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DESIGN AND OPERATION OF A
FLAT LINEAR INDUCTION PUMP

by

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and J. W. Nilsson

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LIST OF SYMBOLS

Optimum Stator Structure

A	rms ampere conductors per unit length
B	instantaneous gap flux density
B_m	amplitude of gap flux density
d	equivalent pump duct diameter
f	frequency
F	instantaneous value of armature mmf wave
F_a	amplitude of armature mmf wave
F_m	magnetizing component of F
g	total magnetic gap
H	head loss
I	rms stator phase current
I_m, i_m	amplitude and instantaneous values of magnetizing component of I_s per unit length
I_r, i_r	amplitude and instantaneous values of liquid current wave, power component of I_s per unit length
I_s, i_s	amplitude and instantaneous values of total equivalent stator surface current density per unit length
I_w, i_w	amplitude and instantaneous values of duct current wave, power component of I_s per unit length
k	kinematic viscosity of fluid
k_d	winding distribution factor
k_p	winding pitch factor

k_w	winding factor, $k_d k_p$
K	pump parameter, $\frac{2f \lambda^2 t}{\rho g}$
K_z	pump parameter to give maximum value of z_h and z_o at constant s and X
K_{ze}	pump parameter for optimum design, $s_o = s_e$, $z_h = z_o$ maximum, X constant
l	distance measured in direction of induced currents
n	number of poles
N_{PH}	total series turns per phase
p	average pressure developed per unit length of pump
$(p)_{actual}$	actual average pressure developed, $(x_p - p_f)$
P_f	hydraulic friction loss (experimentally determined)
P	power loss in the liquid metal per unit length
P_i	input power to liquid metal
P_o	ideal power output in the liquid metal per unit length
$(P_o)_{actual}$	actual power output in the liquid metal (total)
P_w	power loss in the pump duct wall per unit length
q	number of phases
Q	total flow
R_e	Reynolds number
R_r	resistance of liquid parallel to I_r
R_w	combined resistance of top and bottom pump duct walls
s	slip $\frac{v_s - v}{v_s}$

s_e	slip for maximum efficiency at constant X
s_h	slip for maximum pressure at constant I_s , X and K
s_o	slip for maximum developed power at constant X and K
t	internal thickness of pump duct parallel to magnetic field
t_w	thickness of pump duct wall
v	velocity of liquid metal in pump duct
v_r	relative velocity between liquid and synchronous field
v_s	velocity of magnetic field
w	internal width of pump duct parallel to current flow
x	distance measured in direction of flow
X	wall loss parameter, $\frac{\rho^2 t_w}{\rho_w t}$
z_h	head utilization factor, non-dimensional part of formula for p
z_{hk}	maximum value of z_h with variable K
z_{hs}	maximum value of z_h with variable s
z_o	output utilization factor, non-dimensional part of formula for P_o .
z_{ok}	maximum value of z_o with variable K
z_{os}	maximum value of z_o with variable s
α, β	phase angles
$(\eta)_{\text{actual}}$	actual operating efficiency
η_e	electrical efficiency
η_{em}	maximum electrical efficiency
η_h	hydraulic efficiency
η_t	combined theoretical efficiency, $\eta_e \eta_h$

θ	internal phase angle between I_s and $I_r + I_w$
μ	viscosity of liquid metal
λ	wavelength or double pole pitch
ρ	resistivity of liquid
ρ_w	resistivity of pump duct wall material
σ	density of liquid metal
$\cos \theta$	internal power factor

Winding Design

A_B	area of gap orthogonal to magnetic field (i.e., $L w_B$) square inches
A_C	cross sectional area of conductor, square inches
A_S	cross sectional area of copper side bars, square inches
c_p	circuits in parallel
d_1	depth of slot occupied by conductors, inches
d_3	conductor free depth of slot, inches
D_r	equivalent diameter of copper side bars (i.e. $\frac{L}{\pi}$) inches
E_L	line to line voltage, volts
E_ϕ	phase to neutral voltage, volts
f	frequency, cps
g	total magnetic gap, inches
I	rms stator phase current, amperes
I_m	magnetizing current, amperes
k_d	winding distribution factor
k_p	winding pitch factor

L	length of magnetic field, pump or pump duct over wound region, inches
(MLC)	mean length of conductor, inches
n	number of poles
N ₁	primary conductors in series per phase
q	number of phases
R ₁	primary resistance per phase, ohms
R ₂	equivalent resistance of fluid per phase referred to stator, ohms
R _c	equivalent resistance of duct walls per phase referred to stator, ohms
s	slip, $\frac{v_s - v}{v_s}$
s ₁	number of stator slots containing copper
s _s	total slots in iron
t	internal thickness of pump duct parallel to magnetic field, inches
t _w	thickness of pump duct wall, inches
w	internal width of pump duct parallel to current flow, inches
w _B	effective field width (i.e. iron width x stacking factor), inches
w _s	slot width, inches
X ₁	primary leakage reactance, ohms at f
X _m	magnetizing reactance, ohms at f
Z _T	total impedance of equivalent circuit

ρ	resistivity of liquid, microhm inches
ρ_{cu}	resistivity of copper, microhm inches
ρ_{w}	resistivity of pump duct wall material, microhm inches

DESIGN AND OPERATION OF A FLAT LINEAR INDUCTION PUMP*

Delwyn D. Bluhm, R. W. Fisher and J. W. Nilsson

ABSTRACT

A five pole, one slot per phase, full pitch, twelve coil, wye connected, three phase linear induction pump has been designed for operation with 400 C sodium. The design incorporated a graded winding in order to reduce the net pulsating component of the magnetic flux density wave and a pump duct using copper side bars. The small laboratory-size pump was designed in two steps: stator design and winding design. First, the stator and core dimensions were established on the basis of maximum pressure development and maximum power at the design flow and slip. Maximum pressure output was a requirement of the intended pump application. The condition of maximum power output was used in order to provide a good pump efficiency. Second, the parameters for the induction pump equivalent circuit were determined for several winding methods. The winding method which offered 3920 ampere turns per phase (equivalent to a flow rate of 4.88 gpm, a pressure of 60 psi, a line voltage of 250 volts, and a wave length of 21.2 cm as calculated in the first step) was selected. The theoretical performance characteristics of the flat induction pump were computed using the equivalent circuit parameters for a winding with 60 turns per coil. The measurement of developed pressure (0 to 42.13 psi), sodium flow rate (0 to 9.93 gpm), and power input (0 to 103.9 watts) after installation of the pump in an operational sodium circulating loop provided an experimental determination of the pump performance characteristic. In addition, the effect of pump duct temperature on sodium flow rate and a differential transformer system used to locate the sodium level in manometer columns were briefly discussed.

*This report is based on an M. S. thesis submitted by Delwyn D. Bluhm to Iowa State University, Ames, Iowa, July, 1964.

I. INTRODUCTION

Devices which rely on the force that is produced whenever a current carrying solid conductor is imbedded in a magnetic field are well known to today's engineers and scientists. Not so well known are the two groups of devices in which the current is carried either in a conductive gas or a conductive liquid.

An example of a device using a conducting gas is the MHD (Magnetohydrodynamic) generator (1). The gas is made conductive by partial ionization and is forced to cut a magnetic field. Electrodes perpendicular to the field draw off the induced currents. Electromagnetic pumps and flowmeters are two very good examples of devices using conductive liquids. The conductive liquids are usually molten metals at high temperatures. The electromagnetic pump provides the current and magnetic field in the conductive liquid. The resulting force which causes the liquid to be pumped is the useful output. The electromagnetic flowmeter operates on the same principal as the MHD generator except that the conductive mediums are different. The induced EMF at the electrodes is proportional to the rate of flow of the conductive liquid.

This work deals primarily with electromagnetic pumps. These pumps have become increasingly important in the last few years due to the expanded use of molten metals as heat-transfer mediums for nuclear power plants. Pumps developed for this purpose are normally very large. For example, an A. C. linear induction pump for Argonne's Experimental Breeder Reactor II (2) has a capacity of 5,000 gpm of 700 F sodium at 40 psi head with a 43% operating efficiency. Electromagnetic pumps have been

less frequently used to circulate liquid metals and alloys in corrosion test loops (3) and to feed aluminum to a die-casting machine (4).

The object of this work was to design and determine the performance characteristics of a small laboratory-size flat linear induction pump for circulating liquid metals in corrosion test loops.

II. ELECTROMAGNETIC PUMPS

In 1907 Northrup (5) developed a hydrostatic pressure of several inches of water via the interaction of a 600 ampere current and a magnetic field in mercury. He postulated that this effect could be utilized in an ammeter for measuring large currents.

The movement of a liquid metal by using a sliding and rotating magnetic field was proposed by Chubb (6) in 1915. Bainbridge (7) tested an alternating current conduction pump in 1926. Einstein and Szilard proposed an annular linear induction pump for use with alkali metals in 1928. Several working models of this pump rated at about 2 kw were operated with mercury and liquid alloys of sodium and potassium. The design equations for this type of pump using liquid bismuth were presented in a report by Feld and Szilard (8) in 1942.

The flat linear induction pump is only one of many types of electromagnetic pumps for liquid metals. The principal of operation is the same for all electromagnetic pumps. It is the source of the current and magnetic field that changes. Electromagnetic pumps can be divided in two main categories; they are the conduction pumps and the induction pumps.

Conduction pumps may be operated either on alternating or direct current. Either type pump may be identified by the electrodes, brazed to the pump duct wall, which supply the current from an external source to the liquid metal. Conduction pumps have one primary advantage over the other electromagnetic pumps; they can be used to pump even the typical heavy metals such as bismuth or mercury with high resistivity,

high density, and high viscosity. The current flowing through the liquid metal may be made as large as is necessary to overcome the effects of the heavy metals. The biggest disadvantage of these pumps is the requirement of extremely large currents, usually several thousand amperes at voltages of one or two volts.

A direct current conduction pump can be operated either with a permanent magnet or an electromagnet to produce the magnetic field. The disadvantage of the awkward excitation requirements, as mentioned earlier, is partially overcome by the use of a relatively efficient homopolar generator with liquid metal brushes. The use of conventional rectifiers is not recommended for the larger pumps since they are inefficient, bulky, and expensive. Another requirement (and not always a disadvantage) for satisfactory operation of a direct current pump is the wetting of the duct wall by the liquid metal. The performance characteristics of the direct current conduction pump are excellent as the pump is applicable in systems requiring wide variations in power, pressure, and flow. In efficiency too, the direct current pump is unsurpassed by the other types of electromagnetic pumps.

In order to avoid the use of a homopolar generator, an alternating current conduction pump with its associated step-down transformer is sometimes used. The immediate problems encountered with this pump are: lamination of magnetic circuit to reduce eddy-currents and iron losses and thereby improve the normally low efficiency; physically larger pump; poor power factor; and noisy operation. The alternating current conduction pump is limited in size because the power factor decreases rapidly

with size at a constant frequency. However, larger pumps may be built if the frequency is sufficiently lowered. Normally, the reliable step-down transformer is employed near or is part of the pump, while alternate sources of supply may be located remotely.

The second category of electromagnetic pumps, the induction pumps, always have alternating current induced in the molten metal by a traveling or pulsating magnetic field. These pumps are normally, with one exception, used only with the typical light metals with higher conductivities so that the induced currents will be sufficiently large. A classification of the most common types is:

Single-phase annular

Polyphase helical

Polyphase flat linear

Polyphase annular linear

Rotating magnet

The single-phase annular induction pump is similar to a transformer with stationary primary winding but a moveable secondary winding. The liquid metal acts as the secondary winding, and it is pumped by the interaction of radial primary magnetic field in the annular gap and the circulating induced current in the liquid. This pump is generally used only for small pumping requirements because of its inherent low efficiency and poor power factor. For larger pumping capacities the polyphase induction pump must be used.

The polyphase helical induction pump employs a rotating magnetic field produced in the same manner as a squirrel cage induction motor. The squirrel cage is replaced by a cylindrical liquid metal duct in an increased air gap surrounding a "blocked" cylindrical iron core. Helical guide vanes are placed in the duct to impart axial motion to the liquid which is rotating from the action of the rotating magnetic field produced by the three-phase stator winding. The function of end rings is now served partially by the liquid and partially by two copper rings, silver soldered to the duct at each end of the core. A helical induction pump of this type normally is used in applications requiring high pressure and low flow.

The most popular polyphase induction pump is the flat linear type. The resulting linear fluid motion is an improvement upon the helical induction pump when large flows are required. The flat linear induction pump is most easily described as a helical pump which has been cut open axially along one side and finally straightened into a flat rectangular shape. The duct is now located between two axially laminated structures. To improve pump performance, copper side bars are usually added to each side of the duct; they function as end rings in a conventional induction motor. The traveling field is produced by the polyphase multipole stator windings located on either one or both of the laminated iron structures. The losses in this pump may be substantially reduced by grading the magnetic field in the end sections. Grading allows the magnetic field to gradually increase from near zero at the ends to the uniform value over the fully wound region; it is accomplished by using only partially wound end poles.

An important feature of this pump is the possible removal of the stator structures and windings without disturbing the liquid metal system.

The annular linear pump (or Einstein-Szilard pump) can be thought of as being formed from the flat linear type by bending the latter into a cylindrical shape. The radially laminated, cylindrical core, now on the inside of an annular duct, has no windings. The windings, which are pancake-like coils, are located in the circular slots of the stator surrounding the pump duct. Copper side bars are not needed since the induced currents in the liquid metal flow in circular closed-loop paths. The reduction of end effects by field grading is essential for good performance. As compared to the flat linear pump, the annular pump offers lower hydraulic loss, stronger and less complicated construction, lower heat loss, easier field grading, higher leakage reactance, and a more complicated winding removal procedure.

Rotating field induction pumps induce an alternating current in the liquid metal through the use of mechanically rotated permanent magnets or electromagnets. All of the polyphase induction pumps with rotating magnetic fields can be modified to form a rotating magnet pump. The usual arrangement is to use rotating magnets for pumps with loop or helix pump ducts. Copper side bars are required. This type of induction pump has several advantages over the other types, such as a stronger available field providing the possibility of pumping high-resistivity metals, the near unity power factor operation of induction motors, self-cooled magnets, good efficiency and variable pole pitch. The drawbacks include the use of moving parts requiring bearings and problems of pump duct support.

The linear induction type of electromagnetic pump was selected since it has proven its ability to operate continuously at temperatures near 1000 C for as long as 5000 hours (9). This is a very important capability when a pump is to be used in corrosion test loops for circulating metals with high melting points. This selection was also made since it was hoped that this work would justify the use of small flat linear induction pumps in laboratory applications where the pumping requirements are normally very small. In the literature the flat linear induction pump was most commonly employed at large flows for high efficiency while the single phase induction pump and the rotating magnet induction pump were the two types most frequently designed for laboratory use. The efficiencies of these two types of pumps are generally low at the low flow rates where they are employed. The use of the flat linear induction pump offers the advantage of a simpler fabrication technique than with the single phase induction pump; however, the efficiency is slightly less. It offers several advantages over the rotating magnet pump, such as the elimination of moving parts and a simplified pump duct removal procedure. Even though the efficiency of a flat linear induction pump decreases at low flow, proper design of the pump should yield an acceptable efficiency. In fact, this efficiency should compare favorably with the efficiencies of the other small pumps under similar operating conditions.

Normally, the efficiency of linear induction pumps for corrosion loops is unimportant. However, efficiency is a consideration in test loops which are to be operated under design conditions of flow rate and heat transfer. A linear induction pump for use in this type of loop

with an efficiency of 0.12% (9) or lower could cause the heat input to the molten metal to be excessive. This fact necessitated the design of a more efficient flat linear induction pump at a preselected flow rate. The pump was designed specifically for a working fluid of sodium. Sodium is one of the best metals for use with linear induction pumps since it is a typical light metal with low resistivity, low density, and low viscosity.

The availability of an operational sodium loop provided a means of experimentally determining the pump performance characteristics.

III. REVIEW OF DESIGN THEORY

Flat linear induction pumps are sometimes thought to be analogous to squirrel cage induction motors. This is true when considering only the basic principles of the operation of each; however, the similarity ends there. Linear motion rather than rotary motion is required from these pumps, and this causes many different winding design problems. There are many compromises between electrical, physical, and hydraulic requirements which increase the losses and decrease the power factor. For example: high fluid velocity yields a minimum pump size with maximum power factor and larger hydraulic losses; tooth saturation limits the magnetic field strength; deep slots required for cooling of coils reduce the power factor and air gap flux density; and heat insulation surrounding the pump duct increases the air gap and further reduces the power factor and flux density.

The pump design theory is presented in two parts. The first part deals with the development of equations for currents in the duct and liquid, pressure developed, power output, and efficiency after particular values of stator ampere turns or magnetic field strength have been assumed to be available. Some of these equations may be maximized with respect to slip or to a factor called the pump parameter which depends on the physical dimensions of the pump. The stator structure is called an optimum structure for a given set of pumping requirements when it has been optimized with respect to slip or pump parameter for maximum pressure developed, power output or efficiency, depending on what capabilities are needed. The second part of the design theory provides a means of

selecting a winding configuration which will produce the ampere turns or magnetic field strength assumed in the first part.

The design of a flat linear induction pump is quite involved due to the large number of variables. The variables of major concern are liquid metal, pump duct material, temperature of operation, wavelength, duct thickness and width, total magnetic gap, electrical frequency, total number of poles, flow rate, pressure developed, power output in fluid, and overall efficiency. Many of these variables are assumed or selected in order that the other parameters may be calculated from equations derived for maximum pressure, power output or efficiency. If these results are physically realizable, the assumed, selected, and calculated values are used in the design.

It is extremely difficult to theoretically account for the many losses in linear induction pumps. Equations derived for the power losses in the liquid metal, the duct wall, and the stator winding give a reasonable indication of pump efficiency since other losses are normally small in comparison. The "rotor" reactance in the following sections is assumed negligible. It is further assumed that the net pulsating component of the magnetic flux is zero; this is a good assumption if the winding is properly graded (i.e. partially wound at the pump ends causing the field to gradually increase from zero).

A. Optimum Stator Structure

The following is a summary of flat linear induction pump design theory developed by D. A. Watt (10). He has done a great deal of work

in the field of electromagnetic pumping and has written many reports covering the design of various types of electromagnetic pumps.

The traveling wave equation of the equivalent stator surface current density or current sheet can be written in the form

$$i_s = I_s \sin 2\pi \left(\frac{x}{\lambda} - ft \right) \quad (1)$$

The mutual magnetic flux in the liquid metal is derived from a magnetizing current wave

$$i_m = I_m \sin 2\pi \left(\frac{x}{\lambda} - ft - \alpha \right) \quad (2)$$

Since the currents in the liquid and duct walls are in phase, they are

$$i_r + i_w = (I_r + I_w) \sin 2\pi \left(\frac{x}{\lambda} - ft + \beta \right) \quad (3)$$

The rotor and magnetizing currents are different in phase by

$$\alpha + \beta = \frac{\pi}{2} \quad (4)$$

Therefore

$$I_s^2 = I_m^2 + (I_r + I_w)^2 \quad (5)$$

The amplitude of the mmf wave for a three phase distributed fractional pitch winding is

$$F_a = \frac{3}{2} \frac{4}{\pi} \frac{k_w N_{PH}}{n} \sqrt{2} \quad I = 2.70 k_w \frac{N_{PH} I}{n} \quad (6)$$

Since

$$\frac{N_{PH} I}{n} = \frac{\text{amp-turns/phase}}{\text{pole}} = \frac{A}{2} \frac{\lambda}{2q} \quad (7)$$

$$F_a = 2.70 k_w \frac{A}{2} \frac{\lambda}{2} \quad (8)$$

The traveling wave equation of the mmf wave is

$$F = F_a \sin 2\pi \left(\frac{x}{\lambda} - ft - \frac{\pi}{2} \right) \quad (9)$$

The total stator surface current density wave as a function of F_a can be obtained by differentiating Equation 9 with respect to x . The result is

$$i_s = \frac{2\pi}{\lambda} F_a \sin 2\pi \left(\frac{x}{\lambda} - ft \right) \quad (10)$$

Comparing Equation 10 with Equation 1 yields

$$I_s = \frac{2\pi}{\lambda} F_a = \frac{2\pi}{\lambda} \frac{3}{2} \frac{4}{\pi} \sqrt{2} k_w \frac{A}{2} \frac{\lambda}{2q} = 3 \sqrt{2} \frac{k_w A}{q} \quad (11)$$

or using equation (6)

$$I_s = \frac{2\pi}{\lambda} \frac{3}{2} \frac{4}{\pi} \frac{k_w}{n} \frac{N_{PH}}{n} \sqrt{2} \quad I = 12 \sqrt{2} \frac{k_w}{n} \frac{N_{PH}}{\lambda} I \quad (12)$$

If $q = 3$

$$I_s = \sqrt{2} k_w A \quad (13)$$

The correspondence between F_m and I_m (Equation 2) in the fully wound region is similar to that which exists between F (Equation 9) and i_s (Equation 10). Therefore

$$\begin{aligned} F_m &= \frac{1}{\mu_o} B_m g \cos 2\pi \left(\frac{x}{\lambda} - ft - a + \pi \right) \\ &= \frac{\lambda}{2\pi} I_m \cos 2\pi \left(\frac{x}{\lambda} - ft - a + \pi \right) \end{aligned} \quad (14)$$

The flux density is

$$B_m = \frac{\mu_o \lambda}{2 \pi g} I_m \quad (15)$$

The components of I are very important as they show the division of useable and unuseable current between the liquid metal and pump duct wall. It is clear that

$$I_r = \frac{B_m l v_r}{R_r} = \frac{B_m w(v_s - v)}{\rho w/t} = \frac{B_m s v_s t}{\rho} \quad (16)$$

Since

$$v_s = f \lambda$$

Equations 15 and 16 may now be combined to

$$\frac{I_m}{I_r} = \frac{2 \pi g \rho}{\mu_o \lambda s v t} = \frac{2 \pi g \rho}{\mu_o \lambda^2 s f t} = \frac{1}{sK} \quad (17)$$

where the pump parameter, K, is defined

$$K = \frac{\mu_o \lambda^2 f t}{2 \pi g \rho} \quad (18)$$

Likewise

$$I_w = \frac{B_m l v_s}{R_w} = \frac{B_m w v_s}{\rho_w w/2 t_w} = \frac{2 B_m t_w v_s}{\rho_w} \quad (19)$$

From Equations 16 and 19

$$\frac{I_w}{I_r} = \frac{\rho}{s \rho_w} \frac{2 t_w}{t} = \frac{X}{s} \quad (20)$$

where the wall loss parameter, X, is defined

$$X = \frac{\rho}{\rho_w} \frac{2 t_w}{t} \quad (21)$$

Excluding stator leakage reactance, the internal power factor from Equations 4 and 5 is found to be

$$\cos \theta = \cos \tan^{-1} \frac{I_m}{I_r + I_w} \quad (22)$$

where

$$\tan \theta = \frac{I_m}{I_r + I_w} = \frac{1}{sK (1 + \frac{X}{s})} = \frac{1}{K (s + X)} \quad (23)$$

when the values of Equations 17 and 20 are used.

By trigonometric substitutions

$$I_m = I_s \sin \theta = \frac{I_s}{\left[1 + s^2 K^2 \left(1 + \frac{X}{s} \right)^2 \right]^{\frac{1}{2}}} \quad (24)$$

and

$$I_r + I_w = I_r + \frac{X}{s} I_r = I_s \cos \theta = I_s \frac{sK (1 + \frac{X}{s})}{\left[1 + s^2 K^2 (1 + \frac{X}{s})^2\right]^{\frac{1}{2}}}$$

$$I_r = \frac{sK I_s}{\left[1 + s^2 K^2 (1 + \frac{X}{s})^2\right]^{\frac{1}{2}}} \quad (25)$$

Similarly

$$I_w + I_r = I_w + \frac{s}{X} I_w = I_s \cos \theta = I_s \frac{sK (1 + \frac{X}{s})}{\left[1 + s^2 K^2 (1 + \frac{X}{s})^2\right]^{\frac{1}{2}}}$$

$$I_w = \frac{XK I_s}{\left[1 + s^2 K^2 (1 + \frac{X}{s})^2\right]^{\frac{1}{2}}} \quad (26)$$

The average pressure developed in the pump duct per unit of length in the fully wound region of the induction pump is simply

$$p = \frac{i l B}{\text{area}} = \frac{\frac{I_r}{\sqrt{2}} w \frac{B_m}{\sqrt{2}}}{tw} = \frac{I_r B_m}{2 t} = \frac{\mu_o}{4 \pi \text{tg}} I_m I_r = \frac{sv_s B_m^2}{2 \rho} \quad (27)$$

Eliminating I_m and I_r in Equation 27 using Equations 24 and 25 yields

$$p = \frac{\mu_o \lambda}{4 \pi \text{tg}} \frac{sK I_s^2}{1 + s^2 K^2 (1 + \frac{X}{s})^2} \quad (28)$$

When the average pressure p (Equation 28) is maximized with respect to s , the maximum value of p occurs when

$$s = s_h = \left[\frac{1}{K^2} + X^2 \right]^{\frac{1}{2}} \quad (29)$$

The ideal power output per unit length in the liquid metal is

$$P_o = v_p tw = (1-s) v_s p tw \quad (30)$$

and the actual total power output in the liquid metal after accounting for hydraulic losses or after determination of $(p)_{\text{actual}}$ from experimental data is

$$(P_o)_{\text{actual}} = (p)_{\text{actual}} \, twv \quad (31)$$

Thus the actual operating efficiency is

$$(\eta)_{\text{actual}} = \frac{(P_o)_{\text{actual}}}{\text{total input power}} \quad (32)$$

The ideal power loss per unit length in the liquid metal, due to I^2R losses of the induced currents, is

$$P = \left(\frac{I_r}{\sqrt{2}} \right)^2 R_r = \frac{B_m^2 s^2 v_s^2 t^2 \rho w}{2 \rho^2 t} = \frac{B_m^2 s^2 v_s^2 tw}{2 \rho} = sv_s \, ptw \quad (33)$$

The ideal input power to the liquid metal is

$$P_i = P_o + P = (1-s) v_s \, ptw + sv_s \, ptw = v_s \, ptw \quad (34)$$

Also the ideal power loss per unit length in the pump duct, due to the I^2R losses of the induced currents, is

$$P_w = \left(\frac{I_w}{\sqrt{2}} \right)^2 R_w = \frac{4 B_m^2 t_w^2 v_s^2 \rho_w w}{2 \rho_w^2 2 t_w} = v_s \, ptw \frac{X}{s} \quad (35)$$

Hence the ideal electrical efficiency becomes

$$\eta_e = \frac{P_o}{P_i + P_w} = \frac{1-s}{1 + \frac{X}{s}} \quad (36)$$

The combined theoretical efficiency is

$$\eta_t = \eta_e \, \eta_h \quad (37)$$

The maximum ideal electrical efficiency of

$$\eta_{em} = 1 - 2 s_e \quad (38)$$

occurs when the slip is

$$s = s_e = (X^2 + X)^{\frac{1}{2}} - X \quad (39)$$

Likewise the slip for maximum power output, P_o , is

$$s_o = \frac{\left(\frac{1}{K^2} + X^2\right) + \left[\left(\frac{1}{K^2} + X^2\right)^2 + \left(\frac{1}{K^2} + X^2\right)(1 + 2X)\right]^{\frac{1}{2}}}{1 + 2X} \quad (40)$$

The non-dimensional part of Equation 28 is

$$z_h = \frac{sK}{1 + s^2 K^2 \left(1 + \frac{X}{s}\right)^2} = sK \sin^2 \theta \quad (41)$$

and is called the head utilization factor.

When Equation 28 is substituted into Equation 30, the non-dimensional part of resulting expression is

$$z_o = \frac{sK (1-s)}{1 + s^2 K^2 \left(1 + \frac{X}{s}\right)^2} = sK (1-s) \sin^2 \theta \quad (42)$$

which is referred to as the output utilization factor.

The significance of the head utilization, z_h , and output utilization, z_o , factors may not be immediately obvious. Note that both factors may be thought of as functions of two variables (i.e. slip, s , and pump parameter, K); however, it should be remembered that s can vary only over the range of values from 0 to 1.0. The wall loss parameter, X , is constant by prior selection of the duct size, duct material, and working fluid. By maximizing z_h and z_o with respect to s or K , the power output in the liquid metal (Equation 30) and the pressure developed in the duct per unit length (Equation 28) will also be maximized. If the intended application of the pump requires high head,

z_h is maximized. Likewise if high power output is required, z_o is maximized. The requirement of high pressure output was established in this work.

When z_h is maximized with respect to slip by substituting s_h for s in Equation 41, the result is designated by z_{hs} . Similarly maximum z_{os} is used when z_o is maximized with respect to slip by substituting s_o for s in Equation 42.

The maximum values of Equations 41 and 42 with K as the variable occur when

$$K = K_z = \frac{1}{s + \frac{X}{s}} \quad (43)$$

and these values are

$$z_{hk} = \frac{1}{2 \left(1 + \frac{X}{s}\right)} \quad (44)$$

$$z_{ok} = \frac{1-s}{2 \left(1 + \frac{X}{s}\right)} = \frac{1}{2} \eta_e \quad (45)$$

In this work, since a pump with a good pressure rise was required, the design could be based on strictly a maximum z_{hs} occurring at s_h from Equation 29. However, note that basing the design on the optimum K (i.e. K_z) offers a unique method of simultaneously maximizing z_h to z_{hk} and z_o to z_{ok} . A good power output yields a better overall pump efficiency. It should be mentioned that for a given slip, s' , there exists a K_z' and a z_{hk}' ; likewise given s'' there exists a K_z'' and a z_{hk}'' . If s'' is greater than s' , z_{hk}'' will be greater than z_{hk}' . The largest z_{hk} occurs at $s = 1.0$. Of course this is an impractical situation because unity slip is

equivalent to zero flow rate. Figure 1, a plot of z_{hk} and $\frac{X}{s}$ from Equation 44, shows that the design slip is a compromise between the flow rate and pressure required. From Equation 20, $\frac{X}{s}$ is equal to $\frac{I_w}{I_r}$. This shows that the ratio $\frac{I_w}{I_r}$ should be between 0 and approximately 1.0 if the wall loss current is to be kept below the value of the useful liquid metal current. Also note at K_z , $\theta = 45^\circ$, since

$$\tan \theta = \frac{1}{K(s + X)} = 1.0 \quad (46)$$

If the slip for maximum power output (Equation 40) is evaluated at optimum K (Equation 43) and at some preselected X , it is found that this operating point is also one of maximum ideal electrical efficiency, i.e.

$$s = s_o = (X^2 + X)^{\frac{1}{2}} - X = s_e \quad (39)$$

combining Equations 39 and 43 yields

$$K = K_{ze} = \frac{1}{(X^2 + X)^{\frac{1}{2}}} \quad (47)$$

The equation of z_o can now be simplified by substituting for s and K from Equations 39 and 47 respectively, to

$$z_o = z_{os} = z_{ok} = \frac{1}{2} \left[1 + 2X - 2(X^2 + X)^{\frac{1}{2}} \right] = \frac{1}{2} (1 - 2s_e) = \frac{1}{2} \eta_{em} \quad (48)$$

This shows that the optimum value of the output utilization factor z_o is equal to one half of the ideal electrical efficiency and that both z_{os} and z_{ok} occur at the same slip. In the case of the head utilization factor, it can be shown that z_{hs} (occurring at s_h) and z_{hk} (occurring at K_z) cannot occur together; thus there is not a single optimum value of the head utilization factor.

Figure 1. The variation of the maximum head utilization factor, z_{hk} ,

with $\frac{X}{s}$

B. Windings

The proper winding design will provide the field strength required to meet the flow rate and slip used in the stator design calculations. A particular winding configuration is checked by calculating the total theoretical input impedance and the stator phase current at an assumed input voltage and slip. If the resulting ampere turns are sufficient, this particular winding design would be acceptable. In order to find the theoretical input impedance and stator phase current, the equivalent circuit of the pump must be determined. Cage and Collins (11) assumed that the equivalent circuit of an induction motor could be modified to compensate for the case of a liquid metal rotor in a pump duct. They made the following modifications: the rotor leakage reactance was assumed to be negligibly small; the power loss in the fluid channel walls was added in parallel with the magnetizing reactance and the primary iron loss was neglected since saturation in the iron is low. An outline of their results is shown below.

The primary resistance, R_1 , per phase is

$$R_1 = \frac{\rho_{cu} N_1 (MLC)}{A_c c_p} \quad (49)$$

Primary leakage reactance, X_1 , is obtained from

$$X_1 = \frac{2f q w_B N_1^2}{10^7} \left(\frac{\phi_c}{s_1} + \frac{\phi_{s1}}{s_s} \right) \quad (50)$$

where the coil end leakage is

$$\phi_c = \frac{L s_1 k_p^2}{2\pi n^2 w_B} \quad (51)$$

and the stator slot leakage is

$$\phi_{sl} = k_d \left(\frac{d_3}{w_s} + \frac{d_1}{3 w_s} \right) \quad (52)$$

The magnetizing reactance, X_m , is

$$X_m = \frac{E \phi}{I_m} = 6.06 \times 10^{-8} \frac{f (N_1 k_p k_d)^2 A_B}{n^2 g} \quad (53)$$

The equivalent resistance of pump duct walls referred to the primary on a per phase basis is

$$R_c = 3 \rho_w \frac{w}{2 t_w L} (N_1 k_d k_p)^2 \quad (54)$$

When referred to the primary, the equivalent fluid resistance, R_2 , is

$$R_2 = 3 (N_1 k_d k_p)^2 \left[\rho \frac{(w + 0.075 w)}{tL} + \rho_{cu} \frac{0.64 (D_r)}{n^2 A_s} \right] \quad (55)$$

Figure 2 shows the typical single phase equivalent circuit of a polyphase flat linear induction pump with all rotor quantities referred to the stator.

When designing windings for a flat linear induction pump, the grading of the magnetic field must be considered. It has been shown by Blake (12) that end effects (i.e. abrupt changes in the magnetic field at the ends of a linear pump with nongraded windings) cause the flux to have a pulsating component. This component in turn adds a pulsating component of current in the liquid; the current does not increase the output power but does increase the power loss in the liquid metal by the factor $(1 + \frac{1}{s^2})$ over that of a pump with an ideally graded field. This

Figure 2. Single phase equivalent circuit of polyphase flat linear induction pump

added heat loss to the liquid metal is too large even at relatively large slips. For example, at a slip of 0.8 the power loss in the liquid metal is nearly tripled.

These abrupt changes in the magnetic field at the ends of the pump are overcome by many different methods of grading the winding. This gradually increases the flux from zero at the ends to the uniform value in the middle region. The windings may be graded over the entire length of the pump, but this necessitates the separate design of each coil. Windings may also be graded over just the end regions of the pump, thus allowing the flux in the fully wound region to be of the ideal form. A good example of the latter method is the double layer winding with half-wound end poles; in this case only one coil design is required.

IV. THEORETICAL DESIGN OF PUMP

Based on the design theory of flat linear induction pumps and the immediate performance requirements, it was decided that the pump under design should be a five pole, one slot per pole per phase, full pitch, twelve coil, wye connected, three phase pump with a double layer winding on just one stator. This winding configuration automatically offers two half wound end poles and three fully wound central poles.

Initially the designer of a linear induction pump is usually given or must select certain operating conditions such as the working fluid, temperature, duct material, flow rate, hydraulic losses in pump duct acceptable, and approximate output pressure required for the intended application. The restrictions of hydraulic losses and pump duct cavitation almost fix the overall reduction in area from the pipe to the pump duct. And since the pipe area is known, so is the duct area. The ratio of internal duct width and thickness is also restricted; virtually establishing these dimensions and the magnetic gap. From the duct area and flow rate the fluid velocity in the pump region is determined. Now the hydraulic losses can be checked. If these losses are satisfactory, these dimensions may be used. It would be desirable to make the duct thickness and the magnetic gap very small for maximum field considerations; however, the resulting large hydraulic losses could not be tolerated. Conversely, making the duct square and the resulting gap large to reduce hydraulic losses would decrease the magnetic field in the gap and severely limit pump performance.

The dimensions above and equations from design theory may now be used to design an optimum stator structure. The optimum structure offers maximum pressure development, maximum power, or maximum efficiency at the given operating conditions. In this work the requirement of maximum pressure development at a good efficiency was selected on the basis of the intended installation of the pump in an operational sodium loop. The factor z_{hs} was not used since only the pressure output would be maximized. Good efficiency would be obtained only by having a good power output. Therefore, two methods of optimization ($z_o = z_{os} = z_{ok}$, $z_h = z_{hk}$ and $z_o = z_{ok}$, $z_h = z_{hk}$) were checked. Both methods simultaneously offered a maximum z_h for high pressure output and a maximum z_o for good power output after assuming or calculating the operating slip. The calculated slip by the first method was found to be unsatisfactory for these pump conditions. The second method (where $z_o = z_{ok}$, $z_h = z_{hk}$, and $K = K_z$) was the one actually used to design the flat linear induction pump.

The design of the pump was accomplished in two steps. First, as discussed in the two preceding paragraphs, the optimum stator structure for sodium at 400 C was established from calculations for maximum utilization factors. Second, the parameters for the flat linear induction pump equivalent circuit were determined for several winding configurations. Based on these equivalent circuits, the winding method was determined that most nearly offered the ampere turns assumed in the first step.

A. Optimum Stator Structure

The induction pump was designed for operation with sodium metal at 400 C flowing in an inconel pump duct. The reduction in cross sectional

area from 0.75-inch schedule 40 inconel pipe to the duct was made slightly less than three to one in accordance with Watt (13). Larger reductions could cause cavitation (i.e. a condition existing when inlet pressure falls below the vapor pressure of the fluid and resulting small cavities of vapor collapse in the pump duct). The design performance criterion was approximately five gallons per minute flow rate of molten sodium. The various dimensions and material properties used were

$$f = 60 \text{ cps}$$

$$g = 0.794 \text{ cm}$$

$$k = 1/302 \text{ stokes}$$

$$k_d = 1$$

$$k_p = 1$$

$$t = 0.318 \text{ cm}$$

$$t_w = 0.158 \text{ cm}$$

$$w = 3.81 \text{ cm}$$

$$\mu = 0.00284 \text{ poise (14)}$$

$$\rho = 21.93 \text{ microhm cm (14)}$$

$$\rho_w = 98.1 \text{ microhm cm (15)}$$

$$\sigma = 0.857 \text{ grams per cc (14)}$$

In order to obtain a strong field, a small air gap, g , was selected; thus the internal thickness, t , of the pump duct was also small. The internal width, w , was chosen so that the ratio of $\frac{w}{t}$ would be greater than 10.

The resultant fluid velocity was approximately 260 cm per second at the design flow rate using the equation

$$v = \frac{Q}{wt} \quad (56)$$

At this velocity the Reynolds number was 46,250 using the equation

$$R_e = \frac{vd}{k} \quad (57)$$

and the resulting theoretical hydraulic loss was satisfactory at 0.46 psi per foot of pump duct.

Initially an attempt was made to optimize the output utilization factor, z_o , for maximum output at the slip of maximum efficiency and to maximize the head utilization factor, z_h , at K_{ze} . Using Equation 21 the wall loss parameter, X , is 0.223. The slip, found from Equation 39 with $X = 0.223$, is $s = s_e = 0.300$. The pump parameter $K = K_{ze} = 1.91$ is determined from Equation 47 with $X = 0.223$. From Figure 1 with $\frac{X}{s_e} = 0.744$ observe that $z_{hk} = 0.287$ which is a relatively low maximum z_h . Finally the wavelength, λ , is found to be 29.6 cm via Equation 18. The velocity of the magnetic field is computed from the Equation $v_s = f \lambda$; thus $v_s = 1778$ cm per second. At s_e , $v = 1241$ cm per second which greatly exceeds the assumed fluid velocity of 260 cm per second. In addition this high fluid velocity, equivalent to 23.4 gpm, probably could never be attained experimentally due to the severe hydraulic losses. Now assume that this pump was built, but let the operation be such that $v = 260$ cm per second or $s = 0.855$. Thus $K = K_{ze} = 1.91$, $s \neq s_e$, $X = 0.223$, $z_h \neq z_{hk}$, $z_o \neq z_{ok}$, and $\lambda = 29.6$ cm. By using Equations 41

and 42, $z_h = 0.311$ and $z_o = 0.045$. This may appear to be a satisfactory design. However, before selecting a design consider a pump operated at the design value of slip.

At this point it was apparent that for the design to be practical the slip must be increased, but the extent of this increase was not known. In this case both utilization factors were maximized at optimum K since this method allowed different slips to be tried. It was then necessary to find a slip which satisfied the conditions of optimum K and a liquid velocity of 260 cm per second. First note that

$$v_s = \frac{v}{1-s} = f \lambda \quad (58)$$

and therefore

$$\lambda = \frac{v}{f(1-s)} \quad (59)$$

The optimum K is obtained from Equation 43. Combining Equation 18, the pump parameter, with the Equations 43 and 59 yields the following

$$\frac{1}{X+s} = \frac{\mu_o \left[\frac{v}{f(1-s)} \right]^2}{2\pi g \rho} \quad \text{ft}$$

$$s^2 - \frac{2\delta + 1}{\delta} s + \frac{\delta - x}{\delta} = 0 \quad (60)$$

where

$$\delta = \frac{2\pi g \rho f}{\mu_o v^2 t}$$

Equation 60, a quadratic, is easily solved for the slip $s = 0.81$. Since $v = 260$ cm per second was approximate, a design slip of 0.80 was assumed with X still equal to 0.223. Referring back to Figure 1 at $\frac{X}{s} = 0.278$

shows the value of z_{hk} , in this case 0.392, to be very near the maximum value since X is never zero. The corresponding value of z_{ok} was 0.078. The optimum K (Equation 43) is now found to be 0.976. This value of K is shown to produce a maximum z_h in Figure 3 by plotting values of z_h from Equation 41 at various values of K . When this value is substituted into Equation 18, the result is $\lambda = 21.2$ cm. It is obvious now that the design and operation of the pump at the same slip is beneficial. The second design method yielded a higher z_h and increased z_o by almost a factor of two. The wavelength was shorter by the latter method which allowed the five pole pump to be 54.8 cm long rather than 75.8 cm long; this was a very significant result. Thus the overall length of the five pole pump was 54.8 cm with equal slot and tooth dimensions of 1.766 cm. The nominal stator width was determined by the combined thickness of 91 laminations of 24 gauge hot rolled high silicon magnet core iron insulated from each other with 0.004-inch mica sheet.

The stator structure design for the linear induction pump was still not completely determined since the slot depth was unknown. However, all necessary dimensions were available for the design of the pump duct. This is shown in Figure 4.

B. Windings

The next logical step in the design of the flat linear induction pump was the determination of the exact winding configuration (i.e. the required number of turns per coil and the electrical connections). It was assumed that all coils would be identically wound of round 12 gauge

Figure 3. The variation of the head utilization factor with the pump parameter

Figure 4. Pump duct for flat linear induction pump

(0.0808-inch diameter) heavy formvar insulated copper magnet wire. Since the liquid flow can be throttled to the design rate and the ultimate test loop can withstand pressures up to 100 psi, a design gross pressure of 60 psi at 0.8 slip and 250 volts line voltage was used. Based on the performance of other pumps the actual pressure developed by the pump was expected to be from 40 to 80 percent of this design figure.

The required number of ampere turns per phase was determined from the equation formed by the substitution of Equation 12 for I_s into Equation 28. This result, after Equation 28 is multiplied by x to convert p to gross pressure, is

$$N_{PH} I = \left(\frac{4}{288} \frac{p t g n^2 \lambda}{\mu_o x z_h} \right)^{\frac{1}{2}} \quad (61)$$

The solution of Equation 61 with $p = 60$ psi, $t = 0.318$ cm, $g = 0.794$ cm, $n = 5$ poles, $\lambda = 21.2$ cm, $\frac{\mu_o}{4} = 1.451 \times 10^{-7}$, $x = 31.8$ cm (fully wound region), and $z_h = z_{hk} = 0.392$ yields $N_{PH} I = 3,920$ ampere turns per phase.

The various constants, dimensions, and physical properties used in the following winding design calculations are

$$A_c = 0.0049 \text{ in.}^2$$

$$A_s = 0.0625 \text{ in.}^2$$

$$f = 60 \text{ cps}$$

$$g = 0.311 \text{ in.}$$

$$k_d = 1.0$$

$$k_p = 1.0$$

$$L = 21.56 \text{ in.}$$

$$\begin{aligned}
 (\text{MLC}) &= 12.5 \text{ in.} \\
 q &= 3 \\
 s_1 &= 15 \\
 s_s &= 15 \\
 t &= 0.125 \text{ in.} \\
 t_w &= 0.0625 \text{ in.} \\
 w &= 1.5 \text{ in.} \\
 w_B &= (2.660) (0.9) \text{ in.} \\
 w_s &= 0.695 \text{ in.} \\
 \rho &= 8.64 \text{ microhm in. (14)} \\
 \rho_{\text{cu}} &= 1.65 \text{ microhm in. at } 400 \text{ C (15)} \\
 \rho_{\text{cu}} &= 0.827 \text{ microhm in. at } 75 \text{ C (16)} \\
 \rho_w &= 38.6 \text{ microhm in. (15)}
 \end{aligned}$$

The first winding configuration tried was one consisting of 120 turns per coil with one coil in series per phase and four circuits in parallel per phase. Using Equation 49 with $c_p = 4$ circuits in parallel and $N_1 = 240$ series conductors per phase, the primary resistance, R_1 , is 0.127 ohms. The primary leakage reactance, X_1 , is found to be 1.08 ohms from Equations 50, 51, and 52 where $d_1 = 2.16$ in. and $d_3 = 0.34$ in. It is necessary to change the slot depth, $d_1 + d_3$, for different winding designs since a space free of conductors is required for forced air cooling of the pump. When the area of the gap, $A_g = Lw_B = 51.6$ in., is substituted into Equation 53, the magnetizing reactance, X_m , is 2.17 ohms. The equivalent resistance of pump duct walls, R_c , and the

equivalent fluid resistance, R_2 , are 3.72 ohms from Equation 54 and 2.14 ohms from Equation 55 respectively.

It should be mentioned here that the assumption of negligible rotor reactance has been shown by Blake (12) to be good if the ratio of $\frac{t}{\lambda}$ is less than 0.1. The ratio $\frac{t}{\lambda}$ for this particular induction pump is 0.015.

The magnitude of the total pump impedance per phase, $|Z_T| = 2.14$ ohms, is determined from the equivalent circuit, Figure 2, using alternating current circuit analysis at the design slip of 0.8. Dividing the phase voltage, $E_\phi = 144$ volts, by $|Z_T|$ yields the total phase current, I , of 67.4 amperes. Thus the resultant ampere turns per phase for this winding configuration are 8,080 which are more than is required. The current per coil must also be checked since the maximum allowable current for 12 gauge wire is 20 amperes. In this example the current per coil is 16.8 amperes.

Several other winding configurations were investigated before one was found with an ampere-turn rating approaching the design value of 3,920. The results of these investigations are summarized in Table 1.

Based on the data shown in Table 1, the 60 turns per coil winding method was obviously the only one meeting all of the specifications. Even though this method provided slightly more than 3,920 ampere turns per coil, it was adjudged unnecessary to further reduce the turns per coil and to recalculate the pump parameters.

Table 1. Theoretical induction pump parameters for different winding configurations

No. of turns/coil	No. of series turns/phase	c_p	R_1 ohms	X_1 ohms	X_m ohms	R_c ohms	R_2 ohms
120	120	4	0.127	1.08	2.17	3.72	2.14
80	160	2	0.338	2.08	3.86	6.60	3.81
60	240	1	1.01	4.11	8.68	14.87	8.56

No. of turns/coil	s	$ Z_T $ ohms	E_ϕ volts	I amperes	Coil current amperes	Ampere turns/phase
120	0.8	2.14	144	67.4	16.8	8080
80	0.8	4.04	144	35.6	17.8	5700
60	0.8	8.72	144	16.5	16.5	3960

Figure 5 shows the complete flat linear induction pump design. Note that the stator without windings, sometimes called the "core", is almost the same size as the stator with windings; however, it has no slots. The electrical winding connections are shown pictorially in Figure 6.

C. Theoretical Performance Characteristics

The theoretical performance characteristics of the induction pump were computed using the equivalent circuit parameters for a winding with 60 turns per coil. The stator phase current was determined as before except the slip was determined from the equation

$$s = 1 - \frac{Q}{24.35} \quad (62)$$

as the flow rate changed. In this case the phase voltage was also varied. Thus to each value of flow rate and line voltage there was a corresponding unique value of stator phase current.

If the stator phase current, I , is substituted into Equation 12, the value of the stator surface current per unit length, I_s , is obtained. The dimensions and material constants are as listed on page 26. The pump parameter, K , is 0.976. The stator surface current, I_s , is then used in the evaluation of the gross pressure developed over the fully wound region of the pump stator via Equation 28 multiplied by $x = 31.8$ cm (i.e. the length of the fully wound region). When the hydraulic losses, p_f , in the pump duct are deducted from the gross pressure, xp , the resulting value is the actual theoretical pressure developed, $(p)_{\text{actual}}$, by the induction pump. Equations 31 and 32 provide the actual power output in the liquid metal and the actual operating efficiency respectively.

Figure 6. Winding connections for flat linear induction pump

Table 2 is a partial tabulation of flat linear induction pump parameters calculated by the preceding method at various flow rates and line voltages.

Table 2. Theoretical pump performance data

E_L volts	Q gpm	I amperes	Pf psi	(p) _{actual} psi	(P _O) _{actual} watts	(η) _{actual} percent
250	0	17.6	0	71.4	0	0
250	2.44	17.0	0.5	65.5	69.4	1.59
250	4.88	16.5	1.3	59.7	126.7	3.03
250	7.30	15.9	2.6	52.2	166.0	4.20
200	0	14.0	0	45.0	0	0
200	2.44	13.6	0.5	42.3	45.9	1.65
200	4.88	13.2	1.3	37.7	80.0	2.99
200	7.30	12.7	2.6	32.3	102.7	4.08
150	0	10.6	0	25.9	0	0
150	2.44	10.3	0.5	23.7	25.1	1.57
150	4.88	10.0	1.3	21.1	44.7	2.91
150	7.30	9.6	2.6	17.4	55.3	3.84

The theoretical performance characteristics for the induction pump, shown in Figures 7 and 8, clearly verify that the design specifications (i.e. developed pressure of 60 psi when the line voltage is 250 volts

and the slip is 0.8) were attained. Likewise the efficiency is roughly 3% at design conditions which is good for a small laboratory-size induction pump.

Figure 7. Theoretical performance characteristics of induction pump

Figure 8. Theoretical efficiency of induction pump

V. INVESTIGATIONS

The flat linear induction pump was built and installed in an operational sodium circulating loop. The measurement of developed pressure, sodium flow rate and power input provided an experimental determination of the pump performance characteristics.

A. Equipment

During fabrication all materials for the pump were carefully selected for service at high temperatures. In order to reduce the losses caused by eddy currents in the core, each lamination was given a coat of high temperature insulating varnish on one side in addition to the mica insulator. The resulting width of the stator and core was 2.660 inches which was a slight increase over the design nominal width of 2.625 inches. Coils of several different shapes were inserted into a full size wood model of the stator; the shape selected provided the largest free-conductor slot area for force air cooling. Figure 9 shows the pump after installation in the loop along with the duct housings for air cooling of the stator, stator windings, and core.

The hydraulic losses in the pump duct for sodium were calculated from pressure drop measurements made with water. The derivation of equations relating pressure drop and flow rate between sodium and water at constant Reynolds number is summarized in the Appendix. Tap water at 12 C yielded a pressure drop of 12.8 psi and a maximum flow rate of 14.4 gpm while the equivalent values for 400 C sodium were only 0.760 psi and 3.8 gpm. Since the sodium flow rate of 3.8 gpm was not large enough,

Figure 9. Induction pump installed in sodium loop

it was deemed necessary to raise the temperature of the water rather than increase the water flow rate. Future tests were run with water at 57 C using a constant temperature water tank and a mechanical turbine pump. In this case the values of pressure drop and flow rate for 57 C water were reduced only a small amount when converted to the corresponding values for 400 C sodium. The pump duct was cleaned thoroughly with dilute nitric acid after the hot water test since the liquid metal must wet the duct wall for efficient pumping.

The flow rate of sodium was measured with a magnetic flowmeter shown in Figure 10. A small electrical signal is generated by the action of the liquid sodium cutting the lines of force of the powerful permanent magnets. This signal was picked up by the flowmeter electrodes and transmitted to a Brown millivoltmeter stripchart recorder. The flow rate is calculated from the equation

$$Q = E \frac{Y}{B} \quad (63)$$

where E = the voltage output in millivolts, Y = physical constant dependent upon pipe and fluid (Y = 2710 for sodium at 400 C), and B = flux density of magnets in gauss (B = 920 gauss approximately). The flux density was accurately determined from a calibration curve of flux density plotted against magnet temperature. Prior to recording any experimental data the flowmeter was calibrated using a water calibrated venturi that had previously been installed in the loop. The millivoltmeter recorder was calibrated using a Rubicon potentiometer.

Figure 10. Magnetic flowmeter

The pressure developed by the pump was determined using sodium manometer columns located at the inlet and outlet of the pump duct. Each column consists of a single wire heater insulated from a 5/32-inch inconel pipe by alundum beads. A sodium annulus is provided between this tube and the outside 3/8-inch inconel pipe. A cut-away view of the manometer column and heater assembly is shown in Figure 11. The sodium level in the manometer columns was located by Automatic Timing and Controls differential transformers for high temperature application. This transformer, shown in Figure 12, has one primary winding surrounding two displaced secondary windings. The secondary windings are connected in series opposition so that a maximum voltage is obtained when the sodium armature is completely in only one secondary. When both secondaries surround either an empty annulus or sodium, the output is virtually zero. Figure 13 is an electrical block diagram of the sodium level measuring instrumentation. A Hewlett-Packard 200 C.D. audio oscillatory provided a 3600 cps signal to the primary winding of the differential transformer. The difference voltage is stepped up using a Southwestern Industrial Electronics Engineereco transformer with a turns ratio of 1 to 105 and is then presented to a Hewlett-Packard 400 D. vacuum tube voltmeter. Maximum response occurs at 3600 cps; however, the system can detect the sodium level over a range of frequencies.

All three phase line quantities and total input power to the pump were measured using a Westinghouse Industrial Analyzer. The phase voltages were measured by Simpson panel meters. These meters were calibrated before recording experimental data. The input voltage to the induction

Figure 11. Cut-away view of manometer column

Figure 12. Differential transformer

Figure 13. Block diagram of sodium level detection instrumentation

pump was controlled by a Powerstat variable transformer rated at 20 amperes, 240 volts, and 9.7 kva.

B. Experimental Procedure

After installation of the pump duct in the sodium loop, the pump stator and core were assembled to the duct. The loop was insulated at each end of the pump and around the pump duct. The sodium loop and associated equipment are shown in Figure 14.

Initially the empty loop was preheated by a heating transformer while sodium in the sump was melted by resistance heating elements. The single phase heating transformer uses the loop as a one-turn secondary and provides heating currents as high as 2000 amperes. Thermocouples and calibrated potentiometer recorders were used to measure the various temperatures around the loop. After the loop was filled, it was held at a temperature of 450 C for several hours to cause complete wetting of the pump duct walls.

One set of constant-voltage pump data was obtained by fixing the pump voltage and varying the flow rate with a control valve. At one setting of the control valve the sodium levels in the input and output manometer columns were leveled by adjusting the helium pressure over each column of molten sodium. The developed pressure across the pump was then equal to the difference in helium pressure between the columns. Calibrated pressure gauges were used for these measurements. The leveling process was very time consuming, but when it was accomplished the other readings were taken as rapidly as possible. These readings included three phase electrical values, total electrical power input, column height

Figure 14. Sodium loop and associated equipment with induction pump installed

and temperature, column pressure, magnetic flowmeter output, average loop temperature, pump duct temperature, and flowmeter magnetic temperature. All readings were taken with a pump inlet pressure of five pounds per square inch.

Calculations were then performed to determine the actual pressure rise and flow rate. As in the theoretical pump analysis equivalent techniques were used to determine the actual power output in the liquid metal and the actual operating efficiency.

VI. EXPERIMENTAL RESULTS

It was found that reliable data could not be taken at zero flow because the pump duct temperature could not be maintained at 400 C even with the blower off. The fact that duct temperature is critical was verified experimentally. The effect of duct temperature on flow rate, shown in Figure 15, demonstrates that the pressure developed by the pump must also vary with duct temperature. Based on the above observation data were taken at very low flow rates, but the duct temperature was consistently held at 400 C.

Table 3 is a tabulation of experimentally determined parameters of the flat linear induction pump when it is pumping 400 C sodium with an inlet pressure of five pounds per square inch. The experimental performance characteristics of the induction pump are shown in Figures 16 and 17.

Figure 15. Effect of pump duct temperature on sodium flow rate

Table 3. Experimental induction pump performance data

E_L volts	Q gpm	I (phase a) amperes	(p) _{actual} psi	(P _o) _{actual} watts	(η) _{actual} percent
250	0.91	13.3	42.13	16.68	1.11
250	2.65	13.3	40.31	46.47	3.10
250	6.55	13.2	31.38	81.41	6.30
250	9.93	13.2	24.06	103.93	7.53
200	0.65	10.5	27.69	7.83	0.82
200	2.65	10.6	25.56	29.46	3.07
200	5.48	10.5	21.69	51.70	5.62
200	8.05	10.5	17.38	60.86	6.91
150	0.65	7.8	15.94	4.51	0.90
150	2.71	7.8	14.06	16.57	3.60
150	6.24	7.7	10.62	28.83	6.55
114 ^a	4.88	5.9	6.62	14.05	5.11
100	0.77	5.3	6.13	2.05	1.02
100	2.59	5.3	6.06	6.83	3.25
100	4.30	5.3	5.25	9.82	4.91
50	0.97	2.6	1.25	0.53	0.96
50	2.12	2.6	1.25	1.15	1.92

^aDesign flow with control valves 100 percent open.

Figure 16. Experimental performance characteristics of induction pump

Figure 17. Experimental efficiency of induction pump

VII. CONCLUSIONS AND SUMMARY

The good agreement between theoretical analysis and experimental results is clearly shown by comparing the respective performance characteristics. Note that the design condition of approximately five gallons per minute flow rate was easily met in the sodium loop at line voltages above 114 volts.

There were several differences between the two performance characteristics which should be explained or justified. The fact that the actual developed pressure was approximately 60% of theoretical value is a normal occurrence in electromagnetic pumps. The turbulent motion of the liquid metal in the strong magnetic field induces currents in the liquid which interact with the field to produce forces on the liquid opposing turbulence. The result is laminar flow at abnormally high Reynolds numbers and increased hydraulic losses. Further investigations should be conducted in this area so that the reduction in turbulence may be theoretically predicted in future pump designs for any liquid metal used. Another reason for the low experimental pressure rise was that the pump had only three slots per pole per phase. This produces a distorted magnetic flux density wave that dips over each stator slot. A similar stator configuration in an annular induction pump has been shown to have a magnetic field with strong fifth and seventh harmonics which reduced pump output.

The improved experimental efficiency is due to a slightly low theoretical pump impedance. This resulted in larger phase currents and

a greater input power in the theoretical analysis which caused a low theoretical efficiency.

These results demonstrate the feasibility of designing a small flat linear induction pump to meet preselected conditions of flow rate and developed pressure. The proper design will insure the best possible efficiency under these conditions. Pumps of this type for different liquid metals should be designed in the future.

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IX. APPENDIX

The pump duct pressure drop measurements made with water were converted to an equivalent pressure drop for sodium at 400 C. The equivalence of Reynolds number for water or sodium in the same duct provides the basis for equating the friction factor.

When the Reynolds number, Equation 57, for sodium and water are equated at the same equivalent pump diameter, the result is

$$\left[\frac{\sigma v d}{\mu} \right]_{\text{Na}} = \left[\frac{\sigma v d}{\mu} \right]_{\text{H}_2\text{O}}$$

$$v_{\text{Na}} = v_{\text{H}_2\text{O}} \left[\frac{\sigma_{\text{H}_2\text{O}}}{\sigma_{\text{Na}}} \right] \left[\frac{\mu_{\text{Na}}}{\mu_{\text{H}_2\text{O}}} \right] \quad (64)$$

or by Equation 56

$$Q_{\text{Na}} = Q_{\text{H}_2\text{O}} \left[\frac{\sigma_{\text{H}_2\text{O}}}{\sigma_{\text{Na}}} \right] \left[\frac{\mu_{\text{Na}}}{\mu_{\text{H}_2\text{O}}} \right] \quad (65)$$

Equating the friction factors from the Darcy equation yields

$$\left[\frac{2 H d g}{v^2} \right]_{\text{Na}} = \left[\frac{2 H d g}{v^2} \right]_{\text{H}_2\text{O}} \quad (66)$$

where in this case g = acceleration of gravity.

Since the hydraulic friction loss is

$$p_f = H \sigma g \quad (67)$$

the equation relating the pressure drops is

$$\left[p_f \right]_{\text{Na}} = \left[p_f \right]_{\text{H}_2\text{O}} \left[\frac{v_{\text{Na}}}{v_{\text{H}_2\text{O}}} \right]^2 \left[\frac{\sigma_{\text{Na}}}{\sigma_{\text{H}_2\text{O}}} \right] \quad (68)$$

Equations 64 and 68 when combined yield

$$[p_f]_{Na} = [p_f]_{H_2O} \left[\frac{\sigma_{H_2O}}{\sigma_{Na}} \right] \left[\frac{\mu_{Na}}{\mu_{H_2O}} \right]^2 \quad (69)$$

Equations 65 and 69 were the two equations used during the hydraulic loss tests.