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Luminosity Considerations for Colliding
Electron and Proton Beams

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ABSTRACT

The maximum obtainable luminosities for electron-proton colliding beam systems are estimated subject to the constraints that (a) the power radiated by electron synchrotron radiation is fixed (this limits a combination of electron energy and current) and (b) the detuning (beam-beam interaction) of electrons by protons and vice versa must be kept below a certain limit. It is seen that high luminosity can be obtained either with a weakly bunched or unbunched proton beam and many electron bunches, or with a single proton bunch and a single electron bunch, both very short. In the many-bunch mode the optimum luminosity is obtained at lower electron energy than in the single-bunch mode.

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I. INTRODUCTION

Colliding beams of high energy protons and electrons may be expected to produce interesting data on electromagnetic and weak interactions different from what can be obtained with proton-proton or electron-proton and electron-electron collisions. Accordingly, studies of the possibilities of electron-proton colliding beam systems have been carried out at SLAC and Berkeley⁽¹⁾, and also, in connection with the study of the proton-proton collision system "ISABELLE", at Brookhaven⁽²⁾. Both studies envisage a storage ring with protons in the range 70-200 GeV, and an electron ring in the range of 5-15 GeV. The center-of-mass energy in a collision between electrons and protons of energies E_1 and E_2 is

$$W = \left[(E_1 + E_2)^2 - (p_2 - p_1)^2 \right]^{\frac{1}{2}}, \approx 2\sqrt{E_1 E_2} \quad (1)$$

which equals 65 GeV for $E_1 = 15$ GeV, $E_2 = 70$ GeV.

Assume we have two rings, filled with bunches of electrons and protons, respectively, and colliding with an intersection angle 2α . If the rings contain one bunch each, have equal frequencies of revolutions, and the bunches are synchronized for maximum overlap, the luminosity equals⁽³⁾

$$L = \frac{2N_1 N_2}{\pi w^* l^* \alpha} f \quad (2)$$

under the following assumptions. The bunches have gaussian density distributions in all three dimensions, with twice rms length, height, width ℓ_1, h_1, w_1 , and ℓ_2, h_2, w_2 for electrons and protons, respectively; the collision takes place in the vertical plane, and

$$\begin{aligned} w^* &= (w_1^2 + w_2^2)^{\frac{1}{2}} \\ \ell^* &= \left[\ell_1^2 + \ell_2^2 + \frac{1}{\alpha^2} (h_1^2 + h_2^2) \right]^{\frac{1}{2}}, \end{aligned} \quad (3)$$

f is the frequency of revolution.

For uniform density distributions the above formula remains valid except for a factor $\pi/3$, provided ℓ, h, w are again taken to be twice the rms azimuthal, vertical, and radial excursions of the particles from the bunch center, i.e. $1/\sqrt{3}$ times the "total" beam dimensions.

With more than one bunch per ring, f in (2) must be replaced by the frequency of bunch collision; thus if there are B bunches in each ring, all of which collide with the other ring, (2) is multiplied by B . In all cases, N_1 and N_2 are the number of particles per bunch. If the two rings are unequal in radius, but arranged so that $f_1 B_1 = f_2 B_2$ then synchronism is maintained, and (2) remains valid with f replaced by $f_1 B_1 = f_2 B_2$.

Because of radiation damping, the electron bunch usually has much smaller dimensions in all directions than the proton bunch; furthermore the intersection angle α is generally large compared to h/ℓ . Then (3) becomes $w^* = w_2, \ell^* = \ell_2$ and the luminosity is

$$L = \frac{2N_1 N_2}{\pi w_2 \ell_2 \alpha} fB \quad (4)$$

II. LIMITATIONS

1. Radiation

The electron intensity is, in practice, limited by the available radiofrequency power, most of which is needed to compensate for the energy radiated in synchrotron radiation. The radiated power is proportional to $B N_1 E_1^4 / R_1^2$ and, therefore more electrons are available if their energy is lower. For a given RF power it must have

$$N_1 B < P R_1^2 / E_1^4 \quad (5)$$

where P is a constant proportional to the RF power.

2. Beam-Beam Interactions

Instabilities arise from the action of one beam on the other. According to experience with electron-positron rings, and also according to some numerical computational experiments, this effect is measured by the "tune shift" Δv , i.e. the change in betatron oscillation frequency of one beam, caused by the fields produced by the other beam. The maximum allowable tune shift for electrons is approximately $\Delta v < 0.025$; that for protons is not known, but we may (perhaps optimistically) assume that it is also 0.025.

The particles in the center of the bunch experience the largest time shifts.

The time shifts for electrons can be seen ⁽⁴⁾ to be

$$\Delta v_1 = \frac{2e^2}{\pi E_1} \frac{N_2}{w_2 L 2^\alpha} \bar{\beta}_1 \quad (6)$$

where $\bar{\beta}$ is the betatron amplitude function averaged over the interaction region.

The interaction region is of length $h_2/2\alpha$ and the function β_1 varies with distance from the center as

$$\beta_1 = \beta_0 + s^2/\beta_0$$

Using the gaussian distribution, Mohl and Sessler⁽⁵⁾ find that, with given Δv_1 , the luminosity is maximized if one designs the electron ring lattice so that, at the center of the intersection region

$$\beta_0 = h_2/\alpha \quad (7)$$

with the intersection angle α equal to

$$\alpha = \left(\frac{2}{\pi} \frac{N_2 e^2}{E_1} \frac{h_2}{w_2 \ell_2 \Delta v_1} \right)^{\frac{1}{2}} \quad (8)$$

(This represents the best compromise between a short interaction region which minimizes the factor β_1 in (6) and a long interaction region (small α) which increases luminosity.)

We shall henceforth assume that the system is designed with these values of β_0 and α .

If the number of electrons is limited by power considerations, we have then

$$L < PR^2 \left(\frac{2}{\pi} \frac{N_2 \Delta v_1}{w_2 h_2 \ell_2 e^2} \right)^{\frac{1}{2}} E_1^{-7/2} f; \quad (9)$$

the maximum possible luminosity increases rapidly as the electron energy is decreased. Note that (9) does not depend on the number of bunches B.

The SLAC-Berkeley parameters⁽¹⁾ envisage protons of 70 GeV and electrons of 15 GeV. Power considerations limit the electron current to about 30 milliamperes at that energy; to obtain a good luminosity it is then necessary to bunch the proton beam extremely tightly, to a length ℓ_2 of about 25 cm. This requires very large RF power in the proton ring as well as in the electron ring.

Alternatively, we may reduce the electron energy and increase the electron current. Then, in order to obtain a given center-of-mass energy, one would need higher proton energies. But there is another catch:

With a strong electron current, the tune shift of the proton beam may become important. It is

$$\Delta v_2 = \frac{2e^2 N_1}{\pi E_2 w_1 \ell_1 \alpha} \beta_2 \quad (10)$$

where β_2 is the betatron function of the proton ring. Let us assume β_2 is large compared to the length of the interaction region, and is fixed. Then if N_1 is determined by (10), the luminosity becomes

$$L < \frac{N_2 E_2}{e^2 \beta^2} \frac{w_1 \ell_1}{w_2 \ell_2} B f \Delta v_2 \quad (11)$$

This limit also depends on the electron energy E_1 because the dimensions $\ell_1 w_1$ of the electron bunch, which are determined essentially by the quantum fluctuations of synchrotron radiation, depend on the electron energy. According to the theory of quantum fluctuations⁽⁵⁾, we have, for a given structure, transverse dimensions increasing proportionally to energy, while the bunch length depends also on the radiofrequency.

For any electron energy E_1 the voltage must be sufficient so as not only to compensate for radiation loss, but also to confine the bunch in the center of the RF bucket in order to make the lifetime for diffusion out of the bucket large enough. Therefore we may assume that the length of the bunch is always a constant fraction of the RF wavelength, independent of electron energy⁽⁶⁾. This gives

$$w_1 \ell_1 \sim E_1$$

Maximum luminosity is now obtained by adjusting the electron energy so that the luminosity limits (9) and (11) are equal. Taking

$$w_1 \ell_1 = w_0 \ell_0 (E_1/E_0)$$

where $w_0 \ell_0$ and E_0 are reference constants, we write (9) and (11) in the form

$$L_{\text{power limit}} = X E_1^{-7/2} \quad (12)$$

$$L_{\text{detuning limit}} = Y E_1 \quad (13)$$

and therefore the two are equal when

$$E_1 = (X/Y)^{2/9}$$

and the corresponding luminosity is

$$L = X^{2/9} Y^{7/9}$$

with

$$X = f P R^2 \left(\frac{1}{\pi} \frac{N_2 \Delta v_1}{w_2 h_2 l_2 e^2} \right)^{1/2} \quad (14)$$

$$Y = f B \frac{N_2 E_2 \Delta v_2}{e^2 E_0 \beta_2} \frac{w_0 l_0}{w_2 l_2} \quad (15)$$

To obtain the largest possible luminosity subject to the detuning limit Δv_2 it is thus advantageous to make the number of bunches B as large as possible.

If there are a number B_2 of relatively long proton bunches, we may divide the electron beam into trains of short bunches each having a gross length equal to the length of a proton bunch; this gives

$$B = B_2 l_2 / \lambda$$

where λ is the wavelength of the RF in the electron ring. From previous considerations, we saw that the length of an electron bunch is a certain fraction of λ (say $1/20$); then the product $B l_0 / l_2$ appearing in Y equals $B_2 / 20$, independent of the choice of electron accelerating frequency and independent of l_2 .

It is thus seen that the overall maximum luminosity is obtained at an Energy E_1 which depends only on the ninth root of the inverse proton bunch length l_2 , and the luminosity itself also is proportional to $l_2^{-1/9}$.

For illustration, we consider two possibilities: a) parameters of the proton beam derived from the ISABELLE Study⁽²⁾, with the "high luminosity" intersection region described there, the electron beam similar to that described in the PEP study⁽¹⁾; b) proton and electron beams from the PEP study, with a single bunch circulating. The assumed parameters are:

In both cases we take a power limit corresponding to 2.5 megawatts in an electron ring similar to that of Ref. 1, but with radius of curvature in the magnets 150 meters, and circumference 2000 meters.

This gives

$$PR^2 = 1.77 \times 10^{17}.$$

Other parameters are:

	ISA	PEP
N_2	2×10^{13}	7.5×10^{12}
E_2	200 GeV	72 GeV
ℓ_2	1500 cm	23 cm
h_2	0.08 cm	0.12 cm
w_2	0.03	0.06
ℓ_0	6 cm	6 cm
w_0	0.03 cm	0.03 (at $E_0=15$ GeV)
θ_2	70 cm	15 cm
λ_{RF}	120 cm	120 cm
f	0.150 MHz	0.150 MHz
B_2	30	1
Δv_1	0.025	0.025
Δv_2	0.025	0.025

This gives the derived parameters

	ISA	PEP
B	375	1
X	4.65×10^{35}	1.328×10^{36}
Y	1.49×10^{32}	8.152×10^{30}
E_1 (opt)	5.98	14.4 GeV
L (opt)	8.9×10^{32}	1.17×10^{32}
L ($E_1 = 10$ GeV)	1.47×10^{32}	8.2×10^{31}
L ($E_1 = 15$ GeV)	3.56×10^{31}	1.01×10^{32}
Electron current at Opt. E_1	3.32 amp	99 mA
α (half crossing angle)	6.6 mrad	18 mrad
β_{electron}	12 cm	6.7 cm

Thus these two rather different systems have rather similar performance.

It should be noted, however, that:

(a) With the higher proton energy of ISA the center-of-mass energy is higher for a given electron energy; at $E_1 = 6$ GeV the center-of-mass energy of ISA is about the same as PEP at 15 GeV, and the luminosity is about 8 times higher.

(b) The luminosity of PEP cannot easily be increased further by increasing the number of bunches, because the proton bunch length is already shorter than the RF wavelength.

(c) Further increases of the ISA luminosity at 15 GeV can only be obtained by increasing X; this means either increasing the power limit or the beam density.

It may be interesting to estimate the performance of ISA with completely unbunched proton beams. This reduces the line density of N_2/l_2 of the proton beam by a factor $\sim \frac{2\pi R}{30 l_2} \sqrt{\frac{2}{\pi}} = 3.55$ and reduces X by the square root of this factor, i.e. by 1.88. The number of bunches is now 1667 and

and Y is increased by a factor 1.25. For an unbunched ISA we therefore obtain

B =	1667
X =	2.47×10^{35}
Y =	1.86×10^{32}
$E_{1 \text{ opt}} =$	4.94 GeV
$L_{\text{opt}} =$	9.22×10^{32}
L (10 GeV) =	7.8×10^{31}
L (15 GeV) =	1.89×10^{31}
$\alpha =$	3.85 mrad
$\beta_{\text{electron}} =$	21 cm
Electron current	7.13 amperes

Remarks on Beam-Beam Interaction

Keil, Pellegrini and Sessler (unpublished) have pointed out that in an interaction region with finite crossing angle the long-range interaction outside the actual crossing region can contribute substantially to the tune shift $\Delta\nu$. Therefore the results of this paper are valid only if this long-range interaction is somehow inhibited, for example by inserting a shield wall between the two beams as near as possible to the crossing region, or by separating the two beams with bending magnets very close to the crossing region. Studies of the extent to which this can be accomplished will have to be left to the future.

REFERENCES

1. C. Pellegrini et al., Proceedings of 1971 Accelerator Conference, CERN.
2. "ISABELLE" report, (BNL 16716), Section XVII.
3. M. Sands, SLAC-121.
4. This ignores the contribution to $\Delta\nu$ from fields just outside the interaction region. Keil, Sessler and Pellegrini (unpublished ISA note).
5. "D. Mohl and A.M. Sessler, PEP Note 8 (LBL, Nov. 1971).
6. See reference 3.