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Analysis of π^+ p elastic scattering
from 2.74 to 4.0 GeV/c at all angles*

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Abstract

We analyse π^+ p elastic scattering in terms of s-channel helicity amplitudes. We demonstrate that the data are consistent at all angles with the picture of the proton as an absorbing sphere of radius 0.8F, with special edge effects accounting for backward scattering.

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I. Introduction

Recent experiments on the π^+p elastic scattering have provided a new wealth of data for this reaction in the energy range 2.74 to 5.0 GeV/c. Many striking features are to be observed in the data, particularly in the differential cross-sections.¹⁻⁴ Near the forward direction, the differential cross-section (see Fig. 1) shows a very strong diffraction peak which falls off two orders of magnitude in a comparatively small interval of momentum transfer $0 \geq t \geq -.8 \text{ (GeV/c)}^2$. Around $t = -.85 \text{ (GeV/c)}^2$, some structure develops -- there is a substantial change of slope, with a shallow valley occurring here at the lower energies. The subsequent shoulder then falls off to a very deep valley in the vicinity of $t = -2.8 \text{ (GeV/c)}^2$. In the backward direction, there is likewise a sharp peak, followed by a valley which remains at about $u = -.2 \text{ (GeV/c)}^2$ for all the energies considered. At intermediate angles where the differential cross-sections are becoming exceedingly small, there are interesting indications of further structures.

While some of these features have been known for a long time, what is important about the new experiments is that they provide continuous measurements over the whole scattering region, and therefore present a challenge to phenomenologists that is considerably more intriguing than the simpler game of fitting either the forward or the backward scattering data.

There are also some polarization data^{2,5} in this energy range, but these at present cover only the angles near the forward and backward

directions. Near the forward direction (see Fig. 2), the polarization is positive, rising initially to about 20 - 30%. Next there is a shallow dip around $t = -.6 \text{ (GeV/c)}^2$, after which the polarization increases again to more than 50%. Away from the backward direction (see Fig. 3), the polarization is at first positive (rising to 100% for 2.75 GeV/c), then changes sign at about $u = -.24 \text{ (GeV/c)}^2$, roughly the same position at which the striking backward valley in the differential cross-section occurs.

The π^+p total cross-section has also been measured⁶ throughout this energy range (see Fig. 4). It is smoothly decreasing, indicating that there are no really prominent resonances present.

Previous attempts^{7,8,9} have been made to fit and understand separate pieces of this data. In particular, the experimental structures at the ends of the angular region are usually interpreted in terms of exchanges in the t , u channels. However, a clear picture of the pion-nucleon interaction from the point of view of these exchanges has yet to emerge, in spite of the enormous effort put into these models during the past decade. Even the basic question of the dip mechanism, whether due to nonsense wrong-signature zeros or to strong pole-cut interference⁹, is still unresolved!

In the present paper, we suggest another type of model, based on an s -channel approach. This model makes use of the familiar ideas of the optical and absorptive models; however the present model goes far beyond previous models of this type in that it attempts to describe scattering at all angles. In a sense, our model can also be regarded as an extension of the more conventional phase shift analyses¹⁰; however in order to make the expansions in angular momentum states tractable, we inject some

physics (indeed, essentially the same physics as went into the original Regge pole model more than 10 years ago!)

In Section II, we briefly describe the ideas of our model and in Section III state our parametrization of the amplitudes. In Section IV, the fits to the data are presented, and the contributions from the various amplitudes examined in detail. In Section V, a brief comparison is made with Regge pole fits. Lastly in Section VI, we discuss the major evidence for our scattering picture.

II. Model

The authors¹¹ have previously outlined some of the basic ideas of the model in their discussion of the π^-p charge-exchange (CEX) reaction. There, it was shown, through the use of s-channel helicity amplitudes, how the well-known structures in both the forward and the backward directions for CEX could be correlated. (It will be recalled that in the usual Regge approach these directions are treated totally independently.) The essential points of the argument were:

- (i) In the angular momentum expansion of the s-channel helicity amplitudes¹²

$$f_{++}^s = \frac{1}{k} \sum_j (j + \frac{1}{2}) T_{++}^j d_{\frac{1}{2}\frac{1}{2}}^j(\theta)$$

$$f_{+-}^s = \frac{1}{k} \sum_j (j + \frac{1}{2}) T_{+-}^j d_{-\frac{1}{2}\frac{1}{2}}^j(\theta)$$

where k is the c.m. momentum, the low angular momentum states are absorbed (as a result of S_{el}^j being small for low j), while the high angular momentum states likewise give little contribution since they are unscattered. This leads to the conclusion that the major contribution to the scattering for this CEX process comes from a narrow intermediate band of angular momentum states.

- (ii) By considering the oscillations of the corresponding $d_{\lambda\mu}^j(\theta)$ functions, it was shown that this band should be centered around $j = j_c$, where $j_c + \frac{1}{2} = kR$. Here R is the appropriate radius for the interactions, estimated qualitatively to be

about 1f in order to give the forward and backward CEX dips at their observed positions $t = -.6$ and $u = -.2 \text{ (GeV/c)}^2$ respectively. As a consequence, we anticipated that if R is more or less a constant, then j_c should move with energy in proportion to the c.m. momentum k .

We now carry over these same ideas to the case of $\pi^+ p$ elastic scattering, one of the few reactions where good data¹⁻⁶ are available at all angles. Elastic scattering has of course the added complication of diffraction and this must be taken into account. The low angular momentum states which get absorbed out in many of the inelastic reactions such as CEX are just those which give rise to the strong forward diffraction peak in elastic scattering, analogous to Fraunhofer diffraction in optics. In the present situation where the proton has spin $\frac{1}{2}$, this effect might be expected to be much more pronounced in the helicity non-flip amplitude f_{++}^0 than in the helicity flip amplitude f_{+-}^0 which vanishes in the forward direction by angular momentum conservation.

III. Parametrization

The parametrization we choose is intended to reflect the physical ideas discussed in the previous section. We took the angular momentum s-channel helicity amplitudes in the following form:

$$(j + \frac{1}{2}) T_{++}^j = G_{++}^R + iG_{++}^I + \frac{1}{2}\Gamma (v + ix)[(j - j_c) - i\Gamma/2]^{-1} \quad (1)$$

$$(j + \frac{1}{2}) T_{+-}^j = G_{+-}^R + iG_{+-}^I + \frac{1}{2}\Gamma (y + iz)[(j - j_c) - i\Gamma/2]^{-1}$$

Here G^R , G^I were taken as Gaussians of the form $a \exp [-b(j - c)^2]$.

These weight the low angular momentum states (see Fig. 6), in accordance with the relation $T^j = i(1 - S^j)$ and the property that the S-matrix elements for low j are small. It is the Gaussian iG_{++}^I which gives rise to the large forward diffraction peak; loosely speaking, it corresponds in Regge language to the Pomeron contribution, though as we shall see later, the details are markedly different.

The Breit-Wigner terms are meant to represent the peripheral interaction, that is the contribution from the band of angular momentum states being scattered from the edge of the absorbing proton target. The choice of a Breit-Wigner form is of course very restrictive. It correlates both the magnitudes and phases of adjacent angular momentum states within this band, and therefore represents some kind of a collective effect. This form also has a certain amount of appeal however since it corresponds to a simple Regge pole in the complex angular momentum plane. The parameters j_c , Γ were taken to be the same in both helicity non-flip and flip amplitudes.

Altogether therefore, for a given energy in which the differential cross-section at all angles, the available polarization, the total cross-

section and the ratio of the real to the imaginary part of f_{++}^G at $t = 0$ are all fitted simultaneously, there are 18 parameters. (The two Gaussians in the flip amplitude however give fairly small contributions, so that all the basic structures are given essentially by 12 parameters.) We also make sure that the parameters change smoothly on going from one energy to the next (see Table I).

IV. Fits to the Data; Analysis of our Amplitudes

Our fits to the differential cross-sections at all angles for four energies 2.74, 3.0, 3.5 and 4.0 GeV/c are presented in Fig. 1, the corresponding values of the parameters being given in Table I. In Figs. 2, 3, we make comparisons with the available polarization measurements in the forward and backward scattering regions. Fig. 4 shows our fits to the total π^+p cross-sections. The total cross-section of course restricts the imaginary part of the non-flip amplitude at $t = 0$ through the optical theorem. In order to pin down also the real part of this amplitude at $t = 0$, we used the results of Hühler and Strauss¹³ who have calculated the real and imaginary parts of the πN non-flip amplitude at $t = 0$ from forward dispersion relations. Table II shows the agreement.

These fits seem sufficiently reasonable as to provide substantial justification for the simple physical picture behind the parametrization. As far as we are aware, no other model (except of course the standard phase shift analyses at much lower energies) has been able to correlate so much data throughout the whole scattering region in such a unified way, and this is the major achievement of the model.

We turn now to a more detailed analysis of our fits. As we shall show, the experimental features come about in a natural way from the oscillations of the amplitudes. In order to avoid repetition, we shall concentrate on our solution to just one of the energies, namely 4.0 GeV/c, our fits to the other energies being similar.

Non-flip amplitude.

We first of all show in Fig. 5 the contribution to the differential

cross-section from the s-channel helicity non-flip amplitude, indicating also the separate contributions from G_{++}^R and iG_{++}^I . We see that the non-flip amplitude provides most of the differential cross-section¹⁴ all the way out to about $t \approx -1.5 \text{ (GeV/c)}^2$. The forward diffraction peak arises primarily from the imaginary term iG_{++}^I , whose main contributions come from the low angular momentum states. To further illustrate this last point, we have drawn in Fig. 6 the shape of $\text{Im}T_{++}^j$ arising from iG_{++}^I alone (remember to divide out the $j + \frac{1}{2}$ factor on the left hand side of Eq. (1)) as a function of j .

In all our fits, the break at $t \approx -.85 \text{ (GeV/c)}^2$ arises from the first (diffraction) zero of $\text{Im}f_{++}^s$, the hole there being filled up mainly by $\text{Re}f_{++}^s$. The real part of the non-flip amplitude, which is often ignored in many calculations (in spite of the fact that it is known¹³ to be about 30% of the imaginary part at $t = 0$), is therefore very important to us in our fitting scheme.

The role of the non-flip Breit-Wigner is interesting. In contrast to the Gaussian terms, the Breit-Wigner is centred around a higher angular momentum state with $j_c + \frac{1}{2} \approx 5.03$ (for 4.0 GeV/c). The former rapidly die away beyond $t \approx -2.8 \text{ (GeV/c)}^2$, whereas the Breit-Wigner pops up at large angles, helping in particular to fill the backward valley at $u \approx -.2 \text{ (GeV/c)}^2$.

Flip amplitude.

In Fig. 7, we have drawn the contributions from the s-channel helicity flip amplitude, as well as the separate contributions from $\text{Re}(BW_{+-})$, $\text{Im}(BW_{+-})$ and $(G_{+-}^R + i G_{+-}^I)$. The flip amplitude clearly provides the dominant contribution at the larger angles, just as the non-flip amplitude dominated in the forward scattering region as discussed above. The oscillatory

character is provided by the Breit-Wigner term. The width Γ of the angular momentum band is (see Table I) about 1.0, so the band is reasonably narrow; the oscillations correspondingly remain sizeable at all angles. In contrast, the Gaussian terms yield a featureless background (they contain only a few low angular momentum components). They assist mainly in damping out the vigorous oscillations of the flip Breit-Wigner at the center of the scattering region where the differential cross-section is exceedingly small.

One immediate test (see Section II, (11)) of our ideas concerning the flip term is that the band of important angular momentum states described here by the Breit-Wigner should move up in energy as the laboratory momentum increases, according to $j_c + \frac{1}{2} \approx kR$. In our fits to the data, we found (see Table I) that for $p_{lab} = 2.74, 3.0, 3.5$ and 4.0 GeV/c, the corresponding values of $j_c + \frac{1}{2}$ were $j_c + \frac{1}{2} = 4.16, 4.31, 4.89$ and 5.03 . Recalling that for these energies $k = 1.05, 1.10, 1.20$ and 1.30 (GeV/c) respectively, we see that the energy dependence of j_c is indeed indicative of this relation, with the radius of interaction R taken as about $0.8F = 4(\text{GeV}/c)^{-1}$.

Polarization.

As a last comment on our fits, we mention how this model accounts for the observed features of the polarization near the forward and backward directions. One can easily see what happens by noting from Figs. 5,7 which are the dominant amplitudes in the various regions and where their zeros occur. The forward polarization arises primarily from the product of the dominant $\text{Im}f_{++}^G$ and the much smaller $\text{Re}f_{+-}^G$; the interesting dip at $t = -.6 (\text{GeV}/c)^2$ in the polarization then arises from a zero in $\text{Re}f_{+-}^G$ at $t = -.4 (\text{GeV}/c)^2$ and in $\text{Im}f_{++}^G$ at $t = -.85 (\text{GeV}/c)^2$. The zero in the

backward polarization at $u = -.24 \text{ (GeV/c)}^2$ arises from the zero in the dominant Breit-Wigner flip amplitude, which also causes the backward valley in the differential cross-section at this position.

Even a cursory examination of fits shows how the amplitudes are closely tied together. The smaller oscillations of an amplitude which may be dominant elsewhere can be crucially important in fitting the data, and there is not too much room for leverage.

V. Contrast with Regge Models

Those who are familiar with Regge pole analyses^{7,8} will realize that the way the experimental structures are explained, is considerably different for the two models. A detailed comparison of the models is at present under way; we shall therefore content ourselves here with a few brief remarks on some of the main differences between the Regge fits and our own fits (Section IV).

In the usual Regge fits^{7,8}, the imaginary Pomernanchuk contribution is supposed to represent diffraction. However, it is always parametrized as a smooth exponential in t , completely structureless (in both the t -channel and s -channel), with no zero at $t = -.85 \text{ (GeV/c)}^2$, as we have in our $\text{Im} f_{++}^8$. Furthermore, because the Pomernanchuk contribution is large¹⁵, the choice for its parametrization substantially determines many other matters in the subsequent fitting. For example, the dip in the polarization at $t = -.6 \text{ (GeV/c)}^2$ is obtained in the Regge fits through the double zero of a no-compensation coupling for the P' , or through a nonsense wrong signature zero from the ρ coupling together with a vanishing of the relative phase of the ρ and $(P + P')$ contributions. These explanations seem somewhat contrived, but illustrate what has to be done if one chooses a smooth Pomernanchuk.

There are differences also in the backward direction, where Regge fits^{8,9} rely on structure in the A, B amplitudes arising either from a nonsense wrong signature zero of the nucleon coupling or from cuts. In our model, the backward direction is dominated by the s -channel helicity flip amplitude, and this goes over to the A' amplitude there (and not A or B). It is therefore our A' which is most directly associated with the experimental

data, and gives rise to the observed features.

Our model is also antithetical to the one recently proposed by Harari¹⁶. He likewise bases his arguments on a smooth imaginary Pomeranchuk contribution, this assumption being taken from the smoothness of the differential cross-section for elastic proton-proton scattering. Structures in differential cross-sections, such as are observed in π^+p scattering at $t = -.85 (\text{GeV}/c)^2$, then come about from destructive interference with s -channel resonances. However, as is well-known¹⁷, even though the pp differential cross-sections are smooth at lower energies, they begin to develop structure at $t = -.85 (\text{GeV}/c)^2$ above 10 GeV/c , a well-defined shoulder being clearly visible at 20 GeV/c . This trend seems to run counter to the idea of a smooth Pomeranchuk, which should presumably become the more dominant the higher the energy. Again, his smooth Pomeranchuk necessitates a double zero in one of his amplitudes ($\text{Re} f_{+-}^s$) to explain the π^+p polarization dip at $t = -.6 (\text{GeV}/c)^2$.

VI. Energy dependence and concluding remarks

To understand the energy dependence of our fits, it is important to ask the question "What is energy independent?"

So far, we have been able to find three things that are more or less energy independent:

- (i) In Fig. 8a, we have drawn a graph of the contribution of iG_{++}^I to $\text{Im}T_{++}^j$ as a function not of j but of the impact parameter $b = (j + \frac{1}{2})/k$, for the four energies considered. We see that the shape¹⁸ is more or less the same for all the energies. The graph indicates that absorption is substantial for low b , but rapidly decreases around a radius of about $.8F$.
- (ii) In Fig. 8b, there is a similar graph of the contribution of G_{++}^R to $\text{Re}T_{++}^j$ for the four energies. Apart for one point, the shape again seems to be the same for all the energies.
- (iii) The third energy independent feature has already been mentioned in Section IV, where we remarked that the Breit-Wigner j -pole position increased with energy in a manner consistent with scattering from the edge of an object whose radius remains approximately constant at $.8F$. It is important to realize from point (i) above that this is exactly the same radius where the absorption is rapidly decreasing.

Our conclusion therefore is that all the π^+p data we have fit is consistent with the picture of scattering from a sphere which is strongly absorbing out to a radius of about $0.8F$, at which range substantial edge effects (in the form of resonances) are important.

Since the main experimental features we have been discussing (forward diffraction peak and backward peak with a valley at $u = -.2 \text{ (GeV/c)}^2$ occur at higher energies without any new type of feature appearing, one might speculate that all the data we have at present on $\pi^+ p$ elastic scattering is describable essentially in terms of the above physical picture. In one sense, it would of course be good to know that such a simple description is applicable; in another, it would be disappointing since it would mean that we have learned little about the deeper nature of the proton itself. The dynamics are (still) wrapped up in exactly what is causing the internal absorption and the edge effects -- as implied in the previous Section, if this model is correct, the usual Regge pole phenomenology for elastic scattering will require major surgery.

We are at present examining other processes (such as $\pi^- p$, CEX and pp) to see if our model applies equally well in these situations. Our preliminary results indicate that this is indeed the case.

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TABLE I

Values of our parameters for fits in Fig. 1. Dimensions of a^R , a^I , v , x , y and z are $\sqrt{mb}$ (GeV).

p_{lab} (GeV/c)		2.74	3.0	3.5	4.0
G_{++}^R	a_{++}^R	-.46	-.46	-.47	-.49
	b_{++}^R	.94	.45	.32	.26
	c_{++}^R	1.88	1.64	1.72	1.90
G_{++}^I	a_{++}^I	1.16	1.22	1.21	1.28
	b_{++}^I	.28	.24	.15	.14
	c_{++}^I	2.43	2.70	3.28	3.33
G_{+-}^R	a_{+-}^R			-.06	-.05
	b_{+-}^R			1.65	2.99
	c_{+-}^R			1.05	1.54
G_{+-}^I	a_{+-}^I			-.09	-.05
	b_{+-}^I			1.01	2.50
	c_{+-}^I			1.37	0.50
$J_c^{+1/2}$		4.16	4.31	4.89	5.03
Γ		.95	.92	.95	.95
v		.33	.20	-.09	.03
x		.31	.28	.15	.16
y		.55	.52	.29	.27
z		-.24	-.15	-.21	-.14

TABLE II

Comparison of $(\text{Re}/\text{Im}) f_{++}^{\pi}$ at $t = 0$ with values obtained by Höhler and Strauss (Ref. 13) from forward dispersion relations.

P_{lab} (GeV/c)	2.74	3.0	3.5	4.0
R/I (Ref 13)	-.315	-.334	-.312	-.310
R/I	-.31	-.33	-.31	-.31

Figure Captions

- Figure 1 Fits to π^+p elastic scattering differential cross section at $p_{lab} = 2.74, 3.0, 3.5$ and 4.0 GeV/c. Data from Ref. 1 ($\odot, \bar{\odot}$), Ref. 2 ($\frac{1}{2}$), Ref. 3 ($\bar{\odot}$) and Ref. 13 ($\square$).
- Figure 2 Fits to the forward π^+p elastic scattering polarization. Data from Ref. 5 ($\bar{\odot}$) and Ref. 2 ($\frac{1}{2}$).
- Figure 3 Fits to the backward π^+p elastic scattering polarization. Data from Ref. 5 ($\bar{\odot}$).
- Figure 4 Fits (*) to the π^+p total cross section. Data from Ref. 6 ($\odot$).
- Figure 5 Total contribution of the non-flip amplitude ($G_{++}^R + iG_{++}^I + BW_{++}$) to the differential cross section (solid line _____). We also indicate the contributions from G_{++}^R alone (dotted line . . .) and G_{++}^I alone (dashed line - - -).
- Figure 6 Contribution to $\text{Im}T_{++}^j$ (expressed in dimensionless numbers) from iG_{++}^I as a function of j .
- Figure 7 Total contribution of the flip amplitude ($G_{+-}^R + iG_{+-}^I + BW_{+-}$) to the differential cross section (solid line _____). We also indicate the contributions from $G_{+-}^R + iG_{+-}^I$ (dash-dotted line - . - . - .), $\text{Re}(BW_{+-})$ (Dotted line) and $\text{Im}(BW_{+-})$ (dashed line - - -) separately.

- Figure 8** (a) Graph of the contribution of iG_{++}^I to $\text{Im}T_{++}^J \sqrt{mb}$ (GeV) as a function of the impact parameter b (in Fermis) for the energies 2.74 ($\bullet$), 3.0 ($\blacktriangle$), 3.5 ($\blacksquare$) and 4.0 ($\times$) GeV/c.
- (b) Graph of contribution of G_{++}^R to $\text{Re}T_{++}^J \sqrt{mb}$ (GeV) as a function of b (in Fermis); same convention as (a).