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FINAL REPORT ON TEMPERATURE AND STRESS DISTRIBUTIONS
PRODUCED IN LONG CIRCULAR CYLINDERS
COOLED BY EMISSION OF THERMAL RADIATION

Technical Report

ME-19

RELEASED FOR ANNOUNCEMENT
IN NUCLEAR SCIENCE ABSTRACTS

By

Robert S. Pritchard

Maurice W. Wildin

Prepared for Sandia Corporation
Under P.O. 53-0196, Task 4

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FOREWORD

Part I of this report is the master's thesis of Robert S. Pritchard. It presents the experimental results obtained for transient temperature and strain distributions in long solid circular cylinders cooled by emission of thermal radiation. It compares these results with the analytical results reported earlier in reference (1). The equipment and techniques, as well as the auxiliary measurements, used to obtain the experimental results are described in Part I.

Relatively good agreement is found to exist between the experimental and analytical results, as concluded in Chapter 7 of Part I. Due to this agreement, it is felt that a useful answer can be deduced for the problem which motivated the investigation reported in references (1,3) and this report. The evidence and logic which lead to this answer are contained in references (1,3) and Part I of this report. The interested reader may wish to read these reports and arrive at an answer for himself. However, to conserve the reader's time, Part II of this report gives a statement of the problem, an outline of the logic and evidence which lead to an answer, and the answer which we feel follows from this logic and evidence.

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PART I

TEMPERATURE AND STRAIN DISTRIBUTIONS PRODUCED
IN LONG CIRCULAR CYLINDERS COOLED BY EMISSION
OF THERMAL RADIATION

ABSTRACT

This study involves the determination of the temperature and stress distributions produced in long circumferentially symmetric circular cylinders cooled by emission of thermal radiation from the lateral surface.

Temperature and strain responses obtained from an experimental model are presented for a set of parameters which correspond to the highest strain level which could be accurately determined with the strain gages. These responses are in good agreement with the results obtained by solution of an appropriate mathematical model.

Temperature responses obtained from the same model, but having a higher initial temperature, are also presented and compared with analytical results. Reasonably good agreement between experimental and theoretical results was obtained for this set of parameters.

To specify parameters in the mathematical model, certain material properties were needed. Testing to determine thermal conductivity using a transient technique is described. The lateral surface of the model was coated with Krylon Flat Black Spray Enamel to increase the cooling rates, and therefore the stress levels obtained. Testing to determine the value of total hemispherical emittance is also described.

Testing to determine gage factor and false strain was performed to determine these characteristics when the strain gages were embedded in the modelling material.

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Thanks also to Richard Rhorer, William Sedlock, Gaylord Mingo, and Joseph Amode who helped at various times with the experimental work, and to Donald Snyder who drew most of the figures, and to other members of the Mechanical Engineering Department who contributed in various ways to the completion of this work.

A very special thanks to my wife, Barbara, for her patience and understanding during the many months needed to complete the study.

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1.0 INTRODUCTION

1.1 Scope

The scope of this research was to experimentally determine the temperature and stress distributions produced in long circumferentially symmetric circular cylinders cooled by emission of thermal radiation from the lateral surface. The work was initiated to determine if a cylindrical uranium-zirconium fuel rod could be caused to fail by suddenly exposing it to conditions in outer space. It was thought that if the fuel rod were exposed to such an environment while at high enough temperature to cause it to cool rapidly, the thermal stresses produced might be large enough to cause failure. Initial phases of the investigation were completed by Wildin and Putman (1,2,3).¹

This phase of the investigation was divorced from the original question because it was not a fracture study. It was a series of nondestructive tests to experimentally determine temperature and stress distributions produced in a model with the outer space environment simulated in the laboratory. These results were used to verify the temperature and stress distributions predicted by analytical solution in an earlier phase of this investigation.

¹Numbers in parenthesis refer to references at the end of this thesis. See page 100.

1.2 Experimental Work

A cylindrical model, long enough to negate end effects near the axial center, was maintained at a constant, uniform initial temperature. Then, while under sufficiently high vacuum to ensure that radiation was the only significant mode of heat transfer, the lateral surface was exposed to a heat sink at liquid nitrogen temperature. By surrounding the model with a low temperature heat sink, insignificant radiant energy fell on it, thereby simulating the environment assumed in the analytical investigation. As the model cooled, temperature and strain at important positions in the model were monitored to determine the responses as functions of time. The temperature and strain responses at these locations were then compared to the corresponding results obtained from the analytical solution. The mathematical problem from which the analytical solution was obtained is described in Chapter 2.

To evaluate parameters in the boundary value problem, certain material properties had to be known. Testing to determine thermal conductivity and total hemispherical emittance, which were not known at the beginning of this investigation, is described in Appendix A.

Information on design of the model and methods used to embed the thermocouples and strain gages is presented in Chapter 3. Instrumentation used to detect thermocouple and strain gage outputs is described in Chapter 4.

Certain auxiliary equipment was either acquired or developed to simulate the prototype environment. This apparatus is described in Chapter 5. The manner in which the equipment was used to perform the tests is presented in Chapter 6.

Strain gages chosen for use in this testing had not previously been used in the Mechanical Engineering Department. Extensive work was done to determine characteristics of the gages so that accurate strain data would be obtained. These tests are described in Appendix B.

In Chapters 7, 8 and 9, the Results, Conclusions, and Recommendations, respectively, are presented. The manner in which data were reduced to obtain the results is discussed in detail to familiarize the reader with areas in which errors may occur. Experimental temperature and strain responses are presented in graphical form and compared to the corresponding results predicted by the mathematical model. Results of the testing showed that the experimental modelling techniques were satisfactory for use in transient thermal stress studies. For a set of parameters corresponding to $N_R=0.099$ (see page 6), temperature and strain responses in the experimental model were in good agreement with theoretical results. It is concluded that the theoretical solutions may be used to predict the temperature and stress distributions for parameters corresponding to $N_R \leq 0.099$. Testing at a higher temperature level produced reasonably good agreement between experimental and

theoretical temperature responses. Material properties were not known at the higher temperatures, and this fact led to the conclusion that deviations between experimental and theoretical temperature responses were caused by inaccurate prescription of parameters in the mathematical model.

2.0 THEORY2.1 Analytical Solution

Temperature and stress distributions produced in infinitely long circular cylinders cooled by emission of thermal radiation having circumferential and axial symmetry have been predicted by Wildin and Putman (1). The equations corresponded to the maximum rate of cooling which would be encountered by the fuel rod in the prototype environment for a given initial temperature because radiant energy incident on the surface of the fuel rod was neglected.

Temperature distributions were obtained by considering the boundary value problem presented at the end of this paragraph. The equations have been presented here in dimensionless form, assuming constant material properties. The validity of this assumption is discussed in section 7.2.

$$\frac{1}{\xi} \frac{\partial}{\partial \xi} \left(\xi \frac{\partial \varphi}{\partial \xi} \right) = \frac{\partial \varphi}{\partial \tau}$$

$$\varphi(\xi, 0) = 1$$

$$\frac{\partial \varphi}{\partial \xi} (0, \tau) = 0, \quad \tau \geq 0$$

$$\frac{\partial \varphi}{\partial \xi} (1, \tau) = -N_R \varphi^4(1, \tau), \quad \tau \geq 0$$

Variables were defined as:

φ	$= T/T_i$	: dimensionless temperature
ξ	$= r/a$	: dimensionless radial position
τ	$= \frac{kt}{\rho ca^2}$	: dimensionless time
N_R	$= \frac{\epsilon \sigma a T_i^3}{k}$	: dimensionless parameter, constant
T		: temperature ($^{\circ}\text{R}$)
T_i		: initial temperature ($^{\circ}\text{R}$)
r		: radial position (ft)
a		: outer radius (ft)
t		: time (hr)
k		: thermal conductivity (Btu/hr-ft- $^{\circ}\text{R}$)
ρ		: density (lb/ft ³)
c		: specific heat (Btu/lb- $^{\circ}\text{R}$)
ϵ		: total hemispherical emittance
σ	$= 0.1714 \times 10^{-8}$	: Stefan-Boltzmann constant (Btu/hr-ft ² - $^{\circ}\text{R}^4$)

The nonlinear boundary condition made the problem too difficult to solve in closed form, except for small changes in surface temperature. Finite difference techniques were used, and temperature distributions were calculated by digital computer. Development of the finite difference equations is presented in reference (1, Appendix A).

Equations used to predict stress distributions were presented in reference (1) for material properties which varied with temperature. By assuming that material properties were not dependent upon temperature, dimensionless stress equations reduced to:

$$\Sigma_z = 2 \int_0^1 \varphi \xi d\xi - \varphi$$

$$\Sigma_r = \int_0^1 \varphi \xi d\xi - \frac{1}{\xi^2} \int_0^\xi \varphi \xi d\xi$$

$$\Sigma_\theta = \int_0^1 \varphi \xi d\xi + \frac{1}{\xi^2} \int_0^\xi \varphi \xi d\xi - \varphi$$

where

$$\Sigma_j = \frac{(1-\nu)\sigma_j}{\alpha T_1 E}, \quad j=z, r, \theta: \text{ dimensionless stress}$$

$\sigma_j, j=z, r, \theta$: stress (psi)

ν : Poisson's ratio

E : Young's Modulus (psi)

α : coefficient of thermal expansion ($^{\circ}\text{R}^{-1}$)

Dimensionless stresses were evaluated on a digital computer by numerical integration using the dimensionless temperature distributions obtained from the previously-described boundary value problem.

To reduce dimensionless stress to mechanical strain so that the analytical solutions might be compared to the experimental data, the following constitutive relationships were used:

$$E_z = \Sigma_z$$

$$E_r = \frac{\Sigma_r}{1-\nu} - \frac{\nu}{1-\nu} (\Sigma_\theta + \Sigma_z)$$

$$E_\theta = \frac{\Sigma_\theta}{1-\nu} - \frac{\nu}{1-\nu} (\Sigma_r + \Sigma_z)$$

where

$$E_j = \frac{\epsilon_j}{\alpha T_i}, \quad j=z, r, \theta : \text{dimensionless strain}$$

$$\epsilon_j, \quad j=z, r, \theta : \text{mechanical strain}$$

Values for the temperature and strain responses for the parameters used in the experimental work are presented in Chapter 7, Figures 7-20.

2.2 Approximate Temperature Distribution for Small Surface Temperature Changes

Wildin (3, Part II) obtained a solution in closed form for the temperature distribution accompanying small changes in surface temperature by expanding the nonlinear expression appearing in the boundary condition in a Taylor series about the initial temperature, neglecting higher order terms, and thereby linearizing the problem. This approximation took

the form:

$$\varphi^4(1, \tau) \cong \varphi^4(1, 0) + 4\varphi^3(1, 0) [\varphi(1, \tau) - \varphi(1, 0)]$$

Using the initial condition and substituting the approximation into the boundary condition, the boundary value problem describing the temperature field reduced to:

$$\frac{1}{\xi} \frac{\partial}{\partial \xi} \left(\xi \frac{\partial \varphi}{\partial \xi} \right) = \frac{\partial \varphi}{\partial \tau}$$

$$\varphi(\xi, 0) = 1$$

$$\frac{\partial \varphi}{\partial \xi}(0, \tau) = 0, \quad \tau \geq 0$$

$$\frac{\partial \varphi}{\partial \xi}(1, \tau) = 4N_R [1 - \varphi(1, \tau)]^{-N_R}, \quad \tau \geq 0$$

By using superposition, the solution was found in Carslaw and Jaeger (6). The solution became:

$$\varphi(\xi, \tau) = \frac{3}{4} + 2N_R \sum_{n=1}^{\infty} \frac{e^{-\lambda_n^2 \tau} J_0(\lambda_n \xi)}{(16N_R^2 + \lambda_n^2) J_0(\lambda_n)}$$

where eigenvalues, λ_n , were the roots of the equation

$$\lambda_n J_1(\lambda_n) - 4N_R J_0(\lambda_n) = 0$$

Solutions to this problem compared favorably with solutions to the "exact" problem for small changes in surface temperature (small N_R). However, for large changes in surface temperature, deviations between the solutions became apparent. This phenomenon was expected, since the approximation to the boundary condition was not valid in the latter case.

3.0 MODEL FOR TRANSIENT MEASUREMENTS

3.1 Geometry and Material

The model was fabricated as a solid circular cylinder three inches in diameter and fifteen inches long. The length-to-diameter ratio of 5 was used to simulate an infinitely long rod. This ratio was thought to be large enough to minimize end effects in a short section at the axial center of the model. Considerations given to the design may be found in reference (2, section 4.1).

Epocast 10 filled with aluminum powder measuring 50% by weight was used as the modelling material. Composition of this material and proper casting procedures may be found in Putman (2, Appendix B). This material was chosen because it could be cut and then rejoined to form a homogenous model. This property allowed the use of embedded thermocouples and strain gages to determine internal temperature and strain responses. The material also possessed a low thermal diffusivity which effectively expanded the time scale, eliminating the need for fast-response instrumentation.

3.2 Transducers

The model was instrumented in two phases. Putman (2) embedded nine thermocouples to help determine the temperature distribution. The thermocouples were embedded at the same radial locations chosen for the strain gages. In the second phase, strain gages were embedded to read principal strains at these radial positions. In addition to helping determine the temperature distribution the thermocouples were needed

to obtain the false strains so that the desired mechanical strains might be determined. Transducers were located at: the radial center ($\xi=0$), a point just beneath the outer surface ($\xi=0.98$), and two intermediate points ($\xi=0.583$ and $\xi=0.90$). Details of selection of thermocouples and locations, and the method of embedding the thermocouples may be found in reference (2, section 4.2).

3.3 Thermocouples

Six thermocouples were embedded in a plane located at the axial center of the model, section A of Figure 1. They were located at the radial positions shown in Figure 2. Thermocouples 6 and 5 were positioned at the same radial locations as thermocouples 2 and 3 respectively, but on the opposite side of the model, to check circumferential symmetry of the temperature distribution.

Thermocouples 7, 8, and 9 were located at the same radial positions as thermocouple 1, but spaced along the length at sections B, C, and D, respectively, as shown in Figure 1. These were used to help determine end effects caused by the finite length of the model.

Thermocouples 10 and 11 were later embedded near the surface of the model. They were positioned along a diameter approximately 90° circumferentially from the diameter along which the other thermocouples were located (Figure 2). These two thermocouples were used to determine the temperatures of the strain gages near which they were located. An indication of local heating caused by power dissipation in

AXIAL LOCATIONS OF THERMOCOUPLES

Figure 1

(a) Multiple thermocouple section (A)

(b) Single thermocouple sections (B), (C), and (D)

RADIAL LOCATIONS OF THERMOCOUPLES

Figure 2

the strain gages was obtained by comparing output of thermocouples 10 and 11 to the output of thermocouple 1, which was not located near a strain gage, and also by comparing output of thermocouples 10 and 11 when the strain gages were operating with the output when they were not operating.

Lead wires from all thermocouples were brought out of the model as nearly along isothermal surfaces as possible. This precaution was taken to reduce errors in thermocouple output caused by heat flow along the wires. It also reduced the possibility of disturbing the temperature distribution for the same reason. A thorough presentation of considerations concerning lead wires for thermocouples 1-9 may be found in reference (2, pp. 47-49). The same considerations apply to thermocouples 10 and 11. As described by Putman, the lead wires were covered with small drops of epoxy. However, the exposed portions of the wires were later covered with epoxy to reduce the possibility of breaking the fragile wires.

Since thermocouples 10 and 11 were located near the surface, these lead wires were easily brought out along isotherms by embedding them in shallow grooves cut axially along the surface of the model to the end. All lead wires were eventually brought to one end of the model. Here they were soldered to copper tabs which were bonded to the ends of the model. Tabs were used to reduce the possibility of breaking the small diameter wires when connected to heavier lead wires to the readout devices.

3.4 Strain Gage Selection

Strain gages which would accurately indicate the mechanical

strains had to be found. The self-temperature compensating type was sought, since it would produce minimum disturbance in the strain field when embedded. Gages normally used in the Mechanical Engineering Department either would not bond on the top side when embedded, or were not closely temperature compensated for use with the filled epoxy material. The material has a coefficient of thermal expansion of 22.5 micro-inches per inch per degree Fahrenheit.

What were thought to be the best gages for use in this testing were manufactured by Micro-Measurements Inc. They were designated MA-ST-250BG-120. Having been temperature compensated for use with materials having a coefficient of thermal expansion of 20 microinches per inch per degree Fahrenheit, these gages came closer to the proper compensation than any others.

Temperature compensation had to be carefully considered because thermally-induced mechanical strains were expected to be small. Since temperature changes were expected to be as large as 50^oF, the false strains would correspondingly be large unless the gages were closely temperature compensated. Therefore, when the mechanical strains were separated from the total strains, large inaccuracies could be introduced unless false strains were accurately known. To reduce this possibility, an accurate free expansion curve was determined for two of the gages. A description of this testing may be found in Appendix B. The resulting false strain curve, which is based on data for both gages, is presented in Figure 28.

Since no characteristics of these strain gages were accurately known, other testing was performed to ensure that reliable strain data would be obtained. The manufacturer's gage factor ($F_u = 2.11$) was checked to determine if reinforcement was indicated, but the experimental gage factor ($F = 2.09$) was found to be close to the manufacturer's gage factor. The difference between the values was less than scatter between individual tests, justifying use of the manufacturer's specified value for all data reduction. This testing is described in Appendix B.

3.5 Strain Gage Embedding

The model was cut in half close to section (A), Figure 1, on which the thermocouples were located. A rectangular hole was machined into the half in which no thermocouples were embedded so that the gages might be bonded in the proper locations with the desired orientation. Each strain gage was first bonded to a small block of the filled epoxy. The surfaces on which the gages were applied were all flat planes. Although the coordinates along which the strains were to be measured were curvilinear, it was felt that the gages should be mounted on flat planes so that false strain curves would not have to be determined for each gage individually. Dove (4, p. 108), shows that false strain is theoretically dependent on the radius of curvature of the surface on which the gage is mounted.

Strain gages were located as shown in Figure 3. One gage, aligned to indicate axial strain, was located at the center ($\xi=0$). Since radial and circumferential strains at

NOTE 1: Diameter V-W corresponds to same diameter shown in Figure 2.

NOTE 2: Strain gages were located on the sides of the block indicated by the arrows.

NOTE 3: Both gages at $\xi=0.98$ are hidden from view, but solid lines are used for clarity.

STRAIN GAGE LOCATIONS

Figure 3

this location were expected to be small, no gages were aligned in these directions. At the intermediate position ($\xi=0.583$) three gages were mounted, one in each of the principal directions. A triaxial strain field was expected to exist at this radial position. Two gages were embedded near the surface ($\xi=0.98$), one in the circumferential and one in the axial direction. It was physically impossible to locate a gage in the radial direction at a location so near the surface. These six strain gages permitted a check against theory at several important positions.

The strain gages were intended to check principal strains at the points mentioned. However, since the gages were sensitive to strains across a finite area in which gradients existed, errors were introduced. Since circumferential and axial gradients in the model were small, the aforementioned effect was significant only with the gage indicating radial strain. Errors were introduced in the signals from the strain gages aligned circumferentially and axially because these gages were located on flat planes, and not at a constant radius. Although it was known that these errors existed, their effects would have been very difficult to estimate.

A three-wire lead system was used with each strain gage (Figure 4, p. 24). This type of system is described more fully in section 4.2. To be used properly the two bridge lead wires must be exposed to the same temperature distribution. If not, false signals would be produced by different changes in resistance of the wires. Therefore, when the two bridge lead wires for each gage were embedded in the model, great care was taken to locate them as close together as

possible. To further reduce the problem, all lead wires were located approximately along isothermal surfaces. This precaution also helped reduce the possibility of disturbing the temperature distribution by eliminating heat flow along the wires. Small diameter wire (0.004 inch) was used as lead wire within the model to minimize reinforcement caused by different values of Young's Modulus for the wire and epoxy. The lead wires were soldered to small copper tabs at the end of the model to minimize the possibility of breaking them when connecting to the heavier wires leading to the readout devices.

3.6 Surface Preparation

The lateral surface of the model was coated to eliminate the need for careful control of the surface condition of the epoxy substrate. Without a coating, both surface finish and oxidation state played important roles in controlling radiative properties. However, these problems could be reduced considerably if the coating, not the epoxy substrate, controlled surface properties. A coating with a higher total hemispherical emittance than the substrate was desired since this property would increase the rate of heat transfer from the lateral surface, thereby producing higher stress levels in the model for a given initial temperature.

After investigating several materials for coating the surface, Krylon Flat Black Spray Enamel was selected for use. This paint was easy to apply, possessed a high value of total hemispherical emittance ($\epsilon=0.88$), had no undesirable properties under high vacuum, was stable in the temperature range

within which it was to be used, and was opaque to long wavelength radiation when applied in two thin coats. Testing to determine properties of this paint is described in Appendix A.

4.0 INSTRUMENTATION

4.1 Thermocouple Output

Thermocouple voltage signals were recorded outside the working chamber by passing copper and constantan lead wires through the baseplate and connecting them to the indicating devices.

Several recording schemes were tried. First, all lead wires were connected to a multiple-position switch, and a Leeds-Northrup, Series H, strip chart recorder with an "AZAR" attachment recorded output from all thermocouples. However, this method prevented important data from being recorded near the beginning of a test run because approximately 30 seconds were required to switch through all positions. Data for short times were of special interest because a finite length of time was required to expose the model to the heat sink. This could have caused differences between the mathematical and experimental models, since the analysis assumed instantaneous exposure. Later, three recorders (the Leeds-Northrup; a Moseley, Model No. 7000A, X-Y plotter; and a Moseley, Model No. 135, X-Y plotter) were used to continuously record output from three of the thermocouples during each test. Runs for a particular set of parameters were repeated to obtain a complete set of data. This procedure is more completely described in Chapter 6.

Spurious emf's in the lead wires were minimized by using the leadthrough system described by Putman (2, pp. 77-79). This system allowed the wires to be passed through the baseplate without introducing extra junctions.

Originally, all constantan wires were connected to a common lead, which was the constantan wire of the ice bath reference junction. This was done to conserve space and simplify holding the reference junction at 32°F . The copper wire from the reference junction was connected to one pole at each position of the copper-contact switch, and therefore, no extraneous emf's were introduced into the signal at the recorder.

Later, thermocouples 10 and 11 were found to be shorted to the strain gages near which they were embedded. Separate ice bath reference junctions were used for each of them to eliminate the short between the strain gages, which if allowed to remain, would have prevented useful strain data from being obtained.

4.2 Strain Gage Output

To simultaneously and continuously record the output from all strain gages during a test, an eighteen channel Consolidated Electrodynamics Corporation oscillograph, and a William Miller Laboratories twelve channel carrier-wave amplifier system were used.

STRAIN GAGE INSTRUMENTATION

Balanced Wheatstone Bridge Connected to Carrier-wave Amplifier System

Figure 4

The strain gages were connected into separate four-arm balanced Wheatstone bridges using a three-wire lead system (Figure 4). The carrier-wave system had two of the four gages necessary to complete each bridge, mounted internally. The remaining dummy gages (one for each bridge) were bonded to a piece of aluminum and packed in a box filled with thermal insulation to reduce temperature variations. The bridges were powered by a 6000 cps alter-

nating voltage supply provided by the carrier-wave amplifier system. Individual amplification of each channel was provided in this unit, and demodulation of the signal was performed so that only the strain-induced signals were recorded by the oscillograph.

The three-wire lead system was used because the lead wires, having been embedded in the model, would undergo temperature changes. By connecting each active gage to its bridge in this manner, the two wires leading to the bridge appear in adjacent arms of the bridge so that if both wires undergo identical resistance changes no net signal due to these changes will be recorded. Use of a three-wire lead system is described by Dove (4, p. 122).

Consolidated Electrodynamics Corporation, Model No. 7-323, galvanometers were used in the oscillograph because the sensitivity of them provided the best match of characteristics between all parts of the strain gage instrumentation. Noise was minimized by using these relatively insensitive galvanometers and large amplification from the carrier-wave amplifier system.

The output voltages to be recorded by the oscillograph corresponded to the linear superposition of false strains and mechanical strains. However, only the mechanical strains were desired because these may be related to the thermally-induced stresses. Separate testing was performed, as described in Appendix B, to determine the false strains, the knowledge of which allowed the mechanical strains to

be determined from the total strain signals.

Calibrate resistors were used to relate voltage signals to total strains. By placing an appropriate resistor in parallel with the active strain gage, a strain signal of any magnitude could be simulated. The calibrate strain is defined in reference (4, p. 100) and reduces to:

$$\epsilon_{eq} = - \frac{1}{F} \frac{R_g}{R_c + R_g}$$

where

- ϵ_{eq} : strain simulated by calibrate resistor (in/in)
- F : gage factor
- R_g : nominal strain gage resistance (ohms)
- R_c : calibrate resistance (ohms)

To accurately determine sensitivity of the bridge, the calibrate resistor must be placed directly across the tabs of the strain gage. However, the gages were embedded in the model and the model was used inside the bell jar so that it would have been difficult to calibrate at the gage. By calibrating both ways (near the gage and at the bridge) it was found that lead wires were short enough so that strain signals were effectively equal. No correction was needed and the above equation was used to relate strain

to the output when calibrating at the bridge.

A problem was discovered when the output signal was found to be a nonlinear function of strain if the oscillograph was deflected more than about one-third of full scale. The strain level at which nonlinearity began to be evident was determined by using calibrate resistors of different sizes and comparing the deflections. Dove (4, page 128), explains that this effect may be caused by capacitance unbalance in the lead wires. It was decided that amplification should be reduced so that the signals would not exceed the linear range, because the difficulty in reducing the data obtained with a nonlinear signal would be worse than the loss in resolution due to reduction of the amplification.

Although the instrumentation was designed to allow simultaneous recording of all strain data, it was found that two of the gages had become shorted together when embedded in the model and could not be used at the same time. They were apparently in contact at only one spot, which was determined by checking the resistance. Thus, by using only one gage at a time, useful data were obtained from both. Use of these gages will be described more completely in section 7.1.

5.0 AUXILIARY EQUIPMENT

5.1 Introduction

Initial work in developing the necessary auxiliary equipment needed to simulate the prototype environment was performed by Putman. A complete description of his work may be found in reference (2, Chapter 5).

Testing was performed under high enough vacuum to obtain an environment in which radiation was the only significant mode of heat transfer from the surface of the model. A vacuum system was borrowed from Sandia Corporation, and used to produce the desired vacuum in the working chamber. A photograph of the auxiliary equipment assembled for testing is presented in Figure 5.

A cylindrical heat sink was placed around the model to minimize the amount of incident radiation on the lateral surface. The surface facing the model approached a black body so that most of the radiant energy falling on it was absorbed. The unit was cooled with liquid nitrogen so that insignificant radiant energy was emitted.

A collapsible cylindrical heater and shield assembly was designed to simulate the initial condition of the mathematical model. This piece of equipment was used to maintain a constant, uniform initial temperature in the model. It was made collapsible so that at the beginning of each test it could be removed from around the model,

AUXILIARY EQUIPMENT ASSEMBLED FOR TESTING
Figure 5

thereby exposing the model to the heat sink. Piano wire was wound on a set of pulleys geared to a crank outside the working chamber, to permit remote raising and lowering of the heater assembly. The wires were aligned with other pulleys fastened to an aluminum frame, which was also used to support the model.

These pieces of equipment were used as described by Putman (2), except for changes presented in the following sections of this chapter.

5.2 Vacuum System

According to calculations for this apparatus, the level of vacuum needed to negate effect of conduction and convection by the air remaining in the working chamber was 10^{-5} millimeters of mercury. During preliminary testing, a discharge vacuum gage was installed in the baseplate of the chamber to check the accuracy of an ionization gage located in the system near the diffusion pump. Comparison of the two gages indicated that the vacuum differed considerably between the working chamber and the diffusion pump, and that a sufficient vacuum for testing could not be obtained. At one time the ionization gage indicated 10^{-5} mm. Hg when the discharge gage indicated 25×10^{-3} mm. Hg. A constant correction factor could not be used because the pressure gradient in the line connecting the baseplate and the diffusion pump varied with the amount of time the system had been operating. Therefore, the discharge gage in the

baseplate was used to check the vacuum for all further testing.

A systematic investigation of the entire apparatus was performed to determine what improvements were needed to produce the desired vacuum. After determining that the pumping system was capable of producing the desired vacuum in the empty working chamber connected directly to the diffusion pump, work was begun to redesign the line connecting the working chamber to the diffusion pump so that sufficient vacuum might be obtained under test conditions. This work is described in section 5.3.

After this improvement had been completed, the heat sink was found to leak so badly when liquid nitrogen was supplied to it that, even with the increased pumping capacity, the desired vacuum could not be obtained. Consequently, a new heat sink had to be built to obtain a satisfactory test environment.

5.3 Pumping Capacity

To first determine if the pumping system was capable of producing the desired vacuum of better than 10^{-5} millimeters of mercury, all auxiliary equipment was removed from the working chamber, and the baseplate was connected as closely as possible to the diffusion pump. Holes in the baseplate for leadthroughs were sealed with Urethane rubber. In this way, all leaks were reduced to a minimum, and it was found that the pumping system could produce a sufficient vacuum in the working chamber.

As received, the connection between the diffusion pump and the working chamber was a three foot length of one and one-half inch diameter copper pipe. This pipe restricted the flow of air to the diffusion pump and therefore controlled the pumping capacity of the vacuum system. The connection was redesigned, in accordance with information received from Consolidated Vacuum Corporation (7), to increase flow to the diffusion pump.

Stainless steel pipe, three and one-quarter inches in diameter was selected to fabricate the new four foot long connection. The hole in the baseplate was also enlarged to the same diameter as the pipe so that the air flow would not be restricted there. The working chamber was evacuated when this change had been completed and an ultimate vacuum of 7×10^{-6} millimeters of mercury was obtained in several hours. The vacuum system was judged ready to be used for testing.

The baseplate was mounted on a moveable stand of convenient height. The height was chosen so that when a pulley system and balance weight were incorporated to lift the bell jar, it could be easily removed to allow access to the equipment in the working chamber.

5.4 Heat Sink

To simulate the prototype surroundings, a cylindrical heat sink was constructed (Figure 6). The model was to be suspended within this unit. The heat sink surface seen

LIQUID NITROGEN-COOLED HEAT SINK

Figure 6

from the lateral surface of the model was required to have the characteristics of a black body. In addition, the temperature of the heat sink had to be low enough so that radiant energy emitted from it would be negligible compared to the amount of energy emitted by the model.

The black body was made from a large-diameter aluminum pipe. It had an outside diameter of 11 inches, wall thickness of three-eighths inch, and a length of 21 and one-quarter inches. Absorptance of the heat sink was increased by machining a thread into the inner surface. Ten threads per inch, with a 30° included angle were used. A full thread was made, to produce sharp roots and tips.

A reservoir was built around the outside of the pipe to hold liquid nitrogen, for the purpose of cooling the inner surface, thereby negating emitted radiation. The reservoir was formed by placing another aluminum pipe concentrically around the first one, and placing two annular rings around the inner pipe with one butted against each end of the outer pipe. This construction formed four joints which were Heli-arc welded. Outside diameter, wall thickness and length of the outer pipe were 12, one-fourth, and 19 inches, respectively. Two holes were drilled into the outer pipe at the upper end, adjacent to the annular ring. Two small diameter aluminum tubes were brazed into adapter plates, which were Heli-arc welded over the two holes to allow transfer of liquid nitrogen into the reservoir surrounding the heat sink.

An adapter flange was brazed to the other end of each tube to allow passage of the tubes through the baseplate. The flange was bolted to the baseplate and the joint was sealed with a mylar gasket. The reservoir was pressure tested in water at 75 psig to locate leaks, which were sealed by rewelding.

To minimize reflectance of the inner surface of the heat sink, it was anodized, colored with a black analine dye, and then coated with a black paint similar to the type used to finish camera interiors. The paint was applied in thin coats so as not to fill the roots of the threads, which would have reduced the absorptance of the surface. The actual value of absorptance of the heat sink was not determined, but it should be greater than 0.96.

5.5 Collapsible Heater

As designed, the collapsible heater assembly was to be raised and lowered by piano wires wound on two separate pulley systems, which were geared to, and remotely turned by, a shaft passing through the baseplate. The unit was rebuilt to either collapse or extend the heater by using only one set of pulleys, thereby sealing off one baseplate leadthrough and reducing leakage into the bell jar. Movement of the heater was also simplified because only one shaft had to be turned. The new heat sink was elevated from the baseplate, so that no room was available above

it, and the heater assembly had to be collapsed downward.

During early stages of testing the outer shield on the heater assembly became stiff when the heat sink was filled with liquid nitrogen. When this occurred, the heater could not be fully collapsed, and satisfactory test conditions could not be produced. Radiative heat transfer calculations, using the network method, indicated that another shield located between the heater assembly and the heat sink would eliminate this problem.

A shield was made from three circular cylinders of mylar-coated chromium. The diameters differed so they would fit within one another. They were each six inches long so that when connected together with small diameter copper wires and extended, the outer surface of the heater assembly was shielded from the heat sink. The wires were fastened to the top end of each cylinder and to the top end of the cylinder with the next larger diameter. Length of the wires was about five and one-half inches. The smallest cylinder was fastened to the top of the heater assembly by using cantilever sections of piano wire. When the assembly was lowered and collapsed, the shields dropped to the baseplate inside one another. This shield eliminated the stiffness problem.

6.0 PROCEDURE6.1 Test Environment

The model was suspended within the collapsible heater assembly, which was in turn suspended within the heat sink (Figure 5). The heat sink reservoir was kept full of liquid nitrogen during testing to hold the entire inner surface at the boiling point of nitrogen. The vacuum level in the working chamber was between 5×10^{-7} and 2×10^{-6} millimeters of mercury, as read on the discharge gage in the baseplate, for all tests performed.

To heat the model to a uniform initial temperature, the heater assembly was extended, and power was supplied through a variable output transformer (Variac). Thermocouples embedded in the model were monitored to determine the temperature distribution while the model was being brought up to the initial temperature for testing. The model temperature was stabilized as near as possible to the chosen initial temperature. Power to the heater had to be adjusted often to approximate the constant, uniform initial temperature distribution at the axial center of the model. Testing was initiated by dropping and collapsing the heater assembly, thereby exposing the lateral surface of the model to the heat sink. Data recording was begun as soon as the heater assembly began to drop. Approximately 30 seconds was required to expose the entire surface of the model to the heat sink.

6.2 Temperature Recording

Initially, the Leeds-Northrup, Series H, strip chart recorder was used to record output from all thermocouples by using a multiple-position switch. This method proved unsatisfactory because sufficient data were not available for all thermocouples at short times as discussed in section 4.1.

Using this recorder, data reduction took a long time because the signals all had to be plotted point by point on separate graphs. The decision was made to use three recorders (the Leeds-Northrup; a Moseley, Model 135, X-Y plotter; and a Moseley, Model 7000A, X-Y plotter) so that continuous output could be obtained. Now, however, output from all thermocouples could not be recorded simultaneously so that several runs were required to obtain a complete set of data. Therefore, consistency of the data depended upon reproducibility of the initial test conditions. All thermocouple outputs were recorded immediately before each test to establish the initial conditions, and the reproducibility was found to be relatively good.

In later tests the Moseley, Model 135, X-Y plotter was not used because the sensitivity could not be continuously adjusted as with the other two instruments. Eliminating the use of this recorder meant that more tests were needed to obtain a complete set of data. Temperature

data were recorded as soon as the heater assembly began to drop.

6.3 Strain Recording

To prepare to record output from the strain gages, each bridge was balanced when the temperature distribution was near the test initial conditions. A calibrate resistor was switched across each active gage (at the bridge) to determine sensitivity of the channels. Sizes of these resistors were chosen so that the total strain signal would never be larger than the calibrate signal. Amplification was adjusted to keep the signal within the linear range, as discussed in section 4.2. The bridges were rebalanced, if necessary, to remove the signals caused by small temperature changes which occurred while the bridges were being calibrated. The oscillograph trace was started as soon as the heater assembly began to drop.

7.0 RESULTS

7.1 Temperature and Strain Responses

The temperature and strain responses determined from this testing are presented in Figures 7-20, pp. 54-67. In general, these results compare favorably to the corresponding temperature and strain responses obtained from the theoretical analysis. On these graphs, the solid curves represent the analytical solutions, and the circled points define the resultant experimental curves. Dotted lines represent limits on scatter of the experimental results. The dotted lines do not appear everywhere because, in some instances, they were too close to other curves of greater importance.

At least four sets of data were obtained for each thermocouple for each initial temperature. This number was found to be enough to ensure that data were reproducible. Approximately twelve tests were performed to determine that strain data were reproducible. Half this many tests would have been sufficient, except that the gages aligned to indicate circumferential and axial strains at $\xi=0.583$ were shorted together, as mentioned in Chapter 4. During testing these strain gages were used alternately, so that only half as much data were obtained for these positions as for the other positions. The tests lasted no longer than twenty minutes because all mechanical strains had passed through their peak values by this time.

The experimental temperature response curves for both $N_R=0.099$ and $N_R=0.120$ have the same forms as the corresponding theoretical results. However, in general, the experimental temperatures are higher than the theoretical temperatures. Two possible causes of these deviations are discussed in later sections of this chapter. In section 7.4 it is shown that the experimental modelling allows some radiant energy to fall on the model, thereby reducing the rate of temperature change and causing the experimental temperatures to be higher than the theoretical temperatures. In section 7.2 the manner in which the theoretical temperature responses are affected by the choice of material property values is described.

The experimental temperature responses for an initial temperature of 200°F ($N_R=0.099$), presented in Figures 7-10, compare quite closely to the analytical results. A maximum deviation of 2°F occurred at $\xi=0.98$. Since the results compare so well, no explanation of deviations was felt to be needed.

Mechanical strain responses ($N_R=0.099$), presented in Figures 11-16, indicate good comparison between experiment and theory. Except for circumferential strain at $\xi=0.583$, maximum experimental strains were all within 35 microinches per inch of the theoretical values. The largest deviation in absolute value occurred in the circumferential strain at $\xi=0.98$ and corresponded to 10% of the maximum experimental value. The largest percentage deviation occurred in the circumferential strain at $\xi=0.583$ and corresponded to 37% of the maximum experimental value at that gage. The large deviations between the results

for the circumferential strain at $\xi=0.583$ cannot be explained by the errors discussed in this report. It is possible that the strain gage was damaged when it was embedded in the model, or that its characteristics were changed by being shorted to the gage in the axial direction at that location. At any rate, there are no apparent reasons for discrepancies at that location for circumferential strain when reasonably good agreement was obtained for radial and axial strains there.

Reasonably good agreement was obtained between experimental and theoretical temperature responses for an initial temperature of 250°F ($N_R=0.120$). A maximum deviation of 3°F occurred in these results as may be seen on Figures 17-20. Although 3°F is not an extremely large difference, it is larger than the deviations which occurred for testing at the lower initial temperature. It is felt that inaccurate prescription of material properties at this higher temperature level probably caused the larger errors. This problem is discussed in section 7.2.

As shown in the results, tests were performed using both 200°F and 250°F as initial temperatures. For an initial temperature of 200°F both temperature and strain responses were recorded, but for an initial temperature of 250°F temperature responses only were obtained. Strain responses were not obtained in the latter case because mechanical strains could not be accurately determined at temperatures above 200°F . The false strain curve (Figure 28) was not used above this temperature level because the slope of that curve

increased sharply, producing a large amount of scatter in the test data. The effect of the increased slope was to introduce large errors into the false strain values from only small errors in the temperature determination. The upper limit of 250°F for which temperature responses were obtained was selected because this was the highest temperature for which any material properties were known. Although only thermal conductivity and total hemispherical emittance had been determined above 200°F , it was felt that at least the experimental temperature responses should be obtained, and an attempt made to compare with theory.

The highest permissible initial temperatures were used for testing because this choice corresponded to the most important condition when comparing analytical and experimental results. Higher initial temperatures produced larger rates of heat transfer, resulting in greater temperature changes, more significant effects due to nonlinearity of the boundary condition, and larger mechanical strains. Therefore, since the analytical solutions compared favorably to the experimental results for these parameters, it was felt to be a reasonable assumption that the analytical solutions could be used to predict the temperature and stress distributions for lower initial temperatures (smaller N_R). Also, since the results compared favorably at the instrumented locations, it seemed reasonable to assume that the analytical solutions could be used to predict temperature and strain responses throughout the model.

7.2 Theoretical Solution

When the model described in Chapter 3 was used for testing, the only parameter which could conveniently be varied was the initial temperature. Once this value was selected, all parameters in the experimental model were determined. The experimental results could be obtained without knowing any material property values. However, to define the mathematical model, material properties also had to be known. The material property values used to define the mathematical model are presented in Appendix A.

Wildin and Putman (1) showed that maximum principal stress was not greatly dependent upon changes in values of thermal properties with temperature. For this reason, and since most material properties were not well known as functions of temperature, the theoretical solution was obtained considering constant material properties. However, it would be well to note that the theoretical temperature responses would fall at a faster rate if one were to consider the material properties as increasing functions of temperature.

Since both thermal conductivity and total hemispherical emittance were determined in the entire temperature range of testing, and found to be nearly independent of temperature, the values of N_R should be accurate for both initial temperatures used. These values were $N_R=0.099$ for an initial temperature of 200°F , and $N_R=0.120$ for an initial temperature of 250°F . However, specific heat and density were not considered as functions of temperature, and possibly even more important,

had been determined only up to 200°F. If the assumed values of these properties at the initial temperatures were not correct, then the time scale ($t = \frac{\rho c a^2 \tau}{k}$) would not be correct. As mentioned in Appendix A, the material property values used for an initial temperature of 250°F were the same as those used for an initial temperature of 200°F. Had the value of specific heat been obtained by linearly extrapolating the curve presented in reference (9) to 250°F, the value would have been $c = 0.31$ Btu/lb-°R. This choice of specific heat would have increased the time scale for the theoretical temperature responses at $N_R = 0.120$ by about 10%. The result of such a choice would have been to cause the theoretical temperatures at longer times to be higher than the experimental temperatures. Therefore, by using the higher value of specific heat and considering nonconstant material properties the deviations between experimental and theoretical temperature responses may have been reduced. However, since the curve for specific heat did not indicate that linear extrapolation was justifiable, the value of $c = 0.28$ was used in both cases.

7.3 Data Reduction

Thermocouple outputs were reduced to temperature responses in the same manner for all tests. Voltage output was converted to temperature using standard tables.

To determine the resultant experimental temperature response at each radial location, data from the various thermocouples located at a particular radial position were plotted on a single graph. Small deviations from a uniform initial

temperature distribution existed during each test, and these deviations were not the same from one test to the next. To reduce scatter in the temperature response curves for small times, all temperature responses were arbitrarily raised or lowered to the desired initial value. This action was felt to be justifiable because temperature responses from each thermocouple during different tests fell at nearly the same rate. Thus the temperature responses were nearly independent of small perturbations on the assumed uniform initial temperature distribution.

When the temperature responses were plotted on a single graph to determine the best resultant experimental curve, heavy concentrations of data points were considered to be more important than data points which obviously deviated from the rest of the data. By weighting the data thusly, the resultant curves were drawn to depict the most probable temperature responses. The resultant experimental curves were determined before analytical solutions became available so that they were not influenced by the desire to obtain agreement with the analytical results.

The resultant experimental strain responses were determined from the individual strain responses in the same manner as the resultant temperature responses. This procedure was delineated in the preceding paragraph.

To determine the mechanical strain response at each location during a single test, two operations were performed on the record obtained from the oscillograph. First,

deflections of the galvanometers during testing were related to total strain (linearly superposed mechanical and false strains) by comparing the deflections obtained during testing to the deflections produced by calibrate resistors. And since the deflection due to the calibrate resistor could be related to calibrate strain, as discussed in section 4.2, the deflections obtained from testing were expressed in terms of total strain. Calibrate strain was determined using the manufacturer's specified gage factor, $F_u=2.11$. Cross-sensitivity of the strain gages was assumed negligible for all data reduction. These assumptions are considered in Appendix B.

Second, the total strain responses were corrected to account for false strains so that the mechanical strain responses could be determined. False strains were determined by assuming that the resultant experimental temperature responses existed during each test. This assumption was made to simplify data reduction. To accurately determine false strains, the temperature response of each strain gage during each test would have to be known and these values should be used. However, all gages were not instrumented with thermocouples so that individual temperature responses could not be determined. The effects of neglecting deviations between individual temperature responses was estimated by locating a thermocouple near each of the two surface gages ($\xi=0.98$). However, the thermocouple at the axial gage produced erratic output and could not be used. By testing with 15ma. of current

through the gages and testing again with no current through the gages, the temperature of the gage aligned to indicate circumferential strain at $\xi=0.98$ was found to be consistently 5°F higher with current flowing through the gage. This local heating was caused by dissipation of power by the gage. A similar effect probably occurred at the other gages. In the next section an analysis will be performed to determine the errors introduced in the strain responses by using the resultant temperature responses to calculate false strains.

Although the changes in the strain field were determined by the means described previously, the strain level at the beginning of each test had to be arbitrarily chosen. Initial strains in the model were assumed to be zero, and by balancing the bridges immediately before each test, all strain responses began at zero strain. The only way to check this assumption would have been to record the strain responses from some known strain-free reference state. This method was not felt to be warranted due to the difficulty which would have been encountered. Since the initial temperatures were allowed to approach uniformity and since only small axial temperature gradients existed near the test section, it was felt that any existing initial strain distribution would be small and effects of it could be eliminated by assuming a zero initial strain level.

7.4 Error Analysis

The following discussion considers deviations between

theoretical and experimental results introduced by experimental modelling, and errors encountered while attempting to determine the existing experimental temperature and strain fields. The problems discussed here apply, in general, to the comparison of all analytical and experimental results, and no attempt has been made to explain deviations between analytical and experimental responses (temperature or strain) at a particular location. A quantitative estimate of the effect on the temperature and strain responses due to inaccuracies of measuring the existing field quantities is performed. However, only a qualitative effect could be determined for errors introduced by the experimental modelling. It might be well to note that the lengthy consideration of these errors does not imply that the results were not felt to be in good agreement with each other. Instead, the presentation is meant to show what problems can arise when an investigation such as this is undertaken, and to determine the approximate magnitude of inaccuracies which can occur.

In the following four paragraphs deviations between the experimental and theoretical results caused by only approximate modelling of the desired environment will be discussed.

In the mathematical model all radiant energy from the model was assumed to be absorbed, and no energy was incident on the model. In the experimental model, although the heat sink was designed to absorb as much incident radiation as possible, some energy was reflected back to the model during

testing. Also, since the heat sink was at about -320°F , some energy was emitted. Energy emitted from the heat sink was about 0.7% of the amount emitted from the model. Furthermore, the finite lengths of the model and heat sink resulted in some energy falling on the model from the room temperature surroundings. These three approximations tended to reduce the rate of radiative heat transfer from the model, thereby decreasing the temperature changes which occurred during testing.

The mathematical model assumed that uniformity existed in the axial direction. However, in the experimental model a temperature gradient was evident along the axis. The effect on the temperature and strain responses due to this lack of uniformity would have been quite difficult to estimate, and since the results compared favorably, such an investigation was not felt justified. An order of magnitude analysis was performed to determine how much heat was conducted axially compared to the amount of heat transferred from the surface. For an initial temperature of 200°F , this ratio was 5%, and for an initial temperature of 250°F , the ratio was 9%.

In the mathematical model the surface was assumed to be instantaneously exposed to the outer space environment. However, use of the collapsible heater assembly caused a delay of about thirty seconds from the start of a test until the experimental model was fully exposed. This effect was noticeable in testing because all temperature and strain

responses remained zero for a short time after the heater began to move. It was impossible to evaluate the time shift from the data, so the results were plotted with $t=0$ as the time when the heater began to drop.

The initial temperature distribution in the mathematical model was assumed constant and uniform. In the experimental model, the initial temperature distribution was maintained as closely as possible to the assumed distribution, but for any given test the initial temperature varied from uniformity by about $\pm 2^{\circ}\text{F}$ across the test section. The temperatures also changed at a rate of about $0.5^{\circ}\text{F}/\text{min}$ during the time immediately preceding each test. To minimize the effects of the nonuniformity, data were reduced by arbitrarily changing the temperature level of each temperature response to the desired initial temperature. When the data were reduced in this manner, scatter was concentrated at long times as seen from the results. No corrections could be applied to account for the time rate of change of the temperatures at the beginning of a test.

The results presented in this chapter do not exactly represent the temperature and strain responses which existed in the model because errors were introduced while attempting to determine them. The effects of these errors will be analyzed in the following paragraphs.

When transducers are used to determine the values of a field quantity at a point, problems arise because the trans-

ducers possess a finite volume and cannot indicate the desired quantity at only one point. Thus the quantity obtained is an average throughout the volume of the transducer. The effect of this factor on the results had to be ignored because it could not be readily determined.

Voltage outputs from the thermocouples were converted to temperature using standard tables. Since the thermocouples had not been calibrated, output from individual thermocouples could have deviated from that indicated by the standard tables. However, these deviations were felt to be smaller than $\pm 1.1^{\circ}\text{F}$ in the temperature range of testing, as specified by the manufacturer.

Errors which were knowingly introduced when reducing the strain data to mechanical strains produced an uncertainty of +43 and -26 microinches per inch in the experimental strain responses. These values represent the maximum uncertainty at all times during the testing. At short times the uncertainty was smaller than indicated by these values. The largest error was introduced by neglecting local heating of the strain gages. By assuming that the gages were consistently 5°F warmer than indicated by the resultant temperature responses, the resultant strain curves would move toward tension by a maximum of 17 microinches per inch. This value was calculated assuming a temperature change of 52°F .

By using the resultant temperature responses instead of individual temperature responses to determine false strains,

maximum scatter of about 35 microinches per inch was introduced in the data for mechanical strains. This value was calculated assuming a 2°F deviation in temperature responses and a temperature change of about 50°F .

As mentioned previously, the manufacturer's gage factor ($F_u=2.11$) was used to reduce the data. Data presented in Appendix B indicated a mean value of $F=2.09$. Maximum deviation from the true strain field due to this assumption would have been ± 6 microinches per inch.

Finally, minimum resolution in reading the oscillograph was ± 3 microinches per inch.

TEMPERATURE RESPONSE AT $\xi=0$ FOR $T_i=200^\circ\text{F}$

Figure 7

TEMPERATURE RESPONSE AT $\xi=0.583$ FOR $T_i=200^{\circ}\text{F}$

Figure 8

TEMPERATURE RESPONSE AT $\xi=0.90$ FOR $T_i=200^\circ\text{F}$

Figure 9

TEMPERATURE RESPONSE AT $\xi=0.98$ FOR $T_i=200^\circ\text{F}$

Figure 10

AXIAL STRAIN RESPONSE AT $\xi=0$ FOR $T_i=200^\circ\text{F}$

Figure 11

RADIAL STRAIN RESPONSE AT $\xi=0.583$ FOR $T_i=200^\circ\text{F}$

Figure 12

CIRCUMFERENTIAL STRAIN RESPONSE AT $\xi=0.583$ FOR $T_i=200^\circ\text{F}$

Figure 13

AXIAL STRAIN RESPONSE AT $\xi=0.583$ FOR $T_i=200^\circ\text{F}$

Figure 14

CIRCUMFERENTIAL STRAIN RESPONSE AT $\xi=0.98$ FOR $T_i=200^\circ\text{F}$

Figure 15

AXIAL STRAIN RESPONSE AT $\xi=0.98$ FOR $T_i=200^\circ\text{F}$

Figure 16

TEMPERATURE RESPONSE AT $\xi=0$ FOR $T_i \approx 250^\circ\text{F}$

Figure 17

TEMPERATURE RESPONSE AT $\xi=0.583$ FOR $T_i=250^{\circ}\text{F}$

Figure 18

TEMPERATURE RESPONSE AT $\xi=0.90$ FOR $T_i=250^\circ\text{F}$

Figure 19

TEMPERATURE RESPONSE AT $\xi=0.98$ FOR $T_i=250^\circ\text{F}$

Figure 20

8.0 CONCLUSIONS

It is concluded that the temperature and stress distributions may be predicted by solution of the nonlinear boundary value problem described in section 2.1 if $N_R \leq 0.099$. Parameters of the experimental model were known well enough for $N_R = 0.099$ to produce agreement within 2°F between the experimental and analytical results. However, for $N_R = 0.120$ the experimental and analytical temperature responses differed enough to cause the user to be cautious. The maximum deviations of 3°F between these results were probably caused by improperly chosen material properties, thereby causing the parameters in the boundary value problem to be incorrect.

The experimental modelling techniques proved to be trustworthy. The modelling material was easy to work with, and due to the low thermal diffusivity, response times were long enough to make this material excellent for use on this thermal stress study. The problems which arose because the transducers were shorted together can be eliminated by ensuring that a layer of material separates all transducers embedded in the model.

The general method used to simulate the environment assumed in the mathematical model worked very well. The heat sink, in which all possible joints exposed to a vacuum were welded, proved dependable, and it is recommended that all such joints be welded in future work.

Although the strain gages used in this testing produced reliable results for a maximum temperature of 200°F , other gages which are more closely temperature compensated for use with the filled epoxy, should be found. Above about 200°F the slope of the false strain vs. temperature curve increased sharply and could not be accurately determined. Therefore, these strain gages should not be used for low-level thermal stress studies at temperatures above 200°F .

The transient method used to determine thermal conductivity was excellent because it produced the results as a function of temperature in a relatively short time. The value of $k = 0.55 \text{ Btu/hr -ft-}^{\circ}\text{F}$ is felt to be accurate, as shown in the error analysis in Appendix A. This value compares favorably with that obtained by previous investigators (8).

The steady state model developed to determine total hemispherical emittance provided an easy determination of that property. The model was easy to build, and by making it small, the time required for transients to subside was not longer than about a half hour. If the heater wire and thermocouple bead are accurately located, this method should allow an accurate determination of total hemispherical emittance. No other results for Krylon Flat Black Spray Enamel were available to check the value of $\epsilon = 0.88$ obtained in this testing.

9.0 RECOMMENDATIONS

To obtain closer comparison between theoretical and experimental temperature responses using a filled epoxy model with $N_R > 0.099$, either more testing should be performed on a larger diameter model, or material properties should be determined above 200°F. Since material properties are more dependent on temperature at higher temperature levels, the more desirable way to increase N_R would be to increase model diameter.

To determine thermal stresses at higher temperature levels, either new strain gages should be found, or the present gages should be retested above 200°F. If different gages are used, they should be more closely temperature compensated for use with the filled epoxy at temperatures above 200°F.

To perform transient tests under a vacuum environment, the working chamber should be enlarged to accommodate equipment needed to simulate the initial condition without collapsing the heater assembly. By raising or lowering either the model or the heater, the desired conditions could be obtained more quickly than by collapsing the heater assembly.

The techniques used to determine thermal conductivity and total hemispherical emittance proved to be good for testing materials in which thermocouples and heater wires may be embedded. When testing for total hemispherical emittance the models should be increased to about three-eighths inch in diameter to reduce errors caused by mislocation of the heater wire or the thermocouple. Also, the thermocouple should be embedded a known distance under the surface and surface temperature determined using the correction factor presented in Appendix A.

APPENDIX A

MATERIAL PROPERTIESA.1. Introduction

To compare analytical solutions to experimental results, knowledge of certain properties of the filled Epocast 10 modelling material was required. Those properties which had to be known were Poisson's ratio, thermal conductivity, total hemispherical emittance, density, specific heat, and coefficient of thermal expansion. In addition, to relate thermal stresses to strains, Young's Modulus was needed. Data were available for Young's Modulus ($E = 1.11 \times 10^6$ psi), Poisson's ratio ($\nu = 0.33$), density ($\rho = 105.2$ lb/ft³), specific heat ($c = 0.28$ Btu/lb - °R), and coefficient of thermal expansion ($\alpha = 22.5 \times 10^{-6}$ °R⁻¹). This information was presented by Putman (2, p. 140). These values were determined in the temperature range of -60°F to 200°F.

The analytical solutions for the temperature and stress distributions did not vary greatly when nonconstant material properties were considered, so that even though some of the material properties actually were functions of temperature, the above values were used without introducing large errors. These values represent an average of values taken by different investigators (8, 9, 10, and 11). A great deal of variation was found between results, and causes of the variations are not known. Two suspected causes are: cure cycles were not carefully controlled, and the manufacturer did not closely control properties of the Epocast 10 (11).

The value of thermal conductivity presented in reference (10) was not felt to be suitable for use in this testing because the components of the model material were measured by volume, not by weight as in this work. Also, this property was desired as a function of temperature. Therefore, testing was performed to determine thermal conductivity in the temperature range of interest.

Total hemispherical emittance of the lateral surface of the model was also needed. Properties of the paint chosen for use as the surface coating were unknown.

Originally, one model was used to determine both of these properties. A hollow circular cylinder was held at a steady state temperature distribution by an electrical resistance heater. The testing was performed under vacuum within the liquid-nitrogen-cooled heat sink so radiation from the lateral surface was the only significant mode of heat transfer. To determine thermal conductivity, power from the heater was related to gradients within the model by the Fourier equation for conduction of heat. Power was also related to surface temperature by an equation for radiation heat transfer to determine total hemispherical emittance. This method is described by Putman (2, Chapter 6). Further work is presented in reference (3, section III).

However, when testing was attempted, it was found that several major problems prevented reliable data from being obtained. First, circumferential symmetry could not be achieved. Since its existence was assumed in deriving the

analytical expressions for both properties, errors were introduced. The effect of these errors on the results obtained could not be accurately determined. Second, this method required a steady state temperature distribution so that long times elapsed before transient phenomena subsided. Finally, testing under the vacuum environment with the liquid-nitrogen-cooled heat sink caused large end losses. The expressions used to determine both of the unknown material properties did not account for gradients along the axial direction. For these reasons, it was decided that different test procedures and models were needed to obtain the two unknown material properties.

A.2. Thermal Conductivity

Measurement of thermal conductivity using the transient temperature response produced by a line heat source in an infinite medium has been described by many authors. However, Gale (5) is the only one (to the writer's knowledge) who has described the analytical method of solution.

The mathematical problem is formulated by investigating the transient heat flow in an infinite medium when a line heat source is located at the origin. The appropriate boundary value problem is

$$\frac{1}{\alpha_t} \frac{\partial \phi}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi}{\partial r} \right) + \frac{Q}{k} \delta(r)$$

$$\lim_{r \rightarrow \infty} \phi(r, t) = 0$$

$$\lim_{r \rightarrow \infty} \frac{\partial \varphi}{\partial r} (r, t) = 0$$

$$\varphi(r, 0) = 0$$

where

$\varphi(r, t)$	:	temperature ($^{\circ}\text{F}$)
k	:	thermal conductivity (Btu/hr-ft- $^{\circ}\text{F}$)
α_t	:	thermal diffusivity (ft ² /hr)
r	:	distance from origin (ft)
t	:	time (hr)
Q	:	power generated by heat source (Btu/hr-ft)
$\delta(r)$	:	Dirac delta function (ft ⁻²)

Gale shows that by transforming to Cartesian coordinates and utilizing Fourier transform techniques, the temperature response may be given as:

$$\varphi(r, t) = \frac{Q}{4\pi k} \int_0^t \frac{e^{-\frac{r^2}{4\alpha(t-\xi)}}}{t-\xi} d\xi$$

As will be seen after the final solution is obtained, only a solution near the origin is needed. To this end, make the following substitution:

$$u = \frac{r^2}{4\alpha(t-\xi)}$$

The temperature response then becomes

$$\varphi(r, t) = \frac{Q}{4\pi k} \int_{\frac{r^2}{4\alpha t}}^{\infty} \frac{e^{-u}}{u} du$$

According to reference (12), the exponential integral

$$E_1(z) = \int_z^{\infty} \frac{e^{-u}}{u} du$$

may be expressed as an infinite series

$$E_1(z) = -\gamma - \ln z - \sum_{n=1}^{\infty} \frac{(-1)^n z^n}{n n!}, \quad |\arg z| < \pi$$

where $\gamma = 0.5772\dots$ is Euler's constant.

By substituting this expression, the temperature response becomes:

$$\varphi(r, t) = \frac{Q}{4\pi k} \left[-\gamma - \ln \left(\frac{r^2}{4\alpha t} \right) - \sum_{n=1}^{\infty} \frac{(-1)^n \left(\frac{r^2}{4\alpha t} \right)^n}{n n!} \right]$$

for $\frac{r^2}{4\alpha t} < \pi$.

If φ_i is the temperature at time t_i and location r , then by considering the difference between temperatures at two different times, t_1 and t_2 , at a location near the origin:

$$\lim_{r \rightarrow 0} (\varphi_2 - \varphi_1) = \frac{Q}{4\pi k} \left[-\ln \left(\frac{t_1}{t_2} \right) \right]$$

By rearranging, thermal conductivity is expressed as:

$$k = \frac{Q}{4\pi} \frac{\ln\left(\frac{t_2}{t_1}\right)}{(\phi_2 - \phi_1)}$$

The experimental model was designed to simulate the mathematical model as closely as possible. Although the analytical solution was derived by considering an infinite medium, the temperature response in a model having finite dimensions would not be affected by the surface conditions for a certain length of time. With this fact in mind, the model was fabricated in a convenient size with little regard given to surface shape or finish. Since a three-inch-square by six-inch-long mold was available, the model was made this size.

The model was constructed in two sections. First, half of the model was molded under vacuum, and cured at 200°F. Then the upper surface was machined flat and a longitudinal groove cut so that a heater wire and a thermocouple might be embedded. A piece of 0.010 inch diameter Manganin wire was located in this groove. The wire was used to represent the line heat source through the origin. A copper-constantan thermocouple was fastened to the heater wire at the center of the model. It was used to indicate the temperature response. Thermocouple leadwires were located parallel to the heater wire so that negligible temperature gradients would exist in them. Both leadwires exited at one end of the model, near the heater wire. All wires were soldered to copper tabs which

were bonded to the end surfaces of the model. Connections were made to the tabs without fear of breaking the small diameter heater and thermocouple wires. This section of the model was placed back into the mold, and more epoxy was poured on top to fill the mold. The model was then evacuated and cured for 24 hours at 250°F.

The constant line source was closely approximated by using Manganin wire because its electrical resistance is very nearly independent of temperature. By applying a known, constant current through the wire, the power output of the source was easily regulated. By measuring the voltage drop across the wire, the size of the source was readily calculated. A direct current power supply and a digital voltmeter, both manufactured by Hewlett-Packard, were used to regulate and calculate the power of the source.

Since the analysis considered a line source, not a wire having finite diameter, errors were introduced. The effect of the finite wire diameter was to introduce nonlinearity into the temperature-versus-logarithm-of-time curve at small times. However, from the assumptions made in the theoretical analysis the solution is valid only for small values of $\frac{r^2}{4\alpha t}$. Therefore, one is not justified in using extremely short times. For these reasons, useful data were not recorded until the tests were underway for 10 seconds. Graphs of temperature response proved to be linear from this time until the effects of the surface were recorded.

The thermocouple used was copper-constantan model COCO-5, manufactured by Omega Engineering, Inc. It was not calibrated because no accurate means of determining the temperature was available. However, twelve of these thermocouples had been tested for consistency by this writer at a previous date and all proved to be within 0.1°F of one another. This value approximately equalled reading error of the Honeywell "Elektronic 15" used to record output.

To determine thermal conductivity as a function of temperature throughout the range of interest, tests were performed for overlapping temperature ranges by varying initial temperature of the model from test to test. To adjust the initial temperature, the model was placed in an automatically controlled oven. Two cans were used to surround the model, thereby minimizing convective currents on the surface. It had previously been found that this method of shielding allowed the model to be brought very close to a constant initial temperature throughout.

The effect on the results due to end losses was estimated by noting that these losses increased as model temperature increased during each test because the surrounding temperature was held constant. Thus, by comparing the value of thermal conductivity obtained from the beginning of a test at a higher temperature level with that obtained from the end of a test at a lower temperature level, this effect was isolated and an estimate of error was made. For all tests performed, end losses produced no noticeable effect on the results.

A typical graph of data is shown in Figure 21, and a table listing the values of thermal conductivity obtained from all tests is presented in Table I, Figure 22. Upon analyzing the data it was found that for power settings (Q) of 35.0 Btu/hr-ft the temperature response was linear until about 400-700 seconds. An increase in slope appeared in the curve after these times. It was probably caused by the surface effects. However, most tests were run for only 800 seconds so that some of this change in slope may have been caused by experimental errors in the last few temperature readings. Data were not used after this inflection occurred.

The data indicates that the thermal conductivity of this material is nearly independent of temperature from 80°F to 244°F. Therefore, a weighted arithmetic mean value of $k=0.55$ Btu/hr-ft-°F was used. All data taken in twenty-two tests were within plus 4% and minus 8% of this mean value. The small amount of scatter shows the value of this technique for determining thermal conductivity. Tests were simple to set up and run, yet total scatter was only 12%.

It is concluded that thermal conductivity of Epocast 10 filled with aluminum powder measuring 50% by weight is $0.55^{+0.02}_{-0.04}$ Btu/hr-ft-°F in the temperature range of 80°F-244°F.

A.3. Total Hemispherical Emittance

Total hemispherical emittance was determined for the surface coating which was applied to the model for the transient tests. The coating material used was Krylon Flat Black Spray

Enamel.

The model used to determine total hemispherical emittance was a circular cylinder, 1/4 inch in diameter and 8 inches long. A 0.010 inch diameter Manganin heater wire was embedded along the axis. Thus, circumferential symmetry was assumed to exist. A copper-constantan thermocouple was embedded as near as possible to the surface of the model. The material used to make the model was unfilled Epocast 10. The unfilled epoxy was used because it was easier to cast than the filled epoxy. Since the coating had to be opaque to be effective on the stress distribution model, any convenient modelling material could be used.

Tests were performed under steady state conditions so that, neglecting losses, the energy emitted from the surface necessarily was equal to energy supplied by the electrical resistance heater. To ensure that negligible radiation was incident on the model, it was placed within the heat sink described in Chapter 5 of this report. Conduction heat transfer was minimized by testing under a vacuum of 4×10^{-6} mm Hg. Under these conditions, energy transferred from the surface is given by:

$$q = \epsilon \sigma A T_s^4$$

where

q : heat supplied by heated wire (Btu/hr)

ϵ : total hemispherical emittance

σ : Stefan-Boltzmann constant = 0.1714×10^{-8}
(Btu/hr-ft²-°R⁴)

A: lateral surface area (ft^2)

T_s : surface temperature ($^{\circ}\text{R}$)

Heat supplied by the wire resistance heater may be expressed as:

$$q = 3.412VI$$

where

V: voltage drop across wire heater (volts)

I: current flowing through heater (amps)

By equating these two expressions, total hemispherical emittance becomes:

$$\epsilon = \frac{3.412VI}{\sigma A T_s^4}$$

Tests were performed using two different substrate models to determine whether positioning of the heater wire and the thermocouple were critical. Emittance values varied by approximately 15% between the two models. The difference was attributed primarily to mislocation of the thermocouple bead in the second model. In that model the thermocouple was approximately a sixteenth inch from the surface. Because of this fabrication error, data from the second model were not used. In the first model the thermocouple was at the surface.

Initial tests were used to determine how many coats of paint were needed to produce an opaque coating. Two coats were found to be sufficient by: applying a coat of paint, determining the emittance, painting again, redetermining emittance, etc. until no variation in emittance could be detected.

Radiative end losses were held to a minimum by choosing the dimensions of the model so that end areas were quite small relative to the lateral area. In the models tested, these losses were approximately one percent. Leadwires attached to the ends of the model were shielded with highly reflective metal foil so that radiative losses from these surfaces were reduced to a minimum. Maximum heat loss by conduction through the leadwires was approximately four percent throughout the temperature range tested. This value was calculated by assuming a linear temperature distribution along the wire. The effect of these losses was to increase the value of emittance obtained.

Power was supplied and current measured by a Hewlett-Packard direct current power supply. Voltage drop across the heater was measured on a Hewlett-Packard digital voltmeter. Thermocouple output was indicated on a Leeds-Northrup, Model No. 7552, portable potentiometer.

Data for total hemispherical emittance of Krylon Flat Black Spray Enamel are given in Table II, Figure 23. Results seem to be a weak function of temperature so the arithmetic mean value of $\epsilon = 0.88$ was used. All data fell within six percent of this value throughout the temperature range 60 - 250 degrees Fahrenheit.

Since the second model showed that instrumentation of the model was critical, the following method of fabrication is recommended when greater accuracy is needed. Although the one-quarter inch diameter was ideal for reducing end losses, a larger diameter would decrease errors introduced by mislocation of the thermocouple or the heater wire.

Future models should be approximately three-eighths or one-half inch in diameter. To insure purely radial heat flow (circumferential symmetry), the method used to make the models must allow accurate centering of the heater wire. To more accurately determine the surface temperature, the thermocouple bead should be embedded a known distance under the surface. Then, when thermal conductivity is known for the substrate, the surface temperature can be calculated. Assuming radial heat flow only, and using the Fourier equation of heat conduction:

$$T_s = T_i + \frac{Q}{2\pi k} \ln \left(1 - \frac{d}{r_o} \right)$$

where

- T_s : surface temperature ($^{\circ}\text{F}$)
- T_i : indicated temperature at thermocouple location ($^{\circ}\text{F}$)
- Q : radial heat flow per unit length (Btu/hr-ft)
- k : thermal conductivity (Btu/hr-ft- $^{\circ}\text{F}$)
- r_o : radius of model (ft)
- d : distance under surface at which thermocouple is located (ft)

With models constructed in the above manner, more accurate values of emittance could be obtained. However, the value of $\epsilon = 0.88$ was thought to be accurate enough for this work.

RESULTS OF A TYPICAL THERMAL CONDUCTIVITY TEST

Figure 21

TABLE I

THERMAL CONDUCTIVITY OF EPOCAST 10 FILLED
WITH ALUMINUM POWDER MEASURING 50% BY WEIGHT

Test No.	Temperature Range ($^{\circ}\text{F}$)	Power Setting ($\frac{\text{BTU}}{\text{hr-ft}}$)	Thermal Conductivity ($\frac{\text{BTU}}{\text{hr-ft-}^{\circ}\text{F}}$)
1	91-113	35.0	0.53
2	218-239	35.0	0.55
3	203-225	35.0	0.57
4	218-228	15.8	0.53
5	224-244	35.0	0.56
6	193-215	35.0	0.55
7	157-179	35.0	0.55
8	162-183	35.0	0.55
9*	173-195	35.0	0.55
10	185-206	34.9	0.55
11	179-200	35.0	0.54
12	116-136	34.5	0.51
13	168-188	34.5	0.56
14	159-179	34.5	0.57
15	80-90	15.7	0.52
16	92-114	34.1	0.56
17	144-167	34.8	0.56
18	81-91	15.7	0.52
19	149-170	35.0	0.56
20	154-174	35.0	0.55
21	87-104	26.6	0.52
22	91-113	34.1	0.55

* Data for this test are presented in Figure 21.

TABLE II
TOTAL HEMISPHERICAL EMITTANCE
OF
KRYLON FLAT BLACK SPRAY ENAMEL

Coats of Paint	Temperature T_s ($^{\circ}\text{R}$)	Voltage V (volts)	Current I (amps)	Hemispherical Emittance ϵ
2	545	2.25	0.70	0.83
2	565	2.42	0.80	0.88
2	594	2.70	0.90	0.90
2	628	3.03	1.00	0.90
2	664	3.40	1.10	0.89
2	564	2.42	0.80	0.89
2	544	2.24	0.70	0.83
3	545	2.26	0.70	0.83
3	578	2.55	0.80	0.84
3	594	2.70	0.90	0.90
3	622	2.97	1.00	0.91
3	688	3.67	1.20	0.90
3	709	3.90	1.30	0.93
3	517	2.02	0.68	0.89

Figure 23

APPENDIX B - STRAIN GAGE PROPERTIESB.1. Introduction

Strain gages chosen for use in this testing were manufactured by Micro-Measurements Inc. and were designated MA-ST-250BG-120. They had not previously been used in the Mechanical Engineering Department. To ensure that reliable results would be obtained in the thermal stress model, extensive testing was performed to determine the most important characteristics of these gages.

The relationship between change in resistance of the gage and actual longitudinal strain experienced by the gage is termed gage factor, and defined by $F = \frac{\Delta R/R}{\epsilon_t}$. Gage factor was checked to determine if the manufacturer's value ($F_u = 2.11$) could be used when these gages were embedded in the modelling material.

The effect of cross-sensitivity of the gage was neglected in all cases. This assumption was felt to be justifiable because cross-sensitivity was quoted as $K = 0.007$ by the manufacturer.

Strain gages also undergo change in resistance due to the effects of temperature changes. When connected into a bridge system to determine the strain which produces stress, i.e., mechanical strain, the signal caused by this false strain is indistinguishable from the desired signal. Therefore, to separate the effects, false strain as a function of temperature must be determined. Then, by monitoring temperature of the gage during a test, the false strain may be eliminated, and the desired mechanical strain obtained. For this work the

false strain had to be quite accurately known because mechanical and false strains were expected to be of the same order of magnitude.

When using embedded strain gages, care must be taken so that the desired strain field is not disturbed by the presence of the gage. Previous investigators (10) have shown that etched-foil gages do not disturb the strain field when properly embedded. The most important consideration here is to be sure the strain gage is bonded to the material.

These three problems were investigated and the work is described in the following sections.

B.2. Gage Factor

It is possible that when strain gages are embedded in low modulus materials, reinforcement may occur. This problem is discussed in reference (4, section 4.2h). The effect of this reinforcement is to lower the effective value of the gage factor. To determine if the manufacturer's specified value of gage factor was accurate when these gages were embedded in filled Epocast 10, several gages were embedded in the modelling material and subjected to a known strain field.

A straining frame was used to bend a beam in a constant moment section. By loading the beam as shown in Figure 24 (a), the constant moment section was approximated near the center.

After bending, the neutral axis of the beam assumed a constant radius (R) with dimensions shown in Figure 24 (b). The distance (x) was measured with a dial indicator to determine curvature as follows:

(a) Loading

(b) Configuration

CONSTANT MOMENT BEAM

Figure 24

$$\left(R + \frac{h}{2} - x\right)^2 + \left(\frac{l}{2}\right)^2 = \left(R + \frac{h}{2}\right)^2$$

$$R = \frac{4(x - h)x + l^2}{8x}$$

By assuming that plane sections remain plane, longitudinal strain at distance y from the neutral axis is expressed as:

$$\epsilon_l = \frac{(R + y)\theta - R\theta}{R\theta}$$

Therefore, strain may be expressed as a function of measurable physical dimensions.

$$\epsilon_l = \frac{2xy}{x^2 - hx + \frac{l^2}{4}}$$

But x is much less than l for elastic strains so that:

$$\epsilon_l \cong \frac{8xy}{l^2}$$

Strain gages to be tested were located in the beam as shown in Figure 25. Strain gage no. 2 was bonded to a small

- Gage No. 1: FA 12-12 (Mfg. by Baldwin-Lima-Hamilton Corp.)
 Gage No. 2: MA-ST-250BG-120
 Gage No. 3: MA-ST-250BG-120

GAGE FACTOR TEST BEAM

Figure 25

coupon (dotted line), and the coupon was embedded in the beam by machining a rectangular hole, and bonding the block in the hole on all sides. The same procedure was used to embed gages in the thermal stress model. Strain gage no. 3 was embedded by splitting the beam, applying the gage to the free surface and then bonding the beam back together. The beam was relieved in the area above the gage so as not to ruin the gage when it was clamped for rebonding. Strain gage no. 1 was a gage with known characteristics which was used to check the strain field against theory. Output from gage no. 1 did check well against theory, which provided assurance that the two embedded gages were bonded and the strain field was not significantly disturbed at a distance from the embedded strain gages.

Outputs from the gages were indicated on Baldwin-Lima-Hamilton, type N, strain indicators. The gage factor settings on these instruments were adjusted to agree with the manufacturer's value of gage factor in all cases. The beam was strained so that all gages were tested in both tension and compression. Data are presented in Figure 26. The curve was chosen as the best fit of all data.

Slight reinforcement was indicated by the data. The corrected gage factor ($F = \frac{1386}{1400} \times 2.11 = 2.09$) was about 1% lower than the specified value. However, the scatter of data was about $\pm 2\%$ so the manufacturer's specified value of $F_u = 2.11$ was within the experimental error of this work.

B.3. Temperature Compensation

The filled epoxy used for modelling in these experiments

GAGE FACTOR TEST RESULTS FOR MA-ST-250BG-120 STRAIN GAGES
EMBEDDED IN FILLED EPOCAST 10

Figure 26

has a coefficient of thermal expansion of $22.5 \mu\text{in/in} - ^\circ\text{F}$. At present, no manufacturer makes a self-temperature-compensating strain gage for use with this material. Therefore, gages were chosen which were compensated as closely as possible to this value, and accurate tests were performed to determine the false strain produced when they were embedded in the filled epoxy.

Two strain gages were randomly selected from a supply of twenty gages to be used to determine the thermal strains. Each gage was bonded to a small tab made from the modelling material. Small tabs were used because they could be more easily brought to thermal equilibrium than a larger piece. A thermocouple was bonded to the top surface of the gage, and then both transducers covered with more epoxy.

During a temperature compensation test, the tab with thermocouple and strain gage embedded was stabilized at various temperatures in the range of interest by placing it in an oven. Thermocouple outputs were indicated on a Leeds-Northrup millivolt potentiometer.

Each test took about five hours to run, and drift in the available strain indicators proved to be a significant problem. A balanced Wheatstone bridge in which the temperature of all components could be controlled was designed to eliminate this problem. The bridge was waterproofed and immersed in an ice bath (Figure 27) to accomplish the temperature control. Drift was found to be less than 10 microinches per inch during a test.

The active strain gage (R_g) was connected to the bridge using a three-wire lead system. The dummy gages (R_d) were bonded to a piece of aluminum. These gages were closely

compensated for use with aluminum so that output from them caused by a temperature change was minimized. Initial balance was controlled by R_B and R_b which were also immersed in the ice bath to further reduce drift. The three-wire lead system was used to eliminate output due to equal changes in resistance of the lead wires (R_ℓ).

STRAIN GAGE INSTRUMENTATION-FALSE STRAIN DETERMINATION

Figure 27

Batteries supplied current to the bridge, and a voltage dividing circuit allowed constant nominal current to be supplied even though the batteries ran down slightly.

Output was read on a Leeds-Northrup, Model No. 7552, potentiometer and related to strain by using the calibrate resistor (R_C). When balanced, the potentiometer drew very little current and therefore acted as a high impedance load. It was easily rezeroed at any time, and possessed necessary sensitivity and accuracy.

To stabilize temperature in the small tab of epoxy, it was placed in an automatically-controlled oven. Convective

currents were minimized by surrounding the tab with two cans, one inverted.

To begin a test, the bridge was balanced with the tab at room temperature. The temperature of the tab was elevated, allowed to stabilize, and the false strain reading recorded. The calibrate resistor was used at each temperature level.

An analysis was performed to determine the relationship between the resistance change across the active arm of the bridge, and the superposed false strain and calibrate strain. Change in resistance from balance is related to total strain by the definition of gage factor,

$$\Delta R/R_g = F\epsilon_t$$

where

ΔR : change in gage resistance (ohms)

R_g : gage resistance at balance (ohms)

F : gage factor

ϵ_t : total strain (neglecting cross-sensitivity).

If the gage undergoes false strain and the calibrate resistor is also used, the resistance change becomes:

$$\Delta R = \frac{1}{\frac{1}{R_g + \Delta R_f} + \frac{1}{R_c}} - R_g$$

where

R_c : calibrate resistance (ohms)

ΔR_f : change in gage resistance due to false strain (ohms)

Total strain becomes:

$$\epsilon_t = \frac{1}{F} \frac{\Delta R_f}{R_g \left[1 + \frac{R_g + \Delta R_f}{R_c} \right]} - \frac{1}{F} \frac{(R_g + \Delta R_f)}{R_c + (R_g + \Delta R_f)}$$

Now the calibrate strain, ϵ_{eq} is expressed as:

$$\epsilon_{eq} = -\frac{1}{F} \frac{(R_g + \Delta R_f)}{R_c + (R_g + \Delta R_f)}$$

if the calibrate resistor (R_c) is put in parallel with the gage, the resistance of which, after straining, is $(R_g + \Delta R_f)$.

However, the false strain is defined as

$$\epsilon_f = \frac{1}{F} \frac{\Delta R_f}{R_g}$$

and is independent of the calibrate resistance. But for etched foil gages and for small strains, R_c is much greater than $(R_g + \Delta R_f)$, so that

$$\epsilon_t \cong \epsilon_f + \epsilon_{eq}.$$

This indicates that the false strain and calibrate signals may be considered as linearly superposed.

Since the unbalanced bridge was used in its linear output range, the output voltage was calibrated during testing by taking the difference between the total output signals with and without the calibrate resistor. Since the same false strain signal was indicated for both of these cases, the difference between them corresponded to the calibrate strain signal. This signal was then compared to calibrate strain to determine sensitivity of the bridge.

Seven tests were performed on the two gages. Since the strain levels during individual tests were arbitrarily determined by balancing the bridge at the initial temperatures, these strain levels were arbitrarily raised or lowered to give the best correlation of data when plotted on one graph.

No problems arose from the arbitrary choice of strain level because when the stress distribution model was tested, only changes in false strain caused by changes in temperature were needed.

Data from all tests are presented in Figure 29. Below about 200°F the data for both gages compare favorably. However, above this temperature a large amount of scatter in the data was evident and the false strain curve could not be accurately determined. For this reason the strain gages were not used at temperatures above 200°F. The lower portion of the false strain curve is presented in Figure 28 with the temperature and strain scales expanded to allow more accurate determination of false strain. At temperatures below 200°F and for all gages in the lot, it is assumed that the curve is an accurate representation of false strain vs. temperature within , at most, 15 microinches per inch.

B.4. Bond Strength

A strain gage was embedded as if to measure the transverse strains in a beam bent in a constant moment section, as described in section 2 of this Appendix. A tensile load was applied on the bonded top surface of the gage when the beam was loaded. The edge of the gage was located at the outermost tensile fibers of the beam.

Although no accurate quantitative data were taken, the bond withstood a maximum stress of over 800 psi. From theoretical considerations this was felt to be a sufficient bond strength to withstand test conditions in the stress distribution model, so this testing was ended.

FALSE STRAIN VS TEMPERATURE FOR MA-ST-250BG-120 STRAIN GAGES EMBEDDED IN FILLED EPOCAST 10 IN THE TEMPERATURE RANGE OF 85-225°F

Figure 28

FALSE STRAIN VS TEMPERATURE FOR MA-ST-250BG-120 STRAIN
GAGES EMBEDDED IN FILLED EPOCAST 10 IN THE TEMPERATURE
RANGE OF 85-275°F

Figure 29

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PART II

SUMMARY AND CONCLUSION OF THE INVESTIGATION

The question which motivated the investigation reported in Part I and in the first and third references (page 100) is whether a cylindrical fuel rod proposed for use in nuclear auxiliary power systems could be caused to fail mechanically by exposing it to the environment which an orbiting body experiences just prior to entry into the earth's atmosphere. It was reasoned that if the rod had a temperature above the equilibrium temperature associated with that environment, exposure would result in cooling due to emission of thermal radiation. The cooling process would continue until the rate of aerodynamic heating increased to offset the rate of radiative cooling. It was wondered whether the thermal stresses associated with the cooling process would be large enough to produce mechanical failure.

In an attempt to obtain an answer rapidly and easily, the decision was made to consider a limiting case in the initial analytical attack on the problem. In the event that the results for this limiting case indicated a high probability of failure according to a common failure criterion, less extreme cases were to be investigated along with other failure criteria. (It was found that the limiting case produced results from which a useful answer could be obtained.) An experimental investigation was begun at the same time as the analytical investigation; its purpose was to obtain from a physical model data which could be compared with the analytical results.

The limiting case investigated was that which produced the highest stress levels consistent with a brittle condition of the uranium-zirconium hydride fuel rod material. Brittleness of the material was felt to be necessary for high probability of failure during a single cooling process, which would be all that the prototype would encounter. Higher levels of stress were thought, and later proven, to be associated with higher rates of cooling. Hence, the conditions selected for the limiting case were those which would produce the highest rate of cooling consistent with brittleness of the material.

Because the length-to-diameter ratio of the prototype fuel rod is large, the rod was assumed to be infinitely long in the mathematical model. The initial temperature of the rod was assumed to be uniform. The fuel rod material was assumed to be homogeneous and isotropic, although there is no evidence to support the latter assumption, as noted on page 7 of reference (1). From these assumptions it follows that the temperature and stress distributions are functions only of radial position in the rod.

For a material with properties which are essentially constant over the temperature range covered during a cooling process, which is the situation for this investigation, the value of only one parameter need be specified to fix the cooling rate. This parameter, which is dimensionless and may be interpreted as the resistance to conductive heat transfer inside the rod divided by the resistance to radiative heat transfer from the rod, was denoted by N_R . As noted on pages

26 and 27 of reference (1), a value of 0.0395 for N_R corresponds to an extreme upper limit on the rate of cooling of a uranium-zirconium hydride fuel rod in a brittle state. Results were computed for this value of N_R as well as for much larger and somewhat smaller values of N_R . Before discussing the nature of these results and the conclusion which was drawn from them, we shall consider the experimental results which substantiate them.

Experimental results corresponding to $N_R = 0.099$ are presented in Part I, Chapter 7, of this report for both strain and temperature. (Results for strain, rather than stress, are presented in order to avoid errors which might accrue from combining the results arithmetically.) These results were obtained by embedding strain gages and thermocouples in a cylindrical plastic model. The model had a length-to-diameter ratio of five, to reduce end effects.

The experimental results for temperature at four radial positions deviated from the analytical results by a maximum of 2°F for $N_R = 0.099$. The experimental results for principal strain at three radial position were within 35 micro-inches per inch of the analytical values, except for one principal strain at one position. (The results for this strain are anomalous.) The maximum difference between the results for temperature was about 1% while that for strain was about 10% of the maximum principal strain, excluding the anomalous results. Since the agreement between analytical and experimental results for these positions for $N_R = 0.099$

was relatively good, it was concluded that the analytical results satisfactorily predict the temperature and stress (or strain) distributions for $N_R \leq 0.099$. It should be noted that this range includes the value of N_R corresponding to the maximum cooling rate of the prototype.

With confidence established in the analytical results, attention is directed to the nature of the results and the conclusions which can be drawn from them. As stated on page 29 of reference (1), the maximum principal stress which is produced by a cooling rate corresponding to $N_R = 0.0395$ is 1260 pounds per square inch. The ultimate tensile strength reported for uranium-zirconium hydride fuel rod material is 8340 pounds per square inch.^a Although other types of tests yielded lower values for ultimate strength of this material, as noted on page 29 of reference (1), the lowest value reported is almost twice the above value of maximum principal stress. Using the maximum principal stress theory of failure, it follows that the probability of failure of a fuel rod due to cooling prior to entry into the earth's atmosphere is rather low. Unless there is a failure criterion which indicates a much lower stress level for failure of this system--due, perhaps, to the fine cracks in the fuel rod material as mentioned on page 30 of reference (1)--it appears that a fuel rod cannot be expected to fail as a result of radiative cooling.

^a"High Temperature Property Study for Reentering NAP Systems; Final Report," AFSWC TDR 63-17, June, 1963.