CONTROLLING NONLINEAR RESONANCES IN SUPERCONDUCTING STORAGE ACCELERATORS*

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Abstract

There is reason to expect that nonlinear imperfection resonances may be stronger in a high field superconducting storage accelerator than in a storage accelerator using conventional magnets. A rough study is done to estimate the minimum correction magnets required to correct for resonances up to and including the fifth order. It may be sufficient to have roughly one correction magnet in each cell of the storage ring, with each correction magnet having 12 different coils which are independently excited, and with about 36 independent power supplies. It is also suggested that these correction magnets may be used as all purpose correction magnets and correct for the linear resonances, central orbit displacements, and move the working point v_x, v_z as well. In this case, about two correction magnets are required per cell, each correction magnet having 21 different windings, and with about 132 independent power supplies required.

I. Introduction

This paper suggests that it may be necessary for storage rings with superconducting magnets like ISABELLE¹ to have a special set of magnetic field correction elements which can control the effects of near-by nonlinear imperfection resonances. An estimate is given of the number and location of the magnetic field correction elements, and of the different type of field windings needed.

There is some reason to suspect that in a superconducting storage ring like ISABELLE the nonlinear imperfection resonances may be much stronger than in the ISR storage ring at CERN.² This is because in superconducting magnets, the field is determined by the position of the current blocks rather than the iron pole shape as in the ISR, and random errors in the current block positions will introduce random nonlinear terms in the field of each magnet which drive the nonlinear imperfection resonances.

The ISR observed³ effects on the beam due to nonlinear imperfection resonances, including high order resonances like the fifth order which are traditionally regarded as nonharmful. At the same time, the ISR working region is in the vicinity of several fifth order resonances without apparent harm to the beam. Nevertheless, the nonlinear imperfection resonances up to some high order have to be regarded as potentially dangerous. The ISR experience can possibly be interpreted as indicating that if the imperfection resonances were 5 or 10 times stronger, they might seriously hurt the beam.

The above remarks suggest that a superconducting storage ring should have a set of magnetic field correction magnets to control the harmonics which excite the relevant nonlinear resonances. The considerations given below estimate that it may be sufficient to have roughly one correction magnet in each cell of the storage ring. Each correction magnet will have about 12 different coils which can produce different field shapes in the median plane. The correction magnets may

be arranged in groups of azimuthal extent of $2\pi/\nu$ around the ring, where $\nu \approx \nu_x \approx \nu_z$ the integer close to the betatron wave number, and the correction magnets within each group of $2\pi/\nu$ can be independently excited, while magnets in different $2\pi/\nu$ groups have a fixed relationship in their excitation. Altogether about 36 independent power supplies may be needed.

Instead of thinking of the correction magnets as having 12 independent multipole windings, each of which produces a particular multipole field, it may be more convenient to construct the magnet with 12 independently excited wires, the current in each wire being chosen to give the right combination of desired multipoles. The independent wire approach may be more convenient for an approach involving computerized control of the 12 independent currents.

It is also possible to think of constructing an all purpose set of correction magnets which will control not only the nonlinear resonances, but also the linear resonances, the displacement of the central orbit, and the v_x, v_z of the beam. For this set of all purpose correction magnets, it may be sufficient to have two correction magnets in each cell of the ring. Each correction magnet has 21 different field windings, and the correction magnets are arranged in groups of azimuthal extent $2\pi/\nu$ around the ring. Altogether about 132 independent power supplies may be needed.

The considerations given below are rough and intended to give a general idea of the correction magnets required. An actual accelerator is complicated by the existence of straight sections of different lengths, and the various periodicities due to the presence of both cells and straight sections. Some of the consequences of these complications have been ignored for the sake of simplicity. However, it seems likely that the main features of the suggested arrangement of the correction magnets may remain unchanged in a more careful study.

II. Working Region Choice, $v_x \approx v_z$

The number of different nonlinear harmonics around the ring which have to be controlled is considerably reduced if one chooses v_x and v_z to be nearly equal, and to be either both somewhat above the same integer or both below the same integer. For example, one might choose $v_x \approx v_z \approx 16.25$ with a working region which is in the lower left side of the basic square in v_x, v_z space. The 6 fifth order resonances which are nearby are then

$$\begin{aligned} 5v_x &= 81 \\ 4v_x + v_z &= 81 \\ 3v_x + 2v_z &= 81 \\ 2v_x + 3v_z &= 81 \\ v_x + 4v_z &= 81 \\ 5v_z &= 81 \end{aligned} \quad (2.1)$$

One notices that all 6 resonance lines are driven by the 81 harmonic, because $81 = 5 \times 16 + 1$. If v_x and v_z were not close to the same integer, then 6 different harmonics would appear on the right side of Eq. (2.1) making the control of these harmonics more difficult.

* Work performed under the auspices of the U.S. Atomic Energy Commission.

Although the same harmonic appears in all 6 resonance lines, there are two separate field shapes involved. Three of the resonance lines are driven by the fourth order field dependence $r^5 \cos 5\theta$ in the vector potential, in the plane perpendicular to the direction of the beam and the field has only a B_z component and no B_r component in the median plane. These are the three resonance lines in which v_z has an even coefficient. The other three resonance lines are driven by the field dependence $r^5 \sin 5\theta$ in the vector potential and the field has only a B_r component and no B_z component in the median plane.

Thus to control the 6 fifth order resonance lines, four field windings are needed in the correction magnets which are spaced around the ring to generate the correct 81 harmonic with the right phase.

The four separate field windings give the two different fields given by the vector potentials $r^5 \cos 5\theta$ and $r^5 \sin 5\theta$ in the plane perpendicular to the beam motion, and each field winding is needed twice in order to get the correct sine and cosine components of the required 81 harmonic.

The 6 fifth order resonances can also be excited by other harmonics than the 81 because of the periodicity in β -function of the lattice. This is a smaller effect and for the sake of simplicity we will ignore it in this paper in order to arrive at a first approximation of the correction system necessary to control the nonlinear imperfection resonances. A more careful study will have to include the possibility of exciting the fifth order resonance lines by harmonics other than the 81 harmonic.

The 5 fourth order resonances which are nearby are

$$\begin{aligned} 4v_r &= 65 \\ 3v_r + v_z &= 65 \\ 2v_r + 2v_z &= 65 \\ v_r + 3v_z &= 65 \\ 4v_z &= 65 \end{aligned} \quad (2.2)$$

All five resonance lines are driven by the 65 harmonic since $65 = 4 \times 16 + 1$. Three of the resonance lines are driven by the field $r^4 \cos 4\theta$ in the vector potential which has a B_z component in the median plane and two are driven by the field $r^4 \sin 4\theta$ in the vector potential which has a B_r component in the median plane. To control the five fourth order resonance lines, four field windings are needed in the correction magnets which are spaced around the ring to generate the correct 65 harmonic with the right phase.

The four third order resonances which are nearby are

$$\begin{aligned} 3v_r &= 49 \\ 2v_r + v_z &= 49 \\ v_r + 2v_z &= 49 \\ 3v_z &= 49 \end{aligned} \quad (2.3)$$

All four resonance lines are driven by the 49 harmonic since $49 = 3 \times 16 + 1$. Again, to control the four third order resonances, four field windings are needed in the correction magnets which are spaced around the ring to generate the correct 49 harmonic with the right phase.

Altogether, one sees that one needs 12 different field windings to control the third, fourth, and fifth order nonlinear resonances. In the following section,

the possibility of putting all 12 windings in the same correction magnet is discussed.

III. Location of the Correction Magnets Around the Ring

In the previous section, it was shown that 12 different field windings were required to control the fifth, fourth, and third order nonlinear resonances. In this section, the possibility will be considered of putting all 12 windings in one correction magnet which is placed around the ring in order to generate the required 49, 65, and 81 harmonics.

For the sake of simplicity, we will assume that the accelerator has $N = 48$ cells around the ring with v_r and v_z just above $v = 16$, which makes N a multiple of v and the calculations are easier. If, like in the proposed ISABELLE storage ring, one has something like $N = 52$ and $v = 18$, then the calculations are more difficult but something like the scheme suggested here should be possible. It may turn out there are advantages to choosing v to be a submultiple of the N , the number of cells.

We divide the accelerator into 16 sections of azimuthal extent $2\pi/v = 2\pi/16$. Each $2\pi/v$ section will then have three cells since $N = 48$. If one places one correction magnet in each cell, and lets the three correction magnets in each $2\pi/v$ section be independently excited while the correction magnets of each $2\pi/v$ section are excited the same as those in any other $2\pi/v$ section, then one can obtain any specific 48, 64 and 81 harmonic using the three independently excited magnets in each section. If one now lets the excitation of each $2\pi/v$ section vary like $\cos \phi$, where ϕ is the azimuthal angle around the ring, then one can generate any required 49, 65, and 81 harmonic. One will also be generating the 47, 66, and 79 harmonics, but hopefully these will cause no harm. One will also be exciting azimuthal harmonics which are close to other multiples of 16, but these harmonics will excite resonance lines which are much further away.

Thus it appears one may be able to control the nonlinear resonances up to the fifth order for the example shown by putting one correction magnet in each cell where each correction magnet has 12 different field coils. The correction magnets are divided into groups of azimuthal extent $2\pi/v$ around the ring, and the correction magnets within each group of $2\pi/v$ can be independently excited, while correction magnets in different $2\pi/v$ groups have a fixed relationship in their excitation. Thirty-six independent power supplies, roughly $12(N/v)$, are required.

The above considerations are rough. An actual accelerator is complicated by the existence of straight sections of different lengths, and the various periodicities due to cells and straight sections which allow other harmonics besides the three considered above to excite the possibly harmful nonlinear resonances. However, it is likely that the main features of the suggested arrangement of the correction magnets will remain unchanged in a more careful study.

IV. Independent Correction Wires

Each of the correction magnets proposed in the previous sections, have 12 field windings to produce the 6 different field shapes required to control nonlinear resonances up to the fifth order. Instead of thinking of the magnets as having 12 different multipole windings, each of which produce a particular multipole field, it may be more convenient to construct the magnet with 12 independently excited wires as is shown

Fig. 1. Correction wires.

in Fig. 1. Six of the wires carry the independent currents C_1 to C_6 , while six wires carry the independent currents $\bar{C}_1$ to $\bar{C}_6$ as shown in Fig. 1.

One can see that one can generate a pure B_z field or a pure B_r field on the median plane using the coil arrangement shown in Fig. 1 by putting either $\bar{C}_n = C_n$ or $\bar{C}_n = -C_n$. Further for each of the four kinds of fields which correspond to either B_r or B_z components in the median plane and the sine or cosine component of the required harmonic, there are 3 independently excited wires which can be used to obtain the desired third, fourth, and fifth harmonics in the current distribution in order to obtain the required nonlinear multipole fields.

One may note that in addition to the desired third, fourth, and fifth harmonics in the current distribution in each correction magnet, one will also generate harmonics which are lower and higher than the required harmonics, which will create other multipole fields. However, it is possible that the undesired multipoles that are so generated may not be harmful since they will be generated with the wrong harmonic around the ring to excite the closest resonance lines. But if this is not the case, then one will need additional independently excited wires to control the undesired multipole fields.

V. All Purpose Correction Magnets

So far we have thought of the correction magnets as being used to correct the nonlinear fields. One can also think of putting into the same magnets, wires or windings to correct linear resonance lines. Nearby second order linear resonances are

$$\begin{aligned} 2\nu_r &= 32 \\ \nu_r + \nu_z &= 32 \\ 2\nu_z &= 32, \end{aligned}$$

or possibly

$$\begin{aligned} 2\nu_r &= 33 \\ \nu_r + \nu_z &= 33 \\ 2\nu_z &= 33. \end{aligned}$$

Nearby first order resonances which move the central orbit are

$$\begin{aligned} \nu_r &= 16 \\ \nu_z &= 16. \end{aligned}$$

Controlling the above first order and second order resonances will require 8 more windings, 4 for each, in the correction magnets, and in order to control the azimuthal around the ring harmonic 16 and 32 or 33, one will need 2 correction magnets per cell or 6 correction magnets per $2\pi/\nu$ section which would give the possibility of correcting 6 harmonics of $\nu = 16$.

Since we now have 2 correction magnets per cell, one can think of using the correction magnets to control ν_r and ν_z with an additional quadrupole winding in each correction magnet which is different in the 2 correction magnets which are in the same cell in that one has a positive field gradient while the other has a negative field gradient. These quadrupole windings are excited in the same way in every cell, and may be thought of as corresponding to the zero harmonic of $\nu = 16$.

The all purpose correction magnets that control both linear and nonlinear resonance lines and also can move ν_r , ν_z will have 21 windings in each magnet. There are 2 correction magnets per cell, and 132 separate independent power supplies are needed. The 132 is 6×22 since there are 6 correction magnets in each $2\pi/\nu$ section.

Most of the correction currents required are relatively small, and some kind of computerized control of the 132 power supplies seems a possibility.

VI. The $\nu_r = \nu_z = 0$ Resonance Line

The suggestion that $\nu_r = \nu_z$, in order to reduce the number of different harmonics that drive the nearby resonances, also implies that the working region will be quite close to the $\nu_r = \nu_z = 0$ linear coupling resonance. This resonance line is driven primarily by the zero harmonic of a linearly varying field with a B_r component in the median plane. Thus a rather small B_r gradient which is the same in each magnet will produce a large amount of coupling between the r and z motions. It is not clear that this coupling is in any way dangerous. However, to avoid this coupling one would require a rather severe tolerance on the allowed systematic B_r gradient in each magnet or some way of correcting out the B_r gradient.

References

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