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ANALOG ANALYSIS OF FLOW REVERSAL
AND WATER-HAMMER PRESSURE IN
A TYPICAL CLOSED AARR
HYDRAULIC RABBIT-TUBE FACILITY

by

Raymond R. Rohde and Alan L. Winiecki

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Printed in the United States of America
Available from

Clearinghouse for Federal Scientific and Technical Information
National Bureau of Standards, U. S. Department of Commerce
Springfield, Virginia 22151

Price: Printed Copy \$3.00; Microfiche \$0.65

ANL-7336
Engineering and
Equipment (TID-4500)
AEC Research and
Development Report

ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois 60439

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August 1967

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NOMENCLATURE

A = Nominal pipe cross-sectional area	ft ²	Δp_V = Pressure difference across valve	lb/ft ²
a = Acoustic velocity	ft/sec	R = Coefficient of resistance, $f/2D\gamma gA^2$	sec ² /lb-ft ³
b = Pipe thickness	ft	R_n = Coefficient of resistance of nth section of pipe	sec ² /lb-ft ³
C = Coefficient of capacitance in a pipe, $\gamma A \left(\frac{1}{K} + \frac{D}{bE} \right)$	ft	R_V = Resistance of valve	sec ² /lb-ft ²
C_n = Coefficient of capacitance of pipe and water in nth section of pipe	ft	R_{Vm} = Resistance of mth valve	sec ² /lb-ft ²
D = ID of pipe	ft	Sgn = Algebraic sign associated with W^2	
ΔD = Diameter change due to elasticity of pipe	ft	t = Time	sec
E = Modulus of elasticity of pipe material	lb/ft ²	Δt = Increment of time	sec
f = Friction factor		V = Fluid velocity	ft/sec
$f_{1,2,3,4}$ = Functions used in Eqs. 10 and 11		ΔV = Increment in velocity	ft/sec
g = Acceleration due to gravity	32.2 ft/sec ²	$V-n$ = Valve number n	
γ = Specific weight of water	lb/ft ³	W = Water flow rate	lb/sec
K = Bulk modulus of H ₂ O	lb/ft ²	$W_i, W_o, W_{V_1}, W_{V_4}, W_{P'}$ = Water flow rates as designated in Fig. 15	lb/sec
L = Coefficient of inertia of fluid, $1/Ag$	sec ² /ft ³	W_V = Water flow rate in valve	lb/sec
L_n = Coefficient of inertia of fluid in nth section of pipe	sec ² /ft ³	W_{in} = Water flow rate into Section 1 of pipe	lb/sec
p = Pressure	lb/ft ²	W_n = Flow rate in nth section of pipe	lb/sec
p_b = Pressure at inlet to Section 1 of pipe	lb/ft ²	ΔW_n = Increment of water flow rate in nth section of pipe	lb/sec
Δp = Pressure difference across Δx	lb/ft ²	x = Distance	ft
Δp_n = Pressure difference across nth section of pipe	lb/ft ²	Δx = Incremental longitudinal length	ft
Δp_p = (Pump) pressure difference across pump	lb/ft ²	Δx_n = Length of nth section of pipe	ft
p_n = Pressure in nth section of pipe	lb/ft ²		
p_a, p_i, p_o, p_o = Pressures as designated in Fig. 15	lb/ft ²		

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I. INTRODUCTION

The Argonne Advanced Research Reactor (AARR) will include a number of experimental facilities that will permit the rapid insertion of samples into specific regions of the core and reflector and allow rapid removal of the samples from these regions while the reactor is at power. The irradiation experiments performed with these samples involve very short-lived isotopes and therefore require swift removal and processing of the samples after the irradiation.

The samples are referred to as "rabbits." A simple closed* system for performing the irradiation operation (shown in Fig. 1) consists of a piping system and a pump that circulates a coolant through the pipes. Valves allow insertion of the rabbit into the piping system at a sample station, and the flowing coolant propels the rabbit through the system. The position at which the rabbit is irradiated is determined by a mechanical stop that prevents further movement of the sample along the pipe. The flowing coolant holds the rabbit in position and removes the heat generated within the rabbit and the coolant. The rabbit is removed by changing the direction of the coolant flow.

Fig. 1. Simple Representation of
Rabbit-tube Facility

The selection of design parameters and the design of the system are believed to be governed primarily by the transient conditions imposed on the hydraulic rabbit and the associated piping systems. The three problem areas associated with the hydraulic rabbit system, when the reactor is at a steady-state power level, are

*The closed system (i.e., independent of the reactor primary system) was chosen for this study at a time in a course of the AARR project when this system was being considered. A change was made to an open system (i.e., part of the reactor primary system) for reasons unrelated to the analysis presented here. The open system is therefore the only type presently being planned for AARR.

(1) the water-hammer pressures, (2) the rate of change of coolant flow, and (3) the heat-transfer status of the rabbit during reversing of flow. This report is concerned only with the water-hammer pressures and the rate of change of coolant flow, during the flow-reversal process. The first two of the three problem areas were chosen for this initial study to determine if the hydraulic characteristics of a typical rabbit-tube system imposed basic restrictions on the third problem area, the heat-transfer characteristics of the samples.

The design and operation of a hydraulic rabbit system requires verification that the steady-state and transient conditions encountered do not cause voiding of the rabbit tube by nucleate boiling at the surfaces, by coolant vaporization, or by failure of the system from transient pressures generated during operation. Concern over void formation exists because rabbit tubes will be placed in regions in the AARR core that have a positive reactivity worth associated with the creation of voids. Void production within a rabbit tube is possible if adequate pressure is not provided. Voids will occur if the temperature of the rabbit or the coolant exceeds the saturation temperature for the pressure within the rabbit tube. Voids may also occur if valving operations and coolant-flow changes cause transient, low-pressure variations within the piping system.

The maximum water-hammer pressures and minimum flow-reversal times were calculated for a typical AARR rabbit-tube system using the equation for maximum hammer pressure with instantaneous valve closure and with no surge tanks in the system. Parameter values used in this report are listed in Table I. The derivation is based on conservation of momentum and ignores friction.*

$$\text{Maximum pressure} = \frac{\gamma a V}{144g} \text{ psi;}$$

$$a = \frac{\sqrt{\frac{Kg}{\gamma}}}{\sqrt{1 + \frac{KD}{Eb}}} \text{ ft/sec.}$$

For a typical rabbit-tube facility, in which reverse flow is prevented through the pump, with instantaneous valve switching, an initial flow rate of 7 ft/sec, and no surge tanks, the hammer pressure was calculated to be 391 psi. With an acoustic velocity of 4,160 ft/sec, the flow-reversal time would be 12.0 msec.

*Streeter, Victor L., Fluid Mechanics, McGraw-Hill Book Co., Inc., New York (1958).

TABLE I. Typical AARR Rabbit-tube Characteristics

Pipe material	Aluminum
Total pipe length, ft	50
ID of pipe, in.	0.625
Pipe thickness, in.	0.0625
Modulus of elasticity of aluminum, lb/ft ²	1.44×10^9
Coolant	Water
Water temperature, °F	75
Bulk modulus of water, lb/ft ²	4.32×10^7
Water viscosity, lb-sec/ft ²	1.4×10^{-5}
Specific weight of water, lb/ft ³	62.4
Friction factor	0.0213
Normal flow rate, ft/sec	7-14
Normal pressure drop in pipe, lb/in. ²	5-30

These calculations, by their very nature, lead to maximum estimates of the pressure changes and minimum estimates of the flow-reversal times. As a result, we recognized that the design of the rabbit-tube system required a more detailed estimation of these quantities.

Four methods were considered for obtaining more realistic estimates of water-hammer pressures and flow-reversal times: (1) analytical, (2) graphical, (3) digital-computer, and (4) analog-computer.

Analytic techniques did not look promising because of the nonlinearities caused by the inclusion of a frictional-force term, which is proportional to the square of the flow velocity. Graphical solution* was rejected because of its approximate nature. For instance, to facilitate solution, pipe friction is handled by lumping all the pipe friction at the ends of a pipe. An analog- rather than a digital-computer solution was decided upon because of the ease of model development and interpretation.

The objectives of the analog study were:

1. Determine water-hammer pressures as a function of time and system design.
2. Determine flow velocity as a function of time and system design.
3. Provide a basis for qualitative and quantitative analysis of proposed or existing hydraulic rabbit-tube systems.
4. Furnish an insight into the instrumentation requirements for proposed or existing designs of hydraulic rabbit-tube systems.

*Parmakian, John, Waterhammer Analysis, Dover Publications, Inc., New York (1965).

II. SUMMARY

A study of the transient response of a typical rabbit-tube system during flow reversal produced the following results:

1. The maximum water-hammer pressure was approximately 81 psi with instantaneous valve operation for a 1.0-cu ft water-filled* surge tank with backflow prevented through the pump and an initial velocity of 7 ft/sec. The flow-reversal time for these conditions was about 60 msec. Increasing the surge-tank volume to 10 cu ft reduced the maximum water-hammer pressure to approximately 24 psi but increased the flow-reversal time to 200 msec.
2. With the extreme case of unrestricted backflow through the pump, the water-hammer pressures calculated were negligible, but flow-reversal times were in the range of 1 sec or higher.
3. The water-hammer pressures of 81 psi were substantially less than the rough estimate of 391 psi made before the study. Flow-reversal times calculated using the analog model were about a factor of five longer than originally estimated.

The study satisfactorily met all its objectives, which were (1) the calculation of water-hammer pressures and flow-reversal times, (2) providing quantitative and qualitative information about the behavior of the system, and (3) furnishing information regarding instrumentation requirements for the final AARR hydraulic rabbit-tube system.

Several results of the study are of interest. The magnitude of the water-hammer pressure was found to be inversely proportional to the square root of the size of the system surge tanks for a given initial flow rate. For a given surge-tank size, the water-hammer pressure was directly proportional to the initial flow rate. Water-hammer pressures caused the total system pressure to increase where water compression occurred and decrease where water decompression occurred. Flow-reversal times for a given flow rate increased when the surge-tank size increased. For a given surge-tank size, the initial flow rate had a minor effect on the reversal time.

III. SYSTEM DESCRIPTION

Figure 2 is a schematic diagram of a typical AARR hydraulic rabbit-tube system used for estimating water-hammer pressures and flow-reversal times. Surge tanks are shown connected at each end of

* The surge tanks used in the study were assumed to be filled with water with no gas space.

Fig. 2. Typical AARR Rabbit-tube System

operation is considered since it yields maximum water-hammer pressures. The dimensions of a typical AARR hydraulic rabbit-tube system are listed in Table I.

The characteristics of the pump influence the water-hammer pressures and flow-reversal times. Therefore, two extremes in pump behavior were considered. For Case 1 (Fig. 3), the pump is conceived as having a constant head-versus-flow characteristic in one direction with no flow possible in the opposite direction. For Case 2 (Fig. 4), the pump is considered to have a constant head-versus-flow characteristic regardless of flow direction through the pump. Case 1 yields results with favorable (minimum) flow-reversal times but unfavorable (large) water-hammer pressures. Conversely, Case 2 yields data with poor (long) reversal times, but good (small) water-hammer pressures.

Fig. 3. Pump Characteristic: Case 1.

Fig. 4. Pump Characteristic: Case 2.

These two extremes in pump characteristics were chosen to simplify the analysis; the actual system will perform somewhere between the two extreme cases.

IV. DEVELOPMENT OF MATHEMATICAL MODEL

To simplify the simulation of the rabbit-tube system, the parameters involved were reduced to those that contributed significantly to the water-hammer and flow-reversal effects. To achieve this end, the following assumptions were made in developing the mathematical model of the system:

1. The water in the pipe loop is compressible and has mass.
2. The pipes in the pipe loop are elastic.
3. The water does not change phase.
4. The head loss due to fluid friction in the pipes and valves is directly proportional to the square of the flow rate.
5. The surge tanks are water-filled and contain no air.
6. The inertial and compressibility properties of the water in the valves and pump are neglected.
7. The valves and the pump are inelastic.

Based on the above assumptions, the equations necessary to describe the system, which are developed in the appendix, are listed below.

In the tube section, the partial differential equations for pressure and flow rate are

$$-\frac{\partial p}{\partial x} = \frac{L}{g} \frac{\partial W}{\partial t} + RW^2(\text{Sgn})$$

and

$$-\frac{\partial W}{\partial x} = C \frac{\partial p}{\partial t}.$$

For the valve simulation, the nonlinear frictional term is

$$\Delta p_V = R_V W_V^2(\text{Sgn}).$$

The two extremes in pump characteristics are as follows:

Case 1: Δp_p = constant pump head (pressure) greater than zero for flow greater than or equal to zero through the pump. Reverse flow (flow less than zero in magnitude) is not allowed through the pump. (Fig. 3)

Case 2: Δp_p = constant pump head greater than zero regardless of flow direction. (Fig. 4)

Figure 5 is a simplified block diagram of the analog simulation. Because of the nature of solutions on the analog computer, the piping system is considered to be composed of ten sections of pipe. Figure 6 is a more detailed drawing of the actual analog implementation of each piping section and includes the equations that apply to Section 2 of the pipe. The appendix contains a detailed analysis of the development of the analog solution from the original piping equations.

Fig. 5. Simplified Block Diagram of Analog Simulation

Fig. 6. Block Diagram of Analog Circuit for a Typical Lumped-pipe Section

V. QUALITATIVE EXPLANATION OF WATER HAMMER IN SYSTEM

We will now describe the sequence of events that occur when the valves are operated to reverse the coolant flow direction when no surge tanks are present. In Fig. 1, assume that valves V-1 and V-4 are initially open and valves V-2 and V-3 are initially closed. The pump is assumed to be Case 1; that is, it produces a constant head for flow in one direction, but acts as though it were a closed check-valve to flow in the opposite direction. The allowed flow direction will be clockwise from point 5 to point 1 in Fig. 1. Hence the initial flow through the pipe is clockwise from point 2 to point 4.

To reverse the flow direction, valves V-1 and V-4 are closed and valves V-2 and V-3 are simultaneously opened. Instantaneous valve operation is considered since this type of operation results in larger water-hammer pressures and shorter flow-reversal times than slow valve manipulations.

When valve V-4 is closed and valve V-2 is opened, the clockwise flow from point 3 to point 4 attempts to continue through valve V-2. The pump, however, will not allow flow through itself from point 1 to point 5. Hence the water flow out of point 4 is abruptly stopped, and the water that

is still flowing toward point 4 from point 3 compresses the water at point 4 and expands the pipe at that point. This water compression and pipe expansion continues until flow into point 4 is stopped by the positive pressures developed at that point (water hammer). This water compression and pipe expansion occur further and further back from point 4 until a quantity of water, that is sufficient to develop the pressure required to prevent further flow into that location, is accumulated at each successive location. This process is interpretable as a wave of pressure traveling from point 4 back toward point 3. The wave reduces the flow at each location to zero as it passes. It travels from point 4 to point 3 at the acoustic velocity of the piping system.

At point 2 an opposite situation occurs, in that a negative pressure wave is developed that travels from point 2 toward point 3, reducing the flow to zero as it passes.

When the waves from points 4 and 2 meet at point 3, the flow in the entire pipe is zero. The pipe from point 2 to point 3 is shrunk (decreased in diameter), and the water is decompressed. The pipe from point 3 to point 4 is expanded, and the water is compressed.

The water in the expanded pipe from point 3 to point 4 now begins to flow across point 3 into the contracted pipe from point 3 to point 2. This action is again interpretable in terms of the pressure-wave concept. After crossing point 3, the two pressure waves continue in their original directions--the positive pressure wave traveling counterclockwise towards point 2 and the negative pressure wave traveling clockwise towards point 4. As the positive pressure wave travels from point 3 to point 2, the shrunk pipe returns to its normal pretransient size and the water is accelerated at each successive location in the pipe to a velocity equal in magnitude to its original pretransient velocity, but now in the opposite, or counterclockwise, direction. Similarly, as the negative pressure wave travels from point 3 toward point 4, the expanded pipe in successive locations is returned to its normal pretransient size and the water is accelerated to a velocity equal in magnitude to its original pretransient velocity but again in the opposite, or counterclockwise, direction. When the positive pressure wave reaches point 2 and the negative pressure wave, because of the symmetry of the piping system, simultaneously reaches point 4, the entire pipe from point 4 to point 2 contains coolant flowing in a counterclockwise direction. The entire pipe is in its normal initial state, as is the water, except for its reversed direction. Now when the flow from point 3 to point 2 goes through valve V-3, the pump allows the flow through it since it is from point 5 to point 1. The flow continues through valve V-2 and into point 4. Since the flow is now from point 4 to point 3, the pipe at point 4 accepts the flow from the pump and the flow continues around the loop. The cycle is completed, and the flow now goes in the opposite direction.

For Case 2 pump characteristics, water-hammer pressures are not developed since there is no flow stoppage due to pump characteristics. In this case, the flow is gradually reduced because the pressure head in the pump opposes the flow direction during the reversal operation.

The same sequence of events occurs when surge tanks are placed at points 2 and 4. A surge tank allows flow to continue longer in a given direction upon valve closure since it provides a place for the water to accumulate or originate. A surge tank thereby increases the time in which flow reversal takes place.

The magnitude of the water-hammer pressure developed at a point depends on the rate of deceleration of water impinging on that point. Since a surge tank provides a space into which water can continue to flow immediately after valve closure, the water is decelerated more slowly than when no surge tank is present. Hence, a surge tank decreases the water-hammer pressures developed in a system.

VI. RESULTS

All computer runs were made for simultaneous and instantaneous valve operations. Water-hammer pressures and flow velocities were

Fig. 7. Water-hammer Pressure along a Pipe during Flow Reversal

recorded from the analog model for different points along the pipe. These quantities were studied as the sizes of two surge tanks, one on each end of the piping system, were varied. Results for both pump models were obtained.

Figure 7 shows typical results for a computer run. Unless otherwise specified, a Case 1 pump characteristic is assumed. The maximum system hammer pressure occurs at the valves and is positive in magnitude at one of the valves and negative in magnitude at the other valve. The maximum local water-hammer pressure is dependent upon the position along the pipe. Flow versus time also varies around the system, as shown in Fig. 8.

Water-hammer pressures were found to be inversely proportional to the square root of the surge tank size for the Case 1 pump model.

Fig. 8. Flow within a Pipe during Flow Reversal

were observed. Pressure increased on the side of the valve at which the water was compressed and decreased on the side of the valve where the water was decompressed. Negative pressures are, of course, impossible in a physical system since, for example, the water would vaporize when the system pressures approached the vapor pressure corresponding to the water temperature. The computer solutions were based on a zero pressurization of the piping system. In an actual-rabbit tube system, the system would be properly pressurized to prevent vaporization of the water. Hence the analog-computer solutions are interpretable as representing the positive and negative deviations of the system pressure from the steady-state pressurization levels.

The flow-reversal times increased as the size of the surge tanks increased for the Case 1 pump model. Referring to Fig. 10, the reversal time was determined by computing the time required for the flow to reverse at the valve from the time that the valve operation is initiated to the time at which the flow is 90% of its final reversed magnitude. For a 10-cu ft surge tank placed on each end of the pipe, the time required for reversal when the initial flow rate is 7 ft/sec, is 200 msec. With 1-cu ft surge tanks, the reversal time is 60 msec. For an initial flow rate of 14 ft/sec, the reversal time is slightly higher.

Figure 9 shows that the maximum water-hammer pressure developed at the valves for an initial flow velocity of 7 ft/sec was 24 psi when 10-cu ft surge tanks were used and 81 psi when 1-cu ft tanks were used. The water-hammer pressure for a given surge-tank size was directly proportional to the initial flow rate. With 10-cu ft surge tanks on each end of the pipe, the water-hammer pressure for an initial flow rate of 7 ft/sec was 24 psi, while the pressure for an initial flow rate of 14 ft/sec was 49 psi.

Pressures both above and below the normal system pressure

Fig. 9. Water-hammer Pressure as a Function of Water-filled Surge-tank Size for Initial Flow Rates of 7 and 14 ft/sec for a Case 1 Pump Characteristic

Fig. 10. Flow-reversal Time as a Function of Water-filled Surge-tank Size for Initial Flow Rates of 7 and 14 ft/sec for a Case 1 Pump Characteristic

the null occurred closer to the larger surge tank. This is explainable by the fact that the pressure wave travels slower away from the larger surge tank. Hence, the faster wave from the smaller surge tank nulls the slower one closer to the originating point of the slower. Nulling occurs where these waves meet. Figure 11 is a graphic representation of these results.

Changing the pipe friction to take a pipe bend into account near the sample stop had no observable effect on the magnitude of the water hammer at the valves. The pipe friction was increased to ten times the uncurved-pipe friction value, and the pump pressure was increased to maintain identical initial flow rates for this study.

Results with a Case 2 pump characteristic were quite different from the Case 1 solutions. For Case 2 there was no observable water-hammer pressure, even when no surge tanks were present. With no surge tanks the time required for reversal was 1.25 sec for an initial flow rate of 7 ft/sec, and 0.8 sec for an initial flow rate of 14 ft/sec. Figures 12 and 13 show the results for the 7-ft/sec run.

The maximum magnitude of the water-hammer pressure that developed at a point in the pipe was linearly related to the distance between the valve and the point. The maximum system hammer pressure occurred at the valves, with a positive hammer pressure at one valve and a negative hammer pressure at the other. A null in the water-hammer pressure occurred along the pipe. The position of the pressure null depended on the relative sizes of the two surge tanks. If the surge tanks were equal in size and the piping system uniformly sized, the null occurred midway between the valves. If the surge tanks were unequal in size,

Fig. 11. Water-hammer Pressure Distribution for a Case 1 Pump Characteristic and an Initial Flow Rate of 7 ft/sec

Fig. 12. Water-hammer Pressure at the Valve for a Case 2 Pump Characteristic and an Initial Flow Rate of 7 ft/sec

Fig. 13. Flow Pattern at the Valve for a Case 2 Pump Characteristic and an Initial Flow Rate of 7 ft/sec

VII. DISCUSSION

This study has concentrated on Case 1 solutions because these solutions provide insight into the effect of design on conditions of interest in safety analysis. For example, the failure of valve V-3 to open, even though flow was permitted through the pump, could be analyzed by considering Case 1 results.

Initial handbook calculations indicated water-hammer pressures, with no surge tanks, of 391 psi and flow-reversal time of 12.0 msec. Use of surge tanks lowered the water-hammer pressures in the system without increasing the flow-reversal times to unacceptable values. Permissible water-hammer pressures and flow-reversal times were found to be possible with even modest-sized surge tanks. For example, a maximum water-hammer pressure of 24 psi (corresponding flow-reversal time of 200 msec) is possible with two water-filled surge tanks, each with a volume of 10 cu ft, when the initial flow rate is 7 ft/sec. Air-filled tanks, with an energy-absorbing capacity equivalent to 10 cu ft of water, would be considerably smaller.

The magnitude of water-hammer pressure was not uniform along the pipe, but was a function of the position along the pipe. Since negative water-hammer pressures could reduce the total system pressure to the vaporization pressure of water, the system should be designed to prevent such an occurrence in the core region. The location of the system-pressure null can be varied by pipe-length adjustment or by the use of different-sized surge tanks on the ends of the rabbit-tube piping system.

If null-point adjustment is not attempted for the core region; then the system should be adequately pressurized so that when the decompression pressure wave passes through the core region the total pressure in the core is above the vaporization pressure of water. Adequate pressure is required to prevent void formation, which causes positive reactivity addition to the core.

For the acoustically short pipe considered in this report, increasing the length of the pipe will increase the maximum water-hammer pressure for a given initial flow rate and surge-tank size and will increase the time required for flow reversal.

For given pipe dimensions, changing the pipe material can increase or decrease both the maximum water-hammer pressure and flow-reversal time. The study reported here assumed an aluminum system. Zirconium, which has the same modulus of elasticity as aluminum, would exhibit no difference in performance from a water-hammer and flow-reversal standpoint. For a given system, stainless steel, with a modulus of elasticity approximately three times as much as that of aluminum, would yield larger water-hammer pressures and faster flow-reversal times than aluminum would.

The results of the study can be used to establish the requirements of instrumentation and to determine its correct placement, in order to measure flow-reversal or water-hammer pressure behavior in an actual system. For example, a system with the equivalent of a 1-cu ft surge tank, a coolant velocity of 7 ft/sec, and Case 1 pump characteristics, would need instrumentation capable of measuring a pressure change of 81 psi in about 30 msec. The flow measurement device for the same situation must be capable of measuring the complete flow reversal in less than 60 msec, which is the minimum flow-reversal time in the system.

VIII. CONCLUSIONS

The results of this study are sensitive to the system parameters used. Deviations from these assumed conditions for a typical AARR hydraulic rabbit-tube system may alter the results. For example, decreasing the length of pipe will decrease the flow-reversal times. Surge-tanks size and placement, pipe size, and system geometry will change the results presented here and are therefore important in the design of an actual hydraulic rabbit-tube system. It is concluded that reasonable flow-reversal times without excessive water-hammer pressures can be achieved in a typical AARR hydraulic rabbit-tube system by proper design.

The study indicated that a sufficiently high normal system pressure must be maintained if void formation in the rabbit-tube system is to be

avoided. The possibility of voiding is present because negative pressure waves occur in portions of the system and the magnitude and location of the negative pressure regions depends on system design.

The study showed that the placement of pressure sensors in the system for measuring the magnitude of water hammer pressures is important. The water-hammer pressure varies around the system and, hence, we must consider the pressure magnitude attained at different points of the system.

For the typical AARR hydraulic rabbit-tube system with the equivalent of a 1-cu ft water-filled surge tank on both ends of the system, the study showed that pressure-sensing instrumentation must be capable of measuring pressure changes of ~100 psi occurring within about 30 msec.

The two types of pump characteristics used in this study provide a good means of bracketing the limits of flow-reversal times and water-hammer pressures for design and safety considerations.

APPENDIX

Derivation of Equations1. Basic Derivation

Derivation of the two equations for fluid flow in a pipe is based on the application of Newton's Second Law and the conservation of mass.

Fig. 14. Pipe Longitudinal Cross Section

Application of Newton's Second Law to the small slug of fluid in Fig. 14 yields

Sum of forces = mass x acceleration,

or

$$-A\Delta p - \frac{f\gamma\Delta x AV^2(\text{Sgn})}{2gD} = \frac{\gamma A\Delta x}{g} \frac{\Delta V}{\Delta t}.$$

Since $V = W/\gamma A$, we obtain

$$-\frac{\Delta p}{\Delta x} = \frac{fW^2(\text{Sgn})}{2D\gamma g A^2} + \frac{1}{Ag} \frac{\Delta W}{\Delta t}. \quad (1)$$

Taking the limit as Δx and Δt go to zero, we obtain

$$-\frac{\partial p}{\partial x} = \frac{fW^2(\text{Sgn})}{2D\gamma g A^2} + \frac{1}{Ag} \frac{\partial W}{\partial t}. \quad (2)$$

Application of the principle of conservation of mass to the slug of fluid yields

$$\left(\text{Change in flow volume} \right) = \left(\text{Change in fluid volume due to compressibility} \right) + \left(\text{Change in volume due to pipe expansion} \right),$$

or

$$-\frac{\pi D^2}{4} \Delta V \Delta t = \frac{\pi D^2 \Delta p \Delta x}{4K} + \frac{\pi D^3 \Delta p \Delta x}{4bE}.$$

Hence,

$$-\frac{\Delta V}{\Delta x} = \left(\frac{\Delta p}{\Delta t} \right) \left(\frac{1}{K} + \frac{D}{bE} \right), \quad (3)$$

and, taking the limit as Δx and Δt go to zero and also using $V = W/\gamma A$, we obtain

$$-\frac{\partial p}{\partial t} = \frac{1}{\gamma A \left(\frac{1}{K} + \frac{D}{bE} \right)} \frac{\partial W}{\partial x}. \quad (4)$$

If we use the notation of L, R, and C, Eqs. 2 and 4 become

$$-\frac{\partial p}{\partial x} = \frac{L \partial W}{\partial t} + RW^2(\text{Sgn}) \quad (5)$$

and

$$-\frac{\partial p}{\partial t} = \frac{1}{C} \frac{\partial W}{\partial x}, \quad (6)$$

which are the equations for flow in a pipe.

2. Wave Characteristics of Transients

If fluid friction is neglected, Eq. 5 becomes

$$-\frac{\partial p}{\partial x} = \frac{L \partial W}{\partial t}. \quad (7)$$

Combining Eqs. 6 and 7, we obtain

$$\frac{\partial^2 W}{\partial t^2} = \frac{1}{LC} \frac{\partial^2 W}{\partial x^2} \quad (8)$$

and

$$\frac{\partial^2 p}{\partial t^2} = \frac{1}{LC} \frac{\partial^2 p}{\partial x^2}. \quad (9)$$

Equations 8 and 9 have solutions of the form

$$W = f_1(\sqrt{LC}x - t) + f_2(\sqrt{LC}x + t) \quad (10)$$

and

$$p = f_3(\sqrt{LC}x - t) + f_4(\sqrt{LC}x + t). \quad (11)$$

The functions f_1 and f_3 with argument $(\sqrt{LC}x - t)$ represent a wave traveling with a velocity of $1/\sqrt{LC}$ in one direction, and f_2 and f_4 represent a wave traveling in the opposite direction with the same velocity. Hence, under transient excitation, we would expect the flow and pressure variations in the pipe to have wave characteristics. Pipe friction alters the form, but not the wave nature, of the solution.

3. Analog Equation Solution

To make the problem solvable on the analog computer, we must alter the original pipe-section equations so that the partial differential equations (5 and 6) are transformed into a set of ordinary differential equations. This is necessary since the analog has only one independent variable (time). The variable distance is handled by splitting the entire length of pipe into finite pieces, wherein the dependent variables within each section are independent of distance. This is the common finite-difference method of analog solution.

Applying the above to Eqs. 5 and 6 yields

$$-\Delta p_n = L_n \Delta x_n \frac{dW_n}{dt} + R_n \Delta x_n W_n^2 (\text{Sgn}) \quad (12)$$

and

$$-\Delta W_n = C_n \Delta x_n \frac{dp_n}{dt}, \quad (13)$$

where n refers to the pipe-section number. Figure 5 is a simplified block diagram of the model used.

Since the analog computer is essentially a solver of differential equations, Eqs. 12 and 13 are transformed into the following form:

$$-\frac{dW_n}{dt} = \frac{\Delta p_n}{L_n \Delta x_n} + \frac{R_n W_n^2 (\text{Sgn})}{L_n} \quad (14)$$

and

$$-\frac{dp_n}{dt} = \frac{\Delta W_n}{C_n \Delta x_n} \quad (15)$$

The above equations are then programmed on the computer N times, where N is the number of pipe sections used. The number of sections used depends on several factors, such as the nature of the describing equations and the amount of analog equipment available.

Equations for the ends of the pipe representing the equation boundary conditions are derived for the two different pump characteristics. See Fig. 15 for a detail of the valve operation.

Fig. 15

Detail of Valve Operation

Case 2 valve and pump characteristics are more general and therefore will be discussed first.

a. Normal Flow: V-1 and V-4 open; V-2 and V-3 closed

$$W_i = \frac{\sqrt{|P_\ell - P_i|}}{\sqrt{R_{V1}}} (\text{Sgn})_1 \quad \begin{aligned} (\text{Sgn})_1 &= +1 && \text{if } P_\ell > P_i \\ (\text{Sgn})_1 &= -1 && \text{if } P_\ell < P_i \end{aligned} \quad (16)$$

$$W_{V1} = W_p \quad (17)$$

$$W_o = \frac{\sqrt{|P_o - P_a|}}{\sqrt{R_{V4}}} (\text{Sgn})_2 \quad \begin{aligned} (\text{Sgn})_2 &= +1 && \text{if } P_o > P_a \\ (\text{Sgn})_2 &= -1 && \text{if } P_o < P_a \end{aligned} \quad (18)$$

$$W_{V4} = -W_p \quad (19)$$

$$P_\ell - P_a = \Delta p_p = \text{constant} > 0 \quad (20)$$

$$W_{V2} = 0, \quad W_{V3} = 0. \quad (21)$$

b. Reverse Flow (conditions after instantaneous valve operation): V-2 and V-3 open; V-1 and V-4 closed

$$W_i = \frac{\sqrt{|P_a - P_i|}}{\sqrt{R_{V3}}} (\text{Sgn})_3 \quad \begin{aligned} (\text{Sgn})_3 &= +1 && \text{if } P_a > P_i \\ (\text{Sgn})_3 &= -1 && \text{if } P_a < P_i \end{aligned} \quad (22)$$

$$W_{V3} = -W_p \quad (23)$$

$$W_o = \frac{\sqrt{|P_\ell - P_o|}}{\sqrt{R_{V2}}} (\text{Sgn})_4 \quad \begin{aligned} (\text{Sgn})_4 &= +1 && \text{if } P_o > P_\ell \\ (\text{Sgn})_4 &= -1 && \text{if } P_o < P_\ell \end{aligned} \quad (24)$$

$$W_{V2} = W_p \quad (25)$$

$$p_\ell - p_a = \Delta p_p = \text{constant} \geq 0 \quad (26)$$

$$W_{V_1} = 0, \quad W_{V_4} = 0. \quad (27)$$

For irreversible flow in the pump (Case 1), the valve and pump equations are the same as for Case 2 but with the restriction that W_p cannot be less than zero.

Since instantaneous closure was considered, the resistance of a particular valve had only two possible values: a constant finite resistance when open, and infinite resistance when closed. For computational purposes, the standard (open) resistance of each valve was considered to be the frictional equivalent of 25 ft of the system's pipe.