9/13

2

NOVEMBER 1972

MATT-943

ANOMALOUS ABSORPTION OF
RADIATION BY PLASMAS

BY

JOHN M. DAWSON

RECEIVED BY TIC JAN 4 1973

PLASMA PHYSICS
LABORATORY

MASTER

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

This work was supported by U. S. Atomic Energy Commission Contract AT(11-1)-3073. Reproduction, translation, publication, use, and disposal, in whole or in part, by or for the United States Government is permitted.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Anomalous Absorption of Radiation by Plasmas*†

John M. Dawson

Plasma Physics Laboratory, Princeton University,
Princeton, New Jersey 08540

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

ABSTRACT

A discussion of results from theory and computer simulation of a number of topics relevant to the anomalous absorption of radiation by plasma is given. Among such topics are the following: (1) When intense laser radiation falls on a plasma whose plasma frequency is near the laser frequency, it can cause parametric instabilities in which ion density fluctuations and electron plasma waves grow to a saturation level determined by nonlinear processes such as wave scattering and particle trapping. The turbulence generated leads to a strong anomalous absorption. Such absorption deposits most of the energy in the electrons and often leads to highly energetic tails on the electron distribution function. A similar type of enhanced absorption can be produced by two laser beams whose beat frequency is near the plasma frequency. (2) Enhanced ion fluctuations can also lead to anomalous absorption of radiation when the transit time of a thermal electron across a typical ion fluctuation is of the order of the period of the electromagnetic wave. While the enhanced ion density fluctuations might be produced externally or by other

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

instabilities the process giving rise to anomalous absorption tend to amplify them (this is the kinetic instability of Silin¹), and thus to maintain their level. Enhanced absorption of this kind leads to a heating of the main body of the electrons and not to the energetic tails produced by resonant absorption. This form of enhanced absorption can be effective for laser frequencies up to ten times the plasma frequency. (3) Intense laser radiation can interact with enhanced ion fluctuations to produce dc electric currents. Such currents could generate intense magnetic fields which can influence the absorption process.

I would like to discuss some of our work on anomalous ac resistivity which is related to anomalous absorption of electromagnetic radiation by a plasma. I shall first review some of our earlier¹ work, and then discuss some of our more recent results.

Almost ten years ago it was realized that plasmas could exhibit anomalous resistance for frequencies near the plasma frequency.² The cause of this resistance was the generation of plasma waves through the interaction of the oscillating electrons with the ions. Each ion acted as a source for generating plasma waves. If the ions are randomly distributed as they are for thermal equilibrium then the waves generated by different ions are randomly phased and only weak waves are generated which enhance the resistance by only a few per cent. However, if the ions are not randomly distributed, if there are ion waves present, then the waves can add up coherently so as to give a large response. In principal the wave contribution could be enhanced by $n\lambda^3$ for very well ordered ions. This number is perhaps 10^9 so even rather mild ion waves can enhance the resistance by many orders of magnitude.

The problem is how to get the ion waves. DuBois and Goldman³ showed that in generating the plasma oscillation the ion waves themselves are amplified. The momentum given to the plasma wave must be balanced by an equal and opposite momentum given to the ions which amplifies the ion wave. Thus, the ion waves and the plasma oscillations grow together. If the driving intensity is sufficiently large so that this growth exceeds the natural damping

then the ion fluctuation and plasma oscillations grow together - the system is unstable. The growth is ultimately halted by nonlinear effects and a turbulent state exists which gives rise to the anomalous damping.

I might give some simple derivations of these effects. Let us consider an electron plasma oscillation and an ion acoustic wave of the same wave number. Without the driving field they satisfy the equations

$$\frac{\partial^2 n_{ek}}{\partial t^2} + \omega_{ek}^2 n_{ek} = 0, \quad (1)$$

$$\frac{\partial^2 n_{ik}}{\partial t^2} + \omega_{Ac}^2 n_{ik} = 0. \quad (2)$$

If we now subject the plasma to a long wavelength electric field oscillating at frequency ω_0 these two modes become coupled.

$$\frac{\partial^2 n_{ek}}{\partial t^2} + \omega_{ek}^2 n_{ek} = \frac{ikn_{ek} eE_0}{m} \sin \omega_0 t, \quad (1a)$$

$$\frac{\partial^2 n_{ik}}{\partial t^2} + \omega_{Ac}^2 n_{ik} = \frac{ikn_{ek} eE_0}{M} \sin \omega_0 t. \quad (2a)$$

It is easy to see where the coupling term in the first equation comes from.

Consider the plasma to have an ion density fluctuation (Fig. 1) so that

$$n_i = n_0 + n_i(x). \quad (3)$$

The driving E field tends to displace the electrons relative to the ions. If there were no self-consistent restoring force (shielding) this would give the electron charge density

$$\delta n_e = \frac{\partial n_e}{\partial x} \Delta x = \frac{\partial n_e}{\partial x} \frac{e E_o}{m \omega_o^2} \sin \omega_o t \approx \frac{\partial n_i}{\partial x} \frac{e E_o}{m \omega_o^2} \sin \omega_o t. \quad (4)$$

You can see that if n_i is taken to have an e^{ikx} dependence and we drop the ω_{ek}^2 term from Eq. (1) (this term gives the shielding) then Eq. (4) results.

Including the ω_{ek}^2 term gives the effects of shielding.

The derivation of Eq. (2) is more subtle, but the following arguments will make it plausible. The coupling force comes from the pressure associated with the electric field. The total electric field is

$$E = [E_o \sin \omega_o t + E_e(x)]. \quad (5)$$

The electric field pressure is

$$\frac{E^2}{8\pi} = \frac{E_o^2 \sin^2 \omega_o t + 2 E_o E_e(x) \sin \omega_o t + E_e^2(x)}{8\pi}, \quad (6)$$

Now the force is the gradient of this pressure or

$$\frac{\partial E^2/8\pi}{\partial x} = \frac{E_o \partial E_e / \partial x \sin \omega_o t}{4\pi} = -E_o e \delta n_e(x) \sin \omega_o t. \quad (7)$$

We have neglected the $E_e^2(x)$ term as being small. Since δn_e will have a high frequency part which can beat with the $\sin \omega_o t$ term, there is a low-frequency force which the ions must support. This force is very much like the term appearing in the ion equation and in fact, if in the derivation of the equation for the ion density we include this force, we get just the coupling term shown in Eq. (2a).

If one looks for solutions to Eqs. (1) and (2) that go like

$n_e = n_e e^{i(\omega \pm \omega_o)t}$, $n_i = n_i e^{i\omega t}$, one finds the dispersion relation

$$(\omega^2 - \omega_{Ac}^2)[\omega^2 - (\omega_o - \omega_{ek})^2] + \frac{k^2 e^2 E_o^2}{4mM} \frac{(\omega_o - \omega_{ek})}{\omega_{ek}} = 0 \quad (8)$$

This equation predicts two branches of instability for $\omega_o \gtrsim \omega_p$ as illustrated in Fig. 2. In the model we have taken there is no threshold for instability since we have included no damping. If we had included damping then there is a threshold below which the waves do not grow.

We have done numerical simulation of this instability to check the linearized theory to determine the saturation amplitude and to determine the magnitude of the anomalous resistances.¹ The early stages agree well with the linearized theory. There appear to be a number of nonlinear mechanisms which saturate the wave. If E_o is not too large then the waves grow until $\langle E_W^2 \rangle^{1/2} \approx E_o$; at this point the unstable waves can cause secondary instabilities which are as strong as the primary one. Recent work by Valeo and Kruer⁴ shows this nicely. The waves can also scatter off the ion fluctuation which has been produced, thus robbing the unstable waves of energy. Ultimately, the energy must end up in particles. For this region, this seems to be accomplished by a quasi-linear diffusion of particles into the region of unstable waves. This gives some enhanced Landau damping of the waves which also helps limit their amplitude.

For strong E_o the unstable waves grow so large that they can actually reach back and pull particles out of the distribution function. This value is given by¹

$$\frac{e}{m} \left(\langle E_W^2(k)/k^2 \rangle \right)^{1/2} \approx \frac{m}{2} (v_p - 2v_T)^2. \quad (9)$$

Because the number of particles that can be trapped rises very rapidly with E_W , it is almost impossible to push the unstable waves above this level.

One can compute the heating rate by computing the rate at which energy is fed into the unstable waves assuming it is given by the formula which applies to the initial stages of the instability $(\langle 2\gamma(k) W(k) \rangle)^{1/2}$. That is, we assume that the only effect of the nonlinearity is to provide a mechanism for extracting energy from the unstable modes. For both the above cases, this gives an effective collision frequency of

$$\nu^* = \frac{4 \langle \gamma E_W^2 \rangle}{E_o^2}. \quad (10)$$

One of the characteristics of this type of heating is that it produces energetic tails on the distribution function. Work at Livermore by De Groot and Katz⁵ indicates this tail goes like $1/v^2$. Some simple quasi-linear calculations by Kruer⁶ show that such a form is just what is required to stabilize the long-wavelength modes by Landau damping. Of course, the distribution cannot go as $1/v^2$ all the way to infinity, because such a distribution has infinity energy. Long wavelengths suffer from enhanced resistive damping due to the enhanced ion fluctuations. It is the same process which extracts energy from the driver. This damps the energy in long-wavelength modes and cuts off the $1/v^2$ at about $10 \sim 15 v_T$. By and large,

most of the above energy goes to the electrons. Very little goes to the ions, although they also develop energetic tails which go out to a couple of times the acoustic velocity.

Using the above parametric instability has the disadvantage that it deposits the energy in a relatively energetic tail where it is unavailable to the rest of the plasma. One would like to deposit the energy in the main body of the distribution, if possible, or failing in that at only a few v_T . This led us to investigate a number of possibilities for accomplishing this. The first of these I call quasi-resonant mode coupling.⁷ This is a mechanism for transferring energy from plasma waves with high-phase velocity to the ones with lower phase velocity. This mechanism works as follows:

We consider a plasma in which we have an ion-density fluctuation and a long-wavelength plasma oscillation

$$n_i = n_o + \delta n_i \cos kx ,$$

$$E = E_o \exp(i\omega_p t)$$

Since the ions are heavy, we consider them immobile, although, we could allow them to move if we desired. We can now compute the response of the electrons to the interaction of the plasma oscillation with the ion density fluctuations. We can use our Eq. (1), and we get

$$n_{ek} = - \frac{eE_o}{m} \frac{k\delta n_i \exp(i\omega_p t)}{(\omega_p^2 - \omega_{ek}^2)} \cos kx . \quad (11)$$

The associated electric field is given by

$$\left. \begin{aligned} \frac{\partial E_r}{\partial x} &= -4\pi e n_e = \frac{4\pi e^2 n_o}{m} \frac{k E_o \delta n_i / n_o}{(\omega_p^2 - \omega_{ek}^2)} (\cos kx) \exp(i\omega_p t) \\ E_r &= \frac{\omega_p^2}{\omega_p^2 - \omega_{ek}^2} \frac{\delta n_i}{n_o} E_o (\sin kx) \exp(i\omega_p t) \end{aligned} \right\} \quad (12)$$

We see that the interaction with the ions leads to a standing wave which can be decomposed into the two traveling waves

$$\exp[i(\omega_p t + kx)] , \quad \exp[i(\omega_p t - kx)] .$$

The amplitude of the response wave is

$$E_r = \frac{\omega_p^2}{\omega_p^2 - \omega_{ek}^2} \frac{\delta n_i}{n_o} E_o . \quad (13)$$

Now, $\omega_{ek}^2 = \omega_p^2 (1 + 3k^2 \lambda_D^2)$ so this becomes

$$E_r = \frac{\delta n_i / n_o}{3k^2 \lambda_D^2} E_o . \quad (14)$$

If $\delta n_i / n_o$ is as large as $k^2 \lambda_D^2$ then E_r is as large as E_o . In this case all the energy must be transferred from the primary wave to the response, and our treatment is inconsistent; we must include the change in E_o also.

P. Kaw⁸ has carried out such an analysis and he finds

$$\left. \begin{aligned} \epsilon &= \delta n_i / n_o \\ \text{for } \epsilon / \frac{3}{8} k^2 v_T^2 &\gg 1 \\ E(nk, t) &\propto J_n(\epsilon \omega_p t) \end{aligned} \right\} \quad (15)$$

while for $\epsilon / \frac{3}{8} k^2 v^2 \ll 1$

$$\left. \begin{aligned} E(0, t) &= E_0 \cos \omega_p t + \frac{1}{18} \left[\left(\frac{\delta n}{n} \right)^2 \frac{\cos(\omega_p + \Delta \omega) t}{(k^2 \lambda_D^2)^2} \right] \\ \Delta \omega &= \omega_p \frac{3k^2 \lambda_D^2}{2} \end{aligned} \right\} \quad (16)$$

We have simulated this coupling for a number of cases and the results are shown in Figs. 3 - 10. Figures 3 to 5 show results for a case in which $\delta n/n$ is much larger than $k^2 \lambda_D^2$. Figure 3 shows the field oscillations in the primary wave and we see that it is being damped by the coupling to other modes. Figure 4 shows the field oscillations in mode $k - k_0$. We see that this rises as energy is transferred from the primary mode to it, and then it is damped as it feeds energy into higher modes. Figure 5 shows the field oscillations for mode $2k - k_0$. They also rise and then are weakly damped. These results are in very good agreement with the analytic calculations. Figures 6 - 8 show similar results for a case where $k^2 \lambda_D^2$ is comparable to $\delta n/n$. In this case the energy transferred to the short wavelength modes is less, but it is still quite considerable. If the shorter wavelength modes ($2k - k_0$, $3k - k_0$) are Landau damped, then the energy can be fed directly to the particles. W. Kruer has simulated this effect

and has shown that very efficient energy transfer to the main body of the electrons can occur by this mechanism. Kruer's results are shown in Figs. 9 and 10.

Now it is possible to choose k , so that the response wave is directly damped. This happens if $\pm\omega/k$ falls in the electron distribution function as shown in Fig. 11. In this case the response wave is directly Landau damped which leads to a damping of the primary wave (detailed calculations are contained in the Appendix). In this way we can couple a wave with high-phase velocity to one with low-phase velocity, and hence to the plasma particles. The long-wavelength wave suffers enhanced damping.

For plasma heating, how could we get the ion fluctuations? We can get them through the parametric instability of DuBois and Goldman.³ However, in that case the phase velocity of the beating waves is so fast that long energetic tails are pulled out on the electron distribution function and this is what we should like to avoid. One possibility is that the ion waves are fossil waves, that is, waves generated in an early stage of the heating and which still persist. If the waves were generated by parametric instabilities in the early stages of laser heating when the temperature were low (say 100 eV) then the dominant wavelengths will be relatively short with

$k\lambda_D$ (cold) ≈ 0.1 . As the plasma heats up the $k\lambda_D$ for these waves becomes much larger $[k\lambda_D$ (hot)] and the beat disturbance has phase velocity ω_p/k only a few times v_T . Such ion waves should live a long time if the electrons become hot relative to the ions since their Landau damping becomes small. In addition the absorption itself leads to amplification of the ion waves so that it tends to maintain them (this is the same mechanism which leads to the growth of the Silin⁹ instability for $\omega \gg \omega_p$, also see Appendix). It might also be possible to generate the ion fluctuations by a separate method as for example by passing dc current through the plasma, or by exciting sound waves in the plasma before one starts to strongly heat it.

There are a number of other intriguing possibilities for anomalous absorption. It is possible for it to lead to negative absorption, i.e., to amplification. To achieve this consider the situation shown in Fig. 12. Here we have drifted the electrons relative to the ions. We have also imposed upon the ions a sinusoidal density fluctuation of wave number k . We now consider imposing an oscillating electric field on the plasma of very long wavelength and frequency ω . The interaction with the ion fluctuation produces beat disturbances at $\pm \omega/k$. For the situation shown in Fig. 12, the Landau damping of the ω/k disturbance is negative (more fast electrons are slowed down than slow ones are accelerated). Thus, energy is fed into

the beat disturbance and then back into the driving wave leading to its amplification. Since there are none, or at most very few particles at the $-\omega/k$ disturbance, this disturbance plays little role (it would give a small positive absorption). Detailed calculations of this are found in the Appendix.

We have simulated this process on our one-dimensional plasma model. We have considered two situations; in the first the electrons are drifting through the ions with a velocity large compared to their thermal velocity (cold electron beams). In the second case, the drift velocity is three times the thermal velocity. For both cases the ion to electron mass ratio was 10^4 . For the first case the beam velocity satisfies the condition $k_i v_o = 2\omega_p$, where k_i is the wave number of the ion fluctuations. The waves which are amplified are long-wavelength plasma oscillations. They are coupled by the ion fluctuations to short-wavelength negative energy plasma oscillations of the beam (waves propagating slower than the beam). The short-wavelength negative energy waves and the positive energy waves grow together conserving energy. The beam is slowed down providing the energy and the ions take up the momentum. The growth rate of the instability was equal to $\gamma = (\omega_{pe}/2)(\delta n_i/n_o)$.

Figures 13-18 show the results from simulating this case. Figure 13 shows the growth of the electrostatic energy. You see it grows quite rapidly and saturates at about $\omega_p t = 130$. Figures 14 and 15 show the growth of the electric field energy in modes 1 and 3 (1 wavelength and 3 wavelengths in the

system; the initially excited ion mode was 16). Figure 16 shows the decay of the current. At first, there is no decay which is followed by a rapid decay and then a steady decay of the current. During this run the current decreases by about 20%. Figure 17 shows the velocity distribution at the end of the run, while Fig. 18 shows a portion of the electron phase space. One can see that a large number of particles have been slowed down supplying the energy for the instability. However, it is also interesting to note that a number of particles have been pulled out to very high velocity, a few to even three times the initial beam velocity. These particles are accelerated by the unstable waves with high phase velocity.

The second case which was run was that in which the electrons had a drift velocity which was 3 times their thermal velocity. The ion density fluctuation which was initially excited had $k_i v_b = 1.5 \omega_p$. The beat disturbance between long-wavelength plasma oscillations and the ions had a phase velocity of $0.66 v_b$, so that they fell well within the distribution, and the instability was caused by negative Landau damping.

Figures 19-22 show some of the results of these simulations. Figure 19 shows the growth of the electric field energy. The growth is somewhat slower here, but is still substantial and it saturates at $\omega_p t \approx 600$. Figure 20 shows the growth of mode 1; it is typical of the growth of many long-wavelength modes. Figure 21 shows the decay of the current. The decay is steady and the total decay is about 35% over the length of the run. Figure 22 shows the distribution function at the end of the run. The plateau at the velocity of the beat wave is quite clear.

While the amplified waves in these simulations were longitudinal plasma oscillations the same mechanism can be used to amplify electromagnetic waves. In that case ω would be the frequency of the EM wave, and need not be the plasma frequency. In fact it would probably be best to work with frequencies sufficiently separated from the plasma frequency so that interference from the growth of longitudinal plasma oscillations is not encountered. However, amplification of very high frequencies would require very short wavelength ion fluctuations so that the mechanism could probably be made to work best at around $2\omega_p$. To maintain the ion fluctuations one will want the electrons hotter than the ions. The current can be induced by applying sufficient E field to the plasma. One must worry about the competition of this growth process with the growth of ion acoustic instabilities. It appears possible to make this growth exceed that of the ion acoustic instability. One can gain something also by preferentially exciting the mode one wants to grow.

Finally, I should like to discuss still another phenomenon which may be encountered in laser discharges. This is the generation of dc currents by ac fields. The mechanism is similar to that just discussed. Consider the situation shown in Fig. 23. Again we have a plasma containing ion density fluctuation of wave number k , and subjected to an oscillating electric field of frequency ω , and very long wavelength. For the situation shown, the two beat waves are Landau damped and accelerate slower electrons to higher positive and negative velocities. By the symmetry of the situation no net current is generated. Now, let me consider the situation shown in Fig. 24.

The electrons are now drifting through the ions with positive velocity. Now the beat waves with positive phase velocity are more heavily Landau damped than are those with negative phase velocity. Thus, more electrons are accelerated on the positive side than on the negative side, and this enhances the current. (A derivation of this effect is contained in the Appendix.)

We simulated this effect. The situation considered had the electrons initially drifting over the electrons at $1/2$ the thermal velocity, the frequency of the high-frequency wave was $2\omega_p$, the oscillating velocity was $5 v_T$, and ion waves with a range of wave numbers between $k\lambda_D = 0.4-0.8$.

Figures 25-27 show the results of these simulations. Figure 25 shows the ratio of the electron drift velocity to the thermal velocity. It starts with a mean value of $v_T/2$. You can see that the mean value increases with time, the total increases over the run being about $0.1 v_T$. Figure 26 shows the increases in energy of the plasma as the electrons are accelerated and heated by the beat wave. A large fraction of the increase in energy is accounted for by the increase in the mean streaming velocity of the electrons. Finally, Fig. 27 shows the electron distribution fluctuation at the end of the run. You can see how the electrons have pulled out on the positive side.

This calculation brings up the question of whether a plasma containing strong ion fluctuations and subjected to a strong ac field is unstable to the development of a current since here the current is amplified. However, to answer this question one must solve for the fully self-consistent electron

distribution function. This is probably most simply done by numerical simulation and we are looking into this at the present time.

If an ac field does produce dc drifts of even a small fraction of the thermal velocities, extremely large B fields would be produced which might greatly affect the laser discharge.

APPENDIX

The Interaction Between an Oscillating Electric Field and Ion Density Fluctuations

Here we consider the interaction produced in a plasma between a long-wavelength electric field oscillating at frequency ω_0 and ion density fluctuations of wave number k . This interaction can give rise to parametric instabilities of plasma oscillations and ion waves to anomalous absorption, and dc currents.

We take the ion density to be given by

$$n_i = n_0 + \delta n_i \cos kx = n_0 + \delta n_i \operatorname{Re} \exp(ikx) \quad (\text{A1})$$

and the driving electric field to be given by

$$E_0 = E_0 \sin \omega_0 t = E_0 \operatorname{Im} \exp(i\omega_0 t) . \quad (\text{A2})$$

We shall describe the electrons in terms of dielectric functions of k and ω ; thus, we will include both the Vlasov and fluid approximations for electrons.

As in earlier work we work in the frame oscillating with the electrons.²

Thus, we make the transformation

$$\left. \begin{aligned} \xi &= x - \epsilon \sin \omega_0 t \\ u &= v - \omega_0 \epsilon \cos \omega_0 t \\ t &= t \\ \epsilon &= eE_0 / m\omega_0^2 . \end{aligned} \right\} \quad (\text{A3})$$

Here ξ, u are positions and velocities relative to coordinates oscillating with the electrons while x, v are those relative to a frame at rest with

respect to the ions. For the time being we shall consider the ions as fixed although we shall later allow them to respond to low frequency forces. We consider only the one-dimensional case here; the more general case can be built up by allowing a spectrum of ion waves with different directions for k . As shown in Ref. 2, the ion density in the oscillating frame is given by

$$n_i(\xi) = n_o + \delta n \cos[k(\xi + \epsilon \sin \omega_o t)] . \quad (A4)$$

Expanding in ϵ and keeping the lowest order gives

$$n_i = n_o + \delta n \operatorname{Re} \exp(ik\xi) + \frac{k\epsilon\delta n}{2} \operatorname{Re} \left\{ \exp[i(k\xi + \omega_o t)] - \exp[i(k\xi - \omega_o t)] \right\} . \quad (A5)$$

Now, as shown in Ref. 11, the ion charge can be considered a source charge for the electric field; the electric field is given by

$$ikD(k, \omega)E(k, \omega) = 4\pi\rho_s(k, \omega) , \quad (A6)$$

$$E(k, \omega) = 4\pi\rho_s(k, \omega) / ikD(k, \omega) ,$$

where $D(k, \omega)$ is the dielectric for the plasma electrons and is determined by the model used (Vlasov or fluid), $\rho_s(k, \omega)$ is the ion source charge density. There are two parts to $\rho_s(k, \omega)$, a static part ($\omega = 0$) and parts with frequencies $\pm \omega_o$.

Solving for the static parts we find

$$E(\xi, 0) = \frac{4\pi e \delta n D_r(k, 0)}{k |D(k, 0)|^2} \sin k\xi - \frac{4\pi e n D_i(k, 0)}{k |D(k, 0)|^2} \cos k\xi \quad (A7)$$

Here D_r and D_i are the real and imaginary parts of the dielectric.

Solving for the part of E , which varies like ω_o , gives

$$E(\xi, \omega_o) = 2\pi e \delta n \epsilon \left(\frac{D_r(k, \omega_o) \sin(k\xi + \omega_o t)}{|D(k, \omega_o)|^2} - \frac{D_i(k, \omega_o) \cos(k\xi + \omega_o t)}{|D(k, \omega_o)|^2} - \frac{D_r(k, -\omega_o) \sin(k\xi - \omega_o t)}{|D(k, -\omega_o)|^2} + \frac{D_i(k, -\omega_o) \cos(k\xi - \omega_o t)}{|D(k, -\omega_o)|^2} \right) \quad (A8)$$

Equation (A8) gives the strength of the two response waves at $\pm \omega_o/k$. As we see if $|D|^2$ is near zero these will be large. We now calculate a number of interesting quantities which give the various phenomena listed earlier.

First, we calculate the space averaged force on the ions due to the electrons which has frequency ω_o . By the equality of action and reaction this must be the negative of the force on the electrons due to the ions. This force

gives the conductivity at frequency ω_o^2 . This force is given by

$$\begin{aligned} F_i(k=0, \omega_o) &= \frac{e}{2L} \int_{-L}^L [n_i(x) E(x, t)]_{\omega_o} dx \\ &= \frac{e}{2L} \int_{-L}^L [n_i(\xi, t) E(\xi, t)]_{\omega_o} d\xi. \end{aligned} \quad (A9)$$

Substituting in n_i and E and keeping only the terms of order ϵ and δn^2 gives

$$\begin{aligned} F_i(k=0, \omega_o) &= 4\pi e^2 \delta n^2 \epsilon \left(\frac{D_r(k, \omega_o)}{4|D(k, \omega_o)|^2} + \frac{D_r(k, -\omega_o)}{4|D(k, \omega_o)|^2} - \frac{D_r(k, 0)}{2|D(k, 0)|^2} \right) \sin \omega_o t \\ &\quad - \pi e^2 \delta n^2 \left(\frac{D_i(k, \omega_o)}{|D(k, \omega_o)|^2} - \frac{D_i(k, -\omega_o)}{|D(k, \omega_o)|^2} \right) \cos \omega_o t. \end{aligned} \quad (A10)$$

The $\sin \omega_o t$ terms in (A10) are in phase with the driving field and hence produce currents which are out of phase with it. They give a reactive contribution. The $\cos \omega_o t$ terms are 90° out of phase with the driving field and produce currents which are in phase or 180° out of phase with the driving electric field. These terms give resistance. Negative $D_i(k, \omega_o)$ and positive $D_i(k, -\omega_o)$ lead to positive resistance (absorption of energy) while positive $D_i(k, \omega_o)$ and negative $D_i(k, -\omega_o)$ lead to negative resistance (laser action).

Next we compute the space and time averaged force on the ions due to the electrons. Again by the equivalence of action and reaction this must be the negative of the force on the electrons. This force gives rise to the generation of dc currents by the interaction of ac fields and ion density

fluctuations. Substituting in n_i and E , keeping only lowest order terms (order ϵ^2 and δn^2), gives

$$F_i(\omega=0, k=0) = - \frac{2\pi e^2 \delta n^2 D_i(k, 0)}{|D(k, 0)|^2} \quad (A11)$$

$$- \frac{\pi \epsilon^2 e^2 \delta n^2 k}{2} \left(\frac{D_i(k, \omega_o)}{|D_i(k, \omega_o)|^2} + \frac{D_i(k, -\omega_o)}{|D_i(k, -\omega_o)|^2} \right)$$

If $D_i(k, -\omega_o) = -D_i(k, \omega_o)$ as is the case for symmetric distributions then no current is generated. For non-symmetric distributions these terms are not equal and in general the last two terms will give a net force which will either generate a current or cause an existing current to damp away. The first term is a dc resistive term and only arises if $f'(0)$ is non zero.

Finally, we compute the force on the ions which has zero frequency, but which has a k dependence. This is the force which gives rise to the growth of parametric instabilities.

$$F_i(k, \omega=0) = \left(\frac{4\pi e^2 \delta n D_r(k, 0)}{k |D(k, 0)|^2} + \frac{\pi k \epsilon^2 e^2 \delta n D_r(k, \omega_o)}{|D(k, \omega_o)|^2} \right. \\ \left. + \frac{\pi k \epsilon^2 e^2 \delta n D_r(k, -\omega_o)}{|D(k, -\omega_o)|^2} \right) \sin kx$$

$$- \left(\frac{4\pi e^2 \delta n D_i(k, 0)}{k |D(k, 0)|^2} + \frac{\pi k \epsilon^2 e^2 \delta n D_i(k, \omega_o)}{|D(k, \omega_o)|^2} \right. \\ \left. + \frac{\pi k \epsilon^2 e^2 \delta n D_i(k, -\omega_o)}{|D(k, -\omega_o)|^2} \right) \cos kx. \quad (A12)$$

We can find the growth of the parametric instability from this by the following procedure. We assume the ion fluctuations evolve slowly in time in response to the force (A12). We, therefore, write the equations for the ion motion as follows

$$\frac{\partial v_i(k)}{\partial t} = F_i(k, 0) , \quad (A13)$$

$$\frac{\partial n_i(k)}{\partial t} + \frac{n_o}{\partial x} \frac{\partial v_i(k)}{\partial x} = 0 . \quad (A14)$$

Eliminating $v_i(k)$ gives

$$\frac{\partial^2 n_i(k)}{\partial t^2} + \frac{n_o}{\partial x} \frac{\partial F_i(k, 0)}{\partial x} = 0 . \quad (A15)$$

Writing $n_i(k) = \delta n_i \text{Re } e^{i(\Omega t + kx)}$, where Ω is assumed small compared to ω_o , and substituting in $F_i(k, 0)$ from (A12) gives the dispersion relation

$$\Omega^2 = \frac{n_o}{M} (\alpha + i\beta) \quad (A16)$$

where

$$\alpha = \frac{4\pi e^2 D_r(k, 0)}{|D(k, 0)|^2} + \frac{\pi k^2 e^2 \epsilon^2 D_r(k, \omega_o)}{|D(k, \omega_o)|^2} + \frac{\pi k^2 e^2 \epsilon^2 D_r(k, -\omega_o)}{|D(k, -\omega_o)|^2} \quad (A17)$$

$$\beta = - \left(\frac{4\pi e^2 D_i(k, 0)}{|D(k, 0)|^2} + \frac{\pi k^2 e^2 \epsilon^2 D_i(k, \omega_o)}{|D(k, \omega_o)|^2} + \frac{\pi k^2 e^2 \epsilon^2 D_i(k, -\omega_o)}{|D(k, -\omega_o)|^2} \right) \quad (A18)$$

For $\epsilon = 0$ this gives the normal dispersion relation for ion acoustic waves with cold ions, or the ion acoustic instability for drifting electrons. For $\epsilon \neq 0$ it gives the modification of the ion acoustic waves by an ac field and the parametric instability when it arises. This result is not good if $D(k, \pm \omega_o)$ is close to zero because then Ω becomes large and cannot be considered small with respect to ω_o .

The expression shows that we can get instability from the Landau damping terms $D_i(k, \pm \omega_o)$. For Symmetric distributions these two terms are equal and opposite, and cancel out. However, for non-symmetric distributions they do not cancel and instability can arise. This is similar to the kinetic instability of Silin.⁹ However, he considered only symmetric distributions and so he had to go to higher order in ϵ to find an instability.

The ions are cold in the above treatment and so there is no ion Landau damping. This would, of course, set a threshold for instability and influence the growth rate.

REFERENCES

* The author would like to acknowledge the contributions of W. L. Kruer, P. Kaw, and T. Lin, whose efforts produced many of the results reported here.

Work was supported jointly by U. S. Atomic Energy Commission Contract AT(11-1)-3073, and National Science Foundation Contract NSF-GJ-33098 X. Use was made of computer facilities supported in part by National Science Foundation Grant NSF-GP 579.

† Paper presented at the US-Japan Seminar on Laser Interaction with Matter, September 23-28, 1972, Kyoto, Japan.

¹ W. L. Kruer and J. M. Dawson, Phys. Fluids 15, 446 (1972).

² J. M. Dawson and C. Oberman, Phys. Fluids 5, 517 (1962); 6, 394 (1963).

³ D. F. DuBois and M. V. Goldman, Phys. Rev. Letters 14, 544 (1965); Phys. Rev. 164, 207 (1967).

⁴ W. L. Kruer and E. Valeo, Phys. Fluids (to be published).

⁵ J. S. De Groot and J. I. Katz, Bull. Am. Phys. Soc. II, 16, 1282 and 1292 (1971).

⁶ W. L. Kruer, private communication.

⁷ W. L. Kruer, Princeton University Plasma Physics Laboratory Report no. MATT-907 (1972).

⁸ P. K. Kaw, private communication.

⁹ V. P. Silin, Zh. Eksp. Teor. Fiz. 51, 1842 (1966); Sov. Phys.-
JETP 24, 1242 (1967).

¹⁰ P. K. Kaw and J. M. Dawson, Phys. Fluids 12, 2586 (1969).

¹¹ J. M. Dawson in Plasma Physics edited by A. Simon and W.
Thompson (Interscience, New York, 1969) Vol. I, p.1.

722183

Fig. 1. Ion density.

722184

Fig. 2. Growth rate vs $k\lambda_D$ for the parametric instability.

722185

Fig. 3. Electric field oscillations associated with a long-wavelength plasma oscillation in a cold plasma containing ion fluctuation.

722186

Fig. 4. Electric field oscillations for mode $k - k_0$.

722187

Fig. 5. Electric field oscillations for mode $2k - k_0$.

722188

Fig. 6. Electric field oscillations associated with a long-wavelength plasma oscillation in a warm plasma containing ion fluctuations.

722189

Fig. 7. Electric field oscillations for mode $k - k_0$.

722190

Fig. 8. Electric field oscillations for mode $2k - k_0$.

722091

Fig. 9. Electric field energy vs time for various modes due to quasi-resonant mode coupling.

722092

Fig. 10. Electric field energy vs time for various modes due to quasi-resonant mode coupling.

722191

Fig. 11. Electron distribution function showing the phase velocity of the beat disturbances.

722192

Fig. 12. Electron and ion distributions for the plasma laser.

722149

Fig. 13. Growth of the electric field energy for a plasma laser using a cold electron beam.

722146

Fig. 14. Growth of the field energy in the first mode for a plasma laser using a cold electron beam.

722147

Fig. 15. Growth of the field energy in the third mode for a plasma laser using a cold electron beam.

722125

Fig. 16. The decay of the current in a plasma laser using a cold electron beam.

722193

Fig. 17. The electron distribution function at the end of the run of a plasma laser using a cold electron beam.

722194

Fig. 18. Electron phase space for the plasma laser using a cold electron beam.

722195

Fig. 19. The growth of the electric field energy for a plasma laser using a warm electron beam.

722196

Fig. 20. The growth of the field energy in the first mode for a plasma laser using a warm electron beam.

722197

Fig. 21. Decay of the current for a plasma laser using a warm electron beam.

722198

Fig. 22. The electron distribution function at the end of the run for a plasma laser using a warm electron beam.

722199

Fig. 23. Symmetric electron distribution function with the phase velocity of the beat disturbance.

722200

Fig. 24. Non-symmetric distribution function with the phase velocity of the beat waves.

722201

Fig. 25. Amplification of the current by a ac field.

722202

Fig. 26. Plasma heating.

722203

Fig. 27. Distortion of the electron distribution by the beat wave.

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, express or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission to the extent that such employee or contractor prepares, handles or distributes, or provides access to, any information pursuant to his employment or contract with the Commission.