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Dynamic Stabilization of Transverse Kelvin-Helmholtz Instability*

by

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Abstract

The transverse Kelvin-Helmholtz instability is driven by non-uniform $\underline{E} \times \underline{B}$ motion of a plasma. An a.c. electric field near ion cyclotron frequency, applied across the shear layer resulting from the nonuniform d.c. electric field, can be used to stabilize the transverse Kelvin-Helmholtz instability. The stabilizing effect is shown to arise from high-frequency excursion of ions near the shear layer.

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A cylindrical plasma column confined by uniform axial magnetic field and possessing a radial electric field rotates with the $\underline{E} \times \underline{B}$ frequency. If the electric field does not vary linearly with radius, the plasma rotates nonuniformly and velocity shear exists between radially adjacent layers. When adjacent layers mix, the directed kinetic energy decreases and the Kelvin-Helmholtz mode thus may become unstable¹.

A transverse a.c. electric field, near ion cyclotron frequency, is used in the present model to dynamically stabilize the low-frequency Kelvin-Helmholtz mode. We select a two-region plasma, Fig. 1. At equilibrium, there exists a discontinuity in $E_0(x)$ so that a velocity shear exists in the x direction.

To follow the relative motion of electrons and ions due to the applied field of frequency ω_0 , we define the following transformations of particle displacement and velocity:

$$\underline{\rho}_\alpha = \underline{x}_\alpha + \underline{\epsilon}_\alpha \sin \omega_0 t \quad (1)$$

$$\underline{u}_\alpha = \underline{v}_\alpha + \omega_0 \underline{\epsilon}_\alpha \cos \omega_0 t \quad (2)$$

$$\alpha = i, e$$

and

$$\phi_{k_y, \alpha}^n(\underline{\rho}_{\alpha, x}) = \int d\underline{\rho}_{\alpha, y} dt \phi(\underline{\rho}_{\alpha, x}, \underline{\rho}_{\alpha, y}, t) \exp[-ik_y \underline{\rho}_{\alpha, y} + i(\omega + n\omega_0)t] \quad (3)$$

of the function $\phi(\underline{\rho}_{\alpha, x}, \underline{\rho}_{\alpha, y}, t)$. It follows that a function f in both the electron and ion frames is related by

$$f_{k_y, e}^n = \sum_p J_p(\epsilon_x \frac{d}{dx} + k_y \epsilon_y) f_{k_y, i}^{n-p} \quad (3)$$

$$f_{k_y, i}^n = \sum_p J_{-p}(\epsilon_x \frac{d}{dx} + k_y \epsilon_y) f_{k_y, e}^{n-p} \quad (4)$$

$$\underline{\epsilon} = \underline{\epsilon}_e - \underline{\epsilon}_i \quad (5)$$

where J_p is the Bessel function of order p .

The dispersion relation^{2,3,4} is, therefore

$$\nabla^2 \phi_i^n = L_i^n \phi_i^n - \sum_p J_{-p} L_e^{n-p} \phi_e^{n-p} \quad (7)$$

where

$$L_i^n = \frac{\omega_{pi}^2}{(\bar{\omega} + n\omega_o)^2 - \Omega_{ci}^2} \left[\frac{n_o'}{n_o} \frac{k_y \Omega_{ci}}{\bar{\omega} + n\omega_o} - k_y^2 + \frac{1}{n_o} \frac{d}{dx} n_o \frac{d}{dx} - \frac{(\omega' n_o)'}{n_o (\bar{\omega} + n\omega_o)} \right] \quad (8)$$

$$L_e^n = \frac{\omega_{pe}^2}{(\bar{\omega} + n\omega_o)^2 - \Omega_{ce}^2} \left[\frac{n_o'}{n_o} \frac{k_y \Omega_{ce}}{\bar{\omega} + n\omega_o} \right] \quad (9)$$

and $\omega_{p\alpha}^2 = 4\pi e^2 n_o / m_\alpha$, $\bar{\omega} = \omega + k_y U$, $U = \frac{cE_o}{B}$, $\Omega_{c\alpha} = \frac{eB}{cm_\alpha}$, where

prime denotes differentiation with respect to x . In obtaining Eqs. (8) and (9) for the Kelvin-Helmholtz mode, we have neglected finite Larmor radius effect and terms smaller by a factor of $(\frac{\omega}{\Omega_{ci}})^2$. We have also

assumed $k_{||} = 0$ and no collision and wave-particle resonance effects.

For $(\bar{\omega}) < \omega_o \sim \Omega_{ci}$ and small high-frequency field so that

$k_y \epsilon < 1$, Eq. (7) gives

$$\nabla^2 \phi_i^o = L_i^o \phi_i^o - L_e^o \phi_i^o - \frac{1}{4} \left(\frac{d}{dx} + k_y \epsilon_y \right)^2 L_e^o \phi_i^o \quad (10)$$

$$\nabla^2 \phi_i^{\pm 1} = L_i^{\pm 1} \phi_i^{\pm 1} - L_e^{\pm 1} \phi_e^{\pm 1} \pm \frac{1}{2} \left(\epsilon_x \frac{d}{dx} + k_y \epsilon_y \right) L_e^o \phi_e^o \quad (11)$$

where $\phi_i^{\pm 1}$ are associated with frequencies $(\bar{\omega} \pm n\omega_o)$.

For the two-region model in Fig. 1, the differential equations governing ϕ_i^o and $\phi_i^{\pm 1}$ reduce to

$$\left(\frac{d^2}{dx^2} - k_y^2 \right) \phi_i^o = 0 \quad (12)$$

$$\left(\frac{d^2}{dx^2} - k_y^2 \right) \phi_i^{\pm 1} \approx 0 \quad (13)$$

The boundary condition is

$$\frac{\phi_i^n}{\bar{\omega} + n\omega_o} \quad \text{continuous across interface} \quad (14)$$

The solutions to Eqs. (12) and (13) are

$$\phi_{i1}^o = A\bar{\omega}_1 \exp(k_y x) \quad (15)$$

$$\phi_{i1}^{\pm 1} = B^{\pm} \exp(k_y x) \quad x < 0 \quad (16)$$

$$\phi_{i2}^o = A\bar{\omega}_2 \exp(-k_y x) \quad (17)$$

$$\phi_{i2}^{\pm 1} \approx B^{\pm} \exp(-k_y x) \quad x > 0 \quad (18)$$

Multiplying Eqs. (10) and (11) by $\bar{\omega}$, integrating across the interface, we obtain the following dispersion relation when ω_o is close to Ω_{ci} ,

$$\bar{\omega}_1^2 + \bar{\omega}_2^2 = \omega_{\alpha} \omega + \omega_{\alpha} \frac{1}{2} k_y (U_1 + U_2) + \beta \omega_{\alpha} k_y (U_2 - U_1) \quad (19)$$

where

$$\omega_{\alpha} = k_y^2 \epsilon_x^2 \Omega_{ci} \left[-\frac{2\kappa_n}{k_y} - \frac{\Delta n_o}{n_o} \right] + k_y^2 \epsilon_y^2 \Omega_{ci} \frac{\Delta n_o}{n_o} \quad (20)$$

$$\omega_{\alpha} \beta = i k_y^2 \epsilon_x \epsilon_y \Omega_{ci} \frac{\Delta n_o}{n_o} \quad (21)$$

and $-\kappa_n = n'_o/n_o$, Δn_o = density change across the shear-layer interface.

The stability criterion is

$$\left[\omega_{\alpha} + 4\beta k_y (U_2 - U_1) \right]^2 > (16\beta^2 + 4) k_y^2 (U_2 - U_1)^2 \quad (22)$$

Thus, for sufficiently large a.c. electric field strength, the high-frequency excursion of ions near the shear layer dynamically stabilizes the transverse Kelvin-Helmholtz mode. For laboratory plasmas of

$k_y (U_2 - U_1) \approx 10 \text{ kHz}$, $\Omega_{ci} \approx 250 \text{ kHz}$, then $k_y^2 \epsilon_x^2 \sim 0.4$, i. e., the ion excursion is of the order, or a fraction of, the inverse of the instability wavenumber.

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