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**THE DEVELOPMENT AND APPLICATION OF A COUPLED
MONTE CARLO NEUTRON-PHOTON TRANSPORT CODE**

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Abstract

A three-dimensional, temperature dependent Monte Carlo photon transport code, HUCK FINN, and a three-dimensional coupled neutron-photon transport code, POGO, have been developed and used to study the generation and effect of a photon flux in a controlled thermonuclear reactor (CTR). Potentially inverse Compton scattering, wherein a photon gains energy as a result of scattering from a moving electron within the plasma, provides a mechanism for a photon flux to rob energy from the plasma, thereby affecting the operation of the CTR. Photon fluxes are generated within the CTR by reactions in plasma itself and inelastic neutron events in the blanket surrounding the plasma. The POGO code was used to calculate the photon flux incident on a CTR plasma, and the code HUCK FINN was used to calculate the energy robbed from the plasma via inverse Compton events. These calculations show that photon fluxes generated in the blanket and in the plasma do not significantly affect the operation of currently anticipated CTR's.

1. INTRODUCTION

If the "affluent society" now common in the United States were world-wide, could conventional energy sources supply the necessary energy? If so, for how long and at what price? In the answer to these and similar questions lies the motivation for all the current research into other ways of producing energy on a large commercial scale. The controlled thermonuclear reactor (CTR) effort over the last twenty years is one such search for an alternate energy source. It has been argued that, whereas fuel sources such as oil or uranium were limited, and at current efficiencies would be exhausted in some fairly short time, ocean water could supply a virtually inexhaustible fuel source for fusion reactors.^{1,2} It has also been argued that fusion reactors, once they were built, would be safer, cheaper, and pollute less than other means of energy production.^{1,2} Such great rewards, however, are not attained without considerable technical effort. The heart of CTR research has been the quest for the stable confinement of a plasma which is hot enough and dense enough to produce more energy than it took to produce and confine the plasma in the first place.

Unless stable confinement can be obtained, all of the related problems of cost, fuel sources, pollution, and safety are moot. However, once confinement is obtained, these other factors will become central issues. A recent review article suggests that within the next ten years or so a working CTR will be developed.³ Anticipating this, we have begun to look at two effects that may play a role in the economics and safety of a CTR. The two effects we are interested in are first, the conversion of a neutron flux from the plasma into a photon flux as it interacts with the radiation shield surrounding the plasma; and second, the effect of a photon flux on the plasma itself.

The first problem is found in many diverse applications, but has been ignored until recently. In many transport problems one type of particle in the process of interacting with matter creates other types of particles. Fourteen MeV neutrons leaving the plasma in a CTR will create photons, alpha particles, and protons, in addition to other neutrons in the radiation shield (commonly called a blanket) surrounding the plasma. These secondary particles, other than neutrons, are usually

ignored. It is argued that they only deposit their energy "locally"; that is, within a short distance from where it was created compared to the average distance a neutron travels. This, however, may not be the case in a CTR, where the high energy neutrons may create photons that can travel distances comparable to the neutron's mean free path.

To calculate this effect one needs a code that can follow both the neutrons and the subsequent photons. Thus, a "coupled" neutron-photon calculation is necessary. Our primary concern then becomes the development of a coupled neutron-photon transport code. We have chosen to use Monte Carlo techniques to solve this problem because they offer the greatest flexibility in handling the many interactions possible, and because the data sets required for this type of calculation are available to the author. Once a coupled Monte Carlo neutron-photon transport code is developed it can be used for a variety of problems. Although the specific problem we have had in mind is the calculation of neutron-photon processes in a CTR blanket, the code itself is not intended to be limited to that single application. Other applications, such as the analysis of neutron-photon experiments, nuclear weapons effects, radiation therapy, fission reactor shielding, and other shielding problems are envisioned. Some have already been begun.⁴

The second question (the effect of a photon flux on the plasma in the CTR) is concerned with the effect of inverse Compton scattering, wherein a moving electron loses energy by interacting with a photon. If the plasma is hot enough, photons passing through the plasma can *become an effective cooling mechanism for the plasma, thereby changing* the operating conditions of the CTR and possibly even quenching it!

To calculate the effect of a photon flux on the plasma in the CTR we have developed a temperature dependent photon transport code called HUCK FINN. The temperature dependence is required because, as shall be shown later, the mechanism (inverse Compton scattering) by which photons rob energy from the plasma depends on the fact that electrons in a plasma have appreciable velocities. A convenient way of describing the velocity distribution of the electrons is to specify a temperature and to assume that the electrons are distributed according to the Maxwell-Boltzmann distribution at that temperature. Again it is convenient to use a Monte Carlo method, as inverse Compton scattering is nonlinear and far from isotropic.

The present study is divided into several areas. Section 2 explains the Monte Carlo techniques used in transport calculations. The specific CTR problem is presented in Section 3. The HUCK FINN code (temperature dependent Monte Carlo photon transport) is described in Section 4. Section 5 describes the POGO code (coupled neutron-photon transport). The results of applying these codes to the CTR question are presented in Section 6. Finally, Section 7 presents a summary of the work and some conclusions and suggestions for further study.

When this work was started, the author was not aware of any existing coupled Monte Carlo codes. Since that time a group at Oak Ridge has published a description and some calculations done on a coupled neutron-photon code called MORSE.⁵ By that time, however, our development was fairly far along and we felt it best to just continue along our line of development as planned. There are a multitude of interesting problems awaiting the development of coupled neutron-photon codes. We therefore felt that our efforts could still be productive. As will be seen later, the heart of the coupled calculation lies in the gamma production cross sections used. In one sense the coupled calculation is simply a useful tool for discovering the proper gamma production rates for the material under study. In that sense, at least, our effort has been productive.

2. MONTE CARLO TECHNIQUES FOR TRANSPORT PROBLEMS IN CTR'S

The basic idea of the Monte Carlo technique is that the cumulative effect of many independent events can be calculated by summing over a finite number of events chosen at random but in accord with the appropriate probability distribution. Specifically, the effect of a photon or neutron flux on a CTR can be calculated if the probabilities for all individual events are known. For neutrons and photons, these probabilities are expressed in terms of cross sections for the various processes involved. The idea of using Monte Carlo techniques in this manner originated with, or at least found its strongest modern advocates in J. Von Neuman, S. Ulam, and E. Fermi.⁶ In 1954 Herman Kahn outlined most of the techniques still used in Monte Carlo codes today.⁷ Our purpose here is not to review all of this work but merely to present the basic idea behind all Monte Carlo techniques and to illustrate these techniques with a few examples from our work.

For transport calculations, the Monte Carlo method consists of calculating specific neutron trajectories and then summing over these to obtain average quantities of interest. Each trajectory is determined independently and in accord with the assumed known microscopic cross sections. For example, if a photon undergoes a Compton event, its new frequency and direction are determined in such a way that a sum over all such Compton events will yield the known Compton frequency and directional distributions. This is accomplished in the following manner. If the probability distribution of a certain variable x is $p(x)$, and the cumulative probability distribution is defined by

$$P(y) = \int_{x_{\min}}^y p(x) dx, \quad (2-1)$$

then the value of x for a given event is given by

$$x = P^{-1}(\xi), \quad (2-2)$$

where ξ is a random number between zero and one, and we assume $P(x)$ can be inverted. The probability distribution is normalized so that

$$\int_{x_{\min}}^{x_{\max}} p(x) dx = 1, \quad (2-3)$$

where $x_{\min}$ and $x_{\max}$ define the range of x . As before, ξ is a random number uniformly distributed from 0.0 to 1.0.

This basic idea can be restated as follows: Let $p(x)$ be the probability density for some variable of interest, $x_{\min} \leq x \leq x_{\max}$, so that

$$\int_{x_{\min}}^{x_{\max}} p(x) dx = 1.$$

Then $\int_{x_{\min}}^x p(x) dx = P(x)$ is a number between 0.0 and 1.0. Now let ξ be a number chosen at random between 0.0 and 1.0. If the equation $\xi = P(x)$ can be inverted to yield $x = P^{-1}(\xi)$, then to each randomly chosen ξ there corresponds a value of x . Furthermore, a large (but finite) set of random numbers $\{\xi\}$ will yield a set of values $\{x\}$ that is distributed between $x_{\min}$ and $x_{\max}$ in accordance with the probability density $p(x)$.

For example, the probability of a neutron traveling a distance x without a reaction and then reacting in the next increment dx is $\frac{e^{-x/\lambda} dx}{\lambda}$, where λ is the mean free path for the neutron. Thus

$$p(x) = \frac{e^{-x/\lambda}}{\lambda} \quad (2-4)$$

and

$$P(x) = \int_0^x p(x') dx' = 1 - e^{-x/\lambda}. \quad (2-5)$$

Hence, if for each randomly chosen ξ , the value of x is determined as $x = P^{-1}(\xi) = -\lambda \ln(1 - \xi)$, the resulting values of x will be distributed

in accord with Eq. (2-4). Since $1 - \xi$ is as valid a random number as ξ , most computer codes use the relation

$$x = -\lambda \ln(\xi). \quad (2-6)$$

The distance x determined by Eq. (2-6) is called the distance to collision. In a Monte Carlo code this distance is compared with the distance the particle will travel before it reaches a region characterized by a different mean free path, and the distance the particle will have to travel to reach the final census time. The shortest distance determines the event that will happen to the particle. For example, if a particle is 15 cm from a boundary (along the direction it is traveling) and Eq. (2-6) happens to yield a distance to collision of 20 cm in a time independent calculation, then the particle is assumed to cross the boundary. Once across the boundary, the distance to collision is readjusted so that

$$\frac{x_1}{\lambda_1} + \frac{x_2}{\lambda_2} = -\ln(\xi),$$

where x_1 is the distance already traveled in the material with mean free path λ_1 , and x_2 is the distance to collision in the material with mean free path λ_2 . Clearly this can be generalized to the case of many different materials. Thus, only one random number is used to determine the distance to collision, and the particle's position is advanced across material boundaries until either the distance to collision is the shortest distance, or the particle has escaped from the problem. When the distance to collision is the shortest distance, the particle is advanced to the collision point, and the scattering law for the material is used to determine the new direction and energy of the particle. If time dependence is desired, then the time coordinate of the particle and any other quantities (cross sections of the materials, etc.) are advanced as the particle travels along, and the distance to census must also be calculated. Again, the new direction and energy of the particle are chosen from the distribution laws for the collision by using a random number in an equation of the form of Eq. (2-2).

If the straightforward application of the ideas so far presented does not lead to a form that is easily or conveniently inverted, either numerical methods of solving the equation

$$\xi = P(x)$$

can be used or a technique known as the rejection technique can be used. The rejection technique compares the cumulative probability of a randomly chosen x with a second random number, and if $p(x)$ is greater than or equal to this second random number, the value x is accepted. If it is not, two new random numbers are chosen and the process repeated until an acceptable value of x is obtained.

For some transport problems the distance-to-boundary calculation given above is very inefficient in that little information is gained per unit computer time. For example in the plasma in a CTR (density about 10^{16} particles/cc) relatively few collisions take place. However, it may be that the few collisions that do are very interesting, and in order to get a good statistical sample of what effect these collisions have, one would like to modify the standard approach to make it more efficient. In this case we have forced the particles to collide within the plasma, but reduced the importance or weight of the particle by the probability that the particle would have escaped. That is, if R is the distance to the boundary, then $e^{-R/\lambda}$ is the escape probability, and $1 - e^{-R/\lambda}$ is the probability that the particle will collide in the plasma. Therefore we set

$$[1 - e^{-R/\lambda}] \xi = \int_0^x \frac{e^{-x/\lambda}}{\lambda} dx,$$

and choose the distance to collision as

$$x = -\lambda \ln \{1 - \xi [1 - e^{-R/\lambda}]\},$$

and reduce the weight of the particle by the factor $(1 - e^{-R/\lambda})$. Again, ξ is a random number uniformly distributed from zero to one and λ is the mean free path for the particle.

This forced collision method is of great advantage in two types of problems. One is the situation mentioned above, in which the system is so thin that only by forcing particles to collide within the system can good statistics be rapidly obtained. The second is when the system is so thick that the probability of a particle escaping is very small, but the particle leakage rate is of interest. In this second case, an expected leakage rate is calculated with every collision. (The expected leakage rate is $e^{-R/\lambda}$ and it is tallied in the leakage zone each time it is calculated.)

Sometimes in a Monte Carlo calculation one needs to depart entirely from a random selection method and use a very deterministic method. If, for example, one is choosing from a probability distribution for energies that behave exponentially in a given region, most of the choices determined by random numbers will be in the regions of high probability, and a very few will be in the rest of the distribution. If, in addition, it is the region of low probability that plays a significant role (as in the case of deep penetration problems), then serious errors can be made by simply following the recipe given earlier for generating a probability distribution. In this case a better method is to choose the variable uniformly and let the weight of the particle be determined by the probability distribution. In the plasma calculation to be described later, this method played a very important role. The energies of the photons born in the plasma were to be chosen from a distribution function that drops off exponentially with energy. The total amount of energy generated in the plasmas could be reproduced by sampling directly from the probability density. However, the resulting distribution of photons in the high energy tail of the function was badly reproduced. Since these are the photons that penetrate furthest into the shield, they are the ones that are most important in the shield calculation. The first idea for remedying this situation was to pick the energies uniformly and assign a weight proportional to the probability density. This resulted in better statistics in the high energy region. However, small deviations from a perfectly uniform energy selection resulted in large fluctuations in the total energy in the distribution. The most workable solution of this problem appears to be the use of a completely deterministic method of choosing the particle's energy in equal increments and assigning a weight proportional to the area under the probability curve in that increment. This means that if $p(E)$ is the probability density, and N particles are to be generated, E is chosen for the i^{th} particle as

$$E = i (E_{\text{max}} - E_{\text{min}}) / N$$

and the weight of the particle is taken as being proportional to

$$\int_{E-\Delta E}^E p(E) dE,$$

where ΔE is $(E_{\max} - E_{\min})/N$ and $E_{\max}$ and $E_{\min}$ are the maximum and minimum values of the energy to be considered. This method gave good results for both the total energy and the distribution in the high energy tail. Other corrections, such as choosing the energy at either the midpoint of the energy group or even randomly in the group did not appear to greatly affect the final result as long as N was reasonably large.

3. NEUTRON-PHOTON TRANSPORT CONSIDERATIONS IN A CTR

The two questions we are interested in answering are first, what is the photon flux generated by neutron collisions in the blanket of a CTR, and second, what is the effect of a photon flux on the plasma in the CTR. Before beginning to answer these questions in detail we want to outline some of the considerations that have already gone into CTR's. The principal reactions that might be used in a fusion reactor are

From the point of view of energy recovery it would be convenient to have as much as possible of the reaction energy appear as kinetic energy of charged particles, because the particles would still be contained by the magnetic field that confines the plasma. In addition, methods of direct conversion, such as those being developed by Post *et al.*^{8,9} could be used effectively. In addition, of course, all the problems connected with neutrons would be neatly circumvented. The deuterium-tritium reaction clearly does not meet this criterion. Almost all of the energy produced in the reaction occurs as kinetic energy of the neutron. The second reaction, the deuterium-deuterium reaction, mitigates the neutron problem, but at the cost of a large reduction in the energy produced by the reaction. The third reaction, between deuterium and ${}^3\text{He}$, seems to be the most desirable. In a real system even this reaction will result in some neutrons, since there is nothing to prevent the deuterons from producing tritium via (D, D) reactions, and the tritium in turn producing 14 MeV neutrons via (D, T) reactions.

The likelihood of each of these reactions occurring is shown as a function of deuteron energy in Fig. 1. Clearly at deuteron energies less

Fig. 1. Cross sections for D-T, D-D (total) and D-³He reactions (compiled from Oak Ridge National Laboratory Rept. ORNL-3113, revised August, 1964).

than 100 keV the (D, T) reaction is the most probable. Consequently, the first working CTR will most likely be based on the deuterium-tritium reaction. Even if CTR's are built which are based on the other reactions, high energy neutrons are likely to be present. Therefore, it is useful to investigate the effects of photons produced by these neutrons.

Now that the fusion reactions have been defined it is interesting to look at the power densities of a CTR. For a plasma comprised of nuclei of type one at density n_1 and nuclei of type two at density n_2 , the power density is

$$p_{12} = n_1 n_2 \langle \sigma v \rangle_{12} W_{12},$$

where W_{12} is the energy released per reaction, and $\langle \sigma v \rangle_{12}$ is the product of the cross section and the relative velocity, averaged over the respective velocity distributions of the plasma ions. For a given fuel mixture, the power density varies as the square of the plasma density. The symbol W_{12} is commonly known as the Q value of the reaction in nuclear physics, but in plasma physics that symbol is reserved for another quantity.

The confinement time required before a CTR becomes a valid energy source can be estimated from the power density given above. The estimation is made by assuming that the energy released per unit volume should balance the plasma ion kinetic energy in that volume. The time so calculated is known as the Lawson time, and it will be useful in comparing our results on the effects of a photon flux on the plasma. The Lawson time is calculated by setting

$$P_{12} t_L = n_1 n_2 \langle \sigma v \rangle W_{12} t_L = \frac{3}{2} (n_1 + n_2) kT,$$

and solving for the product nt_L , where t_L is the Lawson time, and n is the plasma density. For a 50-50 mixture of type one and type two nuclei, $n_1 = \frac{n}{2}$. The average of the cross section and relative velocity is usually taken over a Maxwell-Boltzmann distribution at T_i , where T_i is the ion temperature. Substituting the calculated values¹ for $\langle \sigma v \rangle$ results in numbers like those shown below for a 50-50 D, T plasma:

$$\begin{array}{ll}
 T = 10 \text{ keV} & nt_L = 2.84 \times 10^{13} \text{ sec/cm}^3 \\
 T = 20 \text{ keV} & nt_L = 1.55 \times 10^{13} \text{ sec/cm}^3 \\
 T = 50 \text{ keV} & nt_L = 1.78 \times 10^{13} \text{ sec/cm}^3
 \end{array}$$

If a CTR uses the deuterium-tritium reaction, its efficiency as an energy source will largely depend on what is exterior to the plasma itself. To be efficient the energy of the neutrons must be utilized. In fission reactors, neutrons are used either to produce fuel or to generate heat, or both. Similarly a CTR should use its neutrons either to generate heat or to produce some useful fuel, or both. This fact has led to two possible approaches. One is to surround the CTR with low grade fertile isotopes and manufacture high grade fission fuel for use in a nuclear reactor. A second approach is to surround the reacting plasma with some fairly light isotopes which, when exposed to neutrons, produce tritium. In either case heat would also be generated in, and extracted from, this fuel-producing region.

Most studies^{2, 10-15} envision a blanket layer which surrounds the plasma. The purpose of the blanket is to produce tritium and to extract heat. This blanket region should be as close as possible to the reacting plasma so that the neutrons seen by the blanket are not degraded in energy by any intervening material. The importance of exposing the blanket to an undegraded neutron flux becomes clear by considering a blanket using lithium as both the tritium breeder and coolant. Natural lithium is made up of two isotopes, ^6Li (7%) and ^7Li (93%). Both isotopes produce tritium when bombarded with neutrons; however, only one reaction is exothermic.

If only a degraded neutron spectrum were available, one might need to use lithium with more than 7% ^6Li . This would naturally increase the price of the lithium.

Another reason for having the blanket as close to the plasma as possible is that the magnetic field coils, which produce the magnetic field

which traps the plasma, are in all likelihood going to be superconducting magnetics. Therefore they will have to be maintained at fairly low temperatures. If the magnetic field coils were closer to the plasma, a larger fraction of the energy produced would be required for refrigerating the coils because of the increased neutron and photon heating in the coils.

Finally, since one wishes to extract heat from the blanket at as high a temperature as possible, the blanket should be as close as possible to the reacting plasma, so that most of the kinetic energy of the escaping neutrons can be converted into high temperature heat.

If lithium is to be used as the coolant and tritium breeder in the reactor, niobium is the most likely metal for use in the structure of the blanket.¹⁶ Niobium appears to be able to withstand the highly corrosive lithium if it is carefully fabricated. Although some other metals and alloys are being considered, niobium appears to be the best choice.

Our main concern is with the photon flux produced in the blanket region, and therefore we will be concerned with the nuclear properties of lithium and niobium. The actual structure of the blanket will be a niobium structure with the lithium coolant inside. For simplicity in our problems we have considered the CTR to be spherical and to be made up of four basic sections. These sections are shown in Fig. 2. We have already discussed the blanket and the plasma; the remaining two regions perform two distinct functions. The region just outside the blanket is a neutron and photon shield, which is required to protect operating personnel and the environment, and to minimize the heat load on the cryogenics associated with the field coils. The fourth region is, of course, the magnetic field coils and the cryogenic system required to make them economically operable.

Figure 3 is a much more detailed design of a spherical CTR used by Fraas¹³ in his consideration of future CTR's. We intend to use this design as a typical CTR. Tables 1-4 present some of the pertinent neutron data for each of the blanket regions. The photon cross sections for niobium and lithium are shown in Figs. 4 and 5. The corresponding neutron cross sections are shown in Figs. 6 and 7. In Section 5 we will describe our efforts to determine the gamma production cross section and the resulting gamma spectrum.

Fig. 2. The basic structural areas of a CTR.

Fig. 3. Detailed CTR design. This figure is taken from Oak Ridge National Laboratory Repts. ORNL-TM-2822 and ORNL-TM-3094.

Table 1. Lithium photon cross sections.

2. 5 LI

ABSORPTION EDGES ABOVE 1.0 KEV. IN KEV.

γ E.	L1 O.	L2 G.	L3 O.	M1 O.	M2 O.	M3 O.	M4 O.	M5 O.	M6 O.	M7 O.	M8 O.	M9 O.
ENERGY (KEV)	TOTAL (BARN)	COHERENT (BARN)	INCOHERENT (BARN)	PHOTOELECTRIC (BARN)	PAIR PRODUCTION (BARN)	TOTAL (CM ² /GRAM)						
1.00000E-03	0.21682E-04	0.45970E-01	0.63110E-05	0.21850E-04	0.	0.18996E-05						
0.16660E-02	0.48359E-03	0.25660E-01	0.87320E-03	0.17520E-05	0.	0.42015E-02						
0.20000E-02	0.28225E-03	0.31576E-01	0.96910E-03	0.27810E-05	0.	0.21501E-02						
0.25000E-02	0.15555E-03	0.26770E-01	0.10850E-01	0.14975E-05	0.	0.13327E-02						
0.30000E-02	0.65453E-02	0.23031E-01	0.11785E-01	0.86975E-02	0.	0.78522E-01						
0.33330E-02	0.65516E-02	0.22090E-01	0.12310E-01	0.62988E-02	0.	0.57570E-01						
0.50000E-02	0.18957E-02	0.13940E-01	0.14210E-01	0.15840E-02	0.	0.16198E-01						
0.66660E-02	0.75095E-01	0.99862E-00	0.15340E-01	0.49769E-01	0.	0.65140E-02						
0.70000E-02	0.66054E-01	0.93876E-00	0.15510E-01	0.41154E-01	0.	0.57522E-03						
0.75000E-02	0.56135E-01	0.85992E-00	0.15740E-01	0.31796E-01	0.	0.48751E-03						
0.80000E-02	0.48965E-01	0.79222E-00	0.15940E-01	0.25120E-01	0.	0.42627E-03						
0.83330E-02	0.45470E-01	0.75010E-00	0.16060E-01	0.21350E-01	0.	0.39473E-03						
0.90000E-02	0.39704E-01	0.67444E-00	0.16280E-01	0.16679E-01	0.	0.34468E-03						
1.00000E-02	0.59981E-01	0.58510E-00	0.16550E-01	0.11600E-01	0.	0.29499E-03						
0.12500E-01	0.26743E-01	0.42964E-00	0.17000E-01	0.54979E-02	0.	0.23216E-03						
0.15000E-01	0.23448E-01	0.32280E-00	0.17270E-01	0.29566E-02	0.	0.20356E-03						
0.20000E-01	0.20566E-01	0.22560E-00	0.17470E-01	0.10800E-02	0.	0.18800E-03						
0.25000E-01	0.16384E-01	0.14358E-00	0.17550E-01	0.50106E-02	0.	0.16527E-03						
0.30000E-01	0.10664E-01	0.10492E-00	0.17590E-01	0.26800E-02	0.	0.16203E-03						
0.40000E-01	0.17741E-01	0.63720E-01	0.17610E-01	0.93770E-02	0.	0.15401E-03						
0.50000E-01	0.17081E-01	0.42820E-01	0.16610E-01	0.42500E-02	0.	0.14628E-03						
0.60000E-01	0.16530E-01	0.30770E-01	0.16200E-01	0.22595E-02	0.	0.14590E-03						
0.70000E-01	0.16046E-01	0.23173E-01	0.15801E-01	0.13200E-02	0.	0.15529E-03						
0.80000E-01	0.15609E-01	0.18090E-01	0.15420E-01	0.89558E-03	0.	0.15551E-03						
1.00000E-01	0.14843E-01	0.11910E-01	0.14720E-01	0.30500E-03	0.	0.12889E-03						
0.15000E-00	0.13788E-01	0.67350E-02	0.13659E-01	0.13162E-03	0.	0.11926E-03						
0.15000E-00	0.13366E-01	0.59020E-02	0.13280E-01	0.91000E-04	0.	0.11577E-03						
0.20000E-00	0.12212E-01	0.31540E-02	0.12180E-01	0.34082E-04	0.	0.10601E-03						
0.30000E-00	0.10624E-01	0.14350E-02	0.10610E-01	0.84600E-05	0.	0.92242E-04						
0.40000E-00	0.99062E-02	0.81500E-03	0.94682E-02	0.30772E-05	0.	0.82525E-04						
0.50000E-00	0.86793E-03	0.52500E-03	0.86740E-02	0.14000E-05	0.	0.75546E-04						
0.60000E-00	0.80287E-03	0.35600E-03	0.80250E-02	0.72220E-05	0.	0.68466E-04						
0.80000E-00	0.70501E-03	0.26720E-03	0.70480E-02	0.26660E-05	0.	0.61293E-04						
0.10000E-01	0.65373E-03	0.13500E-03	0.63360E-02	0.12000E-05	0.	0.55015E-04						
0.15000E-01	0.51516E-03	0.54990E-04	0.51470E-02	0.80602E-07	0.	0.44722E-04						
0.20000E-01	0.44071E-03	0.33400E-04	0.43910E-02	0.60600E-07	0.	0.38256E-04						
0.30000E-01	0.34589E-03	0.14920E-04	0.34530E-02	0.40002E-07	0.	0.30374E-04						
0.40000E-01	0.29556E-03	0.84000E-05	0.28800E-02	0.30000E-07	0.	0.25658E-04						
0.50000E-01	0.25891E-03	0.53800E-05	0.24860E-02	0.24002E-07	0.	0.22476E-04						
0.60000E-01	0.23220E-03	0.37800E-05	0.21970E-02	0.20002E-07	0.	0.20156E-04						
0.80000E-01	0.19670E-03	0.21040E-05	0.17970E-02	0.15000E-07	0.	0.17076E-04						
0.10000E-02	0.17400E-03	0.13470E-05	0.15300E-02	0.12000E-07	0.	0.15105E-04						
0.15000E-02	0.14090E-03	0.59960E-06	0.11310E-02	0.80000E-08	0.	0.12232E-04						

Table 2. Niobium photon cross sections.

2. 4. 18

ABSORPTION EDGES ABOVE 1.0 KEV. IN KEV.

K 0.996	L1 2.700	L2 2.467	L3 2.572	M1 0.	M2 0.	M3 0.	M4 0.	M5 0.	M6 0.	M7 0.	M8 0.	M9 0.
ENERGY (KEV.)	TOTAL (BARN/CM)	COHERENT (BARN/CM)	INCOHERENT (BARN/CM)	PHOTOELECTRIC (BARN/CM)	PAIR PRODUCTION (BARN/CM)	TOTAL (CM=2.7 GRAM)						
1.0000E-03	0.14030E 07	0.14030E 07	0.20070E 01	0.14030E 07	0.	0.90979E 04						
0.16500E-02	0.35335E 06	0.35335E 06	0.31730E 01	0.35335E 06	0.	0.22913E 04						
0.25719E-02	0.13415E 06	0.85029E 03	0.43130E 01	0.13330E 06	0.	0.86992E 03						
0.25720E-02	0.34695E 06	0.85029E 03	0.43134E 01	0.34610E 06	0.	0.22498E 04						
0.24668E-02	0.32384E 06	0.85802E 03	0.44563E 01	0.32300E 06	0.	0.20599E 04						
0.24570E-02	0.35694E 06	0.85817E 03	0.44567E 01	0.35670E 06	0.	0.20502E 04						
0.25000E-02	0.42419E 06	0.85400E 03	0.45060E 01	0.42360E 06	0.	0.20502E 04						
0.26997E-02	0.34701E 06	0.80981E 03	0.48014E 01	0.34620E 06	0.	0.22502E 04						
0.27000E-02	0.43951E 06	0.80977E 03	0.48018E 01	0.43870E 06	0.	0.20502E 04						
0.33300E-02	0.24532E 06	0.75560E 03	0.56800E 01	0.24450E 06	0.	0.15908E 04						
0.50000E-02	0.79306E 05	0.57170E 03	0.76940E 01	0.78810E 05	0.	0.51484E 03						
0.66900E-02	0.35929E 05	0.49840E 03	0.93540E 01	0.35451E 05	0.	0.25298E 03						
0.75000E-02	0.25894E 05	0.41370E 03	0.10690E 02	0.25470E 05	0.	0.16791E 03						
0.85200E-02	0.19443E 05	0.37690E 03	0.10760E 02	0.19070E 05	0.	0.12621E 03						
1.00000E-02	0.11754E 05	0.31480E 03	0.11860E 02	0.11428E 05	0.	0.62202E 02						
0.15000E-01	0.39079E 04	0.20050E 03	0.14990E 02	0.38920E 04	0.	0.25302E 02						
0.16990E-01	0.20819E 04	0.14909E 03	0.15748E 02	0.19770E 04	0.	0.13499E 02						
0.16990E-01	0.14099E 04	0.14899E 03	0.15751E 02	0.13870E 04	0.	0.91007E 02						
0.20000E-01	0.12205E 05	0.13910E 03	0.16030E 02	0.12090E 05	0.	0.75145E 02						
0.20000E-01	0.40639E 04	0.78730E 02	0.17680E 02	0.39659E 04	0.	0.25340E 02						
0.40000E-01	0.18519E 04	0.50960E 02	0.18750E 02	0.17822E 04	0.	0.12009E 02						
0.50000E-01	0.10088E 04	0.35740E 02	0.19130E 02	0.95191E 03	0.	0.68289E 01						
0.60000E-01	0.61392E 03	0.28480E 02	0.19230E 02	0.58781E 03	0.	0.39764E 01						
0.80000E-01	0.28450E 03	0.16250E 02	0.19040E 02	0.24951E 03	0.	0.18495E 01						
1.00000E-01	0.16041E 03	0.11001E 02	0.18910E 02	0.15080E 03	0.	0.10402E 01						
0.15000E-00	0.62043E 02	0.52960E 01	0.17310E 02	0.59450E 02	0.	0.40242E 00						
0.20000E-00	0.36259E 02	0.31100E 01	0.16120E 02	0.35000E 02	0.	0.25493E 00						
0.30000E-00	0.20749E 02	0.14460E 01	0.14210E 02	0.19174E 02	0.	0.15845E 00						
0.40000E-00	0.15653E 02	0.83300E 00	0.12820E 02	0.13000E 02	0.	0.10345E 00						
0.50000E-00	0.12524E 02	0.54110E 00	0.11750E 02	0.12332E 02	0.	0.87698E 01						
0.60000E-00	0.10059E 02	0.37950E 00	0.10890E 02	0.79000E 00	0.	0.78199E 01						
0.80000E-00	0.10269E 02	0.21620E 00	0.95880E 01	0.45924E 00	0.	0.64594E 01						
0.10000E-01	0.90774E 01	0.13940E 00	0.86300E 01	0.30800E 00	0.	0.58862E 01						
0.15000E-01	0.75500E 01	0.62650E 01	0.70220E 01	0.17638E 00	0.89000E-01	0.47661E 01						
0.20000E-01	0.64768E 01	0.35410E 01	0.59940E 01	0.11735E 00	0.53000E-00	0.41598E 01						
0.30000E-01	0.56868E 01	0.16220E 01	0.47160E 01	0.64974E 01	0.89000E 00	0.35876E 01						
0.40000E-01	0.54171E 01	0.80250E 02	0.39350E 01	0.42209E 01	0.14300E 01	0.35127E 01						
0.50000E-01	0.53227E 01	0.57220E 02	0.35970E 01	0.30000E 01	0.18960E 01	0.34513E 01						
0.60000E-01	0.52974E 01	0.34780E 02	0.30020E 01	0.21424E 01	0.22700E 01	0.34513E 01						
0.80000E-01	0.53907E 01	0.22410E 02	0.24550E 01	0.15501E 01	0.29200E 01	0.34596E 01						
0.10000E-02	0.55314E 01	0.14390E 02	0.20900E 01	1.00000E-02	0.34300E 01	0.55868E 01						
0.15000E-02	0.60232E 01	0.65890E 03	0.15460E 01	0.66000E-02	0.44700E 01	0.59058E 01						

Table 3. Lithium neutron cross sections, ${}^7\text{Li}$.

Energy	Elastic	(n,n' γ)	(n,2n)	(n,n't)	(n, γ)	Total
116 2.074E-01	3.417E+00 0.	0.	0.	0.	0.	3.417E+00
117 2.417E-01	8.940E+00 0.	0.	0.	0.	0.	8.940E+00
118 2.710E-01	3.508E+00 0.	0.	0.	0.	0.	3.508E+00
119 2.940E-01	2.403E+00 0.	0.	0.	0.	0.	2.403E+00
120 3.345E-01	1.510E+00 0.	0.	0.	0.	0.	1.510E+00
121 3.776E-01	1.186E+00 0.	0.	0.	0.	0.	1.186E+00
122 4.234E-01	1.033E+00 7.762E-04 0.	0.	0.	0.	0.	1.034E+00
123 5.227E-01	9.810E-01 2.279E-02 0.	0.	0.	0.	0.	1.004E+00
124 5.324E-01	1.001E+00 4.966E-02 0.	0.	0.	0.	0.	1.050E+00
125 7.526E-01	1.050E+00 8.005E-02 0.	0.	0.	0.	0.	1.149E+00
126 9.633E-01	1.152E+00 1.409E-01 0.	0.	0.	0.	0.	1.303E+00
127 1.024E+00	1.238E+00 1.680E-01 0.	0.	0.	0.	0.	1.406E+00
128 1.176E+00	1.304E+00 1.789E-01 0.	0.	0.	0.	0.	1.483E+00
129 1.338E+00	1.329E+00 1.819E-01 0.	0.	0.	0.	0.	1.511E+00
130 1.511E+00	1.344E+00 1.838E-01 0.	0.	0.	0.	0.	1.528E+00
131 1.693E+00	1.359E+00 1.856E-01 0.	0.	0.	0.	0.	1.545E+00
132 1.897E+00	1.385E+00 1.879E-01 0.	0.	0.	0.	0.	1.573E+00
133 2.091E+00	1.445E+00 1.901E-01 0.	0.	0.	0.	0.	1.636E+00
134 2.305E+00	1.591E+00 1.929E-01 0.	0.	0.	0.	0.	1.783E+00
135 2.530E+00	1.714E+00 1.949E-01 0.	1.874E-05 0.	0.	0.	0.	1.909E+00
136 2.765E+00	1.795E+00 1.921E-01 0.	5.217E-03 0.	0.	0.	0.	1.994E+00
137 3.011E+00	1.850E+00 1.806E-01 0.	1.971E-02 0.	0.	0.	0.	2.066E+00
138 3.267E+00	1.919E+00 1.654E-01 0.	3.800E-02 0.	0.	0.	0.	2.118E+00
139 3.533E+00	1.971E+00 1.494E-01 0.	5.703E-02 0.	0.	0.	0.	2.178E+00
140 3.810E+00	2.040E+00 1.370E-01 0.	7.795E-02 0.	0.	0.	0.	2.250E+00
141 4.078E+00	2.228E+00 1.053E-01 0.	1.145E-01 0.	0.	0.	0.	2.447E+00
142 4.395E+00	2.391E+00 7.581E-02 0.	1.597E-01 0.	0.	0.	0.	2.616E+00
143 4.704E+00	2.282E+00 4.372E-02 0.	2.154E-01 0.	0.	0.	0.	2.541E+00
144 5.023E+00	2.151E+00 2.625E-02 0.	2.663E-01 0.	0.	0.	0.	2.444E+00
145 5.352E+00	2.011E+00 1.456E-02 0.	3.131E-01 0.	0.	0.	0.	2.343E+00
146 5.692E+00	1.866E+00 1.271E-02 0.	3.612E-01 0.	0.	0.	0.	2.240E+00
147 6.042E+00	1.750E+00 1.000E-02 0.	3.956E-01 0.	0.	0.	0.	2.156E+00
148 6.403E+00	1.651E+00 1.000E-02 0.	4.212E-01 0.	0.	0.	0.	2.082E+00
149 6.774E+00	1.564E+00 9.084E-03 0.	4.469E-01 0.	0.	0.	0.	2.010E+00
150 7.155E+00	1.491E+00 6.486E-03 0.	4.676E-01 0.	0.	0.	0.	1.965E+00
151 7.547E+00	1.439E+00 2.514E-03 0.	4.874E-01 0.	0.	0.	0.	1.925E+00
152 7.950E+00	1.393E+00 3.139E-03 0.	4.998E-01 0.	0.	0.	0.	1.884E+00
153 8.363E+00	1.337E+00 1.800E-03 0.	5.745E-03 0.	0.	0.	0.	1.843E+00
154 8.786E+00	1.287E+00 1.000E-03 0.	9.974E-03 0.	0.	0.	0.	1.795E+00
155 9.220E+00	1.215E+00 1.000E-03 0.	1.398E-02 0.	0.	0.	0.	1.717E+00
156 9.664E+00	1.143E+00 1.000E-03 0.	1.902E-02 0.	0.	0.	0.	1.639E+00
157 1.012E+01	1.149E+00 1.000E-03 0.	2.216E-02 0.	0.	0.	0.	1.629E+00
158 1.058E+01	1.135E+00 1.000E-03 0.	2.540E-02 0.	0.	0.	0.	1.643E+00
159 1.106E+01	1.194E+00 1.000E-03 0.	3.076E-02 0.	0.	0.	0.	1.639E+00
160 1.155E+01	1.194E+00 1.000E-03 0.	3.522E-02 0.	0.	0.	0.	1.610E+00
161 1.204E+01	1.170E+00 1.000E-03 0.	3.970E-02 0.	0.	0.	0.	1.589E+00
162 1.255E+01	1.152E+00 1.000E-03 0.	4.422E-02 0.	0.	0.	0.	1.559E+00
163 1.307E+01	1.135E+00 1.000E-03 0.	4.882E-02 0.	0.	0.	0.	1.539E+00
164 1.359E+01	1.124E+00 1.000E-03 0.	5.233E-02 0.	0.	0.	0.	1.518E+00
165 1.385E+01	1.117E+00 1.000E-03 0.	5.470E-02 0.	0.	0.	0.	1.506E+00
166 1.413E+01	1.109E+00 1.000E-03 0.	5.709E-02 0.	0.	0.	0.	1.494E+00
167 1.441E+01	1.102E+00 1.000E-03 0.	5.940E-02 0.	0.	0.	0.	1.483E+00
168 1.468E+01	1.100E+00 1.000E-03 0.	6.000E-02 0.	0.	0.	0.	1.480E+00
169 1.494E+01	1.100E+00 1.000E-03 0.	6.000E-02 0.	0.	0.	0.	1.480E+00
170 1.518E+01	1.100E+00 1.000E-03 0.	6.000E-02 0.	0.	0.	0.	1.480E+00
171 1.639E+01	1.100E+00 1.000E-03 0.	6.000E-02 0.	0.	0.	0.	1.480E+00
172 1.698E+01	1.100E+00 1.000E-03 0.	6.000E-02 0.	0.	0.	0.	1.480E+00
173 1.758E+01	1.100E+00 1.000E-03 0.	6.000E-02 0.	0.	0.	0.	1.480E+00
174 1.819E+01	1.100E+00 1.000E-03 0.	6.000E-02 0.	0.	0.	0.	1.480E+00
175 1.882E+01	1.100E+00 1.000E-03 0.	6.000E-02 0.	0.	0.	0.	1.480E+00
176 2.009E+01	0.	0.	0.	0.	0.	0.

Table 3, ${}^6\text{Li}$.

Energy	Elastic	(n,2n)	(n,n'd)	(n,n'd) level	(n,p)	(n,t)	(n, γ)	Total
111 2.074E-01	6.724E+00 0.	0.	0.	0.	0.	2.217E+00 0.	0.	6.941E+00
112 2.417E-01	7.932E+00 0.	0.	0.	0.	0.	2.701E+00 0.	0.	1.003E+01
113 2.710E-01	7.653E+00 0.	0.	0.	0.	0.	2.350E+00 0.	0.	8.013E+00
114 2.940E-01	3.497E+00 0.	0.	0.	0.	0.	1.770E+00 0.	0.	5.269E+00
120 3.345E-01	2.270E+00 0.	0.	0.	0.	0.	1.254E+00 0.	0.	3.545E+00
121 3.776E-01	1.713E+00 0.	0.	0.	0.	0.	9.869E-01 0.	0.	2.697E+00
122 4.234E-01	1.428E+00 0.	0.	0.	0.	0.	6.595E-01 0.	0.	2.086E+00
123 4.727E-01	1.105E+00 0.	0.	0.	0.	0.	4.269E-01 0.	0.	1.705E+00
124 5.124E-01	1.191E+00 0.	0.	0.	0.	0.	3.669E-01 0.	0.	1.557E+00
125 5.525E-01	1.105E+00 0.	0.	0.	0.	0.	3.395E-01 0.	0.	1.445E+00
126 5.933E-01	1.049E+00 0.	0.	0.	0.	0.	3.245E-01 0.	0.	1.374E+00
127 1.024E+00	1.013E+00 0.	0.	0.	0.	0.	3.040E-01 0.	0.	1.322E+00
129 1.175E+00	7.972E-01 0.	0.	0.	0.	0.	2.789E-01 0.	0.	1.278E+00
127 1.339E+00	7.791E-01 0.	0.	0.	0.	0.	2.522E-01 0.	0.	1.231E+00
130 1.511E+00	7.753E-01 0.	0.	0.	0.	0.	2.390E-01 0.	0.	1.210E+00
131 1.693E+00	4.997E-01 0.	0.	0.	0.	0.	2.371E-01 0.	0.	1.240E+00
132 1.987E+00	1.126E+00 0.	0.	0.	0.	0.	2.339E-01 0.	0.	1.390E+00
133 2.091E+00	1.249E+00 0.	0.	0.	0.	0.	2.212E-01 0.	0.	1.527E+00
134 2.305E+00	1.354E+00 0.	0.	0.	0.	0.	2.037E-01 0.	0.	1.598E+00
135 2.530E+00	1.312E+00 0.	0.	0.	0.	0.	1.893E-01 0.	0.	1.630E+00
136 2.795E+00	1.321E+00 0.	0.	0.	0.	0.	1.612E-01 0.	0.	1.678E+00
137 3.011E+00	1.330E+00 0.	0.	0.	0.	0.	1.403E-01 0.	0.	1.757E+00
138 3.267E+00	1.332E+00 0.	0.	0.	0.	0.	1.220E-01 0.	0.	1.850E+00
139 3.535E+00	1.343E+00 0.	0.	0.	0.	0.	1.030E-01 0.	0.	1.984E+00
140 3.810E+00	1.357E+00 0.	0.	0.	0.	0.	8.438E-02 0.	0.	2.070E+00
141 4.093E+00	1.375E+00 0.	0.	0.	0.	0.	8.000E-02 0.	0.	2.096E+00
142 4.345E+00	1.334E+00 0.	0.	0.	0.	0.	7.999E-02 0.	0.	2.081E+00
143 4.704E+00	1.321E+00 0.	0.	0.	0.	0.	7.812E-02 0.	0.	2.058E+00
144 5.023E+00	1.314E+00 0.	0.	0.	0.	0.	7.532E-02 0.	0.	2.037E+00
145 5.392E+00	1.292E+00 0.	0.	0.	0.	0.	7.379E-02 0.	0.	2.006E+00
146 5.942E+00	1.244E+00 0.	0.	0.	0.	0.	7.139E-02 0.	0.	1.977E+00
147 6.403E+00	1.238E+00 0.	0.	0.	0.	0.	6.905E-02 0.	0.	1.949E+00
148 6.774E+00	1.228E+00 0.	0.	0.	0.	0.	6.535E-02 0.	0.	1.908E+00
149 7.155E+00	1.215E+00 0.	0.	0.	0.	0.	6.190E-02 0.	0.	1.879E+00
151 7.647E+00	1.215E+00 0.	0.	0.	0.	0.	5.876E-02 0.	0.	1.866E+00
152 7.950E+00	1.174E+00 0.	0.	0.	0.	0.	5.672E-02 0.	0.	1.838E+00
151 8.353E+00	1.179E+00 0.	0.	0.	0.	0.	5.463E-02 0.	0.	1.780E+00
154 8.785E+00	1.159E+00 0.	0.	0.	0.	0.	5.248E-02 0.	0.	1.740E+00
155 9.220E+00	1.145E+00 0.	0.	0.	0.	0.	5.044E-02 0.	0.	1.715E+00
156 9.664E+00	1.130E+00 0.	0.	0.	0.	0.	4.895E-02 0.	0.	1.689E+00
157 1.012E+00	1.117E+00 0.	0.	0.	0.	0.	4.695E-02 0.	0.	1.668E+00
158 1.059E+00	1.104E+00 0.	0.	0.	0.	0.	4.754E-02 0.	0.	1.645E+00
159 1.105E+00	1.073E+00 0.	0.	0.	0.	0.	4.639E-02 0.	0.	1.614E+00
160 1.159E+00	1.073E+00 0.	0.	0.	0.	0.	4.541E-02 0.	0.	1.571E+00
161 1.204E+00	1.028E+00 0.	0.	0.	0.	0.	4.352E-02 0.	0.	1.534E+00
162 1.259E+00	9.919E-01 0.	0.	0.	0.	0.	4.095E-02 0.	0.	1.502E+00
163 1.307E+00	9.243E-01 0.	0.	0.	0.	0.	3.793E-02 0.	0.	1.468E+00
164 1.359E+00	9.044E-01 0.	0.	0.	0.	0.	3.544E-02 0.	0.	1.422E+00
165 1.386E+00	8.972E-01 0.	0.	0.	0.	0.	3.376E-02 0.	0.	1.425E+00
166 1.411E+00	8.707E-01 0.	0.	0.	0.	0.	3.207E-02 0.	0.	1.406E+00
167 1.441E+00	8.537E-01 0.	0.	0.	0.	0.	3.037E-02 0.	0.	1.386E+00
168 1.469E+00	8.333E-01 0.	0.	0.	0.	0.	2.833E-02 0.	0.	1.367E+00
169 1.524E+00	8.171E-01 0.	0.	0.	0.	0.	2.571E-02 0.	0.	1.344E+00
170 1.581E+00	7.953E-01 0.	0.	0.	0.	0.	2.305E-02 0.	0.	1.321E+00
171 1.639E+00	7.534E-01 0.	0.	0.	0.	0.	2.034E-02 0.	0.	1.274E+00
172 1.699E+00	7.254E-01 0.	0.	0.	0.	0.	1.758E-02 0.	0.	1.250E+00
173 1.758E+00	6.975E-01 0.	0.	0.	0.	0.	1.478E-02 0.	0.	1.225E+00
174 1.819E+00	6.692E-01 0.	0.	0.	0.	0.	1.192E-02 0.	0.	1.189E+00
175 1.882E+00	6.292E-01 0.	0.	0.	0.	0.	7.74E-03 0.	0.	1.189E+00
175 2.007E+00	0.	0.	0.	0.	0.	0.	0.	0.

Table 4. Niobium neutron cross sections.

Energy (MeV)	Elastic	(n,n'γ)	(n,2n)	(n,γ)	Total
116 2.074E+01	8.611E+00	0.	0.	7.754E+02	8.689E+00
117 2.417E+01	8.535E+00	0.	0.	7.437E+02	8.609E+00
118 2.710E+01	8.472E+00	0.	0.	7.175E+02	8.544E+00
119 2.940E+01	8.396E+00	0.	0.	6.857E+02	8.464E+00
120 3.345E+01	8.295E+00	0.	0.	6.439E+02	8.360E+00
121 3.776E+01	8.189E+00	0.	0.	5.995E+02	8.249E+00
122 4.234E+01	8.012E+00	0.	0.	5.270E+02	8.065E+00
123 5.227E+01	7.696E+00	0.	0.	4.224E+02	7.738E+00
124 5.324E+01	7.318E+00	6.428E-05	0.	3.075E+02	7.349E+00
125 7.526E+01	6.749E+00	1.496E-01	0.	2.274E+02	6.922E+00
126 8.833E+01	5.922E+00	5.195E-01	0.	1.823E+02	6.460E+00
127 1.024E+02	5.191E+00	9.463E-01	0.	1.335E+02	6.151E+00
128 1.176E+00	4.555E+00	1.344E+00	0.	8.108E+03	5.917E+00
129 1.338E+00	3.957E+00	1.695E+00	0.	2.538E+03	5.665E+00
130 1.511E+00	3.475E+00	1.921E+00	0.	0.	5.397E+00
131 1.693E+00	3.052E+00	2.053E+00	0.	0.	5.115E+00
132 1.887E+00	2.649E+00	2.182E+00	0.	0.	4.831E+00
133 2.091E+00	2.402E+00	2.240E+00	0.	0.	4.642E+00
134 2.305E+00	2.195E+00	2.283E+00	0.	0.	4.466E+00
135 2.530E+00	1.982E+00	2.300E+00	0.	0.	4.282E+00
136 2.765E+00	1.790E+00	2.300E+00	0.	0.	4.090E+00
137 3.011E+00	1.672E+00	2.293E+00	0.	0.	3.965E+00
138 3.267E+00	1.620E+00	2.280E+00	0.	0.	3.900E+00
139 3.533E+00	1.556E+00	2.266E+00	0.	0.	3.832E+00
140 3.810E+00	1.515E+00	2.252E+00	0.	0.	3.768E+00
141 4.098E+00	1.543E+00	2.238E+00	0.	0.	3.781E+00
142 4.396E+00	1.576E+00	2.223E+00	0.	0.	3.819E+00
143 4.704E+00	1.651E+00	2.207E+00	0.	0.	3.858E+00
144 5.023E+00	1.708E+00	2.191E+00	0.	0.	3.898E+00
145 5.352E+00	1.755E+00	2.174E+00	0.	0.	3.940E+00
146 5.692E+00	1.827E+00	2.157E+00	0.	0.	3.983E+00
147 6.042E+00	1.897E+00	2.139E+00	0.	0.	4.036E+00
148 6.403E+00	1.978E+00	2.121E+00	0.	0.	4.099E+00
149 6.774E+00	2.051E+00	2.102E+00	0.	0.	4.163E+00
150 7.155E+00	2.137E+00	2.089E+00	0.	0.	4.226E+00
151 7.547E+00	2.215E+00	2.077E+00	0.	0.	4.291E+00
152 7.950E+00	2.295E+00	2.064E+00	0.	0.	4.359E+00
153 8.363E+00	2.377E+00	2.051E+00	0.	0.	4.428E+00
154 8.786E+00	2.450E+00	2.038E+00	3.305E-07	0.	4.487E+00
155 9.220E+00	2.449E+00	2.015E+00	1.545E-02	0.	4.479E+00
156 9.654E+00	2.429E+00	1.971E+00	5.532E-02	0.	4.455E+00
157 1.012E+01	2.422E+00	1.909E+00	1.049E-01	0.	4.436E+00
158 1.059E+01	2.420E+00	1.807E+00	1.971E-01	0.	4.414E+00
159 1.106E+01	2.397E+00	1.730E+00	2.508E-01	0.	4.367E+00
160 1.155E+01	2.333E+00	1.681E+00	2.925E-01	0.	4.306E+00
161 1.204E+01	2.275E+00	1.636E+00	3.306E-01	0.	4.242E+00
162 1.253E+01	2.222E+00	1.595E+00	3.579E-01	0.	4.176E+00
163 1.307E+01	2.175E+00	1.554E+00	3.797E-01	0.	4.109E+00
164 1.359E+01	2.137E+00	1.530E+00	3.922E-01	0.	4.059E+00
165 1.386E+01	2.110E+00	1.517E+00	3.983E-01	0.	4.026E+00
166 1.413E+01	2.093E+00	1.505E+00	4.045E-01	0.	3.993E+00
167 1.441E+01	2.057E+00	1.493E+00	4.105E-01	0.	3.960E+00
168 1.469E+01	2.050E+00	1.490E+00	4.120E-01	0.	3.952E+00
169 1.524E+01	2.050E+00	1.490E+00	4.120E-01	0.	3.952E+00
170 1.581E+01	2.050E+00	1.490E+00	4.120E-01	0.	3.952E+00
171 1.639E+01	2.050E+00	1.490E+00	4.120E-01	0.	3.952E+00
172 1.698E+01	2.050E+00	1.490E+00	4.120E-01	0.	3.952E+00
173 1.758E+01	2.050E+00	1.490E+00	4.120E-01	0.	3.952E+00
174 1.819E+01	2.050E+00	1.490E+00	4.120E-01	0.	3.952E+00
175 1.882E+01	2.050E+00	1.490E+00	4.120E-01	0.	3.952E+00
176 2.009E+01	0.	0.	0.	0.	0.

Fig. 4. Photon cross section for lithium.

Fig. 5. Photon cross section for niobium.

Fig. 6. Neutron cross section for lithium isotopes.

Fig. 7. Neutron cross sections for niobium.

4. THE DEVELOPMENT AND SOLUTION OF THE TEMPERATURE DEPENDENT TRANSPORT EQUATION

We approached the answer to the second question, the effect of a photon flux on the plasma itself, by the construction of the temperature dependent photon transport code HUCK FINN. The temperature influences all of the scattering and absorptive events of photons. In a CTR the usual considerations of effects such as photoelectric absorption and Compton scattering are altered by the ionic state of the plasma and the velocity distribution of the free electrons.

The HUCK FINN code is a three-dimensional, time independent, Monte Carlo, temperature dependent transport code which uses the expected value forced collision technique described in Section 2. It is designed to predict both the photon flux entering any specified zone and the energy deposited by photons in all zones. It requires as input the temperatures, materials, densities, and boundaries of each zone. This section is devoted to a development of the transport equation in terms of free-free and bound-free absorption coefficients and temperature dependent Compton cross sections, and the equation's solution by the HUCK FINN code.

For our calculations it is convenient to separate radiation-matter interactions into two categories: one in which the energy levels of the atom must be considered, and one in which the electrons can be considered as free from the atoms. The first group of reactions is well described by a semi-classical treatment of radiation. The second group of reactions includes Rayleigh scattering, pair production, and Compton scattering.

Turning our attention to the first group of reactions, $h\nu_{nm}$ is the energy difference between two states n and m in an atom. The probability that in an interval of time dt an atom in the state n will spontaneously emit a quantum of energy $h\nu_{nm}$ (the atom returning to the state m) into an increment of solid angle dw can be written as

$$\frac{4e^2(2\pi\nu_{nm})^2}{3\hbar c^3} |(r)_{nm}|^2 dw dt,$$

where $(r)_{nm}$ is the matrix element $\int \bar{u}_m r u_n d\tau$ for states u_n and u_m . This type of emission is isotropic.^{17,18} The same atomic state, in the presence

of radiation of frequency ν_{nm} , will have an increased likelihood of emitting a quantum of radiation $h\nu_{nm}$. The probability for this stimulated, or induced, radiation is given by

$$\frac{4\pi^2 e^2}{3\hbar^2 c} \left| (r)_{nm} \right|^2 I_{\nu_{nm}} d\omega dt. \quad (17)$$

Here $I_{\nu_{nm}}$ is the specific intensity of the external radiation. If $dE_{\nu_{nm}}$ is the total radiation energy in the frequency interval between ν and $\nu + d\nu$ passing per unit time through an area dA with a normal $\vec{n}$ into a solid angle $d\omega$ centered about a direction $\vec{s}$, then the specific intensity is defined by the following equation

$$dE_{\nu} = I_{\nu} (\vec{n} \cdot \vec{s}) dA d\omega dt d\nu. \quad (4-1)$$

Stimulated emission is in exactly the same direction as the external radiation.

The total emission involves both the spontaneous and induced radiation. One can represent the total emission into an increment of solid angle $d\omega$, time dt , and frequency $d\nu$ as $j_{\nu} m d\omega dt d\nu$. Here, j_{ν} is the coefficient of emission and m is the emitting mass. Thus,

$$j_{\nu_{nm}} d\omega = N_n \left(\frac{4e^2 (2\pi\nu_{nm})^3}{3\hbar^2 c^3} \left| (r)_{nm} \right|^2 + \frac{4\pi^2 e^2}{3\hbar^2 c} \left| (r)_{nm} \right|^2 I_{\nu_{nm}} \right) \times h\nu_{nm} d\omega / \rho, \quad (4-2)$$

where N_n is the number of atoms in the state n , and ρ is the mass density. The factor $h\nu_{nm}$ comes from the definition of the radiation intensity I_{ν} .

The inverse process to stimulated emission is absorption. The probability that an atom in the state m absorbs a quantum of energy $h\nu_{nm}$ in a time dt is

$$\frac{4\pi^2 e^2 \left| (r)_{mn} \right|^2}{3\hbar^2 c} I_{\nu_{nm}} d\omega dt,$$

as one would expect from the theorem of reciprocity. The absorption coefficient is

$$K_{v_{nm}} = \frac{N_m}{\rho} \frac{4\pi^2 e^2}{3\hbar^2 c} |(r)_{mn}|^2 h\nu_{nm}. \quad (4-3)$$

Here $K_{v_{nm}}$ is the absorption coefficient for photons of frequency ν_{nm} , and N_m is the number of atoms in state m . The amount of radiation absorbed is $\rho K_{v_{nm}} I_{v_{nm}} ds$ when photons of frequency ν_{nm} travel a distance ds in a material of density ρ . The radiation emitted along the distance ds and in that direction is $j_v ds$. Thus, the transport equation calculated along a photon ray in an absorbing medium is

$$\frac{dI_v}{\rho ds} = j_v - K_v I_v. \quad (4-4)$$

If we assume that the populations of state n and state m are related by a simple Boltzmann factor

$$\frac{N_n}{N_m} = e^{-h\nu_{nm}/kT}, \quad (4-5)$$

where k is Boltzmann's constant, and T is the temperature of the gas, and that $(r)_{nm} = (r)_{mn}$; then the transport equation becomes

$$\begin{aligned} \frac{dI_{v_{nm}}}{\rho ds} = & \frac{N_n}{\rho} \frac{h\nu_{nm}^3}{c^2} \frac{4\pi^2 e^2}{3\hbar^2 c} |(r)_{nm}|^2 h\nu_{nm} \\ & - K_{v_{nm}} \left(1 - e^{-h\nu_{nm}/kT}\right) I_{v_{nm}}. \end{aligned} \quad (4-6)$$

The form of the second term of Eq. (4-6) suggests that the transport equation might be simplified if we redefine the absorption coefficient to be

$$K_{v_{nm}} = K_{v_{nm}} \left(1 - e^{-h\nu_{nm}/kT}\right). \quad (4-7)$$

Dropping the redundant subscripts n and m , Eq. (4-6) becomes

$$\frac{dI_v}{ds} = K'_v B_v - K'_v I_v, \quad (4-8)$$

where

$$B_v = \frac{hv^3}{c^2} e^{-hv/kT} (1 - e^{-hv/kT})^{-1}. \quad (4-9)$$

The value B_v is the Planck function and it is the solution for I_v in the case of thermal equilibrium. To obtain Eq. (4-8) many authors write Eq. (4-4), "correct" K_v for induced emission [by multiplying it by the factor $(1 - e^{-hv/kT})$], and assume local thermodynamic equilibrium (LTE) by setting $j_v = K_v B_v$. The LTE assumption is embodied in Eq. (4-5) and means that locally a temperature is defined in the Boltzmann sense.

The energy eigenvalues of an atom fall into two classes: discrete and continuous. There are three distinct classes of absorptive processes involving energy levels n and m . First there are transitions between discrete states. These are sometimes referred to as "bound-bound" transitions, or lines (since the energy $h\nu_{nm}$ is so well determined). For hydrogen the Lyman, Balmer series is an example of bound-bound transitions. The second possible type of transition is from a discrete energy value to a continuous one. This is called a photoelectric event, or bound-free transition. Finally there are transitions from one continuous eigenstate to another. These are called free-free transitions or "inverse Bremsstrahlung." In general the absorption coefficient for both free-free and bound-free events can be put in the form

$$K_v = \frac{1}{u^3} \left[D_{ff} + \sum_i D_{bf}(i) \right], \quad (4-10)$$

where D_{ff} is the coefficient for free-free events, $D_{bf}(i)$ is the coefficient for the i^{th} level's bound-free events, and u is $h\nu/kT$. The sum is taken over all discrete levels accessible to radiation of energy $h\nu$.

For hydrogen, the constants $D_{bf}(i)$ and D_{ff} (or equivalent parameters) are tabulated.¹⁹ In a 20-keV hydrogen plasma, the main

absorptive effect is free-free absorption. For our calculations we have neglected lines and assumed that K_V was well represented by Eq. (4-10).

In a 20-keV hydrogen plasma, the only important process in the second group (those reactions which do not depend on the energy levels of the atoms) is Compton scattering. Pair production, which has a threshold of 1.02 MeV, is not important for energies below 14 MeV. Figure 8 shows photon cross sections for hydrogen in the range from 1 keV to 14 MeV. Except in the region below 5 keV, Compton scattering dominates; however, for a plasma at 20 keV, Compton scattering differs from the usual Klein-Nishina cross section shown in Fig. 8.

For free electrons at rest, the differential Compton scattering cross section is the Klein-Nishina cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{KN} = \frac{1}{2} r_0^2 \left(\frac{k}{k_0}\right)^2 \left[\frac{k}{k_0} + \frac{k_0}{k} - \sin^2 \theta_s \right], \quad (4-11)$$

where $k_0 = \frac{2\pi\nu_0}{c}$ is the wave number of the incident photon, $k = \frac{2\pi\nu}{c}$ is the wave number of the scattered photon, θ_s is the scattering angle, and $r_0 = e^2/m_0c^2$ is the classical electron radius. The values ν_0 and ν are the incident and scattered frequencies respectively. For scattering from electrons in a 20-keV plasma, however, one must take into account the motion of the electrons. The Klein-Nishina cross section is correct for electrons at rest, and in considering a group of monoenergetic and mono-directional electrons one can transform to the rest frame of the electrons where the Klein-Nishina cross section applies. By using this fact and some well-known relativistic invariants, we will derive expressions in the next few paragraphs for the total and differential cross sections in the laboratory frame for photons colliding with monoenergetic and mono-directional electron beams. Once these results are known one can numerically average them over the velocity distribution of the electrons in a plasma.

The total reaction rate is a relativistic invariant

$$\frac{d\sigma}{d^3k'} d^3k' F_\gamma d^3k f d^3p dt d\tau = \text{invariant}, \quad (4-12)$$

where

Fig. 8. Photon cross section for hydrogen.

$F_{\gamma} d^3k$ is the incident flux of photons in the momentum space element d^3k ,

$f d^3p$ is the density of electrons in the momentum space element d^3p ,

$\frac{d\sigma}{d^3k'} d^3k'$ is the differential cross section for scattering a photon from d^3k into momentum space element d^3k' ,

dt is an incremental time element, and

$d\tau$ is an incremental volume element.

The two frames of interest to us are the rest frame of the electron, in which we know the differential and total cross sections; and the lab frame, in which we wish to find the differential and total cross sections. The rest frame of the electrons defines proper time, so that $dt d\tau$ is an invariant. (The incremental element of time dilates in the same way that the incremental element of space contracts.) Thus, we may write

$$\frac{d\sigma}{d^3k'} F_{\gamma} f d^3k' d^3k d^3p = \text{invariant}. \quad (4-13)$$

The integration over variables d^3k and d^3p can be easily carried out in this case, since the momentum distribution of the electrons and the photons are delta functions of direction and energy. After integration over d^3k and d^3p , Eq. (4-13) is

$$\frac{d\sigma}{d^3k'} n^3 F n = \text{invariant}, \quad (4-14)$$

where

$$n = \int f d^3p$$

and

$$F = \int F_{\gamma} d^3k.$$

More specifically,

$$\left(\frac{d\sigma}{d^3k'} d^3k' F n \right)_{\text{lab}} = \left(\frac{d\sigma}{d^3k'} \right) (d^3k')^* F^* n^*, \quad (4-15)$$

where the starred quantities are in the rest frame of the electron. In particular $(d\sigma/d^3k')^*$ is the Klein-Nishina cross section, and n^* is the density of the electrons in their rest frame.

The transformation laws for the flux and density are well known²⁰

$$F^* = \gamma(1 - \beta \cos\theta)F$$

and

$$n^* = \gamma^{-1} n, \quad (4-16)$$

where

$$\gamma \text{ is } (1 - \beta^2)^{-1/2},$$

$$\beta \text{ is } v/c,$$

θ is the angle the electrons make with the incident photons in the laboratory frame, and

v is the velocity of the electrons in the laboratory frame.

The total cross section in the laboratory frame may now be obtained by integrating both sides of Eq. (4-15) over all possible final momenta (k') . The result is

$$\sigma_{\text{lab}} = \sigma_{\text{KN}} (1 - \beta \cos\theta). \quad (4-17)$$

The differential cross section is obtained by using the invariant²⁰ $\frac{n^3 d^3k'}{h\nu'}$ and the transformation law for frequencies

$$\nu^{*'} = \nu' (1 - \beta \cos\alpha)\gamma, \quad (4-18)$$

where α is the angle between the scattered photon and the incident electron. In terms of laboratory quantities the cross section we seek is therefore

$$\frac{d\sigma}{d^3k'} = \frac{(1 - \beta \cos\theta)}{(1 - \beta \cos\alpha)\gamma} \left(\frac{d\sigma}{d^3k'} \right)^*. \quad (4-19)$$

This expression can now be averaged over the appropriate electron momentum distribution to obtain the desired effective scattering cross section. In the case of a hot, fully ionized plasma at temperature T , a reasonable momentum distribution for the electrons is the relativistic Maxwell-Boltzmann distribution

$$f(p)d^3p = \frac{c^3 e^{-E/kT}}{4\pi(m_0c^2)^2 kTK_2(m_0c^2/kT)} d^3p, \quad (4-20)$$

where $K_2(m_0c^2/kT)$ is a modified Bessel function of the second kind of order two. Stone and Nelson²¹ have numerically integrated the differential and total cross section expressions just obtained over the relativistic Maxwell-Boltzmann distribution. Their results have been fitted by the expression²²

$$(\sigma_c)_{lab} = 665.2 \frac{[1 - (1.58 - 0.0012kT)hv]}{(410 - kT + 2hv)} \text{ mb.} \quad (4-21)$$

The units of kT and hv are keV.

It has also been shown that a Monte Carlo method attributed to Von Neumann reproduces the desired differential cross sections reasonably well.²³ Von Neumann's method consists of the following steps:

- Choose an electron at random from the relativistic Maxwell-Boltzmann distribution.
- Transform to the electron's rest frame.
- Scatter according to the Klein-Nishina cross section.
- Transform the scattered quantities back to the laboratory frame.

This method, which is reasonably fast, is well suited for a Monte Carlo calculation, and it does not require that large tables of numerically integrated cross sections be available as the calculation proceeds. In the HUCK FINN code we have used Von Neumann's method and Eq. (4-21) for scattering.

The full radiation transport equation (time independent) is

$$dI_v = -K_v \rho I_v ds - \rho \sigma_s I_v ds + \rho J_v ds + \int \sigma_s(v' \rightarrow v; \Omega' \rightarrow \Omega) d\Omega' dv' \rho I_{v'} ds, \quad (4-22)$$

where

$\rho K_V I_V ds$ is the amount absorbed in a distance ds ,
 $\rho \sigma_S I_V ds$ is the amount scattered out of the ray in a distance ds ,
 $\rho J_V ds$ is the amount emitted along the ray in the distance ds , and
 $\int \sigma_S(v' \rightarrow v; \Omega' \rightarrow \Omega) d\Omega' dv' \rho I_{V'}$ is the in-scattering term.

We have already covered the basic ideas of simulating this equation by the Monte Carlo method in Section 2. Particular emphasis was put on how one samples from the emission term

$$J_V = K_V B_V \propto e^{-hv/kT}. \quad (4-23)$$

One of the first tests of the HUCK FINN code was to neglect the scattering term in Eq. (4-22) and to compare the Monte Carlo solution to the analytic solution. Assuming LTE ($J_V = K_V B_V$) and no scattering term, the transport equation becomes

$$-\frac{dI_V}{\rho(K_V + \sigma_S)ds} = I_V - \frac{B_V}{(1 + \sigma_S/K_V)}. \quad (4-24)$$

The solution is

$$I_V(s) = I_V(0) e^{-\rho(K_V + \sigma_S)s} + \frac{B_V}{(1 + \sigma_S/K_V)} \{1 - e^{-\rho(K_V + \sigma_S)s}\}. \quad (4-25)$$

In the case of a sphere, the energy flux emitted from the surface of the sphere can be calculated by taking

$$|s| = 2R \cos \theta, \quad (4-26)$$

where R is the radius of the sphere, and θ is the angle between the normal to the sphere and the direction of the ray at the surface of the sphere. We assume that there is no external flux incident on the sphere [$I_V(0) = 0$]. Then

$$I_V(\cos \theta) = \frac{B_V}{(1 + \sigma_S/K_V)} \{1 - e^{-\rho(K_V + \sigma_S)2R \cos \theta}\}, \quad (4-27)$$

and the energy leaving an element of surface area $d\Sigma$ is

$$dE_V = I_V(\cos \theta) dv dw dt d\Sigma. \quad (4-28)$$

The total energy leaving the surface of the sphere is

$$E_v = \int_0^1 d\mu \, I_v \mu 2\pi \, dt \, dv \, 4\pi R^2, \quad (4-29)$$

where $\mu = \cos\theta$. Using Eq. (4-27) in (4-28), one obtains

$$E_v = \frac{B_v 8\pi^2 R^2}{(1 + \sigma_s/K_v)} \left\{ \frac{1}{2} + \frac{1}{A} e^{-A} - \frac{1}{A^2} (e^{-A} - 1) \right\}, \quad (4-30)$$

where A is $2\rho R(K_v + \sigma_s)$.

Figure 9 plots this equation and the Monte Carlo results for the following hydrogen parameters: $R = 50$ cm, $dt = 10^{-8}$ sec, $T = 1.0$ keV, $\rho = 1.0$ g/cm³, and $dv = 0.2$ keV. These results are quite encouraging, although they only test those areas of the code that are not affected by scattering. The really interesting effects, of course, involve scattering. In Figs. 10 and 11 we show a comparison of the escaping flux in two cases. In Fig. 10 a point black body source was placed at the center of a sphere and the sphere was assumed to be a cold electron gas. No absorption was allowed. Figure 11 presents the results of a calculation when a 20-keV point black body was assumed. For comparison, in Fig. 11 a 20-keV Planck spectrum is also plotted. These calculations do not correspond to any real physical situation, but they do make a convenient case to study.

The really interesting effects, of course, come from including the correct relativistic scattering. In Figs. 12, 13, and 14 we have plotted the escaping flux from a point black body at 10 keV through a 2 Thompson mean free path hydrogen sphere. For these calculations we have assumed that the hydrogen sphere does not emit and that scattering is done only by free electrons. Figure 12 shows the gradual attenuation of the black body spectrum as the gas heats up. Figure 13 shows the escaping flux when the hydrogen gas is at 10, 20, and 50 keV. As expected the leakage rate and shape of the escaping energy flux are strong functions of temperature when the gas temperature exceeds the source temperature. Figure 14 shows the fractional energy escaping from the sphere as a function of gas temperature. Numbers above 1.0

Fig. 9. Comparison of Monte Carlo and analytic solutions for LTE and no scattering.

Fig. 10. Flux spectrum at two mean-free paths from 8-keV point source.

Fig. 11. Flux spectrum at two mean-free paths from 20-keV point source.

Fig. 12. Escaping flux from point black body source through a two Thompson mean free path hydrogen sphere.

Fig. 13. Escaping flux from point black body source through a two Thompson mean free path hydrogen sphere.

Fig. 14. Transmission as a function of electron temperature.

indicate that energy is being transferred from the gas to the photon flux in the sphere. (That is how more than 100% of the energy in the source escapes.) These and other calculations using the HUCK FINN code agree with the studies of Kalos for electron gases at some temperature T .²⁴

All of these comparisons are for one dimensional calculations; HUCK FINN is a three dimensional code. To check for three dimensional effects the one dimensional calculations mentioned above were checked for isotropy. Since the other codes are all one dimensional, no convenient three dimensional checks exist.

The geometry input to HUCK FINN consists of conic sections and planes. It is a generalized version of a scheme long in use at Lawrence Livermore Laboratory and embodied in codes like TART.²⁵ The HUCK FINN geometry differs from the standard procedure in that planes assume the completely general form

$$A(X - X_0) + B(Y - Y_0) + C(Z - Z_0) = 0.$$

The conic sections are normally of the form

$$(X - X_0)^2 + (Y - Y_0)^2 + C(Z - Z_0)^2 = K$$

and may be rotated about an arbitrary axis through an arbitrary angle.

The input routines themselves were designed by the author to allow easy input from a teletype. There are no rigid formats. Instead the user types designators which indicate what the data following the designator are meant to be. In this way all redundancy usually found in formatted input is eliminated, and a problem can be composed as it is thought, rather than in a prescribed manner. The specific input of HUCK FINN has been described in detail elsewhere.²⁶ Detailed calculations of the CTR problem are presented in Section 6.

5. THE DEVELOPMENT OF THE POGO CODE

One of the prime goals in this research was the construction of a coupled neutron-photon Monte Carlo transport code. In a coupled code, neutrons would be followed in the normal Monte Carlo manner, but instead of assuming that all of the energy lost by the neutron was deposited locally as heat, allowance was made for the transport of some (or all) of this energy as photons. Once photons have been generated they too are followed in the normal Monte Carlo manner. There are, of course, many conceivable uses for a coupled neutron-photon code. In the case of a CTR, such a code is needed to calculate the expected photon flux into the plasma, arising from neutron reactions in the blanket.

The neutron Monte Carlo code TART²⁵ has been successfully used in many applications at Lawrence Livermore Laboratory. Therefore, a logical choice in constructing a coupled Monte Carlo neutron-photon transport code would be to start with the TART code and build on the needed subroutines, using whatever might be useful from the HUCK FINN code.

Figure 15 shows the general structure of the TART code. The first routine labeled POTL includes several subroutines, and essentially generates the neutron problem. In POTL the input deck is read in, and the data are put in a suitable form for the remainder of the code. The zoning of the problem is checked by the subroutine PPOTLV. The neutron cross sections to be used in the calculation are read in and processed in the routine GEN2. These routines form the first chain of the TART code.

The second chain is made up of the routines TRACKER, PCORD, UBANG, and DIGEST. PCORD is the source routine and generates the initial neutrons. TRACKER calculates the distance to collision, the distances to the zone boundaries, and the distance to census for each neutron. Then it advances the neutrons through the problem. As each neutron is advanced through the problem any exit quantities of interest are recorded for later use (for example, the amount of energy deposited by the neutrons as they travel or the amount of neutron flux entering a zone). When a neutron escapes, it either reaches the final census time,

Fig. 15. The TART code.

or it is captured and no further neutrons are generated. The history of that neutron is then considered terminated and TRACKER calls PCORD to get another neutron. After the prescribed number of source neutrons have been run, DIGEST is called and the problem is terminated. DIGEST edits the calculated quantities and prints them out in a useable form. UBANG is the collision routine in TART. Whenever a neutron collides, TRACKER calls UBANG, which determines what kind of collision occurs and calculates the new direction(s) and energy(s) of any neutrons that emerge from the collision event.

The TART code can be run in two ways. First, it can be run in a reaction mode, where the neutron is allowed to undergo only one event per collision. Second, it can be run in the emission mode, where individual events are lost in an averaging of all possible events at a given energy. The reaction mode is how one conceives events actually proceed. If a neutron causes a fission, one expects that the spectrum of neutrons emerging from the event is the result of that fission. In the emission mode all one knows is that the neutron collided, and that some neutrons have emerged from the event. The emerging neutrons are sampled from a conglomerate spectrum that is constructed by summing the weighted spectra arising from all possible types of reactions. The spectrum for each specific reaction is weighted by the reaction cross section. Both methods yield good results for neutron transport calculations. Either method could be used as the first step in a coupled calculation.

Similarly there are two ways in which photons arising from neutron reactions could be generated by a code. For each neutron collision, photons could be generated by sampling from an average overall photon emission spectrum weighted by the photon production cross section. This would be an emission mode. Alternatively, photons could be generated by particular neutron reactions. This would be a reaction mode. The emission mode would yield a better statistical result for photon emissions for a given number of neutrons tracked, but one would need to generate tables of photon production cross sections and emission spectra. In the emission mode, the cross section set for photon production can be independent of the neutron cross section set, which may be an advantage in some cases. In the reaction mode, the photon production is tied to the neutron cross section set, and spectra for each photon producing event are needed.

Once photons have been generated (that is, their initial position, direction, and energy determined) they can be treated in the regular Monte Carlo manner. In the energy range characteristic of photons generated by inelastic neutron reactions, the important photon reactions are Compton scattering and pair production. The Compton scattering need not be temperature dependent in the CTR problem, since the blanket is not going to be at a very high temperature compared to the energy of the photons. In our blanket calculations we have assumed that the Klein-Nishina cross section is adequate. Pair production is handled by using the data of Briggs and Lighthill²⁷ and assuming that when a pair production event occurs the positron and electron deposit their energy locally. The annihilation radiation produced by the positron is generated at the point of the pair production event. The two 511-keV photons produced are assumed to be isotropic in the laboratory frame. This method has been used successfully in other Monte Carlo codes.²⁸

The POGO code is the TART code modified to produce and follow photons in the manner outlined above. The neutronics of the original TART code are unchanged. Figure 16 shows the general structure of the POGO code. Chain 1 is identical to the TART code. In chain 2 every routine but PCORD (the neutron source routine) was modified. A new routine, POT3, was added. POT3 follows the photons, using the expected value forced collision techniques used in HUCK FINN. UBANG (the neutron collision routine) was modified to return the energy available for photons. TRACKER was modified to determine if one or more photons were produced by a neutron reaction and then to call POT3 to follow them. DIGEST (the edit routine) was modified to output the additional photon data. The POGO code uses the reaction mode for both neutrons and photons. Thus, we do not need to calculate and store photon production data, thereby keeping the total program small enough so that it can all be stored in the computer.

One of the most useful aids to an understanding of both the transport process and the reactions that occur in a particular material is an integral measurement. The pulsed sphere work of recent years²⁹⁻³² has enabled the TART code to be checked in a number of interesting cases. The experiments are relatively easy to perform, and the geometries can be made simple enough so that the experiment can be easily modeled. During

Fig. 16. The POGO code.

some recent neutron experiments photon data were also taken.³³ The first real tests for the POGO code were the calculation of some of these experiments.

The energetics of the reaction of a 14-MeV neutron in carbon are shown in Fig. 17. The only photon observed arises from the decay of the 4.43-MeV level shown. Based on this fact, the POGO code was used to calculate the photon flux on the surface of a 20.98-cm carbon sphere due to a deuterium-tritium neutron source at the center of the sphere. Measurements of the total photon energy flux on the surface of the sphere were made using thermoluminescent dosimeters (TLD's).³⁴ The calculated and experimental fluxes agree within the experimental and statistical uncertainties. POGO calculated a photon flux of 0.728 γ -MeV/source neutron. The TLD measurements indicated a value of 0.87 ± 0.17 γ -MeV/source neutron.

As mentioned above, two materials of interest in the CTR problem are niobium and lithium. Measurements by the group at LLL,³³ and others,³⁵ do not indicate any photons produced by 14-MeV neutron reactions in lithium. Such reactions in niobium appear to produce a significant photon spectrum. However, at neutron energies above 1 MeV there do not appear to be any strong lines as is the case with carbon. For this reason a scheme like the one suggested by Howerton and Plechaty^{36,37} for continuum photons from a number of neutron reactions appears to be reasonable.

The Howerton and Plechaty formalism consists of the following postulates:

- The emerging photon spectrum from neutron events is of the form

$$N(E) dE \propto E e^{-RE} dE, \quad (5-1)$$

- The constant R is a linear function of neutron energy in the range from 4 to 14 MeV.
- The average total photon energy is

$$\overline{E}_{\gamma} = \frac{(E_N - 1)\sigma_{nn'} + (E_n - T_{n,2n} - 1)\sigma_{n,2n} + \dots}{\sigma_{nn'} + \sigma_{n,2n} + \dots}, \quad (5-2)$$

Fig. 17. Energetics of carbon-12 plus a neutron.

where

$\overline{E}_\gamma$ is the average energy available for photon generation,

E_n is the energy of the incident neutron,

$\sigma_{n,n'}$ is the inelastic neutron cross section,

$\sigma_{n,2n}$ is the $(n,2n)$ cross section, and

$T_{n,2n}$ is the threshold for the $(n,2n)$ reaction.

By normalizing Eq. (5-1) and finding the average of the normalized expression, the average number of photons generated per photon producing event can be determined. The average photon energy from the distribution (5-1) is

$$\langle E \rangle = \frac{\int_0^u N(E) E dE}{\int_0^u N(E) dE} = \frac{2}{R} - \frac{Ru^2 e^{-Ru}}{1 - (1 - Ru)e^{-Ru}} \quad (5-3)$$

where u is the maximum photon energy permissible. Therefore the multiplicity (that is, the average number of photons produced per neutron event) is

$$M = \overline{E}_\gamma / \langle E \rangle. \quad (5-4)$$

Measurements^{38,39} of the photon spectra from neutrons for several neutron energies are available; they appear to verify the Howerton-Plechaty formalism.⁴⁰

To confirm the Howerton-Plechaty formalism at 14 MeV, a niobium ring was exposed to 14-MeV neutrons at Livermore's ICT. A schematic drawing of the experimental setup is shown in Fig. 18. The actual experiment was carried out by John Kammerdiener during his thesis research.⁴¹ His primary concern was with the neutron scattering, but (with very little additional effort) he was able to collect the photon data for these measurements. Measurements were taken at several angles from 25° to 145°. The results consist of a pulse height spectrum (counts versus channel number) for each angle. Pulse height is a measure of

Fig. 18. Setup for niobium ring experiment.

the energy of the photon incident on the detector. Each channel number represents a photon energy group. When a photon enters the detector and reacts, the resulting electron causes fluorescence in the detector, and that light is detected by a photomultiplier tube. The amount of light generated is proportional to the energy of the incident photon. Thus the size of the pulse generated by the photomultiplier tube and the following electronics determines the appropriate energy group, or channel number, for the incident photon. Each time a photon is detected in this manner, one count is added to the number of counts already assigned to that channel number.

The detector used in these measurements was designed to detect neutrons and was very inefficient for photon detection. In the energy range of interest (0.5 to 5 MeV), photons react with the low atomic number atoms of the detector almost exclusively by Compton scattering. Thus incident photons of the same energy could and did result in counts in different channels, due to the variation in recoil electron kinetic energy with the photon scattering angle. The result of this is that the detector has a response function, or fractional distribution of counts recorded as a function of channel number, for an incident photon of any given energy. Although the response function of the detector used in these measurements is unknown for any photon energy, empirical observations showed that a monoenergetic incident photon flux results in a plateau in the pulse height spectrum.

The raw data for several angles, corrected for background and solid angle factors, are shown in Figs. 19 and 20. The total counts in all channels as a function of angle are shown in Fig. 21. The solid line represents the expression $1.8 (1.0 + \cos^4 \theta)$.

Since there are no plateaus in Figs. 19 and 20, there are no strong lines in the niobium spectrum at 14 MeV. From Fig. 21 one can conclude that the photon spectrum is symmetric as a function of the cosine of the scattering angle in the laboratory frame. Therefore, in the case of niobium, the continuum assumption of Howerton and Plechaty appears to be reasonably accurate. For carbon, the simple model of a single 4.43-MeV photon appears adequate. Thus, there is sufficient information available to perform the intended calculation on a CTR blanket to determine the escaping photon flux from the blanket. The next section outlines the results of our calculations on a CTR using the POGO and HUCK FINN codes.

Fig. 19. Counts vs channel number as a function of angle in niobium ring experiment. Data have been corrected for background and solid angle.

Fig. 20. Counts vs channel number as a function of angle in niobium ring experiment. Data have been corrected for background and solid angle.

Fig. 21. Total counts vs angle for niobium ring experiment. Data have been corrected for background and solid angle.

6. RESULTS OF CTR BLANKET CALCULATIONS WITH POGO AND HUCK FINN

The HUCK FINN and POGO codes have been developed to answer two questions about CTR's. First, what is the photon flux generated in, and escaping from, the CTR's blanket? Second, what is the effect of a photon flux on the plasma in the CTR? In this section we will present the results of our CTR calculations, dealing first with the effects of a photon flux on the plasma in the CTR.

The most serious effect that a photon flux could have on a CTR (cooling of the plasma) arises from the inverse Compton effect. By inverse Compton scattering we are referring to the process whereby a photon gains energy in scattering off of a moving electron. If there were no inverse Compton events, any radiation generated outside of the plasma could only heat the plasma. Radiation losses from the plasma itself would be limited to the fraction of the total plasma energy in the radiation field. Inverse Compton scattering provides a coupling mechanism between the electrons in the plasma and the radiation field by which energy can flow from the electrons to the radiation. The inverse Compton process makes it possible for an externally generated radiation field to cool the plasma, and for radiation generated by the plasma to rob energy from the electrons in the plasma before escaping, thereby affecting both the radiation energy and the kinetic energy of the electrons in the plasma. Here we will examine each of these effects and predict the importance of each in a practical CTR device.

In these calculations, carried out using HUCK FINN, we will consider a spherically symmetric, homogeneous plasma of a given density, radius, composition, and temperature. Our main concern is with the plasma itself. The actual operating conditions of a working CTR are yet to be determined; however, a reasonable guess can be made. If one assumes that the most likely fuel is a deuterium-tritium mixture, then reasonable temperatures are in the range of 20 keV or more.⁴² At densities of 10^{15} particles/cm³, a radius of 500 cm is also reasonable. For other densities other radii are appropriate. In our calculations we have kept the radius constant and varied the density. However, our

results are applicable to other radii, since a simple radius-times-density scaling law holds. For transport calculations such as this one, the important parameter is a typical distance in the plasma (such as the radius) in terms of the mean free path ($\frac{R}{\lambda}$). The mean free path is inversely proportional to the density ($\lambda = \frac{1}{n\sigma}$), so the calculations scale as the radius times the density ($\frac{R}{\lambda} \propto nR$). For radiation losses from the plasma itself, the total radiation generated in the plasma is proportional to the density times the volume ($\frac{4}{3} \pi R^3 n$). The surface over which this energy leaks out is $4\pi R^2$, so that again a radius-times-density scaling applies to the radiation energy generation-to-leakage ratio. Our calculations assume that the radius is fixed and the density changes. In this way the effect of inverse Compton scattering is emphasized as the density increases.

First let us consider the radiation generated by the plasma itself. Some authors have considered the loss of radiation due to Bremsstrahlung and synchrotron radiation in the plasma.^{2,43} We have instead chosen to represent the radiation generated in the plasma by using the assumption of local thermodynamic equilibrium as explained in Section 4. These two approaches should give very similar results since, in the case of a hydrogen plasma at 20 keV, the absorption coefficient is determined primarily by free-free absorption (i.e., Bremsstrahlung).

We have calculated the problem of the radiation generated in the plasma twice. In both calculations the plasma parameters are the same; however, in one, only Klein-Nishina scattering from electrons at rest is allowed, while in the other the electrons had a RMB energy spectrum, so that inverse Compton scattering of the photons was possible. The total scattering cross section used in both of these problems is the correct temperature-dependent one. The results are summarized in Table 5.

Several points should be made:

- Klein-Nishina scattering from electrons at rest predicts a net energy deposit in the plasma, whereas allowing the photons to scatter from moving electrons results in energy being robbed from the plasma by the radiation field.

- In the Klein-Nishina scattering case, all of the energy initially in the radiation field, except at the highest densities, escapes from the plasma.
- Only at very high densities (10^{21} or greater) will the inverse Compton effect be noticeable. Then, however, it makes a very large difference.

Table 5. The effect of inverse Compton processes on photons generated in a 20-keV plasma.

Density, n (particles/cm ³)	Klein-Nishina scattering		Relativistic scattering	
	Energy deposited per unit source energy,	Energy escaping per unit source energy, ^a	Energy deposited per unit source energy,	Energy escaping per unit source energy, ^b
	$\frac{E_{\text{dep}}}{E_0}$	$\frac{E_{\text{leaked}}}{E_0}$	$\frac{E_{\text{dep}}}{E_0}$	$\frac{E_{\text{leaked}}}{E_0}$
10^{13}	9.96×10^{-11}	1.0	-3.68×10^{-10}	1.0
10^{14}	9.96×10^{-10}	1.0	-3.68×10^{-9}	1.0
10^{15}	9.70×10^{-9}	1.0	-3.44×10^{-8}	1.0
10^{16}	9.78×10^{-8}	1.0	-3.45×10^{-7}	1.0
10^{17}	9.99×10^{-7}	1.0	-3.84×10^{-6}	1.0
10^{18}	1.01×10^{-5}	1.0	-3.34×10^{-5}	1.0
10^{19}	9.94×10^{-5}	1.0	-3.37×10^{-4}	1.0
10^{20}	1.02×10^{-3}	0.999	-3.83×10^{-3}	1.004
10^{21}	1.02×10^{-2}	0.999	-4.26×10^{-2}	1.04
10^{22}	1.00×10^{-1}	0.899	-0.604	1.604
10^{23}	0.476	0.523	-5.59	6.59

^a 1.0 is listed when $E_{\text{leaked}} = 0.999$ or greater.

^b 1.0 is listed when $E_{\text{leaked}} = 1.001$ or less.

This third point should not be construed to imply that radiation losses themselves are not important. In Table 6, we compare the Lawson times derived in Section 3 for various plasma densities to the times required for radiation losses to equal the initial kinetic energy of the plasma ions. The radiation loss time is calculated assuming that the initial steady state loss rate is maintained as the plasma cools. If the time required to radiate all the energy in the plasma in this crude rate calculation were comparable to the Lawson time, then time-dependent calculations would be justified. Since this is not the case, our steady state calculation is a reasonable estimate of the initial radiation losses due to naturally generated radiation in the plasma. It should be emphasized that this is only an initial rate; as reactions in the plasma progress, a helium "ash" will build up. One effect of the presence of helium or any other atom of higher atomic number is to

Table 6. Radiation loss times (time required for radiation losses to equal initial particle energy) for a 20-keV, 500-cm-radius plasma.

Density (particles/cm ³)	Lawson time ^a (sec)	Relativistic Compton loss time (sec)	Klein-Nishina Compton loss time (sec)
10 ¹³	1.56	2.12×10^2	2.12×10^2
10 ¹⁴	1.56×10^{-1}	2.12×10^1	2.12×10^1
10 ¹⁵	1.56×10^{-2}	2.12	2.12
10 ¹⁶	1.56×10^{-3}	2.12×10^{-1}	2.12×10^{-1}
10 ¹⁷	1.56×10^{-4}	2.12×10^{-2}	2.12×10^{-2}
10 ¹⁸	1.56×10^{-5}	2.12×10^{-3}	2.12×10^{-3}
10 ¹⁹	1.56×10^{-6}	2.12×10^{-4}	2.12×10^{-4}
10 ²⁰	1.56×10^{-7}	2.12×10^{-5}	2.12×10^{-5}
10 ²¹	1.56×10^{-8}	2.11×10^{-6}	2.12×10^{-6}
10 ²²	1.56×10^{-9}	1.37×10^{-7}	2.44×10^{-7}
10 ²³	1.56×10^{-10}	3.3×10^{-9}	4.2×10^{-8}

^aThese values were derived from a value for $\langle\sigma v\rangle$ given on page 315 of Ref. 2.

increase the Bremsstrahlung radiation from the plasma. (The free-free absorption coefficient is proportional to Z^2 .)⁴⁴ Nevertheless, it is encouraging to note that the radiation losses are not so severe that they would prohibit the functioning of a CTR. Had the cycle time for radiation losses calculated in Table 6 been less than the Lawson time, one would have serious reservations about the economic operation of a steady state CTR. (Even in the case of the cycle time being shorter than the Lawson time, CTR's in pulsed operation might be possible. In a pulsed mode, fusion reactions take place in so short a time that radiation losses are not significant.)

The second area in which inverse Compton processes play a role is in the energy deposited in the plasma by an externally generated photon flux. In a CTR such a flux might be generated by inelastic neutron collisions and other nuclear events in the structure surrounding the plasma, or by radiation generated in the plasma which leaks out and is then reflected back into it. It is uncertain what the incoming flux due to these sources might be in a given situation. However, some general conclusions can be drawn by considering several monoenergetic sources. Again we have calculated the effect of an external radiation source using both the Klein-Nishina scattering law and the more correct relativistic scattering law. Figure 22 and Table 7 show the results of the calculation assuming the Klein-Nishina scattering law. In this case there is always

Table 7. Energy deposited in the plasma per unit source energy — Klein-Nishina scattering (plasma temperature = 20 keV).

Density	E = 0.1	E = 0.3	E = 0.5	E = 1.0	E = 3.0	E = 5.0	E = 10.0	E = 30.0	E = 50.0	E = 100.0
10^{13}	6.6×10^{-13}	2.0×10^{-12}	3.3×10^{-12}	6.6×10^{-12}	1.94×10^{-11}	3.2×10^{-11}	6.1×10^{-11}	1.6×10^{-10}	2.26×10^{-10}	3.33×10^{-10}
10^{14}	6.6×10^{-12}	2.0×10^{-11}	3.3×10^{-11}	6.6×10^{-11}	1.94×10^{-10}	3.2×10^{-10}	6.1×10^{-10}	1.6×10^{-9}	2.26×10^{-9}	3.33×10^{-9}
10^{15}	6.3×10^{-11}	1.9×10^{-10}	3.2×10^{-10}	6.3×10^{-10}	1.84×10^{-9}	3.0×10^{-9}	5.8×10^{-9}	1.5×10^{-8}	2.21×10^{-8}	3.29×10^{-8}
10^{16}	6.4×10^{-10}	1.9×10^{-9}	3.2×10^{-9}	6.3×10^{-9}	1.89×10^{-8}	3.1×10^{-8}	6.0×10^{-8}	1.5×10^{-7}	2.27×10^{-7}	3.43×10^{-7}
10^{17}	6.5×10^{-8}	1.9×10^{-8}	3.2×10^{-8}	6.4×10^{-8}	1.88×10^{-7}	3.1×10^{-7}	5.9×10^{-7}	1.5×10^{-6}	2.21×10^{-6}	3.32×10^{-6}
10^{18}	7.2×10^{-8}	2.0×10^{-7}	3.4×10^{-7}	6.7×10^{-7}	1.98×10^{-6}	3.2×10^{-6}	6.2×10^{-6}	1.6×10^{-5}	2.28×10^{-5}	3.4×10^{-5}
10^{19}	1.07×10^{-6}	2.0×10^{-6}	3.2×10^{-6}	6.3×10^{-6}	1.87×10^{-5}	3.1×10^{-5}	5.9×10^{-5}	1.5×10^{-4}	2.25×10^{-4}	3.27×10^{-4}
10^{20}	4.9×10^{-5}	2.5×10^{-5}	3.6×10^{-5}	6.8×10^{-5}	1.94×10^{-4}	3.2×10^{-4}	6.1×10^{-4}	1.6×10^{-3}	2.26×10^{-3}	3.41×10^{-3}
10^{21}	4.32×10^{-3}	6.7×10^{-4}	4.9×10^{-4}	7.1×10^{-4}	1.97×10^{-3}	3.0×10^{-3}	6.1×10^{-3}	1.5×10^{-2}	2.23×10^{-2}	3.20×10^{-2}
10^{22}	0.278	4.5×10^{-2}	1.9×10^{-2}	1.0×10^{-2}	1.9×10^{-2}	2.85×10^{-2}	5.2×10^{-2}	0.124	0.17	0.238
10^{23}	0.95	0.484	0.32	0.14	7.98×10^{-2}	9.6×10^{-2}	0.15	0.283	0.36	0.477

Fig. 22. Energy vs energy deposit per unit source energy for Klein-Nishina scattering for an external monoenergetic source on a 20-keV, 500-cm-radius plasma.

a net energy deposit in the plasma due to the external radiation. Figure 23 and Table 8 show the results of a similar calculation when the correct relativistic scattering law is used. In this case, for photon energies below 100 keV, there is a net energy loss in the plasma. Thus, for an external flux, there are real differences even at the lowest densities between the case when the correct relativistic scattering law is considered and the case when the Klein-Nishina law is used. The question then becomes how large an external flux is required to seriously inhibit the CTR's operation. An answer to this question can be obtained by assuming that, since the functional dependence of the energy loss on photon energy is relatively flat over a large energy range, a critical flux can be defined as that flux which would rob an amount of energy equal to the initial kinetic energy of the plasma ions in one Lawson time. Thus we define the critical flux, ϕ_c , so that

$$4\pi r^2 \phi_c \left(\frac{E_{\text{dep}}}{E} \right) t_{\text{Lawson}} = \frac{4}{3} \pi r^3 \frac{3}{2} nkT,$$

where

r is the radius of the plasma,

k is Boltzmann's constant,

T is the temperature of the plasma,

t_{Lawson} is the Lawson time, and

$\frac{E_{\text{dep}}}{E}$ is the average energy robbed from the plasma per unit source energy.

Table 8. Energy deposited in the plasma per unit source energy--relativistic scattering.

Density	E = 0.1	E = 0.3	E = 0.5	E = 1.0	E = 3.0	E = 5.0	E = 10.0	E = 30.0	E = 50.0	E = 100.0
10^{13}	-4.71×10^{-10}	-4.73×10^{-10}	-4.7×10^{-10}	-4.64×10^{-10}	$(-5. \times 10^{-10})$	-4.32×10^{-10}	-3.69×10^{-10}	-2.35×10^{-10}	-9.57×10^{-11}	1.02×10^{-10}
10^{14}	-4.71×10^{-9}	-4.73×10^{-9}	-4.7×10^{-9}	-4.64×10^{-9}	$(-5. \times 10^{-9})$	-4.32×10^{-9}	-3.69×10^{-9}	-2.35×10^{-9}	-9.57×10^{-10}	1.02×10^{-9}
10^{15}	-4.68×10^{-8}	-4.90×10^{-8}	-4.91×10^{-8}	-4.88×10^{-8}	-5.0×10^{-8}	-4.36×10^{-8}	-3.95×10^{-8}	-2.36×10^{-8}	-5.20×10^{-9}	1.27×10^{-8}
10^{16}	-4.92×10^{-7}	-4.96×10^{-7}	-4.89×10^{-7}	-4.83×10^{-7}	-5.0×10^{-7}	-4.6×10^{-7}	-3.75×10^{-7}	-1.95×10^{-7}	-1.04×10^{-7}	1.02×10^{-7}
10^{17}	-4.30×10^{-6}	-4.88×10^{-6}	-4.86×10^{-6}	-4.61×10^{-6}	-5.1×10^{-6}	-4.2×10^{-6}	-3.48×10^{-6}	-2.13×10^{-6}	-1.10×10^{-6}	9.95×10^{-7}
10^{18}	-5.27×10^{-5}	-5.27×10^{-5}	-5.26×10^{-5}	-5.19×10^{-5}	-5.1×10^{-5}	-4.45×10^{-5}	-3.90×10^{-5}	-2.41×10^{-5}	-7.09×10^{-6}	1.33×10^{-5}
10^{19}	-5.51×10^{-4}	-5.37×10^{-4}	-5.36×10^{-4}	-5.01×10^{-4}	-5.4×10^{-4}	$-4.4. \times 10^{-4}$	-4.11×10^{-4}	-2.21×10^{-4}	(-7.7×10^{-5})	1.29×10^{-4}
10^{20}	-5.38×10^{-3}	-5.28×10^{-3}	-5.26×10^{-3}	-5.60×10^{-3}	-5.2×10^{-3}	-4.51×10^{-3}	-3.82×10^{-3}	-2.69×10^{-3}	-8.672×10^{-4}	9.68×10^{-4}
10^{21}	-5.23×10^{-2}	-5.49×10^{-2}	-5.59×10^{-2}	-5.91×10^{-2}	-5.2×10^{-2}	-5.17×10^{-2}	-4.61×10^{-2}	-2.42×10^{-2}	-1.08×10^{-2}	8.16×10^{-3}
10^{22}	-5.59×10^{-1}	-5.9×10^{-1}	-9.5×10^{-1}	-8.19×10^{-1}	-7.23×10^{-1}	-5.673×10^{-1}	-4.31×10^{-1}	-0.187	-8.89×10^{-2}	7.91×10^{-2}
10^{23}	-0.16	-6.183	-8.511	-7.239	-3.44	-2.406	-1.377	-0.425	-1.30×10^{-1}	1.81×10^{-1}

Fig. 23. Energy vs energy deposit per unit source energy for relativistic scattering law for an external monoenergetic source on a 20-keV, 500-cm-radius plasma.

The critical flux is therefore

$$\phi_c = \frac{1}{2} \frac{r nkT}{\left(\frac{E_{\text{dep}}}{E}\right) t_{\text{Lawson}}}.$$

We have assumed that the average value of (E_{dep}/E) is given by the value shown in Table 5 and have calculated the corresponding critical fluxes. The results are shown in Table 9. For comparison, the radiation fluxes calculated assuming LTE in the plasma are also given. Even assuming an albedo of one for the shield surrounding the plasma, losses of this kind do not become serious until the density reaches 10^{23} (at a radius of 500 cm). Although the inverse Compton effect does cool the plasma and its effect can be calculated, it should not make any significant difference in the operating conditions of a CTR.

Table 9. Critical flux for typical CTR densities for a 20-keV, 500-cm-radius plasma.

Density	E_{dep}/E	t_{Lawson} (sec)	ϕ_c (keV/cm ² -sec)	ϕ_{nat}	Photon flux due to neutron reactions in blanket
10^{13}	3.58×10^{-10}	1.56	8.71×10^{25}	2.276×10^{14}	1.46×10^{14}
10^{14}	3.68×10^{-9}	1.56×10^{-1}	8.71×10^{26}	2.276×10^{16}	1.46×10^{16}
10^{15}	3.44×10^{-8}	1.56×10^{-2}	9.32×10^{27}	2.276×10^{18}	1.46×10^{18}
10^{16}	3.45×10^{-7}	1.56×10^{-3}	9.29×10^{28}	2.276×10^{20}	1.46×10^{20}
10^{17}	3.84×10^{-6}	1.56×10^{-4}	8.35×10^{29}	2.276×10^{22}	1.46×10^{22}
10^{18}	3.34×10^{-5}	1.56×10^{-5}	9.6×10^{30}	2.276×10^{24}	1.46×10^{24}
10^{19}	3.37×10^{-4}	1.56×10^{-6}	9.51×10^{31}	2.276×10^{26}	1.46×10^{26}
10^{20}	3.83×10^{-3}	1.56×10^{-7}	8.37×10^{32}	2.276×10^{28}	1.46×10^{28}
10^{21}	4.26×10^{-2}	1.56×10^{-8}	7.52×10^{33}	2.276×10^{30}	1.46×10^{30}
10^{22}	6.04×10^{-1}	1.56×10^{-9}	5.31×10^{34}	2.276×10^{32}	1.46×10^{32}
10^{23}	5.59	1.56×10^{-10}	5.73×10^{35}	2.276×10^{34}	1.46×10^{34}

The POGO code was constructed to determine the photon flux generated in the blanket by neutrons escaping from the plasma. For these calculations we again assumed a spherically symmetric case. The actual dimensions were taken from the proposed design of Fraas *et al.*¹³ of Oak Ridge. In their model the blanket is made of three materials—lithium, niobium, and carbon. The configuration is shown in Fig. 3. Using the gamma production models presented in Section 5, we have calculated the expected photon flux from the CTR's blanket. The spectrum of photons leaving the surface of the blanket is shown in Fig. 24. Figure 25 shows the photon flux generated in the blanket and returning to the plasma region of the CTR. In both cases the flux is in units of keV/cc/source neutron. Assuming the reaction rates given by Rose and Clark,² a CTR at a density of 10^{16} will produce 9.17×10^{20} n/sec. This means that the total photon flux below 100 keV returning from the shield onto the plasma is 1.47×10^{18} keV/cm²-sec which is small compared to the critical flux. The values of the photon flux below 100 keV calculated by the POGO code are shown in Table 9. They are comparable to the photon flux generated by the plasma itself for photon energies below 100 keV. The photon flux emerging from the blanket (Fig. 24) clearly shows the effect of the graphite moderator. The increase in flux in the energy group from 4 to 4.5 MeV shows the presence of the 4.43-MeV photon from carbon. At least half of the energy in that group is due to the carbon. This photon is not seen in the photon flux incident on the plasma.

Fig. 24. Escaping photon flux from CTR blanket.

Fig. 25. Photon flux generated in blanket and incident on CTR plasma.

7. CONCLUSIONS AND SUMMARY

The purpose of this investigation was to determine the effect of a photon flux on the reacting plasma in a CTR, and to quantify the conversion of a neutron flux into a photon flux in the blanket of the CTR. These effects were studied by developing two appropriate Monte Carlo transport codes. The effect of a photon flux on the plasma was investigated by the development of the temperature dependent, time-independent Monte Carlo photon transport code HUCK FINN. Calculations with this code on a series of homogeneous spherical plasmas of various densities showed that the inclusion of temperature dependent photon scattering (rather than scattering from electrons at rest) was important from theoretical point of view, but was probably of no practical importance. Only at density-times-radius values of 10^{25} ions/cm² were differences between the two scattering models significant.

The conversion of a neutron flux into a photon flux in the blanket of the CTR was investigated by the development of the coupled neutron-photon Monte Carlo transport code POGO. Calculations with this code have been done on the CTR proposed by Fraas and his co-workers at Oak Ridge. These calculations predict tolerable levels of radiation escaping from the blanket. By tolerable we mean the expected photon flux from the outer surface of the blanket is not large compared to those encountered in fission reactor shielding problems, and that the photon flux which is generated in the blanket and returned toward the plasma should not adversely affect the operation of the reactor.

In the process of developing useful models for photon production reactions, the Howerton-Plechaty formalism was experimentally verified for niobium, and a single level model for carbon. Lithium apparently produces no significant photon radiation.

Much work remains to be done in the area of photon production cross sections from inelastic neutron collisions, and integral measurements with existing apparatus seems a feasible method of obtaining some of this information. I feel that no further work is required in the specific area of CTR radiation loss problems until some radically different CTR designs are being considered.

The codes developed in the course of this research have a much broader area of applicability than solely the CTR problem. Some of the immediate possibilities are the calculation of shielding geometries for radiation therapy situations, and the calculation of experimental integral measurements as bench mark problems for cross section sets.

The inclusion of charged particle transport in coupled calculations may be of interest in both the radiation therapy calculations and in detector response function analysis.

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9. REFERENCES

1. R.G. Mills, Fusion Power (Princeton University Summer Institute in Plasma Physics, Princeton, N.J., 1964).
2. D.J. Rose and M.L. Clark, Plasmas and Controlled Fusion (The M.I.T. Press, Cambridge, 1961).
3. R.F. Post, Ann. Rev. Nucl. Sci. 20, 509 (1970).
4. P. Robinson, Jr. and S.D. Bloom, Preliminary Neutron Shielding and Collimation Calculations for Fast Neutron Radiation Therapy, to be published.
5. E.A. Straker, P.N. Stevens, D.C. Irving, and V.R. Cain, The Morse Code—A Multigroup Neutron and Gamma-Ray Monte Carlo Transport Code, Oak Ridge National Laboratory, Tenn., Rept. ORNL-4585 (1970).
6. N. Metropolis and S. Ulam, J. Amer. Stat. Assoc. 44, 335 (1949). Also see A.S. Householder, Principles of Numerical Analysis (McGraw-Hill, New York, 1953).
7. H. Kahn, Applications of Monte Carlo, Rand Corporation, Rept. AECU-3259 (1956).
8. R. W. Werner, Mirror Machines and Direct Conversion, Lawrence Livermore Laboratory, Rept. UCRL-72487 (1970).
9. R. F. Post, Mirror System: Fuel Cycles, Loss Reduction and Energy Recovery, Lawrence Livermore Laboratory, Rept. UCRL-71753 (1969).
10. D.J. Rose, On the Feasibility of Power by Nuclear Fusion, Oak Ridge National Laboratory, Rept. ORNL-TM-2204 (1968).
11. J.D. Lee, R.W. Werner, B. Myers, P.B. Mohr, G.A. Carlson, and N.C. Christofilos, Some Engineering Aspects of an Astron Fusion Power Reactor System, Lawrence Livermore Laboratory, Rept. UCRL-72336 (1970).
12. G.I. Bell, Neutron Blanket Calculations for Thermonuclear Reactors II, Los Alamos Scientific Laboratory, Rept. LA-4131-MS (1969).
13. A.P. Fraas and H. Postma, Preliminary Appraisal of the Hazards Problems of a D-T Fusion Reactor Power Plant, Oak Ridge National Laboratory, Rept. ORNL-TM-2822 (1970).

14. R.W. Werner, B. Myers, P.B. Mohr, J.D. Lee, and N. C. Christofilos, Preliminary Design Considerations for an Astron Power Reactor System, Lawrence Livermore Laboratory, Rept. UCRL-71768 (1969).
15. G.I. Bell, Neutron Blanket Calculations for Thermonuclear Reactors, Los Alamos Scientific Laboratory, Rept. LASL-LA-3383-MS (1965).
16. J. O. Cowles and A. D. Pasternac, Lithium Properties Related to Use as a Nuclear Reactor Coolant, Lawrence Livermore Laboratory, Rept. UCRL-50647 (1969).
17. L. I. Schiff, Quantum Mechanics (McGraw-Hill, New York, 1955).
18. P. A. M. Dirac, The Principles of Quantum Mechanics (Oxford University Press, Oxford, 1958).
19. C. W. Allen, Astrophysical Quantities (The Athlone Press, London, 1955).
20. L. D. Landau and E. M. Lifshitz, The Classical Theory of Fields (Addison-Wesley Publishing Co., Reading, Mass., 1962).
21. S. Stone and R. G. Nelson, Compton Scattering from Relativistic Electrons, Lawrence Livermore Laboratory, Rept. UCRL-14918-T (1966).
22. A. G. Cole, Lawrence Livermore Laboratory, private communication (1971).
23. A. M. Winslow, Lawrence Livermore Laboratory, private communication (1967).
24. S. Hui, M. H. Kalos, H. Steinberg, and E. Troubetskay, Penetration of Photons Through a Hot Electron Gas, Defense Atomic Support Agency, Rept. DASA 2556 (1970).
25. E. F. Plechaty and J. R. Kimlinger, TART Monte Carlo Neutron-Transport Code, Lawrence Livermore Laboratory, Rept. UCRL-522 (1971).
26. P. Robinson and R. E. Dye, Lawrence Livermore Laboratory, private communication (April, 1970).
27. F. Briggs and R. Lighthill, Analytical Approximations for Total Pair-Production Cross Sections, Sandia Laboratories, Albuquerque, N. Mex., Rept. SC-RR-68-619 (1968).

28. J. Kimlinger and E. F. Plechaty, SORS Monte Carlo Neutron Transport Code for the CDC-6600, Lawrence Livermore Laboratory, Rept. UCRL-50532 (1968).
29. L. F. Hansen, J. D. Anderson, E. Goldberg, E. F. Plechaty, M. L. Stelts, and C. Wong, Time Spectra from Spheres Pulsed with 14-MeV Neutrons and Their Analysis, Lawrence Livermore Laboratory, Rept. UCRL-71124 (1968).
30. L. F. Hansen, J. D. Anderson, E. Goldberg, J. L. Kammerdiener, E. Plechaty, and C. Wong, Predictions for Neutron Transport in Air Based on Integral Measurements in Nitrogen and Oxygen at 14 MeV, Lawrence Livermore Laboratory, Rept. UCRL-17838 (1969).
31. M. L. Stelts, J. D. Anderson, L. F. Hansen, E. F. Plechaty, and C. Wong, Spectra of Fast Neutrons from Water Pulsed with 14-MeV Neutrons, Lawrence Livermore Laboratory, Rept. UCRL-71580 (1969).
32. C. Wong, J. D. Anderson, P. Brown, L. F. Hansen, J. L. Kammerdiener, C. Logan, and B. Pohl, Livermore Pulsed Sphere Program: Program Summary Through July 1971, Lawrence Livermore Laboratory, Rept. UCRL-51144 (1971).
33. J. L. Kammerdiener, Lawrence Livermore Laboratory, private communication (1971).
34. G. Goldberg, Lawrence Livermore Laboratory, private communication (1971).
35. I. L. Morgan, J. B. Ashe, R. W. Benjamin, O. M. Hudson, Jr., S. C. Mathur, D. O. Nellis, C. U. Parker, and W. E. Parker, Total Absorption Anti-Coincidence Gamma Ray Spectrometer, Texas Nuclear Division, Austin, Annual Progress Report (1963).
36. R. Howerton and E. Plechaty, A Recipe for Gamma Ray Production Cross Sections for Nuclides with $A \geq 20$, Associated with 4 to 14 MeV Induced Reactions, Lawrence Livermore Laboratory, Rept. UCRL-50185 (1967).
37. R. J. Howerton and E. F. Plechaty, A Formalism for Calculation of Neutron Induced Gamma-Ray Production Cross Sections and Spectra, Lawrence Livermore Laboratory, Rept. UCRL-70734 (1967).

38. D. M. Drake, J. E. Hopkins, C. S. Young, and H. Conde, Nucl. Sci. Eng. 40, 294 (1970).
39. D. M. Drake, J. E. Hopkins, C. S. Young, and H. Conde, Reports to the AEC Nuclear Cross Sections Advisory Committee Meeting held at Columbia University Oct. 21-23, 1968, Los Alamos Scientific Laboratory, Rept. WASH-1124 (1968).
40. W. H. Collins, SEHEN: A CDC-6600 Code to Display Experimental Gamma Production Cross Sections Data in Forms that Facilitate Evaluation, Lawrence Livermore Laboratory, Rept. MOR-4 (1972).
41. J. L. Kammerdiener, Neutron Spectra Emitted by ^{239}Pu , ^{238}U , ^{235}U , Pb, Nb, Ni, Al, and C Irradiated by 14 MeV Neutrons, Ph.D. thesis, University of California, Davis (1972).
42. E. A. Straker, P. N. Stevens, D. C. Irving, and V. R. Cain, The Morse Code—A Multigroup Neutron and Gamma-Ray Monte Carlo Transport Code, Oak Ridge National Laboratory, Tenn., Rept. ORNL-4585 (1970).
43. I. Spitzer, Physics of Fully Ionized Gases (Interscience Publishers, New York, 1962).
44. W. Heitler, The Quantum Theory of Radiation (Oxford University Press, Oxford, 1954).