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MEASURED DOPPLER COEFFICIENT OF THORIUM
DISPERSED IN GRAPHITE

by

J. R. Brown and J. B. Sampson

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GENERAL ATOMIC
DIVISION OF
GENERAL DYNAMICS

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GA-3422

MEASURED DOPPLER COEFFICIENT OF THORIUM
DISPERSED IN GRAPHITE

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U. S. Atomic Energy Commission
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SUMMARY

The Doppler coefficient in thorium dispersed in graphite has been measured in a reactor spectrum developed in the HTGR critical assembly. This Doppler coefficient has been defined to be $(1/A_{\text{res}})(dA_{\text{res}}/dT)$, where A_{res} is the resonance capture rate of neutrons in thorium. The principal method of measurement employed has been a reactivity technique, which compares the reactivity of a single cold element and a single hot element heated to 700°K. The considerable correction due to the small perturbation in neutron temperature resulting from the heating, which is termed the thermal-base effect, has been experimentally determined by auxiliary measurements.

In addition to this reactivity technique, an activation technique has been used to check the results. In this activation technique, the thermal-base effect has also been experimentally subtracted by measuring the $1/v$ component of the captures at each temperature in vanadium. This vanadium-subtraction technique has been developed in connection with an extensive set of room-temperature measurements of thorium resonance capture, which are described in separate reports. (1)(2)

Measurements of the Doppler coefficient obtained by these two different techniques agree to within 10%. This agreement, demonstrated by applying both techniques for the first time in the same laboratory, shows that the previously reported spread of results and apparent disagreement between results of these two techniques are avoidable.

The average Doppler coefficient for the temperature interval from 300°K to 700°K, for a carbon-to-thorium ratio of about 50 in the thorium-bearing region, was measured to be $3.6 \times 10^{-4}/^{\circ}\text{C}$ to within about 10% uncertainty.

The measured results are in satisfactory agreement with calculated values.

ACKNOWLEDGMENTS

Many persons whose names have not been included in the "Work done by" list on the title page have been most helpful in aiding and guiding the work reported here. Among these persons are Dr. L. W. Nordheim, Dr. H. B. Stewart, and Dr. R. S. Stone. We are indebted to L. J. Todt for performing earlier analysis of the Doppler coefficient, and to K. R. Van Howe for making available his analysis of the Dragon measurements.

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I. INTRODUCTION

The thorium Doppler coefficient measurements to be described constitute part of a program for the quantitative study of thorium resonance-capture phenomena characteristic of conditions in the HTGR (High Temperature Gas-Cooled Reactor), particularly the Peach Bottom 40-Mw(e) HTGR being built for the Philadelphia Electric Company. The thorium in the Peach Bottom reactor is to be dispersed through annuli which also contain graphite and U^{235} . The thorium is mixed homogeneously with graphite, and this mixture constitutes about one-quarter of the core volume.

The experimental program has also included a thorough study of thorium resonance integrals at room temperature. These studies have been reported separately. ⁽¹⁾⁽²⁾

The Doppler coefficient is defined, in accord with general practice, to be the logarithmic derivative with respect to temperature of A_{res} , the resonance capture rate, in Th^{232} in the reactor spectrum. A_{res} is specifically defined in Eq. (10), below.

A quantitative knowledge of the thorium Doppler coefficient, which is an important contributor to the negative temperature coefficient of reactivity in the HTGR, is of utmost importance for a safe and economical design for the Peach Bottom reactor. Before these measurements were undertaken, the experimental data for the thorium Doppler coefficient showed a range of values ⁽³⁾ that varied over a factor of about two. This uncertainty, clearly, was intolerably large. Furthermore, no measurements had been made formerly for thorium dispersed in graphite at a concentration similar to that to be found in the Peach Bottom reactor. The measurements for this program were undertaken to provide the essential experimental confirmation of the magnitude of the Doppler coefficient for the Peach Bottom reactor.

These measurements have been made principally by the reactivity-change technique; with this technique, observations are made of the reactivity change that occurs when a cold fuel tube containing thorium is replaced by a hot fuel tube. A special oscillation procedure for doing this has been developed. This oscillation procedure greatly increases the number of observations that can be made in a given period of time. The reactivity-change technique has been applied in a number of cases of somewhat varied composition and geometry. In addition, the activation technique, by which the change with temperature of the activations resulting from thorium resonance capture is measured, has been applied in one case. Of these two methods of measurement, the activation method is the most straightforward and therefore is less subject to systematic error; however, being inherently less precise, it can give larger random error from measurement to measurement. The satisfactory agreement reported here between both the two methods of measurement and calculation has given a clear and verified understanding of the Doppler coefficient for the Peach Bottom reactor.

In this report are collected results of all measurements and calculations of the Doppler coefficient that have been made in the HTGR program. Many of these results have already been reported,⁽⁴⁾⁽⁵⁾ at least partially. More thorough calculations that have been made recently are also reported. These recent calculations have resulted in several minor revisions of the calculated values reported previously.⁽⁴⁾

The facility used and the principle of measurement are described in a general way in Sections II and III, respectively. A detailed description of the reactivity-change technique of measurement is given in Section IV, and the results obtained by this method are given in Section V. The results obtained from measurements by the activation technique are described in Section VI. Section VII summarizes the results of calculations of the Doppler coefficient. In Section VIII, comments are made on the comparison of measured and calculated results, and a description of the concentration and geometric dependence of the measured Doppler coefficient is given.

II. THE HTGR "LATTICE-TESTING" CRITICAL-ASSEMBLY

LOADING USED IN DOPPLER COEFFICIENT

MEASUREMENTS

Measurements of the thorium Doppler coefficient were made as part of an extensive program that was carried out in a critical facility built at General Atomic to obtain neutronic information essential to the design of the 40-Mw(e) Peach Bottom power reactor. Other phenomena studied in this facility include thorium resonance capture at 300°K, ⁽¹⁾⁽²⁾ the worth of an individual control rod, flux distributions, reactivity coefficients, ⁽⁶⁾ gross criticality, ⁽⁶⁾ and the temperature-dependence of reactivity of a uniformly heated assembly. ⁽⁶⁾ A general view of this facility, referred to locally as the HTGR critical assembly, is shown in Fig. 1. Built in two halves that are separable at a midplane, this facility simulated neutron events of the Peach Bottom reactor with a small assembly, which was therefore economical and sensitive to reactivity changes. As in the Peach Bottom reactor, the thorium and fuel were distributed quasi-homogeneously through about one-quarter of the volume of the core graphite. Two types of loadings of this assembly were made; these loadings are described in detail elsewhere. ⁽⁶⁾⁽⁷⁾

The Doppler measurements, as well as the resonance-absorption and control-rod measurements were made on loadings of an earlier assembly, known as the "lattice-testing" HTGR critical assembly. ⁽⁷⁾ Each such loading contained fuel and thorium in several different types of core regions designed to simulate the correct cold neutron spectrum at the center of the assembly. Measurements were made in the central region having fuel tubes and geometry very similar to that of the Peach Bottom reactor at one stage of its design.

The other type of HTGR critical assembly, ⁽⁶⁾ designated Core 7, was

Fig. 1--Over-all view of Peach Bottom HTGR critical assembly

used principally to study criticality and its temperature-dependence. This assembly had a single-region core which did not simulate the Peach Bottom geometry.

The salient features of the lattice-testing loadings are summarized in the following several paragraphs. All these loadings had the same regions, of a basic composition and of fixed dimensions; however, the poison content of boron and copper contained in the form of strips in the voids of the assembly was different in each loading. The particular poison distribution determined the specific loadings, which have been designated Cores 3 through 6. Doppler coefficient measurements were made principally in Core 4, which contained considerable boron poison in the inner regions, resulting in a radial flux distribution that was quite flat.

The geometry of the lattice-testing loadings had the basic pattern of a rectangular parallelepiped of stacked graphite which also contained other materials that varied with the region. The horizontal axis perpendicular to the plane of separation of the two halves was an axis of symmetry; the various regions surrounded this axis of symmetry. In the direction of this axis, the assembly was bare-ended and 6 ft in length. The composition of the assembly along any line parallel to this axis was characteristic of the particular region in which the line lay. A sketch of the geometry in a vertical plane perpendicular to the axis of symmetry is shown in Fig. 2.

The innermost region, termed the "exact" region, was a hexagonal arrangement of 19 cylindrical fuel tubes which simulated the reactor geometry. The central annulus of each tube contained graphite, U^{235} , and Th^{232} . The distance between flats of this hexagon was about 16 in.

Surrounding the exact region was a "buffer" region built of 2 in. by 4 in. by 12 in. graphite blocks, each with a pair of holes for accommodating compacts consisting of a mixture of graphite and either U^{235} or Th^{232} . The composition of the buffer was such that the neutron reaction rates were essentially the same as those of the exact region, although the geometry was different, being of a more convenient block, rather than tubular,

* BRIDGE BARS ARE LOADED WITH THE SAME CONCENTRATION OF URANIUM AND THORIUM AS THE SURROUNDING REGIONS.

APPROXIMATE SCALE: 1 TO 9

Fig. 2--Core cross section of the "lattice-testing" HTGR critical assembly (transverse plane perpendicular to the axis; reflector not shown)

construction. The outer boundary of the buffer region was a square about 3 ft on a side.

The next region, called the "driver," was a 10-in. -thick arrangement made of the same blocks and holes as the buffer, but with the thorium omitted. The function of the driver was to provide excess neutrons to the inner regions so that criticality was achieved in a smaller volume.

A reflector (not shown in Fig. 2) of 1-ft thickness formed the outermost region. This thickness was increased to 18 in. on the top side to compensate for an observed flux asymmetry caused by reflection from the floor. The effective reflector thickness, thus, should probably be considered to be 12 in. on the sides and 18 in. on the top and bottom.

Figure 3 shows the cylindrical fuel-tube elements of the exact region and the rectangular blocks of the buffer region. The cylindrical-element dimensions shown in Fig. 3 apply to the normal, or small-annulus, fuel tube; the 18 noncentral tubes were all of these dimensions. The inner spine and the outer sleeve were composed of graphite. One hole of each buffer block was loaded with a U_3O_8 graphite mixture, and the other hole with a ThO_2 graphite mixture.

The flux distribution in the longitudinal direction was observed to have the expected cosine distribution, which extrapolated to zero at a distance about 2 cm beyond the bare ends of the assembly.

All Doppler measurements were carried out at the center of the exact region, with the Core 4 composition^{*} of the surrounding 18 fuel tubes of the exact region. The thorium composition in the annuli of these 18 tubes was such that the atomic ratio of carbon to thorium (C/Th) was 55.7. Actually, one measurement was made with the thorium concentration increased by about 20% in the six fuel tubes immediately surrounding the central fuel tube. No significant change in the Doppler coefficient was observed in this modified loading.

The fuel tube whose Doppler coefficient was measured was in all

^{*}Atomic compositions for Core 4 are given in Ref. 7.

Fig. 3--Fuel-bearing units of construction of the "lattice-testing" HTGR critical assembly

cases at the center of the core. In the reactivity-change type of measurements, an entire central cold fuel tube was replaced by an entire hot fuel tube. Each of these fuel tubes was 6 ft long and 3.57 in. in outside diameter. Figure 4 shows one end of the assembly with the central fuel tube partially removed. Central tubes of several types of internal geometries were studied, including those having thorium in a cylindrical annular geometrical arrangement of several different sizes and those having thorium in the central cylindrical spine. For most cases, U^{235} fuel was present in the annulus. Detailed descriptions of the geometries of these various elements are given in Section V.

Fig. 4--Central fuel tube in a position partially removed from the core of the critical assembly

The activation measurement of the Doppler coefficient was carried out by heating a 4-in. length of the central fuel tube containing the activation sample. The heated portion of the fuel tube was located near the midplane of the assembly.

III. PRINCIPLE OF THE REACTIVITY-CHANGE MEASUREMENT OF THE THORIUM DOPPLER COEFFICIENT

Measurement of the Doppler coefficient by the reactivity-change method have been made by carrying out several types of modifications in a portion of the assembly, and observing the change in reactivity caused by these modifications. Since only a portion of the assembly was altered, the interpretation of these changes must be based on the theory of reactivity changes which is derived from perturbation theory.⁽⁸⁾

The following two types of measurements were made:

1. The reactivity change, R_{res} , caused by adding thorium resonance captures to the system at room temperature.
2. The further change with temperature of R_{res} .

The definitions used were as follows:

Let $\phi(u)$ designate the neutron-flux spectrum at the center of the reactor dependent on lethargy, u . Let $\sigma_{res}(u)$ be the resonance component of the thorium absorption cross section with the $1/v$ component of this cross section subtracted:

$$\sigma_{res}(u) = \sigma(u) - \sigma_{1/v}(u) . \quad (1)$$

Here, σ is the total thorium absorption or capture cross section, and $\sigma_{1/v}$ is a component having a $1/v$ energy-dependence such that it contributes the entire 2200-meter/sec value of the cross section. Perturbation theory⁽⁸⁾ states that the addition of absorbing material to the system gives the following reactivity change, R , measured in dollars:

$$R = -K \int du \phi(u) F(u) \sigma(u) , \quad (2)$$

where $F(u)$ is neutron importance which is mathematically adjoint to $\phi(u)$, and K is

$$K = \frac{N}{\beta f} \quad (3)$$

Here, N is the number of atoms of absorber added, β is the delayed neutron fraction, and f is the total fission rate over the entire assembly.

The addition of thorium, having both a resonance and a $1/v$ capture component, contributes both a resonance and $1/v$ component to the reactivity change, R :

$$R = R_{\text{res}} + R_{1/v} \quad (4)$$

where

$$R_{\text{res}} = -K \int du \phi(u) F(u) \sigma_{\text{res}} \quad (5)$$

$$R_{1/v} = -K \int du \phi(u) F(u) \sigma_{1/v} \quad (6)$$

Since $F(u)$ is a slowly varying function of u , a group-averaged value of F can be applied in a manner similar to that of multigroup theory. The average neutron importance, $\bar{F}$, is defined as the average of $F(u)$ weighted by the lethargy over a particular energy band:

$$\bar{F} = \frac{\int_{\text{band}} du \phi(u) F(u) \sigma(u)}{\int_{\text{band}} du \phi(u) \sigma(u)} \quad (7)$$

Equations (5) and (6) can then be approximated as follows:

$$R_{\text{res}} = -K \bar{F}_{\text{res}} \int_{\text{res band}} du \phi(u) \sigma_{\text{res}}(u) = -K \bar{F}_{\text{res}} A_{\text{res}} \quad (8)$$

$$R_{1/v} = -K \bar{F}_{1/v} \int_{1/v \text{ band}} du \phi(u) \sigma_{1/v}(u) = -K \bar{F}_{1/v} A_{1/v} \quad (9)$$

where A_{res} , for example, is

$$A_{\text{res}} = \int_{\text{res band}} du \phi(u) \sigma_{\text{res}}(u) \quad (10)$$

For the special case of the $1/E$ spectrum normalized so that $\phi(u) = 1$ for all u , A_{res} coincides with the resonance integral.

The F values for the $1/v$ band and for the resonance band can be evaluated to sufficient accuracy by assuming that an energy of about 20 ev is the lower energy bound of the resonance band of thorium and also is the upper energy bound of the $1/v$ band. Equations (8) and (9) are valid to a good approximation at all temperatures with the same value of $\bar{F}_{\text{res}}$; that is, $\bar{F}_{\text{res}}$ of Eq. (8) can be assumed to be constant with temperature. Therefore, $(1/R_{\text{res}}) (dR_{\text{res}}/dT)$ is very nearly equal to $(1/A_{\text{res}}) (dA_{\text{res}}/dT)$:

$$\frac{1}{R_{\text{res}}} \frac{dR_{\text{res}}}{dT} \cong \frac{1}{A_{\text{res}}} \frac{dA_{\text{res}}}{dT} \quad (11)$$

Calculations, summarized in Section VII, have been carried out to verify the accuracy of Eq. (11) for HTGR application. These calculations show that the two sides of Eq. (11) are equal to within about 1%. That is, the error made in application of Eq. (11) is considerably less than the experimental error of these measurements.

$(1/R_{\text{res}}) (dR_{\text{res}}/dT)$ was measured by the reactivity-change method. The result of this measurement has been compared with $(1/A_{\text{res}}) (dA_{\text{res}}/dT)$ determined for the same case by the activation-technique measurement. Comparison of measurements with calculations of $(1/A_{\text{res}}) (dA_{\text{res}}/dT)$ has also been made for all cases for which $(1/R_{\text{res}}) (dR_{\text{res}}/dT)$ was measured.

Equations (8) and (11) show that in carrying out measurements by the reactivity-change method the changes to be measured must be restricted to the resonance capture of thorium. This was accomplished by measuring the reactivity of a thorium-containing fuel tube as compared with that of a specially designed auxiliary fuel tube. This auxiliary fuel tube had no thorium, but had boron in its place. The boron concentration was chosen so that its captures were equal to the captures in the $1/v$ component of the thorium cross section of the thorium-containing fuel tube. In all other respects the auxiliary boron-containing fuel tube was identical to the thorium-

containing fuel tube. This design of the boron fuel tube caused its $R_{1/v}$ to be identical with that of the thorium fuel tube. The word "compensating" has been applied to the boron concentration and to the auxiliary boron fuel tube.

The thorium and compensating-boron fuel tubes were used in each of the two types of measurements listed at the beginning of this section.

To measure R_{res} at room temperature, T_0 , which is the first type of measurement, a compensating-boron fuel tube was replaced by a thorium fuel tube, and the difference in reactivity was noted.

In the second type of measurement, the incremental change in reactivity with temperature was measured for each type of fuel tube. This was done by replacing a cold fuel tube with a hot fuel tube of the same type and measuring the reactivity change resulting from this temperature change. Let ΔR_{Th} be the reactivity difference resulting from replacing a cold thorium-containing fuel tube with a hot thorium-containing fuel tube. The reactivity identity of two fuel tubes of the same type was sufficiently good that the absolute values of ΔR_{Th} and of ΔR_B were both small for $\Delta T = \text{zero}$. Then, dR_{res}/dT could be obtained from the measurements from the relation

$$\frac{dR_{res}}{dT} = \frac{dR_{Th}}{dT} - \frac{dR_B}{dT} = \lim_{\Delta T \rightarrow 0} \frac{\Delta R_{Th}}{\Delta T} - \frac{\Delta R_B}{\Delta T}, \quad (12)$$

where ΔT is the change in temperature.

The ΔR values were measured by a technique of repetitive replacement; this was done to improve the statistical accuracy of the determination of small changes.

The quantity dR_B/dT of Eq. (12) is called the thermal-base effect. It results from the perturbation in local neutron temperature which results from heating the central element. In the HTGR critical assembly, the absolute value of dR_B/dT has been found to be a significant fraction of that of dR_{Th}/dT . It was necessary to measure dR_B/dT as carefully as dR_{Th}/dT in order to obtain a meaningful result for the measured Doppler coefficient. The cause of the thermal-base effect and the experimental procedures developed to minimize it are discussed in Section 4.5 and in Appendix A.

IV. MEASUREMENT OF THE DOPPLER COEFFICIENT BY THE REACTIVITY-CHANGE TECHNIQUE

4. 1. THORIUM RESONANCE REACTIVITY AND CHANGE WITH TEMPERATURE

R_{res} has been measured either from the change in assembly period or from changes in position of a calibrated control rod. In the HTGR critical assembly, the limit of sensitivity to which reactivity can be so determined is 0.0001 dollar ($8 \times 10^{-7} \delta k/k$). R_{res} was obtained from the difference in reactivity thus measured at room temperature, T_0 , when a fuel tube (the central one in the exact region) containing thorium replaced one containing compensating boron.

To determine the amount of compensating boron to be used, the $1/v$ component of thorium has been assumed to be responsible for the entire 2200-meter/sec cross section of 7.45 barns. The grams of compensating boron required, $g(\text{boron})$, were obtained from the atomic weights and cross sections:

$$g(\text{boron}) = g(\text{thorium}) \times \frac{A(\text{boron})}{A(\text{thorium})} \times \frac{\sigma_{2200}^{(\text{thorium})}}{\sigma_{2200}^{(\text{boron})}}, \quad (13)$$

where A is the atomic weight of each element. *

The second type of measurement, that of the temperature-dependence of R_{res} , requires determining $\Delta R_{Th}(\Delta T)$ and $\Delta R_B(\Delta T)$, where ΔT is the

* It can be objected, since the low-energy thorium cross section deviates from a $1/v$ energy dependence, ⁽¹⁾⁽⁹⁾ that Eq. (13) overestimates the compensating boron by about 5%. Observations described in Section 4.5 show that the thermal-base effect is insensitive to this error. The small effect of the error on the measured Doppler coefficient is discussed in Section VIII.

change in temperature above T_0 , and T is the hot temperature:

$$T = T_0 + \Delta T \quad (14)$$

From a combination of measurements of the first and second type, $R_{\text{res}}(T)$ was obtained as follows:

$$\begin{aligned} R_{\text{res}}(T) &= R_{\text{res}}(T_0) + \Delta R_{\text{res}}(\Delta T) \\ &= R_{\text{res}}(T_0) + \Delta R_{\text{Th}}(\Delta T) - \Delta R_{\text{B}}(\Delta T) \quad (15) \end{aligned}$$

Smoothed curves were fitted to the measured data for ΔR_{Th} and ΔR_{B} versus temperature. Typical results are shown in Fig. 5. From these curves, ΔR_{res} values were determined for intervals of 100°F (55.56°K), or less. Derivatives were approximated by $\delta R / \delta T$, where δ indicates a difference with temperature, determined from the smoothed data, this difference being that of one of these 55.56°K intervals. Results for dR_{res}/dT and $(1/R_{\text{res}})(dR_{\text{res}}/dT)$ thus obtained from the data of Fig. 5 are tabulated in Section V. These results were obtained by careful and repeated visual curve-fitting to the observed points. The accuracy of the fitting and differentiation derived therefrom is judged to be within $\pm 2\%$.

4.2. OSCILLATOR DESCRIPTION

Since the increase in reactivity caused by replacing a cold fuel tube with a hot fuel tube was typically 0.01 dollar or less, it was desirable to make repetitive measurements of ΔR_{Th} and $\Delta R_{1/v}$ to improve the statistical accuracy of the measurement. The device used in these measurements should probably be more appropriately termed a repeater rather than "oscillator," although it is referred to in this report as an oscillator. Its function was to periodically interchange the hot and cold central fuel tubes and to automatically record the reactivity change. Many repetitions of a reactivity measurement, by the device, thus improved the statistical accuracy.

Fig. 5--Dependence of reactivity changes on temperature change, with a radially flat flux

The oscillator mechanism used for these measurements consisted of a double-acting pneumatic cylinder with a piston with an 8-ft stroke; this device was equipped with hydraulic snubbers at both ends of travel. By means of stainless-steel cables through pulleys attached to the walls of the room, this mechanism moved a 14-ft-long oscillator rod, termed an "oscillator element," horizontally through the core. This oscillator element consisted of two 6-ft-long fuel tubes separated by a 2-ft-long section of Transite (thermal insulation) which was coupled to the graphite by aluminum screw-on sleeves. All sections had the same diameter (3.57 in.) as the fuel tubes. The outer ends of the oscillator element were attached to the cables by stainless-steel fittings. Full travel of the pneumatic cylinder exchanged one 6-ft-long fuel tube in the core for the other, accomplishing this motion in about 1 second.

A 6-ft-long cylindrical oven was located horizontally off one end of the core in such a position that when one end of the oscillator element (the cold fuel tube) was in the core, the other end (the hot fuel tube) was in the oven. When the hot end was in the core, the cold end rested in a graphite trough situated horizontally off the other end of the core. Figure 6 shows the placement of the oven.

The six fuel tubes surrounding the central position were milled with a flat to allow a 1/8-in. air gap at the points where these fuel tubes would otherwise touch the central oscillator element. Transite runners were attached to the two fuel tubes located immediately below the oscillator element, and the oscillator element slid on these runners. With this air-and-Transite insulation, the heat transfer to the surrounding six fuel tubes was found to be less than 10°F per minute at 800°F . Over a period of several hours, the Transite insulation between the hot and cold fuel tubes was sufficient to prevent a temperature rise of more than about 10°F at the end of the cold fuel tube. Thermocouples were placed along the axis for the entire 14-ft length of the oscillator element, and the leads were brought out at the cold end. These long flexible leads permitted the

Fig. 6--External oven placement near critical assembly

oscillator element to move without damaging the thermocouple connections. The thermocouple attachment at the cold end is shown in Fig. 7.

The temperature distributions over the hot fuel tube of the oscillator element were fairly uniform. In the longitudinal direction, the maximum variation from end to center was about 40°F at the highest temperature (800°F). This variation was a result of the oven temperature not being uniform because of losses at the ends. The radial distribution was not measured but was judged to be essentially uniform. The hot fuel tube was initially brought to a desired temperature by allowing it to bake in the oven at the desired temperature for at least one-half hour. During oscillation, the oven temperature was raised to compensate for any heat loss from the hot fuel tube.

Whenever possible, the hot and cold ends of the oscillator element were loaded identically, thus giving a reactivity difference of nearly zero between the two fuel tubes when they were both at the same temperature. Since the cold fuel tube was merely a reference in the oscillations, however, loading of this fuel tube did not have to be identical to that of the hot end; the only requirement was that the reactivity of the cold-end loading be nearly equal to that of the hot-end loading at room temperature. In addition, the uranium content of the hot and cold ends had to be nearly the same so as to minimize any effects due to the differences between delayed neutron precursors in the two fuel tubes.

During the measurements of the Doppler coefficient, the oscillator mechanism was usually programmed to move every 30 sec, each movement requiring about 1 sec, so that each end of the oscillator element alternately had a dwell time of about 29 sec in the core.

The variation in positioning of the oscillator element when placed in the core by the oscillator is, at maximum, $1/16$ in. Measurements indicated that a displacement of $1/2$ in. changed the reactivity by 0.07 cent. Hence, the uncertainty in reactivity measurements resulting from variations in oscillator-element position was at most 0.01 cent, and it was random.

Fig. 7--Thermocouple attachment at cold end of fuel tube

4. 3. SERVO CONTROL ROD AND REACTIVITY CALIBRATION

Rather than record the variations of the neutron flux as the oscillator element was moved, the flux level was maintained constant automatically by means of a servo-driven control rod. This servo rod was actually a small boron strip of 3-ft length, placed in the reflector, and having a total worth of about 6 cents.

The servo system was actuated by the average current output of two ion chambers (gamma-compensated) located in the reflector. A servo-amplifier operating at high gain drove the servo-rod positioning motor. The speed of travel of the servo rod was such that it changed reactivity at the rate of 3 cents per second over the most effective portion of its position.

The servo rod was calibrated from asymptotic period measurements. The position-dependence of the servo-rod sensitivity was obtained from a set of period measurements made using a boron-graphite absorber as an auxiliary aid; this absorber was worth about 1 cent. The reactivity of this auxiliary absorber was always measured by period changes for extreme settings of the servo control rod, together with compensating positioning of the critical-assembly control rod. No differences within the errors of the measurement were ever observed. By this means a differential calibration curve for the servo rod in terms of cents per inch at various positions was obtained, and an integral curve of cents versus position was also obtained.

To ascertain reproducibility, the reactivity of the small 1-cent absorber was measured several times; its reactivity effect was determined to within ± 0.01 cent. The entire calibration of the servo rod was repeated several times with the result that the reactivity change between any two positions of the servo rod was known to within ± 0.01 cent. The servo-rod calibration was nearly linear ($\pm 10\%$) over about 3 cents of its travel. The positional uncertainty of the rod was about 0.1%, or approximately 0.005 cent. Repeated measurements of one reactivity sample showed that the standard deviation for six measurements was not greater than ± 0.01 cent and, in some cases, it was ± 0.005 cent for stable reactor conditions (those of temperature, etc.).

4. 4. MEASUREMENT OF REACTIVITY CHANGES FROM SERVO-ROD TRACES

Figures 8 and 9 show the traces of the position of the servo rod versus time for an oscillator element having small-annulus HTGR fuel tubes at each end of the oscillator element. A trace consists of repeated half cycles. During each half cycle, one of the pair of fuel tubes that are being compared is present in the reactor for a particular dwell time, with alternate tubes present during each succeeding half cycle. The trace during each half cycle is a record of the reactivity-compensating motion of the servo rod after the insertion of one of the pair of fuel tubes. The dwell times of 5 min and of 30 sec for Figs. 8 and 9, respectively, are actually half periods, from which the dwell times can be obtained by allowing for the approximately 1-sec time required for the oscillator mechanism to replace one fuel tube with the other. In general, there is a difference in position of the trace from one half cycle to the next, this difference being caused by a reactivity difference of the two fuel tubes. This can be read directly on the trace, although for large differences it is necessary to make a zero adjustment by the instrumentation in order to keep the two traces on the same piece of paper. The displacement of the trace from one half cycle to the next is therefore the sum of the actual displacement recorded, plus a well-known adjustment of the relative position.

The reactivity difference between any two fuel tubes that were compared was measured by the displacement of the trace between the two half cycles. In order to apply this method quantitatively, it is necessary to note several characteristics of the trace during each half cycle.

The rapid fluctuations during a typical time constant of a few seconds are excited by noise, probably chiefly that in the reactor flux level. It was first necessary to obtain the average trace about which these fluctuations occurred. This was done visually.* The average trace thus obtained

*The damping time of the recording instrumentation was about 1 sec, so that some instrumental averaging has already been applied to the traces reproduced in Figs. 8 and 9.

Fig. 8--Servo-rod trace: 5-min dwell time; no temperature difference

Fig. 9--Servo-rod trace: 30-sec dwell time; no temperature difference

(due to the previously described measurement of the position-dependence of reactivity of the servo rod) is equivalent to a determination of reactivity versus time of the reactor under a condition of the compensating servo rod being inoperative. The accuracy of the averaging is such that reactivity versus time was thus obtained to within ± 0.01 cent at any time.

Each averaged trace during any half cycle can be divided into two parts; initially, there is a rapidly changing, or transient, portion, which is followed by an essentially constant portion at the end of the half cycle. The duration of the transient depends markedly on the half period. For a short half period, the level approached in the essentially constant portion was also found to differ from that which is approached in true equilibrium corresponding to a very long half period. The feature observed which is useful in evaluating these measurements is that a condition of dynamic equilibrium typical of the particular half period was reached and maintained for a significant portion of the half cycle at the end of the half cycle. Furthermore, the same difference between the equilibrium levels for the two half cycles was observed for all half cycles longer than 20 sec.

The reactivity difference which resulted from interchanging one fuel tube with the other has therefore been measured from the observed difference in equilibrium levels of the two half cycles. The level for each half cycle was determined from the averaged trace during the final 10 sec of the half cycle. The half-cycle time was taken to be at least 30 sec, since this method, for half periods shorter than 20 sec, was found to give uncertainties greater than the ± 0.01 -cent tolerance desired.

The following checks of this technique were made, using pairs of fuel tubes that were both at room temperature. The pairs used have included fuel tubes of nearly identical reactivity, as well as those having considerable difference in reactivity. Measurements by the oscillation and compensating-servo techniques were made for a series of half periods in excess of 20 sec. Measurements with the servo inoperative were also made, utilizing asymptotic-period measurements. In the latter, the

oscillator mechanism was used only to exchange fuel tubes that were left in the reactor for long times, and, with each fuel tube in the reactor, asymptotic periods were determined and compared. The reactivity difference for a specific pair of fuel tubes was measured to be the same within ± 0.01 cent in all of these cases.

An additional complication arises when one of the fuel tubes is heated. The hot tube being in the reactor for half of the time of oscillation resulted in a gradual warming of the surrounding assembly. This caused a reactivity drift of a typical value of 0.01 cent per minute. This drift was found to be readily measurable; its effect on one trace is shown in Fig. 10. For each trace, a correction due to this warming was measured. To minimize this correction, the half period should be as short as possible; therefore, a 30-sec half period was chosen for nearly all measurements. This was long enough to give a reliable measure of the displacement of the trace, and short enough that the correction due to the temperature rise in the assembly could be well made.

The observed features of the traces are in agreement with theoretical expectations.

The identical shape of the trace during each half cycle occurs because the same changes in both reactivity and delayed-neutron level occurred, except for the constant difference in reactivity between the fuel tubes in the fully inserted position. The same coupling section of the oscillator element passed through the reactor each time the fuel tubes were interchanged. This replacement process caused a disturbance in prompt reactivity for about a second, this disturbance being very nearly the same for each half cycle. Each fuel tube of the pair contained the same amount of fissile material. It was exposed to the same neutron flux for the same length of time while in the reactor, and was also out of the reactor for the same length of time. The same deficit of delayed neutrons, therefore, occurred after either fuel tube was moved in, and the subsequent buildup of delayed neutrons was also the same for each fuel tube.

Fig. 10--Servo-rod trace: 30-sec dwell time; 370°F temperature difference

The true equilibrium in delayed-neutron level was approached at the end of half cycles having a half period that was long compared with the e-folding time constants of all delayed neutrons. For half oscillation periods long compared with the time constants of important delayed neutrons, that is, long compared with 10 sec, the equilibrium can be expected to be well-enough approximated so that a difference between the levels approached at the ends of the half cycles was a good measure of the displacement of the trace from one half cycle to the next.

It is clear from these observations and theoretical arguments that the described method of measuring reactivity difference between the two fuel tubes is reliable.

A final question about the oscillation technique can be raised: whether the 10-sec time during which the equilibrium level is observed at the end of a half cycle is long enough to provide a measure of the reactivity corresponding to that level accurate to within ± 0.01 cent. This question is essentially concerned with the sensitivity of the servo-mechanism.

A limit of sensitivity of 0.01 cent in the oscillation-technique measurement of reactivity change caused by fuel-tube replacement is consistent with the estimated sensitivity of the servo-mechanism. Let Δt be the period of time during which the servo rod can be considered to remain at the equilibrium position during the final portion of each half cycle. Let δ designate the reactivity of the system determined by the servo-rod position near the end of a half cycle. Therefore, δ is the limit of sensitivity of the reactivity measurement and is limited by the ability of the sensing instruments and servo-mechanism to detect and correct for any deviation from a uniform power level during the time Δt . Let T be the pile period which corresponds to δ . The relative change in power level, $\Delta n/n$, that can occur during Δt is

$$\begin{aligned}\frac{\Delta n}{n} &= \frac{\Delta t}{T} \\ &= C \delta \Delta t ,\end{aligned}$$

* We are indebted to A. M. Perry for pointing out this relation.

where C is the reciprocal of the constant that connects reactivity and pile period, namely, 0.069 dollar/sec.* Let the sensitivity of detection of the servo-mechanism be ϵ . That is, if $(\Delta n/n) < \epsilon$, the servo-mechanism will make no correction during time Δt . The value of δ , therefore, is obtained by taking $\Delta n/n$ to be ϵ :

$$\delta = \frac{\epsilon}{C \Delta t}$$

ϵ for this system is judged to be 10^{-4} or less, and Δt of these measurements is about 10 sec. These figures give

$$\delta = 1.4 \times 10^{-4} \text{ dollar}$$

$$\cong 0.01 \text{ cent}$$

4.5. MEASUREMENTS OF THERMAL-BASE EFFECT

The thermal-base effect mentioned previously resulted from the perturbation in local-neutron temperature which occurred as a result of heating the central fuel tube (containing graphite, uranium, and thorium). This perturbation occurred because, in this more homogeneous reactor, a significant amount of graphite was heated along with the fertile material--in fact, about 5% of the graphite moderator contained in the exact region (16-in. diameter) was heated. A significant thermal-base effect was observed when a tube containing only graphite was heated. This indicates that some modification in local-neutron temperature occurred from the heating. This small change in neutron temperature caused a reactivity change which may be a significant fraction of the Doppler effect. The difference between a good and poor measurement of the Doppler effect by means of the technique involving reactivity changes depends to a large extent on correctly understanding the thermal-base effect.

Two separate steps were taken in the experimental attack on the problem of the thermal-base effect: first, the effect was suppressed to a minimum; second, the minimized effect was carefully measured.

* See, for example, Ref. 10.

The physical nature of the thermal-base effect is discussed semi-quantitatively in Appendix A. It is concluded that theoretically the effect very nearly vanishes in a spatially uniform neutron distribution. For this reason, the radial flux distribution in the assembly was flattened by introduction of poison in the triflutes (the triangular-like gaps between the cylindrical fuel tubes; see Fig. 2). However, poison could not be inserted in the position where it is particularly important to achieve a flat flux, i. e., in the central six triflutes surrounding the oscillating element. Poison in these positions could be accidentally moved by the oscillating element. Consequently, it was necessary to provide for a more complicated distribution of poison, which led to a residual spatial-spectral effect that was difficult to calculate. The final distribution of poison to give an acceptably low thermal-base effect was determined experimentally. This poison was in the form of thin strips of boron-stainless steel. Measurement of the thermal-base effect in both the unflattened and the flattened surrounding flux are shown schematically in Fig. 11. These curves are typical, although the thermal base was usually not entirely linear over the entire temperature range.

The presence of U^{235} in the annulus was found to increase the thermal-base effect; however, it was decided to retain the U^{235} for these measurements in order to simulate realistic reactor conditions more accurately:

Since it was observed that the measured thermal-base effect for various cases was different for the same quantity of compensating boron located in different positions, the boron was always placed at the location of the thorium that it was compensating.

It was observed experimentally that $\Delta R_B / \Delta T$ was surprisingly insensitive to the boron concentration of the compensating boron for the range of concentration of interest in these measurements. For example, the small HTGR annulus of graphite, U^{235} , and boron was found to have the same thermal-base effect, to within the accuracy of the measurements, for both 0.68 g and 1.09 g of boron contained in the 6-ft length of annulus. It can be

Fig. 11--Thermal-base effect as a function of element temperature

seen from Fig. 11 that this thermal-base effect is quite different, however, from that of an annulus of graphite and U^{235} without any boron. Therefore, $\Delta R_B / \Delta T$ apparently changed rapidly with concentration at low concentrations, but saturated to a value that was essentially constant over the range of concentrations which compensated the thorium loadings of interest in these measurements. This saturation effect gave assurance, according to Eq. (13), that the thermal-base effect was well measured, even if the correct compensating boron concentration was only approximately achieved.

V. RESULTS OF REACTIVITY-CHANGE MEASUREMENTS

OF DOPPLER COEFFICIENT

5.1. DESCRIPTION OF CASES MEASURED

The cases measured and the relation of the local geometry to that of the entire critical assembly were discussed at the end of Section II. In this section, specific information about the composition and geometric cross section of these cases is given.

In all cases, the thorium was homogeneously distributed in the thorium-bearing region in particles of less than 44-micron diameter. The grain self-shielding, being not more than 1%,⁽¹¹⁾ has been neglected.

Small HTGR Annulus

All of the fuel tubes having the small HTGR annulus had the following dimensions and compositions:

Thorium-graphite and uranium-loaded annulus		
Outside diameter, in.	2.25	
Inside diameter, in.	1.50	
Outer graphite		
Outside diameter, in.	3.57	
Inside diameter, in.	2.25	
Density, g/cm ³	1.65	
Inner graphite		
Outside diameter, in.	1.50	
Density, g/cm ³	1.90	
Atomic density, atoms/b-cm	N_{Th}	N_{25}
C/Th = 55.7	0.001,490	0.001,401
C/Th = 48.7	0.001,673	0.000,1405
C/Th = 31.5	0.002,379	0.000,1381

Large HTGR Annulus

All of the fuel tubes having the large HTGR annulus had the following dimensions and composition:

Large HTGR Annulus--continued

Thorium-graphite and uranium-loaded annulus

Outside diameter, in.	2.75
Inside diameter, in.	1.75
Outer graphite	
Outside diameter, in.	3.57
Inside diameter, in.	2.75
Density, g/cm ³	1.65
Inner graphite	
Outside diameter, in.	1.75
Density, g/cm ³	1.90

Atomic density, atoms/b-cm	N_{Th}	N_{25}
C/Th = 117	0.000,762	0.000,090

Cylinder

In one case, a thorium-graphite mixture in cylindrical form was measured in a fuel tube having the following dimensions and composition:

Thorium-graphite-loaded region		
Outside diameter, in.	1.50	
Uranium-graphite-loaded annulus		
Outside diameter, in.	2.25	
Inside diameter, in.	1.50	
Outer graphite		
Outside diameter, in.	3.57	
Inside diameter, in.	2.25	
Atomic density of central		
cylinder, atoms/b-cm	N_{Th}	N_{25}
C/Th = 30.2	0.002,547	0

Dragon Elements

Three sets of thorium-loaded elements were obtained from the Dragon Project in England. These elements contained no uranium. The thorium-loaded region was annular, but smaller than both of the HTGR annular cases listed above. The Dragon elements had the following dimensions and compositions:

Thorium-graphite loaded annulus	
Outside diameter, in.	1.43
Inside diameter, in.	0.91
Outer graphite	
Outside diameter, in.	3.57
Inside diameter, in.	1.43
Density, g/cm ³	1.80

Dragon Elements--continued

Inner graphite		
Outside diameter, in.		0.91
Density, g/cm ³		1.80
Atomic density of the three elements, atoms/b-cm	<u>N_{Th}</u>	<u>N₂₅</u>
C/Th = 88.5	0.001,247	0
C/Th = 76.6	0.001,081	0
C/Th = 66.4	0.000,936	0

5.2. RESULTS

The results for ΔR , $R_{\text{res}}(T)$, and dR_{res}/dT (or $\delta R_{\text{res}}/\delta T$) for 55.56°K temperature intervals are listed in Table 1 for six of the cases.

5.3. ESTIMATE OF EXPERIMENTAL ERROR

An over-all experimental uncertainty of $\pm 10\%$ is probably conservative for the reactivity measurement of the value of $(1/I)(dI/dT)$ averaged over the 300° to 700°K temperature range. This estimate of probable accuracy is based partly on statistical uncertainties and partly on experimental reproducibility.

As discussed in Section IV, the measurement of reactivity at any temperature has an uncertainty of not more than ± 0.01 cent. This uncertainty is increased somewhat by the requirement of taking the difference of two reactivities. The percentage uncertainty in obtaining the slope $\delta R_{\text{res}}/\delta T$ can be related to a ± 0.01 -cent uncertainty in a single measurement at each of two temperatures separated by 100°F. This uncertainty varies from 10% at low temperatures to 5% at high temperature. In several cases that have been repeated, a maximum uncertainty of $\pm 4\%$ can be assigned for reproducibility. Thus, an over-all uncertainty of $\pm 10\%$ on the average value of $(1/I)(dI/dT)$ appears to be conservative.

Table 1
RESULTS OF REACTIVITY MEASUREMENTS

ΔT (°F)	T (°K)	ΔR_{tot} (\$)	ΔR_{base} (\$)	ΔR_{res} (\$)	R_{res} (\$)	$\frac{dR_{res}}{dT}$ $\times 10^5 / ^\circ C$	$\frac{1}{R_{res}} \frac{dR_{res}}{dT}$ $\times 10^4 / ^\circ C$
Small HTGR Annulus; C/Th = 55.7							
0	300	0	0	0	-0.1398		
100	355	-0.00460	-0.00156	-0.00304	-0.1428	-5.59	3.96
200	411	-0.00900	-0.00299	-0.00601	-0.1458	-5.22	3.62
300	467	-0.01334	-0.00432	-0.00902	-0.1488	-5.40	3.66
400	522	-0.01740	-0.00554	-0.01186	-0.1517	-5.22	3.47
500	578	-0.02100	-0.00665	-0.01435	-0.1541	-4.32	2.82
600	633	-0.02430	-0.00762	-0.01668	-0.1565	-4.32	2.78
700	689	-0.02736	-0.00849	-0.01887	-0.1587	-3.96	2.51
							3.3 Average
Small HTGR Annulus; C/Th = 48.7							
0	300	0	0	0	-0.1444		
100	355	-0.00472	-0.00156	-0.00316	-0.1476	-5.69	3.90
200	411	-0.00942	-0.00299	-0.00643	-0.1508	-5.89	3.95
300	467	-0.01416	-0.00432	-0.00984	-0.1542	-6.14	4.03
400	522	-0.01882	-0.00554	-0.01328	-0.1577	-6.19	3.97
500	578	-0.02312	-0.00665	-0.01647	-0.1609	-5.74	3.60
600	633	-0.02687	-0.00762	-0.01925	-0.1636	-5.00	3.08
700	689	-0.03025	-0.00849	-0.02176	-0.1662	-4.52	2.71
							3.6 Average
Small HTGR Annulus; C/Th = 31.5							
0	300	0	0	0	-0.1692		
100	355	-0.00577	-0.00156	-0.00421	-0.1734	-7.58	4.42
200	411	-0.01108	-0.00299	-0.00809	-0.1773	-6.98	3.98
300	467	-0.01619	-0.00432	-0.01187	-0.1811	-6.80	3.79
400	522	-0.02114	-0.00554	-0.01560	-0.1848	-6.71	3.67
500	578	-0.02595	-0.00665	-0.01930	-0.1885	-6.66	3.58
600	633	-0.03047	-0.00762	-0.02285	-0.1920	-6.39	3.36
700	689	-0.03472	-0.00849	-0.02623	-0.1954	-6.08	3.14
							3.7 Average

Table 1--continued

ΔT (°F)	T (°K)	ΔR_{tot} (\$)	ΔR_{base} (\$)	ΔR_{res} (\$)	R _{res} (%)	$\frac{dR_{res}}{dT}$ $\times 10^5 / ^\circ C$	$\frac{1}{R_{res}} \frac{dR_{res}}{dT}$ $\times 10^4 / ^\circ C$
Large HTGR Annulus; C/Th = 117							
0	300	0	0	0	-0.1409		
100	355	-0.00465	-0.00139	-0.00326	-0.1442	-5.87	4.13
200	411	-0.00930	-0.00278	-0.00652	-0.1474	-5.87	4.03
300	467	-0.01377	-0.00416	-0.00961	-0.1505	-5.56	3.74
400	522	-0.01785	-0.00554	-0.01231	-0.1532	-4.86	3.21
500	578	-0.02155	-0.00693	-0.01462	-0.1555	-4.16	2.70
600	633	-0.02490	-0.00832	-0.01658	-0.1575	-3.53	2.26
700	689	-0.02815	-0.00970	-0.01895	-0.1593	-3.37	2.12
							3.2 Average
DRAGON Element No. 2; C/Th = 76.6							
0	300	0	0	0	-0.04976		
100	355	-0.00147	-0.00044	-0.00103	-0.05079	-1.85	3.69
200	411	-0.00294	-0.00089	-0.00205	-0.05181	-1.84	3.59
300	467	-0.00442	-0.00133	-0.00309	-0.05285	-1.87	3.58
400	522	-0.00587	-0.00178	-0.00409	-0.05385	-1.80	3.38
500	578	-0.00711	-0.00222	-0.00489	-0.05465	-1.44	2.66
600	633	-0.00817	-0.00267	-0.00550	-0.05526	-1.10	2.00
700	689	-0.00916	-0.00312	-0.00604	-0.05580	-0.97	1.75
							3.0 Average
Cylinder; C/Th = 30.2							
0	300	0	0	0	-0.1340		
100	355	-0.00580	-0.00235	-0.00345	-0.1374	-6.21	4.59
200	411	-0.01126	-0.00470	-0.00656	-0.1406	-5.60	4.04
300	467	-0.01665	-0.00705	-0.00960	-0.1436	-5.47	3.86
400	522	-0.02200	-0.00941	-0.01259	-0.1466	-5.38	3.72
500	578	-0.02735	-0.01177	-0.01558	-0.1496	-5.38	3.64
600	633	-0.03237	-0.01415	-0.01822	-0.1522	-4.75	3.15
700	689	-0.03700	-0.01653	-0.02047	-0.1545	-4.05	2.65
							3.7 Average

VI. ACTIVATION MEASUREMENT OF THE DOPPLER COEFFICIENT

As a check on the reactivity method of determining the Doppler coefficient, an activation technique was used for one measurement. The result of a direct activation measurement of $(1/A_{\text{res}})(dA_{\text{res}}/dT)$ was $(3.7 \pm 0.8) \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$ averaged over the range 300° to 700°K . This is to be compared with values over the same temperature range of $(3.6 \pm 0.4) \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$ and $3.4 \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$ for the same case obtained, respectively, by the reactivity oscillation method of measurement and a theoretical determination. The activation measurements were made in a fuel tube having the small HTGR annulus with a C/Th ratio of 48.7. The agreement is quite satisfactory and shows that there are no serious systematic errors in the measurement by the reactivity method. This is the first case known to us in which both reactivity and activation techniques have been applied in the same laboratory and consistent results have been obtained. Discrepancies reported in the literature⁽³⁾ between results measured by these two techniques have thus been shown to be avoidable.

The activation method of measurement utilized the $1/v$ subtraction technique that was developed for measurements of the resonance integral of thorium, as described in References 1 and 2. The measurements of the Doppler coefficient were made in a small vacuum-insulated oven at the center of the critical assembly. Temperatures up to 1000°F were thus obtained.

The oven was a thin aluminum shell, 4-3/4 in. long, with an outside diameter of 3-3/4 in. This design enables it to be positioned in a fuel-element location. The oven contained a 4-in. length of fuel element. Heating was accomplished by Nichrome wire in the graphite sleeve. The oven was evacuated to a pressure below 20 microns. The shell temperature was

found to be 130°F with the oven operating at 1000°F . Internal thermocouples showed less than a 5°F variation from center to edge, at a center temperature of 1000°F . Helium was circulated through the oven after exposure of the foils in order to give a quick cooldown without oxidation of the foils. It was necessary to remove the vanadium foils used in this technique, for the purpose of counting them, within 5 to 10 min after shutdown.

The activation technique used to measure the Doppler coefficient was based on activation of thorium and vanadium detectors in three positions, namely, in a fuel element heated in the oven, in a cool fuel element outside the oven, and in a Maxwellian spectrum. The detectors exposed inside and outside the oven were placed at the same positions in their respective fuel elements. The Maxwellian flux was developed in the thermal column on the side of the TRIGA reactor. The detection method, in the case of thorium, utilized material actually exposed in the annulus;⁽¹⁾⁽²⁾ after exposure, the material was ground up, collected, and its thorium activations counted. A foil detector⁽¹⁾⁽²⁾ was used for vanadium.

It was not necessary to include Au^{197} detectors in contrast to applications⁽¹⁾⁽²⁾ of this technique made at room temperature to measure A_{res} . It was sufficient to obtain an unnormalized measurement of $A_{\text{res}}(T)$. The resonance capture rate in a thin foil of Au^{197} , which is essentially unchanged with temperature and which is of use only to determine the normalization of A_{res} , was therefore not required.

Measurements with thorium and vanadium in the three positions were required, however, to determine the temperature-dependence of both the $1/v$ and the resonance cross-section components. The vanadium was used to determine the $1/v$ component, from which the thermal-base contribution to the Doppler coefficient was determined in the case of the activation method of measurement.

By an extension of the analysis⁽¹⁾ applied in connection with the activation measurement of resonance integrals, it can be shown that thorium resonance captures in the heated oven are related to the activation of the

various detectors as follows:

$$\left[\frac{A_{\text{res}}^S(T)}{A_{\text{res}}^S(T_0)} \right]_{\text{oven}} = \frac{\frac{A_{\text{Th, oven}}^S(T)}{A_{\text{Th, out}}^S(T_0)} - C \frac{A_{\text{V, oven}}^S(T)}{A_{\text{V, out}}^S(T_0)}}{\frac{A_{\text{Th, oven}}^S(T_0)}{A_{\text{Th, out}}^S(T_0)} - C \frac{A_{\text{V, oven}}^S(T_0)}{A_{\text{V, out}}^S(T_0)}} \quad (16)$$

The quantities in Eq. (16) have the following meaning: A_{res}^S has been defined in Eq. (1). The A^S values on the right side of the equation are the activations of various detectors made in the critical-assembly spectrum (S). Subscripts to the A^S 's designate the type of detector, and whether the exposure was made in the oven or outside the oven (abbreviated as "out"). The temperatures are T , for that of the heated oven, or T_0 , the ambient outside temperature. The oven temperature, being continuously variable, includes the temperature T_0 in its range. Oven-temperature measurements were made for $T = T_0$, and the quantities $A_{\text{Th, oven}}^S(T_0)$ and $A_{\text{V, oven}}^S(T_0)$ occur in the analysis of Eq. (16) at all temperatures. C is the constant;

$$C = 0.96 \left[\frac{A_{\text{V, out}}^S(T_0)}{A_{\text{V}}^M} \bigg/ \frac{A_{\text{Th, out}}^S(T_0)}{A_{\text{Th}}^M} \right], \quad (17)$$

where the A^M 's are the activations in the Maxwellian spectrum, and $0.96^{(1)}$ is the ratio of the $1/v$ component captures to the total captures in vanadium (this total includes a small resonance component). The remaining factors of C have been measured to be 2.763.

Let the ratio of the activation of a detector heated in the oven to that of a detector of the same material outside be r :

$$r_{\text{Th}} = \frac{A_{\text{Th, oven}}^S(T)}{A_{\text{Th, out}}^S(T_0)}, \quad r_{\text{V}} = \frac{A_{\text{V, oven}}^S(T)}{A_{\text{V, out}}^S(T_0)} \quad (18)$$

Equation (16) can be expressed in terms of r_{Th} and r_V , together with the ratios

$$\frac{A_{Th, oven}^S(T_0)}{A_{Th, out}^S(T_0)} \text{ and } \frac{A_{V, oven}^S(T_0)}{A_{V, out}^S(T_0)},$$

which have been measured to be 0.982 and 0.990, respectively. The result obtained from applying these modifications to Eq. (16) is

$$\left[\frac{A_{res}(T)}{A_{res}(T_0)} \right]_{oven} = 1.539(r_{Th} - 0.350 r_V) \quad (19)$$

Equation (17) may be differentiated to obtain the following expression for the Doppler coefficient:

$$\left[\frac{1}{A_{res}} \frac{dA_{res}}{dT} \right]_{oven} = \frac{1.539 \left(\frac{dr_{Th}}{dT} - 0.350 \frac{dr_V}{dT} \right)}{\left[\frac{A_{res}(T)}{A_{res}(T_0)} \right]_{oven}} \quad (20)$$

Figure 12 shows results of the measurements of r_{Th} and r_V . Despite the considerable scatter in experimental points for these measurements, which were essentially of a preliminary nature, each r was determined to within about $\pm 5\%$, which is sufficient to determine the Doppler coefficient to within $\pm 20\%$.

A linear fit, indicated by the straight-line fits shown in Fig. 12, has been made to the measured values of the r 's:

$$r_{Th} = 1 + a_{Th} \Delta T \quad (21)$$

$$r_V = 1 - a_V \Delta T, \quad (22)$$

where ΔT is the temperature difference between oven and outside location. The values of the empirical constants, a , as determined by a visual fit of

Fig 12--Activation ratios versus temperatures, measured in the internal oven

Figure 12 and essentially confirmed by a subsequent least squares determination, were found to be

$$a_{Th} = 0.136 \times 10^{-3} / ^\circ F = 0.245 / ^\circ K ,$$

$$a_V = 0.31 \times 10^{-3} / ^\circ F = 0.056 / ^\circ K .$$

The insertion of these values into Eqs. (19) to (22) gave results for $A_{res}(T)/A_{res}(T_0)$, $[1/A_{res}(T)] [dA_{res}(T)/dT]$, and the average value of $(1/A_{res})(dA_{res}/dT)$ over the 300° to $700^\circ K$ temperature interval given in Table 2.

Table 2
RESULTS FOR A_{res} AND $(1/A_{res})(dA_{res}/dT)$ FROM
ACTIVATION MEASUREMENTS

T ($^\circ K$)	$A_{res}(T)/A_{res}(T_0)$	$\frac{1}{A_{res}} \frac{dA_{res}}{dT}$
300	1.000	$3.96 \times 10^{-4} / ^\circ C$
400	1.040	$3.81 \times 10^{-4} / ^\circ C$
500	1.079	$3.67 \times 10^{-4} / ^\circ C$
600	1.119	$3.54 \times 10^{-4} / ^\circ C$
700	1.158	$3.42 \times 10^{-4} / ^\circ C$
AVERAGE		$3.68 \times 10^{-4} / ^\circ C$

VII. CALCULATIONS OF THE DOPPLER COEFFICIENT

The best available resonance parameter data and computational procedures in current use at General Atomic have been applied to obtain calculational values of $(1/A_{\text{res}})(dA_{\text{res}}/dT)$ for comparison with the measured values. The results so calculated are in good agreement with the measurements, although there are several differences worth noting. The calculations are described in this section of the report. In the next section, Section VIII, a comparison of calculated and experimental results, and comments on the significance of the comparison, are given.

The derivative has been determined by numerical differentiation of $A_{\text{res}}(T)$, calculated at a series of values of T ($\sim 100^\circ\text{K}$ increments). This numerical differentiation procedure appears to be more logical than the often-followed procedure of fitting an empirical function to the A_{res} data. The popular $T^{1/2}$ dependence has no theoretical basis for the homogeneous HTGR geometry and, in fact, deviates from the facts, which show that A_{res} varies approximately as $T^{0.13}$ for the large HTGR annulus. Numerical differentiation can be applied with more confidence than any one-parameter or two-parameter fit to the function $A_{\text{res}}(T)$, particularly for a fit having no discoverable theoretical basis. The accuracy of the numerical differentiation procedure which has been used is such that $(1/A_{\text{res}})(dA_{\text{res}}/dT)$ has about $\pm 3\%$ uncertainty due to the differentiation.

$A_{\text{res}}(T)$ has been calculated by methods which have been described in detail.⁽¹⁾ The method of calculation has been compared with both measurements and with several calculational refinements in order to evaluate the effect of several approximations. Moreover, several small corrections to the approximate calculations have been added to correct for these approximations. The resulting calculational procedures that have been applied are somewhat indirect.

The following several paragraphs indicate the significant points of this analysis.

The completely correct analysis of $A_{res}(T)$ should contain the following features:

1. Calculations based on experimentally verified resonance parameters.
2. A well-determined reactor neutron spectrum.
3. Proper treatment of the effect of the annular lattice geometry.
4. Determination of captures by solution of the integral equation in a manner such as that formulated by Nordheim⁽¹²⁾⁽¹³⁾ and coded by Kuncir.⁽¹⁴⁾

The neutron spectrum and thorium resonance captures have been determined by the GAM-I⁽¹⁵⁾ slowing-down code. For another "lattice-test" loading, namely, Core 3, this GAM-I spectrum has been found to agree well⁽¹⁶⁾ with an iterated multigroup solution for the entire reactor. Calculations have been made for the small HTGR annulus in both a $1/E$ spectrum and in the reactor spectrum determined by GAM-I. Only a small spectrum sensitivity was found, $(1/A_{res})(dA_{res}/dT)$ being about 4% less in the GAM-I reactor spectrum.

The resonance captures in the annulus were calculated⁽¹¹⁾ as though the annulus were an equivalent cylinder embedded in an infinitely extended moderator. The diameter, $\bar{\ell}$, of the appropriate equivalent cylinder was calculated by the following expression, in accordance with a suggestion of L. W. Nordheim:⁽¹⁷⁾

$$\bar{\ell} = \frac{4V}{S_o(1 - C_o) + S_i(1 - C_i)} \quad (23)$$

where V , S_o , and S_i are the volume, external surface, and internal surface, respectively, of the annulus; and C_o and C_i are external and internal Dancoff factors. It has been shown⁽¹⁸⁾ that the use of Eq. (23) to predict resonance

captures in the lattice of annuli should be correct to within a few per cent.

Two alternative sets of thorium resonance parameters have been considered in the analysis made of both the resonance integrals⁽¹⁾ and the Doppler coefficient. These parameters are listed in Appendix B. The two sets, designated "high" and "low" differ only in the value of Γ_n of the two lowest resonances. The "high" set was that once recommended by the Neutron Cross Section Evaluation Group of the Brookhaven National Laboratory.⁽¹⁹⁾ The "low" set was once proposed by Garrison.⁽²⁰⁾ The "low" set is in better agreement⁽¹⁾ with the best, recent integral-experiment measurements of the thin-limit resonance integral of thorium.

It has therefore been assumed in the analysis of the Doppler effect that the "low" parameters are correct. It might seem that the choice of the "low" set of resonance parameters is contrary to the indications of the Doppler measurements, since according to Figs. 13 and 14 the "high" parameters give calculated Doppler coefficients that are in better agreement with measured values, the difference being about 5%. However, the calculated Doppler coefficient is less sensitive to the choice of "high" or "low" sets than is the resonance integral, particularly the thin-limit resonance integral. Furthermore, both the measurements and the calculations of the Doppler coefficient are more difficult to carry out quantitatively than are those for A_{res} . The "low" parameters are therefore applied to the analysis of the Doppler effect. Calculations based on both sets have been carried out for the small and for the large HTGR annulus. The difference found for these cases can be applied to make corrections for those cases which have been calculated for the "high" parameters only; this was done in the case of Dragon elements.

A correction is also required to bring the calculations in the approximate manner in which they were done by GAM-I* into agreement with the integral-equation solution.^(12, 13, 14) To determine A_{res} , GAM-I had to be instructed to treat each resolved resonance by the Wigner resonance-

*A revised version of GAM, designated GAM-II, which has recently been completed, would make such corrections unnecessary in future work of this type.

Fig. 13--Doppler coefficient for a small HTGR annulus with C/Th = 55.7

Fig. 14--Doppler coefficient for a large HTGR annulus with C/Th = 117

integral approximation in either an N. R. or I. M. (I. M. is sometimes called N. R. I. A.) manner. A comparison has been made of calculations of a component of $(1/A_{\text{res}})(dA_{\text{res}}/dT)$ for the large HTGR annulus as carried out both by this approximate procedure and by the ZUT coding⁽¹⁴⁾ of the accurate integral-equation solution. The component chosen was the contribution by the resolved resonances in a $1/E$ spectrum, since ZUT can be applied to that condition. It was found that the ZUT integral-equation calculations give a larger $(1/A_{\text{res}})(dA_{\text{res}}/dT)$ than that determined by the GAM-I approximate method by an amount which has a relative error between 300° and 700°K of 3.5% $\pm$ 1%. The principal reason for this error is the approximate treatment of the 69.2-ev resonance, for which GAM-I as used in this analysis had been instructed to apply the I. M. approximation. This gave too large resonance capture and too little temperature-dependence of this capture. This GAM-I-ZUT correction has been extrapolated to other cases, and it has been assumed that for all cases $(1/A_{\text{res}})(dA_{\text{res}}/dT)$ computed by GAM-I is to be increased 3.5% to obtain the value that would be computed by the more accurate integral-equation method, and in the actual reactor spectrum.

It should be noted that the "high" to "low" resonance parameter correction and the GAM-I to ZUT correction very nearly cancel one another.

The large-annulus HTGR element and the Dragon element differed in geometry and composition from the foreign surrounding lattice (which was composed of small-annulus HTGR elements) in which the oscillator measurements were made. Corrections of a few percent have been made to the calculated Doppler coefficients to correct to the conditions of actual measurement. These corrections have been estimated from the effect of the difference in geometry on the effective carbon-to-thorium ratio, $(C/Th)_{\text{eff}}$. This quantity, which includes the effect of lattice geometry through the well-known equivalence principle,⁽¹²⁾⁽²¹⁾ is:

$$\left(\frac{C}{Th}\right)_{\text{eff}} = \frac{C}{Th} + \frac{1}{4.71} \left(22 + \frac{1}{N_{Th} \ell}\right), \quad (24)$$

where C/Th is the actual carbon to thorium ratio, 4.71 b is the scattering per carbon atom, 22 b is the potential scattering per ThO_2 molecule, N_{Th} is the atomic density of thorium in the thorium-containing region, and $\bar{l}$ is defined by Eq. (14).

Measurements shown in Fig. 17, page 54, show that the relative changes in $(C/Th)_{eff}$ and $(1/A_{res})(dA_{res}/dT)$ are related approximately as follows:

$$\frac{1}{A_{res}} \frac{dA_{res}}{dT} = - \frac{1}{5} \frac{\Delta(C/Th)_{eff}}{(C/Th)_{eff}} \quad (25)$$

Equation (25) has been used to apply corrections in which the $(C/Th)_{eff}$ of the calculated condition differs by about 10% from the measured condition, or to accomplish the small corrections of about 2%. The semiquantitative estimates made of $\bar{l}$ for these mismatches to the surrounding lattice are considered to be sufficiently well known to determine these small corrections. These estimates of $\bar{l}$ are given in Table 3, and the nature of the mismatch corrections can also be seen in the data of this table.

The procedures described have been applied to determine the four corrections stated near the beginning of this section. The experimental data given on Figs. 13 through 16 are $(1/R_{res})(dR_{res}/dT)$, which have been assumed to be equal to $(1/A_{res})(dA_{res}/dT)$, corrected to the conditions of measurement. A semiquantitative calculation has verified the assumption stated in Section III that $(1/R_{res})(dR_{res}/dT)$ can be taken to be equal to $(1/A_{res})(dA_{res}/dT)$. The comparison was based on contributions of the resolved resonances only, and on an importance function at the center of the reactor, $F(u)$, calculated for the critical assembly. These resolved resonances contribute about 90% of the Doppler coefficient. $F(u)$ increases by about 10% for an increase in lethargy, u , between the value corresponding to 5000 ev and the value corresponding to 20 ev. It was found that including $F(u)$ as the weighting function in determining R_{res} increased $(1/R_{res})(dR_{res}/dT)$ by less than 1% above $(1/A_{res})(dA_{res}/dT)$. This correction is therefore entirely negligible, and the equality sign can be validly considered to apply to Eq. (11).

Table 3

GEOMETRIC CHARACTERISTICS OF ANNULI WHOSE DOPPLER COEFFICIENTS WERE STUDIED

Central Element	Adjacent Lattice	Condition*	N_{Th} (b-cm)	C_o	C_i	$\bar{l}$ (cm)	$(C/Th)_{eff}$
Small HTGR annulus: C/Th = 31.5 C/Th = 48.7 C/Th = 55.7	Critical assembly Critical assembly Critical assembly	Matched Matched Matched	0.002,379 0.001,673 0.001,490	0.123 0.123 0.123	0.248 0.248 0.248	2.30 2.30 2.30	75.0 108.6 122.3
Large HTGR annulus: C/Th = 116.6 C/Th = 116.6	Critical assembly Own lattice	Mismatched Matched	0.000,7625 0.000,7625	0.20 0.273	0.203 0.203	3.18 3.18	208.8 203.9
Measured Dragon annulus: C/Th = 66.4 C/Th = 76.6 C/Th = 76.6 C/Th = 88.4	Critical assembly Critical assembly Own lattice Critical assembly	Mismatched Mismatched Matched Mismatched	0.001,247 0.001,081 0.001,081 0.000,936	0.05 0.05 0.168 0.05	0.41 0.41 0.41 0.41	1.63 1.63 1.78 1.63	175.5 201.7 191.6 232.3
Dragon annulus, calculated by Van Howe ⁽²²⁾ C/Th = 72.5	Critical assembly	Mismatched	0.001,142	0.05	0.41	1.63	191.2
Dragon annulus, calculated by Monte Carlo method ⁽²³⁾ C/Th = 72.5	Own lattice	Matched	0.001,142	0.168	0.41	1.78	181.6

* Condition of matching between central element and its adjacent lattice.

Fig. 15--Doppler coefficient for Dragon element having C/Th = 76.5 (453 g/72-in. length)

Fig. 16--Doppler coefficient for small HTGR annular fuel elements of several different C/Th ratios

Fig. 17-- $(1/A_{res}) (dA_{res}/dT)$ for several geometries; results averaged over the temperature range of 300° to $700^\circ K$

VIII. COMPARISON OF CALCULATED DOPPLER COEFFICIENT WITH VALUE MEASURED BY THE REACTIVITY- CHANGE METHOD

The calculated results for $(1/A_{res})(dA_{res}/dT)$ given on Fig. 13 are based on both "high" and "low" resonance parameter data, for the small HTGR annulus having $C/Th = 55.7$. These calculations have been corrected so that they correspond to the more accurate integral-equation solution.^(12,13,14) The results for $(1/R_{res})(dR_{res}/dT)$ measured by the reactivity oscillation method and considered equal to $(1/A_{res})(dA_{res}/dT)$ are given also. Figure 14 gives similar results for the large HTGR annulus. Figure 15 gives similar results for the Dragon elements, determined for "low"* resonance parameters⁽²²⁾ and also shows the Doppler coefficient calculated by the Dragon Project using an entirely independent Monte Carlo method.⁽²³⁾ These Monte Carlo calculated results are corrected to the conditions of measurement and also are revised to be consistent with "low" resonance parameters. Since it is less certain that these corrections to the Monte Carlo calculations have been well made, they should probably be given less emphasis in comparing with measurement.

A comparison of calculated and measured values for several small HTGR annuli are shown in Fig. 16.

Figure 17 summarizes the effect of changes in geometry on $(1/A_{res}) \times (dA_{res}/dT)$, which is the average value of $(1/A_{res})(dA_{res}/dT)$ from 300° to 700°K. The curve on Fig. 17 gives a fit to the experimental points, showing a smooth dependence of the measurements upon the abscissa. The abscissa chosen is an effective C/Th ratio, $(C/Th)_{eff}$, defined in Eq. (15).

* Actually, Van Howe,⁽²²⁾ who made the original calculations of $(1/A_{res}) \times (dA_{res}/dT)$ for the Dragon elements, assumed "high" resonance parameter values. Figure 15 gives Van Howe's calculations reduced by a few percent. This reduction was estimated by differences of Figs. 13 and 14, so that Fig. 15 results are consistent with "low" parameters.

The following two features of the comparison between the calculated and the measured results for $(1/A_{\text{res}})(dA_{\text{res}}/dT)$ are evident from Fig. 13 through 16:

1. The energy dependence is different. The calculated values rise considerably above the measured values at the lowest temperatures considered.
2. Above about 450°K , the energy dependence of calculated and measured values are quite similar, but the calculated values are 10% to 15% below the measured values.

There is little evident reason for this second difference. There is one known systematic error in the measurements, and this error is small. It causes the measured results to err in the direction of being large. Therefore, removal of this error would bring the measured values of $(1/A_{\text{res}}) \times (dA_{\text{res}}/dT)$ (or actually $(1/R_{\text{res}})(dR_{\text{res}}/dT)$) into somewhat closer agreement with calculations. This error is connected with the application of Eq. (7), which overestimates the compensating poison by about 5%. This causes $R_{\text{res}}(T_0)$ to be underestimated by about 2%. It has been pointed out in Section 4.5 that this has no observable effect on (dR_{res}/dT) . Therefore, this error can be expected to cause $(1/R_{\text{res}})(dR_{\text{res}}/dT)$ of the measurement to be about 2% too large.

The difference in energy-dependence of the Doppler effect at the lower temperatures is probably the result of crystalline binding. The calculated values are based on treatment of the Doppler broadening which assumes a Maxwellian distribution model of velocities of absorber atoms applicable to the case of unbound absorber atoms. However, the thorium is present in the form of a ceramic-like ThO_2 crystal for which the 300°K specific heat has probably⁽¹⁾ not yet saturated to the high-temperature value of the specific heat. If this is so, the Doppler broadening at 300°K can be more than that calculated on the basis of a Maxwellian distribution (see Ref. 24 and p. 33 of Ref. 21). At higher temperature, the values based on the Maxwellian and the actual crystalline binding come into agreement. The

deviation at low temperatures caused by crystalline binding increases the effective temperature. This, in turn, increases the 300°K value of thorium resonance captures, and decreases the 300°K value of the Doppler coefficient.

The importance of crystalline binding on the thorium resonance capture has been discussed in some detail in Ref. 1 for HTGR-type reactors. The measurements of thorium resonance capture indicate a value at 300°K larger by probably 5% with thorium contained in the oxide, compared with the metallic form. This difference is to be expected as a result of crystalline binding in a crystal at a temperature at which the specific heat has not yet saturated (see Ref. 24 and p. 33 of Ref. 21). That is, observed deviations at low temperatures between calculations which neglect crystalline binding and measurements occur for resonance captures and also for the Doppler coefficient of thorium contained in the oxide. These deviations are consistent with each other and are both believed to be due to crystalline binding effects which modify the low-temperature Doppler broadening.

No measurements of the specific heat as a function of temperature have been found in the literature for either ThO_2 or ThC . The experimental data found to be most pertinent to this question was the specific heat versus temperature measured for UO_2 . UO_2 is a very similar material to these thorium compounds. For UO_2 , the 300°K value of the specific heat has been reported⁽²⁵⁾ to be about two-thirds of the high-temperature value which is reached above 900°K . It is highly desirable that the specific heat of ThO_2 and ThC be measured so that these suggestions about the modifications caused by crystalline binding can be confirmed. Meanwhile, the data of Figs. 13 through 16 can be interpreted to indicate that crystalline binding does cause these modifications, and therefore to indirectly indicate some information about the specific heat. That is, this is a case of qualitative information on solid-state physics having been obtained from a neutron reactor physics measurement made in a critical assembly.

IX. CONCLUSIONS

Above 500°K, the measured Doppler coefficient of ThO₂ dispersed in graphite is in satisfactory agreement with the values calculated by methods for determining resonance captures that are in current use at General Atomic. (12, 13, 14) The overestimate of the 300°K calculated value can be expected to be improved if the effects of crystalline binding on Doppler broadening in ThO₂ and ThC are included in the calculations. For temperatures above about 500°K, the calculated energy dependence of the Doppler coefficient agrees reasonably well with measurements and is about 10% less than the measured value. This 10% difference is approximately the experimental error of these measurements.

Since an activation measurement of the Doppler coefficient confirmed the reactivity measurement of the Doppler coefficient in the case studied, self-consistent results between these two methods of measurement and with calculations have been obtained.

Appendix A

THE PHYSICAL NATURE OF THE THERMAL-BASE EFFECT

The thermal-base effect can be large due to the following two effects, each of which is proportional to the central neutron leakage:

1. Reactivity perturbation caused by a local change with temperature in the reaction rates in the heated region.
2. A change in local leakage due to modification of local diffusion lengths with temperature.

Both of these effects would be absent if the local neutron temperature were not affected by heating the central element. However, heating this element, with its associated graphite, modifies the local neutron temperature slightly. Because of the sensitivity of these two local effects to the neutron temperature, the small perturbation caused by the heating can cause a thermal-base effect in the HTGR critical assembly which is comparable in magnitude to the Doppler effect itself.

It is shown below that both of these effects are approximately proportional to the neutron leakage; therefore, the thermal-base effect is greatly reduced by carrying out measurements in an assembly having a completely flat flux distribution. The bare-ended HTGR critical assembly has a cosine axial flux distribution at all positions which does not change with temperature. The effect of axial leakage is thus essentially the same at all temperatures, so that this leakage does not contribute significantly to the thermal-base effect. The important flux to flatten in the HTGR assembly is the radial component. This was done by adding boron, placed in void spaces. The degree of success accomplished by the flattening that was achieved is indicated by the result that the thermal-base effect was only one-third as large in the flux-flattened assembly as in the unflattened assembly.

Since a uniform distribution of poisoning could not be completely achieved, some local radial leakage remained and contributed to this residual thermal-base effect.

The nature and magnitude of the two contributions to the thermal-base effect is readily estimated with the aid of perturbation theory, together with values of several characteristics of the assembly. Consider first the reactivity effect due to the reaction rates of these two contributions in the central region which is subject to heating, but which is at the moment at the same temperature as its surroundings.

The reactivity effect of absorptions and fissions in the central element is determined by considering the change which results when the element is removed from the assembly. For a fission cross section, Σ_f and a low energy absorption cross section, $\Sigma_{a,low}$, in this central element, two-group perturbation theory⁽⁸⁾ gives the following reactivity change for this removal:

$$\left(\frac{\delta k}{k}\right)_{a,f} = -C(\nu F_H \Sigma_f - F_L \Sigma_{a,low}) \quad (A-1)$$

where C is proportional to the volume of the element removed and inversely proportional to the fission rate in the assembly. F_H and F_L are the high- and low-energy neutron importances, respectively, at the center of the assembly, and ν is the number of neutrons per fission. The negative sign of Eq. (A-1) occurs because a removal has been made. The value of $(\delta k/k)_a$ of Eq. (A-1) has been measured to be about -0.09 dollar (-0.36 dollar for removal of U^{235} and +0.27 dollar for removal of thorium).

The reactivity value of neutrons lost by radial thermal leakage from the central element can be estimated from a knowledge of the buckling, diffusion length, composition, and the measured distributed reactivity coefficient of boron. The effect of leakage is estimated from Eq. (A-1) by considering the absorption which simulates the leakage loss, although it could be more elegantly derived from the perturbations which result from a change in leakage. In determining this loss by leakage, it is sufficient

to consider only the $B^2 L^2$ component of this leakage, since the effect of temperature on leakage is to modify L^2 . Here, L^2 designates the thermal diffusion length, and B^2 the radial buckling.

B^2 , at low energies, can be readily estimated to be about 0.00086. L^2 is about 100 cm², if an average is carried out up to about 1-ev energy. Therefore, $B^2 L^2$ is about 0.086. That is, for each neutron lost by thermal absorption, 0.086 neutron is lost by radial leakage at energies below about 1 ev. From the composition of the fuel rod, the σ_{2200} absorption cross section per U^{235} atom was found to be 786 b. The 6-ft length of central fuel rod contains 0.602 moles of U^{235} . The measured reactivity coefficient of boron, distributed over the 6-ft rod, is 1.85 dollars/mole. By combining these facts, one obtains the following reactivity worth of neutrons lost by low-energy leakage from the central rod:

$$\begin{aligned} \left(\frac{\delta k}{k}\right)_{\text{leak}} &= -0.086 \times (\text{low energy reactivity coefficient of central element}) \\ &= -0.086 \times \frac{786}{755} \times (\text{moles of } U^{235} \text{ in central element}) \times \\ &\quad (\text{dollars/mole for boron}) \\ &= -0.086 \times \frac{786}{755} \times 0.602 \times 1.85 \\ &= -0.10 \text{ dollar.} \end{aligned}$$

Therefore, the removal of a central rod and the normal leakage from this rod cause negative reactivity effects of comparable magnitudes.

Now, consider the effect of leaving the central rod in the assembly but increasing its neutron temperature. This decreases the central-element reaction rate and increases the leakage by increasing L^2 . Each effect, therefore, leads to a decrease in reactivity, and these decreases add to produce the observed thermal-base effect. To obtain an indication of the magnitude of the resulting reactivity change, first consider the hypothetical case having a Maxwellian neutron spectrum, with the absolute temperature doubled. Neutron reaction rates decrease in the ratio $1/\sqrt{2}$, or by about 30%. The resulting perturbation, compared with the room-temperature

reactivity, would be

$$(0.30) (0.09 + 0.10) = 0.075 \text{ dollar}$$

The thermal-base effect that was observed in the original assembly having no flux flattening showed a reactivity change of about 0.01 dollar for a doubling of absolute temperature. This is consistent with the result just estimated if the effective neutron temperature within the central rod is modified by about 15% by the heating. Such a magnitude of change can be expected as a residual spatial-spectral problem.

The dependence of $(\delta k/k)_a$ upon the thermal leakage, described by the low-energy buckling, B^2 , is seen by evaluating Eq. (A-1) by two-group-theory computations. Introducing the usual four-factor quantities, ϵ , η , f , and p , where

$$k_{\infty} = \epsilon \eta p f, \quad (A-2)$$

the quantity $\nu \Sigma_f$ in Eq. (A-1) can be expressed in terms of $\Sigma_{a, \text{low}}$ as follows:

$$\begin{aligned} \nu \Sigma_f &= \epsilon \eta f \Sigma_{a, \text{low}} \\ &= \frac{k_{\infty}}{p} \Sigma_{a, \text{low}} \end{aligned}$$

The general two-group adjoint equations lead to the result

$$F_H = \frac{p}{k_{\infty}} (1 + B^2 L^2) F_L. \quad (A-3)$$

Introduction of Eqs. (A-2) and (A-3) into Eq. (A-1) lead to the result

$$\begin{aligned} \left(\frac{\delta k}{k} \right)_a &= -C \left[\frac{k_{\infty}}{p} \Sigma_{a, \text{low}} \frac{p}{k_{\infty}} (1 + B^2 L^2) F_L - F_L \Sigma_{a, \text{low}} \right] \\ &= -C F_L \Sigma_{a, \text{low}} B^2 L^2, \end{aligned} \quad (A-4)$$

which is proportional to leakage.

Therefore, the development of a completely flat flux loading eliminates both contributions to the thermal-base effect, each of which contributions is proportional to B^2 . This elimination applies accurately in the limit of small

temperature changes. For larger temperature changes, the elimination cannot be expected to be complete, since spatial-spectral effects can occur and can cause leakage and also cause Eqs. (A-2) and (A-3) to be inapplicable. Even for large temperature changes, however, the thermal-base effect can be expected to be much reduced by the flat flux loading.

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Appendix B

RESONANCE PARAMETERS* OF Th²³² USED IN

CALCULATIONS OF THE DOPPLER COEFFICIENT

E_0 (ev)	Γ_n (10^{-3} ev)	Γ_n^0 (10^{-3} (ev) ^{1/2})	Γ_γ (10^{-3} ev)
21.84	2.4 (high) 1.68 (low)	0.51 (high) 0.36 (low)	30
23.48	4.0 (high) 2.86 (low)	0.83 (high) 0.59 (low)	30
59.55	4.6	0.060	30
69.2	39	4.69	30
113.15	11	1.04	42
121.0	18.5	1.68	41
128.5	0.10	0.009	
129.4	3.1	0.26	
146.2	0.10		
154.6	0.22	0.018	
170.8	58	4.45	48
193.0	14	1.01	50
196.8	0.13	0.009	0.01
199.8	9	0.64	
203			
221.8	23	1.54	46
252.3	24	1.51	42
264	15	0.92	
286.8	21	1.22	50
306.5		1.14	
330.4	55	3.02	53
343.3	30	1.62	
366.7	35	1.83	
371.0	24	1.25	
402.8	9	0.45	
413.4			
456.4			
465.0	45	2.09	
491.0	50		
513.0	3.5	0.15	
531.5	15	0.65	
543.0	1.0	0.04	
573.0	25	1.04	
581.0	1.6	0.06	

*Based on the notation of BNL-325.

Appendix B--continued

E_0 (ev)	Γ_n (10^{-3} ev)	Γ_n^0 (10^{-3} (ev) $^{1/2}$)	Γ_γ (10^{-3} ev)
601.5	11	0.45	
621.5	5	0.20	
660.5	32	1.24	
669.0	17	0.66	
679.5	125	4.80	
692.0	38	1.48	
705.0	18	0.68	
717.5	25	0.94	
746.0	160	5.85	
783.0	10	0.36	
810.0	125	4.40	
848.0	23	0.79	
872.0	10	0.34	
896.0	30	1.00	
950.0	40	1.30	
970.0	7.5	0.24	
990.0	24	0.76	
998.0	70	2.20	
1020			
1046			
1072			
1085			
1100			
1118			
1130			
1159			
1204			
1215			
1238			
1253			
1259			
1280			
1303			
1314			
1367			
1390			

Average Values:

$$\bar{D} = 19.8 \text{ ev}$$

$$\bar{\Gamma}_\gamma = 0.041 \text{ ev}$$

$$\bar{\Gamma}_n^0 = 0.00132 \text{ (ev)}^{1/2}$$

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