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Electron Temperatures and Thermal Conduction in High-Energy Θ Pinches

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ELECTRON TEMPERATURES AND THERMAL CONDUCTION IN HIGH-ENERGY θ PINCHES

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ABSTRACT

The common occurrence of electron temperatures of a few hundred electron volts in θ -pinch experiments is shown by simple analytic arguments to be rather tightly determined by a balance between energy gained from collisions with the much higher temperature ions and thermal conduction to material walls. It is also shown that electron thermal conduction should not cause a greater loss of energy from straight θ pinches of any length than ion end loss unless rather large magnetic mirror ratios can be obtained. This applies in particular to straight θ -pinch fusion feasibility experiments and fusion reactors.

Electron temperatures between 550 and 600 eV are predicted for the new 5-meter linear θ pinch now being built at The Los Alamos Scientific Laboratory.

I. INTRODUCTION

It is generally observed that in θ pinches which are sufficiently energetic to produce ion temperatures of the order of one thousand electron volts (1 keV) or more, electron temperatures remain almost constant in time during the plasma lifetime, which is typically from three to eight microseconds.

It has been conventionally assumed that in these experiments the temperatures are sufficiently high that the electron-ion energy equalization time is much longer than the plasma lifetime and hence that the electrons are born colder than the ions and just stay there. However, as the following simple arguments and resulting scaling laws show, this is far from the truth.

II. ELECTRON ENERGY BALANCE

From Spitzer¹ the electron-ion energy equilibration proceeds according to

$$\frac{dT_e}{dt} = \frac{T_i - T_e}{t_{eq}} \quad (1)$$

As long as electron thermal velocity is much larger than that of the ions (which is just about always), then

$$t_{eq}(\text{sec}) = \frac{3.16 A_i T_e^{3/2}}{n \ln \Lambda} \times 10^8 \quad (2)$$

Here A_i is the atomic weight of the ions, n is the electron density per cm^3 , and $\ln \Lambda$ is the Debye logarithm which is between 12 and 13 for most θ -pinch plasmas. The heating of the electrons as described by Eqs. (1) and (2) competes with cooling by electron thermal conduction along magnetic field lines to the material walls at the ends of the pinch tube.

The electron thermal conductivity given by Spitzer¹, when the various corrections are multiplied in, is

$$K = \frac{2.3 T_e^{5/2}}{\ln \Lambda} \times 10^2 \left(\frac{\text{Joules}}{\text{sec cm eV}} \right) \quad (3)$$

There are additional factors when the ion charge is not e . In a region where the heat flux, Q , can be regarded as constant in Z , the coordinate along the field lines, a solution to the equation

$$Q = \text{const} = K \frac{dT_e}{dZ} \quad (4)$$

is

$$T_e = T_{el} \left(\frac{Z}{l_1} \right)^{2/7} \quad (5)$$

The point $Z = 0$ is the wall where $T_e = 0$. The point $Z = l_1$, where $T_e = T_{el}$, is arbitrary. Because T_e rises rapidly near the wall and then flattens out, a reasonable temperature profile near the ends of the pinch tube is obtained by taking T_{el} to be the mean electron temperature of the pinch and l_1 , the distance from the end wall to the point where $T_e = T_{el}$, which is a sort of thermal sheath thickness, to be about 20% of the tube length L (i. e., 40% of the distance from an end to the middle of the pinch tube). The results turn out not to be sensitive to this choice. The real $T_e(Z)$ profile will of course flatten out a bit more toward the center of the tube, where $Z = L/2$, and be flat there and symmetrical about that point, but this is a good approximation for the purposes of this paper. Keeping the l_1 notation for now, and substituting Eqs. (3) and (5) into (4) gives the heat flux out of either end

$$Q_{\text{end}} = \frac{66. T_{el}^{7/2}}{l_1 \ln \Lambda} \left(\frac{\text{Joules}}{\text{cm}^2 \text{ sec}} \right) \quad (6)$$

When $T_i > T_e$ then the rate of electron heating by ions, Eq. (1), depends almost entirely on T_i . The rate of change of the total electron energy, $W_e = \frac{3l_2}{2} nkT_e$, over some length l_2 (usually the pinch tube half length, $L/2$, in what follows) is then given by

$$\frac{dW_e}{dt} = \frac{3n^2 l_2 k T_i}{2 t_{eq}} = \frac{7.65 n^2 l_2 T_i T_e (\text{eV}) \ln \Lambda}{A_i} \times 10^{-28} \left(\frac{\text{Joules}}{\text{sec cm}^2} \right) \quad (7)$$

Figure 1 illustrates the various lengths involved. Equating Q_{end} and dW_e/dt , Eqs. (6) and (7), and thus assuming an equilibrium between ion heating and conduction cooling of electrons, gives

$$T_e (\text{eV}) = \left[\frac{1.16 n^2 (\text{cm}^{-3}) l_1 l_2 (\text{cm}) T_i (\text{eV}) (\ln \Lambda)^2}{A_i} \times 10^{-29} \right]^{1/5} \quad (8)$$

To understand the meaning of Eq. (8) we choose some typical Scylla IV parameters:²

$$\begin{aligned} n &= 3 \times 10^{16} \text{ cm}^{-3} \\ L &= 10^2 \text{ cm} \\ l_1 &= 0.2 L = 20 \text{ cm} \\ l_2 &= 0.5 L = 50 \text{ cm} \\ T_i &= 3 \times 10^3 \text{ eV} \\ A_i &= 2 \text{ (deuterium)} \end{aligned}$$

Substituting into Eq. (8) gives $T_e = 306 \text{ eV}$. This is very close to the electron temperature measured in Ref. 1, i. e., 295 to 300 eV.

Fig. 1. Schematic diagram showing the various lengths in the theory.

The temperatures seen in references 3-6, and indeed in all other high-energy θ pinches of which the author is aware, differ from this by no more than $\pm 50 \text{ eV}$. This is consistent with the important fact that the $1/5$ power dependence of T_e on the various parameters on the right-hand side of Eq. (8) makes T_e rather insensitive to changes in these

parameters of the magnitude of the differences between the various experiments.

Substituting these parameters and $T_e = 300$ eV back into Eq. (2) gives $t_{eq} = 9 \times 10^{-6}$ sec. This is longer than the plasma lifetime in most experiments, but, because the electron energy is only about one tenth of the ion energy, it only takes about one tenth of this time, or $0.9 \mu\text{sec}$, for the electron temperature equilibrium between ion heating and conduction cooling to be established.

Hence, the assumption that this equilibrium is reached and exists during most of the plasma lifetime seems justified for the experiments considered. On the other hand the ions will be cooled at the rate t_{eq} while, from Eq. (8), T_e remains nearly constant, until T_i falls to something of the order of T_e . At this time the T_e term in Eq. (1) must be included, T_i will merge with T_e and the behavior of the system changes to that of very long systems described below.

III. SCALING OF SHORT OR HIGH-TEMPERATURE θ PINCHES

Simple scaling laws can be derived from these results. Since the ratios between l_1 , l_2 and L can be taken to be constant, at least for long pinches with no field line extension beyond the end of the pinch coil, then, all other things being equal, from Eq. (8)

$$T_e \propto L^{2/5} T_i^{1/5} n^{2/5} \quad (9)$$

Also since $t_{eq} \propto T_e^{3/2}$, then

$$t_{eq} \propto L^{3/5} T_i^{3/10} n^{-2/5} \quad (10)$$

The effects of these scalings should be observable from comparisons between data from Scylla IV² for which $L = 1$ meter and the 5 meter straight θ -pinch which is now nearing completion at Los Alamos. The scaling of T_e with length, Eq. (9) above, agrees well with the measured values of

T_e in References 2-6, as can be most conveniently seen from Table II in Ref. 7. The values of T_i are not given there, but they are large enough and not very different, and beyond that do not matter much because of the weak $T_e \propto T_i^{1/5}$ dependence.

IV. LONG θ PINCHES

As L is increased for a given T_i , T_e increases until T_e approaches T_i , and again the T_e term on the right-hand side of Eq. (1) must be included.

The length at which this cross over from short to long behavior occurs is greater if T_i is larger. If we take this condition, somewhat arbitrarily, to be that at which Eq. (8) would predict that $T_e = T_i/2$, then for the parameters of the example above ($T_i = 3$ keV, $n = 3 \times 10^{16} \text{ cm}^{-3}$) the scaling law for T_e , Eq. (9), gives $L \approx 56$ m. On the other hand if $T_i = 10$ keV, which might be more reasonable for a fusion feasibility experiment, then this condition is met when $L \approx 620$ meters. When L is sufficiently longer than this cross over value then T_i and T_e stay together over most of the length of the system, and the time dependence of this common temperature, $T = T_e = T_i$, and the total energy, $W = (2 \times 3/2) n l_2 kT$, is determined from

$$\frac{dW}{dt} = Q_{\text{end}} \quad (11)$$

Substituting the above definitions and Eq. (6) for Q_{end} into Eq. (11) gives

$$\frac{1}{T} \frac{dT}{dt} = \frac{22 T^{5/2} (\text{eV})}{n l_1 l_2 l_3 \Lambda \times 1.6 \times 10^{-19}} \quad (12)$$

Hence the time for the long system to lose its energy by thermal conduction is approximately

$$t_{\text{con.}} = \frac{n l_1 l_2 l_3 \Lambda \times 7.3 \times 10^{-21}}{T^{5/2} (\text{eV})} \text{ sec.} \quad (13)$$

Assuming, as before, a constant ratio between l_1 and l_2 , the long θ -pinch thermal loss scales according to

$$t_{\text{con.}} \propto L^2 T^{-5/2} n. \quad (14)$$

This is very different from the scaling for the ion thermal loss time for short θ pinches, Eq. (10). Equation (13) was obtained by Green et al.,⁷ but probably does not apply to any of the experiments discussed in Ref. 7 because they are all somewhat shorter than the cross over length of $L \approx 56$ meters for $T_i \approx 3$ keV. From here on we refer to both $t_{\text{eq.}}$ and $t_{\text{con.}}$ as the thermal loss time, $t_{\text{t.l.}}$, meaning whichever is applicable; that is

$$t_{\text{t.l.}} = \begin{cases} t_{\text{eq.}}, & \text{equation 2, } L \text{ short} \\ t_{\text{con.}}, & \text{equation 13, } L \text{ long} \end{cases} \quad (15)$$

It is now interesting to compare these thermal conduction cooling times with the time it takes the plasma to stream out of the open ends of a θ pinch. This time is the time required for a thermal ion to travel the distance, l_2 , from the middle to the end of the system or for a rarefaction moving with the acoustic speed to travel from the end to the middle, depending on whether the ion mean free path is longer or shorter than l_2 . In either case, this streaming loss time is approximately

$$t_{\text{s.l.}} = \frac{7.2 l_2 (\text{cm})}{(T_i (\text{eV}) / A_i)^{1/2}} \times 10^{-7} \text{ sec}, \quad (16)$$

$$t_{\text{s.l.}} \propto L T_i^{-1/2}.$$

Since for a given density and ion temperature the loss time, $t_{\text{t.l.}}$, scales like $L^{3/5}$ up to the cross over and then like L^2 , while the streaming loss time, $t_{\text{s.l.}}$, scales like L all the way, then if $t_{\text{t.l.}} < t_{\text{s.l.}}$ for any length it will be near the cross over length. The ratio $t_{\text{s.l.}} / t_{\text{t.l.}}$ near cross over can be estimated by first substituting the (arbitrary) relationship $T_e = T_i / 2$ into equation (8) to obtain the general cross over condition,

$$(l_1 l_2)^{1/2} = \frac{5.2 T_i^2 A_i^{1/2}}{n \ln \Lambda} \times 10^{13}, \quad (17)$$

and then substituting this relationship into the ratio of Eqs. (16) and (2) for $t_{\text{s.l.}}$ and $t_{\text{t.l.}}$ ($t_{\text{eq.}}$). Almost everything cancels out and the result is

$$\frac{t_{\text{s.l.}}}{t_{\text{t.l.}}} = 0.38 \left(\frac{l_2}{l_1} \right)^{1/2}, \quad (18)$$

and since we have been taking $l_1 = 0.4 l_2$, the right-hand side is about 0.6. By now we are at the end of a long string of approximations such that equation (18) is really only a qualitative result. For example if we had used $T_e = 2/3 T_i$ as the cross over condition instead of $T_e = \frac{1}{2} T_i$, then the 0.38 in Eq. (18) would have been 0.78. However, the qualitative conclusion is that thermal conduction loss is of the order of or less than free streaming end loss at the worst, i.e., near the short-long cross over length. When L is less than cross over, as in all contemporary high temperature θ pinches, or longer than cross over, as a fusion reactor would be at a length of over one kilometer, then electron thermal conduction to the end walls should cause distinctly less energy loss than free streaming or expansion out of the ends.

V. DISCUSSION

It should be noted that as energy loss mechanisms free streaming end loss and thermal conduction do not always have the same local effect on the confined plasma. If the pinch tube is long compared to the ion-ion mean free path, then end loss will cause a rarefaction to propagate into the plasma which in addition to decreasing density also decreases T_e and T_i , though not with exactly the same time and space dependence as the temperature decrease caused by thermal conduction. The end loss is not felt in the middle of the pinch tube until the rarefaction has had time to propagate from the ends at the acoustic speed. On the other hand the electron thermal conduction cooling is felt everywhere almost instantly. This

can have a significant effect on the neutron signal because the neutron production rate is $n^2 \langle \sigma v \rangle$ and near $T_i = 3$ keV where most θ pinches operate now $\langle \sigma v \rangle$ varies approximately like T_i^4 for the DD fusion reaction and faster for DT (the dependence is not as strong at fusion reactor temperatures). Therefore, a small decrease in T_i , say 10 to 20%, in the first few microseconds after peak compression is reached will cause a dramatic drop in the neutron flux. The difference between the effects of end loss and thermal conduction are even clearer when the ion-ion mean free path is longer than the pinch tube, as is the case when $T_i \gtrsim 3$ keV, $L = 1$ meter and $n \sim 3 \times 10^{16}$. Then the initial effect of end loss, which still essentially must wait on a thermal ion transit time, is to reduce n only, whereas conduction reduces T_i . Hence, the initial drop in $n^2 \langle \sigma v \rangle$, especially at present θ -pinch temperatures, is caused almost entirely by conduction and must come sooner than a calculation of the effect of ion loss alone would predict.

Where thermal loss is much slower than streaming loss, it is worthwhile to look for ways of reducing streaming loss. Static magnetic mirrors appear to be capable of reducing end loss⁸ by perhaps a factor of five. The limits here are on the magnitude of magnetic field that coil materials will withstand and the average value of β (ratio of plasma pressure to field pressure) in the plasma. The latter is limited by enhanced diffusion in present θ pinches, but theoretical work on micro turbulence now in progress suggests that if the diameter of the plasma is significantly increased (by say a factor of five) then a large inner region of very high β may occur. This would assist end mirrors in reducing streaming loss. If static mirrors are still not sufficient to reduce streaming loss to a point where thermal loss is comparable, then there is another type of approach which has been discussed at Los Alamos in recent years. If the strength of a long mirror

with a length of the order of the ion-ion coulomb mean free path is made to oscillate with a period of the order of the ion-ion collision time then a large fraction of the ions which are mirrored will be thrown back into the plasma with increased axial velocities. The effect of these ions will be to scatter off of other ions moving toward the mirror and increase the spread of perpendicular velocities of these ions and thus reduce the number of ions going into the mirror whose velocities lie in the loss cone. This approach has something in common with transit time pumping and with the moving mirror proposal of Tuck,⁹ except that there, binary collisions did not play a role.

Should it turn out with the addition of strong mirrors and other end flow stopping schemes that thermal conduction is the major remaining energy loss mechanism from a fusion reactor of some desired length then some scheme for reducing thermal conduction would be needed. Mirrors reduce the cross section of a flux tube and therefore increase Q and dT/dZ . However, a rather long high field region would be required to have a significant effect and this would be too expensive unless cryogenic coils improve considerably. The most obvious scheme is to extend the magnetic field lines beyond the end of the regular pinch tube by at least the length L_2 again, and to puff the filling gas into the pinch tube so that this extended region is initially evacuated. Then L_1 is effectively increased by this extension and there is no cold gas in this region to soak up heat. Also if the vacuum is good, thermal conduction through this region may initially be impossible. Electron thermal conduction is independent of density only as long as the effective velocity of thermal transport is small compared with the electron thermal velocity, and this must break down when the density becomes sufficiently small. This line extension scheme could be further improved by taking the extended field lines at a reduced flux density (and hence reduced cost) and closing them in

a sort of race track geometry (see Fig. 2). This would then be toroidal confinement for electron thermal conduction losses, but would make no effort to confine the plasma in the toroidal geometry once free expansion down the straight field line extensions causes a significant plasma density build up at the bends in the race track. The race track scheme would also tend to eliminate end shorting and the resulting rotational instabilities.¹⁰

Fig. 2. Racetrack scheme for reducing thermal loss by closing the magnetic field lines which pass through the plasma.

We could attempt quantitative estimates of the various loss reducing schemes, but it is really pointless to try to patch these things, and other effects which are needed for physical reality, into the simple scaling approach of this paper. The next step in this problem must be a fully space and time dependent numerical treatment. William Gula of this Laboratory has started work on such a numerical treatment.

The author has a long standing affection for the straight θ pinch, both as an experimental tool and a potential fusion reactor and suspects that if as much thought and effort were put into minimizing the end effects in straight θ pinches as has gone into making the various toroidal containment schemes behave, then the results might be at least as rewarding.

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