90
10-12-65

OCT 11 1965

AEC
RESEARCH
and
DEVELOPMENT
REPORT

BNWL-116

MASTER

PRESSURIZING VESSEL PERFORMANCE EQUATIONS

K. D. COUGHREN

AUGUST, 1965

RELEASED FOR ANNOUNCEMENT
IN NUCLEAR SCIENCE ABSTRACTS

RICHLAND, WASHINGTON

PACIFIC NORTHWEST LABORATORY operated by BATTELLE MEMORIAL INSTITUTE

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission.

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

PACIFIC NORTHWEST LABORATORY

RICHLAND, WASHINGTON

operated by

BATTELLE MEMORIAL INSTITUTE

for the

UNITED STATES ATOMIC ENERGY COMMISSION UNDER CONTRACT AT(45-1)-1830

PRINTED BY/ FOR THE U. S. ATOMIC ENERGY COMMISSION

BNWL-116

UC-38, Engineering
and Equipment

FIRST
UNRESTRICTED

TID-4500

SEP 27 1965

45 III

DISTRIBUTION
MADE

EDITION

PRESSURIZING VESSEL PERFORMANCE EQUATIONS

By

K. D. Coughren

Instrument Research and Development Section
Physics and Instruments Department

August, 1965

PACIFIC NORTHWEST LABORATORY
RICHLAND, WASHINGTON

12 : 1 2 3 4 5 6

Printed in USA. Price \$2.00. Available from the Clearinghouse for Federal
Scientific and Technical Information, National Bureau of Standards
U. S. Department of Commerce, Springfield, Virginia

PRESSURIZING VESSEL PERFORMANCE EQUATIONS

INTRODUCTION

Pressurized water systems frequently use steam pressurized vessels to maintain and control system pressure. Such a system employs a large pressure vessel having a steam pressurized upper region and a lower water region used for water surges. The water is maintained in a saturated condition by heaters so that any change in pressure will either cause water to flash into steam or cause steam to condense. Therefore, pressure is controlled by water flashing or water sprays that condense steam. Water surges in and out as a result of temperature changes in the system connected to the pressure vessel.

ANALYSIS

A mathematical description of a pressurizer is based on energy equations that relate internal energy to external energy inputs. These equations predict the performance of the pressurizer vessel. By convention, all inputs to the system are designated positive and all outputs are designated negative. The analysis is done in two parts: insurge and outsurge. An insurge, defined as a water flow into the pressure vessel, includes flow through the connecting line and flow through the spray nozzles. An outsurge is defined as water flow out of the pressure vessel through the connecting line. Water inventory in the vessel considers insurge, outsurge, steam condensing, and water flashing. Water flashing is assumed to occur at only the interface between the steam region and water region. Condensing is assumed to be on the surface of the spray droplets and the pressure vessel walls. The pressurizer vessel is assumed to be well insulated, and the heat transfer to the wall is in terms of heat storage. Each region of the pressurizer vessel and its boundary conditions is considered separately. The top of the vessel contains the steam region and the bottom of the vessel contains the water region. The water region is divided into two halves, thereby limiting the flashing to only the top half. Another region, the

condensate region, is assumed to be between the water region and the steam region. It includes the sprays and steam that have condensed on the spray droplets and on the vessel walls. All energy modes of each region are considered to completely describe the system. These modes are:

$$\text{Kinetic Energy } \dot{\Sigma KW} \text{ (energy/unit time)}$$

Positive work is defined as work done on the system by the environment. This is kinetic work which appears at the system boundary such as shaft work supplied to the system or supplied by the system.

$$\text{Rate of Heat Transfer } \dot{\Sigma Q} \text{ (energy/unit time)}$$

The rate of heat transfer is defined as positive into the system. More than one source may be available; therefore, a summation is required.

$$\text{Convected Energy Rate } \dot{\Sigma Mh} \text{ (energy/unit time)}$$

The energy convected across a system boundary is defined as positive when mass flow is directed into the system. The convected energy has four primary thermodynamic components. These are specific internal energy u , specific kinetic energy of directed motion $1/2V^2$, flow work Pv , and specific potential energy Z . Therefore, the net convective energy to the region is:

$$\dot{\Sigma Mh} = \dot{M}_{in}(u + Pv + 1/2V^2 + Z)_{in} - \dot{M}_{out}(u + Pv + 1/2V^2 + Z)_{out} \quad (1)$$

$$\text{Stored Energy } \dot{\Sigma MU} = \Sigma [\dot{MU} + \dot{MU}] \text{ (energy/unit time)}$$

Stored energy includes the increase of thermal energy of the stored matter and the energy added with the addition of matter.

$$\text{System Expansion Energy } \dot{PV} = P \frac{dV}{dt} + V \frac{dp}{dt} \text{ (energy/unit time)}$$

The boundaries of a system for thermal dynamic analysis need not remain fixed. When the boundary of the system expands, work must be done by the system on the external environment that limits the system. The energy in the expansion energy term must be appropriate to the particular

moving boundary. The expansion energy would be zero if the boundary moved so rapidly that the pursuing fluid could exert no pressure on the boundary. Assuming that the pressure of the expansion energy is not zero or constant, then any change in this pressure would show up as a change in expansion energy. Therefore, the total derivative of the PV term is used to take into account the change in the movable boundary and the change in pressure at the movable boundary. If the pressure were constant, then the expansion energy would be given as $p\dot{v}$. A completely described system must include all the above energy modes, which are embodied in a mass balance, volume balance, and an energy balance. The equations take the form as follows:

Mass Balance

$$\dot{M}_a + \dot{M}_b + \dots + \dot{M}_j = \dot{m}_1 + \dot{m}_2 + \dot{m}_3 + \dots + \dot{m}_i \quad (2)$$

Volume Balance

$$\dot{V} = \dot{M}_a v + \dot{M}_b v + \dots + \dot{M}_j v \quad (3)$$

Energy Balance

$$\Sigma \dot{Q} - \Sigma \dot{KW} + \Sigma \dot{M}h = \dot{M}u + \dot{P}V \quad (4)$$

where,

$$\dot{M}_a = \frac{dM_a}{dt} \quad (5)$$

$$\dot{M}u = M \frac{du}{dt} + u \frac{dM}{dt} \quad (6)$$

It is assumed that all of the terms of the above equations are in the same, or consistent, units. The common conversion factor to such equations is:

$$\begin{aligned} J &= \text{mechanical equivalent of heat} = 778.2 \text{ ft}\cdot\text{lb/Btu} \\ &= \text{ " " " " } = 252 \text{ cal/Btu} \\ &= \text{ " " " " } = 4.1858 \text{ J/cal} \end{aligned}$$

The application of this method of system analysis will begin by applying the equations to each region.

I. INSURGE

A. Steam Region

The steam region is defined as containing only steam at any instant of time and is bounded by the vessel walls and the movable interface between the steam and water. Steam is condensed during an insurge due to condensation on spray droplets and condensation on the vessel walls. The steam is compressed, becoming hotter than the vessel walls and the spray droplets. Since this is the loss of latent heat from the steam, no temperature gradient exists. This is true for the wet region and approximately true for the superheat region. The remaining mass of steam remains undisturbed by removing steam mass by condensation.

Applying the energy equation

FIGURE 1

$$\dot{\Sigma Q} - \Sigma KW - \dot{M}_s h_g = \dot{M}_g u_g + P_g V_g \quad (\text{Figure 1}) \quad (7)$$

where,

$$\dot{\Sigma Q} = 0 \text{ since there is no net heat added to the region}$$

$\Sigma KW = 0$ because there is no shaft work done on the region

$\Sigma \dot{M}_s h_g$ = the net energy convected to the region.

A mass balance shows

$\dot{M}_s$ (steam leaving the region) = $-\dot{M}_g$ (net negative change of steam mass in the region).

A volume balance gives

$$\dot{V}_g = \frac{\dot{M}_g v_g}{\rho_g} \quad (8)$$

Equation (7) then becomes

$$\dot{M}_g h_g = \dot{M}_g \dot{u}_g + \dot{M}_g \dot{u}_g + P_g \dot{V}_g + \dot{P}_g V_g \quad (9)$$

or

$$\dot{M}_g h_g = \dot{M}_g \dot{u}_g + \dot{M}_g \dot{u}_g + P_g (\dot{M}_g \dot{v}_g + \dot{M}_g v_g) + \dot{P}_g M_g v_g \quad (10)$$

Collecting terms then

$$\dot{M}_g h_g = \dot{M}_g (\dot{u}_g + P_g v_g) + M_g (\dot{u}_g + P_g \dot{v}_g + \dot{P}_g v_g) \quad (11)$$

where,

$$h_g = u_g + P_g v_g \quad (12)$$

$$\dot{h}_g = \dot{u}_g + P_g \dot{v}_g + \dot{P}_g v_g \quad (13)$$

then,

$$\dot{M}_g h_g = \dot{M}_g h_g + M_g \dot{h}_g \quad (14)$$

or,

$$\dot{h}_g = \dot{u}_g + P_g \dot{v}_g + \dot{P}_g v_g = 0 \quad (15)$$

which indicates the internal energy of the remaining steam mass is unaffected by the mass of steam leaving the region as a result of an insurge.

B. Spray Plus Condensate Region

An energy balance is taken of the region containing only the mass of the spray and condensate after it has passed through the steam region. The mass of the condensate is the change in mass of the steam while the spray receives energy from the steam. The condensate region is the region where the phase change from steam to condensate occurs. Therefore, it can be considered as a massless region with zero volume having the steam-water interface as the upper boundary. Condensate leaving this region enters the water region. The energy is transferred to the spray by steam condensing on the spray droplets, thus increasing the temperature of the spray. Steam also condenses on the vessel walls during a pressure rise; but, this effect can usually be neglected because the transients involving the sprays are much faster and of greater magnitude; however, the analysis will include this effect. The analysis of the spray and condensate region begins with writing an energy equation for that region.

FIGURE 2

$$-\dot{Q}_w - \dot{M}_g h_g + \dot{M}_{sp} h_{sp} = \dot{M} u + \dot{P} V \quad (\text{Figure 2}) \quad (16)$$

or

$$-\dot{Q}_w - \dot{M}_g h_g + \dot{M}_{sp} h_{sp} = \dot{M}u + \dot{M}\dot{u} + \dot{P}_g V + P_g \dot{V} \quad (17)$$

but

$$V = Mv \quad (18)$$

Then

$$-\dot{Q}_w - \dot{M}_g h_g + \dot{M}_{sp} h_{sp} = \dot{M}u + \dot{M}\dot{u} + \dot{P}_g Mv + P_g (\dot{M}v + M\dot{v}) \quad (19)$$

Collecting terms gives

$$-\dot{Q}_w - \dot{M}_g h_g + \dot{M}_{sp} h_{sp} = \dot{M}(u + P_g v) + M(\dot{u} + \dot{P}_g v + P_g \dot{v}) \quad (20)$$

where,

$$h = u + P_g v \quad (21)$$

$$\dot{h} = \dot{u} + \dot{P}_g v + P_g \dot{v} \quad (22)$$

Then assuming saturated condensate Equation (20) becomes

$$-\dot{Q}_w - \dot{M}_g h_g + \dot{M}_{sp} h_{sp} = \dot{M}h_f + M\dot{h}_f \quad (23)$$

Writing a mass equation gives

$$\dot{M} = -\dot{M}_g + \dot{M}_{sp} \text{ and } M = -M_g + M_{sp} \quad (24)$$

where,

$$-\dot{M}_g = \dot{M}_c = \text{rate of change of condensate mass.} \quad (25)$$

Substituting then gives

$$-\dot{Q}_w - \dot{M}_g h_g + \dot{M}_{sp} h_{sp} = (-\dot{M}_g + \dot{M}_{sp}) h_f + (M_c + M_{sp}) \dot{h}_f \quad (26)$$

or

$$-\dot{Q}_w - \dot{M}_g(h_g - h_f) - \dot{M}_{sp}(h_f - h_{sp}) = (M_c + M_{sp})\dot{h}_f \quad (27)$$

If

$$\dot{h}_f = \frac{dh_f}{dt} \cdot \frac{dP_g}{dP_g} = \frac{dh_f}{dP_g} \cdot \frac{dP_g}{dt} \quad (28)$$

then,

$$-\dot{Q}_w - \dot{M}_g(h_g - h_f) - \dot{M}_{sp}(h_f - h_{sp}) = (M_c + M_{sp}) \frac{dh_f}{dP_g} \cdot \frac{dP_g}{dt} \quad (29)$$

Solving for $\dot{M}_g$ gives

$$\dot{M}_g = \frac{-\dot{Q}_w - \dot{M}_{sp}(h_f - h_{sp}) - (M_c + M_{sp}) \frac{dh_f}{dP_g} \cdot \frac{dP_g}{dt}}{(h_g - h_f)} \quad (30)$$

In many cases dh_f/dP_g can be linearized and dP/dt is available through differentiation. The heat transferred from the condensate region is seen as a heat loss to the vessel walls as the pressure changes. Heat transferred to the walls may be considered since the insurge causes a higher pressure, which in turn, causes a higher temperature than that of the vessel walls. Because of condensation on the walls, the coefficient of heat transfer between the wall and the steam is large compared to the coefficient when the walls are hotter than the steam (during outsurge). Therefore, heat transferred to the walls during an insurge is considered, while during an outsurge it is neglected. An expansion of the energy equation assuming constant pressure for the spray plus-condensate region during an insurge is shown in Appendix A.

The time constant for heat transfer depends on the mass of the vessel walls and the heat transfer film coefficient. For large vessels the wall can be assumed to be a flat plate. The heat transfer coefficient at the wall can be approximated by the addition of an imaginary thickness to the

wall. A good approximation of the temperature gradient inside the wall can be shown using one node of heat capacity (Figure 3). The heat transferred through the inside surface must be equal to the heat stored in the wall since it is assumed none is transferred outside the wall.

FIGURE 3

The heat transferred into the wall is

$$\dot{Q}_w = -KA_w \frac{\Delta T}{\Delta X} \quad (31)$$

where,

K = thermal conductivity

A_w = steam region wall area, it is assumed the change in wall area due to water level change can be neglected

$$\Delta T = T_g - T_w \quad (32)$$

$$\Delta X = -(x_0 - x_1) \quad (33)$$

The imaginary thickness added to the wall can be defined as the approximation

$$a = K/h_T \quad (34)$$

where

K = thermal conductivity of the wall

h_T = film coefficient of the condensate

then,

$$\Delta X = -(K/h_T + X) \quad (35)$$

Substituting into the heat transfer equation gives

$$\dot{Q}_w = KA_w \frac{T_g - T_w}{X + K/h_T} \quad (36)$$

The heat transferred to the wall must be equal to the heat stored in the wall since the vessel is assumed to be well insulated. The stored heat is

$$\dot{Q}_w = mC_p \Delta T \quad (37)$$

where,

$$m = \rho_w A_w X \quad (38)$$

X = vessel wall thickness since the imaginary thickness cannot store heat

ΔT = change in T_w with respect to time

or

$$\Delta T = \frac{dT_w}{dt} \quad (39)$$

Then

$$\dot{Q}_w = \rho_w A_w X C_p \frac{dT_w}{dt} \quad (40)$$

Therefore, equating the heat storage and heat transfer gives

$$KA_w \frac{T_g - T_w}{X + K/h_T} = \rho_w A_w X C_p \frac{dT_w}{dt} \quad (41)$$

If

$$R_w = - \frac{X + K/hT}{KA_w} \quad (42)$$

$$C_w = \rho_w A_w X C_p, \quad (43)$$

then

$$\dot{Q}_w = \frac{T_g - T_w}{R_w} = C_w \frac{dT_w}{dt} \quad (44)$$

C. Water Region

During an insurge, water enters the water region from the insurge flow, from the sprays, and from steam condensed either by the sprays or vessel walls. The water region was divided into two nodes or sections (Figure 4) in order to more accurately describe mathematically the thermo-

FIGURE 4

dynamics involved. A major part of the thermodynamics was confined to the upper water region while the lower region was used basically as an accumulator or piston action for the water region. This is true especially during

an outsurge when water flashing to steam occurs only at the upper water surface. Heaters may also be employed to enhance the flashing rate which again is confined to the upper region. Heat transfer between the water region and the vessel walls is small and can be neglected. Also, for condensation purposes, the condensation between the steam region and the water region is neglected because the area involved is small compared to the total wall surface area and the spray droplet area. A variable mixing factor, γ , is applied to the insurge to accommodate mixing of water between the two water sections.

During the insurge, condensate and saturated spray water enter the water region at the upper surface.

The energy equation is applied to the upper section of the water region or section(a).

$$\dot{Q}_h + \dot{M}_{sp} h_f - \dot{M}_g h_f + \gamma \dot{M}_{su} h_{su} = \frac{\dot{M}_{wa}}{P} u + \frac{\dot{M}_{wa}}{P} V_{wa} \quad (\text{Figure 5}) \quad (45)$$

or

$$\dot{Q}_h + \dot{M}_{sp} h_f - \dot{M}_g h_f + \gamma \dot{M}_{su} h_{su} = \dot{M}_{wa} h_{wa} + \dot{M}_{wa} h_{wa} \quad (46)$$

but

$$h_{wa} = h_f \quad (47)$$

FIGURE 5

Then

$$\dot{Q}_h + (\dot{M}_{sp} - \dot{M}_g)h_f + \gamma\dot{M}_{su}h_{su} - \dot{M}_{wa}h_f = \dot{M}_{wa}\dot{h}_{wa} \quad (48)$$

Solving for $\dot{h}_{wa}$ gives

$$\dot{h}_{wa} = \frac{1}{\dot{M}_{wa}} \left[\dot{Q}_h + (\dot{M}_{sp} - \dot{M}_g)h_f + \gamma\dot{M}_{su}h_{su} - \dot{M}_{wa}h_f \right] \quad (49)$$

If the spray water is taken from the surge water such that a percent α of the surge becomes spray flow, then a compensating factor is applied to the surge flow giving the following equation for $\dot{h}_{wa}$

$$\dot{h}_{wa} = \frac{1}{\dot{M}_{wa}} \left[\dot{Q}_h + (\dot{M}_{sp} - \dot{M}_g)h_f + \gamma(1 - \alpha)\dot{M}_{su}h_{su} - \dot{M}_{wa}h_f \right] \quad (50)$$

From this equation the condition of the water in section (a) can be calculated at all times. For some types of pressurizing systems, it may be possible to partially linearize $\dot{h}_{wa}$ because for saturated conditions, it is also equal to

$$\dot{h}_{wa} = \frac{dh_{wa}}{dP_g} \cdot \frac{dP_g}{dt} \cdot \frac{dh_f}{dP_g} \cdot \frac{dP_g}{dt} \quad (51)$$

where

$$\frac{dh_f}{dP_g} = \text{constant for small pressure changes.}$$

The pressure of the water region is assumed to be that of the steam region, and the pressure gradient due to water level has been neglected.

II. OUTSURGE

A. Steam Region

During an outsurge, the volume of the steam region is increased by the moveable boundary between the water and steam. The increase in

volume reduces the pressure, and flashing of the saturated water occurs to increase the steam mass. As the steam expands, it cools to a temperature below that of the vessel walls. Due to a much lower heat transfer coefficient from the walls to the steam, the heat transfer in this direction may be neglected. During the expansion there is no kinetic energy supplied to the system.

Since $\dot{Q} = 0$ and $KW = 0$, the energy equation applied to the steam region gives

$$\dot{M}_g h_g = \dot{M}_g u_g + \dot{P}_g V_g \quad (\text{Figure 6}) \quad (52)$$

or

$$\dot{M}_g h_g = \dot{M}_g h_{g1} + \dot{M}_g h_{g2} \quad (53)$$

Reducing the equation gives

$$h_g = u_g + P_g v_g + P_g v_g = 0 \quad (54)$$

which again shows that the internal energy of the original mass of steam is unaffected by flashing steam. This is the same result obtained for the steam region during an insurge.

B. Spray Region Plus Condensate Region

During an outsurge, no steam is condensed on the vessel walls; however, the outsurge may occur when the pressure is high enough to require the sprays to be on until some setpoint in pressure is reached. At this point, the sprays are off and this mode of heat transfer may be neglected.

FIGURE 6

FIGURE 7

Applying the energy to the spray region gives

$$\dot{M}_{sp} h_{sp} - \dot{M}_g h_g = \dot{M}u + \dot{P}V \quad (\text{Figure 7}) \quad (55)$$

which gives

$$\dot{M}_{sp} h_{sp} - \dot{M}_g h_g = \dot{M}h_f + \dot{M}h_f \quad (56)$$

A mass balance gives

$$\dot{M} = \dot{M}_{sp} - \dot{M}_g \quad (57)$$

and

$$M = M_{sp} - M_g \quad (58)$$

Then substituting into Equation (56) gives

$$\dot{M}_{sp} h_{sp} - \dot{M}_g h_g = (\dot{M}_{sp} - \dot{M}_g)h_f + (\dot{M}_{sp} - \dot{M}_g)h_f \quad (59)$$

or

$$\dot{M}_{sp}(h_{sp} - h_f) - \dot{M}_g(h_g - h_f) = (\dot{M}_{sp} - \dot{M}_g)h_f \quad (60)$$

Again, the rate of change of steam mass is equal to the rate of change of condensate; therefore,

$$\dot{M}_c = -\dot{M}_g \quad M_{sp} > 0 \quad (61)$$

Then

$$-\dot{M}_{sp}(h_f - h_{sp}) - \dot{M}_g(h_g - h_f) = (\dot{M}_{sp} + \dot{M}_c)h_f \quad (62)$$

Solving for $\dot{M}_g$ gives

$$\dot{M}_g = \frac{-\dot{M}_{sp}(h_f - h_{sp}) - (\dot{M}_{sp} + \dot{M}_c)h_f}{h_g - h_f} \quad M_{sp} > 0 \quad (63)$$

where,

$$\dot{h}_f = \frac{dh_f}{dP_g} \cdot \frac{dP_g}{dt} \quad (64)$$

The results shown here are the same as for an insurge except the heat transferred to the vessel walls has been eliminated. This equation is valid only as long as the sprays are on.

C. Water Region

Proper baffling at the water inlet causes stratification as water surges into the water region. The water in the top portion is hotter than that of the lower region; therefore, when the water surges out again, steam flashes from the upper region while the cooler water at the bottom surges out without flashing. Proper accounting of this water is necessary so that the pressurizer performance estimates are not unduly pessimistic. This water accounting is done by dividing the water region into two sections, as mentioned before, and shown in Figure 4. An ideal temperature distribution of sections (a) and (b) is shown in Figure 8.

FIGURE 8

An approximation to the ideal temperature distribution is to assume a uniform temperature distribution in each section (a) and (b) as shown in Figure 9. The water in section (a) of Case I is cooler than that of Case II so that flashing will occur sooner in Case II on an outsurge. Case II more nearly

approximates the probable ideal situation at the beginning of the outsurge. As the outsurge continues, Case II will flash more water than the probable case because of the higher temperature of the water near the surface. This would give the pressure vessel a more optimistic performance. In Case I, initiation of flashing is delayed, but the subsequent amount of flashing is more nearly in keeping with the probable case. Figure 10 shows the superposition of the three temperature distributions.

FIGURE 9

FIGURE 10

The energy balance on the water while section (b) is surging out is slightly different than when (b) has completely surged out and (a) is surging out. Since the water in section (a) is hotter than that of section (b), section (a) can flash while the water in (b) is surging out with no flashing in section (b). Thermodynamically, the outsurge of water (b) acts as if it just increases the volume occupied by the steam and water (a). As such, the interface between section (a) and section (b) can be considered as the top of a piston whose length of travel is the volume of section (b) divided by the cross-sectional area of the pressurizer vessel. When the piston reaches this limit of travel, water in section (a), which may be flashing, starts to surge out. The energy equation must then be applied to section (a) and section (b) separately.

The energy equation as applied to section (a) while section (b) is surging out must obey the following constraint

$$V_{wb} > 0 \quad (65)$$

Then

$$\dot{Q}_h - \dot{M}_g h_g = \frac{\dot{M}_{wa}}{P_g} u + \frac{\dot{P}_g V_{wa}}{P_g} \quad (\text{Figure 11}) \quad (66)$$

FIGURE 11

If a mass equation gives

$$\dot{M}_{wa} = -\dot{M}_g \quad V_{wb} > 0 \quad (67)$$

then Equation (66) becomes

$$\dot{Q}_h - \dot{M}_g h_g = -\dot{M}_g h_{wa} + \dot{M}_{wa} h_{wa} \quad (68)$$

or

$$\dot{Q}_h - \dot{M}_g (h_g - h_{wa}) = \dot{M}_{wa} h_{wa} \quad (69)$$

Solving for flashing rate gives

$$\dot{M}_g = \frac{\dot{Q}_h - \dot{M}_{wa} h_{wa}}{h_g - h_{wa}} \quad (70)$$

Equation (70) assumes that flashing is taking place in section (a); therefore, the water and steam are in equilibrium so that

$$h_{wa} = h_f \text{ at } P_g \quad (71)$$

$$v_{wa} = v_f \text{ at } P_g \quad (72)$$

Then

$$\dot{h}_{wa} = \frac{dh_{wa}}{dP_g} \cdot \frac{dP_g}{dt} = \frac{dh_f}{dP_g} \cdot \frac{dP_g}{dt} \quad (73)$$

Then Equation (70) becomes

$$\dot{M}_g = \frac{\dot{Q}_h - M_{wa} \frac{dh_f}{dP_g} \cdot \frac{dP_g}{dt}}{h_g - h_f} \quad V_{wb} > 0 \quad (74)$$

After the water in section (b) has surged out, then the water in section (a) begins to surge out.

FIGURE 12

The following energy equation is valid for section(a) when

$$V_{wb} \leq 0 \quad (75)$$

Then

$$\dot{Q}_h - \dot{M}_g h_g - \dot{M}_{su} h_{wa} = \frac{\dot{M}_{wa}}{V_{wa}} u + \frac{P_g \dot{V}_{wa}}{V_{wa}} \quad (\text{Figure 12}) \quad (76)$$

which reduces to

$$\dot{Q}_h - \dot{M}_g h_g - \dot{M}_{su} h_{wa} = \dot{M}_{wa} h_{wa} + M_{wa} \dot{h}_{wa} \quad (77)$$

The mass equation gives

$$\dot{M}_{wa} = -\dot{M}_g - \dot{M}_{su} \quad (78)$$

Substituting into Equation (77) gives

$$\dot{Q}_h - \dot{M}_g h_g - \dot{M}_{su} h_{wa} = (-\dot{M}_g - \dot{M}_{su}) h_{wa} + \dot{M}_{wa} h_{wa} \quad (79)$$

or

$$\dot{Q}_h - \dot{M}_g (h_g - h_{wa}) - \dot{M}_{su} (h_{wa} - h_{wa}) = \dot{M}_{wa} h_{wa} \quad (80)$$

Solving for flashing rate gives

$$\dot{M}_g = \frac{\dot{Q}_h - \dot{M}_{wa} h_{wa}}{h_g - h_{wa}} \quad (81)$$

Since saturated conditions have been assumed, then

$$\dot{M}_g = \frac{\dot{Q}_h - \dot{M}_{wa} \frac{dh_f}{dP_g} \cdot \frac{dP_g}{dt}}{h_g - h_f} \quad V_{wb} \leq 0 \quad (82)$$

From this it can be seen that the flashing rate is unchanged by water surging out of section (a).

III. VOLUME BALANCE

During both the insurge and outsurge, the volume of the pressurizer vessel must remain constant. A volume balance equation gives

$$\dot{V}_T = \dot{V}_g + \dot{V}_{su} + \dot{V}_{ex} - \dot{M}_g v_f = 0 \quad (83)$$

where,

$\dot{V}_g$ = change in steam volume due to condensation of steam from steam region

$\dot{V}_{su}$ = change in volume due to surge flow

$\dot{V}_{ex}$ = change in volume due to expansion from heat being added by heaters

$\dot{M}_g v_f$ = change in volume due to addition of condensate from steam region.

It should be noted that while steam is leaving the steam region on an insurge, this change in steam volume is partially offset by a volume of condensate because the condensate stays in the vessel. Also, on an outsurge, the change in steam volume due to flashing is partially offset by the change in water volume that has been flashed to steam. If

$$\dot{V}_g = \frac{\dot{M}_g v_g}{\rho_g} = \dot{M}_g v_g + M_g \dot{v}_g \quad (84)$$

then

$$\dot{V}_T = \dot{M}_g v_g + M_g \dot{v}_g + \dot{V}_{su} + \dot{V}_{ex} - \dot{M}_g v_f = 0 \quad (85)$$

solving for $\dot{v}_g$

$$\dot{v}_g = \frac{1}{M_g} \left[\dot{M}_g (v_f - v_g) - \dot{V}_{su} - \dot{V}_{ex} \right] \quad (86)$$

or

$$\dot{v}_g = - \frac{\dot{M}_g}{M_g} (v_g - v_f) - \frac{\dot{V}_{su} + \dot{V}_{ex}}{M_g} \quad (87)$$

During an insurge,

$$\dot{V}_{ex} = 0 \quad (88)$$

Since $\dot{M}_g$ and $\dot{V}_{su}$ have an assigned direction, then Equation (87) is valid for both insurge and outsurge where

$$\dot{V}_{su} = V_o \beta \dot{T} \quad (89)$$

β = coefficient of thermal expansion $FT^3/ft^3 \text{ } ^\circ F$

$\dot{T}$ = temperature rate of change of the pressurized system
excluding the pressurizer vessel

$$\dot{V}_{ex} = \dot{Q}_h (dv_f/dh_f) \quad (90)$$

If all of the spray water is taken from the surge water, then

$$\dot{M}_{sp} = \alpha \frac{\dot{M}_{su}}{v_{sp}} \quad (91)$$

where

α = percent of surge flow.

IV. SUMMARY OF EQUATIONS

A. Insurge

$$\dot{M}_g = \frac{-\dot{Q}_w - \dot{M}_{sp}(h_f - h_{sp}) - (M_c + M_{sp}) \frac{dh_f}{dP_g} \cdot \frac{dP_g}{dt}}{h_g - h_f} \quad (30)$$

$$\dot{M}_c = -\dot{M}_g \quad (25)$$

$$\dot{h}_f = \frac{dh_f}{dP_g} \cdot \frac{dP_g}{dt} \quad (28)$$

$$\dot{Q}_w = \frac{T_g - T_w}{R_w} = C_w \frac{dT_w}{dt} \quad (44)$$

$$R_w = - \frac{X + K/hT}{KA_w} \quad (42)$$

$$C_w = \rho_w A_w X C_p \quad (43)$$

$$\dot{h}_{wa} = \frac{1}{M_{wa}} \left[\dot{Q}_h + (\dot{M}_{sp} - \dot{M}_g) h_f + \gamma \dot{M}_{su} h_{su} - \dot{M}_{wa} h_f \right] \quad (49)$$

$$\dot{h}_{wa} = \frac{dh_f}{dP_g} \cdot \frac{dP_g}{dt} \quad (51)$$

$$\dot{Q}_h = \text{EXTERNAL INPUT}$$

$$M_g = \int \dot{M}_g dt + M_{go}$$

B. Outsurge

$$\dot{M}_g = \frac{-\dot{M}_{sp}(h_f - h_{sp}) - (M_{sp} + M_c)\dot{h}_f}{h_g - h_f} \quad \dot{M}_{sp} > 0 \quad (63)$$

$$\dot{M}_c = -\dot{M}_g \quad \dot{M}_{sp} > 0 \quad (61)$$

$$\dot{M}_g = \frac{\dot{Q}_h - M_{wa} \frac{dh_f}{dP_g} \cdot \frac{dP_g}{dt}}{h_g - h_{wa}} \quad V_{wb} > 0 \quad (74)$$

$$\dot{M}_{wa} = -\dot{M}_g \quad V_{wb} > 0 \quad (67)$$

$$h_{wa} = h_f \text{ at } P_g \quad (71)$$

$$v_{wa} = v_f \text{ at } P_g \quad (72)$$

$$\dot{M}_g = \frac{\dot{Q}_h - M_{wa} \frac{dh_f}{dP_g} \cdot \frac{dP_g}{dt}}{h_g - h_f} \quad V_{wb} = 0 \quad (82)$$

C. Insurge

$$\dot{u}_g + P_g \dot{v}_g + \dot{P}_g v_g = 0 \quad (15)$$

$$\dot{v}_g = -\frac{\dot{M}_g}{M_g} (v_g - v_f) - \frac{\dot{V}_{su} + \dot{V}_{ex}}{M_g} \quad (87)$$

$$\dot{V}_{su} = V_o \beta \dot{T} \quad (89)$$

$$\dot{V}_{ex} = \dot{Q}_h \frac{dv_f}{dh_f} \quad (90)$$

$$\dot{M}_{sp} = \alpha \frac{\dot{V}_{su}}{v_{sp}} \quad (91)$$

$$h_f = f(P_g)$$

When

$$P_g > P_{sat} \text{ (saturation pressure)}$$

then

$$\dot{M}_g = 0$$

$$h_f = h_w$$

The criterion for flashing is

$$P_g > P_{sat} \quad \dot{M}_g = 0$$

$$P_g < P_{sat} \quad \dot{M}_g \neq 0$$

NOMENCLATURE

a	imaginary wall thickness
A _w	pressure vessel wall area in the steam region
C _p	vessel wall specific heat
C _w	heat capacity of vessel wall
h _f	saturation enthalpy
h _g	steam enthalpy
h _{sp}	spray enthalpy
h _T	film heat transfer coefficient of vessel wall

h_{wa}	enthalpy of water region (a)
$\dot{h}_f$	rate of change of saturation enthalpy
$\dot{h}_g$	rate of change of steam enthalpy
$\dot{h}_{wa}$	rate of change of water region (a) enthalpy
J	unit of conversion
K	thermal conductivity of vessel walls
KW	kinetic energy
m	mass
M_c	condensate mass
M_g	steam mass
M_{sp}	spray mass
M_{wa}	water mass of water region (a)
M_{wb}	water mass of water region (b)
$\dot{M}_c$	condensate mass rate of change
$\dot{M}_g$	steam mass rate of change
$\dot{M}_s$	steam mass rate of change
$\dot{M}_{sp}$	spray mass rate of change
$\dot{M}_{su}$	surge mass rate of change
$\dot{M}_{wa}$	water mass of region (a) rate of change
$\dot{M}_{wb}$	water mass of region (b) rate of change
P_g	vessel steam pressure
$\dot{P}_g$	vessel steam pressure rate of change
$\dot{Q}$	heat transfer rate into system
$\dot{Q}_h$	heater heat transfer rate of change
$\dot{Q}_w$	wall heat transfer rate of change

R_w	wall thermal resistance
T_g	steam temperature
T_w	wall temperature
$\dot{T}$	external process temperature rate of change
ΔT	temperature difference
u_g	steam internal energy
$\dot{u}_g$	steam internal energy rate of change
v_f	saturation specific volume
v_g	steam specific volume
$\dot{v}_g$	steam specific volume rate of change
V_g	steam volume
V_o	external process initial volume
V_{wa}	water region (a) volume
V_{wb}	water region (b) volume
$\dot{V}_{ex}$	water expansion volume rate of change
$\dot{V}_g$	steam volume rate of change
$\dot{V}_{su}$	surge volume rate of change
V_T	vessel total volume
$\dot{V}_T$	vessel total volume rate of change
$\dot{V}_{wa}$	water region (a) volume rate of change
X	vessel wall thickness
α	percent of surge flow
β	thermal expansion coefficient
γ	percent split between water regions
ρ_w	wall density

APPENDIX

If the system expansion is assumed to be at constant pressure, the expansion energy term of the energy equation may be given as

$$\text{System Expansion Energy} = P\dot{V}$$

The energy equation for the spray plus condensate region during an insurge is Equation 16,

$$-\dot{Q}_w - \dot{M}_g h_g + \dot{M}_{sp} h_{sp} = \dot{M}u + P_g \dot{V} \quad (A-1)$$

Expanding the equation gives

$$\dot{V} = \dot{M}v \quad (A-2)$$

Then

$$-\dot{Q}_w - \dot{M}_g h_g + \dot{M}_{sp} h_{sp} = \dot{M}u + \dot{M}u + P_g (\dot{M}v + \dot{M}v) \quad (A-3)$$

Collecting terms gives

$$-\dot{Q}_w - \dot{M}_g h_g + \dot{M}_{sp} h_{sp} = \dot{M}(u + P_g v) + \dot{M}(\dot{u} + P_g \dot{v}) \quad (A-4)$$

A term can be added to the right side that does not change the equation or

$$-\dot{Q}_w - \dot{M}_g h_g + \dot{M}_{sp} h_{sp} = \dot{M}(u + P_g v) + \dot{M}(\dot{u} + P_g \dot{v} - \dot{P}_g v - \dot{P}_g v) \quad (A-5)$$

Since enthalpy is defined as

$$h = u + P_g v \quad (A-6)$$

then

$$\dot{h} = \dot{u} + P_g \dot{v} + \dot{P}_g v \quad (A-7)$$

Equation (15) becomes

$$-\dot{Q}_w - \dot{M}_g h_g + \dot{M}_{sp} h_{sp} = \dot{M}h + \dot{M}(\dot{h} - \dot{P}_g v) \quad (A-8)$$

but $\dot{h}$ is also defined as

$$\dot{h} = \frac{dh}{dt} \cdot \frac{dP_g}{dP_g} = \frac{dh}{dP_g} \cdot \frac{dP_g}{dt} \quad (A-9)$$

Then Equation (A-8) becomes

$$-\dot{Q}_w - \dot{M}_g h_g + \dot{M}_{sp} h_{sp} = \dot{M} h + M \left[\frac{dh}{dP_g} - v \right] \frac{dP_g}{dt} \quad (A-10)$$

But v is small compared to dh/dP_g so it can be neglected, then

$$-\dot{Q}_w - \dot{M}_g h_g + \dot{M}_{sp} h_{sp} = \dot{M} h + M \frac{dh}{dP_g} \cdot \frac{dP_g}{dt} \quad (A-11)$$

A mass equation gives

$$\dot{M} = -\dot{M}_g + \dot{M}_{sp} \text{ and } M = -M_g + M_{sp} \quad (A-12)$$

where

$$-\dot{M}_g = \dot{M}_c \quad h = h_f \text{ (saturation enthalpy)} \quad (A-13)$$

Then equation (A-11) becomes

$$-\dot{Q}_w - \dot{M}_g h_g + \dot{M}_{sp} h_{sp} = (-\dot{M}_g + \dot{M}_{sp}) h_f + (M_c + M_{sp}) \frac{dh_f}{dP_g} \cdot \frac{dP_g}{dt} \quad (A-14)$$

or

$$-\dot{Q}_w - \dot{M}_g (h_g - h_f) - M_{sp} (h_f - h_{sp}) = (M_c + M_{sp}) \frac{dh_f}{dP_g} \cdot \frac{dP_g}{dt} \quad (A-15)$$

Solving for $\dot{M}_g$ gives

$$\dot{M}_g = \frac{-\dot{Q}_w - \dot{M}_{sp} (h_f - h_{sp}) - (M_c + M_{sp}) \frac{dh_f}{dP_g} \cdot \frac{dP_g}{dt}}{(h_g - h_f)} \quad (A-16)$$

The result here is shown to be the same as in Equation (30).

BIBLIOGRAPHY

Brown, D. H. Transient Thermodynamics of Reactors and Process Apparatus, DF-r56GL316. General Electric Company, Schenectady.

Zemansky, Mark W. Ideal and Thermodynamics, McGraw-Hill Book Company, Inc., New York, 1957. 4th Edition.

Lee, John F. and Francis W. Sears. Thermodynamics, Addison-Wesley Publishing Company, Inc., Cambridge 42, Massachusetts, 1955.

ONSITE DISTRIBUTIONCopy NumberPacific Northwest Laboratory

1	E. R. Astley
2	R. D. Benham
3	L. J. Chockie
4	E. D. Clayton
5-6	K. D. Coughren
7	F. G. Dawson
8	P. J. Dionne
9	G. E. Driver
10	E. A. Eschbach
11	J. C. Fox
12	J. J. Fuquay
13	G. F. Garlick
14-23	R. A. Harvey
24	R. T. Jaske
25	B. R. Leonard
26	R. S. Paul
27	L. T. Pedersen
28	W. C. Roesch
29	J. T. Russell

ONSITE DISTRIBUTION (contd)Copy Number

30	J. C. Spanner
31	W. G. Spear
32	C. D. Swanson
33	G. R. Taylor
34	G. G. Thieme
35	D. C. Worlton
36-40	Technical Information Files
41	Technical Publications, 300 Area
42-43	Technical Publications, 700 Area

General Electric Company, Richland

44	J. M. Batch
45	H. G. DeVoss
46	E. A. Grimm
47	D. W. Leiby
48	C. F. Poor
49	G. L. Swezea
50-51	GETA File

Richland Operations Office

52	R. K. Sharp
53	Technical Information Library