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Subroutines for Computing
and Graphing Level Lines
of Functions of Two Variables

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**Subroutines for Computing
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by

H. Ralph Lewis

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SUBROUTINES FOR COMPUTING AND GRAPHING
LEVEL LINES OF FUNCTIONS OF TWO VARIABLES

by
H. Ralph Lewis

ABSTRACT

FORTRAN subroutines for computing and graphing level lines of an arbitrary function of two variables are described. Instructions for using the subroutines are presented with an example and listings of the computer programs.

The FORTRAN subroutines described herein numerically solve an equation $z(x,y) = c = \text{constant}$ for y as a function of x . The curve in the $x - y$ plane that solves the equation and passes through a specified point, (x_0, y_0) , is computed and graphed. The accuracy of each point on the curve is determined by the requirement that the fractional deviation of $z(x,y)$ from $z(x_0, y_0)$, i.e.,

$$\frac{|z(x,y) - z(x_0, y_0)|}{|z(x_0, y_0)|},$$

be less than a specified amount. [$z(x,y)$ should be so defined that $z(x_0, y_0) \neq 0$.*] The subroutines require that the user supply subprograms that compute $z(x,y)$, $\partial z(x,y)/\partial x$, and $\partial z(x,y)/\partial y$.

In addition to the subprograms that compute z , $\partial z/\partial x$, and $\partial z/\partial y$, there are four subroutines: two major subroutines, CONPL and ZCON, and two auxiliary ones, SUB1 and SUB2. CONPL reads in the input data, draws and labels the graph axes, and prints the

headings for the output listing; CONPL also calls ZCON, SUB1, and SUB2. ZCON computes, lists, and graphs the level lines through specified points. SUB1 may be used for reading input data needed for computing the function and its derivatives. SUB2 may be used for plotting additional curves on the same film frame after the level lines have been plotted. Both SUB1 and SUB2 may, if desired, consist of the two statements RETURN and END. Communication between SUB1, SUB2, and the subprograms that compute z , $\partial z/\partial x$, and $\partial z/\partial y$ can be effected through COMMON storage. There are no COMMON statements in CONPL and ZCON.

Each point is computed by Newton's method of iteration, and the value of the function at each point differs by no more than the specified error from the value at the starting point. If that accuracy cannot be achieved, then eventually the step size will be reduced to a value less than a certain minimum value. In that case, the statement "DT = 0. STOP THIS CONTOUR." will be printed and the next contour will be started. The step size is never larger than a maximum value, DTST, specified by the user. Also, the step size is never smaller than $DTM = .001 * DTST$. (This limit can be changed by

* Instead of bounding the fractional error, any other quantity, such as the absolute error, may be bounded by changing one card in subroutine ZCON, as is described by a comment statement. If the absolute error is bounded, then $z(x_0, y_0)$ may be allowed to vanish.

changing the value of DTM at the beginning of subroutine ZCON.) However, insofar as is consistent with the maximum and minimum step sizes, the step size is adjusted such that $2 \leq (\text{number of iterations to achieve specified accuracy}) \leq 4$. If four iterations have been performed without achieving the specified accuracy, then the step size is halved and the iteration process is begun anew. If one iteration achieves the specified accuracy, then the step size is doubled, except that the doubling test is not made if the step size was halved during the current step. The independent variable (x or y) is chosen at each step. If $|dy/dx| \geq 1$, then y is the independent variable; if $|dy/dx| < 1$, then x is the independent variable.

The number of points on a contour is limited in three ways. Firstly, the user specifies a number, JM, such that no more than JM X 100 points will be computed. Secondly, the curve may not cross one of the specified boundaries. If a point lies outside a boundary, it will be rejected, and the program will return to the starting point and resume in the other direction along the curve. If it again happens that a boundary is crossed, then the computation is stopped. Thirdly, the user specifies a rectangle enclosing the starting point such that, if the curve once passes out of the rectangle and later enters it, the computation will stop.

The program that computes $z(x,y)$ must be a function subprogram, whose name may be chosen arbitrarily. The arguments of the subprogram must be the coordinates x and y, respectively [for example, ZVAL(X,Y)].

The program that computes $\partial z(x,y)/\partial x$ and $\partial z(x,y)/\partial y$ must be a subroutine whose name may be chosen arbitrarily. If the name chosen is ZDER, then the calling sequence must be

CALL ZDER(X,Y,DER,K),

where DER(1) = $\partial z/\partial x$ at point (X,Y), and DER(3) = $\partial z/\partial y$ at point (X,Y). The meaning of the index K must be

$$K = \begin{cases} -1, & \partial z/\partial x \text{ is computed.} \\ 0, & \text{both } \partial z/\partial x \text{ and } \partial z/\partial y \text{ are computed.} \\ 1, & \partial z/\partial y \text{ is computed.} \end{cases}$$

To compute level lines it is only necessary to use the statement

CALL CONPL(ZVAL,ZDER),

where ZVAL is the name of the function subprogram that computes z, and ZDER is the name of the subroutine that computes $\partial z/\partial x$ and $\partial z/\partial y$. The data required for CONPL is presented in Table I.

Appendix I lists FORTRAN IV versions of CONPL and ZCON for use with the CDC 6600 computers at the Los Alamos Scientific Laboratory.* As an example, versions of programs MAIN, ZVAL, ZDER, SUB1, and SUB2 for graphing level lines of

$$z = \frac{1}{\sqrt{(x-1)^2 + y^2}} + \frac{1}{\sqrt{(x+1)^2 + y^2}}$$

are listed in Appendix II. Sample data for this problem are listed in Appendix III.

ACKNOWLEDGMENT

Mr. F. E. Dunn assisted in working out the details of CONPL and ZCON.

* CONPL and ZCON call subroutines that are standard at the Los Alamos Scientific Laboratory for producing graphical output on 35-mm film with the SC-4020 device.

TABLE I
Data for CONPL

Card No.	Name of quantity to be read	Format	Description
1	NFRM	I4	Number of sets of curves to be plotted = number of frames of film to be used for plotting.
2	S	8A10	First 72 characters written on first frame of film.
3	A	8A10	First 72 characters used for horizontal axis label.
4	B	8A10	First 72 characters used for vertical axis label.
5	C	6A10	First 54 characters written at bottom of each graph.
6,7	D	8A10/8A10	First 72 characters on each of <u>two cards</u> printed at top of listing.
8	$\left\{ \begin{array}{l} \text{XNAME} \\ \text{YNAME} \\ \text{ZNAME} \end{array} \right\}$	3A8	Name of abscissa on listing. Name of ordinate on listing. Name of function on listing.
9	$\left\{ \begin{array}{l} \text{XMAX} \\ \text{XMIN} \\ \text{YMAX} \\ \text{YMIN} \\ \text{ER} \\ \text{NLIN} \end{array} \right\}$	1P5E12.7, I6	Maximum value of abscissa. Minimum value of abscissa. Maximum value of ordinate. Minimum value of ordinate. Maximum error in function (ZNAME). Number of starting points for this frame = number of lines to be plotted on this frame.
----- Read data for SUB1 here. -----			
10 through 9 + NLIN	$\left\{ \begin{array}{l} \text{XO} \\ \text{YO} \\ \text{DTO} \\ \\ \text{ERX} \\ \text{ERY} \\ \text{JM} \\ \text{IPLT} \\ \text{ICON} \end{array} \right\}$	1P5E12.7, I6, 2I1	Abscissa of starting point. Ordinate of starting point. $ \text{DTO} $ = maximum step size. If $\left \frac{dy}{dx} \right _{\text{XO,YO}} \geq 1$, DTO = initial step in y. Otherwise, initial step in x = $\text{DTO} \times (\text{sign of } \frac{dy}{dx} \text{ at XO,YO})$. If curve re-enters rectangle defined by $(\text{XO} \pm \text{ERX}, \text{YO} \pm \text{ERY})$, computation will stop. JM $\times$ 100 = maximum number of points to be computed. IPLT $\left\{ \begin{array}{l} = 0, \text{ no points listed except starting point.} \\ \neq 0, \text{ all points listed.} \end{array} \right.$ ICON $\left\{ \begin{array}{l} = 0, \text{ points on graph not connected.} \\ \neq 0, \text{ points on graph connected.} \end{array} \right.$
----- Read data for SUB 2 here. -----			

For each of the NFRM sets of curves to be plotted, there must be a set of cards corresponding to cards 2 through 9 + NLIN.

Appendix I. Listings of CONPL and ZCON

```

SUBROUTINE CCNPL(ZVAL,ZDER)
  DIMENSION      A( 8),B( 8),C(6),S( 8),D(16)
  EXTERNAL  ZVAL  , ZDER
15 REAC (10,16)NFRM
16 FORMAT (I4)
C 17 NFRM=NUMBER OF SETS OF CURVES TO
C 18 BE PLOTTED=NUMBER OF FRAMES OF
C 19 FILM TO BE USED FOR PLOTTING
20 REAC (10,21)S
21 FORMAT(8A10)
C 22 S IS WRITTEN ON THE FIRST FRAME
23 CALL ADV(1)
24 CALL WLCH(72,400,72,S,1)
25 K=1
26 REAC (10,27)A,B,C,D,XNAME,YNAME, ZNAME,XMAX,XMIN,YMAX,YMIN,ER,NLIN
27 FORMAT(8A10/8A10/6A10/8A10/8A10/3A8/1P5E12.7,16)
C 28 A=HORIZONTAL AXIS LABEL
C 29 B=VERTICAL AXIS LABEL
C 30 C=GRAPH LABEL,GOES AT BOTTOM OF FRAME
C 31 D=HEADING FOR PRINTOUT
C 32 XNAME,YNAME=NAMES FOR X AND Y ON PRINTOUT
C 33 NLIN=NUMBER OF LINES TO BE PLOTTED ON
C 34 THIS FRAME=NUMBER OF STARTING POINTS (X0,Y0)
35 CALL ADV(1)
36 CALL WLCH (16,1000,54,C,2)
37 CALL WLCH (85,970,72,A,1)
38 CALL WLCV (5,940,72,B,1)
39 CALL DGA (110,990, 20,900,XMIN,XMAX,YMAX,
  YMIN)
40 CALL DLNIN(10,10)
41 CALL SLLIN(10,13)
42 CALL SBLIN(10,13)
43 WRITE (9,44)C
44 FORMAT (1H1,8A10/1H ,8A10)
45 CALL SUB1
46 WRITE (9,47)ZNAME,ER,XNAME, YNAME,ZNAME,ZNAME
47 FORMAT (1H0,25HMAXIMUM FRACTIONAL ERROR ,
  13HIN ,A8,4H IS ,F12.8//1H0,7X,A8,14X,
  2A8,14X,A8,16X,2HDT,11X,10H ERROR IN ,A8)
48 N=1
C 49 SUB1 CAN BE USED TO READ IN PARAMETERS
C 50 AND SET THEM FOR ZVAL BY USE OF THE
C 51 COMMON REGION
52 READ (10,53)XU,YU,DU,ERX,ERY,JM,IPLT,ICON
53 FORMAT(1P5E12.7,16,211)
54 CALL ZCON(X0,Y0,ZVAL,ZDER,ER,DT0,XMAX,XMIN,YMAX,YMIN,ERX,ERY,
  IJM,IPLT,ICON)
55 N=N+1
56 IF (N-NLIN) 52,52,57
57 CALL SUB2
C 58 SUB2 CAN BE USED TO PLOT ADDITIONAL
C 59 LINES ON THE GRAPH
60 K=K+1
61 IF (K-NFRM) 26,26,62
62 RETURN
63 END

SUBROUTINE ZCON(XST,YST,ZVAL,ZDER,ER,DTST,XMAX,XMIN,YMAX,YMIN,
  IERX,ERY,JM,IPLT,ICON)
  DIMENSION XPLT(100),YPLT(100),ZPLT(100),X(3),
  IDER(3),DTPLT(100),DZPLT(100)
C 16 (XPLT,YPLT) IS A POINT TO BE PLOTTED AND
C 17 PRINTED. ZPLT AND DTPLT ARE VALUES TO
C 18 BE PRINTED.DZPLT=Z0-Z(XPLT,YPLT)
  ER=ABS(ER)
  ERX=ABS(ERX)
  ERY=ABS(ERY)
  DTM= .001*ABS (DTST)

```

```

28 Z0=ZVAL(XST,YST)
   Z00=ABS(Z0)
41 WRITE (9,42)XST,YST,Z0,DTST
42 FORMAT(1H0,1P4E20.7)
47 JBNC=0
   KBNC=0
   KDER=1
   DA=ABS(DTST)
48 DT=CTST
49 X(1)=XST
50 X(3)=YST
51 J=1
   XPLT(J)=X(1)
   YPLT(J)=X(3)
   ZPLT(J)=Z0
   DZPLT(J)=0.
   DTPLT(J)=DT
   J=J+1
52 JJ=0
53 JSTP=0
C 54 DER(1)=DZ/DX,DER(3)=DZ/DY
55 I=1
   I1=2+I
   I2=2-I
56 CALL ZDER(X(1),X(3),DER,C)
63 IF(ABS (DER(I1))-ABS (DER(I2)))67,661,64
64 I=-I
   I1=2+I
641 I2=2-I
65 IF(-DER(I1)/DER(I2))66,67,67
66 DT=-DT
661 IF(DER(I2))67,662,67
662 KDER=0
   IS=I
   XS=X(1)
   YS=X(3)
   CS=CT
67 L=0
68 K=1
69 XF=X(1)
70 YF=X(3)
72 XT=X(I1)+DT
   IF(ABS (DT)-DTM)721,73,73
721 WRITE (9,722)
722 FORMAT(26H0DT=0. STOP THIS CONTOUR.)
   JSTP=JM
   GO TO 1262
73 X(I1)=XT
   IDER=-I
74 CALL ZDER(X(1),X(3),DER,IDER)
   IF(DER(I2))75,751,75
751 IF(KDER)94,752,94
752 X(1)=XS
   X(3)=YS
   I=-IS
   I1=2+I
   I2=2-I
   DT=CS
   KDER=1
   GO TO 67
75 X(I2)=X(I2)+(Z0-ZVAL(X(1),X(3)))/DER(I2)
84 ZCAL=ZVAL(X(1),X(3))
85 DZ=Z0-ZCAL
   W=ABS(DZ)/Z00
C   TO BOUND THE FRACTIONAL ERROR, THE ABOVE STATEMENT SHOULD BE
C   W=ABS(DZ)/Z00
C   TO BOUND THE ABSOLUTE ERROR, THE ABOVE STATEMENT SHOULD BE
C   W=ABS(DZ)
87 IF(W-ER)101,101,88
88 IF (K-4) 91,94,94
91 K=K+1

```



```

93 GO TO 74
94 DT=CT/2.0
   K=1
97 L=1
98 X(1)=XF
99 X(3)=YF
100 GO TO 72
101 IF(ABS(X(1)-XF)-ABS(DA))1010,1010,94
1010 IF(ABS(X(3)-YF)-ABS(DA))1011,1011,94
1011 IF (X(1)-XMAX)102,102,105
102 IF (X(1)-XMIN)105,103,103
103 IF (X(3)-YMAX)104,104,105
104 IF (X(3)-YMIN)105,110,110
105 IF (JBND) 1051,106,1051
1051 JSTP=JM
   GO TO 1262
106 JBNC=1
   KBND=1
107 DT =-DTST
   X(1)=XST
   X(3)=YST
   JJ=0
   I=1
   I1=2+I
   I2=2-I
   WRITE (9,108)
108 FORMAT(1H0)
109 GO TO 1262
110 XPLT(J)=X(1)
111 YPLT(J)=X(3)
112 ZPLT(J)=ZCAL
113 DZPLT(J)=DZ
114 DTPLT(J)=DT
115 IF (L) 119,116,119
116 IF (K-1) 119,117,119
117 IF (DT*DT-DTST*DTST) 118,119,119
118 DT=2.0*DT
119 J=J+1
   IF(JJ)125,121,125
121 IF(ABS(X(1)-XST)-ERX)123,123,122
122 JJ=1
   GO TO 125
123 IF(ABS(X(3)-YST)-ERY)1261,1261,124
124 JJ=1
125 IF(ABS(X(1)-XST)-ERX)126,126,1261
126 IF(ABS(X(3)-YST)-ERY)1260,1260,1261
1260 JSTP=JM
   GO TO 1262
1261 IF(J-100)56,56,1262
1262 N=J-1
   IF(N)132,132,1263
1263 IF(IPLT)127,129,127
127 WRITE (9,128)(XPLT(M), YPLT(M),ZPLT(M),DTPLT(M),DZPLT(M), M=1,N)
128 FORMAT(1P5E20.7)
129 CALL PLOT(N,XPLT,1,YPLT,1,42,ICON)
130 JSTP=JSTP+1
   IF(JSTP-JM)131,135,135
131 J=1
   IF(KBND)1311,1312,1311
1311 KBNC=0
   GO TO 56
1312 XPLT(J)=XPLT(100)
   YPLT(J)=YPLT(100)
   ZPLT(J)=ZPLT(100)
   DZPLT(J)=DZPLT(100)
   DTPLT(J)=DTPLT(100)
   J=J+1
132 IF(JSTP-JM)1321,135,135
1321 GO TO 56
135 RETURN
136 END

```

Appendix II. Listings of the Other Codes for a Special Problem

```

PROGRAM MAIN(INPUT,OUTPUT,FILM,TAPE10=INPUT,TAPE9=OUTPUT)
EXTERNAL ZVAL , ZDER
CALL CONPL(ZVAL,ZDER)
CALL EXIT
END

```

```

FUNCTION ZVAL(X,Y)
R1=SQRT((X-1.0)**2+Y**2)
R2=SQRT((X+1.0)**2+Y**2)
IF(R1)70,70,90
70 ZVAL=1.E37
RETURN
90 IF(R2)70,70,100
100 ZVAL=1.0/R1+1.0/R2
RETURN
END

```

```

SUBROUTINE ZDER(X,Y,DER,K)
DIMENSION DER(3)
R1=SQRT((X-1.0)**2+Y**2)
R2=SQRT((X+1.0)**2+Y**2)
C DER(1) = DZ/DX
C DER(3) = DZ/DY
IF(R1)1,1,2
1 DER(1)=1.0E+37
DER(3)=1.0E+37
RETURN
2 IF(R2)1,1,3
C IF K=-1, ONLY CZ/DX IS COMPUTED.
C IF K=1, ONLY DZ/DY IS COMPUTED.
C IF K=0, BOTH CZ/DX AND DZ/DY ARE COMPUTED.
3 IF(K)4,4,5
4 DER(1)=-(X-1.0)/R1**3 -(X+1.0)/R2**3
IF(K)6,5,5
5 DER(3)=-Y/R1**3 -Y/R2**3
6 RETURN
END

```

```

SUBROUTINE SUB1
RETURN
END

```

```

SUBROUTINE SUB2
RETURN
END

```

Appendix III. Sample Data

1
THIS IS A TEST OF CONPL AND ZCON.

ABSCISSA
ORDINATE

POTENTIAL DISTRIBUTION
TEST OF CONPL AND ZCON WITH
A POTENTIAL DISTRIBUTION

X	Y	Z	
+10000000+01-10000000+01+10000000+01-10000000+01+10000000-03			17
-05000000+00+00000000+00+03000000+00+03000000+00+03000000+00			200
+05000000+00+00000000+00+03000000+00+03000000+00+03000000+00			200
+00000000+00+00000000+00+03000000+00+03000000+00+03000000+00			200
+00000000+00+03000000+00+03000000+00+03000000+00+03000000+00			200
+00000000+00+10000000+00+03000000+00+03000000+00+03000000+00			2011
+00000000+00+20000000+00+03000000+00+03000000+00+03000000+00			2011
+00000000+00+30000000+00+03000000+00+03000000+00+03000000+00			2011
+00000000+00+40000000+00+03000000+00+03000000+00+03000000+00			2011
+00000000+00+50000000+00+03000000+00+03000000+00+03000000+00			2011
+00000000+00+60000000+00+03000000+00+03000000+00+03000000+00			200
+00000000+00+70000000+00+03000000+00+03000000+00+03000000+00			200
+00000000+00+80000000+00+03000000+00+03000000+00+03000000+00			200
+00000000+00+90000000+00+03000000+00+03000000+00+03000000+00			200
-80000000+00-80000000+00+03000000+00+03000000+00+03000000+00			200
-80000000+00+80000000+00+03000000+00+03000000+00+03000000+00			200
+80000000+00+80000000+00+03000000+00+03000000+00+03000000+00			200
+80000000+00-80000000+00+03000000+00+03000000+00+03000000+00			200