Invited paper to be presented at the American Nuclear Society meeting, November 14, 1972, Washington, D.C.

PAPER OUTLINE AND SLIDES

RECOMMENDED CONSTITUTIVE EQUATIONS FOR FFTF USE AND
APPLICATION TO THE ANALYSIS OF STRUCTURES*

W. L. Greenstreet

Oak Ridge National Laboratory
Oak Ridge, Tennessee 37830

INTRODUCTION

Since Liquid Metal Fast Breeder Reactor systems are to be operated at relatively high temperatures, the inelastic behaviors of these systems under time-varying temperature and load conditions must be considered. Thus, extensions to the present nuclear structural design technology are required to meet the demands imposed in terms of structural analysis capability and design criteria. Particular attention is being given to this aspect for the Fast Flux Test Facility (FFTF) as well as for follow-on reactor systems. To this end, a High Temperature Design Technology Program was established at the Oak Ridge National Laboratory about three years ago.

Briefly, the Oak Ridge program is addressed to the study of materials behavior, the development of mathematical analogs, or constitutive equations, to describe the essential features of this behavior, the development and assessment of structural analysis methods, and the establishment of appropriate design criteria. In this paper, discussions are given on constitutive equations, and the application of these equations in a structural analysis method. The latter is examined through comparisons between calculated and experimental results for selected structures. The materials of interest are 304 and 316 stainless steel and temperatures of interest range to approximately 1200°F.

Small deformation theory is considered, and it is postulated that the total strain tensor can be decomposed into time-independent (elastic and plastic) components and time-dependent (creep) components. Thus, constitutive equations for elastic-plastic and creep behavior are treated separately in

---

*Research sponsored by the U.S. Atomic Energy Commission under contract with the Union Carbide Corporation.

MASTER

the discussion. The equations for current use in FFTF, as well as early demonstration plant, design analyses were selected on the following bases:

1. They were to be based on current technology and possess the capability of accounting for time-varying load and temperature conditions.

2. They were to account for prior permanent deformation (both time-independent plastic and time-dependent creep).

3. They were to be compatible with existing analytical capabilities, that is, they were to be capable of being incorporated into current computational methods, and they were not to cause numerical instabilities in those structural analysis programs presently available for FFTF design use.

ELASTIC-PLASTIC BEHAVIOR

(Slide No.)

A. Material Response

1. Monotonic loading

- 1 a. Stress-strain curves for 304 stainless steel as a function of temperature

2. Cyclic loading

- 2 a. Room temperature cycling of 304 stainless steel over fixed strain range ($\sim 0.9\%$)
- 3 b. Cycling of 304 stainless steel at 1100°F between near constant strain limits ($\Delta\epsilon_t \approx 0.6\%$)

3. Effect of prior creep on cyclic behavior

- 4 a. Creep strain of 1% (1200°F and various stress levels to 15,000 psi) followed by room temperature cycling

4. Combined stress behavior

- 5 a. Program of loading in stress space, with probable path of the loading surface center, and locus of final loading surface

B. Mathematical Representation

1. Kinematic hardening model is appropriate

2. Bilinear stress-strain representations

- 6 a. Monotonic loading

- 7 b. Cyclic loading

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

(Slide No.)

3. Equations

8

- a. Loading surface
- b. Normality relation
- c. Translation tensor
- d. Stress-plastic strain relation

TIME-DEPENDENT BEHAVIOR

A. Material Response

1. Creep curves (type 304 stainless steel)

9
10

- a. Constant load (1200°F)
- b. Step-change in load (1200°F; increasing load)

2. Prior plastic deformation effects

11

- a. Cycled at room temperature ($\Delta\epsilon_p \approx 2.5\%$ and 20 cycles) and creep tested at 1200°F

B. Mathematical Formulations

12

1. Creep formulations

- a. Constant-uniaxial-stress equation
- b. Multiaxial equation
- c. Strain-hardening equation

13

2. Strain hardening representation of uniaxial response

3. Auxiliary rules - presumed behavior

14
15

- a. Change in positive stress
- b. Stress reversal

ANALYSIS RESULTS

A. Elastic-Plastic Behavior

16

- 1. Beam of 304 stainless steel tested at room temperature. Simply supported and loaded at center; 26 in. in length, 2 in. high, and 0.97 in. thick, with 24 in. distance between support points.

B. Reversed Creep Loading

17

- 1. Uniaxial specimen of 304 stainless steel tested at 1100°F, under stress conditions of $\pm 15,000$ psi.

C. Elastic-Plastic-Creep Behavior of Beam (B3)

18

- 1. Simply supported and loaded at center; test temperature of 1200°F (beam was not cycled prior to creep testing)

SUMMARY

Discussions of constitutive equations for FFTF component design use have been given. These equations are based on the current state-of-the-art and, along with the auxiliary rules established for their use, will model essential and important features of the material behavior. The inelastic analysis results described indicate that the finite-element analysis method provides an adequate tool for design analyses. In the particular examples used for discussion, the differences between calculated and experimental results were seen to be at least partially attributable to inaccuracies of the material behavior descriptions. However, the analysis method must be examined further before it can be evaluated. Combined stress conditions are much more revealing, and tests are now underway to provide combined stress data for analysis assessment purposes.

Needs and directions for improvement of the constitutive equations were indicated in the discussion. Since changes in these equations must be guided by experimental evidence, experiments to provide such evidence are now underway at ORNL in addition to those being carried out at other places.

Although the tools for inelastic analyses are imperfect and in need of additional development, the technology that exists today can be used to assess behaviors of structures which exhibit inelastic behavior. Behavioral predictions obtained will depict essential features to be expected. Thus, problem areas can be identified, design adequacy can be assessed in terms of existing criteria, and methods for improving designs can be established in a rational, basically sound manner. The deficiencies of the inelastic analysis tools can be minimized by coupling the use of these tools with engineering judgement in a manner commensurate with the present state-of-the-art.

ORNL-DWG 72-12325

Cyclic Stress-Strain Behavior Type 304 Stainless Steel
Room Temperature.

TYPE 304 STAINLESS STEEL, ANNEALED (9T2796)
TEMPERATURE: 1100°F
STRAIN RATE: 0.005 / min.

ORNL DWG 72-12280

SLIDE NO. 4

ORNL—DWG 71—10566

SLIDE NO. 6

SLIDE NO. 7

ORNL-DWG 71-11388R

SLIDE NO. 8

PLASTIC BEHAVIOR

$$1. \quad f^*(\sigma'_{ij}, \epsilon_{ij}^P, T) = f(\sigma'_{ij} - \alpha_{ij}) - \kappa(T) = 0$$

$$2. \quad d\epsilon_{ij}^P = \Lambda \frac{\partial f^*}{\partial \sigma'_{ij}}$$

$$3. \quad d\alpha_{ij} = c \, d\epsilon_{ij}^P$$

$$4. \quad d\epsilon_{ij}^P = g_{ij} \neq 0 \text{ (during loading)}$$

where

$$g_{ij} = \frac{1}{2\kappa c} \frac{\partial f^*}{\partial \sigma'_{ij}} \left(\frac{\partial f^*}{\partial \sigma'_{kl}} d\sigma'_{kl} + \frac{\partial f^*}{\partial T} dT \right)$$

when the initial yield condition (formulation for f)
used is that of von Mises.

UNIAXIAL CONSTANT-STRESS CREEP CURVES FOR 304
STAINLESS STEEL AT 1200°F

TO DESCRIBE TIME- DEPENDENT CREEP BEHAVIOR REQUIRES:

1. UNIAXIAL CREEP RELATION
2. "FLOW" RULE
3. HARDENING LAW

Time-Dependent (Creep) Behavior.

TYPE 304 STAINLESS STEEL AT 1200 °F
ORNL PRELIMINARY HEAT (NO. 8043813)

SLIDE NO. 12

TIME-DEPENDENT (CREEP) BEHAVIOR

1. Constant-uniaxial-stress creep equation:

$$\epsilon^c(\sigma, T, t) = \epsilon_t(\sigma, T) \left[1 - e^{-r(\sigma, T)t} \right] + \dot{\epsilon}_m(\sigma, T)t .$$

2. Multiaxial constitutive equations:

$$\dot{\epsilon}_{ij}^c = \lambda \sigma'_{ij} .$$

Define:

$$\bar{\sigma}^2 = 3J'_2 = \frac{3}{2} \sigma'_{ij} \sigma'_{ij} ,$$

$$\bar{\epsilon}^2 = \frac{4}{3} I_2 = \frac{2}{3} \epsilon_{ij}^c \epsilon_{ij}^c .$$

Write:

$$\dot{\epsilon}_{ij}^c = \frac{3}{2} \frac{\dot{\bar{\epsilon}}(\bar{\sigma}, t, T)}{\bar{\sigma}} \sigma'_{ij} .$$

3. For strain-hardening law:

$$\dot{\epsilon}_{ij}^c = \frac{3}{2} \frac{\dot{\bar{\epsilon}}(\bar{\sigma}, \bar{\epsilon}, T)}{\bar{\sigma}} \sigma'_{ij} .$$

SLIDE NO. 14

GMHL-DWG 71-10678

SLIDE NO. 15

ORNL-DWG 71-10680

SLIDE NO. 16

SLIDE NO. 17

SLIDE NO. 18

ORNL-DWG 72-10066A