OCT 26 1964

SwRI-1228-40

FUNDAMENTAL INVESTIGATION OF LIQUID-METAL
LUBRICATED JOURNAL BEARINGS

Quarterly Technical Report No. 2
Contract No. AT(11-1)-1228
Project Agreement No. 8
Modification No. 7

MASTER

R. A. Burton
H. J. Carper

to

U. S. Atomic Energy Commission
Washington 25, D. C.

October 1, 1964

Report No. RS-430

SOUTHWEST RESEARCH INSTITUTE
SAN ANTONIO HOUSTON

11/3

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

SOUTHWEST RESEARCH INSTITUTE
8500 Culebra Road, San Antonio, Texas 78206

FUNDAMENTAL INVESTIGATION OF LIQUID-METAL
LUBRICATED JOURNAL BEARINGS

Quarterly Technical Report No. 2
Contract No. AT(11-1)-1228
Project Agreement No. 8
Modification No. 7

R. A. Burton
H. J. Carper

to

U. S. Atomic Energy Commission
Washington 25, D. C.

October 1, 1964

APPROVED:

P. M. Ku, Director
Department of Aerospace
Propulsion Research

Report No. RS-430

FOREWORD

This report was prepared by Southwest Research Institute under Contract No. AT(11-1)-1228, Project Agreement No. 8, Modification No. 7. The work was administered by the Engineering Development Branch, Division of Reactor Development, U. S. Atomic Energy Commission, Washington 25, D. C with Mr. Nicholas Grossman as Project Engineer.

This report covers work performed from July 1 to October 1, 1964.

TABLE OF CONTENTS

	<u>Page</u>
I. INTRODUCTION AND HIGHLIGHTS	1
II. EXPERIMENTAL PROGRAM	7
A. Introduction	7
B. Velocity Measurements	7
1. Instrumentation and Calibration	7
2. Experimental Velocity Profiles	9
3. Computer Analysis of Velocity Measurements	12
4. Results of Analysis of Profile Data	17
C. Friction Coefficient Measurements	20
1. Empirical Wall-Law Equation	20
2. Other Universal Velocity Profile Equations	29
3. Friction Coefficient Results	30
D. Pressure Measurements	30
1. Development of Pressure Measuring Device	30
2. Calibration Equation	37
3. Results of Pressure Measurements	38
III. ANALYTICAL PROGRAM	40
A. Introduction	40
B. Long-Bearing Calculations	40
1. Dynamic Equation	40
2. Wall Shear Term	43
3. Momentum Flux	46
4. Continuity	47
5. Extreme Conditions	49
6. Summary	52
C. Estimation of Performance of Long Journal Bearings	53
1. Results of Calculations	54

TABLE OF CONTENTS (Cont'd)

	<u>Page</u>
D. Short-Bearing Equations	58
1. Pressure Gradient	58
2. Continuity Considerations	59
3. Integration of Film Equation	60
4. Trial Approximation	60
IV. CONCLUSIONS	63
V. FUTURE PROGRAM	64
VI. REFERENCES	65

LIST OF SYMBOLS

a_1	Slope of anemometer calibration curve
b	Half-thickness of film
C_f	Friction coefficient
h	Film thickness
h_o	Bearing radial clearance
I	Anemometer current reading
I_o	Anemometer current reading for zero flow velocity
m	Velocity profile parameter
n	Friction coefficient variation parameter
r	Journal radius
R	Reynolds number ub/ν based on centerline velocity
u	Velocity in direction of turning
Δu	Incrementation of velocity relative to Couette flow
U^+	Dimensionless velocity
x	Coordinate in direction of turning
y	Coordinate normal to boundary
Y^+	Dimensionless wall distance
z	Axial coordinate
ϵ	Eccentricity ratio
ζ	Parameter in full bearing equation

θ	Angular coordinate on bearing
μ	Absolute viscosity
ν	Kinematic viscosity
ξ	Parameter in full bearing equation
ρ	Fluid density
τ	Time-mean shear stress on wall
χ	Parameter in full bearing equation
ζ	Parameter in short bearing equation
$()_b$	Refers to midchannel
$()_{bc}$	Refers to Couette flow at midchannel
$()_{bh}$	Refers to midchannel velocity relative to moving wall
$()_{bo}$	Refers to midchannel velocity relative to stationary wall
$()_c$	Refers to Couette flow
$()_{crit}$	Refers to point of inflection or maximum in velocity profile
$()_h$	Refers to moving wall
$()_o$	Refers to stationary wall (except for h_o and R_o)
$()_s$	Refers to surface velocity of journal

I. INTRODUCTION AND HIGHLIGHTS

Progress Report No. 1 has outlined the general objectives of this investigation, pointing out that one of the principle areas of interest is the behavior of the turbulent bearing film. It has outlined experimental and analytical considerations which affect the behavior of this film. It is important that the reader recognize the orientation of the analyses, so that they may be viewed in perspective, relative to the work in progress. It is not our purpose to devise assumptions and proceed toward solutions (this we understand is being carried out by other groups); it is rather our objective to arrange and simplify the pertinent variables relative to the problem in such a way that "experimental questions" may be asked precisely. In such a frame of reference the question would not be: "what is the overall behavior of a test bearing?", but, rather, "how do certain critical parameters in the flow vary?" Because of the complexity of the problem, many of the important considerations must be presented in relatively detailed derivations. It is only through careful consideration of the individual steps of the derivations that the degree with which they meet their objectives can be evaluated.

Because we have chosen to deal with three separable levels of analysis, it is important that the reader attempt to separate these. On

one level are general statements such as conservation of momentum or conservation of energy, adapted to apply specifically to the bearing film, and written in terms of parameters which deal most effectively with the turbulence problem. For example, the dimensionless pressure which is convenient to use in this particular work, is $p/\rho u^2/2$, rather than the dimensionless pressure which is used in viscous bearing investigations.

Once derived, the basic equations may be dealt with in terms of approximate (or not yet well founded) estimates of the quantities involved in the equations. There is no harm in making such estimations so long as it is recognized that one can always return to the basic equation later, with improved data. The advantage of such approximate calculations is that they do provide some insights into which variables are strongly affected by details of the flow, and which variables are relatively insensitive.

In addition to the above trial calculations, it is also of interest to insert approximate quantities where possible and to carry through, in terms of the same formulation outlined above, complete approximate solutions for bearing configurations. Again, these solutions must not be thought of as final, but as investigations of the behavior of the various quantities in the equations.

In the application of various approaches to the bearing film equations, we shall continue the basically unprejudiced stance adopted in Progress Report No. 1, and shall attempt to assess both the accuracy and ease and applicability of different methods of attack. For example, now that it has become recognized where the earlier sources of error have been in the mixing-length approach, it is possible to improve this to provide more confident answers.

There are newer methods of attack which have recently been proposed (and, which we do not wish to discuss until they have been presented publicly). These also promise to give good results in bearing calculations. In comparing such methods, a given one may very well be "right" in the sense that it can give a reasonably accurate prediction of bearing behavior, but another may be "better" in certain critical situations. One approach may be of broader applicability than the other, or one may simply be easier to use, where the other one offers very little improvement in accuracy.

Ease of use of any of the relationships involved is of primary concern in developing the "engineering approach" to any aspect of technology. Since much of the literature has different orientation, this may lead to the appearance of going contrary to "generally accepted practice." If, for example, a two term expression of simple form can describe a function within accepted accuracy, it may be recommended over an expression of more complex form, though the latter offers either greater theoretical

justification or slightly improved accuracy. In so doing, however, we do not intend to reject other approaches. As a case in point, we call to your attention a very simple wall law (or universal velocity profile expression) that is offered in this report, simply as an 'empirical function' which generates this profile with acceptable accuracy. In using this we do not reject any of the half-dozen other formulations of the wall law. The fact that we do not report all of these at present should not be taken as indicating that we do not wish to investigate the special advantages of each. In order to avoid proliferation, we have limited the discussion in this report, and we hope that it serves as an integrated whole, which presents closely related aspects of the program.

In order to guide the reading of this report, we would like to point out certain highlights which may be somewhat obscured by the "details" in discussion of each, in their own paragraphs. In the experimental program it has been possible to apply hot-wire anemometer techniques in the 6-ft bearing model to obtain preliminary friction coefficient and velocity profile data. Whereas these data are offered as tentative, and are to be improved by statistical and experimental techniques, they are nonetheless provocative even in their preliminary form. Comparisons with data of other early workers for both friction coefficient and velocity profiles have been made possible. We have elected to report, at this time, only those data which relate to Couette flow acting alone, since this is a background case which sets the frame of reference for viewing later results. It is expected

in the forthcoming quarter that we shall be able to present data for both crossed pressure and Couette flows and codirectional pressure and Couette flows, and to examine the differences between these flows and the simple Couette flow.

Derivations are reported for both the extreme long bearing (plane flow) and for the extreme short bearing. Though the same techniques will apply to the bearing with simultaneous tangential and axial pressure flow, we have elected to defer discussion of this more complex case until the forthcoming quarter, so that the required additional assumptions do not confuse the very elementary concepts to be introduced at this point. In addition, we have deferred discussion of Taylor vortex flow until the forthcoming quarter, since this is essentially a separate topic from those which are given maximum emphasis here.

It has been possible to predict some aspects of overall bearing behavior by the use of selected approximations. In the case of the long full-bearing the role of inertia in very large film thicknesses is discussed, and some evidence is offered to show its range of influence. Though similar derivations have been carried out for sectors (where there is an approach to the stagnation pressure at the entry to the sector) these are not reported here since it is planned that this pressure recovery phenomenon will be discussed separately in the forthcoming quarter. In advance we note that,

for the experimental apparatus, the pressure recovery at bearing entrance is a major effect in determining the pressure distribution in a pad or partial arc bearing, and is not simply a minor correction on the overall flow.

II. EXPERIMENTAL PROGRAM

A. Introduction

As indicated in Progress Report No. 1, the experimental program is being carried out with an apparatus, wherein air is used as the working fluid rather than liquid metal. This "test bearing" is approximately 6 ft in diameter, and runs at speeds up to 120 rpm, generating Reynolds numbers in the 0.5 in. film thickness which are comparable to those expected in the much smaller bearings to be used with liquid metals. To give some idea of the relative proportions involved, Figure 1 has been included, showing the assembled bearing model in its air conditioned laboratory, and a typical prototype bearing which this model might simulate.

B. Velocity Measurements

1. Instrumentation and Calibration

Velocities were measured with a Flow Corporation Model HWB 3, hot-wire anemometer. The sensing elements used on the hot-wire probe were 0.0001 in. -diameter tungsten filaments. The hot-wire probe, itself, is a special design, made with extra long needles to which the filament is attached. The long needles allow the complete channel width to be traversed without introducing the 1/8 in. -diameter barrel of the probe into the channel, thus minimizing the probe's effect on the flow.

Calibration of the probe filaments was accomplished by setting the probe to the center of the channel and varying the journal speed (with

FIGURE 1. ILLUSTRATION OF EXPERIMENTAL BEARING
MODEL (6-ft diameter) COMPARED WITH TYPICAL
PROTOTYPE BEARING (1-in diameter)

journal and bearing concentric). Assuming the centerline velocity to be half of the journal surface speed, data were then obtained relating anemometer current reading to velocity. The known characteristics of the anemometer are such that when $(\text{current})^2$ is plotted against $(\text{velocity})^{1/2}$ for incompressible (i. e., low mach-number) flow, the result is a straight line. The results of several calibrations are shown in Figure 2. Since the set of data for the "good" filament do in fact follow a straight line, the assumption that the centerline velocity is half the journal surface speed is verified. Also shown in Figure 2 is a calibration curve for a "damaged" filament. It is believed that such damage occurs when the filament is overheated while being soldered to the probe, among other factors. Because of this aberrant behavior, over 20 filaments have been tested and used or rejected in terms of curves of this type. In some cases a miniature welding torch was used to solder the filaments. In other curves filaments were soldered using a small soldering iron. The latter more often yielded straight line calibration curves. This problem does not arise in high-speed work but is serious in the low-speed range of interest here.

2. Experimental Velocity Profiles

Some preliminary results of the effect of Reynolds number on velocity profile, for zero journal eccentricity, are shown in Figure 3. Here u is the velocity in the direction of journal rotation, u_s the surface velocity of the journal, y the radial distance measured from the bearing surface, and h the channel width (or film thickness). Three current

FIGURE 2. REPRESENTATIVE CALIBRATION CURVES FOR HOT-WIRE ANEMOMETER

FIGURE 3. EFFECT OF REYNOLDS NUMBER ON VELOCITY PROFILES FOR ZERO ECCENTRICITY

readings were obtained and averaged for each data point. It will be seen in the figure that the profiles near the "bearing" continue to change as Reynolds number exceeds 1150, while the profiles near the "journal" do not. It is believed that the flow is somehow slightly affected by the anemometer probe access hole, the diameter of which is the same order of magnitude as the coordinate y . This effect will be investigated more thoroughly during the forthcoming period.

To investigate the effect of journal displacement on the velocity distribution, data were obtained at several Reynolds numbers for an eccentricity ratio ϵ of 0.5. The results are shown in Figures 4, 5, and 6. It will be seen that the displacement has little effect on the velocity profiles. It has not yet been determined if the small differences noted are due to the displacement or simply experimental scatter. These results show that there is little pressure flow in the circumferential direction; and, for reasons of continuity, the flow in the axial direction must have increased so as to result in end leakage. It may then be concluded that the experimental setup tends to approximate the "short bearing."

3. Computer Analysis of Velocity Measurements

Before any velocity data were taken, a program was written to compute velocities from the hot-wire anemometer current readings and plot the velocity profiles on the computer print-out sheet. The steps in the program are as follows:

FIGURE 4. EFFECT OF ECCENTRICITY ON VELOCITY PROFILE
AT THE MINIMUM FILM THICKNESS LOCATION
 $(u_s h_0)/2\nu = 255$ LOW REYNOLDS NUMBER

FIGURE 5. EFFECT OF ECCENTRICITY ON VELOCITY
 PROFILE AT THE MINIMUM FILM THICKNESS
 LOCATION $(u_s h_0)/2\nu = 1150$ (INTER-
 MEDIATE REYNOLDS NUMBER)

FIGURE 6. EFFECT OF ECCENTRICITY ON VELOCITY PROFILE
 AT THE MINIMUM FILM THICKNESS LOCATION
 $(u_s h_0)/2\nu = 2350$ HIGH REYNOLDS NUMBER)

(1) Inputs *

- (a) NO Number of data points.
- (b) H Channel width at point of measurements.
- (c) XC Wall distance to point of inflection or
maximum in velocity profile.
- (d) I_0 Anemometer reading at zero velocity.
- (e) I_1, U_1 Anemometer reading and corresponding
velocity at any point on calibration curve.
- (f) N Journal speed, rpm.
- (g) X_j, J_j Wall distance and anemometer reading for
each data point.

(2) Computer Calculations

- (a) Using calibration equation, $I^2 = a_1 \sqrt{u} + I_0^2$
evaluate a_1 for I_1, u_1 calculate journal surface
speed u_s from N, and calculate dimensionless
velocity $\frac{u}{u_s}$ for all values of dimensionless wall
distance $\frac{Y}{h}$.
- (b) Plot the above results on print-out sheet.
- (c) Calculate $\frac{u}{u_{crit}}$ for all values of $\frac{Y}{Y_{crit}}$ from
 $y = 0$ to $y = y_{crit}$ and plot on print-out sheet.

*These symbols are used only in this particular program and do not necessarily correspond to the symbols in the remaining analyses or in the list of symbols.

- (d) Calculate $\log \left(\frac{u}{u_{\text{crit}}} \right)$ and $\log \left(\frac{y}{y_{\text{crit}}} \right)$ for all values of wall distance from $y = 0$ to $y = y_{\text{crit}}$, obtain a best fit curve by least squares method, and print out slope of this curve. (This will be the exponent of the equation $\frac{u}{u_{\text{crit}}} = \left(\frac{y}{y_{\text{crit}}} \right)^m$.)
- (e) Calculate $\frac{u_s - u}{u_{\text{crit}}}$ for all values of $\frac{h - y}{y_{\text{crit}}}$ from $y = y_{\text{crit}}$ to $y = h$ and plot on print-out sheet.
- (f) Calculate $\log \left(\frac{u_s - u}{u_{\text{crit}}} \right)$ and $\log \left(\frac{h - y}{y_{\text{crit}}} \right)$ for all values of wall distance from $y = y_{\text{crit}}$ to $y = h$, obtain a best fit curve by least squares method, and print out slope of this curve. (This will be the exponent of the equation $\left(\frac{u_s - u}{u_{\text{crit}}} \right) = \left(\frac{h - y}{y_{\text{crit}}} \right)^m$.)
- This exponent is symbolized by c in the program.

By way of illustration a computer print-out of the program is shown in Figure 7, where the program has been prepared in General Electric WIZ language for use with the G. E. 225 computer. Figure 8 illustrates a sample computer plot of a typical velocity profile.

4. Results of Analysis of Profile Data

It was expected that for the case of the concentric journal, the velocity profiles might be symmetrical about the channel centerline since, at this point, the velocity was half the surface speed of the wheel. In this case the exponents, m , of the equations in (d) and (f) above would be the same for a given Reynolds number. However, the results of the computer analysis of the preliminary velocity measurements show that the profile

```

EQU IOR(400), SORT(2070), SIN(2060), COS(2064), C
### LN(2074), EXP(2100), ATAN(2104), ABS(2110), C
### INT(2116), SWT(2122), SKIP(2012), MODF(2126)
I UP8(4096)
D PLOT,(220),TOLP,(30)
D X(60),I(60),V(60),Y(60),WN(60),YN(60),W(60)
**** SL DATA REDUCTION SWR1 PROJECT 09-1525
RD NO,H,XC,I0,I1,V1,I2,N,Z,THETA,E
VS=N*68.25*3.14159/720
PVLZ,THETA,E,NO,H,XC,I0,I1,V1,I2,N,VS
PL J X I C
### V
J=1,X(I0)=-1,KEY=0
* CRDX(J),I(J)
X(J)=X(J-1)
X(J)=XC
IC=I(J),KEY=1
[J=J+1]-NO
10 KEY
P=[I1-I0*I0]/SORT.(V1)
Q=I2/I2/P
T=IC/IC/P-Q
WC=T/V/VS
YC=XC/H
WC-1
YC-1
J=1
WMAX=0
* T=[J]*[J]/P-Q
V(J)=T+T
W(J)=V(J)/VS
Y(J)=X(J)/H
WMAX=W(J)
WMAX=W(J)
15 PV J,X(J),I(J),V(J)
[J=J+1]-NO
NO=NO+1
V(NO)=VS,W(NO)=Y(NO)=1.0
W(0)=Y(0)=0
SL PLOT OF W VS Y
J=0,L=-1,M=0
PL 0-----C
###5-----1C
### Y W
* TOLP.(Y(J),W(J),0,1,0,1,30)
Y(J)=YC
JSAVE=J
25 WN(J)=W(J)/WC
YN(J)=Y(J)/YC
27 WN(J)=[1-W(J)]/[1-WC]
YN(J)=[1-Y(J)]/[1-YC]
30 [J=J+1]-NO
M
31 PL SCALE DISTORTED OVER 2 PER CENT
SL FIRST PLOT OF WN VERSUS YN
PL 0-----C
###5-----1C
### YN WN
J=0,L=-1,SUMXY=SUMXX=0,M=0
* TOLP.(YN(J),WN(J),YN(1),YN(JSAVE),0,1,30)
YN(J)
T=LN.(YN(J))
U=LN.(WN(J))
SUMXY=SUMXY+T*U
SUMXX=SUMXX+T*T
32 [J=J+1]-JSAVE
M
33 PL SCALE DISTORTED OVER 2 PER CENT
C=SUMXY/SUMXX
PVLG
SL SECOND PLOT OF WN VERSUS YN
PL 0-----C
###5-----1C
### YN WN
L=-1,SUMXY=SUMXX=0,M=0
J=JSAVE
WN(J)=[1-W(J)]/[1-WC]
YN(J)=[1-Y(J)]/[1-YC]
* TOLP.(YN(J),WN(J),YN(JSAVE),YN(NO),0,1,30)
YN(J)
T=LN.(YN(J))
U=LN.(WN(J))
SUMXY=SUMXY+T*U
SUMXX=SUMXX+T*T
35 [J=J+1]-NO
M
40 PL SCALE DISTORTED OVER 2 PER CENT
C=SUMXY/SUMXX
PVLG
90 PL DATA NOT IN ASCENDING ORDER
J=NO
* CRDX(J),I(J)
[J=J+1]-NO
91 PL XC NOT FOUND IN DATA
92 PL WC OR YC EQUALS 1

```

FIGURE 7. VELOCITY-PROFILE DATA-ANALYSIS PROGRAM LISTING

* here $W \approx u/u_s$
 $Y \approx y/h$

FIGURE 8. SAMPLE COMPUTER PLOT OF VELOCITY PROFILE

on the bearing side of the centerline is different than that on the journal side of the centerline.

Table 1 shows the variation of the exponents with Reynolds number for $\epsilon = 0$ and $\epsilon = 0.5$ where m_b is the exponent of the bearing side profile, m_j is that of the journal side profile, and $\bar{m}$ is the average of the two. Figure 9 is a plot of the data from Table 1 where the average exponent $\frac{1}{\bar{m}}$ has been plotted against Reynolds number. The figure also shows some of the data of Reichardt⁽¹⁾ and Robertson⁽²⁾ for plane-Couette flow and of Nikuradse (from H. Schlichting,⁽³⁾) for pipe flow. As shown in the figure, the exponent m for laminar plane-Couette flow is equal to unity, while for pipe flow the profile is a parabola at low Reynolds numbers and the best approximation must have a value different from unity. Thus it is not surprising that the experimental data tend toward a lower asymptote than do the data of Nikuradse. Reichardt's and Robertson's data for plane-Couette flow give evidence of following a curve which is much like those in the present experiment. It is possible that the difference noted is due to the effect of b/r (i. e., curvature) on the present results. This may be resolved when we become able to operate at a different film thickness.

C. Friction Coefficient Measurements

1. Empirical Wall-Law Equation

Since the wall shear stress is extremely small, being about 10^{-3} lb/ft², it cannot be measured directly. Therefore, a generally accepted alternative procedure has been used, involving application of the

FIGURE 9. VARIATION OF PROFILE PARAMETER m WITH REYNOLDS NUMBER

TABLE 1. VELOCITY PROFILE EXPONENTS

 $\epsilon = 0$

R_e	m_b	m_j	$\bar{m}$	$\frac{1}{\bar{m}}$
2,360	0.168	0.075	0.121	8.26
1,820	0.197	0.115	0.156	6.40
1,160	0.307	0.120	0.213	4.69
520	0.410	0.329	0.369	2.71
256	0.659	0.602	0.630	1.59
150	0.749	0.827	0.788	1.27
108	0.910	1.420	1.165	0.85

 $\epsilon = 0.5$

1,180	0.228	0.130	0.178	5.62
585	0.362	0.218	0.290	3.45
195	0.764	0.458	0.611	1.64
125	0.882	0.676	0.779	1.28

"wall law" or "universal velocity profile" to a region in the vicinity of the solid boundary surface, to permit computation of the shear stress. Because this procedure appears to assume the shear-stress profile relationship, and because of certain experimental considerations in its application, it is well to review the considerations which are involved. To begin with, in a field of uniform flow in the vicinity of a single wall, where any other boundaries are very far distant, one may assume that, for purposes of dimensional analysis, only the following variables describe the flow: y distance out from the wall, τ_0 wall shear stress (which is also the shear stress throughout the flow), ρ fluid density, μ fluid viscosity and u local velocity. These variables fully specify both the fluid properties and the kinematic and geometric properties of the flow, whether it be turbulent or laminar. They may be grouped dimensionlessly in several ways; but, irrespectively, two dimensionless groups will result in each case. For example, the two dimensionless groups which characterize the above assembly of variables may be U^+ and Y^+ , where $U^+ = u\sqrt{\rho/\tau}$ and $Y^+ = \frac{y}{\nu} \sqrt{\frac{\tau}{\rho}}$. Otherwise, the two variables may be C_f and R , where C_f is defined as $2\tau/\rho u^2$ and R is defined as $\rho y u/\mu$. To show that these groups have a unique relationship to each other we simply note that in terms of the definitions: $U^+ = \sqrt{2/C_f}$ and $Y^+ = R\sqrt{C_f/2}$. Thus we may write $Y^+ = f_1(U^+)$ or $C_f = f_2(R)$ and if the functional relationship is known for one case, it may be uniquely calculated for the other. One may plot U^+ vs. Y^+ , or C_f vs. R , and either will shed special light on certain aspects of the flow. On these two

types of plot the representation of U^+ vs. Y^+ is normally called the "universal velocity profile," or "wall law profile." The reason for this is that its appearance corresponds geometrically to the shape of the equivalent turbulent or laminar velocity profile in flow with uniform shear stress, and in the vicinity of a wall. Numerous experimenters have attempted to determine this fundamental function or profile from measurements in actual flows. It has been fairly well established by such experiments that, for pipe flow, a single universal velocity profile function describes turbulent flow, for at least a third of the pipe radius out from the wall, and is a good first approximation right out to the center of the pipe. It has further been shown, though less exhaustively, that plane-Couette flow also follows the same universal velocity profile for a sizeable portion of the channel distance out from the wall, and also that boundary layer velocity profiles do the same.

Noting again that the "universal velocity profile" also corresponds uniquely to a C_f vs. R plot, it is not surprising that (as observed in Progress Report No. 1) the correspondence between Couette flow and pressure flow velocity profiles is echoed by a similar correspondence between the C_f plots for the two types of flow.

As was pointed out also in the earlier report, this profile in the turbulent regime may be approximated to a fair degree by a simple power law, and also by logarithmic expression. In the laminar regime very near to the wall the universal velocity profile approaches $U^+ = Y^+$ and

there is a transition region in the vicinity of $Y^+ = 10$ where neither of the simple expressions holds. Irrespective of this, if a sufficiently good empirical equation of the profile were available, in terms of existing data, the profile could be generated and could be used to produce both friction coefficient data and velocity profile data under circumstances similar to those which are involved in the definition of this universal velocity profile.

In actual flows, such as those of interest here, the universal velocity profile would be expected to represent the "true one" only in the very near vicinity of the wall. Further out in the stream some departure from this shape would be expected (due to the presence of the opposite boundary), even though this departure may not be extremely great in the half channel. If measurements indefinitely close to the wall could be made (i.e., where the wall distance y is much less than the channel dimension b), one would expect that the velocity at the point y would be related to the shear stress at the wall as in the universal velocity profile. Experimentally, some distance y must be chosen which is a finite fraction of the channel width. This choice is influenced by an uncertainty relationship, where for very small y the universal velocity profile becomes an exact representation of the situation; yet, at the same time, the experimental error in y produces sizeable error in the ultimate results. Similarly, at very large y , this dimension can be known accurately; but departure from the universal velocity profile may be significant. For the present set of measurements it has been found that the best balance between experimental accuracy and applicability of the wall law is at $y/b = 0.2$.

In order to apply the wall law to calculation of shear stress from known velocity, density and viscosity data at the given dimension, it has been necessary to use some mathematical expression to represent known experimental data. If one were totally unprejudiced and approached the scatter of experimental data on the U^+ vs. Y^+ plot it is clear that the following type of expression could be used to represent this over any desired range with any degree of accuracy to be chosen

$$Y^+ = f(U^+) = U^+ + b(U^+)^2 + c(U^+)^3 + d(U^+)^4 \dots$$

Actually, the scatter of the data from different authors is such that the selection of the coefficients is arbitrary relative to the functionality in the most critical range around $U^+ = 10$. For the present work this was side-stepped momentarily until the available data could be evaluated more carefully, and instead we have used the simple expression

$$Y^+ = U^+ + \left(\frac{U^+}{a}\right)^n = U^+ + \left(\frac{U^+}{8.74}\right)^7 \quad (1)$$

It was found that the numerical constants used in the conventional power law gave a very adequate fit over the range of interest. Consequently this simple expression was used in our computer program to relate U^+ and Y^+ for measured values of velocity, wall distance, and fluid properties.

Figure 10 shows these on an semilog plot of U^+ vs. Y^+ , and Figure 11 shows a similar plot, expanded in the vicinity of $Y^+ = 10$, where a very critical comparison in the range of transition can be seen. This also shows the

FIGURE 10. UNIVERSAL VELOCITY PROFILE SHOWING LOGARITHMIC APPROXIMATION AND PROPOSED APPROXIMATION AS COMPARED WITH NIKURADSE'S DATA

FIGURE 11. UNIVERSAL VELOCITY PROFILE EXPANDED TO SHOW TRANSITION ZONE

discrepancy between the different experimental data including those of Nikuradse⁽³⁾, Reichardt⁽⁵⁾, and Laufer (as in Ref. 6).

In the program of experimentation reported here, care has been taken to check the values of U^+ and Y^+ to make sure that no measurements are outside the range of applicability of the simple profile equation which has been selected for this preliminary work. At the highest Reynolds number of operation chosen, the Y^+ was found not to be greater than 30. Correspondingly, at the extremely low Reynolds numbers run, where transition was very likely of the vortex flow type, the value of Y^+ was as low as 2.28, which is very deep in the laminar sublayer. Because of this, one may have confidence in the applicability of the shear stress measurements even in the latter case since the sublayer would not be expected to be influenced greatly by detailed changes in the momentum transfer effects in the layers above. Thus even if the flow were laminar in the extreme case or were of the vortex type, it is believed that even at the lowest Reynolds numbers the measurements can be viewed with reasonable confidence.

2. Other Universal Velocity Profile Equations

The classical representation of the universal velocity profile breaks it into parts as illustrated in Figure 10, where $Y^+ = U^+$ up to the intersection with the function $U^+ = 2.5 \ln 9Y^+$. A similar two piece velocity profile may also be generated with the power law where the $U^+ = Y^+$ curve continues upward until it intersects the power law $Y^+ = \left(\frac{U^+}{8.74} \right)^7$. Neither of these represent the data accurately in the transition range and furthermore

they are more difficult to use than the simple continuous profile provided above in equation (1). By way of reference we cite other approaches to the velocity profile system as follows: Miles⁽⁷⁾, van Driest⁽⁶⁾, Szablewski⁽⁸⁾, Reichardt⁽¹⁾, and Ng⁽⁹⁾. If further work suggests that it is advisable, careful comparisons of these will be undertaken. At present, however, experimental scatter appears to be such as to eliminate the possibility of a critical comparison in the transition range, thus simplicity becomes a useful guide in selecting one to use.

3. Friction Coefficient Results

Preliminary results of measurements in concentric flow (such as would be represented by the velocity profile in Fig. 3) have been used with the empirical wall law equation, Equation 1, to obtain friction coefficient data. These are plotted in Figure 12, where comparisons with other work are possible. Note that the data follow Couette's very thin film data and Fuller's data for thicker films. They fall below Taylor's data for very thick films, but at the highest values of R , correspond more closely to Taylor's results than to Couette's results. Irrespective of the scatter indicated, it is gratifying that the data follow a reasonable trend, and are substantiated by other work as to general magnitudes.

D. Pressure Measurements

1. Development of Pressure Measuring Device

It was stated in the first quarterly report that gage pressures of the order of 10^{-6} psi were to be expected, and that no known instruments

FIGURE 12. RELATIONSHIP BETWEEN COEFFICIENT OF FRICTION AND REYNOLDS NUMBER FOR COUETTE FLOW

are available that accurately resolve pressures of this order. It was then necessary to develop an apparatus to measure the static pressure distribution around the bearing. The following paragraphs describe the steps in the development of the device.

Since the pressures were of extremely small magnitude, it was considered impractical, if not impossible, to amplify the pressure mechanically through the use of some sort of bellows or diaphragm arrangement. Therefore the problem was exchanged for one of measuring a flow in a tube connected to a static pressure tap on the bearing surface. This problem has been solved through the use of a halogen detector with the sensing element placed in the flow at the downstream end of the tube. In its earliest arrangement, a small amount of a halogen was injected into the bearing clearance, and the elapsed time to the detector response was measured. Using the Poiseuille flow equation, the unknown pressure could then be computed.

Preliminary trials of this method produced encouraging results; however, these results were not repeatable for all orientations of the tube containing the sensing element of the halogen detector. Examination of the properties of the halogen compound (Freon 12) showed its density to be approximately four times that of air; and further calculation showed that it was possible that the static pressure in the bearing was not great enough to lift a 1-in. static head of the halogen. Modifications were then made to assure that the tube was always in the horizontal position to cancel the effects of gravity, and to control the size of the halogen sample.

A schematic of the complete system is shown in Figure 13. As shown in the figure, pressure of the halogen (Freon 12 in this case) is admitted into the antechamber by a regulator. The pressure difference ($p_1 - p_2$) then causes a flow in the direction shown in the tube across the sensing element of the halogen detector. The pressure p_2 can be either atmospheric, for absolute measurement of p_1 , or a pressure in the bearing for measurement of the gradient between two points.

The procedure for operating the device is best understood by referring to Figures 13 and 14, where Figure 14 is a schematic of valves A and B. First the 3-way valve B is turned so that it blocks one end of the capillary, and the 2-way valve A is opened so as to charge the capillary with Freon as shown in Figure 13-A. Next, valve A is shut and B is turned so that the capillary exhausts the Freon into the antechamber as shown in Figure 13-B. Now valve B is turned so that the antechamber containing the Freon is directly connected to the tube as shown in Figure 13-C. At this point a stop-watch is started and is stopped when a response is obtained from the halogen detector.

A typical response of the halogen detector is shown in Figure 15, which is a recorder chart of detector voltage versus time. The time required for the detector voltage to increase from the background level to the peak is approximately 3 seconds while the total time for the halogen to reach the detector is of the order of at least 30 seconds so that the error in measuring the total time is probably less than 10%.

FIGURE 13. SCHEMATIC DIAGRAM OF PRESSURE MEASURING DEVICE

A - CHARGE CAPILLARY

B - CHARGE ANTECHAMBER

C - ESTABLISH FLOW FOR
MEASUREMENT

FIGURE 14. SCHEMATIC DIAGRAM OF VALVES IN
PRESSURE MEASURING DEVICE

FIGURE 15 RECORDING OF DETECTOR RESPONSE

2. Calibration Equation

The volume rate of flow in the tube, if the flow is laminar, is given by the Poiseuille equation

$$Q = \frac{\pi R^4}{8\mu} \left(- \frac{dp}{dx} \right) \quad (2)$$

where

Q = volume rate of flow, $\frac{ft^3}{sec}$

R = radius of tube, ft

μ = viscosity of fluid, $\frac{lb-sec}{ft^2}$

$\frac{dp}{dx}$ = pressure gradient, $\frac{lb}{ft^2-ft}$

replacing Q by $\frac{L}{t} \pi R^2$ (where L = length of tube, (ft) and t = time required for a point in the flow to traverse the length L , (sec), and integrating equation (1) we obtain

$$\Delta p = \frac{8\mu L^2}{tR^2} \quad (3)$$

If time t is known, then Equation (2) can be used to calculate the pressure difference ΔP for a given fluid and tube size. The only requirement is that the flow in the tube be laminar. A Reynolds number of $R = 60$, definitely in the laminar regime, is the highest observed thus far in the tube.

3. Results of Pressure Measurements

Preliminary results of the pressure measurements are shown in Figure 16, where dimensionless pressure is plotted against the angular position θ for an eccentricity ratio ϵ of 0.5. The direction of rotation of the journal with respect to the displacement is also shown in the figure. As seen in the figure, the profile is not quite symmetrical about the point of maximum pressure--which in later studies may or may not prove to be significant. The consistency of the pressure measurements is rather surprising (and gratifying) since the order of magnitude of the pressure is extremely small.

FIGURE 16 PRESSURE VARIATION IN TEST APPARATUS
WITH JOURNAL SET AT 0.5 ECCENTRICITY RATIO

III. ANALYTICAL PROGRAM

A. Introduction

Both to aid in the interpretation of experimental results and to provide a perspective for viewing analytical results, a series of derivations has been undertaken. In these we shall not try to "justify" expedient assumptions, but to ask, rather, just how accurately certain aspects of the problem must be dealt with in order to obtain accurate predictions of overall bearing performance. The converse is of interest experimentally, since those aspects of the flow which produce little effect on overall pressure gradient, etc., are consequently difficult to detect or measure and, at the same time, do not need to be measured accurately.

We shall attempt to clarify at each step of the derivation whether a quantity introduced is the consequence of an arbitrary but permissible "definition," an "approximation," or is simply an "example" for numerical estimation of general magnitudes. The initial derivation will be restricted to the no-end-leakage case or "plane flow," with steady turning and fixed shaft position, in order to reduce the number of concepts that must be introduced as the basic approach is outlined.

B. Long-Bearing Calculations

1. Dynamic Equation

Since the assumptions which simplify the Navier Stokes equations for the case of a thin lubricant film are not peculiar to the

present case, these need not be discussed here. An expression of Newton's second law for such a film* may be written as follows (under the assumption of steady flow with the local velocity u in the x direction, and where pressure is assumed to be independent of y , the coordinate measured across the film thickness, which ranges from 0 to h):

$$\int_0^h \rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = \tau_h - \tau_o \quad (4)$$

It is now possible to introduce the definition

$$C_f = \frac{2|\tau|}{\rho u_b^2} \quad (5)$$

such that C_f is defined relative to the wall on which the shear stress is to be measured, and u_b is defined as midchannel velocity relative to that wall. Substituting into Eq. (4)

$$\int_0^h \rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = \left[\frac{C_{fh} \rho (u_{bh})^2}{2} \right]_h - \left[\frac{C_{fo} \rho (u_{bo})^2}{2} \right] \quad (6)$$

Refer to Figure 17A for which the definition of the relative velocities u_{bh} and u_{bo} are defined.

At this point it is desirable to refine further the definition of u_b in order to examine the case of a pressure flow, acting codirectional with a Couette flow. As illustrated in Figure 17-B, a Couette flow with centerline velocity u_{bc} is incremented to the right by a pressure flow, where the incrementation of the centerline velocity is of the magnitude Δu_b .

*For turbulent or laminar film.

FIGURE 17. NOMENCLATURE FOR ANALYSIS OF BEARINGS WITHOUT END LEAKAGE

In view of the definitions of u_{bo} and u_{bh} , these become

$$|u_{bo}| = |u_c + \Delta u_b| \quad (7)$$

$$|u_{bh}| = |u_c - \Delta u_b|$$

Here it is recognized that the Couette flow bears such symmetry relative to each wall that the mid-channel velocity is, in either case, designated by the same quantity u_{bc} , relative to either wall, for the Couette flow acting alone. In terms of this, Equation (6) becomes

$$\underbrace{\int_0^h \rho u \frac{\partial u}{\partial x}}_{\text{I}} + \underbrace{\frac{\partial p}{\partial x}}_{\text{II}} = \underbrace{\frac{C_{fh} \rho (u_c - \Delta u_b)^2}{2} - \frac{C_{fo} \rho (u_c - \Delta u_b)^2}{2}}_{\text{III}} \quad (8)$$

The equation has been divided into three parts for purposes of discussion, part I being the "momentum flux" term, part II being the "pressure gradient" and part III being the "wall shear" term.

2. Wall Shear Term

If C_{fc} is the wall shear stress for the Couette flow before its incrementation by Δu , it is possible to write for other conditions the relation

$$\xi = \frac{C_f}{C_{fc}} \quad (9)$$

Hence ξC_{fc} will permit the computation of actual friction coefficient C_f when ξ is known. To obtain some idea of the type of variation expected of ξ we may write the approximate expression:

$$\phi = \frac{C_f}{C_{fc}} = \left(\frac{R_c}{R} \right)^n \quad (10)$$

In this case we have assumed that the C_f vs R relationship is continuous and can be approximated by suitable selection of n . This would not necessarily be the same n that would best represent the Reynolds number variation for the Couette flow alone, though evidence has been presented (see Progress Report No. 1) which suggests that n will not differ greatly from that magnitude. In terms of this, Term III of Equation (8) becomes

$$(III) = \rho C_{fc} \left(\frac{u_{bc}}{u_{bc} + \Delta u_b} \right)^n \frac{(u_{bc} + \Delta u_b)^2}{2} - \rho C_{fc} \left(\frac{u_{bc}}{u_{bc} - \Delta u_b} \right)^n \frac{(u_{bc} - \Delta u_b)^2}{2} \quad (11)$$

Or using Equation (10)

$$(III) = \rho \frac{C_{fc} u_{bc}^2}{2} \left[\left(1 + \frac{\Delta u_b}{u_{bc}} \right)^{2-n} - \left(1 - \frac{\Delta u_b}{u_{bc}} \right)^{2-n} \right] \quad (12)$$

If $n = 0$ (in which case $C_f = C_{fc} = \text{Const.}$), Equation (12) would reduce to

$$(III) = \rho \frac{C_{fc} u_c^2}{2} \left[\frac{4\Delta u_b}{u_c} \right] \quad (13)$$

This would be expected to prevail where wall roughness causes C_f to depart from smooth-wall behavior and level out at a nearly constant value at high Reynolds number.

Using the results of Equation (13) and Equation (12), a useful comparison is the consequence when the following quotient is determined:

$$\xi = \frac{\left[\left(1 + \frac{\Delta u_b}{u_{bc}} \right)^{2-n} - \left(1 - \frac{\Delta u_b}{u_{bc}} \right)^{2-n} \right]}{4(\Delta u_b/u_{bc})} \quad (14)$$

The quantity ξ would compare the effect of C_f varying, with that of constant C_f as to their influence on term III. Table II shows comparisons of the variation of ξ with $\Delta u_b/u_{bc}$ for $n = 1/2$ and $n = 1/4$, which would be expected to represent a realistic range of magnitudes.

Table II

	ξ	
	$n = 1/2$	$n = 1/4$
$\Delta u_b/u_{bc} \rightarrow 0$	0.75	0.876
$\Delta u_b/u_{bc} = \pm 1/2$	0.741	0.866
$\Delta u_b/u_{bc} = \pm 1$	0.70	0.84

Note that for $\Delta u_b/u_{bc} \rightarrow 0$, $\xi \rightarrow 1 - \frac{n}{2}$. Also note that $\Delta u_b/u_{bc} = 1$, represents the limit of applicability of Equation (8), as written, since exceeding of this magnitude would require a change of the sign in Term III of Equation (8), since the shear stress at one wall would reverse direction.

In terms of ξ , and Equation (13), Term III becomes

$$(III) = \rho \frac{C_{fc} u_c^2}{2} \xi \left[\frac{4\Delta u}{u_c} \right] \quad (15)$$

In recognizing that $\rho C_{fc} u_c^2/2$ is the wall shear stress τ_c of the original Couette flow acting alone

$$(III) = \tau_c \xi \left[\frac{4\Delta u}{u_c} \right] \quad (16)$$

This is a useful form of the term since ξ is not widely variable, as is seen in Table II. As a first approximation, it could well be $\xi = 1 - \frac{n}{2}$. If more accuracy is desired any reasonable choice of n will give a fair approximation since, for the wide range indicated, the ratio of ξ for the two values of n is about 0.85; or, only a 15% error in Term III would have resulted if one value of n had been used for computation when the other should apply. Somewhere between these extremes even better agreement should result.

Should we, in measurement, be able to determine n within 20% of its best value, only a very small error in Term III would result. This suggests the feasibility of obtaining realistic predictions of the "wall shear" term by reasonably accurate experimentation.

3. Momentum Flux

In Equation (8) the momentum flux term, Term I, may be written as

$$(I) = \frac{\int_0^h \rho \frac{u^2}{2} dx}{dx} = \rho \frac{d \left[\left(\xi \frac{\rho u_{bc}^2}{2} \right) h \right]}{dx} \quad (17)$$

It is of interest to determine the general magnitude of ξ and its sensitivity to the shape of the velocity profile. We note that

$$\xi \equiv \frac{\int_0^h u^2}{h u_{bc}^2} = \frac{\int_0^h (u_c + \Delta u)^2}{h u_{bc}^2} \quad (18)$$

Though some additional analysis is possible, we shall proceed directly to an order-of-magnitude investigation of factors influencing the term by the assumption that both u_c and Δu vary out from a wall into either half channel according to an equation of the form $u/u_b = (y/b)^m$. In addition to approximating the flow in each half channel by a power-law profile, it assumes that the law is identical for each. In terms of this, Equation (17) becomes

$$\zeta h u_c^2 = \int_0^b (u_{cb} + \Delta u_b)^2 (y/b)^{2m} dy + \int_b^h [2u_{cb} - (u_{cb} - \Delta u_b)(y/b)^m]^2 dy \quad (19)$$

To obtain some information on the variation of ζ , it is desirable to insert some numerical value of m , which for the present has been chosen as $1/7$. In addition to ζ , two additional parameters have been included in the calculations, and these have been defined as follows

$$A \equiv \frac{\int_0^h \rho u^2 dy}{\rho u_{bc}^2 h} \quad (19)$$

$$B \equiv \frac{\rho (u_{bc} + \Delta u_b)^2}{\rho u_{bc}^2} \quad (20)$$

and

$$A/B = \zeta \quad (21)$$

4. Continuity

The continuity equation for an incompressible lubricant film may be written as follows:

$$(h_o - h) u_{bc} = \int_0^h \Delta u_c dy \quad (22)$$

Here there is no normal motion between surfaces, and h_o is film thickness where only Couette flow would prevail. It may, in fact, be a hypothetical thickness which does not exist in a bearing, but nevertheless serves as a parameter to characterize the flow.

To determine how the integral $\int_0^h \Delta u dy$ is influenced by velocity profile form, let us use, as previously, a profile in the half channel (for velocity relative to the nether wall) $u/u_b = (y/b)^m$. Equation (22) becomes

$$(h_o - h) u_c = 2 \int_0^b \Delta u_b (y/b)^m dy \quad (23)$$

or integrating

$$(h_o - h) u_c = \frac{2\Delta u_b b}{m+1} = \frac{\Delta u_b h}{m+1} \quad (24)$$

Had the velocity Δu_b prevailed across the channel as a "constant" increment to the flow the result would have been

$$(h_o - h) u_c = \Delta u_b h \quad (25)$$

The ratio of the right-hand term in Equation (24) to the equivalent term in Equation (25) may be designated by χ and would be, for $m = 1/7$,

$$\chi = \frac{1}{m+1} = 0.875 \quad (26)$$

Again the critical quantity is close to unity. Hence the correction due to the choice of m tends not to be a major factor. Irrespective of the particular value of m , the continuity equation written in the following form must be correct:

$$(h_o - h) u_c = \chi \Delta u_b h \quad (24a)$$

5. Extreme Conditions

It has been mentioned that the preceding analysis cannot be applied without alteration outside the range $-1 < \Delta u_b / u_{bc} < +1$. Furthermore, the approximate power law expressions for C_f and velocity profile must breakdown when $\frac{u_{bc} - \Delta u_b}{u_{bc}} \rightarrow 0$. On the other hand, shear stress tends also to go to zero in this case, which makes the precise magnitude relatively unimportant.

To show how insensitive a typical quantity might be to gross departures of velocity profile from the chosen form, let us examine the somewhat more realistic power law profile approximation shown in Figure 18, with respect to the continuity equation.

$$\begin{aligned} (h_o - h) u_c &= \int_0^h \Delta u dy = \int_0^h \frac{2u_c}{h^m} y - u_c h = \frac{2u_c h}{m+1} - \\ &\quad - u_c h = \frac{(1-m) u_c h}{m+1} \end{aligned} \quad (27)$$

Table III compares A , B and ζ for different values of $\Delta u_b / u_{bc}$.

FIGURE 18. ILLUSTRATION OF APPROXIMATE VELOCITY PROFILE WHEN $(\Delta u_b)/u_{bc} \rightarrow 1$

Table III

$\Delta u_b/u_{bc}$	<u>A</u>	<u>B</u>	<u>ζ</u>
0	1.006	1	1.006
+1	3.789	4	.947
-1	0.012	0	∞

Note that though $\zeta = \infty$ for $\Delta u_b/u_{bc} \rightarrow -1$, this is not to be misinterpreted as making ζ a useless parameter. Comparison of A and B shows that the momentum flux prediction using the approximate profile, follows the simple prediction reasonably well. Thus if ζ were assumed to be unity and applied to B, it would predict $A = 0$ rather than the very small value of 0.012, computed by the more accurate estimate of the profile.

Obviously, some different assumption of m would change the numbers in Table III somewhat, however, evidence has been cited that the use of $1/7$ is not completely unrealistic. Furthermore, lack of symmetry in the distributions would also affect the results. Nevertheless, reassurance is derived from the closeness of correspondence between Equation (24) and Equation (27) since this implies that refinement of description of the velocity profile will only refine the 5 percent or so discrepancy between the simple uniform-velocity approximation and the effect of the true profile. Hence, any improved accuracy in the experimental profiles will provide a very close estimate of the true momentum-flux term.

When this is compared with the prediction of Equation (25), we note that

$$\chi = \frac{1 - m}{(m + 1)} \quad (28)$$

If, for instance, $m = 1/7$, $\chi = 0.75$, which is not greatly different from the value of 0.875 given by the previous computation in Equation (24).

6. Summary

Though additional comparisons can be made, it is futile to do so without further knowledge of actual velocity profiles. There is nevertheless reassurance in the observation that ζ , ξ and χ are all near unity and are all relatively insensitive to the details of the velocity profile or the C_f variation function.

Again it should be reiterated that the defining-equations for ζ , ξ and χ are not dependent on special assumptions as to the particular velocity profile or C_f variations. They remain critical parameters in the film equation for determining the influence of turbulence quantities. They may be used to compare various turbulence "theories" as to differences in their predictions. They are also the quantities to be evaluated by our experimental measurements. The defining equations for these quantities may be rewritten

$$\xi \equiv \frac{C_{fh}(u_{bc} + \Delta u)^2 - C_{fo}(u_{bc} + \Delta u_b)^2}{4 C_{fc}(u_{bc}) \Delta u_b} \quad (29)$$

$$\chi \equiv \frac{2 \int_0^b \Delta u \, dy}{\Delta u_b h} \quad (30)$$

$$\zeta \equiv \frac{\int_0^h (u_c + \Delta u) \frac{\partial \Delta u}{\partial x}}{2bu_b \left(1 + \frac{\Delta u_b}{u_c}\right) \frac{\partial \Delta u_b}{\partial x}} \quad (31)$$

C. Estimation of Performance of Long Journal Bearings

Rewriting Equation (5) in terms of the above defined quantities

[Eqs. (29), (30), (31)]

$$\rho \zeta (u_{bc} + \Delta u_b) \frac{\partial \Delta u_b}{\partial x} + 2C_{fc} \rho \xi \frac{(u_{bc}) \Delta u_b}{h} = - \frac{dp}{dx} \quad (32)$$

if $\frac{\Delta u_b}{u_{bc}} \equiv U$, $x/h_o \equiv X$, $h/h_o \equiv H$, $p/\rho u_c^2 \equiv P$, $C_{fc} \equiv C$, $R = r/h_o$

Then

$$\zeta (1 + U) \frac{\partial U}{\partial X} + 2C \xi \frac{U}{H} = - \frac{dP}{dX} \quad (33)$$

The continuity equation becomes

$$\frac{(1 - H)}{H} = U\chi \quad \text{or} \quad \left(\frac{1}{H} - 1\right) = U\chi \quad (34)$$

Substituting back

$$\frac{-dP}{dx} = \frac{\zeta}{H\chi^2} d \left(\frac{1}{H} - 1\right) + \frac{2C_f \xi}{H\chi} \left(\frac{1}{H} - 1\right) \quad (35)$$

Let

$$H = 1 - a \sin(bX + c) + a \sin c \quad (36)$$

where

$$b = h_o/r = 1/R \quad (37)$$

This equation may be reduced further, mathematically, as will be discussed in Progress Report No. 3, however, there is little gain in this since a computer was readily available, and since the analytical solution is not really very simple to work with. Consequently, a solution was programmed where dP/dX was found at each value of X and then integrated from the starting point. The value of C was adjusted in a process of successive approximation until the condition is reached where $P_{\theta=2\pi} = P_{\theta=0}$. The program also integrated $P \sin \theta$ and $P \cos \theta$ and obtained the resultant of force for the full bearing (pressure was allowed to go negative wherever it chose), and both the magnitude and direction of the resultant were printed. The machine also rendered out a plot of the pressure distribution function.

The program in WIZ language is shown in Figure 19. In Figure 20, a typical plot of pressure is shown, where $B*X$ corresponds to bx , the angular position, in Equation (36), and $P(X)$ corresponds to the dimensionless pressure. $\$FX$, $\$FY$ and $\$R$ are the components and modulus of the resultant force and $\$PHI$ is the angle at which it acts.

1. Results of Calculations

Table IV shows the relationship between the parameters of Equations (35, (36) and (37), for different conditions. The quantity a is a measure of eccentricity, b is a measure of film thickness relative to circumference ($b = 0.01$ is actually a thicker film than in our test bearing,


```

EQU IOR(400), SQRT(2070), SIN(2060), COS(2064), C
### LNI(2074), EXP(2100), ATAN(2104), ABS(2110), C
### INT(2116), SWT(2122), SKIP(2012), MODE(2126) ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
I UP8(4096) ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
000 $ PROJECT 09 - 1525 ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
010 $ NUMERICAL SOLN. OF A LINEAR DIFF. EQN. ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
020 $ BY EULERS METHOD ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
030 $ JULY, 1964 P.A. HUNTER ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
040 D PLOT.[220] ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
050 D P(100),X(100) ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
060 I PI(3.1415927) ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
065 RAD = PI/180. ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
070 ** CRDSI,A,B,C,CF,XO,PO,ITRX,DLX,ITRC ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
080 DLX = [2.0*PI + B*X0]/[ITRX*B], IC = 0 ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
082 TCF = 2.0*CF ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
085 *** C1 = C/RAD ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
090 SL ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
100 PV , ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
110 PL 1 ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
120 ### PROJECT 09 - 1525 ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
130 PVL,,SKIP,SKIP,SI,A,B,, ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
140 PVLSKIP,SKIP,CF,ITRX,ITRC,, ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
150 PVLSKIP,XO,PO,DLX,DLX,, ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
155 PVLSKIP,SKIP,SKIP,C$ ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
160 SL 1 ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
### PLOT OF W(X) VS B*X ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
170 PL 0-----1
180 ###1-----22
190 B*X H(X) ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
200 YI = PO , FY = FX = 0 ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
210 MIN = 1T76, MAX = -1T76 ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
220 I = 0, SINC = SIN.(C1), C1COSC1 = C1*COS.(C1) ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
240 * XI = XO + I*DLX ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
245 BXI = B*XI ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
250 RXPC = BXI + C1 ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
260 CORX = COS.(RXPC), SIBX = SIN.(BXPC) ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
270 TEMP = SIBX - SINC, HI = 1.0 - A*TEMP ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
275 YIP1 = YI -DLX*A*[SI*(B*CORX - C1COSC1)/HI 1
280 ***+ TCF*TEMP]/[HI*HI] ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
295 SWT.[1] ( 12 ) ( ) ( ) ( ) ( ) ( ) .MAIN
296 PVL YIP1,YI,XI,I$ ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
300 12 FX=YI*COS.[BXI]+FX,FY=FY+YI*SIN.[B1
305 ###XII, P[II] = YI, X[II] = BXI ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
320 PLOT.[BXI,HI,0.2,0] ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
330 YI - MIN ( 11 ) ( ) ( 11 ) ( ) ( ) .MAIN
340 MIN = YI ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
350 11 YI - MAX ( 10 ) ( ) ( ) ( 10 ) ( ) .MAIN
355 MAX = YI ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
360 10 YI = YIP1 ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
370 [I = I + 1] - ITRX ( * ) ( ) ( ) ( * ) ( ) .MAIN
380 SL 1 ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
390 ### PLOT OF P(X) VS B*X ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
400 PV MIN,SKIP,SKIP,SKIP,SKIP,MAX$ ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
410 PL -----+1
420 ###-----2
430 B*X P(X) ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
440 I = 0 ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
445 * PLOT.[X[II],P[II],MIN,MAX] ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
460 [I = I + 1] - ITRX ( * ) ( ) ( ) ( * ) ( ) .MAIN

470 C = C +DLX ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
472 R = SQRT.[FX*FX + FY*FY] ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
473 PHI = ATAN.[FY/FX]/RAD ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
475 PVL,,FX,FY,R,PHI$ ( ) ( ) ( ) ( ) ( ) ( ) .MAIN
480 [IC = IC + 1] - ITRC ( *** ) ( ) ( *** ) ( ** ) .MAIN
490 END ( ) ( ) ( ) ( ) ( ) ( ) .MAIN

```

FIGURE 19. LONG-BEARING-ANALYSIS PROGRAM LISTING

FIGURE 20. SAMPLE PRINT-OUT OF LONG-BEARING-ANALYSIS PROGRAM

Table IV

<u>a</u>	<u>b</u>	<u>R</u>	<u>β</u>	<u>ζ/χ^2</u>
0.1	0.01	4.91	- 88.8°	0
0.5	0.01	17.96	- 88.2°	0
0.9	0.01	21.29	- 88.07°	0
0.1	0.01	7.07	-132.67°	1
0.5	0.01	44.52	-153.00°	1
0.9	0.01	529.60	-177.00°	1
0.5	0.001	20	-101.00°	1
0.5	0.005	30	-146.00°	1

whereas $b = 0.001$ is a realistic film thickness). $\zeta/\chi^2 = 0$ when inertia is neglected and is approximately unity in the actual case. R is the resultant force and β is the angle at which it acts. For the moment there need be no concern with its actual magnitude, but comparing the second series of calculations with the first, note that inertia considerably alters the force resultant. Furthermore, it swings the force to an almost 180° relationship with displacement. For inertia neglected in the thin film or for inertia included in the thicker films the force acts more nearly at 90° to the displacement as might be expected.

D. Short-Bearing Equations

Our experimental apparatus represents the "short bearing" regime of film behavior since the L/D ratio is approximately 0.3. Thus, it is pertinent, in the paragraphs which follow, to set up equations especially for turbulent film behavior applicable exactly to the limiting case of $L/D \rightarrow 0$, and presumably applicable with diminishing accuracy as L/D becomes larger.

1. Pressure Gradient

The short-bearing limiting case may be characterized by a tangential Couette flow and an axial pressure flow. The combined flow may be characterized by a friction coefficient acting on each wall of the type

$$\tau_{\text{wall}} = C_f \rho V^2 / 2 \quad (38)$$

where

$$V \equiv \sqrt{w_b^2 + u_b^2}$$

Note that the symmetry of the flows (at least in the limiting case of $b/r \ll 1$, is such that V would bear the same magnitude relative to either surface. The component of wall shear stress opposing the pressure flow would be (on the assumption that τ_{wall} is codirectional with V) $\tau_{\text{wall}} \frac{w}{V}$. Allowing for the fact that such codirectionality may not prevail, and that C_f may depart from C_{fc} which would prevail for the Couette flow, we may introduce a factor ϕ to be evaluated experimentally, and write

$$-h \frac{\partial p}{\partial z} = \frac{2\phi C_{fc} \rho V^2}{2} \frac{w}{V} \quad (39)$$

Recognizing that w and V are orthogonal

$$2C_{fc} \rho u^2 \left(\frac{w}{u} \right) \sqrt{1 + \left(\frac{w}{u} \right)^2} = -\frac{h}{h_o} \left(\frac{\partial p}{\partial z / h_o} \right) \quad (40)$$

One approach to nondimensionalization of this equation is provided by the following definitions; $P \equiv p / (C_{fc} \rho u^2 / 2)$; $H \equiv h / h_o$; $Z \equiv z / h_o$; $W \equiv w / u$, and

$$2\rho W \sqrt{1 + W^2} = -H \frac{\partial P}{\partial Z} \quad (41)$$

2. Continuity Considerations

If pressure flow in the tangential direction is assumed to be zero, the continuity equation for an incompressible flow in a lubricant film is

$$zu \frac{dh}{dx} = wh \quad (42)$$

where it is assumed that $\partial h / \partial z = 0$.

In the above dimensionless terminology, this becomes

$$\frac{Z}{H} \frac{dH}{dX} = -W \quad (43)$$

Substituting back into Equation (41)

$$2\phi \frac{Z}{H} \frac{\partial H}{\partial X} \sqrt{1 + \left(\frac{Z}{H} \frac{\partial H}{\partial X} \right)^2} = H \frac{\partial P}{\partial Z} \quad (44)$$

3. Integration of Film Equation

Let

$$a \equiv \frac{1}{H^2} \frac{\partial H}{\partial X} \quad \text{and} \quad b \equiv \left(\frac{\partial H}{\partial X} \right)^2 \frac{1}{H^2} \quad (45)$$

In terms of this, Equation (44) becomes

$$\phi \left(\frac{a}{b} \right) 2bZ \sqrt{1 + bZ^2} = \frac{\partial P}{\partial Z} \quad (46)$$

4. Trial Approximation

Integrating Equation (46) with respect to Z (assuming ϕ constant)

$$\frac{2}{3} \phi \left(\frac{a}{b} \right) \left(1 + bZ^2 \right)^{3/2} = P + f(x) \quad (47)$$

For a bearing of uniform width, $p \equiv 0$ at $Z = Z_m$. Thus

$$f(x) = \phi \frac{2}{3} \left(\frac{a}{b} \right) \left(1 + bZ_m^2 \right)^{3/2} \quad (48)$$

Continuing Equation (38) and Equation (48), and substituting for a and b

$$P = \frac{2}{3} \phi \left(\frac{1}{dH/dX} \right) \left\{ \left[1 + \left(\frac{dH}{dX} \right)^2 \frac{Z^2}{H^2} \right]^{3/2} - \left[1 + \left(\frac{dH}{dX} \right)^2 \frac{Z_m^2}{H^2} \right]^{3/2} \right\} \quad (49)$$

If $H = 1 + \epsilon \sin \beta X$, then $\frac{dH}{dX} = \beta \epsilon \cos \beta X$, and near the point where the measured pressure maximum was indicated $\cos \beta X = 1$. This maximum value of dH/dX would also locate the maximum of P in Equation (49).

Thus at $Z = 0$, the additional condition for locating the maximum pressure,

$$-\frac{2\phi}{3(\beta\epsilon)} \left[1 + (\beta\epsilon) \left(\frac{Z_m}{H} \right)^2 \right]^{3/2} = P_{\max} \quad (50)$$

To evaluate this further note that $X = x/h_o$, and $\beta X = 2\pi$ when $x/h_o = \frac{2\pi r}{h_o}$ for the bearing. Thus

$$\frac{\beta X}{2} = \frac{x/h_o}{2r/h_o} = \frac{X}{2R} \quad (51)$$

or

$$\beta = 1/R \text{ where } R = r/h_o \quad (52)$$

In the present bearing-model experiments $\epsilon = 0.5$ and $r \doteq 3$ ft, and

$$Z_{\max} = 5/6 \text{ ft}$$

$$4 \left[1 + \left(\frac{1}{6} \right) \left(\frac{5}{6} \right)^2 \right]^{3/2} = P_{\max} \quad (53)$$

Thus

$$\phi 4(1.174) = 4.7 = P_{\max} \quad (54)$$

Remembering that $P_{\max} \equiv p_{\max}/(C_{fc}\rho u^2/2)$ then, for the measured $P_{\max}$

$$\phi C_{fc} = \frac{P_{\max}}{2.35 \rho u^2} \quad (55)$$

since $p_{\max}/\rho u^2 \doteq 300 \times 10^{-5}$, $C_{fc} \doteq 1.27 \times 10^{-3}$. Though this does not correspond precisely to the C_f for Couette flow it is sufficiently near the proper range to indicate reasonable accuracy for the analysis.

In the above analysis, it may be observed that in our existing configuration the second term in the bracket is indeed relatively small, [Eq (50)], indicating that the axial flow is, truly, small in relation to the Couette flow. In the next quarterly report we shall take advantage of this simplification and examine the full three dimensional problem, when pressure flow is weak, but may act both tangentially and axially. In addition, further experimental data will be obtained to relate local wall shear stress to the components of the velocity profile - which in the long run will provide more information for general analyses.

IV. CONCLUSIONS

We conclude that the chosen experimental technique is, indeed, workable. It has worked in the preliminary stages and provided useful and provocative results.

We conclude also that it is possible to set up analytical procedures for bearing analysis, which result in single differential equations with only a small number of empirical parameters to be evaluated from our experimental measurements.

V. FUTURE PROGRAM

Analytical work will continue with emphasis on the flows expected in the moderate-length bearing with end leakage. The partial bearing derivations with initial "impact pressure" at entrance will be reported, and initial work on coupled flows in the vortex-flow regime will be undertaken.

VI. REFERENCES

1. Reichardt, H., "Über die Geschwindigkeitsverteilung in einer geradlinigen turbulenten Couette strömung, Z., Angew, Math and Mech, Sonderheft, 1956, pp 526-529.
2. Robertson, J. M., "On Turbulent Plane-Couette Flow, " 6th Mid-western Conference on Fluid Mechanics, University of Texas, 1959.
3. Schlichting, H., Boundary Layer Theory, McGraw Hill Book Co. Inc., New York, 1960, p 505.
4. Schlichting, H., Op. Cit., p 507.
5. Reichardt, H., "Vollständige Darstellung der Turbulenten Geschwindigkeitsverteilung in glatten Leitungen, " Z. Angew Math and Mech, 31, No. 7, July, 1951, pp 208-214.
6. van Driest, E. R., "On Turbulent Flow Near a Wall, " Journal of the Aeronautical Sciences, 23, No. 11, Nov. 1956, pp 1007-1011.
7. Miles, J. W., "On the Velocity Profile for Turbulent Flow Near a Smooth Wall, " Journal of the Aeronautical Sciences, 24, Sept. 1957, p 704.
8. Szablewski, W., "Berechnung der turbulenten Strömung in Rohr auf der Grundlage der Mischungsweghypothese, " Z. Angew. Math and Mech, 31, No 4/5, April/May, 1951, pp 131-138.
9. Ng, C. W. and Pan, C. H. T., "A Linearized Turbulent Lubrication Theory, " Paper, International Lubrication Conference, Oct 1964.