarew technique is quite useful in accounting for convection and diffusion of particulates whose mass and size are small. However, effects of inertia and gravity are influential in the transport of most real pollutants. To incorporate these effects in the program, a dynamic equation that includes Stokes drag, gravity and diffusion is solved for each particle. This equation reduces to Eq. (5) in the event that the size or mass of particles becomes negligibly small. In this more recent particle treatment, up to eight different particle species are allowed, each species being characterized by its own size and mass. The reward for the inclusion of gravity and inertial effects is the capability of predicting fallout or deposition patterns of pollutants on the ground as well as on and around structures. The complete details of this particle treatment are given in Ref. (4). By way of illustrating the calculation of particulate transport, Fig. (2e) shows a deposition pattern on the ground caused by a short smoke stack (not drawn) on top of a building. The wind is blowing from left to right through the computational region. This calculation was performed with the basic Sklarew technique, except that the particles were additionally assigned a small downward component of velocity to approximate gravity effects and to make some of them deposit on the floor. Notice that the deposition density is highest in the wake of the building and lowest at the far right side of the calculational region.

Fig. (3) shows another problem that has been calculated using the Sklarew diffusion method, but in this case the sizes and masses of particles were also considered. Air is flowing from left to right over the top of a canyon. A fluid jet with 4 species of particulates is injected from the canyon floor in the back of the mesh, viz, smoke from a power plant in a canyon. The back wall of the calculational region is a plane of symmetry, implying that a mirror image of the geometry viewed lies behind that plane. One sees the heavy deposition of particles on the canyon floor as well as on the top of the short mesa on the right side of the perspective view. Furthermore, notice that particulates deposit on the wall of the mesa. Although it is not evident in this figure, the particulates leaving the canyon comprise one species almost entirely, while the remaining three species deposit on the floor at varying rates.

The basic Sklarew technique can be used to predict accurate concentration distributions only if a large number of cells is used to provide proper resolution of the concentration gradients and if a large number of particles is used to give statistical accuracy. However, the computer limitations on the total

number of cells that can be used in this method does not always afford the luxury of such resolution. The consequences of these limitations are later discussed in some detail. The point is that it would be desirable to have an alternate scheme of modeling particulate transport that is less dependent on cell resolution. A scheme of this nature has been devised and is explained in detail in Ref. (4). Results from this newest scheme are not reported here.

III. COMPUTER USAGE

All of the calculations described in this paper were performed on the CDC 7600 computer with a 68,536 word small core memory. The calculation time per cycle for the computer program averages between one and four seconds depending upon the complexity of the problem and the types and frequency of data output.

The present program allows for a maximum of 3375 computational cells, which is equivalent to a computational region with 15 cells on an edge. In order to impose proper boundary conditions, as previously explained, a full layer of boundary or fictitious cells is necessary around the total periphery of the computing region. This means that the real fluid region has to be revolved with a mesh equivalent to 13 cells on a side. In addition to these cells, the program can have up to 10,000 particles with the partial use of extended core storage.

As the examples show, quite complex and interesting problems can be investigated, even with this limited cell and particle resolution.

Eventually, full use of extended core storage will allow approximately 20,000 computational cells along with approximately 30,000 particles in the computer program. This is roughly equivalent to increasing the resolution by a factor of two. Since it takes a computer additional time to retrieve external data and since there would be a considerable amount of additional data to retrieve, an order of magnitude increase in calculation time per cycle is not difficult to envision.

IV. DISPLAY TECHNIQUES

The calculation of three-dimensional flows is indeed an exciting adventure. However, the excitement soon disappears when one finds himself trying to comprehend the vast amount of data that result from these calculations. Efforts to mentally superimpose three orthogonal planes of data may allow comprehension of local flow fields, but does not readily allow one to recognize the overall structure of a flow.

The difficulties are greatly reduced when the data are displayed in true perspective. For this purpose, a newly developed perspective view technique⁽⁵⁾ has been incorporated into the program. The majority of figures used in this paper illustrate the technique. All of the computer results shown have been processed on the Stromberg-Carlson 4020 microfilm recorder and have not been retouched or altered in any way.

The technique is based on a theory of linear perspective. Fig. (4) illustrates what the eye sees when viewing an object from a distance. Visual rays, emerging from all points on object (ABCDE), converge to a point at the eye (O) and are focused onto the retina, forming an inverted image. The image is the same as that formed on an imaginary view plane (FGHI), placed between the eye and the object, by the piercing points (abcde) of the projecting lines; this plane does not necessarily have to be normal to the central line of sight.

The calculation of the points where the lines of sight pierce the plane are easily made with the computer. For this calculation two points need to be specified in space, an eye point and a point through which the "view" plane must pass. The line connecting these two points is then chosen to be the view plane's normal. In some situations it is better to use a slightly different view plane than that just mentioned. This is fully described in Ref. (5).

Points within the computing mesh that are vertices of rigid free-slip, and no-slip cells are the only ones that get projected onto the view plane. Visibility of these points is determined initially in the calculation by passing a line of sight between the eye point and the point in question. If no obstacle cells, other than the surrounding boundary cells, intersect this line, the point is visible. The plotting is then accomplished by drawing line segments between adjacent pairs of visible vertices that lie on obstacle edges, and between visible vertices on the outer edges of the computing region.

The plot routine used in this fashion consumes relatively little computer time while producing pictures of good quality. Small gaps between lines that should intersect can be seen in some of the figures. An extra step could be incorporated to eliminate these, but it would require additional computing time. Since the present scheme produces very acceptable plots that can be used in many ways, such a refinement has not been considered necessary at this time.

After a perspective plot of the obstacles and the outside of the computing mesh has been made, other features such as velocity vectors and particle distributions can be plotted to show how they appear with respect to the obstacles. Contours of temperature, pressure, concentration, vorticity, etc., can also

be shown in perspective, although this feature has not yet been fully included in the program.

When two slightly different eye positions are used to make separate perspective views of the same object the results may be viewed in a manner to yield depth perception, i.e., stereoscopic vision. For best results the two plots should be viewed in a special stereo viewer. However, another way to view stereo is illustrated in Figs. (5) and (6). Fig. (5) shows two perspective views in which the flow is from left to right around obstacles. A source of particulate matter is located upstream and to the left of the first structure. The left view is for the right eye and the right view is for the left eye. To see in stereo the observer must make each eye focus on the corresponding view by crossing the eyes. In so doing the stereo image appears as a virtual image located between the paper and the eyes. Fig. (6) displays a plane of velocity vectors indicating the flow field near the top of a complicated obstacle (viz, an apartment complex). The flow again is from left to right. In this figure the right view is now for the right eye and the left view for the left eye. Stereo can be seen if the eyes are only slightly crossed making the virtual image appear behind the paper.

Admittedly, viewing stereo in either manner can be quite frustrating, for not everyone seems to be able to bring the superimposed images into focus. Practice and patience are the only remedies and are usually worth the effort.

All of these viewing aids are helpful in comprehending the results of three-dimensional calculations. However, each view must be taken at a certain point in the time history of the calculation. This then requires that many views, taken as time progresses, need to be studied to appreciate the transient features of a problem. Again the computer and the above mentioned plotting devices can be used to alleviate the difficulty, making a perspective view each time step, to produce a motion picture.

An even more interesting movie can be made by moving the eye coordinate each time step along a prescribed path in space. The movie so obtained gives the observer the sensation of flying over the computing region, if in an airplane. Fully three-dimensional transient effects are better appreciated in this way.

Perspective pictures can also be made in color. This form of output is extremely useful in displaying certain types of results. An example of this involves the particle transport method in which more than one species is allowed. Each species of particulates can be plotted in a different color to exhibit the response each has to its environment.

The ultimate in displays are motion pictures in stereo. Unfortunately, these require special viewing aids and, thus, are not easily presented to large audiences.

A final note should be made here to indicate the fact that the outputs available from the computer are vast and are not limited to the types illustrated in this paper. For example, eddy structure can be shown plotting vorticity, and fluid motions can be traced by plotting streamlines. Furthermore, the detailed flow structure is available in the form of complete listings of flow variables for each cell in the computing mesh and the computer can be programmed to display these values in any fashion desired.

V. RESULTS

The majority of examples that are presented here have been performed to illustrate the qualitative capabilities of this new computing method.

To assess the quantitative accuracy of the technique it is necessary to examine its predictions for problems in which either analytical or detailed experimental data are available. For example, the problem of a symmetric plume in a steady, uniform flow field has a well known solution. The simulation of this problem is shown in Fig. (7) which is a perspective view of the particle distribution obtained from the Shuler particle transport approximation.

A source is located at the lower left. The mesh consists of cubical cells with unit edge lengths. A uniform, steady wind left to right convects approximately 6000 particles downstream. The bottom plane, a plane of symmetry, allows the calculation of only half the problem.

The steady state analytic solution for the convection of a plume emanating from a point source is

$$c(x,y,z) = \frac{C_0}{\sqrt{\pi^2 + y^2 + z^2}} \exp \left[-\frac{u}{2h} \left(\sqrt{\pi^2 + y^2 + z^2} - \pi \right) \right]$$

where $C_0 = Q/4\pi h$, Q is the rate of mass input at the source, u is the wind speed and h is the coefficient of turbulent diffusion.

In the finite difference scheme, a point source is not strictly allowed. If many particles were input into the mesh at one point, they would continue forever to move as a single point. To prevent this, the particles to be input each cycle are randomly placed in a small but finite volume on the order of a cell size. In this case, in a cube with edge length 0.4 cell edges.

Even with this approximation, however, concentration gradients near the source will be in error because of poor finite difference resolution. The effects of this localized error can be seen in Fig. (8). This figure shows the normalized concentration that exists along the centerline of the plume. The dashed line represents the concentration obtained by evaluating Eq. (6). The solid line in Fig. (8) indicates the concentration obtained numerically. The normalizing factor in each case was chosen to give a unit value of concentration at the first cell center. The errors are largest in the vicinity of the source, but after particles have moved 4 or so cells downstream (i.e., $n > 4$), the finite difference solution approaches and closely approximates the analytic distribution.

In Fig. (9) transverse concentration distributions at $x = 0$ are compared at various downstream positions. The dashed lines represent the theoretical distributions obtained from Eq. (6). In each case the normalizing factor was chosen to give unit concentration at the centerline. The solid lines represent the corresponding numerical results. Again the worst results are obtained near the source. As one goes further downstream the calculated distributions steadily improve. The middle curve of Fig. (9) is the transverse distribution 5 cells downstream from the source. The outer curve is the distribution 10 cells downstream. The dip at the center of this last curve results from a local fluctuation in particle concentration; the centerline cell had three fewer particles than its right neighbor. Considering the limited resolution used for this problem, the overall agreement is good.

It may be concluded from this example that the simple particle transport model is not accurate in regions where the transverse dimension of the plume is confined to less than four or five computational cells. This is expected, since particle diffusion depends on concentration gradients between cells, and these are not accurate where the cross section of the plume is resolved by only a few cells. In air pollution modeling near structures, point sources are a must, as the size of the sources is usually very small compared to nearby structures. Hence, an alternative particle treatment that is independent of cell size would be of considerable value. Such a treatment has been devised and is described in Ref. (4). In this new treatment diffusion of particles is accomplished by moving each particle with a suitable random velocity that is locally Gaussian. The new technique also accounts for the dynamic influences of Stokes drag and gravity on each particle.

VI. SUMMARY

The development of a computing technique capable of solving the fluid dynamics equations and simultaneously predicting the behavior of particulate matter offers a valuable tool for research in air and water pollution. The variety of examples given here indicate the potentialities and scope of applicability of this new tool to highly complex fluid flow problems.

In the future, new features will be incorporated in the program to permit considerably greater accuracy, more versatility, and the introduction of new capabilities, such as better turbulence models, chemical reactions, and layered atmospheres.

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Figure 1. Location of variables on a typical cell having dimensions l_x , l_y , and l_z

Figure 2a. View of the velocity vectors in a plane taken through the center of the top eddy in Fig. 2b

Figure 2b. Perspective view of the velocity field about a single structure

Figure 2c. Plan view of the velocity field that exists directly behind the structure as seen from downstream

Figure 2d. Plan view of the velocity vectors as seen from above Fig. 2a and magnified

Figure 2e. Perspective view of the particulate distribution in the flow field shown in Figs. 2a through 2d

Figure 3. Perspective view of the distribution of four species of particulates emitted from a jet in the bottom of a canyon

Figure 7. Perspective view of the particulate distribution in a symmetric plume

Figure 6. Illustration of linear perspective (from Brook Taylor, "The Principles of Linear Perspective," 1811)

Figure 8. Comparison of the numerical and analytic solutions for the centerline distribution of a symmetric plume

Figure 9. The two perspective views appear in stereo when viewed cross-eyed

Figure 5. Comparison of the numerical and analytic solutions for the transverse distributions of a symmetric plume

Figure 6. The two perspective views appear in stereo when viewed only slightly crossed

A Three-Dimensional Numerical Model for Calculating the Spread and Dilution of Air Pollutants

R. B. Lantz and K. B. Coats

INTERCOMP Resource Development and Engineering Ltd., Houston, Texas

and

C. V. Kleeper, Aquitaine Company of Canada Ltd.

INTRODUCTION

General

Effective air pollution abatement necessitates having a quantitative evaluation of the effect of environmental factors--wind velocity, temperature and humidity--on the movement and dilution of a pollutant. A comprehensive mathematical model can assist both government and industry in defining these environmental influences and in answering such questions as (1) how many monitoring locations are required (2) where are the critical monitoring sites (3) what severe environmental factors will result in pollution buildups, and (4) where should a new plant be located.

Perhaps the most widely used mathematical models are the "Gaussian plume" models(1,2,3). These models assume a normal distribution of concentration in the vertical and crosswind directions. To match measured pollutant concentrations, the standard deviation of the concentration distribution increases in nearly direct proportion to the downwind distance from the source. The Gaussian plume models have a number of restrictions which limit their use--for example, uniform winds, no chemical reaction, or ground adsorption. The uniform wind field requirement restricts use of the plume models to problems where topographic effects are unimportant.

Two recent articles have described methods which can include topographic effects(4,5). These techniques are in many ways similar to the model we are presenting; however, the numerical methods used are quite different. The methods used in our pollution model have been developed in the petroleum industry to solve multi-phase, multidimensional fluid flow problems. These numerical techniques result in an efficient comprehensive calculational method for assessing the spread and dilution of an air pollutant.

Description of the Model

The model is a numerical solution of the three-dimensional material balances for pollutant flow and for the entire air

stream. The pollutant material balance requires solution of the three-dimensional convection-diffusion partial differential equation. Wind flow over uneven topography or around buildings is calculated by numerically solving a modified form of the three-dimensional equation for velocity potential. The modification includes (1) inviscid potential flow at high elevations, and (2) height-dependent coefficients which account for surface friction (viscous effects) in the boundary layer. The empirical modification causes the horizontal velocities to vary with height, e.g., either of the two familiar forms, logarithmic or a power law.

The material balance on pollutant includes ground adsorption (or vegetative absorption) and chemical reaction in the atmosphere. The eddy diffusivities can be space-dependent. Our experience with the model indicates variation of velocity and eddy diffusivity with height is essential to give agreement with the simple Gaussian plume models of Sutton(1) and Pasquill(3). The plume models are in general agreement with experimental observations.

The numerical schemes used in the solution of both the pollutant and the air material balances are efficient ones requiring minimal computer time. There are no time step limitations involved in the numerical calculations. The steady-state solution, which many urban diffusion models utilize and which should be a good approximation to many practical situations can be obtained accurately in a single time step, or the entire progression to steady-state can be calculated in a series of time steps.

Grid blocks corresponding to terrain features or buildings are removed from the calculations. Storage requirement for 7000 grid blocks, in core, is approximately 100,000 words. A disked version is also available which allows a virtually unlimited number of blocks. Computing time, based upon a CDC 6600, for a single prediction can range from a few seconds for a steady-state case (where the wind distribution is specified) to ten minutes

or more for a highly transient wind condition (where the wind velocity is changing significantly over a few hour time period). Computing times of only a few seconds for a steady-state problem with 7000 grid blocks reflects the fact that an efficient, stable numerical scheme has been used to solve the three dimensional, partial differential equations.

MATHEMATICAL FORMULATION

Wind Calculation Considering Topography

Material balance on the entire flowing air stream, assuming constant density, gives

$$\nabla \cdot \bar{U} = q \quad (1)$$

where $\bar{U}$ = the time averaged air velocity vector consisting of components u , v and w in the x , y and z directions respectively.

q = the source and sink volume rate per unit volume which will generate the desired wind distribution.

If a velocity potential defined by

$$\bar{U} = \nabla \phi \quad (2)$$

is introduced into Equation (1), a form of Laplace's equation is obtained

$$\nabla^2 \phi = q \quad (3)$$

The assumption that the fluid motion is irrotational is contained in the definition of potential given by Equation (2). Solution of Equation (3) for boundary conditions of constant potential in the vertical direction and no terrain feature results in horizontal velocities which do not vary with height. However, near the surface, air viscosity becomes important and measurements of turbulent flow over a flat plate are found to give horizontal velocities which vary with height in a logarithmic or power law fashion. To account for this known behavior, the definition of velocity potential in Equation (2) was modified to include a "potential coefficient," K , given by

$$\bar{U} = K \nabla \phi \quad (4)$$

where K has components K_x , K_y and K_z .

As a consequence, over flat terrain, the horizontal velocities will have the same variation with height as do the corresponding potential coefficients in that direction. Over uneven terrain, vertical and crosswind flow will modify this power law or logarithmic variation. Additionally these coefficients can be varied spatially to indicate different amounts of surface roughness.

Substituting Equation (4) into Equation (1) gives

$$\nabla \cdot K \nabla \phi = q \quad (5)$$

Solution of Equation (5) for the potential, ϕ , along with Equation (4) gives the three-dimensional velocity field $\bar{U}$. A variety of boundary conditions can be used for Equation (5). One possibility is that the wind is one-dimensional along the external boundaries. That is, u is specified and v and w are zero. The horizontal velocity at the boundary can of course be variable with height. Other boundary variations can be created with sources and sinks. No flow boundaries representing terrain are created by setting the appropriate potential coefficient to zero.

Formulation of the air flow in terms of a modified velocity potential represents a significant simplification of the real problem. Neglected in such an approach are such factors as (1) the change in wind direction with increasing elevation (Ekman spiral), (2) formation of eddies on the downwind side of obstacles to flow, and (3) thermal or any density instability which can cause vortices in the flow.

The change in wind direction from that of the geostrophic wind to that of the surface flow appears to vary from 10° over water to perhaps 20° over land [6]. The vertical elevation at which the wind approaches geostrophic may range from 1500 to 3000 feet. Variation of wind direction with elevation could be included in the velocity potential model if comparison with measured concentrations requires it.

Our model for describing wind flow over irregular terrain features has been kept simple for two reasons. First, we did not want to consume a disproportionate share of computer time in solving the wind flow problem compared to that required for the turbulent diffusion solution. Second, the mathematical description of individual eddies downstream from obstacles in the flow is probably not essential to the pollutant dilution problem. In a gross sense, the increased dispersion of pollutant due to eddy formation can be approximated by increasing the eddy diffusivity downstream of obstacles. As more measured data become available on pollutants, accounting for the formation of eddies may be desirable or even required.

The finite difference representation used for Equation (5) was

$$\Delta(T \Delta \phi)_{ijk} = q_{ijk} \quad (6)$$

where

$$\Delta(T\Delta\phi) = \Delta_x(T_x\Delta_x\phi) + \Delta_y(T_y\Delta_y\phi) + \Delta_z(T_z\Delta_z\phi)$$

$$\Delta_x(T_x\Delta_x\phi) = T_{i+1/2}(C_{i+1}-C_i)_{jk} - T_{i-1/2}(C_i-C_{i-1})_{jk}$$

$$T_{i+1/2,jk} = (K_x\Delta y\Delta z/\Delta x)_{i+1/2,jk}$$

Equation (5) is solved by one of two methods depending upon the boundary conditions. These methods are (1) for constant potential in the vertical at the boundaries--line successive over-relaxation (LSOR) and (2) for no flow boundaries with internal sources and sinks--the strongly implicit procedure (SIP) of Stone[7].

Turbulent Diffusion

Material balance on the pollutant flowing in the air stream with velocity field, $\bar{U}$, defined in the previous section gives:

$$\nabla \cdot EVC - \bar{U} \cdot \nabla C + r = \frac{\partial C}{\partial t} - q_s + q_a \quad (7)$$

where C = pollutant concentration, lbs. pollutant/lb. air

E = eddy diffusivity with components E_x , E_y and E_z in ft.²/sec.

r = rate of disappearance of pollutant due to reaction, lb. pollutant/sec./lb. air

q_s = pollutant source rate, lbs. pollutant/sec./lb. air

q_a = pollutant ground adsorption rate, lbs. pollutant/sec./lb. air

t = time, sec.

The rate of disappearance of pollutant by chemical reaction, r , is assumed to be first-order and thus is proportional to the concentration. The pollutant source rate, q_s , is the pollutant mass source rate per unit volume of the total emission, divided by the air density. The ground adsorption rate, q_a , is the mass rate of pollutant absorbed (or adsorbed) at ground level per unit surface area multiplied by the specific surface of the ground and divided by the air density.

Equation (7) can be solved once the velocity field, $\bar{U}$, and the boundary conditions are specified. Calculation of the velocity profile is discussed in the previous section. Boundary conditions for diffusion are that the flux normal to the ground surface and at the upper

boundary is zero. This upper boundary can represent a temperature inversion if desired.

The finite-difference approximation used for Equation (7) is of the form

$$(\Delta T \Delta C)_{ijk,n+1} - \Delta_y(v_{A_y} C)_{ijk,n+1} - \Delta_z(w_{A_z} C)_{ijk,n+1} + v_{ijk}(q_{s,ijk} - \frac{\ln 2}{\tau} C_{ijk,n+1})$$

$$= \Delta_x(u_{A_x} C)_{ijk,n+1} + \frac{1}{\Delta t}(V_{ijk} + \frac{A_z \Delta C_s}{\rho \Delta C}) (C_{ijk,n+1} - C_{ijk,n}) \quad (8)$$

where

$$\Delta(T \Delta C) = \Delta_x(T_x \Delta_x C) + \Delta_y(T_y \Delta_y C) + \Delta_z(T_z \Delta_z C)$$

$$\Delta_y(T_y \Delta_y C) = T_{ij+1/2,k}(C_{j+1}-C_j)_{ik} - T_{ij-1/2,k}(C_j-C_{j-1})_{ik}$$

$$T_{ij+1/2,k} = (E_y \Delta x \Delta z / \Delta y)_{ij+1/2,k}$$

$$\Delta_y(v_{A_y} C) = [(v_{A_x} \Delta z)_{j+1/2} C_j - (v_{A_x} \Delta z)_{j-1/2} C_{j-1}]_{ik}$$

$$V_{ijk} = \text{block volume } (\Delta x \Delta y \Delta z)_{ijk}$$

$$\frac{\Delta C_s}{\rho \Delta C} = \text{slope of the adsorption isotherm, ft.}^3/\text{ft.}^2$$

$i, j, k = x, y, z$ indices

n = the time level index

$\Delta x, \Delta y, \Delta z, \Delta t = x, y, z$ and the time increments

τ = the reaction half-life

The convection terms in each direction are actually weighted toward the upstream flow. That is, if $v_{j+1/2}$ is negative, C_{j+1} is used. The numerical procedure used to solve Equation (8) is the familiar alternating direction procedure (ADI) of Douglas-Rachford[8]. If the primary wind direction is in the x -direction, the magnitude of the x -diffusion is unimportant. Note that for these cases, a two-dimensional ADI procedure can be used since concentrations in each vertical plane are affected only by those upwind rather than by both upwind and downwind directions.

Because of the first-order derivatives in Equation (7), truncation error caused by the finite difference approximations both in space and time can be important. When the physical diffusion is small relative to convection, truncation error gives rise to a numerical diffusivity which can dominate the physical diffusivity. Generally, this happens in cases where the physical diffusion is molecular and consequently small rather than when turbulent diffusion is the predominant dispersive mechanism. Our experience from comparison of the numerical model with analytical solutions indicates truncation error will not be a significant limitation for the majority of the problems of interest.

Plume Rise

Since we have assumed constant density, the wind equations will not describe thermal rise of the plume. Instead, we have used the concept of effective stack height. A number of procedures have been developed to estimate the effective height of stack emitting into the air. Briggs[9] has compared nine of the available techniques as well as his own with existing plume rise data from sixteen different sources. We have used his procedure to estimate the effective stack height in the numerical model.

RESULTS

Comparison with Gaussian Plume Models

No general analytical solution exists for the three-dimensional diffusion-convection equation where eddy diffusivities and the horizontal velocity have a power law or logarithmic height dependence.

Figure 1 illustrates a dimensionless comparison of the often-used Gaussian plume models with computed ground concentrations for the velocity and eddy diffusivities chosen as power law functions of height. The $1/7$, $6/7$ power law models for velocity and diffusion, respectively, generally give good agreement with Sutton's model[1] for $n = 0.5$ (stable, inversion). Other power law forms for the diffusivities were used retaining the $1/7$ power variation of velocity with height. Results for a power of one and $1/2$ are also shown in Figure 1. The $6/7$ power form is in excellent agreement with the Pasquill-Gifford-Pearson model[1]. These particular power law forms give computed results which range from the class C to the class D stability defined by Pasquill[11] and evaluated in terms of standard deviations by Gifford[12].

A dimensionless comparison of ground concentrations was used in Figure 1 to better reflect the functional dependence of concentration on downwind distance. The actual maximum concentrations calcu-

lated by any of the methods differ little as long as the ratio of the crosswind diffusivity to the downwind diffusivity is the same. Unfortunately, the recommended values for this diffusivity ratio to be used with each method are seldom consistent. Thus the dimensionless comparison is more meaningful.

Note that there is a substantial difference in the behavior of concentrations using height dependent velocity and diffusivity compared to the constant velocity-diffusivity case. Agreement of the power law forms for velocity and diffusivity with the Gaussian plume models appears to be well within the accuracy of these models to describe measured behavior.

Conceivably many different power law forms for velocity and diffusivity could be used in the numerical solution and still get reasonable agreement with the Gaussian plume models provided the appropriate dependence of concentration on downwind distance was obtained. However, we would recommend that until more experience is obtained in the use of the model, velocity profiles consistent with Frost's[11] measurements on the effect of temperature gradient be used. We have approximated his measurements in terms of the more commonly used Pasquill-Gifford temperature stability classes.

Table 1 lists values of the powers for both velocity and diffusivity dependence on height which will give fair agreement with the various stability classes of Pasquill-Gifford. The value of the eddy diffusivity at a standard reference plane of 100 feet and a velocity of 1 ft./sec. is also listed for each stability class. The ratio of the crosswind diffusivity to that in the vertical direction must vary substantially to match the maximum concentrations given by the Pasquill method and summarized in Turner[13].

TABLE 1

POWER LAW FORMS APPROXIMATELY EQUIVALENT TO PASQUILL-GIFFORD STABILITY CLASSES

Power	A	B	C	D	E	F
U_z						
u	0.14	0.14	0.14	0.3	0.3	0.6
K_z						
u	1.76	1.38	1.14	1.0	1.0	1.0
U_{ref}	687	186	61	15	6	3
K_y/K_z	0.1	0.5	1.5	6	15	45

For an increase in wind velocity, the reference eddy diffusivity value should be increased in direct proportion to retain agreement with the Pasquill plume model. Also, if the reference plane is increased the reference diffusivity should be increased as the ratio raised to the power, $n-m$.

Calculations for a Large Urban Area

Calculated SO_2 concentrations for the city of Chicago have been compared with both measured values and with values calculated by the Argonne National Laboratory urban diffusion model[13]. The ANL urban calculational technique is based upon the plume models. The source inventory for Chicago is summarized in the reference. The report, for the most part, presents statistical summaries of monthly and seasonal averages. However, the prediction and input data for one winter day, January 15, 1967, is included. We have made a prediction with our numerical model using the input source rates defined in the report.

The sources include six large power plants--four of which directly affect the measuring stations and were included in our model. Additionally, sixteen of the largest industrial sources were isolated and included separately. The remaining sources of SO_2 consisted of high and low rise residential buildings along with the remaining industrial plants. These were also included.

Figure 2 illustrates the areal distribution of calculated ground concentrations overlaying a map outline of the city. The concentrations in Figure 2 are plotted normalized on the basis of 1 ppm. That is, the "ones" represent 0.05 to 0.10 ppm. The "twos" represent 0.15 to 0.20 ppm and the "blanks" between "ones" and "twos" correspond to 0.10 to 0.15 ppm. The concentration contours clearly show the effect of the major sources of SO_2 .

Figure 3 compares the measured SO_2 values at four stations on January 15, 1967 with calculated values from both our numerical model and the ANL urban diffusion model. The diffusion coefficients used were those representative of neutral stability--the equivalent of the Pasquill stability class D indicated in Table I. Note there is good agreement between the two calculational models for three of the stations. For the remaining station our numerical model was in better agreement with the measured data. Computing time was 1.5 minutes on a CDC 6600 for the six 3-hour time steps with the 4800 blocks used in our Chicago grid.

Comparison with Wind Tunnel Tests

Local obstructions to flow whether due to terrain or large buildings can substantially affect the dilution of a pollutant in the air. The total effect is generally due not only to a change in wind speed and direction but is also due to increased turbulence, and, as a consequence, increased diffusion.

We have compared calculated concentrations with the wind tunnel measurements of Strom, et al[14] for downwash from a stack located on top of a building. These model tests included measuring SO_2 concentrations downwind of an isolated stack and comparing them with concentrations downwind from the same size building-stack combination.

Two different types of calculations were made for the isolated stack case. In one, the observed vertical plume rise velocity was specified and the turbulent diffusion of SO_2 was calculated. In the second, the concept of the effective stack height with no plume rise was used. The latter calculation is shown in Figure 4 to be in good agreement with measured concentrations on a vertical line through the plume axis 72' downwind of the stack.

With the building present, the downwash velocities downwind from the building were calculated in the numerical model using a vertical potential coefficient, K_z , of one. The observed vertical plume rise velocities from case 1 above were then added to the calculated downwash velocities. This total velocity still satisfies continuity (Equation (5)).

Diffusion coefficients equal to those used for the isolated stack case gave calculated results in poor agreement with the measured values. Increasing the diffusion in both directions by a factor of four gives reasonable agreement between calculated and measured concentrations. This increase in eddy diffusion undoubtedly reflects the increased turbulence caused by the presence of the building.

Similar model wind tunnel tests were reported by Hino[14] with topographic features present. Results presented in his paper do not show a significant effect of the terrain, however, and we have not attempted to compare our calculations with them.

Other Applications

The effect of chemical reaction and ground adsorption for a typical case of a stack emitting pollutant into a light wind has been calculated. For a pollutant half-life of 1000 seconds, the maximum steady-state ground concentration was reduced by roughly 50%. Half-lives of two to four hours more characteristic of SO_2 , will give 10 to 20% reductions in the steady-state values for light winds and a lesser effect for moderate winds.

Ground adsorption in our formulation does not affect the steady-state concentration distribution. Only the transient concentrations are modified. This is undoubtedly true for actual adsorption processes where the surface sites can all be occupied. Deposition, often synonymously termed as ground adsorption, probably would need to be handled differently.

CONCLUSIONS

Described herein is a three-dimensional numerical model for calculating the movement and dilution of a pollutant emitted into the air. The technique includes modification of the three-dimensional wind field for flow over topographical features and around buildings. Simple chemical reaction and removal by ground adsorption are included in the material balance on pollutant.

Our experience using the numerical model indicates the following:

- 1) Height dependent velocity and eddy diffusivities give calculated results in good agreement with the classical Gaussian plume models for flat terrain.
- 2) Calculated concentrations compare favorably with results from wind tunnel experiments for complicated flow patterns.
- 3) There are no time step limitations in the numerical calculation. The steady-state solutions utilized in the Gaussian plume models and in many urban diffusion models can be generated in a single time step.
- 4) Computing times based upon a CDC 6600 for a large 7000 grid block system will range from less than half a minute for a steady-state problem with no terrain feature to ten minutes or more for a complex transient problem.
- 5) Truncation error should not be a serious limitation for the usual terrain features encountered.

The numerical model has been checked against measured data but even more comparisons are needed to guarantee its applicability to a wide range of terrain flow problems. We feel, however, that the numerical model represents a substantial improvement in the state-of-the-art for calculating pollutant diffusion emitted from a single stack or from multiple sources.

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Comparison Numerical Model
with Gaussian Plume Models

FIGURE 1

Comparison with Measured Concentrations

FIGURE 2

FIGURE 3

Comparison Measured and
Calculated SO2 Concentrations
in Chicago

FIGURE 4

LIDAR OBSERVATIONS IN TURBULENCE AND DIFFUSION STUDIES

R. T. H. Collis

Stanford Research Institute, Menlo Park, California

INTRODUCTION

The lidar technique of remotely mapping and tracking subvisible concentrations of particulates can be of considerable value in turbulence and diffusion studies. By revealing variations in the vertical distribution of the "natural" aerosol it provides information on the stability of the lower atmosphere and its changes with time. In addition, lidar observations can show the motion and dispersal of smoke, or the like, making it possible to study transport and diffusion processes conveniently in three dimensions, even when the concentrations of the material are too low to be seen or photographed.

Although the technique has obvious possibilities for eventual use in routine air pollution control operations, this paper emphasizes the use of lidar in turbulence and diffusion research.

BASIC LIDAR CONCEPT

In its basic form, lidar employs a laser simply as a source of pulsed energy. Typically, Q-switched ruby or neodymium doped glass systems are used to generate pulses, having peak power of tens of megawatts and durations of 10 to 20 nanoseconds, that are directed in highly collimated beams by suitable optical systems. Energy backscattered by the atmosphere is detected by a photomultiplier after collection by a lens or reflector system. The resulting signal is evaluated as a function of time from the transmission of the pulse, in radar fashion, and is typically displayed on a CRT oscilloscope either directly or after storage in a magnetic-disc video recorder. Polaroid or other photographs may be used to record the displayed data.

Taking advantage of the monochromatic nature of the laser energy, one may use narrow-band optical filters in the receiver system to minimize "noise" caused by extraneous light. The nature of the received signal, P_r , is given by the lidar equation:

$$P_r(R) = P_t L^2 \beta(R) A \mu^{-2} \exp \left\{ -2 \int_0^R \sigma(r) dr \right\}$$

where

P_r is instantaneous received power

P_t is the power of the pulse transmitted at time t_0

L is effective pulse length ($L = c\tau/2$, where c is the velocity of light and τ is pulse duration; it is the range interval from which signals are simultaneously received at time t_0)

β is the volume backscattering coefficient of the atmosphere ($\text{ster}^{-1} \text{ km}^{-1}$)

R is range ($R = c(t - t_0)/2$), where t_0 is the time of transmission pulse

σ is the volume extinction coefficient (km^{-1})

A is the effective receiver aperture.

The magnitude of β and σ depend upon the wavelength of the incident energy, and the number, size, shape, and refractive properties of the illuminated particles per unit volume. These range from the hydrometeors of precipitation, cloud, or fog, through the particulate components of the "clear" aerosol, to the molecules of the gaseous atmosphere. The scattering and absorption mechanisms involved have, of course, been widely studied

For the moment it will suffice to note that at the wavelengths commonly employed (for example, 0.6943 microns for ruby lasers and 1.06 microns for neodymium lasers) the energy is readily scattered by particulate material of even very clear atmospheres, and typical lidar systems can detect returns from the clear gaseous upper atmosphere.

In relatively clear atmospheres the extinction coefficient is small, and variations in backscattering as a function of range can readily be interpreted. For example, the presence of a sudden increase in scattering

Observations of Smoke Stack Plumes

In connection with studies of the dispersion of effluent from the tall stacks (some 330 m high) of power plants in western Pennsylvania, a series of lidar observations were made to show the changes in cross section of the downwind plume in various meteorological situations. The lidar was positioned so that vertical cross sections could be observed by scanning in elevation across the beam (Figure 1) at different distances downwind, as shown in Figure 3. Figure 3 also shows how readily the envelope and internal structure of the plume can be mapped, in this case with contours of relative signal intensity (which indicate the approximate relative volume or mass concentration although, due to attenuation, they will underestimate these values on the further side of the denser plumes). On the occasion shown (25 May 1975), the plume initially rose from the stack output at 245 m to approximately 300 m, as reasonably well predicted by the Briggs/ASHRAE plume rise formula. By some 3 km downwind, however, marked fumigation had occurred and the plume had become approximately 1.5 km wide with considerable dispersion into the lower levels at a downwind distance of 5 km. (The density of the plume was such that it was only barely visible to the eye within the first few kilometers downwind.)

The data obtained by lidar revealed in detail the important plume characteristics that must be included in a realistic diffusion model for plumes from tall stacks: i) fanning and tilting due to wind veering with height, ii) fumigation, which brings high concentrations to ground level in a pattern which, because of the tilting, progresses with time from left to right of the plume looking downwind, and iii) plume trapping by elevated stable layers.

Observations of Urban Aerosol Structure

During an investigation of how urban pollution might cause weather modification a series of lidar observations were made in St. Louis, Missouri. Figure 4 shows a typical example of a cross-section developed by making continuous observations from a site approximately 0.5 km north of the Gateway Arch at a rate of 2 pulses/min from 0734 to 1600 local time (13 August, 1971) with the lidar pointed vertically. The height/time cross section which resulted is compared with available radiosonde data, and shows how material is transported upward from the surface in a daily life cycle of the boundary layer. The data presented in Figure 4(b) started at 0703 and used a lidar pulse rate of 10/min. The data from a radiosonde launched near the Arch at 0700 are shown in Figure 4(a). The sky was overcast with the cloud base at 2.5 km and a dense haze layer below. Visibility was estimated to be on the order of 2 mi. The A-scope presentation (received signal as a function of range) of the 0721 lidar firing (Figure 4(c)) reveals that the signal return from beyond the cloud was below the receiver noise level, so the relative density of the aerosol above the cloud is unknown. A dense aerosol layer extended from the surface to a height of 0.25 km and most likely represented particulate matter newly injected or forced in the atmosphere. The vertical profiles of temperature and dew point (Figure 4(a)) show a low-level temperature inversion extending to a height corresponding to the top of this near-surface aerosol layer. A change of lapse rate also marks the cloud base.

After 0734, the height/time cross section was continued with a lidar pulse rate of 3/min (Figure 4(e)). The winds throughout the day remained from the south at 5 to 7 mi/hr. As the day progressed, higher level cloud layers were detected by the lidar. Dark areas above a cloud result from the increased attenuation of the laser energy along the round-trip path through the cloud. The lower overcast "burned off" at about 1030, and the aerosol optical density within the haze layer gradually decreased. This decrease was probably caused by a general decrease in size of hygroscopic particles with the decrease of relative humidity that accompanies warmer temperatures. During the day, depth and optical density of the near-surface layer gradually increased. The top of the aerosol layer at 2.5 km dissipated, and a new upper boundary was established near 2 km. Beginning at 1215 active thermal convection increased the height of the lowest aerosol layer at a rate of 0.75 km/hr. A temperature sounding made at 1230 (Figure 4(d)) from the Arch shows the near adiabatic lapse rate associated with the heating of the lowest atmospheric layer and a change of lapse rate at the tops of haze layers. Clear air convective cells are revealed by consistent wave structure of all three aerosol layers during the 1310-1530 time period (Figure 4(f)). Convection carried the relatively warm and dirty air up to the 2-km boundary and into the relatively clean and cool air, causing convective cloud formation. These clouds, identified in Figure 4(f) by very bright areas with very dark areas above, occurred at maximum heights of the haze layer, which seem to show a periodic nature during the interval from 1300 to 1600.

CONCLUSIONS

Even with a relatively simple lidar system capable of delineating variations in particulate concentration in relative terms, it is possible to reveal the changing structure of the natural aerosol and to track and map such indicators as smoke plumes. With more sophisticated data processing, it is possible to provide quantitative information on diffusion processes, by monitoring changes in particle concentration within such plumes.

Lidar can thus make valuable contributions to air pollution investigations and observation studies. However, it can provide a far broader data base of the mechanisms involved, showing, by reference to individual such as nuclei, meteorological details of the manner in which pollutants are transported and diffused. Similarly, it can provide more completely and consistently than visual or in situ measurements, information on atmospheric structure and its changes with time, such as is needed in the verification of urban pollution models. Therefore, lidar can provide an input data to transport and diffusion models, particularly in connection with vertical mixing, by extending the space and time measurements data.

Finally, the potential role of lidar for weather and air pollution control operations should be noted. Provided that lidar systems of moderate cost can be developed that can be easily operated on a routine, non-attended basis, they could be easily useful in monitoring the conditions in the diffusion volume over a city. In addition to providing a continuous check on the build up of pollution concentrations in the boundary layer, they could provide input data to forecasts or changes of the mixing depth or of stability, by revealing directly or indirectly the dynamic, physical and radiative processes in action. Such inputs would be particularly valuable in anticipating conditions in which air pollution episodes occur.

REFERENCES

This note is based very largely upon the work of my colleagues at JPL, particularly Dr. Marvin S. Johnson, Jr. (see ref. 1) and Dr. Harold E. Hays.

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FIGURE 3 LIDAR OBSERVATIONS OF SMOKE PLUME FROM KEYSTONE STAGE, 25 MAY 1963 IN CONDITIONS OF MODERATE DISCUSSION

FIGURE 4 HEIGHT-TIME CROSS SECTION OF THE AEROSOL STRUCTURE OVER ST. LOUIS, MISSOURI, ON 13 AUGUST 1961 AS OBSERVED BY SRI-GPA MARK VII LIDAR SYSTEM

RECENT RESULTS IN GROUND-BASED SENSING
OF ATMOSPHERIC TEMPERATURE PROFILES

J. B. Snider

Wave Propagation Laboratory
Environmental Research Laboratories
National Oceanic and Atmospheric Administration
Boulder, Colorado 80302

INTRODUCTION

The application of ground-based radiometric measurements of emission from atmospheric oxygen to the problem of estimating the variation of temperature with height has been discussed in recent literature. (1-3) In the majority of measurements reported, temperature profiles have been determined by inverting single frequency measurements of brightness temperature as a function of elevation angle. An alternate technique consists of inverting observations of brightness temperature as a function of frequency at either a fixed or variable elevation angle. The second technique has received relatively little attention.

An obvious advantage in favor of the first, or angular scan method, is that the instrumentation is fairly simple, consisting of a single frequency radiometer with an antenna that is steerable in elevation; however, conversion of antenna to brightness temperature may be complicated and subject to error since the stray radiation into antenna back and side lobes changes with elevation angle. In the second, or multi-spectral method, the antenna is not moved with the result that stray radiation is relatively constant thus simplifying the determination of brightness temperature.

In principle, the multi-spectral system offers the advantage that a profile can be measured in a time period set mainly by the radiometer time constant. With present radiometer sensitivities, the angular scan method requires 15 to 30 minutes to complete a temperature profile measurement. The major disadvantage of the multi-spectral system is that the instrumentation is complicated and relatively expensive.

This paper describes instrumentation developed to permit experimental evaluation of both techniques and presents the results of an experiment performed to compare quantitatively the relative merits of the two techniques.

THEORY

In this section we briefly consider the theory of the estimation of temperature profiles from measurements of microwave emission by oxygen. Emission by oxygen is utilized because this gas is well-mixed in the atmosphere and its absorptive properties in the microwave region are reasonably well understood. Absorption of electromagnetic energy by O_2 occurs in a series of resonant lines centered at 40 GHz and in a single line near 110 GHz; here we consider only the 40 GHz absorption complex. At the earth's surface the individual absorption lines merge into an absorption continuum due to pressure broadening and a ground-based radiometer observes a smooth variation of brightness with frequency. The brightness temperature, T_b , measured at frequency, ν , and elevation angle, θ , is given by

$$T_b(\nu, \theta) = T_b^{\text{ext}}(\nu, \theta) \exp(-\int_0^\infty \alpha_\nu \csc \theta \, dh) + \int_0^\infty T(h) \alpha_\nu [\exp(-\int_0^h \alpha_\nu \csc \theta \, dh')] \csc \theta \, dh \quad (1)$$

where T_b^{ext} = brightness temperature caused by a source external to the earth's atmosphere,
 $T(h)$ = absolute temperature as a function of height, h , above the earth's surface, and
 α_ν = absorption coefficient at frequency ν .

Here we assume that the absorption is a function of oxygen, water vapor, temperature, and pressure only and that scattering is negligible; these assumptions are valid except when clouds or precipitation is present. The problem at hand is to estimate the quantity, $T(h)$, from measurements of the quantity, $T_b(\nu, \theta)$.

Study of (1) shows that input data may be generated in three ways:

1. Angular scan, i.e., measurements of T_b at a given frequency versus elevation angle,
2. Multi-spectral, i.e., measurements of T_b versus frequency at a fixed elevation angle,
- and 3. A combination of angular-scan and multi-spectral techniques.

The method employed to invert the radiative transfer equation (1) has been described in detail by Westwater⁽¹⁾ and will only be briefly described here. The method involves replacing (1) with a Fredholm integral equation of the first kind. The Fredholm equation is then solved by means of a statistical estimation technique that yields estimates of the atmospheric temperature structure based upon a priori knowledge of the temperature, pressure, and humidity statistics for the particular measurement location and the given set of brightness temperature measurements. The solution is the set of atmospheric temperature versus height values that results in the minimum departure (in a least-squares sense) from the expected value.

MULTI-FREQUENCY RADIOMETER

We now describe the instrument developed to compare the relative usefulness of the methods described above in making accurate estimates of atmospheric temperature profiles. Major objectives in the design of the radiometer were capability of operation in both angular-scan and multi-spectral modes, simultaneous operation on 3 or 4 frequencies from a common antenna, high angular resolution in the antenna system, good spectral resolution, and the greatest possible sensitivity consistent with the other design goals.

A block diagram of the final radiometer design is given in Figure 1. The basic system is the well known Dicke radiometer in which the receiver is switched between the antenna and a temperature-controlled reference termination. In the present system, the Dicke switch (modulator) is a rotating vane antenna or that alternately transmits and absorbs the energy from the antenna. The components located between the antenna and rotary modulator serve the following functions: during calibration the radiometer is connected to one of the temperature controlled terminations through the waveguide switch to establish a known temperature reference. (At 0° elevation angle and in the 60 GHz spectral region the atmosphere is essentially a blackbody having an effective temperature very nearly equal to the surface temperature; hence, atmospheric emission may also be used as a calibration reference point.) Incremental calibration of the radiometer output is accomplished by injecting noise from the gas discharge noise source into the signal channel via the precision attenuator and auxiliary arm of the directional coupler. During measurements, the

antenna is connected to the radiometer and noise is injected into the signal channel to allow operation in a "semi-balanced" condition. Since a true balance cannot be obtained at more than a single frequency, an appropriate amount of noise is injected to obtain an approximate balance at all frequencies.

The first mixer translates a 4 GHz band of frequencies in the 50 GHz region to a 4-8 GHz first intermediate frequency. After a stage of amplification, separation into four individual channels takes place in a strip-line multiplexer; center frequencies are 4.5, 5.5, 6.5, and 7.5 GHz. A second conversion to a 60 MHz center frequency follows the multiplexer; the noise bandwidth of the 60 MHz IF amplifiers is approximately 30 MHz to achieve a fair degree of spectral resolution. Following the second IF amplifiers are four identical stages of video detection, synchronous detection using a lock-in amplifier, and analog output recording.

All components before the lock-in amplifiers are located in an equipment enclosure mounted at the antenna. The waveguide compartment is heated to maintain the ambient temperature reasonably constant. Since ohmic losses in the waveguides generate noise, waveguide temperatures are measured by thermistor probes located at several points and remotely recorded for subsequent use in computing the antenna temperature.

The radiometer enclosure and its 1.2 m diameter conical horn antenna are installed on an elevation over azimuth antenna mount which provides the capability of angular scan operation. The antenna position is read out remotely at the data recording position. A relatively large antenna aperture was employed to obtain the high angular resolution mentioned earlier, while the conical horn configuration was selected to minimize stray radiation into the back and side lobes and thereby simplify the conversion of antenna temperature to brightness temperature.

A summary of the essential radiometer characteristics is presented in Table 1.

Table 1. Multi-frequency radiometer characteristics

Operating frequency	- 52.5, 53.5, 54.5, 55.5 GHz
Antenna characteristics	- 1.2 meter diameter conical horn, 3 dB beamwidth 0.3°
Receiver type	- Dicke switching radiometer, dual-conversion superheterodyne 1st IF: 4 to 8 GHz 2nd IF: 60 MHz center frequency, noise bandwidth approx. 30 MHz
Sensitivity	- Approx. 1K for 60 s integration time

TABLE MOUNTAIN EXPERIMENT

During April 1971, an experiment was conducted at Table Mountain near Boulder, Colorado, primarily to evaluate the performance of several radiometric systems in yielding atmospheric temperature profiles; however, as the planning for the experiment evolved, it became clear that additional sensors could not only benefit from but supplement the basic objectives of the main experiment. As a result, the final scope of the program greatly exceeded the original plan. The list of participants along with their respective measurements follows:

<u>Participant</u>	<u>Equipment</u>
Air Pollution Control Office	54.5 GHz radiometer, angle-scan mode
Sperry Rand Corporation	54.5 GHz radiometer, angle-scan mode
Wave Propagation Laboratory, NOAA	4-frequency radiometer, angle-scan and multi-spectral modes
	Acoustic sounder
	Infra-sound array
	Lidar
	Infrared radiometer
	Radiosonde data
	Aircraft temperature measurements
	Surface T, RH, P, Sky photographs
National Center for Atmospheric Research	Boundary layer profiler (tethered balloon with radiosonde)

In the remainder of this paper, we shall limit our discussion to the WPL radiometric observations to provide temperature profiles and a comparison of the inversion results with the temperature profiles measured by the radiosondes.

Data were recorded three times daily beginning at 0400, 0900, and 1500 during a one-week period in April 1971. These observation times were selected in order to acquire data for a variety of profiles. The radiometric observations were made at a series of 10 discrete antenna elevation angles ranging from 0° to 90° ; a complete run required about one hour. During the radiometer measurements, a radiosonde was released and data were recorded to an altitude of about 10 km; the aircraft also recorded temperature-height variations to an altitude of 3 km. Before and after recording of the radiosonde data, the boundary layer profiler made a sounding from the surface to about 500 meters and back to the surface. In addition to these supporting data, sky photographs were made at each antenna elevation angle so that possible contamination of data by clouds could be detected.

The majority of measurements recorded showed lapse temperature profiles; however, 3 temperature inversions were present. We shall now examine a few sets of data from two viewpoints:

1. What is the optimum set of input data?, and
2. What are the relative effects of different statistics upon the final inversion result?

INVERSION RESULTS

The theoretical accumulated standard deviation of the estimated difference in inferred and actual (measured) temperature profiles is shown in Figures 2 and 3; the two figures illustrate the effects of using different statistics as well as different sets of input data. Figure 2 is based upon the 5 year mean all month radiosonde data for Denver, Colorado, while Figure 3 is calculated using the 5 year mean April radiosonde data. The various curves are plotted for different sets of input data; the symbol for a particular set is shown in the legend. Combinations 1A, 1, and 2 refer to the multi-spectral input data combinations listed in Table 2. Combinations 1A and 1 are equivalent except that 1A is computed for a simulated measurement accuracy of 0.1K while combination 1 is based upon the actual observed differences between radiometric measurements and the brightness temperature calculated from radiosonde information. Angle scan designations are simply measurements of brightness temperature versus elevation angle at the frequency indicated.

The curve labeled a priori is the theoretical inversion accuracy that would be obtained if only the long-term statistics and the surface temperature were known. In all the results presented here, the measured surface temperature is used as a constraint. The theoretical curves for spectral combinations 1 and 2 are nearly identical; the curves for combination 4 have been omitted from Figures 2 and 3 for clarity.

Table 2. Explanation of spectral combinations employed to estimate temperature profiles

Combination No.	Input Data	
	Elevation Angle (deg)	Frequency (GHz)
1	5	54.5, 55.5
	10	54.5, 55.5
	15	54.5, 55.5
	30	53.5, 54.5, 55.5
	60	52.5, 53.5, 54.5
	90	52.5, 53.5, 54.5
2	90	52.5, 53.5, 54.5, 55.5
3	60	52.5, 53.5, 54.5, 55.5
4	5	53.5, 54.5
	10	at all angles
	15	
	20	
	30	
	60	
	90	

Several important points are brought out in Figures 2 and 3. First, it is clear that spectral combination 1 results in the smallest standard deviation over the full 10 km height interval. Second, the accumulated error in the angular scan mode is about the same at both 53.5 and 54.5 GHz and is smaller than for multi-spectral measurements at a fixed elevation angle, i.e., combination 2. Thus we reach the not surprising conclusion that a combination of angular scan and multi-spectral input data yields a better inversion result than either set of input data alone. It is somewhat surprising that a fairly large reduction in measurement error (from approx. 2.5K to 0.1K) does not result in a substantial decrease in accumulated error for combination 1. This result may indicate that multi-spectral angular scan measurements need not be extremely accurate to give good inversion results except during serious departures from horizontal stratification.

The rather poor performance of the multi-spectral inversion method may be partly caused by measurement error; however, the main reason is believed to be the smaller number of input measurements in the multi-spectral case. It is also possible that a more optimum selection of frequencies would result in a smaller difference between inferred and actual profiles.

As for the comparative merits of the statistics used to invert the data, it appears that all-month and April statistics produce comparable results over the first 3 or 4 km for all input

combinations. Above 4 km, use of April statistics results in a slightly smaller error when angular scanning or spectral combination is used. Thus, for this rather limited sample at least, the inversion result is rather insensitive to the statistics employed.

We shall now examine some of the actual inversion results and compare the measured and inferred profiles for a few samples. Figure 4 is an example of a profile containing a surface based temperature inversion. The curves are the profile measured by the radiosonde and the profile inferred from the radiometric data; inversion results are shown for four different combinations of input data. All sets of data, except for spectral combination 2, show the presence of the temperature inversion. Although the magnitude of the inversion is slightly underestimated in this example, the height of inflection is fairly well defined. As predicted by theory, spectral combination 1 gives the best overall agreement with the radiosonde profile. Note that combination number 4, for which a theoretical curve is not shown, results in agreement comparable to that obtained with combination 1 and the angular scans at both frequencies. This trend was observed consistently in all the samples considered in this study which suggests that a two-frequency radiometer operated in the angular scan mode yields results comparable in accuracy with those obtainable with a 4-frequency device; however, it does not necessarily follow that this behavior would be true for all geographic areas.

Figures 5 through 7 show inversion results for three other samples using spectral combination 1 and April statistics to illustrate recovery of several different types of profiles. The radiosonde data in Figure 5 show an elevated temperature inversion at about 500 m altitude; the inversion is also seen in the aircraft and boundary layer profile recordings. The inferred profile does not detect the intensity of the elevated inversion although there is a change in the estimated profile lapse rate in the vicinity of the temperature inversion. This inability to detect elevated temperature inversions has also been observed in angular scan measurements.⁽³⁾ In both instances, it is believed that the temperature structure is smoothed out because of integration effects and/or overlapping of weighting functions. In principle, the multi-spectral approach should be more successful in recovering elevated inversions if the frequencies of operation are chosen so as to cause the weighting function to give more information in the vicinity of the elevated inversion. It is clear that the optimum frequency problem should receive additional study.

CONCLUSIONS

Based upon the very limited sample available for this study, it is concluded that a combination of angular scan and multi-spectral input data produces more accurate inferred profiles than do other input data combinations. It further appears that the angular scan method is superior to multi-spectral techniques especially when detecting surface based temperature inversions. However, the possibility exists that a more optimum selection of frequencies would result in improved accuracy for the multi-spectral method. For the measurements reported here, a 2 frequency-angular scan combination yields results nearly as good as those obtained with the 4 frequency angular scan combination. The implication regarding simplification of equipment is obvious; thus, the extent to which this result applies to other geographic areas should be carefully evaluated.

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FOUR FREQUENCY 5mm RADIOMETER

2ND IF-DETECTOR CHANNEL (4 IDENTICAL UNITS)

Figure 1. Block diagram of multi-frequency radiometer.

Figure 2. Theoretical accumulated standard deviation of estimated profile using all-month statistics.

Figure 3. Theoretical accumulated standard deviation of estimated profile using April statistics.

Figure 4. Inferred and measured temperature profiles using four different combinations of input data.

Figures 5, 6, 7. Inferred and measured temperature profiles using combination of angle-scan and multi-spectral input data.

APPLICATION OF AN ACOUSTIC SOUNDER TO AIR POLLUTION MONITORING

D. W. Beran, F. F. Hall, Jr., J. W. Wescott, W. D. Neff*

Wave Propagation Laboratory
NOAA, Environmental Research Labs.
Boulder, Colorado

INTRODUCTION

Control measures for air pollution in most large cities, are more in the nature of an effort to lessen the pain, rather than to cure the patient. In this unhappy state, the role of monitoring devices, which make possible some prediction of hazardous conditions, becomes even more important.

Effective monitoring requires instruments which provide information on the concentration of both gaseous and particulate matter, and instruments for monitoring meteorological conditions which control the dissipation of the noxious elements. This paper concentrates on a relatively new device, the Acoustic Echo Sounder, which is particularly well suited for the task of monitoring those meteorological parameters required for making intelligent decisions concerning the degree of hazard during a particular event.

It is common knowledge that the low level temperature structure is a prime factor in the creation of air pollution events. Severe problems are associated with the low level capping inversion off the coast of California and the Los Angeles basin, an area where quantities of polluted air can be trapped under the advected inversion. While Los Angeles may claim pride of place for these conditions, it is not, unfortunately, unique and many other cities can be found where similar topography - temperature structure interaction leads to severe pollution conditions.

While the temperature lapse rate is of prime concern in air pollution meteorology, secondary, and at times equally important, effects can be attributed to the wind pattern over a city. Clearly, wind direction plays an important role in determining the path pollutants will take after emission. In addition, the strength of the wind is important in determining how rapidly effluents will be carried away; a necessary input when determining if a given emission rate will produce critical concentration levels over a region. Other meteorological parameters, such as the presence of humidity and solar radiation, (both of which induce chemical reactions of certain pollutants) are important, but to a lesser degree.

A monitoring system which could provide a continuous detailed picture of the wind and temperature structure in the lowest 1 km over a city would be of much more value than the in situ meteorological sensors presently used. In general, most forecasters must rely on instrumentation designed to sample the larger synoptic scale, and as such, provide continual data only on surface parameters, upper air soundings being made, at most, four times daily. In addition, aviation requirements have often dictated the location of these sensors, a situation not necessarily in the best interests of the urban meteorologist.

It is suggested that the acoustic sounder is a device which could collect continuous mesoscale measurements in critical regions at a cost significantly below other proposed methods. The remainder of this paper will present a brief discussion of the theory of acoustic sounding followed by the results from recent field tests which demonstrate the potential of the device for measuring wind and temperature structure. In addition, problems relating to operating in the urban environment under all types of conditions will be covered.

THE ACOUSTIC ECHO SOUNDER

The acoustic echo sounder is, in principle, not unlike Sonar. A sound wave is transmitted into the medium to be interrogated, the atmosphere. Turbulent wind and temperature fluctuations scatter some of this wave energy, a part of which can then be collected at a receiver. This process is described by the radar equation

$$P_r = P_t \sigma \frac{C\tau}{2} \frac{A^2}{R^2} L \quad , \quad (1)$$

*Assigned to NOAA/ERL from the NOAA Commissioned Officer Corps.

where the acoustic power returned P_r , is a function of the power transmitted, P_t , the speed of sound, C ; the length of pulse, τ ; the area of the receiving antenna, A ; the range to the area where the return is generated, R ; the losses, L due to absorption and equipment efficiencies, and, finally, the scattering cross section, σ . Theoretical work^{1,2,3} has established the relationship of the scattering cross section per unit volume, σ , to the turbulence parameters. The final form of the scattering equation is,

$$\sigma(\theta) = 0.03k^{1/3} \cos^2\theta \left[\frac{C_v^2}{C^2} \cos^2\frac{\theta}{2} + 0.13 \frac{C_t^2}{T^2} \right] \left(\sin\frac{\theta}{2} \right)^{-\frac{11}{3}}, \quad (2)$$

where $\sigma(\theta)$ is the scattered power, per unit volume, per unit incident flux, per unit solid angle at an angle, θ , from the initial direction of propagation. The wave number of the acoustic wave is represented by $k = 2\pi/\lambda$; C and T are the mean speed of sound and temperature in the scattering volume and C_v^2 and C_t^2 are the structure constants for velocity fluctuation and temperature fluctuations respectively.

The potential of acoustic echo sounding, first analyzed by C. G. Little⁴, has been demonstrated by the work of several investigators^{5,6,7} concentrating mainly on vertically pointing monostatic (colocated transmitter and receiver) systems. This early work clearly showed one of the possible roles that the acoustic echo sounder might play in air pollution monitoring⁸. One of these records made by the repeating traces of a facsimile recorder, synchronized with the pulses from the sounder is shown in Figure 1. This type of record gives a height time cross section of the low level

Figure 1 - Acoustic sounder record taken under stable conditions.

Figure 2 - Acoustic sounder record taken under unstable conditions.

boundary layer structure. Absolute temperature and gradients cannot be directly determined from this type of presentation, but the known interaction of C_n with various stability parameters i.e., temperature gradient and Richardson's number does make it possible to derive a qualitative assessment of the low level temperature structure. For example, regions which are dark (i.e. showing strong echoes) can, in general, be associated with either stable or unstable lapse rates and white areas are produced where conditions are neutral.

It is then logical to ask, how stable regions can be distinguished from those which are unstable? Here, experience has shown that distinctive patterns are associated with the two stability regimes. When acoustic returns are generated in a stable region, the pattern will be horizontally stratified, usually with a wave structure superimposed on the layers. Conversely, when unstable conditions are present, the record will contain predominantly vertically oriented structure. Figure 1 is an example of the return from a stable atmosphere. This can be compared with an unstable case shown in Figure 2. During the morning transition periods, when surface heating after sunrise starts to lift an inversion surface, both patterns can be observed during the same time period. An example of this is shown in Figure 3 where the quasi-horizontal dark region is associated with the remains of the surface based nocturnal inversion. This layer is progressively lifted as the low level lapse rate becomes more unstable. Later the unstable lapse rate predominates, and all remnants of the inversion layer are gone.

Where a particular application requires only that the presence and depth of an inversion are known, this type of system, consisting of a single monostatic antenna and receiver, would suffice. As indicated earlier, however, the wind structure under the inversion is also a major concern. The acoustic echo sounder can be used to provide this type of information through a slightly more complex configuration.

A component of the total wind can be sensed by analyzing the Doppler shift of the returned signal at a single receiver. For monostatic configurations, this component is directed along the axis of the beam and for bistatic systems, the component is along the bisector of the angle formed by the intersections of the two beams. Recent work^{9,10,11} has tested many different configurations and assessed the accuracy with which the wind can be measured with the Doppler technique. The most recent of these experiments used three antennas, one pointing vertically and the other two positioned along orthogonal axes through the first, and tilted in such a way that the three beams intersected at some height above the ground (see Figure 4). In tests with this configuration, the wind measured by

Figure 3 - Acoustic sounder record taken during the transition from stable to unstable conditions. The stable layer is associated with the upper portion of the dark areas; Unstable conditions are forming vertical convective elements below the stable layer.

Figure 4 - Antenna configuration used to measure total wind vector.

the Doppler system was compared with that measured by a Boundary Layer Profiler (BLP), a kytoon mounted anemometer supplied by the field facilities group of NCAR. Results of this comparison are given in the scatter diagram shown in Figure 5. For comparable 30 minute samples, the average value for the Doppler method was 2.33 m/s and that for the BLP was 2.43 m/s. The observed very close agreement indicates the ability of the acoustic Doppler to measure the total wind vector at any height within range of the system.

By using the same configuration as that shown in Figure 4, but with all three systems operating in the monostatic mode, it is possible to measure a vertical profile of the total wind vector. This is done by gating the signal from each of the systems at distances which correspond to the same height above the terrain. The components measured at each of the three gates at a given level are then averaged for periods of the order of 1 or 2 minutes. The resulting components are then resolved into the total wind vector at that height. (The averaging process, assuming Taylor's Hypotheses is valid, minimizes the error that might be produced because of the spatial separation of volumes along the beams away from the point of intersection.) An example of winds measured in this way is shown in Figure 6, a height time section of the horizontal isotachs derived from the total wind vector at ten elevations.

Where absolute temperature information is required, it is at least conceptually feasible to use the wind information and boundary layer theory to calculate the temperature gradient. Greater refinements might also be possible if the measured intensity of C_v^2 and C_t^2 , the structure constant for wind and temperature fluctuations are incorporated. These parameters and the measured wind profile can be combined through empirical boundary layer expressions of stability to estimate lapse rates between very unstable and neutral, and very stable and neutral. These concepts show promise, but have not yet been field tested.

Figure 5 - Scatter diagram of winds measured by Doppler (DOP) and Boundary Layer Profiler Anemometer (BLP).

Figure 6 - Isotachs of horizontal wind derived from acoustic Doppler measurements.

A second approach, also untested, would use the dependence of sound absorption on the frequency of the carrier wave and on temperature and humidity. If it can be assumed that all absorption is due to the classical and molecular portions of the attenuation coefficient, this method should work well, and would require only that 2 or 3 separate frequencies be generated; the difference in signal strength at each frequency is used to derive the temperature and humidity profiles. The assumption of no excess absorption(i.e. that due to processes other than molecular or classical) may prove to be limiting^{12,13} and more information on the nature of the attenuation is needed before the method can be demonstrated.

MONITORING DURING AN AIR POLLUTION EVENT

Our best example, to date, of the potential of an acoustic sounder for monitoring meteorological structure during an air pollution event was obtained during April of 1971. The acoustic sounder was located at Table Mountain, a field site some 50 km northwest of downtown Denver. It was being operated on a round-the-clock basis in a monostatic mode with the antenna pointed vertically.

Fortuitously, the sounder recorded the low level pattern of C_m^2 , indicative of the temperature structure on the 13th and 14th of April, a period when effluent concentrations in Denver reached the "pollution alert" level. The 24 hour long acoustic record for this period is shown in Figure 7.

TABLE MOUNTAIN, COLORADO
April 13-14, 1971

Figure 7 - Acoustic sounder record showing the evolution of the boundary layer structure before, during and after an air pollution alert.

The late afternoon portion, 1700 to 2000 (local time) on the 13th, shows the typical pattern which occurs after thermal plume activity has subsided and the lapse rate is near neutral. Following 2000, we see evidence of a surface based radiation inversion starting to form and deepen. This process continues throughout the nocturnal hours. Other layered structure, probably the result of advected inversion layers, also appear throughout the night. These layers all show the typically horizontally stratified structure with a variety of wave modes superimposed. Of particular interest is the subsiding layer which appears at about 800 meters shortly before 0300 on the 14th. This layer continues to descend until just before 0800 where it merges with the other lower layers and a very strong surface based inversion results. Shortly after this time, surface heating begins to have an effect and the lower regions start to take on the vertically oriented structure indicative of convective activity. This process tends to lift and intensify the capping inversion which was responsible for the air pollution alert. By 1200 on the 14th, the convective activity has destroyed the inversion layer and deep mixing has started. Two hours later at 1400, the air pollution alert was cancelled in Denver.

The temperature structure during this period was recorded by conventional radiosonde soundings at Denver's Stapleton Airport. The temperature, T , and dew point, T_d , profiles for 1700 LT on the 13th, and 0500 LT on the 14th are shown in Figure 8.

Figure 8 - Denver radiosonde records corresponding to the record shown in Figure 7.

The temperature and dew point profiles taken at 1700 LT on 13 April, 1971 show the typical subsidence inversion associated with a large high pressure region. These profiles, shown as dashed lines in Figure 8, were taken near the beginning of the record shown in Figure 7. The second set of profiles (solid lines) were taken at 0500 on the 14th, near the right side of the top portion of the 24 hour record. Looking at the radiosonde records alone, one can only speculate that the elevated inversion, near 3.3 km above sea level (1700 meters above the ground) at 1700 on the 13th had lowered and helped to intensify the inversion at 2.4 km on the 14th. The acoustic sounder record, on the other hand, clearly shows this evolution with the subsidence inversion entering the picture just before 0300 and lowering to about 600 meters above the ground. It is difficult to make a direct comparison of the inversion height at the two locations in question. However, the difference of less than 200 meters at 0500 LT appears very good despite such factors as the variation in surface elevations, local flow patterns and ground cover.

While this record does not tell the entire story, i.e., the concentration level of pollutants and the low level winds are absent, it is a simple matter to trace the events, even with data taken at a point far removed from downtown Denver. Using only this basic type of display, the air pollution meteorologist would have far more information than the present four per day temperature soundings provide.

Before placing an acoustic sounder in the urban environment, it is important to address certain practical questions relating to the ambient background noise in a city. The issue is really two sided; 1) can the sounder be successfully operated in the high noise environment of an urban area, and 2) will the operation of the sounder in this environment prove to be an annoyance to the inhabitants?

The first problem has been extensively analyzed with the conclusion reached that proper side-lobe suppression can be achieved by shielding the antennas¹⁴. It is also interesting to note that acoustic systems designed for other purposes, but essentially operating on the same basic principle, are now being successfully used in the extremely high noise environment of major airports.

Solution of the first part of this problem also eliminates the worst aspect of the second part, the annoyance factor. As most of the sound which might bother people is transmitted in the side-lobe of the antenna, shielding for sidelobe suppression will also greatly reduce the annoyance factor.

CONCLUSIONS

While full scale testing in an urban environment remains to be completed, the results to date clearly indicate the potential of an acoustic sounder as a monitor of important meteorological parameters that contribute to undesirable air pollution situations. At present it appears that the worst effect of a high noise environment on the operation of a sounder will be periodic losses of record, a factor which can be significantly reduced by improved anechoic shielding and proper selection of operating frequencies. In the unlikely event that as much as half of the record is lost, the amount of usable information would still be orders of magnitude greater than is available from conventional measuring techniques. This point is well amplified by comparing the information in Figures 6 and 7 of the previous section.

The importance of knowing the low level winds associated with an inversion also suggests that the Doppler capability should be incorporated into any operational system. The alternative would be to produce a backlog of acoustic records taken under various types of boundary layer structure and then use a pattern recognition technique to relate the observed records to conditions which might produce hazardous air pollution events. The Doppler capability would require slightly more complex equipment, but the additional information would seem to be worth the extra investment. Analog techniques, now under development, will allow rapid extraction of the Doppler shift at pre-determined intervals above the ground. This information can then be converted into a real time display of the wind profile, updated every two or three minutes.

The number and location of monitoring stations in an urban environment must be a function of such factors as, the local terrain, the pattern of residential vs. commercial zoning, and the meso-scale climatology of the city, each case being analyzed individually. If, for example, convergence into the urban heat island is a factor, stations could be placed at strategic locations around the perimeter of a city and the rate of inflow monitored continuously. In the final analysis, especially when compared with other types of remote sensors, the inexpensive acoustic sounder, combining its wind and temperature measuring capability, seems well suited to the task of monitoring the mesoscale urban environment.

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THE USE OF REMOTE RAMAN SPECTROMETRIC TECHNIQUES IN METEOROLOGICAL DIFFUSION STUDIES

Stanley M. Klainer and Elliot R. Schildkraut

Block Engineering, Inc.
Cambridge, Massachusetts

INTRODUCTION

We have been evaluating the applicability of the remote Raman spectrometer to meteorological diffusion studies. Our study to date consists of selecting tracer compounds and determining their detectability in the atmosphere as a function of concentration and distance from the Raman instrument. The final operational scenario will be to release one or more tracer materials from a point source and observe the resultant tracer cloud as it diffuses into the atmosphere. The data can be presented as a three-dimensional plot of cloud concentration vs. time.

The concept of remote Raman spectroscopy is not new. In fact scientists have been trying to make practical use of this potent technique for more than five years. It is surprising that even with the concentrated effort being exerted, progress has been relatively slow. It is our opinion that this lack of growth can most probably be attributed to the fact that the method has not been properly and thoroughly exploited.

If it is recognized at the outset that the value of remote Raman detection is its specificity and not sensitivity then a first step has been taken. The lack of extreme sensitivity derives from the fact that the Raman effect is a very weak second-order light scattering process. The advantages of the technique are the simplicity of the data interpretation, and the ability to present three-dimensional data in absolute concentration. Once this is truly understood, the proper use of the technique becomes imminent.

The problem as we see it, is that most experimentors, including at times ourselves, have tried to use the remote Raman system for applications where high sensitivity is required. This can only be accomplished with very sophisticated instrumentation or by brute force. The latter requires the use of high powered lasers, which need large power supplies and unrealistic input powers, and by using telescopes as large as 60 inches to collect the Raman photons. This results in a very expensive, unwieldy, impractical instrument. There are only two conditions under which the Raman system will have exceptional sensitivity:

1. When a compound is an exceptionally good scatterer, which is rare; and
2. When a compound exhibits the enhancement phenomenon known as resonance Raman, but this latter process is most often not implementable either because of instrument considerations or because of atmospheric absorption.

The application of the remote Raman approach to meteorological diffusion studies does require sensitivity. In this application, however, the compound to be measured is hand-picked for greatest Raman scattering efficiency and low concentrations of this material can be detected at a distance with a practical spectrometer. Coupled with the need for specificity, concentration data, and range information (which are all required for proper diffusion measurements) it can be concluded that the meteorological diffusion studies are a very practical application for the remote Raman technique if an appropriate tracer is selected

CLASSICAL RAMAN SPECTROSCOPY

The Raman effect is a light scattering process. It is similar to Rayleigh scattering but the interaction is inelastic. In Rayleigh scattering, light incident on a molecule is absorbed and immediately re-emitted. The absorbed light does not raise the molecule to an excited energy level as in classical absorption but instead, after absorption the molecule is in a virtual excited state. This has two consequences:

1. The molecule must emit the energy in a very short time, about 10^{-12} seconds,

and

2. The absorption and scattering cross-sections are very small.

It has been estimated that for a typical liquid irradiated at $6328 \text{ \AA}$, 1 part in 10^4 of incident light will be scattered in a path length of 1 centimeter. In the case of gases and aerosols, which are typical sample states for diffusion studies, there will be fewer molecules per unit volume, and the fraction of light scattered will be even less.

The efficiency of the scattering is known to increase at shorter wavelengths and is proportional to the fourth power of the frequency. Considerably more scattering occurs in the ultraviolet than in the infrared. The dependence of scattering efficiency on wavelength can best be exemplified by the fact that the sky is blue. This is so because the blue light is scattered more efficiently than the yellow and red light from the sun.

A small portion of the excited molecules, 10^{-2} to 10^{-6} will emit light at a slightly different wavelength than the incident light, giving rise to what is called Raman scattering (Figure 1). Here the light is absorbed by a molecule in the ground state and the molecule re-emits by returning to an excited vibrational level. The emitted light is at a lower energy, or lower frequency, than the incident light. The difference in frequency is equal to the natural vibration frequency of the molecule. Several such lines will normally be observed and they compose what is known as the Raman spectrum. Each line corresponds to a different vibration of the molecule and each molecule has its own set of vibration frequencies. These are directly related to the structure of the molecule and are thus characteristic of a specific compound. The Raman spectrum thus gives positive identification of the scattering molecule.

FIGURE 1
ILLUSTRATION OF THE RAMAN PROCESS

There is also a Raman component which has higher frequency than the exciting light. This comes from those few molecules which initially absorb radiation while they are in an excited state and decay to a lower energy level. The complete scattering spectrum, therefore, will be made up of the predominant Rayleigh scattering, which is at the same frequency as the incident light, and the two (2) Raman components one at a frequency lower than the incident radiation, the Stokes lines, and one at a frequency higher than the incident radiation, the anti-Stokes lines. At normal temperatures, the Stokes lines are stronger than the anti-Stokes lines because it is more probable for a molecule to be at the ground energy level which gives rise to the lower frequencies. Thus the stronger Stokes lines are usually used for Raman measurements.

THE REMOTE RAMAN SPECTROMETER

The purpose of a Raman spectrometer is to determine the frequency and intensity of the light scattered by a molecule. From this it is possible to identify the scattering material and determine how much is present. The frequency is used for identification and the strength of the line is used to measure concentrations. Inasmuch as the Raman displacement due to a particular molecule is independent of

the illuminating frequency, the displacement from the incident light frequency can be measured in absolute units, usually wavenumbers, and this value is called the Raman shift, $\Delta\nu$. A table of Raman shifts may be considered analogous to a table of absorption or emission frequencies for molecular identification.

From the preceeding discussion it becomes obvious that each incident illuminating frequency will produce a series of Raman lines. In order to make use of these lines, however, they must be related to a specific irradiation frequency and therefore, for all practical purposes, the illuminating source must be monochromatic. Lasers are the best sources of high intensity monochromatic light and are used in all modern Raman spectrometers.

A block diagram of a simple remote spectrometer configuration is shown in Figure 2. In the remote system the monochromatic source is a pulsed laser. The laser beam illuminates the atmosphere and scattering is caused by all of the atmospheric constituents.

FIGURE 2

This is observed through a telescope which is throughput matched to a polychromator or filter system. The polychromator (or filters) sorts the Raman lines into distinct frequencies and its output is detected by one or more photomultiplier tubes, the data is processed and displayed on a chart recorder. Use of a pulsed laser and time gating permits location, that is, measurement of linear distance of the scattering molecules relative to the light source and collector.

Instrument Specifications

The remote Raman spectrometer being used for these measurements has the specifications listed in Table 1. This is a large cumbersome system which gives good sensitivity through the brute force approach. It was designed for maximum versatility as a precursor to a field hardened system.

TABLE 1
SPECIFICATIONS FOR A REMOTE RAMAN SPECTROMETER

LASER	PULSED (FREQUENCY DOUBLED RUBY)
AVERAGE LASER POWER AT 3471.5 Å	0.4 WATTS MINIMUM
PULSE RATE	5 PULSES PER SECOND
PULSE LENGTH	30 NANOSECONDS
OPTICAL COLLECTOR	36 INCH DIAMETER CASSEGRAIN TELESCOPE
POLYCHROMATOR	GRATING, WITH 2 ADJUSTABLE FREQUENCY CHANNELS AND 1 FIXED FREQUENCY CHANNEL (H_2)
RESOLUTION	5.3 Å
DISPERSION	5.5 Å/MM
STRAY LIGHT	10^{-6}
ELECTRONICS	PHOTON INTEGRATION SYSTEM
INTEGRATION TIME	SELECTABLE FROM 0.5 TO 100 SECONDS
CALIBRATION	INTERNAL CALIBRATION AGAINST ATMOSPHERIC H_2
SAMPLE PATH LENGTH	10 TO 100 METERS
SAMPLE	GAS, VAPOR, OR AEROSOL CLOUD
OUTPUT	RAMAN CONTINUANT

Instrument Sensitivity

The sensitivity of the remote Raman spectrometer must be defined in terms of the operational scenario. It is a complex function of instrument parameters and operating conditions. Table 2 lists many of the variables which must be considered. Many of these variables are fixed. Others, such as those presented in Table 3 may possibly be changed to increase the sensitivity for a particular set of measurements.

TABLE 2

FACTORS WHICH DETERMINE RESPONSE IN A REMOTE RAMAN SPECTROMETER

1. LASER FREQUENCY	10. RANGE
2. LASER AVERAGE POWER	11. SAMPLE CONCENTRATION
3. LASER PULSE LENGTH	12. SAMPLE CROSS-SECTION
4. LASER PULSE REPETITION RATE	13. SAMPLE DEPTH
5. COLLECTOR SIZE	14. BACKGROUND (DAY/NIGHT OPERATION)
6. DEPTH OF FIELD	15. WIND VELOCITY
7. FIELD-OF-VIEW	16. WIND DIRECTION
8. LENGTH OF GATE	17. ATMOSPHERIC TRANSMISSION
9. SIGNAL INTEGRATION TIME	

TABLE 3

<u>FACTORS CONTRIBUTING TO INCREASED RAMAN SENSITIVITY</u>			
<u>CONTRIBUTING FACTORS</u>	<u>HOW SENSITIVITY IS INCREASED</u>	<u>LIMITING PRACTICAL FACTORS</u>	<u>RANGE OF INCREASED SENSITIVITY (FACTOR)</u>
SELECTED LASER FREQUENCY	RESONANCE	AVAILABILITY OF LASER AT PROPER FREQUENCY	1 to 10^5
INCREASED LASER FREQUENCY	ν^4 DEPENDENCE	AVAILABILITY OF SUFFICIENT AVERAGE LASER POWER IN ULTRAVIOLET	1 to 60
COLLECTOR EFFICIENCY	INCREASES AS THE SQUARE OF THE RADIUS	SIZE, HEIGHT	1 to 60
INCREASED AVERAGE LASER POWER	LINEAR WITH AVERAGE POWER	(1) AVERAGE POWER IN ULTRAVIOLET (2) SIZE, HEIGHT, AND POWER	1 to 100
LONGER INTEGRATION TIMES	INCREASES AS THE SQUARE ROOT OF THE TIME	(1) PRACTICAL OBSERVATION TIME (2) SAMPLE DECOMPOSITION	1 to 25
RANGE	INVERSELY PROPORTIONAL TO SQUARE OF DISTANCE	LENGTH OF TRACKING RANGE	1 to 10^4

Determination of Sample Concentration

An important asset of the remote Raman spectrometer is its ability to give absolute concentrations.

The actual volume of samples measured can be determined from the divergence of the laser beam, the length of the laser pulse, and the time-gated return of the laser pulse. If it is assumed that the aperture of the laser is circular, as is usually the case, then the shape of the laser beam is a truncated cone (Figure 3). The use of a pulsed laser permits the beam to be dissected into several truncated

cones each of height ct . Here c is the speed of light and t is the duration of the laser pulse. The diameter of the circles at the top and bottom of each cone depends on the beam divergence, which is known, and how far these circles are from the laser. The distance can be determined from the time-gated return.

FIGURE 3

DEFINITION OF SAMPLE VOLUME
FOR A REMOTE RAMAN SPECTROMETER

In normal atmospheric measurements the number of nitrogen molecules in any sample volume is known as is its cross-section. Thus, the nitrogen Raman spectrum can be used as a concentration standard. The Raman spectrum of the tracer material (whose cross-section is also known) is obtained from the same sample volume; then a comparison of the two spectra makes it possible to calculate the number of tracer molecules in that volume.

TRACERS

There are eight (8) basic parameters which are evaluated when selecting a tracer material. These are shown in Table 4. The general rules for evaluation are: good for Raman, harmless to the environment, easy to disseminate, and reasonable for the pocketbook.

TABLE 4

DESIRED CHARACTERISTICS OF A TRACER MATERIAL

1. HAVE LARGE EFFECTIVE RAMAN CROSS-SECTIONS.
2. BE RELATIVELY NON-TOXIC IN BOTH ORIGINAL STATE AND ANY REACTION PRODUCT RESULTING FROM DISSEMINATION.
3. BE SUITABLE FOR DISSEMINATION AS A VAPOR, LIQUID OR SOLID AEROSOL OR SUSPENSION IN LIQUID SLURRY.
4. HAVE PARTICLE SIZE AND SPECIFIC GRAVITY SUCH THAT SETTLING AFTER DISSEMINATION IS MINIMIZED.
5. BE CAPABLE OF UNIFORM SIZE DISTRIBUTION ON DISSEMINATION.
6. HAVE SCATTERING FREQUENCIES (RAMAN SHIFTS) WHICH DO NOT HAVE SPECTRAL INTERFERENCES FROM NORMAL ATMOSPHERIC CONSTITUENTS AND POLLUTANTS.
7. BE READILY AVAILABLE, INEXPENSIVE AND EASY TO USE.
8. CHEMICAL STABILITY.

Spectral Considerations

A most significant consideration when selecting tracer materials is their spectral characteristics. Table 5 shows the spectral information that must be known about each candidate material. This data relates directly to the specificity and sensitivity which can be expected.

TABLE 5

RAMAN SPECTRAL INFORMATION REQUIRED

1. RAMAN SHIFTS
2. RAMAN CROSS-SECTIONS
3. RAMAN LINE WIDTHS

When considering the detection of tracer materials it is desired to choose compounds whose Raman lines are specific to the structure of these compounds and lines which will not be present from atmospheric constituents or in the common pollutants as background. Table 6 shows the Raman shifts for the common pollutants, the normal atmospheric species, and some possible classes of tracer materials. The data in this table gives an initial indication of the spectral separation among the compounds. It shows that there is sufficient spectral separation for specific detection of some tracers disseminated in the atmosphere.

TABLE 6

RAMAN DISPLACEMENT FOR ATMOSPHERIC
SPECIES MAJOR POLLUTANTS AND TRACERS

MOLECULE	RAMAN DISPLACEMENT $\Delta\nu$, cm^{-1}	MOLECULE	RAMAN DISPLACEMENT $\Delta\nu$, cm^{-1}
CO_2	668	NO^+	2248
O_3	712	N_2	2273
SF_6	775	NO^+	2277
NH_3	950	H_2	2302
ALIPHATICS	967	NO^+	2305
AROMATICS	992	N_2	2331
O_3	1043	CO	2349
SO_2	1151	H_2S	2611
CO_2	1242	O_3	2800
CO_2	1265	ALIPHATICS	2857
H_2O	1285	CH_4	2914
CO_2	1286	O_3	3050
NO_2	1320	PRIMARY AMINES	3189
CO_2	1388	PRIMARY AMINES	3256
CO_2	1409	CH_3	3331
CO_2	1430	PRIMARY AMINES	3343
CO_2	1528	CO_2	3609
O_3	1556	H_2O	3652
O_3	1740	CO_2	3716
NO	1876	NO^+	4422
O_3	2105	H_2	4459
CO	2145	NO^+	4478
H_2^+	2175	H_2	4517
NO^+	2221	NO^+	4534
H_2O	2223	H_2	4575
H_2	2244	NO^+	4580
		H_2	4633

The second important piece of information accurate spectral data yields is relative Raman cross-section. This is the term used to define the efficiency with which a molecule scatters the incident radiation. The Raman cross-section is thus the term which determines sensitivity for a particular instrument configuration and a set of operational parameters. As previously stated Raman cross-sections are dependent on the fourth power of the frequency of the monochromatic light; there is also the possibility of non-linear resonance enhancement.

Another piece of spectral data required is spectral line width. In considering a remote Raman spectrometer searching for relatively low tracer concentrations in the atmosphere, spectral band width becomes a major consideration for several reasons: (1) The Raman line has a finite shape and width. It is not a delta function as would be inferred by looking at a table of Raman shifts. There must be separation between these lines if the specificity of the Raman system is to be maintained. Typical aerosols will have line widths on the order of 10 to 15 cm^{-1} . (2) The contribution of the vibration-rotation lines must, therefore, be considered. These are typically 1000 times or more weaker than the pure vibrational line. In samples where the concentration range between the constituents does not vary by more than a factor of 100 to 1000 these lines do not represent an interference problem. In the present scenario, however, a high sensitivity spectrometer is used to detect part-per-million of a specific material in the presence of 78% atmospheric nitrogen, a concentration differential of as much as 10^6 . This means that the very weak nitrogen vibrational-rotation lines may still be more intense than the sample lines. If this is so, either the sample band must be located at a frequency which is clear of the nitrogen vibration line and its vibration-rotation branches or at a frequency in the valley between the individual peaks. (3) The final consideration regarding line width is related to determining the bandpass of the instrument. Ideally for maximum signal to noise it is desirable to collect all of the Raman photons generated and none of the background photons. In practice one designs an instrument whose bandpass and shape is equivalent to the line width. This maximizes the signal from the sample and limits the background contribution.

Detection Limits of Tracer Materials

Our work to date has been directed at selection of tracer materials which have large Raman cross-sections per se. We plan to investigate resonance enhancement shortly. Based on experimental results and measured Raman cross-sections we can project detection limits for several tracer materials, atmospheric constituents, and some of the common pollutants. There are summarized in Table 7.

TABLE 7

TYPICAL DETECTION LIMITS OF THE REMOTE RAMAN
SPECTROMETER (RANGE: 250 METERS)

COMPOUND	(FORMULA)	PRESENT SYSTEM
		S/N = 5 T = 20 SEC (PPM)
SULFUR DIOXIDE	SO ₂	3
NITRIC OXIDE	NO	56
AMMONIA	NH ₃	8
HYDROGEN CHLORIDE	HCL	9
HYDROGEN SULFIDE	H ₂ S	4
HEXANE	C ₆ H ₁₄	1
CARBON MONOXIDE	CO	25
PETROLEUM	CH ₄	4
HYDROGEN	H ₂	10
BENZENE	C ₆ H ₆	2
ORGANIC PHOSPHATES	(-POC-)	<0.1
NITROGEN	N ₂	24
OXYGEN	O ₂	17
SULFUR HEXAFLUORIDE	SF ₆	6
ALIPHATIC HYDROCARBONS (GENERAL)	C _n H _{2n+2}	<3
PRIMARY AMINES (GENERAL)	CH ₃ N	<6

CONCLUSIONS

The work presently underway has demonstrated the feasibility of tracking known materials which are disseminated into the atmosphere. The range over which we can follow the tracer is restricted to less than 1/2 mile because of a limited line-of-sight at our present location. We are currently preparing to take the system to Fort Devens, Massachusetts, where measurements over nearly 2 miles are feasible. The use of the remote Raman spectrometer for meteorological measurements appears to be an excellent match between system capability and data requirements.

We would like to note, in conclusion, that: (1) We are using the remote Raman system for remote fluorescence measurements also. (2) We are working on a second remote Raman system which will have considerably greater sensitivity and be less than 1/3 the size of the present unit. Extreme versatility is the only sacrifice that is being paid for this major improvement. The next generation of instruments should prove even more thoroughly the validity of the remote Raman technique.

The remote Raman spectrometer program is being supported by Edgewood Arsenal. The tracer measurements are being performed for Deseret Test Center.

SAMPLING METHOD FOR AIR OR WATER POLLUTION PROFILE MEASUREMENT USING A SINGLE ANALYZER

T. R. Morrison

W. A. Solomon

Westinghouse Electric Corporation
Defense and Electronic Systems Center
Baltimore, Maryland

1. INTRODUCTION

The sampling method discussed in this paper was developed in the course of carrying out a program for the Maryland State Bureau of Air Quality Control of the Department of Health and Mental Hygiene. The customer Project Leaders were Miss Ann Marie DeBiase and Mr. Felipe Lebron, under the supervision of Mr. William Bonta.

The intent of the program was to introduce an allowance for traffic turbulence into the standard line source - Gaussian plume representation of vehicular traffic emissions. The data are now being reduced, and the results will be reported later.

2. THE PROBLEM

It was desired to obtain vertical profile measurements of the levels of carbon monoxide alongside a roadway. Economic necessity ruled out using separate analyzers for each sample port. This, at first, seemed to rule out simultaneous sampling. However, the literature reveals the folly of attempting to reconstruct profiles from sequential measurements at different ports. Figure 1 shows the variations in the ratios between peak values, three minute and half-hour means, and one hour mean values as found by McCormick and Xintaras.⁽¹⁾ From this graph it can be estimated that if three minute consecutive samples were drawn through eight ports, variations by a factor of two or more could be expected from the true profile levels. Thus, sequential sampling was also not feasible.

3. OUR SOLUTION

The answer to the dilemma would be a means to sequentially analyze samples which had been drawn simultaneously. The ports were distributed on a tower situated in the median strip of a divided highway. The equipment was located in a trailer alongside the highway. The sample lines connecting the ports to the equipment were fairly long, and therefore would constitute a significant volume. It was decided to use the sample lines as reservoirs to store the samples until they could be analyzed.

4. OPERATION

The basic elements of the system are shown in figure 2. The length and diameter of each sample line is controlled so that all volumes are equal. Each line is fitted with a needle valve, and a solenoid valve to regulate and control the flow. Another needle valve between the manifold and pump allows regulation of the total flow. The inclusion of a gas bottle, regulator and solenoid provides for purging the system and calibrating the zero level.

In the first stage of the operating cycle, all sample line solenoids are open and samples are drawn into the lines at equal rates. Because of variations in the lines it is necessary to balance the flow during sampling, so that each line has the same flow rate. This stage ends when the lines are full of new sample. During the next stage the system is purged and the analyzer is zero-calibrated on dry nitrogen.

During the analysis portion of the cycle, each sample line is pumped down individually and the sample analyzed. Of course, as each sample is analyzed a new "sample" is drawn, but this new sample is discarded during the sample portion of the next cycle.

5. RESULTS AND DISCUSSION

On the whole, the system performed as expected and the data for determining the profiles were obtained. Some comments on the experience with the system are in order.

One question which might be raised is whether diffusion during the storage period would erase level fluctuations contained in the original sample. Within the smoothing resulting from the volume of the analyzer sample cell, no effects attributable to sample diffusion could be seen.

Some difficulty was experienced in balancing sample flow rates. It is not sufficient to adjust each line to the desired flow when it alone is being pumped. That is a good start, but it is mandatory that the flows be measured and adjusted while the other lines are also open. Since our system contained only one flow gauge which could not be easily moved, balancing was accomplished by introducing a "pulse" of high-CO gas into each port and timing the transit to the analyzer. The sample line valves were adjusted until all transit times were equal within the desired tolerance.

By introducing additional solenoid and needle valves it is possible to extend the versatility and ease of adjustment of the system. It is desirable that total sample flow, individual sample flow and individual analyze flows all be separately and independently adjustable. In some instances it may be desirable for the analyze flow to be higher than the total sampling flow. By adding an additional solenoid valve, the analyzer can also be span calibrated during each measurement cycle. The improved system is diagrammed in figure 3.

6. AN UNFORESEEN PROBLEM

It may be helpful to other workers in this field to report a difficulty we ran into. The system was originally set up using clear vinyl tubing* for the sample lines. When the first operation was attempted, the analyzer gave concentrations of nearly 100 ppm as the air which had been in the tubes passed through the system (see figure 4). Investigation showed that the contamination was roughly proportional to storage time and was worse during periods of bright sunlight, but was still evident during nighttime. Evidently the plasticizer of the tubing was baking out and contaminating the sample. Substitution of copper tubing eliminated the problem, as the second record in figure 4 illustrates.

Reference

1. McCormick, R.A. and C. Xintaras, 1962: Variation of Carbon Monoxide Concentrations as Related to Sampling Interval, Traffic and Meteorological Factors, Journal of Applied Meteorology, 1, No. 2, 242.

* Brand name Transtube, manufactured by Extron Division of Pipeline Service Company.

Figure 1. Ratio of maximum to 1-hr mean CO concentration as function of averaging time and time of day, Cincinnati, Ohio, 10-26 May 1960. (1)

Figure 2. Basic System Configuration

Figure 3. Improved System Configuration

Figure 4. Records illustrating spurious response from plastic tubing, and valid trace from copper tubing.

OVER THE GREAT LAKES IN LATE FALL

D. H. Lenschow
National Center for Atmospheric Research*
Boulder, Colorado 80302

1. INTRODUCTION

The passage of relatively cold air of Arctic origin over water in late fall and winter can result in extensive modification of the air by the underlying surface. In the past most studies of this phenomenon have been based on radiosonde soundings and surface observations. Lettau⁽¹⁾ and Craddock⁽²⁾, for example, studied the modification of cold air passing out over the North Atlantic. Manabe⁽³⁾ studied similar cold air outbreaks over the Japan Sea. In these studies, the measurements of air modification were obtained by comparing upwind station measurements with measurements obtained downwind after the air had passed over the sea.

Measurements of the direct transfer of heat and moisture into the atmosphere, with the air much colder than the water, by the eddy correlation technique have been obtained from an aircraft by Bunker^(4,5) over the North Atlantic Ocean and⁽⁶⁾ over the Mediterranean Sea. Lenschow⁽⁷⁾ measured the vertical flux of sensible heat and the resultant air modification over Lake Michigan. It is desirable in airborne studies of air modification to select a horizontal scale that is large enough to eliminate the effect of individual convective elements (on the scale of 1 km) on the averaging process and yet small enough to be able to fly across the region at several altitudes in a period of time during which synoptic changes are small. A body of water the size of Lake Michigan (≈ 190 km) seems to be a reasonable compromise.

This paper presents the results of a set of aircraft measurements over Lake Michigan on a day in late fall. The measurements included mean horizontal wind, turbulent fluctuations of all three components of air velocity, temperature, humidity and radiation surface temperature. From these variables, vertical fluxes of heat, moisture and momentum were calculated. Since the mean values of temperature and humidity at flight level were also measured, the important terms in the equations for the rates of change of temperature and humidity can be evaluated. In addition, variations in vertical fluxes and in the mean profiles of wind, temperature and humidity along the trajectory of the air are discussed.

The NCAR Buffalo aircraft measuring system as described by Lenschow⁽⁶⁾, was used for this experiment. This system contains an inertial navigation system to measure the aircraft acceleration, velocity, position and angular orientation. The velocity of the air with respect to the aircraft is measured with vanes and a pitot-static tube mounted on the tip of a boom 5 m in front of the nose of the aircraft. Temperature is measured at the same location with two 0.0025-cm diameter resistance wire thermometers—one, well protected from the airstream, is used for a reference mean temperature measurement, and the other, less well protected but capable of faster response, is used for eddy flux measurements. The refractive index of the air, which is measured with a microwave cavity refractometer mounted on the boom, is used along with temperature and static pressure to measure fluctuations in humidity. A dew-point hygrometer provides an absolute reference for humidity. Surface temperature is measured with an infrared radiation thermometer.

The aircraft was flown between navigational radio stations at Manitowoc, Wisconsin, on the upwind shore and Muskegon, Michigan, on the downwind shore, as shown in Fig. 1. These points were selected because the line between them is almost parallel to the mean boundary layer wind. The azimuth of the aircraft flight path was 136° , while the mean direction of the wind was about 145° . The surface pressure contours on this figure are for 1800Z, 5 November 1970; aircraft measurements were taken from 1500 to 1900Z. The flight path of the aircraft plotted on a vertical cross section across the lake, is shown in Fig. 2. As can be seen, two legs were flown all the way across the lake, one at 180 m and the other at 350 m above the lake. Two other legs were flown about one third of the way across the downwind region of the lake—one just below (and sometimes just above) cloud base, and the other at the top of the mixed layer, which coincided with the top of the clouds. Stratocumulus completely covered the downwind region of the lake, while the clouds were scattered over the upwind region. No precipitation was observed. In addition, several five minute legs at various heights, both parallel and perpendicular to the wind, and vertical soundings up to 2.8 km were flown over both the upwind and downwind sections of the lake.

The measurements along these various flight paths were used to construct the potential temperature vertical cross section shown in Fig. 3. The net heating of the air in the mixed layer amounted to about 3C and the height of the mixed layer increased from about 750 m to about 1200 m from one side of the lake to the other. Above 1700 m, the potential temperature at constant altitude does not change with distance across the lake.

* The National Center for Atmospheric Research is sponsored by the National Science Foundation.

3. VARIATIONS ACROSS THE LAKE

As shown in Fig. 4, the surface temperature varies considerably from one side of the lake to the other. Some of this variation is reflected in the mean temperature and humidity profiles at 180 m. The vertical fluxes of heat and moisture and the turbulent kinetic energy, each divided into five sections with an averaging distance of about 30 km length, are seen to be very closely tied to variations in the surface temperature, or more precisely, the difference between surface temperature and air temperature at 180 m. The vertical fluxes of heat and moisture vary by more than a factor of 2 from the 30 km section having the lowest values of vertical fluxes to the section having the highest values. This indicates the importance of taking measurements all the way across the lake in order to determine the total amount of heat and moisture added to the atmosphere above the airplane flight level.

Such large variations in surface temperature may not, however, exist at other times of the year. Measurements over Lake Michigan by Lenschow⁷⁾ indicate that later in the winter (mid-December through January) the surface water cools to 4°C or lower and is quite uniform across the lake. A new complication may arise under these conditions, however. At this time of the year large areas of the lake may be covered with rafts of broken ice which effectively damp the surface waves and consequently suppress the vertical fluxes of heat and moisture as compared to ice-free areas of the lake.

Measurements of the horizontal variation of temperature were obtained from the five-minute runs parallel and perpendicular to the wind a few km downwind from the upwind shore at heights of 30, 90, 180 and 350 m above the lake. The average horizontal gradient of temperature in this region was measured to be 0.066°C/km at an azimuth of 75°. Integrating the thermal wind equation from the highest level 350 m, downwards,

$$(\Delta \vec{V}_g)_z = (g/f) \int_z^{350} T^{-1} \left(\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} \right) \times \vec{k} dz \quad (1)$$

we obtain $(\Delta \vec{V}_g)_{30} = 6.5$ m/sec at an azimuth of 165°. Thus, the effect of the thermal wind is to increase the surface wind and rotate it counterclockwise. As a result, the measured wind backs with height and decreases in magnitude, as shown in Fig. 5.

After removing the thermal wind below 350 m, the wind profile in Fig. 5 appears to be about as expected if it were in geostrophic equilibrium. With the exception of the lowest level, the wind then increases slightly and veers slightly with height. It is not surprising that the wind at the lowest level is somewhat greater than the next level above it since, as pointed out later, the air is accelerating in this region. Since several hours

are required for the wind to come to geostrophic equilibrium after being perturbed, a part of this temperature gradient very likely existed in the air upwind of the lake. Values of air temperature at the surface at several locations upwind of the lake indicate that this was probably the case. It appears that the air farther to the north was increasingly modified by a progressively longer trajectory over Lake Superior. Furthermore, perhaps for the same reason, the cloud cover increased to the north, which may have reduced radiational cooling in the boundary layer during the preceding night.

The wind maximum in the lower boundary layer in this region has interesting implications on the stress, measured by the eddy correlation method,

$$-\vec{\tau}/\rho = \overline{\vec{u}'w'} + \overline{\vec{v}'w'} \quad (2)$$

where u' , v' and w' are the fluctuations of air velocity in the horizontal directions and the vertical direction, respectively.

The value of $\vec{\tau}$, averaged over the four levels from 30 m to 350 m, was 1.1 dyn/cm² at an azimuth of 101°. Considering that the azimuth of the mean wind was 145°, this seems to be a reasonable direction for the stress. Since the magnitude of the wind decreased with height, however, the production of turbulent kinetic energy by the shear term was negative, i.e.,

$$\vec{\tau}/\rho \cdot \frac{\partial \vec{V}}{\partial z} = -20.9 \text{ cm}^2/\text{sec}^3 \quad (3)$$

Thus, the shear term was removing energy from the turbulent kinetic energy of the flow and consequently tending to increase the energy of the mean flow. The horizontal rate of increase of kinetic energy of the mean flow, averaged over the same four levels of measurement, was found to be

$$\overline{u} \partial \overline{E} / \partial x + \overline{v} \partial \overline{E} / \partial y = 37.6 \text{ cm}^2/\text{sec}^3 \quad (4)$$

where $E = \frac{1}{2}(\overline{u}^2 + \overline{v}^2 + \overline{w}^2)$. Thus, the mean flow was accelerating downstream over the lake at a rate that resulted in an increase of energy that exceeded by almost two times the rate at which energy was being removed from the turbulent kinetic energy by the shear term.

At the same time, the turbulent kinetic energy was decreasing over the lake by

$$\overline{u} \frac{\partial \overline{e}'}{\partial x} + \overline{v} \frac{\partial \overline{e}'}{\partial y} = -3.5 \text{ cm}^2/\text{sec}^3 \quad (5)$$

where $\overline{e}' = \frac{1}{2}(\overline{u'}^2 + \overline{v'}^2 + \overline{w'}^2)$. This is a significant fraction of the buoyant energy production term,

$$\frac{g}{T} \overline{(w'T')} + 0.61g \overline{(w'q')} = 8.5 \text{ cm}^2/\text{sec}^3. \quad (6)$$

where q' is the fluctuation in the mixing ratio of water vapor and g is the gravitational acceleration. Obviously, one consequence of the sudden change in both surface roughness and surface temperature (the water was about 5°C warmer than the adjacent land) as the air passes out over the lake is that the horizontal advection terms become important in the kinetic energy equations.

4. AIR MODIFICATION BY THE LAKE

a. Budget equations

The amount of heat added to the atmosphere as the air crosses the lake can be estimated from the equation for the local rate of change of temperature with time (see Businger and Deardorff⁽³⁾). Lenschow⁽⁷⁾ showed that many of the terms in this equation are negligible for airplane measurements in the planetary boundary layer. Using his results, the equation for the remaining terms is

$$C_p \bar{\rho} \frac{\partial \bar{T}}{\partial t} = -C_p \bar{\rho} \left[\bar{u} \frac{\partial \bar{T}}{\partial x} + \frac{\partial \bar{w}'T'}{\partial z} \right] + \frac{dh}{dt} \quad (7)$$

where C_p is the specific heat of air at constant pressure and dh/dt is the rate of heating per unit mass. The local time rates of change of temperature and humidity were measured to be small during the time of flights and consequently are neglected. Both radiation and water phase changes are important terms in this equation for the case considered here, so that

$$\frac{dh}{dt} = L \frac{dq_g}{dt} - \frac{\partial F}{\partial z} \quad (8)$$

where q_g is the liquid water mixing ratio, L is the latent heat of condensation, and $\partial F/\partial z$ is the radiation flux divergence.

Integrating (7) with height from the aircraft reference flight level, Z_0 , up to just above the mixed layer, Z_1 , [such that $(\bar{w}'T')_1 \approx 0$] and along the flight path of the aircraft, x , divided by the total length, X , across the lake,

$$C_p \left[\frac{1}{X} \int_0^X \int_{Z_0}^{Z_1} \bar{\rho} \bar{u} \frac{\partial \bar{T}}{\partial x} dz dx - (\bar{\rho} \bar{w}'T')_0 \right] = \\ = L \int_{Z_0}^{Z_1} \bar{u} \bar{q}_g dz - (F_1 - F_0) \quad (9)$$

Similarly, the amount of latent heat added to the atmosphere can be estimated from

$$L \left[\frac{1}{X} \int_0^X \int_{Z_0}^{Z_1} \bar{\rho} \bar{u} \frac{\partial \bar{q}_g}{\partial x} dz dx - (\bar{\rho} \bar{w}'q')_0 \right] = \\ = -L \int_{Z_0}^{Z_1} \bar{u} \bar{q}_g dz \quad (10)$$

where q is the mixing ratio of water vapor, and δq_g is the change in liquid water between the downwind and upwind soundings.

A major problem that arises in trying to evaluate (9) and (10) is to determine how to deal with the rate of change of the inversion level with time. The change in height of the inversion level may be due to both mixing through the inversion and mean vertical motion of the air at the inversion level. Since the air is observed to be essentially nonturbulent above the inversion, the air above the inversion will be entrained by the turbulent air below, while the interface remains sharp. The net result is heating of the air below the inversion, since the potential temperature is greater above the inversion than below. Models of cloud-topped mixed layers under a strong inversion have been developed and discussed by Lilly⁽¹¹⁾.

If the mean vertical velocity at the inversion level is assumed to be zero, then the limit of integration, Z_1 , in (9) and (10) is chosen to be just above the maximum height of the modified air, which is the top of the mixed layer at the downwind end of the lake. On the other hand, if the change in inversion height is assumed to be due entirely to lifting, with no mixing through the inversion layer [in this case, $\bar{w}_1 \approx (450\text{m})/(2.14 \times 10^4 \text{ sec}) = 2.1 \text{ cm/sec}$], the limit of integration, $Z_1(x)$, is just above the height of the mixed layer from one side of the lake to the other.

Radiation from the top of the cloud layer was estimated, using the method discussed by Lenschow⁽⁷⁾, to be about 7 mw/cm^2 . However, the lake was only about 65% cloud covered, resulting in an upward flux at the cloud top of $F_1 \approx 5 \text{ mw/cm}^2$. Since the water surface was considerably warmer than the cloud base, this radiative heat loss at the top of the cloud layer was largely offset by the absorption of upward radiation by the cloud layer from the water surface plus the direct absorption of short wave solar radiation by the cloud layer. Thus, the net radiation of the entire layer of modified air, $(F_1 - F)$, was probably quite small, and is neglected in the following discussion.

Values of the measured terms of (9) and (10) are presented in Table 1. The subscript "0" refers to the lower flight level across the lake of 180 m above the surface, which is the lower limit of integration. Since $\bar{w}_1$ is not known, H_2 and L_2 (defined in Table 1), are evaluated for the two extreme conditions outlined above. The large difference in H_2 between the two cases is due to the 8°C higher temperature of the air above the inversion as compared to the air below the inversion, as shown in Fig. 6. This means that in the first case (i.e., for $\bar{w}_1 = 0$), the air above the inversion, up to 1.3 km, all the way across the lake is included in the calculation of H_2 . For the second case, ($\bar{w}_1 = 2.1 \text{ cm/sec}$), this air is not included in the area integration. The mixing ratio of the air above the

mixed layer was between 1 and 2 gm/kg; below the inversion it was about 5 gm/kg. Thus, the difference in L_2 for the two cases is much smaller.

Substituting the measured values for the two cases into (9) and (10) and assuming that $\frac{\partial H_2}{\partial \bar{v}_1}$ and $\frac{\partial L_2}{\partial \bar{v}_1}$ are constants, we obtain a set of three simultaneous equations, presented in Table 1, that can be solved for H_2 , L_2 and $\bar{v}_1$. We can then evaluate Q_2 as the residual term in either (9) or (10). Thus, all the terms in (9) and (10), neglecting radiation, are evaluated above a minimum height of 180 m and presented in Table 1. The value of Q_2 implies an increase in the cloud water mixing ratio of about 0.25 gm/kg over a height interval of 400 m. This does not seem unreasonable, considering the accuracy of the measurements, since Q_2 is obtained as a residual in the budget equations. Since the mean vertical velocity at the top of the mixed layer was calculated to be 1.4 cm/sec, which implies a mean boundary layer divergence of $-0.26 \times 10^{-4} \text{ sec}^{-1}$, the entrainment rate was $2.1 - 1.4 = 0.7 \text{ cm/sec}$.

Further evidence for this entrainment rate is provided by the observation that the mean vertical fluxes of heat and moisture across the lake at 350 m were almost zero. Thus, with the exception of the addition of $Q_2 = 2.0 \text{ mw/cm}^2$, the heating of the mixed layer above 350 m must have come from mixing of warmer air from above the inversion into the mixed layer. Using (9) to compare the amount of heat added to the atmosphere between the 180 m and 350 m flight levels with the sensible heat flux divergence, we obtain $H_2 = 3.8 \text{ mw/cm}^2$ and $C_p \bar{p} (w''T'') = 3.0 \text{ mw/cm}^2$. Thus the residual term is 0.8 mw/cm^2 . Part of this may be due to the absorption of radiation, emitted by the water surface, in this layer of air.

b. Transfer coefficients

The transfer coefficients of heat and moisture over the sea are still subject to some uncertainty. Direct measurements over water surfaces are difficult and consequently not very common. Therefore these coefficients were evaluated for the flight across the lake at 180 m above the surface for comparison with the commonly used values derived primarily from bulk aerodynamic methods. The transfer coefficients are defined by

$$\overline{w''T''} = C_H \bar{u} (\theta_s - \theta) \quad (11)$$

$$\overline{w''q''} = C_E \bar{u} (q_s - q)$$

where the subscript "s" refers to values immediately above the water surface, and the unsubscripted variables refer to the height of measurement, which in this case was 180 m. Solving for the coefficients for this case, we obtain

$$\begin{aligned} C_H &= 0.00067 \\ C_E &= 0.00058 \end{aligned} \quad (12)$$

Although the ratio of the transfer coefficients is close to unity, implying that the mechanisms for the transfer of water vapor and heat are similar, the value of the coefficients is considerably smaller than the value of $C_E = 0.0022$, obtained by Jacobs¹² by the bulk aerodynamic method. Werner¹³ obtained relatively good agreement between airborne eddy flux measurements and bulk aerodynamic calculations of heat and water vapor fluxes over the tropical ocean by using a value of $C = 0.0014$. He did not, however, have a direct measurement of surface temperature, but obtained the surface temperature by fitting a straight line to a set of heat flux measurements at several wind speeds. Clearly this is an area that requires further investigation, since the bulk aerodynamic method is based on gradient measurements within about 10 m of the water surface.

5. CONCLUSIONS

The feasibility of using an aircraft for measuring air modification in the planetary boundary layer has been demonstrated for a case of relatively cold air flowing over Lake Michigan during late fall with the lake temperature about 5°C warmer than the air 180 m above the surface. The important terms in the temperature and water vapor budget equations have been evaluated; the net rate of increase of sensible and latent heat above 180 m above the surface were found to be 5.0 and 5.5 mw/cm^2 respectively. This was not a particularly extreme case. In an earlier study over Lake Michigan, Lenschow⁷ obtained values as large as 33 and 9 mw/cm^2 for sensible and latent heat later in the winter. Over the Sea of Japan, Manabe⁹ obtained values of 48 and 22 mw/cm^2 for sensible and latent heat.

The water surface temperature was found to vary by as much as 3°C from the mean value averaged across the lake. This was reflected in the measured variation of heat and water vapor fluxes, which varied by as much as a factor of 2 between regions of large and small differences between surface and flight level temperatures. Measurements taken over the lake just downwind of the upwind shore demonstrated that the horizontal advection terms in the kinetic energy equations become important under these conditions.

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Figure 1. Surface pressure map of the Great Lakes region at 1800Z, 5 November 1970. The flight path, indicated by a dashed line, lies between Manitowoc, Wisconsin and Muskegon, Michigan

Figure 2. Vertical cross section across Lake Michigan parallel to the flight legs across the lake showing the flight path of the aircraft. Circled points represent five-minute legs perpendicular to the long flight legs. Cloud cover is based on visual observations from the aircraft

Figure 3. Vertical cross section across Lake Michigan with isentropes based on the aircraft flight path shown in Figure 2

Figure 4. Variation of surface temperature and air temperature, absolute humidity, sensible and latent vertical heat fluxes and turbulent kinetic energy at 180 m above Lake Michigan, from one side of the lake to the other

Figure 5. Wind profile obtained from five-minute legs both parallel and perpendicular to the wind at four different levels, a few km downward from the upwind shore. The dashed lines are the wind profile with the thermal wind removed.

Figure 6. Vertical soundings at the upwind and downwind ends of Lake Michigan

TABLE I

Values of the Energy Fluxes Over Lake Michigan on 5 November 197C
 Measured from an Aircraft. The Lower Limit of
 Intergration, Z_0 , is 180 m Above the Surface

$$H_2 = \frac{c_p}{\pi} \int_0^{\pi} \int_{Z_0}^{Z_1} \bar{\rho} \bar{u} \frac{\partial T}{\partial x} dz dx \quad L_2 = \frac{L}{\pi} \int_0^{\pi} \int_{Z_0}^{Z_1} \bar{\rho} \bar{u} \frac{\partial q}{\partial x} dz dx \quad Q_2 = L \int_{Z_0}^{Z_1} \bar{u} \delta q_s dz$$

Case (1): $\bar{w}_1 = 0$

Case (2): $\bar{w}_1 = 2.1 \text{ cm/sec}$

$$H_2(1) = -9.8^{\circ}$$

$$H_2(2) = 12.9$$

$$L_2(1) = 5.3$$

$$L_2(2) = 5.7$$

$$c_p (\bar{\rho} \overline{w'T'}) = 3.0$$

$$L(\bar{\rho} \overline{w'q'}) = 7.6$$

$$H_2 - c_p (\bar{\rho} \overline{w'T'})_0 = Q_2$$

$$L_2 - L (\bar{\rho} \overline{w'q'})_0 = -Q_2$$

$$H_2 = \frac{H_2(2) - H_2(1)}{\bar{w}_1(2)} \bar{w}_1 + H_2(1)$$

$$L_2 = \frac{L_2(2) - L_2(1)}{\bar{w}_1(2)} \bar{w}_1 + L_2(1)$$

$$H_2 - c_p (\bar{\rho} \overline{w'T'})_0 = -L_2 + L(\bar{\rho} \overline{w'q'})_0$$

$$H_2 = 5.0$$

$$L_2 = 5.5$$

$$Q_2 = 2.0$$

$$\bar{w}_1 = 1.4 \text{ cm/sec}$$

*Units of heat flux are mw/cm^2 .

THE SIZE SPECTRUM DETERMINATION OF SPHERICAL AEROSOLS BY LIGHT SCATTERING

PART I: METHOD

Ariel Cohen
Wave Propagation Laboratory
Environmental Research Laboratories
National Oceanic and Atmospheric Administration
Boulder, Colorado 80302

INTRODUCTION

The scattering functions for spherical particles with size parameters $\alpha = 2\pi r/\lambda \ll 1$ are given by the Mie theory.^(1,2) The theory shows that unless the scattering medium is composed of mono-dispersed particles, the fluctuation of the scattering intensities with the variation of either the incident wavelength or with the radius of the particles cannot be easily predicted.⁽³⁾ The usual method to overcome this difficulty is to assume an experimentally-based size spectrum function (such as the Junge distribution) with, normally, two parameters.⁽⁴⁾ These parameters are then calculated from the behavior of the scattering or the extinction of light as a function of wavelength by the examined particles.⁽⁵⁾ Polarization properties of the scattered light as compared with the degree of polarization of the incident light can also be used for the size spectrum definition but their use requires certain assumptions, among others - the number densities - and usually does not have a unique solution.⁽⁶⁾

In this article a method will be described for the determination of the size spectrum of spherical particles with a distribution of indices of refraction, which are present in an atmospheric volume containing aerosol particles (possibly both spherical and unspherical) of unknown nature *a priori* based on the properties of the scattering intensity as a function of α . The method does not require any assumption on the size spectrum function, nor does it demand any knowledge of the type (refractive index) of the scattering spheres.

Method

The scattering intensity of unpolarized light by a spherical particle having a size parameter $\alpha = 2\pi r/\lambda$ and a known index of refraction $n = f(\lambda)$ can be calculated by the use of Mie scattering functions.⁽²⁾ For this calculation, the polarized component (I_{11}) of the scattered light at $\theta = 90^\circ$ by a water droplet normal to the scattering plane, was chosen as an example (see Figure 1). In Table I a series of α_i values is given for which $I_{11,i}$ have maximal values. A corresponding series of α_j values for which $I_{11,j}$ have minimal values can also be given (13 pairs of minimal and maximal values are given in Table II).

Let us now assume that the relative number densities $\{\rho_1^{(n)}; \rho_2^{(n)}\}$ of spherical particles having radii $r_1; r_2$ and refractive index n (out of many other particles) may be defined by measuring the dependence of $I_{11}(\theta = 90^\circ)$ on λ (wavelength). The method suggested here is as follows: each $\rho_i^{(n)}$ determines two values $\lambda_i^{(n)}(r_1); \lambda_i^{(n)}(r_2)$, for the above mentioned spheres:

$$\lambda_i^{(n)}(r_1) = 2\pi r_1 / \alpha_i^{(n)}; \lambda_i^{(n)}(r_2) = 2\pi r_2 / \alpha_i^{(n)}$$

We now measure the scattering intensities of the sample as a continuous function of wavelength, and, thus for each of the above calculated $\lambda_i^{(n)}$, the measured intensity $I_{11}^{(n)}(\lambda_i^{(n)}[r])$ is found. (In this work, the measured intensities are divided by λ_i^2 , see Example.)

If we define:

$$A_n(r) = \sum_{i=1}^N I_{||}^{(n)}(\lambda_i^{(n)}(r)) - \sum_{i=1}^N I_{||}^{(n)}(\lambda_i'^{(n)}(r)) \quad (1)$$

$$A_n(r_1) = \sum I_{||}^{(n)}(\lambda_i^{(n)}(r_1)) - \sum I_{||}^{(n)}(\lambda_i'^{(n)}(r_1)) \quad (1a)$$

$$A_n(r_2) = \sum I_{||}^{(n)}(\lambda_i^{(n)}(r_2)) - \sum I_{||}^{(n)}(\lambda_i'^{(n)}(r_2)) \quad (1b)$$

(where N is the chosen number of the $\alpha_i^{(n)}$ and $\alpha_i'^{(n)}$ values* for which the theoretical $I_{||}(\alpha)$ have maximum [and minimum] values), then $A_n(r_1) / A_n(r_2) = \rho_1^{(n)} / \rho_2^{(n)}$ with an increased accuracy with the increasing of N . The function $A_n(r)$ is therefore, proportional to the size spectrum of the particles with refractive index n , acting as a filter with 100 percent transmission for one particle and almost zero percent transmission for any other particle.

Fig. 1. Computer plotting of $\log_{10} I_{||}$ ($I_{||}$ is the polarized light, normal to the scattering plane), as a function of α . The α for 13 of the maximum values of $I_{||}$ are noted on the abscissa. Dots and plus signs on the graph correspond to the above α values multiplied by 1.2. It can be seen that their ordinate values are "random" with respect to the maxima and minima of the curve.

To prove this assertion, one has to check the contribution of any other particle in the two summations appearing in the definition of $A_n(r)$ (Equation 1) and to show that it can be approached to zero. For example, the contribution of the particles having radius r_2 in Equation 1a, depends on the $\alpha_i^{(n)}(r_2)$ values corresponding to the $\lambda_i^{(n)}(r_1)$ and $\lambda_i'^{(n)}(r_1)$:

*Within a range defined by the measuring system, i.e. if the measurements can be made only in the visible range, the α_i values are limited: $2\pi r_j / 0.7 \leq \alpha_i \leq 2\pi r_j / 0.4$ (r_j is the radius of the sphere, the number density of which is to be defined).

$$\alpha_1^{(n)}(r_2) = 2\pi r_2 / \lambda_1^{(n)}(r_1) = (r_2/r_1) \times \alpha_{MAX,1}^{(n)} \quad (2a)$$

$$\alpha_i^{(n)}(r_2) = (r_2/r_1) \times \alpha_{MIN,1}^{(n)} \quad (2b)$$

It can be seen from Table I, that the ratio $\alpha_{i,MAX}^{(n)}/\alpha_{i+1,MAX}^{(n)}$ is not constant. Nor is the ratio of $\alpha_{i,MAX}^{(n)}/\alpha_{i+j,MAX}^{(n)}$, for any j.

It follows that the particles with radii r_2 contribute to the first summation of Equation 1a, practically random values in respect to the appearance of maxima and minima in Figure 1. Since the same effect of "randomization" appears in the second summation in Equation 1a, the submmations** for the r_2 particles can be approached to the same value

$$A_n(r_1) = \underbrace{\sum_1 I_{||,r_1}^{(n)}(\lambda_1(r_1)) - \sum_1 I_{||,r_1}^{(n)}(\lambda_1'(r_1))}_{\text{Contribution of particle having radius } r_1} + \underbrace{\sum_1 I_{||,r_2}^{(n)}(\lambda_1(r_1)) - \sum_1 I_{||,r_2}^{(n)}(\lambda_1'(r_1))}_{\text{Contribution of particle having radius } r_2} \quad (3)$$

$$\approx \sum_1 I_{||,r_1}^{(n)}(\lambda_1(r_1)) - \sum_1 I_{||,r_1}^{(n)}(\lambda_1'(r_1)) \approx \sum_{MAX,r_1} - \sum_{MIN,r_1}$$

(see example, Section III). In the case that N (- the number of maxima and minima values), derived from Figure 1, is not sufficient in order to achieve a desired accuracy, this number can be approximately doubled by repeating the experiment at a different scattering angle or at a different polarization angle; or even by extinction measurements.

Now:

$$\sum_{MAX,r_1} - \sum_{MIN,r_1} = V \times \rho_1 \times (\sum_{MAX,r_1} - \sum_{MIN,r_1})_{\text{per one particle}} \quad (4)$$

where V is the scattering volume.

Also, we surely have:

$$(\sum_{MAX,r_1} - \sum_{MIN,r_1})_{\text{per one particle}} = F \times \sum_1 (I_{||}(\alpha_{MAX,1}) - I_{||}(\alpha_{MIN,1})) \quad (5)$$

$$\frac{\partial}{\partial \alpha} F \times \sum = F \times \text{Constant}$$

where $I_{||}(\alpha)$ is the theoretical (relative) Mie scattering intensity and F is a factor of the specific experimental system for rationalizing the units of intensity.

By elimination between (4) and (5), and substitution into (3), we get:

$$A_n(r_1) = V \times \rho_1 \times F \times \sum \Rightarrow \rho_1 = A_n(r_1) / (V \times F \times \sum) \quad (6)$$

or dropping the index 1, we get

$$\rho_r = A_n(r) / (V \times F \times \sum) \quad (6a)$$

If the incident light intensity I_0 , is a function of the wavelength so that $I_0 = \int_0^\infty f(\lambda) d\lambda$ one would have to weigh the $I_{||}^{(n)}(\lambda)$ values (appearing in Equation 1) by the factors $f(\lambda)$

Example

To illustrate the method, let us assume that we have a sample of molecular air containing spherical water particles of only two sizes $r_1 = 2.0 \mu m$ and $r_2 = 2.4 \mu m$. In Table III, the

**Each recorded scattering intensity at a specific wavelength ($I_{||}(\lambda_i)$) is the summation of the scattering intensities of the individual particles having radii r_1 and r_2 :

$$A_n(r_k) = \sum_i \sum_j (I_{r_j}(\lambda_i) - I_{r_j}(\lambda_i'))$$

This example indicates that for a sample of water particles with the above mentioned sizes having a size distribution varying within one order of magnitude, 13 maxima and 13 minima are sufficient for determining the size distribution to a high degree of accuracy. When the size distribution covers a variation of more than one order of magnitude, the more maxima and minima values, the better precision is achieved.

$I_{11}(\alpha)$, with $\alpha_{MAX,i}$ and $\alpha_{MIN,i}$, were calculated on the CDC 6400 computer, at the Hebrew University of Jerusalem.

Conclusions

The method described above requires, in principle, only experimental facilities of recording (preferably, by means of a multi-channel recorder) scattering (or extinction) intensities as a function of wavelengths. It can be seen from Table III, that for water droplets with radii varying

(2.0 μm) $\lambda_{MAX,i}$ Å	11120	9590	8550	7660	5930	5490	5130	4820	4470	4220	3470	3320	3170
(2.4 μm) $\lambda_{MAX,i}$ Å	13340	11510	10260	9200	7110	6580	6160	5780	5370	5060	4160	3980	3810
(2.0 μm) $\lambda_{MIN,i}$ Å	12320	10470	8910	7900	5790	5360	4930	4590	4300	4030	3440	3220	3080
(2.4 μm) $\lambda_{MIN,i}$ Å	14780	12570	10700	9480	6950	6430	5910	5500	5160	4830	4130	3870	3700

Table III. Wavelengths used in the determination of the number densities of water droplets of radii 2.0 μm and 2.4 μm .

Note: Since any random size distribution may be divided into sets of two radii each (r_{i1}, r_{i2}) so that $r_{i1}/r_{i2} = 1.2$, the respective λ values appearing above multiplied by the factor ($r_{i1}/2.4$) can be used for the determination of the number densities of any random sample. In case of the available apparatus not covering the required range of λ values, see Conclusions.

between 1.0 μm and 5.0 μm , the α_{MAX} and α_{MIN} values, given as an example in this work (Figure 1, Table II) correspond to wavelengths near and within the visual range. Since the experimental methods of wavelength-scanning are limited, it is suggested that other α_{MAX} and α_{MIN} values be selected (not appearing in Figure 1) rather than change the wavelength range for droplets with radii > 5.0 μm . The only requirement on the selection of these α values is that the Mie scattering intensities corresponding to the $\alpha_{MAX,i}$ or $\alpha_{MIN,i}$ should not vary by more than an order of magnitude (preferably much less).

The required accuracy in the definition of the scattering angle, for this purpose, was calculated to be within ± 0.2 degrees.

It would be well to note that since no theoretical limitations are imposed on the accuracy in calculating the values α_{MAX} and α_{MIN} , it is always possible to calculate these values to an accuracy compatible with the wavelength resolution of the experimental system.

corresponding λ_1 and λ_1' values are listed for the calculation of $A(r_1)$ and $A(r_2)$, respectively. Since $1(\alpha = 2\pi r_1/\lambda_1)/\lambda_1^2 = 1(\alpha = 2\pi r_2/\lambda_2)/\lambda_2^2$ when $r_1/\lambda_1 = r_2/\lambda_2$, the experimental values are assumed to be divided by λ^2 (see Figure 2a, b). This is done to permit the use of one set of $I = f(\alpha)$ values for all wavelengths and radii of particles. 13 maxima and 13 minima were chosen for both the calculation of $A(r_1)$ and $A(r_2)$; $A(r)$ was calculated for a single particle. It can be seen from Table II that the contribution of the particle with radius $r_2 (= 2.4 \mu\text{m})$ in the value of $A(r_1 = 2.0 \mu\text{m})$ is less than two percent.

The same calculations for the contribution of the particle with radius $r_1 (= 2.0 \mu\text{m})$ in the value of $A(r_2 = 2.4 \mu\text{m})$ show that it is less than 1.3 percent.

The first contribution was calculated for

$$\alpha_i = \frac{2.4}{2.0} \alpha_{\text{MAX},i} ; \alpha_i' = \frac{2.4}{2.0} \alpha_{\text{MIN},i}'$$

and the second for

$$\alpha_i = \frac{2.0}{2.4} \alpha_{\text{MAX},i} ; \alpha_i' = \frac{2.0}{2.4} \alpha_{\text{MIN},i}'$$

Figure 2a.

Figure 2. The expected 90° scattering intensities measured as a function of wavelength ($3000 \text{ Å} \leq \lambda \leq 15000 \text{ Å}$) of the component normal to the scattering plane, by a sample including water droplet particles having radii $r_1 = 2.0 \mu\text{m}$ and $r_2 = 2.4 \mu\text{m}$ (2a). In figure 2b all "measured" values are multiplied by $(7000/\lambda)^2$.

Figure 2b.

i	$\alpha_{MAX,i}$	Log I_{11}	1.2 α	$1.2^{-1} \alpha$	i	$\alpha_{MAX,i}$	Log I_{11}	1.2 α	$1.2^{-1} \alpha$
1	2.0	.456	2.4	1.7	26	32.4	1.675	38.9	27.0
2	3.0	.177	4.6	3.2	27	33.2	1.345	39.6	27.7
3	5.9	.599	7.1	4.9	28	34.1	1.818	40.9	28.4
4	7.6	.904	9.4	6.5	29	34.4	1.798	41.3	28.7
5	9.6	1.202	11.5	8.0	30	36.2*	2.047	43.4	30.2
6	11.3	1.428	13.6	9.4	31	37.3	1.389	44.8	31.1
7	13.1*	1.583	15.7	10.9	32	37.9*	2.224	45.5	31.6
8	14.7*	1.665	17.6	12.3	33	38.9	1.396	46.7	32.4
9	16.4*	1.673	19.7	13.7	34	39.6*	2.290	47.5	33.0
10	18.1	1.467	21.7	15.1	35	40.5	1.291	48.6	33.8
11	18.8	1.219	22.6	15.7	36	41.2	2.209	49.4	34.3
12	19.6	1.296	23.5	16.3	37	41.6	1.814	49.9	34.7
13	21.2*	1.579	25.4	17.7	38	42.1	1.500	50.5	35.1
14	22.9*	1.844	27.5	19.1	39	42.9	2.114	51.5	35.6
15	23.1	1.026	27.7	19.3	40	43.3	1.868	52.0	36.1
16	24.5*	1.964	29.4	20.4	41	44.0	1.708	52.8	36.7
17	24.8	1.418	29.8	20.7	42	44.5	1.662	53.4	37.1
18	26.1*	1.794	31.3	21.8	43	45.1	1.851	54.1	37.6
19	26.4	1.631	31.7	22.0	44	45.9	1.887	55.1	38.3
20	27.7	1.362	33.2	23.1	45	46.9	1.753	56.3	39.1
21	28.1*	1.710	33.7	23.4	46	47.7	2.115	57.2	39.6
22	29.8*	1.702	35.8	24.8	47	47.9	1.765	57.5	39.9
23	30.5	1.532	36.6	25.4	48	49.3	2.119	59.2	41.1
24	31.0	1.140	37.2	25.8	49	49.6	2.087	59.5	41.3
25	31.5	1.601	37.8	26.3					

Table I. Maxima of scattering function I_{11} , as a function of size parameter α (θ scattering = 90° , $n = 1.33$).

*(see Figure 1 and Table II)

	α_{MAX}	$I(\alpha_{MAX})$	$I(1.2\alpha_{MAX})$	α_{MIN}	$I(\alpha_{MIN})$	$I(1.2\alpha_{MIN})$
1	11.3	26.4	2.3	10.2	1.09	0.24
2	13.1	38.4	3.2	12.0	0.04	3.02
3	14.7	45.0	7.7	14.1	0.07	12.0
4	16.4	47.0	16.4	15.9	1.25	16.5
5	21.2	37.9	0.57	21.7	0.21	23.7
6	22.9	70.0	0.087	23.45	0.15	47.0
7	24.5	96.2	2.5	25.9	0.16	31.6
8	26.1	58.3	6.2	27.4	0.06	19.0
9	28.1	52.4	19.7	29.2	1.27	9.9
10	29.8	50.6	52.4	31.2	1.95	23.7
11	34.15	114.3	70.0	33.5	1.00	43.7
12	37.85	172.6	52.4	39.0	1.51	54.3
13	39.6	199.5	80.9	40.8	0.40	47.0
$\Sigma I_1 =$		1007.5	316.4	$\Sigma I_1 =$	9.16	332.4

$$\sum I_1(\alpha_{MAX}) - \sum I_1(\alpha_{MIN}) = 590.3 = \sum I_1(\alpha_{MAX}) \propto A$$

$$\sum I_1(1.2\alpha_{MAX}) - \sum I_1(1.2\alpha_{MIN}) = 16.0 \propto \alpha \propto A$$

Table II. Demonstration of the validity of the assertion that $A(r_1) \approx \Sigma_{MAX,r_1} - \Sigma_{MIN,r_1}$ (Equation 3) for 2 water droplets of radii $r_2/r_1 = 1.2$ with a randomly chosen set of 13 pairs of α_{MAX} , α_{MIN} . (I_{11} - relative units)

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Mandel'shtam-Brillouin Scattering of Laser Light as a Remote Sensing Tool

Anders Daniels
Department of Meteorology
University of Hawaii, Honolulu, Hawaii

1. INTRODUCTION

Remote sensing of atmospheric parameters is truly one of the most pertinent problems in meteorology of today. With the advent of the laser several attempts were made to use this unique source of light for atmospheric sensing.

In the first experiments one determined the total intensity of laser light scattered back from a standard atmosphere and the result was interpreted as an aerosol and water droplet density profile.¹ However, it is also desirable to determine the molecular density by remote sensing methods, and a number of schemes to achieve this were developed. One scheme used a laser which could be made to change its radiating frequency from a spectral region where an atmospheric constituent absorbed radiation to another frequency where no absorption took place², or to a frequency corresponding to the Raman component of the scattered light³; the difference in signals could then be interpreted as representing a density profile of the constituent.

Another method⁴, is to determine the frequency spectrum of the returned radiation, since the difference in velocity between molecules and aerosols should separate the return signal when viewed in frequency space; with this technique the temperature of the medium could also be inferred from the frequency distribution of the light scattered by molecules. This distribution should exhibit a broadening characteristic of molecules moving according to a Maxwellian velocity distribution. The only type of laser that could be used in an actual experiment is a gas laser since the relatively broad frequency spectrum of the solid state lasers would completely mask any change in the frequency of the returned light. It has been suggested⁴ that the most powerful gas laser--the CO₂ laser--would be the best choice for an experiment to verify this theory. However, if one uses a CO₂ laser with a wavelength of 10.6 μ , the individual molecules will not act as scatterers of the laser light since here the light wavelength is two orders of magnitude larger than the mean free path of the molecules. In this case the scattering will no longer be produced by individual molecules, but from larger random density fluctuations in the medium. This theory of scattering--known as Mandel'shtam-Brillouin scattering (MB)--will now be summarized and the results interpreted in terms of meteorological parameters, and results of the experiment which verify the theory are then presented.

2. INTENSITY OF SCATTERED LIGHT

Following Fabelinskii⁵, consider a plane monochromatic light wave incident on a homogeneous medium of fixed molecules. These molecules will oscillate with the incident radiation and therefore emit secondary or scattered radiation with definite amplitudes and phases relative to each other. This situation is depicted in Figure 1 below, where a-a' is the wavefront and V_1^* and V_2^* are two volume elements with equal volume and with linear dimensions small compared with the wavelength of the incident radiation. Since the medium is homogeneous, the two volumes will contain the same number of molecules.

Figure 1. Scattering from stationary scattering volumes

At some direction θ it will always be possible to find a volume element V_2^* such that the scattered radiation from this volume will be of the same amplitude, but of opposite phase to the scattered radiation from another volume element V_1^* since all molecules radiate with definite phases and amplitudes to each other and are stationary. The two waves will therefore cancel each other in the direction θ and no net scattered intensity will be observed in this direction. The above discussion

can, of course, be generalized to any direction, except forward and to any position of the wave-front. It can therefore be concluded that it is necessary to have inhomogeneities in a medium to get a net scattered intensity of incident radiation.

If we next let the previously fixed molecules execute random thermal motion but do not allow inhomogeneities on a scale larger than the mean free path for the molecules we get two different scattering situations depending on whether the wavelength of the incident light is smaller or larger than the molecular mean free path.

With an incident wavelength much less than the mean free path, the scattering situation is described by the Rayleigh⁶ or molecular scattering theory where individual molecules produce the necessary inhomogeneities.

If on the other hand the incident wavelength is much larger than the mean free path, the thermal motion of the molecules do no longer produce inhomogeneities on a scale large enough to produce any significant amount of noncoherent scattered radiation. However, considerable radiation is still observed from a medium with a molecular mean free path less than the wavelength of the incident radiation, and this scattering is explained by the Mandel'shtam-Brillouin (hereafter designated as M-B) scattering theory.

For an explanation of M-B scattering let us return to the previous scattering situation (Figure 1). Let the number of molecules in one of the volumes, N , increase by ΔN , while the number in the other volume remains constant. In addition to the initial scattering from the two volumes (equal amplitudes but opposite phase), one of the volumes will now radiate an additional wave, E' , caused by density change ΔN . The total electric field of the scattered radiation from the volumes is now $(\vec{E}_1 + \vec{E}' + \vec{E}_2) = \vec{E}$; since $\vec{E}_1$ and $\vec{E}_2$ are of equal amplitude but of opposite phase. The intensity of light scattered by the addition to the electric field, E' , can be calculated by assuming that the density fluctuations, ΔN , will, under the influence of the incident light $E = E_0 \cos(\omega t)$, yield an additional polarization, $\vec{P}'$, in the volume V_1^* given by,

$$\vec{P}' = \frac{\Delta \epsilon}{4\pi} V_1^* E_0 \cos(\omega t) \quad (1)$$

where $\Delta \epsilon$ is the change in the dielectric constant due to the changed density. Thus we get the scattered intensity $I_{V_1^*}$ from the volume V_1^* ,

$$I_{V_1^*} = \frac{I_0 \pi^2 (V^*)^2 \overline{\Delta \epsilon^2}}{2 \lambda_0^4 L^2} \quad (2)$$

where λ_0 is the wavelength of the incident light and I_0 its intensity. If the dimensions of the density fluctuations are smaller than the scattering volumes, the additional scattering from one volume will not be correlated with that from other volumes and one simply adds the scattered intensities from each individual scattering volume, V^* . The total intensity of light scattered, I_s , from the entire volume, V , thus becomes,

$$I_s = \frac{V}{V^*} I_{V^*} = \frac{I_0 \pi^2 V^* V \overline{\Delta \epsilon^2}}{2 \lambda_0^4 L^2} \quad (3)$$

where (V/V^*) is the number of approximately equally sized scattering volumes. For a gas at low pressures the dielectric constant is related to the number of molecules in a scattering volume, by

$$(c-1) = C N_L = CN/V^* \quad (4)$$

and thus

$$\overline{\Delta \epsilon^2} = \frac{(c-1)^2 \overline{\Delta N^2}}{N_L^2 (V^*)^2} \quad (5)$$

where C is some constant and N_L is Loschmidt's number. With these last two expressions, the scattered intensity from the entire volume becomes

$$I_s = \frac{I_0 \pi^2 v (\epsilon-1)^2}{2 L^2 \lambda_0^4 N_L} \quad (6)$$

As can be seen, this expression is identical to the one predicting Rayleigh scattering.

3. NATURE OF DENSITY INHOMOGENEITIES IN AIR

In the previous section we merely assumed the existence of density inhomogeneities. In this section we will discuss the origin and subsequent behavior of such density inhomogeneities in air.

From a statistical viewpoint, it can be realized that the probability of having a uniform density in a gaseous medium is extremely small, i.e., there will always be differences in the density caused by the always present thermal motion of the molecules. Since these density fluctuations are nonequilibrium perturbations they are unstable and will instantaneously start to dissipate and give rise to new density fluctuations. In a viscous medium such as air the dissipation will follow two processes, a mechanical one involving a net motion of molecules but no change in the thermal energy of the individual molecules and a thermal one resulting in a change in the thermal energy of the molecules but no net transport of molecules⁷. The mechanical process will thus create isothermal pressure waves or phonons which propagate in all directions of the medium. The phase velocity of these random pressure waves can be predicted by arguments identical to those used to determine the phase velocity of ordinary or driven sound waves and their speed of propagation will therefore be the same as that for ordinary "driven" sound waves. The random sound waves will manifest themselves as a continuous "noise" sound if one were to expose a very sensitive microphone to the medium. The number of such isothermal pressure waves or phonons, dz , within a certain range Ω to $\Omega + \delta\Omega$ in a volume V was first given by Debye⁸. His expression was developed for thermal waves in a solid, but can be extended to our situation.

$$dz(\Omega) = \frac{3\Omega^2 V}{2\pi v} d\Omega \quad (7)$$

where v is the phase velocity for the pressure waves in the medium. The minimum wavelength of these fluctuations can be shown to be

$$\lambda = \left(\frac{4\pi}{3}\right)^{1/3} \bar{l} \quad (8)$$

where $\bar{l}$ is the mean free path of the molecules in the medium. For air at standard temperature and pressure $\lambda \sim 0.5\mu$. At the larger wavelengths these density fluctuations probably proceed into the turbulence region. The isobaric thermal wave produced by the second process relieving the density perturbations, will be stationary and decay in time by transferring the excess energy to the surrounding medium as a damped wave.

4. SCATTERING FROM MOVING ISOTHERMAL PRESSURE FLUCTUATIONS

To determine the effect of the motion of the scattering inhomogeneities consider Figure 2 below where $\vec{k}_0$ is the wave vector of the incident light, $\vec{k}_s$ that for the scattered light and $\vec{q}$ that for the travelling isothermal pressure wave.

Figure 2. Scattering from a moving inhomogeneity

Conservation of energy in this situation requires that

$$\hbar \omega_s = \hbar \omega_o \pm \hbar \Omega \quad (9)$$

where $\hbar$ is the Planck constant divided by 2π and ω_s , ω_o are the angular frequencies for the light waves and Ω that for the pressure wave. A positive sign preceding q implies the creation of phonon, a negative sign the annihilation of one. It will, throughout this section, where we treat spontaneous scattering, be assumed that this addition or subtraction of energy from the density field does not significantly effect the density wave picture. Conservation of momentum, furthermore, requires

$$\vec{k}_s = \vec{k}_o \pm \vec{q} \quad (10)$$

The magnitudes of these vectors can be expressed as

$$k_s = \frac{\omega_s n}{c} \quad \text{and} \quad k_o = \frac{\omega_o n}{c} \quad \text{and} \quad q = \frac{\Omega}{v} \quad (11)$$

where v is the phase velocity of the phonons and n is the index of the refraction. This discussion is also valid for other types of inelastic scattering such as Raman and Compton scattering while for elastic scattering processes such as Rayleigh and Thompson scatter, $\Omega = 0$. The absolute value of Ω can be determined from Figure 3 below.

Figure 3. Scattering geometry for inelastic scattering.

From Figure 3 we can see that

$$q = (k_o^2 + k_s^2 - 2 k_o k_s \cos \theta)^{1/2} \quad (12)$$

For $k_o \sim k_s$ this expression becomes

$$q = 2 k_o \sin (\theta/2) \quad \text{and} \quad \Omega = 2 n \frac{v}{c} \sin (\theta/2) \quad (13)$$

The relative frequency change is given by

$$\frac{(\omega_s - \omega_o)}{\omega_o} = \frac{\Delta \omega_{MB}}{\omega_o} = \pm \frac{\Omega}{\omega_o} = \pm 2 n \frac{v}{c} \sin(\theta/2) \quad (14)$$

which thus predicts that scattering will occur at two frequencies, one shifted up corresponding to a phonon annihilation and one shifted down with the same amount corresponding to phonon creation. This net shift, $\Delta \omega_{MB}$, will be;

$$\Delta \omega_{MB} = \pm 2 n v \frac{\omega_o}{c} \sin(\theta/2) \quad (15)$$

In this section we have so far assumed that these isothermal pressure waves do not dampen out as they travel through the medium. This will in reality not be the case where viscous damping will occur resulting in a broadening of the returned light. This broadening can be determined from Stokes law for the propagation of sound in a viscous medium⁵ which predicts a half width for the scattered radiation $\delta \omega_{MB}$;

$$\delta \omega_{MB} = q \Gamma \quad (16)$$

where

$$\Gamma = \frac{1}{\rho} \left\{ \frac{4}{3} \eta + \eta' + \frac{\kappa}{c_p} (\gamma - 1) \right\} \quad (17)$$

and ρ is the density, η the shear viscosity, η' the bulk viscosity, κ the thermal conductivity and $v = c_p/c_v$.

5. SPECTRAL DISTRIBUTION OF SCATTERING FROM STATIC ISOBARIC DENSITY FLUCTUATIONS

As stated in section 3 there will be in addition to moving isothermal pressure fluctuations, stationary isobaric temperature fluctuations which also will give rise to scattered radiation. Since these fluctuations do not move there will be no net shift of the scattered radiation. These are also damped in time and this again produces a broadening of the scattered radiation. This broadening can be shown to produce a halfwidth $\delta\omega_c$ given by

$$\delta\omega_c = \kappa \quad (18)$$

where κ is the coefficient of thermal conductivity.

6. SUMMARY OF M-Z SCATTERING AND ITS APPLICATION TO ATMOSPHERIC SENSING

Spontaneous M-Z scattering, as shown in the sections 4 and 5, results in three peaks in the scattered return, a center peak formed from light scattered by thermal fluctuations, unshifted but broadened by the thermal conductivity of the medium, and two symmetrical peaks, one shifted up in frequency corresponding to phonon annihilation and one shifted down caused by the creation of phonons. These two shifted peaks will also be broadened, but by the viscosity of the medium rather than the thermal conductivity. The total intensity of the peaks is identical that predicted by Rayleigh scattering theory although the physical processes are quite different. The intensity of the center peak I_c is related to that of the shifted peaks, $I_{\pm}$, by the following relation⁷:

$$\frac{I_c}{2I_{\pm}} = \frac{c_p - c_v}{c_v} = v - 1 \quad (19)$$

The net shift of the side peaks caused by M-Z scattering is proportional to the velocity of sound. It seems reasonable to assume that also for sound in the ultrasonic range there should be a relation to atmospheric parameters similar to that for normal sound velocity given by⁸

$$v = 33.05 (T^{1/2} + 0.16 \frac{e}{p}) + W \text{ (m/s)} \quad (20)$$

where T is the absolute temperature, e , the water vapour pressure, p , the atmospheric pressure, and W , the wind speed. If the wind component is neglected the sound velocity is approximately proportional to $T^{1/2}$ since the humidity effect is only of secondary importance. One can therefore write,

$$\pm \omega_{\pm} = \omega / T \quad (21)$$

The broadening of the M-Z satellites is indirectly proportional to the density, while the quantities inside the bracket of the expression for $I_{\pm}$ (16) and (17)—the viscosity and the thermal conductivity—are independent of density but proportional to $T^{3/2}$. One can therefore write,¹⁹

$$\pm \omega_{\pm} \propto T^{-1/4} / \rho \quad (22)$$

Thus if temperature is determined from a measurement of $I_{\pm}$, the expression for $\pm \omega_{\pm}$ could be used to determine the density. The broadening of light scattered by isobaric density fluctuations $\pm \omega_{\pm}$ is proportional to $\pm \omega_{\pm}$ and this quantity would therefore not furnish any further information about the medium. If it were necessary to correct for the wind velocity in (20), a measurement of the frequency of the scattering from aerosols would yield this information.

7. PREDICTED FREQUENCY DISTRIBUTION OF SCATTERED CO₂ LASER LIGHT NEAR THE SURFACE ($\bar{h} \sim 10^{-3}$ cm)

When the atmosphere is irradiated by CO₂ laser light ($1/\mu \sim 10^{-3}$ cm), the requirements for the scattering theory previously discussed are fulfilled i.e. $h/\mu \gg 1$. Typical numerical values for the parameters involved in the expressions for $\pm \omega_{\pm}$, $I_{\pm}$, I_c are listed below (CGS):

$$v = 3.4 \cdot 10^{10} \text{ cm/sec}$$

$$\kappa = 1.0400$$

$$\rho = 1.3 \cdot 10^{-3} \text{ g/cm}^3$$

$$\begin{aligned}
\rho_b &= 1.7 \cdot 10^{-4} \text{ gr/cm}^3 \\
\sigma_b^i &= 0 \text{ (small enough to be neglected)} \\
\mu &= 2.4 \cdot 10^3 \text{ erg/sec, cm, C}^\circ \\
c_p &= 10^7 \text{ cm}^2/\text{sec, C}^\circ \\
\gamma &= 10/7 \\
\Gamma &= 0.25 \text{ cm}^2/\text{sec} \\
\kappa &= 0.20 \text{ cm}^2/\text{sec} \\
\lambda/n &= 1.06 \cdot 10^{-3} \text{ cm}
\end{aligned}$$

With these values inserted into (15), (16) and (18) we get the following values,

$$\begin{aligned}
\Delta\omega_{MB} &= 68 \cdot 2\pi \cdot 10^6 \text{ rad/sec} = 68 \text{ MHz} \\
\delta\omega_{MB} &= 4\pi^2 \cdot 10^6 \text{ rad/sec} = 2\pi \text{ MHz} \\
\delta\omega_c &= 3.2\pi^2 \cdot 10^6 \text{ rad/sec} = 1.6\pi \text{ MHz}
\end{aligned} \tag{23}$$

Figure 4 thus depicts the frequency distribution of CO₂ laser light scattered back from the atmosphere.

Figure 4. Expected frequency distribution of CO₂ laser light scattered from the atmosphere

The integrated intensities of the above three peaks are about equal. There will be a relatively large component in the total return scattered by aerosols. Since these move with the wind velocity the incident frequency will be shifted according to Doppler's theory by an amount $\Delta\omega_{\text{aerosol}}$ equal to,

$$\Delta\omega_{\text{aerosol}} = \frac{4\pi nW}{\lambda_0} \tag{24}$$

where n is the index of refraction and W is wind velocity, which for vertical velocities of the order of cm/sec will be about a few KHz. This return will also be broadened with a halfwidth $\delta\omega_{\text{aerosol}}$ of,

$$\delta\omega_{\text{aerosol}} = \frac{2\pi\Gamma}{3\rho\lambda_0^2 a} \tag{25}$$

With a particle radius, a , of about 1 micron or larger this broadening will be of the order of KHz or less and will therefore not interfere with the other scattering distributions except in a narrow interval near the frequency of the incident light.

8. EXPERIMENTAL VERIFICATION

To demonstrate M-B backscattering from the atmosphere at STP, an experiment was conducted using an elaborate CO₂ laser system in combination with a sensitive homodyne detection system for the backscattering light. The signal to noise calculations preceding the experiment¹² revealed that the instantaneous return signal would be buried in noise which would require time integration to recover the signal. It was therefore necessary to design a laser system that would produce very stable high quality radiation and a subsequent laser amplifier to provide the power needed. The actual system built at the High Altitude Laboratory at the University of Michigan, produced about 50 watt CW of stable plane polarized, single frequency, single mode radiation. The backscattered radiation traversed the laser amplifier before it was beaten together with unshifted light from the laser oscillator on a Ge:Cu detector operating at 4K⁰. The beat signal, which is proportional to the M-B backscattering return, was then analysed by a tunable VHF receiver with a range covering those frequencies where the M-B peaks were expected. The result of an experimental run is shown in figure 4 where the difference between the return from the atmosphere and that from a chopper blade (coated with black paint) divided by chopper return is plotted versus frequency. A negative value of this quantity depends on a computer setting rather than on any real sign difference in the returns. The curve, which is representative for all the runs, shows a broad peak about 4 MHz wide centered at 70 MHz as predicted by the M-B scattering theory. Though the peak is not too pronounced the fact that it was present at all the experiments using different chopping and signal processing techniques makes it safe to conclude that M-B scattering indeed was detected.

Another much more prominent feature, also present in all the experiments, is a sharp increase in the return around 62 MHz when the chopper was blocking the light. This feature is tentatively interpreted as resulting from stimulated¹¹ M-B scattering mainly taking place in the air volume between the end of the laser amplifier and the chopper blade where the light intensity is at a maximum. In a blocking position the chopper blade would reflect the induced pressure wave back into this region of high intensity light where the wave would greatly enhance the backscattering process. There were also other features supporting this interpretation⁽¹²⁾.

9. CONCLUSION

It is concluded that M-B scattering of laser light from the atmosphere has an outstanding potential for remote atmospheric sensing and it is recommended that further research is done employing a more powerful CO₂ laser system to increase the intensity of the return signal. Through the use of stimulated M-B scattering, the problem of minute returns would be alleviated and altitude resolution could be achieved by focusing the beam at some altitude since stimulated scattering only occurs where the intensity of the light is very high.

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Fig. 4) (Backscattered atmospheric return -
Chopper blocked return)/(Chopper block-
ed return)--(D/C)--versus frequency.

A MULTIPLE-SOURCE DIFFUSION MODEL FOR NEW YORK CITY[†]

L. J. Shieh

P. K. Halpern

B. A. Clemens

H. H. Wang

F. F. Abraham

IBM Corporation

Palo Alto Scientific Center

Palo Alto, California 94304

†

A more comprehensive description of this study is to appear in a special ecology issue in the IBM Journal of Research and Development in early 1972.

INTRODUCTION

An experimental multi-source air pollution diffusion model based on the Gaussian plume formulation is described. The model is capable of incorporating point and area sources, time and space dependence of source strengths, and time and space dependence of meteorological variables. Numerical experiments simulating the SO_2 concentration distribution for New York City are presented for January 11, 1971. The numerical results agree favorably with experimental measurements.

BASIC EQUATIONS

The surface concentration resulting from a fixed point source of effective height H emitting a gaseous pollution at the rate of $Q(\text{gm/sec.})$ can be computed by the following formula:

$$x = \frac{Q}{\pi \bar{u} \sigma_y \sigma_z} \exp \left[-\frac{y^2}{2\sigma_y^2} - \frac{H^2}{2\sigma_z^2} \right] \quad (1)$$

where x - and y -axis are chosen as downwind and crosswind directions, respectively, $\bar{u}$ is the average wind speed, and σ_y and σ_z are the horizontal and vertical diffusion coefficients. For a derivation of this formula and a discussion of the assumption invoked (for example, neglect of diffusion in the x -direction) see Gifford (1). The ground surface is assumed to act as a perfect reflector to the pollutants. The effective stack height H used in the model has to include the plume rise due to the buoyancy force of the plume, i.e.,

$$H = h + \Delta h$$

where h is the physical stack height of point source and Δh is the plume rise. We use one of the more widely accepted empirical formulas (Concawe (2)) for plume rise calculations

$$\Delta h = k \frac{Q_h^r}{\bar{u}^s} \quad (2)$$

where k , r , and s are experimentally determined constants.

It is not practicable to construct a diffusion model for a large region by treating all of the pollution sources as point sources. To make the computation feasible, one represents the small sources within a chosen area as an "area source" with a uniform emission rate. In our model, we shall adopt the so-called integrated plume formula. The computation of the concentration at a given point (x, y) contributed by an area source is obtained by integrating equation (1) over the source region:

$$x = \int_{y'=0}^{2a} \int_{x'=0}^{2a} \frac{Q'}{\pi \bar{u} \sigma_y \sigma_z} \exp \left[-\frac{(y-y')^2}{2\sigma_y^2} - \frac{H^2}{2\sigma_z^2} \right] dx' dy' \quad (3)$$

where (x', y') is a point in the source area bounded by $0 \leq x' \leq 2a$ and $-a \leq y' \leq 2a$ is the dimension of a square area source, σ_y and σ_z are functions of $(x - x')$ and atmospheric stability class, and Q'

is the source strength per unit area. The area sources are oriented according to the mean horizontal wind direction $\bar{u}$ in the integration of equation (3).

Our model incorporates the σ_y and σ_z values of Pasquill⁽³⁾ and Gifford⁽⁴⁾ (i.e., see Gifford⁽¹⁾, pp. 102-103, Figures 3.10 and 3.11). Furthermore, it is assumed that an inversion layer of height H_i acts as a physical barrier to the dispersion of the pollutant and the value of σ_z is taken as a constant σ'_z and equal to

$$\sigma'_z = \frac{H_i}{2} \quad (4)$$

$$\text{if } \sigma_z > \frac{H_i}{2} \quad (5)$$

COMPUTATIONAL PROCEDURE

In order to reduce the computational time for real-time predictions of the concentration fields, an independent program calculates the $\propto \bar{u}/Q$ function of equation (3) for various source sizes, effective stack heights and atmospheric stability classes. The existence of an inversion layer in the lower atmosphere is also considered as a variable. The results of these computations are stored in a peripheral memory device of the computer.

The main program is divided into four major parts: 1) analysis of source inventory data; 2) analysis of meteorological data; 3) calculation of the surface pollutant concentration field; and 4) graphical display of computed results.

INPUT DATA FOR MODEL

The method of producing a SO_2 source emission inventory for a specific region is well documented. Studies such as MAPCA⁽⁵⁾, describe in detail the procedure for obtaining an annual emission inventory. However, they do not discuss procedures for utilizing the annual emissions as hourly emissions for use as input in an air pollution diffusion model.

The diffusion model requires emission data at two-hour intervals in order to predict the average ground level concentration for that period. There are various approaches to obtain this two-hourly data from annual emission data. The method that we use has been successfully used in a study by Stroh⁽⁶⁾.

SIMULATION EXAMPLE

The simulation experiments discussed in this section demonstrate the capability of the diffusion model. Figure 11 shows the source emission inventory grid with 400 area sources for the New York metropolitan area. The simulation area is 10.5 x 10 miles and the area sources sizes are 0.5 x 0.5 mile.

1 ± 1 mile, and 2 ± 2 mile. The source size distribution is related to the geographical gradient of emissions. The two-hour average emissions for the area sources were obtained by using the annual emissions using procedure described by Chisholm⁽⁶⁾. The simulations presented below exclude point source data.

The simulation for the hours 02-04 is shown in Figure (2). The prevailing wind for this area is from the southwest at 7 mph. The stability class used is 4. The peak concentration occurs on the west side of mid Manhattan. There is a very steep concentration gradient across the Hudson River. The general isopleth pattern in Brooklyn, Queens, and the Bronx shows elongation of the closed isopleths in a direction parallel to the mean wind direction.

The observations of ground level SO_2 at station 5 in Manhattan and at station 1 and 3 in the Bronx verify the simulation for these two areas. In Brooklyn (station 16) the simulation indicates a relative maximum. The .08 ppm isopleth falls just to the right of the observed .09 ppm. However, in Queens (at station 14) the simulation underestimates the concentration.

The 06-10 simulation is shown in Figure (3). The mean wind direction has changed to northwest at 4.5 mph and the stability class is 3. There are three distinct areas of maximum concentration, two in Manhattan, and one in southeast Bronx. All three areas are elongated in the direction of the mean wind. The change in wind direction has resulted in the movement of the Manhattan maximum (02-04) from the west shore line southward to the East River. A tongue of high concentration extends from the East River into Queens. A second Manhattan maximum appears on the southeastern part of Manhattan. Here again a region of high concentration extends southeastward to Brooklyn with two relative maxima appearing in Brooklyn. The regions of high concentrations are directly related to upwind emissions from areas in Manhattan which have the largest annual emission of SO_2 for all of New York City. The northwest wind has also shifted the region of maximum SO_2 gradient (02-04) along the Hudson River to the western shore line of Manhattan. The observations at station 1, 3, and 5 verify the simulation quite well. In Brooklyn (station 16) and in Queens (station 14) an underestimation by the simulation occurs.

In Figures (4) and (5), the twenty-four hour variation of the SO_2 concentration is presented for two stations. We note that there is reasonable agreement between the observed and computed concentration variation for this period. In Table I, six-hour averages of the concentration for the 24-hour period are presented for five stations. In general, we note good agreement.

Period	Central Park (5)		Brooklyn (16)		Bronx (1)		Manhattan (3)		Queens (14)	
	Obs.	Comp.	Obs.	Comp.	Obs.	Comp.	Obs.	Comp.	Obs.	Comp.
00-06 EST	.11	.10	.12	.09	.15	.10	.17	.12	.10	.09
06-12 EST	.13	.14	.13	.14	.17	.10	.14	.15	.14	.09
12-18 EST	.14	.14	.09	.08	.14	.11	.16	.10	.12	.09
18-24 EST	.14	.16	.14	.14	.11	.13	.19	.20	.14	.13

Table I. Six-Hour Concentration Averages (ppm) for Five Stations

The computer program[†] for our diffusion model is written in Fortran IV(H) and takes 200K bytes of core storage. For an IBM S/360 Model 91 computer, the c.p.u. execution time for a two hour simulation is 15 seconds.

CONCLUSIONS

The numerical experiments described indicate that our model has the capability of predicting the general pattern of the ground level SO₂ concentration in an urban region for a time scale of 2 hours. Furthermore, limited comparisons of the simulated values and observed data in the New York City area give a positive indication of the useful potential of this model. We are presently extending our study to include (1) the spatial dependence of meteorological input, (2) the point source inventory, (3) the complete systematic evaluation of various physical approximations assumed in the model.

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[†] This is an experimental computer program developed for research purposes only and is not available outside IBM.

AREA-SOURCE GRID

AREA SOURCE GRID OF NEW YORK CITY METROPOLITAN AREA

Figure 1

BRONX HIGH SCHOOL STATION 1

JAN. 11, 1971

24 HOUR VARIATION OF SO₂ CONCENTRATION

Figure 4

CENTRAL PARK ARSENAL STATION 5

JAN. 11, 1971

24 HOUR VARIATION OF SO₂ CONCENTRATION

Figure 5

A NUMERICAL MODEL OF THE NOCTURNAL URBAN BOUNDARY LAYER

James L. McIlroy*

Division of Meteorology, Environmental Protection Agency
Research Triangle Park, North Carolina

INTRODUCTION

The structure of the atmosphere over cities differs most significantly from that over adjacent rural areas during the nocturnal period when the heat-island phenomenon usually is developed to its maximum extent. The difference primarily should be the result of momentum fluxes due to the enhanced roughness of the urban surface and heat fluxes due to the varying thermal characteristics of the urban surface and the production of artificial heat from the activities of man.

Experimental studies of the three-dimensional structure of the nocturnal urban heat-island have been conducted recently over several cities (1, 2, 3). Basically, these studies demonstrate the existence and growth of a thermal boundary layer, at air with a rural history traverses the urban environs. The rate of growth and detailed structure of this thermal boundary layer seem to be related to the magnitude of the ambient wind speed and the thermal stability in the upwind rural settings. Downwind of the urban complex, particularly under moderate wind speeds and very stable conditions in the upwind rural area, advection of warmer urban air apparently occurs rapidly; this phenomenon has been termed by Clarke (4) as "the urban heat plume."

Models of airflow over heated islands (5, 6, 7) may be used to simulate some features of the nocturnal urban heat-island and identify many factors important for its development and maintenance. However, such models cannot be applied directly to the urban situation without undergoing considerable modification.

A number of models of the urban heat-island have been developed (8, 9, 10). These models are either purely thermodynamic or extended versions of surface energy balance equations; at such, they cannot realistically simulate the thermal structure except the entire extent of the urban complex. Thus, there exists a need for a model which has this capability.

THE MODEL

The model geometry and basic equations are shown in Figure 1 where $x = (x' + y')/L$, $y = (y' + z')/L$, and h is the gradient Richardson number; the overbars refer to mean values over a time step, and the other symbols have their usual meanings. The model is 2-dimensional and is solved numerically on a digital computer. It was developed from a 1-dimensional, time-dependent boundary layer model of Csanady (11). The constant-flux and transition atmospheric layers are retained but the sub-surface layer is discarded. Instead, the sub-surface thermal characteristics are specified at boundary conditions for inclusion into a surface energy balance equation; surface temperature is computed iteratively from this equation which includes as components longwave radiation and sensible, latent, sub-surface, and artificial heat fluxes. Equality of the eddy exchange coefficients for heat, moisture, and momentum is assumed. The coefficient in the constant-flux layer is based on the local stability and aerodynamic surface roughness. That in the transition layer is specified as decreasing from a maximum value at the top of the constant-flux layer linearly to zero at the top of the assumed planetary boundary layer. Radiational cooling due to water vapor and carbon dioxide only is considered; limited evidence (12) indicates that the radiative effects of pollutants may be insignificant during nocturnal periods. Finally, the pressure gradient are those of the overall synoptic situation.

Here, the city is considered to be a source for surface processes the thermal and aerodynamic roughness characteristics of the mean city. This is not a realistic assumption close to the ground, but should be a satisfactory approximation for the region of flux away from the complex of the local structures. For a nearby flat city, a low constant-flux layer is reasonable. It also allows for considerable mathematical simplification in the equations of boundary conditions.

The primary modification to the basic model of Csanady consists of replacing the local time derivative by local or Lagrangian time derivatives. In the resulting Lagrangian model, vertical advection is ignored and steady-state conditions are assumed. Information from a brief study of the urban heat-island using this model of Csanady (13) indicates that the horizontal advection term is small in order of magnitude larger than the vertical advection term in the thermodynamic energy equation; in the momentum equations, the vertical advection term may be more significant. The Lagrangian model calculations are made in the time domain. The actual horizontal distance covered during a time step is taken to be the minimum distance travelled by the air over all steps of the vertical levels. The values of the variables at each of the vertical levels are adjusted for this

*On assignment from the National Oceanic and Atmospheric Administration, U.S. Department of Commerce.

distance prior to a subsequent time step according to the equalities in Figure 1 assuming linear variations in their values with time. For the short time steps normally used (a fraction of a minute), this assumption should be reasonable.

The resulting model is applied to a traverse across a city in the direction of the mean flow. Information on the aerodynamic surface roughness, surface relative humidity, sub-surface thermal characteristics, and artificial heat is specified at finite horizontal distance intervals. The initial data are those appropriate for the upwind rural environs. In the model, the thickness of the constant-flux layer is either held constant or allowed to vary across the city. Finally, the vertical spacing of levels in the transition layer is determined from a linear stability analysis of the centered finite difference forms of the equations for the layer (11).

MODEL PARAMETERS AND DATA INPUT FOR COLUMBUS, OHIO

The model was used to simulate the (thermal) structure of the nocturnal boundary layer over Columbus, Ohio. The experimental data were collected over the metropolitan area on each of four dates in both September 1968, and March 1969. Specifically, vertical profiles of temperature and moisture were obtained during helicopter flights, and measurements of temperature were taken near the surface during automobile traverses. Measurements of surface (skin) temperature were made remotely using a radiometer flown in a fixed-wing aircraft, and those of winds aloft were acquired from theodolite observations of slowly rising balloons.

In 1968, according to population, Columbus was the 28th largest city in the United States; the population was estimated to be in excess of 580,000 and that of the metropolitan area (Franklin County) in excess of 850,000. Columbus itself is relatively flat as is shown in Figure 2; terrain height above mean sea level is denoted at 20m intervals. According to land-use and building type, the metropolitan area of Figure 2 is partitioned approximately into four zones. The small central zone labeled 1 consists of the downtown area with a mean building height of about 30 m. Zone 2 represents less built-up commercial and industrial areas and dense residential areas, each with a mean building height of about 15 m. The main residential area with a mean building height of about 7 m is represented by zone 3. Finally, zone 4 consists of the rural environs.

Numerical simulations were made for the near steady-state period just before sunrise. The experimental data generally extended to 300-600 m above ground. Above this elevation, initial profile information for wind, moisture, and temperature was estimated from radios taken at 0530 a.m. at Dayton, Ohio, 116 km west south-west of Columbus.

Information concerning the aerodynamic surface roughness, surface relative humidity, artificial heat flux, and sub-surface thermal properties was specified at horizontal distance intervals of 1 km across Columbus. Values of the surface roughness parameter (z_0) ranged from 5 cm in the rural area to 200 cm at the center of the downtown district and were based on the geometry of the local structures. The sub-surface thermal diffusivity (K_s) varied from 0.002 cm²/sec for the loam soil in rural Columbus to 0.020 cm²/sec for (assumed) pure concrete and other such material at the center of the downtown area. Values of K_s for the other areas were interpolated between these extremes in proportion to the percentages of soil versus concrete and other such material. The product of the soil density and soil specific heat was considered to vary much less across the city and hence was specified at a reasonably average value of 0.5 cal/cm³ deg. For each simulation, an average sub-surface temperature profile was formulated for each of the four land-use zones. The profiles were computed using the 1-dimensional model of Estoque from initial conditions at solar noon; the initial soil temperature profiles were estimated from the data taken at the Ohio State University Experimental Farm in rural Columbus along with profile shape information found in Geiger⁽¹²⁾. Estimates of surface relative humidity were made using the experimental data in conjunction with the previously mentioned percentages of soil versus concrete and other such material; only the soil was assumed to hold moisture. Finally, artificial heat fluxes were calculated from the results of an emission inventory⁽¹³⁾ for the dates of the experimental study.

In the model, the maximum value of the eddy exchange coefficient is found at the top of the constant-flux layer. The experimental data showed the most unstable portion of the urban-modified layer, at least over the downtown area, to be confined to about the average height of local structures. The constant-flux layer over the rural environs under stable nocturnal conditions was expected to be very shallow. Thus, as a compromise, a constant value of 15 m was used for the thickness of this layer across Columbus.

During experimental periods in Columbus, the depth of the urban-modified layer seldom exceeded 100 m. A linear stability analysis of the finite difference forms of the equations in the transition layer showed that computational stability could be maintained with a 35 m grid spacing in conjunction with a 15 sec time step. This represented a compromise in obtaining detail in vertical resolution without unduly increasing the computational effort through the use of excessively short time steps.

COMPARISONS BETWEEN NUMERICAL SIMULATIONS AND COLUMBUS, OHIO, DATA

Data were used for the nearly steady-state period around sunrise on two of the four September dates and on three of the four March dates. Overcast sky conditions prevailed during most of the other March period. Each of the other September periods experienced conditions very similar to those on one of the periods utilized.

In the observed and simulated cross-sections, isopleths of air temperature are provided at 1C intervals, except in the vicinity of the urban-modified layer where they are provided at 0.5C intervals. Cross-sections are limited to the lowest 300 m to better show the detail in this relatively shallow layer. Actual values of data are presented only in the observed cross-sections. For the simulations, computed values were used at horizontal increments of 1 km at the levels denoted on the height scale to construct cross-sections. Topographic variations relative to locations where upwind rural soundings were taken also are shown in the observed cross-sections.

The results for 23 March 1969 are taken as typical and are presented here. Figures 3 and 4 consist, respectively, of the simulated and observed cross-sections of temperature across Columbus for this period. As is shown in these figures, the simulations agree well with the observed data with respect to both the spatial changes in the lapse rates and the development of the urban-modified layer. That is, in both, the thermal boundary layer begins to develop in the upwind suburban district and increases in depth into the downtown area. Beyond the downtown sector, air restabilizes, commencing at the surface. Above this layer, advection of warmer urban air in the form of a giant heat plume is simulated. Finally, immediately downwind of the urban area, the thermal stratification in the simulation returns to its upwind rural character, except for the zone occupied by the heat plume. The most obvious general discrepancy between the observed and simulated data is in the detailed thermal structure near the ground in the rural environs; the use of a thinner constant-flux layer may be appropriate over these environs and thus might help to alleviate this discrepancy.

It is not easy to determine how well the urban heat plume was modeled due to the lack of detail in the observed data. However, significant enhancement of temperature in the lower layers is observed in the downwind rural soundings in comparison to those of their upwind rural counterparts.

Precise comparisons between the simulated and observed depths of the urban mixed-layers also were difficult due to the lack of detail in the observed data and the vertical resolution of the simulated data. Nevertheless, some gross comparisons for each of the experimental dates were made for the downtown area. They are presented in the table below. As is shown in this table, the observed and simulated depths are in good agreement.

DATE	MIXED-LAYER DEPTH, m	
	OBSERVED	SIMULATED
9/13/1968	70-80	85
9/21/1968	70-90	85-120
3/17/1969	50	50
3/18/1969	30-40	vertical grid spacing
3/23/1969	70-80	50-85

Observed and simulated mixed-layer depths in the downtown area during experimental periods in Columbus, Ohio.

CONCLUDING REMARKS

The model currently is being used to estimate the effects of alternate land-use strategies on the nocturnal urban thermal structure and hence on the relative atmospheric dispersive ability. It also may have applicability as a parametric tool in ascertaining the relative effects of city size and building geometry on the nocturnal thermal boundary layer.

While the model grossly oversimplifies the actual situation, it is a significant improvement over those in the literature. Hopefully, it is a step in the right direction.

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Figure 1

Figure 2

Figure 3

Figure 4

A PROPOSED MESOMETEOROLOGICAL NETWORK FOR EL PASO

WILLIS L. MOHR

Atmospheric Sciences Laboratory
US Army Electronics Command
White Sands Missile Range, New Mexico

INTRODUCTION

Mesometeorology refers to those dynamic aspects of atmospheric processes which have intermediate scales, smaller than the synoptic phenomena of common weather maps and larger than those parameters sampled at a local weather station. Included are small scale phenomena, such as drainage winds and lee waves induced by local terrain effects, and convective systems, such as thunderstorms and tornadoes. These dimensions are of the same order as those of a metropolitan area, so mesometeorology has its principal future application in those populated regions.

Air pollution problems have focused attention on the mesometeorological characteristics of our modern cities. From the crude first look in effect today for elimination of gross sources of atmospheric pollution, it is clear that application of mesometeorology must expand to provide input for systematic control of all industrial effluents (gases, particles, heat), control of high-speed freeway traffic and aircraft, and safety applications associated with accidents involving toxic materials, fires, epidemics and other disasters. A modern city requires advanced planning and sophisticated application techniques to assure that atmospheric effects are efficiently considered in regulating development and applying safety measures. In addition, future efforts at weather modification will raise innumerable problems in applying controls to this most important aspect of man's environmental resources.

Meteorology has played an important role in the scientific development of the El Paso area. The presence of White Sands Missile Range has fostered a rapid expansion of this scientific field until today the atmosphere of the El Paso area is the best observed in the world. A notable shortcoming in the meteorological program of the orographically complex El Paso area is the lack of an adequate boundary layer observational and forecast system. The need for and an outline of such a system are presented here.

Until the Elephant Butte Reservoir was completed in 1915 floods were matters of general concern for residents of the Rio Grande Valley. Even today local flooding may result from heavy rains in the surrounding mountain areas. Floods, along with their alternate severe weather phenomenon, dust storms, were the focus of meteorological attention in the early days of meteorology in El Paso.

Both of these extreme weather phenomena are very infrequent, with sunshine the principal weather characteristic of the El Paso area. This good weather has resulted in strong shifts in emphasis in local meteorology, first from an agrarian orientation to aviation, and more recently to rocketry. These technological developments have produced a wealth of meteorological experience in the El Paso area, with the locally initiated Meteorological Rocket Network focusing international attention on progress in the atmospheric sciences in the local area.

In the process of expanding our meteorological frontiers upward in the global sense, small-scale phenomena of significant impact on the ecology of the El Paso area have been somewhat neglected. Social pressure relative to air pollution and weather modification has recently reoriented our attention to the surface layers to assure that the technological system which we enjoy does not excessively impair the human characteristics of our environment.

The following pages summarize past events of meteorological significance in the El Paso area, illustrate the state of knowledge of local atmospheric structure, and depict the structure of a local meteorological observational system which will be adequate for control of future metropolitan growth. This latter effort at experimental design is necessarily based on estimates of future technological demands, and there have been no intentional underestimates incorporated into the plan. It is clearly time to take care of the atmospheric aspects of our minimum ecological needs in an energetic fashion. The plan presented here is not necessarily the best which can be devised. It does have the virtue of offering a solid framework on which improvements can be implemented.

Figure 1. Map of the Paso del Norte area.

Mexico as is illustrated in Figure 1. For an excellent historical development of the region the reader is referred to C. L. Semtchen's book *Paso of the North*. The Franklin Mountains extend southward to the edge of downtown El Paso, physically separating the city into a northeast upper valley section and a northeast branch which stretches toward the Tularosa Basin. The Franklin provide an excellent transmitter location (the highest in Texas), but, from a meteorological point of view, they introduce gross complications into the meteorological processes which establish the atmospheric structure of the Paso region.

The water which flows through the Rio Grande Channel is of primary importance to the ecology of this high-altitude desert region. Strong fluctuations in the atmosphere were principal characteristics of the early uncontrolled days. Construction of the Elephant Butte Dam approximately one hundred miles north has removed the fun of river forecasting, reduced the general flooding of lowlands to negligible proportions and cast most of the atmosphere into concrete irrigation ditches. The impact of this major change in physiography on the ecology of the area has been complicated by intensive irrigation efforts, and the terminal ecological situation which current technological changes will impose is yet to be determined.

Meteorology in the El Paso area shifted focus away from the early agricultural emphasis with arrival of the aviation age in the 1930's. Atmospheric observational programs expanded from the surface upward through the troposphere by means of balloon-borne sensors which attained mean burst altitudes of 30 km (~100,000 feet) in the 1930's. In the mid-1940's the rocket age arrived in El Paso with initiation of rocket testing at White Sands Missile Range. The tools for exploration of the upper atmosphere came hand in hand with the requirements for more knowledge of that region. SAC Corporals, V-2s, Vikings and Aerobees quickly made White Sands Missile Range the world center for upper atmospheric exploration. Start of the International Geophysical Year period in 1958 expanded rocket exploration to Fort Churchill, Canada, and other locations and precipitated a general interest in synoptic exploration of the upper atmosphere.

This interest, along with technological advances, primed the El Paso area in the late 1950's for initiation of the global synoptic Meteorological Rocket Network (MRN) which effectively doubled the observed volume of the atmosphere. Balloon peak altitudes for meteorological observations were quickly increased to roughly 60 km (~200,000 feet) by introduction of small rocket systems. The MRN has grown during the first decade of its existence to include some 29 stations scattered around the world, with White Sands Missile Range the center for development of this global synoptic network^{2,4}.

The thesis that man's control of his environment can be restored by legislation for or against specific sources of atmospheric modification is a gross simplification. Man's desire for material things and his rapidly increasing capabilities with force and industrial technology to press ever closer to the upper limits of acceptable heat and global environmental pollution with combustion gases, pollutants, radiation and noise.

Uncontrolled control and use of this modification of the atmosphere can come only from an adequate knowledge of the impact of pollution on the El Paso ecology system and an adequate knowledge of the atmospheric processes which control the contamination cycle. We are gaining the former knowledge through trial and error. We should get the latter in a vigorous scientific manner. An adequate worldwide meteorological network in the El Paso area is a technological necessity.

El Paso (del Norte) is located on the Texas-Mexico border above the Rio Grande where the southward course from the Southern Colorado headwaters and flows southward across the foothills of the Rockies toward the Gulf of

The Atmospheric Sciences Laboratory of White Sands Missile Range has worked with the Schellenger Research Laboratories of the University of Texas at El Paso to play a leading role in development of the required small rocket measurement techniques, data analysis and publication formats which have facilitated this growth of the WSN. Small rocket systems of the Arco and Jakt types have formed the backbone of this new synoptic network, and the wind and temperature sensors, which have been developed for use with these rocket systems have provided revolutionary new concepts of the structure of the upper atmosphere. A total of 303 stratospheric meteorological soundings had been completed at White Sands Missile Range by the end of 1958, out of a total of more than 13,000 soundings collected from all over the world by the WSN. By the end of 1968 a total of 59 volumes containing more than 11,000 soundings had been assembled, reduced to a common format and published in digital and graphical formats by the Schellenger Research Laboratories. All of these data are also available on punched cards and magnetic tapes for automatic data processing.

Graduate training of scientists of the Meteorological Rocket Network effort was accomplished by the Physics Department of the University of Texas at El Paso. Evening classes were conducted for more than 100 local atmospheric science students, and summer institutes were conducted (during the summers of 1960, 1967, 1968, 1970 and 1971) for representatives of the several groups involved in the North American Meteorological Rocket Network. This latter effort was expanded by the United States National Academy of Sciences through the Committee on Space Research of the International Council of Scientific Unions to include seminars at the Imperial College of Science and Technology, London, in 1967 and at the Japan Meteorological Agency, Tokyo, in 1968 to get the Global Meteorological Rocket Network under way. In addition, the University of Texas at El Paso hosted five national meetings of the American Meteorological Society (1953, 1961, 1963, 1966 and 1968) which were designed to consider techniques, air results and foster development of synoptic exploration of the upper atmosphere.

These efforts at exploration of the upper atmosphere through synoptic techniques have made the El Paso area the center of meteorological exploration of the upper atmosphere. This approach is progressing at an accelerated rate with development of new measuring techniques such as the meteor trail radar system which will push the upper limit of synoptic wind observations upward another 30 km to the 110 km level. These accomplishments have made the El Paso area the best observed atmospheric region in the world from a meteorological viewpoint.

While the pollution of the atmosphere has important global aspects, the growing importance of urban air pollution precipitated by our advancing technology and growing population has brought to the forefront yet another aspect of the complex problems of the atmospheric sciences. There are now developing far more stringent requirements for aerospace meteorological data related to the urban air pollution control problem, and it is generally true that meteorological data which has been satisfactory for agricultural and transportation problems are totally inadequate for resolution of the complex physical processes which determine air quality.

Limited rainfall and temperature data are available for several locations in the metropolitan El Paso area, but these records generally lack the continuity and comparable exposure which is so necessary for systematic study of the atmosphere. As an example, rainfall records have been maintained at several Fire Department station locations, but generally only for those circumstances in which the rainfall amounts to more than one-quarter inch. While these records are adequate for their basic purpose of evaluating flooding, they are hardly adequate for a climatological analysis of the impact of terrain or air pollution on the precipitation process.

Temperature and wind data in the El Paso area are even less satisfactory, primarily as a result of problems in exposure. While such records do exist, a reasonable evaluation is that they provide for only a crude estimate of the synoptic variability and climatological structure of the atmosphere in the El Paso area.

ATMOSPHERIC OBSERVATIONS IN THE EL PASO AREA

Routine records of meteorological observations were begun at El Paso in August 1850. These records were maintained by the army at various locations in the Paso del Norte area until this work was turned over to the Weather Bureau on 1 July 1891. The Weather Bureau was a part of the Department of Agriculture during this early period, and the El Paso station operated during this period as a climatological and six-hourly map observational point, which was located at various sites in the downtown area.

Meteorological requirements changed with the coming of air transportation to El Paso, so the Weather Bureau established an hourly observational station at the airport (see point A, Figures 1 and 2) on 20 November 1931. The downtown station was closed on 1 July 1939, and the Weather Bureau was transferred to the Department of Commerce on 30 June 1940. Pilot balloon observations of the winds aloft began at the airport station on 1 April 1932, and radiosonde observations were begun on 21 July 1939. Airplane soundings of the free atmosphere were initiated on 1 July 1935, at El Paso and carried out until balloon-borne radiosondes were initiated in 1939.

An intensive meteorological observational program was conducted at Biggs Field (see point B, Figures 1 and 2) in association with Air Force operations there during the 1950's and early 1960's.

Figure 2. Location of El Paso surface meteorological observation points providing data used in this study. The solid (Rio Grande) and dashed lines indicate the city limits.

Figure 3. Stations which were used for an air pollution study at El Paso in 1968 (Air Quality Survey, El Paso Metropolitan Area, October 15-November 15, 1968. Texas State Health Department, Austin, Texas). ASARCO STATIONS are indicated by triangles.

This station was initiated in April of 1941, and, for a period during the height of Air Force activity (1959-1966) at that location, the weather detachment included complete surface and special observational units along with a forecast unit. This meteorological station was closed in April 1966 with the termination of intensive flight operations at Biggs Field.

Meteorological observations were initiated at White Sands Missile Range in 1945 with the beginning of rocket testing at that site. Surface observations of a variety of types have been carried out at numerous sites on the range, with a complete observational and forecast station located at the headquarters area at the south end of the range (see point C, Figure 1). This locale also includes a radiosonde station (with five other intermittent radiosonde stations on the range), and in 1958 a rocketsonde station for sounding of the free atmosphere routinely to altitudes as high as 80 km was added.

Current plans call for activation in 1971 of a meteor trail radar at White Sands Missile Range which will provide wind observations to 110 km, making it possible to provide wind profiles on a daily basis from the surface to 110 km over El Paso area. Additional special atmospheric observational equipments employed in the headquarters area include an ionospheric sounding station (ES modified down to 230 kHz) which has operated continuously since 1947 and two magnetometer stations which were initiated in 1969.

These major centers for meteorological observations in the El Paso area are supplemented by numerous locations at which limited observational programs are conducted. The known observational points of this type are indicated in Figure 3, and an abbreviated listing of the observational program and period of record for these locations is presented on the following pages.

The American Smelting and Refining Company (ASARCO) has established a network for continuous sampling of sulfur dioxide (SO_2) emissions in the El Paso area. Conductivity-type samplers of special construction with telemetry to the Environmental Sciences Department office are used to control the emission of SO_2 from their tall stack so that concentrations in the metropolitan area will stay within acceptable limits.

The ASARCO SO_2 network consists of 18 stations. This SO_2 network represents the most comprehensive air pollution sampling network to be deployed in the El Paso area thus far. It also represents the first attempt to make industrial use of these techniques in adjusting El Paso's technology to a working basis with the metropolitan population.

The ASARCO sulfur dioxide sampling network is tied directly by telephone line to an IBM 1802 computer which provides 15 second average values from the 18 sensors for display at the control center. The computer is also programmed to provide control personnel with wind and temperature data which are applicable to the dispersion problem.

Measurements of air pollution parameters have been made by the El Paso City-County Public Health Department. The distribution of sampling stations used in the 1968 intensive survey of local air pollution characteristics is illustrated by the dots in Figure 3. Some of the Health Department's sampling equipment is mobile, and the majority of the sites indicated in Figure 3 are used intermittently.

DESIGN OF A MESOMETEOROLOGICAL NETWORK FOR EL PASO

The nature of the data acquisition and processing problems which are associated with meteorological data required for ecological purposes in our modern economy demands that the entire process be automated. The mesonet model outlined here incorporates use of a digital computer for collection, evaluation, storage and presentation of the data and, where possible, involves automation of the sensors and telemetry systems to reduce manual operation of the system to a minimum.

Acquisition of more sophisticated sets of meteorological data for application to ecological problems would be of little value unless those data are efficiently applied toward solution of those ecological problems. It is essential that parallel with the development of such a data system an educational system be implemented which will assure an awareness by our technological managers of the virtue and usefulness of the data. For these reasons it is proposed that the expanded mesometeorological system outlined here include a data acquisition and processing system which is directly associated with the public school system of the El Paso metropolitan area. Wherever possible the system would be located at and operated by the science departments of these institutions, with the resulting data presentations provided to those law enforcement, medical and control organizations as well as others in our technology who have need for environmental data.

To facilitate development of such a unified system of data storage, processing and calculation for ecological purposes, a unique grid system for mesometeorological work is presented for the El Paso metropolitan area. This grid is illustrated in Figure 4, along with an outline of the city and the location of the principal observational locations which would make up the initial network. Kilometer units are used in the coordinates of this grid system.

The network design specifies that each of the public high schools plus certain other locations in the metropolitan area have standardized observational stations located on their grounds and that these basic network stations be tied to a central point by telephone data links. Those parameters observed will initially be meteorological measurements of wind, temperature, precipitation, thermal stability and wind shear, with provision for later incorporation of other environmental measurements as required. Since the sampling rates for these data needs to be highly variable, the data transmission and processing system should be able to provide for routine (say at hourly intervals) pickup of the several parameters being measured with a capacity of increased sampling rates (say at one-minute intervals) during periods of special import.

The network design specifies that the central point for collection, storage, processing and presentation of the data from this mesonet network be the Computation Center at the University of Texas at El Paso. The Central Data Corporation 3150 computer in operation there can accept data from the mesonet and special observational stations and process these raw measurements into forms which are optimum for application to particular ecological problems. An example of the initial mode of operation of the entire system is presented in this Chapter, using the design locations and altitudes of the mesonet network stations presented in Table 1 to derive the altitudes of the grid points and to present a topographic map of the area derived from these data. Finally, an air pollution problem is evaluated in the usual Pasquill-Gifford mode to delineate the nature of the additional computations which will be required when more sophisticated mesometeorological data become available for this application. These examples are designed to illustrate the limitations of current methods of handling the air pollution problem and indicate the improvement which may be expected with implementation of the more comprehensive meteorological observational system proposed here.

The techniques presented here for processing and applying atmospheric data to El Paso's ecological problems must be regarded as an extreme simplification of the programs which will finally be required. The approach does represent a firm step, however, and thus lends itself to improvements which can best be derived through experience.

Raw meteorological and/or air pollution data will be provided to the computer from the mesonet stations illustrated in Figure 5 and listed in Table 1. In view of the many uses to which these data might be applied, these inhomogeneous data should be interpolated and extrapolated onto the standard mesonet grid, and this form of the data should be used for preparation of presentations or for additional ecological calculations. This procedure will have the virtue of providing the best meteorological data available in a unified format ready for computer application to a variety of environmental problems.

Figure 4. The El Paso mesometeorological grid. Coordinates are in kilometers.

Table 1. Mesometeorological network stations for El Paso. Locations are in kilometers on the mesonet grid and elevations above NSL are in meters.

Station	X	Y	Z
Address	21.7	34.0	1204
Ascarate	22.6	16.4	1122
Austin	18.5	21.6	1200
Bel Air	28.2	15.8	1219
Bowie	16.4	16.7	1131
Burges	24.8	20.2	1198
Club	5.1	26.2	1143
Coliseum	19.2	17.9	1128
Coronado	9.4	25.8	1225
Dam	11.2	20.1	1138
Eastwood	27.5	18.7	1207
El Paso	14.6	18.8	1226
Irvin	21.9	30.7	1189
KROD	15.6	21.2	1529
KTSN	17.4	22.4	1713
Park	18.5	28.2	1244
Sunland	8.1	22.4	1136
Tech	16.5	19.7	1214
Thomas	25.6	14.4	1115
UTEP	13.4	18.4	1165
Water	20.6	24.8	1180
Weather	24.1	21.6	1195
Ysleta	28.7	10.7	1122

The first step, then is to process the raw mesonet data onto the mesonet grid. This is accomplished after the mode developed by Inman³ for the National Severe Storms Forecast Center's mesometeorological network. The procedure involves first estimates of the grid point values of the parametric field being analyzed. In general, the overall mean of the observational data provides a satisfactory initial estimate. The field value (z_p) at an observational point is then interpolated from the relation

$$z_p = z_1 + (z_2 - z_1) \frac{\Delta x}{b} + (z_2 - z_1) \frac{\Delta y}{a} - (z_2 - z_1 + z_3 - z_1) \frac{\Delta x}{a} \frac{\Delta y}{b} ,$$

where the grid points of the segment in which p is located are numbered consecutively counterclockwise from the point from which Δx and Δy are measured and a and b are the x and y grid point spacings (1 km) respectively.

The difference (Δz_p) between this computed value and the observed value is then used to derive grid point corrections ($C_{i,j}$) based on the relation

$$C_{i,j} = \frac{\sum_{k=1}^n w_k \Delta z_k}{\sum_{k=1}^n w_k}$$

where k is the number of observations involved. w is a distance-dependent weighting factor defined by

$$w = R^2 - d^2 / R^2 + d^2 ,$$

where d is the distance between the grid point and the observation point and R is the radius of influence. In our case R is set equal to 10 km.

A new estimate of the parametric value at each nearest grid point is then obtained from the relation

$$z' = z + C'.$$

This process is iterated until C' becomes negligible and the parametric field is considered to be a best estimate.

Derivation of parametric values for individual grid points includes information contained in the data from each observational station within 10 km, with emphasis on those stations which are located closest to the grid point. Clearly, any such process of surface fitting to the data will tend to eliminate small-scale features contained in the original atmospheric structure. Since the

severe limitations which are always inherent in meteorological observational systems will already have filtered much of this detail structure out of the information available to the computer, it is desirable to minimize any further filtering effects. The Fortran program used to accomplish this processing is available at the University of Texas at El Paso Computer Center for conversion of any inhomogeneous set of data onto any uniform grid.

Figure 5. Topographic map at 100-meter intervals of the El Paso area derived from the station altitude data of Table 1. Altitudes in meters MSL are printed at km grid points.

As an example of the technique, the altitude data for the mesonet stations listed in Table 1 were processed through the program. The computer output for each 1 km grid point of these input station altitude data was then used to derive the contour map presented in Figure 5. Comparisons with detailed topographic maps illustrates the degree to which this total process of data handling produces a surface which represents the topography of the El Paso area. While these results are clearly disturbing, it must be remembered that the step which we are attempting to take is to advance from the current practice of employing one value of a required parameter to represent the entire El Paso area to a more sophisticated use of a complex environmental surface.

Since the atmosphere is known to exhibit occasional complexities of the same order as that illustrated in surface topography, it is obvious that certain small scale aspects of atmospheric structure cannot be studied through data obtained with the mesonet network outlined here. Improvements in resolution will generally mean more observational points with their attendant economic problems. It seems probable at this point that the data processing techniques presented here will provide quite satisfactory for the observational network outlined in Figure 4 and Table 1.

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AIR POLLUTION EFFECTS ON VISIBILITY IN THE NON-URBAN ENVIRONMENT

Bruce Nicholson
Environmental Improvement Agency
Santa Fe, New Mexico

George White III
Harvard University
Cambridge, Massachusetts

The waste products of our atmosphere must be disposed of. Receptacles for this debris are the earth's land masses, water bodies, and atmosphere. Wastes that are released to the atmosphere consist of particles and gases. Atmospheric residence times for some of these materials may be short, hours or even minutes. For other materials residence times may be measured in terms of years.⁽¹⁾ Some flyash from coal-burning power plants may even circle the globe before being removed.⁽²⁾ Regardless of the residence time, the movement of gases and particles in the atmosphere will be, in large measure, governed by the motions of the atmosphere. Some atmospheric motions dictate the paths to be followed by airborne contaminants; other motions determine the extent to which the contaminants will be diluted.⁽¹⁾ In any event, these atmospheric contaminants will have their effect on the atmospheric visibility. In some cases, small effects take place over large areas, and in others, large effects take place over a small area.

DISPERSION EQUATIONS

The first step in estimating the effects of air pollution on visibility in a general non-urban environment is to be able to estimate the concentration of pollutants downwind from various types of sources. This begins with a mathematical model. There are many such models, but the Pasquill-Gifford gaussian distribution model is easy to use and has experimental support. Modified by the authors to take into account ground deposition, plant uptake, and plume reactions, it has the general form:

$$X_i(x, y, z; H) = \frac{Q_i \exp\left[-\frac{.693}{T_{1/2}} \frac{x}{u}\right]}{2\pi u \sigma_z \sigma_y} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] \times \left\{ \exp\left[-\frac{1}{2}\left(\frac{z-H}{\sigma_z}\right)^2\right] + h_1 \exp\left[-\frac{1}{2}\left(\frac{z+H}{\sigma_z}\right)^2\right] \right\} \quad (1)$$

where X is the concentration in grams/m^3 , x, y, z are the Cartesian coordinate system centered at ground level at the source in which x is along the mean wind direction, y is along the cross wind direction, and z is directed vertically; H is the effective height of the source release above ground; Q_i is the source release strength in grams/sec ; u is the mean wind speed, σ_y and σ_z are the standard deviations of the cross wind and vertical concentration distributions; $T_{1/2}$ is the half life reaction of the gaseous component in the plume; h_1 is the ground absorption and plant uptake coefficient.

An equation can be written for the reaction products in the plume:

$$X_j(x, y, z; H) = \frac{W_{ij} Q_i (1 - \exp\left[-\frac{.693}{T_{1/2}} \frac{x}{u}\right])}{2\pi u \sigma_z \sigma_y} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] \times \left\{ \exp\left[-\frac{1}{2}\left(\frac{z-H}{\sigma_z}\right)^2\right] + h_1 \exp\left[-\frac{1}{2}\left(\frac{z+H}{\sigma_z}\right)^2\right] \right\} \quad (2)$$

Figure 1. Plume Constituent Concentration

where W_{ij} is the weight gain factor from component i to component j . Of course, if there is only particulate matter present which does not undergo coagulation or chemical reaction, the half life term is not present.

A complex of power plants, which includes the Farmington Four Corners Power Plant, is planned for the Southwestern region. These coal-burning power plants will be supplying power, primarily to the coastal regions of the country, and will have a tremendous impact on the air quality in the Southwestern region. The effect of the Farmington Four Corners Power Plant can be seen from Figure 1. The figure shows the estimated particulate concentrations 100 km downwind from the plant under winter E stability (Pasquill stability class) based upon a 1974 emission control level proposed by the Environmental Improvement Agency. Maximum plume concentrations occur at some elevation above ground while background is a maximum at ground level. The plume contributes little to ground level concentrations but is quite significant at higher elevations.

THEORY OF VISIBILITY

In order to determine the impact of pollution on visibility, it is necessary to have a model to estimate the effect of each type of atmospheric pollutant on visibility. In general, the number of particulates in each size range, the size range found, the type of particle (index of refraction), and for absorbing gas, the type of gas, are necessary information to evaluate the effects on visibility. For a random assortment of particles and gases, the meteorological visibility (i.e. the range at which the contrast transmission of the atmosphere is reduced to 2%) will be given by an equation of the form:

$$I_v = \frac{3.912 \times 10^3}{1.207 \sum_i \sum_j \frac{C_{ij} Q_{ij}}{r_{ij} \rho_{ij}} + 7.18 \sum_{\alpha} k_{\alpha\lambda} C_{\alpha}} \quad (3)$$

where ρ_{ij} is the density of the i particle in gm/cm^3 in the size range j ; Q_{ij} is the scattering extinction coefficient for particle i in the size range j ; r_{ij} is the radius of particle i in μm in the j size range; and C_{ij} is the concentration of the i particles in $\mu\text{g/m}^3$ in the size range j ; $k_{\alpha\lambda}$ is the α gas absorption coefficient in liters/mole-cm, and C_{α} in ppm is the concentration of gas α . This leads to the concept of the scattering effective radius which is "that monodisperse particle radius which lies within the original sample size distribution and which produces the same scattering effect (visibility) as the original sample under similar conditions and with the same density and index of refraction as the original sample."⁽³⁾ This definition leads to a single parameter representation of meteorological visibility. Equation 3 now takes the form:

$$L_v = \frac{800}{\sum_i K_i C_i} \quad (4)$$

where C_i is the mass concentration of pollutant (i) in $\mu\text{g/m}^3$ and K_i is a constant which is a function of the particle density, scattering effective radius and extinction coefficient for that scattering radius and takes the form:

$$K_i = .2477 \frac{Q_i(r_{si})}{\rho_i r_{si}} \quad (5)$$

where ρ is the density of pollutant in gm/cm^3 (i); r_{si} is the scattering effective radius of pollutant (i) in μm and $Q_i(r_{si})$ is the extinction coefficient of pollutant (i) at r_{si} . It should be emphasized that visibilities should not be computed using mass median diameter as is often done in the literature since this has almost no relationship to light scattering effects.

Data presented by Ettinger⁽⁴⁾ produced a scattering effective radius of $0.457\mu\text{m}$ for background particulates. Scattering effective radii have been calculated for flyash, based upon measured particle distributions, sulfuric acid mist, and sulfates.^(3,5) These lead to K_i values of 1.0 for background, 1.46 for sulfates, 1.2 for acid mist at 20% relative humidity and for flyash, .50 in 1971, 1.0 in 1974, and 1.5 in 1977 based upon different proposed New Mexico Air Pollution Regulations. The change in the K values for flyash occur as the result of shifts in the size distribution of the particles. For NO_2 gas a value of $K = .034$ was determined.

When there is a plume distributed according to equation (1), the components must be integrated across the plume. The equation given by Middleton⁽⁶⁾ for the apparent luminance of an object viewed from a distance y is

$$\frac{dB(y)}{dy} = -\sigma(y)B(y) + B_a(y) \quad (6)$$

where $B(y)$ is the apparent luminance of the object at y , $\sigma(y)$ is the extinction coefficient of the atmosphere and $B_a(y)$ is the apparent luminance of a lamina of air. If absorption of the atmosphere is negligible compared to scattering, then we can assume that σ and B_a are similar functions of y or:

$$\sigma(y) = \sigma(o) f(y)$$

$$B_a(y) = B_a(o) f(y)$$

(7)

Then, defining contrast in terms of luminance and noting that for objects viewed on the horizon, a lamina of air viewed at any range has the same brightness and letting $f(y)$ take the form

$$f(y) = Q_1 + C \frac{1}{\sqrt{2\pi} \sigma_y} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]$$

(8)

where Q_1 is the background concentration and C is given by:

$$C = \sum_{i=2}^n \frac{K_i X_i}{\frac{1}{\sqrt{2\pi} \sigma_y} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]}$$

(9)

and X_i is given by equation (1) or (2) and K_i by equation (5), it can be shown, after some manipulation and transformation of variables that the meteorological visibility for vision across a plume with a constant background concentration is given by equation 10.

$$\frac{800}{2C} + \frac{1}{2} = \frac{\sigma_y}{C} \int_0^{L_v/2\sigma_y} Q_1 dr + \int_{-\infty}^{L_v/2\sigma_y} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}r^2} dr$$

(10)

Figure 2. Cross Plume and Background Visibility

The cross plume meteorological visibilities using equation 10 are shown as a typical mountain valley situation in Figure 2. These represent the level sight meteorological visibilities across a dispersed plume that is located in the valley and traveling out of the plane of the paper. The case presented is the estimated visibility at 100 kilometers downwind from the Four Corners Power Plant in summer under E stability (Pasquill stability class) for a proposed level of emission control by 1974. The solid line is the background visibility and the dashed line is the combined plume and background visibility. It should be noted here that although the background visibility continually increases with height, the visibility across the plume at first decreases, then reaches a minimum, and then increases until it eventually converges with the background. Some points of possible confusion should be clarified at this point. Remember that the dashed line represents the level line of sight between an observer and an object, both at the same elevation, and not what an observer at say 1000 feet might estimate the visibility to be looking up at a point 2000 feet at the mountain. Also, at about 1700 feet, the cross plume visibility line crosses the mountain outline. This means that if the observer has good eyes, he will just be able to distinguish the mountain at that point. The image will not be clear. Now, as the observer goes up in height, still looking across in a level plane, the mountain will go from completely obscure to barely visible, to hazy looking, and gradually to a clear outline. What will the observer standing at 1000 feet see as he looks from straight across up to the top? He will never see a crisp, clear outline of the mountain top. It will look hazy because he must look through part of the dispersed plume, but it will not be as clear as if he were going up in height looking straight across. The estimation of visibility on a slant path is a more difficult problem to solve, for it depends on the observer's position relative to a plume which has a gaussian variation in the cross plume direction and in the vertical direction, and if the plane of the slant path is not normal to the plume centerline, then the cross plume and vertical standard deviations change with up or down plume distance. Further effort will be directed towards developing means of estimating visibility on a slant path.

VISUAL RANGE VS. METEOROLOGICAL VISIBILITY

There is a problem in defining or using the meteorological visibility as the visual limit. For a black body object with inherent contrast of $C_o = -1$, the apparent contrast at the meteorological visibility would be .02. It has been shown by Middleton⁽⁶⁾ that there are few people who can distinguish this low a contrast; and if one wished to include 90% of a random sample of people who would be able to distinguish an object, the apparent contrast would have to be .08. This represents many miles when translating it into practice. Furthermore, the meteorological visibility is defined in terms of a per cent contrast transmission or $\frac{C(R)}{C(O)} = .02$. If you are viewing a colored object with inherent contrast of $-1/2$, then $C(L_v) = .01$. The term visual range as used by Middleton⁽⁶⁾ is more appropriate; and in physical terms, this is the range at which an object can just be distinguished. Putting this in equation form

$$V = \frac{1}{\sigma} \log_e \left| \frac{C_o}{\epsilon} \right| \quad (11)$$

where ϵ is the liminal contrast, σ is the atmospheric extinction coefficient and V is the visual range, and C_o is the inherent contrast. Now, we can use $\epsilon = .08$, the value to ensure that 90%

of the sample of people would be able to see the object. A series of curves can be constructed which, for a given inherent contrast, relates the per cent of total extinction to the per cent of visual range at $\epsilon = .08$. It should be noted at this point that because an object can be seen, does not mean that the effects of pollution will not be noticed. Indeed, a distinct effect on visibility will be seen. Whether it is a widespread effect or a local effect makes no difference. In general, color tones of objects viewed closer up will be lost and the crispness of the objects seen from close in on out to the visual limit will decrease.

MEASURING VISUAL RANGE

There are several ways of measuring the visual range or the meteorological visibility if this definition is preferred. These can be broken down into (a) instruments for measuring extinction (telephotometers), (b) instruments for measuring scattering and (c) meters using artificial haze. These categories have been discussed by Middleton but should not be considered a complete list. The first two categories deserve mention. The first, telephotometers measure the amount of extinction, or more exactly, they measure the brightness of the object being viewed and the background horizon sky. From this, the extinction coefficient of the atmosphere may be calculated; or knowing the intrinsic contrast of the object and the sky, the fraction of the visual range (or if preferred, the meteorological visibility) may be calculated, and knowing the range of the object, the visibility can be estimated. The telephotometer may be either visual or photoelectric and is for all intensive purposes a light meter. The field of view should be small in order to eliminate any scattered light which might affect the results.

The second class of instruments are nephelometers. These instruments measure the light scattering properties of an atmospheric sample at the point at which the instrument is located. Integrating nephelometers measure the total integrated scattering of the sample and give a direct measure of the scattering coefficient. The extinction coefficient is a direct measure of the visibility, and either the visual range (based on some agreed upon liminal contrast) or the meteorological visibility may be estimated. A problem arises using the integrating nephelometer. Suppose you are operating on top of a mesa. It is incorrect to use the value of the extinction coefficient determined from the nephelometer with equation 11 to estimate the visibility across the valley (because the background concentration decreases with altitude and the intervening space above the valley where the visibility is being estimated has fewer particles than where the nephelometer is sampling). The visibility calculated may be correct at ground level provided the number of particles does not change at ground level over the distance that L_v is being estimated. The same problem arises if there is a smoke plume between you and the object for which the visibility is being estimated. The telephotometer does not have this limitation since it is directly measuring the total extinction caused by the atmosphere between the observer and the object and not on the extinction based upon an air sample located at the instrument.

VISIBILITY STANDARDS

Mention has been made that if there is no visibility degradation, there will be no adverse health or other adverse effects caused by air pollution. The present trend is to assign an ambient air standard for particulates. How does this relate to visibility? The New Mexico ambient air standard for suspended particulates is $150 \mu\text{g}/\text{m}^3$ for a 24-hour average. This

translates roughly to a ground level visibility of only 5.3 miles. In a location where natural ground-level visibility is around 40 plus miles, this is not saying much. The states tend to follow the federal standards and federal standards apply to the coastal regions which have poor visibility to begin with. In general, visibility increases with altitude and this is why persons in New Mexico can see significantly beyond 40 miles; but as indicated previously, where there are sources which insert pollutants high in the atmosphere, the visibility is decreased when a plume crosses the line of sight. A plume may not contribute significantly to ground level concentrations, depending on how much it has dispersed, but it will contribute to visibility reduction. Controlling particulates alone does not completely solve the problem. NO_2 which results from the combustion process is also present but is masked by particulates. Unless this is controlled, you are trading grey skies for brown skies even though you can see further. A need exists for establishing the relation of visibility to particle concentrations and should be developed to the point where either a measurement of visual range or particle concentration will be sufficient to relate the two. This is by no means an easy thing to do; but in our opinion, it definitely needs to be tackled.

It is important to preserve or maintain a good visibility from an aesthetic and psychological point of view. Because of this, officials in the New Mexico agency are definitely planning to adopt and propose such a standard within the next twelve months just as for the other standards that make up the determination of its "quality environment."

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TURBULENT DIFFUSION IN A CITY STREET

P. C. Chang, P. N. Wang and A. Lin
University of Utah Fluid Mechanics and Hydraulics Laboratory
Salt Lake City, Utah 84112

INTRODUCTION

With the general public's awareness of environmental pollution, the impact of the accumulation of emission from automobiles on the livability in city streets has raised grave concerns among people who work in metropolitan areas. The recirculation and inhalation of pollutants in the "city well" formed between high-rise buildings may make the air quality in the close vicinity of street ground and building exterior drastically different from that indicated by the ambient air. Studies of urban air pollution meteorology in general do not yield sufficiently detailed information about this important small-scale phenomenon. The purpose of this paper is to deepen our understanding for small-scale circulation and diffusion patterns in city streets with special emphasis on the effects of building height and street width on ventilation of the pollutants produced from the ground source.

In this paper circulation and diffusion in a city street is investigated through a laboratory and computer simulation study by considering transverse wind over a simplified city street, i.e., a rectangular trough. The problem of flow over a rectangular trough is, of course, not new and has been previously studied by many investigators either in laboratories or through numerical calculations. (1,2,3,4,5,6) These studies were mostly motivated to determine the effect of cavity flow on pressure drag, skin-friction and heat transfer. So far, fundamental studies of flow over trough in terms of small-scale circulation and diffusion in city streets are far from sufficient. (7,8,9,10)

EXPERIMENTAL FACILITY AND PROCEDURES

The laboratory study was performed in an atmospheric wind tunnel and a water tunnel on three simplified street models with building height-street width ratios $H/L = 0.5, 1.0, \text{ and } 2.0$. The atmospheric wind tunnel, which has been described in detail by Lin and Chang (11) is a closed circuit tunnel with a twenty-six foot test section measuring two feet by three feet in cross section. The city street models in the form of rectangular troughs were made of smooth iron plate with a fixed base dimension of one foot. The top of the models was flush with the tunnel floor, which simulates the building roof. In all measurements the oncoming boundary layer had an ambient velocity of 8 fps and a boundary layer thickness, δ , of 8 in., which yielded a Reynolds number, $Re = 3.1 \times 10^4$. This ambient velocity is so selected as to allow detailed mapping of mean velocity field inside the cavity with a special low velocity anemometer.

It has long been recognized as an extremely difficult task to measure the velocity in the interior of the cavity flow, where the flow directions are not prescribed and the fluctuation velocities are comparatively large, so that the conventional application of pressure probe or hot-wire is not straightforward. (1,2) In this study a directional low velocity anemometer (Disa 55D 80/81) was employed to do such measurements. The anemometer contains, as its sensing element, a hot-wire probe which is vibrated at fixed amplitude and frequency (≈ 250 Hz). Through an AC coupling and rectification of the signal, the final output is linearly proportional to velocity. The basic measuring range of the anemometer is 0-30 cm/sec., but the range may be extended by operating the instrument as a constant current anemometer. The unit was calibrated with an equipment which consists of a translucent cylinder with an orifice at the bottom and a small tube connected to the top of the cylinder. Allowing water to drain through the orifice will create a calculative air flow in the small tube where the probe is calibrated.

The structure of turbulent flow outside of troughs were also measured. The mean velocity was measured with a standard pitot-static tube in conjunction with a Rouse precision manometer. The measurements of longitudinal turbulence intensity were made with a linearized constant temperature hot-wire anemometer (Disa Model 55001).

For direct visualization of various flow circulation patterns, additional observations were made in a 30 foot long and 2 foot wide water tunnel. The plexiglass trough model, with a fixed 6-inch height and variable width, was attached to the side wall of the flume. The building block was extended two feet in both upstream and downstream directions. The flow patterns inside the trough were taken with a Nikon camera by photographing the floating glitters stream over the water surface.

Diffusion of the effluent from automobiles was simulated in the water tunnel by releasing Rhodamin WB dye from the street base through a 1/16 inch tap hole. The source rate regulated with a constant head tank was selected as 5 cc/min with the strength of 38 ppm to avoid disturbing the existing flow field. The detailed concentration inside the cavity and downstream flow is measured with a Turner Model III Flurometer. The sampling was made discretely with a syringe through tapered glass tube having a 1/16 inch I.D. tip. To compare the relative ventilation efficiency for various conditions, the mean and probability distribution of resident time

in city streets was also determined by measuring the resident time of small floating glitters released from the street base. For each case some 200 samples were recorded.

COMPUTER SIMULATION

To explore the nature of flow in troughs comprehensively, it will be very useful to investigate the dependence of flow pattern in troughs on Reynolds numbers. Part of the experimental results presented in this paper is devoted to the study as to how strongly such dependence prevails. The computational study serves as a supplementary investigation to the experiments where Reynolds numbers are moderately high. The computational schemes are adopted from the simplified Marker and Cell (SMAC) method developed in Los Alamos.(12) The flows over a trough approaching from a uniform channel were investigated for three different trough configurations, i.e., $H/L = 1:2, 1:1, 2:1$, respectively. At the entrance of the approaching channel uniform unit velocities were imposed and the flow exits continuatively. All the walls and floors are non-slip boundaries except the upper boundary of the flow region which has been designated as a free slip ceiling. The calculations with Reynolds numbers of the order 3 were carried out in UNIVAC 1108 and the velocity vector fields which reached the steady state were plotted through Gerber plotter.

RESULTS AND DISCUSSION

As a transverse wind flows over a city street, the oncoming wind separates from the edge of upstream roof and forms a recirculating cavity flow in the street depression. The recirculating cavity which is separated from the ambient wind through a free shear layer shall generate a thin boundary layer along the street surface and building walls. The whole entity of cavity flow obviously depends upon the geometry of the trough and the parameters of the oncoming boundary layer, e.g., boundary layer thickness and level of turbulence intensity. The resulting different scales of air motion dictate the air pollution in city streets -- the recirculating cavity flow transports and distributes the pollutants in the street trough and the turbulence motion in the free shear layer provides mixing process to diffuse them to the external flow. We shall thus begin by presenting aerodynamic characteristics of flow over a rectangular trough.

Flow in the Rectangular Trough

The detailed velocity fields in the rectangular trough measured in the wind tunnel accompanied with the results of flow visualization from water tunnel and numerical calculations with the simplified MAC method are summarized in Figure 1. Although the results from different simulations which have different ambient conditions are not expected to be the same, they are complements to each other to show various aspects of flow characteristics. The velocity vectors of the wind tunnel experiments were constructed from the measured horizontal and vertical components. Due to the limit of the physical size of the low velocity anemometer, no attempts were made to extend our measurements into the thin boundary layer along the wall.

The most well defined and most frequently studied case is the square trough. As expected, the experimental results indicate the existence of a single recirculating vortex. The photographic pattern in particular shows informatively this vortex motion and its sweeping into the external flow. The thin boundary layer with "dead water" zone formed in the corners are also clearly observed from this visualized pattern. The recirculating vortex measured in the wind tunnel is, however, skewed towards the upstream wall and not symmetrical as being assumed in many models. For large Reynolds number, apparently there is more downward transport of momentum near the downstream wall. The fact that zero velocity region is not at the centroid of geometry will make the velocity maximum and wall shearing stress unsymmetrically distributed. The single vortex pattern formed in the square cavity is found to be quite stable from flow visualizations. This is to say that, while the flow reaches a moderate Reynolds number, the recirculating vortex will show up and persistently maintain its pattern. The vortex motion from numerical calculations seems not to have fully developed due to its low Reynolds number. Although some calculations were carried out for Reynolds number up to 100, the results were still essentially the same as those presented. It is also noted these low Reynolds number flows consistently show a "dipping" phenomenon in the upper limb of the rectangular trough.

As the building height is reduced to only one-half of the street width, the separated flow will reattach somewhere on the downstream wall or even on the street ground. This type of wake interference flow is thus a less smooth flow as compared to that over a square trough. Since the separation streamline may reach the lower level of the city street, better mixing between dissipative vortex flow and the external flow may be expected. Although an average flow pattern can be obtained, as shown in Figure 1, it should be noted this flow is basically unsteady in nature. From flow visualization the separation cavity is sluggish back and forth in the trough with occasional "filling" and "emptying" of the cavity with fresh ambient fluid. Again the discrepancy between experimental and computed results is attributed to the difference in Reynolds number.

When the building height is equal to or greater than twice that of the street width, as the case frequently encountered in metropolitan areas, the flow patterns may take various forms which depend upon the characteristics of oncoming wind. As shown in the photographic pictures taken

within a narrow band of Reynolds number, there exists three distinct circulation patterns for the trough with $H:L = 2:1$. The most interesting pattern is the well-defined double-vortices rotate alternately in opposite directions inside the trough. Apparently, the upper vortex which rotates in clockwise direction is maintained through transmission of shear stress from the ambient fluid at the upper limb. The upper vortex subsequently transmits energy to rotate the lower part of the fluid into counter-clockwise vortex motion. As the Reynolds number increases, it appears the upper vortex will grow in size and intrude into the lower vortex. At a still higher Reynolds number, the lower vortex is completely depressed and an elongated and inclined vortex forms in the trough. This is also the only pattern observed in our wind tunnel experiments. From Figs. 1c, 1h and 1k, it is noted the center of vortex extends downwards as Reynolds number increases. It may be generally stated that the existence of double-vortex is essentially a moderately low Reynolds number phenomenon. However, a critical Reynolds number cannot be uniquely determined since the structure of the oncoming boundary layer is also an important controlling parameter.

Flow Over the Rectangular Trough

Figure 2 shows the distributions of longitudinal component of the mean velocity $\bar{U}$, and turbulence intensity $\sqrt{u'^2}$ across several sections for the rectangular troughs. The complete mean velocity profiles were obtained by combining data above the trough measured with a pitot static tube and the data within the trough measured with the low velocity anemometer. The turbulence intensity measurements were extended slightly into the lower part of the mixing region by assuming the mean velocity was essentially horizontal.

It is noticed that the basic structure of turbulent wind over the rectangular trough is quite similar for different ratios of building height to street width. The oncoming wind has the typical characteristics of a turbulent boundary layer over a flat plate. As the oncoming boundary layer separates at the edge of the upstream roof, there develops a free shear layer over the trough. This is characterized by the existence of a region of strong turbulence intensity where mixing between recirculating cavity flow and external flow occurs. In the downstream direction this region of strong turbulence is seen to diverge in width and decrease in its maximum magnitude. It is important to note that, while the point of maximum intensity remains at the level of the building roof for square trough, it projects downward for the other troughs. Since the maximum turbulence coincides with the separation streamline, the aforementioned fact is the consequence that the flow over the troughs of $H/L = 2/1$ and $H/L = 1/2$, reattaches somewhere on the downstream building wall. The flow over the square trough which may be regarded as having a pseudo roof, is thus smoother than the other cases. The perspective picture is further supported by the wake-type mean velocity profiles and excess turbulence intensities in the disturbed boundary layer over the downstream roof of the non-square troughs.

Diffusion From the Ground Source

During rush hours the emissions from automobiles in metropolitan areas may be considered as a continuous area source from the street surface. In this study the transport and diffusion of pollutant inside the trough is examined with a continuous point source located at the street center. The detailed transverse concentration field is measured in the plane of source location with the understanding that the pollutant may diffuse along the city street. Some typical concentration measurements are shown in Figure 3. These measurements were made after the concentration fields reached the quasi-steady state, i.e., the incoming source was balanced by the diffusion out of the trough. The dye concentration is expressed in terms of the area of circle centered at the point of observation. It should be noted that different scales are used for different geometries for the convenience of presentation.

During experiments it was observed that the dye released from the ground was initially confined in the thin boundary layer in the close neighborhood of the source, but immediately immersed to and was subsequently transported by the recirculating vortex flow. It may thus be observed from Figure 3 that the concentration field strongly correlates with the velocity field presented in Figure 1.

For shallow troughs ($H < L$), the high concentrations appear along a path close to the upstream wall and between source and upper mixing zone. Once the source is transported to the upper limb, the strong mixing in the free shear layer will quickly diffuse the pollutant partly into the ambient fluid and partly back to the recirculating vortex. The latter yields a fairly uniform concentration over more than half of the trough near the downstream wall. Accumulation of pollutants near the street corner of the upstream wall is believed to be associated with "dead water" phenomena.

For the trough of $H/L = 2/1$, two concentration fields, corresponding respectively to the cases with double vortices and a single vortex, are shown in Figure 3c and Figure 3d. For the flow with double vortices the concentration maxima are clearly confined to the street level close to the downstream wall. For the flow with a single vortex, the concentration maxima distributed over the whole street level, even the "average" flow recirculates in the clockwise direction. This is believed to be the consequence of unsteady nature of flow. The concentrations over the remaining area are fairly constant, but much higher than those in the shallow trough.

From the above concentration fields, it becomes possible to calculate the inhaling capacity of various troughs. For a constant source strength, the amount of pollutant could be retained in different rectangular troughs is compared in Table I. In these calculations, the case of square trough is used as a comparison standard and all data are converted as if they had a constant street width. As shown in Table I, the effect of high-rise buildings on the amount of pollutant inhaled to the city trough is very substantial. In terms of mean concentration, it is also seen that air quality in business districts may be harshly deteriorated by the construction of tall buildings.

Table I

Height (H): Width (W)	Reynolds No. R_L	Amount of Retained Pollutant	Mean Concentra- tion in Trough	Mean Resident Time	Remarks
1:1	2.1×10^4	1	1	1	Sgle.vortex
1:2	3.6×10^4	0.32	0.64	0.25	
2:1	1.1×10^4	13.76	6.88	2.97	Dble.vortex
2:1	2.6×10^4	8.24	4.12	0.80	Sgle.vortex

The mean resident time of pollutant in a city trough, which is defined as the average duration of time the pollutant remains within the trough, is also shown in Table I. It is an informative index of the ventilation efficiency of city streets. The drastic difference in resident time for the deep trough due to different recirculation patterns should be particularly noted.

The nondimensionalized probability distribution of resident time in city troughs, as shown in Figure 3e, is found to follow a similarity curve. The fact that probability distribution of the resident time may be uniquely specified by its mean value suggests that it may be used to continuously predict the air quality resulting from complicated traffic patterns and diurnal variation of meteorological conditions. If the mean resident time as a function of wind speed and direction, and atmospheric stability is determined, a forecasting method may be developed by considering the traffic pattern and meteorological parameters as the informative input and the probability distribution of residence time as the overall transfer function. This method to be elaborated and refined later, shall be useful in urban planning and traffic control.

Comparison of the above simulation studies with some field experiments or the available field data^(8,10) will be very desirable. It is hoped that the combined study will provide some guidance to the further refinements of the existing models.⁽¹⁰⁾

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VELOCITY FIELD ($H/L=1/2$)

$R_L = 4.7 \times 10^4$

$R_L = 3.6 \times 10^4$

VELOCITY FIELD ($H/L=1/2$)
COMPUTED

0.04fps

$R_L = 3$

VELOCITY FIELD ($H/L=1/1$)

$R_L = 4.7 \times 10^4$

$R_L = 2.1 \times 10^4$

VELOCITY FIELD ($H/L=1/1$)
COMPUTED

0.001fps

$R_L = 3$

VELOCITY FIELD ($H/L=2/1$)

$R_L = 4.7 \times 10^4$

$R_L = 1.1 \times 10^4$

$R_L = 2.0 \times 10^4$

VELOCITY FIELD ($H/L=2/1$)
COMPUTED

0.21fps

$R_L = 3$

† Lines w/o arrows are not on scale

Fig. 1. (a), (b), (c): Measured velocity fields in the troughs from wind tunnel experiments.
(d), (e), (f), (g), (h): Visualized flow pattern from water tunnel experiments.
(i), (j), (k): Computed velocity fields with the simplified MAC method.

Fig. 2. Mean velocity profiles and longitudinal turbulence intensity profiles for turbulent wind over the trough.

Fig. 3.
 (a), (b), (c), (d): Measured concentration fields.
 (e): Nondimensionalized probability distribution
 of resident time in the troughs.

URBAN DIFFUSION OF CARBON MONOXIDE*

Walter F. Dabberdt, F. L. Ludwig, and Warren B. Johnson**
Stanford Research Institute, Menlo Park, California

INTRODUCTION

The measurement programs and analyses presented in this report have been used to test and improve a diffusion model developed during earlier phases of the program.^(1,2) The basic model, based on existing experimental data and previous research results, is a receptor-oriented Gaussian diffusion model and is designed to calculate carbon monoxide concentrations from routinely available traffic and meteorological data.

Model predictions were initially compared with measured data from Continuous Air Monitoring Program (CAMP) stations; calculated and observed values often differed significantly in magnitude, although they showed similar trends. An extensive field program in San Jose, California showed that these differences are primarily the result of local effects in the street canyons and around buildings that can sometimes cause CO concentrations to vary by as much as a factor of 3 from one side of the street to the other. The basic model failed to predict observed concentrations accurately partly because it treated the diffusion process only on the urban mesoscale, whereas concentrations observed at monitoring locations may reflect two predominant scales of motion: the urban mesoscale and the "street" microscale. One of the principal accomplishments of the program is the development of a new submodel that describes the microscale effects and substantially improves the basic model.

The program has also resulted in refinements to other components of the model, thereby further improving the model's performance. The nature and performance of the basic diffusion model and the various submodels will be presented in addition to the scope and some preliminary results of a recent field measurement program in St. Louis, Missouri, designed to further evaluate the total model.

ELEMENTS OF THE MODEL

The Basic Model

The basic model is a modified form of the receptor-oriented Gaussian plume formulation developed by Clarke.⁽³⁾ The concentration at a receptor is determined by the emissions from logarithmically spaced angular segments aligned with the mean transport wind. The innermost sector extends 125 m from the receptor--roughly comparable to the size of a city block. The four inner sectors have a width of 45°, and the outer sectors are 22.5°. The latter corresponds to the plume width ($\pm 2\sigma$) predicted by Gifford⁽⁴⁾ for slightly unstable conditions; the broader sectors near the receptor allow for large initial dispersion as observed by Pooler⁽⁵⁾ and McElroy.⁽⁶⁾

Because nearly all urban CO emissions are from internal combustion vehicles, the model assumes zero source height. Emissions within each sector are assumed to be uniformly distributed. The smaller sectors near the receptor provide the higher resolution required for nearby sources.

The surface concentration (C , gm/m³) from a continuous, ground-level point source (Q , gm/sec) with perfect reflection at the ground is given by

$$C = \frac{Q}{\pi \sigma_y \sigma_z u} \exp(-y^2/2\sigma_y^2) \quad , \quad (1)$$

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** Current affiliation: Division of Meteorology, Environmental Protection Agency, Research Triangle Park, North Carolina.

where σ_y and σ_z are the lateral and vertical standard deviations (m) of concentration, y is the lateral distance (m) from the plume axis, and $\bar{u}$ is the mean transport wind speed (m/sec). Formulation of the surface concentration from a continuous ground-level line source (such as a highway) follows upon integration of Eq. (1) over y , where

$$C = Q_L (2/\pi)^{1/2} (\bar{u}\sigma_z)^{-1} \quad (2)$$

On the basis of experimental data, ^(4,5,6,7) σ_z is taken to have the form

$$\sigma_z = ax^b, \quad (3)$$

where x is the downwind distance from the source and the parameters a and b depend on atmospheric stability. In operation, the traffic links and portions of links falling within each area segment are identified, CO emissions from the individual links accumulated, and the total emission is then assumed to be uniform over the segment. The contributions from each of the various upwind area sources to the CO concentration at the receptor point are computed individually for one-hour periods from the integration of Eq. (2) using (3) and are then summed to give the total concentration. A simple box model is substituted for the calculations at distant segments when there is a limiting mixing depth. Under these conditions, pollutants tend to become uniformly distributed in the vertical after sufficient travel time. The model changes from Gaussian to box form at the distance where the two formulations give equal surface concentration values.

Street Effects Submodel

In an earlier phase of the program, ⁽²⁾ an intersection in downtown San Jose, California, was instrumented to obtain the data necessary to describe and subsequently model street effects. Analysis of the data showed that the rooftop wind direction is the singularly most important meteorological factor in determining the distribution of CO in the street. When the roof-level wind blows within about $\pm 45^\circ$ of the cross-street direction, a helical circulation develops in the street. At street level the cross-street component is opposite the roof-level wind direction, causing a downflow of relatively clean air in front of the downwind buildings that face the roof-level wind, and an upflow across the street. The resulting low-level CO concentrations in front of the downwind buildings are often only half those observed across the street. Cross-street gradients of CO are small for winds approximately parallel to the street.

A simple model based on physical principles has been developed that describes the observed distribution of CO in the street canyon. First, the model presumes that the emissions from the local street traffic are added to the CO already present in the air that enters from roof level. These additive concentrations are proportional to the local street emissions and inversely proportional to the roof-level wind speed, which is augmented by a small amount to account for the mechanically induced air movement caused by traffic. "Box model" reasoning indicates that the CO concentration is inversely proportional to street width (W) and uniform with height in front of the downwind buildings. Thus, in front of the downwind buildings the CO concentration that is added by street sources (ΔC) is given by

$$\Delta C = \frac{kQ}{W(u + 0.5)} \quad (4)$$

On the upwind side of the street the model is also based on box model reasoning, but the volume into which the emissions can be mixed is limited by the helical circulation that transports street emissions toward the buildings and upward. As the air moves from the source, the volume into which the pollutants are mixed increases, so the concentration is taken to be inversely proportional to the slant distance (L) between the receptor and the nearest traffic lane. For concentrations in front of the upwind buildings, the equation is height dependent:

$$\Delta C = \frac{kQ}{L(u + 0.5)} \quad (5)$$

When the wind is nearly parallel to the street, the additive concentration, ΔC , is described by the average of the values from the two equations and is the same on both sides of the street. Figure 1 is a schematic representation of the cross-street circulation between buildings.

Specification of Input Parameters

The input variables required for the model are: (1) transport wind direction, (2) transport wind speed, (3) mixing depth, (4) atmospheric stability, and (5) vehicular CO emissions. The model is designed to be generally applicable to any city where conventional (airport) weather observations might be the only meteorological data available. None of the required input variables are directly observed. The airport surface wind speed and direction have been applied as a first approximation to the transport wind, but suitable adjustments may need to be made to account for urban effects. The San Jose field measurement program provided data for the initial validation and refinement of the various algorithms used to specify the input variables, and also tested the total model. Some of the results of that program are summarized in the following discussion.

FIGURE 1 SCHEMATIC OF CROSS-STREET CIRCULATION BETWEEN BUILDINGS

made from a CO mass budget analysis that was based on upper level wind measurements and CO concentrations measured around the central business district with helicopter- and van-borne instruments.

Two methods are used for determining the mixing depth. The morning temperature sounding combined with the afternoon maximum surface temperature is used for the calculation of the afternoon (maximum) mixing depth. Early morning mixing depths make use of a simple model of the mixing layer over urban areas proposed by Summers⁽⁸⁾ and an empirical relationship between city size and urban and rural nighttime temperatures developed by Ludwig.⁽⁹⁾ An interpolation scheme is used for the estimation of hourly mixing depth values. For daylight hours the interpolation is based on observed hourly temperatures, and for evening hours the interpolation is based on time. Atmospheric stability is estimated from hour of the day, wind speed, and amount of cloud cover. This is essentially the well-known Pasquill-Gifford formulation but has been modified to reflect the more stable conditions observed during the hours immediately after sunrise and before sunset.

PERFORMANCE OF THE MODEL

The performance of the total model has been evaluated in two ways with data collected in downtown San Jose. First, predicted concentrations were compared with observations on an hourly basis; second, histograms of observed concentration were compared with calculated values.

For the hourly concentrations, values calculated at heights from 3 m to rooftop were within ± 3 ppm for nearly 80 percent of the hours at the midblock stations; observed values ranged to 16 ppm. Correlation coefficients between observed and calculated values were as high as 0.71 for these stations. Results at the intersection stations were not quite so good. Figure 2 illustrates both observed and calculated concentrations for a three-day period at one of the midblock stations.

FIGURE 2 CALCULATED AND OBSERVED CO CONCENTRATIONS FOR STATION 7 AT SAN JOSE FOR 9, 10, AND 11 DECEMBER 1970

The frequency of occurrence of observed and calculated CO concentrations was determined for five geometrically spaced class intervals. For the midblock sites, the distributions differed by less than 6 percent of the total sample in any class interval. Intersection sites showed differences up to 21 percent. Figure 3 depicts calculated and observed frequencies of one-hour average CO concentrations for a midblock station.

FIGURE 3 CALCULATED AND OBSERVED FREQUENCIES OF ONE-HOUR AVERAGE CO CONCENTRATIONS, SAN JOSE

FIELD STUDY FOR FURTHER EVALUATION OF THE MODEL

Scope

The development and initial validation of the new street submodel during the San Jose program represents a potentially significant contribution in the modeling of microscale urban diffusion. It was imperative that the model be tested further under substantially different "canyon" conditions, i.e., with a larger ratio of building height to street width compared with the conditions in San Jose. St. Louis, Missouri, has the well-developed street canyons that were required.

Selection of a suitable field site required the elevation of three basic criteria: (1) relatively uniform building heights, (2) sufficiently heavy traffic flow, and (3) availability and suitability of the buildings for equipment installation. Accepting the premise that a perfect field site does not exist, we selected the intersection of Broadway and Locust Street as the site most suitable for providing data that would fulfill the goals of the program. The equipment was deployed to measure both the horizontal and vertical variations of carbon monoxide and wind in two adjacent street canyons that are perpendicular to each other. The locations of the air sampling intakes are schematically illustrated in Figures 4 through 7. Three-component wind sensors were installed at the lower and upper sampling heights on all vertical arrays in the street canyon to provide detailed information on the three-dimensional structure of the canyon circulation. A vertical array of temperature sensors was installed adjacent to the south Locust air sampling array. Another temperature profile system, together with CO and wind sensors, was mounted on a tall TV tower located atop one of the buildings at the site to provide background CO and meteorological data.

Traffic data were routinely collected on the two streets throughout the program; the city provided us with hourly traffic data collected over two-day periods at a variety of locations throughout the city in addition to historical data. With this configuration of equipment, we were able to study simultaneously and in considerable detail the concentration patterns in two orthogonal street canyons.

FIGURE 4 FIELD SITE IN ST. LOUIS

Installation of the equipment began during early August 1971, and actual data collection started on 24 August and continued through 15 October. The equipment was operated around the clock whenever possible, and data were collected regularly throughout the entire period except for two weeks during September. An electrical storm at that time rendered the system inoperable for the period. Sensor interrogation and data recording were controlled by a dedicated minicomputer. All sensors (CO, wind, temperature, and traffic) were regularly interrogated over an eight-minute period, and average values were typed on a teletypewriter to serve as a real-time data check; the computer also wrote the raw data on magnetic tape. These data were later analyzed on a larger computer and provided hourly values of the mean and variance of all parameters.

In addition to the fixed data network, mobile van and helicopter measurements of CO and temperature were also made. The helicopter was used to provide vertical profile measurements up to 2000 ft as well as horizontal traverses around the downtown area at several altitudes up to 1100 ft. The van collected data on spatial variations of CO around downtown and downwind of a major freeway, as well as measuring both temporal and spatial variations on the streets where the fixed installation was maintained.

Preliminary Data Presentation

Cross sections of the distribution of CO and winds in the two street canyons are presented for three one-hour periods in Figures 5 through 7; winds are given relative to "street north (Broadway)." Figure 5 corresponds to a situation where a helical circulation in Locust Street might be expected from our previous (San Jose) results. This appears to be the case upon consideration of the CO distribution, although the winds reveal the complexities of the canyon circulation. The strength of the along-street flow is noteworthy in view of the cross-street orientation of the tower winds. In contrast, the horizontal CO distribution in the Broadway canyon is relatively flat.

Figure 6 depicts the patterns in the two canyons several hours later when the tower wind has veered and now makes the same angle relative to both streets. It is interesting to note the differences in the two CO fields--especially near street level. The winds in the Locust Street canyon are very light, and the resulting distribution of CO appears to reflect the traffic pattern, that is, concentrations are highest near midstreet and taper off toward the buildings. Although the along-street winds predominate on Broadway, the moderate cross-street winds at the lower level appear to support the observed concentration pattern.

The patterns reflected in Figure 7 are for a period only three hours later than those given in Figure 6. The traffic density has not changed substantially. Significant meteorological changes have taken place, namely frontal passage has occurred in the interim, and CO concentrations are now relatively low and are comparable in both canyons.

These data have been presented to illustrate the nature of the measurements that were made, as well as some of the complexities of the problems we are investigating. The analyses are preliminary and, as yet, we have only tentative explanations for the observed patterns. The analysis phase of this continuing program has only recently begun, and no evaluation of the basic or street effects models has yet been made with the St. Louis data.

FIGURE 7 CARBON MONOXIDE CONCENTRATIONS (ppm) AND STREET—
ORIENTED WIND VECTORS (m/sec), DOWNTOWN ST. LOUIS

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A NUMERICAL MODEL FOR CARBON MONOXIDE CONCENTRATIONS NEAR A HIGHWAY

Maurice B. Danard, Rao S. Koneru and Peter R. Slawson

University of Waterloo
Waterloo, Ontario, Canada

1.

INTRODUCTION AND SUMMARY

A numerical approach to the problem of urban air pollution has many advantages. Probably the most important is that complex models may be studied. That is, it is unnecessary to make simplifying assumptions, whose physical justification may be questionable, merely to render the problem tractable. On the other hand, if an analytical solution is desired, such simplifications are often essential. Another advantage of numerical models is that if the parameters influencing pollution become better known in the future (e.g. by direct measurement of winds and diffusivities) these improved estimates may readily be incorporated into the existing model.

This report describes the design and testing of a numerical model to compute carbon monoxide concentrations in a vertical plane orthogonal to Highway 401 at Keele St. in Toronto, Canada. The approach is to integrate in time the non-steady two-dimensional diffusion equation (Section 2). Diffusivities vary both horizontally and vertically. Winds vary in the vertical. The highway is treated as an area source. On the upwind boundary, concentrations are assumed zero. Background concentrations could also be used as a boundary condition. Starting from zero initial concentrations, computations are continued until the efflux out the downwind boundary is equal to the influx from the highway. Wind speeds are assumed to vary logarithmically with height below 10 m and linearly above this level. The roughness length is taken as 1 m. Larger diffusivities are employed over the highway in the lowest 3 m ($20 \text{ m}^2 \text{ sec}^{-1}$) than to the sides. Above 10 m the diffusivity is greater for cold air advection ($9 \text{ m}^2 \text{ sec}^{-1}$) than for warm ($3 \text{ m}^2 \text{ sec}^{-1}$). Mixing heights and upper winds are computed using upper air data at Buffalo, New York. Winds at 10 m and surface temperatures are assumed the same as at Toronto International Airport (Malton).

The model is tested on 22 cases in 1969 for which concentrations have been measured at the Keele St. fixed site (Slawson ^{6}). Results are described in Section 3. Steady state is attained after at most 10 min. for these cases. The computed concentration has a simple correlation coefficient of 0.82 with the observed concentration. Background effects are also significant.

2.

THE MODEL

2.1 General Description

The diffusion equation may be written, assuming $w = \partial/\partial y = 0$,

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x} K_x \frac{\partial C}{\partial x} - Cu + \frac{\partial}{\partial z} K_z \frac{\partial C}{\partial z} \quad (1)$$

The xz plane is vertical and orthogonal to the highway. The origin is at ground level at the center of the highway. Given the eddy diffusivities K_x and K_z , the cross-wind component u , and initial and boundary conditions, Eq. (1) is to be solved for the concentration C .

The lower boundary conditions at $z = 0$ are

$$\begin{aligned} K_z \frac{\partial C}{\partial z} &= -S & \text{for } |x| \leq l \\ &= 0 & \text{for } l < |x| \leq d \end{aligned} \quad (2)$$

Here, $2l$ is the width of the highway and $2d$ is the width of the domain of the solution. The areal source term S is obtained from the traffic density (see Section 2.2). The upper boundary condition at $z = h$ is

$$K_z \frac{\partial C}{\partial z} = 0 \quad (3)$$

At the upwind boundary ($x = -d$ if $u > 0$, $x = d$ if $u < 0$), C is kept equal to the background concentration (i.e., the hypothetical concentration if the highway were absent). For simplicity the background concentration is assumed zero although actual values could be used if known. At the downwind boundary it is assumed that

$$\frac{\partial}{\partial x} \left(K_x \frac{\partial C}{\partial x} \right) = 0 \quad (4)$$

Substituting (4) into (1), one obtains the equation actually applied at the downwind boundary:

$$\frac{\partial C}{\partial t} = -\frac{\partial}{\partial x} Cu + \frac{\partial}{\partial z} \left(K_z \frac{\partial C}{\partial z} \right) \quad (5)$$

In this model, K_x and K_z may both be arbitrary functions of x and z , u may be an arbitrary function of z , and S may be an arbitrary function of x .

Initially, C is set equal to zero. Eq. (1) is integrated in time by the method of Dufort and Frankel^{1} (see also Richtmyer and Morton^{5}, p. 190) until a steady state is attained. The initial conditions merely effect the time (1-10 min.) required to reach a steady condition.

2.2 Computing the areal source term

As reported by Slawson^{6} (p. 87), the average car is assumed to emit 36.3 gm of carbon monoxide per mile when travelling at 60 mph. The width of Highway 401 at Keele Street is 320 ft. or approx. 100 m. If n cars hr^{-1} pass a given point, then the emission per unit area is

$$\begin{aligned} S &= \frac{n \text{ cars hr}^{-1} \times 36.3 \text{ gms car}^{-1}}{1 \text{ mile} \times 100 \text{ m} \times 1609 \text{ m mile}^{-1} \times 10^{-3} \text{ gms mg}^{-1} \times 3600 \text{ sec hr}^{-1}} \\ &= 6.27 \times 10^{-5} n \quad \text{mg n}^{-2} \text{ sec}^{-1} \end{aligned} \quad (6)$$

Here it has been assumed that the traffic is uniformly distributed over all lanes. At speeds less than 60 mph the emission per mile would increase (Ludwig et al^{4}, p. 27) and (6) may then be an underestimate. In particular, if the traffic has stopped and all cars are idling, Eq. (6) would give $S = 0$ since $n = 0$. However, apart from this case, it is thought that (6) is sufficiently accurate for the purposes of this investigation.

2.3 Variation of V , K_x and K_z

Hourly values of the surface wind (V_t) at Malton are assumed to apply at a height $Z_t = 10$ m. The x-component of the 10 m wind is not permitted to be less than 1 m sec^{-1} in magnitude. Winds at 950, 900, 850, 800, 750 and 700 mb are linearly interpolated in time from 1200 and 0000 GMT data at Buffalo. Below 10 m the wind speed is assumed to vary logarithmically with height from zero at $Z_o = 1 \text{ m}$ to V_t at 10 m:

$$V = V_t \frac{\ln(Z/Z_o)}{\ln(Z_t/Z_o)} \quad (7)$$

The wind direction is assumed constant between Z_o and Z_t . Above 10 m both u and v are assumed to vary linearly with height. Use is made here at the Buffalo upper air wind data mentioned above.

Consistent with (7), K_z is provisionally assumed to vary linearly with height between Z_o and Z_t (Haltiner and Martin⁽²⁾, pp. 230-232):

$$K_z = k_v^2 \frac{V_t^2}{\ln(Z_t/Z_o)} \quad (8)$$

Here k_v is the von Karman constant (0.4). For $V_t = 10 \text{ m sec}^{-1}$ and $z = 10 \text{ m}$, (8) gives $K_z = 3.5 \text{ m}^2 \text{ sec}^{-1}$.

So far, no variation of K_z with x has been introduced. However, mechanical mixing by traffic results in a much larger value of K_z over the highway than at the same height off the roadway. Therefore, K_z is set equal to $20 \text{ m}^2 \text{ sec}^{-1}$ in the lowest 3 m over the highway. This figure is obtained by noting that K_z may be expressed as the time average product of the perturbation velocity and a mixing length. The former is taken as 4 m sec^{-1} (15% of the average car speed) and the latter is assumed to be 5m (about one car length). To the side of the highway, K_z is assumed to decrease linearly with distance to the value given by Eq. (8) 50 m from the edge of the roadway.

Lettau and Hoeber⁽³⁾ computed K_z (actually, momentum diffusivity) for two days in August, 1960, over Helgoland in Germany. On the day with warm air advection, they obtained a value at 100 m of $3 \text{ m}^2 \text{ sec}^{-1}$. On the day with cold air advection, this had increased $9 \text{ m}^2 \text{ sec}^{-1}$. These two values are used here for heights above 10 m. In the neutral advection case, a value of $6 \text{ m}^2 \text{ sec}^{-1}$ is employed. The sign of the temperature advection can easily be deduced from a synoptic chart.

In the absence of other information K_x is assumed to have the same dependence of x and z as K_z .

3. COMPARISON WITH MEASUREMENTS AT THE FIXED KEELE ST. CO MONITORING SITE

The model is run for 22 cases and a comparison is made between the actual concentration at the fixed Keele St. measuring station (15 m north of the edge of the pavement and 2 m above the ground) and the value predicted by the model at that location. Results are shown in Table 1. With the exception of July 14, the times chosen are at the peak morning and afternoon traffic periods. For example, 1130 GMT refers

to conditions between 7 and 8 AM EDT, and 2030 GMT to 4 and 5 PM. The values of observed concentration C_o and traffic density n are taken from Slawson⁽⁶⁾.

Generally speaking, the afternoon periods have higher mixing heights and wind speeds, and lower values of C_o and C_p than do the morning intervals. An exception is Aug. 1 where the onset of rain shortly before noon prevented the normal diurnal rise in mixing height and wind speed. Both C_o and C_p are higher at 2030 GMT than at 1130 on the 1st. On the average, C_p is less than C_o . This is to be expected since C_p accounts for the highway only. Other sources (i.e., background concentration) are not included.

For the most part, high concentrations are associated with low values of h and V_t , low positive values of u_t , and warm air advection.

Table 1. Comparison between actual (C_o) and predicted (C_p) carbon monoxide concentrations at the fixed Keele St. site. Cold, neutral and warm air advection are denoted by C, N and W. respectively. Times are in GMT. The year is 1969

Time	C_o (ppm)	C_p (ppm)	n (cars hr ⁻¹)	h (m)	V_t (m sec ⁻¹)	u_t (m sec ⁻¹)	Advection
2030, Jul. 8	1	2.2	14080	1193	6.0	3.4	N
1230, Jul.14	2	0.1	13360	306	4.3	-4.2	W
1130, Aug. 1	10	6.2	13040	10	2.9	2.6	W
2030, Aug. 1	13	9.5	13790	10	2.7	1.7	W
1130, Aug. 7	14	7.1	12830	10	3.4	2.2	W
2030, Aug. 7	5	1.7	13730	353	7.8	6.3	W
1130, Aug.15	17	14.0	12750	10	1.3	1.1	W
2030, Aug.15	7	2.2	14710	1231	6.0	5.5	W
1130, Aug.18	13	14.1	13880	10	2.0	1.2	W
2030, Aug.18	6	2.9	13870	1481	5.1	2.2	W
1130, Aug.19	6	0.5	13210	10	2.9	-1.9	C
2030, Aug.19	1	0.0	13730	343	5.4	-5.2	C
1130, Aug.20	9	1.3	12580	10	1.6	-1.3	C
2030, Aug.20	2	0.1	13830	965	6.9	-6.1	C
1130, Aug.21	10	0.8	12880	10	2.0	-1.7	C
2030, Aug.21	1	0.0	13590	1360	6.5	-6.0	C
1130, Aug.25	8	1.3	13620	10	2.9	-1.0	N
2030, Aug.25	1	0.0	14110	1068	7.2	-5.9	N
1230, Aug.26	8	0.1	13260	10	4.7	-4.5	C
2030, Aug.26	2	0.0	14020	1367	6.7	-6.5	C
1130, Aug.28	13	11.9	12680	10	4.7	1.2	W
2030, Aug.28	8	2.6	14470	1587	6.3	4.4	W

Using the results of Table 1, correlation coefficients between C_o and C_p , n , h , u_t , n/h and $n/u_t h$ were computed. These are shown in Table 2. The concentration predicted by the model, C_p , has the highest correlation with C_o . The quotient n/h is also highly correlated with C_o . When equations for C_o are computed by stepwise regression, the first variable selected in C_p followed by n/h . The resulting equation is

$$C_o = 2.6 + 0.569 C_p + 0.0037 \frac{n}{h} \quad (9)$$

The standard error of (9) is 2.0 ppm and the multiple correlation coefficient is 0.92. Assuming the traffic density in adjacent streets is proportional to n , one would expect the concentration due to these other sources to be proportional to n/h . Thus we can interpret the high correlation of n/h with C_o as reflecting the importance of the background concentration C_b . From (9) we have

$$C_b = 2.6 + 0.0037 \frac{n}{h} \quad (10)$$

The constant 2.6 appears to be a bit high. The possibility exists that in addition to representing the background concentration, Eq. (10) corrects other systematic errors in the model. One might expect the coefficient of C_p in (9) to be unity. However, the fact that it is smaller than one may mean that the figure of 36.3 gms car⁻¹ mile⁻¹ at 60 mph (see Section 2.2) is an overestimate. Actually, the value given by Ludwig et al.⁽⁴⁾ (page 27) for 1969 automobiles is 34 gms. The negative correlation between C_o and n is explainable simply by the fact that the afternoon intervals have higher traffic densities and, for other reasons, lower concentration than the morning periods.

Table 2. Correlation coefficients between the observed concentration at the Keele St. fixed site and various parameters.

Variable	C_p	n	h	V_t	u_t	n/h	$n/(u_t h)$
Corr. coeff.	.82	-.51	-.61	.73	.49	.80	.45

4.

CONCLUDING REMARKS

Numerical simulations have also been carried out on the influence of various parameters on computed concentrations. However, space limitations prevent describing these results in detail. Wind speed and direction are very important since they determine the low-level diffusivities and the transport perpendicular to the highway. Mixing height and type of advection are less significant.

Thus, to take full advantage of the numerical method described here, an accurate knowledge of the winds and diffusivities near the highway is desirable. Unfortunately, this was not available. Malton Airport, from which wind data were taken, is about 8 miles west-southwest of Keele St. and Highway 401. Despite this short-

coming, reasonably good results are obtained (see Section 3).

Work is continuing on the model with particular attention being paid to the decrease of concentration with distance from the highway. The model is also being adapted to a depressed highway.

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WIND PROFILES AND CHANGE OF TERRAIN ROUGHNESS AT RISØ

by

H. A. Panofsky
Pennsylvania State University
University Park, Pennsylvania, U.S.A.

and

E. L. Petersen
Atomic Energy Commission
Risø, Denmark

INTRODUCTION

The 125 m meteorological tower at AERISØ is unique in that it is located on a narrow tongue of land, with water surface at distances from 100 to 300 m in various directions from the tower, and land trajectories in other directions.

The tower carries anemometers at 7 levels, thermometers at 8 levels and moisture equipment at two levels. Ten-minute average winds, centered on every hour, and recorded over a period of ten years are now available on magnetic tapes.

As a result, the Risø data are ideal for assessing the effect of roughness change on the wind profiles, for various stability conditions. Further additional information can be brought to bear on the so far controversial question of the shape of the wind profile over homogeneous terrain up to 100 m or so.

THE OBSERVATIONS

Wind speeds and directions are obtained from cup anemometers and vanes, built by G. Schultz according to specifications by Risø personnel. Temperatures are sensed by platinum resistance thermometers, and humidity by Lithium Chloride humidimeters.

Wind speeds are measured at seven heights: 7 m, 23 m, 39 m, 56 m, 96 m, and 123 m; temperatures at the same heights and at 2 m; wind directions at 7 m, 56 m, and 123 m; moisture at 2 m and 123 m.

The booms are directed toward the Southwest, so that winds between North and East were not used in this study. Only the wind on the top of the tower was not sensed on a boom but directly above the tower. The mean profiles indicated significant differences between the behavior of the anemometer at this level, and those at the lower levels, probably due to the difference in aerodynamic environment. Therefore, the wind at 123 m was omitted in the detailed analysis.

The tower is located on a small mound, 7 m above the water. The terrain in the neighborhood of the tower is indicated in Fig. 1. Air trajectories reaching the tower from various directions (excepting 005° - 085°) have been divided into 3 general classes.

Class A: Wind directions 05° - 125°, inc: trajectory almost entirely over land, but with somewhat variable roughness as function of distance and direction.

Class B: Wind directions 125° - 245°, inc: trajectory first over land, then over water, and then over land again, the stretch over water varying from 300 to 800 m in length.

Class C: Wind directions from 265° - 005°, inc: long water trajectory followed by about 300 m land trajectory.

Cases with wind directions between 245° and 265°, inc, were not included in any of the major groups because of possible influence of land areas 2500 m from the tower.

REDUCTION OF THE OBSERVATIONS

Originally, all the data were separated according to wind direction into 11 groups as listed in Table 1. Observations were used only if at least two of the three measured wind directions fell within the range of wind directions specified for the particular group. If any wind direction, speed or temperature was missing at any hour, no observations for this hour were used. Also, observations were omitted if the wind speed at 7 m was less than 2.5 m sec⁻¹.

For each wind direction group, the wind data were also classified according to the gradient Richardson number computed from differences between winds and temperatures at 7 m and 23 m. Due to the short averaging times for wind and temperature, the Richardson numbers used in the categorization were computed from centered three-period averages of temperature and wind speed differences.

Five Richardson number categories were defined for each wind direction group, separated by Richardson numbers -0.03, -0.005, +0.005, and +0.25.

The central category will be called the "neutral" category.

For each wind direction group, and each Richardson number category, mean wind speeds were computed at all seven levels, as well as standard deviations of ten-minute wind speed, minimum and maximum profiles and mean Richardson numbers.

PROPERTIES OF THE MEAN WIND PROFILES IN NEUTRAL AIR

For each wind direction group and Richardson number category, mean wind profiles were plotted on semilog paper. The number of individual profiles averaged in each mean profile varied from 37 to more than 2600.

In order to avoid confusion, the 55 individual mean profiles are not shown here, but mean profiles for the three principal trajectory types will be shown and discussed later. However, many of the properties of the individual profiles are listed in Table 1.

Before considering the data in Table 1, we must remember that theories of wind profiles apply to individual neutral profiles rather than to average profiles.

In particular, in neutral air, over homogeneous terrain, we have the logarithmic profile, known to be valid up to 30 m or so:

$$u = \frac{u_*}{k} \ln \frac{z}{z_0}$$

where u_* is the friction velocity, k von Karman's constant, z the height, z_0 the roughness length and u the wind speed.

As long as we treat narrow sectors of wind directions, z_0 can be taken as a constant for an ensemble of profiles and we can write

$$\bar{u} = \frac{\bar{u}_*}{k} \ln \frac{z}{z_0} \quad (1)$$

where an overbar denotes an ensemble mean.

But if z_0 varies significantly within a sector, we have

$$\bar{u} = \frac{\bar{u}_*}{k} \ln \frac{z}{\bar{z}_0} - \frac{\bar{u}_*'}{k} \ln \bar{z}_0'$$

where $\bar{z}_0$ is the geometric mean roughness length. A prime denotes deviations from means. If the last term is small as is to be expected, the roughness lengths determined from mean logarithmic profiles are then geometric means in a given sector.

Qualitatively, all profiles show the properties expected from theories of the effect of roughness change on wind profiles. As suggested first by Elliott (1958), when air under neutral conditions travels from smooth to rough terrain, the wind profile over the rough terrain essentially consists of two logarithmic parts with different slopes; the lower part of the wind profile is characteristic for wind flow over rough terrain, with large wind shears and roughness lengths, and the upper part reflects the flow over smooth terrain with smaller wind shears and roughness lengths. At the height of the intersection of the two logarithmic sections, h , a kink appears. Although this kink is probably not really a discontinuity of wind shear, it can be treated as such for many purposes.

The level of the kink, h , effectively separates air near the ground, which has felt the rough terrain, from air higher up, which has not yet come under the influence of the rough terrain. All wind profiles with wind directions from 125° through 195° to 005° show such kinks. The heights of the kinks as determined by eye from the mean profiles are listed under h in Table 1.

There exists no quantitative theory describing wind profiles in air which has undergone two large changes of roughness, as it has when the wind directions vary from 125° through 255°. In such cases, one might qualitatively expect that the wind profiles can be divided into three logarithmic sections; the lowest responds to the local roughness around the tower; the second is

characteristic for the air flow over the water, and the highest has wind shears typical for the flow over land at considerable distances. Thus, there are two kinks in the profiles. Such profiles were actually observed. The heights of the lower and upper kinks are listed as h and h' , respectively.

Table 1 shows that single kinks are indeed found for water $\rightarrow$ land trajectories, and double kinks for land $\rightarrow$ water $\rightarrow$ land trajectories. Typically, with fetches over land of 300 m, the lowest kink occurs at about 39 m; for significantly shorter fetch (group 5) h is smaller. In the cases with land $\rightarrow$ water $\rightarrow$ land trajectories, the upper kink occurs at 56-72 m when the distance of the far shore from the tower is of order of 600 m.

Table 1. Profile characteristics in neutral air.

Group	Direction deg	Number of profiles averaged	Distances in m to:		Roughness lengths, cm			Kink h	Heights, m h'
			water	far shore	z_0	z_0'	z_0''		
1	85-105	37	-	-	3.5	-	-	-	-
2	105-125	56	-	-	1.5	-	-	-	-
3	125-145	56	300	600	5	small	6	35	56
4	145-165	58	100 ^{*)}	700 ^{*)}	9	imag.	20	39	56
5	165-195	48	100	900	14	0.3	*)	23	65
6	195-215	81	200	600	2	0.01	*)	39	72
7	215-245	264	300	700	14	small	10	39	72
8	245-275	587	300	2500	30	small	-	39	72
9	275-305	354	300	-	22	small	-	40	-
10	305-335	213	280	-	28	small	-	39	-
11	335-005	64	280	-	6	small	-	33	-

*) Also islands in the water. *) Uncertain.

Table 1 also gives roughness lengths. These are determined by fitting straight lines to separate sections of the profiles by eye and noting the intercepts with the ordinate axis. For Class A (over land trajectories, group 1 and 2) only a single roughness was determined for each group which is given in the column headed z_0 . For class B (land $\rightarrow$ water $\rightarrow$ land trajectories, groups 3-8), three roughness lengths could be defined, which are listed under headings, z_0 , z_0' and z_0'' , respectively. The first of these refers to the terrain closest to the tower; the second to the water and the third to the terrain beyond the far shore. Usually the wind shears of the sections of profile responding to the water surface were so small that reasonable roughness estimates were not possible; in those cases, the values of z_0' were listed as "small". In one case, the wind shear was even negative, and the roughness lengths imaginary. For class C (water $\rightarrow$ land trajectory, groups 9-11), two roughness lengths are given, with again the first, z_0 , referring to the land next to the tower, and the second, z_0' , to the water.

The roughness lengths z_0 in Table 1, which correspond to the terrain near the tower, vary significantly with wind direction. Largest roughness lengths are observed with West and Northwest winds, in agreement with the location of large buildings in these directions (see Fig. 1). Roughness lengths are generally smaller when the winds come from the North, Southwest or Southeast, again in qualitative agreement with the terrain conditions in these directions.

As mentioned above, the roughness lengths z_0' , corresponding to the water area, are smaller than expected. They do, in fact, correspond to aerodynamically smooth flow, which may have occurred some of the time. However, average winds in the neutral category are so strong that it is difficult to justify the hypothesis of smooth flow. Perhaps the difficulty comes from having to determine the over-water roughness lengths from portions of the wind profiles from relatively high levels. In any case, similarly low roughness lengths were obtained on shore towers by Echols (1970).

Similarly, the roughness lengths given for the far shore are quite uncertain, particularly since they are usually determined from only two wind observations. The variations of z_0'' with wind direction is quite possibly not significant.

Typically, the height of the lower kink is 39 m. This value can be compared with theoretical estimates. We will use Elliott's theory, which is based on the assumption of logarithmic profile segments. There exist now theories and observations showing that this assumption is not quite correct (see e.g., Peterson, 1969); nevertheless the interface heights h computed from Elliott's

theory agree well with observations and with estimates from the more complex, and presumably more correct, theories.

Elliott gives an approximate formula for h which should be valid under the conditions of the observations used here:

$$h/z_0 = \{0.75 + 0.03 \ln (z_c'/z_0)\} (x/z_0)^{0.8} \quad (2)$$

Here, x is the distance from the tower to the point of terrain change. Fortunately, h is only slightly affected by the roughness of the water surface, z_0' , which is poorly known. We use here $z_c' = .01$ cm, a typical value derived from the over-water wind profiles (see e.g. Brooks and Kruegermeyer, 1970). With this, z_0' and with $z_0 = 20$ cm, h , the height of the interface, comes out as 36 m, in good agreement with the observed values. The agreement is, in fact, surprising in view of the height of the base of the tower and the displacement of the flow due to the finite height of the roughness elements, which have been disregarded in the theory.

It is much more difficult to explain the magnitude of h' in group B, the upper height limit of the part of the wind profiles with the small wind shear typical for over-water flow. From Elliott's theory (eq. 1) with the water roughness as .01 cm, and the roughness of the land on the other side of the fjord as 10 cm, the height of the kink at a distance of 600 m from the beginning of the water surface comes out as only 25 m, lower than the kink produced by the nearer water-land boundary. If Elliott's theory could be applied independently at the two lines of surface discontinuity, the internal boundary layers would intersect before they reach the tower. Clearly, the theory does not allow for such interactions, and it is recommended that a new theory be developed which can describe wind profiles following two distinct roughness changes.

The difference in the mean profiles within each major class were not extreme, so that only mean wind profiles for each major class are shown here. They illustrate well the differences between the classes, and are presented as Fig. 2.

As mentioned previously, roughness discontinuities are unimportant in class A, and the mean profile is essentially logarithmic, with a mean roughness length of 3 cm. On the other hand, the profile for class C shows one kink separating straight lines, and that for class B, two kinks.

It is possible to use the profiles in neutral stratification at Risø to contribute to the discussion about wind profiles up to 100 m over homogeneous terrain. Such wind profiles are solutions of the differential equation:

$$\frac{du}{dz} = \frac{u_*}{\lambda} \quad (3)$$

which is considered to be a definition of a "mixing length", λ . The other quantities have their usual meaning: u , the mean wind speed, z the height and the u_* the local friction velocity proportional to the square root of the local Reynolds stress. In order to determine the wind profile, we must integrate eq. 3. This means that we must specify the variation of u_* and λ with z . Of course, if u_* is constant and $\lambda = kz$ (where k is von Karman's constant) we get the familiar logarithmic formula.

The vertical variation of u_* can be derived quite rigorously, if the vertical variation of wind direction is small - which it tends to be in neutral air up to 100 m or so. Under those conditions (see, e.g., Fiedler and Panofsky, 1972):

$$u_* = u_{*0} - fz \quad (4)$$

where u_{*0} is the friction velocity at the surface, f is the Coriolis parameter.

If the mixing length remained equal to kz as z increases, and the friction velocity followed eq. 4, the wind profile, as plotted in the figures, would slowly bend to the left with increasing height. However, this effect is quite small below 100 m, particularly with the strong winds typical with small Richardson numbers at Risø.

The behavior of λ is much more uncertain. It is clear from wind models extending through the whole planetary boundary layer that the linear vertical increase cannot continue, but λ must eventually reach a constant value or even decrease upward. The effect of this change is to make the wind profile bend to the right with increasing height.

Several models for λ have been proposed. A model suggested by Blackadar (1962), which is quite satisfactorily successful in explaining planetary boundary layer profiles, can be satisfactorily approximated up to 100 m height by:

$$u = \frac{u_{*0}}{k} \ln \frac{z}{z_0} + 144 fz \quad (5)$$

This equation includes also the small vertical variation of friction velocity prescribed by eq. 4.

Very few towers with instruments are located in even nearly homogeneous terrain. That at Cape Kennedy, Florida, satisfies these conditions reasonably well. Average wind profiles from this tower (unpublished) show considerable irregularities, but do not suggest any systematic deviation from the logarithmic wind profile in neutral air up to 150 m.

We can now consider the mean profiles for classes A and C in this context. The mean profile for class A shows no systematic deviation from the logarithmic profile, even though the roughness increases somewhat with distance, a fact which should cause larger shears at greater heights. There is no evidence that $du/d(\ln z)$ increases significantly with height, as predicted by eq. (5). Similarly, class C shows no evidence that the logarithmic wind shear increases significantly with height near the top of the tower*. The last term in eq. (5) alone would produce a wind difference between 96 m and 72 m greater than $.4 \text{ m sec}^{-1}$, in contradiction to the observations.

Although the Risø site is not perfect for this purpose, the observations there again suggest that logarithmic wind profiles are valid to at least 100 m height in neutral air, implying that the vertical variation of mixing length is essentially linear up to that level.

THE PROFILES IN UNSTABLE AIR

Wind profiles in unstable air over homogeneous terrain in the surface layer are quite well understood, on the basis of Monin-Obukhov similarity theory. The normalized logarithmic wind shear, ϕ , is given as a universal function of z/L , where L is the Monin-Obukhov length, which depends on the vertical fluxes of heat and momentum and is usually assumed to be independent of height, at least up to 20 m or so.

There is now general agreement about the universal function in unstable air. An expression well fitting most carefully measured wind profiles is:

$$\phi = (1 - \gamma \frac{z}{L})^{-1/4} \quad (6)$$

Here γ is a constant, which has a value of about 16 (Paulson, 1970). Frequently, heat and momentum flux are not measured, so that L is not given. It is, therefore, convenient to use the working hypothesis proposed independently by Businger, Dyer and Pandolfo that, in unstable air, $z/L = Ri$. We will use this equation to determine L by use of temperature and wind differences between 7 m and 23 m, so that:

$$L = z/Ri \quad (7)$$

where z will be taken as the geometric mean height between 7 m and 23 m, or 13 m.

Equations (6) and (7) are valid for individual profiles. Only if the range of Richardson numbers is small in a given category, can these equations properly be applied with ϕ and Ri interpreted as averages. Also equation (6) has never been tested above the surface layer. We shall try such a test on the basis of wind direction class A, for which the neutral wind profile was essentially logarithmic. Since $du/d \ln z$ varies only very little with height, we can then assume that $\phi = 1$ for the profile, provided we use u_{*0} in the definition of ϕ .

We now consider the ratio of wind differences between two levels in unstable and stable air:

$$\frac{\Delta u_u}{\Delta u_n} = \frac{u_{*0u}}{u_{*0n}} \frac{\phi_u}{\phi_n} \quad (\phi_n = 1) \quad (8)$$

Subscripts n and u here stand for neutral and unstable, respectively. The ratio of the friction velocities in neutral and unstable air is nearly equal to the corresponding ratio of mean winds at 7 m.

Thus we can compute ϕ_u over various layers from

$$\phi_u = \frac{u_n}{u_u} \frac{\Delta u_u}{\Delta u_n} \quad (9)$$

Observed ϕ 's computed from eq. (9) are then compared with ϕ 's computed from eq. (6) under the assumption that L is independent of height. The results are shown in Fig. 3. Observed and computed nondimensional wind shears agree tolerably well, except for the top layer under most unstable conditions, when the observed wind shear is slightly negative. Thus, in general, eq. (6) will be assumed to be valid up to 100 m height, for mean profiles. In a sense, the lack of systematic disagreement in Fig. 3 justifies the use of eq. (6) for mean profiles.

*The slight apparent curvature can be completely removed by a small change in the azimuths defining class limits.

Fig. 4 shows mean wind profiles in the three direction sectors in slightly unstable air (Richardson number between 7 m and 23 m exceeds -0.03). The average Richardson number in each sector between the same height turns out to be -0.016 . This number differs so little from zero that straight-line sections still fit the profiles well. The heights of the kinks are not significantly different from those in neutral air, as would be expected under these conditions.

If eq. (6) is accepted, roughness lengths can be estimated from profiles in unstable air by writing the wind profiles in the form (see, e.g. Paulson, 1970):

$$u = \frac{u_*}{k} \ln \frac{z}{z_0} - \psi(Ri) \quad (10)$$

where ψ has the form

$$\psi = \ln \left(2 Ri \frac{1+x}{1-x} \right) - 2 \tan^{-1} x + \pi/2 \quad (11)$$

and x is given by:

$$x = (1 - 16 Ri)^{1/4} \quad (12)$$

Thus if the wind is plotted as functions of $\ln z - \psi$, straight-line profile segments should result whose intersections with the ordinate axis give the roughness lengths.

Again eq. (1) has been developed for individual profiles. If we take an ensemble average of expression (10) holding z/z_0 constant, we get:

$$\bar{u} = \frac{\bar{u}_*}{k} \left[\ln \frac{z}{z_0} - \bar{\psi}(Ri) \right] - \frac{\bar{u}_*^+ \bar{\psi}^+}{k} \quad (10a)$$

It is quite legitimate to replace $\bar{\psi}(Ri)$ by $\psi(\bar{Ri})$ because, within each Richardson number category, ψ is nearly linear with Ri . The last term, however, is not obviously negligible. In unstable air, ψ is positive, and likely to be larger the smaller the wind and u_* . In fact, some preliminary calculations suggest that the last term in eq. (10a) is positive and of order 10-15% of the terms containing $\bar{\psi}$. The last term was neglected in the applications of eq. (10a) here, because its effect would have been small, and the assumptions that L is invariant with height, even through a "kink", is much more serious.

Thus, $\bar{u}$ was plotted as function of $\ln z - \psi(\bar{Ri})$ for the profiles of Fig. 4, and the geometric mean roughness lengths came out as 2, 3, and 20 cm for sectors A, B and C, respectively, in fair agreement with the corresponding roughness lengths in neutral air, 3, 7 and 23 cm (Fig. 2).

Fig. 5 shows the profiles in the most unstable Richardson number category, in which the Richardson number average -0.195 , -0.215 , and -0.140 in sectors A, B and C, respectively. As is to be expected, the plotted points show some curvature, concave to the right. It is difficult to locate the exact heights of the kinks on such curved profiles. Therefore, on the same diagram straight-line segments are drawn which represent the relation of wind with $\ln z - \psi(Ri)$. As before, these lines can be used to infer roughness lengths near the tower in the three sectors. Their values come out as 2, 3, and 20 cm in the three sectors, exactly the same as in slightly unstable air. Altogether the roughness lengths are quite consistent, showing that the effect of instability on wind profiles is handled satisfactorily.

There appears no very significant difference in the heights of the kinks in neutral and unstable air, except that the slopes are so small in unstable air that the portions of the profiles in class B produced over water and those over the far shore cannot be distinguished.

There is a slight indication on Fig. 5, as well as on graphs representing wind profiles from narrower sectors that the interface separating air produced by the rough terrain near the tower from over-water air is slightly steeper in unstable than in neutral air. Whereas the intersection of this interface with the tower typically occurs near 39 m, in neutral air, it appears to occur nearer 45 m in unstable air. However, this estimate must be taken as extremely uncertain.

Qualitatively, the effect of a change of surface roughness should spread more rapidly in the vertical in unstable than in stable air. This was first shown by Miyake (unpublished Masters' thesis, University of Washington), who suggested that the slope of the interface should be proportional to the ratio of the standard deviation of vertical velocity and the wind speed, σ_w/u . In unstable air, this slope can then be written:

$$\ln dx = \frac{\phi_3(Ri)}{\ln z/z_0 - \psi(Ri)} \quad (13)$$

where ϕ_3 is the universal function representing the ratio σ_w/u_* . The average Richardson number between the ground and 40 m on the tower is about -0.3 for the profiles under discussion. For this Richardson number, ϕ_3 exceeds its neutral value by 15% or so (see e.g. Haugen et al, 1972). This estimate, though uncertain, would suggest an increased interface height by about 15% for the unstable over the neutral profiles. An additional increase should result from ψ in

Figure 1. Map of Risø tower and surrounding area, with major sectors of wind direction indicated

Figure 2. Mean wind profiles in neutral air for sectors A, B, and C

Figure 3. Comparison between observed and computed normalized logarithmic wind shear in unstable air

Figure 4. Mean wind profiles in slightly unstable air for sectors A, B, and C

Figure 5. Mean wind profiles in strongly unstable air for sectors A, B, and C

Figure 6. Mean wind profiles in moderately to slightly stable air (Ri below 0.25) for sectors A, B, and C

Figure 7. Mean wind profiles in quite stable air for sectors A, B, and C

the dominator of eq. (13). We must conclude then that an increase of interface height with decreased Richardson number is likely, but has not been well demonstrated by wind profiles at Risø.

WIND PROFILES IN STABLE AIR

Fig. 6 shows average wind profiles for the three classes in stable air, but with Richardson numbers below 0.25. Surface layer analysis has been reasonably successful in stable air if ϕ is assumed to be linear in z/L , and L is taken constant with height. Abortive attempts were made with this type analysis here. They failed, presumably because it is unrealistic to assume that L is independent of height from the surface to 100 m. Therefore, Fig. 6 will be discussed qualitatively only.

As is to be expected, the profiles are basically made of sections concave to the right on semilogarithmic paper. Classes B and C generally indicate kinks between profile sections influenced by land and water at about 33 m, a little lower than those in neutrally stratified air. The wind shear in class B is greater than that in class C near the top of the tower due to effect of the land beyond the water; and the wind shear at low levels is greater for class C than for class B, because of the greater roughness near the tower for class C. Finally, class A has large shears all through the profile because all of it is influenced by land.

For completeness, Fig. 7 shows mean profiles for Richardson numbers above 0.25, when, presumably, turbulence was weak or absent. Not much can be said about these profiles except that the curvature remains concave to the right, and there appears to be little, if any effect of surface roughness change on the profiles.

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BOUNDARY LAYER WIND PROFILE MODEL IN A STEADY STATE, DIABATIC, BAROCLINIC ATMOSPHERE

by

Alfred K. Blackadar
The Pennsylvania State University
University Park, Pennsylvania

and

Jon H. Cooper
American Smelting and Refining Company
El Paso, Texas

SYMBOLS

$\vec{A}, A_x, A_y$	Thermal wind and components
E_M	Rate of mechanical production of turbulent energy
E_{II}	Rate of convective production of turbulent energy
f	Coriolis parameter
g	gravity
H	vertical flux of heat
K_m	exchange coefficient for momentum
$\vec{k}$	unit vertical vector
k	von Karman's constant
L	Monin-Obukhov length
l	mixing length
S	wind shear
T	temperature
u_*	friction velocity
u_g, v_g	geostrophic wind components
$\vec{v}, u, v$	wind vector and components
z	height
z_0	roughness length
ρ	air density
$\vec{\tau}, \tau_x, \tau_y$	stress vector and components

INTRODUCTION

The flow in the planetary boundary layer is normally governed by a bewilderingly large number of parameters. In order to reduce the number of significant parameters we may observe that the rate of heating within the boundary layer is largely controlled by, and thus correlated with, the temperature advection. In many situations the surface exerts no strong control on the maintenance of the overlying temperature structure, so that we can, as a first approximation, postulate that the vertical distribution of heat flux is regulated by the horizontal advection of temperature so as to maintain a locally constant temperature. Although this postulate is admittedly an ideal, it simplifies the problem greatly by removing the vertical distribution of turbulent heat flux as an independently specified function, leaving it to be an implicit function of the other parameters. Moreover, it sets up a physically consistent locally steady-state system with diabatic heating. The purpose of this paper is to discuss a model of this kind of layer.

DESCRIPTION OF MODEL

The following parameters of the problem are defined

$$R_0 \equiv u_{a0}/fz_0 \quad \text{Drag Rossby Number}$$

$$\sigma_0 = u_{a0}/fL_0 \quad \text{Planetary Richardson Number}$$

$$\vec{A}/f \quad \text{Normalized thermal wind vector, independent of height}$$

In addition, we specify the following dimensionless variables

$$\begin{aligned} K &\equiv f K_m / u_{a0}^2 & U &\equiv (u - u_g)/u_{a0} & Z &\equiv f(z - z_0)/u_{a0} \\ R &\equiv \int_0^Z V dZ & l_a &\equiv fL/u_{a0} & V &\equiv (v - v_g)/u_{a0} \\ Q &\equiv \int_0^Z U dZ \end{aligned} \quad (1)$$

It is assumed that the flow is horizontally uniform and steady. The equations of motion may be written

$$V + \frac{\partial}{\partial z} \left\{ \frac{\tau_x}{\rho u_{a0}} \right\} = 0; \quad -U + \frac{\partial}{\partial z} \left\{ \frac{\tau_y}{\rho u_{a0}} \right\} = 0 \quad (2)$$

By choosing the x-axis to be parallel to the surface wind stress, we obtain, as boundary condition

$$\tau_{x0} = \rho u_{a0}^2 \quad \tau_{y0} = 0 \quad (3)$$

At infinite heights it is assumed that the stress disappears. Upon integrating (2) from 0 to Z, we obtain

$$\frac{\tau_x}{\rho u_{a0}^2} = 1 - R \quad \frac{\tau_y}{\rho u_{a0}^2} = Q \quad (4)$$

which, with the introduction of the exchange coefficients and boundary shear values, becomes

$$\frac{\partial U}{\partial Z} = \frac{1 - R}{K} - \frac{A_x}{f}; \quad \frac{\partial V}{\partial Z} = \frac{Q}{K} - \frac{A_y}{f} \quad (5)$$

The use of exchange coefficients is advantageous in facilitating the extrapolation of empirical relations from the surface layer into the deeper planetary layer. A reasonable way to do this, suggested by Ching (1) is to use Panofsky's relation

$$K_m = (E_m + \gamma E_H)^{1/3} \ell^{4/3} \quad (6)$$

where E_m and E_H are the rates of mechanical and convective production of eddy kinetic energy, respectively, and ℓ is best represented by

$$\ell = \frac{b u_{*0}}{f} \tanh(kzf/b u_{*0}) \quad (7)$$

After non-dimensionalization, insertion of appropriate functions for E_m and E_H , and substitution into (5), we obtain eventually the quartic equation for K :

$$K^4 + \frac{\gamma \sigma \ell_*^4}{K} K - [(1 - R)^2 + Q^2] \ell_*^4 = 0 \quad (8)$$

where ℓ_* stands for

$$\ell_* = b \tanh\left(\frac{b}{k} Z + \frac{1}{R_0}\right) \quad (9)$$

Next, the vertical distribution of heat flux H will be determined. For locally steady temperature, the following equation is required

$$\vec{v} \cdot \nabla_p T = - \frac{1}{\rho c_p} \frac{\partial H}{\partial z} \quad (10)$$

and with the introduction of the thermal wind equation, it becomes

$$\frac{\partial H}{\partial z} = - \frac{c_p \rho T f^2}{g} \left(\vec{v} \times \frac{\vec{A}}{f} \right) \cdot \vec{k} \quad (11)$$

Accordingly, one may obtain

$$\sigma = \sigma_0 + k_f \int_0^Z \left(\frac{\vec{v}}{u_{*0}} \times \frac{\vec{A}}{f} \right) \cdot \vec{k} dz \quad (12)$$

This equation was used to evaluate σ in (8) during the integration of the model. At this time only cases with $\vec{A}$ parallel to the geostrophic wind have been studied.

Numerical Solution

It has been found convenient to use two height coordinates in this one layer model. That is, close to the surface the height is scaled by z_0 such that $\xi \equiv \ln z/z_0$, while above that region the height coordinate Z is used. Using the logarithmic height coordinate equations (4) are altered by the multiplication of e^ξ/R_0 , since $\ln z/z_0 = \ln R_0 Z$. That is,

$\partial U/\partial \xi = e^\xi/R_0 \partial U/\partial Z$ and $\partial V/\partial \xi = e^\xi/R_0 \partial V/\partial Z$. The solution of the equations begins by selecting values for the external parameters σ_0 , R_0 , A/f , and by estimating surface values of U and V which is equivalent to estimating u_{g0}/u_{*0} and v_{g0}/u_{*0} . The surface values at

hand, successive values of U, V, Q, R, and K may be computed by using the following equations where subscripts refer to height levels.

$$U_{i+1} = U_{i-1} + (\partial U / \partial Z)_i (Z_{i+1} - Z_{i-1})$$

$$V_{i+1} = V_{i-1} + (\partial V / \partial Z)_i (Z_{i+1} - Z_{i-1})$$

$$Q_{i+1} = Q_i + 1/2 (U_{i+1} + U_i) (Z_{i+1} - Z_i)$$

$$R_{i+1} = R_i + 1/2 (V_{i+1} + V_i) (Z_{i+1} - Z_i)$$

For $\partial U / \partial Z$ and $\partial V / \partial Z$ equations (4) are used. In solving for K a Newton-Raphson convergence scheme using $[(1-R)^2 + Q^2]$ as the original estimate has been used with great success. It is fortunate here that Descartes's rule of signs guarantees for K at most one real positive root. Which, it is assumed, is the correct answer. In the above equations

$$\bar{U}_{i-1} = (U_{i-2} + 2U_{i-1} + U_i)/4$$

$$\bar{V}_{i-1} = (V_{i-2} + 2V_{i-1} + V_i)/4$$

to assure stability in the computation. Naturally, a similar set of equations would be used for the logarithmic coordinate.

If one has selected the correct surface values of U and V and Ekman-like spiral results. If not, the surface values must be refined by computing new curves using separate test alterations to the surface values and computing the correct change to make in the surface values from the following equations

$$dU_i = (\partial U_i / \partial U_0)_{V_0} dU_0 + (\partial U_i / \partial V_0)_{U_0} dV_0$$

$$dV_i = (\partial V_i / \partial U_0)_{V_0} dU_0 + (\partial V_i / \partial V_0)_{U_0} dV_0$$

This scheme is iterated until the desired degree of accuracy is reached in placing a reference height or level on the origin. Successive heights are treated the same way until the computed values of U_0 and V_0 to place a reference level on the origin are essentially the same as the previous values of U_0 and V_0 required to place the former level at the origin. At that point an Ekman-like spiral results.

RESULTS

Since wind profile data above the tower layer is scarce, the predicted values of the geostrophic drag coefficient, u_{*0}/V_g , have been compared with Clarke's (2) observed values via a generalized form of the Kazanski-Monin equation derived by Blackadar (3) and Zilitinkevich (4). That is

$$\ln R_0 = A(\sigma) + \left[\left(\frac{k V_g^2}{u_{*0}} \right)^{1/2} - B^2(\sigma) \right]$$

The results appear in figures one and two showing plots of A and B as functions of stability. The results are encouraging, with the plot of A suggesting additional experimentation with values of the asymptotic mixing length, b, required. Although not reproduced here, indications from this model suggest that the A and B functions are not independent of baroclinicity. Rather, B decreases linearly from approximately 4.5 in the neutral barotropic case to slightly negative at A/f equal to 200, and A increases with slight curvature from about 2 under neutral barotropic conditions to about 6.5 for A/f equal to 200. In all cases in this report the thermal wind is in the same direction as the surface geostrophic wind.

The hodographs in figures three and four show a decreasing angle between the surface geostrophic wind and true wind, and decreasing geostrophic drag with increasing instability or thermal wind. In the baroclinic case, the angle becomes negative at A/f equal to 200, indicating the true wind blowing from low to high pressure. The balance of forces has been checked and is still maintained under these circumstances. This phenomenon occurs with lesser values of baroclinicity as the instability increases. Ellison (5) obtained this same characteristic in a wind profile model in which $K_m = k u_* z$ throughout the boundary layer. The two wind profiles shown, figures five and six, show in the neutral barotropic case the wind damping to the geostrophic value. In the baroclinic example the wind is seen as becoming geostrophic and adjusting thereafter as rapidly as the geostrophic wind increases.

Finally, although not shown in this report, the mixing coefficient behaves according to Prandtl's relationship in the constant stress layer and then decreases from a maximum above that layer in a neutral barotropic atmosphere. With increasing instability the mixing coefficient approaches a maximum asymptotically, a maximum which increases as the instability increases. With increasing baroclinicity this asymptotic behavior occurs less rapidly, such that with low values of the thermal wind there is still some reduction of the mixing coefficient above the constant stress layer. This decrease then becomes less pronounced as the thermal wind increases.

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Figure 1

Figure 2

Figure 3

Neutral Baroclinic $R_0 = 5 \times 10^5$
 Thermal Wind in Same Direction as Surface
 Geostrophic

Figure 4

Figure 5

Figure 6

TURBULENCE STRUCTURE NEAR THE GROUND

A. C. Neuendorffer* and T. H. Pries
Atmospheric Sciences Laboratory
White Sands Missile Range, New Mexico

INTRODUCTION

Similarity theory has been enormously useful in our understanding of fluid flow primarily because it can classify similar types of flow quantitatively and unambiguously. Similarity theory also acts as a logical structure within which one may develop simple physically based assumptions or approximations about turbulent flow into a precise quantitative hypothesis. Unfortunately, the results of this type of approximate similarity like the elegant and classical logarithmic velocity profile are, at best, qualified successes. Their limitations lead back to the simplicity or ambiguity of the initial physical assumptions and not to a deficiency in similarity itself. In other words, although defining non-dimensional numbers is always unambiguous, equating non-dimensional numbers is not.

Perhaps what is needed are more elaborate physical assumptions based upon the qualitatively observed aspects of turbulent structure. No doubt the basis of such assumptions should be simple eddy motion. However, one must avoid treating turbulence as a specific random kinetic assortment of eddies because these eddies must continually interact and reinteract with the ever present wind shear.

More important still, one should avoid treating all measured atmospheric fluctuations as basically turbulent or chaotic in nature. This is especially true for space-stationary temporal fluctuations which run into a few minutes duration or longer so that we are dealing with massive movements of air.

With this in mind, six independent, long duration wind-temperature records from a 32-meter mast at White Sands Missile Range, New Mexico⁽¹⁾ were examined using simple spectral techniques in time alone. The results are surprisingly consistent within the framework of what follows.

MICROMETEOROLOGICAL OBSERVATIONS AT WHITE SANDS MISSILE RANGE

Turbulent Fluctuations

Horizontal, and basically lateral, mechanical rolls are produced by the wind shear under Helmholtz, i.e. shear, instability⁽²⁾. Although these include large and long-lived but extremely weak eddies, they show up best as the small strong short-lived fluctuations of the night w' spectra (Figure 1). But the large weak long-lived eddies suppressed in the w' spectra leave behind marked indentations on the wind and temperature stratification. At night these large kinks, observable in the u' and T' spectra of Figure 1, are of such dimensions that their peak temporal frequency roughly equals $7/Ri^*$ cycles per hour independent of the height above ground. As the critical Ri of 0.25 is approached from zero, and stability attained, the peak is lost in the long term variations and the spectra becomes featureless. Probably very similar kinks exist during the day but these do not dominate the spectra.

The kinks left in u and T stratification must undergo stress from wind shear until such that the lowest protruding parts of the kinks stack up against the higher down-wind parts of the corresponding kinks underneath. The result is a strong directional asymmetry of stratification so often observed⁽³⁾ as a skewed sawtooth characteristic of u' and T' temporal measurements. This is also consistent with everyday experience. Windy days are well described as gusty because one is repeatedly subject to sharp bursts of wind usually preceded by rather extended periods of calm. Such conditions are the direct consequence of sheared stratification kinks. This heaving, or piling up, of a vertical set of kinks along their windward face might best be described as a velocity-temperature discontinuity or pseudo-shock front. This pseudo-shock front is seldom exactly perpendicular to the wind direction. As such, strong directional wind shifts can be produced within the shock front which in turn may be used by us to define the front. Therefore the v' fluctuations appear to be at three times the kink frequency which indicates a shock thickness roughly one third the width of the kinks. It is interesting that in spite of the strong resemblance at night between the v' and the w' spectra they are the furthest removed from each other.

The existence of shocked kinks both day and night is borne out in a plot of the spectral phase delays between the five separate levels of the 32-meter mast (Figure 2). Being angled over by wind

*Present affiliation National Environmental Satellite Service, NOAA, Suitland, Maryland.

** Ri is the gradient Richardson Number, a stability measure.

shear, the pseudo-shock front hits the mast at the top first, then hits the lower points successively. This is especially prominent in the large, well defined phase lags of the v' fluctuations.

This is to be expected if the v' fluctuations are formed in the thin but penetrating shell of the shock front. The disturbances travel down the pole as if they were transferred by a simple diffusion wave of extremely large diffusion coefficient. All this means is that the phase shifts are proportional to the square root of the frequency. This fact plus the measured ratio of approximately $\sqrt{3}$ between v' and u' phase shifts indicates a constant ratio of 1 to 3 in $v'-u'$ structure size even during the day.

From such plots as Figure 2 it is easy to determine that the pseudo-shock front is of the form:

$$\text{Shock displacement downwind} = C \sqrt{\lambda_v} (\ln^2 Z + 1/4 \ln Z)$$

where λ_v (in meters) is the width of the shock defined by the v fluctuation wavelength. The constant C is $\sim 1/6$ during the day and $\sim 1/4$ at night. It should be noted from the λ_v dependency that thin shocks are relatively steep while thick shocks are more inclined.

The character of these pseudo-shock fronts is made apparent during dust storms. The blowing sand and dust appear as smooth streaks visible along road surfaces. These smooth streaks are often broken by thin sharp twisted discontinuities immediately preceding a strong gust of wind. The gust usually carries considerable amounts of sand aloft. This flying sand has been picked up by the strong Bernoulli pressure forces which exist within the shock front. These Bernoulli forces are also the forces which preserve the shocks' integrity in the presence of wind shear. It is this suction effect on sand and not eddy transport that is responsible for dust storms. Figure 2 was taken during a March dust storm, however, it is unusual from other phase plots only in the steepness and penetration of its shock front.

Non-turbulent Fluctuations

Not all measured fluctuations can properly be described as turbulent. For example there are two simple types of gravity waves of importance to the lower troposphere: 1) Internal gravity waves resulting from a vertical stratification of entropy and 2) The special class of deeply penetrating normal gravity waves resulting from a vertical stratification of density under certain boundary conditions. To visualize this special type of normal gravity wave consider two tall adjacent columns of air connected only at their bottoms. If the left column falls, it flows into the right column and gains gravitational potential energy and with it a restoring force. That the connection be only at the bottom makes this the only gravity wave with strong lateral movements in the lower troposphere. This wave is complemented by internal gravity waves in which the two columns of air must be completely unconnected and setting on a free lower boundary. For each independent column there is a balance between gravitational and thermal energy such that the falling column heats and expands while the rising column cools and contracts. This process and not a transference of mass provides the gravitational potential energy storage necessary for internal gravity waves. This "Brunt" oscillation is the only gravity wave with strong vertical movements in the lower troposphere. Combined, the lateral movements of these normal gravity waves and the vertical movements of the internal gravity waves might easily have a dominating effect on the important longitudinal convective rolls of the lower troposphere⁽⁴⁾.

Do these waves exist? Fortunately both types of gravity waves would be unaffected by wind shear since their wave fronts lie along the direction of the wind, paralleling the longitudinal rolls. Simple linear waves perpendicular to the wind would be torn apart by wind shear. Yet we already know that large perpendicular gravity waves do exist in the lee of many mountain ranges in spite of wind shear. Might not the simpler parallel waves be a more common if less visible occurrence?

All that would be required for such oscillations is a constant frequency throughout the troposphere in addition to the already satisfied free lower boundary conditions. The frequency of the

normal gravity wave $\omega_G = \sqrt{\frac{g}{\rho}} \frac{\partial \rho}{\partial z}$ is clearly of constant periodicity three minutes. The frequency of internal gravity waves⁽⁵⁾:

$$\omega_B = \sqrt{\frac{g}{\rho}} \frac{\partial \rho}{\partial s} \frac{ds}{dz} = \sqrt{\frac{g}{T}} \left(1 + \frac{1}{\Gamma} \frac{dT}{dz} \right)$$

where $\Gamma = g/c_p$ is constant only when the vertical temperature profile is of the exponential form:

$$T(z) = \frac{g\Gamma}{\omega_B^2} [1 - A \exp(\frac{\omega_B^2}{g} z)]$$

Fortunately this form is nearly a perfect fit for atmospheres approaching the U. S. Standard Atmosphere not only in the troposphere ($\omega_B = 2\pi/9$ minutes, $A = 0.614$) but the stratosphere ($\omega_B = 2\pi/4 \frac{1}{2}$ minutes, $A = -.044$) and even the lower mesosphere ($\omega_B = 2\pi/6$ minutes, $A = .037$).

The simple ratios of these periods at 3 minutes and at 9, 4 1/2, and 6 minutes must be more than mere coincidence. Gossard⁽⁶⁾ has observed precisely this set of frequencies in the oscillations of the ionosphere. Vali⁽⁷⁾ observes sharp 9 and 4 1/2 minute periodicities in atmospheric density with the mile long Stevens Pass interferometer. Even in the short lifetimes of cumulus clouds Anderson⁽⁸⁾ notes discrete periodicities right around 9 minutes.

Most important to boundary flow, are the two gravity oscillations of the troposphere at $\omega_B = 2\pi/9$ minutes and $\omega_B = 2\pi/3$ minutes = $3\omega_B$. Are the convective longitudinal rolls coupled to these oscillations?

Figure 3 is one of many unstable spectra published by Cramer and Record⁽⁹⁾ with prominent peaks near 3.7 and 9.25 millihertz precisely two and five times the tropospheric Brunt frequencies, respectively. Of course, one and three ω_B and not two and five ω_B was expected but remember we treated the two types of gravity waves as independent and uncoupled. Simple combinations of one and three will give two and five.

At the lower measuring heights of the 32-meter mast we also consistently obtained the same peaks of 2 and 5 ω_B in daytime heat flux, as during April 7 (Figure 4, bottom).

In contrast the much smaller low frequency peak in nighttime heat flux at $3/2 \omega_B$ (Figure 4, top) is at an antiresonance frequency of gravitational waves. The effect of this anomaly is most noticeable as a strong dip in nighttime UT coherences (Figure 5, top). This dip is not apparent in the daytime UT coherences (Figure 5, bottom). It is doubtful that this antiresonance is a simple gravitational oscillation and yet it must have an origin.

Surprisingly simple acoustical waves with pressure variations of a few millibars do require mass motions on the order of a meter per second ($v \sim C \cdot \rho' / \rho$). It is therefore convenient that the entire White Sands' Tularosa Basin constitutes a closed acoustical, or more properly Lamb wave, resonance cavity 37 miles wide giving it a fundamental harmonic of 10 C.P.H. or $3/2 \omega_B$. Although this solves our nighttime anomaly the argument does not stop there.

The Tularosa Basin is just one of a multitude of mountain locked basins located near the origins of strong mountain lee waves. Each of these basins is of such dimensions as to act as an acoustical resonator of fundamental frequency 10 C.P.H., $3/2$ the Brunt frequency of the troposphere. A partial list of such basins appears in Table 1.

The $3/2 \omega_B$ acoustic oscillation probably has nothing to do with the singular normal gravity wave frequency $3 \omega_B$. Instead it acts to couple the shearing frequency of the stable troposphere,

$$\omega_S = \frac{du}{dz} \sim 2\pi/18 \text{ minutes} = 1/2 \omega_B$$

with the internal gravity wave of frequency ω_B . This allows the internal gravity wave to range in oscillation from $1/2 \omega_B$ to $3/2 \omega_B$ in order to keep up with a three fold velocity differential in tropospheric winds and thus form lee waves of constant wavelength.

This added complication of fractional as well as whole Brunt harmonics in the wind spectra of certain mountain locked valleys must be accepted as still further evidence for the existence of discrete frequencies.

TABLE 1

Probable Lee Wave Sources

[10 C.P.H. acoustical (Lamb wave) wavelength $\lambda_A = 74$ miles (st.)]

<u>Closed Basins of $1/2 \lambda_A = 37$ miles</u>	<u>Location</u>
Jordan River Valley	Middle East
Brahmaputra River Valley	Tibet
Durance River Valley	France
Trondheim Fjord	Norway
The Basin of Southern Chile	Chile
Tularosa Valley	New Mexico
Mackenzie (Lake Tekapo) Basin	New Zealand
Moray Firth	Scotland
<u>Open Basins of $1/4 \lambda_A = 18 \frac{1}{2}$ miles</u>	<u>Location</u>
Tiber River and other Italian Alps Valleys	Italy
Rhone Valley	Switzerland
Owens Valley	California
Shasta - Scott Basins	California
Lake Granby Basin	Colorado
Pechora River Valley and other Ural Mt Valleys	Austria
Buenventura River Valley	Mexico

Any daytime spectra taken at White Sands such as Figure 6 is thus an amalgam of the gravity resonances and acoustical resonances, plus beat frequencies, with the beat frequency itself usually acting as a low frequency cutoff at 6 2/3 C.P.H.

This concentration of spectral energy at discrete frequencies is especially clear in the w' spectra under very unstable conditions (Figure 7). The high frequencies still retain the general trend of similarity theory with a steady decrease in frequency as z increases. This trend comes to a stop as we approach z_{wg} , z_{wg} and beat frequencies. Most striking of all is the remarkably sharp acoustical cavity resonance of 3/2 w_g allowed under such nearly windless daytime conditions.

CONCLUSION

The emphasis of this paper is that one cannot ignore and need not ignore a number of somewhat involved interactions in explaining atmospheric turbulent fluctuations. The most important of these are: 1) The interactions between mechanically produced eddies and wind shear, and 2) The interactions between longitudinal convective rolls and gravitational oscillations.

Sometimes even the compressibility of air is a factor to be considered in understanding the complex interentangled motions from which boundary flow cannot be extricated.

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Figure 1: Typical nighttime spectra, October 3, 1970.
($\Delta t = 0.01$, Ω is in cycles per hour).

Рис. 1: Зависимость параметров от Q . 1, 2, 3, 4, 5 — значения U для $Q = 0, 0.001, 0.002, 0.003, 0.004$ соответственно. Сплошная линия — расчет, пунктирная — эксперимент.

Рис. 2: Зависимость параметра $\frac{nS(m)}{\sigma^2}$ от Q . Сплошная линия — расчет, пунктирная — эксперимент. Значения U для $Q = 0, 0.001, 0.002, 0.003, 0.004$ соответственно. Сплошная линия — расчет, пунктирная — эксперимент.

Figure 4: Spectra of WTT for day and night cases as a function of frequency (cycles per hour) at five levels (top October 5-6, 1973, bottom April 7, 1973).

Figure 5: Power spectra of u , v , w and T as a function of frequency (cycles per hour) for daytime case. (March 11, 1973. u = cross frequency, AB = associated frequency).

Figure 7: Power spectra of the vertical component of the vertical wind fluctuations as a function of frequency (cycles per hour) for a very unstable case (November 6, 1970. B = Brunt frequency, AR = Acoustical resonance).

RATE OF GROWTH OF THE NOCTURNAL BOUNDARY LAYER

James U. Deardorff
National Center for Atmospheric Research⁺
Boulder, Colorado 80502

1. INTRODUCTION

The planetary boundary layer is almost always stably stratified over land at night, when it is also typically much shallower than in the previous afternoon. For this reason its height is then of special importance to those concerned with the vertical dispersion of pollutants. If contaminants were emitted from a city at night at the same rate as in the day, the pollution levels could become unbearably large in the absence of a heat-island effect. The extent of intensification would equal or exceed the ratio of the height of the boundary layer in the daytime to that at night.

It is surprising that even without considering the complications of a city's roughness elements and heat sources, the expected height, h , of the nocturnal boundary layer is a subject of speculation and controversy. On the one hand, supporters of the similarity theory^(1,2) assume that

$$h = (cu_s/f)F\left(\frac{u_s}{\ell}\right) \quad (1)$$

where c is a constant of about 0.3, u_s is the surface friction velocity, f is the Coriolis parameter, and ℓ is the Monin-Obukhov length. The function $F(u_s/\ell)$ is unity with neutral stratification and less than unity with stable stratification. Note that (1) predicts h to be independent of time in circumstances when u_s/f and ℓ are nearly constant.

On the other hand, inspection of temperature and wind soundings (cf Blackadar⁽³⁾) so observe that h typically increases with time during the night, starting from a small value shortly after sunset. This result can also be noticed from measurements of the Great Plains Turbulence Program⁽⁴⁾ although the vertical resolution of 100m in the reported data leaves much to be desired. Better resolution was obtained from the Cedar Hill (Dallas) tower in a study by Tamm and Banta⁽⁵⁾ who also found h to increase with time. However, as the particular night of their study the horizontal pressure gradient intensified significantly, and there may be some doubt whether this caused h to increase with time, through increased values of u_s . Recently available Australian data of Clarke et al⁽⁶⁾ also indicate that h decreases with time at night, judging from temperature measurements at 50m height intervals. However, the data is not dissimilar from earlier observations from which Clarke⁽⁶⁾ concluded that (1) is appropriate.

A recently developed experimental tool, the acoustic echo sounder,⁽⁷⁾ promises to be a most reliable method for determining local values of h as a function of time. Although no comprehensive field study has yet been undertaken using this device, preliminary results⁸ indicate that the height

is usually sharply defined and often surmounted by internal gravity waves.

One other new tool offering promise is the direct numerical calculation of energy containing eddies in three space dimensions. However, when this method⁽⁹⁾ was applied with stable stratification in a model containing 32,000 grid points, most of the vertical transport of heat and momentum was accomplished by the subgrid-scale eddies rather than by the directly calculated eddies. Effects of the subgrid-scale eddies were parameterized by means of (space-time dependent) eddy coefficients, and it seems unlikely that this parameterization was good enough to produce trustworthy mean wind and temperature profiles under these conditions. Furthermore, a relatively small time step had to be taken because of the large vertical wind shear which develops with stable stratification, and it was not feasible to study the possible time dependence of h occurring over an interval of 8-10 hours. There were four qualitative results of interest, however:

(a) The boundary layers for heat and momentum are of comparable height and can be considered a single entity of height, h ;

(b) The height, h , was well marked by small values of temperature variance and turbulence intensity above as compared with below;

(c) It was also marked by rapidly increasing values of the local Richardson number (Ri) with height; and

(d) Some turbulence as well as internal waves exists above h due to large scale pressure fluctuations centered near the surface and extending to heights well above $z = h$.

Numerical results (b) and (d) above are compatible with preliminary results from the acoustic sounder. Results (c) is comparable with data⁽¹⁰⁾ which suggests to us that the top of the stable boundary layer can be defined as the lowest level where Ri , as measured from large scale horizontal averages, exceeds some "quenching" value lying between 1 and 2. This result should in no way contradict the body of observations and theory which indicate that turbulence cannot develop within a laminar shear layer unless $Ri < 1/2$.

The purpose of the present paper is to make use of the above findings in a relatively simple one-dimensional time-dependent model of the stable planetary boundary layer in order to investigate the behavior of h with time and compare against the prediction of similarity theory (1). The situation to be modeled is the case study of Tamm and Banta⁽⁵⁾ and one of the questions to be answered numerically is whether h would have increased with time in that case had the horizontal pressure gradient remained constant. Another topic to be treated concerns the possible parameterization of h for use in global circulation studies. If u_s/f in (1) is found to be an irrelevant height scale.

⁺O. Soren and P. Hoff, private communication.

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2. THE NUMERICAL MODEL

A model could be chosen for which the horizontally averaged Reynolds stresses and fluxes are themselves predicted from height and time-dependent equations, and closure assumptions are made for the triple velocity correlations and dissipation rate.⁽⁹⁾ The more conventional closure at the Reynolds stress level, using eddy coefficients, is preferred here so that realistic wind and temperature profiles can be more easily obtained and emphasis placed upon the rate of change of h with time. Although it is fairly well known that the use of (positive) eddy coefficients is inadequate within the convective planetary boundary layer,^(10,11,12) their use appears less unsatisfactory with stable stratification and downward directed heat flux.

a. Eddy coefficients

These were prescribed empirically, and the eddy coefficient for heat, K_H , was allowed to differ from that for momentum, K_M . Near the surface, expressions for K_H and K_M were taken to be compatible with assumed log-linear velocity and temperature profiles.⁽¹³⁾ Thus for small z ,

$$K_M = k u_* z / (1 + \beta z / L) \quad (2a)$$

and

$$K_H = 1.35 k u_* z / [1 + \beta z / (1.35 L)] \quad (2b)$$

where k is Karman's constant (taken as .35), and β is the "linear" constant for which the value 4.7 is recommended.⁽¹⁴⁾ The general expressions used are:

$$K_M = k u_* z / [1 + 2.7 z / L + 6 z / h + 5 (z / L)^2] \quad (3a)$$

$$K_H = 1.35 k u_* z / [1 + 5 z / L + 3 z / h + a (z / L)^2] \quad (3b)$$

The constant, a , in (3b) was chosen such that

$$(K_H / K_M)_{z=h} = 2; \quad (4)$$

a ranged in magnitude from .026 to 0.4. In (2,3) it may be noted that for $L \rightarrow \infty$ (neutral case), K_H and K_M approach constant values at large z , a formulation first used by Blackadar.⁽¹⁵⁾ Then (3a) gives the maximum value for K_M to be

$$(K_M)_{\max} = .024 u_*^2 / \epsilon \quad (5)$$

which agrees with the detailed numerical study⁽¹⁶⁾ for neutral stratification when $h = .35 u_*^2 / \epsilon$. For finite L , and for $h/L = 1$, the two linear terms in the denominator of (3a) and (3b) combine to give the approximate value for β in (2) of 4.7. The last terms in the denominators of (3a,b) are empirically derived to give best agreement with the study of Ismail and Davenport⁽¹⁷⁾ to be presented. The vast difference in magnitudes of these terms indicates that K_H exceeded K_M at low and moderate heights, with the reverse predicted by (4) for $z=h$. Above $z = h$, K_H and K_M were arbitrarily prescribed to decay as $\exp[-(z-h)/h]$, which is in qualitative agreement with the three-dimensional numerical results mentioned earlier; i.e.,

K_H and K_M decrease very slowly above $z = h$ even in the absence of any upper level sources of background turbulence.

The level h was taken as the lowest height at which $Ri = 1.7$, where

$$Ri = (g/\theta) \partial\theta/\partial z / [\partial u/\partial z]^2 + (\partial v/\partial z)^2 \quad (6)$$

is the local Richardson number, θ is the potential temperature, and u, v are the downstream and cross-stream velocity components based on surface wind direction.

An example of K_H and K_M from (3a,b) for the Dallas tower data for 0140 local time, Feb. 23, 1961 is shown in Fig. 1. At this time h was calculated to be 343m and L to be 142m. Effects of the dissimilarity in K_H and K_M upon the shapes of the wind and temperature profiles will be discussed in Section 3.

b. Monin-Obukhov length and surface fluxes

The log-linear surface-layer expressions for wind and temperature contain L as follows:

$$u_1 = (u_* / k) [\ln(z_1 / z_0) + \beta z_1 / L] \quad (7a)$$

$$\theta_1 - \theta_s = [-w_*^* \theta_s^* / (1.35 k u_*)] [\ln(z_1 / z_0) + 1.35 \beta z_1 / L] \quad (7b)$$

where θ_s is the average potential temperature at the surface, $w_*^* \theta_s^*$ is the kinematic heat flux extrapolate to surface level, and subscript 1 refers to a height within the surface layer. z_0 is the roughness length, the effective value of which has been taken the same for temperature as for momentum. Uncertainty in this assumption is compensated by uncertainty in the proper definition and measurement of θ_s for a rough surface. A value for z_0 of 10cm was adopted in this study, although the value 3cm was also tested.

If (7b) is divided by the square of (7a) and multiplied by $\beta z_1 / L$, one obtains

$$\begin{aligned} z_1 / L &= k z_1 (\beta / L) (-w_*^* \theta_s^*) / u_*^2 \\ &= [1.35 \beta z_1 (\beta / L) (\theta_1 - \theta_s) / u_*^2] \cdot [\ln(z_1 / z_0) + \beta z_1 / L]^2 / [\ln(z_1 / z_0) + 1.35 \beta z_1 / L] \end{aligned} \quad (8)$$

In (8), β was set to 4.7. All quantities except z_1 / L on the right side of (8) are known at the beginning of each time step. Accurate values of z_1 / L were obtained by iterating (8) four times on each time step, starting with the initial guess $z_1 / L = 0$ on the first time step of the numerical integration.

Subsequently, (7a,b) were separately solved for u_z and then for $w'\theta'_s$. Values obtained were used in the equations of motion and potential temperature at the lowest level; u_z was further needed in formulations (3a,b) for K_m and K_H .

c. Governing equations and boundary conditions

Equations of motion and heat were utilized with assumptions of horizontal homogeneity and no mean vertical motion, w:

$$\frac{\partial U}{\partial t} = fV - \frac{1}{\rho_0} \frac{\partial P}{\partial X} + \frac{\partial}{\partial z} (K_m \frac{\partial U}{\partial z}) \quad (9a)$$

$$\frac{\partial V}{\partial t} = -fU - \frac{1}{\rho_0} \frac{\partial P}{\partial Y} + \frac{\partial}{\partial z} (K_m \frac{\partial V}{\partial z}) \quad (9b)$$

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} (K_H \frac{\partial \theta}{\partial z} - R) \quad (9c)$$

where U and V are the eastward (X) and northward (Y) pointing velocity components, and R is the long wave radiative flux. R was specified from

$$\partial R / \partial z = -K_R \partial^2 T / \partial z^2 + 2 \text{ deg K day}^{-1} \quad (10)$$

where T is the absolute temperature and K_R is a radiative diffusivity which was assigned the value $600 \text{ cm}^2 \text{ sec}^{-1}$ from analysis of the studies of Elliott (11) and Kondo (12). For this particular study the wind speed in the surface layer was of order $4-6 \text{ m sec}^{-1}$ and the choice of K_R was not critical.

In (9a,b) the horizontal pressure gradients, when normalized by a constant density, were specified as a function of time as determined from figures in Izumi and Barad (5) as follows:

$$\frac{1}{\rho_0} \frac{\partial P}{\partial X} = .079 [1 + 1.1 \times 10^{-5} (t - 1.44 \times 10^4)] \quad (11a)$$

$$\frac{1}{\rho_0} \frac{\partial P}{\partial Y} = -.123 [1 + 1.1 \times 10^{-5} (t - 1.44 \times 10^4)] \quad (11b)$$

in units of cm sec^{-2} where t is the time in seconds after 1720 local time. The pressure gradient thus intensified by a factor of nearly 1.5 during the evening and night, although little change with time in geostrophic flow direction was evident and none was modeled. In (9a,b) f was set to $7.7 \times 10^{-5} \text{ sec}^{-1}$.

In addition, a thermal wind associated with vertical temperature gradient and sloping terrain (16,17) was incorporated. However, the estimated crosswind slope of 1:1000 caused the pressure gradients to vary by less than 3% in the lowest 700m.

The surface stress was assumed to be in the direction of the wind at level z_1 . Stress components extrapolated to surface level in the (X,Y) frame were then specified by

$$-(\overline{u'u'})_s = u_z^2 \cos \alpha \quad (12a)$$

$$-(\overline{v'u'})_s = u_z^2 \sin \alpha \quad (12b)$$

where

$$\alpha = \tan^{-1}(v_1/u_1) \quad (12c)$$

is the direction of the surface wind relative to east. This direction was set to about 50° initially and was a product of the integration subsequently.

The surface temperature, θ_s , needed in (7b) and (8) was prescribed by

$$\theta_s - 292.3 = -2.9 \ln[(t+10)/10] \text{ deg K} \quad (13)$$

with t in seconds. The constant 2.9 was estimated from the temperature profiles presented by Izumi and Barad (5).

Boundary conditions at the top of the model were taken as

$$\partial U / \partial z = 0 = \partial V / \partial z = \partial \theta / \partial z.$$

d. Finite-difference procedures

The height of the model was 700m, and a uniform vertical grid was used with interval $\Delta z = 10\text{m}$. Thus $z_1 = 10\text{m}$.

Centered finite differences of second order accuracy were employed in the vertical direction, while forward time steps of first order accuracy were used in (9a,b,c). Grid values of U,V , and θ were located at the same levels, while K_m and K_H were located at intermediate heights.

At the level $z = \frac{1}{2}\Delta z$, the quantities $K_m \partial U / \partial z$, $K_m \partial V / \partial z$ and $K_H \partial \theta / \partial z$ in (9a-c) were interpolated between respective surface values of the fluxes and interior values centered at $z = (3/2)\Delta z$. The surface fluxes were taken from (12a,b) and from use of (?). In this manner, no explicit lower boundary conditions on U,V and θ are needed in the solution of (9a-c).

The simulated integration period extended from 1720 to 0500 local time, and used time steps which varied between 47 sec and 8 sec.

3. COMPARISON OF MODEL WITH DALLAS TOWER STUDY

a. Temperature and wind profiles

These are shown in Fig. 2 at the five times during the evening and night of Feb. 22-23, 1961, for which profiles were presented by Izumi and Barad (5). Notice the increased thermal stability which usually occurred in the upper third of the thermal boundary layer. This feature was reproduced in the numerical model, but only after K_H was assumed to decrease rapidly with height in the upper half of the boundary layer as in Fig. 1 and Fon. (3b). The nearly isothermal temperature profiles in the early morning within the lower third of the boundary layer were not quite satisfactorily reproduced by the model, suggesting that the maximum value of K_H should have been even larger. The observed temperature profile for 0300 exhibits a peculiar kink aloft which did not occur numerically. It suggests that sometime between 0140 and 0300 RI became greater than the quenching value at a level somewhat below 300m and caused h to decrease abruptly to this lower level.

The assumed surface temperatures are also shown in Fig. 2 and were only 1 to 2°C less than the 10m temperatures. Greater differences would have occurred if α_s for temperature had been taken smaller than α_s for wind.

The numerically calculated wind profiles agree satisfactorily with the observed ones, using formulation (1a) for K , but only because, in addition, the pressure gradients were prescribed to increase with time at all levels by (11a,b). Half of the wind speed increase is seen to have been caused by the increase of the pressure gradient, and the other half by the inertial overshoot mechanism of Blackadar.⁽¹³⁾ However, the model did not reproduce a local maximum in wind speed by means of this mechanism because both the initial wind speed and the assumed pressure gradient were essentially constant with height in the region in which the jet developed. Blackadar's mechanism may have been appropriate, however, if the geostrophic wind had possessed or developed a significant maximum at a level of 350m or less. The inertial overshoot is seen to have commenced as early as 2200, even though the pressure gradients were increasing with time.

Various shapes for K were tested that more resembled K_p in Fig. 1; however, these led to wind-speed profiles with insufficient or even reversed curvature compared with those observed. The cubic polynomial profile⁽¹⁴⁾ was also tested, but gave K values too large at intermediate heights compared with the specified values in the surface layer.

The fact that the most realistic wind profiles were achieved when K was not distinctly confined within the boundary layer (see Fig. 1) may seem surprising. However, the larger assumed values of K_m than of K_p aloft could represent the capability of internal gravity waves to transport momentum.

The wind direction, α of Eqn. (12c), is shown in Fig. 3 for both the observations (solid curve) and the calculations (dashed curve). Also shown is the geostrophic flow direction of 13° north of east. The rapid increase in α , followed by a slow decrease, was fairly well reproduced and is explainable as follows: Around sunset when a shallow stable layer begins to form, the turbulence aloft becomes mostly decoupled from the ground and decays rapidly. The stress components are then confined to a very shallow layer near the ground in that the frictional forces, which equal their vertical derivatives, integrate away from the ground. This in turn causes increased cross-isobaric flow through low pressure near the surface for compensation. This situation gradually relaxes as the stable boundary layer gradually thickens, as the surface stress gradually decreases due to decreased wind speeds near the surface.

5. Blackadar number

Profiles of h are shown in Fig. 4, and indicate that throughout most of the boundary layer h was of order 0.1. The procedure of defining h as the lowest level where $h \geq 0.1$ is seen to have given reasonable results, although it caused a small kink in the h profile at the threshold value which may not be realistic. If K_p had been prescribed to become small at some fixed height such as 2 km,^(15,16) in-

stead of at the height $h(t)$ determined by Eq. (1), the distinctiveness of the thermal boundary layer would have been lost.

c. The boundary layer height

The change in h with time calculated by the model is shown by the solid curve in Fig. 5 along with estimates (circles) of h with error bars obtained from the actual temperature profiles. The rate of increase of h with time is related not only to values of K_m and K_p at $z = h$, but also to the maximum values at lower levels. Even if K_m and K_p had been set to zero at and above $z = h$, h would still have increased with time (though at a decreased rate) due to existence of terms of the type $(\partial K / \partial z) (U^2 / \alpha)$ on the right side of (9a-c).

Numerically obtained values of the Monin-Obukhov length, L (see Eqn. (8) for its definition) are shown in Fig. 5 by the dotted curve. L was calculated to reach a minimum value of about 30m at 1815 and to increase thereafter because, roughly speaking, the cube of the wind speed increased faster thereafter than did the product of wind speed and air-surface temperature difference. The dashed curve in the figure is explained in Section 5.

d. Momentum fluxes

Profiles of the u and v wind components (with respect to axes along and normal to the surface wind direction) and profiles of uw and vw are shown in Fig. 6. The absence of any wind or stress spiral aloft is apparently associated both with the rate of growth of h and with the increasing values of geostrophic wind speed with time. In a separate numerical integration when the latter was held constant, a weak spiral in u and uw did develop but only after about 0400 local time.

e. Heat flux

The kinematic heat flux at the various times is shown in Fig. 7 by the thinner curve of each set. The heavier curves give a combined turbulent and radiative heat flux defined by

$$H = \overline{w\theta} + \bar{q} - R(h) \quad (16)$$

where R is the radiative flux defined in (10). Since only an increase of H with height leads to an increase of temperature with time, the profile of H is smooth, whereas that of $\overline{w\theta}$ is not necessarily so, especially near the surface. In a situation with weaker winds and turbulence, the difference between H and $\overline{w\theta}$ would become much more pronounced at small z and one must then question the use of $\overline{w\theta}$ alone in boundary-layer formulations for the temperature profile. The continued increase of H with height well above $z = h$ in Fig. 7 reflects the net radiative cooling rate assumed aloft in (10). The concrete nature of H versus z which develops at later time has been traced to the decreasing rate of surface cooling as the night progresses.

4. VARIATION OF PARAMETERS

Several parameters or governing conditions were altered one at a time with respect to those utilized in the above numerical integration, to be called run (a).

In run (b), the time dependence was removed from the pressure gradients prescribed in (11). The height, h , nevertheless increased with time at almost the same rate as for run (a). This result thus adds even more evidence to those observations which indicate that the height, h , must be considered a time-dependent quantity in general which cannot be obtained from steady state similarity arguments in the stable atmosphere. (The same conclusion has been found earlier for the case of unstable stratification, even when very weak.⁽¹¹⁾)

In run (c), the roughness length, z_0 , was decreased from 10 cm to 3 cm, which is a value recommended for this site with southwest winds.⁽²¹⁾ The main difference in results was an increased wind speed (by about 20%) in the surface layer at z_1 , giving values considerably larger than observed. This change was accompanied by values of u_A averaging 6% less than in run (a), and an average value of $\partial h/\partial t$ about 8% less.

In run (d), the quenching value for Ri aloft was changed from $Ri_q=1.7$ to $Ri_q=1.0$. This caused h to rise at a rate about 10% less. Thus, a more precise value for Ri_q , and confirmation of this concept, is desirable.

In run (e), the surface temperature was held constant with time after 2200. Values of the total flux, H , thereafter became more constant with height in the lower portion of the boundary layer. However, h was calculated to grow even faster with time because of the decreased stratification which developed. Hence the growth of the boundary layer seems in no way proportional to the rate of surface cooling.

In run (f), the initial value of $\partial\theta/\partial z$ was prescribed to be 1/3 closer to the adiabatic than in run (a), while θ_1 was held the same as in run (a). It was thought that h might then increase at a correspondingly greater rate. However, h increased only 5% more rapidly.

In run (g), $\partial U/\partial z$ and $\partial V/\partial z$ above 100 m were each increased from zero initially to 1 m sec⁻¹ per 400 m. The initial wind speed was held the same as in run (a) at the height of 200 m. There was no significant change in the rate of increase of h with time.

In run (h), the pressure gradients and initial wind components were cut in half. Interestingly, values of Ri exceeding 1.7 then developed within the boundary layer by 2000 and caused h to decrease from 100 m initially to 60 m. Thereafter, h was calculated to grow at a rate of about 1/3 of that of run (a).

5. PARAMETERIZATION OF $\partial h/\partial t$

If the height, h , is determined by the level where Ri first exceeds a quenching value, Ri_q , then it can be shown that

$$\partial h/\partial t = Ri_q \left[\frac{\partial}{\partial t} (Ri) \right]_h / \left[\frac{\partial}{\partial z} (Ri) \right]_h \quad (15)$$

Unfortunately, it appears more difficult to obtain simple or reliable expressions for the time and height dependencies of Ri at $z = h$ than to obtain one for $\partial h/\partial t$ itself. Furthermore, runs (f) and (g) above imply that $\partial h/\partial t$ is not very sensitive to externally imposed conditions on the vertical gradients provided Ri above $z = h$ exceeds Ri_q significantly.

The dashed curve in Fig. 5 which correctly gives $\partial h/\partial t$ is given simply by

$$\partial h/\partial t = .025 \left[1 - h/(.35u_A/f) \right] u_A - \psi \nabla h + u_h \quad (16)$$

where ∇h and u_h were assumed zero. The factor in brackets has been included to indicate the belief that the similarity theory for a neutral boundary layer should at least provide an upper limit to h in a stable atmosphere. In this study the height, $.35u_A/f$ (of order 2km), exceeded h so much that the form of the correction factor in (16) could not be tested. It is uncertain if a stability factor containing some function of h/L also needs to be incorporated into (16), although run (h) suggests this possibility. Of course, the use of (16) presupposes that some method for obtaining u_A or for relating it to the geostrophic flow speed is available.

The observations of Clarke et al.⁽⁶⁾ suggest to me that the factor in (16) may be closer to .04 or .05 than to .025. On the other hand, the analysis by Lettau and Dabbert⁽²²⁾ of data from the Antarctic Plateau station indicates that a steady value for h of only about 32 m occurred. It should therefore be apparent that (16) is tentative and subject to revision as more observations become available and analyzed.

The initial value for h which is appropriate immediately after the surface becomes cooler than the air above is another uncertainty which this study could not clarify. It seems unlikely that this value can be assumed zero when large scale roughness elements, of perhaps heterogeneous surface temperatures, exist.

6. CONCLUSIONS

The main conclusion is that the height of the nocturnal boundary layer is generally time dependent and incapable of being described by any steady state similarity theory, as in (1). The same conclusion has been obtained by Gill⁽²³⁾ from different reasoning.

The rate of growth of h with time which occurred in the Dallas Tower Study⁽⁵⁾ was proportional to the friction velocity, u_A , as in (16).

About the same rate of growth was calculated to occur if the pressure-gradient field had remained constant with time during the observations of that study instead of increasing in intensity. However, it seems likely that the proportionality factor in (16) should incorporate a stability function before (16) can be used with confidence.

It is also concluded that the concept of a stable boundary layer being limited by the height at which Ri first exceeds a certain quenching value is a useful one for numerical studies which predict Ri as a function of height and time. A quenching value of $Ri_q = 1.7$ gave satisfactory results for the rate of growth of h with time.

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Figure 1. Example of assumed eddy coefficients for momentum, K_m , and for heat, K_H

Figure 2. Measured⁽⁵⁾ (upper half) and numerically calculated (lower half) profiles of temperature and wind speed at five selected times during the evening and night of Feb. 22-23, 1961, at the Dallas tower

Figure 3. Surface flow angle, α , relative to east as a function of time

Figure 4. Vertical profiles of Richardson number calculated by the model at the five selected times. Arrow denotes the quenching value, $Ri_q = 1.7$

Figure 5. Dependence of h upon time. Solid line: numerically calculated dependence; circles with error bars: observed height of thermal boundary layer; dashed line: empirical form 1a; dotted line: calculated dependence for the Monin-Obukhov length, L .

Figure 6. Wind-component profiles (upper half) and stress component profiles (lower half) at the five selected times. Components refer to axes directed along and across the surface flow direction.

Figure 7. Profiles of kinematic heat flux, $\overline{w\theta}$ (thin curve), and of total eddy plus radiative heat flux, H (thick curves), for the five selected times.

BUOYANCY EFFECTS IN A TURBULENT BOUNDARY LAYER

S. P. S. Arya

University of Washington, Seattle, Washington

INTRODUCTION

The importance of buoyancy forces in affecting the structure of atmospheric turbulence has long since been recognized.¹ But, only during the last two decades, such effects have been considered realistically in theoretical relations of mean wind and temperature.² Investigations on the effect of buoyancy on the detailed structure of turbulence are even of more recent origin.³⁻⁶ Most of the studies have concentrated on the lower 5% or so of the planetary boundary layer, whose structure is now fairly well understood. For the outer region, the observations are fewer and rarely in agreement with the theories based on the simplified equations of motion in which horizontal advection terms are usually ignored. The main reasons for this disagreement may be the inherent non-stationarity and non-homogeneity of the planetary boundary-layer flow caused by diurnal variations of heat flux, surface inhomogeneities, large scale secondary flows such as rolls, etc. It can easily be shown that the requirements for assuming stationarity and homogeneity are far more strict and difficult to satisfy for the outer part of the boundary layer than these are for the surface layer.

A less rigorous but simpler treatment than afforded by the use of equations of motion, can be obtained by applying the concept of local similarity of equilibrium boundary layers.⁵ This has been used with great advantage for small scale developing boundary layers in which horizontal acceleration terms are indeed significant. The assumption is that the distribution of mean velocity and various statistics of turbulence in an undisturbed boundary layer depends only on the local variables such as the height z , the ambient velocity U_∞ , the boundary layer thickness δ , the surface roughness z_0 , fluid density ρ , viscosity ν , etc., and not on the prior history of the flow. The same concept has also been applied to the planetary boundary layer,⁶ although a sufficiently detailed and precise evaluation of various similarity functions representing the dimensionless characteristics of the same has not been possible from the limited observations that are now available. We have done this on a smaller scale for the case of a wind tunnel boundary layer in which buoyancy effects were simulated by cooling or heating of a part of the tunnel floor, and at the same time, heating or cooling of the ambient air. The results of experiments are presented and their relevance to the atmospheric boundary layer will be discussed.

SIMILARITY CONSIDERATIONS

Based on the concept of local similarity⁵, we assume that all the mean flow and turbulent characteristics at a certain location in the boundary layer depend on the following variables: ($z, U_\infty, \theta_\infty, \nu, \alpha, \delta, \theta_0, g/T_0$) (1)

Here, θ_∞ is the temperature difference between the surface and the ambient air, α is the molecular diffusivity of heat, δ_0 is the thermal boundary layer thickness and g/T_0 is the buoyancy parameter. Then, from dimensional analysis, the normalized quantities like $U/U_\infty, \theta/\theta_\infty, u'^2/U_\infty^2, v'^2/U_\infty^2, w'^2/U_\infty^2, \theta'^2/\theta_\infty^2, z'w'/U_\infty^2, w'\theta'/U_\infty\theta_\infty, u'\theta'/U_\infty\theta_\infty$, etc., must be universal functions of the following dimensionless parameters:

$$(z/\delta, Re_\delta, Ri_\delta, \sigma, \delta_0/\delta) \quad (2)$$

Here, $Re_\delta = U_\infty \delta / \nu$, is the Reynolds number; $Ri_\delta = g \theta_\infty \delta / (T_0 U_\infty^2)$, is a bulk Richardson number; and $\sigma = \nu / \alpha$ is the Prandtl number, U and θ denote local mean velocity and temperature; u', v' and w' are velocity fluctuations in x, y and z (vertical) directions, respectively; θ' denotes temperature fluctuations; and overbar represents averaging with respect to time.

Of the listed parameters in (2), σ may be dropped from consideration if one is concerned only with the flow of air e.g., in the atmosphere, or in a wind tunnel. For a fully developed and well mixed thermal layer, the ratio δ_0/δ is expected to be close to unity and therefore may also not be important. If δ and δ_0 are substantially different it would be more appropriate to normalize z by the latter for representing $z/\theta_\infty, \theta'^2/\theta_\infty^2$, etc. So basically we have z/δ or $z/\delta_0, Re_\delta$, and Ri_δ , whose effect on the structure of the stratified boundary layer we wish to investigate.

Wall Similarity

The so-called universal law of wall similarity is based on the observation that the external variables $U_\infty, \theta_\infty, \delta$, etc., are not directly relevant to the similarity of the flow in the surface layer, if velocity and temperature are scaled by $u_\tau = (\tau_0/\rho)^{1/2}$, and $\theta_\tau = H_0/\rho C_p u_\tau$, where, τ_0 and H_0 are the surface shear stress and the heat flux. The scales u_τ and θ_τ are, however, determined

by the universal parameter α

$$u_0/u_* = f(Ri_*, Ri_*), \quad (1)$$

$$\phi_0/\theta_* = f_\phi(Ri_*, Ri_*). \quad (2)$$

The surface layer similarity hypothesis¹⁰ predicts that with above the viscous sublayer, the non-dimensional characteristics such as $u_0 = \frac{u_*}{\theta_*} \frac{\partial u}{\partial z}$, $Ri_* = \frac{u_*^2}{\theta_*} \frac{\partial \theta}{\partial z}$, u_*^2/θ_* , u_*^2/θ_*^2 , u_*^2/θ_*^3 , etc., must be universal functions of the stability parameter

$$z = \frac{z}{l} = \frac{\partial^2 u_*^2}{k \theta_*} \quad (3)$$

where, k is Von-Karman's constant and l is the Obukhov length. Many observations in the atmosphere^{7, 8} and a few in the laboratory⁹ have generally established the validity and usefulness of the above similarity hypothesis.

For the atmospheric boundary layer, u and θ are usually ignored in the similarity analysis⁷; in (1) these would be replaced by u_0 and the Coriolis parameter f . Then, in Eqs. (2) - (4), Ri_* will be replaced by the surface Rossby number $R_0 = U_*/fz_0$, where U_* denotes the geostrophic wind speed. An examination of the role of R_0 and R_0 in the laboratory and the atmospheric boundary layers, respectively, for determining u_0/u_* and ϕ_0/θ_* , suggests an interesting analogy between the two parameters. This is not very surprising if one considers them both as representing the ratio of the outer boundary layer scale δ ($\delta \sim u_*/f$, for the atmosphere), to the surface layer scale z_0 ($z_0 \sim u_*/\theta_*$, for the flow over a smooth plate). It seems that the effect of Rossby number on turbulent intensities and fluxes may largely be simulated by properly matching u_0/u_* and ϕ_0/θ_* in the laboratory and the atmospheric boundary layers.

As for the buoyancy effects, we will use Ri_* only as a convenient parameter for presenting our experimental results. Because of its heavy dependence on δ , it may not be a suitable parameter for comparison with atmospheric results. A better criterion would be to compare the distributions of the gradient or flux Richardson number with z/l , in the two cases.

THE WIND TUNNEL AND INSTRUMENTATION

Experiments were made in the meteorological wind tunnel at Colorado State University. The closed circuit tunnel has a $1.8 \times 1.8 \text{ m}^2$, 39 m long test section. After the initial 13 m, the floor consists of a 12 m aluminum plate which can be cooled or heated to any desired temperature between -5 and 20°C . The air-conditioning system allows for the ambient air temperature to be maintained between 5 and 65°C . Complete details of the tunnel and its heating and cooling system are given elsewhere.¹⁰

Mean velocities were measured using a standard pitot-static tube and mean air temperatures with a copper-constantan thermocouple. Plate temperatures were monitored by thermocouples embedded in the plate at 30 cm intervals along the centerline. Velocity and temperature fluctuations were measured by a resistance thermometer (cold wire), and three hot-wires operated in various configurations with respect to the flow. The hot wires were sensitive to velocity as well as temperature fluctuations and an elaborate calibration and measurement technique¹¹ was employed to obtain all turbulent intensities and fluxes of interest, within estimated errors of $\pm 10\%$ and $\pm 20\%$, respectively.

OBSERVATIONS AND RESULTS

After the hot or cold air entered the test section, a constant pressure boundary layer developed along the tunnel floor. A thermal boundary layer also developed over the cooled or heated section of the floor. The measurements reported here were made at two stations near the downstream end of the plate where both the momentum and thermal boundary layers were fairly thick and well developed. The various boundary layer parameters are given in Table 1. The letter S or U before the run number indicates stable or unstable stratification; various runs have been classified into different stability groups, such that the runs in each group represent approximately the same conditions.

The boundary layer thicknesses δ and δ_θ represent the lowest heights for which U and θ become indistinguishable from their values in the free stream. There is an uncertainty of 5-10% in their determination because of very small gradients of U and θ in the outermost region, specially in unstable conditions. For stable runs, u_0 and θ_0 have been determined from hot-wire measurements of fluxes at several heights close to the surface, which indicated the existence of a constant flux layer.⁹ No such detailed measurements were made for the unstable runs for which u_0 has been

measured by the so-called 'profile method'. The wall heat flux has been determined from measurements of electric power supplied to the plate after allowing for the losses due to radiation, convection to supports etc.

MEAN VELOCITY AND TEMPERATURE PROFILES

For the lowest 15% of the boundary layer, the observations are plotted in Figure 1. The satisfactory collapse of data from different runs supports the surface-layer similarity hypothesis. The function $u_0(z)$ is not much different from that observed for the surface layer of the atmosphere but $\theta_0(z)$ is quite different, especially in unstable conditions. The main reason for this lack of similarity may be that in our heated plate experiments, θ_0 and θ_1 are much too large (an order of magnitude greater than the normal values for the atmosphere), which were counteracted by our choice of simulating Richardson number of 1 of the same magnitude as found in the atmosphere's surface layer. For stable runs, the heat fluxes were reasonably small and the observations have shown good similarity with the atmospheric results.

For the boundary layer as a whole, the velocity and temperature distributions are presented in Figures 2 and 3, in the usual defect form of similarity considerations. In this way, the variability due to changes in Re_τ , especially within the narrow range covered in our experiments should all be eliminated. Therefore, most of the variation is due to thermal stratification, which is quite large indeed. The gradients of U/U_τ and θ/θ_τ decrease in the surface layer and increase in the outer region, as stability (Ri_τ) increases.

Figure 2 shows that the quantities U_0/U_τ and θ_0/θ_τ both decrease with increasing stability. A small part (< 15%) of this variation is due to different values of Re_τ , which are also indicated. For the near-neutral conditions, the assumption of the defect-law similarity of profiles in the outer layer and the log-law similarity in the surface layer leads to the following relations:

$$U_0/U_\tau = A^{-1} \ln(Re_\tau U_0/U_\tau) + A \quad (6)$$

$$\theta_0/\theta_\tau = A^{-1} \ln(Re_\tau U_0/U_\tau) + A_2 \quad (7)$$

in which, A and A_2 are universal constants. Analogous relations have been derived for the atmospheric boundary layer, which have also been extended to include the effect of stability.² In the latter case, A, A_2 , etc., are assumed to be universal functions of the stability parameter. Adopting the same approach here, A and A_2 are determined from Eqs. (6) and (7) after substituting for the measured values of U_0/U_τ , θ_0/θ_τ and Re_τ . These are given for various runs in the last two columns of Table 1. Note that A and A_2 vary over a wide range, especially in stable conditions. Similar trend has been found for the planetary boundary layer,² in which case R_ρ must replace Re_τ in Eqs. (6) and (7).

TURBULENCE STRUCTURE

The results of measurements of turbulent intensities in stable conditions are presented in Figures 4, 5 and 6. The graphs of normalized r.m.s. fluctuations vs. z/δ show the effect of changes in both Re_τ and Ri_τ , which could not be varied independently of each other. In order to show the effect of buoyancy alone, some data have also been presented (see Figures 4 and 5) for neutral conditions. Note that the velocity fluctuation intensities decrease monotonically with increasing stability and also, to some extent, with decreasing Reynolds number. The surface layer is highly anisotropic as indicated by the relative magnitudes of u , v and w - components. In the outer part these become roughly equal, indicating a strong tendency towards isotropy for $z/\delta > 0.6$.

In contrast with the velocity fluctuations, the normalized r.m.s. temperature fluctuations are seen to be very little affected by stratification (see Figure 5). Fluctuations in temperature occur as a consequence of the fluctuations in velocity in the presence of a temperature gradient. It is not clear why the former should simply be scaled by θ_τ and remain more or less independent of stability. The same behaviour is indicated by our measurements in unstable conditions, which are also presented in Figure 5. The variation of $(\theta')^2/1/2$ with height is much more pronounced than that of velocity components. For unstable runs, the former varies roughly as $z^{-1/3}$ in the surface layer, as predicted by the free convection similarity theory. Mean temperature and velocity profiles, however, are not adequately described by the same theory.²

In Figure 6 are shown the measured distributions of turbulent energy and turbulent shear stress in the boundary layer for three different stability conditions. In this and the following figures, actual data points have been omitted for the sake of clarity, and only eye-fitted smooth curves drawn through them are shown. Buoyancy appears to affect shear stress more than it affects the turbulent energy. The ratio $\overline{u'^2 v'}/q^2$, which is an important parameter in certain analytical formulations, decreases with increase in stability, especially in the outer layer. In the surface layer it remains roughly constant.

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一、本行自成立以來，承蒙各界愛護，業務日見發達。茲為擴大服務起見，特在
 上海、南京、漢口、廣州、香港、汕頭、廈門、福州、長沙、重慶、成都、昆明、貴陽、西安、蘭州、西寧、迪化、烏魯木齊、拉薩、台北、高雄、基隆、台中、台南、新竹、嘉義、屏東、花蓮、台東、澎湖、金門、馬祖等處，均設有分行或辦事處，以便各界光臨。

[illegible]

一、政治：政治是国家的根本，是人民生活的保障。政治的好坏，直接关系到国家的兴衰和人民的幸福。因此，我们必须高度重视政治，不断加强政治建设和政治体制改革。

一、本報自創辦以來，承蒙各界愛護，不勝感荷。茲為擴大宣傳，特在各地設立分報處，以便讀者訂閱。凡欲訂閱者，請向各分報處接洽。本報宗旨，在於報導事實，傳播知識，服務社會。凡有新聞、評論、文學、藝術等稿件，均可寄交本報編輯部。本報地址：上海南京路。電話：一二三四。

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一、凡我同胞，如有任何不法行為，請即向當地政府或有關部門舉報，我們將全力配合調查，絕不姑息。
 二、本會將定期開展法治宣傳活動，提高廣大市民的法律意識，營造良好的法治氛圍。
 三、本會將加強與各級政府、司法機關的溝通與合作，共同維護社會的公平正義。
 四、本會將關注社會弱勢群體，提供法律援助服務，保障他們的合法權益。
 五、本會將加強自身建設，提高工作人員的專業素養，確保各項工作高質量開展。
 六、本會將定期向社會各界報告工作進展，接受社會監督。
 七、本會將積極參與社會公益活動，為社會做出更多貢獻。
 八、本會將加強與其他社會組織的交流與合作，共同推動社會法治建設。
 九、本會將關注國際法治動態，學習借鑒先進經驗。
 十、本會將加強與媒體的聯繫，及時發布相關信息，提高社會影響力。

三、四、五、六、七、八、九、十、十一、十二、十三、十四、十五、十六、十七、十八、十九、二十、二十一、二十二、二十三、二十四、二十五、二十六、二十七、二十八、二十九、三十、三十一、三十二、三十三、三十四、三十五、三十六、三十七、三十八、三十九、四十、四十一、四十二、四十三、四十四、四十五、四十六、四十七、四十八、四十九、五十、五十一、五十二、五十三、五十四、五十五、五十六、五十七、五十八、五十九、六十、六十一、六十二、六十三、六十四、六十五、六十六、六十七、六十八、六十九、七十、七十一、七十二、七十三、七十四、七十五、七十六、七十七、七十八、七十九、八十、八十一、八十二、八十三、八十四、八十五、八十六、八十七、八十八、八十九、九十、九十一、九十二、九十三、九十四、九十五、九十六、九十七、九十八、九十九、一百。

1. 本報告は、昭和二十一年四月一日から三月三十一日までのものである。
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 10. 本報告は、昭和二十一年四月一日から三月三十一日までのものである。

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一、政治
 二、经济
 三、文化
 四、教育
 五、军事
 六、外交
 七、宗教
 八、法律
 九、艺术
 十、科学
 十一、医药
 十二、农业
 十三、工业
 十四、商业
 十五、交通
 十六、能源
 十七、环境
 十八、社会
 十九、家庭
 二十、个人

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一、政治
 二、經濟
 三、文化
 四、教育
 五、社會
 六、宗教
 七、藝術
 八、科學
 九、法律
 十、道德
 十一、哲學
 十二、歷史
 十三、地理
 十四、生物
 十五、物理
 十六、化學
 十七、醫學
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 二十三、經濟學
 二十四、社會學
 二十五、人類學
 二十六、心理學
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 二十八、美學
 二十九、音樂學
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 三十一、電影學
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 三十四、建築學
 三十五、園林學
 三十六、環境學
 三十七、生態學
 三十八、地質學
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1. 凡在本行开立存款账户的存款人，均可向本行申请开立支票。

[illegible]

一、政治
 二、經濟
 三、文化
 四、教育
 五、社會
 六、宗教
 七、藝術
 八、科學
 九、法律
 十、軍事
 十一、外交
 十二、內政
 十三、財政
 十四、稅收
 十五、金融
 十六、交通
 十七、郵政
 十八、電信
 十九、新聞
 二十、出版
 二十一、印刷
 二十二、攝影
 二十三、繪畫
 二十四、音樂
 二十五、戲劇
 二十六、電影
 二十七、體育
 二十八、衛生
 二十九、醫藥
 三十、農業
 三十一、牧業
 三十二、漁業
 三十三、蠶桑
 三十四、林業
 三十五、礦業
 三十六、工業
 三十七、商業
 三十八、貿易
 三十九、銀行
 四十、保險
 四十一、證券
 四十二、期貨
 四十三、期權
 四十四、債券
 四十五、股票
 四十六、基金
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 五十二、委託
 五十三、授權
 五十四、轉讓
 五十五、繼承
 五十六、遺贈
 五十七、遺產
 五十八、債務
 五十九、債權
 六十、契約
 六十一、合同
 六十二、協議
 六十三、決定
 六十四、裁決
 六十五、判決
 六十六、裁定
 六十七、命令
 六十八、指示
 六十九、通知
 七十、公告
 七十一、聲明
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 七十八、批准
 七十九、同意
 八十、贊成
 八十一、反對
 八十二、支持
 八十三、擁護
 八十四、保護
 八十五、維護
 八十六、保障
 八十七、確保
 八十八、安全
 八十九、危險
 九十、風險
 九十一、機會
 九十二、挑戰
 九十三、困難
 九十四、問題
 九十五、矛盾
 九十六、衝突
 九十七、爭端
 九十八、糾紛
 九十九、訴訟
 一百、官司

2. 我 們 的 工 作 是 在 黨 的 領 導 下 進 行 的 ， 我 們 的 工 作 是 在 黨 的 領 導 下 進 行 的 ， 我 們 的 工 作 是 在 黨 的 領 導 下 進 行 的 。
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1. 凡在本行開辦之各項業務，均應遵守本行所定之規章制度，並應隨時注意業務之改進，以期提高服務品質。

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Figure 1
Similarity functions for the surface layer

Figure 2
Effect of buoyancy on the velocity defect profiles

Figure 3. Effect of buoyancy on the temperature-defect profiles

Figure 4. Effect of stability on u_{*}/U_{∞} and $\theta_{*}/\theta_{\infty}$

Figure 5. Distribution of $(u'^2)^{1/2}/U_\infty$ for different stability conditions

Figure 6. Distribution of $(v'^2)^{1/2}/U_\infty$ for different stability conditions

STABILITY GROUP	$R_{i\theta}$	Re_θ
○ I	.010	3.0×10^5
△ II	.025	2.0×10^5
□ III	.101	1.1×10^5
○ NEUTRAL	0	3.0×10^5
□ NEUTRAL	0	1.1×10^5

Figure 7. Distribution of $(w'^2)^{1/2}/U_\infty$ for different stability conditions

Figure 8. Distribution of $(\theta'^2)^{1/2}/\theta_\infty$ for different stability conditions

Figure 9. Distributions of turbulent energy and shear stress stable conditions

Figure 10. Distributions of the horizontal and vertical fluxes of heat in stable conditions

Figure 11. Distributions of the correlation coefficients of heat fluxes

Figure 12. Distributions of $-\gamma_{uv}$ and ϵ_h/ϵ_m in stable conditions

MODELING THE ANISOTROPY IN STRATIFIED SHEAR FLOW

by

C. G. Justus and J. E. Hicks
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332

INTRODUCTION

Several recent random walk models have been developed for the study of turbulent diffusion. Patterson and Corrsin⁽¹⁾ developed a one dimensional model in which a random velocity field was constructed on a grid in x space. Three dimensional models have been developed by Kraichnan⁽²⁾ and by Riley and Corrsin⁽³⁾ in which the random field is synthesized from wave-vector components and realized in x space only along the particle trajectories. However, there is no interaction among the different wave numbers in these models, and the computed ratio of Lagrangian to Eulerian time scale is unrealistic [Riley and Corrsin⁽³⁾]. Thompson⁽⁴⁾ has developed a model which, like that of Riley and Corrsin, considers the random motion of "alien" particles in the gas due to Stokes forces produced by differences between particle and fluid velocities. Thompson's model allows for stable stratification and topographic effects.

The model presented here is a three dimensional random walk of a fluid element or a fluid element pair each of which moves according to the acceleration equation

$$\ddot{A}(t) = d^2 y(t)/dt^2 = \ddot{a}(t) + (\rho'/\rho_o)g + \nu \nabla^2 \underline{u} \quad (1)$$

where $\ddot{a}(t)$ is a randomly fluctuating pressure force acceleration which is correlated over space and time. The buoyancy acceleration is taken to act in the z direction and is modeled by the relation

$$(\rho'/\rho_o)g = \Gamma[z(t) - z(o)] \quad (2)$$

where the factor $\Gamma = (g/\theta) \partial\theta/\partial z$ is determined from the profile of potential temperature θ with altitude z .

The fluid element velocity $\underline{U}(\underline{x}, t)$ is made up of a mean velocity field $\bar{\underline{U}}(\underline{x})$, which is assumed known, plus a turbulent velocity $\underline{u}$. The contribution to the viscous term in (1) is due to the turbulent velocity modeled by the relation

$$\nu \nabla^2 \underline{u} = -\omega \underline{u} \quad (3)$$

where ω is assumed constant. The total velocity is determined at successive time steps by a finite difference solution of (1) of the form

$$\underline{U}_{j+1} = \underline{U}_j + \underline{A}_j \Delta t \quad (4)$$

The turbulent velocities are then computed by subtraction

$$\underline{u}_{j+1} = \underline{U}_{j+1} - \bar{\underline{U}}(\underline{y}_{j+1}) \quad (5)$$

where the coordinates of the fluid elements are determined in a similar fashion from

$$\underline{y}_{j+1} = \underline{y}_j + \underline{U}_j \Delta t + \frac{1}{2} \underline{A}_j (\Delta t)^2 \quad (6)$$

The values of the random pressure acceleration are computed such that their distribution is Gaussian with standard deviation σ and proper correlations are maintained over the space and time displacements between successive time steps. The Eulerian correlation ρ_1 of the random pressure acceleration component a_1 is modeled by the isotropic relation.

$$\rho_1(\underline{r}, \Delta t) = [f(r) - g(r) (r_1/r)^2 + g(r)] h(\Delta t) \quad (7)$$

where the separation vector $\underline{r} = \underline{y}_j + 1 - \underline{y}_j$ and $f(r)$ is a Gaussian longitudinal correlation with integral scale L , $g(r)$ is the corresponding transverse correlation required by isotropy, and $h(\Delta t)$ is a Gaussian time correlation with integral scale T_E . Note that an assumption of an isotropic acceleration field does not require an isotropic velocity field.

RESULTS

The model which has been used in these preliminary results is $\Gamma = \text{constant}$ and a constant shear $\partial \bar{U} / \partial z = S$ in the x component mean wind. The y and z component mean winds are taken to be zero. All parameters have been non-dimensionalized in terms of the length L and time T_E . For the results reported here $\Delta t = 0.1$, and $\omega = 1$ were used and the initial separation of the fluid element pairs was taken as 0.1. The fluid elements were released with an rms turbulent velocity which would insure that their rms velocity remained stationary.

Some typical results of the diffusion of a fluid element pair are shown in Figure 1, where the mean square separation is plotted versus time. The x - component separations at times after release between about 10 and 100 time scales follow the relation

$$x^2 = C_3 t^3 \quad (8)$$

because of the shear influence on this coordinate. Because of the particular choice of parameters for Figure 1 the mean square separation after about 100 time scales follows the relation

$$x^2 = C_2 t^2 \quad (9)$$

In Figure 1 the y - component separations at times after about 10 time scales follow approximately an eddy diffusion relation

$$y^2 = 2K_y t \quad (10)$$

while the z - component separations after about 100 time scales level off at a fixed vertical separation. It is obvious from these results that the diffusion in stratified shear flow is highly anisotropic. The crosswind dispersion of the eddy diffusion type is observed under all stable conditions modeled, however the exact behavior of the diffusion in the vertical and downwind directions depends considerably on the values of shear S and stability Γ .

With certain values of Γ and S the downwind dispersion retains its third power behavior at all observed times and the vertical diffusion, under certain values of Γ and S , follows an eddy diffusion relation for all times considered. Figure 2 shows the variation with Γ and S of the power law exponent which describes the long time x - component diffusion. The region where the second power law behavior was observed is at the higher stability values, but the transition value of Γ becomes lower as the shear S increases. Eddy diffusion in the x direction prevails in the case of no shear for all stability conditions. A similar variation of the power law exponent which describes the vertical diffusion is seen in Figure 3. The zone in which vertical eddy

diffusion prevails is seen to be identical with the zone of third power downwind diffusion. In the region where the vertical diffusion approaches a constant value at large times, the x - component diffusion follows a second power law at large times. It is interesting, however, that at zero shear the vertical diffusion exponent at long times is continuous with the values at non-zero shear.

The anisotropy of the turbulence field model is evidenced in the velocity components also, as exhibited by Figure 4, which shows the variation with shear of the ratio of the rms velocity component v' in the crosswind direction to the rms velocity component u' in the downwind direction. As would be expected, the degree of anisotropy increases with increasing shear. The velocity field appears from Figure 4 to be slightly nearer isotropy at stable conditions than at neutral stability ($\Gamma = 0$). The vertical rms velocity was essentially identical with the crosswind rms velocity for all values of S and Γ . It should be emphasized again that these results revealing anisotropic velocity and diffusion characteristics were obtained from a model which has an isotropic acceleration field for the turbulent pressure variations.

RECOMMENDATIONS

In view of the distinctly different character of the diffusion in the various component directions, it appears that the plume type of dispersion models in current use could be improved upon considerably by a model which would reflect the major anisotropy features revealed here. In particular the results revealed here indicate that the vertical stability, as reflected by Γ , and the horizontal "stability" as represented by S must both be considered in determining the diffusion characteristics. (Most current diffusion models employ only the Pasquill-Gifford stability index, which is a measure of the convective or vertical stability.)

It is suggested that the best approach would be an "integrated puff" model, similar to that reported by Roberts et al.⁽⁵⁾, illustrated schematically in Figure 5. However, the integrated puff model must be generalized to account for the fact that the Gaussian puff must have σ_x and σ_y unequal because of the different downwind and crosswind characteristics. Presumably with sufficient observational data the present model, using the integrated puff approach, can be calibrated against actual diffusion situations in the atmosphere. Hopefully a set of standard reference curves similar to the Pasquill-Gifford curves can be developed which would give values of σ_x , σ_y , and σ_z versus time.

ACKNOWLEDGEMENTS

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Figure 1. Mean square separations versus time. Component 1 is along the wind and shear direction. Component 2 is crosswind and component 3 is vertical

Figure 2. Variation of the long time power law exponent of the x - component diffusion with shear and stability

Figure 3. Variation of the long time power law exponent of the vertical diffusion with shear and stability

Figure 4

Variation of the ratio of the cross-wind and downwind rms velocity components with shear and stability

INTEGRATED PUFF' DIFFUSION
WITH $\sigma_x = \sigma_y$

Figure 5

Schematic illustration of the "integrated puff" diffusion model with $\sigma_y = \sigma_x$, but with allowance for time varying wind

AN INVESTIGATION OF THE APPLICATION OF TAYLOR'S HYPOTHESIS TO ATMOSPHERIC BOUNDARY LAYER TURBULENCE*

D. C. Powell and C. E. Elderkin

Atmospheric Sciences Department
Battelle, Pacific Northwest Laboratories
Richland, Washington 99352

INTRODUCTION

According to C. C. Lin,⁽¹⁾ the vertical shear of the mean wind in the boundary layer places an upper limit to the wave lengths to which Taylor's hypothesis, equating time correlations and space correlations in turbulence, applies. The primary purpose of this paper is to evaluate the upper wave length limit by analysis of field data taken from a line of turbulence sensors, logarithmically spaced in the approximate direction of the mean wind in the atmospheric boundary layer.

THEORETICAL FRAMEWORK

Consider a regime of wind tunnel turbulence for which the mean advective velocity is $\bar{U}$ and for which the characteristic turbulence velocity is u . Taylor's hypothesis predicts that if the ratio, $\hat{u}/\bar{U}$, is small, the covariance functions of the turbulence in time at a fixed location and in space in the longitudinal direction are, to a good approximation, the same function and may be written,

$$\bar{R}_{ij}(\tau) = \bar{R}_{ij}(\xi), \quad \xi = \bar{U}\tau \quad i = 1, 2, 3; \quad j = 1, 2, 3 \quad (1)$$

In Equation (1) τ is time lag, ξ is space lag, and the two covariance tensors are unnormalized.

According to C. C. Lin,⁽¹⁾ there is no justification, in general, for extending Taylor's hypothesis to shear flow, such as is found in the atmospheric boundary layer. However, Lin goes on to say that Taylor's hypothesis may still be a good approximation for those wave numbers, κ , of the turbulence such that

$$\frac{\kappa \bar{U}}{d\bar{U}/dz} \gg 1 \quad (2)$$

holds. The corresponding equation using frequency rather than wave number is

$$\frac{2\pi n}{d\bar{U}/dz} \gg 1 \quad (3)$$

It is desirable to replace this equation by one expressing a more exact relation employing the symbol $\geq$ with respect to some experimentally determined constant rather than $\gg 1$.

FIELD ARRAYS OF SENSORS

The data for the three tests analyzed for this writing were all taken from arrays of sensors horizontally dispersed at a single level. The levels for the three tests were 15, 30, and 58 m, respectively. The 15 and 30-m tests were each conducted from sensors mounted on seven portable towers logarithmically spaced on a line that was alongwind to winds from 290°. The six separation distances between consecutive tower positions on this line were 128, 64, 32, 16, 8, and 4 m, progressing from the upwind to the downwind direction. The four upwind positions were instrumented by Gill anemometers, and the three downwind positions (separated by the least distances) were instrumented by sonic anemometers. This arrangement is illustrated by a figure and is explained in greater detail in another paper⁽²⁾ presented by C. E. Elderkin. The 58-m test was conducted from four towers on the Hanford diffusion grid, equally spaced at separation distances of 223 m. Data from two of these positions are analyzed herein. The along-line direction between the two positions was 208°.

CHARACTERISTIC DESCRIPTION OF THE DATA

Specifics regarding the three tests analyzed are given in Table 1.

* This paper is based on work performed under United States Air Force Contract No. F33615-M-5009, "Take-Off and Landing Critical Atmospheric Turbulence (TOLCAT)."

TABLE 1

CHARACTERISTIC DESCRIPTION OF THE DATA

	Test Number		
	T603	T703	T105
Date of recording	19 Jan 71	24 Mar 71	2 Dec 68
Time of recording	1323-1351	0945-1013	1345-1426
Averaging interval (sec)	0.2	0.2	0.3
Height (m)	15	30	58
Stability	Stable	Unstable	Stable
Azimuth angle (deg)	9	11	4
$\bar{U}$ (mps)	6.4	7.0	8.3
$u^* = \sqrt{-u'w'}(z)$ (mps)	0.47	0.40	0.41
$(u^*/\bar{U})^2$	0.0054	0.0033	0.0024
$d\bar{U}/dz$ (sec ⁻¹)	0.066	0.020	0.022
u^*/kz (sec ⁻¹)	0.078	0.033	0.018

In Table 1, u^* is assumed to be the characteristic velocity of the turbulence. When stress is measured at 15 or 30 m, the assumption is more reasonable than when the stress is measured at 58 m, giving a value that is probably low. The k in the table is Von Karman's constant, taken as 0.4. The azimuth angle is the angle between the direction defined by the sensor line and the mean wind direction.

Each set of two towers in the logarithmic array is separated by a unique distance. Therefore, genuine spatial covariances, computed by averaging over space, are not available. Instead, the pseudo-spatial covariances were computed by averaging over time, the products of winds for two points in space. When space correlations are compared with the time correlations from an array using sonic and Gill anemometers jointly, the time correlations should be from the same instrumental combination that the space correlations are from. Therefore, when time correlations and space correlations are compared, space correlations for 4, 8, or 12 m should be compared to sonic time correlations; space correlations for 32, 64, 96, 128, 192, or 224 m should be compared to Gill time correlations; and space correlations for 16, 24, 48, or 252 m should be compared to intermediate values between the two time correlations.

CORRELATION ANALYSIS

Comparison of the time correlations and space correlations for each of the three turbulence components of Test T603 is shown in Figure 1. Each of these graphs exhibits four correlation functions, two scales for the abscissa and one scale for the ordinate. The four correlation functions are: 1) Solid line - time autocorrelation from sonic data; 2) Dashed line - time autocorrelation from Gill data; 3) Large dots - spatial correlations; and 4) Dotted line - exponential function $R(\xi) = \exp(-\xi/L_0)$ with L_0 so determined that the exponential function matches the spatial autocorrelation where the value is e^{-1} .

The scale for the abscissa along the top of each figure is for time lag. The scale for the abscissa along the bottom of the figure is for space lag, which is scaled to match the time lag according to Equation (1). The scale along the ordinate is for the correlation values.

Figure 1 shows that for u'_a , the turbulent wind component parallel to the measurement line, the model holds quite well for separation distances up to 128 m. The spatial correlations for 4, 8, and 12 m are on or very near the time correlation line for the sonic data, as expected. The spatial correlations for 16 and 24 m are intermediate between the sonic and Gill autocorrelation curves although closer to the sonic autocorrelation curve. The spatial correlation for 32 m separation is very close to the Gill autocorrelation curve, and the spatial correlation at 48 m is between the two lines, as would be expected. The spatial correlation for 64 m is slightly low, that for 96 m is right on the Gill autocorrelation curve as expected, and that for 128 m is slightly high. The last three spatial values follow the sonic rather than the Gill autocorrelation curve and are conspicuously high. Similar evaluations can be made for the correlation functions for v'_a , the horizontal turbulent wind component normal to the measurement line, and for the vertical wind component, w' . The weakest verification is for v'_a although a smoothed curve through the spatial correlations compares reasonably well with the temporal autocorrelation functions for both v'_a and w' . Exponential form is followed best by the experimental curve for the vertical component.

The space lag in the alongwind direction for which the spatial correlation value for a particular test is e^{-1} may be defined as a characteristic length for that test. Correlation lengths

of 51, 16, and 5 m for u'_a , v'_a , and w' , respectively, are indicated for this particular data sample. The lengths are functions of T and ΔT , which are 28 minutes and 0.2 seconds, respectively.

The only data analyzed from Test T703 were the Gill data. The azimuth angle for this test was 11° (compared to 9° for Test T603), and the conditions were unstable. For u'_a the spatial correlations fall off somewhat more rapidly than the corresponding time correlations; for v'_a the correspondence was quite strong; and for w' the spatial correlations were slightly higher than the time correlations. Correlation lengths of 77, 176, and 23 m for u'_a , v'_a , and w' , respectively, are indicated for this particular data sample.

For Test T105 there was only one separation distance, 223 m, which corresponds to a transport time of 27 seconds with $\bar{U}$ at 8.3 mps. The spatial correlations at zero time lag and the time correlations at 27 seconds were 0.342 and 0.333 for u'_a , and 0.444 and 0.447 for v'_a . Because the spatial correlation for w' was 0.007 and because the time correlation for w' became negative at 9 seconds, no comparison was made in the vertical. The horizontal comparisons are obviously very strong.

PHASE SPECTRAL ANALYSIS

The required data for this type of analysis may be obtained by recording from at least two sensors, one of which is placed downwind of the other. For these data, Taylor's hypothesis predicts that the phase spectrum calculated by cross-analyzing parallel components from both sensors will be

$$\phi(n) = 360n\xi/\bar{U} \quad (4)$$

in degrees.

For Test T603 data for u'_a , v'_a , and w' are available from the entire logarithmic line of seven sensors. The phase spectral analysis to be presented draws chiefly on the results from this test. Investigation of height dependency is initiated by introducing results from Tests T703 and T105, based on analysis of data from fewer sensors.

The results from Test T603 can best be seen by examining the set of eight phase-spectral plots as a group, see Figures 2 and 3. There is one graph plotted for each separation distance where phase spectra show any organization. These distances are 4, 8, 12, 16, 24, 32, 48, and 64 m. Items requiring explanation on the graphs are as follows: 1) The heavy diagonal line across each figure is the theoretical phase spectrum predicted by Taylor's hypothesis; 2) The equation of this line, Equation (4) is given elsewhere on the figure, first in general form and then in final calculated form, using $\bar{U} = 6.4$ mps; 3) The dashed vertical line indicates the lower frequency limit at which the phase spectra for the three components appear organizable; 4) The horizontal dashed line is an upper limit of coherency, the significance of which is discussed below.

Consider once more the passage of turbulence from the location of one sensor to that of another in the downwind direction. Assume that the sensors are close enough together so that the larger scale variations in the turbulence maintain their characteristics, or individuality, over the path of advection. We say that this part of the turbulence is coherently advected from one location to the other. Intuitively there must be a lower limit to the size of the variations that can be coherently advected over the given path and hence an upper limit to the frequencies representing these variations in the time series. This is important in phase-spectral analysis because the phase spectra might not be expected to show organization at frequencies above the upper limit of coherency.

Because the coherency function is continuous, some arbitrary cutoff value of coherency must necessarily be selected before an upper frequency limit can be defined. Although the maximum frequency of eddies considered to be coherently advected will depend on what cutoff value of coherency is arbitrarily chosen, much may be learned from an extended study in which one cutoff value is uniformly applied. The cutoff value adopted for these studies is e^{-1} , or about 0.37. The upper limit of coherency is defined as that frequency of the u'_a spectrum at which the coherency function is e^{-1} . This value was chosen because the slope of the coherency function is usually quite steep through this value.

The best way to learn from the phase spectral graphs (Figures 2 and 3) is to examine them in sequence in order of increasing separation distance, observing certain changes. As the separation distance increases, the line function for the ideal phase spectrum moves to a higher phase location on each succeeding graph, associating a larger value of the phase spectrum with the same frequency. Theory predicts that the vertical line, indicating the lowest frequency at which the phase spectrum is organized, should be at the same frequency regardless of separation distance. This frequency, n_0 , appears to be reasonably estimated at 0.05 Hz from the analysis. The horizontal line, indicating the upper limit of coherency, appears at approximately the same phase angle, ϕ_0 , on each diagram. The placement varies slightly, but not significantly, about 240° . Phase-spectral

data have been plotted above this value where organized results were found. Therefore, as the graphs are viewed in order of increasing separation distance, the diagonal line rises and its intersections with the fixed cutoff lines for n_0 and ϕ_0 converge, thereby decreasing the portion of the diagonal line that is followed by organized phase-spectral functions. The graphs show 64 m as the separation distance for which the horizontal and vertical lines intersect the diagonal at almost the point of convergence of these intersections. For greater separation distances the phase spectrum shows little tendency to correlate with the diagonal. Also for the 64-m separation distance, only the phase spectra of the two horizontal components were well enough organized to constitute functions of frequency. If any of these functions are drawn by connecting consecutive data points for each wind component with straight lines, the difference in behavior between the organized portion contained above n_0 and below ϕ_0 , in comparison to the unorganized portion below n_0 , is made even more apparent by the extremes of slope variation in the phase-spectral function in the unorganized region of low frequency.

The summary of results from the 15-m data is:

1. The phase spectra are organized from a frequency of 0.05 Hz up to or beyond the frequency of the coherency limit. This implies that during this test Taylor's hypothesis was valid for all frequencies above the lower limit, notwithstanding the upper limit of coherency.
2. The frequency limit of coherency was approximately 240° . This means that during this test eddies were coherently advected over a maximum distance of about $2/3$ of their own alongwind dimension.
3. Most of the experimental data points are slightly below the line of the ideal spectrum. This indicates that the eddies are traveling faster than the mean wind speed. The ratio of the eddy translation speed to the mean wind speed may be estimated by computing for each separation distance, ξ , the quantity $\xi/U\tau_m$ where τ_m is the time lag of the maximum cross correlation between the parallel components separated by distance ξ . When this was done for all ξ from 4 to 32 m, the average results for u'_a , v'_a , and w' were 1.12, 1.11, and 1.16, respectively. Also, the phase-spectral data fall below the theoretical curve at a distance consistent with a translation speed about 12 percent higher than the mean wind speed.

It may be argued that the first two conclusions depend on the length of the sample used. For example, if 7 minutes of data are analyzed rather than 28, banded spectral estimates for a given frequency from the 7-minute analysis will be based on approximately one-fourth as many degrees of freedom as from the 28-minute analysis. Therefore, if 7-minute data had been analyzed, the lower limit frequency at which Taylor's hypothesis applies and the upper limit frequency of coherency might be found at higher frequencies than in the 28-minute analysis.

To explore this, one 7-minute segment was analyzed for the 4, 8, and 12-m separation distances. This was accomplished by omitting the averaging of the data together, four points at a time, as was done for the 28-minute analysis. The results are shown in Table 2.

TABLE 2

COMPARABLE PHASE SPECTRAL AND COHERENCY ANALYSIS FOR SEVEN-MINUTE DATA

<u>Separation Distance</u> (m)	<u>Lower Limit for T.H.</u> (Hz)	<u>Upper Limit of Coherency</u> (deg)
4	0.05	310
8	0.063	190
12	0.063	230

Altogether, the figures in the table cannot be interpreted to show any reliable tendency of the limit frequencies to shift upward for 7-minute analysis versus 28-minute analysis. In general, the phase spectral functions for the 7-minute analysis follow the ideal phase spectrum with only slightly more deviation than do those for the 28-minute analysis.

Other similar phase and coherency spectral information is available from tests at 30 and 58 m elevation. The tentative conclusions that may be drawn from analysis of all three tests are as follows:

1. The lower limit frequency, n_0 , for Taylor's hypothesis appears to vary inversely with height up to 60 m, if stability is maintained constant. The frequency is higher for unstable conditions.
2. Lin's expression, Equation (2), may be replaced by asserting that Taylor's hypothesis holds in boundary layer for those wave numbers, κ , such that $\kappa \bar{u} \geq 5 d\bar{u}/dz$ for the stable case and such that $\kappa \bar{u} \geq 10 d\bar{u}/dz$ for the unstable case.
3. The statistic obtained by dividing n_0 by u^*/kz appears more stable than Lin's expression, using $d\bar{u}/dz$ in the denominator. This is probably because there is more certainty in measuring u^* than $d\bar{u}/dz$. Adopting $n_0' (u^*/kz)$ for modeling because of its lesser variability,

we find these lower limits for Taylor's hypothesis:

Stable case: $n_o/(u^*/kz) \geq 2/3$

Unstable case: $n_o/(u^*/kz) \geq 1$

4. The length of the longest eddies advected according to Taylor's hypothesis is about ten times the measuring height. The figure is indicated to be greater for stable conditions than for unstable conditions.
5. Computations for all three heights consistently indicate an eddy translation speed slightly greater than the mean wind speed.
6. If a value of e^{-1} is accepted as a lower limit of coherency, eddies are coherently advected over paths not greater than their own alongwind dimension. Height and stability dependence are not yet clear.

Pertinent numerical results are given in Table 3.

TABLE 3

SUMMARY OF PHASE SPECTRAL AND COHERENCY ANALYSIS

	Test Number		
	T603	T703	T105
Height (m)	15	30	58
Stability	Stable	Unstable	Stable
n_o (Hz)	0.050	0.030	0.012
$n_o/(d\bar{U}/dz)$ (cycles)	0.88	1.5	0.55
$n_o/(u^*/kz)$ (cycles)	0.64	1.0	0.67
$\lambda = \bar{U}/n_o$ (m/cycle)	128	233	692
$\lambda/z = (n_o z/\bar{U})^{-1}$	8.5	7.8	11.9
$\xi/\bar{U}\tau_m$ (average value)	1.13	1.07	1.09
ϕ_o (deg)	240	240	340

Analytically it can be shown that the ratio λ/z should increase with height. In keeping with the results of this writing, Equation (2) can be altered to indicate that Taylor's hypothesis holds for all wave numbers, κ , such that

$$\kappa \bar{U} \geq A \, d\bar{U}/dz \quad (5)$$

where A is a constant for a given stability. We may continue by substituting $2\pi/\lambda$ for κ , and $(u^*/k) \ln(z/z_o)$ for $\bar{U}$, from the logarithmic wind law for neutral conditions. The result is that Taylor's hypothesis holds for all wave lengths, λ , such that

$$\lambda/z \leq (2\pi/A) \ln(z/z_o) \quad (6)$$

Thus, the increase of λ with height is slightly more than linear. However, the results thus far do not allow detailed definition of height and stability dependence.

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Figure 1. Time and space correlations, Test T603

Figure 2. Phase spectra - separation distances 4 to 16 m, Test T603

Figure 3. Phase spectra - separation distances 24 to 64 m, Test 603

MEASUREMENTS AND GRAPHS OF TURBULENCE AUTOCORRELATIONS IN SPACE AND TIME*

C. E. Elderkin and D. C. Powell
Atmospheric Sciences Department
Battelle, Pacific Northwest Laboratories
Richland, Washington 99352

INTRODUCTION

In 1967 Gifford⁽¹⁾ elucidated a point of view holding that there are four distinct coordinate reference systems in which turbulence description is pertinent. The Eulerian fixed-point system (I) refers to turbulence as experienced by fixed-point measuring instruments. The Eulerian space system (II) is a "snapshot" of turbulence. The quasi-Lagrangian system (III) refers to turbulence as measured at a point moving with the mean motion. The Lagrangian system (IV) refers to the motion of a single fluid particle. Gifford also identified several writers who, from an intuitive basis, consider systems (III) and (IV) to be equivalent. Csanady⁽²⁾ also accepts this as being approximately true.

Because system (I) is a convenient frame of reference for measurement, inferences about turbulence in the other reference frames are often made from the Eulerian fixed-point description. By extension of Eulerian fixed-point measurements to include a line of sensors logarithmically spaced in the direction of the mean motion, a description of turbulence in system (III) may be simulated. The primary purpose of this paper is to describe this experimental approach and to indicate some initial results therefrom.

The assumption of Taylor's hypothesis is involved in relating turbulence in system (I) to turbulence in system (II). Details of this problem are the subject of another paper⁽³⁾ by the same authors.

Relating systems (I) and (IV) we have the Hay-Pasquill beta hypothesis

$$R(\tau_E) = R(\tau_L) \quad ; \quad \tau_L = \beta \tau_E \quad (1)$$

In Equation (1), R is a correlation function for any of the three turbulence components, τ is time lag, the subscripts E and L indicate Eulerian and Lagrangian, respectively and β is assumed to be a constant.

The assumption expressed by Equation (1) is more restrictive than the following expression given by Gifford.

$$\beta = n_T/n_L \quad (2)$$

where n_T and n_L are characteristic frequencies in systems (I) and (IV). Equation (1) states that the spectra for a given turbulence component in systems (I) and (IV) are the same function distinguished only by transposition. Equation (2) does not involve this assumption. The β defined therein can be an average value of a proportionality factor, i.e., a function of frequency. Neither Equations (1) or (2) preclude the likelihood, expressed by F. B. Smith,⁽⁴⁾ that β is different for each turbulence component.

The assumption that systems (III) and (IV) are approximately equivalent may be expressed by

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$$R(\tau_L) = R(\tau, \xi) \quad , \quad \xi = \bar{U}\tau \quad (3)$$

The space lag, ξ , is measured in the longitudinal direction. By combining Equations (1) and (3) we get another expression relating systems (I) and (III) assuming that the Eulerian correction function differs from the quasi-Lagrangian correlation function by only the scale factor, β . This corresponds to the investigation that is of interest in the present paper and is expressed by

$$R(\tau_E) = R(\tau, \xi) \quad , \quad \begin{array}{l} \text{requiring } \xi = \bar{U}\tau \\ \text{assuming } \tau = \beta\tau_E \end{array} \quad (4)$$

INSTRUMENTATION AND FIELD ARRAY

The multi-point turbulence measurement system, providing the data for this study, utilizes sensors mounted on an array of towers for measurement of turbulence fluctuations in the three components of the wind at various heights and horizontal spacings, and under various meteorological conditions. The measurement array is shown in Figure 1. The data are collected in the field on analog recorders, played back into a medium-range computer for preprocessing the data, and are re-recorded on industry standard tapes for later detailed processing and analysis on a UNIVAC 1108 computer.

Two types of turbulence sensors are utilized: fast response sonic anemometers for the most accurate determinations of momentum and heat fluxes used in characterizing the stability of the atmosphere and for spatial correlations over small separation distances; and less expensive Gill three-propeller anemometers for the greater number of measurements at large separation distances where their slower response is adequate.

The mountings for both types of instruments are constructed in a manner to assure true sensor leveling and optimum orientation into the wind at inaccessible remote locations on tower tops and on the ends of tower booms. This has been accomplished by utilizing remotely monitored electrolytic levelers on each sensor and an antenna rotor base on each sensor for remote reorientation from centrally located control trailers.

The sensors are mounted on four permanently erected 62-m towers, and eight 30-m expandable portable towers which can be repositioned for various measurement configurations to meet different experimental requirements. The sensor booms can be moved to various heights on the 62-m towers and the sensors on the tops of the 30-m towers can be raised and lowered to measure the turbulence at selected heights.

The two control trailers, downwind of the tower array, contain the signal conditioning and data recording equipment. Two 14-channel analog magnetic tape recorders are generally used to record the three wind component signals from each of three sonic anemometers and five Gill anemometers, and the temperature signals from each of two sonics. One channel on each recorder is used for a coordination timing signal.

For the experiment to be discussed in this presentation, data from seven of the sensors were utilized in the analysis. The sensors were located on the expandable towers adjusted to 15-m height and logarithmically spaced along the mean wind direction at 4, 8, 16, 32, 64, and 128-m separations between consecutive sensors. Sonic anemometers were located at the shortest intervals — 4 m separating the first and second and 8 m between the second and third. Gill anemometers were located at the remaining four sampling positions.

DATA PROCESSING

After each experiment, tape recordings of the field data are taken to a computer laboratory where the data are played back from one of the analog tape units into an analog-to-digital converter

which is interfaced to an SEL 840A medium-range computer. The computer is programmed to test the data for reasonableness and to perform preliminary editing; average a selectable number (usually 5) of the 100 to 200 basic samples per second together to minimize aliasing and provide a range of optional digitizing rates; make instrument corrections such as conversion of sonic nonorthogonal measured components to cartesian wind components; perform scaling functions to account for gains and offsets used during measurements; and finally, to record digital values of the three orthogonal wind components and the temperature on an industry standard digital tape recorder interfaced with the SEL 840A.

The UNIVAC 1108 programs perform more refined editing and correct Gill anemometer data for deviations of the propellers from a true cosine response, which is necessary for adequate turbulence statistics, before re-recording the true wind components in cm/sec and temperature in hundredths of a Celsius degree on "master tapes." These are then processed by the probability sub-routines which, for selectable time periods within the total data segment, calculate and remove means for each of the variables; reorient the coordinate system in the direction of the mean wind, if desired; provide up to three different high-pass filterings of the data and the probability density functions — individual and joint — for each when requested; and calculate variances for each component and covariances between them, as well as the third and fourth moments. An option to plot any of the time series in the data segment being analyzed is also available. The unfiltered turbulence variables are then selected, two at a time, for analysis by the spectral sub-routines. Here, the data are optionally detrended with the best fitting first or second order polynomial for each time series and tapered to prevent ringing oscillations in the fast Fourier transform calculated spectra. Then the Fourier coefficients are calculated from which banded power spectral and cross-spectral estimates are formed. The auto- and cross-correlation functions can, in turn, be transformed from these estimates if required. Variances and covariances for the detrended time series are also available from this program.

ESTIMATION AND CALCULATION OF THE EULERIAN SPACE COVARIANCES

Each set of two towers in the logarithmic array is separated by a unique distance. Therefore, genuine spatial covariances, computed by averaging over space, are not available from these data. However, by cross-analyzing parallel components measured at distinct separation distances, a pseudo-spatial covariance may be defined as given below

$$R_{\alpha\alpha}(\xi) = E[u_{\alpha}(t, x_0) u_{\alpha}(t, x_0 + \xi)] \quad , \quad \alpha = 1, 2, 3, \text{ no summation} \quad (5)$$

Here the expectation is a time average; and x_0 and $x_0 + \xi$ are the locations of two measuring points. Pseudo-spatial correlations computed in this manner will be referred to simply as spatial correlations hereafter. From a line of 7 positions, 21 distinct separation distances are available for calculating spatial correlations. The correlations for both space and time may be defined by extending the same principle to a correlation function of two arguments. The result, normalized, may be written

$$R_{\alpha\alpha}(\tau, \xi) = E[u_{\alpha}(t, x_0) u_{\alpha}(t + \tau, x_0 + \xi)] / E[u_{\alpha} u_{\alpha}] \quad (6)$$

An example of this type of turbulence analysis is given by Lumley and Panofsky⁽⁵⁾ in their Figure 5.22. Such two dimensional correlation function analyses have been calculated for each turbulence wind component for the one experiment discussed here where the array of sensors was at 15 m height, the wind direction was 9° to the line of sensors at 6.4 mps, and slightly stable conditions prevailed. The analysis was for a 28 minute data sampling period with each variable sampled at 0.2 second intervals.

PROPERTIES OF TURBULENCE ILLUSTRATED IN DIAGRAMS OF TIME-SPACE CORRELATIONS

The time-space correlation functions for the three turbulence wind components: u'_a , the horizontal component along the tower line; v'_a , the horizontal component normal to the tower line; and w' , the vertical component, are given in Figures 2, 3, and 4, respectively. These diagrams may be interpreted in several interesting ways. First, it will be noted that the function marked off by the isopleths on the abscissa is a pure time correlation, and is necessarily symmetrical. The function marked off by the isopleths on the ordinate is the spatial correlation. The scales of the two axes are adjusted to be equal when $\xi = \bar{U}\tau$. The correlation at a point moving with the mean motion is the function marked off by the isopleths on the 45° line labeled $\xi = \bar{U}\tau$.

Also, Taylor's hypothesis predicts that the same identical function is marked off on the abscissa and the ordinate. The isopleths of correlation in Figures 2, 3, and 4 should be ellipses with their major axes along the line $\xi = \bar{U}\tau$. If the major axes of the ellipses fall, for instance, above the 45° line, an eddy transport speed greater than $\bar{U}$ is indicated.

Assuming that the quasi-Lagrangian correlation function is a good approximation to the true Lagrangian correlation function, Equation (4) predicts that the function marked off on the 45° line will differ from that marked off on the abscissa by no more than a proportionality factor, β . Comparing lag time, τ , for equal values of correlation, the first of which is on the 45° line and the second of which is on the abscissa, provides a ratio which is an estimate of β . If Equation (4) holds for system (III), the value of β should be approximately the same regardless of where on the correlation diagram it is determined. A large volume of these graphs should show reliably whether or not β varies systematically with correlation length and from one turbulence component to another, as well as demonstrate its dependence as a function of stability or turbulence intensity.

The eccentricity of the ellipses indicates to what extent the frozen field hypothesis holds. The strict frozen field hypothesis requires that the isopleths in the diagrams are straight lines parallel to the line, $\xi = \bar{U}\tau$, and symmetrically located on each side of that line. The eccentricity should, in general, be inversely proportional to the relative turbulence intensity and directly proportional to the value of β .

The principle axis of the correlation contours appears to be slightly above the $\xi = \bar{U}\tau$ line for all three components but is most clearly shown for w' in Figure 4, indicating an effective eddy translation speed exceeding the mean wind speed. This feature is more obvious from the analysis discussed in the accompanying paper given by D. C. Powell.⁽³⁾

From Figures 2, 3, and 4, we see that the eccentricity (and thus β) is least for the u'_a component graph and the greatest for v'_a . This does not correspond to an inverse ranking of the relative turbulence intensities since $\overline{u'^2} = 1.80 \text{ m}^2/\text{sec}^2$, $\overline{v'^2} = 0.77 \text{ m}^2/\text{sec}^2$, and $\overline{w'^2} = 0.36 \text{ m}^2/\text{sec}^2$. This indication that β is not the same for each component is more clearly shown in Figure 5. It is also seen that some tendency toward smaller β at larger τ is suggested. However, this suggestion that the quasi-Lagrangian and Eulerian fixed-point correlation functions are not truly similar is premature, since sensor characteristics could account for the small variations (smaller than between components) observed. The β values of about 5.3 for the lateral wind component and 3.9 for the vertical component are consistent with other evaluations of β . Using the turbulence variances given above and the mean wind speed, 6.4 mps, turbulence intensities of 0.138 and 0.056 can be calculated for the lateral and vertical components, respectively. Using these values and the values of β just given, comparisons may be made with values of β versus the reciprocal of the intensity for the lateral component on a graph given by Gifford⁽¹⁾ and of β versus the intensity of the vertical component on graphs given by Angell et al.⁽⁶⁾ In both cases the positions on these graphs found by the method used in this investigation are intermediate when compared to those of

other investigators. This suggests that the technique for studying β described here is a useful approach.

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Figure 1. Measurement array

Figure 2. Autocorrelation function of u' in time and space

Figure 3. Autocorrelation function of v' in time and space

Figure 4. Autocorrelation function of w' in time and space

Figure 5. Estimates of β for the three wind components as a function of Eulerian time lag

ANALOG, DIGITAL AND HYBRID SIMULATION OF A PLANETARY

BOUNDARY LAYER METEOROLOGICAL FORECAST MODEL

M. Don Merrill
Associate Professor
Electrical Engineering
New Mexico State University
Las Cruces, New Mexico

Gary A. Ethridge
Mathematician
Atmospheric Sciences Laboratory
White Sands Missile Range, New Mexico

Norman E. Arnold
Second Lieutenant
United States Army
Ft. Bliss, Texas

INTRODUCTION

During the period from 1959 through 1969 Dr. William H. Clayton was principal investigator for work being done on a Low-Level Meteorological Model (LLMM). This work took place at Texas A&M University with funds provided by the United States Army Signal Corps.

During this time a mathematical format of the LLMM was developed and implemented on four Comcor analog computers. A twelve hour forecast test was developed for the LLMM and the results of this test were reported in (3).

The above analog method is a moderately complex atmospheric model and requires approximately 500 amplifiers when implemented on an analog computer.

Since this model was developed and implemented, funding has not been available for thorough verification. But even if the mathematical format is not totally accurate, it is a sufficiently complex problem to provide a means for comparing the analog, digital and hybrid computer solutions techniques for problems of this type.

The equations that were used in the simulation are a rearrangement of the original mathematical format for the Low-Level Meteorological Model as given in (2).

The mathematical format as stated in (1) was used as the starting point for writing the digital, analog, and hybrid computer programs for the LLMM. This format is given in Appendix A.

The LLMM is divided into three parts, all of which are solved simultaneously to obtain information primarily about winds, temperature, and vapor pressure from ground level to one thousand meters.

The first part is the soil temperature model which gives a temperature gradient from two meters below ground level to ground level. The surface soil temperature from this section is used in the second part of the model.

The second part is the transition between the ground and the atmosphere. The result is an adjusted ground level air temperature.

The third part is the atmospheric model which extends from ground level to one thousand meters and forecasts winds, temperatures, and vapor pressures.

DIGITAL SIMULATION

The Low-Level Meteorological Model is described by a series of ordinary differential equations. To solve these equations, an IBM system/360 program for the simulation of continuous systems was first used. The Continuous System Modeling Program (CSMP) has an input language that will accept a system of ordinary differential equations.

Once the equations are implemented, the solution is reached by iteration from problem time zero through some forecast interval by some arbitrary time step. By this technique it is possible to simulate a continuous system. The size of the time step has an effect on both the stability and the accuracy of the solution.

CSMP provides seven integration routines. Milne and Runge-Kutta are variable-step routines; a second Runge-Kutta, Simpson's, trapezoidal, Adams, and rectangular are fixed-step integration routines. To get a cross-section of the effect of the integration method on computation time and the

primary system variables, four of the seven integration methods were used: Milne, both Runge-Kutta's, and trapezoidal.

Effect of Integration Routines

The digital simulation of the LLMM uses Test Case A (3) as a basis for all evaluations. Test Case A is a twelve hour forecast test case that was originally used in the work with the analog model. From each run, plots were made of the following variables: u versus v ; and T , e , S_d , RN , $q_{e,o}$, $q_{c,o}$, $\cos \zeta$, T' , $K_{m,8}$, τ_o , $\tau_{y,o}$, S_2 , S_8 versus time.

These plots were compared point by point with each other and found to match exactly. It can be concluded that for the integration step size of 30 seconds and the integration interval of 12 hours, the integration method has no effect on the primary variables. These integration methods did effect the execution time of the digital computer program as shown in Table 1. The conclusion drawn here is that Milne, one of the most sophisticated and least time consuming integration methods, is a good choice until such time as a more exhaustive check can be made using a wide range of integration step sizes and intervals.

One other observation was made concerning the integration step size. When the integration step size using the rectangular integration method is increased above 60 seconds the resulting time solution is unstable.

ANALOG SIMULATION

The equations of Appendix A were amplitude scaled by Dr. Clayton for use in their analog simulation and are found in (1). These amplitude scaled equations were used in both the analog and hybrid computer simulations of this paper.

An initial attempt to duplicate the meteorological forecast model solution at White Sands Missile Range was made during the summer of 1969. Approximately one week was spent at the Texas A&M College with Dr. W. Clayton learning the analog format. The programming was then begun on the analog computers made available by flight simulation at White Sands Missile Range. The three analogs required to meet the demands of the model consisted of two EAI-231R analog computers and one EAI-24D analog computer. Just after the first of the year, 1970, the model at White Sands Missile Range successfully duplicated the results of W. Clayton on the A&M computers.

HYBRID SIMULATION

The analog solution of the meteorological forecast model, which Dr. Clayton developed, requires a considerable amount of analog computing equipment.

The objective of the hybrid approach is to replace all such analog gear with one self-contained general purpose analog computer together with one medium sized digital computer.

The hybrid computing system used in this study consists of an EAI 8800 electronic analog computer linked to a DDP-24 solid state digital computer. The 8800 is a 100 volt solid state general purpose analog computer with extensive logical computing capacity and logic control. It has a capacity of 338 computing amplifiers, 48 quarter square multipliers, 6 resolvers, and in addition to manual set-up function generators, there are also 6 card set-up function generators. All of the 240 pots are servo-set either from the analog console or through the digital pot setting program.

The digital computer has 8k memory locations all directly addressable. The word length is 24 bits, arithmetic operations are parallel binary, and the memory cycle time is 5 microseconds. Input-output is by typewriter or papertape. The DDP-24 can be programmed in symbolic assembly language or

in Fortran.

The equations of the boundary layer model have been treated by means of the method first described in the early 1900's as the "method of lines". The given partial differential equation is reduced, by discretization of the space variable, to a system of ordinary differential equations in the time variable. This system may be solved by analog, digital, or hybrid computing methods.

Given a set of differential equations, the digital solution time is proportional to the number of equations due to its serial nature of calculation. The analog computer run time for a system of equations, given enough computing elements, is independent of the number of equations since it is a parallel computing machine.

For a large problem the requirements for analog gear can easily tax most analog systems beyond their capability. Hybrid computing methods can handle these larger problems. The hybrid bridges the gap between analog and digital computers. It maintains the analog function of rapid integration and yet provides for mass data storage capability. Also many problems call for operations like matrix manipulations, solution to transcendental equations and other arithmetic operations which are more suitable to digital techniques.

The hybrid solution method utilizes a procedure known as cell multiplexing, or component time sharing. The time sharing method involves another hybrid function known as sample hold and playback. In this hybrid feature the analog is run for a very small time period and upon a command from the digital the analog is put into the hold mode during which time the output voltages on the amplifiers are sampled and converted to digital form and stored in memory.

Before the analog is returned to the operate mode, the digital inputs to the analog are updated with more timely data.

As an example of the DSCT method using component time sharing, let us compare the analog and hybrid techniques for a one dimensional diffusion equation. (fig. #1). After the partial differential equation is transformed into the set of ordinary differential equations one observes that the so called analog cells are simply linked together with one cell for each discrete space point x_j . The analog solution can be obtained on one run.

The hybrid diagram reflects the time sharing of one common cell, based on the analog approach. For each x_j the sample hold and playback functions are performed. Upon completion of the x_j -th level the digital inputs to the analog cell are reformulated to account for the next level, x_{j+1} , which is to be integrated. Cycling through all the discrete levels generates the function $f(x,t) = y$. Whether or not y is a sufficiently accurate solution to the problem must be determined. The above formulated procedure is repeated using the stored values $f(x,t)$ as new inputs to the analog cell at respective levels. The latest solution is then compared to the last one to determine if the functional values exhibit much change. The solution is said to have converged when the changes are less than some prescribed value.

Application to the Forecast Model

There are five parameters of the forecast model which require the DSCT approximations. The equations in the DSCT format are equations 1-5 of Appendix A. Hence if time sharing were used on the five parameters U , V , T , E , T' , only five integrators would be necessary. However the soil equation (4) is completely hardwired as in the analog solution. Only U , V , T , and E are time shared. In addition the entire 8-meter level for all these parameters are also hardwired as in the all analog solution. Figure #2 shows the four cells to be multiplexed over the remaining levels z_j , $j = 2, 3, \dots, 12$.

COMPARISONS OF ANALOG, DIGITAL, AND HYBRID MODELS

In Table I, the analog, digital and hybrid computer computation times are given which indicate the relative speed at which each of these computer types was able to solve the problem of a twelve hour forecast using the LLM. The analog has the fastest run time is to be expected because of the completely parallel operation. The hybrid solves each level in a parallel manner but is slower because level to level is solved serially. The digital is much slower because the entire solution is serial.

There is more to be considered than just computation time. System costs, availability, and difficulty of programming must be considered. System cost by the hour is approximately \$200 per hour for the analog and hybrid and \$360 per hour for the digital. The availability of systems large enough to handle this size of problem is in reverse order of computation time. Digital computers of the size of the IBM System/360 Model 50 and larger are readily available. Hybrid systems that have 120 amplifiers are somewhat less available, and analog systems that contain 500 amplifiers are not at all readily available. The time necessary for development of a model this large is also the computation time. The digital simulation treated in this paper required approximately three months to develop. The hybrid model that was developed required four months to complete by starting with the mathematical format already scaled. It would have required approximately another three months to generate the scaled equations from the original mathematical format, or a total of seven months to do the equivalent of what was done in three months on the digital. The analog model development time was estimated to require nine months.

CONCLUSION

The results for some of the parameters of the forecast model are shown in Fig. 3, 4 and 5. In general there is excellent agreement between the digital and analog simulating but only for certain parameters is there good agreement in the Hybrid solution. The Hybrid plot is for only four hours of forecast time because the solution for the momentum exchange coefficient yielded a completely unstable system for times greater than this. It is the opinion at this time that a programming error has caused the unstable solution for this parameter.

From the author's experience in programming this problem, the course that would be followed in future problems would be to use digital simulation methods for initial program development and after a successful model has been achieved, convert the problem to a Hybrid solution to make use of the faster solution time for production runs.

APPENDIX A

The mathematical format of the LLM that was used in the computer simulations is as follows (1).

$$u_j = (1/d) * [-g \partial h / \partial x|_{z_j} + f v_j + (1/\rho \Delta z_j) (\tau_{x,j,k} - \tau_{x,i,j}) - \alpha_j u_j + \beta_j v_j]^\dagger \quad (1)$$

$$v_j = (1/d) [-g \partial h / \partial y|_{z_j} - f u_j + (1/\rho \Delta z_j) (\tau_{y,j,k} - \tau_{y,i,j}) - \alpha_j v_j - \beta_j u_j] \quad (2)$$

* $(1/d)$ is the integral operator

†Subscripts and meaning of attached subscripts:

- j = 1, 2, 3, 4, ..., 12 = index of layer or level
- i = $j - 1$
- k = $j + 1$
- Δz_j = thickness of j th layer

u_j = East-West wind component at level j
 $\tau_{x,j,k}$ = x component of shearing stress at interface of j th and k th layers
 z_j = elevation of the j th level
 $z_{i,j}$ = mean elevation between i th and j th levels

$$T_j = (1/d) [R - (1/\rho C_p \Delta z_j) (q_{c,j,k} - q_{c,i,j}) - u_j \partial T / \partial x|_{z_j} - v_j \partial T / \partial y|_{z_j}] \quad (3)$$

$$T'_j = (1/d) [(1/\lambda \Delta z_j) (q_{s,j,k} - q_{s,i,j})] \quad (4)$$

$$e_j = (1/d) [- (P_j / \alpha \rho L \Delta z_j) (q_{e,i,j}) - u_j \partial e / \partial x|_{z_j} - v_j \partial e / \partial y|_{z_j}] \quad (5)$$

$$E = (1/d) q_{e,o} / L \quad (6)$$

$$\tau_o = \rho D_8 S_8 \quad (7)$$

$$\tau_{x,o} = \rho D_8 u_8 \quad (8)$$

$$\tau_{x,16} = \rho K_{m,16} (u_{32} - u_8) / 1600 \ln(32/8) \quad (9)$$

$$\tau_{x,64} = \rho K_{m,64} (u_{100} - u_{32}) [1/6800 + 1/6400 \ln(100/32)] / 2 \quad (10)$$

$$\tau_{x,i,j} = \rho K_{m,i,j} (u_j - u_i) / (z_j - z_i) \quad (11)$$

$$\tau_{y,o} = \rho D_8 v_8 \quad (12)$$

$$\tau_{y,16} = \rho K_{m,16} (v_{32} - v_8) / 1600 \ln(32/8) \quad (13)$$

$$\tau_{y,64} = \rho K_{m,64} (v_{100} - v_{32}) [1/6800 + 1/6400 \ln(100/32)] / 2 \quad (14)$$

$$\tau_{y,i,j} = \rho K_{m,i,j} (v_j - v_i) / (z_j - z_i) \quad (15)$$

$$q_{c,o} = -\rho C_p D_8 (T_8 - T_o) K_n / K_m \quad (16)$$

$$q_{c,16} = -\rho C_p K_{h,16} (T_{32} - T_8) / 1600 \ln(32/8) \quad (17)$$

$$q_{c,64} = -\rho C_p K_{h,64} (T_{100} - T_{32}) [1/6800 + 1/6400 \ln(100/32)] / 2 \quad (18)$$

$$q_{c,i,j} = -\rho C_p K_{h,i,j} (T_j - T_i) / (z_j - z_i) \quad (19)$$

$$q_{e,o} = -\rho L a D_8 (e_8 - e_o) K_v / P_o K_m \quad (20)$$

$$q_{e,16} = -\rho L a K_{v,16} [P_8 (e_{32} / P_{32}) - e_8] / P_8 1600 \ln(32/8) \quad (21)$$

$$q_{e,64} = -\rho L a K_{v,64} [P_{32} (e_{100} / P_{100}) - e_{32}] [(1/6800) + 1/6400 \ln(100/32)] / 2 P_{32} \quad (22)$$

$$q_{e,i,j} = -\rho L a K_{v,i,j} [P_i (e_j / P_j) - e_i] / P_i (z_j - z_i) \quad (23)$$

$$q_{s,o} = (T_o - T'_o) / G \quad (24)$$

$$q_{s,i,j} = \mu_{i,j} (T'_j - T'_i) / (z_j - z_i) \quad (25)$$

$$f = 2\omega \sin \phi \quad (26)$$

$$P_j = P_o - \rho g z_j \quad (27)$$

$$K_h = 1.4 K_m \quad (28)$$

$$K_v = 1.6 K_m \quad (29)$$

$$K_m = (133 K_{m,8}) (z/2) (1 - z/2)^2 \quad (30)$$

$$K_{m,8} = 800 k^2 S_8 [1 - C \sqrt{800} (T_8 - T_o) / k S_8^2]^2 / \ln(800/z_o) \quad (31)$$

$$D_8 = k^2 S_8 [1 - C \sqrt{800} (T_8 - T_o) / k S_8^2]^2 / (\ln(800/z_o))^2 \quad (32)$$

$$\partial h / \partial x|_j = \partial h / \partial x|_i + \Delta h_{i,j} \partial T / \partial x|_{k+1} / 293 \quad z_j \geq 100m \quad (33)$$

$$\partial h / \partial y|_j = \partial h / \partial y|_i + \Delta h_{i,j} \partial T / \partial y|_{k+1} / 293 \quad z_j \geq 100m \quad (34)$$

$$\partial h / \partial x|_o = ICG \sin(ICG) + [FCG \sin(FCG) - ICG \sin(ICG)] t / \Delta t \text{ forecast} \quad (35)$$

$$\partial h / \partial x|_o = ICG \cos(ICG) + [FCG \cos(FCG) - ICG \cos(ICG)] t / \Delta t \text{ forecast} \quad (36)$$

$$R_N - q_{c,o} - q_{s,o} - q_{s,o} = 0 \quad (37)$$

$$R_N = (1 - j) S_d - \epsilon \sigma [T_o^4 - T_g^4 (m + n \sqrt{e_o})] \quad (38)$$

$$S_d = F_c I \Psi \cos \zeta e^{-N / \cos \zeta} \quad (39)$$

$$\cos \zeta = \sin \phi \sin \delta + \cos \phi \cos \delta \cos H \quad (40)$$

$$e_o = e_{o,s} - q_{e,o} / S_o \text{ if } q_{e,o} > 0 \quad (41)$$

$$e_o = e_{o,s} \text{ if } q_{e,o} \leq 0 \quad (42)$$

$$e_{o,s} = 6.11 \times 10^{10} [7.5 T_o / (237.3 + T_o)] \quad (43)$$

$$S_2 / S_g = (e_2 - e_o) / (e_g - e_o) = (T_2 - T_o) / (T_g - T_o) = [\ln(200/z_o) + C\sqrt{200} \theta_{*}/u_{*}^2] / [\ln(800/z_o) + C\sqrt{800} \theta_{*}/u_{*}^2] \quad (44)$$

$$\theta_{*}/u_{*}^2 = K_m q_{c,o} \sqrt{\rho/c_p} K_h \tau_o^{3/2} \quad (45)$$

$$S_g = \sqrt{u_g^2 + v_g^2} \quad (46)$$

$$u_{geo} = -g \partial h / \partial y|_o / 2\omega \sin \phi \quad (47)$$

$$v_{geo} = g \partial h / \partial x|_o / 2\omega \sin \phi \quad (48)$$

$$H = (1/d) \dot{H} \partial t = (1/d) (1/240) \frac{\text{deg}}{\text{sec}} dt \quad (49)$$

$$T_2 = (T_g - T_o) [(\ln \frac{200}{z_o} + C 200 \theta_{*}/u_{*}^2) (\ln \frac{800}{z_o} + C 800 \theta_{*}/u_{*}^2)] - T_o \quad (50)$$

$$e_2 = (e_g - e_o) [(\ln \frac{200}{z_o} + C\sqrt{200} \theta_{*}/u_{*}^2) / (\ln \frac{800}{z_o} + C\sqrt{800} \theta_{*}/u_{*}^2)] - e_o \quad (51)$$

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TABLE I - Execution Time for Analog, Digital and Hybrid Computers (12 hours Forecast)

Analog		.5 minutes
Digital*	Mil- Runge-Kutta Variable Step	8.18 9.37
	Runge-Kutta Fixed Step	15.35
	Trapezoidal	10.82
Hybrid (Projected from a 4 hour forecast)		5 minutes

*12 hour forecast using 30 second step size - Time includes compilation time

DSCV

$$\frac{1}{\partial^2} \frac{\partial \varphi}{\partial t} = \frac{\partial^2 \varphi}{\partial x^2}$$

$$\left. \frac{d\varphi}{dt} \right|_{x=x_i} \approx \frac{1}{\Delta x^2} [\varphi_{i+2\Delta x} - 2\varphi_{i+\Delta x} + \varphi_i]$$

(a)

(b)

(c)

(a) PARABOLIC DIFFUSION EQUATION WITH DSCV APPROXIMATION. (b) ANALOG SOLUTION. (c) HYBRID SOLUTION.

DIGITAL TERMS

$$\sum_i q_i, U \frac{\delta E}{\delta x}, V \frac{\delta E}{\delta y}$$

DIGITAL TERMS

$$\sum_i q_i, U \frac{\delta T}{\delta x}, V \frac{\delta T}{\delta y}$$

DIGITAL TERMS

$$\sum_i \tau_i, \alpha V, BU$$

DIGITAL TERMS

$$\sum_i \tau_i, \alpha U, DV$$

Fig. 2 Basic diagram of Hybrid solution

Fig. 1 Example of DSCV

A DIFFERENTIAL METHOD FOR THE PREDICTION OF THE EFFECTS OF ATMOSPHERIC BOUNDARY
LAYER TURBULENCE USING THE TURBULENCE KINETIC ENERGY EQUATION

William M. Byrne, Jr.
Project Engineer
Beech Aircraft Corporation

and

Shen C. Lee
Associate Professor of Mechanical and Aerospace Engineering
University of Missouri at Rolla

NOMENCLATURE

a, b, c, d	coefficients in the generalized parabolic equation
a_1, a_2	empirical constants
c_j	volume density of fluid j
D_k	dissipation of turbulence kinetic energy
F	blowing parameter
h	static enthalpy
$\bar{h}$	stagnation enthalpy
h_j	chemical energy released by fluid j
k	turbulence kinetic energy
p	static pressure
r	distance from centerline of axisymmetric body
R_j	production of species j due to chemical reaction
u	mean velocity in the streamwise direction
u_∞	free stream velocity
v	mean velocity normal to the wall
x	distance in the streamwise direction
y	distance normal to the wall
α	exponent differentiating planar and axisymmetric flow
ϵ	effective viscosity
ρ	density
σ_{c_j}	exchange coefficient of fluid j
σ_k	exchange coefficient of turbulence kinetic energy
σ_h	exchange coefficient of enthalpy
τ	shear stress
τ_m	maximum shear stress
φ	general dependent fluid property (u, k, h, c_j)
ψ	stream function
ψ_I	the wall at a given x location
ψ_E	the outer edge of the boundary layer at a given x location
w	nondimensional cross stream distance

I. INTRODUCTION

The effect of air pollution on the health, economic, and social environments throughout the world is certainly one of the most pressing problems imaginable. The extent of the problem has been recently dramatized when a court order was issued requiring shutdown of various industries in Birmingham, Alabama, because the pollutant level had exceeded an acceptable health standard. The subsequent loss of production and wages, as well as legal costs, demands that the fluid mechanics of the problem be attacked from every available resource to help prevent the problem from getting entirely out of hand. Since it is difficult (if not impossible) to control atmospheric conditions, the atmospheric surface layer where the majority of the present air pollution problems exist should be studied in detail to gain an understanding of those processes which control the dispersion of any pollutants allowed to enter the atmosphere. Significant progress toward achieving such an understanding has been made by Cermak⁽¹⁾ and others in developing wind tunnel facilities and techniques for the laboratory simulation of atmospheric motions. It is felt that a general technique for predicting fundamental turbulent transport processes would provide a useful tool for coordinating laboratory atmospheric simulations, field measurements, and available data on turbulent mixing. It is hoped that the development of such a tool could then lead to a greater understanding of the dispersion processes for the objective of more reliably modeling the atmosphere so that reasonable and effective legal requirements can be established to control the pollution of our environment.

The authors have taken a step toward this objective by focusing on the atmospheric surface layer where the majority of the pollution effects presently exist while searching for a universal tool to solve the overall problem. The phenomena of interest here is known to be turbulent in nature. A survey of the field indicated that there was no universally accepted theory of turbulence and that several different theories have proved their usefulness for the solution of actual problems. The most generally used theory was found to be the mixing length theory originally proposed by Prandtl⁽²⁾. It is probably safe to say that this theory (and modifications thereof) has been applied to solve more engineering problems successfully than any other and has in any case played an important role in the solution of turbulent flow field problems. It can be applied to any turbulent flow problem for which a sufficient amount of turbulent flow field data is available. However, a great deal of detailed data on atmospheric turbulence is not presently available nor is it likely to be because of the extreme cost of such measurements. Therefore, we found ourselves asking the same question Mr. Donaldson⁽³⁾ has asked, "Can we not find a somewhat more all-embracing method of turbulent transport computation so that one can compute the general transport properties of a large variety of turbulent flows for which empirical data may not be available through the proper use of the data on turbulent mixing that are already in our possession?" The method we have settled on is one related to one proposed by Bradshaw et al⁽⁴⁾ in which the governing system of equations is closed by solving the turbulence kinetic energy equation along with the other equations of interest and making use of an empirical relationship between the magnitude of the local turbulence kinetic energy and the local Reynolds shear stresses. The body of this paper then presents the governing equations, some insight into the method of solution including a "slip" boundary condition for the turbulence kinetic energy equation, and some comparisons between predicted and measured turbulent boundary layers showing that the computation method is applicable to fully developed equilibrium boundary layers, a "relaxing" boundary layer without pressure gradient, boundary layers in the presence of favorable and positive pressure gradients, and favorable pressure gradient turbulent boundary layers with blowing or suction.

II. GOVERNING EQUATIONS

The governing equations of the two-dimensional turbulent boundary layer are those of continuity, momentum, turbulence kinetic energy, total enthalpy, and species. These equations will simply be stated here in the interest of brevity since their derivation may be found elsewhere⁽⁵⁾. The "steady" continuity equation:

$$\frac{\partial}{\partial x} (r^\alpha \rho u) + \frac{\partial}{\partial y} (r^\alpha \rho v) = 0 \quad (1)$$

α is equal to zero in the case of planar flows and unity in the case of axisymmetric flows.

The longitudinal equation may be condensed from the Navier-Stokes equations using order of magnitude arguments by assuming that: (1) distances in the cross stream direction are small compared to longitudinal distances, (2) the mean velocity in the direction normal to the plane in which the streamwise velocity vector lies is small, and (3) the velocity in the streamwise direction is large compared to the velocity in the cross stream direction, in which case it becomes:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = - \frac{dp}{dx} + r^{-\alpha} \frac{\partial}{\partial y} (r^\alpha \epsilon \frac{\partial u}{\partial y}) \quad (2)$$

where the effective viscosity is defined by:

$$\epsilon = \frac{\tau}{\partial u / \partial y} \quad (3)$$

The turbulence kinetic energy equation may be written as⁽⁵⁾:

$$\rho u \frac{\partial k}{\partial x} + \rho v \frac{\partial k}{\partial y} = r^{-\alpha} \frac{\partial}{\partial y} \left(r^{\alpha} \frac{\epsilon}{\sigma_k} \frac{\partial k}{\partial y} \right) + \epsilon \left(\frac{\partial u}{\partial y} \right)^2 - D_k \quad (4)$$

The quantity D_k represents the dissipation of turbulence kinetic energy and was expressed as follows:

$$D_k = a_2 \rho k^{3/2} / y_1$$

where y_1 is the surface layer thickness. In the case of profiles with a shear peak τ_m located at a distance of $y/y_1 > 0.25$, a_2 was computed from

$$\begin{aligned} a_2 &= 1.8 & \text{for } y > y(\tau_m) \\ a_2 &= 1.8 y(\tau_m)/y & \text{for } y \leq y(\tau_m) \end{aligned}$$

When no shear peak occurred at a distance of $y/y_1 \approx 0.25$, a_2 was computed from

$$\begin{aligned} a_2 &= 1.8 & \text{for } y > y_1/4 \\ a_2 &= 1.8 y_1/4y & \text{for } y \leq y_1/4 \end{aligned}$$

The quantity ϵ/σ_k represents the diffusion coefficient for turbulence kinetic energy, where σ_k was assumed to equal .7. It was found that the solutions were relatively insensitive to the value of σ_k . This indicated that diffusion of turbulence kinetic energy did not play a major role in the surface layers investigated.

If the boundary layer is composed of more than one fluid, a conservation of species equation may be written as:

$$\rho u \frac{\partial c_j}{\partial x} + \rho v \frac{\partial c_j}{\partial y} = r^{-\alpha} \left[\frac{\partial}{\partial y} \left(r^{\alpha} \frac{\epsilon}{\sigma_{c_j}} \frac{\partial c_j}{\partial y} \right) \right] + R_j \quad (5)$$

where c_j is the j th species, R_j is the production of the species j due to chemical reaction, and ϵ/σ_{c_j} is the diffusion coefficient for the concentration flux of the species j . When analyzing a boundary layer composed of a group of fluids, a species equation may be written for all but one constituent which is then handled implicitly by the continuity equation.

A total energy equation may be derived from the first law of thermodynamics as:

$$\begin{aligned} \rho u \frac{\partial \bar{h}}{\partial x} + \rho v \frac{\partial \bar{h}}{\partial y} &= r^{-\alpha} \frac{\partial}{\partial y} \left\{ \frac{r^{\alpha} \epsilon}{\sigma_h} \left[\frac{\partial \bar{h}}{\partial y} + \left(\frac{\sigma_h}{\sigma_k} - 1 \right) \frac{\partial k}{\partial y} \right] \right. \\ &\quad \left. + \sum_{j=1}^n \left(\frac{\sigma_h}{\sigma_{c_j}} - 1 \right) h_j \frac{\partial c_j}{\partial y} + \left(\sigma_h - 1 \right) \frac{\partial (u^2/2)}{\partial y} \right\} \end{aligned} \quad (6)$$

where ϵ/σ_h is the diffusion coefficient for the stagnation enthalpy which is defined as:

$$\bar{h} = h + \frac{u^2}{2} + k + \sum_{j=1}^n h_j c_j \quad (7)$$

The governing equations are closed if one has a method for determining the effective viscosity and the various diffusion coefficients. It is this point where the firm physics of the fluid ends and empiricism takes over. To solve equations (1), (2) and (4) simultaneously, it is necessary to have empirical models relating the local turbulence kinetic energy to the local shear stress, to be able to compute the dissipation of turbulence kinetic energy, and to have an acceptable model for the diffusion of turbulence kinetic energy. Unfortunately, there is a paucity of information either analytical or experimental to guide the selection of these models as the large difference in models of this kind in the literature reveals. We have chosen to use modifications of the models of Bradshaw et al.⁽⁴⁾. Correlations of turbulence kinetic energy and turbulence shear stress data from the measurements of Bradshaw, Klebanoff⁽⁶⁾ and Lee and Harsha⁽⁷⁾ are shown in Figure (1). These correlations indicate that in general,

$$\tau \approx .3 \rho k$$

This correlation must, of course, fail at a surface since turbulence kinetic energy must go to zero at a surface while a shear stress exists at the surface. A "slip" condition has been assumed at a surface. The "slip" value at the surface is determined based on Equation (8) using the measured surface shear stress. This manipulation is justified since the goal in solving the turbulence kinetic energy equation is to provide a means for determining the shear stress throughout the surface layer, not to determine the turbulence kinetic

energy profile. In other words, modification of the turbulence kinetic energy equation is justified if it leads to acceptable results for the remaining dependent variable profiles and hence a better understanding of the turbulent surface layer.

III. SOLUTION OF THE GOVERNING EQUATIONS

The finite-difference procedure described by Patankar and Spalding⁽⁸⁾ for solving simultaneous parabolic partial differential equations was used to solve the governing equations. A streamline coordinate system was established by defining the stream function ψ such that

$$\rho u r^\alpha = \partial \psi / \partial y \quad (9)$$

and

$$\rho v r^\alpha = -\partial \psi / \partial x \quad (10)$$

thus satisfying the continuity equation. A dimensionless stream function w was defined as

$$w = \frac{\psi - \psi_I}{\psi_E - \psi_I} \quad (11)$$

where the subscripts I and E designate the internal and external boundaries, respectively. Equations (9), (10) and (11) are used to transform the governing equations into the following general form:

$$\frac{\partial \phi}{\partial x} + (a + bw) \frac{\partial \phi}{\partial w} = \frac{\partial}{\partial w} \left(c \frac{\partial \phi}{\partial w} \right) + d \quad (12)$$

where ϕ represents the dependent variable of interest and the coefficients a , b , and c are defined as

$$a = - \frac{1}{(\psi_E - \psi_I)} \frac{d\psi_I}{dx}$$

$$b = - \frac{1}{(\psi_E - \psi_I)} \frac{d(\psi_E - \psi_I)}{dx}$$

$$c = \frac{\rho u r^{2\alpha} \epsilon}{(\psi_E - \psi_I)^2 \sigma_\phi}$$

The quantity $\sigma_\phi = 1$, σ_k , σ_h or σ_{c_j} for $\phi = u$, k , h or c_j , respectively. The expressions for the coefficient d for various flow parameters are listed in Table I.

ϕ	d
u	$-\frac{1}{\rho u} \frac{\partial p}{\partial x}$
k	$\frac{y^{2\alpha} \rho u \epsilon}{(\psi_E - \psi_I)^2} \left(\frac{\partial u}{\partial w} \right)^2 - \frac{D_k}{\rho u}$
h	$\frac{\partial}{\partial w} \left\{ \frac{\rho u r^{2\alpha}}{(\psi_E - \psi_I)^2} \frac{\epsilon}{\sigma_h} \left[\sum_{j=1}^n \left(\frac{\sigma_h}{\sigma_{c_j}} - 1 \right) h_j \frac{\partial c_j}{\partial w} + \left(\frac{\sigma_h}{\sigma_k} - 1 \right) \frac{\partial k}{\partial w} + (\sigma_h - 1) \frac{\partial}{\partial w} \left(\frac{u^2}{2} \right) \right] \right\}$
c_j	$\frac{R_j}{\rho u}$

Table I. Coefficient d of Equation (12)

IV. DISCUSSION OF PREDICTIONS

A logical progression of increasingly complex turbulent boundary layers was used in developing the mathematical models described in this paper. A wide range of boundary conditions were investigated since the prediction scheme being sought was to have as broad a range of application as possible. The mathematical

models finally selected are those which provided the best predictions with accelerated, neutral, and decelerated boundary layers and with positive, zero and negative mass addition at the wall. Several of the test cases considered to be significant in the development of the mathematical models will be discussed in this paper. A matrix of these test cases is given in Table II shown below.

The first test case was the impermeable flat plate in zero pressure gradient. The experimental data of Klebanoff⁽⁶⁾ was used to define the initial mean velocity and turbulence kinetic energy profiles. This case was chosen as a starting point because it represents the most fundamental form of the turbulent boundary layer and because the measurements of Klebanoff provided enough fluctuating velocity information to determine turbulence kinetic energy and turbulent shear stress profiles. The predicted mean velocity and turbulence kinetic energy profiles remained similar as expected. Results of this first test case are presented by Byrne⁽⁵⁾.

Next, the models were tested against an initially disturbed relaxing boundary layer on an impermeable flat plate in zero pressure gradient. The measurements of Levitch⁽⁹⁾ were used to determine initial distributions of mean velocity and turbulence kinetic energy. The predicted mean velocity and turbulent shear stress distributions are compared to the experimental results of Levitch in Figure (2). The predicted mean velocity distributions are in agreement with the measured data over most of the boundary layer and are approximately 10 percent lower than the measured data in the inner 20 percent of the boundary layer. The predicted values of wall shear stress are within 10 percent of the measured values at all downstream locations. The hook in the predicted shear stress distributions which develops in the first 24 inches is gradually damped out and good agreement is evident at the downstream location of 72 inches.

Since the prediction scheme provided reasonable results for boundary layers without free stream pressure gradients, it was next applied to boundary layers in the presence of pressure gradients. Figure (3) shows predicted mean velocity and turbulent shear stress profiles in the presence of a positive or adverse pressure gradient caused by a decelerating free stream velocity. The experimental measurements of Bradshaw et al⁽⁴⁾ were used to determine initial mean velocity and turbulence kinetic energy distributions. The experimental free stream velocity varied exponentially with distance as,

$$u_{\infty} \propto x^{-.255}$$

The predicted non-dimensional velocity profile remained essentially unchanged for 30 boundary layer thicknesses. The predicted shear stress profile exhibited similarity characteristics at all downstream locations.

Figures (4) and (5) compare predicted distributions of mean velocity and turbulent shear stress to the measurements of Thielbahr et al⁽¹⁰⁾ for a boundary layer with a negative pressure gradient caused by an accelerating free stream velocity. Initial mean velocity distributions were obtained from the measurements of Thielbahr. Thielbahr did not measure shear stress or turbulence kinetic energy across the boundary layer; however, wall shear stresses were reported. Initial turbulence kinetic energy distributions were obtained by using profiles geometrically similar to the measured turbulence kinetic energy profiles of Klebanoff⁽⁶⁾ and matching the wall value of turbulence kinetic energy with the measured wall shear stress of Thielbahr through equation (8). Figure (5) represents a more rapidly accelerating boundary layer than Figure (4). Both figures show very good agreement between predicted and measured mean velocity profiles. Good agreement is also achieved between the predicted and measured wall shear stresses.

It is significant to note the differences in the structure of the boundary layer caused by negative and positive pressure gradients. Figures (4) and (5) show that for a negative pressure gradient the maximum shear occurs at the wall and increases in the downstream direction, and the mean velocity profiles are flat near the wall. Figure (3) shows that for a positive pressure gradient the maximum shear moves away from the wall and decreases in the downstream direction, and the mean velocity profiles do not exhibit the flat behavior near the wall.

Turbulent boundary layers are often influenced by mass transfer at the wall. Figures (6), (7) and (8) compare predicted distributions of mean velocity and turbulent shear stress to the measurements of Thielbahr et al⁽¹⁰⁾ for accelerating boundary layers with varying amounts of mass transfer at the wall. A blowing

TABLE II. MATRIX OF TEST CASES

Figure	Case	Ref.	$dp/dx(lbf\ ft^{-2}\ ft^{-1})$	$\bar{F}$
2	5	9	0	0
3	4	4	.607	0
4	6	10	-.635	0
5	7	10	-.787	0
6	10	10	-.787	-.001
7	8	10	-.787	+.001
8	9	10	-.787	+.004

Note: Case numbers refer to discussion in Reference (5).

parameter F has been defined as,

$$F = \frac{\rho_I v_I}{\rho_\infty u_\infty}$$

where the subscript I indicates conditions at the wall; thus, v_I is the gas transpiration velocity at the wall. The three values of F investigated were -.001, +.001 and +.004. Initial distributions of mean velocity and turbulence kinetic energy were obtained from the mean velocity and wall shear measurements of Thielbahr et al in the same manner as described for Figures (4) and (5). The predicted mean velocity distributions are within 5 percent of the measured data for all three values of F. The predicted wall shear values agree with the trend of the measured data for all three values of F.

The effects of mass transfer on the structure of the boundary layer can be seen by comparing Figures (6), (7) and (8) with Figure (5). Figure (6) shows that suction tends to increase the wall shear stress and flatten the mean velocity profile near the wall. Figure (7) shows that a slight addition of mass decreases the wall shear stress and tends to move the mean velocity profile away from the wall. Figure (8) shows that a larger mass addition decreases the wall shear stress further, moves the maximum shear stress away from the wall and moves the mean velocity profile further from the wall.

V. CONCLUSIONS

It has been shown that the simultaneous solution of the turbulence kinetic energy equation with the equations of continuity and momentum provides an effective engineering tool for predicting the behavior of turbulent boundary layers. The analytical results have been compared with available experimental data for turbulent boundary layers in the presence of negative, positive or zero pressure gradients and turbulent boundary layers with blowing or suction with generally satisfactory agreement for both mean velocity and shear stress distributions. It is felt that the method in its present form could be used to predict mean velocity and shear stress distributions in atmospheric boundary layers if sufficient data were available to define initial distributions of mean velocity and turbulence kinetic energy. Extension of this method to more complex atmospheric problems involving transport of mean energy and species concentration will require additional laboratory experiments to develop empirical models for use in the mean flow energy equation and species concentration equation.

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Figure 1. Correlation between shear stress and turbulent kinetic energy

Figure 2. Prediction of a perturbed boundary layer

Case 5, Levitch

Figure 3. Prediction of a Boundary Layer with Moderate Adverse Pressure Gradient

Figure 4. Prediction of a Boundary Layer with Slight Favorable Pressure Gradient

Experiment	Prediction
	————— $x = 0$ in
○	— · — · — $x = 15.89$ in.
□	 $x = 23.91$ in.
△	- - - - - $x = 31.86$ in.

Figure 5. Prediction of a Boundary Layer with Stronger Favorable Pressure Gradient

Experiment	Prediction
	— $x = 0 \text{ in.}$
○	— · — $x = 15.89 \text{ in.}$
□	····· $x = 23.91 \text{ in.}$
△	- - - - $x = 31.86 \text{ in.}$

Case 10, Thielbahr et al

$$\frac{dp}{dx} = - .787$$

$$F = -.001$$

Figure 6. Prediction of an Accelerated Boundary Layer with Slight Suction

Figure 7. Prediction of an Accelerated Boundary Layer with Slight Injection

Figure 8. Prediction of an Accelerated Boundary Layer with Stronger Injection

A NUMERICAL MODEL FOR PREDICTING MESOSCALE WINDS ALOFT[†]

John S. Cornett
NOAA, Air Resources Laboratory
Las Vegas, Nevada

INTRODUCTION

Modeling efforts described in this report were designed to satisfy meteorological needs of the Atomic Energy Commission nuclear testing program. Tests are designed to contain all radioactivity beneath the ground surface. Control plans for detonations are predicated on maximum possible release, therefore of particular importance is the mesoscale wind field over and around the Nevada Test Site (NTS). Wind field predictions must be provided to the Test Manager so that he can anticipate any radiological hazard that might result from an unplanned release of radioactivity to the atmosphere.

This report describes the initial attempt to develop a simple numerical model which could eventually satisfy economical restraints and yet yield a usable product in terms of predicting the three-dimensional wind field for operational requirements on the NTS.

Since many mesoscale wave components have periods of less than 10 hrs and wave lengths of approximately 10 km(1), the total turn-around time after the data are input should be kept to less than 10 minutes and preferably less than 5 min to permit time for the duty forecaster to digest the computer output and prepare his mesoscale forecast.

MODEL DESIGN

Hjorthorn(2) has suggested that above the planetary boundary layer the temporal variation of wind velocity is primarily a function of the inertial terms in the equation of motion. This result indicates that it would be feasible to construct an advective model which is strongly dependent upon the inertial force due to horizontal motion. Based upon this presumption, Haden(3) developed a model capable of predicting wind profiles. Following Haden's formulation, one obtains a set of steady prognostic equations for predicting temporal changes in the wind field;

$$\frac{\partial S_u}{\partial t} + u \frac{\partial S_u}{\partial x} + v \frac{\partial S_u}{\partial y} + w \frac{\partial S_u}{\partial z} = 0 \quad (1)$$

$$\frac{\partial S_v}{\partial t} + u \frac{\partial S_v}{\partial x} + v \frac{\partial S_v}{\partial y} + w \frac{\partial S_v}{\partial z} = 0 \quad (2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (3)$$

where u , v , and w represent the x , y , and z components of the wind, and S_u , S_v represent the vertical shear of u and v , respectively. Equations (1) and (2) are explicitly time dependent and represent the complete set of expressions for the wind-prediction model. Equation (3) is the continuity equation which is always satisfied by the model and which is used to calculate vertical motion from the initial and predicted u and v fields. The vertical motion field is acquired from (3) by integration using a special numerical technique(4) which implicitly accounts for sloping terrain.

In the interest of computational convenience the model, which is limited in horizontal extent to 160 n. mi. by 160 n. mi., was formulated in a cartesian coordinate system. On a mesoscale the dynamic effects due to this geometric distortion are negligible as pointed out by many authors(5).

The three-dimensional grid latticework utilized by both models was oriented with the x -axis extending east-west, the y -axis extending north-south, and the z -axis extending vertically, normal to the xy -plane. The grid points on the x - and y -axes were spaced 20 n. mi. apart, along the z -axis grid points were spaced 1000 ft (0.164 n. mi.) apart. Fig. 1 shows the x , y grid system and indicates the positions of frequently used wind-sounding stations. Notice, in Fig. 1 that the NTS (indicated by the dashed area) is located near the center of the grid framework.

[†] This work was supported by the United States Atomic Energy Commission under Nevada Operations Office contract no. SF54-331.

Figure 1. Topography in the grid network. Contours are drawn for each 1000-ft level above mean sea level.

The wind-prediction model was set up to predict wind profiles between the 7000- and 14000-ft levels above mean sea level (MSL).

Numerical integration of (1) and (2) is carried out in the following manner. First, the initial values of u and v are objectively prescribed as functions of x , y , and z . Then the vertical windshear field is calculated from the initial wind field. Next (3) is integrated numerically to determine the vertical motion field based on u and v . Equations (1) and (2) are then integrated for a small time increment, Δt , yielding the predicted components of the wind shear field time $t_0 + \Delta t$. The horizontal components of the wind are then calculated at each grid point. This is accomplished by solving the equations

$$\begin{aligned} u(z) &= S_u^* \Delta z - u(z + \Delta z) \\ v(z) &= S_v^* \Delta z - v(z + \Delta z) \end{aligned} \quad (4)$$

where S_u^* and S_v^* are the predicted u - and v -component vertical wind shears at $t_0 + \Delta t$ between 7000 and approximately 18000 ft MSL, Δz is the layer thickness, and $u(z)$ and $v(z)$ are the components of the horizontal wind velocity. The wind velocity at the top of the model, 500 mb, is specified by the National Weather Service baroclinic or barotropic model and the wind components at lower levels are determined through (4) and (5) by building downward, level by level, to 7000 ft above MSL.

To achieve rapid solutions to (1)-(2), to simplify calculations near the boundaries, and for practical expediency, the simplest, and perhaps the least accurate, finite-difference analogs have been used in the initial model. In particular, upstream differencing analogs are used and are considered to provide representative solutions to the corresponding partial equations. A theoretical advantage of the upstream-difference equations is that by its use one can rigorously prove stability(6).

Solutions to (3) are achieved with the ground ($z=0$) boundary condition,

$$u(z) = v(z) = w(z) = 0 \quad (5)$$

Figure 1 shows that a small percentage of the terrain lies above 7000 ft MSL. Consequently, only a few of the grid points in the model lie on or below the ground. For these grid points the Dirichlet condition(7) is used; namely,

$$u(i,j,k,t) = v(i,j,k,t) = w(i,j,k,t) = 0 \quad (6)$$

so that no calculations are performed on or below the ground boundary.

For grid points located on any windward side boundary,

$$\frac{\partial S_u}{\partial t} = \frac{\partial S_v}{\partial t} = 0, \quad t > 0 \quad (7)$$

so that there is no flux of wind shear across this boundary.

For $t > 0$, the Dirichlet condition specifies,

$$\begin{aligned} u(\text{windward boundary}) &= \text{constant} \\ v(\text{windward boundary}) &= \text{constant} \\ w(\text{windward boundary}) &= \text{constant} \end{aligned} \quad (8)$$

Along any leeside boundary, wind shear is permitted to flow across the boundary so that for $t > 0$ the Neumann condition(8), is used; namely,

$$\frac{\partial S_u}{\partial x} = \frac{\partial S_v}{\partial x} = 0 \quad (9)$$

Upstream differencing schemes are especially suitable for handling these type boundary conditions because no special forms of the difference equations are required(6).

The boundary conditions specified by (6) and (9) are meteorologically realistic; however, the restraints placed on the models by conditions (7) and (8) are not as realistic as they are expedient from a practical standpoint. Nevertheless, for the short time-period forecasts (<10 hrs) for which the model is used, perhaps (7) and (8) are not too restrictive since the model is capable of only resolving mesoscale flows with wavelengths greater than about 40 n. mi. (=75 km).

Along the top boundary of the wind-prediction model a special boundary has been created. Between the 14000-ft level and the 500-mb level, the wind-prediction model contains a special

computational domain for predicting the wind shear in this region. The wind shear between these two levels is calculated from the 500-mb and 14000-ft winds and from the mean thickness of the layer. Along the top boundary of the model, specified as the 500-mb level, a combination of Dirichlet and Neumann conditions are used;

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 0 \quad (10)$$

for $w < 0$,

$$\frac{\partial S_u}{\partial z} = \frac{\partial S_v}{\partial z} = 0 \quad (11)$$

$$u = u(t), v = v(t) \quad (12)$$

Condition (10) requires that no horizontal wind-shear can exist at the 500-mb level within the horizontal area occupied by the wind-prediction model (160 n. mi. by 160 n. mi.).

Although some arguments can be presented concerning the restrictive nature of this condition, no data are available to permit a more realistic assumption. Equation (11) specifies a linear change in wind-shear across the top boundary. Reference to (1) and (2) illustrates that this condition is imposed on the model to prevent a flux of wind shear into the model from outside the model boundaries; a condition analogous to (7). Condition (12) requires that the 500-mb wind components be variable with time. This variability is designated by the changes in wind velocity predicted by the NWS baroclinic or barotropic models. The wind velocity is varied linearly with time between the initially observed wind velocity and the final predicted 500-mb wind. Consequently, through (12), it is possible to put kinetic energy into the wind-prediction model through the top boundary.

DATA COLLECTION AND MODEL INITIALIZATION

A flexible system is utilized by ARL-LV for taking winds aloft soundings in support of the AEC nuclear testing program on the NTS. Normally at least six or seven one-man sounding units take standard, Pibal wind soundings in the vicinity of the NTS, plus one or more radar soundings near the test location on the NTS. Sounding sites that are used frequently are plotted in Fig. 1 which shows that most of these sites are located off the NTS and are scattered rather uniformly around the NTS. Usually the sounding network is activated five to six hours before "shot time" and wind soundings normally are taken every hour. The plotted analyses and predicted wind flow are available about 30 min after receipt of the observations.

The wind data are objectively analyzed over the grid network by the method of Bergthorssen and Doos(9), and Cressman(10) in order to obtain the initial wind conditions for the model.

The analysis is performed separately on each u - and v -component of the wind in a series of four passes over the data. A first guess of the wind field is required at each level and this is obtained at the 14000-ft level by taking an average of the observations of the wind component being analyzed. The final analysis at 14000 ft then becomes the first guess at the next lower level (13000 ft above MSL) and this process is then continued downward to the lowest level (7000 ft above MSL).

Error detecting procedures for the wind data have also been incorporated into the analysis using a method by Glahn, Hollenbough, and Lowry(11).

DISCUSSION OF RESULTS

Ten independent cases were used to verify the model's predictions, such that, various times of the year and upper wind flow regimes are represented. One case is treated in detail as an illustration of the type of predictions that the model is capable of producing. The general flow in the vicinity of the NTS for this case was southwesterly at slow to medium speeds throughout the low and middle troposphere. The general weather reported at the NTS during the prediction period was fair with scattered altocumulus and broken cirrostratus clouds. A special radiosonde (12) taken near the test area at 0900 PST showed that the atmosphere was generally unstable from the ground to near the 8000-ft level above MSL. Between 8000 and 10000-ft a strong temperature inversion separated the low-level southerly flow from the upper level southwesterly flow.

The initial wind flow at 0400 PST is shown in Fig. 2 using streamlines and isotachs for the 7000-, 10,000-, and 14,000-ft levels above MSL. In addition to the streamline-isotach analyses the conventional wind shafts and barbs are plotted at each station to represent the observed winds.

Figure 2 shows that, initially, slow speeds prevailed at all three levels with southerly flow

Figure 2. The initial streamline-isotach analysis for test case number 2. Observed winds are indicated by conventional wind shafts and barbs at each station. The NTS is outlined near the center of the figure.

at the lowest level veering into southwest flow at the middle level and westerly flow at the top level.

The predicted wind flow for the same levels, six hours later (1000 PST), is shown in Figure 3. However, the plotted wind shafts and barbs show the observed winds at these levels for 1000 PST. In general, the model predicted a substantial increase in speeds at all levels. At 14,000-ft above MSL the flow was predicted to become more southwesterly. At 10,000-ft above MSL the general flow was predicted to be mainly southwesterly except in the SE portion of the grid area the winds were predicted back from the initial northwest flow to southwest flow. At 7000-ft the winds were predicted to veer into the southwest. Comparison of the streamline-isotach field with the plotted winds at the 10,000-ft level shows there is good agreement between the predicted flow and the observed winds. At the 14,000-ft level above MSL the agreement is not as good as at 10,000-ft but the trend from slow speeds increasing to moderate speeds was predicted. Also, the backing trend from westerly flow to southwesterly flow was predicted although the model did not back the winds quite far enough. The poorest predictions were made at the 7000-ft level where the model predicted wind speeds greater than observed and flow much more southwesterly than actually occurred. A possible explanation for the poor prediction at the 7000-ft level was a decoupling of the low-level winds from the winds at higher levels. This could have been caused by the stable layer between 8000- and 10,000-ft above MSL indicated by the radiosonde.

A time cross section of the winds at the NTS station is shown in Figure 4. The observed and predicted wind flow at 2-hour intervals for each level is indicated by arrows and digits at the head of the arrows indicating speed. Observed and predicted winds are indicated by an "O" and a "P", respectively. However, if the observed and predicted wind direction were the same, only one arrow is plotted. In general, as may be seen in Figure 4, there was excellent agreement between the observed and predicted winds except at the 7000- and 8000-ft levels. Again the explanation for the poorer predictions at these low levels may be due to the previously mentioned decoupling effect.

The verification statistics for the model were computed by determining the root mean square errors (RMSE) of the wind profiles at three of the observing stations in the grid network. These stations are indicated by the TTA, UCC, and LAS identifiers given in Figure 1. These stations were selected because they are spread across the grid network and they are located at varying elevations. Namely, 6000 ft, 4000 ft and 2000 ft above MSL for TTA, UCC, and LAS, respectively.

Averages of the RMSE from the three stations were computed for each test case at one-hour prediction intervals. The resulting average RMSE of the magnitude of the error vector at each prediction time are plotted in Figure 5 where they are compared with the RMSE of a persistence forecast using the initial wind profiles. Figure 5 shows that the model is worse than persistence for the first three hours of the forecast period but is better than persistence for the last three hours.

The poor predictions for the first three hours may be explainable in terms of study done by Wurtele⁽¹³⁾. Wurtele concludes that an upstream finite differencing scheme simulates phase speeds that are greater than the actual wave speeds as the wavelength approaches the smallest possible wavelength resolved by the prediction grid network. This could lead to the prediction of wind changes which are more rapid than those which actually occur, especially for the early forecast periods.

CONCLUSIONS AND FUTURE PLANS

It is thought that the results obtained by this initial and extremely simple numerical model are encouraging enough to pursue modifications that could lead to improved predictions. However, it should be explicitly pointed out that the basis for the underlying assumption, i.e., conservation of vertical shear, has not been shown by this study.

These modifications could be the following:

1. It can be shown that the velocity divergence term ($\hat{S} \cdot \hat{\nabla} \cdot \hat{v}$) in the equations of motion is roughly equal in magnitude to the inertial term for mesoscale waves. Thus, the addition of the divergence term should be considered in the prediction equations.
2. The bottom boundary of the model could be lowered from 7000 ft to, say, 4000 above MSL, in order to induce more terrain effects into the model.
3. Investigate the possibilities of using a finite-difference scheme which is less dissipative and diffusive than the current upstream differencing scheme.
4. Utilize the available data to investigate the concept of the conservation of vertical wind shear on the mesoscale.
5. Allow shear to vary with time on the bottom boundary by using statistically derived shear variations for each observing station. This modification may allow a shear sink on the ground.

Figure 4. Time cross-section of the winds at an NTS station. The observed and predicted flow is indicated by arrows and digits at the head of the arrows indicate speed. Only one arrow is plotted if the observed and predicted directions were the same.

Figure 2. Root Mean Square Error (RMSE) of the numerical model wind profile predictions and observations based on persistence of the wind profile as a function of forecast period.

boundary when a breakdown occurs in the decoupling between the boundary layer and upper winds such as that caused by diurnal heating.

All but the last modification are being currently pursued since they involve relatively simple and straight-forward computer programming.

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A NOTE ON OBSERVATIONS OF TURBULENCE AND MEAN FLOW IN VEGETATIVE CANOPIES

Joseph H. Shinn and Ronald M. Cionco

Lower Atmospheric Research Technical Area
Atmospheric Sciences Laboratory
White Sands Missile Range, NM

It is the purpose of this note to present observations on the peculiar nature of turbulence and mean flow in vegetative canopies. A long term research effort on canopy flow has recently been concluded and two lengthy reports on aspects of canopy flow have been published^(1,2). It was our good fortune to obtain a wealth of experimental data from widely varying vegetation types, and this data enables us to draw conclusions and summaries far ahead of many of our colleagues. Because the work has been terminated it is important that an overview be given of our conclusions, that some of the unexplained problems be pointed out, and that some unpublished tentative conclusions be presented for your evaluation and comments.

The scope of our study included data analyses from ten forests, ten crops and two wind tunnel simulations. Most of the data had been previously unreported, and is as yet by no means fully analyzed.

ISOLATION OF THE PROBLEM AREAS

It is practical to separate the problem of air flow through an exposed canopy wall from the problem of established flow, since in the latter case one can assume there are no horizontal variations of time-averaged variables. It was shown by Shinn⁽³⁾ that the established equilibrium flow is not attained unless the horizontal extent of the vegetation is more than 25% the height of the vegetation. Complete equations for the horizontal and vertical variation of the canopy's wind velocity and turbulent energy are available in our recent reports^(1,2) and will not be discussed here.

In order to get an overall picture of canopy flow we will first discuss the gross flow features of the canopy, parameterization of vegetation density, and the properties of the complex versus the "ideal" canopy. Following this we will present observations on turbulent intensity, anisotropy, the scales of turbulence, the effects of thermal stability, and comment on the origin and local dissipation of turbulence.

THE GROSS FLOW FEATURES

The flow field in vegetative canopies is made of many local divergences within the wake of obstacles, but it can be shown that a velocity sensor placed on a tower will be representative of the gross time-average flow, providing that the tower is not placed in either an opening or in proximity to a plant. Furthermore, even the most sensitive mechanical sensors are near their low-speed design limits, but they do give data in agreement with the probability distributions of smoke-puff tracers.

The most useful parameterization of the vegetation density is the canopy index, α ,⁽¹⁾ which is defined as the non-dimensional mean velocity shear taken at the canopy top so that:

$$\text{for "ideal" canopies} \quad \alpha = (\ln U/U_h) / ((Z/h) - 1) \quad (1)$$

$$\text{for complex canopies} \quad \alpha = (U/U_h) (dU/dZ) \quad (2)$$

where h is canopy height, U is wind vector speed, and Z is height. According to Cionco⁽¹⁾, canopy index values (from least squares fitting with equation 1) less than about 2.5 define an "ideal" canopy in which the atmospheric boundary layer above is still sufficiently coupled to the flow within the canopy. The gross flow, therefore, is a simple one-dimensional, exponential function of height as shown in Figure 1a. Figure 1b shows wind profiles from the more complex canopies. A compilation of canopy indices^(1,2) for various types of vegetation shows that factors such as element flexibility and canopy density affect the values as given in Table 1.

Figure 2 graphically describes the functional relationship of the index to canopy density, element flexibility and wind speed. We see that the index increases as density and flexibility increase and also that as elements streamline with flow the index varies with wind speed. Other parameters of the plants such as leaf area density, index (also drag coefficient) and the canopy lateral wind parameters are all related to the index.

Figure 1: Exponential plot of wind profiles within a) simple and b) complex canopies.

Figure 2: Variation of canopy index with canopy density, element flexibility and wind speed at $z = H$.

Table 1. Compilation of canopy index values for simple and complex canopies.

for "ideal" canopies:	$\bar{\alpha}$	for forest canopies (mean $\pm$ standard error):	
Immature corn	2.82	Gum-maple	4.42 $\pm$ 1.05
Oats	2.80	Maple-fir	4.03 $\pm$ 0.69
Wheat	2.45	Jungle	3.84 $\pm$ 1.52
Corn	1.97	Spruce	2.74 $\pm$ 1.29
Plastic strips	1.67	Oak-gum	2.68 $\pm$ 0.66
Rice	1.62		
Sunflower	1.32		
Christmas trees	1.06		
Larch plantation	1.00		
Wooden pegs	0.79		
Citrus orchard	0.44		
Bushel baskets	0.36		

From Shinn's canopy index values ⁽²⁾ greater than 2.5, a two-dimensional flow is shown in Figure 3 to develop because of decoupling with the flow above. There is also evidence that the wind vector turns to the right with height under normal conditions in the Northern Hemisphere for complex forest canopies. See Figure 4.

Figure 3: Predicted and observed canopy velocity components for two-dimensional flow in forests.

Figure 4: Predicted and observed canopy wind turning angle for the gum-maple forest; pressure gradient force (P) and cloud drift (C) directions are shown.

OBSERVATIONS OF TURBULENCE

Intensity of Turbulence

Turbulence intensity is defined as the ratio of the RMS characteristic speed to the time-averaged speed:

$$I = \frac{\sqrt{\overline{u'^2} + \overline{v'^2} + \overline{w'^2}}}{\bar{V}} \quad (3)$$

or individually:

$$I_u = (\overline{u'^2})^{1/2} / \bar{V} = u' / \bar{V} \quad (4)$$

$$I_v = (\overline{v'^2})^{1/2} / \bar{V} = v' / \bar{V} \quad (5)$$

$$I_w = (\overline{w'^2})^{1/2} / \bar{V} = w' / \bar{V} \quad (6)$$

As guidance, we know that the horizontal vector intensities approach a value of one in the canopy. This value is high for fluid flows. Figure 5 exhibits a wide range of intensity levels for several canopy types. For simple canopy elements the intensities are essentially constant with height, while complex elements encourage considerable variations with height. The simple wooden pole elements of the wind tunnel indicate that mean intensity values increase as canopy density increases:

Density	Sparse	Moderate	Dense
$\bar{I}$	0.11	0.14	0.22

There is also a mean density effect upon the intensity of a canopy element progressing from simple arborescent crops through to complex "jungle" forests. Table 2 is a compilation of mean intensity values for five different canopy types.

Table 2: Turbulence intensity vs canopy complexity.

CANOPY TYPE	$\bar{I}$
Arborescent Crops	.14
Tree Plantations	.14
Conifer Trees	.16
Deciduous Trees	.16
"Jungle" Forest	.17 to .22

Figure 1: Intensity of turbulence within and above simple and complex canopies.

Anisotropy

The individual components of equations 4, 5 and 6 (longitudinal, lateral and vertical) of the RMS speeds typically have different values so that their ratios also define coefficients of anisotropy as

$$\sigma_v / \sigma_u = \sqrt{(\overline{v'^2}) / (\overline{u'^2})} \quad (7)$$

$$\sigma_w / \sigma_u = \sqrt{(\overline{w'^2}) / (\overline{u'^2})} \quad (8)$$

In simple canopies σ_v is quite small compared to σ_u , but the comparable magnitudes of u and v in several forests permit several computations to be made both within and above the trees. See Table 3.

Table 3: Coefficients of anisotropy above and below the tree crowns compared to boundary observational (Panofsky et al.), and theoretical (Taylor) results.

Above the trees; $1 < Z/h < 1.3$		
	$\frac{\sigma_v}{\sigma_u}$	$\frac{\sigma_w}{\sigma_u}$
Spruce	0.83	0.69
Hardwoods (2)	0.89	0.63
In the trunk space; $Z/h \approx 0.4$		
Spruce	0.55	0.40
Hardwoods (2)	0.63	0.36
In the boundary layer; $Z/h > 1$		
Observed	0.80	0.54
Theory (local isotropy)	1.15	1.15

Scales of Turbulence

In a limited amount of spectral analysis performed on the time series of the speed of the vector wind we determined the fixed point (Eulerian) time scale by integration of the autocorrelation function over time. The time scales τ_E are related to characteristic Eulerian length scales L_E by the well-known Taylor hypothesis: $\tau_E = L_E / \bar{V}$. Curiously enough, the Eulerian length scales are constant with height in the lower 80% of the canopy, and above this they are given by the mixing length hypothesis. This is, above $Z/h = 0.8$, $L_E = 0.4 (2-d)$, and $d = 0.7h$. For example, below $Z/h \approx 0.8$ we found $L_E = 100$ cm for a gum-maple forest, while in a separate study of a larch plantation Allen ⁽⁴⁾ found $L_E = 150$ cm. In the only high frequency (high wave-numbers) spectral analysis we performed, see (5), a curious effect was that for wave-numbers greater than 1.5×10^3 cycles cm^{-1} the slope of the log spectra changed from $-5/3$ to $-3/10$. The latter is probably an artifact, but no confirmation was attempted. Hanna has pointed out (personal communication) that this indicates much more than normal energy at these frequencies and could be a result of the data analyses, although we took careful steps to avoid aliasing. It is more likely due to excessive variances at high frequency from inertia of the mechanical sensor. High frequency spectra using hot-wire anemometers have been taken by Allen ⁽⁴⁾ and Meroney ⁽⁶⁾, and the results indicate that all of the spectra have a minimum energy (speed variance) at a wave-length equal $0.3h$. At least we qualitatively agree with this observation.

Estimation of the eddy diffusivity in the complex canopy has usually failed wherever mean velocity profiles were used. This is due to the fact that the shear often vanishes because the velocity profiles have maxima and minima. There is a serious question about using the commonly accepted Reynolds analogy that stress is proportional to strain. We would rather define eddy diffusivity from $K = \bar{u}'L$ where $\bar{u}'$ is the RMS vertical speed and L is a Lagrangian length scale. As a working hypothesis we assume equality between the Lagrangian L and the Eulerian scale L_E . Using this method we computed a tentative universal profile for eddy diffusivity in the complex canopy which agrees with independent observations by Meroney and Yang ⁽⁷⁾, and by Donald ⁽⁸⁾, who also did not use shear measurements. Figure 6 shows comparable results.

Origin and Local Dissipation of Turbulence

The local origin of turbulence is unknown; it may be residual from downward locally penetrating gusts originating within the boundary layer or it may be due to large scale momentum subsidence. Shinn's calculations ⁽⁹⁾ have shown that in at least one case the turbulence energy dissipation rate was surprisingly conservative with height, which should be expected from our observations of turbulence scales.

Figure 6: Relative diffusivities by methods other than shear measurements.

Figure 7: The lack of influence of temperature gradient on lateral turbulence intensity in the canopy.

Thermal Stability Effects

It is common that in daylight a temperature inversion exists below the vegetation and temperature lapse above. The very opposite is true at night. In either case the temperature inversion suppresses buoyant mixing. We therefore feel that in large part thermal effects are always present to the same degree in the canopy and can therefore be ignored. For example, we have found no correlation between the canopy temperature gradient and the canopy lateral turbulence intensity. Figure 7 shows the lack of influence of the canopy temperature gradient upon lateral intensities, σ_A .

Further work should be done on the spectral analysis of velocity components and on thermal effects in the canopy. We are not pursuing the subject further at this time.

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ECOSYSTEM SIMULATION: A REQUIREMENT FOR SURVIVAL

Amos Eddy

Meteorology Department
University of Oklahoma
Norman, Oklahoma

A. INTRODUCTION

Although the survival of the human species is probably irrelevant on the cosmic scale, and likely undesirable for the well-being of our own biosphere, it is a matter of some concern to most of us who are its members. We do not want to become the backwash of some moribund stochastic process. However, if we are to maintain our position as a life process in the mainstream of biological evolution, we are going to have to take a more responsible position concerning the consequences of our actions. We waste resources far out of proportion to our value to the system which sustains our existence: it cannot continue indefinitely to support us in the style to which we have become accustomed. The above statements are considered to be neither optimistic nor pessimistic but rather to be a synthesis of the scientific facts as I see them. This paper presents the outline of a method of attack which can give us the capability we need to assume a position as a responsible member of the earth's biosphere. The key concept is to provide a useful and adaptable computer ecosystem simulation model as a focal point of information acceptance, processing and dissemination which is accessible to all relevant decision making members of our society.

Technology can produce such a device; however, it is not at all clear that any desired readjustments in life styles will result. Long before the model reaches any high degree of perfection, its potential for manipulation of resources and human events will make it many times more susceptible to misuse than has been the ability to release nuclear energy. There is, of course, always the possibility that it will be used in a creative manner.

The following discussion is state-of-the-art in all aspects except those of co-operation and co-ordination. There is no doubt that some such model-device will exist within the next decade in the United States or elsewhere.

B. GENERAL MODELLING CONCEPTS

The problem is restated: how can those processes which are critical to the stability of the world ecosystem be described by computer simulation? The hypothesis central to the model predicates that the activities of man constitute very powerful "short" term perturbations in the system. Forces exist which draw this animal into semi-stationary herds or conglomerations called cities which feed somewhat like parasites on the surrounding rural areas. Thus, cast in the role of a primary building block of our model, the "man-centered urban-rural" eco(sub) system.

In designating this building block as central to our problem we have effectively selected a time-space scale range within which we expect a significant amount of system-affecting action to take place.

Actions (processes) are assigned to identifiable time-space scale ranges and the manner in which they are represented in the model depends on their position in the spectrum relative to the urban-rural scale size. Relatively small scale events are considered more as random processes the smaller they become; whereas, relatively large scale events are amenable to deterministic representation.

Actions become "significant" when the stability of the ecosystem depends on their varying within certain anticipated bounds whereas at the same time there is a non-zero probability that they can venture outside these bounds.

"Stability" is of necessity a subjective and probabilistic concept. Clearly any model which would drive the entire biosphere into nothing but wolves, rabbits and carrot patches would be deemed an unstable system. Also one which evolved world temperature patterns fluctuating around mean surface values of 10,000°C would be suspect. A system which would show no perceptible change in its evolution if one man were removed would be called stable even though this man could have been a perverted genius who would have set up a chain of events which destroyed the system, or possibly have been the man who solved the world food problem by making kelp into hamburgers. A stable model is one, then, which does nothing scientifically offensive over the time period of interest in the prediction.

Consider a 50-year prediction. Consider further, initial conditions which comprise a "complete" set of 1920 data, and the model to predict adequately the state of the ecosystem for 1970. Such circumstances would support the supposition that the model is adequately stable to undertake a 50-year prediction given a "complete" set of initial conditions and the appearance of NO NEW SIGNIFICANT PROCESSES during the period of the prediction. While we have some hope of satisfying the condition of "complete" initial conditions for certain scales of interest, we are frustrated when we try to predict the explicit occurrences of new (hence unpredictable) processes.

We now re-examine "significance" in the following way. Suppose we remove just the perturbations from the wind field caused by the mid-latitude thunderstorms in 1920 and rerun the 50 years of evolution to find no detectable change in the 1970 prediction in any part of the whole system. We could then consider not representing explicitly such wind perturbations in our initial conditions on the grounds that they were not significant. Next suppose we eliminated all thunderstorm precipitation from our initial conditions (1920 data). In certain areas of the world, this could cause botanical disasters from which the system could not recover within 50 years. Such an experiment might even show the elimination of entire species from certain critical biological niches and a consequent uncontrolled proliferation of others which would completely upset the evolution of the ecosystem. Thus our model must exhibit the following characteristics:

- (1) "Complete" representation of initial conditions for all significant parameter fields.
- (2) Stable representations of all processes over the time period of interest.
- (3) Consideration of all processes relevant to the existence of the human component of the biosphere on the space-time scale of the urban-rural ecosystem.

C. SPECIFIC MODELLING COMMENTS

In order to facilitate discussion, the overall model will be considered to have three main segments. These will concern themselves with:

- (1) DATA ANALYSIS
- (2) RESOURCE MANAGEMENT
- (3) PREDICTION

It is clear that these segments are quite interdependent and that attempts to structure them independently will lead to unstable evolution of the ecosystem.

1. Data Analysis

An analysis is considered to be a maximum likelihood statement of the structure of a parameter field in time and space with an accompanying statement of its probable sample variability.

Observational data are quantified measures of the state of the ecosystem. The task of this segment of the model is to see that observation set is both valuable and complete within imposed constraints. The data are valuable if they assist in effecting a profitable increase in the stability of significant processes in the prediction model. The profit is assessed by first separating the signal (relevant information) from the noise (that portion of the observation set not required for the stable evolution of the prediction model), and then comparing the increase in utility (benefit) obtained through incorporation of the signal into the structure fields of initial conditions to the total cost of data gathering, analysis and incorporation.

A complete observation set is one which leads to a representation of the initial conditions which is adequate to make the prediction model stable.

Constraints are imposed on the observation gathering and analysis by the geophysics, the biology, the economics and the relevance to the problem.

One of the purposes of this analysis component is to probe the prediction model for instabilities which could be reduced by better definition in the initial conditions, to design field experiments to obtain new data in an optimal manner, and then to incorporate the consequent new signal information into the statement of initial conditions.

Clearly the analysis must make use not only of new observations but also of existing structure models which often take the form of mathematical "laws". "Order" in the observations is sought, wherever possible, by making the null hypothesis that the new observation sample comes from a population whose properties are known. The best expression of these population properties can be provided by specialists in each associated discipline who must contribute state-of-the-art knowledge if the initial conditions are to represent reality in the most effective manner.

Many statistical tools are available for the building of this subsection: information theory, pattern recognition, experimental design, Bayesian Decision Theory, multiple discriminant analysis, spectrum analysis and multivariate regression analysis represent a few examples of the craft available to the applied statistician of today.

2. Resource Management

The primary object of the overall ecosystem simulation model is to optimize the long-term use of resources to the benefit of man. Since events proceeding from decisions made by man cause more instability than any other processes, an ability to test new laws, resource management procedures, or the consequences of effective planning must be built explicitly into the system. This model segment provides to decision makers the capability to manipulate the system. Their decisions and constraining laws must be accepted IN ENGLISH and then translated by the computer into the esoteric languages of mathematical programming which in turn impose the necessary critical controls on the scientific subsystems. Optimal, probable and possible concatenations of events which result from trial management decisions can be displayed (for example) in time sequence on banks of output scopes for detailed study of critical points in the system's consequent evolution.

Such mathematical computer games are quite commonplace in the urban planning profession. Richard M. Duke has developed what is probably the best known urban game, M.E.T.R.O., which incorporates simulations of social and economic characteristics, budgets, voters, and business segments of the community of LANSING, Michigan.

John R. Hamburg has combined the current land use and traffic analyses of Chicago to develop his transportation game: CATS. Jay Forrester has his generalized URBAN DYNAMICS MODEL and is now busily modelling the world for the CLUB of ROME. One model of particular interest to this symposium is the Chicago Air Pollution System Model developed at Argonne National Laboratories. It has used real data, real people and real decision making to test its "war game" strategies of optimal response to actual air pollution episodes.

Water resource management practices are becoming very sophisticated in many cases in this country. Some state agencies have computer programs which keep track of nearly every drop of water in their states both quantitatively and qualitatively. They not only know how many acre feet of water are stored in each reservoir, what is the flow rate of each river and stream, and the current status of the ground water supply, but they also know the past and current usage and the traditional and legal constraints put on usage. Water pollution monitoring devices add to this wealth of information to make the total amount of data so large that computer handling becomes less and less a luxury. A steady stream of demands for permits to build new dams, undertake new water-demanding industrial development, expand urban areas, and set up new irrigation systems makes computer planning models a necessity. Numerical simulation permits the testing of the consequences on the entire water system of a spectrum of possible strategies BEFORE one is chosen as the most likely optimal course to follow.

Many other sectors of the community use computer simulation games to assist their decision making: range management, transportation systems, gas and power dispatchers, merchandisers, and architects are a few of these.

There will be no lack of experience encountered when the management section of the ecosystem simulation model is built.

3. Prediction

Successful prediction of future states of the ecosystem depends on the adequate description of all relevant processes as well as on the initial conditions. One method used to keep within bounds any system in which one is not certain of the processes involved is to exercise a hierarchy of budgetary controls. These can run from strict adherence to the "laws" of conservation of energy, mass, and momentum to the conservation of water, biomass, and food chains (an example of entropy destroying processes) and on down to an insistence that each parameter in the ecosystem vary only within empirically established bounds. It is common practice for modellers of predictive systems to begin with very tightly controlled processes and gradually to relax the restrictions as the description of each process is refined to the point where it describes reality adequately.

K.E.F. Watt of the University of California at Davis has produced an extremely interesting model explicitly concerned with the fact that the vital processes which transform the ecosystem are energy consuming. Watt's ecosystem simulation comes closer to the comprehensive concept I am attempting to describe in this paper than any of the ones I have examined to date. George Van Dyne of Colorado State University is constructing a detailed model of a grasslands biome which is biomass conscious. Mints of UCLA, Leith of NCAR, and Kasahara and Washington of NCAR have all produced comprehensive atmospheric general circulation models which conserve energy (in a more precise but more restrictive sense than Watt) and momentum. Although these atmospheric models display a highly skilled virtuosity with the computer manipulation of equations, they are fundamentally deterministic. Fleming of the USAF has begun work on a stochastic-dynamic prediction model which might closer approach the meteorological subsystem needed in an ecosystem simulation model.

Other subsystems concern the transportation of power, material, and people from one place to another on a variety of scales. Also, a comprehensive economic model is clearly necessary not only to act as a tracer commodity but also to introduce memory and mobility into the value of actions. From another point of view, changes in the distribution of wealth can be considered as indicators of probable changes in processes involving the human component.

Population dynamics, urban dynamics and industrial dynamics represent aspects of experimental social psychology of vital concern to our ecosystem model.

It can be seen from the above discussion that an impressive arrays of highly sophisticated submodels have evolved in most cases as the result of the rather unorganized curiosity of individual specialists. In many instances someone has seen a particular use for these games and provided support to the specialist to "tune" his model to a subset of reality. It seems to me that the time has come to hook together the products of all these individual efforts and to produce an all inclusive ecosystem simulation program to permit a more balanced overall planning of human endeavour.

B. A. PRIMITIVE PASTORAL

Work on ecosystem modeling in the Hydrology Department of the University of Oklahoma began in 1963. A small group of enthusiastic students assisted and faculty set about a self-education program. This has not only produced a succession of operating numerical predictive models, but also has developed a great many valuable contacts with scientists, planners and industrialists in a variety of fields of direct interest. Cross-disciplinary programs have been planned and carried out and the results reported in papers and dozens of tables to interested groups across the country. The model at present has the following characteristics.

Several interlocking scientific subroutines are used to simulate the basic ecosystem. Parameters which limit or influence the ecosystem-determining processes of the system are controlled by an executive management routine. The model is stochastic. Real data are incorporated where possible and eliminated where necessary. Heavy-duty processing was used wherever they could be found and were the product of our intuition when they were unobtainable from authoritative sources. The complete model has run for over a decade of simulation on the main-frame computer.

Our ecosystem is considered to be about 100 miles square encompassing a city, farming areas, a lake, and a river system. Some of our scientific subroutines simulate: the hydrology, the atmosphere, the botany, the zoology, and the urban activity.

The atmospheric submodel is rather complete and simulates climatology through a combination of the use of Markov chains and conventional physical relationships. Real data have been used to obtain all the relevant statistical parameters. Precipitation comes from both widespread atmospheric clouds and restricted cumulonimbus phenom activity. These two types produce different responses in the hydrology subroutines. Surface solar radiation and soil temperature are accounted for here and are of obvious importance in the botany sub-routine. An urban heat island model and a simple Fahrenheit-Celsius air pollution diffusion model exert a great deal of influence on the urban population. Although the memory in the atmosphere is short, the memory of atmospheric effects in crop conditions, lake levels, population dynamics and health of the people (effects, for example, of air pollution episodes) is considerably longer.

The hydrology submodel keeps a fairly decent accounting of the water of the ecosystem. Soil moisture enters and goes in an acceptable fashion and the evapotranspiration forms part of this segment. Unit hydrograph concepts permit time sequencing of river stages and flow rates in the river system. Variations in water storage in the lake have profound effects on municipal development, agriculture, and industrial development through consequent water resource management decisions made in the executive routine.

Although our biology subroutines are quite incomplete they do conform to some reasonable models. Biomass budgets are kept in the botany subroutines to relate above ground live biomass, standing dead, litter, below ground biomass and the associated transfer processes in an acceptable manner. The zoology subroutines provides consumers for the hydrology and botany routines and food for the urban segment. It provides for range management, and poses land use and water pollution problems.

The city is dynamic and its population mobile. People have identifiable attributes and suburbs have individual statistical characterizations. People and subdivision boundaries move in response to stress. This stress can take a variety of forms, some of which involve the geophysical environment. Air pollution represents one such population dynamics forcing function. Social and economic pressures also induce people to move and change their life-style. It is obviously a great Machiavellian game to control the evolution of an entire city through a series of somewhat subtle manipulations in the executive management routine. The game may even be useful.

Water use, land use and air pollution control games have been played with this model. The results have been encouraging and highly stimulating.

Our work continues.

1. 在 1949 年 10 月 1 日，即中华人民共和国成立的那一天，毛泽东在天安门城楼上向全国人民发表了著名的“开国大典”讲话。这一天，北京城万人空巷，人们都涌上街头，庆祝新中国的诞生。毛泽东在讲话中宣告：“中华人民共和国中央人民政府今天成立了！”

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TIME-DEPENDENT GLOBAL ENERGY MODELING

D. A. Payer and T. Sellers

University of California, Santa
Cruz, California

INTRODUCTION

At the present time, there is a marked need to model the energy transport processes in the atmosphere because of the potential for change that has been developed by man. Over the past years, there have been many attempts to achieve this modeling goal, and the approaches have differed considerably. The most basic approach consists of applying the full equations of motion and energy to the atmosphere (averaged over some space and time scale, References [1], [2], and [3]) and then integrating these equations by some numerical technique. In a global scale, none of the research based on integration of the full equations of motion has yet reached a sufficiently accurate state that it can be used for predicting changes in the global temperatures.

Another approach to global energy modeling is to perform an energy balance on the atmosphere and oceans over large latitudinal belts, and develop empirical transport coefficients based on present-day conditions. A model so constructed would be limited to small changes from present conditions, because it assumes no drastic change in atmospheric or ocean structures. The problem of small deviations from present conditions is however a significant one, since history has shown that major changes in the environment can be caused by only a few degrees change in sea level temperature. Also, this approach is at the present time much more tractable computationally, and does not involve the excessive amounts of computer time demanded by the more fundamental approach.

Much of the basic research for the energy balance approach has been done by Eady (References [4] and [5]) and Sellers (References [6] and [7]). They have defined the most important physical processes in the earth-atmosphere system and have carried out steady state calculations on this system. The paper by Sellers, Reference [8], is the most sophisticated and contains much of the previous work of Eady. Sellers performs an energy balance on latitudinal belts, which included radiation, sensible heat transfer in the atmosphere, sensible heat transfer in the ocean and transport of water vapor. His results agree with present day temperatures in the latitudinal belts (the transport coefficients were chosen to do this), and also predicted realistic changes in sea level temperatures due to changes in atmospheric characteristics such as albedo and solar constant. The major shortcomings of this work was the lack of time in the calculations, and the use of a crude numerical method which gave double values for a solution. The lack of time in the calculations can be very significant, since transients caused by the thermal heat capacity of the atmosphere and oceans could damp out many of the changes in latitudinal temperatures. Also, the removal of the double solutions would dismiss the speculation concerning their meaning for the stability of atmospheric circulations, References [6] and [8].

The model of the global energy balance presented in this paper does include time, and it has been shown that the time dependent integrals do satisfy all interpretations of the results. Many of the problems that Sellers has posed all yield drastically different interpretations when time is considered as a factor. Also, the use of a better numerical solution technique with the time dependent equations shows that double solutions do not exist for a properly posed problem.

BASIC MODEL

As mentioned previously, the model employed is based on the energy equation applied to latitudinal belts in the atmosphere and oceans. The geometry of the belts is shown in Figure 1 where a typical latitudinal belt has been shown on the earth and in terms of spherical coordinates. The belt will be chosen to extend to the top of the atmosphere and to the bottom of the ocean. In order to derive the basic equation for the model the energy equation in its time dependent form is

energy of a turbulent flow. A general form of the energy equation for a finite volume is (reference [1]):

$$\frac{d}{dt} \int_V \rho \left(\frac{1}{2} \mathbf{u} \cdot \mathbf{u} + \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{1}{2} \mathbf{w} \cdot \mathbf{w} + \frac{1}{2} \mathbf{u} \cdot \mathbf{v} + \frac{1}{2} \mathbf{v} \cdot \mathbf{u} + \frac{1}{2} \mathbf{u} \cdot \mathbf{w} + \frac{1}{2} \mathbf{w} \cdot \mathbf{u} + \frac{1}{2} \mathbf{v} \cdot \mathbf{w} + \frac{1}{2} \mathbf{w} \cdot \mathbf{v} \right) dV + \int_V \rho \left(\frac{1}{2} \mathbf{u} \cdot \mathbf{u} + \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{1}{2} \mathbf{w} \cdot \mathbf{w} \right) dV = \int_V \rho \left(\frac{1}{2} \mathbf{u} \cdot \mathbf{u} + \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{1}{2} \mathbf{w} \cdot \mathbf{w} \right) dV$$

(1)

$$\frac{d}{dt} \int_V \rho \left(\frac{1}{2} \mathbf{u} \cdot \mathbf{u} + \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{1}{2} \mathbf{w} \cdot \mathbf{w} \right) dV + \int_V \rho \left(\frac{1}{2} \mathbf{u} \cdot \mathbf{u} + \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{1}{2} \mathbf{w} \cdot \mathbf{w} \right) dV = \int_V \rho \left(\frac{1}{2} \mathbf{u} \cdot \mathbf{u} + \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{1}{2} \mathbf{w} \cdot \mathbf{w} \right) dV$$

where the kinetic energy and work done by atmospheric and ocean currents have been neglected, since they are insignificant for the problem dealt with in this paper. In equation (1) the first volume integral is the rate of change of the internal energy in a hole, while the second is the flux of energy through the area of the latitudinal hole surface, where $\mathbf{u}$ is the velocity and $\mathbf{v}$ is the constant value of the temperature. The last two terms are the molecular heat flux into the volume and the energy absorbed due to heat sources such as radiation and the burning of fossil fuels. The summation sign and indices in equation (1) have been introduced to distinguish between the different constituents being transported such as air, water vapor and ocean water.

Before going into the details of the various terms in equation (1), some simplifications can be accomplished due to approximations which can be justifiably employed. The first approximation is that molecular heat conduction will be ignored compared to turbulent heat transport. Therefore, the third term in equation (1) will be neglected. The second approximation is to neglect the energy fluxes at the surface of the ocean and top of the atmosphere, thus reducing the area integral in the second term of equation (1). The third approximation is to introduce an average temperature and properties for a latitudinal hole which will take the spatial elongations around the hole into account.

Further simplification of equation (1) is required before it can be put into a useful form, and this consists of expressing the velocity and temperature in their mean and fluctuating components. In averaging in this manner, the term in equation (1) can be rewritten as

$$\begin{aligned} \frac{d}{dt} \int_V \rho \left(\frac{1}{2} \mathbf{u} \cdot \mathbf{u} + \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{1}{2} \mathbf{w} \cdot \mathbf{w} \right) dV &= \frac{d}{dt} \int_V \rho \left(\frac{1}{2} \mathbf{u} \cdot \mathbf{u} + \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{1}{2} \mathbf{w} \cdot \mathbf{w} \right) dV \\ &+ \frac{d}{dt} \int_V \rho \left(\frac{1}{2} \mathbf{u} \cdot \mathbf{u} + \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{1}{2} \mathbf{w} \cdot \mathbf{w} \right) dV \\ &+ \frac{d}{dt} \int_V \rho \left(\frac{1}{2} \mathbf{u} \cdot \mathbf{u} + \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{1}{2} \mathbf{w} \cdot \mathbf{w} \right) dV \end{aligned}$$

(2)

$$\begin{aligned} \frac{d}{dt} \int_V \rho \left(\frac{1}{2} \mathbf{u} \cdot \mathbf{u} + \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{1}{2} \mathbf{w} \cdot \mathbf{w} \right) dV &= \frac{d}{dt} \int_V \rho \left(\frac{1}{2} \mathbf{u} \cdot \mathbf{u} + \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{1}{2} \mathbf{w} \cdot \mathbf{w} \right) dV \\ &+ \frac{d}{dt} \int_V \rho \left(\frac{1}{2} \mathbf{u} \cdot \mathbf{u} + \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{1}{2} \mathbf{w} \cdot \mathbf{w} \right) dV \\ &+ \frac{d}{dt} \int_V \rho \left(\frac{1}{2} \mathbf{u} \cdot \mathbf{u} + \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{1}{2} \mathbf{w} \cdot \mathbf{w} \right) dV \\ &+ \frac{d}{dt} \int_V \rho \left(\frac{1}{2} \mathbf{u} \cdot \mathbf{u} + \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{1}{2} \mathbf{w} \cdot \mathbf{w} \right) dV \end{aligned}$$

$$\frac{d}{dt} \int_V \rho \left(\frac{1}{2} \mathbf{u} \cdot \mathbf{u} + \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{1}{2} \mathbf{w} \cdot \mathbf{w} \right) dV = \frac{d}{dt} \int_V \rho \left(\frac{1}{2} \mathbf{u} \cdot \mathbf{u} + \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \frac{1}{2} \mathbf{w} \cdot \mathbf{w} \right) dV$$

$$\begin{aligned}
& - 2 \cos \left[(\bar{v}_y \bar{v}_x \sin \theta + \bar{v}_y' \bar{v}_x' \sin \theta) \right]_{\theta=0} \\
& - (\bar{v}_y \bar{v}_x \sin \theta + \bar{v}_y' \bar{v}_x' \sin \theta) \Big|_0
\end{aligned}$$

where R_s is the net radiation input, S the net energy input from internal heat generation and the last term the energy lost or gain to a belt due to evaporation or condensation of water vapor, respectively. Also, the primed quantities are the fluctuating components of velocity and temperature.

If the differences in θ are now replaced by derivatives, Equation (2) can be written as

$$\begin{aligned}
& \left[\bar{v}_y \bar{v}_x \bar{v}_z + \bar{v}_y' \bar{v}_x' \bar{v}_z' + \bar{v}_y' \bar{v}_x' \bar{v}_z' (1 - \tau) \right] \sin \theta \frac{d\theta}{dz} + R_s \sin \theta + S \\
& - \left[\bar{v}_y \frac{d\theta}{dz} \left(\frac{d\theta}{dz} \bar{v}_x \tau \sin \theta \right) + \bar{v}_y' \frac{d\theta}{dz} \left(\frac{d\theta}{dz} \bar{v}_x' \tau \sin \theta \right) \right] \\
& + \frac{\partial}{\partial z} \left[\frac{d\theta}{dz} \sin \theta + \bar{v}_y \frac{d\theta}{dz} \right] + \frac{\partial}{\partial z} \left[\frac{d\theta}{dz} \tau \bar{v}_x \sin \theta + \frac{d\theta}{dz} \right] \\
& + \frac{\partial}{\partial z} \left[\frac{d\theta}{dz} \sin \theta + \bar{v}_y \frac{d\theta}{dz} \right]
\end{aligned} \quad (3)$$

where the following definitions have been employed

$$\begin{aligned}
\bar{v}_y \bar{v}_x \bar{v}_z & = - \frac{\partial}{\partial z} \frac{\bar{v}_y \bar{v}_x \tau}{\partial z} \\
\bar{v}_y' \bar{v}_x' \bar{v}_z' & = - \frac{\partial}{\partial z} \frac{\bar{v}_y' \bar{v}_x' \tau}{\partial z} \\
\bar{v}_y' \bar{v}_x' \bar{v}_z' & = - \frac{\partial}{\partial z} \frac{\bar{v}_y' \bar{v}_x' \tau}{\partial z} \\
\theta & = \text{specific humidity}
\end{aligned}$$

In the derivation of Equation (3) the following assumptions were made: (1) The sensible heat transport in the ocean is due solely to "turbulent" transport; (2) The net flow of water vapor into a belt is equal to the condensation or evaporation of water vapor; (3) All transport coefficients are related to the average sea level temperature of the belt (1).

To proceed further with Equation (3) data must be found for the transport coefficients, specific humidity, radiation characteristics and internal heat generation for the various latitudinal belts. In the present paper, extensive use of the data of Sellers, Reference [6], will be employed. In that paper the transport coefficients K_s , K_e and K_p were listed as functions of latitude. Also given were the pressure depth (ρ/ρ_0), sea level ocean T , and surface albedo (α) data. The expression for the net radiation R_s was

$$\begin{aligned}
R_s & = Q_s (1 - \alpha) - I_s \\
Q_s & = \text{incident solar radiation} \\
I_s & = \text{infrared emission to space}
\end{aligned}$$

The use of a sea level wind velocity ($\bar{v}_w$) for the air and water vapor is absolutely necessary to avoid the use of negative transport coefficients or diffusivities. A modified version of Sellers' expression is

$$v_{\theta} = -A \left(\frac{\partial T}{\partial \theta} + \left| \frac{\partial T}{\partial \theta} \right| \right) \quad \text{north of 5N}$$

$$v_{\theta} = -A \left(\frac{\partial T}{\partial \theta} - \left| \frac{\partial T}{\partial \theta} \right| \right) \quad \text{south of 5N}$$

where A is a function of latitude and $\left| \frac{\partial T}{\partial \theta} \right|$ is the average value of the absolute value of the temperature gradient in all the belts. It should be clearly recognized that v_{θ} is not a physical velocity, but a velocity chosen to give a logical energy transport structure.

The final information needed to complete Equation (3) is to give the specific humidity for a belt. One solution is to assume that the air is saturated and use the Clausius-Clapeyron equation, and this was employed by Sellers. In the present work a relative humidity of approximately 75% was used throughout the investigation and this is more characteristic of the current conditions.

METHOD OF SOLUTION

The partial differential equation given by Equation (3) is essentially a time-dependent version of the equation given by Sellers. This equation can be described mathematically as a nonlinear, second order, parabolic partial differential equation. The nonlinearity is caused by the water vapor terms with their exponential dependence on temperature. In general, equations of this type can be difficult to solve, and one must be careful in choosing a method of solution. In the present paper a finite difference method will be employed, and it will be shown that this method is superior to the techniques used by Sellers.

In order to solve Equation (3), initial conditions on the temperature distribution at some time and boundary conditions must be known. For all of the problems solved the initial conditions were chosen to be the existing yearly averages, and the boundary conditions were applied at the poles. The boundary conditions at the poles are that the energy transport into the poles is zero. This condition differs from the zero flux condition at the poles, since zero flux would imply $\frac{\partial T}{\partial \theta}$ equals zero, and this is not the condition theoretically or experimentally.

The basic method of solution employed was an implicit finite difference method similar to that used for the unsteady heat condition equation. With this scheme all derivatives with respect to θ are evaluated at the unknown station $t + \Delta t$ (see Figure 2 for a schematic representation), and the time derivative was evaluated as a backward difference from the conditions at $t + \Delta t$. The reason that the implicit scheme was chosen, was because of its superior mathematical stability. Previous experience with highly nonlinear parabolic equations has shown that the stability characteristics of this scheme are necessary to get a valid solution.

One unusual aspect of the calculation was the way that the second derivative terms were evaluated. These derivatives were evaluated as the difference in two first order derivatives because of the uncertainty in the data for the diffusivities. Figure 3 shows a plot of the eddy diffusivity for sensible heat transport in the air as given by Sellers. As can be seen from Figure 3, the derivative of K_a can be quite large or small, and because of uncertainties in the data it was not thought that accurate derivatives could be obtained from this data. Therefore, the second derivative terms were evaluated in the following manner

$$\frac{\partial}{\partial \theta} (K_a \sin \theta \frac{\partial T}{\partial \theta}) = \frac{q_{i+1, j+1/2} - q_{i+1, j-1/2}}{\Delta \theta}$$

where

$$q_{i+1, j+1/2} = K_{aj+1/2} \sin \theta_{j+1/2} \frac{T_{i+1, j+1} - T_{i+1, j}}{\Delta \theta}$$

$$q_{i+1, j-1/2} = K_{aj-1/2} \sin \theta_{j-1/2} \frac{T_{i+1, j} - T_{i+1, j-1}}{\Delta \theta}$$

With this method derivatives of the diffusivities were not required, and accurate solutions were obtained.

In the calculation the nonlinear terms were linearized by evaluating the temperature dependent coefficients at the previous time step. For the method to be accurate iteration must be used or small time steps taken. For the calculations carried out in the present paper it was found that a time step of one year was small enough to obtain converged solutions without iteration. With this scheme the temperatures at the unknown station $t + \Delta t$ had to be solved simultaneously, however, since the matrix of the coefficients of the temperature was tri-diagonal this was accomplished in a direct fashion.

Another point that should be mentioned in regard to the method of solution is the double roots obtained by Sellers. With the present scheme there are no double roots either for steady or the transient conditions. The double roots obtained by Sellers were a direct result of the numerical method he choose to solve for the temperature. Therefore, any speculation concerning the meaning of the alternate solution is without physical basis. In fact, there is not enough physical modeling in Equation (3) to make any statement concerning the stability of the atmosphere.

RESULTS

For the present calculations it was decided that only twenty latitudinal belts should be employed because of the uncertainties in the transport coefficient data. With this number of belts the calculations agreed exactly with present day conditions for the temperatures and thus checked Sellers work. One problem that remained before time dependent calculations could be carried out was to determine the depth of ocean that would contribute to its heat capacity. To determine this depth use was made of the fact that annual oscillations of temperature have been recorded as deep as 300 meters (Reference [5]). Therefore, to be on the conservative side the depth of ocean for the heat capacity was chosen as 600 meters. (Note: This depth is not directly related to the mixed layer involved in daily transport.)

To show the effect of the depth of ocean on the present model, calculations were carried out with various depths, and are shown in Figure (4). Figure (4) shows the time-dependent response of typical equatorial and polar latitudinal belts for a two per cent decrease in the solar constant. Two ocean depths, sixty and six hundred meters, have been employed. In the sixty meter case a steady state is reached at almost fifty years from the initial decrease in solar constant. However, when the six hundred meter ocean is employed steady state is not nearly obtained after two hundred years. Clearly, the heat capacities of the ocean have a dominant effect on changes for the six hundred meter depth. (Note: The present calculations have no phase lag between the ocean and atmosphere incorporated in them.)

One very interesting aspect of Figure (4) is the relatively large changes in sea level temperature (10° to 15°) due to a two per cent change in albedo. These changes have been observed by Sellers, but the important role of the time lags have not been completely comprehended. This long time scale means that the results should be interpreted very cautiously, since changes in the atmospheric radiation vary in time. For example, if a sinusoidal variation of the solar constant with a period of 100 years is employed with a 2% amplitude, the net effect on the belt temperature will be drastically different. Figure (5) shows the time-dependent temperature variation for the same latitude belts as Figure (4), except that a sinusoidal variation of the solar constant is used. In this case the belt temperature changes are on the order of 1° from the minimum to maximum location, and this is an order of magnitude reduction. The oceans with their large heat capacity have damped out the effect of solar constant fluctuations.

Another interesting point in Figure (5) is the phase relationships in time and space. The equatorial belt follows the solar constant fairly rapidly, while in the polar belts a phase lag greater than 90° is observed. The reason for the lag is due to the fact that the polar belts receive most of their energy from the other belts, and not from solar radiation. Therefore, they must wait for changes to occur in the equatorial belts before an effect is felt. The net result is that the polar belts follow the equatorial belts in the temperature fluctuations.

The final calculations which have been completed so far are those with internal heat generation. The source of the heat for the model is that projected to be used by man in the near future. Upper estimates by power specialists indicate that as much as 150 per year may be used by man in the next 50 to 100 years ($1Q = 1 \cdot 10^{18}$ Btu). In order to estimate the influence of this energy source, calculations were carried out with an exponential increase in energy consumption to 15 Q/year. Two

different time scales for the increase were chosen, and these were 50 and 100 years. The results of the calculation are seen in Figures (5) and (7). Figure (6) shows the initial and final sea level temperature distributions along with the distribution curve for the heat generation in the latitudinal belts. The temperature changes were found to be of the order of one to a couple of degrees with the largest changes occurring at the poles. The fact that the largest changes existed near the poles is interesting, since the heat is actually released in the temperate zones. This calculation shows the sensitivity of the poles to changes in the larger and warmer temperate zones. Also, the changes in temperature calculated are substantial enough for internal heat generation to be a factor in future climatic changes. Figure (7) illustrates the time-dependent aspects of the changes for the latitudinal belt near 85S for the two different time constants chosen. For the small time constant case the time phase lag is significant.

CONCLUSIONS

The major results and conclusions which have been obtained from the previous research are:

- (1). A time-dependent model equation of the energy balances on the earth has been derived. The nature of this equation has been analyzed and an efficient method of solution has been developed.
- (2). The heat capacity of the oceans play a dominant role in transient processes in the environment. The neglect of the ocean heat capacity will lead to serious misinterpretation of results.
- (3). The temperature of polar latitudinal belts are very sensitive to changes in the temperate and equatorial zones. Substantial time lags may, however, exist before these changes are felt.
- (4). Internal heat generation by man could cause substantial changes in global temperatures. Also, time lags do exist before these changes may be felt.
- (5). No double solutions for temperature exists for the present method of solution, and the existence of double solutions seems highly unlikely on both physical and mathematical grounds.

LIST OF NOMENCLATURE

a	radius of earth		
C _p	specific heat at constant pressure		
d _a	depth of atmosphere		
d _o	depth of ocean		
d _l	depth of land		
L	latent heat of water		
1'	percent ocean in a latitudinal belt		
ρ	density		
Q _i	source or sink of heat		
$\vec{q}$	molecular heat flux		
q	specific humidity		
T	temperature		
V	volume		
v	velocity		
		<u>Subscripts</u>	
		a	air
		o	ocean
		l	land
		w	water vapor
		<u>Superscript</u>	
		—	average sea level value
		'	fluctuating value

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Figure 1 - Basic Geometry of the Model

BASIC FINITE DIFFERENCE METHOD

Figure 2 - Schematic Finite Difference Grid

Figure 3 - Latitudinal Variation in Transport Coefficient

Figure 4 - Influence of Ocean Depth on the Model

Figure 5 - Sinusoidal Variation in Solar Constant

Figure 6 - Effect of Internal Heat Generation

Figure 7 - Time Dependence of Internal Heat Generation

COMMENTS ON SYMPOSIUM AIMS AND GOALS

Werner Klug
Institut für Meteorologie
Technische Hochschule Darmstadt

It is with great pleasure that I speak to you at this Symposium which seems to be a very interesting and stimulating one.

I have been asked to speak to you about the aims and goals of this Symposium and since I have heard about these--like you--only at the very beginning of this meeting. I shall only be able to talk to you at this hour about the impressions and thoughts I had while I was listening to the papers presented. These will be related to the aims and goals set up by Dr. Luna on the first day.

One of the important goals of this symposium, according to Dr. Luna, was "to provide a medium for communication and discussion in the related fields of air pollution, turbulence and diffusion." Now as I look back to the coffee breaks, the discussion evening and even to the banquet we had last night, I have the impression that this goal was achieved to a hundred percent. This is mostly due to the perfect organization and friendly hospitality which was extended to us by the organizers of this meeting. I take the liberty to thank them for all of us who were able to take part in this Symposium. But besides this general goal there have been more specific points raised and since I was asked to do so, I shall try to discuss and comment on them.

1. "If the solution of air pollution problems is largely a political (enforcement) problem, is there any justification for diffusion modeling and boundary layer studies in the air pollution fields?"

I must admit that I was deeply puzzled when I first heard this question, but after the very first paper presented here, I changed my mind and I thought the question to be a relevant one, at least with regard to some people at this Symposium. I shall therefore discuss it.

From my point of view, diffusion modeling is justified even if there were no practical applications connected with it. If we are in the position to model properly, the boundary layer and the diffusion of material in it, then this would mean that we understand the meteorological processes connected with it at least to that degree that we can supply numbers which are representative for the processes going on in the atmosphere's boundary layer. Now, if there are practical applications involved--and we know there are-- and one is of the opinion that the results produced by such models are not sufficiently accurate enough, the only possible conclusion would be to invest so much brain work and money that the outcome is adequate and accurate enough. But I think the outcome is already--at least for some models--sufficient, that only the users of our results are not willing and/or not in the position to make full use of them. If this indeed is the case, then I agree with Dr. Dabberdt who pointed out that meteorologists may have failed to bring over their ideas and results to the potential users, and we are to blame for this. Let me explain this with an example which is all too well known to air pollution meteorologists and which is representative for other problems in this connection. The meteorologist is often asked what height a stack with specified emission data must have so that the maximum ground level concentration does not, under all circumstances, exceed a specified standard. This is the usual way such questions are put to us. If one sticks to this question as it is raised one can come up with answers which are probably correct but which result in stack heights, which have the height of the 500 mb layer (≈ 6 Km). This is obviously absurd and the point is that the raised question must be modified to a slightly different one: given the specified emission data, what height must the stack have, so that maximum ground level concentrations do not exceed the given standard in x-percent of all cases. The x-percent, however, must be supplied by air pollution authorities and to state this percentage value is not a problem which can be solved by meteorological considerations and therefore by meteorologists. There are other arguments, even political ones, which are pertinent. To supply reliable answers to the rephrased question is, from my point of view, not a too difficult job, if there are not any local conditions (such as deep valleys, local circulations, etc) which prohibit the use of our relatively simple models. But even in the other, more complicated cases, there are ways to obtain reliable estimates.

Urban modeling should also be used in the context of supplying frequency distributions of concentrations at given points and it is here where results have already been observed which are accurate enough for the purposes mentioned. I have heard the argument that the public asks for absolutely clean air--whatever that is--and does not care for any standards which are set up. Here I can only say that air pollution authorities and politicians have obviously failed to educate the public of what is feasible and what is not. Especially the consequences of such demands should be stated clearly.

2. "Is there any hope that the data and theory for flows (atmospheric and wind tunnel), over a vast flat plane, in near-neutral and steady conditions can ever be used to understand conditions in the real world?"

My answer here is there is not only hope but there is already proof for this. The Prandtl-layer model which has been set up by using similarity ideas for even non-neutral conditions is one of the examples where this is true. It is even hard today to test with the most sophisticated instruments which is the better of two formulas. Within the error of measurement, a number of theoretical or semiempirical results are able to predict the "real world" under the conditions mentioned. Other examples are perhaps less accurate but are sufficient for many practical purposes.

3. "Does the direct solution of the Navier-Stokes equations, which is made possible by the current generation of computers, provide realistic solutions to real atmospheric flows of interest in air pollution studies?"

The answer here can be given by any attendant of this symposium who listened to the papers presented. The answer is yes as long as the question refers to numerical solutions of the slightly modified type of the N-S equations. Such "realistic" solutions were presented here, although I have some doubt that these solutions are correct already in each and every detail. But a simultaneous solution of the equations of motion and the diffusion equation where the diffusion coefficients are generated by the characteristics of the flow will solve many of the problems encountered.

4. "Is the spatial and temporal density of measurements required to understand the state of the atmosphere within a city within the financial or physical capability of any organization interested in the data?"

The answer is yes if again it is understood that it is not for example, the concentration at minute x of hour y on day z at a given location which is of importance for our problems, but it is the frequency distribution of concentrations over one or a few years; which is the decisive quantity to look for. And here it is the diffusion model, which allows us to put up a limited number of monitoring stations and additional information can be supplied by the model.

5. "Is there information in the detailed measurements of atmospheric turbulent variables such as, coherence, quadrature spectra, covariance, etc., which is useful in the theory or practice of transport and diffusion prediction?"

This is admittedly a more subtle question than the ones raised before. As you all know, it is the Lagrangian frame of measurements which is needed for diffusion problems and the perfect way to obtain these measurements has not been discovered and probably will never be. However, it has been demonstrated here, that it is possible to obtain from elaborate measurements of cospectra as function of time and space, information about such quantities as β , which relates Lagrangian and Eulerian time scales. There may be other ways to think of, but I believe such information can be extracted from existing or future measurements.

6. "Are present sensor systems for measurements of atmospheric variables adequate for turbulence and diffusion studies on the microscale? mesoscale? global scale?"

I would like to differentiate here between the sensor system as a single instrument which allows us to measure the quantity we are interested in and the sensor system which is given by a number of such instruments, the measuring network. In the first case with measuring devices such as lidar, acoustic soundings, radiometers, constant pressure balloons, aircraft measurements, automatic weather stations, radar, remote sensing from satellites and many more, the meteorologist is in the position to obtain all the measurements he needs, if these devices and the instrument carrier were available to him in a density which is adequate for his purposes. This is not done for many cases, mainly due to lack of money.

In closing, let me make a few remarks about problems which have not been dealt with at this meeting. For modeling the atmosphere to obtain numerical forecasts over more than 3-4 days or modeling the general circulation, it is of great importance to know the turbulent fluxes of momentum, heat and moisture which pass through the boundary layer. These fluxes must be given in relation to the large scale external parameters such as the geostrophic wind vector on top of the boundary layer, the roughness length, the Coriolis parameter and temperature difference as well as mixing ratio difference across the boundary layer. These relationships must be supplied by the boundary layer meteorologist in due time to assure a success of these activities.

The other, from my point of view, important problem not mentioned here is the mesoscale transport of air pollution. In Europe, this is becoming a more and more relevant question and we shall soon be asked for modeling of such processes. I shall not elaborate on the problems connected with such questions but perhaps we shall hear about them at the next meeting which I hope will be as successful, pleasant and interesting as the one we are just having.

COMMENTS ON SYMPOSIUM
AIMS AND GOALS

Niels E. Busch
Danish AEC Risø

The following comments on symposium aims and goals were reconstructed on the basis of rather meager notes and several weeks after the Air Pollution, Turbulence and Diffusion Symposium, December 7-10, 1971, in Las Cruces, New Mexico.

Dr. Werner Klug's talk on the same subject was delivered immediately before mine. He took care of the specifics so adequately and eloquently that on the spur of the moment I decided to change my talk away from the few things I know a little about and towards the more general problems to which we ought to direct our attention before we invest our time and somebody's money in air pollution meteorology.

In his introduction to the symposium Dr. R. E. Luna formulated a series of six questions, which he hoped would be answered or at least thoroughly discussed in the course of the symposium. They were not answered and some of them not even discussed. The organizing committee suggested that I take these questions as a basis for my hopefully not too incoherent remarks.

Question 1. If the solution of air pollution problems is largely a political problem, is there any justification for diffusion modelling and boundary layer studies?

This is, I believe, what is called a leading question. In air pollution abatement and control the major problems are: 1. In which order of priority should the polluters be attacked. 2. In terms of air quality what will the effects be of a given move against these polluters. Irrespective of whether the pollution situation is considered a looming ecological catastrophe or just a major nuisance, it is obvious that without adequate engineering answers to the two questions any air pollution agency will be groping in the smoke.

Are the meteorologists ready and able to provide their part of the answers? I think we are, although many air pollution meteorologists are coming out with insufficient answers to the wrong problems. Too many of us are writing scientific papers instead of solving pollution problems. We give our scientific interests a slight twist and present them as being of immediate importance to abatement and control agencies. I do believe firmly in basic science, but I also believe firmly in calling it basic science and not air pollution research.

It is true that the better we understand the physics the better are our chances for designing the proper solutions to the pollution problems facing us. However, we should not forget that an important part of the work is to weigh needs and possibilities against cost and time. Although only a minute fraction of the funds which must be spent in the future on the fight against air pollution will be invested in meteorology, this discipline holds a key position in the complicated

process of assessing priorities, evaluating results, and devising solutions. It is after all the motions of the atmosphere which link the pollution sources to air quality. If the meteorologists take air pollution seriously they should become members of interdisciplinary teams, learn much more about pollution, and press hard for a say when and where the problems and the projects are formulated.

Question 2. Is there any hope that the data and theory for flows over a vast flat plane, in near neutral and steady conditions can ever be used to understand conditions in the real world?

Science knows no other way of attacking complicated problems than to seek out the simplest nontrivial cases first and then to proceed from the understanding of the simpler cases to the more complex.

The understanding of microscale turbulence in horizontally homogeneous boundary layers is fairly comprehensive. There are admittedly many aspects for which we could desire clarification, but are they relevant to the planning and execution of air pollution programs? As a micrometeorologist I am myself engaged in basic studies of atmospheric turbulence, and I do like to think that this part of my work is useful also in the context of pollution, but I am cautious about referring to it as relevant to air pollution. We should be moderately careful not to overstate our case. We may not be able to keep our promises. "Overselling" meteorology, being more enthusiastic than sober considerations can support, could cause crises similar to those which appear to trouble the computer industry. Our situation is even more precarious. They only have technology and the public to worry about. We have Nature, too.

Question 3. Does the direct solutions of the Navier-Stokes equations, which is made possible by the current generation of computers, provide realistic solutions to real atmospheric flows of interest in air pollution studies?

I do not know of any direct solutions to the exact N.-S. equations which are applicable to atmospheric flows over their entire spectrum, and I do not believe we will ever see any. I should like to make an obvious but important point here. Most attempts to solve the equation of motion in fluid mechanics and all attempts at solving the equations for turbulent flows involve restrictive assumption. The closure problem of turbulence is well known and the closure schemes plenty. Some of these schemes are of more general nature and less empirical than others, but basically the game is empirical. Add to these nonlinear difficulties those connected with the specification of boundary conditions and initial conditions and it seems quite obvious that without appeal to similitude and empirical results simulation of atmospheric flows will not be possible.

We cannot simulate any flow unless we can parameterize the relevant features of the "microscopic," i.e. subgrid, processes. Furthermore we are forced to assume that it is sufficient to specify boundary and initial conditions in terms of averages over "macroscopic" domains in time and space. This in essence means that we take refuge to some kind of statistical similarity.

Instead of "first principles" we have the two "axioms" of practical modelling: 1. A model may describe certain flow characteristics satisfactorily although others are distorted considerably. 2. The less detailed in prediction we require, the better the chances are that we may get away with using a simple model. In the light of these "axioms" it becomes apparent why the experimental verification, which is all too often missing, is so important and why modelling is a dubious affair unless it has rather well defined aims.

Question 4. Is the spatial and temporal density of measurements required to understand the state of the atmosphere within a city within the financial or physical capability of any organization interested in the data?

I must say that this is a typical question, which illustrates the backward way many of us think. One may legitimately be interested in observing, explaining, or predicting specific phenomena and effects, but one does not ask for "understanding of the state of the atmosphere." From a scientific point of view there are many much more fruitful problems to study than the various aspects of flows in and around cities. From an engineering point of view few meteorological problems are more important.

Owing to the complexity of the problem it is highly improbable that a few models, not to speak of one single, unifying model, will ever be conceived which can account for all combinations of meteorological, topographic, aerodynamic, and source parameters. We must define the problem and then look for a model which describes it. Because of the dire consequences which pollution may have, we are forced to be very careful in setting priorities. First things first--there won't be time for second things. Thus the meteorologists must leave their traditional academic attitude and become problem and project oriented.

This brings me to my only complaint about this symposium. We have heard some forty-eight papers. The quality has been high and the contents relevant and useful, but except for one introductory paper we have heard nothing about air pollution problems as the abatement and control agencies see them, nothing about the effects of air pollution, and nothing about the political, legal, social, and economic consequences of air pollution and air pollution abatement. Except for one interesting but somewhat futuristic paper on the methodology of ecosystem simulation, there has been no attempts towards system analysis, establishment of a list of priorities or general guide lines. And then Dr. Eddy's paper scare me no end by making a giant computer and myriads of probably uncheckable subroutines responsible for the survival of mankind.

Question 5. Is there information in the detailed measurements of atmospheric turbulent variables such as, coherence, quadrature spectra, covariance, etc., which is useful in the theory or practice of transport and diffusion prediction?

As far as I see it, the question is not whether the type of statistics mentioned is useful. The questions are: Which spectra, coherences, etc? Are these quantities useful enough to warrant

the investments of time and money it takes to obtain them? Are there other kinds of statistics which are more relevant?

These questions are too complex for us to discuss here, but let me make three observations:

1. Prediction of turbulent diffusion should be based on predictions of turbulence characteristics.
2. It is easier and safer to estimate turbulence characteristics from other turbulence characteristics than from knowledge about the mean flow.
3. The more information we have about the structure of the turbulent flow the better prepared we are to argue for its similarity with other turbulent flows about which we know.

Question 6. Are present sensor systems for measurements of atmospheric variables adequate for turbulence and diffusion studies on the microscale? Mesoscale? Global scale?

An experiment requires that at least two basic conditions be fulfilled. The one is that a sensor exists which directly or indirectly can give us a measure of the quantity we want with the accuracy with which we want it. The other is that the sensor can be brought to where we want the information collected. It does not make much sense to separate the two aspects. Off hand I cannot think of any atmospheric variable which we could not measure, if we could get it into the laboratory or get the laboratory out there.

Some sober thought reveals that even in clear air there are a number of quantities which either cannot be measured in practice or only with unsatisfactory certainty. Microscale density-fluctuations, humidity-fluctuations on all scales, and radiative heat-fluxes are examples.

The major problem, however, is to get soundings which are closely enough spaced in time and space to allow us to follow the evolution of the spatial features of the flow patterns. This is a difficult task on all scales, but particularly in mesometeorology. Mesoscale phenomena fail to obey simple dispersion relations (e.g., Taylor's hypothesis), most of their important features are too small for ordinary synoptic and aerological networks to be of any use, and their characteristic time scales are comparable to the time it takes to fly an airplane across them. Furthermore many mesoscale circulations of particular interest in air pollution studies are so weak that the stability of the platform from which the measurements are taken as well as the sensitivity of the instrumentation become critical. There is hardly any doubt that remote sensing by means of ground based equipment is the only possibility. Fortunately there are substantial reasons to believe that short-wave radars, acoustic radars, and laser systems may be able to provide some of the answers before long.

Concluding remarks. A noted colleague of ours once showed his disrespect for air pollution meteorology by describing it very briefly as "something about smoke coming out of a stack and disappearing in the downstream direction." If I understood and remember correctly Dr. V. H. Sussman in his introductory lecture accused the meteorologists of failing to get the real nature of the

problems across to the public and to the authorities and of wasting time and efforts on providing cover for wrong decisions. Apparently we have got problems.

However, we share these problems with all scientists involved in pollution work. By and large scientists are not used to the kind of political limelight which is flooding the pollution stage. We have a heavy responsibility toward the politicians. As scientists we cannot solve political problems. That is a job for politicians, but as it is one cannot blame the politicians for not understanding much of what is going on. Far too often we present them with detailed, fractional information instead of evaluated alternatives for attack on major pollution problems. We have no reason to be overly optimistic about our ability to devise solutions to pollution problems and should be cautious not to convey the impression that solutions are around the corner and that we know which corner. Don't surf the pollution wave. An honest and well-balanced evaluation of priorities and feasibilities is what is in demand.

COMPARISONS AMONG TRACER, TETROON, AND WIND FIELD DERIVED
TRAJECTORIES AND SOME DIFFUSION ESTIMATES*

G. E. Start C. R. Dickson
 NOAA
Idaho Falls, Idaho

and

P. G. Voilleque
Health Services Lab, USAEC
Idaho Falls, Idaho

A need existed to understand transport, diffusion and plume depletion effects upon airborne material carried to distances of about 100 kilometers and for time periods of many hours. Extensive and painstaking diffusion measurements have been made in the past couple of decades and have been summarized and reported by many authors, including Heffter and Slade, for example.(1),(2) These measurements have, for the large part, described diffusion at the shorter distances or have dealt with large instantaneous sources at long distances, high altitudes, or both. The intent of this study was to better understand the transport and diffusion of quasi-continuous effluent releases dispersed within the planetary boundary layer.

At the initial stages of such a measurement program the safest course of action seemed to be one which included several simultaneous types of measurements a) to determine the consistency of the different measurement techniques, b) to combine the complementary data types to gain a more complete overview of the effluent behavior, and c) to identify and emphasize data measurements which appeared to be the vital ones. At the beginning of such a study it is crucial to have direct measurements of effluent concentration to know the effective plume trajectory and dilution. The key information available from this testing were the measurements of tracer ground-level concentrations at successive downwind distances. Based on these measurements the downwind change of relative axial concentration and lateral tracer dispersion could be examined. Along with these direct tracer measurements, this testing included unique, simultaneous measurements of low level winds from an encompassing network tower of mounted wind sensors and trajectories from serially released tetroons. The information presented in this paper is in no way a statistical climatology. Instead, it illustrates the successful field implementation of a unique combination of complementary and valuable measurements which could be a useful technique for achieving a better understanding of regional dispersion of airborne material.

Two field tests have been conducted to date. The second test conducted on August 31, 1971, was highly successful and will be used to illustrate comparisons of the various transport and diffusion estimation techniques. A third test attempt was aborted due to the failure to achieve suitable testing conditions; the first test was of limited use due to the presence of a wind shear or discontinuity zone across the sampling grid some 40 kilometers downwind.

The measurements described in this study were conducted over the Upper Snake River Plain in southeastern Idaho at the National Reactor Testing Station and adjacent areas. The Upper Snake River Plain is approximately 5000 feet above sea level and is enclosed by mountains which rise 4000 to 6000 feet above the valley floor along all but the southwestern edge of the rectangular shaped study area. The irregular shaped boundary within figure 1 outlines the National Reactor Testing Station.

The tracer used in this investigation was methyl iodide gas labeled with the radioisotope iodine-131. This tracer technique was developed and utilized during a limited number of diffusion comparisons with uranine dye and molecular iodine gas for downwind travel distances less than 3.2 kilometers.(3) This gas is chemically inert, for practical purposes, and allows a control of plume depletion due to dry deposition. Sampling checks have not revealed a tendency for ultra-violet decomposition of the methyl iodide into reactive (depositing) iodine species. Time integrated air samples were collected using a network of high volume air samplers, each loaded with an iodine impregnated charcoal cartridge and a glass fiber type prefilter.(3) The sampler collection efficiency for methyl iodide was 0.75. Four sampling arcs, with from 5 to 8 high volume air samplers per arc, were located at approximately 6, 19, 48, and 80 kilometers downwind. Qualitative and quantitative identifications of iodine-131 in the samples were made using gamma spectroscopy and conventional well scintillation counting techniques. Methyl iodide gas was released over a 65-minute period from the 76 meter stack at the Idaho Chemical Processing Plant.

Twenty-four tower mounted wind sensors were operated within the rectangular area superimposed upon figure 1. Wind directions and speeds measured at somewhat randomly located tower sites within the grid were interpolated to a rectangular mesh of points. Calculated trajectories were derived from these "wind fields" for comparison with observed plume centerline positions and tetroon trajectories.(4)

*The work described in this paper is supported by an agreement with the Division of Reactor Development and Technology, U. S. Atomic Energy Commission.

A transponder equipped tetroom was released about one and one half hours before the methyl iodide release. Other tetrooms were launched at the beginning of the release and at approximately 20 minute intervals during the ensuing hour. In all, five tetrooms were launched. The tetroom-transponder flights were tracked by a M-33 radar. The use of the radar-tetroom-transponder system to obtain estimates of air trajectories and wind statistics over relatively long distances has been given by Pack and Angell.(5),(6) Since a single radar was being used to follow several tetrooms in the air at the same time, positioning of individual tetrooms was accomplished about once every five minutes. Detailed tracking information was not determined for any one tetroom; rather, the general position of each tetroom was determined so that the lateral and vertical envelopes of tetroom positions for the series of launches could be estimated. The tetrooms remained within 600 meters above the surface much of the time. All tetroom positions during vertical height oscillations were measured below 1600 meters above ground level. An examination of the Boise radiosonde data and the 700 and 500 mb analyses for 1200 GMT on August 31 and 0000 GMT on September 1, 1971 was made. From this information and the surface temperatures in the test area a maximum adiabatic mixing depth of 1400 meters was estimated. This value was similar to the 1600 meter depth within which the tetroom oscillations were contained.

Normalized axial concentrations versus downwind distance were plotted in figure 2. NRTS curves for stability classes B and C are plotted for reference. The methyl iodide decrease of slope with distance--a decrease which was not explained simply by sampling inaccuracies or the occurrence of the peak concentrations at positions between samplers--should be noted. Two additional curves, those labeled CLID and BLID, were shown to illustrate the effect upon the climatological curves if vertical tracer dispersion were confined within a layer 1400 meters above ground level. These two additional curves will be discussed in later paragraphs.

Lateral standard deviations measured for methyl iodide were plotted versus downwind distance in figure 3. The NRTS curves for uranine dye, labeled for stability classes B, C, and D are included for reference.(7) Statistically, the methyl iodide slope was the same as the NRTS slopes of 0.85.

The horizontal trajectories derived from the tower wind measurements are compared to the tetroom flights in figure 4. A tetroom trajectory is not plotted for a release time of 1430 MST (the beginning of the methyl iodide release). This flight ended about 60 seconds after launch. Letter symbols are located along the plotted trajectories at successive 15 minute intervals, with times for wind field derived trajectories denoted by upper case letters and tetroom times denoted by lower case letters. The tetroom positions generally led the corresponding calculated trajectory positions by a few kilometers (in agreement with pibal observed wind speeds 2-3 mps greater at tetroom flight altitudes). In every case the tetroom trajectories deviated to the east of the wind field determined trajectories. The smallest lateral separation between a wind field derived trajectory and a tetroom trajectory occurred for the flights begun at 1507 MST. This tetroom consistently flew at lower heights for more of the time than any other tetroom.

The angular displacements between wind field and tetroom trajectories were on the order of 5 degrees. Pibal winds within the first 800 meters above the surface supported a wind direction turning consistent with this separation. These same trajectories are shown in figures 5 and 6. In figure 5 (6) all tetroom (wind field derived) trajectories are plotted together. The dashed trajectory in figure 5 is the wind field determined trajectory corresponding to tetroom release #2. This line seemed to have been a reasonable estimate of its extreme northwestward position. Open circles were plotted to note the points of peak sampled methyl iodide on each arc. At six kilometers (figure 5) the wind field derived trajectories are better locaters of the peak concentration; at greater distances the tetrooms were best (figure 6).

To convert the trajectory data into a form similar to methyl iodide lateral dispersion, the arbitrary assumption was made that the lateral width of the envelope of the trajectories contained 95 percent of all the possible trajectories that might have occurred in this period, or in other words, that they represented $\pm 2\sigma$ about the (undefined) mean. There is no justification for this assumption although intuitively it seemed reasonable. While it rendered the absolute values of the plume spread suspect, the rate of spreading was unaffected by the particular choice of numerical σ value.(6) The line in figure 3 labeled tetrooms is a plot of these envelope widths after division by four. Since all of the tetrooms were tracked short of arc D at 76 kilometers, the data point at 76 kilometers was estimated from extrapolations from the last trajectory end points. The vertical lines through these points illustrated the variability resulting from uncertainties in defining the edges of the envelope of tetroom trajectories. When the variability at each point was considered, the slope of these tetroom derived σ_y values was not significantly differed from either the NRTS curves or the methyl iodide measurements. The envelope derived values are a factor of 2 to 3 less than the methyl iodide measured lateral dispersion. In the study of tetroom trajectories in the Los Angeles Basin the use of the trajectory envelope width divided by four was found to underestimate the lateral dispersion determined by the running mean variance statistic.(6) This systematic difference, always found to be less than a factor of 2, may explain some of the differences in σ_y values. When figures 5 and 6 were reexamined an interesting feature was noted. When the envelopes

of wind field and tetron trajectories were superimposed the width of the combined envelopes was 1-1/2 to 2 times wider at the longer distances. The average difference in angular bearing of the trajectories (and their envelope means) was close to 5 degrees. If new estimates of lateral dispersion were made from the pooled envelopes of trajectories, values of σ_y approaching the methyl iodide measurements resulted. These systematic differences in the near surface wind field trajectories and the corresponding tetron trajectories suggested the effect of vertical wind direction shear. Five sequential pibal observations, made at the release point and moving down the test grid with the plume, substantiated a wind direction change with height of this size.

Using a mixing depth of 1400 meters and limiting the maximum σ_z (the standard deviation of vertical effluent concentration) to be 80 percent of the depth of the mixing layer yielded an estimated maximum of 1120 meters.⁽⁸⁾ When the effects of the vertical lid were incorporated to modify the rate change of normalized axial concentrations with distance (see Smith and Singer, for example) a slower rate of dilution was determined.⁽⁹⁾ When the value of 1400 meters for the lid was used to modify the curves in figure 2 for NRTS stability classes B and C, the value of σ_z became a maximum at 15 and 29 kilometers, respectively. The effect of a lid appeared to reasonably account for the observed slower rate of dilution.

From equation (1), a form of the standard Gaussian plume model for a continuous point source, the effective σ_z was calculated at each arc.

$$\sigma_z \text{ (effective)} = \frac{Q}{\pi U \sigma_y} \frac{1}{x_p} \quad (1)$$

where Q is rate of tracer release, U is the effective wind speed, x_p is the measured (methyl iodide) axial concentration and σ_y is the standard deviation of methyl iodide in the lateral direction. Figure 7 shows the calculated effective σ_z values for methyl iodide. The line labeled tetrons represents the values of σ_z estimated from the envelope of lateral spreading of tetrons. NRTS curves of σ_z for stability classes B and C are plotted for reference.

The dot-dashed line in figure 7 depicted the maximum σ_z value estimated from the depth of the adiabatically mixed layer. The dashed line labeled tetron estimated limit represented a maximum σ_z equal to eighty percent of the height of the extreme tetron vertical oscillation (this treatment was somewhat analogous to the arbitrary treatment of the lateral width of the horizontal trajectory envelopes). Since the tetron envelope width divided by four underestimated the lateral dispersion, the "tetron envelope" effective σ_z derived from equation (1) must have been overestimated by a factor of 2 to 3. Likewise, use of eighty percent of the maximum attained tetron height must probably over estimate the effective height of limited vertical dispersion. The σ_z determined from tetron height oscillations was 516 meters. The mean effective σ_z for methyl iodide tracer was near 650 meters. Since the tetron data are known to slightly underestimate vertical dispersion, the tetron σ_z value was reasonable and as expected.

The various measurements from this test have shown that lateral tracer spreading behaved as expected. The relative axial concentration decreased more slowly with distance than would be expected from an extrapolation of measurements at short distances. The effect of a lid which inhibited vertical dispersion seemed to explain this slowed rate of dilution. The tracer centerline trajectory was well described by both the wind field derived trajectories and the tetron trajectories. At the longer distances in the presence of vertical shear of wind direction, the tetron trajectories were the best indication of transport. The lateral tracer dispersion can probably be well approximated by the running mean variance statistic; less detailed information such as the trajectory envelope width should be used only with considerable caution. The standard deviation of tetron height oscillations provided a reasonable estimate of the mean effective σ_z for the methyl iodide tracer.

New empirical guidelines for estimating transport and diffusion have not been presented; instead a useable methodology has been presented with an illustration of some of the types of information derivable from the techniques. The testing methodology could be enhanced by the addition of suitable vertical sampling.

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Figure 1. Field measurement area with wind tower locations (dots) and tracer sampling arcs (labeled Arc A, etc). The tracer release location is designated ICPP.

Figure 2. Methyl iodide measurements and NRTS climatological curves of relative axial concentrations versus downwind distance.

Figure 3. Lateral dispersion values from the NRTS climatology, methyl iodide measurements, and approximations from envelopes of tetron trajectories.

Figure 4. Paired horizontal trajectories for tetroons (lower case letters) and wind field advected (upper case letters) hypothetical particles. Letters are spaced along the trajectories at 15 minute intervals. Trajectory beginning times are listed above each plot.

Figure 5. Composite plot of tetron trajectories #3, 4, and 5. Dashed line locates wind field derived trajectory beginning at 1430 MST. Locations of peak measured methyl iodide concentrations are denoted by open circles.

Figure 6. Composite plot of wind field derived trajectories. Dashed line locates trajectory for 1 hour before the methyl iodide release. Locations of peak measured methyl iodide concentrations are denoted by open circles.

Figure 7. Vertical dispersion values from the NRTS climatology, and the effective values determined from methyl iodide measurements. Estimates based on horizontal tetron trajectory envelopes used to define lateral dispersion are labeled tetrons.

SOME DIAGNOSTIC APPLICATIONS OF WIND SPEED AND COMPONENT SPECTRA FOR MESOSCALE THROUGH SYNOPTIC SCALE FREQUENCIES*

Larry L. Wendell
NOAA, Idaho Falls, Idaho

INTRODUCTION

The power spectrum of the horizontal wind speed presented by Van der Hoven has been accepted as the classic distribution of atmospheric kinetic energy covering a range of frequencies associated with synoptic scale through micro-scale motion.⁽⁴⁾ The major features of the mesoscale through the synoptic scale region of this spectrum have been substantiated recently in part by Oort and Taylor using data from several stations over a ten-year period and the very powerful Fast Fourier Transform (FFT) technique for obtaining the spectral estimates.^{(1),(2)} This study showed some evidence of the spectral gap. Their results also agreed with those of Van der Hoven in showing that most of the variance of the wind speed is explained by the activity of traveling cyclones and anti-cyclones. Similar results have been reported by Vinnechenko although he also showed that the energy associated with the annual, synoptic and mesoscale variations of the wind is much greater in the free atmosphere than near the surface.⁽⁵⁾

Probably the most significant disagreement with Van der Hoven's spectrum was in the vicinity of the diurnal oscillation. He found no energy peak at a period near 24 hours; instead a minor peak appeared at a period of 12 hours. Oort and Taylor found a major peak at 24 hours and a minor, but noticeable, peak at 12 hours. They attributed this difference mainly to the 90 m height difference in the observations. Their data, observed at 10 m, was felt to be more representative of conditions within the surface layer and therefore more subject to diurnal oscillations. They did a very comprehensive analysis of the diurnal oscillation in the wind speed.

For the purpose of comparison Oort and Taylor presented the separate energy spectra of the north-south (v) and the east-west (u) components of the wind for one of the stations.⁽²⁾ It was their conclusion from this comparison that, at the station involved, there was a lack of an important diurnal cycle in the zonal and meridional wind components and they interpreted this to mean that the diurnal cycle in the wind speed is not accompanied by a similar cycle in direction. They also indicated that the relative importance of an annual periodicity appearing in the u and v components of the wind showed a variation between locations. Another significant point of comparison was that the total variance in the component analysis was a factor of three greater than for the speed analysis over the same data sample. This was attributed to an additional degree of freedom added by the wind direction.

It is the difference between these two types of analysis that is the concern of this work, as well as the insight into the nature of the oscillation which one can gain by their comparison. Also the merit of rotating the axes on which to determine horizontal wind component, in order to determine a principle axis of oscillation is investigated.

ENERGY ANALYSIS OF A SIMPLE OSCILLATING FLOW

The total kinetic energy of the wind, by the nature of its definition, is independent of direction. Considering only the horizontal component of the wind we can write

$$K_t = |V_H| = u^2 + v^2 \quad (1)$$

where u and v are orthogonal components of the velocity, generally taken to be positive to the east and north respectively. We may define the horizontal wind speed as

$$S_H = (u^2 + v^2)^{1/2} \quad (2)$$

which when squared, to obtain the kinetic energy, gives the same result as (1).

Several conventional meteorological wind sensing devices measure the wind in terms of horizontal speed (S_H) and direction (θ). Some, such as cup anemometers, measure speed only. Thus an analysis of the wind in terms of speed alone is not only convenient, but sometimes all that is possible. In view of the fact that the total kinetic energy is the same whether determined from the wind components or the speed would make this seem to be a minor problem. However, one does not have to make a very sophisticated analysis before the possibility of a serious discrepancy arises.

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One of the simplest types of analyses which can be performed on a time series of wind data is the computation of the mean and the variance. When the variable under consideration has units of velocity, the variance is taken to be proportional to the average kinetic energy of the deviations from the mean. The total kinetic energy can then be considered to be composed of the energy of the mean flow and the "eddy" flow

$$\overline{v^2} = \overline{v}^2 + \overline{v'^2} \quad (3)$$

where the bar represents a time mean over a specified interval and the prime indicates a deviation from the mean. That there may be a significant difference in the mean and variance of the speed when compared to these same quantities for the components can be illustrated with a simple example. For the sake of simplicity, consider the horizontal velocity to be composed of a single component defined as

$$\begin{aligned} v(t) &= A \sin t \\ u(t) &= 0 \end{aligned} \quad (4)$$

where t is time and A is the amplitude. In the conventional sense, this would be a wind oscillating from north to south and back again with a period of 2π and peak velocities of $+A$ and $-A$. Wind measuring equipment which senses speed only would report only the speed fluctuation of the wind defined by (4) and can be written as

$$S(t) = A|\sin t| \quad (5)$$

A comparison of the plots of (4) and (5) are shown in figure 1.

If we compute the kinetic energies of the mean flow (K_m) and eddy flow (K_e) from these two series (integrating over 2π is sufficient) we obtain the values shown in table 1. Note that the total kinetic energy ($K_t = K_m + K_e$) is the same computed from either series. However, the division between the energy of the mean and eddy components is quite different as one would expect from the comparison of the plots in figure 1. When direction is considered in this example there is no energy in the mean flow, it is all in the eddy flow. In the analysis of the speed data approximately 80% of this eddy energy appears as energy of the mean flow leaving only 20% as energy of the eddy flow.

Table 1. Comparison of energy values for velocity and speed where $V = A \sin t$.
 K_e = eddy energy, K_m = energy of mean flow and K_t = total energy.

	Velocity V	Speed S
Mean	0	0.6366A
K_m	0	$0.4053A^2$
K_e	$0.5A^2$	$0.0947A^2$
K_t	$0.5A^2$	$0.5A^2$

If we now consider a Fourier analysis of the functions in the above example we find that the appearance of eddy energy as mean energy is not the only problem. For the velocity variable ($V(t)$) we simply find all the energy appearing at the first harmonic with period 2π . For the speed variable ($S(t)$) we find no contribution to the variance at the first harmonic. The contribution to the variance from the rest of the harmonics is given by

$$K(n) = \begin{cases} 0, & n - \text{odd} \\ \frac{4}{\pi} A^2, & n = 0 \\ \frac{8A^2}{\pi^2(1-n^2)^2}, & n - \text{even} \end{cases} \quad (6)$$

where n represents frequency in cycles/ 2π . In evaluating (6) we find that about 95% of the variance of $S(t)$ is accounted for at $n = 2$. Thus we can summarize the effect of using the data from a speed sensor only to measure the wind, as depicted in (4), by saying that about 80% of the eddy energy appears as energy of the mean flow and most of the remaining 20% appears at a frequency twice what it is for the velocity.

As drastic as these results seem to be, we need now to consider the wind of the example in terms of the real atmosphere. The wind defined by (4) would represent, in the atmosphere, a condition in which not only is the speed fluctuating (as defined in (5)), but the direction is undergoing a cycle of complete reversals. This is the condition commonly observed on the shoreline of a large body of water or in a valley with a diurnal cycle in the direction and a semi-diurnal cycle in the speed. A significant wind reversal cycle may also be noted with the passage of cyclones and anticyclones. The frequencies of these cycles would depend, of course, on the size of the systems and the speed at which they were translated. Thus one might expect the contributions to the total variance of these systems to appear over a broad band of frequencies.

Another phenomenon which can cause a strong variation in wind direction is a phenomenon analogous to the cyclone-anticyclone example cited above, but on a much smaller scale. This is the case in which an eddy or group of eddies are imbedded in a translating flow which carries them past the wind sensor.⁽³⁾ If the eddies are strong relative to the translating flow, the direction can undergo as much as a complete reversal. However, if the eddies are relatively weak the major effect will be a speed fluctuation. This can be visualized by superimposing a transporting velocity on to the oscillating velocity of (4) and writing

$$V(t) = A \sin t + B \quad (7)$$

where B is the magnitude of the transporting velocity. Clearly if B is larger than A there will be no reversal of direction and the mean and variance results for $v(t)$ and $S(t)$ will be the same. This is the condition which is sought generally for microscale turbulence analyses. It is the condition which usually prevails if there is more than just a light breeze blowing and the period of interest is less than a few hours. All of this brings us to the question of how one might interpret the differences and similarities in power spectra of horizontal wind speed and of the combined spectra of the components of the horizontal wind.

COMPARISON OF WIND SPEED AND COMPONENT ENERGY SPECTRA

The first comparison of a wind speed spectrum and a wind component spectrum for a long data sample was by Taylor and Oort.⁽²⁾ The comparison was made for a ten year sample of hourly winds for Caribou, Maine. The diagram showing this comparison is shown in figure 2. From the considerations above we may interpret the difference in total variance as a loss of energy of the oscillatory flow to the energy of the mean flow as obtained by the use of speed data only. The real case considered here shows a variance loss of 67% as compared to 80% for the simple example above. In examining the spectral curves in figure 2 we find that the frequency band hardest hit by this loss is for oscillations with periods of about 2 to 7 days. This is the band associated with the passage of cyclones and anticyclones. One might deduce from this result, without knowing the location of the station, that it lies in a location strongly affected by mid-latitude storm systems.

Oort and Taylor pointed out, as the most interesting difference between the speed and component spectra, the lack of an important diurnal cycle in the zonal and meridional wind components. The interpretation was that the wind direction near the surface, at the stations considered, is highly variable and does not follow a systematic diurnal cycle. When both spectra are plotted on the same graph, as in figure 2 we can see that the difference in the contributions to the variance by the band average, including the frequency of one cycle day⁻¹, is not nearly as impressive as the difference in the frequency bands associated with the passing cyclones and anticyclones. The fact that the energy in these bands dropped so significantly in the speed spectra would indicate that these disturbances cause significant variations in the wind direction. It should be mentioned here that Oort and Taylor pointed out that a line by line examination of the band including the diurnal cycle revealed that the energy at one cycle day⁻¹ exceeded the energy at the other frequencies in the band by a factor of about 400.

We may show a contrast to this result by considering the energy spectra for both the wind speed and components for a station located at the National Reactor Testing Station in southeastern Idaho. These two spectra have been computed over a period of 8192 hours in 1969 for the 76 m level and are shown in figure 3. The spectral values have been multiplied by frequency in this diagram in order that area under the curve will be representative of contributions to the variance. The speed spectra is quite similar to that for the Caribou, Maine station especially with regard to the predominant diurnal peak and the minor peak at a period of 12 hours. The differences between the speed spectrum and the component spectrum are also quite similar except in the vicinity of the period of 24 hours. At this geographical location and height above the ground the magnitude of the diurnal peak in the component spectrum is much greater than the height of the peak in the speed spectrum. There is about a 68% discrepancy at this frequency compared to the 80% for the sine wave example. This would indicate that the fluctuation in the velocity is primarily one of direction. This agrees with local observations of a diurnal wind reversal caused by topographic features in the surrounding area. The predominance of the diurnal peak at this location and level in the spectra varies from a maximum for the summer season to hardly noticeable for the winter season.

To indicate the dependence of the long range wind speed and component energy spectra with height, they are shown in figure 4 for the same location and period at a level 6 m above the ground. We note, as would be expected, a much lower variance than for the 76 m level. It may also be noted that the distribution of the peaks is very similar, and that the energy discrepancy between spectra in the band of frequencies associated with synoptic scale disturbances is also similar. However, around the frequency of the diurnal cycle the similarity ends. There is only a small difference between the speed and component spectra instead of the large difference observed at the 76 m level. This would indicate that the wind fluctuations at the 6 m level are primarily speed fluctuations instead of direction fluctuations. This is probably due to the fact that the slope of the ground in the immediate vicinity of the tower is opposite the general slope of the terrain over the Upper Snake River Plain. This means that at night when vertical transfer of momentum is at its minimum, the general downslope flow is effectively opposed by this local topographic anomaly. However, during the day when the momentum of the general upslope flow is transferred downward through convective mixing, the opposition by this local topographic anomaly is not nearly as important. The overall effect at 6 m is a relatively strong afternoon wind, but a very light nighttime wind with no significant reversal of direction as is observed at 76 m.

EFFECT OF AXES ROTATION ON WIND COMPONENT SPECTRA

In the spectral plots of figures 2, 3 and 4 the horizontal velocity spectra are the direct sum of the spectra of the orthogonal components of the wind. The orientation of the axes from which the components are determined has no effect on the sum of the spectra of the components. Only under isotropic conditions, however, will it have no effect on the individual components. How then should we orient the axes? For long period data samples, the accepted convention is for the u and v components of the wind to be oriented in east-west and north-south directions, respectively? This orientation is well suited for the free atmosphere because it facilitates the calculation of meridional transport. However, in the lower layers of the atmosphere where surface effects are important this orientation loses some of its usefulness.

The component spectra presented by Gort and Taylor showed only minor differences between the u and v spectra except at a period of one year.⁽²⁾ At this period there was somewhat more energy in the v (north-south) component than the u component. They pointed out that this difference in the component energy varied at different locations. The individual component spectra for the 76 m level of the NRTS tower are shown in figure 5 (the sum of these spectra are shown in figure 4). We note that these spectra are quite similar, and without prior knowledge or further investigation one might be tempted to assume that near isotropic conditions exist at this level and geographic location. However, the power of the Fast Fourier Transform technique allows us to recompute, very inexpensively, the spectra for as many orientations of coordinate axes as we like.

In this case, rotation of the coordinate axes has some very drastic effects on the component spectra. The spectra for the same time period and level as shown in figure 5 are shown in figure 6, this time with a 49° clockwise rotation of the axes. For this orientation (v - northeast-southwest, u - southeast-northwest) the only portions of the spectra which are similar are for periods less than about 7 hours. For periods longer than 7 hours a comparison of the spectra indicates that practically all the eddy energy is in the v component. In view of this result we might consider 49° to be the orientation of the principle axes of the turbulent energy for disturbances with periods greater than 7 hours at this level and geographical location.

As one might suspect this result is strongly influenced by the topography of the area, some of which has already been discussed. The gently sloping Upper Snake River Plain is approximately 60 miles wide and is bounded on both sides by mountains. These boundaries can be approximated by two parallel lines which are oriented at about 40° to 50°. These mountain barriers tend to channel the flow over the plain to a northeast-southwest orientation as evidenced by the fact that the energy of the synoptic scale disturbances lies almost entirely in the northeast-southwest component. The distribution of energy for the v component shows about 93% is eddy energy and 7% is energy of the mean flow (which is positive). This would indicate that for a continuously released effluent from location at 76 m, one could expect it to stream alternately northeastward and southwestward with the northeastward direction being slightly favored.

We can see from this example that observation of changes in the component spectra, as the orientation of the coordinate system is changed, can be used to advantage in determining the importance of various surface effects on the boundary layer motion. It is conceivable that in some locations, i.e., over the ocean, the component spectra would not change with a rotation of the axes. In other locations the spectra might not behave the same in different frequency bands under rotation. From spectra computed at other locations over the Upper Snake River Plain, it is apparent that there are differences in the behavior of the component spectra in the same geographical region. More extensive analyses of other stations and different years and seasons are being conducted.

SUMMARY AND CONCLUSIONS

The comparison of wind speed and component spectra for long time periods for two different geographical locations have shown that marked differences can occur. An energy analysis was performed on a simple one-dimensional model in which the wind was allowed to vary as a sine wave through both positive and negative values. It was demonstrated that about 80% of the eddy energy of the velocity function appears in the energy of the mean speed and almost all of the other 20% appears in the speed spectra at twice the frequency at which it appeared in the velocity spectra. This is not to say that the speed spectra is mathematically incorrect, because the speed spectra does accurately represent the frequencies at which the speed is varying. The difference is one of a physical nature and becomes important when the variation in the wind direction is through as much as 180° . For sampling periods of more than a few hours this situation can easily be realized, in which case an analysis of the speed alone does not realistically represent the division of the total energy into the eddy energy and energy of the mean flow, as well as the distribution of the eddy energy over the appropriate range of frequencies. This does not mean however, that the computation of the speed spectra over long time periods cannot serve a useful purpose.

The comparison of the speed spectra and the component spectra may be used to determine the nature of the velocity variation in the different frequency bands of the spectrum. If the energy in a given frequency band for the speed spectra is significantly less than the combined component spectra, then the velocity variation for this frequency band involves a direction reversal. If there is little difference between the two types of spectra for a particular frequency band, then the velocity fluctuations in this band are primarily speed fluctuations. With the type of information about the kinematics of the velocity variation provided by this spectral comparison, one has a better chance to find the physical causes involved. This type of insight can also be provided by the component spectra through another comparison technique. This involves examination of the changes in the individual component spectra as the coordinate axes, upon which the components are defined, are rotated. The results indicated that surface effects on boundary layer flow can cause conditions which are far from isotropic. A 49° clockwise rotation of the axes showed a fairly even division of energy between components change to a totally lopsided division of energy. This rotation can be carried out at any geographical location to determine the principle axes, if they exist, for the whole spectrum or any selected frequency band.

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Figure 1. Velocity and speed plots of a simple oscillating wind.

Figure 2. Energy spectra for wind speed and combined spectra of the u and v components of the wind at a height of 19 m for a 10 year data sample (after Cort and Taylor). (2)

Figure 3. Energy spectra for wind speed and combined spectra of the u and v components of the wind at a height of 76 m for a 8192 hr period beginning 1 January 1969.

Figure 4. Same as figure 3 for a height of 6 m.

Figure 5. Energy spectra for u and v components with v the north-south component.

Figure 6. Same as figure 5 with coordinate axes rotated 49° clockwise.

IDENTIFICATION OF AEROSOLS IN THE SOUTHWESTERN UNITED STATES
USING ACTIVATION ANALYSIS AND SCANNING ELECTRON MICROSCOPY*

William A. Sedlacek, Paul R. Guthals, and H. L. Smith
Chemistry and Nuclear Chemistry Division
University of California
Los Alamos Scientific Laboratory
Los Alamos, New Mexico 87544

As a multiple use of USAF RB-57F and C model aircraft during routine training missions and developmental flights for existing AEC programs, a program testing a variety of aerosol sampling equipment has been carried out and a wide variety of particulate samples have been collected. The program has been carried out on a "spare time" basis utilizing a little time from each of the people listed in Figure 1. Also it must be noted here that this is obviously a progress report.

Two types of aircraft are used as shown in Figure 2. The RB-57F is a long winged high-altitude aircraft which we have utilized for stratospheric studies relating to the environmental impact of SST's. The short winged aircraft is high performance RB-57C which is utilized for collection of samples at low altitudes between 500 and 7000 ft above the surface.

Utilizing these aircraft, samples have been collected from about thirty different sources in the region shown in Figure 3. This region is considered to be our aerosol studies laboratory. Within this region are a number of relatively isolated sources emitting particulates into a relatively clean atmosphere. This condition now nearly unique in our modern world, provides an unusually fine opportunity to develop techniques for characterization of individual sources and study of the transport and dispersion of the aerosols produced. Over a dozen types of sources, as listed in Figure 4, have been sampled and in a number of cases it has been possible to sample more than one source of a given type in order to seek individual distinguishing differences.

The principal sampler used is diagramed in Figure 5. The sampler is mounted mid-wing on the RB-57F and below the wing tip on the RB-57C. It has been aerodynamically tested to sample isokinetically at the inlet. The particulates are collected by impaction on an IPC 1478 filter panel which has essentially zero pressure drop. The aircraft velocity is approximately 200 knots, the air velocity through the filter is slightly greater than 2000 ft per minute, the collection efficiency is known to be 95 to 100 percent for at least all particles down to 0.1 microns and about 250 cubic meters of air are sampled per minute.

The collected particulates are examined by activation analysis for comparison of their trace element content. The activation analysis scheme used is shown in Figure 6 and is essentially that put forth by John Winchester et al. The elements listed in Figure 7 are found in the analysis of various samples. For purposes of visual presentation and comparison of samples the activity of a selected gamma ray for each element as listed is presented on a bar graph as seen in Figure 8. Note that the activity scale is a log scale of the activity present at the end of the irradiation. These profiles represent a synopsis of the appearance of the observed gamma spectra.

Figure 9 compares activation analyses of three different types of sources. It can be seen that presence or absence of various elements makes these samples different from one another. Sulfur appears only in the Gypsum sample. Titanium only in the Farmington flyash and Pagosa Springs sawdust burner samples. Iodine is absent in the flyash and barium is not present in the gypsum. There are similar differences among the elements sensed by the long irradiation.

Now let us compare sources of the same type as shown in Figure 10. The elemental compositions of the two carbon black plants are almost identical, except for the presence of iridium in the Borger-Pampa, Texas plant. There is only a trace of iridium present and whether its origin is in the gas used by the plant or if it is introduced by the process used is unknown. However, the high sensitivity of activation analysis to iridium has made this distinguishing difference discernable. Similarly a comparison of three copper smelters in Figure 11 shows similar differences. Observe the presence or absence of magnesium, titanium, tin, iodine, antimony, rare earths, tungsten, and rhenium.

Some samples do not have significant differences in elements present however as illustrated by Figure 12. Ignoring the chromium, iron, gold, and protactinium differences which are at about the limit of the sensitivity threshold, there are no differences in elements present. This is to be expected as the principle source of flyash at the Four Corners Power Plant is dirt mixed in with the coal. There is a variation in the abundance of vanadium and barium, which were concentrated by the coals, plant life precursors; but the accuracy of the analysis has not yet been improved sufficiently for us to believe these ratio differences are distinctive enough for characterization.

The second analytical technique, scanning electron microscopy (SEM) will provide further distinguishing evidence at this point. The power plant's combustion process will drastically change the morphological and size characteristics of the particulate. Figure 13 is an SEM picture of the fibers of an unflowed filter.

Figure 14 shows flyash from the electrostatic precipitators on the power plant at Farmington and Figure 15 shows flyash collected on the filters in the plume at a distance of two miles from the plant. It is quite obvious that the flyash has the physical appearance of small spherical particles. Wind blown dust on the other hand looks like small crystalline-like particles of sand. Figure 16 is a collection of flyash made in a dispersed plume at a distance of fifty miles from the power plant. Again note the spherical particles of flyash.

Figures 17 and 18 show the appearance of particles collected at a distance of 150 miles from the power plant. The activation analysis of these two samples revealed a characteristic fingerprint which could have been flyash, wind blown dirt, or a combination thereof. Figure 17 shows a sample collected on March 24, 1970 in a haze in the Espanola Valley of New Mexico. The SEM picture shows the presence of both flyash and dust particles. Note that not only are the two types of particles distinguishable by their characteristic morphologies, but the particle size distributions for the two types are different. The average diameters of the flyash particles are smaller than the crystalline dust particles. In this sample the fraction of the particles that are flyash is greater than in Figure 18, which is a sample collected in the same location on March 25, 1971. During the night of March 24th a strong wind had blown increasing the dust contribution to the haze.

Thus we have been able to use the trace element content of atmospheric particulates together with their morphological characteristics to fingerprint sources of aerosols. These techniques will now be put to use to study the transport and dispersions of particulates emitted into the atmosphere, for the study of mesometeorology and evaluation of hazard presented by individual sources.

² Work performed under the auspices of the U. S. Atomic Energy Commission.

Personnel Involved

LASL	OTHERS
CNC-11	Officers and men 58th WRS, Kirtland AFB Col. Click Smith commanding
Louise Smith	Maj. Tom Oliver 58th WRS
Bill Sedlacek	Sandia Corp.; Albuquerque, N. M.
Paul Guthals	EG&G; Albuquerque, N. M.
Ted Norris	
John Halagna	
Capt. Russell Glenn (USAF)	
Ken Apt (Postdoctoral)	
CNC-DO	
George Cowan	
H-5	
Harry Ettinger	
Ron Stafford	
Evan Campbell	
GK-2	
Dan Loughran

FIGURE 2.

RB-57C

FIGURE 3.

SOURCES SAMPLED

1. Power Plants		10. Iron Smelters	
a. Farmington, N. M.	(1)	a. Pueblo, Colo.	(12)
b. Rapid City, S. D.	(25)	b. Provo, Utah	(13)
2. Sawmill Waste Burners		11. Oil Refinery	
a. Pagosa Springs, Colo.	(2)	a. El Paso, Tex.	(14)
b. Springerville, Ariz.	(3)	12. Forest Fires	
3. Copper Smelters		a. near El Cajon, Calif.	(15)
a. Globe-Miami, Ariz.	(4)	b. near Fontana, Calif.	(16)
b. Douglas, Ariz.	(5)	c. near Jemez Springs, N. M.	(26)
c. Hurley, N. M.	(6)	13. Chemical Plant	
4. Carbon Black Plants		a. Henderson, Nev.	(17)
a. Pampa-Borger, Tex.	(7)	14. Zinc Smelter	
b. Seagraves, Tex.	(8)	a. Amarillo, Tex.	(23)
5. Gypsum Plant and Mine		15. Borex Plant	
a. Rosario, N. M.	(9)	a. Boron, Calif.	(30)
6. Perlite Plant and Mine		16. Espanola Valley	(24)
a. No Agua, N. M.	(10)	17. Urban Centers	
7. Potash Mines		a. Los Angeles, Calif.	(18)
a. Carlsbad, N. M.	(11)	b. Salt Lake City, Utah	(19)
8. Cement Plants		c. Denver, Colo.	(20)
a. Monolith, Calif.	(29)	d. Phoenix, Ariz.	(21)
b. Laramie, Wyo.	(28)	e. Albuquerque, N. M.	(22)
9. Paper Mill		f. El Paso, Tex.	(14)
a. Panama City, Fla.	(31)	g. St. Louis, Mo.	(27)
		h. Houston, Tex.	(32)

FIGURE 4.

FIGURE 5. Cross Section of LASL Wing Tip Particulate Sampler for RB-57C.

FIGURE 6.

GAMMA RAY USED

<u>5 min Irrad.</u>	
	<u>γ</u>
²⁴ Na	1368 keV
²⁷ Mg	1014
²⁸ Al	1778
³⁷ S	3102
³⁸ Cl	1642
⁴⁰ Ca	3083
⁵¹ Ti	320
⁵² V	1434
⁵⁶ Mn	1811
⁶⁰ Co	1039
¹⁰¹ Mo	192
^{116m} In	417
^{125m} Sb	332
¹²⁸ I	443
¹³³ Ba	166

<u>5 hr Irrad.</u>			
	<u>γ</u>		<u>γ</u>
²⁴ Na	1368 keV	¹⁴¹ Ce	145
⁴² K	1525	¹⁵⁰ Sm	103
⁴⁶ Sc	889	¹⁵² Eu	122
⁵¹ Cr	320	¹⁷⁷ Lu	208
⁵³ Fe	1099	¹⁸¹ Hf	482
⁶⁰ Co	1173	¹⁸⁷ W	686
^{69m} Zn	439	¹⁸⁸ Re	155
⁷² Ga	834	¹⁹² Ir	316
⁷³ As	559	¹⁹⁸ Au	412
⁷⁵ Se	136	²³³ Pa(Th)	312
⁸² Br	777		
^{110m} Ag	658		
¹²⁴ Sb	603		
¹³⁴ Cs	796		
¹⁴⁰ La	487		

FIGURE 7.

FIGURE 8.

FIGURE 9.

FIGURE 11.

FIGURE 10.

FIGURE 12.

2250 X
Unflown IPC Filter Fiber

FIGURE 13.

3750 X
Sample in Farmington, N.M.
Power Plant Plume at 2 miles

FIGURE 15.

2250 X

2250 X
Electrostatic precipitator Sample
from Farmington, N.M. Power Plant

FIGURE 14.

2250 X

2250 X
Sample in Farmington, N.M.
Power Plant Plume at 50 miles

FIGURE 16.

1500 X

1500 X

Espanola Valley Sample Mar. 24, 1970

FIGURE 17.

2250 X

2250 X

Espanola Valley Sample Mar. 25, 1970

FIGURE 18.

DIFFUSION FROM AN INSTANTANEOUS LINE SOURCE IN THE ATMOSPHERIC SURFACE LAYER

S. N. Putta
Air Pollution Control Division
Colorado Department of Health
Denver, Colorado

J. E. Cermak
Professor-in-Charge, Fluid Mechanics Program
Department of Civil Engineering
Colorado State University
Fort Collins, Colorado

INTRODUCTION

An understanding of turbulent diffusion is of great interest to many meteorological problems, particularly to studies of air pollution, evaporation and heat transfer. Since the source of contaminants is in many cases close to the solid boundaries, the study of diffusion in turbulent boundary-layer flows is of special significance. Developments in our knowledge of the statistical properties of the motion of passive particles in turbulent flow have been remarkable in spite of the inherent complexities. G. I. Taylor⁽¹⁴⁾ demonstrated that the characteristics of transport processes are related to the Lagrangian statistical functions of turbulent motion. Aris⁽¹¹⁾ developed moment transformation theory to describe dispersion in turbulent pipe flows. Later such a theory was applied to atmospheric diffusion by Smith⁽¹³⁾, Saffman⁽¹²⁾, Chatwin⁽⁵⁾ and Atesman⁽²⁾

Although the similarity concept for diffusion processes in turbulent shear flows is new, it has been an exceedingly useful idea in earlier developments describing diffusion in jets, wakes and mixing layers. Monin⁽⁹⁾ demonstrated that in a horizontally homogeneous stationary surface layer of air, turbulent diffusion obeys the similarity theory in which the values of shear velocity u_* and the stability length L are the only scales of velocity and length. For neutral flows where the length L is not defined, Batchelor⁽³⁾ proposed the following similarity hypothesis which is Lagrangian in nature. Statistical properties of the velocity of marked fluid particles depend only on u_* and the time of travel t . Important results of this hypothesis are (1) the center of mass of a cloud of diffusing particles $(\bar{x}, \bar{z})$, given by

$$\frac{d\bar{z}}{dt} = bu_*, \quad \bar{z} = bu_*t \quad (1)$$

$$\frac{d\bar{x}}{dt} = U \quad \text{at} \quad z = \bar{z} \quad (2)$$

where b and c are universal constants and (2) the probability density function for positions (x, y, z) of the dispersing particles will be similar when scaled with $\bar{z}$ as length scale and can be given as

$$C(x, y, z|t) = \frac{1}{(bu_*t)^3} f\left(\frac{x-\bar{x}}{bu_*t}, \frac{y}{bu_*t}, \frac{z}{bu_*t}\right) \quad (3)$$

For a two-dimensional case the density function becomes

$$(x, z|t) = \int_{-\infty}^{\infty} c(x, y, z|t) dy = \frac{1}{(bu_*t)^2} F\left(\frac{x-\bar{x}}{bu_*t}, \frac{z}{bu_*t}\right) \quad (4)$$

The forms of the density functions f and F are not known. During the last ten years, considerable work has been done by Ellison⁽⁶⁾, Gifford⁽⁷⁾, Cermak⁽⁴⁾, Pasquill⁽¹⁰⁾ and Klugg⁽⁸⁾ on the validity and application of similarity theory to turbulent diffusion in the atmospheric surface layer.

The purpose of the study by Putta⁽¹¹⁾ reported in this paper was to determine the statistical properties of a dispersing cloud and to construct a probability density function for the concentration of the passive material released instantaneously at the ground level from a crosswind line source. The concept of Lagrangian similarity theory is assumed to be applicable and the theory of

moment transformation is utilized. The flow is assumed to be neutrally stratified and fully developed. The surface is assumed to be uniformly rough and it is assumed that the vertical velocity distribution is logarithmic.

DENSITY FUNCTION FOR CONCENTRATION

For an instantaneously released crosswind line source, the dispersing field of concentrations can be represented by a three-dimensional density function $\chi(x, z, t)$. This function can be decomposed as

$$\chi(x, z, t) = f(x|z, t) \chi_*(z, t) \quad (5)$$

where

$$\chi_*(z, t) = \int_{-\infty}^{\infty} \chi(x, z, t) dx$$

is a marginal density function and $f(x|z, t)$ is a conditional density function for the direction of mean flow, given z and t . These density functions satisfy the following relation for mass conservation:

$$\begin{aligned} \int_0^{\infty} \int_{-\infty}^{\infty} \chi(x, z, t) dx dz &= \int_0^{\infty} \int_{-\infty}^{\infty} f(x|z, t) \chi_*(z, t) dx dz \\ &= \int_0^{\infty} \chi_*(z, t) dz = 1.0 \end{aligned} \quad (6)$$

These two density functions are now in a convenient form and their functional forms can be determined by making use of the equations describing the dispersion.

Consider the density function $f(x|z, t)$. It can be expressed in terms of the statistical moments as follows. The Fourier transform of $f(x|z, t)$ is given by

$$\phi(\alpha|z, t) = \int_{-\infty}^{\infty} e^{i\alpha x} f(x|z, t) dx = \overline{e^{i\alpha x}}(z, t) \quad (7)$$

and the inverse transform of $\phi(\alpha|z, t)$ gives

$$f(x|z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\alpha x} \phi(\alpha|z, t) d\alpha \quad (8)$$

Derivatives of $\phi(\alpha|z, t)$ at the origin $\alpha=0$ are equal to the moments

$$\left. \frac{\partial^n}{\partial (i\alpha)^n} \phi(\alpha|z, t) \right|_{\alpha=0} = \overline{x^n}(z, t) = \mu_n(z, t) \quad n = 1, \dots, n$$

Hence, Taylor's expansion for $\phi(\alpha|z, t)$ about $\alpha=0$ can be written in terms of the moments as

$$\phi(\alpha|z, t) = 1 + \frac{\mu_1(z, t)}{1!} (i\alpha) + \frac{\mu_2(z, t)}{2!} (i\alpha)^2 + \frac{\mu_3(z, t)}{3!} (i\alpha)^3 + \dots + \frac{\mu_n(z, t) (i\alpha)^n}{n!} \quad (9)$$

Moments $\mu_n(z, t)$ can be related to the cumulants $k_n(z, t)$ as:

$$\begin{aligned} \mu_1(z, t) &= \left. \frac{\partial \phi(\alpha; z, t)}{\partial (i\alpha)} \right|_{\alpha=0} = \left. \frac{\partial}{\partial (i\alpha)} \exp[\ln \phi(\alpha; z, t)] \right|_{\alpha=0} = k_1(z, t) \\ \mu_2(z, t) &= \left. \frac{\partial^2 \phi(\alpha; z, t)}{\partial (i\alpha)^2} \right|_{\alpha=0} = k_1^2(z, t) + k_2(z, t) \\ \mu_3(z, t) &= \left. \frac{\partial^3 \phi(\alpha; z, t)}{\partial (i\alpha)^3} \right|_{\alpha=0} = k_3 + 3k_2 k_1 + k_1^3 \end{aligned}$$

$$\text{Similarly, } \mu_4 = k_4 + 4k_1 k_3 + 3k_2^2 + 6k_1^2 k_2 + k_1^4 \quad (10)$$

The coefficient of skewness and kurtosis can now be defined as

$$\lambda_3 = k_3/k_2^{3/2} \quad \text{and} \quad \lambda_4 = k_4/k_2^2 \quad (11)$$

The integration of Eq. (8) can be performed using the results in Eqs. (9) through (11) and the function $f(x|z,t)$ can be obtained in terms of its statistical moments. The result can be shown in its well known form of the Gram-Charlier density function as

$$f(x|z,t) = (2\pi k_2)^{-1/2} \left(1 + \frac{1}{6} \lambda_3 H_3 + \frac{1}{24} \lambda_4 H_4 + \frac{1}{72} \lambda_3^2 H_6 + \frac{1}{1152} \lambda_4^2 H_8 \right. \\ \left. + \frac{1}{144} \lambda_3 \lambda_4 H_7 + \dots \right) \exp(-\xi^2/2) \quad (12)$$

where $\xi = (x-k_1)k_2^{1/2}$ is a normalized variable and

$$H_n(\xi) = \xi^n - \frac{n(n-1)}{1!} \frac{\xi^{n-2}}{2} + \frac{n(n-1)(n-2)(n-3)}{2!} \frac{\xi^{n-4}}{2^2}$$

are the Hermite polynomials.

The statistical properties k_2 , λ_3 and λ_4 which can be obtained from the equations of dispersion are demonstrated in the following sections.

MARGINAL DENSITY FUNCTION $\chi_*(z,t)$

In the absence of mean vertical velocity and horizontal velocity gradient, a differential equation for the concentration downwind of an instantaneous line source can be written as

$$\frac{\partial \chi}{\partial t} + U_1 \frac{\partial \chi}{\partial x} = \frac{\partial}{\partial x} \left(K_x \frac{\partial \chi}{\partial x} \right) + \frac{\partial}{\partial z} \left(K_z \frac{\partial \chi}{\partial z} \right) \quad (13)$$

The boundary conditions are as follows:

$$\chi \rightarrow 0 \quad \text{as} \quad z \rightarrow \infty \\ \text{and} \quad x \rightarrow \pm \infty \quad (14)$$

$$K_x \frac{\partial \chi}{\partial z} \rightarrow 0 \quad \text{as} \quad x \rightarrow \pm \infty \quad (15)$$

$$K_z \frac{\partial \chi}{\partial z} \rightarrow 0 \quad \text{as} \quad z \rightarrow 0 \quad (16)$$

$$\text{The initial condition is } \chi(x,z,t) = \delta(x,z) \quad \text{at} \quad t = 0 \quad (17)$$

Now Eq. (13) may be integrated for the direction of x (using the boundary condition (15)) to obtain the following equation for the marginal density function χ_* :

$$\frac{\partial \chi_*}{\partial t} + \int_{-\infty}^{\infty} U_1 \partial \chi = \int_{-\infty}^{\infty} \frac{\partial}{\partial z} \left(K_z \frac{\partial \chi}{\partial z} \right) dx \quad (18)$$

For dispersion in an adiabatic atmospheric surface layer, a logarithmic distribution for the mean velocity $U_1(z)$ is appropriate and can be written as

$$\frac{U_1}{u_*} = \frac{1}{\kappa} \ln \frac{z}{z_0} \quad (19)$$

When mass and momentum transfers are identical (Reynolds analogy) and when the stress in the surface is constant, it can be shown that

$$K_z = \kappa u_* z \quad (20)$$

Substituting Eqs. (19) and (20) for U_1 and K_z in Eq. (18) and integrating using the boundary conditions of Eqs. (15) and (16) and the initial condition of Eq. (17), it can be shown that

$$\chi_*(z,t) = \frac{1}{\kappa u_* t} \exp(-z/\kappa u_* t) \quad (21)$$

STATISTICAL PROPERTIES OF THE DISPERSING CLOUD

Consider the dispersion equation (13) for large times. Monin (1959) and Saffman (1962) have shown that the longitudinal diffusion term $[\frac{\partial}{\partial x} (k_x \frac{\partial \chi}{\partial x})]$ contributes very little to the horizontal scattering, compared to that caused by the interaction of vertical gradient of mean velocity with the vertical turbulence transfer. When this term is eliminated from (3.1), it can be written as

$$\frac{\partial \chi}{\partial t} + U_1 \frac{\partial \chi}{\partial x} = \frac{\partial}{\partial z} (k_z \frac{\partial \chi}{\partial z}) \quad (22)$$

Defining the center of mass of the cloud $(\bar{x}, \bar{z})$ as

$$\bar{z} = \int_{-\infty}^{\infty} \int_0^{\infty} z \chi \, dz \, dx \quad \text{and} \quad \bar{x} = \int_0^{\infty} \int_{-\infty}^{\infty} x \chi \, dx \, dz \quad (23)$$

and performing the integration on equation (22), Chatwin⁽⁵⁾ has shown that

$$\bar{z} = u_* t$$

and

$$\bar{x} = \frac{u_* t}{\kappa} \left[\ln \left(\frac{\kappa u_* t}{z_0 e^\gamma} \right) - 1 \right] \quad \text{for } t > \frac{4z_0}{u_*} \quad (24)$$

Now the vertical variance of the density function can be written as

$$\begin{aligned} \sigma_z^2(t) &= \int_0^{\infty} \int_{-\infty}^{\infty} (z - \bar{z})^2 \chi(x, z, t) \, dx \, dz \\ &= \int_0^{\infty} (z - \bar{z})^2 \chi_*(z, t) \, dz = (\kappa u_* t)^2 \end{aligned} \quad (25)$$

In the constant stress region of the surface layer, where the turbulent intensities for the three directions are of the same order, it can be assumed that

$$\sigma_x^2 = \sigma_z^2 = (u_* t)^2$$

The standard deviations σ_x and σ_z give estimates of horizontal and vertical extents of the dispersing cloud. Choosing σ_x and σ_z as the longitudinal and vertical length scales, the similarity density function of Eq. (4) can be written as

$$\chi(x, z, t) = \frac{1}{a\kappa(u_* t)^2} F\left(\frac{x - \bar{x}}{a u_* t}, \frac{z}{\kappa u_* t}\right) \quad (26)$$

where a is proportionality constant in the relation $\sigma_x = a u_* t$. By defining the similarity variables as

$$\beta = \frac{x - \bar{x}}{a u_* t} \quad \text{and} \quad \eta = \frac{z}{\kappa u_* t} \quad (27)$$

Equation (26) can be transformed to give

$$\eta \frac{\partial^2 F}{\partial \eta^2} + (\eta + 1) \frac{\partial F}{\partial \eta} + \left(\beta - \frac{1}{a\kappa} \ln \eta e^\gamma \right) \frac{\partial F}{\partial \beta} + 2F = 0 \quad (28)$$

Longitudinal moments of F about $\bar{x}$ are defined as

$$\theta_n(\eta) = \int_{-\infty}^{\infty} \beta^n F \, d\beta \quad \text{for } n = 0, 1, 2, \dots \quad (29)$$

where $\theta_n(\eta)$ is the n th moment about the center of mass of the cloud. Boundary conditions to be satisfied by F and $\theta_n(\eta)$ are:

$$F \rightarrow 0 \text{ as } \beta \rightarrow \pm\infty$$

$$\text{and } \eta \rightarrow \infty \quad (30)$$

$$\theta_n(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty \text{ for } n = 0, 1, 2, \dots \quad (31)$$

$$\eta \frac{d\theta_n}{d\eta} \rightarrow \kappa=0 \text{ as } \eta \rightarrow 0 \quad (32)$$

Multiplying Eq. (30) by $1, \beta, \beta^2, \beta^3, \dots, \beta^n$ respectively and then integrating, using the boundary conditions, the following differential equations are obtained for the moments:

$$\eta \frac{d^2\theta_0}{d\eta^2} + (\eta+1) \frac{d\theta_0}{d\eta} + \theta_0 = 0 \quad (33)$$

$$\eta \frac{d^2\theta_1}{d\eta^2} + (\eta+1) \frac{d\theta_1}{d\eta} = -\frac{\theta_0}{a} \ln \eta e^\gamma \quad (34)$$

$$\eta \frac{d^2\theta_2}{d\eta^2} + (\eta+1) \frac{d\theta_2}{d\eta} - \theta_2 = -\frac{2\theta_1}{a\kappa} \ln \eta e^\gamma \quad (35)$$

$$\eta \frac{d^2\theta_3}{d\eta^2} + (\eta+1) \frac{d\theta_3}{d\eta} - 2\theta_3 = -\frac{3\theta_2}{a\kappa} \ln \eta e^\gamma \quad (36)$$

$$\vdots \quad (37)$$

$$\eta \frac{d^2\theta_n}{d\eta^2} + (\eta+1) \frac{d\theta_n}{d\eta} - (n-1)\theta_n = -\frac{n\theta_{n-1}}{a\kappa} \ln \eta e^\gamma \quad (38)$$

These equations can be interpreted as follows:

- (33) is a vertical transport equation, which gives the vertical distribution of the material,
 - (34) is a distortion equation which gives the distribution of the mean of the density function F in the vertical direction,
 - (35) is a dispersion integral equation which determines the variance of the distribution of the material in each horizontal layer of the cloud,
 - (36) and (37) are skewness and flatness equations of the distribution function F .
- Solutions of these equations are obtained up to the third moment as

$$\theta_0(\eta) = e^{-\eta} \quad (39)$$

$$\theta_1(\eta) = \frac{e^{-\eta}}{a\kappa} [\ln \eta + \gamma - 1] + \frac{E_1(\eta)}{a\kappa} \quad (40)$$

$$\theta_2(\eta) = \frac{1}{a^2\kappa^2} [2E_1(\eta)\eta + \eta^2 e^{-\eta}(\eta+2) - 2\eta E_1(\eta) + 3e^{-\eta} - (\eta+1)(\Gamma''(1, \eta) + 2\gamma\Gamma'(1, \eta) + \gamma^2 e^{-\eta})] \quad (41)$$

and

$$\theta_3(\eta) \Big|_{\eta=0} = -\frac{2.267}{a^3\kappa^3} \quad (42)$$

where

$E_1(\eta)$ is exponential integral given by

$$E_1(\eta) = \int_{\eta}^{\infty} \frac{e^{-\eta}}{\eta} d\eta \quad \text{and} \quad \eta = \ln(\eta e^\gamma)$$

The expressions $\Gamma''(1, n) = \int_n^{\infty} e^{-t} (\ln t)^2 dt$.

$$\Gamma'(1, n) = \int_n^{\infty} e^{-t} \ln t dt, \text{ and}$$

$$\Gamma^n(a, n) = \frac{\partial^n \Gamma(a, n)}{\partial a^n} = \int_n^{\infty} (\ln t)^n e^{-t} t^{a-1} dt \quad (43)$$

are the derivatives of incomplete gamma functions with respect to the parameter 'a'. These derivatives obtained by the authors for all n are available at the Air Pollution Control Division, Colorado Department of Health.

The mean, the variance and the skewness coefficient of the density function can be derived from the above results. The mean is defined as follows:

$$\bar{x}_n = \mu(n) = \frac{\int_{-\infty}^{\infty} x \cdot x dx}{\int_{-\infty}^{\infty} x dx} \quad (44)$$

Transforming the variables into the similarity coordinates as

$$x = au_0 t \theta + \bar{x}, \quad x = \frac{1}{az(u_0 t)^2} F(\theta, n) \quad (45)$$

it can be shown that

$$\mu(n) = (au_0 t) \frac{\theta_1(n)}{\theta_0(n)} + \bar{x} \quad (46)$$

Similarly, the variance of the density function can be shown to be

$$\sigma_x^2(n) = \frac{\int_{-\infty}^{\infty} (x-\mu)^2 x dx}{\int_{-\infty}^{\infty} x dx} = (au_0 t)^2 \left[\frac{\theta_2(n)}{\theta_0(n)} - \left(\frac{\theta_1(n)}{\theta_0(n)} \right)^2 \right] \quad (47)$$

and the skewness coefficient of the density function to be

$$\lambda_3(n) = \frac{E\{(x-x_n)^3\}}{[E(x-x_n)^2]^{3/2}} = \frac{1}{\sigma_x^3(z, t)} \frac{\int_{-\infty}^{\infty} (x-x_n)^3 x dx}{\int_{-\infty}^{\infty} x dx} \\ = \left[\frac{\theta_3(n)}{\theta_0(n)} - 3 \left(\frac{\theta_1(n)}{\theta_0(n)} \right)^2 + 3 \frac{\theta_1(n)\theta_2(n)}{\theta_0^2(n)} \right] / \left[\frac{\theta_2(n)}{\theta_0(n)} - \left(\frac{\theta_1(n)}{\theta_0(n)} \right)^2 \right]^{3/2} \quad (48)$$

At ground level $n = 0$. Substituting the values for $\theta_2(0)$, $\theta_1(0)$, $\theta_0(0)$, $\theta_3(0)$ into Eq. (48), the skewness coefficient at the ground level is found to have the following value:

$$\lambda_3(0) = \frac{0.215}{(0.355)^{3/2}} = 1.02. \quad (49)$$

The variations of the mean and the variance of the density function f with n are given in Figs. 1 and 2. It can be pointed out here, that the variation of this variance with height

above the boundary is similar to that of the shearing stress in a two-dimensional turbulent shear flow. The skewness coefficient is obtained through Eq. (49) for the ground-level distribution only. However, to be consistent, it can be assumed to remain constant with height, as the density function is assumed to be similar in Eq. (26). Integration of the Eqs. (37) through (38) for higher order statistical properties, will define the density function with more sophistication. Such an accuracy is sacrificed at this point due to increased analytical difficulties. However, much significant information on the concentration field can be derived from these results as demonstrated in the next section.

DENSITY FUNCTION

Substituting the results expressed by Eqs. (46), (47) and (48) into the function given by Eq. (12) and multiplying it by the marginal density function of Eq. (11), the concentration field of Eq. (5) can be shown to have the following form:

$$f(x, z, t) = f(x|z, t) \cdot f_z(z, t) = \frac{A}{\sqrt{2\pi} \sigma_x} \exp\left(-\frac{x^2}{2\sigma_x^2}\right) \left[1 - \frac{1}{6}\left(\frac{z^3}{\sigma_z^3} - 3\right) \exp\left(-\frac{z^2}{2\sigma_z^2} - \frac{z}{\sigma_z}\right)\right] \quad (50)$$

where

σ_x^2 = variance of longitudinal density function, given by Eq. (47),

σ_z^2 = variance of marginal density function, given in Eq. (15),

$t = (x - \bar{x})/\sigma_x$, and

$A = 1.01$ is a constant, makes the density integrate to unity.

A typical density curve for the concentration at ground level is presented in Fig. 3. However, the concentrations at other levels above the ground can be obtained from the same figure. The density curve may be slid along the mean curve to a required height, z , and multiplied by the marginal density function, f_z . Concentration versus time curves can also be obtained from the above density function.

SHAPE OF THE CLOUD

A description of concentration field is obtained by Fatta⁽¹¹⁾ from Eq. (50). A typical shape of the cloud is given in Fig. 4 by plotting isoconcentration lines. The shape is asymmetric about the mean due to the presence of shear in the surface layer of the atmosphere. The top of the cloud is stretched forward with a greater speed while it is simultaneously diffusing in the vertical and longitudinal directions. It can be observed from the figure that, the mean of the concentration distribution at any given layer below $z = 0.36z$, lags behind the center of mass of the total cloud ($\bar{z}$), and above this height $z = 0.36z$, it leads the center of mass of the cloud. The shape of the cloud is tested for its physical dimensions by the published experimental results as follows:

Maria⁽⁹⁾ in his approach to determine the vertical spread of smoke within the surface layer defined

$$z = \frac{4z_{max}}{3} = b'z_0 \quad (51)$$

where z_{max} is the maximum height of the cloud. By analyzing the smoke puff in the field experiment, he reported a value for $b' = 0.15$. Pasquill⁽¹⁰⁾ defined z_{max} as the height at which the concentration level $c = 0.1 c_{max}$ while analyzing the experimental data obtained at Porton. In the relation $z_{max} = b'z_0$, he obtained a value for $b' = 1.3$. In the present analysis, utilizing Fig. 4 and defining z_{max} the same way as Pasquill did, the value of b' is estimated as follows:

$$\frac{z_{max}}{z_0} = 1.3 \text{ at } c = 0.1 c_{max}$$

i.e.,

$$z_{max} = 1.3 z_0 = 0.91 z_0 \quad (52)$$

where

$$z_0 = \text{Estima Constant} = 0.4.$$

This value of $b' = 0.91$ lies between the experimental estimates of Maria and Pasquill. It is also predicted that the ratio

$$\sigma_x/\sigma_z = au_0t/ku_0t = 3.75 .$$

(53)

The only observations available are those by Kasanski and Monin, reported by Monin⁽⁹⁾ where, the maximum downwind and vertical diameters of smoke puffs are reported. The observed ratio of these diameters was estimated to be 4.0 after the initial growth had passed. The predicted value of 3.75 in the present analysis is certainly in good agreement with the observed value.

CONCLUSIONS

1. Moment equations are derived from the Eulerian equations by assuming Lagrangian similarity density function for concentration. These equations are solved and the statistical properties, namely the mean, the variance and the skewness coefficient of the longitudinal density function are obtained. The vertical distribution of the longitudinal variance property of the cloud indicates a similarity with the distribution of shearing stress in the surface layer of the atmosphere.

2. The maximum height of the cloud is found to be increasing linearly with time. The proportionality constant b in the relation $z_{max} = b u_0 t$, is found to be equal to 0.92. This value lies in between the experimental observations of Monin (1959) and Pasquill (1966).

3. The length of the cloud is found to be always greater than the height. The length-to-height ratio which is obtained as the ratio of the two standard deviations σ_x/σ_z is shown to be equal to 3.75. This result is in good agreement with the experimental estimation of 4.0, obtained by Kasanski and Monin (Monin 1959). It is apparent from this study that the results can be applied toward understanding of the dispersion of pesticides and insecticides released by low flying aircraft onto vegetation in farm lands and forest areas. Ground-level exposures can be estimated utilizing the proposed density function in this study when the density of the material is not very much different than the density of the air. To be optimistic, the results of this study can be utilized also to estimate ambient air-pollution levels from automobile exhausts when the wind direction is across the highway system. When the frequency of the automobiles passing the highway and the frequency of wind velocities are known, the pollution levels, downwind of the highway can be estimated by summation of the contribution from each automobile. Such estimates can be made using the present high-speed computers.

The foregoing results are applicable only to a cloud dispersing within the constant-stress region of the neutral atmospheric boundary layer.

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Fig.1 Mean of Longitudinal Density Function

Fig.2 Variance of Longitudinal Density Function

Fig.3 Density Function for the Direction of Mean Flow

Fig.4 Shape of Instantaneous Line Source Cloud

WIENER-HERMITE EXPANSION APPLIED TO PASSIVE SCALAR DISPERSION
IN A NONUNIFORM TURBULENT FLOW. II

Martial L. Thiebaux

Advanced Study Program, National Center for Atmospheric
Research,* Boulder, Colorado 80302

INTRODUCTION

The concentration $\theta(\underline{x}, t)$ of a scalar contaminant passively transported by a velocity field $\underline{U}(\underline{x}, t)$ satisfies the advection equation

$$\partial_t \theta = - \underline{U} \cdot \nabla \theta. \quad (1)$$

The form of (1) restricts θ to be a mass mixing ratio in case the flow is compressible. Source terms, reactions, and molecular diffusion are neglected in (1). The purpose of this report is to demonstrate how inhomogeneous and nonstationary statistics describing $\underline{U}$ can be incorporated into the time evolution (integrodifferential) equation for the expectation value $\bar{\theta}(\underline{x}, t)$ of the concentration in an ensemble of flows.

INTEGRODIFFERENTIAL DIFFUSION EQUATION

By (i) separating the velocity field into its mean $\bar{\underline{U}}(\underline{x}, t)$ (generally dependent on the space-time variables as indicated) plus its random part $\underline{U}'(\underline{x}, t)$, (ii) expanding θ and $\underline{U}'$ in stochastic Wiener-Hermite functionals, (1), (2) and (iii) systematically eliminating the higher order expansion coefficients (kernel functions) with (1) and the initial condition that $\theta(\underline{x}, 0) = \bar{\theta}(\underline{x}, 0)$, the exact integrodifferential equation determining $\bar{\theta}$ in terms of Eulerian velocity moments can be derived. (3), (4) We omit repeating the details of the derivation and state the result:

$$D_{t_1} \bar{\theta}(\underline{x}_1, t_1) = \sum_{v=2}^{\infty} A_v(\underline{x}_1, t_1) \quad (2)$$

where

$$D_{t_1} \equiv \partial_{t_1} + \bar{\underline{U}}(\underline{x}_1, t_1) \cdot \nabla \quad (3)$$

and

$$A_v(\underline{x}_1, t_1) = (-1)^v \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 \dots \int_0^{t_{v-1}} dt_v \times \langle \langle \underline{U}'_i(\underline{x}_1, t_1) \nabla_{i1} \underline{U}'_j(\underline{x}_2, t_2) \nabla_{2j} \dots \underline{U}'_k(\underline{x}_v, t_v) \nabla_{vk} \rangle \rangle \bar{\theta}(\underline{x}_v, t_v). \quad (4)$$

The operator D_{t_1} is the substantial derivative that follows an imaginary particle moving with the mean velocity $\bar{\underline{U}}$. In (4), $\underline{x}_\lambda$ is the position at time t_λ of the same imaginary particle that later passes through the point $\underline{x}_1$ at time t_1 . Hence all the $\underline{x}_\lambda$, $1 \leq \lambda \leq v$, in (4) lie on the same mean trajectory; and the coordinates $\underline{x}_\lambda$ of any such point can be considered to be a function of its time t_λ and the space-time coordinates $(\underline{x}_0, t_0)$ of any one other point on the trajectory. The meaning of the gradient operators in (4) can now be understood: $\nabla_{\sigma l} f(\underline{x}_\lambda, t_\lambda)$ is the l -component of the gradient of function f , evaluated on the mean trajectory at time t_λ , but taken with respect to the coordinates of the point labeled by σ . Thus we have

$$\nabla_{\sigma l} f(\underline{x}_\lambda, t_\lambda) = \frac{\partial r_{\lambda m}}{\partial r_{\sigma l}}(\underline{x}_\sigma, t_\sigma, t_\lambda) \cdot \frac{\partial f}{\partial r_{\lambda m}}(\underline{x}_\lambda, t_\lambda) \quad (5)$$

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and hence

$$\nabla_{\sigma\lambda} = d_{\lambda m}(x_{\sigma}, t_{\sigma}, t_{\lambda}) \nabla_{\lambda m}, \text{ for any } \sigma \text{ and } \lambda, \quad (6)$$

where

$$d_{\lambda m}(x_{\sigma}, t_{\sigma}, t_{\lambda}) \equiv \frac{\partial r_{\lambda m}(x_{\sigma}, t_{\sigma}, t_{\lambda})}{\partial r_{\sigma\lambda}} \quad (7)$$

Summation over repeated subscripts is always implied.

The brackets $\langle \dots \rangle$ denote certain statistics defined recursively as follows. Let a_{α} , $\alpha = 1, 2, \dots$ denote a set of zero-mean random variables. Then

$$\langle a_1 a_2 \rangle = \overline{a_1 a_2}, \quad (5)$$

$$\langle a_1 a_2 a_3 \rangle = \overline{a_1 a_2 a_3}, \quad (6)$$

while for $v \geq 4$

$$\langle a_1 \dots a_v \rangle = \overline{a_1 \dots a_v} - \sum_{\mu=2}^{v-2} \overline{a_1 \dots a_{\mu}} \langle a_{\mu+1} \dots a_v \rangle. \quad (7)$$

The statistics that appear in (4) are therefore combinations of ordinary moments of the velocity field. From the preceding discussions these are Eulerian, one-point, multi-time correlations and their space derivatives defined in the frame moving with the mean velocity.

As a first example, we display the structure of A_2 in a horizontal mean velocity $(\bar{u}_x, \bar{u}_y)$ that depends on time and height (z_1) only, as might exist in a transient Ekman spiral. The velocity statistics are assumed to be homogeneous in the horizontal planes. We find

$$\begin{aligned} A_2(x_1, t_1) &= \int_0^{t_1} dt_2 K_{mj}(z_1, t_1, t_2) \bar{\theta}_{mj}(x_2, t_2) \\ &+ \int_0^{t_1} dt_2 K_j(z_1, t_1, t_2) \bar{\theta}_{,j}(x_2, t_2) \end{aligned} \quad (8)$$

where

$$\begin{aligned} K_{11} &= R_{11} + R_{31}N_1, \quad K_{22} = R_{22} + R_{32}N_2, \quad K_{33} = R_{33}, \\ K_{12} &= K_{21} = \frac{1}{2} (R_{12} + R_{21} + R_{32}N_1 + R_{31}N_2), \\ K_{23} &= K_{32} = \frac{1}{2} (R_{23} + R_{32} + R_{33}N_2), \\ K_{13} &= K_{31} = \frac{1}{2} (R_{13} + R_{31} + R_{33}N_1), \\ K_1 &= R_{11,1} + R_{21,2} + R_{31,3} + R_{31,1}N_1 + R_{31,2}N_2, \\ K_2 &= R_{12,1} + R_{22,2} + R_{32,3} + R_{32,1}N_1 + R_{32,2}N_2, \\ K_3 &= R_{13,1} + R_{23,2} + R_{33,3} + R_{33,1}N_1 + R_{33,2}N_2. \end{aligned} \quad (9)$$

In the above we have used

$$R_{ij} = \overline{U_i'(z_1, t_1) U_j'(z_1, t_2)}, \quad (10)$$

$$R_{ij,k} = \overline{U_i'(z_1, t_1) U_{j,k}'(z_1, t_2)}, \quad (11)$$

$$N_1 = \int_{t_1}^{t_2} \frac{\partial \bar{u}_x}{\partial z_1} (z_1, t') dt', \quad N_2 = \int_{t_1}^{t_2} \frac{\partial \bar{u}_y}{\partial z_1} (z_1, t') dt'. \quad (12)$$

If $\bar{\theta}$ varies slowly with time in comparison to the velocity correlation functions, $\bar{\theta}$ can be updated to the current time t_1 under the integrals in (8). Retaining only this updated $v=2$ term in (2), we get a diffusion equation

$$\partial_t \bar{\theta}(\underline{z}, t) = W_j(\underline{z}, t) \partial_j \bar{\theta}(\underline{z}, t) + D_{jm} \partial_j \partial_m \bar{\theta}(\underline{z}, t) \quad (13)$$

in which

$$W_j(\underline{z}, t) = \bar{U}_j(z, t) + \int_0^t dt' K_j(z, t, t') \quad (14)$$

and

$$D_{jm}(\underline{z}, t) = \int_0^t dt' K_{jm}(z, t, t'). \quad (15)$$

Before considering other examples or higher terms in the sum on the right-hand side of (2), it is worth noting that the expanded structure of each A_v can be quite complicated since each V_{λ} in (4) operates on everything to its right. Two simplifying assumptions reduce this complexity for illustrative purposes:

(a) Velocity moments are locally invariant under inversions through points moving with the mean velocity. This implies that even (odd) derivatives of odd (even)-order one-point moments must vanish.

(b) The d_{ij} defined above are space-independent. This restricts nontrivial mean flows to simple shearing or pure straining motion.

We display the resulting reduced A_v for $v = 2, 3, 4$. We write $d_{ij}(t_1, t_2) = d_{ij}(1, 2)$ and $\overline{U_i(\underline{z}_1, t_1) U_j(\underline{z}_2, t_2)} = \overline{U_i(1) U_j(2)}$, etc. without ambiguity.

$$A_2(t_1) = \int_0^{t_1} dt_2 d_{im}(1, 2) \overline{U_i(1) U_j(2)} \bar{\theta}_{,mj}(2). \quad (16)$$

$$A_3(t_1) = \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 \left\{ d_{in}(1, 2) d_{jm}(2, 3) \overline{U_i(1) U_{jn}(2) U_k(3)} \right. \\ \left. + [d_{in}(1, 3) d_{jm}(2, 3) + d_{im}(1, 3) d_{jn}(2, 3)] \overline{U_i(1) U_j(2) U_{k,n}(3)} \right\} \bar{\theta}_{,mk}(3). \quad (17)$$

$$A_4(t_1) = \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 \int_0^{t_3} dt_4 \left\{ d_{ip}(1, 2) d_{jn}(2, 3) d_{km}(3, 4) \langle\langle U_i(1) U_{j,p}(2) U_{k,n}(3) U_l(4) \rangle\rangle \right. \\ + d_{ip}(1, 3) d_{jn}(2, 3) d_{km}(3, 4) \langle\langle U_i(1) U_j(2) U_{k,np}(3) U_l(4) \rangle\rangle \\ + [d_{ip}(1, 4) d_{jn}(2, 3) d_{km}(3, 4) + d_{im}(1, 4) d_{jn}(2, 3) d_{kp}(3, 4)] \langle\langle U_i(1) U_j(2) U_{k,n}(3) U_{l,p}(4) \rangle\rangle \\ + [d_{ip}(1, 2) d_{jn}(2, 4) d_{km}(3, 4) + d_{ip}(1, 2) d_{jm}(2, 4) d_{kn}(3, 4)] \langle\langle U_i(1) U_{j,p}(2) U_k(3) U_{l,n}(4) \rangle\rangle \\ + [d_{ip}(1, 3) d_{jn}(2, 4) d_{km}(3, 4) + d_{ip}(1, 3) d_{jm}(2, 4) d_{kn}(3, 4)] \langle\langle U_i(1) U_j(2) U_{k,p}(3) U_{l,n}(4) \rangle\rangle \\ + [d_{ip}(1, 4) d_{jn}(2, 4) d_{km}(3, 4) + d_{ip}(1, 4) d_{jm}(2, 4) d_{kn}(3, 4) \\ + d_{im}(1, 4) d_{jn}(2, 4) d_{kp}(3, 4)] \langle\langle U_i(1) U_j(2) U_k(3) U_{l,pn}(4) \rangle\rangle \Big\} \bar{\theta}_{,lm}(4) \\ + \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 \int_0^{t_3} dt_4 d_{ip}(1, 4) d_{jn}(2, 4) d_{km}(3, 4) \langle\langle U_i(1) U_j(2) U_k(3) U_l(4) \rangle\rangle \bar{\theta}_{,lmnp}(4). \quad (18)$$

WIENER-HERMITE EXPANSION

In the absence of knowledge of high order velocity moments, some additional assumptions are required to establish rules that relate high order moments to low order ones. We shall now see that a truncated Wiener-Hermite expansion of the velocity field leads to such rules. The Wiener-Hermite expansion for the velocity is

$$U_i^*(\underline{r}, t) = \sum_{n=1}^{\infty} U_{i1}(\underline{r}, t; 12\dots n) H(12\dots n) \quad (19)$$

In which $U_{i1}(\underline{r}, t; 12\dots n)$ is a nonrandom kernel function and $H(12\dots n) \equiv H^{(n)}$ is the n th order Wiener-Hermite functional. (1), (2) In (19), the repeated integer arguments k stand for space-time variables $(\underline{r}_k, t_k)$ paired with vector indexes $i_k (=1, 2, 3)$. We refer to k or $(\underline{r}_k, t_k, i_k)$ as a W-H argument. Integrals or sums over the repeated W-H arguments are implied in (19). The defining properties of the Wiener-Hermite functionals are (i) the covariance of the purely Gaussian first order functional is

$$\langle H(1) H(2) \rangle = \delta(\underline{r}_1 - \underline{r}_2) \delta(t_1 - t_2) \delta_{i_1 i_2} \quad (20)$$

(ii) higher order functionals are polynomials in first order ones, (iii) $\langle H^{(m)} H^{(n)} \rangle = 0$ if $m \neq n$ (statistical independence), and (iv) the functionals (and kernels therefore, are symmetric in the W-H arguments.

Retaining only the first term in (19) would give a purely Gaussian (multivariate-normal) velocity field: all odd-order moments vanish while all even-order moments are products of second order moments. This relation may be written symbolically in the form of a recursion relation

$$\overline{12\dots v} = \sum \overline{0\beta} \cdot \overline{\gamma\delta\dots\lambda} \quad (21)$$

where $\overline{12\dots v}$ is an order- v moment evaluated at v space-time points, $\overline{0\beta}$ is a second order moment at two of the points, and $\overline{\gamma\delta\dots\lambda}$ is an order- $v-2$ moment evaluated at the remaining $v-2$ points. The sum is over all such distinct combinations of two points. Passive scalar dispersion in a Gaussian field is a respectable and unsolved problem that has been tackled by numerical techniques. (5)

It is interesting to consider the consequence of retaining the next term in (19). If we let

$$U_i^*(\underline{r}, t) = U_{i1}(\underline{r}, t; 1)H(1) + \epsilon U_{i1}(\underline{r}, t; 12)H(12) \quad (22)$$

and construct the n th moment of U_i^* to first order in ϵ , we find that all even moments are still related to second moments in the Gaussian way, but odd moments are not necessarily zero. The odd order- v moment at v space-time points may be written symbolically as

$$\overline{12\dots v} = \sum \overline{0\beta\gamma} \cdot \overline{\delta\epsilon\dots\lambda} \quad (23)$$

where $\overline{0\beta\gamma}$ is a third moment evaluated at three space-time points and $\overline{\delta\epsilon\dots\lambda}$ is a moment of even order- $v-3$ evaluated at the remaining $v-3$ points. The sum is over all such distinct combinations of three points. Eq. (23) is a simple extension of the Gaussian relation (21) to odd moments. Second and third moments are now the basic elements out of which all higher moments are constructed. The set of moments defined by (22) is adopted here primarily as an illustrative example and, although it is an open possibility, there is no intention to represent it as a valid approximation to a physical set of velocity moments. In fact, the set is actually unrealizable in the univariate limit where it happens that the probability density function can become negative.

SIMPLE SHEAR MEAN FLOW

In the remainder of this report we restrict the analysis to a uniform shear mean flow with homogeneous and stationary statistics. Since assumption (a) is compatible with this type of flow and (b) is satisfied, the expression (16) - (18) will be used. If the mean flow is in the x -direction and ω is the mean rate of shear in the z -direction, then the elements of $d_{ij}(\tau_1, \tau_2)$ are

$$d_{11} = d_{22} = d_{33} = 1, \quad d_{31}(\tau_1, \tau_2) = -\omega(\tau_1 - \tau_2),$$

and

$$d_{ij} = 0, \text{ otherwise.}$$

Consider the second moment of the velocity field

$$R_{ij}(\underline{r}, t) = \overline{U_i^*(0, 0) U_j^*(\underline{r}, t)}. \quad (24)$$

The statement of shear homogeneity and stationarity is

$$R_{ij}(\underline{r}, t) = R_{ij}(-\underline{r} + \omega t \hat{z}, -t) \quad (25)$$

where $\hat{z}$ is the unit vector in the x-direction. From (24) and (25) the statement of shear homogeneity in differential form becomes

$$\overline{U_i'(\underline{r}, t) U_{j,m}'(\underline{r}', t')} = -d_{mk}(t, t') \overline{U_{1,k}'(\underline{r}, t) U_j'(\underline{r}', t')}. \quad (26)$$

Thus, if we now assume the flow is incompressible, (26) requires that for any i,

$$0 = \overline{U_i'(\underline{r}, t) U_{m,m}'(\underline{r}', t')}$$

and

$$0 = d_{mn}(t, t') \overline{U_m'(\underline{r}, t) U_{1,n}'(\underline{r}', t')}. \quad (27)$$

Eqs. (25) - (27) reduce to familiar results when the shear is turned off and $d_{mn}(t, t') \rightarrow \delta_{mn}$.

The assumptions of shear homogeneity, stationarity, incompressibility, and truncation of the velocity field expansion according to (22) lead to a highly reduced form for A_4 :

$$\begin{aligned} A_4(t_1) = & \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 \int_0^{t_3} dt_4 \left\{ d_{im}(1,4) d_{jn}(2,4) d_{kp}(3,4) \right. \\ & \times \overline{U_i(1) U_{l,pn}(4) \cdot U_j(2) U_k(3)} \bar{\theta}_{,ml}(4) \\ & + d_{ip}(1,4) d_{jn}(2,4) d_{km}(3,4) \left[\overline{U_i(1) U_k(3) \cdot U_j(2) U_l(4)} \right. \\ & \left. \left. + \overline{U_i(1) U_l(4) \cdot U_j(2) U_k(3)} \right] \bar{\theta}_{,kmp}(4) \right\}. \end{aligned} \quad (28)$$

The contribution to A_3 from the isotropic part of the third moments vanishes because first space derivatives of isotropic third moments vanish at the origin.⁽⁶⁾ An anisotropic third moment would have so much space-time structure, even with appropriate reflection symmetries, that we could not easily present here results suitable as an illustrative example. Hence, we shall ignore A_3 , or take $A_3 = 0$ either because third moments are assumed isotropic, or simply because the velocity field is assumed Gaussian.

Next we shall expand the second moment in a power series in ω :

$$R_{ij}(\underline{r}, t) = v^2 R_{ij}^I(\underline{r}, t) + \omega R_{ij}^A(\underline{r}, t) + O(\omega^2). \quad (29)$$

It is especially helpful, in continuing this analysis, to suppose that the turbulence is driven in part by isotropic forces independent of the shear in addition to the anisotropic forces that produce the mean shear. The first term on the right-hand side of (29) is then a purely isotropic second moment with v^2 equal to the mean square of one component of the velocity, and

$$R_{ij}^I(\underline{r}, t) = \phi_1(t) \delta_{ij} + \phi_2(t) (2\lambda^2)^{-1} (r_i r_j - 2r^2 \delta_{ij}). \quad (30)$$

In (30), λ is the Taylor microscale, the time correlation functions are even functions of time, and the space structure is displayed only up to quadratic terms. It is convenient to assume $\phi_1(t) = \phi_2(t) \equiv \phi(t)$, although this is not crucial.

In addition to inversion invariance, we assume further that the turbulent shear flow is statistically invariant under (i) reflection in the x-z plane and (ii) reflection in the x-y plane when $\omega \rightarrow -\omega$. The space-structure of the first anisotropic term in (29), correct to quadratic terms, is then

$$\omega R_{ij}^A(\underline{r}, t) = \omega \times \begin{array}{c} \begin{array}{c} j \rightarrow \\ i \downarrow \end{array} \begin{array}{|c|c|c|} \hline e_1 xz & f_1 yz & a_i + b_i x^2 \\ & & + c_i y^2 + d_i z^2 \\ \hline f_2 yz & e_2 xz & g_1 xy \\ \hline a_2 + b_2 x^2 \\ + c_2 y^2 + d_2 z^2 & g_2 xy & s_3 xz \\ \hline \end{array} \end{array} \quad (31)$$

in which $a_1, a_2, b_1, b_2, \dots, g_1, g_2$ are time-dependent parameters. Shear homogeneity and incompressibility lead to a number of relations satisfied by these parameters. Those relevant to this work are

$$a_1(t) = a_2(-t), \quad b_1(t) = b_2(-t), \quad c_1(t) = c_2(-t), \quad d_1(t) = d_2(-t) - \frac{v^2}{2\lambda^2} \phi(t). \quad (32)$$

It is now possible to evaluate A_2 and A_4 for a weak shear mean flow using (29) in (16) and (28), and consistently retaining only terms linear in ω . Instead of displaying the resulting integrodifferential equation, it is instructive to force it into the form of a long-time diffusion equation as was done in deriving Eq. (13). Stationarity then permits one free integration in A_4 . We find

$$D_t \bar{\theta}(\underline{r}, t) = (A_{2ij} + A_{4ij}) \bar{\theta}_{,ij}(\underline{r}, t) + A_{4ijkl} \bar{\theta}_{,ijkl}(\underline{r}, t) \quad (33)$$

where

$$A_{2ij} = v^2 \int_0^\infty dt \left\{ \phi(t) \delta_{ij} + \omega \left[P(t) - \frac{1}{2} t \phi(t) \right] \beta_{ij} \right\}, \quad (34)$$

$$A_{4ij} = \frac{5v^4}{\lambda^2} \int_0^\infty dt \int_0^t dt' \phi(t) \phi(t') (t' - t) \delta_{ij} + \frac{\omega v^4}{\lambda^2} \int_0^\infty dt \int_0^t dt' \left\{ 2\phi(t) \phi(t') (t^2 - t t') + [\phi(t) P(t') + \phi(t') Q(t)] (t' - t) \right\} \beta_{ij}, \quad (35)$$

and

$$A_{4ijkl} = v^4 \int_0^\infty dt \int_0^t dt' \phi(t) \phi(t') \delta_{ij} \delta_{kl} - \omega v^4 \left\{ \int_0^\infty dt \int_0^t dt' \phi(t) \phi(t') + \frac{1}{2} \left[\int_0^\infty dt \int_0^t dt' \phi(t) \phi(t') \right]^2 + \int_0^\infty dt \int_0^t dt' [t \phi(t) P(t') + t' P(t) \phi(t')] \right\} \delta_{ij} \delta_{kl}. \quad (36)$$

In these expressions,

$$P(t) = (2v^2)^{-1} [a_1(t) + a_2(t)] = (2v^2)^{-1} \frac{\partial}{\partial \omega} [R_{13}(0, t) + R_{31}(0, t)], \quad (37)$$

$$Q(t) = \lambda^2 v^{-2} [b_1(t) + c_1(t) + d_1(t) + b_2(t) + c_2(t) + d_2(t)] = \frac{1}{2} \lambda^2 v^{-2} \frac{\partial}{\partial \omega} \nabla^2 [R_{13}(0, t) + R_{31}(0, t)], \quad (38)$$

and

$$\beta_{ij} = \delta_{i3} \delta_{j1} + \delta_{i1} \delta_{j3}. \quad (39)$$

Finally we note that the anisotropic time correlation functions $P(t)$ and $Q(t)$ may be related to the isotropic time correlation function $\phi(t)$ if we assume the second moment is unchanged when ω and t simultaneously change sign. This operation preserves the mean flow but is neither an obvious nor an unreasonable symmetry of the turbulent field. Under this assumption, however, all the anisotropic time-dependent parameters are odd functions of time, which requires from (32), that

$$P(t) = 0 \quad \text{and} \quad Q(t) = -\frac{1}{2} t \phi(t). \quad (40)$$

For a simple numerical example, we take $\phi(t) = e^{-t/\tau}$, where τ is the Eulerian time integral scale, and easily find

$$A_{2ij} + A_{4ij} = v^2 \tau \left[1 - \frac{5}{2} \left(\frac{v\tau}{\lambda} \right) \right] \delta_{ij} - \frac{1}{2} \omega v^2 \tau^2 \left[1 - 5 \left(\frac{v\tau}{\lambda} \right)^2 \right] \beta_{ij} \quad (41)$$

and

$$A_{4ijkl} = v^4 \tau^3 \delta_{ij} \delta_{kl} - \frac{5}{2} \omega v^4 \tau^4 \delta_{ij} \beta_{kl}. \quad (42)$$

DISCUSSION

Three difficulties have emerged in the foregoing:

(1) The terms proportional to $(v\tau/\lambda)^2$, within the square brackets in (41), are the contributions from A_4 . The smallness of these terms in comparison to unity is then a crude test of convergence of the expansion (2). Since $v\tau$ is a rough measure of the Eulerian space integral scale⁽⁷⁾ which in turn is likely to be of the same order of magnitude as the microscale, $(v\tau/\lambda)$ is of order unity and the convergence test seems to fail. Whatever the correct interpretation of this "test" may be, it is clearly unsatisfactory to stop the procedure at this point. The (non)convergence of expansion (2) presents a serious difficulty, even in the isotropic limit. An obvious approach to resolving the convergence problem is to evaluate A_v in (2) for arbitrary v with explicit and simple assumptions such as isotropy, multivariate-normal velocity statistics, and exponential or Gaussian correlation functions, and then study the analytic properties of the expansion. Such a study would seem necessarily to precede and should shed light on applications to more general flows.

(11) The validity of retaining only the first two terms of the Wiener-Hermite expansion of the random part of the velocity field should be studied. Should Eq. (22) be a valid representation for a real turbulent flow, then it provides a convenient basis for including non-Gaussian moments of the velocity field in expansion (2). Otherwise the Wiener-Hermite expansion would seem to be of little use beyond serving as an intermediate vehicle for deriving the formally exact integrodifferential equation defined by (2), (3), (4).

(11) The third difficulty concerns the presence of higher space derivative terms such as A_{ijkl}^5, i,j,k,l exhibited in Eq. (33). All orders of derivatives are present in the complete expansion in Eq. (2). It is important to know how sensible it is to ignore the higher order derivatives as was done in establishing a diffusion equation like Eq. (13). A rough comparison of the magnitudes of comparable parts of (41) and (42) gives the reasonable criterion that if $(v\tau/L)^2 \ll 1$, where L is a scale length for the variation of $\bar{u}$, then higher space derivative terms are negligible.

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COMPUTATIONAL EVALUATION OF THE GAUSSIAN PLUME MODEL

Paul Michael

Brookhaven National Laboratory
Upton, Long Island, New York

INTRODUCTION

Dispersion calculations are most frequently performed by means of the Gaussian plume model. More important than the assumption of normality in the horizontal and vertical is the assumption that the proper selection of parameters, in particular σ_y and σ_z , will allow for the proper computation of the concentration of a pollutant of interest. Particular applications often make other assumptions; e.g., that σ_y and σ_z are independent of the source elevation, that the proper wind direction is that which exists at source height, etc. The purpose of the work reported here is to evaluate some of these assumptions. Ideally, such an evaluation should be done by careful field experiments, including a wide range of meteorological measurements. This study is a computational one; both because it is simpler to do than an experimental study and it is hoped that guidance for future experimental measurements will result.

The procedure used was to obtain "K" theory solutions for various assumptions concerning the parameters, such as wind and diffusivity profiles, and then to find parameters of the Gaussian plume model that would match these solutions. The thought behind this procedure is twofold. First, that if the Gaussian plume model is able to match conditions in the atmosphere it should be able to simulate solutions to K theory equations. Second, while it is recognized that atmospheric dispersion is not well calculated by K theory it is reasonable to expect that K theory will reproduce trends properly; that is, if two K theory solutions are based upon different assumptions concerning, say, the wind velocity, then differences in the solutions would be similar to the differences that would occur in the atmosphere under an equivalent change in conditions.

K THEORY SOLUTIONS

The starting for the K theory solution is:

$$U \frac{\partial C}{\partial x} + V \frac{\partial C}{\partial y} = K_y \frac{\partial^2 C}{\partial y^2} + \frac{\partial}{\partial z} \left(K_z \frac{\partial C}{\partial z} \right) \quad (1)$$

where $C(x, y, z)$ is the material concentration, $U(z)$ and $V(z)$ are the wind velocities in the x and y directions, and $K_y(z)$ and $K_z(z)$ are diffusivity coefficients in the y and z directions. (Explicit reference to the independent variables has been omitted from the equation for convenience.) The major simplifying assumptions are that x direction diffusion can be neglected, and that off diagonal terms in the diffusivity tensor can be neglected. The neglect of x diffusion allows the equation to be solved as an initial value problem in x . The neglect of off diagonal terms is equivalent, of course, to assuming that the principal axes of the diffusivity tensor coincide with x, y, z axes. (Some discussion of the importance of the off diagonal terms has been given by Calder, ⁽¹⁾ Saffman, ⁽²⁾ Smith, ⁽³⁾ among others.)

The method of solution is closely related to the work of Aris, ⁽⁴⁾ Smith, ⁽⁵⁾ Saffman, ⁽⁶⁾ and

Tyldesley and Hallington.⁽⁷⁾ One first defines moments:

$$C_n(n, z) = \int y^n C \, dy. \tag{3}$$

Equation 1 is multiplied by y^n and integrated by parts yielding:

$$U \frac{\partial C}{\partial n} + \frac{\partial}{\partial z} \left(K_z \frac{\partial C}{\partial z} \right) = n U C_{n-1} + n(n-1) K_y C_{n-2}. \tag{5}$$

C_0 is the crosswind integrated (CWI) concentration. The center of the mass of the distribution, $\bar{y}(n, z)$, and the variance $\sigma_y^2(n, z)$, are given from the moments:

$$\bar{y} = \frac{C_1}{C_0}, \tag{4}$$

$$\sigma_y^2 = \frac{C_2}{C_0} - \bar{y}^2. \tag{5}$$

The equations for the C_n can be solved successively in terms of lower order moments. Theoretical solutions were obtained by approximating the derivatives by central differences, using the Crank Nicholson method to derive a set of implicit difference equations. These equations were solved subject to initial conditions in n and boundary conditions in z by means of matrix factorization, i.e., Gauss elimination.

From the computed moments the y dependence can be calculated by means of a Gram-Charlier expansion; i.e., the distribution is assumed to be given by a Gaussian centered at $\bar{y}$, with a width, σ_y , multiplied by a sum of Hermite polynomials.

Fig. 1. Wind velocity profiles

Fig. 2. Diffusivity profiles

Figure 1 shows the velocity profiles used in an attempt to simulate unstable and neutral atmospheric conditions. The neutral profile is logarithmic. The unstable profile is log-linear at small z (with a Monin length of about 100 meters) and becomes constant at above about 100 meters. Figure 2 shows two diffusivity profiles used for each of the simulated atmospheric conditions. Two profiles were used for each condition because of uncertainty as to the form of the eddy diffusivity profile that would best simulate unstable and neutral atmospheres. The y diffusivity, K_y , was taken to be independent of z .

In Fig. 3 one can see that the σ_z near the source varies from case to case for differing source heights by the same order of magnitude as the variation commonly assumed when one goes from one stability category to another; e.g., Turner. (9) Thus, the indication here is that when one selects σ_z curves to be used in a "model" attention should be given to dependence upon elevation.

A peculiar feature of the curves in Fig. 3 is presence of discontinuities. These occur because in these particular calculations there is so much vertical diffusion that the solutions at no point exceed the maximum that would be predicted by Gaussian plume theory. For the cases with source heights equal to 10, 100 and 200 meters respectively, the maximum GHI from K theory is 25, 15 and 10 percent below that which Gaussian theory would predict.

Figure 4 exhibits a different phenomenon. Here the maximum GHI is greater than would be predicted from Gaussian theory; hence, there are regions, denoted by the horizontal dashed lines, in which there does not exist a σ_z that could allow the Gaussian plume model to match the K theory results. Here the K theory predicts maximum GHI's only about 5 percent greater than Gaussian theory would predict. One might attempt to correct this discrepancy by including in the Gaussian plume model additional image sources to account for the diffuse lid implied by the fact that the diffusivity decreases with height.

However, Fig. 5 is for the stipulated neutral condition with K_z independent of height; i.e., there is no diffusion lid. In this case, also, there are regions in which no σ_z exists. Here the K theory maximum GHI exceeds the Gaussian predicted maximum by about 25 percent. Figure 6 is similar; the K theory prediction of the maximum GHI being about 15 percent greater than the Gaussian theory prediction.

As mentioned, the foregoing results were almost independent of directional shear. Calculation of σ_y , as would be expected, was sensitive to the curving of the wind with height. Figure 7 shows the wind vectors as a function of height that was used in conjunction with neutral cases. In the case illustrated, the angle, θ , between the wind direction at 100 meters and that at ground level was 0.3° .

Fig. 7. Wind vectors at various altitudes

Fig. 8. Calculated σ_y for various amounts of directional shear

Figure 9 shows the computed σ_y as a function of the distance from the source for cases where θ is 0.1° , 0.2° , and 0.3° respectively. One can see that functional dependence of the curves depends upon the amount of directional shear. This is a factor that should be accounted for in the selection of σ_y to be used with Gaussian plume models. This result is consistent with conclusions drawn by Hoffman, (4) Smith, (13) Kennedy, (10) and Pasquill, (11)

Fig. 9

Fig. 10

Figs. 9-10. Trajectory of ground level center of mass

The solid curves on Figs. 9 and 10 show the trajectory of the ground level center of mass from an elevated source. The dotted line indicates the wind direction at source height, the ground level wind direction is along the x axis. (For clarity the y axis is expanded by a factor of 3.) The bars at right angles to the trajectory extend a distance σ_y on each side of the trajectory. At the source the trajectory follows the source height direction but quickly feels the influence of the wind between source height and the ground and deviates toward the left; as the plume progresses further downwind, the influence of the high altitude wind direction is felt and there is a turning to the right. Hence, the trajectory of the center of mass follows a curved trajectory. Connolly⁽¹⁰⁾ observed a similar effect when calculating for ground level releases. The wind direction at source height does seem to be the most reasonable choice for use with the Gaussian approximation, though we note that deviations by the order of σ_y are possible.

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CONCENTRATION FLUCTUATION AND PEAK-TO-MEAN CONCENTRATION RATIOS IN REAL PLUMES^a

J. V. Ramsdell

Atmospheric Sciences Department
Battelle, Pacific Northwest Laboratories
Richland, Washington 99352

INTRODUCTION

There are numerous instances in which time-integrated plume diffusion models are inadequate to solve current air pollution problems. In particular, the models fail when instantaneous concentration predictions are needed. Although Hilst,⁽¹⁾ Davidson⁽²⁾ and Gifford^(3,4) have attempted to model diffusion in instantaneous plumes from a more-or-less theoretical viewpoint, little experimental data is available on which to model the instantaneous plume. In the fall of 1967, however, a series of diffusion tests were conducted at Hanford which provide experimental insight into the concentration fluctuations associated with real plumes. These concentration fluctuations can be ascribed in part to the meandering of the instantaneous plume and in part to random variations of the concentration within the instantaneous plume. This paper is primarily concerned with the description of the magnitude of the fluctuations and the systematic ways in which they vary.

EXPERIMENTAL DESIGN AND DATA DESCRIPTION

During the summer and fall of 1967 a prototype diffusion grid, using ^{85}Kr as a tracer, was installed and tested at Hanford. The layout of this grid was essentially the same as that described by Barad and Fuquay⁽⁵⁾ for the Green Glow Diffusion Program, although reduced in scale. A total of 64 halogen-quenched Gieger-Mueller sensors were located on concentric arcs as shown in Figure 1, and functioned as independent sequential samplers operating in unison. The output from the samplers was accumulated over 38.4 sec sampling periods with the lost time between sampling periods of the order of a few milliseconds. The source consisted of a cylinder from which the ^{85}Kr was released at a constant rate of about 1 Ci min^{-1} at an effective height of approximately 1 m. A comprehensive description of the instrumentation has been published by Ludwick, et al.⁽⁶⁾

Four continuous tracer releases were made during periods of relatively constant atmospheric stability. The significant meteorological and diffusion parameters for these releases are summarized in Table 1. Typically, the maximum activity within the plume was of the order of 170 counts sec^{-1} at 200 m and 25 counts sec^{-1} at 800 m. Calibration of the sensors indicated that 9.2 counts sec^{-1} were approximately equal to $1 \mu\text{Ci m}^{-3}$. The complete data for the prototype tests are extensive and are contained in a separate data volume.⁽⁷⁾ The standard deviations of the crosswind distribution of tracer in the mean plume (σ_y) are included in Table 1. The estimates of the standard deviation of the vertical distribution of the tracer (σ_z) included in Table 1 were obtained using the bivariate-normal diffusion equation, the measured values of the centerline concentration, wind speed and σ_y . Complete reflection of the plume at ground level was assumed for this computation. All crosswind distances were measured from the center of mass of the concentration distribution and vertical distances were measured from the ground. Positions within each plume were normalized to these standard deviations.

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⁸⁵Kr is an artificially produced inert gas having a half-life of 10.76 years which decays by beta emission with a predominant (99%) energy of 0.69 MeV.

TABLE 1

TEST PARAMETERS

TEST	DURATION OF RELEASE*	Ri**	$\bar{U}^\dagger$	$\sigma_u^\dagger$	$\sigma_v^\dagger$	200M		800M	
						$\sigma_y^\ddagger$	$\sigma_z^\ddagger$	$\sigma_y^\ddagger$	$\sigma_z^\ddagger$
1	15+28	+0.20	1.2	0.22	0.14	20	12	67	360
2	14+15	-0.022	7.7	1.5	1.3	42	10	196	25
3	9+58	-0.31	4.1	1.1	0.93	24	35	64	120
4	19+51	+0.046	2.6	0.60	0.33	25	23	83	46

* DURATION OF RELEASE IN MIN AND SEC

** AVERAGE VALUE OF RICHARDSON NUMBER FOR THE RELEASE PERIOD COMPUTED FOR THE LAYER BETWEEN 1.5 AND 12.2 M

† WIND SPEED AND STANDARD DEVIATIONS IN m sec^{-1} BASED UPON OBSERVATIONS TAKEN AT 5 sec INTERVALS DURING THE PERIOD OF TRACER GENERATION

‡ STANDARD DEVIATIONS OF PLUME CONCENTRATION DISTRIBUTIONS IN m

Figure 1. Layout of the ⁸⁵Kr Diffusion Grid

A detailed analysis of the data from 1.5 m level samplers has recently been published.⁽⁸⁾ The results of this analysis are summarized in the next two sections. Following this summary in order are the presentation of a peak-to-mean concentration ratio model and a discussion of absolute peak-to-mean concentrations. Finally, the direction of current research at Battelle, Pacific Northwest Laboratories (PNL) is indicated.

ANALYSIS

On the average the concentration fluctuations tend to follow rather well defined patterns. The analysis and discussion to follow are intended as a description of these mean patterns. No significant differences in the concentration fluctuations could be determined between the data from the 200 and 800 m arcs. In addition, 4 plumes do not provide sufficient data from which to attempt to identify stability effects on the fluctuations. Therefore, all data were combined for analysis.

Concentration Fluctuations

One of the characteristics of diffusion in real plumes is the intermittency of the concentration at any fixed position. The frequency of occurrence of a given concentration level at a position is a function of location with respect to the mean plume axis. This relationship is clearly shown in the cumulative frequency distributions in Figure 2. In preparation of this figure the concentrations for each sensor, averaged over 38.4 sec periods, were normalized to the local mean concentration during plume passage. The normalization compensated for variations in the rate of tracer release, for variations in atmospheric conditions between tests and for diffusion between 200 and 800 m.

A highly skewed distribution of short-term concentrations is obvious in Figure 2. It is of

Figure 2. Cumulative Frequency Distribution of Normalized Short-Term Concentrations

to the smoothing of the concentration data as well as to the position within the plume. For this reason, peak-to-mean concentration ratios (P/M) are examined in relation to both the position in the mean plume and the smoothing of the data.

Maximum values of concentrations, smoothed over increasing time intervals by use of running means, were determined for each sampling position. From these maxima and the local mean concentrations, peak-to-mean ratios were calculated. Individual peak-to-mean ratios are indicated as P/M , and mean ratios by an overbar, $\overline{P/M}$. The ratio of the duration of plume passage (T) to the duration of the peak (t) was computed as a measure of the degree of smoothing of the peak and is called the time ratio (T/t). Large time ratios indicate minimal smoothing. The use of peak-to-mean ratios provides the same normalization benefits described for the short-term concentrations, and the time ratio provides a normalization for the variations in duration of releases.

The relationship between P/M and the normalized crosswind position is shown in Figure 3. The time ratios in the figure indicate the mean time ratio for the group of P/M used to derive each curve. The scatter about each of the curves in Figures 3 and 4 is indicated in Table 2 of Reference 8. At small time ratios the P/M ratios are almost independent of crosswind position. As the time ratio increases the relationship between $\overline{P/M}$ and $|y|/\sigma_y$ approaches those described by Singer⁽⁹⁾ and hypothesized by Gifford.⁽⁴⁾ An interesting feature of this figure is that P/M ratios approach a definite maximum at the edge of the plume. The relationship between $\overline{P/M}$ and T/t is better examined in Figure 4. The increase of $\overline{P/M}$ with T/t is as expected; however, it cannot be adequately described by a power law as has previously been suggested.^(4,10,11) Gifford's estimates of the power law relationship between $\overline{P/M}$ and T/t have been included in Figure 4 for comparison. It is particularly significant that the $\overline{P/M}$ observed near the center of the ^{85}Kr plumes were higher than would have been anticipated from Gifford's centerline curve. It is also interesting to note that in each case the magnitude of $\overline{P/M}$ ratios is less than or equal to the magnitude of T/t .

particular interest that short-term concentrations are less than the local mean concentration 65 percent of the time near the mean plume axis and about 85 percent of the time near the edge of the mean plume. It is assumed that the plume is absent from a position when the short term concentration is less than 1/10 the local mean concentration, then the plume is absent from the mean plume axis more than 35 percent of the time. At the edge of the mean plume this absence increases to about 80 percent.

Peak-to-Mean Concentration Ratios

Frequently, interest is expressed in the upper tails of the cumulative frequency distributions in Figure 2. These tails contain the peak-to-mean concentration ratios. The peak concentration is defined as a maximum short-term concentration at a given location. The peak concentrations are intimately related

Figure 3. Peak-to-Mean Concentration Ratio as a Function of Normalized Cross-Wind Position

Figure 4. Peak-to-Mean Concentration Ratio as a Function of the Time Ratio

A Model for Peak-to-Mean Concentration Ratio

The relationship between $\overline{P/M}$, $|\gamma/z|$, and T/t , as illustrated in Figure 3 and 4, appeared to be sufficiently well defined to justify a modeling. As the initial data analysis was not based upon a strong theoretical foundation, a strictly empirical model was attempted. The shapes of the curves in Figures 3 and 4 were used as a guide in selecting the mathematical functions to be incorporated in the model. Additional inputs were obtained from the following considerations:

1. At $T/t = 1$, $\overline{P/M}$ must equal 1 for all crosswind positions in the plume.
2. $\overline{P/M}$ ratios cannot exceed T/t in magnitude.
3. $\overline{P/M}$ ratios are a minimum at $|\gamma/z| = 0$.
4. $\overline{P/M}$ ratios approach a maximum at large $|\gamma/z|$.

The double curvature in the relationship between $\overline{P/M}$ and $|\gamma/z|$ indicated that the $\tanh^2$ of some function of $|\gamma/z|$ would be an appropriate factor in the model. Similarly, the relationship between $\overline{P/M}$ and T/t indicated that the $\ln(T/t)$ would also be an appropriate factor in the model. These factors were combined in a variety of ways in an attempt to satisfy all of the conditions. Regression coefficients were determined for each of the prospective models using the data from which Figures 3 and 4 were derived (approximately 220 points) and least squares techniques. Each model was judged by comparison of lateral and transverse cross-sections of the model with corresponding curves from Figures 3 and 4. The best $\overline{P/M}$ model was determined to be:

$$\overline{P/M} = 1.4 T^{\frac{1}{2}} E^{\frac{1}{2}} + 0.70 E^{\frac{1}{2}} + 1.0 \quad (1)$$

where

$$T^{\frac{1}{2}} = \tanh^2 \left[\frac{\gamma/z}{\frac{\gamma}{z} + \frac{z}{\gamma}} \right] \quad (2)$$

and

$$E^{\frac{1}{2}} = \ln(T/t) \quad (3)$$

Quantitative estimates of the goodness of fit of the model to the data were not reliable as the ratio of the individual $\overline{P/M}$ ratios about the $\overline{P/M}$ is not homogeneous. Figure 5 shows an isometric view of the model. The model is not entirely satisfactory, because the predicted $\overline{P/M}$ ratios

Figure 5. Isometric View of the Peak-to-Mean Concentration Ratio Model

T/t near the edge of the plume for small T/t . However, this does not appear to be a serious deficiency.

For lack of more definitive data, it is tempting to use the model for prediction of $\overline{P/M}$ ratios beneath elevated plumes. With this prospect in mind, the data from the towers were analyzed for P/M ratios. These P/M ratios were grouped by T/t and normalized position in the plume, where the normalized position is defined by

$$r/c_r = [(y/c_y)^2 + (z/c_z)^2]^{1/2} \quad (4)$$

It was then assumed that r/c_r could be substituted for $|y/c_y|$ in Equation (2).

The peak-to-mean ratios predicted by the model under this assumption are reasonably close to the observed values as shown in Figure 6. However, caution is in order as the vertical extent of the data was limited to $z/c_z \leq 1.1$. The tendency of the initial model to over predict $\overline{P/M}$ ratios at large $|y/c_y|$ and small T/t is evident in Figure 6 as the data tend to drop below the line in the middle of the P/M ratio range. But, the most important feature of Figure 6 is that in no case did the model significantly under predict a $\overline{P/M}$ ratio.

Absolute Peak-to-Mean Concentration Ratios

In many ways it seems advantageous to consider peak concentrations normalized to a common concentration rather than the local mean

Figure 6. Correlation of Predicted and Observed Peak-to-Mean Ratios

concentration. The obvious concentration for use in the normalization of peak concentrations would appear to be the average concentration at mean plume axis. To prevent confusion, peak concentrations normalized to this axial mean concentration will be referred to as absolute P/M ratios ($\overline{P/M}_a$), as opposed to conventional P/M ratios ($\overline{P/M}_L$) in which peak concentrations are normalized to the local mean concentration.

An absolute peak-to-mean concentration ratio might be thought of as the product of a conventional P/M ratio and the ratio between the local and axial mean concentrations. Thus, it is to be expected that $\overline{P/M}_a$ are a function of the same variables as $\overline{P/M}_L$. A summary of absolute peak-to-mean ratios is presented in Table 2 by relative crosswind position and time ratio. The central value in each group is the geometric mean $\overline{P/M}_a$ for the class and the upper and lower values are one geometric standard deviation removed from their respective $\overline{P/M}_a$. The range of values of T/t and y/σ_y for each group is indicated by the arithmetic mean $\pm$ one standard deviation.

TABLE 2
AVERAGE VALUES OF ABSOLUTE PEAK-TO-MEAN CONCENTRATION RATIOS AS A FUNCTION
OF THE NORMALIZED CROSSWIND POSITION AND TIME RATIO

(T/t)	$ y /\sigma_y$				
	0.21 $\pm$ 0.13	0.70 $\pm$ 0.14	1.23 $\pm$ 0.16	1.75 $\pm$ 0.16	2.31 $\pm$ 0.16
1.94 $\pm$ 0.06	1.83	2.13	1.83	0.877	0.239
	1.44	1.26	0.988	0.351	0.060
	1.13	0.743	0.535	0.140	0.015
	n=24	n=28	n=25	n=23	n=15
3.98 $\pm$ 0.22	2.85	3.48	3.12	1.66	0.474
	2.32	2.03	1.70	0.680	0.097
	1.75	1.19	0.923	0.278	0.020
	n=24	n=28	n=25	n=23	n=16
8.04 $\pm$ 0.48	3.65	4.28	3.96	2.51	0.763
	2.81	2.66	2.34	1.09	0.169
	2.16	1.65	1.39	0.475	0.038
	n=24	n=28	n=25	n=23	n=16
20.2 $\pm$ 3.9	4.33	4.95	4.59	3.70	1.02
	3.53	3.56	3.00	1.81	0.268
	2.88	2.55	1.97	0.883	0.070
	n=22	n=26	n=23	n=19	n=15

The relationship between $\overline{P/M}_a$ and normalized crosswind positions is shown in Figure 7. The most striking feature of this figure is that the maximum $\overline{P/M}_a$ ratios occur at the mean plume axis. The relatively flat portion of the curves near the center of the mean plume is the result of plume meander and its extent essentially defines the limit of meandering of the axis of the instantaneous plume. As the edge of the mean plume is approached, the rapid decrease in $\overline{P/M}_a$ is caused by the comparison of peak concentrations at the extreme edge of the meandering plume with the axial mean concentrations. A small crosswind displacement with respect to the mean plume σ_y appears as a significantly larger displacement when compared to the σ_y of the instantaneous plume. The relationship between absolute peak-to-mean concentration ratios and the time ratio is clearly shown in Figure 8. As with conventional $\overline{P/M}$, $\overline{P/M}_a$ monotonically increase with increasing T/t . Unlike conventional $\overline{P/M}$, however, the $\overline{P/M}_a$ do not start from a common point at $T/t = 1$. Rather, $\overline{P/M}_a$ appear to converge at some large value of T/t .

CURRENT AND FUTURE RESEARCH

During the past year the ^{85}Kr system has been greatly expanded. The horizontal coverage of samplers on the 200 and 800 m arc has been increased and an additional arc has been added at 1600 m. The biggest improvement, however, is in the vertical coverage. Two additional towers have been

Figure 7. Absolute Peak-to-Mean Concentration Ratios as a Function of Normalized Crosswind Position.

Figure 8. Absolute Peak-to-Mean Concentration Ratios as a Function of the Time Ratio

instrumented on each of the 200 and 800 m arcs, and the height of the top of the sampling array has been doubled on each arc. The electronics in the data collection system have also been improved so that in future tests it will be possible to examine considerably longer releases and at the same time use shorter sampling increments.

Data from future tests will be analyzed in much the same fashion as the present data, but other more fundamental types of analysis are also planned. In particular, an effort will be made to relate the diffusion directly to turbulence measurements. Analysis of the existing data indicate that at least in some plumes it is possible to identify those frequencies in the turbulence spectra which contribute to plume meander. The diffusion in the instantaneous plumes will be examined following identification and removal of plume meander. A partially completed analysis of existing data indicates that the peak-to-mean ratios in the instantaneous plumes may be only 50 percent of those reported here, and that the scatter of the individual values may also be significantly reduced.

A complete listing of the potentialities of the ^{85}Kr system is impossible, but they go well beyond those mentioned here. It is obvious that this system provides the opportunity to examine details of the diffusion process which until now have been beyond the reach of experimentalists.

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SOME APPLICATIONS OF GENERALIZED SIMILARITY ANALYSIS TO THE THEORY OF ATMOSPHERIC DIFFUSION AND EVAPORATION

Kenneth L. Calder*
Division of Meteorology, Environmental Protection Agency
Research Triangle Park, North Carolina

INTRODUCTION

In an earlier paper, the author (Calder, 1967) has shown how the method of generalized dimensional analysis proposed by Long (1963) which, for well-posed boundary-value problems, frequently provides much stronger results than those obtainable by ordinary dimensional analysis, can be replaced by a simple mathematical transformation technique that is applied to the governing differential equations and boundary conditions. Using this technique the sharper results yielded by Long's method may be obtained directly by a purely mathematical rote that avoids all reference to the concepts of dimensional analysis and the Buckingham Π -theorem.

In this paper the transformation technique is applied to some boundary-value problems that arise in the classical eddy-diffusivity approach to problems of turbulent atmospheric diffusion and evaporation**. An analysis is made of the three-dimensional steady-state concentration field that is produced by the turbulent diffusion of matter released from a continuously emitting point-source located at ground level. Also considered is the related two-dimensional steady-state problem of the concentration field from an infinite crosswind line source. Finally the problem of turbulent evaporation from a crosswind strip at the earth's surface is discussed. For all these problems it is shown that the transformation technique yields much stronger results, i.e., ones containing much more information, than are obtainable using the method of classical dimensional analysis.

CONCENTRATION FIELD OF CONTINUOUS POINT SOURCE

The problem to be considered [cf., Sutton (1953)] is the steady-state field of mean concentration $X(x, y, z)$ produced by a steady continuous point-source of diffusing material, of strength Q , and located at ground level. The coordinate system is chosen with the x -axis in the direction of the (horizontal) mean wind u , the y -axis across wind, and the z -axis vertical. The mean wind velocity is assumed to be unidirectional, steady and dependent only on the height above the ground ($z = 0$). The ground is assumed to be non-absorbent and impervious to the diffusing substance, and to be the only physical barrier to transport and diffusion. As usually considered the three-dimensional equation of turbulent diffusion is written

$$u \frac{\partial X}{\partial x} = \frac{\partial}{\partial y} \left(K_y \frac{\partial X}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_z \frac{\partial X}{\partial z} \right) \quad (1)$$

where K_y and K_z are the eddy diffusivities in the y - and z -directions, respectively, and it is also assumed that turbulent diffusion in the direction of the mean wind can be neglected. We shall assume that the wind speed and the eddy diffusivity functions can be expressed as simple powers of the height, i.e., that

$$u = U z^m, \quad K_y = \lambda_1 z^a, \quad K_z = \lambda_2 z^b \quad (2)$$

The boundary conditions for the problem are (cf., Sutton, loc. cit.)

- $X \rightarrow 0$ as $x, y, z \rightarrow \infty$
- $K_z \frac{\partial X}{\partial z} \rightarrow 0$ as $z \rightarrow 0, x > 0$ (ground impervious to matter)
- $X \rightarrow \infty$ at $x = y = z = 0$.
- The continuity condition:

$$\int_{-\infty}^{\infty} \int_0^{\infty} u X(x, y, z) dz dy = Q \text{ for all } x > 0$$

(*On assignment from the National Oceanic and Atmospheric Administration, U.S. Department of Commerce.)

**Such an application was first suggested as appropriate by the author as a result of some questions raised by Dr. F. A. Gifford, Jr., of the NOAA Atmospheric Turbulence and Diffusion Laboratory, Oak Ridge, Tennessee, concerning similarity theory and these particular atmospheric problems.

which expresses the fact that matter is neither lost nor created as the plume travels downstream. Assuming that the problem is well-posed and that a unique solution exists, then it is evident by inspection from equations (1), (2) and (3) above, that this must be a functional relationship of the form

$$f\{x; x, y, z; U, \lambda_1, \lambda_2; Q\} = 0 \tag{4}$$

where for conciseness we have not explicitly included the quantities m , α and β that are taken to be absolute numerical constants for the given problem.

To consider first the application of ordinary dimensional analysis to this problem and to equation (4), we note that in terms of the three fundamental dimensions of mass (M), length (L), and time (T), the eight quantities that are involved in the functional relationship of the equation have the dimensional matrix

	x	x	y	z	U	λ_1	λ_2	Q
M	1	0	0	0	0	0	0	1
L	-3	1	1	1	(1- α)	(2- α)	(2- β)	0
T	0	0	0	0	-1	-1	-1	-1

Now the so-called Pi-theorem of Buckingham, which is the precise mathematical basis of ordinary dimensional analysis, states that for a functional relationship (like equation (4)) that involves r variables to be "unit-free", i.e., invariant under all changes of the n fundamental units, it must be mathematically equivalent to a unique functional relationship between $(r-m)$ independent dimensionless power-products of the original variables, where m is the "rank" of the dimensional matrix. [If a matrix contains a non-zero determinant of order m , but all determinants of order greater than m in the matrix have the value zero, the matrix is said to be rank of m .] Since the functional relationship between the independent dimensionless power-products involves fewer quantities than the initial relationship, it provides more information and a more powerful result, and herein, of course, lies the primary value of dimensional analysis. In the present example it is seen that the rank of the dimensional matrix is three and hence the relationship between the eight variables of equation (4) must be expressible as a unique functional relationship between $(8-3) = 5$ independent dimensionless power-products. From the dimensional matrix above it is readily verified that the five quantities $\xi_1, \xi_2, \xi_3, \xi_4, \xi_5$ defined below are dimensionless and they are clearly independent

$$\xi_1 = \frac{xU}{Q} \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{m+2}{2-\beta}}, \quad \xi_2 = \frac{x}{z}, \quad \xi_3 = \frac{y}{z}, \quad \xi_4 = \left(\frac{\lambda_2}{U}\right)^{\frac{1}{1-\alpha-\beta}} / z, \quad \xi_5 = \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{2-\beta}} / z \tag{5}$$

It follows that the result of applying ordinary dimensional analysis to the continuous point-source problem is a unique functional relationship that may be written

$$\xi_1 = \phi\{\xi_2, \xi_3, \xi_4, \xi_5\} \tag{6}$$

We now treat the same problem by the transformation technique (Colder, loc. cit.) and first make a simple linear transformation of all the dependent and independent variables of the problem (but not the constant parameters). Thus let (the prime and overbar notation proves very convenient in applications)

$$x' = x/\bar{x}, \quad x' = x/\bar{x}, \quad y' = y/\bar{y}, \quad z' = z/\bar{z} \tag{7}$$

Then equation (1) gives

$$U(\bar{z}z')^m \frac{\bar{x}}{\bar{z}} \frac{\partial x'}{\partial x'} = \lambda_1 (\bar{z}z')^{\alpha} \frac{\bar{x}}{\bar{y}^2} \frac{\partial^2 x'}{\partial y'^2} + \frac{1}{z} \frac{\partial}{\partial z'} \left[\lambda_2 (\bar{z}z')^{\beta} \frac{\bar{x}}{\bar{z}} \frac{\partial x'}{\partial z'} \right] \tag{8}$$

Hence if

$$\frac{U\bar{z}^m}{\bar{x}} = \frac{\lambda_1 \bar{z}^{\alpha}}{\bar{y}^2} = \lambda_2 \bar{z}^{\beta-2} \tag{9}$$

the diffusion equation becomes

$$z'^m \frac{\partial x'}{\partial x'} = z'^{\alpha} \frac{\partial^2 x'}{\partial y'^2} + \frac{\partial}{\partial z'} (z'^{\beta} \frac{\partial x'}{\partial z'}) \tag{10}$$

Also the boundary conditions (3) become

$$\begin{aligned}
 (a) \quad X' &\rightarrow 0 \quad \text{as} \quad x', y', z' \rightarrow \infty \\
 (b) \quad z'^{\beta} \frac{\partial X'}{\partial z'} &\rightarrow 0 \quad \text{as} \quad z' \rightarrow 0, \quad x' > 0 \\
 (c) \quad X' &\rightarrow \infty \quad \text{at} \quad x' = y' = z' = 0 \\
 (d) \quad U \bar{y} \bar{z}^{m+1} \bar{X} \int_0^{\infty} \int_0^{\infty} z'^{-m} X' dz' dy' &= Q
 \end{aligned} \tag{11}$$

The last of the four boundary conditions becomes

$$\int_0^{\infty} \int_0^{\infty} z'^{-m} X' dz' dy' = 1$$

provided that

$$U \bar{y} \bar{z}^{m+1} \bar{X} = Q \tag{12}$$

Hence provided that equations (9) and (12) are satisfied the transformed diffusion equation and boundary conditions will only involve the three transformed spatial variables x', y', z' (we do not explicitly mention the quantities m, α, β as these are absolute numerical constants for the problem) so that

$$X' = \phi_1(x', y', z') \tag{13}$$

where ϕ_1 denotes a universal function. However, from (9) we have

$$\bar{x} = \frac{U \bar{z}^{m+2-\beta}}{\lambda_2}, \quad \bar{y} = \left[\frac{\lambda_1}{\lambda_2} \bar{z}^{\alpha+2-\beta} \right]^{\frac{1}{2}}$$

while from (12)

$$\bar{X} = \frac{Q}{U \bar{y} \bar{z}^{m+1}} = \frac{Q}{U \left(\frac{\lambda_1}{\lambda_2} \right)^{\frac{1}{2}} \bar{z}^{\frac{2m+\alpha-\beta+4}{2}}}$$

It follows from equation (13), which is equivalent to

$$\frac{\bar{X}}{\bar{x}} = \phi_1 \left\{ \frac{\bar{x}}{\bar{z}}, \frac{\bar{y}}{\bar{z}}, \frac{\bar{z}}{\bar{z}} \right\}$$

that

$$\frac{\bar{X} U \left(\frac{\lambda_1}{\lambda_2} \right)^{\frac{1}{2}} \bar{z}^{\frac{2m+\alpha-\beta+4}{2}}}{Q} = \phi_1 \left\{ \frac{\bar{x}}{\frac{U \bar{z}^{m+2-\beta}}{\lambda_2}}, \frac{\bar{y}}{\left(\frac{\lambda_1}{\lambda_2} \bar{z}^{\alpha+2-\beta} \right)^{\frac{1}{2}}}, \frac{\bar{z}}{\bar{z}} \right\} \tag{14}$$

However, this solution must not depend on $\bar{z}$, which does not appear in the original formulation of the problem. To remove the $\bar{z}$ from the left hand side of the equation we multiply it by the variable

$\bar{x} / \frac{U \bar{z}^{m+2-\beta}}{\lambda_2}$ raised to the appropriate power to obtain

$$\frac{\bar{X} U \left(\frac{\lambda_1}{\lambda_2} \right)^{\frac{1}{2}} \bar{z}^{\frac{2m+\alpha-\beta+4}{2}}}{Q} \left[\frac{\lambda_2 \bar{x}}{U \bar{z}^{m+2-\beta}} \right]^{\frac{2m+\alpha-\beta+4}{2(m+2-\beta)}}$$

and hence

$$\frac{\bar{X} U \left(\frac{\lambda_1}{\lambda_2} \right)^{\frac{1}{2}} \left(\frac{\lambda_2 \bar{x}}{U} \right)^{\frac{2m+\alpha-\beta+4}{2(m+2-\beta)}}}{Q} = \phi_1 \left\{ \frac{\lambda_2 \bar{x}}{U \bar{z}^{m+2-\beta}}, \frac{\bar{y}}{\left(\frac{\lambda_1}{\lambda_2} \bar{z}^{\alpha+2-\beta} \right)^{\frac{1}{2}}}, \frac{\bar{z}}{\bar{z}} \right\} \tag{15}$$

However, the only way in which the value of ϕ_1 can be independent of $\bar{z}$ is if it is of the special form (relative to its three variable arguments) in which it reduces to

$$\phi_2 \left\{ \frac{\lambda_2 x}{U \bar{z}^{m+\beta}} \left(\frac{\bar{z}}{z} \right)^{m+\beta}, \frac{\lambda_2 x}{U \bar{z}^{m+\beta}} \left[\left(\frac{\lambda_1}{\lambda_2} \right)^{\frac{1}{2}} \frac{\bar{z}^{m+\beta}}{y} \right]^{\frac{2(m+\beta)}{m+\beta+2}} \right\} \\ = \phi_2 \left\{ \frac{\lambda_2 x}{U \bar{z}^{m+\beta}}, \frac{\lambda_2 x \left(\frac{\lambda_1}{\lambda_2} \right)^{\frac{m+\beta}{m+\beta+2}}}{U y^{\frac{2(m+\beta)}{m+\beta+2}}} \right\}$$

Thus finally we have

$$\frac{X U \left(\frac{\lambda_1}{\lambda_2} \right)^{\frac{1}{2}} \left(\frac{\lambda_2 x}{U} \right)^{\frac{2(m+\beta)}{m+\beta+2}}}{Q \left(\frac{\lambda_1}{\lambda_2} \right)} = \phi_2 \left\{ \frac{\lambda_2 x}{U \bar{z}^{m+\beta}}, \frac{\lambda_2 x \left(\frac{\lambda_1}{\lambda_2} \right)^{\frac{m+\beta}{m+\beta+2}}}{U y^{\frac{2(m+\beta)}{m+\beta+2}}} \right\} \quad (16)$$

The transformation technique has thus resulted in a universal functional relationship between only three variables in comparison with the result obtained by ordinary dimensional analysis and expressed by equation (6) that involves a universal relationship between five variables. The result expressed by equation (16) is thus a much stronger one. In terms of the variables of equation (5) it may easily be shown that equation (16) is equivalent to

$$\xi_1 \xi_5^{\frac{1}{2}(\beta+2m+2)} \left[\xi_4 \xi_2^{\frac{1}{m+\beta}} \right]^{\frac{(m+\beta)(2m+\beta+2)}{2(m+\beta+2)}} \\ = \phi_2 \left\{ \xi_2 \xi_4^{m+\beta}, \xi_2 \xi_4^{m+\beta} \xi_5^{\frac{(m+\beta)(m+\beta+2)}{2(m+\beta+2)}} \xi_3^{\frac{2(m+\beta)}{m+\beta+2}} \right\} \quad (17)$$

A special case arises when $\alpha = m$, i.e., when the crosswind diffusivity varies as the same power of the height as the wind speed. In this case (16) becomes

$$X(x, y, z) = \frac{Q U^{\frac{m+1}{m+\beta+2} - \frac{1}{2}}}{\lambda_1^{\frac{1}{2}} \lambda_2^{\frac{m+1}{m+\beta+2}} z^{\frac{m+1}{m+\beta+2} + \frac{1}{2}}} \phi_2 \left\{ \frac{\lambda_2 x}{U \bar{z}^{m+\beta+2}}, \frac{\lambda_1 x}{U y^2} \right\} \quad (18)$$

For this special case an exact analytic solution was obtained by E. L. Davies (unpublished) in the 1930's who found

$$X(x, y, z) = \frac{M}{z^{\frac{m+1}{m+\beta+2} + \frac{1}{2}}} \exp \left\{ - \frac{U}{z} \left[\frac{y^2}{4 \lambda_1} + \frac{z^{m+\beta+2}}{(m+\beta+2) \lambda_2} \right] \right\} \quad (19)$$

where

$$M = \frac{Q U^{\frac{m+1}{m+\beta+2} - \frac{1}{2}}}{2(m+\beta+2)^{\frac{2(m+1)}{m+\beta+2} - 1} \lambda_1^{\frac{1}{2}} \lambda_2^{\frac{m+1}{m+\beta+2}} \pi^{\frac{1}{2}} \Gamma \left(\frac{m+1}{m+\beta+2} \right)}$$

The special character of this case is shown by determining the partial differential equation that is satisfied by the unknown function ϕ_2 of two variables that appears in (18). In the case where $\alpha = m$ it is possible to separate the variables so that

$$\phi_2 \left\{ \frac{\lambda_2 x}{U \bar{z}^{m+\beta+2}}, \frac{\lambda_1 x}{U y^2} \right\} = \phi_1 \left\{ \frac{\lambda_2 x}{U \bar{z}^{m+\beta+2}} \right\} \phi_3 \left\{ \frac{\lambda_1 x}{U y^2} \right\}$$

and obtain a solution by well-known straightforward methods. When $\alpha \neq m$ separation of the variables is no longer possible and no simple solution can be expected.

CONCENTRATION FIELD OF A CROSSWIND LINE SOURCE

We next consider the closely related two-dimensional problem of the steady-state field of concentration $X_2(x, z)$ from a continuous crosswind line source of strength q at ground level, and as governed by the diffusion equation

$$u \frac{\partial X_2}{\partial x} = \frac{\partial}{\partial z} \left(K_z \frac{\partial X_2}{\partial z} \right) \quad (20)$$

with the boundary conditions

$$\begin{aligned} (a) \quad X_2 &\rightarrow 0 \quad \text{as} \quad x, z \rightarrow \infty \\ (b) \quad K_z \frac{\partial X_2}{\partial z} &\rightarrow 0 \quad \text{as} \quad z \rightarrow 0, x > 0 \\ (c) \quad X_2 &\rightarrow \infty \quad \text{at} \quad x = z = 0 \\ (d) \quad \int_0^\infty u X_2(x, z) dz &= q \quad \text{for all } x \end{aligned} \quad (21)$$

With wind and diffusivity profiles as in equation (2) (but now omitting consideration of K_y) the above equations for the line source require a functional relationship of the form

$$f(X_2; x, z; U, \lambda_2, q) = 0 \quad (22)$$

between six variables. By a straightforward application of ordinary dimensional analysis that exactly parallels the analysis already made for the point-source problem, we find that this can be reduced to a unique functional relationship between $(6-3) = 3$ independent dimensionless power-products of the original variables and readily verify that such a relationship is given by

$$\xi_1 = f(\xi_2, \xi_3) \quad (23)$$

where

$$\xi_1 = \frac{X_2 U z^{m+2}}{q}, \quad \xi_2 = \frac{x}{z}, \quad \xi_3 = \left(\frac{\lambda_2}{U}\right)^{\frac{m+2-\beta}{m+2-\beta}} / z \quad (24)$$

The line-source problem may also be analyzed from the beginning by the transformation technique in exactly analogous fashion to the point source. However, the required result can be simply obtained directly from equation (16) by noting that if X is the point source solution and we put $X' = \int X dy$ then X' satisfies all the line source conditions above, provided we replace the point-source strength Q by q .

In equation (16) let

$$U y^{\frac{2(m+2-\beta)}{m+2-\beta}} / \lambda_2 z \left(\frac{\lambda_2}{U}\right)^{\frac{m+2-\beta}{m+2-\beta}} = 0$$

so that

$$dy = \left(\frac{\lambda_2 z}{U}\right)^{\frac{m+2-\beta}{2(m+2-\beta)}} \left(\frac{\lambda_2}{U}\right)^{\frac{1}{2}} \theta^{\frac{m+2-\beta}{2(m+2-\beta)}} d\theta$$

Then, on integrating (16) with respect to y over the infinite range, we obtain

$$\frac{X_2 U \left(\frac{\lambda_2 z}{U}\right)^{\frac{2(m+2-\beta)+1}{2(m+2-\beta)}}}{q} = \left(\frac{\lambda_2 z}{U}\right)^{\frac{m+2-\beta}{2(m+2-\beta)}} \int_{-\infty}^{\infty} \theta^{\frac{m+2-\beta}{2(m+2-\beta)}} f\left\{\frac{\lambda_2 z}{U z^{m+2-\beta}}, \frac{1}{\theta}\right\} d\theta$$

which is equivalent to

$$X_2 = \frac{q U}{\lambda_2^{\frac{m+1}{m+2-\beta}} x^{\frac{m+1}{m+2-\beta}}} \psi \left\{ \frac{\lambda_2 x}{U z^{\frac{m+1}{m+2-\beta}}} \right\} \quad (25)$$

where ψ is a universal function of its single argument. Thus at $z = 0$, X_2 is known apart from a multiplicative constant. Equation (25) is the line-source result based on utilization of the transformation technique. It is a much stronger result than the one obtained by ordinary dimensional analysis and expressed by equation (23) as a relationship between three variables. In terms of the variables of equation (24) it may be shown that equation (25) is equivalent to

$$\xi_1 \left(\xi_2 \xi_3^{\frac{m+1}{m+2-\beta}} \right)^{\frac{m+2}{m+2-\beta}} = \psi \left\{ \xi_2 \xi_3^{\frac{m+1}{m+2-\beta}} \right\} \quad (26)$$

Determination of the form of the function ψ above requires more complete analysis. This is simple and provided in the Appendix. The final result is the exact analytic solution first obtained by O.F.T. Roberts (unpublished paper). This is

$$X_2(x, z) = \frac{q (m+2-\beta)}{\Gamma \left(\frac{m+1}{m+2-\beta} \right)} \left[\frac{U}{(m+2-\beta) \lambda_2 x} \right]^{\frac{m+1}{m+2-\beta}} \exp \left\{ - \frac{U z^{\frac{m+1}{m+2-\beta}}}{(m+2-\beta)^2 \lambda_2 x} \right\} \quad (27)$$

provided that $(m+2-\beta) \geq 0$

EVAPORATION FROM A CROSSWIND STRIP

As a final example, we consider the two-dimensional problem of evaporation from a saturated plane strip extending indefinitely across wind and of finite length downwind. This is the problem considered in the classical evaporation investigations of Sutton (loc. cit.). The governing diffusion equation is, of course, still equation (20), while for initially clean air blowing over a saturated strip $0 \leq x \leq x_0$, followed by a surface impermeable to vapor, the appropriate boundary conditions are

$$\begin{aligned} (a) & X \rightarrow 0 \quad \text{as } x \rightarrow 0 \quad (z > 0) \\ (b) & X \rightarrow X_0 \text{ (const.) as } z \rightarrow 0 \quad (0 \leq x \leq x_0) \\ (c) & K_2 \frac{\partial X}{\partial z} \rightarrow 0 \quad \text{as } z \rightarrow 0 \quad (x > x_0) \\ (d) & X \rightarrow 0 \quad \text{as } z \rightarrow \infty \end{aligned} \quad (28)$$

With wind and diffusivity profiles as in equation (2) (and now for convenience writing λ instead of λ_2) the above equations for the evaporation problem require a functional relationship of the form

$$f \{ X, X_0; x, z; x_0, U, \lambda \} = 0 \quad (29)$$

between seven variables. Through ordinary dimensional analysis, this may be reduced to a unique functional relationship of the four independent dimensionless power-products of the original variables that may be written

$$X = X_0 \psi \left\{ \frac{x}{x_0}, \frac{z}{x_0}, z / \left(\frac{\lambda x_0}{U} \right)^{\frac{1}{m+2-\beta}} \right\} \quad (30)$$

In the event that $x \leq x_0$, the concentration distribution will evidently be the same as for an infinite saturated strip ($x_0 = \infty$), so that X will not depend on the actual value of x_0 . From equation (30) and noting how the variables involve x_0 , this can only be the case if the function ψ is of the special form that reduces to a function of two variables, i.e.,

$$X = X_0 \psi_1 \left\{ \frac{x}{x_0}, z / \left(\frac{\lambda x_0}{U} \right)^{\frac{1}{m+2-\beta}} \right\} \quad (31)$$

where ψ_1 is another universal function.

To apply now the transformation technique to this problem we transform the variables by equation (7). The diffusion equation then gives

$$U (\bar{z}_2)^m \bar{\lambda} \frac{\partial \bar{X}}{\partial \bar{z}_2} = \frac{1}{2} \frac{\partial}{\partial \bar{z}_2} \left\{ \lambda (\bar{z}_2)^{\beta} \bar{\lambda} \frac{\partial \bar{X}}{\partial \bar{z}_2} \right\} \quad (32)$$

so that if

$$\frac{U \bar{z}^m}{\bar{z}} = \lambda \bar{z}^{\beta-2} \quad (33)$$

then

$$\bar{z}^m \frac{\partial \chi}{\partial z'} = \frac{\partial}{\partial z'} \left\{ \bar{z}^{\beta} \frac{\partial \chi}{\partial z'} \right\} \quad (34)$$

Also provided that

$$\bar{z} = z_0 \quad \text{and} \quad \bar{\chi} = \chi_0 \quad (35)$$

the boundary conditions (28) become

$$\begin{aligned} (a) \quad \chi' &\rightarrow 0 & \text{as} \quad z' \rightarrow 0 \quad (z' > 0) \\ (b) \quad \chi' &\rightarrow 1 & \text{as} \quad z' \rightarrow 0 \quad (0 \leq z' \leq 1) \\ (c) \quad z'^{\beta} \frac{\partial \chi'}{\partial z'} &\rightarrow 0 & \text{as} \quad z' \rightarrow 0 \quad (z' > 1) \\ (d) \quad \chi' &\rightarrow 0 & \text{as} \quad z' \rightarrow \infty \end{aligned} \quad (36)$$

Since the transformed governing equation (34) and the boundary conditions (36) are free of arbitrary parameters (as before α and β are absolute numerical constants) it follows that the solution of the problem must be of the form

$$\chi' = \phi \left\{ z', z' \right\} \quad (37)$$

where ϕ denotes a universal function. Hence, reverting to the original variables,

$$\frac{\chi}{\chi_0} = \phi \left\{ \frac{z}{z_0}, \frac{z}{z_0} \right\}$$

or

$$\chi = \chi_0 \phi \left\{ \frac{z}{z_0}, \frac{z}{(\lambda z_0 / U)^{\frac{1}{\beta-2}}} \right\} \quad (38)$$

Again we see that, although we are still concerned with two space-like variables, this is a stronger result than that obtained by ordinary dimensional analysis and given by equation (33). In the special case that $z \leq z_0$, i.e., when there is no characteristic length z_0 for the problem, still further reduction is possible since z_0 must not occur in the solution. This can only be the case if ϕ is of the special form in which it reduces to a function of just a single variable of the form

$$\chi = \chi_0 \phi_1 \left\{ \frac{z}{(\lambda z_0 / U)^{\frac{1}{\beta-2}}} \right\} \quad (39)$$

where $\phi_1(0) = 1$.

The total rate of evaporation E from the strip of width z_0 is

$$E = \int_0^{z_0} u(z) \chi(z, z) dz = \int_0^{z_0} U z^{-\alpha} \chi_0 \phi_1 \left\{ \frac{z}{(\lambda z_0 / U)^{\frac{1}{\beta-2}}} \right\} dz$$

or with the change of variable

$$\theta = z / (\lambda z_0 / U)^{\frac{1}{\beta-2}}$$

$$E = U \chi_0 (\lambda z_0 / U)^{\frac{\alpha+1}{\beta-2}} \int_0^{\infty} \theta^{-\alpha} \phi_1(\theta) d\theta = C \chi_0 (\lambda z_0)^{\frac{\alpha+1}{\beta-2}} U^{\frac{1-\alpha}{\beta-2}} \quad (40)$$

where C is a constant.

It is of interest to note that for the problem of total rate of evaporation, and starting from a functional relationship of the form

$$f \{ E, \chi_0, z_0, U, \lambda \} = 0$$

between five variables, ordinary dimensional analysis would reduce the problem to one involving two independent dimensionless group-products of three variables and thus expressible explicitly through a universal function of a single variable. By using the transformation technique as above we determine the form of this function to within an unknown multiplicative constant.

Actually the result given by equation (40) above is more general than the classical result obtained by Sutton, who makes use of the so-called Schmidt conjugate power law relation that requires $\beta = i-m$. In this case

$$E = C x_0 (\lambda x_0)^{\frac{m+1}{m-1}} z^{\frac{m}{2m+1}} \quad (41)$$

Following Sutton with $m = \tau/2 - \nu$ we obtain

$$E = C x_0 (\lambda x_0)^{\frac{2}{2+\tau}} U^{\frac{\tau}{2+\tau}} \quad (42)$$

Although the Sutton result is stated as $E \sim U^{\frac{2+\tau}{2+\tau}}$, this is because Sutton takes $\lambda \sim U^{1-\tau}$

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APPENDIX

A Derivation of Roberts' Line Source Formula

The identification of the similarity variables appropriate to the line source problem as given by equation (25) of the main paper, readily permits complete analytic solution of the problem in an elementary fashion. For let us write

$$\chi(x, z) = x^{\omega} F(\theta) \quad \text{where} \quad \theta = z^{m+2-\beta}/x \quad (1)$$

The two-dimensional diffusion equation (2) gives

$$\begin{aligned} U z^m \frac{\partial \chi}{\partial x} &= \frac{\partial}{\partial z} \left\{ \lambda_2 z^{\beta} \frac{\partial \chi}{\partial z} \right\} = \frac{\partial}{\partial z} \left\{ \lambda_2 z^{\beta-1} \left(z \frac{\partial \chi}{\partial z} \right) \right\} \\ &= \lambda_2 (\beta-1) z^{\beta-2} \left(z \frac{\partial \chi}{\partial z} \right) + \lambda_2 z^{\beta-1} \frac{\partial}{\partial z} \left(z \frac{\partial \chi}{\partial z} \right) \end{aligned} \quad (2)$$

The

$$\text{L.H.S.} = U z^m x^{\omega-1} \left[\omega F - \theta \frac{dF}{d\theta} \right]$$

and

$$\text{R.H.S.} = \lambda_2 x^{\omega} z^{\beta-2} (m+2-\beta) \left[(\beta-1) \theta \frac{dF}{d\theta} + (m+2-\beta) \theta \frac{d}{d\theta} \left(\theta \frac{dF}{d\theta} \right) \right]$$

Hence

$$\frac{U}{\lambda_2 (m+2-\beta)} \left[\omega F - \theta \frac{dF}{d\theta} \right] = \left[(\beta-1) \frac{dF}{d\theta} + (m+2-\beta) \frac{d}{d\theta} \left(\theta \frac{dF}{d\theta} \right) \right] \quad (3)$$

so that x no longer appears in the equation.

We try as a solution

$$F = A e^{-B\theta} \quad (4)$$

obtaining by substitution

$$\begin{aligned} &\frac{U}{\lambda_2 (m+2-\beta)} \left[\omega A e^{-B\theta} + \theta A B e^{-B\theta} \right] \\ &= \left[-(\beta-1) A B e^{-B\theta} - (m+2-\beta) A B e^{-B\theta} + (m+2-\beta) A B^2 e^{-B\theta} \theta \right] \end{aligned} \quad (5)$$

The terms in θ disappear if

$$\frac{UB}{\lambda_2 (m+2-\beta)} = (m+2-\beta) B^2$$

or

$$B = \frac{U}{\lambda_2 (m+2-\beta)^2}$$

The remaining terms in equation (5) then give

$$a = - \frac{(m+1)}{(m+2-\beta)}$$

in agreement with equation (25) of main paper. The constant A in equation (4) may be determined from the condition

$$q = \int_0^{\infty} u X(x, z) dz$$

The final result is the line source solution of Roberts as given by equation (27) of main paper.

PREDICTION OF LATERAL DIFFUSION BY
TURBULENT THEORY*

Yasuo Ide, Nagasaki Technical Institute of
Mitsubishi Heavy Industries, Ltd.
Nagasaki-shi, Japan

Abstract - Prediction of lateral diffusion is one of very important factors in air pollution problem as well as diffusion studies, and its prediction method is established by applying Inoue's turbulent theory (1952) to the author's quasi-steady diffusion theory (1971).

It is shown that the main predictor for lateral diffusion is either energy dissipation rate or Eulerian turbulent energy. Experimental verification of this method is also discussed, making use of such diffusion data reported by Fuquay et al. (1964), Peterson (1968) and Slade (1968).

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1. Introduction

It is well known that turbulent diffusion from a continuous source in the atmosphere is in the transient state for a finite observation time and in the ultimate state for an infinite observation time. A plume in the ultimate state and in the transient state is respectively called a steady plume and a quasi-steady plume. Most of practical plumes we have to study in air pollution problem, for example, are quasi-steady plumes in the lateral direction since turbulence of large scale very often exists in the usual horizontal atmosphere except at very stable meteorological condition. Hence lateral diffusion of a quasi-steady plume should be made clear in the terms of an finite observation time and a travelling time as well as its related properties of turbulence. However we have now few methods to predict lateral diffusion on this point although much studies in this field, theoretical and experimental, have been done in the last two decades. This paper describes the method to predict lateral diffusion of a quasi-steady plume, which is formulated by applying Inoue's turbulent theory to the author's quasi-steady diffusion theory.

2. Prediction of Lateral Diffusion

Let us observe lateral spread of a quasi-steady plume at the travelling time scale T in a fixed frame during an observation time s . Then the variance of the plume $\sigma^2(T, s)$ can be given by⁽¹⁾

$$\sigma^2(T, s) = 2 \overline{v^2} \int_0^T (T - \frac{1}{2}) R(\frac{1}{2}, T + s) d\frac{1}{2} \quad (1)$$

where $\overline{v^2}$ is Lagrangian turbulent energy of sufficiently long sampling, and $R(\xi, t)$ is Lagrangian relative function dependent on Lagrangian correlation function $R(\xi)$ as follows:

$$R(\xi, t) = R(\xi) - \frac{1}{t^2} \int_0^t (t-\eta) \{R(\eta+\xi) + R(\eta-\xi)\} d\eta. \quad (2)$$

If we observe a quasi-steady plume for sufficiently long interval, we can obtain a steady plume so that Eq. (1) should be naturally in agreement with the wellknown Taylor's variance formula representing a steady plume. In case of a small travelling time, Eqs. (1) and (2) yield

$$\sigma^2(\tau, s) = \overline{v_s^2} T^2 \quad (3)$$

since $R(\xi) \approx 1$ for small ξ and

$$\overline{v_s^2} \left\{ 1 - \frac{2}{s^2} \int_0^s (s-\eta) R(\eta) d\eta \right\} = \overline{v_s^2} \quad (4)$$

where $\overline{v_s^2}$ is Lagrangian turbulent energy sampled over s-interval.

In case of a large travelling time. Eqs (1) and (2) yield

$$\sigma^2(\tau, s) \doteq 2 \overline{v^2} T \int_0^\infty R(\xi) d\xi = 2 \overline{v^2} T T_L \quad (5)$$

where T_L is Lagrangian integral time.

Hence, we can know, from Eqs. (3) and (5), that a quasi-steady plume should spread in proportion to T near a continuous source and to $\sqrt{T}$ very far downwind as expected from Taylor's theory. However the effect of turbulent energy on quasi-steady diffusion must be remarked as in

Eq. (3), not $\overline{v^2}$ but $\overline{v_s^2}$.

On the other hand Inoue obtained the following remarkable results from his turbulon theory,⁽²⁾

For Lagrangian correlation function

$$R(\xi) = 1 - \xi/\tau_0, \quad (\tau_\infty \ll \xi \ll \tau_0) \quad (6)$$

and for the relation among energy dissipation rate ϵ , turbulent energy and sampling time, he obtained

$$\epsilon \propto \overline{v^2}/\tau_0 \propto \overline{v_s^2}/s \quad (7)$$

where τ_0 : life time of the largest turbulon

τ_∞ : life time of the smallest turbulon.

Furthermore he obtained the sampling time effect upon Eulerian turbulent energy expressed as in

$$\overline{u^2}_{\tau_0} = \left(\frac{\tau_0}{\tau}\right)^m \overline{u^2}_{\tau} \quad (8)$$

and evaluated the m-value ($m=2/3$).

Priestley and Sheppard,⁽³⁾ and Corrsin⁽⁴⁾ also obtained the following Lagrangian correlation function which is quite similar to Eq. (6).

$$R(\xi) = 1 - k\epsilon\xi/\overline{v^2}, \quad (\sqrt{\nu/\epsilon} \ll \xi \ll T_L) \quad (9)$$

and estimated a constant k equal to about 0.5.

Comparing Eq. (6) with Eq. (9) we obtain the proportional constant for Eq. (7)

$$\varepsilon = \frac{1}{R} \frac{\overline{v^2}}{\tau_0} \quad (10)$$

(a) ε - predictor for lateral diffusion

We assume very large T_L in the atmospheric turbulence so that Eqs. (6) and (9) are valid. In reality T_L is very large in the usual meteorology except at very stable condition. Then we obtain the variance $\sigma^2(T, S)$, putting Eq. (6) into Eqs. (1) and (2)

$$\sigma^2(T, S) = \frac{\overline{v^2} S T^2}{3 \tau_0} \left\{ 1 + \frac{2T}{SS} \frac{(1 + \frac{SS}{2T})}{(1 + \frac{SS}{T})^2} \right\} \quad (11)$$

$(T + S \ll T_L)$

Using Eq. (10) we can rewrite Eq. (11)

$$\sigma^2(T, S) = \frac{A\varepsilon}{3} S T^2 g\left(\frac{T}{S}\right) \quad (12)$$

where

$$g(\eta) = 1 + \frac{2}{5} \frac{\eta^2(\eta + 5/4)}{(\eta + 1)^2}, \quad \eta = \frac{T}{S}. \quad (13)$$

Eq. (12) predicts lateral diffusion if ε and a proportional constant k are provided, and yields the approximate relation with respect to a travelling time:

$$\sigma^2(T, S) \propto \begin{cases} T^2, & T \leq S (\ll T_L) \\ T^3, & T > S (\ll T_L) \end{cases} \quad (14)$$

Thus a quasi-steady plume is in the linear zone ($\sigma \propto T$) near a source and in the cuspidal zone ($\sigma \propto T^{1.5}$) at far downwind. (5)

(b) Eulerian energy predictor for lateral diffusion

Eqs. (7) and (12) yield

$$\sigma^2(\tau, s) = k_1 \overline{v_s^2} T^2 g\left(\frac{\tau}{s}\right) \quad (15)$$

where k_1 is a universal constant.

A constant k_1 can be so determined that Eq. (15) agrees with Eq. (3) at small T , and consequently k_1 becomes unity. Usually it seems more difficult to measure Lagrangian energy $\overline{v_s^2}$ rather than Eulerian energy $\overline{u_\tau^2}$. We have

$$\overline{v_s^2} = \overline{u_{s/\beta}^2} \quad (16)$$

for the relation of these energies.⁽⁶⁾ β is the so-called time scale ratio, i.e. the ratio of Lagrangian time to Eulerian time and depends on s and the properties of turbulence. If Eulerian properties of turbulence is provided, β can be determined as function of s after β -equation is numerically solved.

Putting Eq. (16) into (15)

$$\sigma^2(\tau, s) = \overline{u_{s/\beta}^2} T^2 g\left(\frac{\tau}{s}\right) \quad (17)$$

This equation indicates that a predictor of Eulerian turbulent energy should be obtained through sampling over s/β - interval. However $\overline{u_{s/\beta}^2}$ is not usually computed at the field experiments everdone but $\overline{u_\tau^2}$, and τ is often selected equal to the period of emission, for example.

In order to convert $\overline{u_\tau^2}$ to $\overline{u_{s/\beta}^2}$, we make use of Eq. (8) and obtain

from Eq. (17)

$$\sigma^2(\tau, s) = \overline{u_\tau^2} \left(\frac{s}{\tau \beta(s)} \right)^m \tau^2 g\left(\frac{\tau}{s}\right) \quad (18)$$

If standard deviation of the wind direction distribution sampled over τ -interval, $\sigma_{A,\tau}$, is given, Eq. (18) can be rewritten as

$$\sigma^2(x, s) = \sigma_{A,\tau}^2 \left(\frac{s}{\tau \beta(s)} \right)^m x^2 g\left(\frac{x}{\sigma s}\right) \quad (19)$$

with the aid of the following practical relations:

$$\overline{u_\tau^2} = \sigma^2 \sigma_{A,\tau}^2, \quad x = \sigma \tau$$

where σ is mean wind speed and x is downwind distance.

3. Experimental Discussion

In order to discuss the experimental verification of the prediction method mentioned above, the experimental data of diffusion reported by Fuquay et al. (1964), Peterson (1968) and Slade (1968) are employed. Fuquay, Simpson and Hinds (1964)⁽⁷⁾

Fuquay et al. carried out 46 Hanford ground source diffusion experiments from a continuous point source of which release times ranged from 20 to 75 minutes, and they presented the data of lateral plume spread (standard deviation) as function of a travelling time in addition to such the meteorological data as Richardson number, the mean wind speed, the computed standard deviation of the wind direction distribution for the emission period ($\tau = s$) and others. Thus this data seems just suitable to discuss the Eulerian energy predictor. Figs. 1 - 3 shows the comparison and agreement of Eq. (18) for release times of 30, 60 and

75 min. respectively in which the time scale ratios are assumed from unity to 10. From these figures and the similarly compared figures for other release times already shown at the Symposium, it may be concluded that Eq. (18) is practically useful for predicting lateral diffusion if β is appropriately selected.

Peterson (1968)⁽⁸⁾

Peterson reported the analysis of data obtained from an aircraft measurement experiment in which argon-41 gamma radiation intensities were observed as far as 160 n.mi. downwind from the 350-ft stack of the air-cooled nuclear reactor at Brookhaven National Laboratory located on central Long Island. There is a 2 - 3 hr difference between observations taken close to the source vs. those observation farthest downwind, and it took about 10 min. to observe the cross section of the plume ($s \approx 10$ min). He concluded from Fig. 4 that the lateral spread appeared to fit an extension of the Pasquill-Gifford "D" curve with distance where G_y (n.mi.) was approximately equal to two thirds of a travelling time in hours. His data travelling time up to 10 - 12 hr is the most suitable for discussion of the ϵ - predictor although ϵ was not measured in his experiment, unfortunately. Fig. 4 shows the comparison of Eq. (12), indicated by solid lines, in which the ϵ -values of 0.25 to 2 cm²/sec³ are assumed, referring to Lettau's ϵ -data⁽⁹⁾ and k is taken 0.5, following Priestley and Corrsin. Although Peterson arrived at his conclusion through his regression line (dotted line in Fig. 4), it may be more reasonable to conclude that the author's prediction is in good agreement with the data except only the nearest one, and that observed diffusion in this zone ($T \gg s$) should be in the cuspidal zone just as quasi-steady diffusion theory predicts.

Slade (1968)⁽¹⁰⁾

Slade presented a summary of lateral diffusion data for most of the experiments ever done. The σ_y values for each test in each series were divided by the appropriate σ_θ ($=\sigma_A$) values, and an average point for each series was constructed as shown in Fig. 5.

Sampling times range from 10 to 60 min ($s=10 \sim 60$ min), and both surface and elevated sources are considered. Thus his data seems suitable to compare with the Eulerian energy predictor. Eq. (19) is plotted by solid lines in the same figure in which the following values are assumed on the average over all experiments: wind speed $U = 6$ m/s, $\tau = s - 30$ min and β is 1 to 10. Fig. 5 shows that all diffusion in question may be in the linear zone as the quasi-steady diffusion theory predicts and that Eq. (19) is useful for prediction if parameters in the equation are appropriately selected.

4. Conclusion

Prediction method of lateral diffusion of a quasi-steady plume is established and shown to be available for practical prediction, and either the energy dissipation rate or Eulerian turbulent energy are discussed to be main appropriate predictors.

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Figure 1. Comparison of lateral diffusion between the prediction and experiments. A release time is 30 min

Figure 2. Comparison of lateral diffusion between the prediction and experiments. A release time is 60 min

Figure 4. Comparison of lateral diffusion between the prediction and experiments. An observation time is about 10 min

Figure 3. Comparison of lateral diffusion between the prediction and experiments. A release time is 75 min

Figure 5. Summarized lateral diffusion, both surface and elevated sources

THE APPLICATION OF A GENERALIZED LAGRANGIAN DIFFUSION MODEL TO AIR POLLUTION SIMULATION STUDIES

Robert G. Lamb*
Department of Meteorology
University of California
Los Angeles, California

INTRODUCTION

The techniques for describing the statistical properties of the concentrations of marked particles, such as air pollutants, in a turbulent fluid can be divided into two broad categories--Lagrangian and Eulerian. The Eulerian methods attempt to formulate the concentration statistics in terms of the statistical properties of the Eulerian fluid velocities, i.e. the velocities measured at fixed points in the fluid. A formulation of this type would be very useful in air pollution modeling not only because the Eulerian statistics are readily measurable--they could be determined from continuous time recordings of the wind velocities monitored by a fixed network of anemometers--but also because the mathematical expressions are directly applicable to problems in which a number of species of chemically interacting particles are present, such as the problem of simulating photochemical smog. Unfortunately, the Eulerian approaches lead to a serious mathematical obstacle known as the closure problem for which no generally valid solution has yet been found. A number of approximate solutions have been proposed but each leads to an equation for the concentration statistics which is very cumbersome to use and which gives accurate results for only a limited class of problem. Consequently, none of these equations is in current use in atmospheric diffusion studies.

By contrast, the Lagrangian techniques attempt to describe the concentration statistics in terms of the statistical properties of the displacements of groups of particles released in the fluid. The mathematics of this approach are much more tractable than those of the Eulerian methods--no closure problem is encountered--but the applicability of the resulting equations is somewhat limited due to the difficulty of accurately determining the required particle statistics. Moreover, the equations are not directly applicable to problems involving non-linear chemistry, such as the simulation of photochemical smog. Nevertheless, all of the equations in current use in atmospheric diffusion studies are of the Lagrangian type. Even the classical diffusion equation which is usually expressed in terms of an Eulerian coordinate system is based on Lagrangian, i.e. particle oriented, statistics. Recently the present author has shown how the complete statistical properties of the concentrations and the displacements of the marked particles are related.^{(5) (6)} Regarding the mean concentration it was found that

$$\begin{aligned} \langle c(\mathbf{R}, t) \rangle = & \int_{-\infty}^{\infty} Q(\mathbf{R} - \mathbf{R}', t - t_0) \langle c(\mathbf{R}', t_0) \rangle d\mathbf{v}(\mathbf{R}') \\ & + \int_{t_0}^t \int_{-\infty}^{\infty} Q(\mathbf{R} - \mathbf{R}', t - t') S(\mathbf{R}', t') d\mathbf{v}(\mathbf{R}') dt' \end{aligned} \quad (1)$$

where $\langle c(\mathbf{R}, t) \rangle$ is the ensemble mean of the concentration within a small volume element centered at the point $\mathbf{R}$ at time t , i.e. the mean "point" concentration; $S(\mathbf{R}, t)$ is the strength of the source of marked particles at the point $\mathbf{R}$ at time t ; and $Q(\mathbf{R} - \mathbf{R}', t - t') d\mathbf{v}(\mathbf{R}')$ is the probability that a marked particle released at time t' from the small volume $d\mathbf{v}(\mathbf{R}')$ centered at $\mathbf{R}'$ will be found at the point $\mathbf{R}$ at the later time t . The purpose of this paper is to modify this equation so that it can handle problems involving certain types of chemical reactions and to consider the application of the resulting equation to actual atmospheric diffusion studies.

THE MEAN CONCENTRATION OF LINEARLY REACTIVE PARTICLES

A chemical reaction which satisfies the rate equation

$$\frac{d\langle c \rangle}{dt} = A \langle c \rangle, \quad A \neq f(\langle c \rangle) \quad (2)$$

is called a linear reaction. The photolysis of SO_2 and the decay of radioactive substances are examples of linear processes. An important feature of this type of chemical reaction is that the marked particles do not interact with one another. For this reason the effects of the reaction on the mean concentration $\langle c(\mathbf{R}, t) \rangle$ can be accounted for rather easily in equation (1) by modifying the probability density function Q . That is, in the presence of

*Present affiliation: Department of Chemical Engineering, California Institute of Technology, Pasadena, California

linear chemical reaction, the probability of finding a given marked particle at some point (R, t) given that it was at (R', t') , $t' \leq t$, is just the probability that two events will occur, namely (1) that the particle undergoes a displacement $R - R'$ in the time $t - t'$, and (2) that the particle does not lose its initial marking through chemical processes during the time interval $t - t'$. These two events are independent—it is assumed that the marked particles in no way perturb the fluid motions—so the probability that both will occur is just the product of their individual probabilities. The probability of the indicated displacement is just $Q(R - R', t - t') dv(R')$. The probability of a chemical change during $t - t'$ is obtained as follows:

The probability that a particle will not undergo a chemical change during the small time interval dt is $1 - A dt$. (3)

To determine the probability of no change during the arbitrary interval $t - t'$, we can subdivide $t - t'$ into a set of small subintervals dt'' , to which (3) is applicable, and treat the problem as a discrete random process. However, due to the fact that A is not necessarily constant in time, we must first transform the time coordinate t into the nondimensional time frame $\xi(t)$ defined by

$$\xi(t) = \int_{t'}^t A(t'') dt''.$$

Since $d\xi = A(t)dt$, the probability P that the particle will change in the time interval $d\xi$ is equivalent to the length of the interval, i.e. $P = d\xi$. Now divide the period $\xi(t) - \xi(t')$ into N equal subintervals of length $d\xi$, i.e.

$$d\xi = \frac{\xi(t) - \xi(t')}{N}.$$

The probability that the particle will not decay in any of these N intervals is clearly $(1 - d\xi)^N$.

But

$$(1 - d\xi)^N = 1 - Nd\xi + \frac{N(N-1)}{2!} d\xi^2 - \dots$$

and on passing to the limit as $d\xi \rightarrow 0$ we obtain

$$\lim_{d\xi \rightarrow 0} (1 - d\xi)^N = \lim_{d\xi \rightarrow 0} \left[1 - (Nd\xi) + (Nd\xi)^2/2! - \dots \right] = e^{-\xi(t)}.$$

We arrive finally at the conclusion that the probability that the particle will not change chemically in the interval $t - t'$ is

$$\exp\left(-\int_{t'}^t A(t'') dt''\right).$$

With this result we conclude that when the marked particles have a chemistry consistent with (2), the mean concentration is given by

$$\begin{aligned} \langle C(R, t) \rangle &= \int_{-\infty}^{\infty} Q(R - R', t - t_0) \exp\left(-\int_{t_0}^t A(t') dt'\right) \langle C(R', t_0) \rangle dv(R') \\ &+ \int_{-\infty}^{\infty} \int_{t_0}^t Q(R - R', t - t') \exp\left(-\int_{t'}^t A(t'') dt''\right) S(R', t') dt' dv(R'). \end{aligned} \quad (4)$$

APPLICATION OF EQUATION (4) TO GENERAL DIFFUSION STUDIES

The only inherent conditions limiting the applicability of equation (4) are (a) that the marked particles do not affect the fluid motions and (b) that any chemical reactions satisfy equation (2). However, as was mentioned in the introduction the difficulty of obtaining accurate approximations of Q may also impose restrictions on the utility of equation (4). This is especially true in the case of air pollution studies because near the ground, where the turbulence is inhomogeneous, nonstationary, and anisotropic, Q is a function not only of the meteorological conditions, terrain, etc. but also of the four variables R, R', t and t' . These conditions virtually preclude a determination of Q by field experiments. Consequently, one must turn to fluid simulation models to derive estimates of the probability density function Q . We shall demonstrate the technique in this section.

Deardorff⁽²⁾ has developed a planetary boundary layer model which he has used to conduct numerous experiments. Among these is a recent study⁽³⁾ of the dispersion of fluid particles in a three dimensional turbulent flow which roughly approximates atmospheric flow below an inversion base of constant height h . In each of two experiments, representing neutral ($h/L = 0$) and slightly unstable ($h/L = -4.5$) atmospheric conditions, where L is the Monin-Obukhov length, eight hundred particles were released at the same instant from points on a horizontal plane and followed for a given period of time. Using the computed trajectories of each particle, the present author calculated the two dimensional density function $Q(x - x', z - z', t - t')$ in the following way:

First the coordinate system of each particle was transformed to a moving frame

$$\begin{aligned} \xi_i(\tau) &= x_i(\tau) - (x'_i + \bar{u}\tau) \\ \xi_i(\tau) &= z_i(\tau) - z' \end{aligned} \quad i = 1, 2, \dots, 800$$

where $\bar{u}$ is the mean value of the x wind component at the level of release of the particles and $\tau = t - t'$. A new function $Q_{\xi\xi}$ was introduced such that

$$Q_{\xi\xi}(\xi, \xi, \tau) = \begin{cases} N(\xi, \xi, \tau) [800h(2X(\tau))]^{-1}, & | \xi | \leq X(\tau) \\ 0, & \text{otherwise} \end{cases}$$

where $N(\xi, \xi, \tau)$ is the number of particles at time τ which have $\xi - \Delta\xi/2 \leq \xi_i \leq \xi + \Delta\xi/2$ and $\xi - \Delta\xi/2 \leq \xi_i(\tau) \leq \xi + \Delta\xi/2$ and $X(\tau)$ is a distance which is exceeded by the $|\xi_i|$'s of only a negligible fraction of the 800 particles. The intervals $\Delta\xi$ and $\Delta\xi$ are also functions of τ and are made sufficiently large that a minimum number of the intervals $\Delta\xi \Delta\xi$ are void of particles at each τ . Clearly we may approximate the desired function Q by

$$Q = Q(x-x', z-z', t-t') \approx Q_{\xi\xi}(\xi - \bar{u}\tau, \xi, \tau).$$

The results of these analyses for both the neutral and slightly unstable cases were then used in (4) to compute the steady-state mean cross-wind integrated concentrations, $\langle c_{cwi}(x, z) \rangle$, of a chemically inert substance, $A(t) = 0$, arising from a single steady point source, $S(x, t) = M\delta(x - x_s)$ where M is a constant and δ is the three dimensional delta function, when $\langle c(x, t_0) \rangle = 0$. For these conditions (4) gives

$$\langle c_{cwi}(x, z) \rangle = \lim_{t \rightarrow \infty} M \int_0^t Q(x - x_s, z - z_s, t - t') dt'. \quad (5)$$

The values of $\langle c_{cwi} \rangle$ given by this formula for the cases of $h/L = 0$ and $h/L = -4.5$ are shown in Figures 1a and 1b, respectively. In the figure distances and velocities have been made nondimensional using h and the friction velocity, respectively. The values shown have not been smoothed and therefore reflect statistical fluctuations arising from the finite size of the sample of particle trajectories on which the ensemble property Q is based. We should point out that the effective sample contains much fewer than 800 trajectories because the motions of any particular particle are correlated for a time with those of other particles due to the proximity of the particles to one another when they were released.

Figure 1. Relative concentration isopleths computed with equation (4) using Deardorff's data. Upper figure is for neutral stability ($h/L = 0$); lower figure represents slightly unstable conditions ($h/L = -4.5$).

It is of interest to compare the values given by (5) with those derived from the conventional plume formula, namely

$$\langle c_{cwi}(x, z) \rangle = \frac{M}{\sqrt{2\pi} \sigma_z^2} \left[\exp\left(-\frac{(z - z_s)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z + z_s)^2}{2\sigma_z^2}\right) \right] \quad (6)$$

where $\sigma_z = \sigma_z(x - x_s)$ is the dispersion parameter given by the empirical Pasquill-Gifford data as a function of downwind distance $(x - x_s)$ and atmospheric stability.

Figure 2a presents a comparison of the concentrations given by eqn (5)— $z = z_s = 25h$ and $h/L = 0$ —with the corresponding values given by (6) for both the neutral (class D) and slightly stable (class E) conditions. Figure 2b compares the results given by equation (5)— $z = z_s = 0$ and $h/L = -4.5$ —with the corresponding

values given by (6) for both the slightly unstable (class C) and neutral (class D) conditions. In all cases the concentrations $\langle c_{cwi}(x,z) \rangle$ have been scaled by $M/\partial h$, i.e.

$$\langle c_{cwi} \rangle^* = \langle c_{cwi}(x,z) \rangle \left(\frac{\partial h}{M} \right),$$

and distances in the Pasquill-Gifford data have been scaled by $h = 450$ meters, i.e. $x^* = (x - x_s)/h$.

Looking at Figure 2a we see that at small distances x^* from the source, the concentrations given by (5) for the neutral case differ considerably from those predicted by (6) for either the neutral or slightly stable conditions. However, the values given by (5) for the neutral case and those given by (6) for slightly stable conditions come into almost exact agreement as x^* becomes large.

Figure 2. Comparison of downwind concentrations along plume axis as computed with equations (5) and (6) under various stability conditions.

for the same atmospheric stability; and (b) that the values given by (5) do not have a Gaussian distribution in z^* —the skirts of the profiles are higher than those of the Gaussian formula (6).

Similar differences between formulas (5) and (6) are revealed by Figure 3b where the concentrations given by (5) for the slightly unstable case are compared with those given by (6) for neutral conditions. Again we see that the curve representing (5) does not have a Gaussian shape.

The results presented above are only preliminary but nevertheless they demonstrate that a numerical simulation model of the planetary boundary layer can be a valuable tool for determining the particle statistics which are required in atmospheric diffusion studies. The role of the boundary layer model becomes even more important in connection with studies involving the covariances of the concentrations where the statistics of pairs of particles, which are nearly impossible to measure in the field under any conditions, are required.

The behavior exhibited in Figure 2b by the concentration values predicted by (5) for slightly unstable conditions is not as regular as in the case above. Here the curve representing equation (5) wavers between the curves derived from formula (6) for the cases of neutral and slightly stable stabilities.

A further comparison of the results of formulas (5) and (6) is presented in Figures 3a and 3b. (In these figures the concentrations are normalized by their values at the plume axis.) Figure 3a displays the z^* profile of the concentrations given by (5) for the neutral case at two points, $x^* = 3$ and $x^* = 10$, downwind from the source. Also shown in the figure are the z^* profiles of the concentrations at $x^* = 3$ and $x^* = 10$ obtained from (6) for both the neutral and slightly stable cases. On comparing the results of the two formulas we see (a) that the profile described by (5) agrees best with that given by (6) for the slightly stable case—note the radical difference between the ground level values of $\langle c_{cwi} \rangle^*$ given by (5) at $x^* = 10$ and the corresponding value given by (6)

Figure 3. Comparison of vertical concentration distribution in a point source plume given by equations (5) and (6) at two downwind locations, $x^* = 3$ and $x^* = 10$, under various stability conditions.

REDUCTION OF EQUATION (4) TO DISCRETE-TIME CONTINUOUS-SPACE EQUATIONS

Consider the situation where the random part of the velocity of any particle, i.e. $v_i(t)$, where the subscript refers to one of the three coordinate axes, has a correlation function $R_{ij}(t; \tau) = \langle v_i(t) v_j(t + \tau) \rangle$ which vanishes sufficiently rapidly with increasing values of τ that a time scale

$$T_{ij} = \frac{1}{\langle v_i(t) v_j(t) \rangle} \int_0^\infty R_{ij}(t; \tau) d\tau \quad (7)$$

exists for all values of t and all possible points (R', t') of release of the particle. Under this condition it is to be expected that the motion of any particle at any time t will be statistically independent of its motions prior to the time $t - \Delta t$ where

$$\Delta t \gg \max\{T_{ij}\} \quad (8)$$

In other words the particle "forgets" its past and as a result its motions take on the attributes of a Markoff process. In particular, assuming that the positions at the earlier times $t_0 + (n-1)\Delta t$, $t_0 + (n-2)\Delta t$, ... were $\mathbf{R}_{n-1}$, $\mathbf{R}_{n-2}$, ... , respectively, the conditional probability density $Q(\mathbf{R}_n | \mathbf{R}_{n-1}, \mathbf{R}_{n-2}, \dots)$ of the particle position $\mathbf{R}_n$ at time $t_0 + n\Delta t$ satisfies

$$Q(\mathbf{R}_n | \mathbf{R}_{n-1}, \mathbf{R}_{n-2}, \dots) = Q(\mathbf{R}_n | \mathbf{R}_{n-1}). \quad (9)$$

In turn the conditional density $Q(\mathbf{R}_n | \mathbf{R}_r)$ satisfies the Chapman-Kolmogoroff equation

$$Q(\mathbf{R}_n | \mathbf{R}_r) = \int_{\infty} Q(\mathbf{R}_n | \mathbf{R}_p) Q(\mathbf{R}_p | \mathbf{R}_r) d\mathbf{r}(\mathbf{R}_p) \quad (10)$$

where $n > p > r$ are any integers. Making use of (10) in (4), we obtain after some manipulation

$$\begin{aligned} \langle C(\mathbf{R}_n, t_n) \rangle &= \int_{\infty} Q(\mathbf{R}_n | \mathbf{R}_r) \langle C(\mathbf{R}_r, t_r) \rangle \exp\left(-\int_{t_r}^{t_n} A(t') dt'\right) d\mathbf{r}(\mathbf{R}_r) \\ &+ \int_{\infty} \int_{t_r}^{t_n} Q(\mathbf{R}_n | \mathbf{R}') S(\mathbf{R}', t') \exp\left(-\int_{t'}^{t_n} A(t'') dt''\right) dt' d\mathbf{r}(\mathbf{R}'), \end{aligned} \quad (11)$$

where $t_n = t_0 + n\Delta t$. Although this equation lacks the temporal resolution possessed by (4), it requires less computational effort to evaluate.

Looking, in particular, at the case where $\frac{1}{A}$ and the scales of the temporal variations in the source function $S(\mathbf{R}, t)$ are large compared to Δt and where the spatial variations in $S(\mathbf{R}, t)$ have characteristic lengths L_i in the i coordinate direction which satisfy

$$L_i \gg [\Delta t^2 (\langle v_i^2 \rangle + \hat{u}_i^2)]^{1/2},$$

where $\hat{u}_i$ is the i th component of the deterministic fluid velocity, we can approximate (11) by the simpler equation

$$\langle C(\mathbf{R}, n\Delta t) \rangle = \int_{\infty} Q(\mathbf{R} | \mathbf{R}', (n-1)\Delta t) \exp(-A(n\Delta t)\Delta t) \left\{ \langle C(\mathbf{R}', (n-1)\Delta t) \rangle + S(\mathbf{R}', (n-1)\Delta t) \right\} d\mathbf{r}(\mathbf{R}') \quad (12)$$

provided that the spatial and temporal variations in both the turbulence and the deterministic velocity components have sufficiently large scales. A sample calculation made with equation (12) will be described later in this paper.

REDUCTION OF EQUATIONS (4) AND (12) TO OTHER LAGRANGIAN FORMULATIONS

At this point it is useful to point out how equations (4) and (12) fit into the family of diffusion formulas in current use. Equation (4) is the most general expression of the relationship between the mean particle concentrations and the statistics of the particle displacements. All of the diffusion equations in current use can be derived from equation (4) under conditions which we shall now outline.

In the case where (a) the turbulence is completely homogeneous; (b) the ground acts merely as a reflector of the fluid particles; and (c) only a single instantaneous point source has acted, i.e. $S(\mathbf{R}, t) = M\delta(t-t_0)\delta(\mathbf{R}-\mathbf{R}_0)$ where M is a constant, it is easy to show that equation (4) reduces to the Gaussian puff formula. This is a result of the fact that in homogeneous turbulence the density function Q is marginally and perhaps jointly normal in the three coordinates of the displacement vector (Batchelor⁽¹⁾).

Under conditions (a) and (b) above and with uniform, steady winds and only a single continuous point source active, i.e. $S(\mathbf{R}, t) = M\delta(\mathbf{R}-\mathbf{R}_0)$, equation (4) reduces in the limit as $t \rightarrow \infty$ to the well known Gaussian plume formula.

With regard to equation (12), if the temporal variations in A are large compared to Δt and if the turbulence is locally stationary and homogeneous*, it can be shown⁽⁵⁾ that in the limit of large time and space scales equation (12) reduces to the diffusion equation

$$\frac{\partial \langle C \rangle}{\partial t} + \frac{\partial}{\partial x_i} (\hat{u}_i \langle C \rangle) + A \langle C \rangle = S + \frac{\partial^2}{\partial x_i \partial x_j} \left(\frac{1}{2} K_{ij} \langle C \rangle \right) \quad (13)$$

where the diffusivities K_{ij} are proportional to $2 \langle v_i v_j \rangle T_{ij}$. (The convention of summing on repeated subscripts applies in (13) also $x_1 = x$, $x_2 = y$, and $x_3 = z$.)

The relationships among all of the equations mentioned above are summarized schematically in Figure 4.

*By locally stationary, homogeneous turbulence we mean that the spatial and temporal inhomogeneities in the turbulence are of such large scales that the density function Q of a particle released anywhere in the fluid is the Gaussian density function for travel times t in the range $0 \leq t \leq \Delta t$.

Figure 4. Relationships among various Lagrangian formulations of the mean concentration $\langle c(R,t) \rangle$.

As far as the author is aware, the generalized equation (4) and the discrete-time equation (12) have heretofore not been used in atmospheric diffusion studies. In this paper we have already presented a sample calculation performed with (4) and would now like to describe a computation made using equation (12).

The problem we wish to treat is somewhat hypothetical but it will demonstrate the versatility of formula (12). Specifically, our problem is to determine the configuration of green areas for an urban area which would result in a maximum reduction in the pollution levels, assuming that the green areas act as pollutant sinks. We assume (a) that the city is uniform in the direction normal to the mean wind, i.e. along the y axis; (b) that the removal of the pollutants by the green areas can be described by the flux condition $\partial \langle c \rangle / \partial z = \kappa \langle c \rangle$ on the green surfaces with $\kappa = .02 \text{ meter}^{-1}$; (c) that the non-green areas are uniform sources of pollution; (d) that the mean wind is horizontal and uniform in speed along the x and y axes but has a constant shear in the z direction; and (e) that there is an upper level inversion over the city.

The probability density function Q is assumed to have the two-dimensional form

$$Q(x-x', z-z', \Delta t) = [\sqrt{2\pi} \sigma_x]^{-1} \exp\left(-\frac{(x-x'-\hat{u}\Delta t)^2}{2\sigma_x^2}\right) \sum_{p_2} \frac{2(p_2^2 + \kappa^2) \cos(p_2 z') \cos(p_2 z)}{h(p_2^2 + \kappa^2) + \kappa} e^{-\frac{1}{2}\sigma_z^2 p_2^2} \quad (14)$$

where $\sigma_x = 2K_x \Delta t$, $\sigma_z = 2K_z \Delta t$, $\hat{u} = (1.5 + z'/150) \text{ m/sec}$, and the p_2 are roots of the transcendental equation $p_2 \tan(p_2 h) = \kappa$. Equation (14), derived in reference (4), is a modification of the Gaussian density function which accounts for the absorption of the pollutants at the ground and their confinement by the inversion.

Two green belt configurations were tested, one consisting of a single six kilometer wide belt downwind from a city of equal width and one consisting of six belts of one kilometer widths sandwiched between six similar cities. These are shown in Figure 5 together with the steady-state concentrations which were obtained from equation (12) for each case. In both calculations we assumed $K_x = 130 \text{ m}^2/\text{sec}$, $K_z = 5 \text{ m}^2/\text{sec}$, and $h = 210 \text{ m}$. The calculated concentrations for each green belt design are compared in table 1 below.

The results presented above illustrate the wide scope of problems to which equation (12) is applicable. We should also point out that from the standpoint of diffusion modeling this equation also has a number of practical advantages over other equations. For example, equation (12) requires much less computing time to evaluate than the puff formula because in the latter each puff of material is treated from the time that it is produced until the time that the computations end. However, with equation (12) each puff is treated individually for only a time Δt regardless of the length of the period over which the calculations are carried out. In a test calculation⁽⁵⁾ the speed of execution of (12) was found to be approximately 200 times greater than that of a two-dimensional puff formula. Equation (12) also has an important advantage over the diffusion equation. This is because (12) is an integral equation which is very simple to evaluate numerically whereas the diffusion equation is a differential equation which requires finite difference methods to evaluate. As is well known, to obtain an accurate estimate of the solution of a differential equation using finite difference techniques, one must find a difference scheme which is consistent with the differential equation, which is computationally stable, which has a suitably small truncation error, and which satisfies any conservation principles implicit in the problem.

To find a scheme which meets all of these requirements is often a difficult task.

Figure 5. Steady-state values of the mean pollutant concentrations over two cities with different green belt designs. The thick horizontal bars at $z = 0$ denote green areas (pollutant sinks); thin bars denote area sources.

Table 1. Comparison of ground level pollutant concentration in two cities with different green belt designs.

Average surface level concentrations for	single green belt city	multiple green belt city
green areas only	3.70	3.97
developed areas only	10.30	5.63
entire urban area	7.00	4.80
maximum concentration	15.60	8.87
minimum concentration	1.61	1.55
Total mass of pollutant in the air over the entire city	$10.50 \cdot 10^6$	$6.35 \cdot 10^6$

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OPTIMAL VELOCITY DISTRIBUTION AT INDUSTRIAL STACK EXIT

Gee Tsang
School of Engineering
University of Guelph
Guelph, Ontario
Canada

INTRODUCTION

As a hot plume rises, ambient air is entrained into the plume. The entrained air increases the upward mass flux of the plume and reduces the plume velocity and temperature. The reduction in temperature in turn reduces the subsequent velocity increase caused by buoyancy.

For a hot plume rising in an inverted atmosphere and reaching the interface of the inversion, if the entrained air does not dilute the hot gases to a temperature lower than that of the upper layer, the plume will continue rising in the upper atmosphere. If the plume is sufficiently diluted by the entrained air that its temperature at the interface is lower than that of the overlying warm air, it will not be able to rise into the upper atmosphere and cause air pollution to the lower atmosphere.

To reduce air pollution in the lower atmosphere, high stacks are built. When there is no inversion, a plume at a higher altitude will have a longer distance of diffusion to the ground. This will reduce the ground level concentration of the pollutants. When there is an inversion, the smoke from a higher stack will have less time to be diluted by the lower ambient air and thus the chance of rising into the higher atmosphere is better. Efforts have also been made to increase the temperature and velocity of the flue gases. The first method increases the plume rise when there is no inversion and increases the chance of rising in the upper atmosphere when there is an inversion. The second method, while increasing the plume rise when there is no inversion, will not help the plume to rise in the upper atmosphere when there is an inversion; because the negatively buoyant plume will eventually sink back to the inversion after intruding a little further into the upper layer under the action of the increased momentum.

Tall chimneys and the addition of heat and momentum to the flue gases mean high capital and operational costs. One may therefore ask the question of whether the plume rise and the chance of rising across an inversion by a plume can be increased by other means. The present study gives a positive answer to the above question.

As the temperature and velocity of a plume are reduced by ambient entrainment, the temperature and velocity of a plume will be better preserved if the ambient entrainment is reduced to minimum. Entrainment of fluid between two flow layers will occur when the two layers flow at different velocities. Ambient air is entrained into a plume because the vertical velocity of the plume and the ambient air are different. The greater is the velocity difference, the greater will be the entrainment. The present proposition is; that instead of having the ambient air entrained by the hot plume, let the plume entrain also hot gases from the chimney. By this way, the flue gases will not be diluted and the temperature and velocity of the plume will be preserved. The above may be achieved by the mental picture of making the flue gases flowing out from the chimney in concentric layers of progressively decreasing velocities from the centre to the edge. The central layer will then entrain fluid from the layer next to it, which in turn will entrain fluid from the layer further out. In such a fashion each layer entrains fluid from the outer layer until the edge of the chimney is reached. At the edge of the chimney, cold ambient air will be entrained. However, as the velocity of the outermost layer is small, ambient entrainment will be small.

THEORY

In the present study, the usual approximation that the density difference between the plume and the ambient fluids produces only the buoyancy that causes the motion, but does not affect the density of the fluids, will be used. Under this approximation, the equation of motion in the vertical direction for a steady plume from a stack may be written as

$$v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} = \frac{F_z}{\rho} - \frac{1}{\rho} \frac{\partial p}{\partial z} + \gamma \left(\frac{\partial^2 v_z}{\partial r^2} + \frac{\partial^2 v_z}{\partial z^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} \right). \quad (1)$$

In the above equation, v is the velocity, ρ is the density of the ambient fluid, F is the body force, p is the pressure, γ is the eddy viscosity, and the subscripts z and r denote the vertical and the radial direction respectively. In the present study, the origin of the cylindrical

coordinate is placed on the top of the stack and the z axis points in the upward direction and is coincident with the axis of the stack.

For a conventional industrial stack of a long cylinder, the flow inside the chimney is uniform and one-dimensional. For a uniform, one-dimensional flow, the left hand side terms of equation (1) vanish and the equation indicates an equilibrium between the buoyant force, the pressure gradient, and the viscous resistance. For a natural draft chimney, there is no pressure gradient and the equation indicates an equilibrium between the buoyant force and the viscous resistance. After leaving the chimney, the above equilibrium no longer exists. The eddy viscosity will decay as turbulence is no longer maintained by the presence of a solid boundary. The main processes the plume takes after leaving the stack will be

- 1, the mixing and consequently the redistribution of momentum and buoyancy between the plume fluid and the entrained ambient fluid,
- and 2, the acceleration of the mixed plume fluid by the increasingly important unbalanced buoyant force.

The eddy viscous force may be expected to be even less important in the present proposition as the velocity is expected to reduce to a small value at the edge of the stack, which means a small velocity gradient and consequently a small viscous shear. The above discussion thus leads to the following hypothesis;

After leaving the chimney, the turbulence in a plume causes mainly turbulent entrainment. The viscous shear generated by the turbulence is small compared to the buoyant force.

Under the above hypothesis and another approximation that the pressure above the stack exit is atmospheric, so there will be no pressure gradient, and seeing that the body force here is the buoyancy, equation (1) may be written as

$$v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} = \frac{\Delta \rho}{\rho} g, \quad (2)$$

where $\Delta \rho$ is the density difference between the plume and the ambient fluids, and g is the gravitational acceleration.

Although the validity of the hypothesis has yet to be proven, the use of the Eulerian equation of motion (equation (2)) in place of the Navier-Stokes equation (equation (1)) is not really new. It is well known that the Bernoulli equation is obtained by such a replacement. As in the case of the Bernoulli equation, the effect of the eddy viscosity may be taken into account and put into a coefficient of modification later when sufficient experimental data are accumulated.

For a plume in the present proposition, fluid flows in layers of different velocities and entrainment takes place between them. The radial velocity may thus be considered as the entrainment velocity V_e . As V_e is considered positive according to convention when flows towards the axis of a plume, one has $V_e = -V_r$. Using the entrainment velocity V_e , equation (2) may be written as

$$\frac{\partial v_z}{\partial z} = \frac{V_e}{v_z} \frac{\partial v_z}{\partial r} + \frac{\Delta \rho g}{v_z \rho}. \quad (3)$$

At a given height z , the above equation may be integrated between the axis and radius r ,

$$\int_0^r \frac{\partial v_z}{\partial z} 2\pi r dr = \int_0^r \left(\frac{V_e}{v_z} \frac{\partial v_z}{\partial r} + \frac{\Delta \rho g}{v_z \rho} \right) 2\pi r dr. \quad (4)$$

The left hand side of the above equation is the summation of the rates of change of volumetric flux over all the elementary areas, which is equal to the rate of change of volumetric flux through the integrated area $\frac{\partial}{\partial z} \int_0^r v_z 2\pi r dr$. Equation (4) thus becomes

$$\frac{\partial}{\partial z} \int_0^r v_z 2\pi r dr = \int_0^r \left(\frac{V_e}{v_z} \frac{\partial v_z}{\partial r} + \frac{\Delta \rho g}{v_z \rho} \right) 2\pi r dr. \quad (5)$$

For a plume to preserve its temperature and velocity, it is desirable for each layer to entrain as little fluid as possible from the next layer; for the layer further out is always more diluted by the ambient air. As a smaller rate of entrainment means a smaller rate of increase of vertical volumetric flux, the problem becomes how to distribute the velocity in a plume so that

$\frac{\partial}{\partial z} \int_0^r v_z^2 2\pi r dr$ or the right hand side of equation (5) will be minimum.

The right hand side of equation (5) is a function of the radial distance r and may be denoted as $f(r)$. It will be minimum if V_z is arranged in such a way that $\partial f / \partial r = 0$. The differentiation of $f(r)$ with respect to r gives

$$v_e \frac{\partial v_z}{\partial r} + \frac{\Delta \rho g}{\rho} = 0, \quad \text{or} \quad (v_e \frac{dv_z}{dr} + \frac{\Delta \rho g}{\rho})_{z=z_*} = 0, \quad (6)$$

seeing that at a given height z_* , V_z is a function of r alone.

According to the entrainment hypothesis, the entrainment velocity at a point is proportional to the difference between a suitably chosen characteristic axial velocity V_{zc} and the local axial velocity V_z .

$$v_e = \alpha (V_{zc} - V_z), \quad (7)$$

where α is the entrainment coefficient. If the average velocity between the axis and r is chosen as the characteristic velocity, the above equation becomes

$$v_e = \alpha \left(\frac{\int_0^r v_z^2 2\pi r dr}{\pi r^2} - V_z \right). \quad (8)$$

When the velocity distribution in a plume is Gaussian, Rouse et. al. (1952) and Ellison and Turner (1959) found that the entrainment coefficient is a constant of the order of 0.1.

A constant α may also be assumed for the present study. The substitution of equation (8) in equation (6) gives

$$\left(\frac{\int_0^r 2v_z r dr}{r^2} - V_z \right) \frac{dv_z}{dr} + \frac{\Delta \rho g}{\rho \alpha} = 0. \quad (9)$$

Once a plume leaves the stack, it is beyond human control. Thus, if a plume is to be controlled, it has to be controlled at the stack exit. At the stack exit, $\Delta \rho / \rho$ is not a function of r , so the last term of equation (9) is a constant. To satisfy the equation, the first term in equation (9) has also to be free from r . Thus, one has

$$\frac{\int_0^r 2v_z r dr}{r^2} - \frac{dv_z}{dr} - V_z \frac{dv_z}{dr} \propto r^0. \quad (10)$$

From the second term $V_z \frac{dv_z}{dr} \propto r^0$, which leads to $V_z \propto r^{1/2}$, one can see that the following velocity distribution will satisfy equation (10);

$$V_z = A + B r^{1/2}, \quad (11)$$

where A and B are constants yet to be determined.

The substitution of equation (11) into equation (9) results in

$$-\frac{B^2}{10} + \frac{\Delta \rho g}{\alpha \rho} = 0. \quad (12)$$

Thus, equation (11) becomes

$$v_z = A + \left(\frac{10 \Delta \rho g}{\rho} \right)^{1/2} r^{1/2}. \quad (13)$$

Using the boundary condition that at $r = 0$, v_z = the centre line velocity = $v_{z_{cent}}$, the above equation becomes

$$v_z = v_{z_{cent}} + \left(\frac{10 \Delta \rho g}{\rho} \right)^{1/2} r^{1/2}. \quad (14)$$

The negative sign in the above equation should be used as it will give a velocity distribution expected by the present proposition. After writing

$$a = \frac{\alpha \rho v_{z_{cent}}^2}{10 \Delta \rho g}, \quad r_* = r/a \quad \text{and} \quad \phi = \frac{v_z}{v_{z_{cent}}}, \quad (15)$$

equation (14) changes to a non-dimensional form as

$$\phi = 1 + r_*^{1/2}, \quad (16)$$

which is the velocity distribution at the stack exit for a plume of minimum ambient entrainment.

Equation (16) may now be applied to industrial stacks. It is seen from the equation that r_* may not be greater than unity. otherwise ϕ will be negative, which means a reversed flow at the stack exit. When r_* equals unity, one has

$$r = a = \frac{\alpha \rho v_{z_{cent}}^2}{10 \Delta \rho g}, \quad (17)$$

which is the equation relating the size of the stack exit, the initial density difference, and the discharge velocity of a plume of minimum ambient entrainment. The above statement is made under the assumption that the cutting off of the velocity distribution at $r_* = 1$ will not significantly affect the entrainment characteristics of the flow within the plume.

If the discharging velocity of the flux gases from a conventional stack is U and the radius of the stack is R , from the conditions that the volumetric flux (consequently the heat flux) and the momentum flux given by the new stack and the conventional stack are the same, one has

$$v_{z_{cent}}^2 a^2 \int_0^1 \phi^2 2\pi r_* dr_* = \pi R^2 U^2 \quad (18)$$

and

$$v_{z_{cent}}^2 a^2 \int_0^1 \phi^2 2\pi r_* dr_* = \pi R^2 \rho U^2. \quad (19)$$

The simultaneous solution of the above two equations gives

$$\begin{cases} a = 1.29R \\ v_{z_{cent}} = 3U, \end{cases} \quad \text{or} \quad \begin{cases} R = 0.775 a \\ U = \frac{1}{3} v_{z_{cent}} \end{cases} \quad (20)$$

It may be noted that the above relationships are independent of the entrainment coefficient.

LABORATORY STUDIES

The tested plumes were intended to be scaled models of industrial plumes. As ambient entrainment greatly affects the ability of a plume to rise across an inversion, and which in turn,

greatly affects the condition of air pollution in the lower atmosphere, the experiments in the present study were designed to compare the ability of rising across an inversion by a conventional plume and a plume from the present study.

From dimensional analysis, one obtains the following equation to guide the model study,

$$\eta = f_1 \left(\frac{\rho_1 - \rho_i}{\rho_1}, \frac{\rho_1 - \rho_2}{\rho_1}, \frac{Q^2}{D^5 g}, \frac{H}{D} \right). \quad (21)$$

where η is the percentage of the plume fluid that can penetrate the inversion and continue to rise in the upper atmosphere, ρ_i is the density of the plume fluid when released, ρ_1 and ρ_2 are the density of the air in the lower and upper atmosphere respectively, Q is the volumetric rate of discharge of the plume fluid, H is the height of the inversion above the top of the stack, D is the diameter of the stack at exit, and g is the gravitational acceleration.

In the experiments of the present study, a length scale of $k_l = 1/144$ was used, which led to a discharge scale of $k_Q = 1/2.49 \times 10^5$ and a velocity scale of $k_v = 1/12$.

Four experiments were performed with three experiments on conventional plumes and one experiment on the plume suggested by the present study. The model and the corresponding prototype parameters of the tested plumes are shown in Table 1.

Table 1. PARAMETERS OF THE TESTED PLUME

Test No.	Plume Type	Model						Prototype					
		H	D	Q	$\frac{\rho_i - \rho_1}{\rho_1}$	$\frac{\rho_1 - \rho_2}{\rho_1}$	V	H	D	Q	$\frac{\rho_i - \rho_1}{\rho_1}$	$\frac{\rho_1 - \rho_2}{\rho_1}$	V
		in.	in.	in. ³ /sec.	%	%	in./sec.	ft.	ft.	ft. ³ /sec.	%	%	ft./sec.
1	Conventional	24	1	21.5	30	2.3	27.4	288	12	3100	30	2.3	27.4
2	Conventional	34	1	21.5	30	2.3	27.4	408	12	3100	30	2.3	27.4
3	Conventional	35	1	21.5	30	3.0	27.4	420	12	3100	30	3.0	27.4
4	Proposed	35	1.29	21.5	30	3.0	16.45	420	15.5	3100	30	3.0	27.4

V = Average velocity at stack exit.

It is seen from Table 1 that because of equipment limitations, the discharging velocity and the initial density difference of the tested plumes are slightly less than those used in industries and the inversion is slightly larger than that occurred in nature. However, they are of the same order of magnitude so the experimental findings should bear industrial relevance to real industrial plumes.

The plume in Test 4 was intended to be the proposed version of the plume in Test 3. For this reason, the volumetric flux and the momentum flux of these two plumes were the same and the ambient conditions were identical.

As a continuously varied velocity distribution as described by equation (16) was difficult to obtain, a step velocity distribution was used in Test 4 as an approximation. To obtain the step velocity distribution, the model stack was partitioned by 4 thin metal (0.005") concentric cylinders and the required rates of discharge were introduced to each partition.

The experiments were performed in a steel tank 5' wide x 6' high x 3' deep with a glass front window. Descending model plumes were used to simulate the ascending prototype plumes. Fresh water was used to simulate the atmosphere in the low layer and salt solution in water was used to simulate the overlying upper atmosphere. The plume fluid was simulated by a mixture of

salt solution and calcium nitrate solution in water. The plume fluid and the underlying salt solution in the tank (which simulated the overlying atmosphere) were dyed red for visual observation.

To produce the inversion in the tank, a measured amount of salt was dissolved in water. This solution was pumped into the test tank and more water was added until the level inside the tank reached the height determined by the density of the inversion. After leaving the solution to quiet down for an hour or so, a polyethylene slab was laid on the top of it. A grid of 2" x 2" of 1/8 inch perforating holes was drilled on the slab. A foam rubber sheet was then laid over the polyethylene slab. Fresh water was introduced to the tank from above through a shower head. The fresh water seeped through the perforating holes and lay evenly over the salt solution to form the simulated lower layer of the atmosphere. When the water inside the tank reached the desired level, the inflow water was cut off and the polyethylene slab and the sponge rubber were gently lifted from the top of the water.

The experiments were recorded by time sequenced photographs at a time interval of one second. Detailed experimental set-up is not shown here because of space limitation.

EXPERIMENTAL FINDINGS

The experimental results are shown in Figs. 1 to 4. The photographs are turned upside down to show the prototype case. The discussions below are also in terms of the prototype.

Fig. 1 is from Test 1 and shows the rise of a plume from a conventional stack of 12 feet diameter in an inverted atmosphere. The inversion was 2.3% and 288 ft. above the top of the stack. It is seen from Fig. 4 that under this atmospheric condition, the smoke plume was not sufficiently diluted to a temperature lower than that of the upper atmosphere, so the plume penetrated across the inversion and continued to rise in the upper atmosphere. The photographs show that upon reaching the upper solid experimental boundary, the advancement of the smoke was stopped, and the plume spread sideways. In nature, there is no solid boundary on the top and the smoke plume should continue rising.

When the height of the inversion was increased to 408 feet above the top of the stack, the smoke plume coming out from the stack had more time to entrain the colder ambient air in the lower atmosphere, so its temperature was lower than that in the previous case when reached the inversion. It is seen from Fig. 2 that when the plume reached the inversion, the temperature of the smoke at the edge of the plume has been diluted to a temperature lower than that of the upper atmosphere, its velocity was also slowed down by the entrained ambient air, so this part of the smoke could not rise into the upper atmosphere. Smoke therefore slowly accumulated below the inversion and eventually reached the ground. It is seen from Fig. 2, however, that the part of the plume which was filtered by the inversion, was small. Most of the smoke, which was the central part of the plume, still penetrated across the inversion and rose in the upper atmosphere. Upon reaching the upper boundary, it spread to the sides. Besides the above two parts of the smoke, there is also some smoke which, although was diluted to a temperature lower than that of the upper atmosphere, managed to penetrate the inversion into the upper atmosphere by momentum, but fell back later to the inversion under the negative buoyancy.

In Test 3, the height of the inversion was further increased to 420 feet and the inversion was increased to 3%. In this case, the smoke plume at the inversion was diluted by the ambient air in the lower atmosphere to a temperature lower than that of the upper atmosphere. It is seen from Fig. 3 that a large part of the smoke at the edge of the plume was immediately filtered by the inversion. The rest of the plume, although managing to penetrate across the inversion by momentum, did not continue to rise in the upper atmosphere, which would have been detected by the spread of the plume at the upper boundary, but sank back to the inversion under the negative buoyancy. Under such conditions, severe air pollution in the lower atmosphere occurred.

Fig. 4 shows the experiment in Test 4, in which the smoke was discharged from the proposed stack. The atmospheric condition was exactly the same as in the previous case. The rate of discharge, the temperature of the flue gases and the momentum flux of the discharged gases were also the same. It is seen from the photographs that the smoke plume penetrated the inversion and continued to rise in the upper atmosphere, which was detected from the spread of the plume at the upper boundary. Very little smoke was filtered by the inversion and the severe air pollution of the previous case was avoided.

CONCLUSIONS

The present study shows the condition of air pollution at ground level can be improved by better preserving the temperature and velocity of the flue gases, which in turn, can be achieved by minimizing the entrainment of the ambient air into the plume. Using the entrainment hypothesis, equations are obtained which give the optimal size of the stack exit and the optimal velocity distribution at the stack exit. Laboratory experiments showed the theory is workable.

The laboratory experiments in this study were qualitative and were mainly in testing the superiority of the plume from the new stack in rising across an inversion. More laboratory experiments should be performed to quantitatively test the theory under different atmospheric conditions.

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Fig. 1. Record of Test 1.

Fig. 2. Record of Test 2.

Fig. 3. Record of Test 3.

Fig. 4. Record of Test 4.

DIFFUSION OF CONTINUOUS TIME DEPENDENT SOURCES INTO A TURBULENT WAKE

P. C. Chang, J. D. Hooten and S. K. Kao
University of Utah, Salt Lake City, Utah

I. INTRODUCTION

In the incidence of a sudden rupture of a pollutant-containing structure such as a nuclear reactor, the contaminant leakage within the building introduces the problems of expedient ventilation through the building for various outside meteorological conditions. The effluent with time-dependent source strength due to pollutant dilution inside the building will subsequently diffuse downstream into a building-distorted flowfield with a separation cavity and a turbulent wake. Although the fastest ventilation may be desired, precaution must be exercised to prevent the inhalation of pollutant in the vicinity of buildings from reaching hazardous levels, especially the buildup within the cavity. Proper environmental safety control during this type of emergency is thus contingent upon understanding the mechanism involved in the complete diffusion process from the inner buildings to the far wakes.

II. GENERAL CONSIDERATION

a. Characteristics of Flow Around a Sphere

When a blunt obstacle such as a spherical building is introduced into an essentially parallel background flow, the background flow will be distorted in the vicinity and the downstream of the obstacle. In Fig. 1, the schematic diagram for the flow around a suspended sphere is given. As the boundary layer flow developed along the upstream surface of the sphere, it separates from the boundary surface as the consequence of continuous momentum depletion against shearing stresses in the boundary layer with adverse pressure gradient. Right next to the separation point, a slow backflow in the direction of the pressure gradient may set in to form a low pressure cavity zone characterized by a large, sluggish eddy motion and a strong obstacle-generated turbulent mixing. The vorticity field generated behind the obstacle is convected downstream, and diffused at the same time to a confined region known as a wake. Due to the diffusion of vorticity the wake must continually increase in breadth, but ultimately spread out its intensity that the flowfield will recover to that of the undisturbed background flow.

The characteristics of separation, cavity and wake of sphere flow are controlled by the Reynolds number. The detailed structure of flow around sphere for a wide range of Reynolds numbers has recently been comprehensively reviewed by Chang (1). Of course, the situation in the atmosphere is even more complex than what has just been described. Due to the presence of the ground surface, the background flow is a turbulent shear flow. The general characteristics of the building-distorted atmospheric boundary layer flow have been described by Halitsky (2) with a special reference to gas diffusion near buildings.

b. Dilution of Source Inside the Building

In this study, the dilution of source inside the building is considered to be almost completely due to the dynamic excitation of external flows. In other words, the volume source itself does not possess any momentum. As shown in Fig. 1, there may be several openings at different orientations around the building. Dilution of source inside the building may result from the flow exchange due to 1) flow through the compartment because of pressure difference between openings, 2) pulsating flow caused by pressure fluctuations, 3) the penetration of turbulent eddies, and 4) static or molecular diffusion (3).

It is of practical importance to predict the concentration decay inside the building. The initial concentration C_0 may be expressed as

$$C_0 = G_{p0}/V_T$$

where G_{p0} is the initial pollutant mass and V_T which remains to be a constant is the total fluid volume inside the building. For the observed conditions of steady external flow, and opening configuration, the rate of change of the concentration can be obtained by considering the conservation of volume inside the building.

$$(\text{volume flux})_{in} = (\text{volume flux})_{out} = \alpha V_T = \text{const.}$$

where α is a positive value less than one. Then the concentration $C(t)$ at any time t will be

$$C(t) = \frac{1}{V_T} \left[G_{p0} - \int_0^t \alpha C(t') V_T dt' \right] = C_0 - \alpha \int_0^t C(t') dt'.$$

After incorporating the initial conditions, it yields

$$C(t) = C_0 \exp(-\alpha t) \quad (1)$$

This simple derivation indicates the dilution rates follow an experimental decay while these rates depend only upon mass flow rates into or out of the building.

c. Diffusion Pattern in the Building Wake

The effluent from the building opening is a time dependent continuous point-source. Many methods have been used to describe the diffusion pattern of a steady continuous point-source as the pollutant moves with the general flow. Because of its complexity, no analytical model is attempted in this paper to describe the detailed dispersion behavior in the sphere-disturbed field. Instead, we will compare our experimental data with the commonly used diffusion formulas to see how a disturbed flow field modifies the diffusion pattern.

Most of the diffusion theories relate the diffusion to follow the same general format for the Gaussian error distribution, for one-dimensional case, that is

$$x(y,t) = A'(t) \exp \left(-\frac{y^2}{B} \right)$$

For this study, this equation is nondimensionalized by the source strength of the corresponding time to account for the time-dependent behavior of the continuous point-source.

$$\frac{x(y,t)}{C(t)} = \frac{x(y,t)}{C_0 \exp(-\alpha t)} = A \exp \left(-\frac{(y-y_g)^2}{B} \right) \quad (2)$$

where y_g is the y value to the position of the observed maximum concentration. Thus the maximum concentration of the Gaussian distribution occurs at $y-y_g = 0$ and the amplitude of the curve is $x(y_g,t)/C_0 \exp(-\alpha t)$ at this position.

If more than one emission source appears such as shown in Fig. 1, the comparison of the observed concentration profile can be made to the linear superpositions of the multi-Gaussian error distributions.

III. EXPERIMENTAL FACILITIES AND PROCEDURES

The experiments were performed in a recirculating water tunnel having a width of 1 foot, depth of 2.5 feet and a test section of 18 feet. The building model is a 6 inch outside diameter, hollow, transparent, smooth plexiglas sphere with 1/16 inch walls. The sphere had a provision of eight 3/8 diameter orifices along an equator thus allowing the background flow to approach the orifice openings in various configurations. The sphere was placed along the center line of the flume and submerged 0.6 feet from the water surface to be practically free from side wall and surface wave effects. A background flow with an approaching velocity of about 0.55 ft/sec was used for the diffusion experiments. No extensive measurements of the velocity distributions were taken, but the Reynolds numbers calculated from the mean velocity determined by the discharge measurements indicated that the flow field consisted of a turbulent background flow with a laminar separation and a turbulent wake.

A methyl orange dye was used as contaminant in diffusion experiments. Concentrations were determined for specified orifice combinations through discrete sampling at measured time intervals. The samples were obtained by using 6 inch, 20 gauge hypodermic needles in connection with syringes or aspirators. The samples collected from the center of the sphere were used to determine the average concentration within the sphere. Concentration samples were analyzed with a Bausch and Lomb Spectronic 20 spectrophotometer and colorimeter.

To supplement the experiments, a fluorescein sodium dye was used in the visualization of the flow and diffusion characteristics. The visualization technique included illuminating the dye with a "black" ultraviolet light and photographing the flow patterns using a 16 mm Kodak movie camera.

IV. RESULTS AND DISCUSSION

a. Dilution of Source Inside the Sphere

The investigation of pollutant dilution inside the sphere was directed toward determining the relationship of the change of inside concentration to the outside flow condition for various orifice opening combinations. A typical set of experimental results on dilution behavior is shown in Fig. 2. They were obtained by maintaining a downstream hole open at $\phi = 180^\circ$ and varying the positions of two upstream orifices which were symmetric about the x -axis. These results indicate

that the concentration inside the sphere decays exponentially and the decay rates depend upon the orifices orientation. For a constant flow condition, the pressure difference among orifices dictates the volume flow rate into or out of the sphere. Fig. 2 therefore provides an excellent verification of the analytical prediction by Eq. (1).

Summarized in Fig. 3 are the exponential decay rates of concentration within the sphere in terms of parameter α . Two typical cases are presented; case one, with a constant orifice at $\phi = 180^\circ$ as the same condition of Fig. 2, and the second case with a constant orifice at $\phi = 0$. It is evident that the dependence of the decay rates on orifice position for case 1 resembles the pressure distribution around a sphere at a sub-critical Reynolds number as measured by Flachsbarth (4). In other words, the greater the pressure difference, the larger the pollutant discharge and thus the rate of concentration change will be. Some interesting phenomena are noticed for case 2. Since the constant orifice locates at the upstream stagnation point which has the maximum pressure, the varying orifices will always serve as effluent orifices. When the effluent orifices are placed along the fore-body of the hemisphere, the source discharge triggers the formation of a turbulent boundary layer and thus a turbulent separation which occurs further downstream than would a laminar separation on the sphere. Of course, when the effluent orifices are located along the after-body of the hemisphere, normal laminar boundary layer around the sphere is formed as determined by the Reynolds number based upon approaching flow and sphere geometry. The decay rates for case 2 shown in Fig. 3 therefore inverseley resembles the pressure distribution around a sphere at a supercritical Reynolds number for $\phi < 90^\circ$ and at a subcritical Reynolds number for $\phi > 90^\circ$. In both cases, when the varying orifice coincides with the constant orifice, dilution of concentration can only be resulted from the mixing process appeared through the single orifice opening. The extremely slow decaying rate through this mixing process as shown in Fig. 3, indicate the forced ventilation caused by pressure difference between openings is the major mechanism for source dilution inside the building.

b. Concentration Distribution in the Turbulent Wake

All of concentration measurements in the turbulent wake are completed with the upstream stagnation orifice ($\phi=0$) kept open while the position of a pair of symmetrical downstream orifices varies. This enables a sufficiently large mass of pollutant to escape from the sphere and be sampled downstream, and with large concentrations in the samples, more accurate transmissivity readings can be obtained.

However, due to the confined boundary of the experimental tunnel and the limited capacity of the concentration monitoring device, concentration measurements were conducted within the building vicinity and cavity region. In order to avoid further complexities associated with the momentum transfer due to the vertical velocity shear of the background flow, most of the concentration measurements were performed in the horizontal x-y plane.

For a potential application of these results to the practical environmental pollution problems, the experimental data will be discussed as if the x-axis was the ground surface from the method of image viewpoint. Consequently, the source may be considered as being released from various locations from a hemispherical dome building.

Fig. 4 shows three sets of concentration profiles in the horizontal plane (x-y). The angle positions of effluent orifices and the longitudinal locations of the concentration profiles are identified in the figure. The experimental data denoted as solid circles are the average values of concentrations obtained at the shown position and the symmetrical image position about the x-axis. To eliminate the time dependency of source strength so that the concentration measurements at different times can be directly correlated, the concentration profiles are nondimensionalized with respect to the source strength within the sphere at the time of the measurements. The anomalous diffusion pattern associated with a disturbed flow field is examined by comparing the experimental data with the commonly used Gaussian distribution. Since we have a pair of symmetrical sources, a linearly superimposed double Gaussian distribution is plotted as a solid line in each case of Fig. 4 for comparison. These Gaussian curves were obtained by assuming that the axis of maximum observed concentration is the axis of maximum normal probability distribution. In this way a normalization constant is obtained to normalize the maximum concentration. Having made the first approximation of a normal distribution, the standard deviation of the curve is then determined to be the point on the curve at which the area under the curve between the point of maximum concentration and this point includes 34.13% of the total area, by graphical integration.

For $\phi = 90^\circ$, the contaminant source is released from the top of the dome and emitted normally into the boundary layer flow. As the initial concentration is absorbed into the flow, the pollutant is transported along the surface of building and the concentration maxima rapidly converge towards the ground surface. This phenomenon is observed visually and verified by the concentration profiles which indicate the pollutant maxima occurring at about one inch instead of three to four inches from the boundary. This tendency is revealed more clearly by the

concentration isopleths shown in Fig. 5a. As previously discussed the discharge from the effluent orifice causes the boundary layer to become artificially turbulent, the point of separation moves over a considerable distance in the downstream direction. For the effluent orifice location at upstream side of the separation, it may be expected that the concentration maxima will be convected downstream along the separation streamline or wake boundary after they are transported along the boundary layer to reach the separation point.

Since the width of the wake only increases slightly in terms of distance $x(\propto x^{\frac{1}{2}})$, and the separation streamline is the region of maximum turbulence and shearing stress, the pollutant tends to be uniformly distributed in the wake region through strong turbulent mixing. The concentration profiles in the outer portion of the boundary layer, however, follow quite closely the Gaussian distribution even in the vicinity of the building. Unless the jet-like emission is strong enough to penetrate through the boundary layer, the diffusion behavior for a source emitted from the point with $\phi < 90^\circ$ should have similar characteristics as just being described.

The concentration profiles resulting from the downstream orifices open at $\phi = 120^\circ$ indicate strongly the effect of the cavities embedded in the wake flow. The pollutant is emitted from the building directly into the cavities formed as the consequence of a laminar separation near the equator.

The observation taken at a distance of 4 inches downstream from the center of the sphere defines these phenomena extremely well. The slight jet-like emission causes the concentration to peak at the first observation point. Due to the recirculation of the pollutant within the cavity, relatively large concentrations are observed between the peak concentration positions and the ground surface. It is evident that the cavities cause the concentration profiles completely deviate from the normal distribution. The relatively strong mixing in the wake region and the superposition of two pollutant sources seem to cause a converging of the maxima toward the ground surface as indicated by the concentration profiles and the corresponding isopleths shown in Fig. 5b. It is important to note from the isopleths that the maximum ground concentration occurs at the point some one building diameter from the origin.

The concentration build-up inside the cavities not only present a potential hazardous region but also alters the conservation of total pollutant mass along the downstream direction. Although the pollutant inside the cavities will eventually diffuse into downstream wake, no equilibrium condition can ever be obtained in the cavity region due to unsteady nature of pollutant sources.

The last case presented in Fig. 4 has a source emitted from the downstream ground surface ($\phi = 180^\circ$). The momentum carried by the pressure-excited pollutant effluent is counterbalanced by the reverse flow in the wake. In this case, the concentration profiles closely follow the normal distribution curve and the concentration maxima continuously stay on the ground surface.

In summary, the concentration measurements in the building vicinity indicate the concentration behavior at the near building region depends upon the release position of the pollutant and strongly couples with the appearance of separation cavity and turbulent wake. In some cases, the emission of pollutant may even modify the boundary layer flow. The concentration maximum tends to coverage toward ground surface and the concentration in the wake region is smoothed to a nearly uniform distribution due to strong turbulent mixing. Although the present concentration measurements were confined in the near building region, the obtained results do suggest the concentration profiles approach to a normal distribution or an exponential-type similarity distribution as they propagate downstream.

Acknowledgment

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Fig. 1. Schematic diagram for the diffusion from a spherical source into a turbulent wake.

Fig. 2. Dilution of source inside the sphere for various orifice angles with a constant orifice open at $\phi = 180^\circ$.

Fig. 3. Decay rates of concentration within the sphere for various orifice combinations.

Fig. 4. Concentration profiles in the wake.

(a)

(b)

Fig. 5. Concentration isopleths.

A NUMERICAL MODEL OF THE LAKE MICHIGAN LAKE BREEZE

Arthur G. Tingle
GCA Corporation, Salt Lake City, Utah

INTRODUCTION

The objective of this paper is to describe an investigation by the author⁽¹⁾ of a time-dependent numerical model of atmospheric behavior in the shoreline vicinity adjacent to large bodies of water. The scale of motion under consideration, called mesoscale, is on the order of 100 km horizontally and 3 km vertically. Mesoscale atmospheric phenomena must be considered in relation to the overall synoptic scale weather pattern. The mesoscale phenomena to be modeled are the atmospheric fields of wind and temperature created by differential heating of land and water under varying synoptic wind conditions.

The resulting winds, when they locally dominate the synoptic pattern, are known as sea or lake breezes. Sea and lake breezes are similar physically, but may differ in scale and intensity due to the smaller dimensions of the lake surface. Therefore, the qualitative description and physical model of land-water circulations given below and the local climate influences of these circulations are applicable to both sea and lake breezes.

PHYSICAL DESCRIPTION OF LAND-WATER WIND CIRCULATIONS

Land-water wind circulations are initiated by differential heating and cooling of the land and water surfaces. According to Defant,⁽²⁾ if the air mass overlying the shoreline region is initially at rest, shortly after sunrise heated air over land begins to rise creating a pressure gradient directed offshore. This results in an offshore flow aloft that in turn causes an increase in pressure over water and a decrease in pressure over land. The onshore flow known as the sea or lake breeze then begins in a shallow layer near the shoreline, but offshore. The discontinuity or front appears to progress inland in a series of pulses with a period of about 2 hours (Olsson).⁽³⁾ During the intervals when the front is approximately stationary, the depth of onshore flow increases and the onshore wind increases in speed. When the front is moving inland, the onshore flow layer becomes more shallow, but the maximum wind speed remains about the same.

Blerly⁽⁴⁾ has constructed a physical model of land-water wind circulations by synthesizing the observations of several workers. The physical model, illustrated in Figure 1, interprets a lake or sea breeze in terms of atmospheric discontinuities induced by the breeze. The discontinuities referred to are surfaces at which a variable such as temperature or wind direction changes abruptly. The line labeled "Top of Momentum Boundary Layer" is a surface enclosing the lake air characterized at its leading edge or front by a change in wind direction from onshore to offshore, a sharp drop in temperature and an increase in humidity. The top surface may exceed a kilometer in height and is not in general horizontal, as illustrated, but is more likely a wave (see Figure 5). The line labeled "Top of Dynamically Induced Internal Boundary Layer" is a surface characterized by a maximum in the onshore wind field. The line labeled "Top of Thermally Induced Internal Boundary Layer" is a surface at which the temperature lapse rate changes abruptly. Over the lake there is a strong temperature inversion in this layer which changes to a superadiabatic temperature lapse rate as the lake air moves inland. The temperature inversions shown in profile A are caused both by the descending air motion over the lake and the lake temperature. The depth of this layer varies from 20 m to 60 m in Blerly's data.

In particular, the following features should occur as the land temperature is increased:

- o The initial horizontal motion is an offshore flow aloft;
- o A "tongue" of lake air starts flowing inland a couple of hours after the lake and land temperatures become equal;
- o There is a low-level maximum in the velocity field of the onshore flow of lake air;
- o Inland and above the lake air, there is an opposing offshore flow;
- o The velocity in the offshore flow layer aloft is comparable in magnitude to the onshore velocity;
- o The horizontal wind vectors rotate to the right with time in response to the Coriolis force;
- o There is a strong updraft near the boundary between the lake air and the land air (the sea breeze front) as this boundary moves inland;
- o The vertical wind component has alternating cells of positive and negative motion ahead of the front and with negative motion behind the front;
- o The inland temperature profile has a superadiabatic layer near the ground and an adiabatic layer above;
- o The offshore temperature profile has an inversion in the lower layers;

- o The front moves with a pulsing motion inland with a period of a few hours.

THE NUMERICAL LAKE BREEZE MODEL

The numerical mathematical model described in this section is a general model designed to simulate land-water wind circulation systems. Observations of a lake breeze taken near Muskegon, Michigan in June 1965, as reported by Olsson, ⁽³⁾ are used to verify the model for a particular situation.

The model is divided into two layers. The lower layer, called the "Boundary Layer", extends from the surface to 50 m height; the fluxes of heat and momentum in this layer are assumed constant, as suggested by Estoque. ⁽⁵⁾ The upper layer, called the "Transition Layer", extends from 50 m to a maximum height of 3050 m, although 2050 m was used as the upper boundary in the computations. The shoreline is assumed to be a straight line of infinite length and both the lake and land are assumed to have semi-infinite extent. The model is schematically illustrated in Figure 2. It is derived from the General Circulation Model of the National Center for Atmospheric Research by writing the equations in two-dimensional Cartesian coordinates and by ignoring latent heat, radiation processes, and terrain effects. These equations simulate a dry, compressible atmosphere and allow the use of an observed vertical temperature profile with the restriction that superadiabatic layers are not permitted. An arbitrary wind profile in the vertical is incorporated in the model as an initial condition. The observed diurnal surface temperature wave is used as the driving force of the model circulation. The lateral boundary conditions used in the model require that the horizontal partial derivative of all variables be equal to zero. Further, the vertical component is zero at the top of the Transition Layer and all wind components are zero at the surface.

List of Symbols

x = Onshore direction	y = Along-shore direction	z = Height
t = Time	u = x wind component	v = y wind component
w = Vertical velocity	p = Pressure	ρ = Density
T = Temperature	θ = Potential temperature	g = Gravitational acceleration
f = Coriolis parameter	R = Gas constant for dry air	c _p = Specific heat for dry air at constant pressure
c _v = Specific heat for dry air at constant volume	$\gamma = c_p/c_v$	Q = Rate of heating per unit mass
K = Vertical exchange coefficient	K _x = Horizontal diffusion parameter = $10^6 \text{ cm}^2 \text{ sec}^{-1}$	h = Height of internal boundary = 50 m
Z _T = Height of top of model	DX = Horizontal grid spacing	U ₀ = Initial value of u(z)
DZ = Vertical grid spacing	DT = Time step	

Momentum Equations

The lake-breeze model is by hypothesis uniform in the along-shoreline, or y, direction. Therefore, the Cartesian coordinate equations describing the horizontal motions of the model are (see List of Symbols):

$$\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial x} - w \frac{\partial u}{\partial z} + f v - \frac{1}{\rho} \frac{\partial p}{\partial x} + K \frac{\partial^2 u}{\partial x^2} + K_x \frac{\partial^2 u}{\partial x^2} = \text{RHSU} \quad (1)$$

and

$$\frac{\partial v}{\partial t} = -u \frac{\partial v}{\partial x} - w \frac{\partial v}{\partial z} - f(u - U_0) + K \frac{\partial^2 v}{\partial x^2} + K_x \frac{\partial^2 v}{\partial x^2} = \text{RHSV} \quad (2)$$

The vertical exchange coefficient K is assumed to be the same for both heat and momentum. The value for K is determined at the internal boundary, as described later, and is then decreased in the vertical by the relation

$$K_z = K_h \exp \left[-\text{DEC} (z - h)^2 \right] \quad (3)$$

where DEC is a constant, following McPherson. ⁽⁶⁾ The quantity $f U_0$ is a pressure force in the along-shoreline direction that balances the initial wind field, as suggested by Estoque and Dharmalakkar. ⁽⁷⁾

Pressure and Vertical Velocity

The equations of this section are essentially the same as those used in the NCAR Global General Circulation Model of the Atmosphere (Kasahara and Washington). ⁽⁸⁾ This model uses height as the vertical coordinate, uses the pressure tendency equation, and allows the pressure to vary at the model upper boundary.

Assuming Z_T to be the height of the upper boundary and w = 0 at Z = Z_T, then the pressure tendency equation is

$$\frac{\partial p}{\partial t} = B + g w - g \int_z^{Z_T} \frac{\partial(\rho w)}{\partial z} dz \quad (4)$$

where B is the pressure tendency at Z = Z_T. The vertical velocity is obtained from

$$w = - \int_0^z \frac{\partial u}{\partial x} dz - \frac{1}{\gamma} \int_0^z \frac{(B+J)}{p} dz + \frac{1}{c_p} \int_0^z \frac{Q}{T} dz, \quad (5)$$

where

$$J = u \frac{\partial p}{\partial x} - g \int_z^{z_T} \frac{\partial(\rho u)}{\partial x} dz, \quad (6)$$

and

$$B = \frac{\frac{1}{\gamma} \int_0^{z_T} \frac{J}{p} dz - \frac{1}{c_p} \int_0^{z_T} \frac{Q}{T} dz + \int_0^{z_T} \frac{\partial u}{\partial x} dz}{- \frac{1}{\gamma} \int_0^{z_T} \frac{dz}{p}}. \quad (7)$$

Non-Adiabatic Heating and Diagnostic Equations

The only source of heat represented in the model is solar heating of the underlying land surfaces that is transferred upward by eddy exchange processes. This process is simulated in the model by the rate of non-adiabatic heating per unit mass, denoted by Q , which is defined by the relation

$$\frac{Q}{c_p} = \frac{\partial}{\partial z} \left(K \frac{\partial \theta}{\partial z} \right) + K_x \frac{\partial^2 \theta}{\partial x^2} \quad (8)$$

In order to complete the set of transitional layer equations, the following diagnostic equations are respectively used to obtain the density, temperature and potential temperature:

$$\rho = - \frac{1}{g} \frac{\partial p}{\partial z}, \quad T = - \frac{g}{R} \frac{\partial z}{\partial \ln p}, \quad \text{and} \quad \theta = T \left(\frac{p_0}{p} \right)^{\frac{1}{\gamma}}. \quad (9)$$

The expression $(p_0/p)^{-286}$ is calculated from the initial vertical pressure distribution and is not changed during program execution in order to avoid computing exponentials of slowly changing variables.

Surface Boundary Layer

The boundary layer equations are derived using the assumption that there is a constant flux sublayer of 50 m depth below the transition layer. The purpose of the equations is to provide lower boundary conditions at a height of 50 m for the transition layer prognostic and diagnostic equations. The boundary layer processes are divided into three regimes:

- o Free convection;
- o Forced convection; and
- o A minimum stable regime which is an assumed state of the atmosphere when forced convection is small; in this case, a minimum of $K = 10^4 \text{ cm}^2 \text{ sec}^{-1}$ is used.

McPherson⁽⁶⁾ discusses the application of a constant flux boundary layer to a sea breeze model.

Superadiabatic Temperature Adjustment

During the time that the surface temperature increases, some columns in the model may develop layers that are superadiabatic. This leads to instability in a hydrostatic model and must be prevented from occurring. To avoid these superadiabatic lapse rates, a procedure described by Olinger, Welick, Kasahara and Washington⁽⁹⁾ was adopted. This method requires that the temperature lapse rate be adjusted whenever it is less than the dry adiabatic lapse rate, $-g/c_p$, with the following two constraints and one definition utilized for the adjustment process:

- o The internal energy, $\int T \rho dz$, remains constant between adjacent layers.
- o The new lapse rate equals the critical value $\frac{\partial T^*}{\partial z} = - \frac{g}{c_p}$, where the starred temperature is the adjusted value; and,

- o In order to adjust the pressures to correspond to the new temperatures, an invariant mean temperature is defined by

$$\bar{T} = \frac{\int \rho T dz}{\int \rho dz}. \quad (11)$$

This procedure is only necessary over land, since the water temperature does not change.

Finite Difference Procedure

The momentum equations are solved by a modified version of the Lax-Wendroff scheme described by Richtmyer and Morton.⁽¹⁰⁾ Defining three indices (i = horizontal grid counter, j = vertical grid counter, and

n = time step counter), the right-hand side of the u -component equation is computed by central differencing in space the current values of the variables, except that values from the previous time step are used in the diffusion terms for computational stability. This scheme is applied each time a predetermined odd number of time steps are computed (41 in the present case). Therefore, at this point, a decision is made whether the counter has reached its assigned value and then a branch is made to one of two choices:

(1) Central Difference
$$u_{i,j}^{n+1} = u_{i,j}^{n-1} + 2\Delta T \cdot \text{RHSU} \quad (12)$$

(2) Lax-Wendroff
$$u_{i,j}^{n+1} = 0.5(u_{i+1,j}^n + u_{i-1,j}^n) + \Delta T \cdot \text{RHSU} \quad (13)$$

with a similar procedure for the v component.

In computing the pressure and vertical velocity values, an average value for the integrand was calculated for each vertical grid spacing ΔZ , leading to a rapid solution of the series of integrals without sacrificing much accuracy in the lake breeze model.

As the numerical solution progresses, it may break apart in time as well as in space. To diminish this problem, a time smoother was installed in the model so that when its counter reaches a preset value, the three prediction variables are changed by:

$$u_{i,j}^n = 0.5 u_{i,j}^n + 0.25 (u_{i,j}^{n-1} + u_{i,j}^{n+1}) \quad (14)$$

with similar expressions for v and p .

DISCUSSION OF RESULTS

Initial Conditions

An effort was made to simulate observations reported by Olsson. (3) This required considerable interpolation as conditions were changing rapidly near the surface and measurements taken from a light aircraft took over an hour to complete. Temperatures used in the model are listed in Table 1.

TABLE 1. MODEL TEMPERATURES

Height (m)	Initial Vertical Temperature Profile (Invariant Horizontally)										
	0	50	150	250	350	450	550	650	750	850	950
Temperature (°C)	14.0	14.0	14.0	14.0	13.3	12.5	11.8	11.0	10.6	10.1	9.6
Height (m)	1050	1150	1250	1350	1450	1550	1650	1750	1850	1950	2050
	Temperature (°C)	9.1	9.0	8.9	8.6	8.2	7.6	7.5	7.3	7.1	6.9
Time (EST)	Surface Temperature Change (As Observed 24 km Inland)										
	0800	0900	1000	1100	1200	1300	1400	1500			
Temperature (°C)	14.0	16.7	18.3	19.4	20.6	21.7	22.2	22.6			

The pressure was computed from the hydrostatic equation with a surface pressure of 1000 mb. An initial uniform offshore wind field of 1.0 m sec^{-1} was estimated as representative of the early morning observations. The lake temperature was held at 14°C , although this is not necessary in the model.

Comparison with Physical Model

The model was run for six hours real time and all of the features noted in Figure 1 occurred. However, two discrepancies developed that should be corrected if, as accepted here, the physical model is correct. The first discrepancy is that the superadiabatic layer near the ground does not appear inland, except for the first two grid points—3 km and 6 km inland. This happens sometime after the superadiabatic temperature adjustment procedure for the Transition Layer has started operating (about 3 hours). The second discrepancy, which becomes apparent about the same time as the first, is that the low-level wind flow does not change from onshore to offshore at the location of the maximum updraft. It changes at a position that moves increasingly farther inland. There is also, after this time, no connected field of offshore flow surrounding the lake air. Due to the fact that these discrepancies occur at comparable times and locations, they could be related. However, they should not obscure the fact that the model behaves properly for the other features.

The initial horizontal motion attributable to the lake breeze was an increase in the offshore flow aloft at a model time of about 2 hours. The 3 km grid spacing was too coarse to show an inland flow starting offshore, as was observed to begin at 1.6 km offshore. The circulation is well developed at 3 hours model time (Figure 3) with a maximum onshore velocity of 4.3 m sec^{-1} , a depth of onshore flow of 650 m and an offshore flow aloft with a maximum velocity of 2.5 m sec^{-1} . The explanation for the small cell of positive flow aloft is not obvious, but it should be noted by comparison with Figure 3 that this is also the location where the vertical velocity changes sign, implying the possibility of an internal "breaking apart" of the solution. The observed flow

(Figure 8) has a cell of light winds at 800 m height 6 km inland, and the computed cell might therefore be an exaggeration of a real feature. Further development of the flow is shown in Figures 4 through 7 until 6 hours model time when the u component (Figure 7) is still increasing in the speed of the onshore flow and this flow layer has deepened to 950 m. An opposing offshore flow inland as such has disappeared from the figure, but is located at 45 km inland in the computer printout.

Comparison With Observations

The flow features listed in Table 2 were selected for direct comparison of the observations and computational results.

TABLE 2
COMPARISON OF DETAILED MODEL FEATURES WITH OBSERVATIONS

Circulation Feature	Model Time				Observation Time	
	2 hours	3 hours	4 hours	5 hours	1100 EST	1400 EST
Depth of Onshore Flow	350 m	600 m	600 m	700 m	200 m	350 m
Maximum Onshore Velocity	0.9 m sec ⁻¹	4.3 m sec ⁻¹	6.1 m sec ⁻¹	7.9 m sec ⁻¹	1 m sec ⁻¹	3 m sec ⁻¹
Height of Maximum Velocity	150 m	140 m	150 m	250 m	30 m	100 m
Maximum Return Flow Velocity	1.4 m sec ⁻¹	2.8 m sec ⁻¹	4.1 m sec ⁻¹	4.9 m sec ⁻¹	3 m sec ⁻¹	4 m sec ⁻¹
Height of Maximum Return Velocity	1050 m	1050 m	1250 m	1250 m	1000 m	1000 m
Inland Location of Lake Breeze Front As Given By:						
Onshore Flow	7.5 km	14 km	17 km	24 km	3 km	8 km
Maximum Updraft	6 km	9 km	12 km	12 km		
Temperature Gradient	3 km	6 km	9 km	9 km		

The model time of 2 hours appears to correspond best to the observation time of 1100 EST, despite the fact that the lake and land temperatures became equal at about 0800 EST. This is probably explained by the fact that the lake breeze had to overcome the inertia of a strong land breeze that was poorly simulated in the model.

The depth of the model onshore flow and its intensity clearly exceed the observed values. However, the model temperature field, over land and below 1 km, was about 2°C warmer than that observed at 1400 EST. This is most likely related to the fact that the low-level superadiabatic lapse rate was destroyed beyond 6 km inland, as discussed above. Energy exchanges at the air surface interface are of overriding importance to the model and will therefore be studied more carefully in the future. In particular, a surface energy balance equation should probably be used to determine the surface temperature, as the shelter temperatures are almost certainly inappropriate. The formulation for the vertical exchange coefficient (Equation (3)) is also poor because its value is controlled by the modeler and not by the physics of the problem. Nevertheless, the proper treatment of the cumulative effect of essentially subgrid processes in any dynamic model is an unsolved problem.

The height of the maximum onshore velocity corresponds better than might be judged at first glance, since this value is located at either the first or second grid point above the internal boundary and the model does not have the resolution reported in the observations. The return flow features correspond very well, since both in the model and in the observations the return flow is spread over a deep layer, and the average velocities are similar in value and height.

Table 2 shows that two other computational features give reasonable agreement with the observed frontal position and probably should be used in preference to a requirement that the wind changes direction from onshore to offshore flow at the front. These are the location of the maximum updraft and the location at which the temperature shows a distinct drop in value horizontally across the front. The frontal location suggested by the temperature gradient lags one grid point (3 km) behind that suggested by the updraft location. It should be noted that, between 4 and 5 hours model time, the frontal location suggested by these features remains constant, during which time the onshore flow deepens and intensifies. This is in excellent agreement with the observational feature discussed by both Moroz⁽¹¹⁾ and Olsson⁽³⁾ that the front appeared to pulse inland, rather

Figure 1. Schematic drawing of a physical model of land-water wind circulations showing the discontinuities, the accompanying wind pattern and the temperature lapse rates expected at the several cross sections (after Bierly). (4)

Figure 2. Visual representation of lake breeze model.

Figure 3. Model U component at 3 hours. Units are meters per second.

Figure 4. Model W component at 3 hours. Units are centimeters per second.

Figure 5. Model U component at 5 hours. Units are meters per second.

Figure 6. Model W component at 5 hours. Units are centimeters per second.

Figure 7. Model U component at 6 hours. Units are meters per second.

Figure 8. Observed U component at 1400 EST. Units are meters per second.

than progress inland at a uniform rate. Thus, in several instances, an apparent serious discrepancy between the model and the observations can be resolved to indicate that the model is behaving in a proper manner.

The observed and predicted temperature patterns are similar and the model does develop an inversion offshore, as shown in the following table for model time 6 hours at 3 km offshore.

TABLE 3
MODEL TEMPERATURES 3 KILOMETERS OFFSHORE AT 6 HOURS MODEL TIME

Height	Surface	50 Meters	150 Meters	250 Meters	350 Meters
Temperature	14 C	14.9 C	15.7 C	15.4 C	14.6 C

In summary, to the extent that direct comparisons can be made, the model has a more intense circulation than observed. This can be partly attributed to the higher model temperatures over land. When this fault is corrected, the model should simulate quite well many details of observed land-water wind circulations. Future development and improvement of the model will be supported by the Research and Experimental Branch, Meteorology Division, Desert Test Center in Salt Lake City.

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THE DETERMINATION OF DIFFUSION COEFFICIENTS IN A SHEARING FLOW BY
CHEMICAL TRACER TECHNIQUES

A. V. Quesada
M. A. MacLeod

Air Force Cambridge Research Laboratories
L.G. Hanscom Field, Bedford, Massachusetts 01730

1. INTRODUCTION

An exact solution was recently obtained for the diffusion of an initially spherical, neutral, Gaussian cloud in a one-dimensional constant shear flow.⁽¹⁾ The solution cannot be applied directly to derive upper atmosphere diffusion coefficients because experimentally such coefficients are determined by densitometrically scanning photographic images of the clouds. The images are therefore, projections of the cloud radiance on the photographic film plane. Mathematically, they represent the line of sight integral of the cloud radiance. For spherically symmetric clouds all line of sight integrals are equivalent and to determine the diffusion coefficient it is sufficient to take one fourth of the slope of the line that represents the square of the Gaussian half width as a function of time.⁽²⁾ When the cloud diffuses in a shear flow, the line of sight integration depends on the orientation of the cloud with respect to the observer. It is then, necessary to locate the puff relative to the observer's frame of reference. A simplification of the integration along the line of sight direction occurs frequently in practice because the slant range is large enough to make the lines of sight from different parts of the cloud nearly parallel. This circumstance reduces the integration along the direction of the line of sight to a one dimensional integral that requires for its evaluation a general coordinate rotation and a simple scale adjustment to correct for a slowly varying slant range. The net effect of these operations, under the assumption of an optically thin cloud, is to convert the 3-dimensional expression that describes the number density distribution in the cloud into a two-dimensional optical density distribution in the film plane. The two-dimensional film plane irradiance can be reconstructed using for conversion the film response to exposures through a calibrated step wedge. A sequence of cloud photographs analyzed in this manner may be used to derive values for the diffusion coefficient and the components of the shear. The analysis is based on the remarkable result that the integrated distribution is a Gaussian function for which the sum of the squares of the principal half-widths is a cubic polynomial in the time with coefficients depending on the diffusion coefficient, the shear components and the cloud orientation. This result, applied to data from one observation site, allows the simultaneous determination of the diffusion coefficient and the magnitude of the shear components.

II. DIFFUSION OF PUFF IN SHEAR FLOW

The expansion of a gaseous release that has reached pressure equilibrium with the ambient can be described in the absence of chemical reactions by the equation

$$\frac{\partial n}{\partial t} = \text{div} [D \text{ grad } n - n \vec{u}] \quad (1)$$

where

$n(r,t)$ is the number density of the cloud at time t

D is the appropriate two-particle diffusion coefficient

and

$\vec{u}$ is the ambient wind.

The following assumptions, valid in a large number of practical cases, are introduced to simplify Eq (1)

- (i) the radius of the cloud is small in comparison to the scale length of diffusion
- (ii) the ambient wind is horizontal and varies only with altitude
- (iii) the wind shear is constant.

Assumptions (ii) and (iii) lead to an ambient wind of the form

$$\vec{u} = \hat{x}(u_{ox} + S_x z) + \hat{y}(u_{oy} + S_y z) = \vec{u}_0 + \vec{S}z \quad (2)$$

The translation $\vec{r}' = \vec{r} + \vec{u}_0 t$ combined with a horizontal plane rotation through an angle $\theta = \arctan(S_y/S_x)$ refers the diffusion process to a coordinate system in which the center of the puff is at rest and the new x-axis points along the direction of the shear vector [Fig. 1]. A further simplification results from the introduction of dimensionless variables

$$v = n/n_0, \quad x' = x/a, \quad y' = y/a, \quad z' = z/a, \quad \tau = tD/a^2$$

and the dimensionless parameter

$$\mu = Sa^2/D.$$

Dropping the primes the non-dimensional equation reads

$$\frac{\partial v}{\partial \tau} = \nabla^2 v - \mu z \frac{\partial v}{\partial x} \quad (3)$$

This equation was solved in Ref. (1) for a spherically symmetric initial distribution of Gaussian character,

$$v(r, 0) = \exp(-r^2) \quad (4)$$

where $r^2 = x^2 + y^2 + z^2$ and boundary conditions are imposed that require $n(r, t)$ and its first derivatives to approach zero for $r \rightarrow \infty$ and $t \geq 0$.

The solution of Eq. (3) that reduces to Eq. (4) at $t = 0$ is

$$v(x, y, z, \tau) = \frac{\exp[-v^2/(1+4\tau)] \exp[-F/G]}{\sqrt{(1+4\tau)G}} \quad (5)$$

where

$$F = v^2(1+4\tau) + 4\mu v z \tau^2 + z^2(1+4\tau + \frac{4}{3}\mu^2 \tau^3)$$

$$G = (1+4\tau)^2 + \frac{4}{3}\mu^2 \tau^3(1+\tau)$$

$$v = x - \mu z \tau$$

This solution has a number of interesting properties.

(i) The limiting form for zero shear ($\mu \rightarrow 0$) is the well known spherically symmetric Gaussian cloud.

(ii) The surfaces of constant density are generally triaxial ellipsoids. This is immediately apparent when the quadratic form in the exponent of Eq. (5) is referred to principal axes. In these circumstances the equation takes the canonical form

$$Q = \beta_+ x'^2 + \beta_- y'^2 + \beta_- z'^2 = -\log_e(v \sqrt{G/\beta}) \quad (6)$$

where

$$\beta = 1/(1+4\tau)$$

$$\beta_+ = [1/\beta + \mu\tau [\mu\tau(1 + \frac{4}{3}\tau) \sin^2\sigma - (1+2\tau) \sin 2\sigma]] / G$$

$$\beta_- = [1/\beta + \mu\tau [\mu\tau(1 + \frac{4}{3}\tau) \cos^2\sigma + (1+2\tau) \sin 2\sigma]] / G$$

$$\tan 2\sigma = (1+2\tau) / \left(\frac{\mu\tau}{2} (1 + \frac{4}{3}\tau) \right)$$

(iii) The major and minor axes of the ellipsoids lie in the vertical plane that contains the shear vector; the third axis, intermediate in length between these two, lies along the y-axis and is, therefore, perpendicular to the shear.

(iv) The inclination σ of the X-axis to the horizontal plane (the x, y-plane) decreases as time progresses. For positive shear ($\mu > 0$) the initial value of σ is $\pi/4$, while for negative shear ($\mu < 0$) it is $-\pi/4$, provided Eq. (12) is interpreted as defining the principal value of 2σ i.e., $-\pi/2 < 2\sigma < \pi/2$. This behavior is shown schematically in Fig. 2.

(v) At late times the center point density varies inversely as the shear and as the $-5/2$ power of the time. More generally, the variation of the center point density as a function of time is shown in Fig. 3.

III. INTEGRATION OF DENSITY DISTRIBUTION ALONG AN ARBITRARY DIRECTION

To integrate Eq. (6) along lines parallel to some arbitrary direction it is advantageous to refer the quadric Q to a coordinate system in which the arbitrary direction corresponds to the direction of say, the y-axis. This involves performing a general rotation of the axes about the origin, generated by an orthogonal matrix of elements c_{ij} that satisfy the conditions

$$\sum_j c_{ij} c_{kj} = \delta_{ik} \quad \text{and} \quad \sum_j c_{jk} c_{jm} = \delta_{km} \quad (7)$$

Details of the integration are found in a forthcoming report.⁽³⁾ The resultant two-dimensional distribution is

$$N(x, z, \tau) = \sqrt{\frac{\pi}{GB}} \left[\exp \left(-x^2 \left(A - \frac{D^2}{B} \right) - z^2 \left(C - \frac{F^2}{B} \right) - 2xz \left(E - \frac{DF}{B} \right) \right) \right] \quad (8)$$

where

$$A = \beta_+ c_{11}^2 + \beta_- c_{21}^2 + \beta_- c_{31}^2$$

$$B = \beta_+ c_{12}^2 + \beta_- c_{22}^2 + \beta_- c_{32}^2$$

$$C = \beta_+ c_{13}^2 + \beta_- c_{23}^2 + \beta_- c_{33}^2$$

$$D = \beta_+ c_{11} c_{12} + \beta_- c_{21} c_{22} + \beta_- c_{31} c_{32}$$

$$E = \beta_+ c_{11} c_{13} + \beta_- c_{21} c_{23} + \beta_- c_{31} c_{33}$$

$$F = \beta_+ c_{12} c_{13} + \beta_- c_{22} c_{23} + \beta_- c_{32} c_{33}$$

The Gaussian width of this distribution is given by the new quadratic form

$$Q' = x^2 \left(A - \frac{D^2}{B} \right) + z^2 \left(C - \frac{F^2}{B} \right) + 2xz \left(E - \frac{DF}{B} \right) = 1 \quad (9)$$

The two independent invariants of this form are

$$(1/a_+^2) + (1/a_-^2) = A + C - (D^2 + F^2)/B \quad (10)$$

and

$$1/(a_+^2 a_-^2) = AC - E^2 + (2DEF - AF^2 - CD^2)/B \quad (11)$$

where a_+ and a_- denote the semi-major and semi-minor axes of the ellipse represented by Eq. (9). Considerable simplification of these expressions can be accomplished by resorting to the orthogonality conditions Eqs. (7). A rather lengthy algebraic reduction leads to the remarkably simple relation

$$a_+^2 + a_-^2 = 2/\beta + \mu\tau q [c_{32}^2 - c_{12}^2 + (1 + c_{22}^2) \cos 2\tau] \quad (12)$$

where

$$q^2 = (1 + 2\tau)^2 + \left(\frac{\mu\tau}{2}\right)^2 \left(1 + \frac{4}{3}\tau\right)^2$$

This equation will be written in the next section as a function of the diffusion coefficient, the wind shear and the parameters that define the cloud orientation with respect to the observer.

IV. TRANSFORMATION TO THE OBSERVER'S FRAME OF REFERENCE

Let us refer Eq. (6) to a system of coordinates (the observer system) whose y-axis points along the line connecting its origin to the center of the ellipsoid. The relative orientation of the principal axis system and the observer system can be specified by means of three angles that define rotations about a y, z and x-axis, respectively. These angles, denoted by σ , θ and φ , are shown in Fig. 4. We recognize σ as the angle between the long axis of the ellipsoid and the horizontal plane, θ as the angle between the vertical plane that contains the shear vector and the vertical plane that contains the line of sight and φ as the elevation angle of the line of sight.

The rotation matrix elements that appear in Eq. (12) are then

$$\begin{aligned} c_{12} &= -\sin \sigma \sin \varphi - \cos \sigma \cos \varphi \sin \theta \\ c_{32} &= -\cos \sigma \sin \varphi + \sin \sigma \cos \varphi \sin \theta \\ c_{22} &= \cos \varphi \cos \theta \end{aligned}$$

Substituting these values in Eq. (12) we obtain the non-dimensional expression

$$a_+^2 + a_-^2 = 2(1 + 4\tau) + \mu\tau \left[\mu\tau \left(1 + \frac{4}{3}\tau\right) (1 - \sin^2 \theta \cos^2 \varphi) - (1 + 2\tau) \sin \theta \sin 2\varphi \right] \quad (13)$$

To recover the corresponding dimensional relation we recall that

$$\begin{aligned} 1 + 4\tau &= 1 + (4tD)/a^2, & a &= \text{initial radius of the cloud} \\ \mu\tau &= tS \end{aligned}$$

Arranging the right hand side member in increasing powers of the time we find that the sum of the squares of the principal Gaussian half-widths is a general cubic in t , namely,

$$\begin{aligned} a_+^2 + a_-^2 &= 2a^2 + t[8D - a^2 S \tilde{\varphi}] + t^2[a^2 S^2 \Omega - 2DS \tilde{\varphi}] + \\ &\quad + \frac{4}{3}t^3 S^2 D \Omega \end{aligned} \quad (14)$$

where

$$\tilde{\varphi} = \sin \theta \sin 2\varphi, \quad \Omega = 1 - \sin^2 \theta \cos^2 \varphi$$

Eq. (14) is displayed in Fig. 5 for the single values of a, φ , θ and D and the various values of S identified in the diagram.

V. EVALUATION OF D, S AND θ FROM SINGLE SITE DATA

Equation (14) indicates that the quantity $a_+^2 + a_-^2$ grows as a cubic polynomial function of the time. This means that in general we can write

$$a_+^2 + a_-^2 = A + Bt + Ct^2 + Et^3 \quad (15)$$

If such behavior is assumed to be present in the measured sum of principal Gaussian half-widths at different times, knowledge of the experimentally determined values of the coefficients in Eq. (15) can be used to solve for the parameters that appear in Eq. (14).

Measurements on photographs of the expanding cloud are taken at a number of known times. Then, a best estimate of the values of the coefficients is obtained by application of a least squares procedure. Note, however, that since there are four coefficients in Eq. (15) and five parameters in Eq. (14) one of the parameters must be independently evaluated before we can solve for the remaining parameters. It can be readily seen that the elevation angle φ can be independently measured from the observation site. From Eqs. (14) and (15) and letting $z = 4D$ we have

$$S^2 (1 - \sin^2 \theta \cos^2 \varphi) = 3E/z$$

$$zS \sin \theta \sin 2\varphi = -2C + (3AE/z)$$

Substituting these values in

$$8D - a^2 S \sin \theta \sin 2\varphi = 3$$

we obtain a cubic to determine z ,

$$z^3 - \frac{B}{2} z^2 + \frac{AC}{2} z - \left(\frac{A}{2}\right)^2 E = 0 \quad (16)$$

Since A and E are positive quantities, it can be shown readily that Eq. (16) must have (at least) one positive real root. Once z is found the remaining parameters follow from the relations

$$S^2 = \frac{3E}{z} + \left(\frac{3AE - Cz}{z^2 \sin \varphi} \right)^2 \quad (17)$$

and

$$\cos^2 \varphi \sin^2 \theta = 1 - 3E/(zS^2) \quad (18)$$

Eqs. (17) and (18) determine the magnitude of the shear vector and the sine of the angle θ . Thus single site data determines the magnitude of the shear vector and the line along which it acts, but not its direction. This simple reflection ambiguity is easily resolved by utilizing photos from a second site to establish the orientation of the major axis. Geometrically, the ambiguity is a consequence of the impossibility, when presented with an orthogonal projection of an ellipsoid onto a plane, to decide whether the top or the bottom of the ellipsoid is closer to the observer.

The present technique was tested by applying it to data from an experiment carried out at Eglin AFB, Florida on 14 May 1969. Boron dioxide puffs deposited in the lower thermosphere by rocket were photographed from three sites and the Gaussian radii were derived by densitometric techniques from the film exposed at one site. These were then combined to give the values of $(a_+^2 + a_-^2)$ (shown for one puff in Fig. 6) from which the values for (D, S, θ, α) were derived by the least squares technique outlined above. In order to test the capabilities of this method, it is desirable to have independent measures of these parameters. This is possible for the shears, but not for the diffusion coefficient of the initial Gaussian radius. Thus the validation of the analysis must rest on the consistency of the two sets of experimentally derived shears and on a comparison of the experimental D values with those calculated from the appropriate standard

atmosphere. These comparisons are not straightforward for this experiment for a number of reasons. Among the complicating factors are the non-uniformity of the shears over the puffs, the presence of short-period variations in the puff positions, and concurrent probable short-period fluctuations in the temperature and density. Nonetheless, vertical shears were derived from a cubic spline fit made to each of the wind components as a function of altitude using data from five puffs released on the ascent and descent portions of the rocket trajectory. These values are shown in the accompanying Table. The error bounds quoted represent the standard

Table
Comparison of diffusion coefficients and shears calculated by the three techniques described in the text.

Puff Altitude	Technique	D (km^2/sec)	$\bar{u}$ ($\text{m}/\text{km sec}$)	$\bar{v}$ (deg.)
117 km	Conventional	.00391 (long.)	8.9 $\pm$ 3.0	35 $\pm$ 19
		.00083 (trans.)		
		.00237 (mean)		
	This paper	.00102		
	USDA 1966	.00090		
122 km	Conventional	.00324 (long.)	7.3 $\pm$ 2.9	-57 $\pm$ 17
		.00103 (trans.)		
		.00214 (mean)		
	This paper	.00136		
	USDA 1966	.00094		

deviations of the spline shears over an altitude range of 2 km centered on the mean puff positions. Corresponding error bounds were not calculated for the present analysis, but the agreement is seen to be reasonably good. Three of the four values fall within the error bounds quoted, while the fourth is within the 1.5 σ range. The H_2 -air mutual diffusion coefficients derived by the present technique are higher than those found by calculation from the 1966 Supplement to the U.S. Standard Atmosphere by a factor of 2 (at 117 km) and a factor of $\frac{1}{2}$ (at 122 km), but are a factor of 2 lower than the mean of the longitudinal and transverse diffusion coefficients calculated in the usual manner (see Golomb and Macleod⁽²⁾). The large deviations from the USDA 1966 values appear to be due to the short-period waves that are clearly present in the data, but further work is necessary to resolve this question.

VI. CONCLUSIONS

Shear of the ambient wind has been shown to be capable of modifying substantially the values of diffusion coefficients derived from the observation of contaminant clouds in the atmosphere. A relatively simple procedure has been outlined for the experimental determination of the shears and the true diffusion coefficient in the presence of vertical shear of a horizontal wind. Densitometric analysis of the data is only required from one site. The technique was illustrated by applying it to the analysis of an experiment carried out in the lower atmosphere.

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VIII. REFERENCES

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Figure 1. Orientation of coordinate systems. Vector u_0 centered on vector is randomized uniformly in (u,v) plane

Figure 2. Cloud development in each plane for $n = 1.0$. Elliptic axes and eccentric radii of given instant

Figure 3

Figure 4. Orientation of cloud with respect to the observer

Figure 5

Figure 6