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July 2011

U.S. DEPARTMENT OF
ENERGY

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Science

BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
Upton, L.I., N.Y.

AADD-103

ACCELERATOR DEPARTMENT
Internal Report

MASTER

COMBINATION OF CYLINDRICAL CURRENT SHEETS
TO CANCEL EXTERNAL FIELD

H.C. 8 / 1.00; MN 150

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R.A. Beth

March 29, 1966

RELEASED FOR ANNOUNCEMENT
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It can be shown¹ that any required finite two dimensional magnetic field can be produced inside or outside a right circular cylinder by letting the proper current distribution flow along the elements of the cylinder.

By using two coaxial cylinders, we can cancel the external field and still produce an arbitrarily required internal field.

One Cylinder

A cylinder of radius r with its axis perpendicular to the complex $Z = X + iY$ plane will intersect the latter in the circle

$$Z = z = re^{i\theta}, \quad r = \text{constant.} \quad (1)$$

Let the current distribution be given by the Fourier series

$$\frac{dI}{d\theta} = \sum_{n=1}^{\infty} n C_n \cos(n\theta + \theta_n) \quad (2a)$$

1. R.A. Beth, "Fields Produced by Cylindrical Current Arrays", BNL Accelerator Dept. Int. Rpt. AADD-102, March 25, 1966.

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The integrated current flowing in the section of the cylinder between 0 and θ is then

$$I(\theta) = \sum_{n=1}^{\infty} C_n \sin(n\theta + \theta_n) + \text{constant} . \quad (2b)$$

In the complex form, $H = H_Y + iH_X$, the interior and exterior fields are, respectively [Eqs. (10), Ref. 1],

$$H_L(Z) = -2\pi \sum_{n=1}^{\infty} n C_n r^{-n} e^{i\theta_n} Z^{n-1} \quad (3a)$$

$$H_R(Z) = 2\pi \sum_{n=1}^{\infty} n C_n r^n e^{-i\theta_n} Z^{-(n+1)} . \quad (3b)$$

Two Cylinders

Assume we have two coaxial current sheet cylinders of radii $a < b$. We can make the field exterior to both, $|Z| > b$, vanish by choosing the current distribution coefficients in Eq. (2) so that

$$C_n = -\frac{K_n}{2\pi n a^n} \quad \text{for } r = a \quad (4a)$$

and

$$C'_n = \frac{K_n}{2\pi n b^n} \quad \text{for } r = b \quad (4b)$$

with the θ_n the same for both distributions, i.e. $\theta'_n = \theta_n$. By superposition, the field interior to both, $|Z| < a$, is then

$$H_{in}(Z) = \sum_{n=1}^{\infty} K_n \left(\frac{1}{a^{2n}} - \frac{1}{b^{2n}} \right) e^{i\theta_n} Z^{n-1} \quad (5a)$$

and the field between the cylinders, $a < |Z| < b$, is

$$H_{\text{betw}}(Z) = - \sum_{n=1}^{\infty} K_n \left(e^{-i\theta_n} Z^{-n-1} + b^{-2n} e^{i\theta_n} Z^{n-1} \right) \quad (5b)$$

while the external field, for $|Z| > b$, is zero.

For comparison with Eqs. (5) we may write the fields (3) that would be produced if the second cylinder, $r = b$, were removed:

$$H_L(Z) = \sum_{n=1}^{\infty} K_n \left(\frac{1}{a^{2n}} \right) e^{i\theta_n} Z^{n-1} \quad \text{for } |Z| < a \quad (6a)$$

and

$$H_R(Z) = - \sum_{n=1}^{\infty} K_n e^{-i\theta_n} Z^{-n-1} \quad \text{for } |Z| > a \quad (6b)$$

Hence the addition of the second cylinder to cancel the external field reduces each component of the internal field by the factor

$$f_n = \left(\frac{1}{a^{2n}} - \frac{1}{b^{2n}} \right) / \frac{1}{a^{2n}} = 1 - \left(\frac{a}{b} \right)^{2n} \quad (7)$$

compared to what it would be with the inner cylinder alone. If, for example, $a/b = 1/2$, the dipole field ($n = 1$) would be reduced by 25%, the quadrupole component ($n = 2$) by 6¼%, etc.

We can find, via Eqs. (2) and (4), the current distributions on cylinders of radii $a < b$ to produce any nonsingular interior field [Eq. (5a)].

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Example

The current distributions to produce an ideal constant gradient field

$$H = B + GZ$$

inside $r = a$ and zero field outside $r = b$ will be given by

$$K_1 = B \left/ \left(\frac{1}{a^2} - \frac{1}{b^2} \right) \right. \text{ and } K_2 = G \left/ \left(\frac{1}{a^4} - \frac{1}{b^4} \right) \right.$$

namely, in abamperes/radian:

$$\frac{dI}{d\theta} = - \frac{K_1}{2\pi a} \cos \theta - \frac{K_2}{2\pi a} \cos 2\theta \quad \text{for } r = a$$

and

$$\frac{dI'}{d\theta} = \frac{K_1}{2\pi b} \cos \theta + \frac{K_2}{2\pi b} \cos 2\theta \quad \text{for } r = b .$$

If $a = 9.0$ cm, $b = 18.0$ cm, $B = 10,000$ gauss and $G/B = 0.0425 \text{ cm}^{-1}$, the required linear densities are, in amperes/centimeter

$$\frac{10}{a} \frac{dI}{d\theta} = - 21,220 \cos \theta - 6493 \cos 2\theta$$

and

$$\frac{10}{b} \frac{dI'}{d\theta} = 5305 \cos \theta + 812 \cos 2\theta$$

with a maximum density on the high field side of the inner cylinder of about 27,700 amperes/cm.

For the same AGS type of field we found in Ref. 1

$$\frac{10}{a} \frac{dI}{d\theta} = - 15,915 \cos \theta - 6088 \cos 2\theta \text{ amperes/cm}$$

for a single $a = 9.0$ cm cylinder, with the maximum density of about 22,000 amperes/cm.

RAB/ke

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