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ELECTROEXCITATION FORM FACTORS
AND
FIXED-MULTIPOLARITY SUM RULES[†]

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ABSTRACT

Sum rules are derived for form factors describing the longitudinal isoscalar electroexcitation of nuclear levels with definite multipolarity. The sum-rule limits are not constant numbers but functions of the momentum transfer. Their behavior for small and large momentum transfers is discussed. An application is also presented for the excitation of the first-excited state of ${}^6\text{Li}$ by inelastically scattered electrons.

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The derivation of a sum rule traditionally involves the evaluation of the expectation value of the product of two operators (e.g. multipole moments) and its rewriting as a sum, following the insertion of a complete set of states between the operators. Two standard features normally appear in the result: a) the left-hand side is a sum of terms of positive sign, and b) the right-hand side, i.e. the limit, is a number, a physical quantity such as the charge or (a power of) the mean-square radius. The usefulness of a sum rule is related to these properties which provide ^anormalization of the fractional strength of the individual excited states.

In the following, we shall derive sum rules for nuclear isoscalar excitations which do not exhibit these features. They involve operators

$$F^J(q) = [4\pi(2J+1)]^{1/2} \int d^3x j_J(qx) Y_{J0}(\Omega_x) \rho(\vec{x}) \quad (1)$$

where q is the momentum transfer and

$$\rho(\vec{x}) = \sum_{i=1}^A \delta(\vec{x} - \vec{r}_i)$$

is the mass-density operator. If the nuclear ground state is restricted to $J^\pi = 0^+$, the matrix elements

$$F_{n0}^J(q) = \sqrt{4\pi} \langle n || F^J(q) || 0 \rangle \quad (2)$$

represent isoscalar form factors for the longitudinal electroexcitation of the levels $|n\rangle$ with angular momentum J and parity $(-1)^J$. We shall consider two distinct values q and q' of the momentum transfer and evaluate the expectation value of the product $-i\hbar F^J(q') \dot{F}^J(q)$ where $-i\hbar \dot{F}(q) \equiv [H, F(q)]$. Hermitian conjugation and time-reversal invariance establish the relation

$$-i\hbar \langle 0 | F^J(q') \dot{F}^J(q) | 0 \rangle = \frac{1}{2} \langle 0 | [F^J(q'), [H, F(q)]] | 0 \rangle, \quad (3)$$

which makes possible (without charge-exchange difficulties since the F 's are isoscalar) the evaluation of the expectation value. After insertion of a complete set of states we find

$$\begin{aligned} \sum_n^{(J)} E_n F_{on}^J(q') F_{no}^J(q) \\ = \frac{\hbar^2}{2m} q' q \int d^3x \rho_{oo}(x) \{ J j_{J-1}(q'x) j_{J-1}(qx) \\ + (J+1) j_{J+1}(q'x) j_{J+1}(qx) \} \end{aligned} \quad (4)$$

where $\rho_{oo}(x) \equiv \langle 0 | \rho(\vec{x}) | 0 \rangle$ is the ground-state density assumed to be spherically symmetric. Clearly only transitions of multipolarity J are included in the sum. We note also that the individual terms are not restricted to a definite sign and that although the right-hand side involves only one-particle operators it has no particularly simple physical significance. A number of interesting relations can be derived from Eq.4; we shall consider two such special cases. In the one, we regain the simplifying feature of positive terms appearing in the sum by letting $q' = q$; in the second we allow $q' \rightarrow 0$, noticing that this leads to a more meaningful expression in the right-hand side. The results are

$$\begin{aligned} \sum_n^{(J)} E_n |F_{on}^J(q)|^2 \\ = \frac{\hbar^2}{2m} q^2 \int d^3x \rho_{oo}(x) \{ j_{J-1}^2(qx) + \frac{(J+1)}{J} j_{J+1}^2(qx) \} \end{aligned} \quad (5)$$

and

$$\sum_n^{(J)} E_n M_{on}^J F_{no}^J(q) = (-1)^{J-1} \frac{\hbar^2}{2m} J (2J+1) q^J \left(\frac{1}{q} \frac{d}{dq} \right)^{J-1} F_{el.}(q), \quad (6)$$

where $F_{el.}(q) \equiv F_{oo}(q)$ is the elastic form factor, and M_{on} is the unretarded transition matrix element, related to the form factor by

$$F_{on}^J(q) \xrightarrow{q \rightarrow 0} \frac{q^J}{(2J+1)!!} M_{on}^J \quad (7)$$

We shall confine our discussion to a comparison of the two theoretical expressions presented above, and an illustration of their possible usefulness in a specific example. We notice first that the expression on the right side of Eqs. (5) and (6) are functions of q , providing a different limit for each value of the momentum transfer. They are ground-state properties, not required to bear any resemblance in shape to the individual terms in the sum.

The appealing feature in the limit of Eq. (6) is its relation to an observable quantity, viz. the elastic form factor. This is not true for the integral expression in Eq. (5) which is nonetheless easy to evaluate for any given density. A plot of this expression (for several values of J) is given in Fig. 1 for the simple case of a uniform distribution $\rho_{00}(x)$. The main trend in these curves is the almost monotonic increase with increasing q . The oscillations are relatively minor and have been found to become even less pronounced for less abruptly changing densities. The finite limit reached rather promptly for larger momentum transfers should be contrasted with the behavior of Eq. (6) whose limit tends to zero for $q \rightarrow \infty$. More mathematically, we can replace, for sufficiently large q , the spherical Bessel functions in Eq. (5) by their asymptotic expressions. We then find

$$\lim_{q \rightarrow \infty} \sum_n^{(J)} E_n |F_{on}(q)|^2 = \frac{\pi^2}{4m} (2J+1) A \langle r^{-2} \rangle \quad (8)$$

which, apart from the factor $2J+1$ is independent of multipolarity. It is probably worth relating this result to the corresponding limit of a non-energy-weighted sum rule. Using closure we obtain the identity

$$\sum_n^{(J)} |F_{on}^J(q)|^2 = 4\pi(2J+1) \langle 0 | \sum_{ij} j_J(qr_i) j_J(qr_j) Y_{J0}(\Omega_i) Y_{J0}(\Omega_j) | 0 \rangle \quad (9)$$

For large q , the terms with $i \neq j$ become negligible while the remaining terms give

$$\lim_{q \rightarrow \infty} \sum_n \langle J \rangle |F_{on}^J(q)|^2 = \frac{2J+1}{2q^2} A \langle r^{-2} \rangle \quad (10)$$

We notice that multiplication by $\hbar^2 q^2 / 2m$ gives precisely the result of Eq. (8). This is consistent with the expectation that the contributing terms in this case represent quasielastic scattering, as is suggested also by the linear dependence on A of both Eqs. (8) and (10).

For small momentum transfers, we can replace the form factors by their $q \rightarrow 0$ limits given by Eq. (7). We then find that Eqs. (5) and (6) become identical to each other and indeed quite familiar.¹ It is not difficult to demonstrate that they remain essentially identical when the next power of q , viz. q^{J+2} is considered. Their difference for larger q is shown graphically in Fig. 2 for the case of ${}^6\text{Li}$, which we shall now discuss. We want to study in particular, the electroexcitation of the first excited state of ${}^6\text{Li}$ whose longitudinal quadrupole form factor is known experimentally, as is also the elastic form factor F_{el} . A simple theoretical model for the inelastic form factor can be obtained from Eq. (6) by a method previously referred to as the single doorway-dominance approximation (DDA).² This approach relates the radiative widths and the inelastic form factors (for a limited range of q) to the component, in each state, of a fragmented, giant (doorway) excitation. This leads to the expectation, that each form factor is proportional to that of the doorway state. The common functional dependence on q can then be taken outside the summation in Eq. (6) and becomes identified with its right-hand side which, in turn, is available from elastic scattering data. An overall normalization constant can be determined by fitting the experimental value of the radiative width. These instructions lead unambiguously to the solid line in Fig. 2 which agrees remarkably well with the data.

The dashed curve in Fig. 2 represents, on the other hand, the sum-rule limit of Eq. (5). It is calculated for a density consistent with the data on F_{el} for ${}^6\text{Li}$ and is normalized to the radiative-transition strength of the first excited state. Except for the low- q region, it is seen to deviate considerably from the data and from the doorway-dominance prediction. This may illustrate the non-triviality of this prediction, but it also emphasizes an ambiguity in its assumptions: if the DDA is applied to Eq. (5) rather than Eq. (6), it leads to different expressions for the form factors (the dashed line in Fig. 2) which indeed compare poorly with experiment. It is clear (and not really surprising) that we do better by restricting it to Eq. (6). This does not seem unreasonable since the mixed sum of Eq. (6) selects preferentially states with larger radiative strengths and may be more appropriate for bound-state-to-bound-state transitions. This is indicated by the difference in the behavior of these two sum-rule limits for large q . It is also clear that we cannot consistently assume the DDA of Eq. (6) for all states (i.e., all energies). The sum rule of Eq. (5) is also useful, suggesting e.g. the definition of a momentum-transfer dependent oscillator strength, a natural generalization of the oscillator strength in the conventional (unretarded) sum rules. In the case of ${}^6\text{Li}$ we find that approximately 8% of the total, energy-weighted quadrupole strength is contained in the first-excited state at low q but only 0.8% at $q = 1.6 \text{ fm}^{-1}$. This relative decrease in strength should be related to the enhancement, for larger q , of processes not included into the doorway excitation mentioned above e.g. quasielastic scattering. A more critical discussion of these questions and a presentation of additional sum rules for form factors and transition densities will be contained in a forthcoming publication.

REFERENCES

1. See e.g. E. K. Warburton and J. Weneser in "Isospin in Nuclear Physics" ed. by D. H. Wilkinson, Wiley Interscience, p.195 (1969).
2. E. I. Kao and S. Fallieros, Phys. Rev. Letters 25 (1970) 827. T. J. Deal and S. Fallieros, to be published in Phys. Rev. C, April, 1973.
3. Neuhausen, R., Z. Phys. 220 (1969) 456.
4. Eigenbrod, F., Z. Naturforsch. 23a (1968) 1671.
5. Hutcheon, R., and H. Caplan, Nucl. Phys. 127A (1969) 417.
6. Suelzle, L., et. al., Phys. Rev. 162 (1967) 992.

FIGURE CAPTION

- Fig. 1 Sum rule limits of Eq. (5). The ordinate is the ratio of the right-hand side of Eq. (5) to its large q limit, Eq. (8).
- Fig. 2 The data for the quadrupole excitation of ${}^6\text{Li}$ is from Ref. 3 (solid circle), Ref. 4 (open circle), and Ref. 5 (cross). The solid curve is the DDA from Eq. (6), and the dashed from Eq. (5), both using a spherical phenomenological ground state density taken from the elastic scattering of Ref. 6. A slight generalization of the equation presented in the text has been appropriately introduced in order to take into account the non-vanishing spin of ${}^6\text{Li}$. The lifetime given provides the best fit of the solid curve to the data.