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COMPUTATIONAL METHODS OF GAMMA IRRADIATOR DESIGN

B. MANOWITZ, R.H. BRETTON, L. GALANTER, AND F.X. RIZZO

December 1964

BROOKHAVEN NATIONAL LABORATORY

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December 1964

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1. INTRODUCTION

In the last few years a radiation process industry has begun to emerge. Research efforts have uncovered many applications of ionizing radiation for the destruction of bacteria in food and medical supplies, for the initiation of chemical reactions, and for the modification of properties of polymers. Some of the applications have been commercialized. Electron accelerators have been used commercially to improve the thermal resistance of wire insulation and to sterilize medical supplies. Cobalt-60 gamma irradiators have also been used commercially for sterilizing medical supplies and for initiating chemical reactions.

Design principles sufficient to meet the requirements for electron beam generators of low penetration are well known. In general, however, for thick targets gamma ray emitting radioisotopes or x-ray producing machines will be required. For many of the potential applications of gamma emitters a need exists for a set of comprehensive design principles. There is not yet available in the literature a collection of methods for determining dose distribution in finite targets from finite radiation sources to the required degree of accuracy of many of the applications. Nor are there readily available detailed descriptions of the various radioisotope sources procurable in massive quantities, such as their nuclear characteristics, encapsulation technology and testing, heat generation and dissipation characteristics, safety and cost. Nor is there readily available a treatise on the design of chemical reactor vessels in which the reactions are induced by ionizing radiation.

For many years a research program on radiation engineering, physics, and chemistry of large scale radioisotope sources has been in operation at Brookhaven National Laboratory. A similar program having to do with beam characteristics of electron and x-ray generators has been initiated at the National Bureau of Standards. As part of the Brookhaven program a comprehensive design manual for gamma irradiation is in preparation. The writing of this manual has proved to be a formidable task. Work has been in progress for about two years

and it has become obvious to the authors as well as several other BNL staff members that considerably more time will be needed for completion of the entire volume. However, because of gamma irradiator designers and potential users' many requests, this report describing the computational methods for gamma irradiator design has been prepared.

One of the most important parameters in irradiator design is the dose uniformity in the target. In food sterilization and in many other applications dose uniformities expressed as the ratio of maximum dose at any point in the package to minimum dose at any point in the package must be 1.25 or less. If one is to design for a maximum-to-minimum ratio of 1.25, i.e., variations of $\pm 12.5\%$, then it must be possible to predict the dose at any point in the package with a precision much greater than the variation permitted. One would like to be able to predict the dose at any point in a finite target, with an accuracy of $\pm 5\%$. This accuracy is greater than that obtainable by methods typically used for shielding calculations. The methods presented herein do permit accuracies of this sort for dose distribution determinations in finite targets from finite sources, albeit only for thin homogeneous sources (essentially monochromatic radiation incident on the target face) and for homogeneous targets. Thin radioisotope sources are available and, indeed, since they minimize self-absorption, are optimum for many applications. The analytical treatment of heterogeneous sources and heterogeneous targets is under study at Brookhaven and will be described in a subsequent report.

This report includes a brief review of the principles of radiation physics, a description of analytical and empirical computation methods for determining dose distributions from practical cobalt-60 and cesium-137 sources in practical package targets, and the incorporation of these methods into irradiator designs.

2. RADIATION PHYSICS

2.1 Interaction of Gamma Radiation With Matter

The individual processes by which gamma rays may interact with matter, in the range of energies from 10-kev to 10 Mev, are (in the order of their importance): (1) photoelectric effect, (2) Compton scattering, (3) pair production, (4) Rayleigh or coherent electron scattering, (5) annihilation radiation, (6) bremsstrahlung, (7) fluorescence radiation, and (8) other processes of negligible importance.^(1,2)

In the design of irradiators one is concerned mainly with the three primary processes.

The following discussion summarizes the most important aspects of these three primary processes. For more detailed and comprehensive treatment of this subject, the reader is referred to the standard works of Heitler,⁽¹⁾ and Bethe and Askin.⁽³⁾ The discussions by Davisson,⁽⁴⁾ White,⁽⁵⁾ Stephenson,⁽⁶⁾ Evans,⁽²⁾ and Kaplan⁽⁷⁾ may also be instructive.

Photoelectric Effect

The photoelectric effect is one of two primary processes in which the photon energy is completely absorbed by the atom. It is characterized by the disappearance of the incident photon and the emission of an orbital electron. Conservation of momentum is insured by the recoil of the entire atom, and thus it may be stated that the process of photoelectric absorption can take place only with a bound electron. The electron emerges with an energy $E = h\nu - I$, where I is the ionization energy of the electron.

The effect is predominant in the energy range below 100 kev for all medium and high atomic number absorbers. The cross section per atom increases approximately with the fourth or fifth power of the atomic number and decreases with increasing photon energy as $E^{-7/2}$.

Compton Scattering

The Compton process is an inelastic or incoherent interaction of a photon with a free

electron in which part of the energy of the photon is imparted to the electron which leaves the point of impact at some angle relative to the direction of the incident photon. The remaining energy is retained by the photon which scatters at an angle θ after which it may interact again.

The total microscopic Compton cross section may be divided into two components

$$\sigma_e = \sigma_{es} + \sigma_{eca} \quad (2.1.1)$$

where σ_e is the cross section for the amount of energy in the scattered photons, and σ_{eca} is the cross section for the amount of energy transferred to the electrons. These values have been calculated from the Klein-Nishina formula and are tabulated by Klein-Nishina,⁽⁸⁾ Davisson and Evans,⁽⁴⁾ and Gladys White Grödstein.⁽⁹⁾

The microscopic total Compton cross section is independent of the atomic number and therefore the atomic cross section depends directly on Z , the atomic number.

$$\sigma_a = \sigma_e Z$$

The total mass absorption cross section for the Compton process is

$$(\mu/\rho)_K = N_0 Z_e \sigma / A \quad (2.1.2)$$

where N_0 = Avogadro's number, A = atomic mass, and Z = atomic number.

Pair Production

Pair production is not of great importance in the attenuation in low Z targets of gamma photons emitted from most radioisotopes. The process involves the interaction of a photon in the field of electrons or in the nucleus of the atom. All the photon energy is transferred to an electron-positron pair with the subsequent disappearance of the photon. The minimum energy at which the process occurs is 1.022 Mev, which is the total rest energy of the electron-positron pair. The probability for this process increases almost linearly with E and approximately as Z^2 . It is thus most important for the heavier elements and for the more penetrating photons.

Theoretical calculations are quite difficult and involve many approximations. Pair production is always followed by the emission of softer radiation due to the subsequent annihilation of positrons with electrons resulting in two 0.511-Mev photons for each positron. Annihilation radiation has been calculated to be relatively unimportant. According to Goldstein, it accounts for, at most, 7% of the deposited energy, for 6 to 10-Mev photons in targets of atomic numbers from 25 to 85.

2.2 Attenuation of Gamma Radiation

Narrow Beam Geometry

The absorption of a narrow beam of monoenergetic gamma radiation exhibits an exponential attenuation with absorber thickness because each photon is removed from the beam in a single interaction. Thus, the number of photons removed, ΔN , is proportional to the thickness of the absorber, Δx , and the number of incident photons, N , and may be related by the following equation.

$$\Delta N = -\mu N(\Delta x) \quad (2.2.1)$$

where μ is the proportionality constant known as the total gamma absorption coefficient. It is constant for monoenergetic gamma rays and in Eq. (2.2.1) is in units of reciprocal length. If x is expressed in units of mass per unit area (g/cm^2), the mass attenuation coefficient, μ/ρ , is used since the product of the coefficient and the thickness must be dimensionless.

Integration of Eq. (2.2.1) yields the well known Lambert's law:

$$N/N_0 = e^{-\mu x} \quad (2.2.2)$$

The left-hand side of Eq. (2.2.2) may also be replaced by the ratio of energy flux density or intensity (I), or by the ratio of particle flux density. The equations are valid for narrow or collimated beams of monoenergetic photons. Photons are thus removed from the beam both by absorption processes and by scattering, and μ is the total absorption coefficient.

Broad Beam Geometry

The exponential attenuation of radiation as described for narrow beam geometry or for a "pure" absorber does not truly represent the actual dose or flux density variation in a target. Whereas the photoelectric and pair production interactions completely remove a photon from the beam, the Compton process results in the deflection of a photon with resultant lower photon energy and the ejection of an orbital electron. The actual energy imparted to the target is contained in the kinetic energy of the electrons. Therefore, a calculation of the dose based on the total or narrow beam absorption coefficient would result in a value which is too small. A correction factor which is called the buildup factor or buildup is therefore used. It is the ratio of the "true" dose as exactly calculated or measured divided by the "narrow beam" or uncollided dose. The use of the buildup thus allows one to obtain the "true" dose from a relatively simple calculation of the uncollided flux. The buildup factors readily available from the literature^(10,11,12,13,14,15) are for point sources in infinite homogeneous media such as water, iron, aluminum, tin, and lead. Buildup factors are also available for plane isotropic sources, and plane directional sources.^(10,11) Some data are available for plane directional sources and finite rectangular targets.^(16,10,11) For calculations where the geometry approximates the conditions for which the buildup was obtained, the data may be applied directly. However, for more complex geometries the calculation technique requires the inclusion of some analytical form of the buildup factor in an expression for the differential photon flux and the subsequent integration of the resulting function. This is illustrated in the following equation for the dose at a point in a homogeneous scattering medium at a distance, r , from a point source or monoenergetic photons where S = number of photons emitted per second, E_0 = initial energy per photon, K = constant relating energy flux to dose, h_T = target thickness, and μ_0 = total linear absorption coefficient of target based on initial photon energy, E_0 .

$$B(\mu_o h_T, E_o) = \text{dose buildup factor} \quad (2.2.3)$$

$$D = \frac{KB(\mu_o h_T, E_o)}{4\pi r^2} E_o e^{-\mu_o h_T} \quad (2.2.4)$$

Buildup factors for various geometries may be obtained for number flux, energy flux, energy absorption, or dose.

2.3 Source Geometries

Point Source

The formulae used in calculating radiation flux from an extended source to a point in a target are derived by integration of the flux from a point source. This flux, or point "kernel," may be extended to sources and targets of far greater complexity by integrations over source and target.

Consider an isotropic point source of gamma photons in an infinite homogeneous medium emitting S photons per second of energy E_o . The photon flux at P may be expressed as

$$\phi = (S/4\pi)(e^{-\mu_o r}/r^2) \quad (2.3.1)$$

where μ_o is the total narrow beam absorption cross section of the target for photons of energy E_o .

Line Source

The radiation flux at point P_2 at distance a from the line of radiation may be expressed by the following equation:

$$\phi = (S_\ell/4\pi a) \left[\int_0^{\theta_1} e^{-\mu_o a \sec \theta} d\theta + \int_0^{\theta_2} e^{-\mu_o a \sec \theta} d\theta \right] \quad (2.3.2)$$

At P_1 located on the perpendicular bisector of the line (see Figure 2.3.1)

$$\phi = (S_s/2\pi a) \int_0^{\theta_3} e^{-\mu_0 a \sec \theta} d\theta \quad (2.3.3)$$

The secant integral, $\int_0^{\theta} e^{-b \sec \theta} d\theta$, cannot be determined analytically. Various workers⁽¹⁷⁾ have evaluated it numerically. (See the section in the appendix on mathematical functions for a complete discussion, together with graphs and tables of its value.)

Plane Source

The flux at a point P_1 at distance Z from an infinite plane isotropic source emitting S_a photons/cm²/sec may be expressed by the following equation (see Figure 2.3.2):

$$\phi = \frac{S_a}{2} \int_{\mu_0 Z}^{\infty} \frac{e^{-t}}{t} dt \quad (2.3.4)$$

The integral $\int_{\mu_0 Z}^{\infty} \frac{e^{-t}}{t} dt$, where t is a "dummy" variable of integration, is called the exponential integral, $E_1(\mu_0 Z)$, and is quite useful in shielding calculations. Values of the E_1 function are tabulated in the appendix. More extensive tables of E_1 have been compiled by NBS-WPA.⁽¹⁸⁾

Disc Source

At P_1 on the center line of a disc, of radius R , which is emitting S_a photons per unit area per unit time, the uncollided flux is

$$\phi = \frac{S_a}{2} \left[E_1(b_1) - E_1(b \sec \theta) \right] \quad (2.3.5)$$

where $b_1 = \mu_0 Z$ and $\theta = \tan^{-1}(R/Z)$ (see Figure 2.3.3).

Slab Source

The photon flux at P_1 , a distance Z in an infinite homogeneous target from a uniform isotropic slab source of thickness h is given by the following equation (see Figure 2.3.4):

$$\phi = (S_v/2\mu_s) \left[E_2(b_1) - E_2(b_1 + \mu_s h) \right] \quad (2.3.6)$$

FIG. 2.3.1

FIG. 2.3.3

INFINITE PLANE SOURCE

FIG. 2.3.2

INFINITE SLAB SOURCE

FIG. 2.3.4

where S_v = source strength per unit volume (photons/cm³/sec), μ_s = linear absorption cross section of source (cm⁻¹), and $b_1 = \mu_o Z$, where μ_o is the linear absorption cross section of the target and

$$E_2(b_1) = b_1 \int_{b_1}^{\infty} (e^{-t}/t^2) dt \quad (2.3.7)$$

For other source geometries such as cylindrical, trapezoidal, and spherical sources the reader is referred to the standard shielding manuals of Rockwell⁽¹³⁾ and Goldstein.⁽¹¹⁾

2.4 Conversion of Radiation Flux to Dose Rate

To facilitate the conversion of gamma photon flux to radiation dose, Table 2.4.1 was compiled. Values of energy absorption cross section, exposure dose in air, absorbed dose in air, and absorbed dose in water are listed for photon energies from 0.1 to 4 Mev based on a value of W of 34.0 ev/ion pair and Bureau of Standards' energy absorption cross sections.

2.5 Buildup

The buildup factor has been defined as the ratio of the actual gamma flux which would be calculated or measured accurately to that which would be calculated by using simple exponential attenuation based on the total narrow beam absorption coefficient. The above definition results in a number buildup factor. However, it is more usual to use the absorbed dose buildup factor or energy buildup factor.

The absorbed dose buildup factor can be defined as the ratio of the actual dose rate which one could experimentally measure at a point in a medium to the dose rate due to uncollided flux at that point which one could calculate by using simple exponential attenuation with the total absorption coefficient. The energy buildup factor is defined similarly.

Although a great deal is known about the interaction processes of gamma rays in matter, no decision has been reached to date regarding a completely satisfactory method of handling buildup. A popular method, that of J. J. Taylor,⁽¹²⁾ expresses the buildup from a point isotropic source in an infinite homogeneous medium as a sum of two exponentials

Table 2.4.1

Energy Absorption Cross Sections and Radiation Flux to Dose Rate Conversion Factors

Energy	Linear energy absorption cross section (air), cm^{-1} μ_a	R/hr per $\text{Mev}/\text{cm}^2\text{-sec}$ $\frac{D_r}{\Phi}$	Absorbed dose* in air, rads/hr per $\text{Mev}/\text{cm}^2\text{-sec}$ $\frac{D_{ab}}{\Phi}$	Absorption dose in H_2O , rads/hr per $\text{Mev}/\text{cm}^2\text{-sec}$
0.1	0.2994×10^4	0.1526×10^{-5}	0.1338×10^{-5}	0.1486×10^{-5}
0.2	0.3477	0.1772	0.1554	0.1726
0.4	0.3820	0.1947	0.1708	0.1896
0.5	0.3831	0.1953	0.1713	0.1902
0.6	0.3820	0.1947	0.1708	0.1896
0.663	0.3817	0.1945	0.1706	0.1895
0.8	0.3724	0.1903	0.1669	0.1854
1.0	0.3605	0.1838	0.1612	0.1790
1.25	0.3461	0.1764	0.1547	0.1716
1.5	0.3318	0.1690	0.1482	0.1646
2.0	0.3069	0.1564	0.1372	0.1523
3.0	0.2725	0.1389	0.1218	0.1353
4.0	0.2500	0.1274	0.1117	0.1241

*Based upon value of $W = 34.0$ ev per ion pair and density of H_2O equal to unity.

$$B = A_1 e^{-\alpha_1 \mu_0 t} + A_2 e^{-\alpha_2 \mu_0 t} \quad (2.5.1)$$

where μ_0 = total absorption coefficient for the initial gamma energy, E_0 ; B = the buildup at a depth $\mu_0 t$ mean free paths in the medium; $A_1 = 1 - A_2$; and α_1 and α_2 are functions of E_0 .

A table of the functions A_1 , α_1 , and α_2 is given in Appendix I for absorbed dose buildup in several different media.

The most readily available and reliable data on buildup are contained in the report by Goldstein and Wilkins.⁽¹⁰⁾ The results are based on calculations of the gamma transport equation for point and plane sources in an infinite homogeneous medium by using the moments method developed by Fano and Spencer.⁽¹⁹⁾ The results for various target media such as water, aluminum, iron, tin, tungsten, lead, and uranium are tabulated and plotted in Appendix I. The important property of the buildup is that it is a function of photon energy, atomic number, and target depth. Since it is a measure of the effect of multiple Compton scattering, it decreases with increase in energy, and increases with penetration. At photon energies around 1 Mev the buildup decreases with increase in the atomic number of the target because of the shift in the absorption process in favor of the photoelectric effect. It is important to note that most data are available for only point or plane sources in infinite homogeneous targets.

2.6 Incorporation of Buildup Into Equations for Uncollided Flux

The methods of incorporating buildup into calculations of dose may be divided into two groups. In Group A for (1) a point source in an infinite medium or (2) an infinite plane isotropic source in an infinite medium, the appropriate buildup factors of Goldstein and Wilkins⁽¹⁰⁾ may be incorporated directly by multiplying the uncollided dose rate at the required depth by the buildup at that point. Buildup factors are tabulated for point sources, plane monodirectional sources, and plane angular sources in such media as air, iron, lead, tin, and uranium in NYO-3075.⁽¹⁰⁾

In Group B, where one wants the dose rate at a point from an extended source in a finite or infinite medium, the method requires the use of the point source buildup factor in the mathematical expression of the dose rate from a differential area of the source. The total effect of buildup is thus obtained mathematically by integrating over the source area. Because of the exponential attenuation of radiation the use of Taylor's method, where a sum of weighted exponentials is used to express the buildup, does not result in a change in the mathematical function within the integral. For example, for an infinite plane source, the uncollided radiation flux at some depth b (in mean free paths) in the target was given by Eq. (2.3.4) to be:

$$\phi = (S_a/2) \int_b^{\infty} (e^{-t}/t) dt = (S_a/2) E_1(b) \quad (2.3.4)$$

The uncollided dose rate may be obtained by multiplying the energy flux by the appropriate conversion factor K_1 . Thus,

$$D_{\text{uncollided}} = K_1 E_o (S_a/2) \int_b^{\infty} (e^{-t}/t) dt = K_1 E_o (S_a/2) E_1(b) \quad (2.6.1)$$

Since

$$B(E_o, b) = A_1 e^{-\alpha_1 t} + (1-A_1) e^{-\alpha_2 t}$$

it may be shown that the total dose rate

$$D_{\text{total}} = K_1 E_o (S_a/2) \left[\int_{b(1+\alpha_1)}^{\infty} A_1 (e^{-t}/t) dt + \int_{b(1+\alpha_2)}^{\infty} (1-A_1) (e^{-t}/t) dt \right] \quad (2.6.2)$$

$$= K_1 E_o (S_a/2) \left[A_1 E_1[b(1+\alpha_1)] + (1-A_1) E_1[b(1+\alpha_2)] \right] \quad (2.6.3)$$

Another method such as the use of linear buildup where $B = (1+\alpha_3 \mu t)$ would result in the following equation:

$$D_{\text{total}} = K_1 E_o (S_a/2) \left[E_1(b) + \alpha_3 e^{-b} \right] \quad (2.6.4)$$

The number of other ways in which buildup may be incorporated in expressions for dose rate are too numerous to mention here. The reader is referred to Goldstein, or to a paper by Hubbell⁽¹⁴⁾ which summarizes and analyzes the methods used and their accuracy. It has been found by the authors that buildup equations containing two or three exponential terms obtained from Goldstein's data^(10,11) by curve stripping have been the most useful technique.

3. COMPUTATIONAL PROCEDURES

3.1 General Approach

Up to this point we have been concerned with the radiation physics involved in the determination of absorbed dose in a medium from gamma sources ranging in complexity from point source geometry to slab source geometry. The discussion, however, has been confined to idealized geometries in which, for the case of the slab source, the dose was calculated along the center line of the slab in an infinite homogeneous medium. In irradiator calculations, however, the designer is invariably required to determine the dose distribution at any point in space relative to an extended source in a system where both the source activity and the target medium may or may not be homogeneous. Further complications are introduced when one considers that the target (see Figures 3.1.1 and 3.1.2) is always finite in size and is usually bounded by air or some other medium whose density is different from that of the target.

An irradiator usually utilizes an extended slab source which is always finite in thickness and is generally encapsulated in stainless steel (A_b , Figures 3.1.1 and 3.1.2). A source-to-target air gap, G_A , is often necessary to allow for mechanical clearance especially in irradiators in which the target motion is in a direction parallel to the plane of the source.

Since dose-uniformity specifications usually cannot be met when the source area and the target area are equal, source overlap is used as a means of improving the uniformity ratio.

From the foregoing discussion it should become apparent that dose determination to any degree of accuracy in even a simple irradiator system does not lend itself to simple calculational techniques.

The following is a discussion of a method developed as a recommended approach for obtaining the dose distribution in irradiator design problems for real source-to-target

FIG. 3.1.1

FIG. 3.1.2

configurations.

The method involves the following steps:

(1) The dose is determined under static conditions at all important target points assuming that the source is a plane and is imbedded in an infinite homogeneous density-one medium. This assumption permits us to temporarily neglect the complicating factors of the effects of boundaries, air gap, self-absorption, absorber thickness, target density, source and target overlap, source and target inhomogeneities, and target motion. These effects are taken into consideration as a series of correction factors.

Two techniques for determining the infinite medium dose will be described. One is an analytical approach for which a computer program has been written and which is included in the appendix. The other is an empirical approach which is based upon experimental data (see Paragraph 3.2).

(2) The next step is to correct the infinite medium dose for any nonuniform distribution of source activity (see Paragraph 3.7).

(3) Corrections to the infinite medium dose are then made for target densities other than unity and for variations in target density. While the analytical procedure handles these problems directly, the empirical technique, which is based upon experimental data for unit density targets, requires the use of modeling or scale-adjustment techniques. Scaling techniques will be discussed in full detail in a later section of this report.

(4) The dose distribution is further corrected for the effects of source-to-target air gap and absorber thickness, taking into account actual source thickness and density, and the thickness and densities of all absorbers between the source and the target. This is accomplished through the use of experimental data in conjunction with scaling techniques, or computer program FUDGE 4 (see Paragraph 3.4).

(5) Next, a correction is applied to the selected normalized doses for finite boundaries.

This correction is due to the scattered radiation which escapes from the interface between the dense material of the target and the surrounding air, and is thus lost from the target. This loss is greatest at the actual boundary and the effect extends back approximately one mean free path inside the target. Corrections for water-air boundaries have been obtained experimentally for the single boundary case. Approximation methods based upon these data will be presented in this report (see Section 3.5).

(6) Vertical dose distribution is adjusted in the design by source overlap or by specific activity augmentation. Both experimental data and analytical methods are available to make these corrections (see Sections 3.6 and 3.7).

(7) The effect of target motion on the dose distribution is accounted for by the use of static equivalent systems. This is discussed in Section 4.

3.2 Dose in Infinite Medium

The following section describes in detail the two previously mentioned techniques of determining the dose distribution in infinite homogeneous media, i.e., (1) the calculational method which is based upon the IBM computer program FUDGE 3A, and (2) the superposition method which is based upon experimental data obtained for cobalt-60 and cesium-137 and unit density targets. The calculational method may be used for any size rectangular plane isotropic source of any photon emitter in an infinite homogeneous medium of any density. The superposition method, however, is limited to cobalt-60 and cesium-137 sources. For targets of density other than 1.0 the technique of scaling or modeling must be used. Method 2 is most useful where it may be applied directly, i.e., for source sizes and targets for which data are available. The preferential selection of either method 1 or 2 depends upon the requirements of the individual design and the availability of the IBM 7090 computer.

FUDGE 3A

FUDGE 3A calculates the specific dose rate distribution from any finite plane isotropic

gamma source in an infinite homogeneous target at any point in the target, based on a modified 12-point Gauss quadrature integration over the source plane. Buildup due to multiple scattering may be included as the sum of up to three exponentials.

Consider a point P in a homogeneous medium at a distance Z from a plane isotropic gamma source having any generalized shape. For this case let P', the projection of P on the plane, fall anywhere within the boundaries of the source as in Figure 3.2.1. Let S_a = source strength per unit area in photons/cm²/sec. The photon flux at P, using P' as the center of the coordinate system, will then be given by:

$$\phi = \frac{S_a \int dS \exp[-\mu(\rho^2 + Z^2)^{\frac{1}{2}}]}{4\pi(\rho^2 + Z^2)} \quad (3.2.1)$$

Substitution of $dS = \rho d\rho d\theta$ plus conversion to dimensionless units results in:

$$\phi = (S_a/2)[E_1(\mu Z)] - (S_a/4\pi) \int_0^{2\pi} E_1[\mu(Z^2 + \rho_{\max}^2)^{\frac{1}{2}}] d\theta \quad (3.2.2)$$

where $\rho_{\max}$ is an arbitrary function of θ .

The body of the program involves the evaluation of $\rho_{\max}$ and the integration of Eq. (3.2.2).

Appendix V contains a FORTRAN listing of FUDGE 3A and detailed instructions for its use.

Plane Source Superposition Method

The source superposition method was designed to provide a simple, concise, and accurate technique for the synthesis of plane source dose distribution information. The method enables one to obtain information for plane sources imbedded in an infinite homogeneous medium. The basis for the system is largely experimental data and extensions of the data obtained from experimentally and analytically determined empirical relationships. The data have been normalized to zero source thickness and are presented in the form of the dose rate

FIG. 3.2.1

contribution at a point from a small source element as it is placed at various positions in a plane throughout the medium. By summing up or superimposing the dose-rate contributions from the source element in its various positions it is possible to generate dose distribution information for plane sources of any size and shape at any point in the medium.

In Figure 3.2.2, a 10×10-in. source element is shown in numerous positions, lettered "A" through "S." The source positions are arranged symmetrically about the point "X" which is located at the corner of source position "A." The dose rate at "X" from any combination of these source positions can be obtained by simply summing up their contributions to the dose rate at point "X."

For example, the specific dose rate along the central axis of a 20×20-in. plane source at a distance Z from the plane is four times the contribution at point "X" from source position "A" at distance Z (see Figure 3.2.3). Likewise, the specific dose rates at the corner and the edge of a 20×20-in. plane source would be found by adding the contributions from the following positions:

$$\text{Corner dose} = A + (2 \times B) + C \quad (\text{Figure 3.2.4})$$

$$\text{Edge dose} = 2(A + B) \quad (\text{Figure 3.2.5})$$

The specific dose rates at the center, the corner, and the edge of a 20×60-in. slab source would similarly be:

$$\text{Center line dose} = 3 \times (A + B + D)$$

$$\text{Corner dose} = A + 2B + C + D + E + F + G + I + J + N + M$$

$$\text{Edge dose} = 2 \times (A + 2B + C + D + E)$$

(The edge referred to here is the long edge.) In Tables 3.2.1 and 3.2.2 the specific rates from 10×10-in. source elements in positions "A" through "S" in an infinite water medium at distances from the source plane ranging from 1 to 12 in. are tabulated for both cobalt-60 and cesium-137 sources. The target-to-source distance is the perpendicular distance to the

S	R	Q	O	N	M
R	P	N	L	J	I
Q	N	K	H	G	F
O	L	H	F	E	D
N	J	G	E	C	B
M	I	F	D	B	A

FIG. 3.2.2

FIG. 3.2.3

FIG. 3.2.4

FIG. 3.2.5

Table 3.2.1

Dose Rate From 10×10-in. Plane Cobalt-60 Source at Various Plane Positions

Specific dose rate, rads/hr per C/cm ²										
d*	A**	B	C	D	E	F	G	H	I	J
1	4.558×10 ⁴	2.207×10 ³	5.954×10 ²	2.528×10 ²	1.170×10 ²	4.250×10 ¹	2.850×10 ¹	1.500×10 ¹	1.120×10 ¹	8.650×10 ⁰
2	3.117	2.145	5.69	2.508	1.135	4.150	2.793	1.475	1.102	8.523
3	2.296	2.048	5.500	2.500	1.099	4.050	2.736	1.449	1.085	8.396
4	1.744	1.930	5.312	2.400	1.064	3.950	2.678	1.424	1.067	8.268
5	1.347	1.790	5.050	2.295	1.029	3.850	2.621	1.398	1.049	8.141
6	1.052	1.640	4.720	2.169	9.932×10 ¹	3.750	2.564	1.373	1.031	8.014
7	8.272×10 ³	1.494	4.400	1.940	9.579	3.650	2.506	1.347	1.014	7.886
8	6.540	1.324	4.130	1.880	9.225	3.550	2.449	1.322	9.959×10 ⁰	7.759
9	5.200	1.176	3.750	1.810	8.871	3.450	2.392	1.296	9.782	7.632
10	4.149	1.033	3.450	1.709	8.518	3.350	2.335	1.271	9.605	7.505
11	3.320	9.05 ×10 ²	3.090	1.570	8.165	3.250	2.277	1.246	9.427	7.377
12	2.667	7.855	2.810	1.428	7.800	3.150	2.220	1.220	9.250	7.250
d*	K	L	M	N	O	P	Q	R	S	
1	6.800×10 ⁰	5.350×10 ⁰	3.530×10 ⁰	2.870×10 ⁰	2.000×10 ⁰	1.420×10 ⁰	1.180×10 ⁰	6.500×10 ⁻¹	3.450×10 ⁻¹	
2	6.705	5.209	3.486	2.836	1.980	1.406	1.169	6.441	3.405	
3	6.609	5.068	3.443	2.803	1.960	1.391	1.158	6.382	3.359	
4	6.514	4.927	3.399	2.769	1.940	1.376	1.147	6.323	3.314	
5	6.418	4.786	3.356	2.736	1.920	1.362	1.136	6.264	3.268	
6	6.323	4.646	3.312	2.702	1.900	1.347	1.255	6.205	3.223	
7	6.227	4.505	3.268	2.668	1.880	1.333	1.115	6.146	3.177	
8	6.132	4.364	3.225	2.635	1.860	1.318	1.104	6.086	3.132	
9	6.036	4.223	3.181	2.601	1.840	1.304	1.093	6.027	3.086	
10	5.941	4.082	3.137	2.567	1.820	1.289	1.082	5.968	3.041	
11	5.846	3.941	3.094	2.534	1.800	1.275	1.071	5.909	3.000	
12	5.750	3.800	3.050	2.500	1.780	1.260	1.060	5.850	2.950	

*d = distance from target point to source plane, in.

**Positions A to S refer to source positions as shown in Figure 3.2.2.

Table 3.2.2

Dose Rate From 10×10-in. Plane Cesium-137 Source at Various Plane Positions

Specific dose rate, rads/hr per C/cm ²										
d*	A**	B	C	D	E	F	G	H	I	J
1	9.850×10 ³	4.225×10 ²	9.950×10 ¹	4.875×10 ¹	1.630×10 ¹	7.500×10 ⁰	2.840×10 ⁰	1.160×10 ⁰	7.610×10 ⁻¹	5.250×10 ⁻¹
2	6.717	4.025	9.500	4.608	1.574	7.123	2.766	1.133	7.446	5.140
3	4.933	3.830	9.000	4.341	1.517	6.746	2.691	1.103	7.283	5.030
4	3.727	3.565	8.575	4.075	1.461	6.368	2.616	1.073	7.119	4.920
5	2.854	3.298	8.025	3.808	1.405	5.991	2.542	1.051	6.956	4.810
6	2.204	2.983	7.525	3.541	1.348	5.614	2.467	1.024	6.792	4.700
7	1.710	2.666	7.025	3.274	1.292	5.237	2.393	9.964×10 ⁻¹	6.628	4.590
8	1.331	2.353	6.550	3.007	1.236	4.859	2.318	9.691	6.465	4.480
9	1.040	2.065	6.000	2.741	1.179	4.482	2.244	9.418	6.301	4.370
10	8.148×10 ²	1.795	5.530	2.474	1.123	4.105	2.169	9.146	6.137	4.260
11	6.402	1.528	5.092	2.207	1.066	3.728	2.095	8.873	5.974	4.150
12	5.044	1.310	4.365	1.940	1.010	3.350	2.020	8.600	5.800	4.040
d*	K	L	M	N	O	P	Q	R	S	
1	3.620×10 ⁻¹	2.530×10 ⁻¹	1.330×10 ⁻¹	9.200×10 ⁻²	5.100×10 ⁻²	2.940×10 ⁻²	2.250×10 ⁻²	8.350×10 ⁻³	2.700×10 ⁻³	
2	3.548	2.482	1.306	9.030	5.014	2.893	2.214	8.232	2.666	
3	3.476	2.434	1.281	8.860	4.927	2.846	2.177	8.114	2.633	
4	3.405	2.386	1.256	8.696	4.841	2.798	2.141	7.996	2.599	
5	3.333	2.337	1.232	8.527	4.755	2.751	2.105	7.877	2.566	
6	3.261	2.289	1.207	8.359	4.668	2.704	2.068	7.759	2.532	
7	3.189	2.241	1.183	8.191	4.582	2.656	2.032	7.641	2.498	
8	3.117	2.193	1.158	8.023	4.496	2.609	1.996	7.523	2.465	
9	3.046	2.145	1.134	7.855	4.409	2.562	1.959	7.405	2.431	
10	2.974	2.096	1.109	7.686	4.323	2.515	1.923	7.286	2.397	
11	2.902	2.048	1.085	7.518	4.236	2.467	1.886	7.168	2.364	
12	2.830	2.000	1.060	7.350	4.150	2.420	1.850	7.050	2.330	

*d = distance from target point to source plane, in.

**Positions A to S refer to source positions as shown in Figure 3.2.2.

plane from the point "X" as shown in Figure 3.2.3. The units of the dose rate as shown are rads/hr per C/cm².

By using the data in Tables 3.2.1 and 3.2.2, the method is demonstrated below with dose distribution calculations for 20×20-in. and 20×60-in. plane sources.

Example 1

20×20-in. cobalt-60 plane source, depth = 3 in. of water.

$$\text{Specific dose rate}_{(\text{center})} = 4 \times A = 4 \times 2.296 \times 10^4 = 9.194 \times 10^4 \text{ rads/hr per C/cm}^2$$

$$\begin{aligned} \text{Specific dose rate}_{(\text{corner})} &= A + (2 \times B) + C = 2.296 \times 10^4 + 2 \times 2.048 \times 10^3 + 5.5 \times 10^2 \\ &= 2.76 \times 10^4 \text{ rads/hr per C/cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Specific dose rate}_{(\text{edge})} &= 2 \times (A + B) = 2 \times (2.296 + 0.2048 \times 10^4) \\ &= 5.002 \times 10^4 \text{ rads/hr per C/cm}^2 \text{ at the center of the edge} \end{aligned}$$

Example 2

20×60-in. cesium-137 plane source depth = 3 in. of water

$$\begin{aligned} \text{Dose rate}_{(\text{center})} &= 4 \times (A + B + D) = 4 \times (4.933 \times 10^3 + 3.83 \times 10^2 + 4.341 \times 10^1) \\ &= 2.144 \times 10^4 \text{ rads/hr per C/cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Dose rate}_{(\text{corner})} &= A + 2 \times B + C + D + E + F + G + I + J + N + M \\ &= 4.933 \times 10^3 + 2 \times 3.83 \times 10^2 + 9.0 \times 10^1 + 4.341 \times 10^1 + 1.517 \times 10^1 \\ &\quad + 6.746 \times 10^0 + 2.691 \times 10^0 + 7.283 \times 10^{-1} + 5.03 \times 10^{-1} + 8.86 \times 10^{-2} + 1.281 \times 10^{-1} \\ &= 0.586 \times 10^4 \text{ rads/hr per C/cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Dose rate}_{(\text{edge})} &= 2 \times (A + 2 \times B + C + D + E) \\ &= 2 \times 4.933 \times 10^3 + 2 \times 3.83 \times 10^2 + 9.0 \times 10^1 + 4.341 \times 10^1 + 1.517 \times 10^1 \\ &= 1.17 \times 10^4 \text{ rads/hr per C/cm}^2 \text{ at the center of the long edge} \end{aligned}$$

It should be noted that in superimposing the source element contributions it is

necessary to use and carry through the six place numbers as given in the tables while the computation is in progress. When the computation is completed the doses may be rounded off to a reasonable number of significant figures. When the method is used in this manner it is possible to reconstruct the original experimental data to within $\pm 1\%$. The original data are accurate to within $\pm 3\%$. The data as they appear here have been normalized to zero source thickness and therefore do not take self-absorption into consideration. The effect of self-absorption will be dealt with in a later section as a correction factor.

Superposition data using 2.5 and 5-in. source elements have been made available in Appendix IV for both cobalt-60 and cesium-137.

3.3 Density Corrections of Infinite Medium Dose

In Paragraph 3.1 it was pointed out that the computer program handled materials of densities other than unity directly. Since the vast majority of experimental data are available for water targets of density one, the method of empirically evaluating dose rates and dose-rate distributions for material densities other than 1.0, the superposition method described above can only be used in conjunction with a scaling or modeling technique.

The basis for this procedure has been explained by various workers such as Dove⁽²⁰⁾ and Johanson.⁽²¹⁾ Briefly, it depends upon transferring from material of one density to material of another density while maintaining constant attenuation, multiple scattering, and solid angle. For example, to scale from medium 1 (water, $\rho_1 = 1.0 \text{ g/cm}^3$) to medium 2 (material density, $\rho_2 = 4.0 \text{ g/cm}^3$) and assuming that the mass absorption cross sections of both materials are the same, it is obvious that in order to obtain the same dose from a point source in the two media that

$$\frac{S_1}{4\pi r_1^2} \exp[-(\mu/\rho)_1 \rho_1 r_1] = \frac{S_2}{4\pi r_2^2} \exp[-(\mu/\rho)_1 \rho_2 r_2] \quad (3.3.1)$$

if,

$$\frac{S_1}{4\pi r_1^2} = \frac{S_2}{4\pi r_2^2}, \text{ such that } \frac{S_1}{S_2} = \frac{r_1^2}{r_2^2} \quad (3.3.2)$$

Then,

$$\rho_1 r_1 = \rho_2 r_2 \quad (3.3.3)$$

$$\frac{\rho_1}{\rho_2} = \frac{r_2}{r_1} \quad (3.3.4)$$

and

$$S_1 = S_2 \frac{\rho_2^2}{\rho_1^2} \quad (3.3.5)$$

For a disc source of radius R_1 at distance Z from the disc the flux at a point P_1 in medium 1 is

$$\phi = \frac{S_{A1}}{2} \left(E_1[(\mu/\rho)_1 \rho_1 Z_1] - E_1[(\mu/\rho)_1 \rho_1 Z_1 \sec \theta] \right) \quad (3.3.6)$$

In medium 2, when $(\mu/\rho)_1 = (\mu/\rho)_2$ and $\theta_1 = \theta_2$,

$$\phi_2 = \frac{S_{A2}}{2} \left(E_1[(\mu/\rho)_2 \rho_1 Z_2] - E_1[(\mu/\rho)_1 \rho_2 Z_2 \sec \theta_1] \right) \quad (3.3.7)$$

If $S_{A1} = S_{A2}$ and one desires to maintain the dose rate constant, that is, $\phi_1 = \phi_2$, then Z_2 , the distance from point P_2 to the source, is given by

$$Z_2 = \frac{Z_1 \rho_1}{\rho_2} \quad (3.3.8)$$

and

$$\frac{R_1}{R_2} = \frac{Z_1}{Z_2} = \frac{\rho_2}{\rho_1} \quad (3.3.9)$$

It can thus be seen that

$$A_1 = A_2 \frac{R_2^2}{R_1^2} \quad (3.3.10)$$

It may be shown that these principles are valid for all other geometries. The basic principle is that one must maintain constant solid angle and constant attenuation. The method assumes that the multiple scattering in the two media is equal. This is not an unfair assumption for the materials and material thicknesses usually used in irradiator design.

In applying the principle to plane sources, it can be stated, for example, that the dose rate 6 in. away from the center of a 10×10-in. plane source in water density $\rho_1 = 1 \text{ g/cm}^3$ is equivalent to the dose rate 12 in. away from a 20×20-in. source of the same activity per unit area in a medium whose density $\rho_2 = 0.5 \text{ g/cm}^3$. The total activity required for case 2 would be four times the activity of case 1. In this example the scaling factor is $(\rho_1/\rho_2) = 1/0.5 = 2$.

To obtain a dose distribution in a medium whose density is not unity, one would make the determination by using an appropriately scaled model and the superposition data for unit density material.

3.4 Absorber Thickness and Air Gap Corrections

Previously we have been concerned with sources imbedded in infinite homogeneous targets with no inhomogeneities in the target material. This section treats targets which are composed of layers of different materials which in irradiators could comprise the air gap or air space between the target and the source, the source encapsulating material, and the source material itself. In both the calculational and the empirical approach the correction for effects of absorber thickness and air gap are handled as a one-step correction.

Calculational Method

The calculational method of handling problems involving absorber thickness and air gap is carried out by using FUDGE 4. This computer program is basically identical to FUDGE 3A; however, it sums up the total absorber thickness to a point in the target from the source plane and calculates a fictitious mean free path or linear absorption coefficient based on the total linear distance. This calculation is done for all depths of interest in the

target.

The method may be best explained by the following illustrative example.

Let us assume that one has a 0.050-in. thick cobalt-60 source, doubly encapsulated with 0.080 in. of stainless steel. There is a 0.100-in. thick steel absorber between the clad source and the target surface, with a 2.00-in. air gap total between the source and target.

The total mean free path to a point 3.00 in. from the target surface may then be calculated:

$$\begin{aligned}\sum_{i=1}^{i=n} \mu_i t_i &= (\mu t)_{\text{Co}} + (\mu t)_{\text{steel}} + (\mu t)_{\text{absorber}} + (\mu t)_{\text{target}} \\ &= \frac{0.050}{2} \times 1.216 + 0.080 \times 1.067 + 0.100 \times 1.067 + \\ &\quad 3.00 \times 0.163 \\ &= 0.7115\end{aligned}\tag{3.4.1}$$

The total linear distance to the point in the target is:

$$\Sigma t = \frac{0.050}{2} + 0.080 + 0.100 + 2.00 + 3.00 = 5.205 \text{ in.}$$

The effective linear absorption coefficient is therefore,

$$\bar{\mu} = \frac{\Sigma \mu t}{\Sigma t} = 0.1367 \text{ in.}^{-1}$$

The fictitious or effective mean free path, which is the reciprocal of the above, is

$$\bar{\lambda} = \frac{\Sigma t}{\Sigma \mu t} = 7.315 \text{ in.}$$

For the case illustrated the dose rate is calculated by the program to all points at a depth in the target of 3.00 in. assuming a uniform target interposed between the source plane and target plane with a total absorption cross section of 0.1367 in.^{-1} .

The target depth is then incremented as desired and a new value of $\bar{\mu}$ and $\bar{\lambda}$ calculated

and used for calculations in this plane.

A FORTRAN listing of FUDGE 4 is included in Appendix V together with the required input data and input format.

Empirical Methods

Equations are discussed for relating dose rate at a point in a target with interposed absorbing material and air gap to an equivalent water system.

The basic data presented for use with the superposition method are for unit specific gravity (water) material in an infinite medium. In order to use these data directly, any practical system with absorbers and air gap and target material of specific gravity other than unity should be suitably scaled to the unit specific gravity case.

As was previously stated the principles of scaling and their justifications are adequately discussed by Dove⁽²⁰⁾ and Johanson,⁽²¹⁾ and to some extent in Paragraph 3.3.

These principles are applied here to relate an actual source with self-absorption, additional absorbing material, and an air gap to an equivalent water system to permit use of the plane source superposition data presented above.

To obtain equivalence of dose rate at a point in a target for two systems differing in density, one must preserve the total attenuation and solid angle with reference to the source. If the atomic numbers of the media are not greatly different, equivalence of dose rate should be obtained.

The principles are best illustrated by an example. In the cross-sectional views shown in Figures 3.4.1 to 3.4.4, plane angles are considered rather than the actual solid angles for reasons of simplicity.

Figure 3.4.1 shows the cross section of an actual system consisting of absorbers A and B, air gap C, and target D. Figure 3.4.2 shows an arrangement which preserves the angle θ and in which the total attenuation is preserved but the absorbers A and B have been replaced

FIG. 3.4.1

FIG. 3.4.2

FIG. 3.4.3

FIG. 3.4.4

with their water equivalent. Subscript E indicates the water equivalent case. For this case a new source S_{E1} , with a greater height than S , is required to give the same dose rate at X as in Figure 3.4.1. In Figure 3.4.3 the target material (specific gravity assumed < 1) has been replaced by its water equivalent. This results in a new source S_{E2} , shorter than S_{E1} . In Figure 3.4.4 the attenuation in air is assumed negligible, and the air gap has been eliminated keeping total attenuation constant and preserving the angle θ . The new source, S_{E3} , is still shorter than S_{E2} . One can progress from Figure 3.4.1 directly to Figure 3.4.4 by using the relations below.

To maintain total attenuation constant

$$\Sigma \mu_i x_i = \text{constant} \quad (3.4.2)$$

$$\mu_A x_A + \mu_{\text{air}} x_{\text{air}} + \mu_T x_T = \mu_W x_{WE} \quad (3.4.3)$$

$$\frac{\mu_A x_A}{\mu_W} + \frac{\mu_{\text{air}}}{\mu_W} x_{\text{air}} + \frac{\mu_T}{\mu_W} x_T = x_{WE} \quad (3.4.4)$$

The second term in Eq. (3.4.4) usually can be neglected.

To preserve the total solid angle, the plane angle θ is to remain the same. (The other linear dimension is discussed briefly below.) Let

$$x_o = x_A + x_{\text{air}} + x_T \quad (3.4.5)$$

$$\tan \theta = H_S / 2x_o \quad (3.4.6)$$

and for the final configuration

$$\tan \theta = H_{SE} / 2x_{WE} \quad (3.4.7)$$

$$\frac{H_{SE}}{H_S} = \frac{x_{WE}}{x_o} \quad (3.4.8)$$

where H_{SE} = height of equivalent source, and H_S = height of actual source.

It can be seen that changing point X to Y (Figure 3.4.1) does not change the value for H_{SE} . Thus, any point along XY or its extension has the same equivalent source height given by Eq. (3.4.8). However, the final situation in Figure 3.4.4 does not give the complete picture since only one angle θ has been considered. Consideration of other angles would show that in the final configuration the other linear dimensions are changed in the same ratio as that for the source height. Thus, for the given example, the target height of the equivalent system is the same as that of the source shown in Figure 3.4.4 by lines AC and BD.

In general,

$$\frac{H_{TE}}{H_T} = \frac{X_{WE}}{x_o} \quad (3.4.9)$$

For a static plane source a similar relation would hold for the other linear dimension. Thus, in using the static equivalent approach the actual source length should be evaluated from the scaled version by the relation

$$\frac{L_{SE}}{L_S} = \frac{X_{WE}}{x_o} \quad (3.4.10)$$

It will be noticed from Eq. (3.4.4) that for materials with $\mu_i > \mu_W$ the equivalent distance is greater than the actual distance. The reverse is true for materials with $\mu_i < \mu_W$. Thus, the net effect of adding absorbers and air gap may nearly cancel out the effect on source height. Thus, high density absorbers with a water equivalent thickness of 1 in. and a 1-in. air gap would have nearly equal and opposite effects on the equivalent source height, and if the target material were of unit density material the equivalent and actual sources would be the same size. In many instances this is close to the actual case and thus no correction for source size due to air gap and absorbers would be required. In the above case the actual air gap would be given by:

$$x_{air} = x_A \left(\frac{\mu_A}{\mu_W} - 1 \right) \quad (3.4.11)$$

Of course, the total attenuation is not changed for the above.

Note that for position Z in Figure 3.4.1 x_T is changed and the equivalent source height for this position is not the same as at X. Relation (3.4.8) may be written

$$\frac{H_{SE}}{H_S} = \frac{(X_{WE})_{ab+air} + X_{WT}}{(X)_{ab+air} + X_T} \quad (3.4.12)$$

If the target material is equivalent to water then $X_{WT} = X_T$. If, as mentioned above, the effect of absorber cancels out the effect of the air gap, then $(X_{WE})_{ab+air} = (X)_{ab+air}$, and for a target of unit density the actual and equivalent source heights are the same for all positions. This is not true in general for any other case. For example, consider a target material of density 0.5 and let the effect of absorbers and air gap be equal and opposite, and the net distance $(X)_{ab+air} = 1$ in. and $X_T = 4$ in. Then,

$$\frac{H_{SE}}{H_S} = \frac{1+2}{1+4} = \frac{3}{5} = 0.6$$

For a value of $X_T = 2$ in.,

$$\frac{H_{SE}}{H_S} = \frac{1+1}{1+2} = \frac{2}{3} = 0.67$$

Note that if no air gap or absorber were present, $H_{SE}/H_S = 0.5$ for all cases. (See the Appendix for additional information on absorber and air gap corrections.)

3.5 Boundary Corrections

The accurate calculation of dose distributions in a finite target depends upon the knowledge of the buildup at or near the boundaries of the target. Currently the information available is inadequate for the accurate handling of all finite medium problems. The data at hand are primarily applicable to either point source or plane monodirectional sources and are limited to the case of one boundary. For a more complete discussion of finite buildup factors and dose distributions near and at the interface between two media of different densities the reader is referred to Berger and Doggett. (22)

In this report an approximation method is presented which is based upon one-boundary point-source data. These data are given in Table 3.5.1 and can also be calculated from the following empirical equation.

$$\frac{D(b_1)}{D(\infty)} = 1 + 1.1 e^{-0.8b_1} [0.115 e^{-0.16b} - 0.1] \quad (3.5.1)$$

When the point source correction factors in Table 3.5.1 are used for plane source corrections, the following procedure should be followed (see Figure 3.5.1).

(1) Find the effective distance from the plane source to the point in question as per the next section, and call this b .

(2) Find the distance from the point to the interface by measuring along the perpendicular (line 1, Figure 3.5.2) to the source plane if the interface (plane 1) is parallel to the source plane, or along the perpendicular (plane 2). Call this distance b_1 .

By using the values of b and b_1 as determined above, find the correction factor in Table 3.5.1.

The Effective Distance to a Plane Source

The effective distance to a plane source is the distance from the target point in question at which the infinite medium dose from a hypothetical point source containing the same number of curies as the plane source is equal to the infinite medium dose from the plane source.

Calculation of this effective distance is carried out using the superposition method and the curves for the dose, distribution from a point source in an infinite water medium which are presented in Figures 3.5.3 a, b, and c.

As an example consider the effective distance to a 20×20-in. plane source from a point along the center line of the source at a distance of 10 in. in an infinite water medium. (See Figure 3.2.3 in the section on superposition methods.) The dose rate as determined with the superposition method is 1.66×10^4 rads/hr per C/cm^2 . Since the activity per unit area of the source is $1 C/cm^2$, the 20×20-in. plane source contains 2.58×10^3 C. A point source

Table 3.5.1

Air-Water Boundary Correction Factors for Point Cobalt-60 Source*

b, source to detector distance, in.	b ₁ , detector to boundary distance, in.							
	0.0	0.25	0.50	1.00	2.00	3.00	4.00	6.00
0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1	0.9905	0.9923	0.9933	0.9953	0.9979	0.9992	0.9997	1.00
2	0.9811	0.9845	0.9868	0.9906	0.9957	0.9983	0.9994	1.00
3	0.9720	0.9767	0.9802	0.9860	0.9936	0.9974	0.9992	1.00
4	0.9632	0.9692	0.9736	0.9814	0.9915	0.9965	0.9990	1.00
5	0.9544	0.9616	0.9673	0.9772	0.9893	0.9957	0.9989	1.00
6	0.9462	0.9542	0.9608	0.9730	0.9875	0.9947	0.9988	1.00
7	0.9385	0.9472	0.9548	0.9692	0.9858	0.9939	0.9987	1.00
8	0.9315	0.9408	0.9492	0.9658	0.9844	0.9930	0.9984	1.00
9	0.9250	0.9355	0.9444	0.9629	0.9830	0.9924	0.9983	1.00
10	0.9197	0.9318	0.9414	0.9606	0.9819	0.9920	0.9981	1.00
11	0.9161	0.9292	0.9393	0.9589	0.9809	0.9917	0.9980	1.00
12	0.9139	0.9273	0.9373	0.9580	0.9800	0.9914	0.9979	1.00
13	0.9124	0.9260	0.9367	0.9578	0.9795	0.9911	0.9978	1.00
14	0.9113	0.9252	0.9358	0.9575	0.9790	0.9908	0.9976	1.00
15	0.9107	0.9247	0.9353	0.9574	0.9788	0.9905	0.9975	1.00
16	0.9103	0.9243	0.9349	0.9574	0.9786	0.9904	0.9974	1.00
17	0.9100	0.9238	0.9346	0.9573	0.9785	0.9903	0.9973	1.00

*Correction factor, $F_b = D_{(b')}/D_{(\infty)}$, dose rate in finite medium/dose rate in infinite medium.

FIG. 3.5.1

FIG. 3.5.2

DOSE DISTRIBUTION IN RADS/hr FROM A ONE CURIE
COBALT-60 POINT SOURCE IN AN INFINITE WATER
MEDIUM

FIGURE 3.5.3a

DOSE DISTRIBUTION IN RADS/hr FROM A ONE CURIE
COBALT-60 POINT SOURCE IN AN INFINITE WATER

FIGURE 3.5.3b & c

geometry must be found in which the dose rate from a 2.58×10^3 C source will be 1.66×10^4 rads/hr, or

$$\frac{1.66 \times 10^4 \text{ rads/hr}}{2.58 \times 10^3 \text{ C}} = 6.44 \text{ rads/hr per C}$$

in an infinite water medium.

Referring to the curves in Figure 3.5.3, it is found that at a distance of 12.3 in. the dose rate from a 1-C point source is 6.44 rads/hr. Therefore, 12.3 in. is the effective distance, b , to the plane source. The results of similar effective distance calculations, at the center of the plane at a distance of 10 in., from 20×40-in., 20×60-in., and 20×80-in. sources are 14.2, 15.8, and 17.0 in., respectively.

The boundary correction method presented here is estimated to be a fairly good approximation.

3.6 Source Overlap Corrections

When it is required to obtain the dose distribution in a target in an irradiator system in which source overlap is employed, the problem is somewhat more complicated. The dose distribution in the target from that portion of the source which is directly beneath the target, S as in Figure 3.6.1, can be obtained from the plane source superposition method or the computer method together with appropriate correction factors. However, the determination of the dose contribution in the target from those portions of the source which

overlap the target (L_0) is made more complex because most of the gamma rays enter the target through the boundary perpendicular to the source overlap, plane B_1 , and travel partially through a "nonabsorbing" air medium.

The method advocated is to divide the overlap into line sources, and to individually calculate and sum the line contributions. The dose rate due to the overlap is then added to the contribution from the finite plane sources, to obtain the total dose rate.

Calculational Techniques

A detailed illustration of the technique described above when used for computer calculation is given in comprehensive problem No. 1.

Empirical Method

Experimental data for the dose distribution from cobalt-60 source elements which overlap the target on one side are presented in Tables 3.6.1, 3.6.2, and 3.6.3. The data are tabulated for source overlap sections (L_0 in Figure 3.6.2) 42 in. long and ranging in height from 3/4 to 6 in. Vertical dose distributions are given for the back, middle, and front planes of targets (along lines $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$ in Figure 3.6.2) ranging in thickness from 2 to 6 in. with source-to-target spacings from 0 to 3.0 in. Since BNL standard sources were used in this work, source self-absorption and absorption in the encapsulating material have been taken into consideration. BNL standard sources are 0.060 in. thick and are encapsulated in 0.036 in. stainless steel. The tabulated data include the effects of boundaries for all points in the target except those such as A' , B' , and C' , which are located on the edge of the target opposite the overlap (see Figure 3.6.2). These boundary effects can be neglected without seriously impairing the accuracy of the calculation.

Reasonable estimates of dose rates in systems where target thicknesses and air gaps are different from those tabulated can be obtained by plotting and interpolating the data. Extensive extrapolation should be avoided except where rough estimates are required.

Table 3.6.1

Dose Rate Contributions Inside a 2-in. Thick Target Due to Cobalt-60 Source Overlap

(Specific dose rate, 10^4 rads/hr per C/cm²)

Distance from edge of target, in. (see Fig. 3:6.2)	Source overlap, in.	Back plane				Middle plane				Front plane			
		Distance from the front plane of the target to the effective plane of the source, in.											
		0	1	2	3	0	1	2	3	0	1	2	3
0 (at edge facing overlap)	0.75	1.455	0.8085	0.5498	0.3951	3.207	1.763	0.9864	0.6522	3.881	2.587	1.499	1.003
	1.5	2.706	1.613	1.143	0.8371	5.304	3.116	1.849	1.308	5.627	3.989	2.582	1.817
	2.25	3.730	2.356	1.730	1.278	6.759	4.177	2.609	1.935	6.468	4.894	3.396	2.501
	3.0	4.538	2.907	2.269	1.676	7.778	5.024	3.299	2.496	6.916	5.525	4.032	3.072
	3.75	5.169	3.536	2.744	2.031	8.511	5.708	3.929	2.984	7.174	5.991	4.531	3.547
	4.5	5.643	3.976	3.137	2.3403	9.045	6.274	4.501	3.394	7.358	6.349	4.933	3.951
	5.25	5.999	4.329	3.444	2.602	9.441	6.737	5.026	3.743	7.519	6.630	5.259	4.294
6.0	6.2630	4.607	3.6768	2.827	9.740	7.125	5.506	4.031	7.675	6.855	5.523	4.584	
2	0.75	0.5164	0.4193	0.3557	0.3083	0.6845	0.6252	0.5713	0.4635	0.6199	0.9055	0.8085	0.6900
	1.5	0.9665	0.7874	0.6888	0.6101	1.170	1.088	1.031	0.8678	0.9632	1.552	1.445	1.267
	2.25	1.344	1.108	0.9906	0.8915	1.542	1.449	1.401	1.222	1.186	2.038	1.954	1.752
	3.0	1.660	1.387	1.261	1.1475	1.841	1.748	1.711	1.527	1.347	2.415	2.369	2.160
	3.75	1.919	1.627	1.501	1.374	2.091	2.001	1.970	1.788	1.470	2.720	2.719	2.511
	4.5	2.137	1.833	1.711	1.573	2.299	2.217	2.191	2.014	1.565	2.967	3.0131	2.808
	5.25	2.303	2.010	1.893	1.746	2.477	2.405	2.380	2.208	1.652	3.171	3.261	3.065
6.0	2.441	2.160	2.0492	1.891	2.632	2.571	2.546	2.374	1.737	3.341	3.471	3.288	
4	0.75	0.2345	0.2436	0.2598	0.2436	0.2452	0.3331	0.3450	0.3439	0.1865	0.4716	0.4840	0.4851
	1.5	0.4388	0.4538	0.4889	0.4619	0.4349	0.5730	0.6253	0.6242	0.3142	0.8284	0.8893	0.8904
	2.25	0.6156	0.6371	0.6873	0.6570	0.5907	0.7617	0.8506	0.8597	0.4112	1.114	1.230	1.236
	3.0	0.7665	0.7950	0.8592	0.8295	0.7217	0.9164	1.037	1.059	0.4910	1.341	1.525	1.533
	3.75	0.8953	0.9298	1.006	0.9842	0.8344	1.050	1.193	1.231	0.5600	1.540	1.780	1.798
	4.5	1.0068	1.044	1.133	1.120	0.9320	1.164	1.326	1.382	0.6204	1.710	2.001	2.032
	5.25	1.099	1.142	1.240	1.239	1.018	1.265	1.440	1.516	0.6754	1.859	2.191	2.240
6.0	1.179	1.224	1.332	1.343	1.095	1.355	1.540	1.634	0.7260	1.988	2.358	2.425	
6	0.75	0.1310	0.1692	0.1951	0.2021	0.1229	0.1946	0.2452	0.2469	0.0879	0.2792	0.3207	0.3601
	1.5	0.2383	0.3029	0.3514	0.3687	0.2253	0.3471	0.4339	0.4512	0.1547	0.5002	0.5999	0.6582
	2.25	0.3299	0.4129	0.4835	0.5088	0.3132	0.4705	0.5870	0.6226	0.2091	0.6835	0.8435	0.9153
	3.0	0.4108	0.5072	0.5967	0.6317	0.3887	0.5724	0.7164	0.7687	0.2565	0.8403	1.059	1.141
	3.75	0.4890	0.5886	0.6970	0.7400	0.4555	0.6581	0.8258	0.8970	0.2992	0.9767	1.247	1.340
	4.5	0.5553	0.6614	0.7865	0.8381	0.5137	0.7303	0.9217	1.010	0.3390	1.0964	1.416	1.5243
	5.25	0.6167	0.7261	0.8668	0.9281	0.5664	0.7928	1.006	1.111	0.3763	1.2042	1.540	1.6952
6.0	0.6738	0.7843	0.9401	1.010	0.6138	0.8466	1.081	1.201	0.4119	1.3012	1.675	1.852	
10	0.75	0.0528	0.0787	0.1062	0.1412	0.0501	0.0814	0.1283	0.1450	0.0361	0.1234	0.1698	0.1984
	1.5	0.0966	0.1391	0.1908	0.2436	0.0922	0.1499	0.2248	0.2636	0.0673	0.2301	0.3207	0.3757
	2.25	0.1352	0.1887	0.2593	0.3234	0.1288	0.2081	0.3046	0.3655	0.0947	0.3228	0.4571	0.5369
	3.0	0.1701	0.2318	0.3164	0.3902	0.1608	0.2577	0.3741	0.4566	0.1194	0.4069	0.5811	0.6819
	3.75	0.2024	0.2703	0.3671	0.4490	0.1892	0.3008	0.4372	0.5380	0.1420	0.4824	0.6948	0.8140
	4.5	0.2327	0.3058	0.4144	0.5034	0.2149	0.3388	0.4949	0.6129	0.1630	0.5519	0.8010	0.9353
	5.25	0.2618	0.3318	0.4590	0.5551	0.2382	0.3729	0.5477	0.6830	0.1825	0.6155	0.8975	1.0469
6.0	0.2898	0.3677	0.5021	0.6050	0.2597	0.4034	0.5976	0.7482	0.2008	0.6743	0.9875	1.1488	
15	0.75	0.0239	0.0308	0.0485	0.0550	0.0248	0.0330	0.0636	0.0706	0.0165	0.0588	0.0911	0.1019
	1.5	0.0449	0.0575	0.0895	0.1020	0.0436	0.0622	0.1121	0.1337	0.0316	0.1122	0.1703	0.1957
	2.25	0.0632	0.0809	0.1251	0.1430	0.0590	0.0886	0.1533	0.1908	0.0456	0.1607	0.2414	0.2830
	3.0	0.0798	0.1018	0.1564	0.1797	0.0721	0.1126	0.1909	0.2424	0.0586	0.2057	0.3077	0.3649
	3.75	0.0949	0.1204	0.1842	0.2137	0.0839	0.1345	0.2263	0.2906	0.0709	0.2467	0.3702	0.4420
	4.5	0.1089	0.1372	0.2091	0.2455	0.0945	0.1548	0.2607	0.3360	0.0823	0.2850	0.4295	0.5158
	5.25	0.1218	0.1528	0.2318	0.2757	0.1042	0.1736	0.2943	0.3791	0.0931	0.3206	0.4861	0.5853
6.0	0.1338	0.1670	0.2528	0.3047	0.1133	0.1908	0.3277	0.4202	0.1033	0.3539	0.5416	0.6500	
20	0.75	0.0118	0.0144	0.0215	0.0259	0.0103	0.0113	0.0271	0.0388	0.0118	0.0388	0.0501	0.0582
	1.5	0.0223	0.0279	0.0420	0.0507	0.0188	0.0248	0.0524	0.0776	0.0221	0.0743	0.3092	0.1120
	2.25	0.0318	0.0404	0.0614	0.0744	0.0260	0.0397	0.0761	0.1153	0.0313	0.1056	0.3475	0.1627
	3.0	0.0404	0.0518	0.0793	0.0965	0.0324	0.0548	0.0990	0.1500	0.0394	0.1336	0.3833	0.2119
	3.75	0.0482	0.0622	0.0959	0.1174	0.0381	0.0694	0.1211	0.1823	0.0468	0.1583	0.4180	0.2604
	4.5	0.0552	0.0717	0.1110	0.1368	0.0433	0.0829	0.1427	0.2122	0.0535	0.1804	0.4517	0.3078
	5.25	0.0617	0.0802	0.1249	0.1546	0.0480	0.0952	0.1637	0.2391	0.0594	0.1998	0.4851	0.3550
6.0	0.0676	0.0877	0.1374	0.1709	0.0523	0.1061	0.1847	0.2631	0.0648	0.2170	0.5185	0.4016	

Table 3.6.2

Dose Rate Contributions Inside a 6-in. Thick Target Due to Cobalt-60 Source Overlap

(Specific dose rate, 10^4 rads/hr per C/cm²)

Distance from edge of target, in. (see Fig. 3.6.2)	Source overlap, in.	Back plane				Middle plane				Front plane			
		Distance from the front plane of the target to the effective plane of the source, in.											
		0	1	2	3	0	1	2	3	0	1	2	3
0 (at edge facing overlap)	0.75	0.2964	0.2425	0.1741	0.1277	1.439	0.9809	0.4301		3.935	2.857	1.509	1.1211
	1.5	0.6656	0.5756	0.3978	0.3050	2.733	1.849	1.002		5.439	4.474	2.684	2.048
	2.25	1.066	0.9421	0.6554	0.5152	3.865	2.625	1.643		6.247	5.541	3.606	2.824
	3.0	1.470	1.297	0.9249	0.7443	4.830	3.320	2.300		6.736	6.285	4.361	3.482
	3.75	1.856	1.626	1.197	0.9750	5.633	3.951	2.899		7.049	6.824	4.986	4.042
	4.5	2.207	1.927	1.464	1.201	6.279	4.517	3.361		7.253	7.228	5.511	4.517
	5.25	2.518	2.200	1.720	1.415	6.794	5.029	3.674		7.394	7.534	5.953	4.932
2	6.0	2.792	2.448	1.961	1.615	7.192	5.492	3.856		7.491	7.771	6.330	5.287
	0.75	0.2156	0.1784	0.1342	0.1132	0.4797	0.3854	0.2776		0.5929	1.067	0.8624	0.7599
	1.5	0.4490	0.3660	0.2905	0.2533	0.8893	0.7304	0.5471		0.9459	1.817	1.558	1.385
	2.25	0.6858	0.5557	0.4575	0.4098	1.218	1.040	0.7988		1.190	2.377	2.124	1.911
	3.0	0.9120	0.7454	0.6203	0.5713	1.563	1.318	1.024		1.309	3.795	2.587	2.361
	3.75	1.127	0.9340	0.7820	0.7330	1.841	1.566	1.221		1.509	3.126	2.973	2.754
	4.5	1.325	1.117	0.9275	0.8839	2.086	1.786	1.388		1.622	3.388	3.296	3.097
4	5.25	1.507	1.291	1.055	1.0176	2.303	1.983	1.527		1.715	3.598	3.567	3.411
	6.0	1.674	1.452	1.162	1.1319	2.496	2.155	1.641		1.793	3.770	3.796	3.694
	0.75	0.1579	0.1185	0.1234	0.1019	0.2345	0.1946	0.1940		0.2102	0.4527	0.5390	0.5336
	1.5	0.3142	0.2409	0.2419	0.2048	0.4469	0.3590	0.3584		0.3560	0.8165	0.9809	0.9648
	2.25	0.4646	0.3638	0.3550	0.3175	0.6393	0.5045	0.4991		0.4711	1.115	1.348	1.3286
	3.0	0.6064	0.4840	0.4617	0.4302	0.8107	0.6393	0.6220		0.5676	1.362	1.660	1.6455
	3.75	0.7401	0.6025	0.5603	0.5385	0.9622	0.7660	0.7287		0.6517	1.569	1.925	1.9285
6	4.5	0.8640	0.7178	0.6519	0.6414	1.097	0.8862	0.8209		0.7255	1.746	2.151	2.1861
	5.25	0.9771	0.8278	0.7365	0.7373	1.216	1.004	0.9023		0.7907	1.896	2.347	2.4233
	6.0	1.081	0.9345	0.8130	0.8235	1.321	1.119	0.9734		0.8495	2.024	2.514	2.6432
	0.75	0.1131	0.0830	0.0970	0.0927	0.1258	0.1137	0.1342		0.1067	0.2479	0.3719	0.3880
	1.5	0.2204	0.1649	0.1886	0.1882	0.2383	0.2118	0.2544		0.1913	0.4807	0.6926	0.7157
	2.25	0.3212	0.2458	0.2748	0.2755	0.3407	0.2991	0.3617		0.2619	0.6898	0.9648	0.9959
	3.0	0.4139	0.3240	0.3551	0.3590	0.4361	0.3794	0.4539		0.3239	0.8720	1.197	1.2428
10	3.75	0.5001	0.3995	0.4284	0.4382	0.5240	0.4543	0.5326		0.3794	1.025	1.393	1.4654
	4.5	0.5804	0.4728	0.4952	0.5131	0.6049	0.5249	0.5978		0.4300	1.153	1.550	1.6664
	5.25	0.6548	0.5429	0.5561	0.5831	0.6814	0.5934	0.6522		0.4773	1.259	1.699	1.8524
	6.0	0.7238	0.6103	0.6111	0.6483	0.7631	0.6581	0.6964		0.5215	1.345	1.817	2.0254
	0.75	0.0577	0.0469	0.0631	0.0625	0.0571	0.0517	0.0658		0.5066	0.1056	0.1935	0.2145
	1.5	0.1093	0.0865	0.1157	0.1161	0.1088	0.0975	0.1326		0.9243	0.2258	0.3782	0.4150
	2.25	0.1587	0.1232	0.1629	0.1646	0.1513	0.1387	0.1951		1.070	0.3390	0.5433	0.5982
15	3.0	0.2014	0.1584	0.2083	0.2099	0.1931	0.1762	0.2490		1.596	0.4403	0.6894	0.7621
	3.75	0.2434	0.1929	0.2475	0.2530	0.2324	0.2103	0.2942		1.887	0.5265	0.8091	0.9071
	4.5	0.2844	0.2277	0.2874	0.2950	0.2701	0.2415	0.3307		2.163	0.5982	0.9061	1.0354
	5.25	0.3248	0.2633	0.3262	0.3360	0.3069	0.2706	0.3597		2.430	0.6575	0.9821	1.1454
	6.0	0.3647	0.3000	0.3644	0.3755	0.3429	0.2976	0.3822		2.687	0.7052	1.042	1.2403
	0.75	0.0267	0.0225	0.0331	0.0367	0.0172	0.0180	0.0265		0.2334	0.0690	0.0959	0.1132
	1.5	0.0503	0.0419	0.0618	0.0690	0.0344	0.0400	0.0658		0.4636	0.1299	0.1929	0.2310
20	2.25	0.0719	0.0591	0.0874	0.0981	0.0516	0.0606	0.1069		0.6819	0.1843	0.2845	0.3466
	3.0	0.0923	0.0751	0.1102	0.1248	0.0883	0.0794	0.1413		0.8829	0.2342	0.3670	0.4490
	3.75	0.1113	0.0902	0.1308	0.1491	0.0845	0.0950	0.1683		1.066	0.2800	0.4398	0.5396
	4.5	0.1296	0.1048	0.1498	0.1716	0.1000	0.1074	0.1888		1.227	0.3223	0.5007	0.6161
	5.25	0.1474	0.1190	0.1673	0.1924	0.1145	0.1168	0.2044		1.370	0.3610	0.5491	0.6802
	6.0	0.1651	0.1330	0.1835	0.2118	0.1285	0.1237	0.2162		1.493	0.3968	0.5858	0.7328
	0.75	0.0113	0.0103	0.0183	0.0167	0.0076	0.0098	0.0140		0.0113	0.0464	0.0492	0.0550
	1.5	0.0234	0.0210	0.0350	0.0350	0.0148	0.0219	0.0276		0.0253	0.0812	0.1058	0.1245
	2.25	0.0354	0.0312	0.0533	0.0533	0.0214	0.0278	0.0409		0.0399	0.1103	0.1640	0.2000
	3.0	0.0468	0.0403	0.0700	0.0700	0.0273	0.0358	0.0535		0.0534	0.1372	0.2179	0.2728
	3.75	0.0572	0.0482	0.0816	0.0816	0.0324	0.0429	0.0655		0.0652	0.1634	0.2648	0.3369
	4.5	0.0664	0.0547	0.1552	0.0969	0.0368	0.0492	0.0789		0.0748	0.1905	0.3032	0.3886
	5.25	0.0744	0.0598	0.1639	0.1070	0.0406	0.0546	0.0877		0.0823	0.2192	0.3331	0.4284
	6.0	0.0812	0.0641	0.1709	0.1152	0.0438	0.0592	0.0979		0.0881	0.2505	0.3560	0.4572

Table 3.6.3

Dose Rate Contributions Inside a 4-in. Thick Target Due to Cobalt-60 Source Overlap

(Specific dose rate, 10 ⁴ rads/hr per C/cm ²)													
Distance from edge of target, in. (see Fig. 3.6.2)	Source overlap, in.	Back plane				Middle plane				Front plane			
		Distance from the front plane of the target to the effective plane of the source, in.											
		0	1	2	3	0	1	2	3	0	1	2	3
0 (at edge facing overlap)	0.75	0.5120	0.3854	0.2803	0.2479	1.369	0.9055	0.5929	0.4528	4.312	2.695	1.520	1.105
	1.5	1.100	0.8031	0.6010	0.5336	2.555	2.032	1.261	0.9756	5.767	4.296	2.598	2.011
	2.25	1.709	1.245	0.9244	0.8301	3.574	3.002	1.951	1.520	6.457	5.266	3.423	2.754
	3.0	2.285	1.701	1.243	1.121	4.436	3.822	2.598	2.048	6.865	5.951	4.070	3.374
	3.75	2.787	2.132	1.555	1.407	5.158	4.506	3.191	2.538	7.141	6.463	4.598	3.884
	4.5	3.202	2.503	1.857	1.685	5.762	5.072	3.719	2.972	7.346	6.859	5.029	4.299
	5.25	3.530	2.797	2.153	1.954	6.268	5.525	4.180	3.349	7.505	7.180	5.390	4.641
2	6.0	3.784	3.019	2.441	2.213	6.687	5.890	4.579	3.668	7.631	7.444	5.692	4.924
	0.75	0.3056	0.2614	0.2054	0.1833	0.5443	0.4743	0.4032	0.3450	0.7007	1.024	0.8355	0.6099
	1.5	0.6021	0.5174	0.4156	0.3660	0.9863	0.8678	0.7266	0.6425	1.086	1.752	1.509	1.283
	2.25	0.8824	0.7626	0.6204	0.5433	1.355	1.202	1.004	0.9023	1.340	2.291	2.048	1.795
	3.0	1.141	0.9928	0.8171	0.7158	1.670	1.492	1.249	1.139	1.522	2.709	2.499	2.232
	3.75	1.378	1.207	1.005	0.8829	1.942	1.750	1.467	1.349	1.662	3.042	2.87	2.598
	4.5	1.593	1.407	1.178	1.045	2.183	1.978	1.666	1.543	1.775	3.309	3.183	2.911
4	5.25	1.783	1.586	1.340	1.201	2.397	2.182	1.848	1.721	1.867	3.524	3.443	3.175
	6.0	1.952	1.750	1.490	1.350	2.586	2.365	2.017	1.884	1.944	3.701	3.660	3.397
	0.75	0.1887	0.1779	0.1461	0.1401	0.2803	0.2803	0.2587	0.2560	0.2200	0.5067	0.5390	0.5121
	1.5	0.3639	0.3439	0.2932	0.2743	0.5040	0.4986	0.4560	0.4576	0.3585	0.9110	0.9648	0.9352
	2.25	0.5256	0.4941	0.4350	0.3999	0.6927	0.6792	0.6177	0.6274	0.4722	1.241	1.315	1.291
	3.0	0.6749	0.6342	0.5692	0.5185	0.8576	0.8344	0.7578	0.7729	0.5687	1.511	1.606	1.596
	3.75	0.8102	0.7630	0.6932	0.6290	1.003	0.9702	0.8823	0.9017	0.6501	1.734	1.860	1.865
6	4.5	0.9352	0.8816	0.8059	0.7314	1.133	1.093	0.9982	1.018	0.7196	1.920	2.081	2.102
	5.25	1.049	0.9894	0.9067	0.8273	1.251	1.204	1.106	1.124	0.7789	2.075	2.276	2.317
	6.0	1.153	1.087	0.9956	0.9157	1.359	1.307	1.206	1.220	0.8296	2.204	2.449	2.507
	0.75	0.1304	0.1240	0.1083	0.1094	0.1563	0.1752	0.1806	0.1860	0.0943	0.3018	0.3541	0.3838
	1.5	0.2296	0.2318	0.2156	0.2097	0.2851	0.3358	0.3170	0.3310	0.1638	0.5497	0.6425	0.6953
	2.25	0.3164	0.3277	0.3175	0.3019	0.3972	0.4441	0.4297	0.4501	0.2220	0.7594	0.8861	0.9605
	3.0	0.3994	0.4161	0.4134	0.3876	0.4964	0.5373	0.5262	0.5541	0.2746	0.9362	1.1001	1.191
10	3.75	0.4797	0.5002	0.4986	0.4685	0.5870	0.6208	0.6124	0.6457	0.3236	1.087	1.289	1.399
	4.5	0.5595	0.5800	0.5768	0.5467	0.6679	0.6963	0.6906	0.7287	0.3710	1.216	1.460	1.588
	5.25	0.6387	0.6587	0.6458	0.6216	0.7423	0.7664	0.7617	0.8047	0.4174	1.327	1.616	1.761
	6.0	0.7179	0.7342	0.7067	0.6949	0.8086	0.8311	0.8280	0.8748	0.4632	1.423	1.757	1.922
	0.75	0.0588	0.0614	0.0636	0.0690	0.0582	0.0749	0.0862	0.0976	0.0213	0.0738	0.0970	0.1148
	1.5	0.1035	0.1158	0.1224	0.1299	0.1103	0.1374	0.1552	0.1790	0.0407	0.1390	0.1892	0.2237
	2.25	0.1444	0.1657	0.1761	0.1837	0.1572	0.1901	0.2129	0.2491	0.0584	0.1967	0.2754	0.3256
15	3.0	0.1831	0.2120	0.2254	0.2330	0.1998	0.2363	0.2628	0.3116	0.0745	0.2471	0.3557	0.4231
	3.75	0.2199	0.2551	0.2707	0.2779	0.2385	0.2778	0.3075	0.3671	0.0892	0.2909	0.4295	0.5105
	4.5	0.2559	0.2961	0.3127	0.3189	0.2740	0.3155	0.3482	0.4178	0.1027	0.3286	0.4974	0.5908
	5.25	0.2913	0.3354	0.3522	0.3566	0.3066	0.3508	0.3859	0.4642	0.1152	0.3609	0.5588	0.6625
	6.0	0.3258	0.3723	0.3889	0.3914	0.3372	0.3831	0.4204	0.5068	0.1270	0.3889	0.6143	0.7261
	0.75	0.0244	0.0237	0.0473	0.0399	0.0215	0.0288	0.0367	0.0472	0.0391	0.1348	0.1763	0.2097
	1.5	0.0461	0.0496	0.0786	0.0776	0.0420	0.0558	0.0695	0.0903	0.0720	0.2496	0.3326	0.3892
20	2.25	0.0660	0.0757	0.1061	0.1121	0.0614	0.0806	0.0986	0.1291	0.1022	0.3520	0.4717	0.5498
	3.0	0.0843	0.1013	0.1310	0.1426	0.0792	0.1032	0.1245	0.1637	0.1310	0.4436	0.5962	0.6926
	3.75	0.1014	0.1246	0.1541	0.1690	0.0954	0.1236	0.1473	0.1947	0.1581	0.5255	0.7094	0.8230
	4.5	0.1175	0.1451	0.1757	0.1914	0.1099	0.1418	0.1672	0.2220	0.1840	0.5999	0.8129	0.9432
	5.25	0.1326	0.1622	0.1962	0.2102	0.1228	0.1580	0.1847	0.2464	0.2087	0.6673	0.9078	1.055
	6.0	0.1470	0.1757	0.2156	0.2258	0.1341	0.1720	0.2000	0.2677	0.2323	0.7266	0.9951	1.159
	0.75	0.0108	0.0161	0.0195	0.0226	0.0091	0.0124	0.0183	0.0232	0.0200	0.0381	0.0592	0.0700
	1.5	0.0216	0.0323	0.0394	0.0463	0.0180	0.0258	0.0361	0.0475	0.0350	0.0765	0.1154	0.1395
	2.25	0.0323	0.0479	0.0577	0.0687	0.0266	0.0388	0.0520	0.0704	0.0474	0.1121	0.1677	0.2080
	3.0	0.0410	0.0623	0.0740	0.0877	0.0346	0.0511	0.0677	0.0913	0.0581	0.1434	0.2157	0.2738
	3.75	0.0506	0.0747	0.0878	0.1030	0.0421	0.0620	0.0806	0.1096	0.0676	0.1698	0.2596	0.3336
	4.5	0.0584	0.0849	0.0991	0.1152	0.0489	0.0716	0.0915	0.1252	0.0762	0.1913	0.2295	0.3874
	5.25	0.0652	0.0929	0.1077	0.1248	0.0550	0.0797	0.1007	0.1384	0.0842	0.2083	0.3359	0.4338
	6.0	0.0710	0.0989	0.1136	0.1323	0.0606	0.0865	0.1084	0.1492	0.0916	0.2217	0.3685	0.4740

A long narrow source element was chosen as the basis of the method because the technique was intended to be used primarily as the static equivalent of a continuous irradiator system in which the target is moved past a much shorter source with the direction of motion as shown in Figure 3.6.3 (parallel to the source plane). However, the method can also be used to obtain the dose distribution in a "batch-type," semicontinuous irradiator (Figure 3.6.4), where the direction of motion of the target is perpendicular to the plane of the source. In the latter case the dose distribution contribution from overlap in the three important planes along lines $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$ (Figure 3.6.2) can be obtained directly from the data. The doses along the lines $\overline{DD'}$, $\overline{EE'}$, and $\overline{FF'}$ can be obtained with a reasonable degree of accuracy by assuming that the relationship between the dose at a point P (Figure 3.6.5 at the center of the package to the dose at a point P' (having the same Y and Z coordinates as P) located at the end of the target in plane ABCD (Figure 3.6.5) is as follows:

$$\text{Dose at P'} = \frac{1}{2}(\text{dose at P}) - (\text{a single boundary correction})$$

This is a good approximation for all points in plane ABCD except for points such as N' (Figure 3.6.5) which are located on the edge of the target facing the source. These points can either be calculated or determined by the infinite medium superposition method.

3.7 Specific Activity Augmentation

An alternate method which is sometimes used to achieve dose uniformity in a target consists of building up the specific activity of the source along its perimeter. The problem of determining the dose distribution resulting from this nonuniform distribution of source activity can be handled analytically with either FUDGE 3A or FUDGE 4. Empirically the superposition methods which utilize 1, 2.5, and 5-in. elements (see Appendix) can be used in addition to the data in Figure 3.5.3 for the dose distribution from a point source in an infinite medium.

FIG. 3.6.3

FIG. 3.6.4

FIG. 3.6.5

4. IRRADIATOR DESIGN

4.1 Introduction

An irradiator designer usually starts with a given set of parameters having to do with the amount of product to be treated, the treatment desired, and the dimensions of the unit product. From these he determines the amount of source activity required to effect the treatment and then decides upon a geometric configuration between source and target that will most efficiently or most economically satisfy the design requirements.

Parameters important for irradiator design are listed in Table 4.1.1.

The parameters that may be considered independent are mass flow rate, minimum total dose, maximum residence time, and target dimensions. The mass flow rate and minimum total dose determine the capacity of the irradiator and their product is a unit of power, usually pound-rads per hour or megarad-tons per hour. The magnitude of this product will ultimately determine the required amount of source activity.

The mass flow rate is obviously arbitrary. Typically, it has varied from 100 lb/hr to 2000 lb/hr in irradiators ranging from small research facilities to large production units.

The minimum total dose is determined by the treatment desired for the product. Typical minimum total doses are 8,000 to 10,000 rads for insect sterilization; 15,000 to 25,000 rads for grain disinfestation; 75,000 to 500,000 rads for radiopasteurization of shellfish and other marine products; 2 to 2.5×10^6 rads for sterilization of medical supplies; and 4.5×10^6 rads for sterilization of meat products.

Maximum residence time for some food products is usually specified to prevent thawing and refreezing of frozen products. On the average it is usually limited to a maximum of ~1 hr.

The target dimensions of length and height are also somewhat arbitrary. Length is only important when the total target travel is short compared to source length in a moving

Table 4.1.1
Package Irradiator Parameters

<u>Symbol</u>	<u>Definition</u>	<u>Dimensions</u>
1. W	Mass flow rate of target	lb/hr
2. $I_{\min}$	Minimum total dose in target	rads
3. D_t	Minimum dose rate in target	rads/hr
4. U	Dose uniformity, $I_{\max}/I_{\min}$	dimensionless
5. t_R	Residence time	hr
6. V	Linear velocity of package	ft/hr
7. $H_S; L_S$	Source height; source length	ft
8. $H_T; L_T$	Target height; target length	ft
9. h_T	Target thickness	ft
10. ρ_T	Target density	lb/ft ³
11. m	Number of parallel source plaques in multisource irradiator	---
12. S_g	Source activity per unit mass (specific activity)	C/g
13. S_a	Source activity per unit area	C/cm ²
14. E_u	Source utilization efficiency	---

target irradiator. For practical purposes the individual target or package length should be no larger than one-half the source length, otherwise the target travel can only be varied by rather large increments. Since there must be a void space into which the package will move, packages of small length result in a smaller void space and consequently a larger target volume and higher efficiency. Target height will be in multiple increments of individual package heights. The number of increments chosen will depend upon the source-to-target geometry ultimately decided upon.

The maximum target thickness is controlled by the specifications for maximum dose uniformity and is therefore limited by target density, atomic number, and photon energy. These factors, in turn, affect the over-all dose attenuation and absorbed dose variation within the target.

The dependent variables include dose uniformity, target velocity, source dimensions, source specific activity, and source utilization efficiency. The designer must bear in mind that many of these variables are interrelated and thus may not be changed indiscriminately. For example linear target velocity cannot be altered in a continuous irradiator without changing the mass flow rate unless one also changes the target dimensions. For a fixed velocity, the total target travel becomes a function of the minimum dose rate in the target which in turn is related to source specific activity, source-to-target geometry, gamma photon absorption cross section of the target, dose buildup factor in the target, and gamma photon energy of the source.

Dose uniformity depends upon target thickness, target density, and source-to-target geometry. In certain moving target irradiators, horizontal variation of dose is minimized at any point in the target by target motion. However, vertical and depth dose variation are not affected by horizontal target motion. Vertical uniformity may be increased by overlapping the target with additional source or by other flux flattening techniques such as source specific

activity augmentation near top and bottom edges or by the use of variable thickness absorbers. Depth-dose variation may be minimized by decreasing target thickness, decreasing target density, increasing the distance between source and target, by the use of absorbers to decrease the gamma flux gradient within the target, and by "cross-fire" irradiation.

Although the dimension of source length is quite flexible for irradiators where the target moves parallel to the source axis, source height and thickness dimensions are more restricted. Usually the source height is governed by the target height as previously discussed and is some integral multiple of source element length for vertically stacked elements. Source element length is related to nuclear reactor specifications and ease of handling, encapsulating, and shipping. Within reasonable limits (several inches to a foot), source length can be specified by the irradiator designer.

A range of source specific activities may be selected up to the practical maximum for the particular isotope. For reactor produced sources the specific activity is related to neutron flux, isotopic abundance, activation cross section, half-life, and time in the reactor. For separated fission products, the specific activity is governed by isotopic abundance, fission yield, chemical purity, and half-life. The source strength per unit area for a fixed value of specific activity may be varied by adjusting the source thickness or by using more than one layer of source elements. Practical values for attainable specific activities range from 5 to about 400 C/g of cobalt metal for cobalt-60, and activities up to 25 C/g of cesium chloride for the separated isotope cesium-137.

The source utilization efficiency is related to many of the variables such as photon energy, source dimensions, source density, source-to-target geometry, target dimensions, and target density. Utilization efficiency is defined as the ratio of the power absorbed at the minimum dose point in the target divided by the total power emitted by the source.

4.2 General Design of Continuous Irradiators

Irradiators could be classified according to target characteristics and requirements, since the attainment of these requirements is the ultimate goal of the design. The primary classification of irradiators depends upon the physical state of the target material, i.e., fluids or solids.

Another type of classification relative to the target material is given by the nature of the radiation effect produced. At the present time practical systems under consideration involve two predominant effects: (1) biological and (2) chemical. Of the two, the requirements for the former are usually more simply stated and involve a direct proportionality to the dose evidenced by common usage of a combined capacity figure (lb-rad/hr). Chemical effects are much more involved since the formation of active species by radiation is only a single step in the over-all kinetic scheme, and dose rate as well as total dose are significant.

In the development given below the irradiators considered refer to the irradiation of solids (or containers of fluids in which no mixing is assumed) in which biological effects are of prime importance. Usually the irradiator must process a given amount of material over a given time period in such a way as to provide a minimum total dose in the material and accomplish this within certain dose uniformity limitations. It may be accomplished continuously or intermittently. Thus, in any design we must be able to evaluate the dose at any position within the target material. In principle this can be simply stated by the following equation:

$$I = \int_0^{\theta_T} D(\theta) d\theta \quad (4.2.1)$$

which states that the total dose, I , at any position in the target material is equal to the integral of the instantaneous dose rate over the total time, θ_T , of irradiation. The variation of D with time is due to variations in spatial relationships with time, between source and target material. More specific functional forms of the above equation for certain types of motion

of source and target systems are developed below. One of the reasons for such an approach is to focus attention on the main parameters that enter into dose evaluation in a complex system. From such relationships the concepts of an integrated normalized dose or dose rate become more meaningful. Furthermore, the static equivalent system can be readily visualized from such functional relations.

4.3 Equivalence of Static and Moving Systems

Linear Motion

In the design of irradiators it is sometimes convenient for purposes of calculation to substitute for the actual moving system an equivalent one from a standpoint of absorbed radiation. This equivalent system may be static or one in which the motion of the target and the source have been interchanged. Quite frequently experimental data are available for static experiments which can be related to continuous systems.

Consider the simple case of a point source, S, and point target, T, shown in Figure 4.3.1, in which the point target moves with uniform velocity, V, from position X_0 to X_2 past the stationary source, S. It is obvious that the target would accumulate the same dose if it remained stationary at X_1 , and the source moved with velocity, V, from Y_0 to Y_2 . It is seen that this is a simple problem in relative motion and hence it does not make any difference which point is stationary and which is moving.

In a continuous irradiator, material would be introduced at X_0 and removed at X_2 . In the equivalent moving source example the target material would remain stationary at X_1 until the source completed its passage from Y_0 to Y_2 and then would be removed. The source would instantaneously return to Y_0 for the next passage.

The uniform motion of the point source creates an effective line source and it is apparent that the above relative motion of point source and point target may be replaced by a static system consisting of a point target and line source. The equivalence of the two may

FIG. 4.3.1

be established as follows:

The dose received by the target from the moving point source of strength, S_p , can be expressed as:

$$I = \int_0^{\theta} S_p \phi(Y) dt \quad (4.3.1)$$

where θ is the transit time of the source and $\phi(Y)$ is a geometric factor. The corresponding dose received by the target from a line source of strength per unit length, S_L , irradiated for a time equal to τ would be:

$$I = \tau \int_{Y_0}^{Y_2} S_L \phi(Y) dY \quad (4.3.2)$$

The geometric factor is the same in both cases. The relationship between Y and t is given by

$$Y = Y_0 + Vt \quad (4.3.3)$$

If Eq. (4.3.3) is substituted into Eq. (4.3.1) the following equation results.

$$D = \frac{1}{V} \int_{Y_0}^{Y_2} S_p \phi(Y) dY \quad (4.3.4)$$

Equating (4.3.2) and (4.3.4),

$$S_L \tau = \frac{S_p}{V} \quad (4.3.5)$$

and

$$S_L = \frac{S_p}{V\tau} \quad (4.3.6)$$

If the irradiation time for the static case is equal to the transit time ($\theta = \tau$) then

$$S_L = \frac{S_p}{Y_2 - Y_0} = \frac{S_p}{\Delta Y} \quad (4.3.7)$$

Several other simple examples may be considered. In Figure 4.3.1 let T represent a,

line perpendicular to the plane of the paper rather than a point target. The movement of the line target past the point source, S, generates a plane-continuous target. If the relative motion is interchanged, the line target becomes stationary at X_1 and the point source, S, moves from Y_0 to Y_2 . This, as before, generates an effective line source. Thus, the equivalent static case would be a line source and a line target perpendicular and separated in space. Relations (4.3.5), (4.3.6), and (4.3.7) still apply. For this case the accumulated doses would vary along the target length.

If the reverse situation is considered in which a point target moves past a line source (Figure 4.3.1), the equivalent motion of the source would generate an effective plane source for the static case.

For this case it can be shown that

$$S_A \tau = \frac{S_L}{V} \quad (4.3.8)$$

and if $\theta = \tau$,

$$S_A = \frac{S_L}{Y_2 - Y_0} \quad (4.3.9)$$

where S_A = source strength per unit area for the plane source. If in Figure 4.3.1 both S and T are considered to be lines, the equivalent static case (to moving line target and stationary line source) would be a plane source and line target. The above relations (4.3.8) and (4.3.9) would hold.

As an interesting variation in linear motion consider Figure 4.3.2. A point target moves toward and through a plane source as shown. In the equivalent moving source case the point target would be located at B and the plane source would traverse the distance from A to C. This would generate an effective volume source. This volume source would have the absorption properties of the surrounding medium (air in many cases). The foregoing illustrates an important point in that all the above examples are mathematical equivalents. If the results of experimental data with actual sources are being used to evaluate the above

systems, then they must be used with caution. For example, none of the cases above involves self-absorption by source material. While actual plane, line, and point sources can be fabricated with relatively little self-absorption, it would be essentially impossible to fabricate a volume source as that described above. Thus, the static case is useful only as an alternative mathematical representation.

If a continuous plane target moves through a point source (reverse of that shown in Figure 4.3.2), the equivalent static case would be a line source perpendicular to the plane target.

The above serves to illustrate the equivalence of moving and static elementary systems. Many other examples may be cited. In the above discussion, source and targets of finite volume were avoided to simplify the analysis. However, in any practical irradiator both target and source would have to be treated as being finite in three dimensions.

Consider the case in which slabs of materials are continuously passing a point source as shown in Figure 4.3.3.

Mathematically the continuous slab may be considered as being generated by the movement of a plane past the source, S. As before, the motion of source and target may be interchanged with the stationary plane at B and the source moving from D to F. However, in order to maintain equivalence, the geometry and density of the media must be preserved. Note that we are considering a unit mass of material located at a plane section at B but bounded by target medium as in the original slab. In this case there is a dose variation with depth, from G to A, as well as in the dimension not shown. However we may still consider an integrated value at a given cross section or we may focus our attention on any single point in the cross section at B. The movement of the point source generates an equivalent line source and the static case which represents a moving slab and a stationary point source would be the plane sections located at B but contained within a slab of target material and irradiated

FIG. 4.3.2

FIG. 4.3.3

by a line source. The relationships between source strength and irradiation times are given by Eqs. (4.3.5), (4.3.6), and (4.3.7).

If in Figure 4.3.3, S represents a line source instead of a point source, other conditions being the same, the static case would be given as above except the line source would be replaced by a plane source and Eqs. (4.3.8) and (4.3.9) would apply. Thus, experimental data on dose rates in slab targets irradiated by a plane source may be directly used for continuous irradiators in which the slab passes continuously by a line source. According to the above model the plane source should be of length equal to the transit passage in the irradiator and of height equal to that of the line source. In the static model the length of the slab target must represent the longest path in the absorbing medium that would be encountered in the irradiator. If there is no air gap the slab would be of the same length as the source.

In many practical irradiators, targets in slab form move past a plane source. If the motions of target and source are interchanged, the situation is as shown in Figure 4.3.4.

In the original arrangement the target plane, T, enters at A with uniform velocity, V, and exits at C with the source plane of length, N, located at B. To maintain mathematical equivalence for the case with motion interchanged, the center of the source moves from A to C with uniform velocity, V, past the central plane of the target at B. The movement of this plane source generates an equivalent stationary source. However, it will be noted that this stationary source will have a variable source strength in the proposed direction of travel. Assuming that the source strength of the moving source is uniform, then the source strength of the equivalent stationary source at any position would be proportional to the time at that point during which source material is present during passage. Hence, the area represented by the dotted line between D and E in Figure 4.3.5 would be one of uniform intensity since any position here is traversed by the entire source. On the other hand, the regions FAD and ECG will have a variable source strength. For uniform velocity and uniform source strength

FIG. 4.3.4

FIG 4.3.5

of the moving source, these regions will be represented by linear variations, becoming zero at F and G as shown in Figure 4.3.5. Here, N = length of moving source (or stationary source in moving target system), H = distance traversed in irradiator, and S_s = source strength of equivalent stationary sources (and target plane for region DE). The relationship between S_s and S_m (moving source) is given by

$$S_s = \frac{S_m N}{V\tau} \quad (4.3.10)$$

and if τ = transit time, θ , then $V\theta = H$ and

$$S_s = \frac{S_m N}{H} \quad (4.3.11)$$

Experimentally the above arrangement could be closely approximated by overlapping thin plane sources as shown in Figure 4.3.6. As N becomes small, the moving plane source approaches a line source the variable regions in Figure 4.3.5 disappear, i.e., a uniform plane source is obtained. As H becomes large for a given value of N the contributions of the end regions are negligible and again the equivalent stationary source may be approximated by a uniform one.

For the general case it is more convenient to think in terms of a normalized dose rate at a point in a medium centrally located before a source of variable source strength such as that shown in Figure 4.3.5. The source of variable source strength may be considered as the sum of a large number of differential ones of constant source strength each varying in length, as shown in Figure 4.3.5. By definition a normalized dose rate is independent of source strength and for the above case is a function of the length of the differential element. An effective normalized dose rate may be defined as that value which when multiplied by the constant value of the source strength, S_s , shown in Figure 4.3.5 and given by Eq. (4.3.11), will give the same total dose rate, D , at the point in question as that obtained with the actual arrangement with variable source strength. For this definition the effective normalized

dose rate becomes:

$$d_{\text{eff}} = \frac{1}{2N} \int_{H-N}^{H+N} d(L) dL \quad (4.3.12)$$

When using the method of superposition it is easy to keep a running sum of d vs. L for a given height source and a plot may be prepared as shown in Figure 4.3.7.

It can be seen from this figure that for large lengths such as H_1 (targets introduced in a region of negligible intensity), the length of the source N has no effect on the effective normalized dose rate which is equivalent to that for a plane of uniform source strength.

At H_2 the effective normalized dose is equal to that from a plane of uniform source strength if the source length is very small (approaching a line source) as indicated by "ab." For a larger source, "jk," the effective normalized dose rate can be obtained by graphical integration as indicated above.

In setting up an equivalent stationary system for experimental measurements, the geometry in the dimension not shown in the above figures should be the same in both the moving and the stationary systems. For example, this dimension may have an overlap of source plane above the target for better uniformity of dose. This overlap would be the same in both cases. Measurements of absorbed dose would be made in the central plane. This assumes symmetrical passage of target past the source in the irradiator. If the approach and recession of the target are not symmetrical, the experimental plane would not be centrally located but could be easily established from the geometry of the system.

Circular Motion Systems

Other systems of interest involve uniform circular motion. As a simple example, consider the various types of circular motion involving a solid cylindrical target and a line source, as shown in Figures 4.3.8, a, b, and c.

In Figure 4.3.8a, the source, S , moves with constant angular velocity around a stationary target, T . In an actual irradiator, target material would be introduced for a period of

FIG. 4.3.6

FIG. 4.3.7

FIG. 4.3.8a

FIG. 4.3.8b

FIG. 4.3.8c

time (conveniently taken as corresponding to an integral number of revolutions of the source) and then withdrawn, and fresh target material added. If the motion of the target (center of gravity) and source are interchanged, we have the system shown in Figure 4.3.8b. Note that in this representation the target does not rotate on its own axis but nevertheless is irradiated uniformly in any integral number of revolutions. In a continuous irradiator of this type, target material would be continuously inserted at a given position on the circle. After making a complete revolution it would be continuously removed. Systems as in Figures 4.3.8a and b are completely equivalent. Another possibility is shown in Figure 4.3.8c. In this case the target rotates on its own axis in front of a stationary source, S. All the above systems are equivalent for uniform angular velocity. An equivalent stationary system can be visualized by noting that the moving line source in Figure 4.3.8a generates a cylindrical shell source. The relationship between the source strength, S_L , of the moving line source and that of the stationary shell source, S_A , would be given by

$$S_A = \frac{S_L n}{\omega r \tau} \quad (4.3.13)$$

where n = number of revolutions, ω = angular velocity (radians/sec), r = radius, and τ = irradiation time for stationary system.

In the above examples the various sources were assumed to have no self-absorption. Any real source will have a finite volume; hence self-absorption must be taken into account. Furthermore, a real source will also be clad with a metal which will absorb radiation. The physical interchange of motion from target to source can still be made since this does not involve any mathematical approximation. However, difficulty arises in suitably representing the stationary equivalent of a moving system. Visualize the situation shown in Figure 4.3.9, for a moving rod source which has now replaced the mathematical line source. It will be noted that the linear motion of the rod now produces a mathematical volume with properties not easily represented physically. If the motion of the center of the rod is followed, the

FIG. 4.3.9

same amount of source material and cladding is always presented to the target plane. However, the cladding and source thickness would have to be variable in an equivalent stationary plane system to produce the same effect. The situation is more complex if one considers a point off center since now the amount of source material and cladding presented to the target plane are variable.

Since radiation absorbed in source material and cladding never reaches the target, the diameter of rods, thicknesses of slabs, and cladding should be kept as small as possible to increase efficiency. The self-absorption and cladding losses in real sources usually are not negligible. However, the differences in dose in a target plane between a moving rod source and a stationary plane source of uniform cladding and source thickness equal to that of the rod, would be expected to be small. Very little difference would be expected between a moving slab source and a stationary one for the conditions previously described if both had the same thickness of source and cladding.

4.4. Specific Irradiator Design

Introduction

Thus far a simple classification of continuous irradiators has been presented and it has been pointed out that for convenience of calculation it is possible to use data on static source target configurations to predict dose distributions in moving systems. In order to develop specific irradiator design information certain source-target configurations have been chosen for further descriptive treatment. The selection is not exhaustive and has been made primarily on the basis of mechanical feasibility. An attempt has been made to develop a series of generalized equations to describe target motions in these configurations. These equations are not intended for use in detailed irradiator design, but are intended as a mathematical description of complex motion in certain important source-target geometries. The reader may desire to refer directly to the irradiator design examples given in Paragraphs 5.1

and 5.2 which illustrate the generalized equations developed in this section.

Slab Target - Linear Motion of Target

Single Source. Most material to be irradiated for biological effects can be packaged in containers which resemble slabs. In a continuous operation these slabs will pass through the system at a constant rate, V , except at the end of a pass where the direction of travel is reversed. It will be assumed that the time required to shift a package and resume reverse flow is negligible. Since symmetry improves uniformity of dose, it will be assumed that for any given continuous irradiator: (1) there will be an equal number of forward and reverse passes; (2) there will be an equal number of passes on each side of the source; (3) for any given pass on one side of the source, the corresponding pass on the other side will have the opposite side of the slab exposed. This will be the case if the target is not turned at the end of a pass. (4) the source will be centrally located relative to any single pass.

A typical arrangement is shown in Figure 4.4.1. In this arrangement target material enters at A, moves to C, reverses direction and continues in this way to give one forward and one reverse pass on each side of the source. In an actual irradiator one package would immediately follow another, thus filling the entire irradiator for maximum efficiency of radiation utilization. For convenience, only a few key positions are shown in Figure 4.4.1, where the source is a line source perpendicular to the plane of the paper.

The result of passing target material through the irradiator can be represented as follows:

Let $D(x,y,z)$ be the dose rate at any location in the target area due to a single source. If a target package is moved at a uniform velocity, V , in the x direction in front of the source it will receive a total dose $I_n(x_o, y_o, z_o)$ where x_o , y_o , and z_o are the dimensions within the target and the subscript n refers to the particular passage. The nearest passage to the source would be indicated by the subscript 1. A certain amount of simplification can be

FIG. 4.4.1

introduced at this point if it can be assumed that all target material for given values of y_0 and z_0 have the same irradiation time history in the x_0 direction. This would be true for any size target if the target material were introduced and removed from any single passage in regions of negligible dose rate. In view of the assumed symmetry relationships it would also be approximately true for any reasonably sized target. For the case where a large sized target is introduced in a region of significantly nonuniform dose distribution the reader is referred to the Appendix.

Consequently, it is assumed that for continuous irradiators, the dose variation in the target in the direction of travel is negligible. The mathematical expression for the first passage is given by:

$$I_1(y, z_0) = \frac{1}{V} \int_0^{\Delta x} D(x, y, z_0 + a) dx \quad (4.4.1)$$

where a represents the finite distance between the source and actual target material on the inside pass due to air gap, container walls, etc. The quantity Δx is the distance traversed by the target.

Since the target is not turned at the end of the passage, the z distance within the target on the opposite side of the source is equivalent to $(t - z_0)$ on the original side.

$$I_{-1}(y, t - z_0) = \frac{1}{V} \int_0^{\Delta x} D(x, y, t - z_0 + a) dx \quad (4.4.2)$$

where t is the package thickness and the subscript -1 indicates the symmetrically opposite passage to passage 1.

For the next pass one thickness away from the source,

$$I_2(y, z_0) = \frac{1}{V} \int D(x, y, z') dx \quad (4.4.3)$$

For the symmetrically opposite passage,

$$I_{-2}(y, t - z_0) = \frac{1}{V} \int D(x, y, z'') dx \quad (4.4.4)$$

For the n^{th} passage

$$I_n(y, z_0) = \frac{1}{V} \int D(x, y, z') dx \quad (4.4.5)$$

$$I_{-n}(y, t - z_0) = \frac{1}{V} \int D(x, y, z'') dx \quad (4.4.6)$$

where $z' = z_0 + (n-1)t + na$,

$$z'' = n(a+t) - z_0.$$

The total dose accumulated by the target can be obtained by summing all the contributions of the individual passages.

$$I_t(y, z_0) = \sum_1^n I_n(y, z_0) + \sum_1^n I_{-n}(y, t - z_0) \quad (4.4.7)$$

If the detailed flux field about the source is available, including the effects of boundaries such as air gap and container wall, the above integrations may be performed either analytically or numerically, depending on the nature of the information available. At this point it should be emphasized that detailed knowledge of the entire flux field is needed for the above integration.

Let us now derive an equivalent static system. First consider the interchange of motion of target and source. The same accumulated dose going from A to C (Figure 4.4.1) can be obtained by locating the target at B and moving the source from R to T at the same velocity V. Now note that the line source movement from R to T has generated a plane source. Thus, from Eq. (4.3.9) the source strength S_A of the generated plane source is related to the source strength of the line source by the relation:

$$S_A = \frac{S_L}{\Delta x} \quad (4.4.8)$$

where Δx is the length of passage. This is equivalent to the statement that the dose received at a point in the target for a transit time of $\Delta x/V$ due to a line source of strength S_L curies per unit length is the same as that which would be received at the same point in the target

when centrally located before a plane source of S_A curies per unit area of the same height as the line source, with a length equivalent to the passage distance and an irradiation time, $\theta = \Delta x/V$. The amount of experimental data needed for the equivalent static plane source is considerably less than for the actual line source case since only data for the centrally located region of the plane source are needed. This is especially important if there are several passages with gaps and container walls. Since it would be difficult to estimate the radiation flux at all points, direct experimental data would be needed.

Let a normalized dose rate, d , be defined by

$$d = \frac{\text{actual dose rate}}{\text{source strength}} = \frac{D}{S} \quad (4.4.9)$$

For a static system the dose I at a point is given simply by

$$I = D\theta \quad (4.4.10)$$

Now if at a certain point in a target, in a single pass, a dose I_n is required and a height of the line source has been selected, then for the equivalent static case with a plane source, it follows from Eqs. (4.4.9) and (4.4.10) that

$$I_n = S_A \theta d_n \quad (4.4.11)$$

where d_n is the experimental normalized dose rate for a plane source of height equal to that of the actual line source. If the irradiation time, θ , is set equal to the transit time, $\Delta x/V$, then

$$I_n = \frac{d_n S_A \Delta x}{V} \quad (4.4.12)$$

Combining Eq. (4.4.8) with Eq. (4.4.12),

$$S_L = \frac{VI_n}{d_n} \quad (4.4.13)$$

Thus the necessary curie strength of the line source, S_L , for any given linear velocity, V , and specified dose, I_n , can be obtained from the experimental normalized centrally located plane source data, d_n . The magnitude of d_n for a given height and fixed target position will

vary with length Δx which corresponds to the passage distance by the source. For a multiple pass unit as shown in Figure 4.4.1 there would be a different value of d for each pass, and the sum for all passes should be used in Eq. (4.4.13). An actual target is three-dimensional, but because of uniformity in the direction of travel (see symmetry conditions) only variations in dose rate in a plane section perpendicular to direction of travel need be considered. It will be noted that the equivalent static model requires data only in this central plane.

Another practical source is a plane or thin slab source. If we interchange the motion of target and source and consider the equivalent static source as before, we find as pointed out in the section on the equivalence of moving and static systems, that the equivalent static source is one of variable source strength as shown in Figure 4.3.5. As also pointed out in that section, this type of variation can be approximated experimentally by overlapping thin sheets of source material. The same effect can be obtained with a single source plane by changing the position of the source plane at constant time intervals or by changing the position of the detector to simulate the above condition. However, for two important cases only the normalized dose rate in a centrally located position relative to a slab or plane source is needed for design purposes. One case is for a narrow slab source for which the discussion for a line source holds. The other case is represented by a plane source which is small relative to the distance traveled by the target material or for which the dose rate at the initial target position is small compared to the value immediately before the source. For this case the region of variable source strength contributes a negligible amount of radiation to the target material and the assumption of a plane source of uniform strength is permissible. The relation between the strength of the actual source and that of the static equivalent is given by the simple relation given previously:

$$S_s = \frac{S_m N}{\Delta x} \quad (4.3.11)$$

where S_s and S_m are the source strengths of the static and moving sources, and N is the

length of the moving source (or stationary source in moving target system), and Δx is the distance traversed in the irradiator. It should be pointed out that the second case described above is a very practical one since for high efficiency, the initial target position should be in a region of low intensity.

Similar to the development for the line source,

$$I_n = \frac{S_s \Delta x d_n}{V} = \frac{S_m N d_n}{V} \quad (4.4.14)$$

$$S_m = \frac{V I_n}{N d_n} \quad (4.4.15)$$

Thus, for specified values of total dose, velocity of target and source length, the required source strength for a moving target system can be determined from the normalized dose rate obtained in a centrally located position before a plane source of the same height as the moving source and a length equivalent to the travel distance in the irradiator if the approximation discussed above holds. Eq. (4.4.15) is written for a single pass and the sum of the specific dose rates for all the passes in an irradiator should be used to calculate the over-all dose, I .

Multiple Sources. Since any mass within an irradiator absorbs radiation, it is desirable to reduce the mass of nontarget material to a minimum. In this respect it is better to have a single source of high specific activity than a number of sources of similar dimensions but of lower specific activity. However, in some cases it is necessary for various reasons to use multiple sources. A multiple slab source is shown in Figure 4.4.2. In the irradiation of canned foods, the "running bug" problem may require that the food be irradiated simultaneously from both sides, which would necessitate multiple sources. The "running bug" problem refers to the possibility that in a fluid medium bacteria could move in a manner such that they are always in the region of minimum dose rate. Thus, if the material were not irradiated simultaneously from both sides, the bacteria presumably by

FIG. 4.4.2

such movements could avoid a lethal dose.

Obviously, if the entire flux field is known for the arrangement shown in Figure 4.4.2, the dose at any point in a target can be calculated by integrating with respect to time along the path shown. It would be convenient to relate the multiple source case to the single source case described above. It is assumed here that the multiple sources are all identical. It will be noted from Figure 4.4.2 that over-all symmetry exists for the arrangement of sources and passes, but there is a lack of symmetry for any one source. For a system of "m" sources and "p" total passes (previously for a single source "n" represents the number of passes on one side of the source), it can be seen from Figure 4.4.2 that there will be a total of "m" one-sided contributions to a position in a target moving in a passage adjacent to a source. This is also valid for positions to and including $(p-m+1)/2$ (passages away from a source). For the next passage beyond this there will be only m-1 contributions, the next m-2, etc. Thus, as a first approximation one can write

$$I_{ms} = m \sum_{j=1}^k (I_j + I_{-j}) + \sum_{q=1}^{m-1} (m-q)(I_L + I_{-L}) \quad (4.4.16)$$

where I_{ms} = dose for multiple source system,

m = number of identical sources,

k = $(p-m+1)/2$,

p = total number of passes,

L = k+q, and

I_j, I_L, I_{-j}, I_{-L} are one-sided single source contributions.

Eq. (4.4.16) provides a simple way of studying the relative curie and specific activity requirements for different arrangements of sources and passes. Note that for the central position in a slab $I_j = I_{-j}$ and that I_{ms} is to be a fixed quantity. For simplicity, we may compare any arrangement for which "m" = 1 and for which the second term drops out

since "m-1" = 0. Equating the two expressions for the case $I_j = I_{-j}$

$$\sum_{j=1}^n (I_j) = m \sum_{j=1}^k (I_j) + \sum_{q=1}^{m-1} (m-q)I_L \quad (4.4.17)$$

Now note that for given target and source sizes the dose for a given passage is proportional to a specific activity and a normalized dose "i." Regardless of the arrangement (neglecting self-absorption in the sources), i_j would be a constant for a given passage. Thus, the above equation may be arranged to the following form:

$$\frac{S_{ss}}{S_{ms}} = \frac{m \sum_{j=1}^k i_j + \sum_{q=1}^{m-1} (m-1)i_L}{\sum_{j=1}^n i_j} \quad (4.4.18)$$

The subscripts ss and sm refer to single and multiple source, respectively. Both numerator and denominator may be divided by the normalized dose for a particular passage, say i_1 , and thus relative results may be obtained requiring only the knowledge of the ratios of normalized doses in different passages. If one assumes, for example, the attenuation is exponential, a large number of arrangements may be compared without detailed calculations. Since the above form does not include self-absorption, the results should be considered to be a first approximation.

If the absorption of radiation by sources is significant, then contributions to passages the same distance away in general are not equivalent and should be determined separately. For example, in Figure 4.4.2 for passages two target thicknesses away from a source, two involve absorption of radiation by a source while the other does not. The static equivalent case for the above must also include any source which absorbs radiation.

Slab Target - Multiple Pass in Vertical Direction

The multiple pass systems referred to in the previous sections involved only a single

pass of material for any given distance normal to the source plane; multiple here referring to passes at different normal distances from the source. Uniformity of dose in the vertical direction within a target can be improved if the target material makes two passes before a source at the same normal distance from the plane source. This is shown schematically in Figure 4.4.3.

This is an end view of the arrangement at only one normal position on each side of the source. A multiple arrangement with other normal (perpendicular) passages could obviously be used to increase over-all efficiency. As shown, the target material enters at A traveling in a direction into the plane of the paper. Due to the relative position of the source, that portion of the target at "a" will receive a smaller dose than that at "b." At the end of the passage the target is raised to position B and returns by the source in the same normal passage but in the elevated position B. It will be noted that portion "a" of the target which on the forward pass was in a low dose rate zone, now is in the high dose rate zone previously occupied by "b," and vice versa. At the end of the return pass B the target is shifted horizontally to C on the opposite side of the source and the same type of operation is repeated, the target finally leaving the system at D.

For an ideal arrangement of source and target the dose received at a given normal distance from the source as a function of the vertical height y is shown in Figure 4.4.4a. At the end of the forward pass at A the dose distribution is as shown with "a" receiving a small dose and "b" a large one. The opposite arrangement is shown for passage B. If these are added, the total dose at T is seen to be ideally constant with respect to y . If the relative arrangement of sources and target is not properly chosen a situation shown in Figure 4.4.4b will result in which the total dose T is not uniform with respect to y . For best uniformity the source height should be approximately equal to the target height.

The above arrangements are at some particular normal plane "z." Even if the ideal

FIG. 4.4.3

HT. Z
ON
PKG.

FIG. 4.4.4 a

HT. Z
ON
PKG.

FIG. 4.4.4 b

arrangement shown in Figure 4.4.4 is realized at this plane, in general it would not exist at some other normal plane. If the target is not too thick the variation would not be great. This will be discussed in more detail in the discussion of uniformity.

It will also be noted that the same total absorbed dose could have been obtained by passing a combined target equivalent to $A+B$ in a single pass on each side of the source at one-half the velocity. From this analysis it can be seen that the penalty paid for the improved uniformity is a more complex and more rapidly moving conveyor system.

Slab Target - Intermittent Motion of Source and/or Target

Stationary Source - Lateral Target Motion. The irradiator shown schematically in Figure 4.4.1 can be operated in a series of constant time intervals rather than continuously with the same over-all throughput. In this method of operation one or several unit packages move in discrete steps, remaining in each position for a definite incremental exposure time, $\Delta\theta$. If we neglect the time required to move the target material from one position to another, the total dwell time will be $N_T \Delta\theta$, where N_T is the total number of discrete steps in a normal passage. Figure 4.4.5 shows a single passage on one side of a slab source consisting of three discrete positions. The dose accumulated during this passage, for a point (x_o, y_o, z_o) or (x_o, y, z_o) within the target, can be represented as follows:

$$I_1(x_o, y, z_o) = [D_{1,0}(x_o, y, z_o + a) + D_{1,1}(x_o + L_T, y, z_o + a) + D_{1,(-1)}(x_o - L_T, y, z_o + a)] \Delta\theta \quad (4.4.19)$$

z_o is measured from the front surface of the target on pass 1 and $y = y_o$, the vertical distance being measured from the center of the target. Note that $D_{1,1}$ and $D_{1,(-1)}$ are not equivalent but their sum is the same for x_o or $-x_o$. This is similar to the effect of irradiating a slab identically from both sides in the z direction. In the above notation the first subscript on D refers to the passage, and the second to the position along that passage, $N = 0$ being the central position.

FIG. 4.4.5

If the total number of stops in a passage is odd, then from symmetry relations, one stop should be immediately in front of the source as shown in Figure 4.4.5. This position is shown as $N = 0$. Thus the total number of stops will be $2N+1$, where N is the number of stops beyond the central position on one side of the source. If there is an even number of stops there will be no single central stop and stops will be symmetrically located relative to the source as shown in Figure 4.4.6. For this case the $N = 0$ position will not be present and the total number of stops will be $2N$.

Eq. (4.4.19) may be generalized for an arbitrary number of passages and positions. For passage "n" and an odd total number of positions, $N_T = 2N+1$,

$$I_n(x_o, y, z_o) = \left[D_{n,0}(x', y, z') + \sum_{N=1}^{N_T-1} \left\{ D_{n,N}(x', y, z') + D_{n,-N}(x', y, z') \right\} \right] \Delta\theta \quad (4.4.20)$$

where $x' = x_o \pm N(L_T + b)$

$z' = z_o + na + (n-1)t$

a = air gap, container wall thickness, etc.

b = spacing to allow for air gap, container walls, etc., in x direction

t = target thickness

L_T = target length

If n is replaced by $-n$ and z' by z'' in the above relation, the corresponding relation for

$I_{-n}(x_o, y, z_o)$ is obtained:

$$z'' = n(a+t) - z_o \quad (4.4.21)$$

The total dose, I_T , is now given by the relation:

$$I_T = \Sigma(I_n + I_{-n}) \quad (4.4.22)$$

Note that I_T is now a function of all three dimensions.

The corresponding relations for an even number of positions would be identical after

FIG. 4.4.6

omission of the $D_{n,0}$ term and substitution of $x'_0 = x_0 + (L_T/2)$ for x_0 .

Similar to the case for continuous irradiators an average dose rate, $\bar{D}_n$, for a passage may be defined by:

$$I_n = \bar{D}_n N_T \Delta\theta \quad (4.4.23)$$

where the total number of positions, N_T , during passage is $2N+1$ if odd, and $2N$ if even, and the dwell time per stop is $\Delta\theta$.

The definition for $\bar{D}_n$ may be obtained by substituting the corresponding expression for I_n from Eq. (4.4.20) into Eq. (4.4.23).

In terms of a normalized dose rate,

$$I_n = S_A \bar{d}_{nA} N_T \Delta\theta \quad (4.4.24)$$

Completely analogous to the continuous case we may write

$$I_{\min} = 2S_A N_T \Delta\theta \sum_{1}^n \bar{d}_{nA(\min)} \quad (4.4.25)$$

The factor 2 arises from the symmetrical irradiation of the target on both sides of the source and $\bar{d}_{nA(\min)}$ is the normalized average dose rate for the point within the target receiving the minimum total dose.

The capacity relation for continuous irradiators [Eq. (4.4.30)] can also be used for the intermittent case if the velocity is interpreted as being an average value defined by:

$$\bar{V} = \frac{\Delta x}{N_T \Delta\theta} \quad (4.4.26)$$

Relations for S_A and C given by Eqs. (4.4.42) through (4.4.44) are also applicable if $I_{\min}$ and $\bar{d}_{nA}$ are defined by the relations given above.

Withdrawn Source - Target Motion Perpendicular to Source. Another method of operating in an intermittent system is illustrated in Figure 4.4.7. A target is irradiated for a time $\Delta\theta$ at a position in front of a source, the source is then withdrawn to permit move-

FIG. 4.4.7

ment of the target from A to B, then the target C moves into the A position and the process is repeated.

The mass flow rate for this arrangement may be written as:

$$W = \frac{H_T t_p L_T}{\Delta \theta} \quad (4.4.27)$$

This is a special case of the type shown in Eq. (4.4.30) with $N = 0$ being the only contribution for each passage. In Figure 4.4.5 it is assumed that the target will be moved in negligible time to the opposite side of the source, whereas in Figure 4.4.7 the source moves to allow the target to pass. The difference between the two cases is one of dose uniformity. For the case shown in Figure 4.4.7 the uniformity of dose would be poor unless considerable source overlap or source shaping on all sides were provided, whereas in Figure 4.4.5 the lateral motion increases the uniformity of dose in the direction of motion (if more than the $N = 0$ position is occupied).

Stationary Source - Vertical and Lateral Motion of Target. Referring to Figure 4.4.5, it will be noted that for appropriate source and target dimensions the uniformity in the direction of motion can be quite good. The effect of lateral motion is similar to that shown in Figure 4.4.3 for vertical motion in a single passage for a continuous irradiator. An intermittent motion may also be imposed in the vertical direction to further improve uniformity of dose. The special case which is usually referred to as a quadrant irradiation is illustrated in Figure 4.4.8. The motion on one side of the source from a point view above the source plane is shown in Figure 4.4.8. A square target moves into position A where it is irradiated for a time $\Delta \theta$ with corner 1 receiving the maximum dose and corner 4 the minimum. At the end of this time period it is moved to position B and again irradiated for a time $\Delta \theta$. At this position corner 3 receives the maximum dose and corner 2 the minimum. This is followed by movement to C and eventually to D. When the cycle is completed all corners will have received the same dose. By suitable dimension selection, the dose in the middle of the

FIG. 4.4.8

target will be approximately equal to the dose at the corners. The entire process is repeated for the other side of the source so that each target face receives the same dose.

If we assume that the minimum dose is at the center of an individual target then

$$I_{\min} = 8D_c(\Delta\theta) = 8S_A d_c(\Delta\theta) \quad (4.4.28)$$

or

$$S_A = \frac{I_{\min}}{8d_c(\Delta\theta)} = \frac{WI_{\min}}{8(H_T)^2 t \rho d_c} \quad (4.4.29)$$

where d_c is the normalized dose rate for the center of the target. One may assume more than one perpendicular passage on each side of the source to increase the efficiency.

The concept of quadrant irradiation can be successfully used for different arrangements but there will be a variation in the uniformity ratio at different positions depending on the size of the square, the density of target material, and the radiation energy. It is quite evident that the over-all uniformity ratio is strongly affected by the package thickness.

When an actual operating system is considered, clearance between targets must be provided which may significantly affect the geometry. It may therefore be advantageous to make the source and target shapes as squares of different sizes (see Figure 4.4.9).

Even with no provision for target clearance it will be noted that for large squares the only corner receiving a significant amount of radiation is corner 1 near the center of the source. For a single traverse of the target on one side, the net dose received by this corner would be I . The dose at "a" would be $2I_a$ since this point in the target would reside in a radiation zone twice during each traverse.

For perfect uniformity $2I_a$ must equal I , or the dose at "a" should be one-half that at position 1. Measurements show this to be approximately true with experimental ratios of I_a/I for reasonably sized plaques ranging from 0.5 to 0.7. The ratios were measured as a function of depth in a water target for two approximately square-shaped cobalt-60 plaques.

FIG. 4.4.9

The results are listed in Table 4.4.1.

That I_a/I increases with decrease in target size and with increase in depth is clearly shown in this table. For smaller plaques, however, the dose contribution at the other positions becomes significant and the uniformity cannot be deduced from the ratio alone. The figures also indicate that for a multipass system the uniformity ratio between any two positions will be different for different passes.

Capacity Considerations in Irradiator Design

One quantity that must be specified for an irradiator is the mass throughput per unit time. If this is a variable item, then additional information is needed to determine if an average rate with adequate storage holdup to take up surges is satisfactory, or if the design must satisfy the maximum capacity. Where holdup for even a short period of time impairs the quality of the target material, the irradiator should be designed for peak rates. The capacity relation for an irradiator is simply expressed by the following:

$$W = VtH_t\rho_t \quad (4.4.30)$$

or solving for the velocity,

$$V = \frac{W}{tH_t\rho_t} \quad (4.4.31)$$

where W = mass flow rate (lb/hr)

t = target thickness (ft)

H_t = target height (ft)

ρ_t = average target density (lb/ft³)

V = linear velocity of moving target (ft/hr)

For the case shown in Figure 4.4.3 where there are two passages in the vertical direction, the height, H_t , refers to a single passage.

In the above equations W and ρ_t will be known for any given design. The target thickness, t , will either be fixed by the physical size of the unit material being irradiated (e.g.,

Table 4.4.1

Ratio of Central Dose Rate to That at Edge of Midpoint, I_a/I

<u>Depth, in.</u>	<u>Plaque size</u>	
	<u>40 × 33 in.</u>	<u>20 × 16 in.</u>
2	0.511	0.525
3	0.523	0.559
4	0.530	0.590
5	0.546	0.609
6	0.560	0.626
8	0.578	0.650
10	0.589	0.677
12	0.562	0.738

oranges, grapefruit) or by a specified uniformity of dose. Usually an over-all uniformity of dose is specified, but if we assume at this point that vertical nonuniformity will be corrected by one of several techniques available, the nonuniformity will then be due to variation in the z direction or thickness of the target. At this point of consideration we may then assume that the specified uniformity ratio and known ρ_t fixes the thickness, t. Thus the linear velocity of the target system will be fixed if the target height is fixed.

Another specification given for any target-irradiator system is the minimum dose, $I_{\min}$, which the target material must receive. Consider at this point the dose received at a particular position in the target material as a result of one pass through the irradiator. From previous relations (in abbreviated form)

$$I_n = \frac{1}{V} \int D_n dx \quad (4.4.32)$$

where D_n is the dose rate as a function of x in the particular y and z positions selected. An average dose rate $\bar{D}_n$ for the passage may be defined by the following:

$$I_n = \frac{\Delta x}{V} \bar{D}_n \quad (4.4.33)$$

In a similar manner a normalized dose rate is defined by the following:

$$\bar{d}_n = \frac{1}{S \Delta x} \int D_n dx = \bar{D}/S \quad (4.4.34)$$

The dose I_n will be directly proportional to the source strength for a particular geometry; hence the above may be expressed in normalized form.

$$I_n = \frac{\Delta x}{V} \frac{\bar{d}_{na} C}{A} \quad (\text{area source}) \quad (4.4.35)$$

$$I_n = \frac{\Delta x}{V} \frac{\bar{d}_{nL} C}{L} \quad (\text{line source}) \quad (4.4.36)$$

where C = total-curies

A = area of source, cm^2

$\bar{d}_{na}$ = normalized average dose rate (rads/hr per C/cm²)

$\bar{d}_{nL}$ = normalized average dose rate (rads/hr per C/cm²)

or, in terms of specific activity

$$I_n = \frac{\Delta x}{V} S_A \bar{d}_{na} \quad (\text{area source}) \quad (4.4.37)$$

$$I_n = \frac{\Delta x}{V} S_L \bar{d}_{nL} \quad (\text{line source}) \quad (4.4.38)$$

By using the notation in section (1) the total dose received at that particular position which corresponds to the minimum dose will be [from Eq. (4.4.7)]

$$I_{\min} = \sum_1^n (I_{n\min} + I_{-n\min}) \quad (4.4.39)$$

For slab type geometry in which the symmetry conditions discussed previously are met and in which some method is used to reduce the nonuniformity of dose in the y direction, the minimum dose will lie in the central plane of the slab; hence, $I_{n\min} = I_{-n\min}$ and

$$I_{\min} = 2 \sum I_{n\min} \quad (4.4.40)$$

where n is the number of passages on one side of a single source. From Eqs. (4.4.37) and (4.4.40)

$$I_{\min} = \frac{2\Delta x}{V} S_A \sum_1^n \bar{d}_{na} \quad (4.4.41)$$

A similar relation may be written for a line source. If the above relation is combined with Eq. (4.4.30) and solved for the specific surface activity, S_A , the following result is obtained

$$S_A = \frac{WI_{\min}}{2tH_t \rho_t \Delta x \sum_1^n \bar{d}_{na}} \quad (4.4.42)$$

The quantity WI represents a combined capacity term of mass and absorbed radiation and is

frequently designated as a single term, Q , in lb-rad/hr.

$$S_A = \frac{Q_{\min}}{2tH_t\rho_t\Delta x \sum_{n=1}^n \bar{d}_{na}} \quad (4.4.43)$$

Since $S_A = C/A$

$$C = \frac{Q_{\min} H_s L_s}{2tH_t\rho_t\Delta x \sum_{n=1}^n \bar{d}_{na}} \quad (4.4.44)$$

where H_s = height of source, cm

L_s = length of source, cm

It should be noted at this point that by previous convention specific surface activity is reported in units of curies/cm² and thus the source dimensions are usually given in units of cm. On the other hand, velocity and target height are frequently given in units of ft/hr and ft. It is assumed that the reader will use a consistent set of units in applying Eqs. (4.4.43) and (4.4.44).

The above relations apply for a single source arrangement. For multiple source arrangement the following relation for $I_{\min}$ may be used instead of Eq. (4.4.40) [see Eq. (4.4.16)].

$$I_{\min} = 2m \sum_{j=1}^k I_{j\min} + 2 \sum_{q=1}^{m-1} (m-q) I_{L\min} \quad (4.4.45)$$

Eq. (4.4.45) neglects absorption of radiation by the sources.

For a given source arrangement and target passage the integrated dose increases as Δx is increased up to a certain limiting value; further increase in the travel distance Δx will have no effect on the integrated dose. From Eq. (4.4.34) it is seen that when this point is reached, $\bar{d}_n$ will vary inversely as Δx . Hence if the target is introduced into the irradiator in a low flux region the quantity $(\Delta x \sum \bar{d}_{na})$ is essentially constant for a given source

arrangement. For this case it is seen that for a given capacity the specific activity required is inversely proportional to the height of the target and since the height of target is inversely proportional to the velocity for a fixed target thickness, t , the specific activity is also directly proportional to the velocity.

The corresponding relations for a line source would be

$$S_L = \frac{Q_{\min}}{2tH_t \rho_t \Delta x \Sigma \bar{d}_{nL}} \quad (4.4.46)$$

$$C = \frac{Q_{\min} H_L}{2tH_t \rho_t \Delta x \Sigma \bar{d}_{nL}} \quad (4.4.47)$$

From relations (4.4.12) and (4.4.13) given previously for the static equivalent case and Eq. (4.4.38), it can be shown that the relationship between the normalized dose rates for the static equivalent case and the actual case is:

$$d_{ns} = \Delta x \bar{d}_{nL} \quad (4.4.48)$$

and

$$S_L = \frac{Q_{\min}}{2tH_t \rho_t \Sigma d_{ns}} \quad (4.4.49)$$

$$C = \frac{Q_{\min} H_L}{2tH_t \rho_t \Sigma d_{ns}} \quad (4.4.50)$$

For a rectangular area source using the approximate relationships given by Eqs. (4.3.11) and (4.4.14)

$$L_s d_{ns} = \Delta x \bar{d}_{na} \quad (4.4.51)$$

$$S_A = \frac{Q_{\min}}{2tH_t \rho_t L_s \Sigma d_{ns}} \quad (4.4.52)$$

$$C = \frac{Q_{\min} H_S}{2tH_t \rho_t \Sigma d_{ns}} \quad (4.4.53)$$

In the above expression for total curies the normalized dose rate, d_{ns} , approaches a limiting value for a given source arrangement as the travel distance, Δx , is increased. Since $Q_{\min}$ and ρ_t are specified and t will be fixed by uniformity conditions, the above limiting case where Σd_{ns} is constant clearly shows that a reduction in total curies can be obtained by reducing the ratio of source height to target height. It should be noted that a reduction in H_S at constant H_t will also cause a reduction in d_{ns} , whereas an increase in H_t at constant H_S will not significantly affect d_{ns} . A decrease in the ratio H_S/H_t for a uniform source strength will also decrease the uniformity of dose. This may be overcome by a multiple vertical pass arrangement as shown in Figure 4.4.3.

Uniformity Considerations

Uniformity between any two points in a target is defined as the ratio of the respective doses at those two points. If every point in a target could be exposed to the same integrated flux, the dose absorbed would be uniform. For a large slab source the flux pattern in a plane parallel to the slab deviates from uniformity only at the boundaries. This is shown in Figure 4.4.10.

The nonuniformity of dose rate distribution in the x direction of travel will not introduce perturbations in the targets since every point along a line parallel to the source can be made to pass through the same flux field. For a single slab the flux pattern in the z direction (normal to source plane) is obviously nonuniform, decreasing with target depth as shown in Figure 4.4.11. Uniformity in this direction may be improved (1) by passing the target between parallel sources, and (2) by passing the package outside a single source in such a manner that each side of the target receives the same dose. The rule of symmetry discussed

TYPICAL VERTICAL DOSE RATE DISTRIBUTION
AT A FIXED DEPTH IN TARGET.
NO SOURCE OVERLAP
FIGURE 4.4.10

FIGURE 4.4.11

previously is then applicable. As indicated, this may be effected by moving the target sideways without turning at the end of a pass. If the target thickness is a variable, uniformity may also be improved by reducing the thickness. Interposition of absorber between source and target will improve uniformity, at the expense of efficiency, by reducing the flux gradient. Similarly, an increase in source thickness will decrease nonuniformity again at the expense of efficiency for the same reason.

The uniformity with depth may also be altered by the selection of isotopes of different energy for a fixed target density. The total linear and mass absorption cross sections decrease as E^{-1} in the range 100 kev to 3 Mev for low atomic number targets. Therefore, the uncollided flux or dose variation with depth would be flatter with increase in photon energy. However, buildup due to multiple scattering would decrease with increase in energy and the latter process may tend to cancel the effects of the former. Thus it has been the experience at BNL that for broad radiation sources of cesium-137 or cobalt-60 the depth dose variation was more or less independent of the photon energy. In the case of sodium-24 sources with photon energies of 1.38 Mev and 2.76 Mev, some flattening of the depth-dose curve was observed over that obtained for either cesium-137 ($E = 0.662$ Mev) or cobalt-60 ($E = 1.17$ and 1.33 Mev). The remarks made immediately above apply mainly to slab sources.

Uniformity in the y direction can be improved by source shaping, source overlap, or target overlap. The first technique is achieved by increasing the specific activity or thickness of the source boundaries to compensate for the lower flux in this area. Source overlap is defined as

$$\frac{H_s - H_t}{2} \quad (4.4.54)$$

Target overlap may be defined as

$$\frac{H_t - H_s}{2} \quad (4.4.55)$$

H_t is taken to be the total height of target material in a single pass regardless of changes in

direction of individual packages, i.e., for Figure 4.4.3 the effective H_t will be twice that of one package. This particular method has the advantage that it reduces the total curie requirement. As emphasized previously, it does lead to a more complex and faster moving conveyor system.

To summarize, for slab targets moving in continuous linear motion, improvements in uniformity are achieved by:

- (1) maintaining constant linear motion in the direction of travel in a symmetrical source-target system.
- (2) irradiating target materials symmetrically
 - (a) for direction of target travel normal to plane source by irradiating both sides of target under identical conditions.
 - (b) in a direction perpendicular to motion and in a plane parallel to the source by target interchange at the end of a pass in a double target system, or by multiple passages in one normal position as shown in Figure 4.4.3.
- (3) altering source in such a way as to provide a more uniform flux field, especially near boundaries by one of the following techniques:
 - (a) varying the specific activity of the source.
 - (b) varying the amount of source material at a constant specific activity.
 - (c) varying the source-target distance by source shaping or overlap.
- (4) reducing target thickness.
- (5) introducing absorbing material between source and target.

Efficiency

The efficiency of primary interest is that usually referred to as "utilization efficiency" and is defined by the relation:

$$E_u = \frac{\text{energy absorbed in target at minimum dose}}{\text{total electromagnetic energy emitted from source}}$$

Losses in efficiency may be classified separately according to their origin as follows:

- (1) self-absorption in sources and cladding
- (2) absorption in nontarget material outside the source such as container walls, conveyor parts, etc.
- (3) energy escaping the system as unabsorbed radiation
- (4) excess energy absorbed in target material above the required minimum dose

The above outline immediately suggests methods of improving the efficiency of an irradiator, since a reduction in any of the items will accomplish this. For item (1), a reduction in self-absorption for a given unit surface may be effected by reducing the thickness of source material and increasing the specific activity by a corresponding amount. Obviously, minimum cladding commensurate with safety aspects should be used. Since high specific activity isotopes are usually more expensive than low ones, the problem becomes an economic one. The subject of source fabrication will be treated in detail in a subsequent publication. In reference to item (2), structural materials and target containers of low density such as aluminum should be used where conditions permit.

In principle, loss due to item (3) can be essentially eliminated by surrounding the source with target material in all directions. However, to do this for a continuous irradiator would require an extremely complicated and elaborate conveyor system. For a system such as shown in Figure 4.4.1, it would not be difficult to provide enough passages in a direction normal to the source plane to eliminate radiation leakages in this direction. Since the flux decreases more or less exponentially with distance, few passages would be required. For a 6-in. target of unit density no more than three passages would be necessary. However, a large amount of radiant energy will escape in a vertical direction. This loss can be reduced by making the target overlap the source. For a single target system (in the vertical direction) this will decrease the uniformity of dose and loss due to item (4) will be increased.

However, by using a multiple target system in the vertical direction and using either multiple passage (Figure 4.4.3), or target interchange at the end of a pass, the loss of radiation in this direction can be greatly reduced without seriously affecting the uniformity.

An increase in efficiency accomplished by a decrease in radiation leakage usually results in the following:

- (1) possible increased physical size of irradiator
- (2) more intricate conveyor system
- (3) increased dwell time of target material in the irradiator

Due to the nature of the target material, it may be necessary to limit the dwell time. Thus, it may not be possible within the specification of dose, uniformity, dwell time, and capacity to fully utilize this method of increasing irradiator efficiency.

It should be noted that the use of source shaping and the insertion of absorbers between source and target to improve uniformity do so at the expense of loss in efficiency, since any increased activity at the boundaries to compensate for low dose rates also results in an increased radiation leakage, and any radiation absorbed by nontarget material is a direct loss.

A quantitative relation for E_u is as follows:

$$E_u = \frac{k_1 Q_{\min}}{k_2 C} \quad (4.4.56)$$

where Q = capacity, megarad-lb/hr

C = total source strength, curies

k_1 = power absorption conversion factor = 1.26 watts per megarad-lb/hr

k_2 = isotopic gamma power, watts/curie = 0.0148 for cobalt-60

= 0.00324 for cesium-137

5. COMPREHENSIVE IRRADIATOR DESIGN PROBLEMS

5.1 Problem No. 1

The use of available calculation methods and techniques is best illustrated by solving an irradiator design problem with some typical specifications.

A comprehensive theoretical calculation was conducted for the following problem specifications:

W , capacity = 1000 lb/hr

$I_{\min}$, minimum total dose = 4.5×10^6 rads

ρ_T , target density = 1.0 g/cm³

$(D_{\max}/D_{\min})_{\text{over-all}}$, uniformity ratio = 1.25

$\theta_{R(\max)}$, maximum residence time = 90 min

Approach and Assumptions

The parameters and design variables to be determined for this problem are:

1. source size and shape,
2. target size and shape,
3. total source activity and specific activity, and
4. irradiator efficiency.

The problem was approached in the following way:

For simplification, it was decided to use a rectangular source irradiator and rectangular box targets. The target height was fixed at 20 in. and the target height was arbitrarily selected as 16 in. The target thickness and source height are controlled by the uniformity specification. These parameters may only be determined by trial and error; however, a knowledge of the depth dose distribution for infinite plane sources (infinite in length and width) in infinite media of the same density permits one to eliminate target thicknesses >6 in. It was thus decided to select target thicknesses of 4, 5, and 6 in. and to determine the

amount of source overlap required to meet the design specifications of uniformity for a single source-two pass irradiator.

Package velocity, total package travel, and pass length are determined by the mass rate of flow, the target cross sectional area, target density, and the residence time.

Package Velocity. The package velocity is determined by the following equation:

$$V = \frac{W}{\rho H_t t} \quad (5.1.1)$$

for $t = 4.0$ in.

$$V = \frac{1000 \text{ lb/hr}}{62.5 \text{ lb/ft}^3 \times \frac{20.0}{12} \text{ ft} \times \frac{4.0}{12} \text{ ft}} = 28.8 \text{ ft/hr}$$

for $t = 5.0$ in.

$$V = 28.8 \times \frac{4}{5} = 23.04 \text{ ft/hr}$$

for $t = 6.0$ in.

$$V = 28.8 \times \frac{4}{6} = 19.20 \text{ ft/hr}$$

Total Package Travel. The total package travel is given by the product of velocity times residence time, assuming that the time required to push the package from pass 1 to pass 2 is negligible.

Since residence time was fixed at 90 min, the calculations are given as follows:

$$\Sigma L_t = V \times \theta_R$$

$$t = 4 \text{ in.} \quad \Sigma L_t = 28.8 \times \frac{90}{60} = 43.2 \text{ ft} = 518.4 \text{ in.}$$

$$t = 5 \text{ in.} \quad \Sigma L_t = 23.04 \times \frac{90}{60} = 34.56 \text{ ft} = 414.72 \text{ in.}$$

$$t = 6 \text{ in.} \quad \Sigma L_t = 19.20 \times \frac{90}{60} = 28.80 \text{ ft} = 345.6 \text{ in.}$$

Pass Length. The length of travel on each pass is given by $\Sigma L_t/n$, where n is the

number of passes. The pass length is calculated by assuming instantaneous package introduction and removal, and is given by the following equation:

$$L_p = \frac{\sum L_t}{n} + L_t = \frac{\sum L_t}{n} + 16 \text{ in.}$$

Values of pass length for target thicknesses of 4, 5, and 6 in. are tabulated below.

<u>t, in.</u>	<u>L_p, in.</u>
4	275.2
5	223.4
6	188.8

Specific Dose and Dose Rate Distribution

Prior to calculating the dose and dose distribution in the target, the source size and thickness had to be tentatively fixed. From the total minimum dose of 4.5×10^6 rads and a production of 1000 lb/hr, the power input is calculated as 5.67×10^3 w (1 megarad-lb/hr is equivalent to 1.26 w). If one assumes a utilization efficiency of about 25% the total curie requirement will be:

$$C = \frac{5.67 \times 10^3 \text{ w}}{14.8 \text{ w/kC cobalt-60}} \times \frac{1}{0.25} = 1.53 \times 10^6 \text{ C}$$

Assuming that one desires to use a specific activity of about 50 C/g, that the height of the source will be about 30 in., and that the thickness of a single source element is 0.060 in., one can calculate the source length, L_s , for source plaques composed of 1, 2, . . . , j thicknesses of source elements.

$$S_g = \frac{1.53 \times 10^6}{L_s \times 30 \times j \times 0.060 \times (2.54)^3 \times 8.9} = 50 \text{ C/g}$$

Solving for L_s ,

$$L_s = \frac{1.53 \times 10^6}{30 \times 0.060 \times (2.54)^3 \times 8.9 \times 50} = 116.56$$

Thus for $j = 1, L_s = 116.56 \text{ in.}$

$j = 2, L_s = 58.28 \text{ in.}$

$j = 3, L_s = 38.85 \text{ in.}$

The self-shielding thickness in mean free paths for a source composed of three laminations of elements assuming a total cladding thickness per element of 0.020 in. would be (see Figure 5.1.1):

$$\begin{aligned}\Sigma \mu_t z &= \frac{0.060 \times 3}{2} \text{ in. } (1.220 \text{ in.}^{-1}) + 0.020 \times 4 \text{ in.} \times 1.070 \text{ in.}^{-1} \\ &= 0.1954\end{aligned}$$

The water equivalent thickness would thus be:

$$\frac{0.1954}{0.163 \text{ in.}^{-1}} = 1.199 \text{ in., since the value of } \mu_t = 0.163 \text{ in.}^{-1} \text{ for water}$$

at an average photon energy of 1.25 Mev.

However, for the purposes of this illustrative problem it was decided to fix the equivalent self-shielding of the source and cladding at one inch of water.

The calculation of the total dose throughout the target can be done in two ways, either by integrating the dose from the real, finite source throughout the target travel or by using a fictitious source equal in length to the target travel and maintaining the same curie content. Since the maximum target travel length depends on the package thickness and the dose uniformity depends upon source height or overlap as well as target thickness, it was decided to calculate the dose for a target thickness of 4 in. (later it turned out that this is the maximum thickness for which one can meet the uniformity requirements with an infinite source) and the for source overlap of target of 0, 4, and 6 in. The source length was taken as 36 in. Since the target height was 20 in., the basic source size was $20 \times 36 \text{ in.}$ As previously noted, the package travel distance is equal to 518.4 in. for a 4-in. package thickness. The travel on one side of the source in a two pass-one source system is $518.4/2 = 259.2 \text{ in.}$, which was

CROSS-SECTION OF TRIPLY STACKED COBALT 60
SOURCE - EACH SINGLY ENCAPSULATED
FIGURE 5.1.1

rounded off to 260 in. Since the total pass length equals $260 + 16 = 276$ in. to allow for one package to enter the irradiator and one to leave, values of dose rate out to values of $x = 138$ in. from the center of the source are needed for the integration calculation.

The first case to be considered was the zero overlap case. In this calculation, basically all the radiation coming directly from the source must enter the front face of the targets. Therefore, it can be assumed that the conditions of a finite plane source in an infinite medium may be met if the finite thickness of the packages is neglected.

A study of the computer program, FUDGE 3A (Appendix V) shows that one can handle the case of zero overlap directly; however, when the source height extends beyond the target, some of the radiation travels through air and enters the top edge of the target. The portion of the source that overlaps the target is divided into a number of line sources parallel to the target travel and the dose contribution from the lines at each point in the target is added to the results for zero overlap by using FUDGE 3A.

The computer program calculates the dose rate distribution throughout any homogeneous target for any range of x , y , and z coordinates. The program incorporates buildup as the sum of exponentials. Use of this mathematical form allows one to incorporate the exponential factor within the dose integral without affecting the nature of the integration; the exponential factors modify the total linear absorption cross section, μ , by multiplying it by $(1+\alpha_1)$, $(1+\alpha_2)$, etc. The resulting fictitious or modified values of μ are then used for the calculation, weighting the resulting dose rates by the appropriate value of C from the following equation for buildup:

$$B = C_1 e^{-\alpha_1(\mu_0 d)} + C_2 e^{-\alpha_2(\mu_0 d)} + \dots$$

Linear forms of buildup, power series expansions, or linear-exponential forms may also be used. However, as stated in Paragraph 2.5 the integrand may be altered by the higher terms in the expansion and it may be difficult to perform analytical integrations. If buildup

is expressed by

$$B = 1 + k_1(\mu_0 d)e^{k_2(\mu_0 d)}$$

or

$$B = 1 + k_3(\mu_0 d),$$

it is possible to combine the results of the computer program (the constant term does not alter the integration) with an analytical integration resulting from the inclusion of the second term in the integral. This can be done exactly only along the central axis of disc source or for infinite plane or disc sources. Along asymmetrical axes the problem becomes much more difficult and may be handled by other mathematical techniques [Sievert Power Series, Acta Radiol. (Stockholm) 89 (1921)].

Zero Overlap Case, 36 × 20-in. Source, 4-in. Target Thickness

The computer program, FUDGE 3A, was run for this case for points out to 140 in. beyond the source center. The buildup was incorporated into the calculation as the sum of two exponentials.

$$B = C_1 e^{-\alpha_1(\mu_0 d)} + C_2 e^{-\alpha_2(\mu_0 d)}$$

Values of C_1 , C_2 , α_1 , and α_2 were obtained from Rockwell⁽¹³⁾ and from Taylor⁽¹²⁾ for dose buildup in water, interpolating graphically for an average energy of 1.25 Mev. The following values were used:

$$C_1 = 0.00 \quad \alpha_1 = -0.095$$

$$C_2 = -7.90 \quad \alpha_2 = 0.059$$

Values of buildup based on the above constants were checked against values interpolated from moments-method calculations of Goldstein and Wilkins^(10a) and found to be +12% at $\mu_0 d = 1$ and -8.4% at $\mu_0 d = 10$.

Since the incorporation of the buildup into FUDGE 3A is not entirely obvious, it should be pointed out as previously mentioned that the linear absorption cross section is modified by multiplying μ by $(1+\alpha_1)$ and $(1+\alpha_2)$. Thus:

$$\mu_1 = 0.163(1-0.095) = 0.1475 \text{ in.}^{-1}$$

$$\mu_2 = 0.163(1+0.059) = 0.1726 \text{ in.}^{-1}$$

The parameters used as input to the program are λ_1 and λ_2 which are reciprocals of μ .

$$\lambda_1 = 1/0.1475 = 6.779 \text{ in.}$$

$$\lambda_2 = 1/0.1726 = 5.793 \text{ in.}$$

The program output gives both the dose distribution throughout the target and the average dose rate along any horizontal path parallel to the source plane (constant y and z ; variable x coordinate) (see Figure 5.1.2). The integrated total dose may readily be obtained from the average dose rate over the same path. Referring to Figure 5.1.2, dose rates were calculated for $x = 0$ (2) 140 in.; $y = 0$ (1) 10 in.; and $z = 1$ (1) 7 in. for a target of unity density, and for a source surface activity of 1 C/cm^2 . By inspection it can be seen that the minimum dose rates at a fixed depth in the target (constant Z) will be obtained for values of $y = B/2 = 10$ in. as measured from the source center for a target whose height is equal to the source height. Values obtained for this case are illustrated in Table 5.1.1 for $y = 10$ in. and $z = 3.0$ in.

Examination of Table 5.1.1 also shows that the average dose rate over x is almost exactly inversely proportional to x when one is at appreciable distances beyond the source boundary, since the contributions to total dose at these points are essentially negligible. This property of the system is quite useful when working with target travel lengths which are large compared to source length and eliminates the need for a computer calculation for every path length. The illustration which follows should clarify this further.

PLANE SOURCE GEOMETRY
FIGURE 5.1.2

Table 5.1.1

Dose Rate Distribution From Plane Cobalt-60 Source(Source size, 36×20 in.; target, infinite water; $Y = 10$ in.; $Z = 3$ in.; $D =$ specific dose rate, rad/hr per C/cm^2)

<u>X, in.</u>	<u>D</u>	<u>X, in.</u>	<u>D</u>
0	5.375×10^4	48	6.613×10^{-1}
2	5.366	50	4.610
4	5.340	52	3.220
6	5.282	54	2.254
8	5.186	56	1.581
10	5.034	58	1.110
12	4.795	60	7.812×10^0
14	4.392	62	5.505
16	3.721	64	3.885
18	2.748	66	2.745
20	1.768	68	1.942
22	1.104	70	1.375
24	7.005×10^3	72	9.751×10^{-1}
26	4.534	74	6.921
28	2.981	76	4.918
30	1.983	78	3.498
32	1.331	80	2.490
34	9.003×10^2	90	4.609×10^{-2}
36	6.126	100	8.688×10^{-3}
38	4.189	110	1.662
40	2.877	120	3.222×10^{-4}
42	1.983	130	6.317×10^{-5}
44	1.371	140	1.251
46	9.510×10^1		

 $\bar{D}$ from $x = 0$ to 140 in. = 7.070×10^3 rad/hr per C/cm^2

The total dose accumulated along a path of fixed height and depth ($y = y_1, z = z_1$) and parallel to the source from $x = 0$ to $x = 138$ in. can be expressed by the following equation, assuming a constant target velocity, v :

$$I = \frac{1}{v} \int_0^{138} D(x, y_1, z_1) dx \quad (5.1.2)$$

The computer program was written to give the average dose rate over any pass length, X_m , by performing an integration of the calculated dose rate distribution over that path and dividing by the path length. Accurate values of the integration which were done by Simpson's rule require both the selection of small increments in x and the division of path length into an even number of these increments. The average dose rate, $\bar{D}$, may be expressed by the following equation:

$$\bar{D} = \frac{\int_0^{X_m} D(x, y_1, z_1) dx}{X_m}$$

Since the value of the integral in Eq. (5.1.2) remains approximately constant with change of X_m (for this case), then $X_m \bar{D}$ is also constant and is equal to the value of the integral.

Thus for the conditions of Table 5.1.1 where the average dose rate $\bar{D} = 7.070 \times 10^3$ rad/hr, then

$$\int D dx = 140.0(7.070 \times 10^3) = 9.898 \times 10^5 \text{ rad-in./hr per C/cm}^2$$

$\bar{D}$ for the path length of 138 in. equals $7.070 \times 10^3 \times \frac{140}{138} = 7.173 \times 10^3$ rad/hr. The total dose accumulated by a point moving along this path from -122 to +138 in. on each side of the source (this is a point P_1 at the top-center of a target whose cross section is 20×4 in., with one inch of water equivalent shielding in front of the target as in Figure 5.1.3) is given by:

$$I = \frac{2}{v} \int_{-122}^{+138} D dx$$

SOURCE - TARGET GEOMETRY - NO OVERLAP

FIGURE 5.1.3

Since

$$\int_0^{138} Ddx \approx \int_{-122}^0 Ddx \approx \int_0^{140} Ddx,$$

$$I = \frac{4}{v} \times 9.898 \times 10^5$$

As previously calculated for a 4-in. target thickness, $V = 28.8 \text{ ft/hr} = 345.6 \text{ in./hr}$ and:

$$I = \frac{4}{345.6} \times 9.898 \times 10^5 = 1.146 \times 10^4 \text{ rad}$$

Since the required minimum dose $= 4.5 \times 10^6 \text{ rad}$, the surface activity is

$$S_a = \frac{4.50 \times 10^6}{1.146 \times 10^4} = 3.93 \times 10^2 \text{ C/cm}^2$$

The total activity required may then be calculated as

$$C = S_a A = 3.93 \times 10^2 \times 36 \times 20 \times (2.54)^2$$

$$C = 1.824 \times 10^6 \text{ curies}$$

Since the source thickness was assumed to be 0.180 in., the specific activity required will

be $S_g = S_a / t\rho$.

$$S_g = 3.93 \times 10^2 \times \frac{2.54}{0.180} \times \frac{1}{8.9}$$

$$S_g = 96.5 \text{ C/g}$$

The source utilization efficiency may now be calculated from Eq. (4.4.56).

$$E_u = \frac{WI}{0.0148 C} = \frac{10^3 \times 4.5 \times 10^6 \times 1.26 \times 10^{-6}}{0.0148 \times 1.824 \times 10^6}$$

$$E_u = 0.210$$

The dose distribution in the target was calculated next to determine the maximum over-all dose variation for this geometry. Table 5.1.2 was prepared to facilitate the calculation and illustrate the method. Average specific dose rates (averaged over the horizontal or x direction) are tabulated at heights of 0 to 10 in. from the source center, and at depths of 1 to 5 in. For a two pass-one source irradiator and a 4-in.-thick target, assuming one inch of water equivalent shielding between source face and target, the surface doses may be obtained by adding the value for $Z = 1$ in. to the value for $Z = 5$ in. These combined values are listed as $\bar{D}_{(0)}$. The center doses at $Z = 3$ in. are equal on both passes and these values may be doubled to give $\bar{D}_{(2)}$. The ratio of the maximum dose rate in the target, which occurs at the target surface at $y = 0$, divided by the minimum dose rate which occurs at the center of the target at $y = 10$ in., was calculated to be 2.27. This is far in excess of the desired ratio of 1.25. The technique of obtaining smaller ratios by increasing the source height while maintaining the target height at 20 in., will be discussed next. First, however, it should be pointed out that the dose rate ratios at each height, which are listed in the last column of Table 5.1.2, indicate that the dose ratio decreases as y increases up to a maximum of 1.334 at $y = 8$ in., and then it decreases to 1.230 at $y = 10$ in. A plot of the values would indicate that while surface and center doses decrease similarly as one moves vertically away from the source center in the y direction, the surface dose decreases much more rapidly than the center dose near the top of the package, mainly due to the rapid decrease of $\bar{D}_{1.0 \text{ in.}}$ (one pass). This is probably because the dose at surface points which have less shielding between them and the source is more sensitive to solid angle change than points at greater depths in the target.

Equivalent Source Methods - Static Equivalent

Prior to calculating the effect of source overlap, it was decided to compare the results obtained by the above discussed methods involving calculations using the actual source size

Table 5.1.2

Depth-Dose Rate Distribution in Water Targets From Cobalt-60 Source

(Source: 36×20×0.18 in.; Target: 280×20×4 in.)

y, in.	$\bar{D}$ for one pass, 10^4 rad/hr per C/cm ²				
	z, in.				
	1	2	3	4	5
0	2.4242	1.6939	1.2750	0.9903	0.7827
1.0	2.4202	1.6901	1.2715	0.9872	0.7800
2.0	2.4063	1.6770	1.2594	0.9764	0.7706
3.0	2.3835	1.6555	1.2399	0.9593	0.7559
4.0	2.3498	1.6241	1.2118	0.9349	0.7353
5.0	2.3030	1.5813	1.1746	0.9035	0.7095
6.0	2.2352	1.5207	1.1232	0.8614	0.6757
7.0	2.1322	1.4321	1.0510	0.8041	0.6305
8.0	1.9713	1.3044	0.9542	0.7311	0.5750
9.0	1.7164	1.1366	0.8418	0.6527	0.5180
10.0	1.2863	0.91936	0.7070	0.5610	0.4528

$$\bar{D}_{(y=0)}/\bar{D}_{(y=10)} = \begin{matrix} 1.885 & 1.842 & 1.803 & 1.765 & 1.729 \end{matrix}$$

y	D for two passes, 10^4 rad/hr per C/cm ²			
	$\bar{D}_{(0)} + \bar{D}_{(5.0)}$	$\bar{D}_{(2.0)} + \bar{D}_{(4.0)}$	$2\bar{D}_{(3.0)}$	$\frac{\bar{D}_{(0)} + \bar{D}_{(5.0)}}{2\bar{D}_{(3.0)}}$
	$\bar{D}_{(0)} + \bar{D}_{(5.0)}$	$\bar{D}_{(2.0)} + \bar{D}_{(4.0)}$	$2\bar{D}_{(3.0)}$	$\frac{\bar{D}_{(0)} + \bar{D}_{(5.0)}}{2\bar{D}_{(3.0)}}$
0	3.2069	2.6843	2.5500	1.2576
1.0	3.2002	2.6773	2.5430	1.2584
2.0	3.1769	2.6534	2.5188	1.2613
3.0	3.1394	2.6148	2.4798	1.2660
4.0	3.0851	2.5590	2.4236	1.2729
5.0	3.0125	2.4848	2.3493	1.2824
6.0	2.9109	2.3821	2.2464	1.2958
7.0	2.7627	2.2362	2.1020	1.3143
8.0	2.5463	2.0355	1.9084	1.3343
9.0	2.2350	1.7893	1.6836	1.3275
10.0	1.7391	1.4804	1.4140	1.2299

$$\bar{D}_{(y=0)}/\bar{D}_{(y=10)} = \begin{matrix} 1.844 & 1.813 & 1.803 \end{matrix}$$

$$(D_{\max}/D_{\min})_{\text{over-all}} = \frac{3.2069}{1.4140} = 2.268$$

and obtaining averages by integration, with the static equivalent source method. An advantage to be gained by the static equivalent source method would be a great saving in computer time and calculation output without all the detailed information which is probably not needed until one arrives at a final design. Basically, the method, which is discussed in much greater detail in Section 4.3, involves the use of an artificial source of the same height as the real source but whose length is determined by the path distance of the target.

The target which is also taken to be as long as the source is assumed to be stationary and is irradiated for a time equal to the transit time of a real package on one pass of the irradiator. The total dose received at points lying in a plane bisecting the source and target would be identical to that calculated by integration of the "true" dose rates. Thus, only a few points are involved in the calculation, i.e., only important points in the $X = 0$ plane. If the individual target package length is small compared to the path length, and if the path length is large compared to the true source length, end effects or horizontal dose variation in a package should be negligible. A computer calculation was therefore done using the same input parameters as previously; however, the source length was assumed to be 260 in. At the point of minimum dose rate ($x = 0$ in., $y = 10$ in., and $Z = 3$ in.) the total dose rate was calculated to be 5.4995×10^4 rad/hr per C/cm². The total dose that would be accumulated in the irradiator for a residence time of 1.5 hr would therefore be:

$$I' = 1.5 \text{ hr} \times 5.4995 \times 10^4 = 8.250 \times 10^4 \text{ rad}$$

Since the total activity of the "artificial" source is greater than the real source ($S_a = 1$ C/cm² in both cases), the total dose must be corrected by the ratio of source areas:

$$\therefore I = I' \times \frac{36 \times 20}{260 \times 20} = 8.250 \times 10^4 \times \frac{36}{260} = 1.142 \times 10^4 \text{ rad}$$

Comparison of this result with the integrated total dose for the 36×20-in. source shows them to be essentially identical except for the last figure. It was therefore decided to use the

static equivalent source method for all subsequent calculations.

Overlap Calculation - Plane Source Method

The effect of source overlap was next considered for a point P_1 located at the top surface of the target and in a central vertical plane, 2 in. from the target face (see Figure 5.1.4). If one neglects the source self-shielding from the overlap area, the radiation at P_1 may be assumed to come from one source in an infinite water medium and another source in air. For the case of 4-in. overlap, the source planes are 260×24 in. and 260×4 in., respectively, based on the static equivalent method.

Point P_1 has the coordinates $X = 0$, $y = 12$ in., $Z = 3$ in. for the first case, and $X = 0$, $y = 2$ in., $Z = 3$ in. for the second case. Computer calculations were combined as follows:

Source 1: $L_s = 260$ in., $H = 24$ in. Target: water

$$D \text{ at } (0, 12, 3) = 5.549 \times 10^4 \text{ rad/hr per C/cm}^2$$

Source 2: $L = 260$ in., $H = 4$ in. Target: air

$$D \text{ at } (0, 2, 3) = 4.260 \times 10^4 \text{ rad/hr per C/cm}^2$$

$$D_{\text{total}} = (5.549 + 4.260) \times 10^4 = 9.809 \times 10^4 \text{ rad/hr per C/cm}^2$$

$$I_{\text{total}} = 9.809 \times 10^4 \times 1.5 \text{ hr} = 1.471 \times 10^5 \text{ rad per C/cm}^2$$

At point C, $x = 0$, $y = 0$, $z = 3$.

Source: 260×28 in. Target: water

From FUDGE 3A

$$D = 1.0607 \times 10^5 \text{ rad/hr per C/cm}^2 = \bar{D}$$

$$I = 1.591 \times 10^5 \text{ rad per C/cm}^2$$

Basing the design on the minimum dose at P_1 , the curie content may be calculated from the fictitious source

SOURCE-TARGET GEOMETRY-
4" OVERLAP (ONE PASS SHOWN)
FIGURE 5.1.4

$$C = \frac{W L_s H_s}{2 L_T H_T t \rho_T D} = \frac{10^3 \times 4.5 \times 10^6 \times 260 \times 28 \times (2.54)^2}{2 \times \frac{260 \times 20 \times 4}{1728} \times 62.5 \times 9.809 \times 10^4}$$

$$C = 1.432 \times 10^6 \text{ curie}$$

For the real source where $L_s = 36 \text{ in.}$, $H_s = 28 \text{ in.}$, $t = 0.180 \text{ in.}$

$$S_g = \frac{1.432 \times 10^6}{36.0 \times 28 \times 0.180 \times (2.54)^3 \times 8.9} = 54.1 \text{ C/g}$$

Thus it may be seen that by increasing the source height from 20 to 28 in. the total activity requirement will decrease from 1.823×10^6 to $1.432 \times 10^6 \text{ C}$. This calculation method, however, does not allow for the one inch water equivalent self-shielding of the overlap portion of the source and therefore underestimates the required activity. In addition, the dose rates at points below the top edge of the target cannot be calculated and therefore another method based on combined computer and hand calculations must be used. This is illustrated next.

Source Overlap Calculations

The method of calculating the effect of source overlap may best be explained by reference to Figure 5.1.5 which is a side view of the source-target geometry. It may be seen that if one draws straight lines connecting point a to f in the source plane with points in the target plane, some of the lines intersect the target face; however, most of the lines intersect the top plane of the target and travel both through air and the target, suffering less attenuation. For this reason, source overlap can increase dose rates at top points without adding significantly to the center dose, and thus flatten the vertical dose distribution. However, the computer program does not calculate the dose for this geometry and a combination method must be used.

SOURCE - TARGET GEOMETRY-OVERLAP
DIVIDED INTO LINE SOURCES
FIGURE 5.1.5

Let us assume that it is desired to calculate the effect of a 6-in. source overlap at both the top and bottom of the target. Due to symmetry, we can confine our discussion to the top upper half of the target. We may divide the overlap area of the source into convenient increments, depending on the accuracy desired and the number of hand calculations we wish to do. Assume that we can get fairly accurate results using 6 increments, each 1 in. in width. We next assume that the total activity per increment may be concentrated in a line at the center of the increment. Thus we obtain 6 line sources, parallel to the target travel, at heights of 10.5, 11.5, etc., to 15.5 in., which are designated lines a through f, respectively.

To find the dose contribution of these lines to a point in the target one can combine the results of the computer calculation for the source area below the top overlap section, which includes the bottom overlap, with hand calculations for the contributions from lines a to f. With reference to a point in the top half of the target, the assumption is that the source is in an infinite medium. Since the radiation from lines a through f in the lower quadrant travels through much greater distances in the target, the effect of neglecting the small distances traveled in air is considered negligible.

Let us consider point 3 in the central plane of the target, 2 in. below the top surface. We now draw lines connecting each of the line sources a through f with point 3. It can be seen from Figure 5.1.5 that line a-3 intersects the front surface but that lines b-3 to f-3 enter through the top. Recalling that there is one inch of water equivalent shielding in front of the source, the total distances and the distances in the target plus the source shielding were calculated from the geometric arrangement. These distances are listed in Table 5.1.3.

The dose from each of the line sources can then be calculated by first calculating the gamma flux to point 3, incorporating exponential buildup into the integral as follows:

Table 5.1.3

Line Source Calculation

Distances to Line Sources From Point 3: Coordinates (0, 8, 3)

Line source	Line	y_ℓ , in.	$y_\ell - y_3$	$d^2 = (3.0)^2 + (y_\ell - y_3)^2$	Distances in target			Distance in source shielding $t_3 = \frac{1.0}{3.0} d$	Σt
					d	Front face $t = \frac{2.0}{3.0} d$	$t = d \left(\frac{2.0}{y_\ell - y_2} \right)$		
a	a-3	10.5	2.5	15.25	3.905	2.603	---	1.302	3.905
b	b-3	11.5	3.5	21.25	4.610	---	2.634	1.537	4.171
c	c-3	12.5	4.5	29.25	5.408	---	2.404	1.803	4.207
d	d-3	13.5	5.5	39.25	6.265	---	2.278	2.088	4.366
e	d-3	14.5	6.5	51.25	7.159	---	2.207	2.386	4.593
f	f-3	15.5	7.5	65.25	8.078	---	2.154	2.693	4.847

$$\phi_L = \frac{S_L}{2\pi d} \left[\int_0^{\theta_L} c_1 e^{-0.163(1+\alpha_1)(\Sigma t) \sec \theta} d\theta + \int_0^{\theta_L} c_2 e^{-0.163(1+\alpha_2)(\Sigma t) \sec \theta} d\theta \right]$$

Since

$$c_1 = 8.90, c_2 = -7.90, \alpha_1 = -0.095, \text{ and } \alpha_2 = +0.059$$

$$\phi_L = \frac{S_L}{2\pi d} \left[8.90 \int_0^{\theta_L} e^{-0.1475(\Sigma t) \sec \theta} d\theta - 7.90 \int_0^{\theta_L} e^{-0.1726(\Sigma t) \sec \theta} d\theta \right]$$

For source increments of 1.0 in. and $S_a = 1 \text{ C/cm}^2$,

$$S_L = 3.7 \times 10^{10} \times 2 \frac{\text{photons}}{\text{sec-C}} \times 1.0 \text{ in.} \times 2.54 \frac{\text{cm}}{\text{in.}} \times \frac{1 \text{ C}}{\text{cm}^2} = 1.88 \times 10^{11} \frac{\text{photons}}{\text{sec-cm}}$$

From Table 2.4.1, $k = 1.722 \times 10^{-6} \text{ rad/hr per Mev/cm}^2\text{-sec}$. Therefore,

$$\begin{aligned} D &= 1.722 \times 10^{-6} \frac{\text{rad-cm}^2\text{-sec}}{\text{hr-Mev}} \times 1.88 \times 10^{11} \frac{\text{photons}}{\text{sec-cm}} \times \frac{1.25 \text{ Mev}}{\text{photon}} \times \frac{1}{2\pi d(2.54) \text{ cm}} \\ &\times \left[8.90 \int_0^{\theta_L} e^{-0.1475(\Sigma t) \sec \theta} d\theta - 7.90 \int_0^{\theta_L} e^{-0.1726(\Sigma t) \sec \theta} d\theta \right] \\ &= 2.535 \times 10^4 \times \frac{1}{d} \left[8.90 \int_0^{\theta_L} e^{-0.1475(\Sigma t) \sec \theta} d\theta - 7.90 \int_0^{\theta_L} e^{-0.1726(\Sigma t) \sec \theta} d\theta \right] \end{aligned}$$

To facilitate the calculation of line source dose rates and to eliminate the necessity for many interpolations of the above integral function,

$$F(\theta_L, b) = \int_0^{\theta_L} e^{-b \sec \theta} d\theta,$$

a computer program based on a twelve point Gauss quadrature was written. (17b) Values of

$F(\theta, b)$ were calculated for $\theta = 1^\circ(1^\circ)90^\circ$ and $b = 0(0.001)2.000$. The program is available as a Fortran function subroutine and may be easily incorporated into other programs.

The calculation of the doses at point 3 for both 4 and 6-in. overlaps may best be illustrated by Table 5.1.4 which lists the values of all the parameters required to arrive at the dose rate from the line sources. The total contribution to point 3 from the 4-in. overlap (sum of four line sources) was calculated as 2.3487×10^4 rad/hr. The remainder of the source may be assumed to be 24 in. high from the viewpoint of point 3, which is at a height of 10 in. from the center of this source. The results of a FUDGE 3A calculation for a 260×24-in. source at $x = 0$, $y = 10$, $Z = 3$ in. gives us a dose rate of 7.5019×10^4 rad/hr. The total dose rate at point 3 from the 260×28-in. source with 4 in. of source overlap at both top and bottom is therefore 9.850×10^4 rad/hr.

The overlap calculation was performed for points 1 to 8 for both 4 and 6-in. overlap at top and bottom and these values were combined with FUDGE 3A calculations for 260×24-in. and 260×26-in. sources to arrive at the total dose in each case. Calculations for total doses at heights of 6 in. or less in the target were done entirely by the computer calculation using a 260×28-in. source for the 4-in. overlap case and a 260×32-in. source for the 6-in. overlap case. Dose rates at depths in the target of 1, 3, and 5 in. as well as the combined values then gives one an idea of the over-all dose distribution in a target for overlaps of 4 and 6 in. These results may then be compared with values previously obtained for zero overlap (listed in Table 5.1.7 for a 260×20-in. source). Results in Table 5.1.7 may be converted to the results of Table 5.1.2 by the method previously explained under Source Equivalent Method in this section. The values in Table 5.1.7 were obtained by a modified 6-point Gauss calculation using FUDGE 3A which may explain some slight variation between the results of Tables 5.1.2 and 5.1.7. The conversion factor from Table 5.1.2 to Table 5.1.7 is obtained by assuming a constant total dose for long pass lengths by a relatively short source,

Table 5.1.4

Overlap Calculation for Point 3

(Source: Line sources, 260 in. long)

[1] Line source	[2] d, in.	[3] θ , degrees	[4] Σt , in.	[5] $b_1=0.1475\Sigma t$	[6] $b_2=0.1726\Sigma t$	[7] $F(\theta, b_1)$	[8] $F(\theta, b_2)$
a	3.905	88.3	3.905	0.5760	0.6740	0.57793	0.50473
b	4.610	88.0	4.171	0.6152	0.7199	0.54737	0.47426
c	5.408	87.6	4.206	0.6204	0.7260	0.54330	0.47037
d	6.265	87.2	4.366	0.6441	0.7537	0.52583	0.45319
e	7.159	86.8	4.593	0.6776	0.7929	0.50226	0.43010
f	8.078	86.4	4.847	0.7149	0.8365	0.47747	0.40599

[9] $8.9F(\theta, b_1)-7.9F(\theta, b_2)$	[10] [col. 9] [col. 2]	[11] Specific dose rate, 10^4 rad/hr per C/cm ²
1.1562	0.29607	0.7505
1.1429	0.24793	0.6285
1.1194	0.20698	0.52417
1.0997	0.17553	0.4450
1.07232	0.14979	0.3797
1.04216	0.12902	0.3271

$$\sum_a^d D = 2.3487 \times 10^4 \text{ rad/hr per C/cm}^2$$

$$\sum_a^f D = 3.0555 \times 10^4 \text{ rad/hr per C/cm}^2$$

Table 5.1.7

Dose Rate Distribution in Water Target

Source Dimensions: 260×20 in.

(Zero Overlap)

y, in.	Z = 1	Z = 3	Z = 5
10	1.0005×10^5	5.4995×10^4	3.5222×10^4
9	1.3369	6.5640	4.0428
8	1.5460	7.4516	4.5054
7	1.6552	8.1538	4.8910
6	1.7347	8.7014	5.2261
5	1.7901	9.1208	5.5041
4	1.8281	9.4326	5.7246
3	1.8554	9.6562	5.8889
2	1.8729	9.8060	6.0032
1	1.8832	9.8918	6.0687
0	1.8861	9.9199	6.0902

For two-pass one source irradiator:

$$D_{\max} = (1.8861 + 0.6090) \times 10^5 = 2.4951 \times 10^5$$

$$D_{\min} = 2 \times 5.4995 \times 10^4 = 1.0999 \times 10^5$$

$$D_{\max}/D_{\min} = 2.268$$

Table 5.1.5

Depth-Dose Rate Distribution in Water Targets From Cobalt-60 Source

(Source: 260×28×0.018 in.; Target: 260×20×4 in.)

		Specific dose rate, 10^5 rad/hr per C/cm ²								
		Plane source: 260×24 in.			Overlap source: 260×4 in.			Total source: 260×28 in.		
y	y'	Z=1.0	Z=3.0	Z=5.0	Z=1.0	Z=3.0	Z=5.0	Z=1.0	Z=3.0	Z=5.0
10	12	1.7517	0.5549	0.3569	---	0.4185	0.2784	1.7517	0.9703	0.6353
9	11	1.8099	0.6586	0.4072	---	0.3065	0.2094	1.8099	0.9651	0.6165
8	10	1.8524	0.7502	0.4548	---	0.2348	0.1672	1.8524	0.9850	0.6220
7	9	1.8840	0.9931	0.4970	---	---	0.1356	1.8840	0.9930	0.6326
6					---	---	---	1.9078	1.0138	0.6301
5					---	---	---	1.9256	1.0296	0.6429
4					---	---	---	1.9389	1.0416	0.6527
3					---	---	---	1.9484	1.0502	0.6600
2					---	---	---	1.9549	1.0561	0.6649
1					---	---	---	1.9586	1.0595	0.6678
0					---	---	---	1.9598	1.0607	0.6688

$D_{\max}$ occurs at $y = 0$ and $= 1.9598 \times 10^5 + 0.6688 \times 10^5 = 2.6286 \times 10^5$ rad/hr per C/cm².

$D_{\min}$ occurs at $y \approx 9$ and $= 2(0.9651 \times 10^5) = 1.9302 \times 10^5$ rad/hr per C/cm².

$$D_{\max}/D_{\min} = 1.362$$

- Note: 1. y is actual height above midplane of target; y' based on a 260×24-in. target.
 2. At $Z = 1.0$ in., all the radiation enters front face of target and values are therefore calculated from a 260×28-in. plane by FUDGE 3A.

and correcting for the smaller total activity in the smaller source. Thus, to convert the dose rate from the 36×20-in. source to the 260×20-in. source, one multiplies $\bar{D}_{(36 \times 20)}$ by $\frac{\text{total pass length}}{V\theta} \times \frac{260}{36}$. For this case where the total pass length is 560 in. (average horizontal dose for the 36×20-in. source was calculated over a distance of 140 in. from the center of the source), the package velocity is 345.6 in./hr, and the residence time, θ , is 1.5 hr, the value of the conversion factor becomes 7.802.

The effect of source overlap may now be illustrated by comparing both the total curie requirements and the maximum dose variations in each of the three cases: (1) no overlap, (2) 4-in. overlap, and (3) 6-in. overlap. The curie calculations are illustrated below under Curie Calculations for Problem No. 1. For zero overlap, the minimum dose occurs at $y = 10$ in., $Z = 3$ in. and the curies required to obtain the minimum dose of 4.5×10^6 rad is 1.824×10^6 C. For the 4-in. overlap, $D_{\min}$ occurs at $y = 9$ in., $Z = 3$ in.; however, the minimum dose rate increases from approximately 5.50 to 9.65×10^4 rad/hr. Although the height of the source has been increased from 20 to 28 in., the total activity required decreases from 1,824 to 1.455×10^6 . The value of $D_{\max}/D_{\min}$ also improves with overlap, going from 2.27 for zero overlap to 1.36 for 4-in. overlap (see Tables 5.1.5 and 5.1.7).

Upon increasing the overlap to 6 in. the total activity required increases to 1.549×10^6 C; however, this is still better than for no overlap. The position of the minimum dose has shifted to $y = 7$ in., $Z = 3$ in. and it may be seen from Table 5.1.6 that the source overlap has caused the dose rates at ($y = 10$ in., $Z = 3$ in.) and ($y = 7$ in., 8 in., 9 in., and 10 in.; $Z = 5$ in.) to be higher than points at the same depth at $y = 0$. However, the over-all dose ratio, $D_{\max}/D_{\min}$, decreases to 1.27 and is close to the required value of 1.25.

It might be noted that no corrections were made for the fact that these were finite targets in which the "true" buildup may be smaller than the values used which were based on infinite media. It is possible to correct for these differences based on the Monte Carlo

calculations of M. Berger of the National Bureau of Standards; however, it was assumed that the backscattering from concrete shielding walls in a real irradiator would compensate for the finite targets.

Table 5.1.6

Depth-Dose Rate Distribution in Water Targets From Cobalt-60 Source

(Source: 260×32×0.018 in.; Target: 260×20×4 in.)

Specific dose rate, 10^4 rad/hr per C/cm ²										
Plane source: 260×26 in.			Overlap sources: 260×6 in.			Total source: 260×32 in.				
y	y'	Z=1.0	Z=3.0	Z=5.0	Z=1.0	Z=3.0	Z=5.0	Z=1.0	Z=3.0	Z=5.0
10	13	---	5.5627	3.5817	---	5.3807	3.8351	18.559	10.943	7.4168
9	12	---	6.6027	4.0875	---	3.9956	2.9299	18.883	10.598	7.0174
8	11	---	7.5220	4.5669	---	3.0546	2.3337	19.129	10.577	6.9006
7	10	---	---	4.9928	---	---	1.8810	19.318	10.356	6.8738
6								19.463	10.487	6.5934
5								19.574	10.589	6.6792
4								19.658	10.666	6.7451
3								19.718	10.722	6.7938
2								19.760	10.760	6.8273
1								19.784	10.783	6.8468
0								19.792	10.790	6.8533

$D_{\max}$ occurs at $y = 0$ and $= 26.645 \times 10^4$

$D_{\min}$ occurs at $y \approx 7$ and $= 2(10.356 \times 10^4) = 20.712 \times 10^4$

$D_{\max}/D_{\min} = 26.645/20.712 = 1.286$

Source Requirement Summary

Case 1. No overlap; source height = 20.0 in.; $D_{\min}$ occurs at $y = 10.0$ in. and 5.4995×10^4 rad/hr per C/cm².

To obtain a total dose of 4.5×10^6 rad,

$$C' = \frac{4.5 \times 10^6 \times 1000 \frac{\text{rad-lb}}{\text{hr}} \times 260 \times 20 \text{ in.}^2 \times (2.54)^2 \frac{\text{cm}^2}{\text{in.}^2}}{2 \times 5.4995 \times 10^4 \frac{\text{rad cm}^2}{\text{hr C}} \times \frac{260 \times 4 \times 20}{1728} \frac{\text{in.}^3}{\frac{\text{in.}^3}{\text{ft}^3}} \times 62.5 \frac{\text{lb}}{\text{ft}^3}} = 1.824 \times 10^6 \text{ C}$$

For actual source dimensions of 36×20×0.180 in.,

$$S_g = \frac{1.824 \times 10^6}{36 \times 20 \times 0.180 \text{ in.}^3 \times (2.54)^3 \frac{\text{cm}^3}{\text{in.}^3} \times 8.9 \frac{\text{g}}{\text{cm}^3}} = 96.50 \text{ C/g}$$

Case 2. 4-in. source overlap; source height = 28.0 in.; $D_{\min}$ occurs at $y = 9$ in. and $= 9.651 \times 10^4$ rad/hr per C/cm².

$$\therefore C' = 1.824 \times 10^6 \times \frac{28}{20} \times \frac{5.4995}{9.651} = 1.455 \times 10^6 \text{ C}$$

For source dimensions of 36×28×0.018 in.,

$$S_g = \frac{1.455 \times 10^6}{36 \times 28 \times 0.180 \times (2.54)^3 \times 8.9} = 96.50 \times \frac{1.455}{1.824} \times \frac{20}{28} = 54.98 \text{ C/g}$$

Case 3. 6-in. source overlap; source height = 32.0 in.; $D_{\min}$ occurs at $y = 7$ in. and $= 10.36 \times 10^4$ rad/hr per C/cm².

$$\therefore C' = 1.824 \times 10^6 \times \frac{32}{20} \times \frac{5.4995}{10.36} = 1.549 \times 10^6 \text{ C}$$

For source dimensions of 36×32×0.018 in.,

$$S_g = 96.50 \times \frac{1.549}{1.824} \times \frac{20}{32} = 51.22 \text{ C/g}$$

5.2 Problem No. 2

The purpose of this problem is to demonstrate the use of the empirical techniques which were previously discussed in this report. In the interest of brevity, a calculation will be performed to determine the dose distributions and the uniformity ratio for the irradiator parameters which were found to satisfy the specifications of Problem No. 1.

The uniformity ratio is therefore required for a one slab-two pass irradiator employing a cobalt-60 slab source whose real size is 32 in. high, 36 in. long, and 0.180 in. thick. The package targets, which are of specific gravity 1.0, are 20 in. high, 16 in. long, and 4 in. thick, and are "butted" when inside the irradiator. There is a 6-in. source overlap on both the top and bottom of the target. The total distance between the effective plane of the source and the front plane of the target is 1 in. The absorber thickness between the source and the target including the half thickness of the source is equivalent to 1 in. of water.

As was pointed out in Problem No. 1 for the type of irradiator and the source-target geometry used, the ratio $I_{\max}/I_{\min}$ must be determined from the vertical dose distributions in the front, middle, and back planes of the target. The dose distribution is uniform in the horizontal direction, the direction of travel of the target.

The first step in the calculation is to determine the vertical dose distribution in the target in the front, middle, and back planes from an adequate static equivalent source whose height is the same as the target (without overlap).

The static equivalent source length used in Problem No. 1 was 260 in. This calculation will be performed for this source length although it may become apparent as the calculation proceeds that a shorter equivalent source will suffice.

In Figure 5.2.1, twelve target points (numbered 1 to 12) are shown in the three important planes of the target. These are the points at which the dose distribution are required.

The infinite medium specific dose rates at these points will be calculated using the

FIGURE 5.2.1

cobalt-60 superposition methods. It should be noted that target points 1, 2, 3, and 4 lie in the 5-in. plane and therefore superposition data for 5-in. depth must be used. The front plane is the 1-in. plane and therefore points 9, 10, 11, and 12 require the 1-in. superposition data. Likewise, points in the middle plane require the 3-in. superposition data. Superposition methods based upon three different source-element sizes will be used. The data for the 10×10-in. elements can be found in Section 3 of this report and those for the 2.5×2.5-in. and 5×5-in. elements in Appendix IV. The superposition methods as they are presented here permit the construction of planes up to 120 in. long without extrapolation. Therefore, a source length of 120 in. will be used initially. If the contributions from the remaining portions of the plane are later determined to be significant, further calculations will be undertaken.

Figure 5.2.2 illustrates the building blocks necessary to calculate the dose to the three points (4, 8, and 12) which are located at the top edge of the target.

Because of symmetry only one-half of the source need be considered. The answer will be multiplied by 2 when the calculation is completed. In this case it was necessary to use only 10×10-in. source elements. These elements appear as unprimed letters in Figure 5.2.2. By using the appropriate superposition data the doses at the three points in question will now be calculated and tabulated in Table 5.2.1. (Refer to Section 3 for proper use of the superposition method.)

In Figure 5.2.3 the building block pattern required for points 3, 7, and 11 which are located 2.5 in. below the top edge of the target are shown. Because the target point in question is located both off the edge and off the center of the target, it is necessary to use a complicated array of source elements. The source elements identified with a singly primed letter are those which are 5×5-in. squares. Lower case letters are used for the 2.5×2.5-in. elements. In addition to these standard elements, it should be noted that 'half

Table 5.2.1
Work Chart for Superposition Method

α Source section	β Components of α	1-in. plane		3-in. plane		5-in. plane	
		Doses of components	Total	Doses of components	Total	Doses of components	Total
I	M	0.000353	0.031	0.000344	0.0305	0.000336	0.0282
	+I	0.00112		0.00109		0.00105	
	+F	0.00425		0.00405		0.00385	
	+D	0.02528		0.0250		0.02295	
II	N	0.000287	0.0157	0.000280	0.01485	0.000274	0.0140
	+J	0.000865		0.000840		0.000814	
	+G	0.00285		0.002736		0.002621	
	+E	0.0117		0.01099		0.01029	
III	A	4.559	4.780	2.300	2.505	1.347	1.526
	+B	0.2207		0.2048		0.1790	
IV	C	0.05954	0.2803	0.0550	0.2598	0.0505	0.2295
	+D	0.2207		0.2048		0.1790	
V	I	0.031	5.107	0.03048	2.81	0.02819	1.798
	+II	0.0157		0.01485		0.0140	
	+III	4.78		2.505		1.526	
	+IV	0.2803		0.2598		0.2295	
VI	I	0.031	0.0467	0.03048	0.04533	0.02819	0.04219
	+II	0.0157		0.01485		0.0140	
VII	I	0.031	4.811	0.03048	2.536	0.02819	1.554
	+III	4.780		2.505		1.526	
VIII	A'	3.52	4.061	1.41	1.871	0.690	1.048
	+B'	0.413		0.343		0.255	
	+D'	0.0984		0.0907		0.0782	
	+F'	0.0292		0.0272		0.0246	
IX	G'	0.0213	0.08384	0.0202	0.7403	0.0185	0.6219
	+E'	0.0637		0.0594		0.052	
	+C'	0.179		0.159		0.131	
	+B'	0.413		0.343		0.255	
	+H'	0.0125		0.0114		0.0106	
	+F'	0.0292		0.0272		0.0246	
	+E'	0.0213		0.0594		0.052	
	+D'	0.0984		0.0907		0.0782	
X	VIII	4.061	4.8994	1.871	2.611	1.048	1.67
	+IX	0.8384		0.7403		0.6219	
XI	2×VIII	8.122	8.96	3.742	4.482	2.96	3.582
	+IX	0.8384		0.7403		0.6219	
XII	$\frac{1}{3}$ [F']	0.0146	0.0984	0.0136	0.0912	0.0123	0.0808
	+D'	0.0492		0.0454		0.0391	
	+F'	0.0146		0.0136		0.0123	
	+G'	0.01065		0.0101		0.00925	
	+H'	0.00625		0.0057		0.0053	
	+K'	0.00616		0.0028		0.00253	
XIII	f	0.0708	3.094	0.0621	1.113	0.0483	0.556
	d	0.166		0.131		0.0917	
	b	0.517		0.298		0.166	
	a	2.24		0.622		0.250	
XIV	I	0.031	16.28	0.03048	7.72	0.02819	4.70
	+II	0.0157		0.01485		0.0140	
	+X	4.8994		2.611		1.67	
	+XII	0.0984		0.09115		0.0808	
	+XIII	3.094		1.113		0.556	
XV	I	0.031	18.01	0.03048	9.00	0.02819	5.52
	+II	0.0157		0.01485		0.0140	
	+X	4.8994		2.611		1.67	
	+VIII	4.061		1.871		1.048

FIGURE 5.2.2

FIGURE 5.2.3

elements" are used such as $\frac{1}{2}F'$ and $\frac{1}{2}D'$ (see Figure 5.2.3). The notation $\frac{1}{2}F'$ refers to a source element which is located at approximately the same relative position of an F' element but has one-half its area and one-half its curie content. The dose contribution from a $\frac{1}{2}F'$ element is therefore approximately one-half the contribution of an F' element. The approximation of the dose contribution from a source element which cannot be represented directly by one of the available superposition element sizes, can only be used when the total distance between the source and the target point is large compared to the size of the substitute element, and when at least 1 to 2 mean free paths of absorbing material is interposed between the source and target point.

A more accurate method of handling the half elements in Figure 5.2.3 would be to treat them as point sources and use the data in Paragraphs 3.5 and 3.7 on augmentation, or to use the data in the Appendix for the 1×1-in. superposition method.

It should also be noted that the eight 10×10-in. elements (D, E, F, G, I, J, M, and N), representing the 20×40-in. source section which is furthest from the target point, are not in the precise positions called for by the superposition method. The error introduced by a slight discrepancy in position for source elements which are far from the target point is negligible. Point source approximations can be made for the above eight elements.

The success of the superposition method depends upon the use of exact data for those portions of the source which are close to the target point, i.e., within 1 to 2 mean free paths (6 to 12 in.) of the point. Good approximations may be made for source elements which are not in this range.

The building block diagrams for the points 2, 6, and 10 which are 5 in. below the top edge of the target and points 1, 3, and 9 which are at the center of the target are given in Figures 5.2.4 and 5.2.5, respectively.

A table (Table 5.2.1) will now be constructed based upon the use of the superposition

FIGURE 5.2.4

FIGURE 5.2.5

methods from which the required dose distributions have been determined.

The results of the calculation are summarized in Table 5.2.2.

Examination of the results indicates that an additional point in each plane, 7.5 in. below the top edge of the target, would be useful in plotting the vertical dose distributions. The building block diagram for these three points is presented in Figure 5.2.6. These three points will be numbered 13, 14, and 15 for the points in the back, middle, and front planes, respectively. The results of the calculation at these points are also presented in Table 5.2.2.

The dose rates listed in Table 5.2.2 are for a plane source 120 in. long. A quick calculation will now be made to determine whether additional work is needed to take into consideration the dose contributions from elements lying beyond the element M which would be needed to extend the length of the source to 260 in. In each case, to increase the source length from 120 to 260 in., twenty-eight 10×10-in. elements are needed. If twenty-eight O elements were used, the result would be an overestimation of the effect. The maximum effect from this large source section would therefore be 0.0056×10^4 rad/hr per C/cm². Examination of Table 5.2.2 indicates that this alters the dose at any point by a negligible amount (a fraction of 1%). It is therefore evident that the 120-in.-long source is adequate as a static equivalent.

To summarize, the dose rates at the fifteen target points listed in Table 5.2.2 have been calculated by using a 120-in.-long × 20-in.-high × 0.180-in.-thick plane cobalt-60 source imbedded in an infinite water medium.

From the statement of Problem No. 1, however, the actual target is finite — 4 in. thick × 20 in. high — and longer than the source. The total distance from the effective plane of the source to the front plane of the target is 1 in. and the total absorber thickness between the source and target is equivalent to 1 in. of water or 0.163 mean free paths of absorbing material. This total absorber includes the effective self-shielding of the source as well as additional absorber material such as source cladding. The effective plane of the source in

Table 5.2.2

Specific dose rate, 10^4 rads/hr per C/cm²

<u>Target point</u>	<u>Infinite medium</u>	<u>Corrected for effect of boundary</u>	<u>Boundary correction factor (see Table 5.2.3)</u>
1	6.22	6.17	0.991
2	5.52	5.02	0.910
3	4.70	4.22	0.891
4	3.60	2.98	0.828
5	10.15	9.95	0.980
6	9.06	8.86	0.978
7	7.72	7.45	0.965
8	5.62	5.01	0.892
9	19.25	19.21	0.9981
10	18.01	17.96	0.997
11	16.28	16.03	0.985
12	10.22	9.32	0.912
13	5.94	5.41	0.911
14	9.69	9.50	0.980
15	19.21	19.18	0.998

FIGURE 5.2.6

this discussion is taken at the center of the actual source, such that it is shielded top and bottom by 0.090 in. of cobalt metal. The air gap is 1 in. minus the space occupied by the total absorber thickness.

Since both the absorber thickness and the air gap are exactly what was called for in the statement of the problem, it will not be necessary to make corrections for these parameters. If absorber thickness and air gap corrections were necessary the scaling methods presented in Section 3 would be used. If only absorber thickness corrections were required the effect of absorber thickness curves in the Appendix would be used. This method will be demonstrated later when we deal with the effect of overlap.

The dose rates in Figure 5.2.2 must be corrected for the effect of boundaries. These corrections will be undertaken before accounting for source overlap.

The corrections for the effect of boundaries as it is presented in this report are based upon point source data. In Problem No. 1 no boundary corrections were made. This is justified in most irradiator systems because of the presence of shielding walls and other high density materials near the boundaries which serve to "reflect" gamma rays back into the target.

No boundary correction is necessary for the effect of the boundary formed by the front plane of the target. In multi-pass irradiators source material and target material are always present in the immediate vicinity of this plane. Since in a continuous irradiator system the package targets are normally "buted," no boundary corrections are required for the vertical edges of the target. However, corrections are required for the boundaries formed by the back plane of the target and by the top and bottom horizontal edges of the target. Corrections must be made whenever a point lies within one mean free path of a boundary.

To obtain a boundary correction factor from Table 3.5.1 in Section 3, b , the effective distance between the target point and the source plane; and b_1 , the perpendicular distance

from the target point to the boundary, must be determined. A composite correction factor, which is a correction for the effect of all of the boundaries, is obtained by multiplying together all the single boundary correction factors. Values of b_1 for the fifteen target points shown in Figure 5.2.1 have been determined and tabulated in Table 5.2.3. In this table, "B" refers to the back plane, and "T" refers to the plane formed by the top edge (see Figure 5.2.1). Those points which are more than one mean free path from a plane require no correction with respect to that plane.

Values of b , the effective distance to the source must be determined next. These calculations are performed by using the approximation method discussed in Section 3 on boundary corrections. Reference for this calculation must be made to Figure 5.2.1 and to the dose rates in Table 5.2.2.

Point 1 in Figure 5.2.1 is located at the center of the back plane of the target at a depth equivalent to 5 in. of water. As was previously pointed out (see Table 5.2.3) the only correction necessary for this point is that due to the boundary formed by the back plane of the target. The infinite medium specific dose rate to this point from Table 5.2.2 is 6.22×10^4 rads/hr per C/cm². The source size is 20×120 in., the area of the source is 1.548×10^4 cm², and the curie content is 1.548×10^4 C. From the curves in Figure 3.5.3 the dose rate from a point source of 1 C at a distance of 14.2 in. in an infinite water medium is:

$$\frac{6.22 \times 10^4}{1.548 \times 10^4} = 4.018 \text{ rads/hr}$$

Therefore, b (the effective distance to the 120×20-in. plane source) is 14.2 in. This value is listed in Table 5.2.3 along with the values of b calculated for the other fourteen target points in this problem. For each set of values of b and b_1 listed in Table 5.2.3, a correction factor will be found by using the data in Table 3.5.1. For example, for point 1 the correction factor in relation to plane B for which $b_1 = 0$ and $b = 14.5$ in. is found to be 0.911. In this case the data had to be interpolated. For point 6 two correction factors are necessary

Table 5.2.3

Target point	Boundary plane	b_1 , distance to boundary, in.	b , effective distance to source, in.	Single boundary correction factor	Composite correction factor
1	B T	0 >1 mfp	14.5	0.911	0.911
2	B T	0 5.0	14.8	0.911 0.9988	0.910
3	B T	0 2.5	15.4	0.9106 0.9846	0.897
4	B T	0 0	16.2	0.9101 0.9101	0.828
5	B T	2.0 >1 mfp	12.3	0.9799	0.980
6	B T	2.0 5.0	12.7	0.9797 0.9981	0.978
7	B T	2.0 2.5	13.35	0.9793 0.9852	0.965
8	B T	2.0 0	14.35	0.9789 0.911	0.892
9	B T	4.0 >1 mfp	9.85	0.9981	0.998
10	B T	4.0 5.0	10.2	0.9981 0.9990	0.997
11	B T	4.0 2.5	10.5	0.9981 0.9866	0.985
12	B T	4.0 0	12.3	0.9979 0.9134	0.912
13	B T	0 >1 mfp	14.4	0.911	0.911
14	B T	2.0 >1 mfp	12.4	0.9798	0.980
15	B T	4.0 >1 mfp	9.82	0.9981	0.998

(for both planes B and T). These values, from Table 3.5.1, are 0.9797 and 0.9981, respectively. For each target point it is necessary to calculate a composite correction factor. The composite correction factor is the product of all single boundary correction factors at that point. In the case of target point 1 there was only one correction factor, 0.911. This, therefore, is also the composite correction factor. The same is true of points 5, 9, 13, 14, and 15. For target point 2, the two correction factors must be multiplied together. This has been done and the results are listed in the last column of Table 5.2.3.

In the third column of Table 5.2.2 the boundary corrected dose rates are listed.

Since the irradiator system under consideration is a one-slab, two-pass system, the target passes the source twice. On the first pass the target is irradiated in the orientation shown in Figure 5.2.1. On the second pass the target is presented to the source such that the front plane and the back planes are interchanged. The middle plane, of course, remains in the same orientation during both passes. It is necessary therefore to add the dose distributions in the back and front planes and to double the middle plane doses.

For example, the dose at target point 4 must be added to that of point 12 and the dose at point 8 must be multiplied by 2. In Table 5.2.4 the results of this operation are tabulated.

Up to this point dose distributions have been calculated using only that portion of the plane source whose projection on the target is the same height as the target. Source overlap will now be employed to "shape" the dose distribution. The use of additional source material in the vicinity of the low dose rate regions, the top and bottom edges of the target, will serve to "flatten" the dose distribution and improve the uniformity ratio.

Dose Distribution From Source Overlap

In Problem No. 1 it was found that 6 in. of source overlap on the top and the bottom of the target was required to satisfy the specification for uniformity ratio. In the calculation of the dose distribution from the overlap portions of the source the data in Table 3.6.2 in Section 3

Table 5.2.4

Distance from top edge of target, in.	Average specific dose rate after two passes, 10^4 rads/hr per C/cm ²	
	Outside planes	Middle plane
0	12.30	10.02
2.5	20.25	14.90
5.0	22.98	17.22
7.5	24.59	19.0
10.0	25.38	19.9

Table 5.2.5

Distance from edge of target facing the source overlap, in.	Specific dose rate, 10^4 rads/hr per C/cm ²		
	Front plane	Middle plane	Back plane
0	7.45	5.89	3.02
2	3.70	2.37	1.75
4	2.20	1.31	1.09
6	1.42	0.831	0.734
10	0.727	0.383	0.372
15	0.380	0.172	0.1757
20	0.222	0.087	0.099

of this report will be employed. The required data from Table 3.6.2 have been tabulated in Table 5.2.5. Since in this calculation the target thickness was 4 in. and the source plane to target spacing was 1 in., it was possible to obtain the required dose distribution data directly from the tables on overlap. This, however, is not always possible. For target thicknesses between 2 and 6 in. and for source to target spacings between 0 and 3 in., the data can be interpolated with a reasonable degree of accuracy.

The doses in Table 5.2.5 are for the case of 6-in. overlap on one edge of the target and are for a total absorber thickness, $\mu t = 0.112$. The total absorber is made up of 0.060 in. of cobalt metal self-shielding and 0.036 in. of stainless steel source cladding, which is the total absorber used in the collection of the overlap data in Tables 3.6.1, 3.6.2, and 3.6.3. Since the total absorber thickness used in this problem is the equivalent of 1 in. of water ($\mu t = 0.163$), a correction for the effects of 0.051 mfp of absorber must be made. The absorber corrections will be made with the aid of the curves in Figures VI-1, VI-3, and VI-5 in Appendix VI. The data in Figure 5.2.5 were plotted to facilitate the use of convenient values from the curves in the Appendix (see Figure 5.2.7). For the front plane absorber corrections, Figure VI-1 will be used. Figures VI-3 and VI-5 will be used for the middle and back planes, respectively.

From Figure VI-1 the relative dose rate in the front plane at the edge of the target is reduced from 0.93 to 0.89 when the absorber thickness is changed from 0.112 to 0.163 mfp. These values were obtained from the 3-in. curve in Figure VI-1. Since the dose at this point, from Figure 5.2.7, is 7.44×10^4 rads/hr per C/cm^2 , it is reduced to:

$$\frac{0.89}{0.93} \times 7.44 \times 10^4 = 7.13 \times 10^4 \text{ rads/hr per } C/cm^2$$

For a point in the front plane, 9 in. from the edge of the target, the 12-in. curve in Figure VI-1 is used. The same change in absorber thickness at this point reduces the relative dose from 0.515 to 0.355. The specific dose rate is reduced from 0.850×10^4 rads/hr per

VERTICAL DOSE DISTRIBUTION DUE TO 6" SOURCE OVERLAP ON ONE EDGE OF TARGET

FIGURE 5.2.7

C/cm² to:

$$\frac{0.355}{0.515} \times 0.85 \times 10^4 = 0.586 \times 10^4 \text{ rads/hr per C/cm}^2$$

Table 5.2.6 is a tabulation of the dose distribution data from Figure 5.2.7 corrected for absorber thickness.

The corrected specific dose rates from Table 5.2.6 have been plotted in Figure 5.2.8. This is the dose distribution in the 20-in. high target resulting from 6-in. source overlap at one edge of the target only. This distribution is found by using target scale "A." By using scale "B," the dose distribution at the same points can be obtained when 6-in. source overlap is applied to the opposite edge of the target. The vertical distribution from the center of the target to one edge in each of the three important planes, due to source overlap on both the top and bottom edges of the target is computed by adding the dose distribution found by using scale A to that found using scale B. These results are tabulated in Table 5.2.7.

The dose rates tabulated in Table 5.2.7 are the doses in three planes due to overlap during a single pass in the irradiator. For two passes, the doses rates at the front and back planes must be added, and the middle plane doses must be multiplied by 2. These results are found in Table 5.2.8.

In the final step of finding the dose distributions for two passes in the irradiator, including the effects of source overlap, the results from Tables 5.2.4 and 5.2.8 must be combined. To facilitate this the data for the outside planes have been plotted in Figure 5.2.9 and a summation curve has been drawn. The same was done with the middle plane data in Figure 5.2.10.

From the summation curves in Figures 5.2.9 and 5.2.10 it is found that the maximum dose in the irradiator target occurs at the center of the outside plane. This dose is 26.93×10^4 rads/hr per C/cm². The minimum dose lies along the vertical line in the middle plane at a point 2 in. from the top and bottom edges of the target. This dose is 18.9×10^4

Table 5.2.6

Distance from edge of target facing source overlap, in.	Front plane			Middle plane			Back plane		
	Dose* from Fig. 5.2.7	Absorber correction factor	Dose* cor- rected for absolute thickness	Dose* from Fig. 5.2.7	Absorber correction factor	Dose* cor- rected for absolute thickness	Dose* from Fig. 5.2.7	Absorber correction factor	Dose* cor- rected for absolute thickness
0	7.44	0.957	7.13	5.89	0.947	5.58	3.02	0.955	2.89
3	2.75	0.815	2.18	1.72	0.921	1.58	1.37	0.954	1.31
6	1.42	0.771	1.01	0.830	0.883	0.733	0.740	0.933	0.684
9	0.850	0.583	0.586	0.455	0.845	0.385	0.435	0.901	0.392
12	0.555	0.520	0.290	0.272	0.805	0.219	0.272	0.878	0.239
15	0.383	0.463	0.178	0.170	0.772	0.131	0.176	0.854	0.150
18	0.275	0.391	0.1075	0.112	0.728	0.0816	0.122	0.832	0.102
21	0.203	0.329	0.065	0.076	0.695	0.0526	0.090	0.813	0.073

* Specific dose rate, rads/hr per C/cm².

Table 5.2.7

Distance from edge of target, in.	Specific dose rate, rads/hr per C/cm ²		
	Front plane	Middle plane	Back plane
0	7.21	5.61	3.03
2	3.15	2.23	1.78
4	1.87	1.29	1.17
6	1.26	0.873	0.856
8	0.980	0.685	0.689
10	0.890	0.624	0.646

rads/hr per C/cm². The uniformity ratio is 1.42.

Table 5.2.8

Distance from edge of target, in.	Specific dose rate, 10 ⁴ rads/hr per C/cm ²	
	Outside plane	Middle plane
0	10.24	11.22
2	4.93	4.46
4	3.04	2.58
6	2.12	1.75
8	1.67	1.37
10	1.54	1.25

VERTICAL DOSE DISTRIBUTION DUE TO
6" SOURCE OVERLAP
(CORRECTED FOR EFFECT OF ABSORBERS)

FIGURE 5.2.8

VERTICAL DOSE DISTRIBUTIONS IN OUTSIDE PLANES

FIGURE 5.2.9

VERTICAL DOSE DISTRIBUTIONS IN MIDDLE PLANE

FIGURE 5.2.10

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APPENDIX

APPENDIX I
GAMMA RAY BUILDUP FACTORS

TABLE I-1
Energy Absorption Buildup Factor, Ba

Point Isotropic Source				Water			
E_0	μ_{or}						
Mev	1	2	4	7	10	15	20
.255	3.03	6.88	21.8	68.1	154	421	906
.5	2.46	4.93	13.4	36.1	71.5	163	306
1.0	2.13	3.71	7.85	16.8	28.2	51.9	83.8
2.0	1.85	2.82	4.99	8.67	12.9	20.0	28.6
3.0	1.74	2.52	4.10	6.57	9.12	13.4	17.9
4.0	1.59	2.18	3.37	5.18	7.01	10.1	13.0
6.0	1.46	1.89	2.76	3.98	5.18	7.07	8.83
8.0	1.38	1.74	2.42	3.36	4.28	5.70	7.00
10.0	1.31	1.60	2.13	2.88	3.60	4.73	5.76
.662	2.34	4.50	10.5	30.0	-	-	-
1.25	2.06	3.36	6.60	13.0	20.3	35.7	-
Aluminum							
.5	2.61	4.92	11.4	26.9	49.2	104	185
1.0	2.15	3.64	7.40	15.0	23.2	44.1	69.1
2.0	1.80	2.74	4.93	8.63	12.8	20.2	28.3
3.0	1.66	2.37	3.88	6.35	8.94	13.4	18.2
4.0	1.54	2.09	3.24	5.05	6.93	10.2	13.5
6.0	1.40	1.81	2.63	3.92	5.27	7.62	9.92
8.0	1.31	1.62	2.26	3.25	4.28	6.10	7.88
10.0	1.25	1.51	2.02	2.83	3.70	5.22	6.74
Iron							
0.5	2.80	4.84	9.97	20.4	34.2	64.3	101
1.0	2.19	3.58	7.00	13.6	21.9	38.8	58.5
2.0	1.78	2.64	4.53	7.90	11.9	19.3	27.4
3.0	1.58	2.21	3.61	6.05	8.82	14.0	19.8
4.0	1.45	1.95	3.03	4.90	7.10	11.1	16.7
6.0	1.30	1.63	2.38	3.73	5.35	8.67	12.7
8.0	1.21	1.45	1.99	2.99	4.23	6.89	10.4
10.0	1.16	1.35	1.78	2.61	3.69	6.17	9.89
Lead							
.5	1.51	1.80	2.19	2.61	3.01	3.52	-
1.0	1.76	2.37	3.39	4.74	6.01	7.78	9.70
2.0	1.58	2.10	3.14	4.70	6.32	9.05	12.0
3.0	1.37	1.73	2.30	3.85	5.43	8.65	12.6
4.0	1.24	1.49	2.09	3.25	4.79	8.46	13.9
5.11	1.19	1.39	1.89	2.96	4.61	9.38	18.5
6.0	1.14	1.29	1.70	2.63	4.20	9.46	21.5
8.0	1.10	1.21	1.50	2.21	3.50	8.60	23.0
10.0	1.08	1.16	1.38	1.92	2.92	7.15	20.4

NOTE: Data obtained from Reference 11, p. 371. Values for tin, tungsten and uranium not included.

TABLE I-2(a)

Exponential Approximations to Energy Absorption Buildup FactorPoint Isotropic Sources - Water Target

Substance	E_0 , Mev	A	$-\alpha_1$	α_2
Water	1.0	13.5	.100	.010
	2.0	8.1	.068	.0405
	3.0	5.6	.059	.073
	4.0	4.5	.0555	.11
	6.0	3.4	.0525	.156
	8.0	2.8	.05	.17
	10.0	2.5	.0473	.1719
	1.25	8.90	.095	.059

$$B = Ae^{-\alpha_1 (\mu_0 r)} + (1-A)e^{-\alpha_2 (\mu_0 r)}$$

NOTE: Data obtained from Reference 11, p. 378.

TABLE I-2(b)

Exponential Approximation to Energy Absorption Buildup FactorPoint Isotropic Source - Water Target(Cs-137 $E_0 = 0.662$ Mev)

Parameter	Subscript		
	1	2	3
C	3.50	-3.00	.50
α	-.308	.231	1.824

$$B_a = C_1 e^{-\alpha_1 (\mu_0 r)} + C_2 e^{-\alpha_2 (\mu_0 r)} + C_3 e^{-\alpha_3 (\mu_0 r)}$$

NOTE: Calculated by L. Galanter from data of Goldstein (Table I).

TABLE I-3

GAMMA-RAY BUILDUP FACTORS IN WATERPlane Isotropic Source

		E_0 , Mev							
		.5	1	2	3	4	6	8	10
B_E^{Pl}	WATER								
	$\mu_o r = 1$	4.65	3.20	2.37	2.06	1.80	1.64	1.52	1.47
	2	8.43	5.03	3.31	2.76	2.33	2.01	1.82	1.72
	4	19.1	9.69	5.35	4.08	3.30	2.75	2.42	2.20
	7	45.1	18.7	8.65	6.09	4.80	3.78	3.21	2.89
	10	82.9	28.8	12.3	8.20	6.47	4.83	3.97	3.51
	15	176	50.4	18.8	11.8	8.91	6.73	5.15	4.49
B_r^{Pl}	$\mu_o r = 1$	4.74	3.34	2.57	2.23	2.02	1.80	1.66	1.57
	2	8.71	5.24	3.60	3.03	2.66	2.29	2.03	1.87
	4	20.6	9.98	5.87	4.55	3.94	3.12	2.73	2.46
	7	50.3	19.8	9.78	7.00	5.77	4.45	3.65	3.28
	10	94.5	31.9	13.9	9.52	7.62	5.67	4.60	4.04
	15	205	56.8	21.3	13.9	10.7	7.60	6.05	5.21

NOTE: Data obtained from Reference 11, p. 375.

TABLE I-4

Ratio of Scattered Energy Buildup Factors in Fine and Infinite ShieldsPlane Monodirectional Source

$$B_E(X, X) - 1 / B_E(X, \infty) - 1$$

ENERGY (Mev)	MEDIUM	$\mu_0 r \equiv X$				
		1.0	2.0	4.0	8.0	16.0
0.66	Water	0.663	0.713	0.783	0.785	0.784
1.0		0.720	0.754	0.821	0.828	0.830
4.0		0.885	0.912	0.920	0.926	0.933
1.0	Iron	0.821	0.851	0.888	0.895	0.895
4.0		0.910	0.923	0.936	0.932	0.949
10.0		0.959	0.972	0.974	0.978	0.977
1.0	Tin	0.911	0.924	0.935	0.938	0.946
4.0		0.926	0.955	0.967	0.974	0.978
10.0		0.960	0.962	0.973	0.971	0.969
1.0	Lead	0.951	0.969	0.975	0.979	0.982
4.0		0.977	0.982	0.990	0.992	0.994
10.0		0.990	0.995	0.992	0.994	0.995

NOTE: $B_E(X, X)$ = energy buildup factor in semi-infinite target,
 X mean free paths thick, at depth X .

$B_E(X, \infty)$ = energy buildup factor in infinite target at depth X .

Data obtained from Reference 15, p. 54, or from Reference 11, p. 212.
 Also see Reference 22.

TABLE II-1

Exponential Integral Function

$$E_n(x) = x^{n-1} \int_x^{\infty} \frac{e^{-t}}{t^n} dt$$

X	E ₁	E ₂	E ₃
0.00	∞	1.0000000	0.5000000
.01	4.0379296	0.9496705	0.4902766
.02	3.3547078	0.9131045	0.4809683
.03	2.9591187	0.8816720	0.4719977
.04	2.6812637	0.8535389	0.4633239
.05	2.4678985	0.8278345	0.4549188
.06	2.2953069	0.8040461	0.4467609
.07	2.1508382	0.7818352	0.4388327
.08	2.0269410	0.7609611	0.4311197
.09	1.9187448	0.7412442	0.4236096
.10	1.8229240	0.7225450	0.4162915
.11	1.7371067	0.7047524	0.4091557
.12	1.6595418	0.6877754	0.4021937
.13	1.5888993	0.6715385	0.3953977
.14	1.5241457	0.6559778	0.3887607
.15	1.4644617	0.6410387	0.3822761
.16	1.4091867	0.6266739	0.3759380
.17	1.3577806	0.6128421	0.3697408
.18	1.3097961	0.5995069	0.3636795
.19	1.2648584	0.5866360	0.3577491
.20	1.2226505	0.5742006	0.3519453
.25	1.0442826	0.5177301	0.3246841
.30	0.9056767	0.4691152	0.3000418
.35	0.7942154	0.4267127	0.2776693
.40	0.7023801	0.3893680	0.2572864
.45	0.6253313	0.3562291	0.2386625
.50	0.5597736	0.3266439	0.2216044
.55	0.5033641	0.3000996	0.2059475
.60	0.4543795	0.2761839	0.1915506
.65	0.4115170	0.2545597	0.1782910
.70	0.3737688	0.2349471	0.1660612
.75	0.3403408	0.2171109	0.1547667
.80	0.3105966	0.2008517	0.1443238
.85	0.2840193	0.1859986	0.1346581
.90	0.2601839	0.1724041	0.1257030
1.00	0.2193839	0.1484955	0.1096920
1.10	0.1859909	0.1282811	0.0958809
1.20	0.1584084	0.1111041	0.0839347
1.30	0.1354510	0.0964455	0.0735763
1.40	0.1162193	0.0838899	0.0645755
1.50	0.1000196	0.0731008	0.0567395
1.60	0.0863083	0.0638032	0.0499057
1.70	0.0746546	0.0557706	0.0439367
1.80	0.0647131	0.0488153	0.0387157
1.90	0.0562044	0.0427803	0.0341430
2.0	4.89005x10 ⁻²	3.75343x10 ⁻²	3.01334x10 ⁻²

2.5	2.49149	1.97977	1.62954
3.0	1.30484	1.06419	0.89306
3.5	6.97014x10 ⁻³	5.80189x10 ⁻³	4.94538x10 ⁻³
4.0	3.77935	3.19823	2.76136
4.5	2.07340	1.77869	1.55244
5.0	1.14830	0.99647	0.87780
5.5	6.4093x10 ⁻⁴	5.6168x10 ⁻⁴	4.9877x10 ⁻⁴
6.0	3.6008	3.1826	2.8460
6.5	2.0343x10 ⁻⁴	1.8115x10 ⁻⁴	1.6300x10 ⁻⁴
7.0	1.1548	1.0351	0.9366
7.5	6.5831x10 ⁻⁵	5.9353x10 ⁻⁵	5.3970x10 ⁻⁵
8.0	3.7666	3.4138	3.1181
8.5	2.1621	1.9689	1.8057
9.0	1.2447	1.1384	1.0479
9.5	7.1848x10 ⁻⁶	6.5963x10 ⁻⁶	6.0927x10 ⁻⁶
10.0	4.1570	3.8302	3.5488

NOTE: Data obtained from Reference 23, p. 155.
Also see Reference 18 for E₁ values.

TABLE II-2

Secant Integral Function

$$F(\theta, b) = \int_0^{\theta} e^{-b \sec \theta} d\theta$$

b	1°	2°	5°	10°
0	1.745x10 ⁻²	3.491x10 ⁻²	8.727x10 ⁻²	1.745x10 ⁻¹
.50	1.059	2.117	5.290	1.056
1.00	6.420x10 ⁻³	1.284	3.206	6.388x10 ⁻²
1.50	3.894	7.786x10 ⁻³	1.943	3.865
2.00	2.362	4.722	1.178	2.338
3.00	8.688x10 ⁻⁴	1.737	4.328x10 ⁻³	8.558x10 ⁻³
4.00	3.196	6.388x10 ⁻⁴	1.590	3.132
5.00	1.176	2.350	5.843x10 ⁻⁴	1.147
7.00	1.591x10 ⁻⁵	3.178x10 ⁻⁵	7.887x10 ⁻⁵	1.536x10 ⁻⁴
9.00	2.153x10 ⁻⁶	4.300x10 ⁻⁶	1.065	2.059x10 ⁻⁵

b	20°	40°	60°	80°	90°
0	3.491x10 ⁻¹	6.981x10 ⁻¹	1.047	1.396	1.571
.50	2.095	4.046	5.632x10 ⁻¹	6.415x10 ⁻¹	6.437x10 ⁻¹
1.00	1.258	2.350	3.077	3.282	3.283
1.50	7.551x10 ⁻²	1.367	1.703	1.762	1.762
2.00	4.533	7.964x10 ⁻²	9.534x10 ⁻²	9.712x10 ⁻²	9.712x10 ⁻²
3.00	1.635	2.718	3.067	3.085	3.085
4.00	5.896x10 ⁻³	9.330x10 ⁻³	1.013	1.015	1.015
5.00	2.127	3.221	3.407x10 ⁻³	3.409x10 ⁻³	3.409x10 ⁻³
7.00	2.773x10 ⁻⁴	3.894x10 ⁻⁴	4.000x10 ⁻⁴	4.001x10 ⁻⁴	4.001x10 ⁻⁴
9.00	3.618x10 ⁻⁵	4.784x10 ⁻⁵	4.848x10 ⁻⁵	4.848x10 ⁻⁵	4.848x10 ⁻⁵

NOTE: Values obtained from References 17a and 17b.

TABLE III-1

Absorbed Dose Conversion Factors

To Obtain Multiply by	rad	ev/gm	watt-hr/lb	BTU/lb	(Gram calorie)/Gm.
rad	1	$.624 \times 10^{14}$	1.260×10^{-6}	$.430 \times 10^{-5}$	2.389×10^{-6}
ev/gm	1.602×10^{-14}	1	2.019×10^{-20}	$.689 \times 10^{-19}$	3.827×10^{-20}
watt-hr/lb	$.794 \times 10^6$	$.496 \times 10^{20}$	1	3.41	1.897
BTU/lb	2.325×10^5	1.451×10^{19}	.2930	1	.5554
(Gram calorie)/ Gram	$.419 \times 10^6$	$.261 \times 10^{20}$	.528	1.80	1

TABLE III-2

Definitions and Fundamental Constants

1 curie = 3.700×10^{10} disintegrations per second.

1 Rutherford = 10^6 disintegrations per second.

Avogadro's number = 6.023×10^{23} atoms per gram atom.

Charge on electron = 4.80×10^{-10} e.s.u. = 1.602×10^{-19} coulombs.

Compton Wavelength = 2.426×10^{-10} cm.

Gamma-ray Wavelength, $\lambda = \frac{1.239 \times 10^{-4}}{E_0(\text{ev})}$ cm.

1 rad = 100 erg/g.

W = mean energy to produce one ion pair in air = 34.0 ev.

TABLE III-3(a)

X-Ray Attenuation Coefficients

Water							
PHOTON	Scattering		Photo	Pair Production		Total	
ENERGY	With	Without	Electric			With	Without
	Coherent	Coherent	K and L	Nucleus	Electron	Coherent	Coherent
Mev	Barns/ Molecule	Barns/ Molecule	Barns/ Molecule	Barns Molecule	Barns/ Molecule	cm ² /g	cm ² /g
0.01	12.8	6.40	146			5.31	5.10
.015	9.54	6.29	39.6			1.64	1.53
.02	8.19	6.18	15.4			0.789	0.722
.03	6.96	5.97	4.09			.370	.336
.04	6.34	5.78	1.55			.264	.245
.05	5.92	5.61	0.73			.222	.212
.06	5.70	5.46	.40			.204	.196
.08	5.33	5.17	.15			.183	.178
.10	5.05	4.93	.071			.171	.167
.15	4.50	4.44	.020			.151	.149
.20	4.10	4.07	.010			.137	.136
.30	3.55	3.54				.119	.118
.40		3.17					.106
.50		2.89					.0966
.60		2.68					.0896
.80		2.35					.0786
1.0		2.11					.0706
1.5		1.716		0.0029			.0575
2.0		1.464		.011			.0493
3.0		1.151		.033	0.0001		.0396
4.0		0.960		.055	.0004		.0339
5.0		.828		.072	.001		.0301
6.0		.732		.089	.002		.0275
8.0		.599		.116	.003		.0240
10.0		.510		.138	.006		.0219

NOTE: Data obtained from Reference 5, p. 47.

TABLE III-3(b)

Total Gamma-Ray Mass Absorption Coefficient for Air

(Composition: 0.755N, 0.232 O, 0.013 A by Weight)

Photon Energy	With Coherent	Without Coherent
(Mev)	(cm ² /g)	(cm ² /g)
0.01	5.09	4.89
0.015	1.59	1.48
0.02	0.764	0.697
0.03	0.349	0.317
0.04	0.245	0.226
0.05	0.204	0.194
0.06	0.186	0.178
0.08	0.166	0.161
0.10	0.155	0.151
0.15	0.136	0.134
0.20	0.123	0.123
0.30	0.107	0.106
0.40	0.0954	0.0953
0.50	0.0868	0.0868
0.60	0.0804	0.0804
0.80	0.0706	0.0706
1.0	0.0635	0.0635
1.5		0.0517
2.0		0.0445
3.0		0.0357
4.0		0.0307
5.0		0.0274
6.0		0.0250
8.0		0.0220
10.0		0.0202
15.0		0.0178
20.0		0.0166
30.0		0.0158
40.0		0.0156
50.0		0.0157
60.0		0.0158
80.0		0.0160
100.0		0.0164

NOTE: Data from Reference 5, p. 50.

TABLE III-3(c)

Total Gamma-Ray Mass Absorption Coefficients for Concrete

(Composition: 0.56% H, 49.56% O, 31.35% Si, 4.56% Al, 8.26% Ca, 1.22% Fe,
0.24% Mg, 1.71% Na, 1.92% K, 0.12% S) ($\rho = 2.35 \text{ g/cm}^3$)

Photon Energy	Mass Absorption Coefficient	Photon Energy	Mass Absorption Coefficient
(Mev)	(cm^2/g)	(Mev)	(cm^2/g)
0.01	24.6	1.0	.0635
0.015	7.68	1.5	0.0517
0.02	3.34	2.0	0.0445
0.03	1.10	3.0	0.0363
0.04	0.542	4.0	0.0317
0.05	0.350	5.0	0.0287
0.06	0.267	6.0	0.0268
0.08	0.197	8.0	0.0243
0.10	0.169	10.0	0.0229
0.15	0.139	15.0	0.0214
0.20	0.124	20.0	0.0209
0.30	0.107	30.0	0.0209
0.40	0.0954	40.0	0.0213
0.50	0.0870	50.0	0.0217
0.60	0.0804	60.0	0.0222
0.80	0.0706	80.0	0.0230
		100.0	0.0237

NOTE: Without coherent scattering.

Data from Reference 5, p. 50.

TABLE III-4

μ_a , Energy Absorption Coefficient for Gamma Rays in cm^2/g .

E, Mev	H ₂ O	Air	Al
0.01	4.89	4.70	26.1
0.015	1.33	1.30	7.67
0.020	0.520	0.516	3.15
0.030	0.15	0.15	0.877
0.040	0.064	0.064	0.351
0.050	0.0396	0.0388	0.17
0.060	0.0282	0.0270	0.10
0.080	0.0256	0.0238	0.0535
0.10	0.0251	0.0234	0.0378
0.15	0.0278	0.0250	0.0278
0.20	0.0298	0.0264	0.0276
0.30	0.0318	0.0284	0.0280
0.40	0.0331	0.0296	0.0290
0.50	0.0329	0.0296	0.0286
0.60	0.0329	0.0296	0.0286
0.80	0.0322	0.0288	0.0278
1.0	0.0311	0.0280	0.0268
1.5	0.0289	0.0256	0.0249
2.0	0.0265	0.0238	0.0234
3.0	0.0233	0.0212	0.0212
4.0	0.021	0.019	0.0201
5.0	0.020	0.018	0.0193
6.0	0.019	0.017	0.0189
8.0	0.017	0.016	0.0183
10.0	0.0165	0.015	0.0183

NOTE: Data from Reference 11, p. 237.

TABLE III-5
Important Constants for Co⁶⁰ and Cs¹³⁷ Gamma Rays

	<u>Co⁶⁰</u>	<u>Cs¹³⁷</u>
Initial photon energy, E_0 (Mev)	1.17, 1.33	.662
Average initial photon energy, E_0 (Mev)	1.25	.662
Total photon energy per disintegration, E_T (Mev)	2.50	.548
Total linear absorption coefficient in water, μ_t (in. ⁻¹)	0.163	.219
Linear energy absorption coefficients in water, μ_a (H ₂ O) (in. ⁻¹)	0.0759	.0830
Total linear absorption coefficient in cobalt metal, μ_t (Co) (in. ⁻¹)	1.22	--
Total linear absorption coefficient in steel, μ_t (steel) (in. ⁻¹)	1.070	1.449
Specific gamma power (watt/curie)	14.8×10^{-3}	3.24×10^{-3}
Absorbed dose rate constant for water, K_{H_2O} (rad/hr per Mev/cm ² -sec)	1.722×10^{-6}	1.883×10^{-6}

APPENDIX IV SUPERPOSITION METHODS

Relative Source Element Positions

In Figure IV-1 below, the primed capital letters A' through L' are used to symbolize 5 in. x 5 in. source elements. Source elements in the same relative positions denoted by the lower case letters a through l are used to symbolize the 2.5 in. x 2.5 in. source elements. Similarly, the primed lower case letters a' through l' are used for 1 in. x 1 in. source elements.

			L'
K'	H'	G'	F'
H'	F'	E'	D'
G'	E'	C'	B'
F'	D'	B'	A'

FIGURE IV-1

TABLE IV-1

Dose Rate from 1" x 1" Plane Cobalt-60 Sources at Various Plane Positions

Distance*	<u>Specific Dose Rate, 10^3 Rads/hr per curie/cm²</u>											
	<u>Source Position **</u>											
	a'	b'	c'	d'	e'	f'	g'	h'	i'	j'	k'	l'
1.0	8.383	3.815	2.415	1.748	1.364	.9361	.8064	.6255	.5598	.5057	.4613	.4226
2.0	2.821	1.981	1.512	1.214	1.010	.7489	.6612	.5312	.4816	.4399	.4050	.3740
3.0	1.323	1.090	.9221	.7955	.6978	.5562	.5039	.4212	.3877	.3589	.3343	.3119
4.0	.7375	.6522	.5833	.5258	.4779	.4023	.3723	.3221	.3008	.2820	.2655	.2503
5.0	.4548	.4175	.3853	.3568	.3320	.2905	.2732	.2429	.2295	.2174	.2068	.1966
6.0	.2992	.2807	.2642	.2490	.2353	.2115	.2013	.1828	.1743	.1665	.1597	.1530
7.0	.2059	.1959	.1867	.1780	.1701	.1559	.1497	.1381	.1326	.1276	.1231	.1187

* Distance from Target Point to Source Plane (Inches).

**Positions a' through l' refer to source positions as in Figure IV-1.

TABLE IV-2

Dose Rate from 2.5" x 2.5" Plane Cobalt-60 Sources at Various Plane Positions

Distance*	Specific Dose Rate, Rads/hr per Curie/cm ²											
	Source Position**											
	a	b	c	d	e	f	g	h	i	j	k	l
1.0	2.24x10 ⁴	5.17x10 ³	2.58x10 ³	1.66x10 ³	1.18x10 ³	.708x10 ³	.575x10 ³	.417x10 ³	.344x10 ³	.300x10 ³	.259x10 ³	.229x10 ³
2.0	1.11x10 ⁴	3.96	2.22	1.50	1.11	.678	.553	.389	.331	.287	.250	.222
3.0	6.22x10 ³	2.98	1.85	1.31	.976	.621	.512	.367	.311	.272	.239	.211
4.0	3.83	2.23	1.50	1.11	.842	.551	.461	.338	.289	.255	.223	.197
5.0	2.50	1.66	1.19	.917	.717	.483	.407	.303	.263	.233	.207	.181
6.0	1.68	1.23	.930	.742	.593	.414	.352	.268	.234	.210	.187	.167
7.0	1.17	.907	.720	.593	.486	.350	.301	.232	.205	.185	.167	.147
8.0	.845	.676	.557	.471	.396	.292	.256	.201	.179	.161	.146	.131
9.0	.632	.514	.433	.372	.320	.243	.216	.172	.153	.139	.127	.114
10.0	.474	.391	.339	.295	.259	.201	.182	.146	.132	.119	.109	.099
11.0	.357	.307	.266	.237	.210	.167	.152	.124	.113	.102	.094	.085
12.0	.273	.242	.211	.191	.171	.139	.127	.105	.096	.088	.080	.074

* Distance from Target Point to Source Plant (Inches)

** Positions a through k refer to source positions as shown in Figure IV-1.

TABLE IV-3

Dose Rate from 2.5" x 2.5" Plane Cesium-137 Source at Various Plane Positions

Specific Dose Rate, 10^3 Rads/hr per curie/cm²

Source Position**

Distance*	a	b	c	d	e	f	g	h	i	j	k
1.0	4.85	1.08	.600	.391	.282	.166	.132	.088	.076	.063	.053
2.0	2.30	.860	.512	.348	.256	.152	.122	.084	.071	.060	.052
3.0	1.39	.647	.410	.293	.219	.136	.112	.078	.066	.057	.050
4.0	.767	.475	.329	.246	.192	.122	.102	.071	.061	.053	.045
5.0	.528	.350	.256	.196	.155	.103	.089	.063	.054	.047	.041
6.0	.363	.260	.198	.156	.126	.088	.074	.055	.048	.042	.037
7.0	.256	.193	.153	.123	.102	.072	.062	.047	.041	.037	.032
8.0	.186	.146	.118	.097	.081	.059	.052	.040	.035	.031	.028
9.0	.132	.108	.091	.077	.066	.049	.043	.033	.030	.027	.024
10.0	.096	.081	.068	.060	.052	.040	.035	.028	.025	.022	.020
11.0	.071	.061	.053	.047	.041	.032	.029	.023	.021	.019	.017
12.0	.053	.045	.041	.037	.032	.026	.023	.019	.017	.016	.014

* Distance from Target Point to Source Plane (Inches).

**Positions a through k refer to source positions shown in Figure IV-1.

TABLE IV-4

Dose Rate from 5" x 5" Plane Cobalt-60 Sources at Various Plane Positions

Distance*	Specific Dose Rate, rads/hr per curie/cm ²										
	Source Position**										
	A'	B'	C'	D'	E'	F'	G'	H'	I'	J'	K'
1.0	3.52x10 ⁴	4.13x10 ³	1.79x10 ³	.984x10 ³	.637x10 ³	.292x10 ³	.213x10 ³	.125x10 ³	.0982x10 ³	.0780x10 ³	.0616x10 ³
2.0	2.13x10 ⁴	3.82	1.70	.952	.617	.280	.209	.119	.0942	.0743	.0584
3.0	1.41x10 ⁴	3.43	1.59	.907	.594	.272	.202	.114	.090	.0704	.0560
4.0	9.65x10 ³	3.01	1.45	.853	.559	.260	.193	.110	.0862	.0673	.0533
5.0	6.90	2.55	1.31	.782	.520	.246	.185	.106	.0829	.0640	.0506
6.00	5.02	2.13	1.15	.708	.478	.232	.176	.100	.0783	.0610	.0477
7.0	3.73	1.73	.996	.627	.432	.218	.162	.094	.0740	.0578	.0450
8.0	2.80	1.42	.857	.552	.383	.199	.148	.0875	.0683	.0542	.0422
9.0	2.11	1.15	.725	.481	.338	.182	.136	.0809	.0635	.0502	.0394
10.0	1.60	.933	.610	.416	.296	.163	.123	.0745	.0583	.0464	.0364
11.0	1.23	.769	.512	.359	.258	.145	.111	.0686	.0536	.0425	.0336
12.0	.964	.635	.428	.307	.224	.129	.099	.0622	.0488	.0389	.0309

* Distance from Target Point to Source Plane (Inches)

**Positions A' through K' refer to source positions as shown in Figure IV-1.

TABLE IV-5

Dose Rate from 5" x 5" Plane Cesium-137 Sources at Various Plane Positions

Specific Dose Rate, 10^3 rads/hr per curie/cm²

Source Position**

Distance*	A'	B'	C'	D'	E'	F'	G'	H'	I'	J'	K'
1.0	7.58x1	.972	.386	.207	.119	.052	.035	.0179	.0131	.0098	.0074
2.0	4.53	.882	.369	.200	.116	.051	.034	.0177	.0129	.0096	.0073
3.0	3.09	.762	.341	.189	.112	.049	.033	.0173	.0126	.0094	.0072
4.0	2.04	.664	.307	.174	.104	.047	.032	.0166	.0123	.0092	.0070
5.0	1.48	.542	.270	.158	.097	.044	.030	.0160	.0119	.0089	.0068
6.0	1.08	.444	.235	.141	.088	.040	.028	.0152	.0114	.0086	.0065
7.0	.797	.360	.201	.124	.080	.037	.027	.0143	.0108	.0082	.0062
8.0	.603	.290	.170	.107	.070	.04	.024	.0133	.0102	.0077	.0059
9.0	.439	.233	.141	.092	.062	.030	.022	.0124	.0094	.0072	.0056
10.0	.329	.187	.116	.076	.053	.027	.020	.0115	.0087	.0067	.0052
11.0	.243	.149	.096	.065	.046	.024	.018	.0104	.0080	.0062	.0048
12.0	.187	.119	.078	.053	.039	.021	.016	.0095	.0073	.0057	.0045

APPENDIX V

Computer Codes

V-1 FUDGE 3A

1. Name of code: FUDGE 3A
2. Computer for which designed: IBM 7094
Programming system: FORTRAN II
3. Nature of problem solved: gamma dose rate distribution at any desired set of coordinate points in any infinite homogeneous target from a finite rectangular plane isotropic gamma source of arbitrary size. Utilizes a modified 12 point Gauss quadrature integration over the source area. Includes buildup in exponential form.
4. Restrictions on complexity:

Let NX = number of positions along the X or horizontal axis, starting at the source center, at which the dose rate is calculated.

NY = number of positions along the Y or vertical axis, starting at the source center, at which the dose rate is calculated.

NZ = number of positions measured along the Z axis perpendicular to the source at which the dose rate is calculated.

NL = number of terms in the exponential approximation of buildup.

Then $1 \leq NX < 200$
 $1 \leq NY \leq 32,767$
 $1 \leq NZ \leq 32,767$
 $1 \leq NL \leq 10$

5. Typical Running Time: About .065 sec/point per unit value of NL.
6. Features of the code:
 - a. Basically the code calculates the uncollided photon flux at a point in an infinite homogeneous target. The integration over the source uses polar coordinates and is based on rotation about the projection of the desired point on the source plane and the intersection of the radius vector with the source boundary. Exponential forms of the buildup factor may be incorporated since this does not change the form of the function being integrated. Absorbed dose rate or exposure dose rate may be calculated by the use of an appropriate constant. Average values of dose rate along paths parallel to the X axis at all values of Y and Z are also calculated based on a Simpson's Rule integration. Correct averages depend on selection of NX as an odd number and the use of an appropriately fine mesh size.
7. Present status: In use.

8. Input instructions:

Three different cards are required per case if the isotope or target material is varied with one blank card inserted between each set. For a fixed isotope and target material the first case requires three cards and each successive case requires two cards. To terminate a run two consecutive blank cards are used.

The following is a description of the input variables and data cards:

a. Card No. 1.

1. CST - constant to convert energy flux in $\text{Mev/cm}^2\text{-sec}$ to absorbed dose rate in $\text{rad/hr per curie/cm}^2$.
2. ET - total gamma ray energy in Mev.

b. Card No. 2.

1. FA - source width in inches; floating point.
2. FB - source height in inches; floating point.
3. NX - number of mesh points in the X direction; fixed point.
4. NY - number of mesh points in the Y direction; fixed point.
5. NZ - number of mesh points in the Z direction; fixed point.
6. DX - mesh spacing in X direction (inches); floating point.
7. DY - mesh spacing in Y direction (inches); floating point.
8. DZ - mesh spacing in Z direction (inches); floating point.
9. NL - number of terms in the exponential approximation of the buildup factor; fixed point.

c. Card No. 3.

1. AL(I) - mean free path in inches of gamma rays at initial energy, adjusted for buildup by the following equation:

$$AL(I) = \frac{1}{\mu_t (1 + \alpha_I)} \quad \text{V-1.1}$$

$$\text{where } B = \sum_{I=1}^{I=NL} C_I e^{-\alpha_I (\mu_t Z)} \quad \text{V-1.2}$$

$$\text{and } \sum_{I=1}^{I=NL} C_I = 1.00 \quad \text{V-1.3}$$

2. C(I) - weighting factor in Eq. V-1.2 given above.

V-2 FUDGE 4

1. Name of code: FUDGE 4.
2. Computer for which designed: IBM 7094.

Programming system: FORTRAN II

3. Description of code:

Fudge 4 is basically the same as FUDGE 3A except that it was written for composite slab targets which may be infinite in the X and Y directions and finite in Z. Source thickness, cladding thickness, absorber thickness, and finite air gaps between source and target may be incorporated in the calculation. Five different cards are required per case. As in FUDGE 3A, a blank card is required between sets of cards if the isotope or the target material is varied. For consecutive computer calculations involving changes in source dimensions, cladding thickness, absorber thickness, air gap, or target mesh points, only sets of cards number 2 to 5 are required after the first set of cards number 1 to 5. Two blank cards are used at the end of the last set to terminate the computation.

4. Typical running time: About .261 sec/point for each value of NL.
5. Input instructions:

The following is a description of the input variables and the order in which they are to be inserted:

a. Card No. 1.

1. CST - Same as FUDGE 3A.
- ET - Same as FUDGE 3A.

b. Card No. 2.

1. FA)
2. FB)
3. NX)
4. NY)
5. NZ) -- Same as FUDGE 3A
6. DX)
7. DY)
8. DZ)
9. NL)

c. Card No. 3.

1. ALFA(I) - value of α_I in buildup equation defined in FUDGE 3A description, equation V-1.2.
2. C(I) - Same as in FUDGE 3A. NOTE: There are NL sets of ALFA and C values with a maximum of three sets per card.

d. Card No. 4.

1. CLAD1 - source thickness, inches.
2. CLAD2 - source cladding thickness, inches.
3. CLAD3 - absorber thickness, inches.
4. AGAP - air gap between source face and target face, inches.

e. Card No. 5.

1. ESMU1 - total linear absorption coefficient of source, inches⁻¹.
2. ESMU2 - total linear absorption coefficient of source cladding, inches⁻¹.
3. ESMU3 - total linear absorption coefficient of air at S.T.P., inches⁻¹.
4. ESMU4 - total linear absorption coefficient at air at S.T.P., inches⁻¹.
5. ESMU5 - total linear absorption coefficient of target material, inches⁻¹.

CFDGE3A	FUDGE 3A	
	DIMENSIONAMU(24),R(24),DOSE(200,10),FX(200),AL(10),	0001
	1TDOSE(200),C(10),DOSAVE(10)	2
115	FORMAT (E 10.3, F 10.3)	0003
110	FORMAT(2F10.5,3I6,3F10.5,14)	4
601	FORMAT(1H 40X3HZ =F10.5,5X3HY =F10.5//)	0005
602	FORMAT(1H 0PF8.2,1P10E12.4)	6
603	FORMAT(1H 4X1HX58X9HDOSE RATE10X7HLAMBDA=F15.5//)	0007
604	FORMAT(1H140X44HDOSE RATE VS POSITION FOR RECTANGULAR SOURCE)	8
605	FORMAT(6F10.5)	0009
606	FORMAT(1H 4X1HX58X15HTOTAL DOSE RATE//)	10
607	FORMAT(1H 5X,35HTOTAL DOSE SAME AS DOSE, NO BUILDUP //)	0011
810	FORMAT(1H0,10X,20HTOTAL AVG DOSE RATE= 1PE15.5)	12
305	FORMAT(1H08HDOSAVE =1PE15.4//)	0013
C	SET UP COFFS FOR GAUSS QUADRATURE	14
	PI=3.14159	0015
	AMU(1)=.00921968 *.5	16
	AMU(2)=.04794137 *.5	0017
	AMU(3)=.11504866 *.5	18
	AMU(4)=.20634102 *.5	0019
	AMU(5)=.31608425 *.5	20
	AMU(6)=.43733329 *.5	0021
	R(1)=.02358767 *.5	22
	R(2)=.05346966 *.5	0023
	R(3)=.08003916 *.5	24
	R(4)=.10158371 *.5	0025
	R(5)=.11674669 *.5	26
	R(6)=.12457352 *.5	0027
	DO91 N=7,12	28
	J=13-N	0029
	AMU(N)=.5-AMU(J)	30
91	R(N)=R(J)	0031
	DO92 N=13,24	32
	J=N-12	0033
	AMU(N)=0.5+AMU(J)	34
92	R(N)=R(J)	0035
	WRITEOUTPUTTAPE6,604	36
112	READ INPUT TAPE 5,115, CST, ET	0037
	IF (CST) 3,1500,3	38
	3 READINPUTTAPE5,110,FA,FB,NX,NY,NZ,DX,DY,DZ,NL	0039
	IF(FA)112,112,150	40
150	IF(NL)1500,1500,114	0041
114	READINPUTTAPE5,605,(AL(I),C(I),I=1,NL)	42
	IF(AL(1))1500,1500,113	0043
113	WRITE OUTPUT TAPE 6,300,FA, FB, DX	44
300	FORMAT(1H05X3HA =F10.5,5X3HB =F10.5,5X9HDELTA X =F10.5//)	0045
	IF (NL-1) 730 ,730,820	46
730	WRITEOUTPUTTAPE6,607	0047
820	DO700IJL=1,10	48
	DO700IJK=1,200	0049
700	DOSE(IJK,IJL)=0.	50
	DO701IJK=1,200	0051
701	TDOSE(IJK)=0.	52
	DO750 IJL=1,10	0053
750	DOSAVE(IJL)=0.	54
	DO1011K=1,NZ	0055
	FZ=DZ*FLOATF(K)	56
510	DO1011J=1,NY	0057
	FY=DY*FLOATF(J-1)	58
	WRITEOUTPUTTAPE6,601,FZ,FY	0059
	DO703L=1,NL	60

1	ALAMB=AL(L)	61
	WRITEOUTPUTTAPE6,603,AL(L)	0062
	DO1010IX=1,NX	63
	FX(IX)= DX* FLOATF(IX-1)	0064
4	X=ABSF(FX(IX))/ALAMB	65
	A=ABSF(FA)/ALAMB	0066
	B=ABSF(FB)/ALAMB	67
	Z=ABSF(FZ)/ALAMB	0068
	Y=ABSF(FY)/ALAMB	69
	V=X-.5*A	0070
	VC=Y-.5*B	71
	IF(V)301,302,301	0072
302	V=.1E-10	73
301	IF(VC)303,304,303	0074
304	VC=.1E-10	75
303	IF(X)5,6,5	0076
6	IF(Y)7,101,7	77
5	U=Y/X-B/A	0078
	IF(U)8,8,7	79
7	UX=Y	0080
	UY=X	81
	UB=A	0082
	UA=B	83
	X=UX	0084
	Y=UY	85
	A=UA	0086
	B=UB	87
	UV=VC	0088
	UVC=V	89
	V=UV	0090
	VC=UVC	91
8	IF(V)101,9,9	0092
9	IF(VC)102,103,103	93
C	IN ZONE 1	0094
101	TH1=-ATANF(-V/(.5*B+Y))	95
	TH2=ATANF((.5*A+X)/(.5*B+Y))	0096
	TH3=ATANF(-VC/(.5*A+X))+.5*PI	97
	TH4=ATANF(V/VC)+PI	0098
	PHI=0.0	99
	Q=-EXPIF(-Z)	0100
	DO201 I=1,24	101
	TH=2.*PI*AMU(I)+TH1	0102
	IF(TH-TH2)211,212,212	103
212	IF(TH-TH3)213,214,214	0104
214	IF(TH-TH4)215,216,216	105
211	SX=(Y+.5*B)/COSF(TH)	0106
	GOTO217	107
213	SX=(X+.5*A)/SINF(TH)	0108
	GOTO217	109
215	SX=VC/COSF(TH)	0110
	GOTO217	111
216	SX=V/SINF(TH)	0112
217	S=SQRTF(SX**2+Z**2)	113
201	PHI=PHI+R(I)*(Q+EXPIF(-S))	0114
	PHI=.5*PHI	115
	GO TO 1010	0116
C	IN ZONE 2	117
102	TH1=ATANF(V/(Y+.5*B))	0118
	TH2=ATANF((X+.5*A)/(Y+.5*B))	119
	TH3=ATANF(-VC/(X+.5*A))+.5*PI	0120
	TH4=ATANF(-VC/V)+.5*PI	121

PHI=0.0	122
DTH=TH4-TH1	0123
DO236 I=1,24	124
TH=TH1+AMU(I)*DTH	0125
IF (TH-TH2) 231,232,232	126
232 IF (TH-TH3) 233,234,234	0127
231 SX1=V/SINF (TH)	128
SX2=(Y+.5*B)/COSF (TH)	0129
GOTO235	130
233 Q=1./SINF (TH)	0131
SX1=V*Q	132
SX2=(X+.5*A)*Q	0133
GOTO235	134
234 SX1=V/SINF (TH)	0135
SX2=VC/COSF (TH)	136
235 S1=SQRTF (SX1**2+Z**2)	0137
S2=SQRTF (SX2**2+Z**2)	138
236 PHI=PHI+(-EXPIF (-S1)+EXPIF (-S2))*R(I)	0139
PHI=PHI*DTH/(4.*PI)	140
GO TO 1010	0141
C IN ZONE 3	142
103 TH1=ATANF (V/(Y+.5*B))	0143
TH2=ATANF ((X+.5*A)/(Y+.5*B))	144
TH3=ATANF ((X+.5*A)/VC)	0145
TH4=ATANF (V/VC)	146
PHI=0.0	0147
DTH=TH3-TH1	148
PHI=0.0	0149
DO226 I=1,24	150
TH=TH1+AMU(I)*DTH	0151
IF (TH-TH2) 221,222,222	152
221 SX1=V/SINF (TH)	0153
SX2=(Y+.5*B)/COSF (TH)	154
GOTO225	0155
222 IF (TH-TH4) 223,224,224	156
223 Q=1./SINF (TH)	0157
SX1=V*Q	158
SX2=(X+.5*A)*Q	0159
GOTO225	160
224 SX1=VC/COSF (TH)	0161
SX2=(X+.5*A)/SINF (TH)	162
225 S1=SQRTF (SX1**2+Z**2)	0163
S2=SQRTF (SX2**2+Z**2)	164
226 PHI=PHI+(-EXPIF (-S1)+EXPIF (-S2))*R(I)	0165
PHI=PHI*DTH/(4.*PI)	166
1010 DOSE (IX,L)=CST*ET*(3.7E+10)*PHI	0167
1020 DO1030 N=1,NX,10	168
M=N+9	0169
WRITEOUTPUTTAPE6,602,FX(N),(DOSE (I,L),I=N,M)	170
1030 CONTINUE	0171
C CALC OF AVERAGE HORIZONTAL DOSE RATE, NX MUST BE ODD	172
N1X=NX-1	0173
IF (N1X) 410,410,500	174
410 WRITEOUTPUTTAPE6,320	0175
320 FORMAT (1H02X27HAVERAGE DOSE NOT CALCULABLE)	176
GO TO 703	0177
500 ODD=0.	178
EVEN=0.	0179
DO400 I=2,N1X,2	180
400 EVEN=EVEN+DOSE (I,L)	0181
N2X=NX-2	182

	DO401 I=3,N2X,2	183
401	ODD=ODD+DOSE(I,L)	0184
	DOSAVE(L)=(DOSE(1,L)+4.*EVEN+2.*ODD+DOSE(NX,L))/(3.*FLOATF(N1X))	185
	WRITEOUTPUTTAPE6,305,DOSAVE(L)	0186
703	CONTINUE	187
	IF(NL-1)1011,1011,720	0188
720	DO 704 IX=1,NX	189
	TD0SE(IX)=0.	0190
	DO704 L=1,NL	191
704	TD0SE(IX)=C(L)*DOSE(IX,L)+TD0SE(IX)	0192
	WRITEOUTPUTTAPE6,606	193
	DO710 N=1,NX,10	0194
	M=N+9	195
710	WRITEOUTPUTTAPE6,602,FX(N),(TD0SE(I),I=N,M)	0196
C	CALC OF TOTAL AVG DOSE RATE	197
	TD0AV=0.	0198
	DO 800 L=1,NL	199
800	TD0AV=TD0AV+C(L)*DOSAVE(L)	0200
	WRITEOUTPUTTAPE6,810,TD0AV	201
1011	CONTINUE	0202
	GOTO3	203
1500	CALL EXIT	0204
	END	205

*****0206*****

CFDGE4	FUDGE4	1
	DIMENSIONAMU(24),R(24),DOSE(200,10),FX(200),AL(10),	0002
	1TDOSE(200),C(10),DOSAVE(10)	3
	DIMENSION ALFA(6),B1MU(6)	0004
115	FORMAT (E 10.3, F 10.3)	5
110	FORMAT(2F10.5,3I6,3F10.5,I4)	0006
601	FORMAT(1H 40X3HZ =F10.5,5X3HY =F10.5//)	7
602	FORMAT(1H 0PF8.2,1P10E12.4)	0008
603	FORMAT(1H 4X1HX58X9HDOSE RATE10X7HLAMBDA=F15.5//)	9
604	FORMAT(1H140X44HDOSE RATE VS POSITION FOR RECTANGULAR SOURCE)	0010
605	FORMAT(6F10.5)	11
606	FORMAT(1H 4X1HX58X15HTOTAL DOSE RATE//)	0012
607	FORMAT(1H 5X,35HTOTAL DOSE SAME AS DOSE, NO BUILDUP //)	13
810	FORMAT(1H0,10X,20HTOTAL AVG DOSE RATE= 1PE15.5)	0014
305	FORMAT(1H08HDOSAVE =1PE15.4//)	15
620	FORMAT(4F10.7)	0016
621	FORMAT(5F10.7)	17
630	FORMAT(1H 14HSOURCE THICK.=F12.7,3X5HCLAD=F12.7,3X8HABSORB.=F12.7,	0018
	13X8HAIR GAP=F12.7//)	19
631	FORMAT(1H 4HMU1=F12.7,3X4HMU2=F12.7,3X4HMU3=F12.7,3X4HMU4=F12.7,3X	0020
	14HMU5=F12.7//)	21
C	SET UP COFFS FOR GAUSS QUADRATURE	0022
	PI=3.14159	23
	AMU(1)=.00921968 *.5	0024
	AMU(2)=.04794137 *.5	25
	AMU(3)=.11504866 *.5	0026
	AMU(4)=.20634102 *.5	27
	AMU(5)=.31608425 *.5	0028
	AMU(6)=.43733329 *.5	29
	R(1)=.02358767 *.5	0030
	R(2)=.05346966 *.5	31
	R(3)=.08003916 *.5	0032
	R(4)=.10158371 *.5	33
	R(5)=.11674669 *.5	0034
	R(6)=.12457352 *.5	35
	DO91 N=7,12	0036
	J=13-N	37
	AMU(N)=.5-AMU(J)	0038
91	R(N)=R(J)	39
	DO92 N=13,24	0040
	J=N-12	41
	AMU(N)=0.5+AMU(J)	0042
92	R(N)=R(J)	43
	WRITEOUTPUTTAPE6,604	0044
112	READ INPUT TAPE 5,115, CST, ET	45
	IF (CST) 3,1500,3	0046
3	READINPUTTAPE5,110,FA,FB,NX,NY,NZ,DX,DY,DZ,NL	47
	IF(FA)112,112,150	0048
150	IF(NL)1500,1500,114	49
114	READINPUTTAPE5,605,(ALFA(I),C(I),I=1,NL)	0050
	IF(C(1))1500,1500,113	51
113	WRITE OUTPUT TAPE 6,300,FA, FB, DX	0052
300	FORMAT(1H05X3HA =F10.5,5X3HB =F10.5,5X9HDELTA X =F10.5//)	53
	READINPUTTAPE5,620,CLAD1,CLAD2,CLAD3,AGAP	0054
	READINPUTTAPE5,621,ESMU1,ESMU2,ESMU3,ESMU4,ESMU5	55
	WRITEOUTPUTTAPE6,630,CLAD1,CLAD2,CLAD3,AGAP	0056
	WRITEOUTPUTTAPE6,631,ESMU1,ESMU2,ESMU3,ESMU4,ESMU5	57
	IF (NL-1) 730 ,730,820	0058
730	WRITEOUTPUTTAPE6,607	59
820	DO7001JL=1,10	0060
	DO7001JK=1,200	61

700	DOSE(IJK,IJL)=0.	62
	DO701IJK=1,200	0063
701	TDOSE(IJK)=0.	64
	DO750 IJL=1,10	0065
750	DOSAVE(IJL)=0.	66
	DO910I=1,6	0067
910	AL(I)=0.	68
	SMUT=ESMU1*CLAD1/2.+ESMU2*CLAD2+ESMU3*CLAD3	0069
	+ESMU4*AGAP	70
	SCLAD=CLAD1/2.+CLAD2+CLAD3+AGAP	0071
	DO1011K=1,NZ	72
	FZ=DZ*FLOATF(K-1)	0073
	SMFP=SMUT+ESMU5*FZ	74
	ZDIST=SCLAD+FZ	0075
	BARMU=SMFP/ZDIST	76
	DO901I=1,NL	0077
	DIMU(I)=BARMU*(1.+ALFA(I))	78
901	AL(I)=1./DIMU(I)	0079
510	DO1011J=1,NY	80
	FY=DY*FLOATF(J-1)	0081
	WRITEOUTPUTTAPE6,601,FZ,FY	82
	DO703L=1,NL	0083
	ALAMB=AL(L)	84
	WRITEOUTPUTTAPE6,603,AL(L)	0085
	DO1010IX=1,NX	86
	FX(IX)= DX* FLOATF(IX-1)	0087
4	X=ABSF(FX(IX))/ALAMB	88
	A=ABSF(FA)/ALAMB	0089
	B=ABSF(FB)/ALAMB	90
	Z=ABSF(ZDIST)/ALAMB	0091
	Y=ABSF(FY)/ALAMB	92
	V=X-.5*A	0093
	VC=Y-.5*B	94
	IF(V)301,302,301	0095
302	V=.1E-10	96
301	IF(VC)303,304,303	0097
304	VC=.1E-10	98
303	IF(X)5,6,5	0099
6	IF(Y)7,101,7	100
5	U=Y/X-B/A	0101
	IF(U)8,8,7	102
7	UX=Y.	0103
	UY=X	104
	UB=A	0105
	UA=B	106
	X=UX	0107
	Y=UY	108
	A=UA	0109
	B=UB	110
	UV=VC	0111
	UVC=V	112
	V=UV	0113
	VC=UVC	114
8	IF(V)101,9,9	0115
9	IF(VC)102,103,103	116
C	IN ZONE 1	0117
101	TH1=-ATANF(-V/(.5*B+Y))	118
	TH2=ATANF((.5*A+X)/(.5*B+Y))	0119
	TH3=ATANF(=VC/(.5*A+X))+.5*PI	120
	TH4=ATANF(V/VC)+PI	0121
	PHI=0.0	122

Q=-EXPIF(-Z)	123
DO201 I=1,24	0124
TH=2.*PI*AMU(I)+TH1	125
IF (TH-TH2) 211,212,212	0126
212 IF (TH-TH3) 213,214,214	127
214 IF (TH-TH4) 215,216,216	0128
211 SX=(Y+.5*B)/COSF(TH)	129
GOTO217	0130
213 SX=(X+.5*A)/SINF(TH)	131
GOTO217	0132
215 SX=VC/COSF(TH)	133
GOTO217	0134
216 SX=V/SINF(TH)	135
217 S=SQRTF(SX**2+Z**2)	0136
201 PHI=PHI+R(I)*(Q+EXPIF(-S))	137
PHI=.5*PHI	0138
GO TO 1010	139
C IN ZONE 2	0140
102 TH1=ATANF(V/(Y+.5*B))	141
TH2=ATANF((X+.5*A)/(Y+.5*B))	0142
TH3=ATANF(-VC/(X+.5*A))+.5*PI	143
TH4=ATANF(-VC/V)+.5*PI	0144
PHI=0.0	145
DTH=TH4-TH1	0146
DO236 I=1,24	147
TH=TH1+AMU(I)*DTH	0148
IF (TH-TH2) 231,232,232	149
232 IF (TH-TH3) 233,234,234	0150
231 SX1=V/SINF(TH)	151
SX2=(Y+.5*B)/COSF(TH)	0152
GOTO235	153
233 Q=1./SINF(TH)	0154
SX1=V*Q	155
SX2=(X+.5*A)*Q	0156
GOTO235	157
234 SX1=V/SINF(TH)	0158
SX2=VC/COSF(TH)	159
235 S1=SQRTF(SX1**2+Z**2)	0160
S2=SQRTF(SX2**2+Z**2)	161
236 PHI=PHI+(-EXPIF(-S1)+EXPIF(-S2))*R(I)	0162
PHI=PHI*DTH/(4.*PI)	163
GO TO 1010	0164
C IN ZONE 3	165
103 TH1=ATANF(V/(Y+.5*B))	0166
TH2=ATANF((X+.5*A)/(Y+.5*B))	167
TH3=ATANF((X+.5*A)/VC)	0168
TH4=ATANF(V/VC)	169
PHI=0.0	0170
DTH=TH3-TH1	171
PHI=0.0	0172
DO226 I=1,24	173
TH=TH1+AMU(I)*DTH	0174
IF (TH-TH2) 221,222,222	175
221 SX1=V/SINF(TH)	0176
SX2=(Y+.5*B)/COSF(TH)	177
GOTO225	0178
222 IF (TH-TH4) 223,224,224	179
223 Q=1./SINF(TH)	0180
SX1=V*Q	181
SX2=(X+.5*A)*Q	0182
GOTO225	183

224	SX1=VC/COSF(TH)	184
	SX2=(X+.5*A)/SINF(TH)	0185
225	S1=SQRTF(SX1**2+Z**2)	186
	S2=SQRTF(SX2**2+Z**2)	0187
226	PHI=PHI+(-EXP1F(-S1)+EXP1F(-S2))*R(1)	188
	PHI=PHI*DTH/(4.*PI)	0189
1010	DOSE(IX,L)=CST*ET*(3.7E+10)*PHI	190
1020	DO1030N=1,NX,10	0191
	M=N+9	192
	WOT6,602,FX(N),(DOSE(I,L),I=N,M)	0193
1030	CONTINUE	194
C	CALC OF AVERAGE HORIZONTAL DOSE RATE, NX MUST BE ODD	0195
	N1X=NX-1	196
	IF(N1X)410,410,500	0197
410	WRITEOUTPUTTAPE6,320	198
320	FORMAT(1H02X27HAVERAGE DOSE NOT CALCULABLE)	0199
	GO TO 703	200
500	ODD=0.	0201
	EVEN=0.	202
	DO400 I=2,N1X,2	0203
400	EVEN=EVEN+DOSE(I,L)	204
	N2X=NX-2	0205
	DO401 I=3,N2X,2	206
401	ODD=ODD+DOSE(I,L)	0207
	DOSAVE(L)=(DOSE(1,L)+4.*EVEN+2.*ODD+DOSE(NX,L))/(3.*FLOATF(N1X))	208
	WRITEOUTPUTTAPE6,305,DOSAVE(L)	0209
703	CONTINUE	210
	IF(NL-1)1011,1011,720	0211
720	DO 704 IX=1,NX	212
	TD0SE(IX)=0.	0213
	DO704L=1,NL	214
704	TD0SE(IX)=C(L)*DOSE(IX,L)+TD0SE(IX)	0215
	WRITEOUTPUTTAPE6,606	216
	DO710N=1,NX,10	0217
	M=N+9	218
710	WRITEOUTPUTTAPE6,602,FX(N),(TD0SE(I),I=N,M)	0219
C	CALC OF TOTAL AVG DOSE RATE	220
	TDAV=0.	0221
	DO 800 L=1,NL	222
800	TDAV=TDAV+C(L)*DOSAVE(L)	0223
	WRITEOUTPUTTAPE6,810,TDAV	224
1011	CONTINUE	0225
	GOTO3	226
1500	CALL EXIT	0227
	END	228

*****0228*****

APPENDIX VI

ABSORBER CORRECTIONS

The data presented in the following curves have been included for use by the methods of section 3.0 of this report.

The presentation of the data can best be understood by referring to Figures VI a and b. For each curve the plane source size is denoted by A and B. A is the dimension in the x direction and B the dimension in the y direction. The center of the coordinate system is located at the center of the source at a distance of one inch from the source plane. (As in Figure VI b.) This point is located at the air-water interface, facing the source. Z is the depth in the target. In the curves the relative dose rate is plotted against the absorber thickness in mean free paths. The absorber thickness referred to is A_b as shown in Figure VI b.

ALL DATA IN CURVES BELOW ARE FOR
COBALT-60 PLANE SOURCES

FIGURE VIa

FIGURE VIb

FIGURE VI-1

FIGURE VI-2

FIGURE VI-3

FIGURE VI-4

FIGURE VI-5

FIGURE VI -6

FIGURE VI-7

FIGURE VI-8

FIGURE VI-9

FIGURE VI-10

FIGURE VI-11

FIGURE VI-12

FIGURE VI-13

FIGURE VI-14

FIGURE VI-15

FIGURE VI-16

FIGURE VI-17

FIGURE VI-18

FIGURE VI-19

FIGURE VI-20

FIGURE VI-21

FIGURE VI-22

FIGURE VI-23

FIGURE VI-24