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Absence of Second-Order Correction to the Triangle Anomaly in Quantum Electrodynamics*

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We report on the investigation of possible ambiguities involved in calculating the second-order correction to the axial-vector anomaly. The testing ground is finite-mass quantum electrodynamics where the anomaly is defined using Pauli-Villars regularization. The result supports Adler's previous conclusion that there exist no such corrections. Furthermore, the mass renormalization is explicitly shown to be consistent in this order.

I. INTRODUCTION

A remarkable aspect of the Adler-Schwinger modification[1] of the axial-vector divergence in quantum electrodynamics is the conclusion that it seems to be known exactly to all orders in perturbation theory. The anomaly is apparently isolated in the calculation of the lowest-order triangle graph, following general arguments in a cutoff theory summarized recently by Adler.[2]

As an explicit check of this result, Adler and Bardeen[3] have found in a second-order calculation that, indeed, no modification of the anomaly arises. This involved insertions of <u>renormalized</u> vertex and propagator corrections into the skeleton triangle graph of the pseudoscalar-photon-photon matrix element and their check was specifically of the low-energy theorem related to the anomaly. In detail, the matrix element was Taylor-expanded around zero photon-four-momentum and the mass of the fermion circulating through the triangle was kept finite. No cutoff appears in this calculation because the insertions were renormalized. (This is equivalent to the case where the infinite cutoff limit is taken before stringing the insertions together with their counter-terms along the triangle skeleton.)

However, the question concerning the absence of higher-order corrections remains controversial. One may ask whether the order of momentum integrations matters, whether the photon cutoff limit can be taken indiscriminately, whether the photon-momenta expansion involves any ambiguity, and even whether the massless-fermion case changes anything. For example, we are aware of cases in which (superficially divergent but actually finite) double momentum-integrals change value upon change of integration order.[4]

In an effort to uncover possible ambiguities in second-order calculations, Abers, Dicus, and Teplitz[5] have evaluated double-integrals (representing anomalies which would vanish if integration shifts were allowed) taking into account both integration-order possibilities. They expanded in external four-momenta, and found no problem in the Ward Identity related to π-decay, although more general situations were labeled ambiguous. Another possible way to examine this second-order problem in which there would be no need for a cutoff is by a calculation in the Landau gauge and in a massless-fermion theory.[6]

We present here the results of another extensive test of the aforementioned ambiguities in the second-order calculation relevant to π-decay. We use Pauli-Villars regularization in order to define the anomaly according to a prescription given earlier.[7] The regulator masses and a photon cutoff are kept finite until the end of the calculation. For all of the following cases, no new anomaly was found:

i) Integrate insertions before triangle; keeping the photon momentum dependency.

ii) First, expand in photon momenta; then integrate insertions before triangle.

iii) First, expand in photon momenta; then integrate the triangle loop before insertions.

An interesting by-product was the consistency of mass renormalization in case (i) above.

The second-order corrections are introduced in Sec. II and the renormalization is discussed in Sec. III. We address ourselves to the anomaly in Sec. IV. Section V contains a short summary.

II. SECOND-ORDER CORRECTIONS

We consider spinor electrodynamics regulated according to the Pauli-Villars prescription[8] for the fermion propagators and according to the usual photon propagator cutoff $[1/q^2 \rightarrow 1/q^2 - 1/(q^2-\Lambda^2)]$. In our specific calculation, the only explicit role of the Pauli-Villars regularization is in the definition of the axial-vector Ward-Identity (WI) anomaly. The cutoff Λ^2 and the regulator masses will be kept finite until the end of the calculation.

We need only concern ourselves with the pseudoscalar-vector-vector (PVV) triangle graph in our discussion of the existence of an anomalous term in the spinor axial-vector WI. This is because such an anomaly can be identified as the coefficient of the m^{-1} term in an inverse-mass expansion of the PVV vertex function.[7] Thus we concentrate on the same quantity considered by Adler and Bardeen[3] in their low energy theorem check, but for somewhat different reasons.

In terms of the bare fermion mass m_0, the actual form of the canonical unrenormalized WI involves m_0 PVV. Therefore the relevant Feynman diagrams (see Figs. 1 and 2) in our calculation include a $-\delta m\gamma_5$ vertex counterterm where $m = m_0 + \delta m$ is the physical mass. We need not consider the vacuum polarization corrections to the external photons since they affect only the separate issue of charge and wave function renormalization.[9]

The explicit contributions of the diagrams (1)-(6) of Fig. 1 and their crossed-photon counterparts look like[10]

$$\Gamma_{\mu\nu}^{(1)}(p,q) \equiv 2e_0^2 m\Lambda^2 \int_\ell \int_t \sum_{j=1}^{6} \frac{N_j}{t^2(t^2-\Lambda^2)(\ell^2-m^2)D_j} \tag{2.1}$$

where

$$N_1 = \mathrm{Tr}[\gamma_5(\not\ell+\not p+m)\gamma_\lambda(\not\ell+\not p+\not t+m)\gamma_\mu(\not\ell+\not t+m)\gamma^\lambda(\not\ell+m)\gamma_\nu(\not\ell-\not q+m)]$$

$$D_1 = [(\ell+p)^2-m^2][(\ell+p+t)^2-m^2][(\ell+t)^2-m^2][(\ell-q)^2-m^2],$$

$$\text{etc.} \tag{2.2}$$

The factor 2 in (2.1) represents the fact that the crossed contributions obtained by $p \leftrightarrow q$, $\mu \leftrightarrow \nu$ are identical to the uncrossed terms. In addition, we have from the diagrams (7)-(9) in Fig. 2 and their crossed versions,

$$\Gamma_{\mu\nu}^{(2)}(p,q) \equiv 2im\delta m \int_\ell \sum_{j=7}^{9} \frac{N_j}{D_j} \frac{1}{\ell^2-m^2} \tag{2.3}$$

with

$$N_7 = \mathrm{Tr}[\gamma_5(\not{\ell}+\not{p}+m)(\not{\ell}+\not{p}+m)\gamma_\mu(\not{\ell}+m)\gamma_\nu(\not{\ell}-\not{q}+m)] \ ,$$
$$D_7 = [(\ell+p)^2-m^2]^2[(\ell-q)^2-m^2] \ ,$$
$$\text{etc.} \tag{2.4}$$

Diagram (10) and its crossed counterpart yield

$$\Gamma_{\mu\nu}^{(3)}(p,q) = 2i\delta m \int_\ell \frac{\mathrm{Tr}[\gamma_5(\not{\ell}+\not{p}+m)\gamma_\mu(\not{\ell}+m)\gamma_\nu(\not{\ell}-\not{q}+m)]}{(\ell^2-m^2)[(\ell+p)^2-m^2][(\ell-q)^2-m^2]} \tag{2.5}$$

in which the ℓ-integral is finite and unambiguous.[7]

The total contribution of second-order corrections to m_0PVV is

$$\Gamma_{\mu\nu}(p,q) = \sum_{i=1}^{3} \Gamma_{\mu\nu}^{(i)}(p,q) \quad . \tag{2.6}$$

We restrict ourselves to the case: $p^2 = q^2 = 0$ and $\eta < 4$ where

$$\eta \equiv (p+q)^2/m^2 \quad . \tag{2.7}$$

The condition on η is somewhat of a formality designed to avoid worrying about cuts in the η-variable.

From Lorentz covariance and dimensional considerations,

$$\Gamma_{\mu\nu}^{(i)}(p,q) \equiv \epsilon_{\mu\nu\sigma\tau}p^\sigma q^\tau \Gamma^{(i)}(\eta,R) \tag{2.8}$$

with

$$R \equiv \Lambda^2/m^2 \quad . \tag{2.9}$$

The problem is now at hand. We address ourselves next to integrating over t and then ℓ in the anomaly investigation. Later we reverse this order of integration in the course of looking for ambiguities in this investigation.

III. THE RENORMALIZATION QUESTION

The evaluation of $\Gamma^{(1)}$ doing the t-integral first, keeping Λ finite, and keeping η nonzero is quite tedious but straightforward. We present the result only:

$$\Gamma^{(1)}(\eta,R) = -2ie_0^2(2\pi)^{-4}[F(\eta,R)-F(\eta,0)] \tag{3.1}$$

where

$$\begin{aligned}
F(\eta,R) = \int_0^1 & dydzdwdudv \Bigg\{ \bar{y}\,\bar{z}^{-1}\left[-\frac{\bar{y}\bar{u}}{B_1}+\frac{2}{B_2}+\frac{y\bar{w}}{B_3}+\frac{\bar{z}^2}{B_4}\right] \\
& - \bar{y}^2\bar{z}^{-1}v\Bigg[\frac{1+z+z^2}{B_5}+\frac{1}{B_5^2}\,[2+\eta\bar{y}v\bar{u}(wy+uv\bar{y}) \\
& -\eta(1+z)v\bar{y}\bar{u}]+\frac{1}{B_6^2}\,[\bar{z}^2-\eta z\bar{z}w(y\bar{w}+v\bar{u}\bar{y}) \\
& -\eta z\bar{z}\bar{w}(yw+uv\bar{y})-\eta(z^2w\bar{w}-zw+\tfrac{1}{2})+\eta\bar{z}(y\bar{w}+v\bar{u}\bar{y})]\Bigg] \\
& +z\bar{y}^2y^{-1}v\left[-\frac{1}{B_5}+\frac{2}{B_6}\right]+\bar{y}^2y^{-1}v\Bigg[\frac{1}{B_7}\,[z+(1+z)(y+uv\bar{y})] \\
& -\frac{1}{B_8}\,[(1+z)v\bar{u}\bar{y}-z]\Bigg]-\bar{y}^3y^{-1}(1+z)\left[\frac{2uv^2}{B_7^2}+\frac{v\bar{v}}{B_8^2}\right]\Bigg\}\,.
\end{aligned} \tag{3.2}$$

In (3.2) we have employed the notation $\bar{y} \equiv 1-y$, $\bar{z} \equiv 1-z$, etc. and have introduced the denominators

$$B_1 \equiv B-\eta\bar{y}\bar{u}(yw+u\bar{y})$$

$$B_2 \equiv B-\eta yw\bar{y}\bar{u}$$

$$B_3 \equiv B-\eta y\bar{w}wz\bar{z}^{-1}-\eta y\bar{w}(yw+u\bar{y})$$

$$B_4 \equiv B-\eta y\bar{w}wz\bar{z}^{-1}-\eta(yw+\bar{y}u)(y\bar{w}+\bar{y}\bar{u})$$

$$B_5 \equiv B-\eta\bar{y}v\bar{u}(yw+uv\bar{y})$$

$$B_6 \equiv B-\eta y\bar{w}wz\bar{z}^{-1}-\eta(yw+uv\bar{y})(y\bar{w}+\overline{vyu})$$

$$B_7 \equiv B-\eta v\bar{y}\bar{u}(uv\bar{y}+y)$$

$$B_8 \equiv B-\eta uv^2\bar{y}^2\bar{u} \tag{3.3}$$

with

$$B \equiv \bar{y}+y\bar{z}^{-1}+Ryz^{-1} \,. \tag{3.4}$$

We may summarize the salient properties of $F(\eta,R)$:

(1) As expected, $F(\eta,R)$ [and hence $\Gamma^{(1)}$] is logarithmically divergent as $R \to \infty$.

(2) Although $F(\eta,R)$ has a logarithmic infrared-like divergence from the $y = 0$ integration region, this divergence is independent of R. Thus $\Gamma^{(1)}(\eta,R)$ is free of this problem [which arises only because of the separation (3.1)].

(3) Apart from the R-independent infrared divergence which is unrelated to zeros in the B_j, the quantity $\Gamma^{(1)}(\eta,R)$ is well-defined for $0 \leq R < \infty$ and $0 \leq \eta < 4$. To see this, note first that the coefficients of η in (3.3) always include at least one $y\bar{y}$, $u\bar{u}$, or $w\bar{w}$ factor. These factors $\leq 1/4$. After rationalizing the denominators B_j it would appear that the regions around $y = 0$, $z = 0$ and $y = 0$, $z = 1$ (where we have zeros in the rationalized denominators) produce divergences. Nevertheless, aside from the aforementioned infrared problem, no further divergences

do develop (as can be seen, e.g., by transforming y and z into polar coordinates).

Next we can address ourselves to the part of $F(\eta,R)$ which is infinite in the limit $R \to \infty$. We have isolated this part in the second term of an expansion in R^{-1}. [The infrared divergence itself is isolated in the first term of this expansion which is cancelled completely in (3.1).] The result is

$$\Gamma^{(1)}(\eta,R) = -i3 \frac{e_0^2}{(2\pi)^4} \eta \, \ell n R \int_0^1 du dv \frac{v^3 \bar{u}u}{\beta^2} + O(R^o) \qquad (3.5)$$

where $\beta \equiv 1-\eta v^2 \bar{u}u$. The results for the mass counterterms are

$$\Gamma^{(2)}(\eta,R) = i \frac{1}{\pi^2} \frac{\delta m}{m} \int_0^1 du dv \left[\frac{\eta v^3 \bar{u}u}{\beta^2} + \frac{v}{2\beta}\right] \qquad (3.6)$$

and

$$\Gamma^{(3)}(\eta,R) = -i \frac{1}{\pi^2} \frac{\delta m}{m} \int_0^1 du dv \frac{v}{2\beta} \quad . \qquad (3.7)$$

But we know that

$$\delta m = \frac{3e_0^2}{16\pi^2} m \, \ell n \, R + O(R^o) \quad . \qquad (3.8)$$

Hence, from Eqs. (3.5)-(3.8), we see that the ℓn R terms completely cancel in the second-order calculation. The renormalization in this order is demonstrated.

It is important to observe, however, that the existence of an anomaly in the lowest order calculation implies that the usual renormalization will not suffice to obtain cutoff-independent results in higher orders. This point, made earlier by Adler,[2] implies that the renormalization constant for the $\gamma_5\gamma_\mu$ vertex is not Z_2 to fourth order and higher.

So far, Pauli-Villars regularization has played no part in the renormalization discussion since the second-order corrections are rendered finite individually with a photon cutoff. Its part comes next in the anomaly discussion; on the other hand, the preceding discussion holds equally well for regulator Fermion fields.

IV. THE SECOND-ORDER ANOMALY QUESTION

Finding that the second-order corrections were finite in the last section, we proceed with the investigation concerning whether or not an anomaly exists in this order. According to the definition given in reference (7), the WI anomaly is proportional to the mass-independent terms in the dimensionless $\Gamma^{(i)}(\eta,R)$. For the lowest order calculation we could pick up the anomaly by considering the regulator masses go to infinity with impunity since Λ did not enter. In the present second order calculation, since both regulator masses and Λ can go to infinity independently, the value of the parameter $R \equiv \frac{\Lambda^2}{m^2}$ will depend on the order of the two limit in case m is a regulator mass. Therefore we shall keep $R \gtrsim 0$ and otherwise arbitrary in order to avoid any ambiguity in the limiting procedure.

In view of the above remark, we need not consider any specific order for $\Lambda \to \infty$ and the regulator masses $\to \infty$, nor do we have to distinguish explicitly whether m is a regulator mass or the mass of the spinor field, as long as $R \geq 0$ and arbitrary.

The anomaly is identified as the part of $\Gamma(\eta,R)$ which depends only on the mass independent parameter $\sigma \equiv \eta/R$. Let us write

$$\bar{\Gamma}^{(i)}(\eta,\sigma) \equiv \Gamma^{(i)}(\eta,\eta/\sigma) \quad .$$

From (3.1), (3.2) and (3.3), by examining the various denominator factors B_i, it can be shown that the following expansion exists,

$$\bar{\Gamma}^{(1)}(\eta,\sigma) = f_1(\sigma)+\eta g_1(\eta,\sigma) \tag{4.1}$$

where $g_1(\eta,\sigma)$ is well defined for $0 \leq \eta < 4$ and $R \equiv \eta/\sigma \geq 0$ in terms of the Feynman parameter integrations like those in (3.2). Similar expansions exist for $\Gamma^{(2)}$ and $\Gamma^{(3)}$:

$$\bar{\Gamma}^{(2)}(\eta,\sigma)+\bar{\Gamma}^{(3)}(\eta,\sigma) = \eta g_{23}(\eta,\sigma) \tag{4.2}$$

where the term depending only on σ is absent. Therefore the mass counter terms do not contribute to the anomaly. The anomaly, if any, is proportional to

$$f_1(\sigma) = \bar{\Gamma}^{(1)}(0,\sigma) = \Gamma^{(1)}(0,0) \equiv 0.$$

Hence we conclude that the anomaly vanishes. We emphasize that although the last equation looks trivial, this result comes from the non-trivial form of $\bar{\Gamma}^{(1)}(\eta,\sigma)$ in (4.1).

In order to have a further check of the above result we also calculated the anomaly in the limit $\Lambda^2 \to \infty$ with R arbitrary but non-negative. The anomaly is then contained in $\Gamma^{(1)}(0,R)$, without any photon-momenta expansion.

From Eqs. (3.1)-(3.4), we have

$$\Gamma^{(1)}(0,R) = -2ie_0^{\,2}(2\pi)^{-4}F(R) \tag{4.3}$$

where we could perform the w,u, and v integrations easily so that

$$F(R) = \int_0^1 dydz \left\{ \bar{y}\bar{z}^{-1} \left[-\frac{5}{2}\frac{y}{B} - \frac{\bar{y}}{B^2} \right] + \frac{1}{B}\left[(z+\frac{3}{2})\bar{y}^2 + (1-\frac{5}{2}z)\bar{y} - \frac{3}{2}z \right] + \frac{1}{B^2}\left[z\bar{y}^2 + \frac{1}{2}(1+z)\bar{y} + \frac{1}{2}(1+z) \right] + \frac{1}{2}y^{-1}\left[1-2z+\frac{3z}{B} - \frac{1+z}{B^2} \right] \right\} - \frac{1}{4} \quad . \tag{4.4}$$

We can employ the type of argument used by Adler and Bardeen[3] to show that F(R) vanishes. Recalling the remarks made after Eq. (3.4), one can see that F(R) is analytic in R except for a possible cut along the negative R axis where B can vanish. But the discontinuity across this cut vanishes and, moreover, F(R) is bounded for $|R| \to \infty$ in any direction in the complex-R plane. Since a bounded entire function must be a constant and since F(0) = 0, we have $F(R) \equiv 0$ everywhere. Finally, we have

$$\Gamma^{(1)}(0,R) = 0 \quad . \tag{4.5}$$

The result (4.5) supports the claim that the low energy theorem for the $\pi^0 \to 2\gamma$ decay amplitude found by Adler and Bardeen is unchanged by the second-order radiative corrections to the nucleon triangle loop. We shall discuss other ambiguities next.

We examine several interchanges in the order of doing things (integrations and limits) in the arbitrary R case:

(A) Eq. (4.4) was obtained by directly setting $\eta = 0$ in the integrand, after the t and ℓ integration, but before the Feynman parameter integration. The identity $(B+\eta f)^{-1} = B^{-1}-B^{-1}\eta f(B+\eta f)^{-1}$ can be used to show that the order of the limit $\eta \to 0$ and the Feynman parameter integration can be reversed.

(B) By keeping R arbitrary, the order of the limits $\Lambda \to \infty$ and $m \to \infty$ is immaterial.

(C) If we set $\eta = 0$ <u>before</u> doing the ℓ and t integrals of Eq. (2.1) and the ℓ integrals in Eqs. (2.3) and (2.5), we find that indeed $\Gamma^{(2)}+\Gamma^{(3)} = 0$ and that

$$\Gamma^{(1)'}(0,R) = -ie_0^{\ 2}(2\pi)^{-4}G(R) \tag{4.6}$$

where

$$G(R) = \int_0^1 dz\bar{z}^{-1}\left\{\frac{1+\hat{B}}{\hat{B}^3}\,\ell n(1+\hat{B})[2+\hat{B}-z(1+\hat{B})] - \frac{1}{\hat{B}^2}\,[2+2\hat{B}-z(1+\tfrac{3}{2}\hat{B})]\right\}$$

where $$\hat{B} = Rz^{-1}+\bar{z}^{-1}-1 \quad . \tag{4.7}$$

It is gratifying to see that G(R) is proportional to the expression given in Eq. (92) of Adler and Bardeen[3,11] and hence is zero by their arguments. We again find no anomaly. Presumably by a judicious choice of Feynman parametrization, one would be able to show explicitly that F(R) = G(R) even in integral form. We might remark that we have also obtained (4.6) in a different way by calculating $\frac{\partial}{\partial p_\lambda}\frac{\partial}{\partial q_\sigma}\Gamma_{\mu\nu}^{(1)}(p,q)$ at $p_\lambda = q_\sigma = 0$.

(D) We have also performed the momentum-integrations in the reverse-order for $\eta = 0$. Doing the ℓ-integral first, we obtain

$$\Gamma^{(1)}(0,R) \propto H(R) \tag{4.8}$$

where

$$H(R) = 1 + \int_0^1 dydzy\bar{z}\bar{z}^{-1}\left\{\frac{\bar{z}-z}{[Ry\bar{z}\bar{z}+y]^2} - \frac{1}{Ry\bar{z}\bar{z}+y}\right\}. \tag{4.9}$$

We find $H(R) = 0$ by the same sort of argument used in the evaluation of (4.4).

V. DISCUSSION

By defining the axial-vector anomaly in the framework of Pauli-Villars regularization, one is able to eliminate a number of immediate problems. Equations involving infinite quantities are replaced by equations involving cutoffs so that one can get a "handle" on the infinities. For finite cutoffs, the corresponding momentum-integrals are absolutely convergent: variable shifts and momentum assignments are then unambiguous. Thus, gauge invariance is maintained from the start and the anomaly is quite clearly defined.

We have seen that second order insertion with finite photon cutoff leaves the Adler-Schwinger anomaly uncorrected, without recourse to the usual photon-momenta expansion. This result agrees with the Adler-Bardeen renormalized insertion and the Abers-Dicus-Teplitz unrenormalized shift calculations (both of which expanded in photon-momenta and both of which used a different identification of anomaly than ours). The possible ambiguities involving massless fermions remains, but at this stage the absence of higher-order corrections continues to be substantiated.

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11. By carrying out the u-integration in their expression, one obtains a result explicitly proportional to G(R). We thank Professor Adler for pointing this out to us.

FIGURE CAPTIONS

Fig. 1 Vertex and self-energy corrections to the mPVV vertex function. There are six more such corrections to the crossed triangle graph obtained by $p \leftrightarrow q$, $\mu \leftrightarrow \nu$.

Fig. 2 Mass counterterm corrections to the mPVV vertex function. There are four more crossed corrections corresponding to $p \leftrightarrow q$, $\mu \leftrightarrow \nu$.

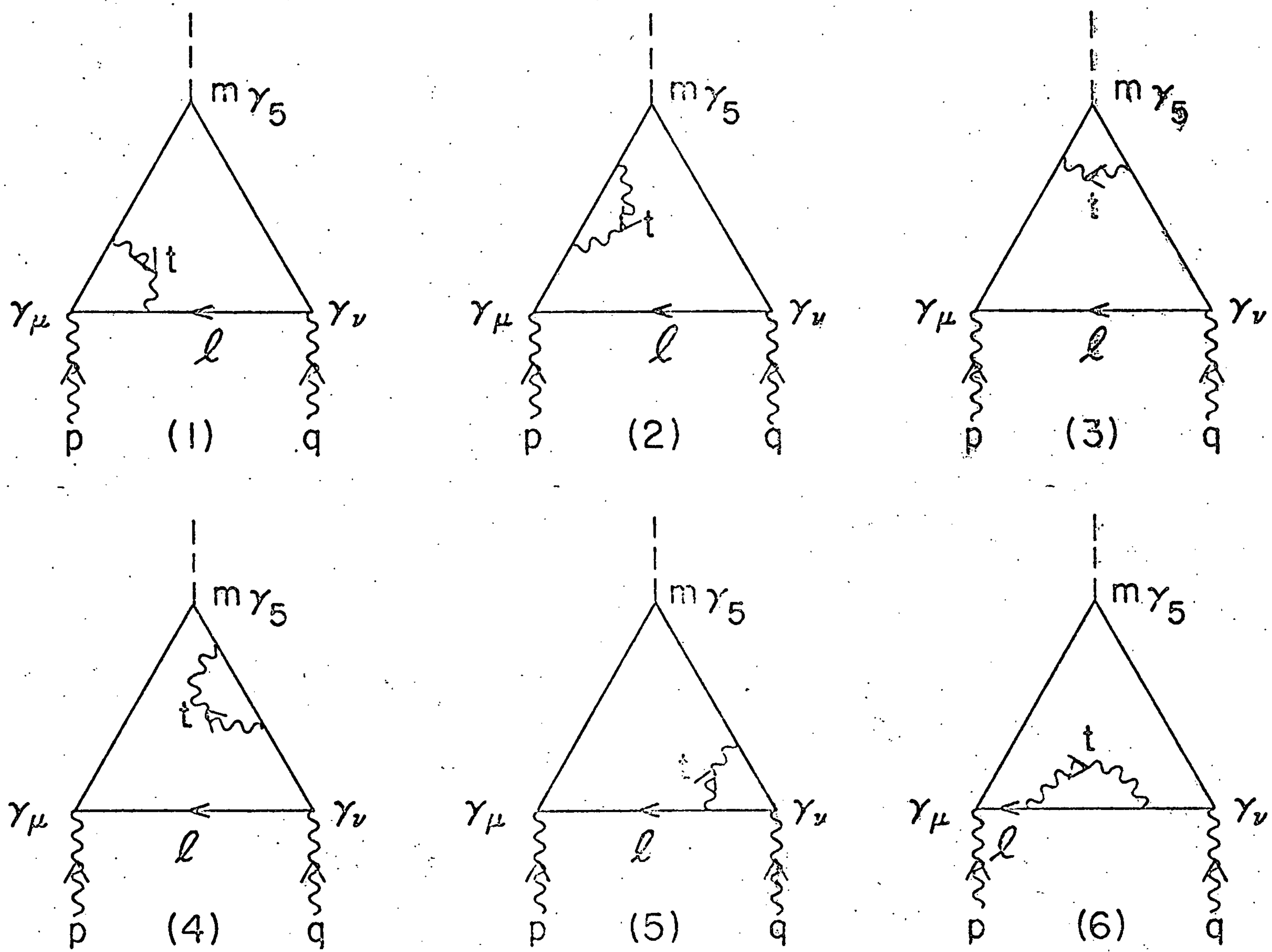
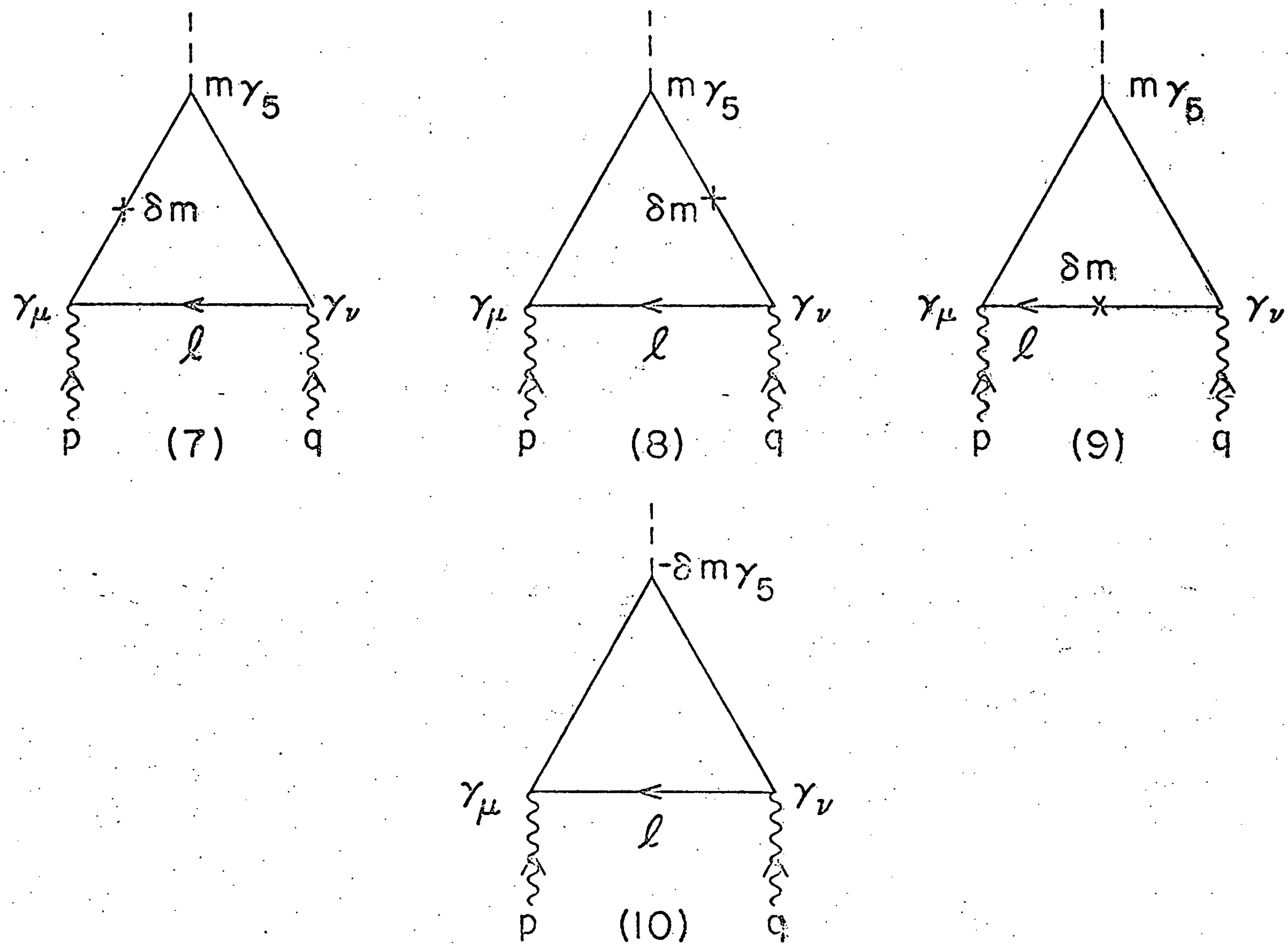


FIG. 1



$m\gamma_5$
δm
γ_μ
γ_ν
ℓ
p
(7)
q
$m\gamma_5$
δm
γ_μ
γ_ν
ℓ
p
(8)
q
$m\gamma_5$
δm
γ_μ
γ_ν
ℓ
p
(9)
q
$-\delta m\gamma_5$
γ_μ
γ_ν
ℓ
p
(10)
q

FIG. 2