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PLASMA DIAGNOSTICS
WITH
SHORT ELECTROMAGNETIC PULSES

by

H J Schmitt

Harvard University

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SANDIA CORPORATION MONOGRAPH

PLASMA DIAGNOSTICS WITH SHORT ELECTROMAGNETIC PULSES

by

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July 1963

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ABSTRACT

The propagation of short electromagnetic pulses in an ionized medium and the reflection of transient pulses from bounded ionized regions is discussed. Analytical expressions for transient oscillations in reflected and propagated electromagnetic fields excited by a unidirectional pulse are derived for certain simple cases. The transient oscillations show a periodicity which is directly related to the plasma frequency and are useful for nearly instantaneous plasma diagnostics.

An experiment is described in which the reflection and transmission of unidirectional pulses of approximately 1-nanosecond duration is observed in the afterglow of a coaxial discharge tube. In addition to the dispersion of the quasi TEM mode in the coaxial test tube a transient oscillation with much lower periodicity is also observed and interpreted as arising from a surface-wave propagating near the interface between glass walls and the annular ionized region. This surface-wave propagates only at frequencies below the plasma frequency of the ionized region and its application for diagnostics of dense plasmas is indicated.

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PLASMA DIAGNOSTICS WITH SHORT ELECTROMAGNETIC PULSES

Introduction

The dispersion of electromagnetic signals in the simplest type of plasma, a collision-free neutral and isotropic electron gas is characterized by a sharp cut-off phenomenon at the plasma frequency. The propagation constant k for a plane wave is given by

$$k^2 = \frac{\omega^2}{c_o^2} - \frac{\omega_p^2}{c_o^2}.$$

The group velocity is zero at the angular plasma frequency ω_p and rises with increasing angular frequency ω of the electromagnetic signal asymptotically to the velocity of light in free space c_o . For frequencies lower than ω_p a monochromatic electromagnetic wave decays exponentially in the direction of propagation. A completely similar dispersion relation is encountered in hollow cylindrical waveguides, where the angular cut-off frequency ω_c is used instead of the angular plasma frequency.

A complex electromagnetic signal which contains significant frequency components around and beyond the plasma frequency, is set up in an ionized medium. At a distant point the highest frequency components will contribute to the observed signal first and the components near ω_p last. In general, this dispersion leads to a transient ringing with a frequency near ω_p . From a circuit point of view the plasma may be compared to a filter ringing near its cut-off frequency when excited by a pulse or a similar shock phenomenon. A transient ringing can also be interpreted as resulting from a momentary force acting on the electrons, which subsequently oscillate with the plasma frequency around their stable rest positions.

The dispersion of short unidirectional pulses in a waveguide and the resulting transient ringing at the frequency of transverse resonance has been verified experimentally.¹ A theoretical discussion of transient phenomena on re-entry plasmas has been reported by Johnson.²

It will be shown in this report that the oscillatory response resulting from dispersion in a plasma is indeed experimentally observable. Since the frequency of the transient oscillations is intimately related to the electron density, a measurement of the time sequence of the transient response can be used as a diagnostic tool. In principle, a measurement of $\omega_p = 2\pi/T_p$ can be accomplished in the time necessary to observe one period of the transient ringing, hence $T = T_p$. The almost instantaneous nature of the measurement is demonstrated in the observation of the decay of electron density in the diffusion controlled afterglow of neon gas.

The dispersion of electromagnetic signals is also of interest for the transmission of electromagnetic pulses or broadband signals through natural or artificially created ionized regions. Here considerable dispersion may impair the recognition of the true signal.

Plane Wave Transmission and Reflection

For a few particularly simple situations the dispersion of idealized transient signals in a simple isotropic ionized medium described by a real macroscopic relative dielectric constant

$$\epsilon_r = 1 - \frac{\omega_p^2}{\omega^2} \quad (1)$$

can be calculated rigorously and the response expressed in analytical form. Specifically, the dispersion of a unidirectional plane-wave signal with a step-function time-dependence has been solved for the analogous case of a waveguide.³ Hence, if in an unbounded plasma a step-like rise of transversely polarized current, or magnetic field strength $\vec{H} = \hat{y}H_y$ occurs at the time $t = t_0$ in a plane $z = 0$

$$H_y(z = 0, t) = \frac{-iH_0}{2\pi} \int_{c-j\infty}^{c+j\infty} \frac{e^{p(t-t_0)}}{p} dp \quad (2)$$

the response for the electric field strength $\vec{E} = \hat{x}E_x$ in a plane at z is given by

$$E_x(z, t) = \frac{-i}{2\pi} Z_0 H_0 \int_{c-j\infty}^{c+j\infty} \frac{e^{p(t-t_0) - \frac{z}{c_0} \sqrt{p^2 + \omega_p^2}}}{\sqrt{p^2 + \omega_p^2}} dp = \begin{cases} 0 & t - t_0 < \frac{z}{c_0} \\ Z_0 H_0 J_0 \left(\frac{2\pi}{\lambda_p} \sqrt{c_0^2 (t - t_0)^2 - z^2} \right) & t - t_0 \geq \frac{z}{c_0} \end{cases} \quad (3)$$

with $\lambda_p = 2\pi c_0 / \omega_p$ the free-space wavelength at the plasma frequency and $Z_0 = \sqrt{\mu_0 / \epsilon_0}$ the wave impedance of free space. The initial transient response is shown in Figure 1 for a value of $z = 3$ meters and three different values of the plasma frequency (full lines). The first signal in the form of a sharp spike arrives with the speed of light, i.e., 10^{-8} sec after the switching operation at $z = 0$. The spike results from a superposition of the highest frequency components contained in the infinitely sharp step assumed as excitation. With increasing distance of the point of observation, or with increasing plasma frequency, it decreases in width as even small differentials in the group velocity become significant at large distances. Subsequently the transient response changes into a damped oscillatory form, rapidly approaching a period of oscillation equal to T_p . For sufficiently large times the Bessel function finally can be approximated by

$$J_0(\xi) \sim \frac{\cos\left(\xi - \frac{\pi}{4}\right)}{\sqrt{\frac{\pi}{2}\xi}} \sim \frac{\cos\left(\omega_p(t - t_0) - \frac{\pi}{4}\right)}{\sqrt{\frac{\pi}{2}\omega_p(t - t_0)}} \quad (4)$$

A second simple situation arises for the reflection of a plane linearly polarized signal in the form of a unidirectional step in the electric field strength from a plane boundary at $z = 0$ separating free space and a simple isotropic ionized medium, Figure 2. In practice this assumption requires the transition to be rapid with respect to the shortest wavelength considered significant in the spectrum of the

Figure 1. Transient electric field strength in simple plasma due to sudden rise in magnetic field strength ($z = 3m$; full lines - step function rise; broken line - pulse shaped rise).

Figure 2. Reflection of electromagnetic signal from simple plasma.

incident signal. The plane-wave reflection coefficient is

$$r = \frac{1 - \sqrt{\epsilon_r}}{1 + \sqrt{\epsilon_r}}. \quad (5)$$

The transmission coefficient is

$$t = 1 + r = \frac{2}{1 + \sqrt{\epsilon_r}}. \quad (6)$$

For the electric field strength in the transmitted signal, it follows that

$$E_x(0, t) \Big|_{0^-} = \frac{-2iE_0}{2\pi} \int_{c-j\infty}^{c+j\infty} \frac{e^{pt}}{p \left(1 + \sqrt{1 + \frac{\omega_p^2}{p^2}} \right)} dp. \quad (7)$$

This integral is listed in Ref. 4. Its value is

$$E_x(0, t) \Big|_{0^-} = \frac{2J_1(\omega_p t)}{\omega_p t}. \quad (8)$$

Hence, for the reflected wave, the continuity relation yields

$$E_x^r(0, t) = \frac{2J_1(\omega_p t)}{\omega_p t} - 1. \quad (9)$$

The time behavior of the reflected wave amplitude is plotted in Figure 3 (full line) as a function of the normalized time $\omega_p t = \tau$. For the reflected wave Figure 3 displays a relatively smooth initial rise of the signal strength due to the nearly complete transmission of high frequency components, followed by a damped oscillatory behavior superimposed on the steady-state response of $E_x^r(0, t \rightarrow \infty) = -1$. The oscillation frequency quickly approaches the plasma frequency. It is to be noted that the oscillations ultimately decay as $t^{-3/2}$ in the observation of the reflected transient signal, as compared to a $t^{-1/2}$ decrease in the case of pulse transmission. Consequently, in an experimental observation it should be easier to measure these oscillations for propagation through a plasma. This is clearly demonstrated in the experiment described later on.

The practically important situation in which a plane wave of complex time dependence propagates into a plasma medium or through a slab of an ionized medium has apparently not yet been solved rigorously. Since it leads to an inversion integral which is not listed, its solution will not be attempted here. It may safely be assumed that qualitatively similar transient oscillations will persist after the high frequency components, which determine the detailed nature of the initial response, have passed by. In practice the detailed leading part of the transient response is difficult to observe for a number of reasons. Specifically, the limited resolution of available oscilloscopes as well as the limited rise times of practical transient steps or pulses may determine the appearance of the detailed response. The effect of collisions between electrons and neutral particles as well as ions has to be included in the analysis. Although collisions can in principle be accounted for, they have been neglected here because of mathematical difficulties. The detailed structure of the beginning of transient responses will be altered since the ionized medium probably is neither uniformly distributed in space nor even isotropic. These are precisely the points which have to be considered in a comparison with experimental results.

For the experimental observation of a transient propagation it is more convenient to use short pulses than step-function signals; in this way any possible steady-state response is eliminated and a clearer separation is made between partial signals which have travelled through different optical path lengths. For a perfect rectangular pulse the response is immediately obtained from the known response to a step-function signal by subtracting the response to a negative step at the proper time. The excitation used in the experiment consists of a short spike with a fast rise time of 0.4×10^{-9} sec (as observed on the oscilloscope) followed by an exponential drop e^{-t/τ_0} where the time constant $\tau_0 = RC = 2.5 \times 10^{-10}$ sec is determined by a series capacitor $C = 5 \mu\mu f$ and a resistance of 50 ohms. For the two idealized cases open to analytical solutions the transient response due to such an excitation has been computed by graphical integration with the assumption of a linear initial rise. For a plasma frequency of 100 Mc/sec the response is shown in Figure 1 and Figure 3 as dotted lines. While the response varies in detail compared to the case of a step-function excitation or excitation by a Dirac pulse, the introduction of purely passive pulse-shaping elements does not significantly vary the nearly monochromatic damped oscillations of the persisting transient response. In particular, from Figure 1, it may be concluded that for a distance of observation of 3 meters, the ringing frequency is approximately 10% larger than the plasma frequency for times of observation about 10^{-8} sec after the arrival of the signal, or at twice the time needed for the wave to traverse this distance at the speed of light.

Figure 3. Transient electric field strength reflected from plane, semi-infinite simple plasma (full line - step function incidence; broken line - pulse incidence; $f_p = 100 \text{ Mc/sec}$, lower time scale).

Experimental Setup

The object of the experiment is the observation of pulse dispersion in an ionized medium and the reflection from an ionized region. To allow comparison with theoretical results, the plasma was confined to the annular space between the electrodes of a coaxial transmission line which normally supports a TEM-wave with plane phase fronts. The plasma is generated by a repetitive direct-current pulse of a few microseconds duration applied to two auxiliary electrodes. With a controlled delay after termination of the ionization pulse a short test pulse is generated and the dispersion of this test pulse upon reflection and transmission in the discharge tube is observed on a sampling scope.

The experimental system is schematically shown in Figure 4. The coaxial discharge tube consists of an inner Pyrex tube with precise boring 9.45 mm and an outside diameter of 12.5 mm. When erected vertically it is supported on the top by a spring holder connected to the annular nickel cathode and on the bottom by a flange directly extending from the outer tube. The outer cylinder is a standard Pyrex tube with an inside diameter of 25 mm and an outside diameter of 28 mm. It is flared out on top to hold the slightly oversized cathode.* A nickel anode is placed directly at the bottom of the tube through a small extension with a diameter of approximately 1.5 cm.

To support electromagnetic waves along the coaxial line a precisely fitted brass tube is slid into the inner Pyrex tube to a distance of approximately 15 cm beneath the cathode. The outer Pyrex wall is painted with a highly conducting colloidal silver solution to the extent of the inner metallic conductor. The choice of transverse dimensions of the discharge tube is controlled by the aim to avoid any reflections at the junction to the connecting coaxial cable. With a dielectric constant of $\epsilon'_r = 5$ for Pyrex glass, the characteristic impedance of the discharge tube section for the TEM mode in the absence of ionization is 54 ohms, compared to 50 ohms for RG9U cable used as connecting transmission line. The junction is accomplished by a standard FXR coaxial transition, after the outer conductor is flared out slightly from 25 mm to 28 mm in the form of a silver painted paper cone of about 3 cm length.

The discharge pulse is generated in a microsecond-pulse generator with a repetition rate of about 90 pp/sec. The pulse is amplified to a level of between 1000 v and 3000 v in a single stage 3E29 amplifier. The exact ignition voltage depends on the gas pressure in the tube and the pulse duration. A fraction of the pulse energy supplied by the generator is used to trigger a Tektronix 585 oscilloscope. The delayed trigger output of this scope is accurately adjustable in time delay and free of jitter. It is used to trigger the time base of the main sampling scope (Tektronix Type 611). The delayed trigger output of the 585 scope as well as the plasma tube ignition pulse are shown on the 585 scope as a measure of the elapsed time in the afterglow of the discharge.

Upon triggering the Tektronix sampling scope provides a short rise-time unidirectional pulse (rise time ~ 0.4 nanoseconds) with a duration of 0.8 microseconds and a pulse amplitude of about 300 mv.

*In earlier constructions the cathode was off-set sideways. These tubes were found more difficult to ignite.

Figure 4. Experimental setup for observation of transients in plasma.

This pulse is differentiated by a $5\ \mu\text{f}$ series capacitor looking into a 50 ohm load and the first resulting spike is used as the actual test pulse.* It is led through a section of 100' coaxial RG9U cable to the sampling scope input. From here it branches into a 50' section of cable leading to the discharge tube. The direct pulse is eliminated in the observation by restricting the measurement to the later time interval when pulses reflected in the discharge tube return to the input terminals. In addition, a 200' long section of coaxial cable, short circuited at its end, is connected across the input of the sampling scope to block out slowly varying signals resulting from charge accumulation on the walls of the discharge tube during the actual discharge. All delay cables are chosen in length so that a period of about 30 nanoseconds after observation of reflection from the junction to the discharge tube are free of spurious responses.

The length of the coaxial discharge tube which is made conducting and acts as a transmission line is 1.50 meter. It is chosen so that with the resolution limit of the oscilloscope (about 1000 Mc/sec), (i.e., plasma frequencies of about 1000 Mc/sec) the tube would be at least a few wavelengths long. The pulse amplitude, as it appears at the junction to the discharge tube is approximately 40 mv, small enough to not upset the electron distribution.† If no ionization is present the pulse proceeds essentially undisturbed along the test tube and is completely reflected at the termination of the metallic walls. It re-emerges at the junction with a time delay corresponding to about 3 meters travel, or about 10^{-8} sec. This response is shown in Figure 5 (time scale from right to left, pulse amplitude negative). The initial small disturbance 10^{-8} sec in front of the main pulse is the residual reflection from the junction zone, a subsequent disturbance 10^{-8} sec after the main pulse constitutes the second reflection from the junction after total reflection at the termination occurred twice. Higher-order reflections are negligible.

Figure 5. Reflection of pulses from coaxial discharge tube without ionization. Initial small disturbance represents residual reflection at junction. Main pulse is reflected from open ended terminals of discharge tube. Upper exposure 5 nsec/div, lower exposure 2 nsec/div.

*Earlier it was attempted to use nanosecond pulses derived from a mercury switch pulser. Although the time jitter in the mechanical switch could be reduced to about $10\ \mu\text{sec}$, in short afterglows significant changes may occur in such a time interval, and responses obtained from repetitive ignitions do not quite overlap.

†Measurements made with a mercury pulser in conjunction with a fast conventional oscilloscope (Tektronix 517, observing directly on the plates) showed that the response for test signals about 10 volts in amplitude or more depend on the polarity of the pulse with respect to the asymmetric electrodes of the coaxial test tube.

In order to approximate a plasma with low collision rate, neon was used as the principal gas. The collision frequency in neon is approximately $\nu \sim 3 \times 10^7$ collisions/sec at 1 mm pressure, hence, for the lowest plasma frequency of interest, $\omega_p \sim 2\pi \times 10^7$ 1/sec, the ratio of ν/ω_p is still small enough to be neglected in first order. For ease of ignition a mixture of 99.8% neon and 0.2% argon was used. Before assembling the conducting linings the discharge tube was evacuated to about 4×10^{-7} mm Hg for 12 to 24 hours, baked out by means of heating tapes at 400°C and the electrodes fired at white glow. It was sealed permanently after filling. It was found that the duration of the afterglow decreases rapidly in the presence of spurious contaminations.

The electron distribution in this tube builds up first in the form of a cloud around the cathode. For the duration of the excitation the electron cloud is rapidly pulled towards the anode. The progress of the electron cloud can be measured by observing the Doppler shift of a continuous wave signal applied to the terminals of the discharge tube and reflected from the moving cloud of charged particles during the discharge, Figure 6. Measurements have been made at a frequency of 457 Mc/sec and at a pressure of 1 mm Hg for various driving conditions. With increasing pulse voltage the speed of the electron cloud increases until ultimately a much more rapid ionization perhaps in the form of an avalanche results. From the value of the Doppler shift at the beginning (time scale from right to left) and at the end it is also apparent that the electrons are slowed down slightly as they progress towards the anode. The point where the Doppler response stops, represents the arrival of the cloud at the anode. The terminal speeds are of the order of $1 - 3 \times 10^5$ m/sec, roughly in accord with values calculated from a formula given by Seeliger.⁵

In angular direction the homogeneity of the ultimate electron density appeared excellent, while in the radial direction the distribution at pressures below a few mm Hg will be controlled by ambipolar diffusion. The homogeneity in the axial direction can, in principle, be determined from the transient response to short pulses by comparison of reflected return which involves largely the region near the anode, and the transmitted signal, which involves nearly the entire length of the tube. No discrepancies have been observed, so that reasonable homogeneity can be inferred.

Figure 6. Doppler shift of continuous wave signal during ignition of discharge tube. $f = 457$ Mc/sec. Amplitude of exciting pulse: (a) 2480 v, (b) 2850 v, (c) 3220 v, (d) 4210 v. 1 mm Hg neon, time scale 10 μ sec/div.

The experimental system deviates from the theoretical model discussed before essentially in the presence of the glasswalls containing the ionized particles. The effect of these dielectric walls manifests itself, except for determining the diffusion mode, in the appearance of an additional propagation branch for electromagnetic waves in the form of a surface wave, which propagates only at frequencies below the plasma frequency.

Experimental Results and Discussion

Measurements on the reflection of transient pulses from the ionized region and transmission through the discharge tube were taken at 0.2 mm Hg pressure and at 1 mm Hg pressure for the neon-argon gas mixture for various pulse durations and pulse amplitudes of the ionizing pulse. The experimentally observed responses as a function of elapsed time in the afterglow are reproduced in Figure 7 for 0.2 mm Hg and in Figure 8 for 1 mm Hg pressure. It should be noted again, that the time scale is from right to left.

The fact that smooth response curves are observed, indicates the precise reproducibility of the discharge, since in the sampling system each dot in the curve results from a new ignition. For 100 samples/cm total actual time to construct one response curve is about 11 seconds, in view of the repetition rate of 90 ignitions per second. The equivalent time scale is noted in the caption, usually 2×10^{-9} sec/div, except for measurements early in the afterglow where it is 5×10^{-9} sec/div.*

Two main features are clearly recognizable in the experimental results: early in the afterglow the incident electromagnetic test pulse is partially reflected at the ionized medium with a negative reflection coefficient and partially transmitted, reemerging with a considerable time delay after reflection from the end of the coaxial conductors in the form of a relatively smooth pulse. The reflected pulse as well as the transmitted pulse are followed by a slow ringing. Later in the afterglow, at about 100 μ sec for a pressure of 0.2 mm Hg and at about 500 μ sec for a pressure of 1 mm Hg a different oscillatory response becomes visible. The leading part of this response has travelled with nearly the speed of light. At the same time in a few exposures small oscillations with about the same frequency become visible in the signal reflected from the plasma medium. With increasing time elapsed in the afterglow the amplitude of these oscillations increases and the periodicity of the oscillations decreases while the slowly varying response dominating at earlier times, although in principle superimposed, becomes insignificant.

At early times in the afterglow, the leading part of the slowly varying signal has travelled with a speed approximately equal to the speed of light in the glass walls. The wave propagation in this mode is interpreted as a surface wave similar to the case considered by Tamir and Oliner.⁶ Transmission in this mode occurs only for frequency components below the plasma frequency. Early in the afterglow the electron concentration is high and the electromagnetic field concentrated in and near the glass walls. With

*The sweep is magnified by a factor 5 and 2 respectively, so that only 20 dots and 50 dots per cm are shown respectively on the photographs.

Figure 7. Transient electromagnetic response for reflection of unidirectional pulse from plasma medium and transmission through plasma slab, observed in the afterglow of neon gas at 0.2 mm Hg pressure.

time scale: 10 μ s - 50 μ s 5×10^{-9} sec/div
 60 μ s - 400 μ s 2×10^{-9} sec/div
 sensitivity: 5 mv/div

400 μ s

Figure 8. Transient electromagnetic response for reflection of unidirectional pulse from plasma medium and transmission through plasma slab observed in the afterglow of neon gas at 1 mm Hg pressure.

time scale: $60 \mu s - 400 \mu s$ $5 \times 10^{-9} \text{ sec/div}$
 $400 \mu s - 2000 \mu s$ $2 \times 10^{-9} \text{ sec/div}$
sensitivity: 5 mv/div

Figure 9. Plane geometry of plasma confined by metal-backed glass walls.

decreasing electron concentration the field extends more into the ionized medium and the signal velocity for this mode is increased. Higher frequency components are progressively cut-off in this mode as the electron concentration and hence the plasma frequency decrease in time. A theoretical solution of pulse dispersion in this mode will not be attempted here. The existence of this surface mode and some of its more pronounced features are, however, evident by considering the simpler planar geometry sketched in Figure 9. Let b be the thickness of each of the metal-backed glass walls with a real relative dielectric constant ϵ_r' , and let a be the thickness of an idealized homogeneous plasma region with a macroscopic dielectric constant ϵ_r . It is easily shown that if the system is transversely infinitely extended and for physical reasons variations of fields in the y -direction can be excluded, TE modes cannot exist if $k_0(a+b) \ll 1$ where $k_0 = 2\pi/\lambda_0$, the free space propagation constant. All possible modes are transverse magnetic and are derived from a E_z component. Straightforward application of the field equations and boundary conditions for the tangential components of the electric vector $\vec{E} = \hat{x}E_x + \hat{z}E_z$ and the magnetic vector $\vec{H} = \hat{y}H_y$ leads to the characteristic equation for the propagation constant $\gamma = jk_0\sqrt{\epsilon_r'}\alpha$,

$$\frac{\sqrt{1-\alpha^2}}{\sqrt{\epsilon-\alpha^2}} \epsilon \tan k_0 b \sqrt{\epsilon_r'} \sqrt{1-\alpha^2} = -\tan k_0 b \sqrt{\epsilon_r'} \frac{a}{2b} \sqrt{\epsilon-\alpha^2} \quad (10)$$

where the definition $\epsilon = \frac{\epsilon_r}{\epsilon_r'}$ is introduced. A plot of real solutions for α is shown in Figure 10 where $a > 2b$ has been assumed as in the actual discharge tube. The branch for negative values of ϵ_r corresponds to the surface wave which exists only if $\epsilon = \epsilon_r/\epsilon_r' < -1$, i.e., the frequency has to be sufficiently low compared to the plasma frequency. A similar result has been derived by Tamir and Oliner. Actually, the major change in the propagation constant occurs when $\epsilon_r = \epsilon_r'(a/2b)$. From a circuit point of view this corresponds precisely to the condition that the capacitive impedances resulting from the series connection of the glass walls considered as capacitors (capacitance C) and the plasma medium balance out, hence $C_{\text{total}} \rightarrow \infty$. It should be noted, though, that rigorously the propagation constant here is finite. However, it is likely that the ringing observed early in the afterglow occurs with periodicities according to this sharp rise in dispersion, the period slightly modified to account for the different geometry and the nonhomogeneous distribution of charged particles in radial direction. It should be pointed out that the existence of this type of surface wave and its propagation characteristic below the plasma frequency opens in principle the possibility to measure electron densities which correspond to plasma frequencies considerably beyond the highest frequencies resolved by the oscilloscope.

Figure 10. Low frequency approximation for propagation branches in plasma slab (ϵ_r , thickness a) confined by glass walls ($\epsilon_r' = 5$, thickness b , $a/2b = 2$)
Slope of dashed continuation depends on frequency.

Figure 11. Transient electromagnetic response for reflection of unidirectional pulse from plasma medium and transmission of unidirectional pulse through plasma slab.

time scale: 2×10^{-9} sec/div

sensitivity: 2 mv/div

1 mm Hg pressure neon gas

700 μ s in afterglow

While the surface wave arises only in bounded ionized regions, the cut-off phenomenon associated with $\epsilon_r = 0$, hence $\omega = \omega_p$ is independent of the radial distribution of electron density. In a transmission line it may be understood as the condition for zero transverse capacitance and corresponds exactly to the plane wave cut-off phenomenon discussed in the theory. It manifests itself in the partial reflection from the plasma medium and the oscillatory response dominating at later times in the afterglow. The leading part of the transmitted signal has traveled with the speed corresponding to the TEM mode in the empty guide and it represents the dispersion of the transmitted pulse in the plasma filled guide in this mode.

The frequency spectrum of the reflected pulse contains essentially only components below and slightly above the plasma frequency. With increasing time in the afterglow the reflected pulse broadens and its amplitude is ultimately reduced with decreasing plasma frequency. At the same time the period of oscillation in the transmitted wave, and where visible in the reflected wave, is lengthened in a manner proportional to the decrease in plasma frequency. As higher and higher frequency components are able to propagate through the ionized medium, the original pulse shape is ultimately restored in the transmitted wave. The detailed structure of the transient response is most correctly shown for low plasma frequencies, e. g., Figure 7, where the instrumental limitations are insignificant and the reflection at the interface, which was not considered in the theory, negligible. It is clearly seen to agree precisely with the computed result for the pulse shape used in the experiment, Figure 1. A detailed picture with increased gain of the oscilloscope is reproduced in Figure 11. It clearly shows the presence of oscillations expected from theory in both the reflected wave and the transmitted wave.

At the pressures employed and in view of the narrow dimensions of the discharge tube, the decay of the electron concentration is primarily controlled by ambipolar diffusion to the glass walls. In the absence of contaminations the computed rate of electron annihilation by recombination is smaller by a factor of several orders of magnitude. This expected behavior is somewhat substantiated by plotting the plasma frequency on a logarithmic scale over time on a linear scale. By a direct measurement of the periodicity of transient oscillations the decay of the electron concentration shown in Figure 12 is obtained. Account has been taken of the fact, that the measured period early after arrival of the leading signal (i. e., in the time interval from 10^{-8} sec - $2 \cdot 10^{-8}$ sec after the first reflection becomes visible) is shorter by about 10% than the period of the plasma frequency. A reasonable fit to a straight line is obtained except for those

Figure 12. Plasma frequency and electron concentration in the afterglow of neon (0.2% argon) gas.

points measured relatively early in the afterglow. According to the diffusion theory,⁷ the decay of the electron number density n [electrons/cm³] is determined by

$$n = n_0 e^{-D_a t / \lambda^2} f \quad (11)$$

where D_a is the coefficient of ambipolar diffusion and λ the smallest eigenvalue to the wave equation

$$\left(\nabla^2 + \frac{1}{\lambda^2} \right) f = 0. \quad (12)$$

The boundary condition is that f vanish at the container walls. In the coaxial system let b' denote the outer radius of the inner glass tube and a' the inner radius of the outer glass tube. If no azimuthal asymmetry is present, solutions of (12) are linear superpositions of cylinder functions of zeroth order and the time constants of possible diffusion modes are determined from the sequence of zeros of the characteristic equation

$$J_0\left(\frac{b'}{\lambda}\right) N_0\left(\frac{a'}{\lambda}\right) - J_0\left(\frac{a'}{\lambda}\right) N_0\left(\frac{b'}{\lambda}\right) = 0. \quad (13)$$

The lowest values for λ determines the diffusion mode persisting for the longest time, while higher modes quickly become insignificant. The numerical value⁸ is $\lambda_1 = (a' - b')/3.122 \sim 0.2$ cm. For the coefficient of ambipolar diffusion the values, normalized to 760 mm Hg are found

$$\begin{aligned} D_a &= 0.172 \text{ cm}^2/\text{sec} && \text{(measured at 0.2 mm Hg neon)} \\ D_a &= 0.205 \text{ cm}^2/\text{sec} && \text{(measured at 1 mm Hg neon).} \end{aligned}$$

These values may be compared to the ionic diffusion coefficient. At 0°C the value of the diffusion coefficient of Ne^+ ions in neon, as obtained from the tabulated mobility⁹ is $D_i = 0.107 \text{ cm}^2/\text{sec}$. It may be assumed that at times of more than several hundred microseconds in the afterglow the electrons are in thermal equilibrium with the neutral gas molecules. This assumption bases on the time constant for the decay of velocity anisotropies by collisions ($\tau_1 \sim$ fractional microseconds) and the somewhat longer time constant for energy exchange between electrons and neutral constituents ($\tau_2 \sim \tau_1 M/2m \sim$ few hundred microseconds for neon at 1 mm Hg) (M = mass of neutral particle, m = electron mass for thermal equilibrium (electron temperature T_e = ion temperature T_i and for large electronic mobility compared to the ionic mobility). Since D_a is related to D_i by

$$D_a \sim D_i \left(\frac{T_e}{T_i} + 1 \right) \sim 2 D_i$$

A possible reason for the measured difference might be that the electrons were not yet sufficiently cooled down in the time period of measurements. A slight difference might also be explained by the possible presence of Ne^{++} ions with about 40% higher mobility and accordingly larger diffusion coefficient. Also possibly residual contaminations led to attachment processes which can change the length of the afterglow considerably.

Conclusion

In any linear system no more information can be retrieved from a study of transient responses than can be obtained by a series of measurements with continuous waves at all frequencies contained in the pulse. For plasmas, such measurements can yield, in principle, a great amount of information about the number density of electrons, their collision rates, their temperature and their distribution in space. An advantage of the operation with short transient signals merely lies in the fact that the whole measurement is accomplished in extremely short time and thus is particularly applicable in rapidly changing situations.

Although so far only the simplest characteristic of a reasonably good approximation to a simple plasma has been determined, it should, in principle, be possible to include in the evaluation such phenomena as collisions, for example by a measurement of the decay time of transient oscillations or inhomogeneities in the distribution of ionized particles by considering reflection and transmission characteristics.

The possible extension to include observation of electron densities resulting in plasma frequencies beyond the limit of oscilloscopes (~ 1000 Mc/sec) is intriguing and might be investigated in more detail on simpler geometric arrangements.

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