Radiation Coupling of Bernstein Modes in Dense Plasmas

Interim Progress Report

by

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July 1972

Introduction

Microwave radiation emitted from both laboratory and astrophysical plasmas¹ has been observed to be very intense at harmonics of the local electron cyclotron frequency. Landauer,² who first reported observing the spectrum of radiation from a Penning discharge as the $\vec{B}$ field was progressively increased, found that radiation peaks through the 45th harmonic were of large magnitude but that there was no peak at the fundamental. Others³ have reported similar results. In the past ten years there have been many attempts at solving this problem,⁴⁻¹⁴ however, all theories presented to date are incomplete.

*Research supported in part by the U. S. Atomic Energy Commission under Contract AT(11-1)-2081 with the University of Colorado.

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We attempt a solution of the above problem and adopt the following model. We assume we can neglect ions as $\omega \sim \Omega_e$. Further, we assume the plasma to be of the loss cone variety. However, as a first approximation to the loss-cone distribution we assume a delta function distribution for electrons, i.e. $f_0 = \delta(v_z) \delta(v_\perp - u)/2\pi u$. This distribution obviously will simulate the anisotropy in pitch angle that is characteristic of the loss-cone distribution. We also assume wave propagation $\perp$ to the static magnetic field; the density inhomogeneity weak and constant in direction; $\vec{B} = B_0(1+\epsilon x) \hat{z}$, with

$$\epsilon \equiv \frac{1}{B_0} \frac{dB}{dx} \Big|_{x=0};$$

and the following geometry

Using the above model, our aim will be to investigate the coupling of a longitudinal mode (the Bernstein mode which propagates at frequency $\omega \sim m \Omega_e$, $m = 1, 2, 3, \dots$) to an electromagnetic mode which can then propagate out of the plasma. The main part of the investigation will be a study to determine which E-M mode is involved, the efficiency of coupling between the Bernstein mode and E-M mode, and finally a study of the characteristics of the E-M mode at the plasma-vacuum boundary to see if escape is possible and efficient. With this model and information we can equally well evaluate the coupling of external r. f. sources into the plasma at these frequencies to see whether

efficient enhanced containment might be suggested.

The procedure that we follow is similar to the method outlined by Stix¹⁰ in studying the mode conversion of a Bernstein wave to the QT-x electromagnetic mode in a Maxwellian plasma. Stix proposed that Bernstein waves are excited by thermal or suprathermal electrons and propagate inward to the upper hybrid critical layer where they reflect and mode convert to the QT-x mode which can tunnel past the right hand cutoff and then propagate out of the plasma. Figs. 1a,b,c depict this scheme. The plasma is approximated in three regions: cold plasma ($k \rho_L \ll 1$), warm plasma ($k \rho_L \sim 1$), hot plasma ($k \rho_L \gg 1$) where ρ_L is the Larmor radius. Stix found that efficient mode conversion does take place.

Of great importance in this study is the Bernstein mode which was theorized in 1958 by Bernstein.¹⁵ Certain overall statements can be made about this particular mode:

- (i) The waves are electrostatic.
- (ii) For propagation vectors oblique to the $\vec{B}$, cyclotron damping can occur with severity depending upon the phase velocity of the wave and the detailed shape of the particle velocity distribution function.
- (iii) For propagation perpendicular to $\vec{B}$ such damping cannot occur and any damping that occurs must be purely collisional.
- (iv) Although the exact dispersion characteristics of propagation depend on the detailed shape of the particle velocity distribution function, cutoffs ($k \rightarrow 0$) always occur

at the cyclotron harmonics ($M\Omega_e$) with $M = 2, 3, \dots$

and at the upper hybrid frequency $\omega_{UH} = (\omega_{pe}^2 + \Omega_e^2)^{1/2}$

characteristic of cold plasmas. Resonances ($k \rightarrow \infty$)

occur at $M = 1, 2, 3, \dots$ "¹⁶

Harris¹⁸ has shown that for a delta function electron distribution of the form $f(v_r, v_z) = \delta(v_z - v_{zo}) \delta(v_r - v_{ro}) / 2\pi v_{ro}$ the dispersion relation is given by

$$0 = 1 - \frac{\omega_{pe}^2}{\omega^2} \sum_{n=1}^{\infty} \frac{\frac{\partial}{\partial \mathcal{H}} J_n^2(\mathcal{H})}{\mathcal{H}/2 \left[\left(\frac{\omega}{n\Omega_e} \right)^2 - 1 \right]}$$

where $\vec{k} \perp \vec{B}_0$, $\mu \equiv \frac{kv_{ro}}{\Omega_e}$. Graphical solutions of this equation have been presented by Crawford¹⁶ and are shown in Figure (2). The solution plotted is that of complex ω and real k . Note that Figure (2) depicts an oscillatory solution which is such that at a high enough value of the quantity $(\omega_{pe}/\Omega_e)^2$ neighboring modes can couple giving rise to an imaginary part of ω (assuming k real). This imaginary part is represented by the dotted lines in Figure (2). Figure (3) is obtained from Figure (2) by setting $\omega = 3\Omega_e + \delta$ where $3\Omega_e \gg \delta > 0$ and then plotting kv_o/Ω_e versus ω_p^2/Ω_e^2 . This figure shows how the mode will propagate in a mildly inhomogeneous media. Note that there are two nonconnected loops. The number of loops depends upon the magnitude of δ .

In addition to the above longitudinal wave which we can assume to be present in any delta function type plasma, we must investigate the presence of any transverse modes. Hopefully, we want to find a T-V mode that would, on a graph such as Fig. (3), pass very close to a hump

where the possibility of efficient coupling is present (i. e. where $\frac{dk}{dx} \rightarrow \infty$). The T-V mode must also be required to propagate out of the plasma and become the normal E-M mode outside of the plasma ($\omega = kc$).

We begin our analyses by neglecting ions and assuming the electrons to be distributed in velocity space as

$$f_0(v_z, v) = \delta(v_z) \delta(v - u)/2\pi u.$$

Further, we assume $\omega \sim m r_e$ and include a static magnetic field

$B_0 = B_0 \hat{e}_z$, and look for T-V waves propagating in the y direction.

Assuming variations like $e^{i\vec{k} \cdot \vec{r} - i\omega t}$, Maxwell's equations yield

$$\vec{D}(\vec{k}, \omega) \cdot \vec{E}(\vec{k}, \omega) = 0$$

where

$$\vec{D}(\vec{k}, \omega) = \vec{x}(\vec{k}, \omega) + \begin{pmatrix} 1 - k^2 c^2 / \omega^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 - k^2 c^2 / \omega^2 \end{pmatrix}$$

and

$$\vec{x} \cdot \vec{E}(\vec{k}, \omega) = -4\pi/i\omega \vec{j}(\vec{k}, \omega).$$

Using the standard Green's function technique on the Vlasov equation the electric susceptibility, $\vec{x}$, becomes

$$x_{xx} = (-\omega_{pe}^2 / 2\omega) \sum \frac{1}{\omega^*} \left[MJ_{M-1}^2 \left(\frac{ku}{\Omega_e} \right) - MJ_{M+1}^2 \left(\frac{ku}{\Omega_e} \right) \right]$$

$$x_{xy} = (-\omega_{pe}^2 / 2\omega) \sum \frac{1}{\omega^*} \left[-2MJ_M^2 + MJ_{M-1}^2 + MJ_{M+1}^2 \right]$$

$$\omega^* \equiv \omega - M\Omega_e$$

$$x_{yy} = (-\omega_{pe}^2 / 2\omega) \sum \frac{1}{\omega^*} \left[-2 \frac{ku}{\Omega_e} J_M J_{M-1} + 2 \frac{ku}{\Omega_e} J_M J_{M+1} \right]$$

$$+ MJ_{M-1}^2 - MJ_{M+1}^2$$

$$X_{yx} = -X_{xy}$$

The dispersion relation for T-V waves is found by setting the determinant of $\overleftrightarrow{D}$ equal to zero (neglecting the part which gives long waves).

$$\begin{pmatrix} 1 - \frac{k^2 c^2}{\omega^2} + X_{xx} & X_{xy} \\ -X_{xy} & 1 + X_{yy} \end{pmatrix} = 0$$

Assuming convergence of the sum, we can keep only the highly resonant term if ω is very close to $m\Omega_e$

$$\begin{aligned} 1 - \frac{k^2 c^2}{\omega^2} &\sim \frac{\omega_{pe}^2}{2\omega} \frac{1}{\omega - m\Omega_e} \{2m(2J_m^2 + J_{m-1}^2 - J_{m+1}^2)\} \\ &+ m \frac{k^2 c^2}{\omega^2} \{4J_m^2 - J_{m-1}^2 + J_{m+1}^2\} - \frac{\omega_{pe}^4}{4\omega^2} \frac{1}{(\omega - m\Omega_e)^2} \\ &\{2m \frac{ku}{2} J_m (J_{m+1}^3 - J_{m-1}^3 + J_{m+1}^2 J_{m-1} - J_{m-1}^2 J_{m+1}) \\ &+ 2m^2 (J_{m-1}^4 + J_{m+1}^4) - 4m^2 J_m^2 (J_{m+1}^2 J_{m-1}^2 - J_m^2)\} , \end{aligned}$$

which is of the form:

$$\omega^2 - A \cong B \frac{\omega}{\omega - m\Omega_e} \{c + D/\omega^2\} - \frac{E}{(\omega - m\Omega_e)^2} \{F+G-H\}.$$

Noting Figure (3) we see that near a hump, the Bernstein mode has $\frac{ku}{\Omega_e} \sim m \Rightarrow k \sim \frac{m\Omega_e}{u} e$. Thus for coupling inside the plasma we need

$$\frac{m\Omega_e}{kc} \sim \frac{m\Omega_e u}{m\Omega_e c} \sim u/c \ll 1.$$

So we expect $m\Omega_e \ll kc$. Now look at the RHS of the previous equation. The value of the coefficients c, D, F, G, H are determined by $\frac{ku}{\Omega}$ and by m . Thus, we can change the value and sign of the constants by choosing different m and $\frac{ku}{\Omega_e}$.

We have investigated these functions, $c, D, F \dots$, and have shown that there does exist a T-V wave with frequency near $m\Omega_e$.

We now investigate the possibility of the T-V mode passing close to a Bernstein hump on the $\frac{ku}{\Omega_e}$ vs density graph. For simplicity we look at $m = 3$ only and we find with $\frac{\omega-3}{\Omega_{ee}} \approx \frac{1}{5}$ we get the requirement $c^2/u^2 \sim 13.6$. Thus, a T-V mode does pass near the hump if $c^2/u^2 \sim 14$ for $\omega \sim 3\Omega_e$ and

$$\frac{\omega - m\Omega_e}{\Omega_e} \sim \frac{1}{5}.$$

Using this equation we have also investigated how the T-V mode propagates as density decreases, and it is easily shown to evolve into the free space wave as the density adiabatically goes to zero.

In summary then we have shown the existence of transverse waves with frequencies near harmonics of the cyclotron frequency and it appears that for a sufficient ratio of u/c , the wavevector of these modes approach that of the Longitudinal Bernstein mode where $\frac{dk}{dx} = \infty$, but k is finite. This implied breakdown of the WKB condition implies that the transverse wave which, in fact, does couple to the vacuum can carry energy out of, or into, these unstable Bernstein modes.

QT-X And QT-O in inhomogeneous medium

$$\begin{aligned} n_{UH}^2 &= \omega_{pe}^2 + n_e^2 \\ \omega_{pe}^2 &= \frac{4\pi n_e e^2}{m_e} \end{aligned}$$

Bernstein Mode in Maxwellian
Plasma

index of refraction

 m_z^2

$$\omega = m\omega_e + \delta$$

 $m_e = \text{density}$

FIG. 1. The square of the index of refraction (n_x^2) is plotted against density. The frequency, which is fixed, is equal to the hybrid frequency at the value of density indicated. For an inhomogeneous plasma with linearly increasing density, the abscissa is proportional to distance (x). Dotted lines represent extrapolations of approximate dispersion relations beyond their regions of validity. Thus the extrapolated cold-plasma hybrid resonance ($n_x^2 \rightarrow \infty$) point is seen to coincide with the cut-off ($n_x^2 \rightarrow 0$) point for the warm-plasma-approximation plasma wave. QT-X designates the quasitransverse extraordinary (electromagnetic) mode. The parallel wave number, k_z , is assumed small or zero. At the top of the figure the bulge out to the right corresponds to the maximum of the right-hand side in Eq. (11) which occurs under the conditions of Eq. (12).

FIGURE 1C.

FIG. 2. Dispersion characteristics of perpendicularly propagating cyclotron harmonic waves—delta-function transverse velocity distribution.

FIGURE 2

FIGURE 3

Bernstein Mode in Beam Plasma

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