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STEADY FLOW CHARACTERISTICS OF A SINGLE-PHASE NATURAL CIRCULATION LOOP *

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ABSTRACT

One-dimensional steady state single-phase natural circulation flow in a circular loop was analyzed. The resulting correlations are presented for both laminar and turbulent flows for wide ranges of the dimensionless parameters involved. In applying these correlations to a particular case it is necessary to carry out iterative calculations. Example calculations of this kind are given. Experiments were carried out using water, and the measured data were compared with the analytical predictions.

INTRODUCTION

Natural circulation heat transfer loops have been a subject of investigation for many years. Recent applications have focused considerable attention on single-phase operation at high temperatures and pressures (2, 3). Due to the technical importance of these applications the acquisition of new information is desirable for purposes of design, and its validity needs to be established experimentally. This report is concerned with single phase natural convection in a circular loop. Analytical results for steady state operation are presented in terms of the pertinent system parameters. Experimental data are given in support of the analytical predictions obtained.

Published analytical work (2, 4, 5) dealing with a number of individual natural circulation loops has been successful in predicting the experimental observations of steady state, transient, and oscillatory behavior of those particular systems. The analytical technique used in these instances consisted of finite-difference solution, by digital computer, of the one-dimensional equations of continuity, momentum, and energy together with the equation of

state. Since the analyses were not made dimensionless, general correlations in terms of dimensionless system parameters were not obtained. In this report the analytical results are given in terms of dimensionless parameters so that wide ranges of all system variables are conveniently taken into account.

The physical system treated is shown in Fig. 1. The circular loop is situated in a vertical plane, and the enclosed fluid is heated over the bottom half of the loop ($\pi < \theta < 2\pi$) while being cooled over the top half ($0 < \theta < \pi$). The wall heat flux is considered to be constant over the interval $\pi < \theta < 2\pi$ and the wall temperature is constant over the interval $0 < \theta < \pi$. Due to the unbalance of buoyant forces produced by density variations, flow of the fluid inside the circular loop takes place. This flow, in turn, is impeded by viscous stresses at the solid boundaries of the loop. The object of this study was to determine the fluid flow rate and temperature distribution as functions of the imposed thermal conditions, the dimensions of the system, and the properties of the flowing fluid.

ANALYSIS

In order to keep the analysis concise a number of simplifying conditions are applied. Fluid density is taken to be a linear function of temperature, but independent of pressure. It is assumed that the flow is one-dimensional

Figure 1. Physical System

and all properties of the fluid at any section are assumed constant across that section. Axial heat conduction in the fluid is neglected and the convection coefficient is assumed to be constant over the upper portion of the loop.

For the element of fluid in the control volume of Fig. 1 the governing equations are:

$$\rho = \rho_o [1 - \beta (T - T_o)] \quad (1)$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{R} \frac{\partial (\rho u)}{\partial \theta} = 0 \quad (2)$$

$$\frac{-g}{g_c} \rho \cos \theta - \frac{2}{r} \tau_w - \frac{1}{R} \frac{\partial p}{\partial \theta} = \frac{1}{g_c} \left[\frac{\partial}{\partial t} (\rho u) + \frac{1}{R} \frac{\partial (\rho u^2)}{\partial \theta} \right] \quad (3)$$

$$\rho c_p \frac{\partial T}{\partial t} + \frac{c_p}{R} \rho u \frac{\partial T}{\partial \theta} = \frac{2}{r} h [T_w - T] \quad \text{for } 0 < \theta < 2\pi \quad (4)$$

$$\rho c_p \frac{\partial T}{\partial t} + \frac{c_p}{R} \rho u \frac{\partial T}{\partial \theta} = \frac{2}{r} q_w'' \quad \text{for } \pi < \theta < 2\pi \quad (5)$$

For steady flow, equations (2) through (5) reduce to

$$\rho u = \text{constant} = G \quad (6)$$

$$\frac{-g}{g_c} \rho \cos \theta - \frac{2}{r} \tau_w - \frac{1}{R} \frac{dp}{d\theta} = \frac{1}{g_c R} \frac{d(\rho u^2)}{d\theta} \quad (7)$$

$$\frac{C_p G}{R} \frac{dT}{d\theta} = \frac{2}{r} h [T_w - T] \quad \text{for } 0 < \theta < \pi \quad (8)$$

$$\frac{C_p G}{R} \frac{dT}{d\theta} = \frac{2}{r} q_w'' \quad \text{for } \pi < \theta < 2\pi \quad (9)$$

At this point, it is convenient to define the two dimensionless groups, $N_L = 2\pi\beta R q_w'' / r G C_p$ and $z = h / q_w'' \beta$.

Integrating equation (8) yields

$$T(\theta) = T_w - [T_w - T(0)] \exp \left[- \left(\frac{N_L z}{\pi} \right) \theta \right] \quad \text{for } 0 < \theta < \pi, \quad (10)$$

and integrating equation (9) results in

$$T(\theta) = T(\pi) + \frac{N_L}{\pi\beta} (\theta - \pi) \quad \text{for } \pi < \theta < 2\pi. \quad (11)$$

Now,

$$T(\pi) = T_w - [T_w - T(0)] \exp [-N_L z] \quad (12)$$

from equation (10). Imposing the condition $T(0) = T(2\pi)$ on equation (11) results in

$$T(0) = T(\pi) + \frac{N_L}{\beta} \quad (13)$$

Solution of equations (12) and (13) for $T(\pi)$ and $T(2\pi)$ yields

$$T(\pi) = T_w + \frac{N_L}{\beta} \left[\frac{1}{\exp [N_L z] - 1} \right] \quad (14)$$

and

$$T(2\pi) = T_w + \frac{N_L}{\beta} \left[\frac{\exp [N_L z]}{\exp [N_L z] - 1} \right] \quad (15)$$

Integrating equation (7) over θ from 0 to 2π results in

$$-\frac{g}{g_c} r \int_0^{2\pi} \rho(\theta) \cos \theta d\theta - 2 \int_0^{2\pi} \tau_w d\theta = 0 \quad (16)$$

since $\int_0^{2\pi} dp = 0$ and $\int_0^{2\pi} d(\rho u^2) = 0$. Equation (16) represents a steady state torque balance about the center of the loop, point 0 in Fig. 1.

At this stage in the analysis, it is appropriate to employ expressions for the wall shear stress, τ_w , which represent this stress in terms of the fluid properties and the flow velocity. The fact that temperature differences exist in the fluid creates a practical difficulty, since actual fluid properties are temperature dependent. In reality, the radial temperature variation in the fluid at any section of the tube distorts somewhat the actual velocity profile of the fluid at that section from what it would be under isothermal conditions, and this distortion varies from section to section in general. The lack of

isothermal conditions requires that the properties of the flowing fluid (for example, viscosity) be evaluated at an average temperature. Further, previous experiments leading to the standard expressions for laminar and turbulent shear stress have involved forced flow in straight tubes as opposed to natural convection flow in curved tubes. Since no practical alternative to using these relationships existed, they were employed in the present analysis. Kreith (6) gives the expression

$$\tau_w = \frac{8}{Re} \frac{\rho u^2}{g_c} \quad (17)$$

for fully-developed laminar flow, and

$$\tau_w = \frac{0.023}{Re^{0.2}} \frac{\rho u^2}{g_c} \quad (18)$$

for fully-developed turbulent flow. Substitution of these expressions into equation (16) gives

$$- gr \int_0^{2\pi} \rho(\theta) \cos \theta \, d\theta - \frac{8\mu G}{r} \int_0^{2\pi} \frac{d\theta}{\rho(\theta)} = 0 \quad (19)$$

for laminar flow, and

$$- gr \int_0^{2\pi} \rho(\theta) \cos \theta \, d\theta - 0.04 \frac{G^{1.8} \mu^{0.2}}{r^{0.2}} \int_0^{2\pi} \frac{d\theta}{\rho(\theta)} = 0 \quad (20)$$

for turbulent flow.

At this point, the equation of state, equation (1), and equations (10), (11), (14) and (15) are used to evaluate the integrals $\int_0^{2\pi} \rho(\theta) \cos \theta d\theta$ and $\int_0^{2\pi} d\theta/\rho(\theta)$ which appear in equations (19) and (20). It can be demonstrated that

$$\begin{aligned} \int_0^{2\pi} \rho(\theta) \cos \theta d\theta &= -\rho_0 \beta \int_0^{2\pi} T(\theta) \cos \theta d\theta \\ &= -\rho_0 \cdot \frac{N_L}{\pi} \cdot \left[2 + \frac{N_L z}{\left(\frac{N_L z}{\pi}\right)^2 + 1} \left[\frac{\exp \left[\frac{N_L z}{\pi} + 1 \right]}{\exp \left[\frac{N_L z}{\pi} - 1 \right]} \right] \right] \end{aligned} \quad (21)$$

and

$$\begin{aligned} \int_0^{2\pi} d\theta/\rho(\theta) &= \int_0^{\pi} \frac{d\theta}{\rho_0 [1 + \beta(T_0 - T_w)] + \rho_0 \beta [T_w - T(0)] \exp \left[-\left(\frac{N_L z}{\pi}\right) \theta \right]} \\ &+ \int_{\pi}^{2\pi} \frac{d\theta}{\rho_0 [1 + \beta(T_0 - T(\pi))] - \rho_0 \frac{N_L}{\pi} (\theta - \pi)} \end{aligned} \quad (22)$$

It is convenient to consider $T_0 = T_w$. This removes the dimensionless parameter $\beta (T_0 - T_w)$ from the integrand in equation (22), and hence, from the results of the analysis. Results tabulated by Dwight (7) and by Gröbner and Hofreiter (8) were used to integrate equation

(22). When $\rho(\theta)$ is positive, equation (22) becomes

$$\int_0^{2\pi} \frac{d\theta}{\rho(\theta)} = \frac{\pi}{\bar{\rho}_o} \left[1 + \left[\frac{1}{N_L z} + \frac{1}{N_L} \right] \ln \left[\frac{1 - \frac{N_L}{\exp(N_L z) - 1}}{1 - \frac{N_L \exp(N_L z)}{\exp(N_L z) - 1}} \right] \right] \quad (23)$$

Substituting equations (21) and (23) into equation (19) gives the laminar flow dimensionless steady-state torque balance as

$$\frac{Re}{Gr} = \frac{1 + \left[\frac{N_L z/2}{(N_L z/\pi)^2 + 1} \right] \left[\frac{e^{N_L z} + 1}{e^{N_L z} - 1} \right]}{1 + \left[\frac{1}{N_L z} + \frac{1}{N_L} \right] \ln \left[\frac{e^{N_L z} - N_L - 1}{e^{N_L z} (1 - N_L) - 1} \right]} \equiv f(N_L, z) \quad (24)$$

where Gr represents a modified Grashof number as defined in the nomenclature. In the same manner equation (20) gives rise to the turbulent flow result which is

$$\frac{Re^{1.8}}{347.8 Gr} = f(N_L, z) \quad (25)$$

The function $f(N_L, z)$ was evaluated for various values of N_L and z , and plots were generated as shown in Figs. 2 through 6. Results for both laminar and turbulent flows can be determined from these curves as indicated by equations (24) and (25). For this reason the ordinates of the

Figure 2. Analytical Representation of Laminar and Turbulent Steady States ($0 < N_L \leq 0.0018$; $0 < Z \leq 2500$)

Figure 3. Analytical Representation of Laminar and Turbulent Steady States ($0 < N_L \leq 0.05$; $0 < Z \leq 500$)

Figure 4. Analytical Representation of Laminar and Turbulent Steady States ($0.05 \leq N_L \leq 0.15$; $0 < Z \leq 500$)

Figure 5. Analytical Representation of Laminar and Turbulent Steady States ($0.15 \leq N_L \leq 0.999$; $0 \leq Z \leq 100$)

Figure 6. Analytical Representation of Laminar and Turbulent Steady States ($0 \leq N_L \leq 0.999$; $0 < Z \leq 10$)

plots have been doubly labeled with Re/Gr (for laminar flow determinations) and with $Re^{1.8}/347.8 Gr$ (for turbulent flow situations).

APPLICATION OF ANALYTICAL RESULTS TO A PARTICULAR SYSTEM

The flow rate of a fluid in a particular loop subjected to a prescribed heat flux can be determined by means of an iterative calculation procedure. Such a procedure is used here because the mass flux, G , appears in two of the three dimensionless groups in the results of the preceding section, and the heat flux, q_w'' , appears in all three of these groups. That is, G cannot be evaluated directly for a given value of q_w'' from equations (24) and (25) or from the plots given in Figs. 2 through 6. The same statement applies to a determination of q_w'' for a given value of G .

An example iterative calculation is presented here for the loop used in the experimental portion of this investigation. The loop was filled with liquid water at atmospheric pressure and had the dimensions $R = 1.25$ ft. and $r = 0.0492$ ft. The object at this point is to predict the mass flux, G , for given values of q_w'' and T_w . For instance, consider $q_w'' = 500$ Btu/hr-ft² and $T_w = 70$ F.

The resulting flow is assumed to be laminar due to the low value of q_w'' , and the dimensionless parameters are written as:

$$N_L = \left[\frac{2 \pi R \beta q_w''}{r C_p} \right] \cdot \frac{1}{G}$$

$$z = h/q_w'' \beta$$

$$\frac{Re}{Gr} = \left[\frac{2 \pi C_p \mu}{g R r \beta \rho_o^2 q_w''} \right] G^2$$

The reference temperature is 70 F, and the reference density is taken here as the density of water at 70 F (9), or $\rho_o = 62.3 \text{ lbm/ft}^3$. For the purpose of evaluating property values the temperature of the circulating water is assumed to be in the neighborhood of 80 F. The properties of the water (9) are: $C_p = 1.0 \text{ Btu/lb}_m \text{ F}$, $k = 0.350 \text{ Btu/hr-ft-F}$, $\mu = 2.08 \text{ lb}_m/\text{hr-ft}$, $\beta = .000161 \text{ F}^{-1}$. The heat transfer coefficient for laminar flow in the cooled section is estimated from the forced convection fully developed correlation for constant wall temperature (10). This correlation is $Nu = 3.65$, and so $h = 3.65 \times 0.35/2 \times 0.0492 = 13 \text{ Btu/hr-ft}^2\text{-F}$. With $g = 32.2 \text{ ft/sec}^2$ the dimensionless parameters become

$$N_L = .00357/G$$

$$z = 161.5$$

$$\frac{Re}{Gr} = .0211 G^2$$

where G has the units, lbm/sec-ft^2 .

It is appropriate at this point to note from Figs. 2 through 6 that $Re/Gr \leq 1.0$. From the above relationship for Re/Gr in terms of G , the upper limit for G is approximately 7 lbm/ft^2 -sec.

Now, to illustrate an iterative procedure for obtaining G the following preliminary steps are taken:

initial trial value for $Re/Gr = 0.80$

corresponding value of $G = \sqrt{0.80/0.0211} = 6.15 \text{ lbm/sec-ft}^2$

corresponding value of $N_L = 0.00357/6.15 = 0.00058$

value of N_L from Fig. 3 = 0.04

The last two results are incompatible, but it is clear from Fig. 3 that they can be brought into closer agreement by increasing the value of Re/Gr with z held constant at 162.5. This step produces a much lower value of N_L from Fig. 3. To achieve a resulting N_L in the neighborhood of 0.00058 from Fig. 3, Re/Gr must be very close to unity. A value of 0.9925 is chosen and the procedure illustrated above is repeated.

second trial value of $Re/Gr = 0.9925$

corresponding value of $G = \sqrt{0.9925/0.021} = 6.87 \text{ lbm/ft}^2$ -sec

corresponding value of $N_L = 0.00357/6.87 = 0.00052$

value of N_L from Fig. 3: between zero and 0.001

For this trial the last two steps are compatible, and the problem is essentially solved. However, it is still necessary

to verify that the flow is laminar. The predicted value of Re is $6.87 \times (2 \times 0.0492) \times 3600/2.08 = 1170$, a value low enough to give a reasonably good probability that laminar flow would be achieved during operation of an actual loop under the conditions prescribed here. The flow rate of $6.87 \text{ lbm/ft}^2\text{-sec}$ corresponds to a bulk fluid velocity of $6.87/62.3 = 0.11 \text{ ft/sec}$.

Fortunately, the solution of the above problem was expedited by the fact that Re/Gr was not very sensitive to changes in N_L once the approximate region of operation was established (see Fig. 3). For this reason an acceptable iterative solution was obtained with only two trials. In general more trials would be needed in regions where the curves for various N_L values are farther apart.

In cases where property variations make it difficult to estimate suitable values initially, a calculated average temperature can be determined once the calculated values of N_L and z have been obtained by means of the procedure indicated above. This average temperature can then be used to re-evaluate the fluid properties. The entire iterative procedure can then be repeated, giving an improved value of G . This procedure can be repeated as many times as may be necessary for obtaining sufficiently accurate results.

The work published by Pomerantsev (3) provides an interesting comparison with the analysis and iteration procedure presented here. Pomerantsev carried out an analysis

of steady free-convection flow in a loop and obtained an equation associated with a momentum balance which made it possible to solve graphically for the flow rate in a particular loop under prescribed conditions. The analysis was one-dimensional and the method for obtaining a solution to the steady flow equation is equivalent to the iterative procedure described here. Values of fluid properties are assumed and after graphical determination of the flow-rate, a mean temperature is available which can be used to redetermine the properties. This graphical procedure also may be repeated as many times as may be necessary for achieving a desired accuracy.

In order to obtain calculated performance data for the experimental loop used in this study, a digital computer program was developed to systematically perform the iterative calculations previously described and to re-evaluate fluid properties until proper convergence of property values was achieved. Fig. 7 shows the flow chart for this program.

The tube making up the top half of the experimental loop was surrounded by a pyrex jacket through which cooling water was passed. Even though the heat input to the loop was varied, the average cooling water temperature was kept fairly constant during the experiments. However, the radial temperature drop across the upper tube wall varied with heat input rate and varied slightly with distance along the tube. The latter variation was neglected in carrying out the computer calculations, but the inner tube wall temperature was

Figure 7. Flow Chart for Iterative Computer Program

estimated by accounting for the thermal resistance of the pyrex tube and by using the fact that the average cooling flux must be equal to the imposed heat flux along the bottom half of the loop. The resulting expression obtained for predicting tube wall temperature was

$$T_w = \left[(T_c)_{\text{average}} + 0.015 q_w'' \right] F \quad (26)$$

The average coolant temperature was written as the mean of the inlet and outlet values and was assumed equal to 62.5 F in the computer calculations performed.

In carrying out the computer calculations the heat transfer coefficient for the cooled section was obtained from the correlations developed by Hausen (10). These expressions are shown in Fig. 7. They give the average coefficient between the tube entrance and any position along the tube. They were developed for forced flow conditions, but since they represented the best information conveniently available for calculating h in the iterative procedure, there was no more attractive alternative to using these correlations. The symbols μ_w and μ_B represent the fluid viscosity evaluated at wall and at the bulk temperature respectively. All other properties involved in forming the dimensionless parameters appearing in the correlations were evaluated at the bulk fluid temperature.

The computed results for the experimental loop are shown in Figures 8, 9, and 10. The laminar flow calculations

Figure 8. Predicted Values for Experimental Loop

Figure 9. Predicted Temperatures for Experimental Loop

Figure 10. Locus of Predicted Operating Conditions

produced $Re = 2300$ at a value of q_w'' close to 1000 Btu/hr-ft^2 . For the turbulent flow calculations, $Re = 2300$ was found to occur at a value of q_w'' very close to 1200 Btu/hr-ft^2 . Transition from laminar to turbulent flow in the loop does not actually occur in a simple manner at a single specific value of Re , but the division between laminar and turbulent flow regimes was assumed to occur at $Re = 2300$. The smooth locus of the calculated steady states (Fig. 10) appears favorable because it possesses essentially no distortion near points B and C, which mark the region of separation between the laminar and turbulent calculated results. It can be noted that the ordinate in Fig. 10 was only slightly below unity throughout the operating range studied for the experimental loop used in this investigation.

The predicted performance characteristics exhibited a relatively small unbalance between the prescribed rates of heat input to the loop for the heater (as specified by q_w'') and the calculated rates of heat rejection from the circulating fluid. This unbalance in calculated results was never in excess of 3.5%, and arose from the approximations inherent in the iterative procedure.

It is interesting to compare the result achieved in the more involved iterative calculations performed by computer with that obtained in the simple example iteration presented at the beginning of this section. The computer determined value of G for $q_w'' = 500 \text{ Btu/hr-ft}^2$ was found to

be 6.5 lbm/sec-ft^2 as compared with the value of about $6.9 \text{ lbm/ft}^2\text{-sec}$ which was determined by simple hand calculations. The rather favorable agreement may be somewhat fortuitous in this particular case, but it does serve to point out the possibility of achieving reasonably good results without resorting to the additional detail of a lengthy computer iteration procedure. A major advantage of using such a detailed calculation procedure, however, is that errors due to inaccuracies in reading the graphical results from Figs. 2 through 6 are avoided. These errors can be considerable in operating ranges where the numerical values are difficult to determine easily from these curves.

EXPERIMENTAL MEASUREMENTS AND COMPARISON WITH PREDICTED RESULTS

An experimental apparatus was constructed as shown in the schematic diagram of Fig. 11. Dimensions of major importance were given in the previous section. The system was designed to incorporate the features of the original problem statement as closely as possible.

The loop was made of pyrex glass, and a ground joint at the bottom of the loop was added to allow for expansion in the heated lower half. Power was supplied through a variable transformer to heating elements wound on the lower half of the loop surface. The three heaters consisted of Chromel A ribbon and were wired in parallel. A

Figure 11. Schematic Diagram of Experimental Apparatus

reservoir was provided which was connected by a vent tube to the circulating flow passage. This reservoir was used in filling the loop and also served to maintain pressure in the loop at a safe level during operation. The fluid expansion during heating might otherwise have forced the ground joints open or broken the glass. Also, the reservoir would have accommodated the resulting volume change of the fluid if local boiling had occurred anywhere in the heated section. The pyrex loop was covered with fiberglass wool insulation to minimize heat exchange with the environment. Instrumentation for temperature measurements was arranged as shown in Fig. 11.

The parallel connection of the heaters permitted use of heater ribbon of considerable length with a relatively high resistance (1.4 ohms/ft). The ribbon was thin enough to be wound around the glass quite easily. The large length used made it possible to cover a large portion of the glass surface area with the heater element, and this resulted in a fairly good approximation to the desired constant heat flux at the inner surface of the tube. Each of the three heaters had a resistance of about 23 ohms at room temperature. This permitted the variable transformer to deliver its maximum power in the range from 90 to 120 volts at the output terminals.

The temperature of the circulating fluid was measured at $\theta = \pi$ and at $\theta = 2\pi$ by means of thermocouple probes. These were mounted in rubber stoppers which were

inserted in the glass nipples installed at right angles to the circulating flow path (see Fig. 11). By establishing the length of the thermocouple probe accurately, it was possible to place the thermocouple junction at any desired radial location in the tube. The copper-constantan junctions of these probes were located at a distance of 0.445 r from the inner surface of the tube based on information given in Appendix A. The coolant fluid inlet and outlet temperatures were also measured, and a flowmeter was used to determine the coolant flow rate. A Minneapolis-Honeywell Rubicon potentiometer and a Leeds & Northrup null detector were used for all temperature measurements. The Rubicon potentiometer was stated by the manufacturer to be accurate to $\pm 0.0015\%$. The thermocouple wire used was precision copper-constantan guaranteed to agree with standard conversion tables to within $\pm 0.5\%$.

The procedures used for calculating the input heat flux and the circulating fluid flow rate are given in Appendix B. Data obtained from experimental measurements and calculated results based on these data are given in Appendix C. These results are shown in Figs. 12 and 13 along with plots of the corresponding predicted results from the computer calculations. Possible errors in the values of input heat flux, as determined from measured data, were estimated to be no larger than about 3%. Temperatures in the circulating fluid were carefully measured, and the plotted results show

Figure 12. Comparison of Predicted and Observed Mean Temperatures

Figure 13. Comparison of Predicted and Observed Mass Flow Rates

relatively little scatter (Fig. 12). For any prescribed heat flux in the range investigated, the fluid temperature difference existing between the two measurement locations ($\theta = \pi$ and $\theta = 2\pi$) was comparatively small. Thus, a small measurement error at either location could produce a sizeable percentage error in the calculated temperature difference and hence, in the calculated value of G (see Appendix B). For this reason the plotted points in Fig. 13 show considerable scatter despite the fact that they follow a rather well ordered pattern.

It is apparent (Figs. 12 and 13) that the predicted and experimental results are in agreement with respect to qualitative behavior and in terms of order of magnitude of the temperature and flow rate values obtained. However, certain discrepancies exist.

In the laminar region the observed flow rates were somewhat lower than predicted (Fig. 13). This might have been due to the fact that a constant heat transfer coefficient was assumed in the analysis, while the actual local coefficients were probably very high close to the cooler inlet due to entrance effects (8) and diminished with distance away from the inlet. These conditions would have caused the entering hot fluid to cool more rapidly and thus generate a smaller buoyant torque than predicted. The smaller buoyant torque in turn would have led to actual flow rates smaller than those obtained from the analysis (see Fig. 13). Another possible explanation for the low flow

rates observed in the laminar region is that the viscous effects might have been larger than those accounted for in the analysis. (The latter were based on the usual forced convection friction factor.) This situation might have been produced by local free convection phenomena not normally present in flows dominated by forced convection conditions. In fact, slowly moving large scale eddies in the fluid were observed visually through the walls of the pyrex loop even for low heat fluxes where the general nature of the flow could be qualitatively described as laminar. Despite the evidence of somewhat complicated flow patterns in the laminar region the experimentally determined average heat transfer coefficients (obtained from calculated values of input heat flux) were only slightly larger, on the average, than the corresponding values obtained from forced convection theory and used in the computer program (see Fig. 7).

When the heat flux for the experimental system was increased so as to exceed the limiting conditions for the laminar range, the flow was observed to become truly turbulent and exhibited an appearance of considerable local mixing in the regions that were visible through the pyrex tube. For the largest values of input heat flux where the flow was highly turbulent, the experimentally determined heat transfer coefficients were found to be considerably lower (by as much as 30 to 40% in some instances) than the corresponding values used in obtaining the computer-calculated

predictions (Fig. 7). It appears that this was the primary reason for the experimentally observed temperatures being generally higher than the calculated values (Fig. 12), by as much as 15 F at the maximum input heat flux used. The measured flow rates in the same range of operation were somewhat larger than predicted (Fig. 13). These discrepancies were attributed primarily to the fact that the fluid properties at the higher observed temperatures were significantly different from the values associated with the computer-calculated fluid temperatures.

SUMMARY AND MAIN CONCLUSIONS

In this investigation single-phase natural circulation flow in a circular loop was studied both analytically and experimentally. The analytical results for both laminar and turbulent flows were obtained in a straightforward manner and were presented graphically in dimensionless form. The application of these graphical results to a particular situation was shown to require an iterative procedure in general due to the nature of the dimensionless parameters involved. The iterative procedure was programmed so that the details could be carried out using digital computer calculations.

A digital computer was used to determine predicted performance characteristics for the system used in the experimental portion of the investigation. It was shown that

in some instances the analytically obtained graphical information can be used in conjunction with manual iterative calculations to produce reasonably good predictions. However, the computer program provided the advantage of carrying out large numbers of iterative calculations rapidly, thus avoiding excessive human labor. The manual calculations were found to produce poor accuracy for some operating ranges, a difficulty that was easily rectified by using the more accurate computer calculations. The lesser accuracy of the manual method in these regions is due to the fact that numerical values cannot be obtained precisely enough by visual inspection of the curves representing the generalized dimensionless correlations.

The computer program was capable of accounting for fluid property variations as well as providing a means for using improved correlations to obtain values of the convection coefficient over the top portion of the loop. Despite these advantages, the program was limited by the fact that there was no convenient alternative to using the standard forced convection correlations for calculating values of the convection coefficient. This procedure really amounted to an assumption which was found to be reasonable, based on the experimental observations, except for higher input heat fluxes.

An experimental apparatus was used to obtain measurements for comparison with the computer-calculated

predictions. The qualitative agreement was found to be favorable with respect to general trends and order of magnitude of the numerical results obtained. Both predicted and measured results illustrated increases of circulating fluid temperatures and flow rates with increasing input heat flux.

Moderate deviations of the analytical results from the corresponding experimentally obtained values were found. Predicted flow rates in the laminar range were somewhat larger than observed, the difference probably being due to the simplicity of the analytical model. In particular, the model did not account for a reduction in the actual buoyant torque produced by variations in the local convection coefficient. For the largest rates of input heat transfer used, where the flow was definitely turbulent, the correlation used to calculate the convection coefficient produced values considerably larger than those observed experimentally. This caused the fluid temperature predictions to be lower than the measured values. Predicted flow rates in this region were somewhat lower than observed, most probably due to fluid property variations occurring over the range of the temperature deviations.

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NOMENCLATURE

C_p	Specific heat at constant pressure, Btu/lb _m -F
$(C_p)_c$	Specific heat of coolant at constant pressure, Btu/lb _m -F
d	Tube inside diameter, ft
f	A function of N_L and z , defined in equation (24), dimensionless
g	Local acceleration of gravity, ft/sec ²
g_c	Universal gravitational constant, $32.174 \frac{\text{ft-lb}_m}{\text{lb}_f\text{-sec}^2}$
G	Steady flow mass flux, pu, lb _m /ft ² -sec
Gr	Modified Grashof number, $\frac{R g \rho_o^2 r^2 \beta q_w}{\pi \mu^2 G C_p}$, dimensionless
h	Average heat transfer coefficient for cooler, Btu/hr-ft ² -F
k	Thermal conductivity, Btu/hr-ft-F
$\dot{m}_c$	Coolant mass flow rate, lb _m /sec

N_L	Loop number, $\frac{2\pi \beta R q_w''}{r G C_p}$, dimensionless
Nu	Nusselt number, $\frac{hd}{k}$, dimensionless
p	Pressure, lb_f/ft^2
Pr	Prandtl number, $\frac{\mu C_p}{k}$, dimensionless
q	Input heat transfer rate to fluid, Btu/hr
q_w''	Heat flux through heater wall, Btu/hr-ft^2
r	Tube inside radius, ft
R	Loop radius, ft
Re	Reynolds number, $\frac{\rho u d}{\mu}$, dimensionless
t	Time, sec
T	Mean temperature across tube section, F
T_o	Reference value of temperature, F
T_w	Wall temperature in cooler, F

T_c	Coolant temperature, F
ΔT_c	Coolant temperature rise, F
u	Mean velocity across tube section, ft/sec
v	Local velocity in tube, ft/sec
y	Radial distance from tube wall, ft
z	Ratio, $\frac{h}{q_w''\beta}$, dimensionless
β	Density variation coefficient, $(F)^{-1}$
θ	Position coordinate in loop, radians
μ	Dynamic viscosity, $lb_m/hr\text{-}ft$
ρ	Mean density across tube section, lb_m/ft^3
ρ_o	Reference value of density corresponding to T_o , lb_m/ft^3
τ_w	Wall shear stress, lb_f/ft^2
ϕ	Local fluid temperature, F

APPENDIX A

RADIAL LOCATION OF PROBES FOR MEAN TEMPERATURE MEASUREMENTS

Eckert and Drake (10) consider a tube of inside radius r and designate distance measured radially inward from the inner tube wall as y . It is shown that the velocity and temperature profiles,

$$\frac{v}{v_s} = 2\left(\frac{y}{r}\right) - \left(\frac{y}{r}\right)^2 \quad (\text{A } 1)$$

and

$$\frac{\phi - \phi_w}{\phi_s - \phi_w} = \frac{6}{5}\left(\frac{y}{r}\right) + \frac{3}{5}\left(\frac{y}{r}\right)^2 - \frac{4}{5}\left(\frac{y}{r}\right)^3, \quad (\text{A } 2)$$

satisfy the required boundary conditions and adequately describe fully-developed laminar flow in a tube. The subscript s is used to indicate the centerline of the tube and the subscript w indicates the wall of the tube. v and ϕ are the local velocity and temperature, respectively, which are functions of y/r .

Use of the above profiles gives the mixed mean temperature across the tube cross section as

$$T = T_w + 0.583 \left(\phi_s - \phi_w \right) \quad (\text{A } 3)$$

It can be verified by substitution into the temperature profile expression that the local temperature at $y/r = 0.445$ is the same as T according to the analysis given here. The thermocouple probes in the experimental apparatus were located on this basis. It was assumed that the resulting local temperature measurements gave sufficiently accurate values of the corresponding mixed mean temperatures. Some error might have resulted from the fact that the actual laminar flow temperature profiles may have deviated somewhat from that given by the idealized temperature profile expression. This is plausible in view of the fact that large eddies in the flow were visually observed within the laminar operating range. There was no apparent necessity to relocate the probes for measurements in the turbulent range since temperature profiles under these conditions are known to be relatively flat over the entire central portion of the tube. Thus, small movements of the probe either inward or outward would have caused no appreciable change in the measured temperature values under turbulent flow conditions.

APPENDIX B

CALCULATION OF HEAT FLUX AND FLOW RATE FROM MEASUREMENTS

In calculating the experimental results it was assumed that the heat transfer to the coolant passing through

the cooling jacket was equal to that supplied by the heaters to the circulating fluid. This condition was not completely achieved in the experiments due to a small amount of heat conduction in the pyrex tube from the heated section to the cooled section, and due to a small amount of heat transfer from the environment to the cooling jacket. It was found that these deviations were not significant and did not produce sizeable errors in the calculated results. Fiberglass insulation was used to minimize the heat transfer from the environment to the cooling jacket and to reduce the external losses from the heaters. The latter were appreciable, but did not affect the accuracy of the results because the calculation procedure involved no assumptions concerning losses from the heaters to the surroundings.

The input heat transfer to the circulating fluid (see Fig. 1) is given by

$$q = q''_w (2\pi r)(\pi R) \quad . \quad (B 1)$$

An energy balance performed on the lower portion of the loop yields

$$q = (\pi r^2) G C_p [T(2\pi) - T(\pi)] \quad . \quad (B 2)$$

Under steady state conditions the heat transfer rate from the circulating fluid to the coolant is the same. Performing an energy balance on the cooling jacket gives

$$q = \dot{m}_c (C_p)_c \Delta T_c \quad (B\ 3)$$

Combining the three equations above produces the two relations used to calculate the input heat flux and the circulating fluid flow rate. They are

$$q_w'' = \frac{\dot{m}_c (C_p)_c \Delta T_c}{2\pi^2 r R} \quad (B\ 4)$$

and

$$G = \frac{\dot{m}_c (C_p)_c \Delta T_c}{\pi r^2 C_p [T(2\pi) - T(\pi)]} \quad (B\ 5)$$

q_w'' and G were calculated from these relationships using measured values of the coolant flow rate ($\dot{m}_c$), the coolant temperature rise (ΔT_c), and the circulating fluid temperatures at the heating section inlet and outlet. The coolant flow rate was measured by means of a flowmeter while the coolant temperature rise and circulating fluid temperatures were obtained using thermocouples. All other quantities needed for calculating q_w'' and G from equations (B 4) and (B 5) were known.

APPENDIX C

TABULATION OF EXPERIMENTAL DATA AND RESULTS

The experimental data and calculated results are

tabulated separately in Tables 1 and 2 respectively. Data shown for the individual experimental runs were obtained while the loop was in steady state operation. Each value given in Table 1 represents the average of 4 or more readings taken at time intervals of 1 minute.

TABLE 1

Observed Data

Run No.	Power kw	Coolant Flow		(T _C) inlet		ΔT_C		T (π)		T (2 π)	
		scale	lb/min	μV	$^{\circ}F$	μV	$^{\circ}F$	μV	$^{\circ}F$	μV	$^{\circ}F$
1.	0.173	7.08	9.85	628	60.35	10	0.48	801.8	68.8	902.6	73.4
2.	0.229	6.99	9.8	621.6	60.08	13.1	0.62	844.1	70.5	951	75.3
3.	0.277	6.79	9.5	605.8	59.3	17.0	0.8	968.4	76	1064.2	80.2
4.	0.346	6.76	9.5	600.9	59	20.9	1.0	1029	79	1134	83.8
5.	0.385	6.7	9.4	600.5	59	24	1.15	1081.3	81.7	1188.5	86.1
6.	0.4716	6.7	9.35	596.7	58.8	28.7	1.37	1163.7	84.7	1279	90.2
7.	0.559	7.09	9.85	588.8	58.4	31	1.48	1232.8	88	1348.3	93
8.	0.581	7.1	9.85	595.7	58.8	32	1.52	1252	88.9	1385	95
9.	0.695	7.1	9.85	598	58.9	37.8	1.8	1347.8	93.2	1485.8	99.1
10.	0.759	7.14	10	596.8	58.9	40.8	1.95	1399.3	95.3	1547.3	102
11.	0.841	7.36	10.3	611	59.6	44	2.1	1499.8	100	1623.8	105
12.	0.865	7.31	10.25	618.5	59.9	46	2.19	1540	101.6	1657	106
13.	0.947	7.28	10.25	616.3	59.8	51	2.43	1617	104.5	1738	110
14.	0.671	7.37	10.3	650	61.3	37	1.76	1379.5	95	1497.8	99.7
15.	0.764	7.4	10.4	644.3	61.1	42	2.0	1465	98	1593.5	103.9
16.	1.08	7.35	10.25	637	60.7	58.9	2.8	1756.8	110.5	1180.3	116
17.	1.70	8.21	11.5	641	60.9	83	3.95	2266	132	2401	137.5
18.	1.75	8.18	11.5	641.3	60.9	85.7	4.07	2295.3	133	2440	139.1
19.	1.29	8.66	12.05	604.7	59.2	58.7	2.78	1860.3	115	1983.7	120
20.	1.42	8.62	12	618.7	59.9	65	3.1	1971.7	119.5	2094.7	125
21.	1.54	8.61	12	625.3	60.3	70.3	3.34	2073	124	2198.7	129
22.	1.04	8.61	12	617.3	59.8	48	2.28	1640	106	1763	110.9
23.	1.22	8.58	12	615.7	59.7	56.7	2.68	1810	112.9	1930.7	118
24.	1.16	8.39	11.85	611.7	59.6	54.7	2.59	1766.7	111	1895	116.8
25.	1.32	8.4	11.85	605.3	59.3	62	2.95	1905	116.2	2029	122

TABLE 2

Calculated Results

Run No.	q''_w Btu/hr-ft ²	Mass Flow Rate lb/min
1.	234	1.03
2.	300	1.265
3.	376	1.81
4.	468	1.98
5.	533	2.41
6.	632	2.34
7.	720	2.92
8.	738	2.46
9.	876	3.01
10.	962	2.91
11.	1068	4.33
12.	1110	5.11
13.	1230	4.54
14.	893	3.86
15.	1028	3.53
16.	1417	5.23
17.	2240	8.25
18.	2310	7.68
19.	1655	6.7
20.	1840	6.77
21.	1972	8.0
22.	1352	5.58
23.	1590	6.32
24.	1518	5.29
25.	1728	6.03