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**A WAVE PROPAGATION MODEL
FOR IMPULSE PREDICTION**

Otis R. Walton

July 30, 1971

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Contents

Abstract	1
Introduction	1
Qualitative Features	2
Free-Surface Configuration	3
Tamped Configuration	5
Quantitative Features	6
Free-Surface Configuration	7
Tamped Configuration	7
Summary of Equations	8
Discussion	9
Comparison With Data and Hydrocode Calculations	9
Normalized Results	10
Impulse vs Fluence	12
Summary	16
Acknowledgment	16
References	17

A WAVE PROPAGATION MODEL FOR IMPULSE PREDICTION

Abstract

Melt-dominated impulse produced by blowoff from a thin region of material that is instantaneously heated above melting may be estimated by considering propagation of the initial stress profile as an acoustic wave in a semi-infinite medium. The time integral of the stress profile as it passes some point is shown to be proportional to the total energy deposited. The constant of proportionality depends only on the depth of material melted and a simple initial shape param-

eter for depositions near a free surface. If a material with a low relative shock impedance is placed over the heated region, the impulse is the same as for the free-surface case. If a material with a high relative shock impedance is placed over the heated region, the proportionality constant depends only on the relative shock impedance and is independent of melt depth and deposition shape. The model fails when the vapor pressure of the heated material becomes significant.

Introduction

A simple model has been developed to predict the melt-dominated impulse (change in momentum per unit area) generated when a thin region of material is instantaneously heated above the melting temperature. The model is applicable to front-surface energy depositions (as in the case of electron beams or x rays) or to energy depositions interior to the material or at an interface between two materials (as may be the case for exploding foils or x rays). The basic assumptions in the model are violated at high energy densities where the vapor pressure of the liquid is large (i.e., on the order of kilobars).

The model is based on the following assumptions:

- (1) The equation of state is linear, i.e.,

$$P = \gamma \rho \epsilon, \quad (1)$$

where

P = pressure,

γ = constant (usually between 1 and 2 for metals),

ρ = density, and

ϵ = specific energy density.

- (2) We assume acoustic waves. That is, the initial wave shape is assumed to propagate at constant velocity, unattenuated through the material.

- (3) Material heated above its melting temperature cannot sustain a tension and also cannot expand significantly without having its pressure go essentially to zero.

The theory does not really depend on the assumption (1) of a linear equation of

state; however, such an assumption allows the integrals to be evaluated in an almost trivial manner, and it also allows the results to be summarized by very simple equations. The acoustic wave assumption (2) is not as unrealistic as it might appear on first reading. We are concerned with the total momentum in the pressure wave as it travels through the material. If we have an anisotropic shock wave (as is usually the case), the wave will interact with the material as it passes and will leave the material in a different thermodynamic state after passage of the shock; thus, the total kinetic energy of the wave (or the material in the wave) is constantly being dissipated and turned into both thermal energy and a residual kinetic energy of the material

behind the wave. However, during this process the total momentum in the problem is conserved. This is analogous to an inelastic collision of two bodies; kinetic energy is not conserved, but the net momentum of the two bodies after the collision is just the sum of their initial momenta. Therefore the total momentum that is initially contained in the material in a pressure wave is equal to the momentum that will be eventually transferred to all of the material the wave passes through as it is dissipated. Thus, to find the impulse at late times we only need to deal with the initial wave profiles and determine how they are split between the "blowoff" material and the remainder of the sample.

Qualitative Features

We can easily describe the qualitative features of the model by examining just how the initial pressure wave does get split between the blowoff material and the remainder of the sample. Consider a pressure profile in the interior of a slab of material that is initially at rest. The propagation of this initial pressure profile may be treated by considering it to split into two waves, one traveling in each direction, each with half the initial amplitude¹ (see Fig. 1). The total momentum in the material is zero; however, the two waves have equal and opposite momenta that are given by

$$\begin{aligned} \text{momentum/unit area} &= I_R = -I_L \\ &= \int_0^\infty P(t) dt, \quad (2) \end{aligned}$$

where $P(t)$ is the value of the pressure observed as the wave travels by some

Fig. 1. Initial stress distribution in a material treated as two waves traveling in opposite directions, each with half the initial amplitude.

fixed position to the right of the initial profile, I_R is the impulse contained in the right-moving wave, and I_L is the impulse in the left-moving wave.

If the initial pressure profile is located just to the right of an interface between two materials of differing shock impedances, then the wave moving to the left will immediately encounter the interface with this "tamper" and be split into two waves, one transmitted, the other reflected. The transmitted and reflected profiles are determined by the relations

$$\sigma_T = \frac{2Z_2}{Z_1 + Z_2} \sigma_i$$

and

$$\sigma_R = \frac{Z_2 - Z_1}{Z_1 + Z_2} \sigma_i, \quad (3)$$

where Z_j is the shock impedance² of material j . ($Z = \rho c$.)

If we have a free surface, then $Z_2 = 0$ and the reflected wave is just the inverse of the incident wave.

FREE-SURFACE CONFIGURATION

Consider a monotonically decreasing initial energy deposition in which some of the material is heated into a liquid state. Assume that the initial energy deposition is located at a free surface, and examine the propagation of the resultant initial stress profile. We shall ignore the difference in the equation of state between the liquid and the solid and treat the liquid as a solid-like material that has zero tensile strength. If we treat the initial stress profile as two waves and allow them to propagate for some small

time, Δt , we find upon examining the resultant stresses that the reflected negative stress wave from the free surface is always larger in magnitude than the positive stress in the wave moving toward that surface (because of the assumption of a monotonically decreasing initial profile). Thus, the resultant stress is negative for a distance $\Delta d = c\Delta t$. If the material is liquid, then it is physically impossible for it to support such negative stresses, and the material will relieve to zero stress by separating or spalling away from the underlying structure, thus creating a new free surface at the depth $c\Delta t$. For each successive time increment, Δt , the above process is repeated until the end of the melted region is encountered and the negative stress is physically able to exist in the material without causing separation or spall. We may make the time increments as small as we like to describe the continuous process. The velocity with which the free surface propagates into the liquid is the wave propagation velocity, c , of the material, so the melt depth is encountered at a time $t = \frac{d}{c}$. Any portion of the initial stress profile (moving toward the free surface) that is still in the solid material at the time $t = \frac{d}{c}$ will be reflected from the free surface at the melt depth and become a tensile wave following the inward moving half of the initial stress profile. Since the wave has already moved toward the free surface a distance d when the propagating free surface has reached the depth d , we find that the reflected tensile wave that propagates into the solid is just the reflection of that portion of the wave that was initially beyond the depth, $2d$ (where d is the melt depth). Figure 2(b) represents the

(a) Initial distribution adjacent to a free surface

(b) Resultant distribution after time $t = d/c$ if negative stress could exist in the liquid

(c) Resultant distribution at late times $t > d/c$

(d) An equivalent stress distribution with the same total impulse as stress wave shown in 2(c)

Fig. 2. Initial stress distribution adjacent to a free surface treated as two waves traveling in opposite directions, each with half the initial amplitude.

situation just as the propagating free surface reaches depth d (for illustration purposes the liquid region is fictitiously allowed to

show a net tension). The net stress profile is shown as the dashed line, and the various solid lines represent the four

components of this net stress profile. After the liquid region has separated from the solid and a new free surface has been generated at the melt depth, wave 4 is totally reflected; the resultant stress profile in the solid is shown in Fig. 2(c). The total impulse contained in this pressure wave is

$$I_{\text{net}} = I_1 - I_4, \quad (4)$$

where

I_1 = impulse in the right-moving half of the initial pressure distribution, and

I_4 = impulse in that portion of the right-moving wave that was initially beyond the depth, $2d$.

A hypothetical stress wave with the same net impulse is shown in Fig. 2(d). This stress profile is just that portion of the initial right-moving wave contained between the initial free surface and the depth, $2d$.*

TAMPED CONFIGURATION

Now if instead of a free surface, our initially heated sample is in contact with a solid material with a shock impedance Z_2 , the left-moving wave will interact with the interface to produce transmitted and reflected waves according to Eq. (3). The reflected wave may be either positive or negative depending on the shock impedance of the tamping material. Figure 3(a)

*The descriptions and the equations presented in this paper assume that the initial pressure profile decreases monotonically with depth in the material. However, a similar analysis can be done for any initial profile; a nonmonotonic profile will be treated in a later paper covering time-dependent energy depositions.

(a) Resultant distribution after time $t = d/c$ for a low-shock impedance tamper (i.e., tension occurs in the liquid)

(b) Resultant distribution after time $t = d/c$ for a high-shock impedance tamper (no tension occurs in the liquid)

Fig. 3. Resultant stress distributions if the initial distribution is adjacent to a tamping material.

shows the resultant stress and the various component waves for the tamped configuration in the case where the reflected wave is negative. Comparing Fig. 3(a) with 2(a), we see that the main differences are the addition of the transmitted wave (5) in the tamper and the reduction in amplitude of the reflected wave (3).

If the reflected wave is positive, then its momentum just adds to the momentum of the initial wave (1) moving to the right. If the reflected wave is negative (i.e., tension), then we must examine the total net pressure in the liquid region to see if any net tensions are present. If we find

that a net tension is present in the liquid, we assume that a new free surface is created, and the process then becomes identical to the free-surface case for the remainder of the left-moving wave which is then completely reflected from that new surface.

Whenever a net tension would result in the liquid we "spall immediately," and the net impulse is the same as that for the free-surface case. If no net tension is created in the liquid, the entire left-moving wave interacts with the material

interface, and the impulse in the reflected portion wave (3) gets added to the impulse in wave (1). In the case of no immediate "spall" of the liquid, it is assumed that the waves will travel to the respective free surfaces of the sample and tamper and then be reflected as tension waves. As soon as one of these tension waves encounters the liquid region, "spall" or separation occurs, and the waves are "trapped" in the respective materials so that the net momenta in these materials are just those of the initial waves entering them.

Quantitative Features

The impulse delivered to the sample may be calculated by taking the time integral of the net stress wave in the sample as it passes some point, i.e.,

$$I = \int_0^{t_f} P \, dt.$$

But we know P as a function of position, $P = P(x)$ —not as a function of time; consequently a change of variables is necessary. The wave passes a point with a velocity s so that x and t are related by $\frac{dx}{dt} = s$. Thus,

$$dt = \frac{dx}{s}.$$

By choosing the appropriate limits of integration, we now have

$$I = \int_0^{x_f} \frac{P}{s} \, dx.$$

(Note that s is the wave velocity, not the particle velocity.)

If we assume a linear equation of state $P_0 = \gamma \rho_0 \epsilon$, we have

$$I_r = \int_0^{x_f} \frac{\gamma \rho_0 \epsilon}{2s} \, dx$$

as the impulse in the right-moving wave (1) (where the 2 is inserted because we are considering only the half of the initial profile moving to the right). If we let $s \approx c_0$ and $\gamma \approx$ a constant, then they may be taken out of the integral, or

$$I_r = \int_0^{x_f} \frac{\gamma \rho_0 \epsilon}{2s} \, dx \approx \frac{\gamma}{2c_0} E_{\text{total}}, \quad (5)$$

where E_{total} is the total energy absorbed in the material.

(For other derivations of this equation, or very similar expressions, see Refs. 3 and 4. The discussion here follows closely that of Ref. 4.)

Now in order to calculate the total net momentum, we need to add to I_r that

fraction of the momentum in the left-moving wave that gets reflected back into the material on the right. The reflection occurs either at the original front surface or at the melt depth "surface," which was created by the spalling or splashing off of material.

FREE-SURFACE CONFIGURATION

The expression for I_r , the impulse contained in the right-moving half of the initial pressure distribution, is independent of the shape of the deposition profile and depends only on the total energy absorbed. This expression represents an upper limit on the melt-dominated impulse for the free-surface case, since we must subtract the impulse of the small reflected tensile tail to obtain the net impulse [Eq. (4)]. The impulse contained in the tensile tail [wave (4) in Fig. 2] is due to that portion of the initial wave that was below the depth, $2d$.

$$I_4 = \frac{\gamma}{2c_0} \int_{2d}^{x_f} \rho_0 \epsilon \, dx = \frac{\gamma}{2c_0} E_{2d}^{x_f},$$

where $E_{2d}^{x_f}$ is the total energy absorbed beyond depth $2d$. Subtracting I_4 from I_r , we obtain the net impulse for the free-surface configuration:

$$\begin{aligned} I_{\text{free surface}} &= I_r - I_4 \\ &= \frac{\gamma}{2c_0} \left(E_{\text{total}} - E_{2d}^{x_f} \right) \end{aligned}$$

or an equivalent expression [e.g., see Fig. 2(d)].

$$I_{\text{free surface}} = \frac{\gamma}{2c_0} E_0^{2d}, \quad (6)$$

where $E_0^{2d} = \int_0^{2d} \rho_0 \epsilon(x) \, dx$ is the energy deposited between the front surface and depth $2d$.

TAMPED CONFIGURATION

As has already been pointed out, the impulse in the tamped configuration will be identical to the free-surface impulse whenever we spall immediately, i.e., whenever a net tension results in the liquid region. Using acoustic waves we may qualitatively determine if a net tension will occur. When the reflected wave has just reached the melt depth, we can compare the value of its peak negative stress to the positive stress in the left-moving wave at the melt depth to determine if a net tension has occurred in the liquid. Thus if

$$\frac{Z_2 - Z_1}{Z_1 + Z_2} P_0 < -P_{2d},$$

then a net tension has occurred in the liquid region and we have an immediate spall situation. Since $P \propto \epsilon$, we may rewrite the condition for immediate spall as

$$\frac{Z_2 - Z_1}{Z_1 + Z_2} < -\frac{\epsilon_{2d}}{\epsilon_0},$$

where

ϵ_{2d} = initial specific energy at depth $2d$,

ϵ_0 = initial specific energy at the front surface,

Z_2 = shock impedance of the tamper, and

Z_1 = shock impedance of the sample.

Note that it is also possible for no net tension to occur as the peak of the

reflected wave moves through the liquid and still have a net tension in the liquid at some later time if

$$-\frac{Z_2 - Z_1}{Z_1 + Z_2} \epsilon(x) > \text{any } \epsilon \text{ between } x \text{ and } x + 2d.$$

Thus the shape of the deposition curve should be checked to see if such a situation exists. This situation probably exists only for a small percentage of real configurations, and if it is encountered, the acoustic wave idea can still be applied by considering both the part of the wave reflected from the interface and the part of the wave reflected from the new free surface that enters the solid region. The actual expressions depend on the location of the net tension and will not be presented here.

If no net tension results in the liquid upon reflection of the left-moving wave from the interface, the entire reflected portion of this wave goes into the solid material. The value of the pressure in this reflected wave is the initial pressure in the left-moving wave multiplied by the reflection coefficient at the interface. Thus the net impulse in the reflected portion is the impulse contained in the original wave multiplied by the reflection coefficient, i.e.,

$$\begin{aligned} I_{\text{reflected}} &= -\frac{Z_2 - Z_1}{Z_2 + Z_1} I_L \\ &= \frac{Z_2 - Z_1}{Z_2 + Z_1} I_r, \end{aligned} \quad (7)$$

and the net impulse in the total wave that enters the solid becomes

$$I_{\text{tamped}} = I_r + \frac{Z_2 - Z_1}{Z_2 + Z_1} I_r.$$

Recall that I_r [Eq. (5)] is independent of the shape of the deposition profile, so the tamped impulse in the case of no immediate spall is independent of the deposition profile and depends only on the shock impedances of the tamper and sample.

SUMMARY OF EQUATIONS

The above results may be summarized in the following expressions:

$$I = \begin{cases} I_{\text{free surface}} & \text{for } \frac{Z_2 - Z_1}{Z_2 + Z_1} < -\frac{\epsilon_{2d}}{\epsilon_0} \\ & \text{(immediate spall)} \\ I_r \left(1 + \frac{Z_2 - Z_1}{Z_2 + Z_1} \right) & \text{for } \frac{Z_2 - Z_1}{Z_2 + Z_1} > -\frac{\epsilon_{2d}}{\epsilon_0} \end{cases} \quad (8)$$

(no immediate spall)

where

$$I_{\text{free surface}} = \frac{\gamma}{2c_0} E_0^{2d},$$

$$I_r = \frac{\gamma}{2c_0} E_0^\infty,$$

and

$$E_a^b = \int_a^b \rho_0 \epsilon \, dx. *$$

These equations indicate that if spall occurs immediately, the total impulse is independent of what the tamping material

* In the expressions presented for the calculation of impulse we have assumed that consistent units are used. Thus, for the expression

$$I = \frac{\gamma}{2c} E,$$

the value for I will be given in taps if E is in ergs/cm² and c is in cm/sec. In order to use more conventional values where E is given in cal/cm² and c is in cm/μsec, we must multiply the right-hand member of our equation by a factor of 41.8, i.e.,

$$I = 41.8 \frac{\gamma}{2c} E,$$

which will give a value for I in taps.

is and depends only on the energy deposition profile parameter E_0^{2d} , the total energy from the surface to a depth $2d$. On the other hand, if there is no imme-

diated spall of the liquid (i.e., no net tension), then the impulse depends only on the shock reflection coefficient $\frac{Z_2 - Z_1}{Z_2 + Z_1}$ and the total energy absorbed E_0^∞ .

Discussion

COMPARISON WITH DATA AND HYDROCODE CALCULATIONS

A test of the validity of the formula for I_r may be made by comparing the free-surface impulse produced in experiments or calculated by other techniques for shallow deposition profiles to the value given by Eq. (5). J. Reaugh at Physics International has compared electron beam experimental data with an expression equivalent to Eq. (5) and also to the expanding vapor MTBBAY theory. His comparison indicated that a linear fluid model (or the present acoustic wave model) fit the experimental data reasonably well while the MTBBAY theory overpredicted melt-dominated impulses by more than a factor of two at high fluences.³

Both tamped and untamped configuration impulse values predicted by the present model have been compared to hydrocode calculations. An initial energy deposition profile was assumed to exist in a nickel sample. Various tamping materials with different shock impedances were arbitrarily put in front of the nickel sample to act as tampers. Hydrocode calculations were done with the PUFF hydrocode using the Grover three-phase equation of state for the nickel. The energy densities in the nickel were such that most of the heated region was in the liquid vapor two-phase region; however,

on expansion the residual vapor pressure (along the release adiabats) was less than 0.1 kbar, so the assumptions of the acoustic wave model were approximately satisfied. The results of the comparison are summarized in Table 1 and Fig. 4.

The spall criteria for the acoustic wave model assume that no attenuation or broadening of the stress waves occurs as they propagate through the material. The hydrocode calculations, on the other hand, should represent a more realistic behavior of the actual wave shapes. In a real situation, attenuation of the reflected wave may prevent net tensions from occurring, so there is no immediate spalling unless $\frac{Z_2 - Z_1}{Z_2 + Z_1}$ is considerably less than $-\frac{\epsilon_{2d}}{\epsilon_0}$. The hydrocode calculations verified that the reflection coefficient had to be significantly less than $\frac{\epsilon_{2d}}{\epsilon_0}$ for immediate spall to occur. An attenuation coefficient, $\alpha \lesssim 1.0$, could be included in the spall criteria to account for this deviation from acoustic behavior, e.g., the expression $\frac{Z_2 - Z_1}{Z_2 + Z_1} < -\frac{\epsilon_{2d}}{\alpha\epsilon_0}$ is probably a better representation of the condition for immediate spall. However, more work needs to be done before an accurate value of α is established or its dependence on deposition parameters determined.

Close examination of the hydrocode output indicated that in cases near the transition from spall-to-no-spall net tension occasionally occurred in the liquid

Table 1. Comparison of the acoustic wave model and hydrocode calculations.

Fixed parameters: $\frac{\epsilon_{2d}}{\epsilon_0} = 0.127$; $E_0^\infty = 177$ cal; $E_0^{2d} = 100$ cal; $Z_1 = 4.138$; $\gamma_1 = 1.83$.

Tamping material	Shock impedance	$\frac{Z_2 - Z_1}{Z_2 + Z_1}$	Impulse (ktaps) ^a		
			Acoustic model		PUFF
			Spall	No spall	
Free surface	0	-1	8.22	—	8.04
Be	1.495	-0.469	8.22	7.73	8.26
Th	2.57	-0.305	8.22	10.1	(8.3) ^b 11.6
V	3.111	-0.142	8.22	12.5	13.08
Brass	3.211	-0.126	8.22	12.7	13.42
Ni	4.138	0	8.22	14.6	14.82
W	7.63	0.297	8.22	18.9	16.86

^a A tap is a e.g.s. unit of momentum per unit area:

$$1 \text{ tap} = 1 \text{ g} \left(\frac{\text{cm}}{\text{sec}} \right) \left(\frac{1}{\text{cm}^2} \right) = 1 \frac{\text{dyne sec}}{\text{cm}^2}.$$

^b The hydrocode run with Th/Ni showed "immediate" spall as predicted by the acoustic wave model with an impulse of 8.3 ktaps. However, as the problem progressed, the liquid region expanded enough to close the void space and recombine with the solid, allowing waves again to propagate through the liquid region from solid to tamper and back; the final impulse obtained at late times was 11.6 ktaps.

Fig. 4. Comparison between the acoustic wave model and the hydrocode calculations.

after the peak tensile wave had entered the solid. Also, recombination of the void space created by immediate spall was observed when the shock impedances were near the spall-to-no-spall transition (see the footnote in Table 1).

NORMALIZED RESULTS

Normalized plots of the acoustic wave model are presented in Fig. 5. The diagonal line with a slope of 45° in Fig. 5 represents the impulse in the case where no immediate spall of the liquid occurs. The horizontal lines are representative of the family of lines corresponding to the impulse in the cases where immediate spall occurs. The

Fig. 5. Normalized plot of results from the acoustic wave model.

vertical lines in Fig. 5 represent the locations of typical transitions from a horizontal line to the diagonal line. That is, the vertical lines indicate the location of the transition from spall-to-no-spall conditions.

The particular deposition profile used for the hydrocode comparison had the transition from spall-to-no-spall to the right of the intersection of the corre-

sponding curves, indicating that the tamped momentum is always equal to or greater than the free surface momentum. The actual location of the transition from spall-to-no-spall is a function of the shape of the deposition profile, i.e., the value of $\frac{\epsilon_{2d}}{\epsilon_0}$. It is possible for a deposition profile to have the transition from spall-to-no-spall located to the left of the intersection of their respective curves on

Fig. 5, in which case the tamped impulse could be considerably lower than the untamped impulse for appropriate tamper shock impedances. If the deposition profile satisfies the condition

$$\frac{\epsilon_{2d}}{\alpha\epsilon_0} > \frac{E_{2d}^\infty}{E_0^\infty},$$

and if the shock impedance of the tamper is such that

$$-\frac{\epsilon_{2d}}{\alpha\epsilon_0} < \frac{Z_2 - Z_1}{Z_2 + Z_1} < -\frac{E_{2d}^\infty}{E_0^\infty}, \quad (7)$$

then the tamped impulse will be lower than the free surface impulse.

If we assume that the attenuation coefficient in the spall criteria is approximately 1 ($\alpha \approx 1$), we may easily relate these conditions [Eq. (7)] to the shape of the deposition profile. If the deposition profile can be represented as a straight line on semi-log paper ($\ln \epsilon$ vs x), then

$$\frac{\epsilon_{2d}}{\epsilon_0} = \frac{E_{2d}^\infty}{E_0^\infty}$$

and the transition from spall-to-no-immediate-spall occurs when the spall and no immediate spall impulses are identical. If the deposition profile on semi-log paper has positive curvature, the transition from spall-to-no-spall occurs when the no spall impulse is higher than the immediate spall impulse (to the right of their intersection on Fig. 5). And if the deposition profile has negative curvature on semi-log paper, the transition occurs when the no spall impulse is lower, and a tamped configuration could have a lower impulse than an untamped one.

If the energy deposition profile is generated by photoelectric absorption of photons, we find that a monoenergetic source would give an exponential deposition profile (a straight line on semi-log paper), while any spectrum consisting of at least two energies of photons would result in a profile that had positive curvature on semi-log paper. Thus, for initial profiles generated by photon absorption, the tamped impulse will always be equal to or greater than the untamped impulse (unless α is considerably less than 1).

IMPULSE VS FLUENCE

Examination of the impulse as a function of the total energy absorbed yields another interesting prediction of the acoustic wave model. At low total energies the tamped configurations will have much higher impulses than untamped configurations with the same deposition profile. However, as the total energy absorbed is increased, the difference between tamped and untamped impulse is reduced; when fairly high total energies are absorbed with a low-shock-impedance tamper the tamped and untamped impulses will eventually become identical. This changing of tamping effect with total energy absorbed is a result of the fact that for monotonically decreasing deposition profiles the parameters pertinent to impulse production, namely E_{2d}^∞/E_0^∞ and ϵ_{2d}/ϵ_0 , change as the amount of material melted changes. It should be noted that this phenomenon depends on the assumption of a monotonically decreasing deposition profile, so it would not be observed in exploding foil type experiments, where the deposition of energy is uniform across

the heated region. We should also keep in mind that if the higher total energy configurations have a significant vapor pressure or vaporized material in the heated region, the model fails and the tamped impulse will again be higher than the untamped one. As an example of the variation of tamping factor with fluence we can consider an initial deposition profile given by $\epsilon(x) = Ae^{-bx}$ and assume that variations in the total energy may be represented by changes in the value of A only. Figure 6 shows several $\epsilon(x)$ curves given by changing the value of A from 0.5 to 10, with $b = 0.3$ (arbitrary scales are used for both ϵ and x). If we assume a melt energy (in arbitrary units) of 0.5, the melt depth is given by

$$d = \frac{\ln \frac{\epsilon_m}{A}}{-b} = \frac{\ln \frac{0.5}{A}}{-0.3} .$$

Table 2 gives the values of the pertinent parameters for various values of A along with the impulse predicted by the acoustic wave model for a tamper/sample configuration with a $\frac{Z_2 - Z_1}{Z_2 + Z_1}$ of -0.1 and for an untamped configuration with identical deposition. Figure 7 is a plot of the tamped and untamped impulse vs E_0^∞ . The untamped impulse curve goes to zero at $E_0^\infty = 1.67$ because no material is melted at energies lower than this. The tamped

impulse, on the other hand, has a finite value even when no material is melted because the pressure induced in the solid material can still "throw off" the tamper (if we assume zero bond strength). The curves drawn in Fig. 7 are for a straight line (on semi-log paper) deposition profile. For other deposition profiles the curves would be very similar, except that there would be a discontinuity in the tamped impulse curve when the impulse-producing mechanism changed from no immediate spall to immediate spall. The locations of such transitions are shown qualitatively on Fig. 7 by the vertical lines labeled "positive" and "negative," representing deposition profiles with positive and negative curvatures on semi-log paper, respectively. For example, if the deposition profile had positive curvature, then as the total energy absorbed was increased the tamped impulse would increase until at some value of E_0^∞ the mechanism would change from no spall to immediate spall and there would be a decrease in impulse followed by a different slope to the impulse vs E_0^∞ curve. For extremely high fluences, i.e., when material is vaporized, the tamped impulse again will be greater than the untamped by as much as a factor of 5 or 10. Such high energy density calculations are beyond the range of this model and must be done by other techniques.⁵

Fig. 6. Typical deposition profiles given by $\epsilon(x) = Ae^{-bx}$.

Table 2. Acoustic wave model impulse from a deposition profile given by $\epsilon(x) = Ae^{-bx}$. Arbitrary units are used for energy, position, and impulse (e.g., let melt energy = 0.5, $\gamma = 2$, $c = 1$, $b = 0.3$, and $\frac{Z_2 - Z_1}{Z_2 + Z_1} = -0.1$).

E_0^∞	$\frac{E_{2d}^\infty}{E_0^\infty}$	ϵ_0	$\frac{\epsilon_{2d}}{\epsilon_0}$	I_r	$I_{\text{Free surface}}$	I_{Tamp}
33.3	0.0025	10.0	0.0025	33.3	33.25	33.25
16.7	0.01	5.0	0.01	16.7	16.5	16.5
10.0	0.0278	3.0	0.0278	10.0	9.97	9.97
6.67	0.0625	2.0	0.0625	6.67	6.25	6.25
5.0	0.111	1.5	0.111	5.0	4.44	4.5
3.33	0.25	1.0	0.25	3.33	2.5	3.0
2.5	0.44	0.75	0.44	2.5	1.39	2.25
2.0	0.694	0.60	0.694	2.5	0.611	1.80
1.67	1	0.50	1	1.67	0	1.5 ^a
0.833	1	0.25	1	0.833	0	0.75 ^a

^aZero bond strength assumed between tamper and sample.

Fig. 7. Tamped and untamped impulse vs total energy absorbed.

Summary

The simple model presented can be used to calculate the melt-dominated impulse for instantaneous energy depositions in tamped or untamped configurations. The results agree reasonably well with hydrocode calculations and available experimental data. Unforeseen discontinuities in impulse vs fluence and in the impulse vs shock impedance of tamper curves have been predicted by this model. Not all of these discontinuities have been verified by hydrocode calculations yet, and almost no experimental data is available for tamped configurations, so at this point these discontinuities in value and slope remain predictions of the model.

The basic limitation of the model is that it cannot be applied to material that is vaporized, since such material may

still maintain significant pressures even after expansion by more than an order of magnitude. For such cases, impulse estimates based on the principle that a given fraction of the deposited energy will end up as kinetic energy through $P dV$ work of the vapor on the surrounding material have been developed. Those applicable to free surfaces, BBAY and MTBBAY, have been in use for some time. For tamped configurations, techniques recently developed at LLL look promising.⁵

Modifications to the acoustic wave model to allow for finite deposition times including the propagation of the waves and the expansion of material during deposition are being developed. These extensions of the model will be the subject of a future report.

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