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BYPASSING THE INSTABILITY-INDUCED THROUGHPUT LIMIT IN CALUTRON ISOTOPE SEPARATORS

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BYPASSING THE INSTABILITY-INDUCED THROUGHPUT LIMIT
IN CALUTRON ISOTOPE SEPARATORS

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Igor Alexeff*

ABSTRACT

In this report we discuss a way of bypassing the instability-induced throughput limit in calutron (magnetic mass-analysis) type isotope separators.¹ We demonstrate both experimentally and theoretically that the isotron (time-of-flight) type of isotope separator is relatively immune to this type of instability, although still susceptible to the limitation due to the granular nature of the ion beam.² Depending on the severity of this latter limit, the throughput limit of an isotron could be up to several orders of magnitude greater than that of a calutron.

INTRODUCTION

In our previous report on instabilities causing a limit to the throughput in calutron (magnetic mass-analysis) isotope separators, we derived an instability condition that was sufficient to quantitatively explain the phenomenon.¹ We showed that one of a family of instabilities driven by an ion stream moving across the required magnetic field had sufficient gain to disrupt the separation process. Such instabilities were studied intensively in our now-defunct study of plasma heating via turbulence.³

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In this report we wish to note that the family of instabilities discussed above only exist if the magnetic field is present in the region of the calutron ion beam. If the magnetic field is removed, the electrons in the ion beam can flow freely to neutralize the ion clump creating the instability. As a result, in the one-dimensional case, the instability can be made to disappear for all values of beam density. In a three-dimensional treatment, instabilities can still occur due to waves propagating at large angles across the beam, but the gain is low and the effect should not be serious, as will be shown later. Thus, we can expect an isotope separator having no magnetic field (an isotron, or time-of-flight device) to possibly have a much larger throughput than one depending on a magnetic field.

DISCUSSION

Instability Threshold

We now discuss theoretically the one-dimensional case, using results from my lecture notes at the University of Tennessee. In our previous discussion² we considered two ion streams of equal density, and in a coordinate system moving with the center of mass derived the following dispersion relation,

$$1 = \frac{\omega_p^2}{2(\omega - k\Delta v)^2} + \frac{\omega_p^2}{2(\omega + k\Delta v)^2}$$

Here the variables are more defined as follows:

$$\omega_p^2 = \left(\frac{ne^2}{\epsilon_0 m_i} \right),$$

where n is the ion density (m^{-3}), e is the ion charge (coulomb), ϵ_0 is the permittivity of space (8.85×10^{-12}), and m_i is the ion mass (kg). ω and k are the frequency and wavenumber of the disturbance (sec^{-1} and m^{-1} , respectively), and $2\Delta v$ is the velocity difference between the two ion species (meter per second).

If we now assume that the ion streams are moving through a Maxwellian sea of electrons of temperature T , and are moving sufficiently slowly that the electrons can follow and neutralize the ion clumps, a dramatically different dispersion relation is obtained,

$$\left(\frac{m_i}{KT} \right) = \frac{1}{2(\omega/k - \Delta v)^2} + \frac{1}{2(\omega/k + \Delta v)^2}.$$

Here all variables are defined as before, except for KT , which is Boltzmann's constant (joules per $^\circ\text{K}$) times the electron temperature ($^\circ\text{K}$). This derivation assumes that the wavelength is large compared with the Debye radius—the opposite assumption results in lower gain for the instability and need not concern us here.

A graphical investigation of the instability threshold demonstrates that the system first goes unstable (if it does) for $\omega/k = 0$, and that if $(m_i/KT) > 1/(\Delta v)^2$, the system is always stable. This is in contrast to the calutron case² where we found that an instability threshold existed in terms of ion beam density and that in ordinary calutron operation the system was always operating in the unstable (but not catastrophically unstable) regime.

Let us compute the values of the velocities in the above inequality, assuming typical calutron parameters.

$$m_i = 238 \times 1.67 \times 10^{-27} \text{ kg}$$

$$K = 1.60 \times 10^{-19} \text{ joule per electron volt}$$

$$T = 1.0 \text{ electron volt}$$

$$\Delta v = \frac{1}{2} \text{ the splitting in velocity between a } U^{235} \text{ and } U^{238} \text{ ion}$$

$$\begin{aligned} & \text{beam at 10 keV} \\ &= \frac{1}{2} \times v_{\text{beam}} \times \left(\frac{1}{2} \frac{\Delta m}{m} \right) \\ &\approx 3 \times 10^2 \text{ meter sec}^{-1} \end{aligned}$$

Using our criterion above, we find $(2 \times 10^{-6})^2 > (10^{-5})$, or our system is unstable. However, increasing $(\Delta v)^2$ by about a factor of 5 (increasing the beam voltage to 50 kV) puts us into the stable regime. Reducing the electron temperature by placing thermally-emitting electrodes near the beam⁴ also enhances stability. We have in the past experimentally reduced the electron temperature to 0.2 eV in the laboratory.⁴

An interesting computational and experimental study pertaining to this subject is contained in our paper discussing ion waves in cesium plasmas.⁵ Here we show that if the ion temperature is sufficiently large $(\Delta v)^2 > (KT/m_i)$, no ion-acoustic waves (collective effects) exist, but the ions merely free-stream down the system.

Unstable Operation

We now discuss a most pertinent part of this theory for practical isotope separation—how far into the unstable region we can push the operation of the system and still get effective separation. We know, for example, that calutron separators operate well into the unstable regime.¹ There are two considerations to discuss here, rate of growth (can the instability grow fast enough to be apparent?) and saturated

amplitude (does the instability at maximum amplitude cause enough of a perturbation to be a problem?).

First we compute the rate of growth. Putting the two-stream equation in the same canonical form as before, we obtain

$$1 = \frac{\left(\frac{KT}{m_i}\right) k^2}{2(\omega - k\Delta v)^2} + \frac{\left(\frac{KT}{m_i}\right) k^2}{2(\omega + k\Delta v)^2} .$$

In our previous case we computed the maximum temporal growth rate to be

$$\omega_i = \frac{1}{2} \omega_p .$$

(An error of $\sqrt{2}$ was and is present but is not serious.) The analogous expression here is

$$\omega_i = \left(\frac{KT}{m_i}\right)^{1/2} k .$$

For maximum growth we use the maximum possible value for k which is

$$k_{\max} \approx \frac{1}{\lambda_D} = \left(\frac{ne^2}{\epsilon_0 KT}\right)^{1/2} .$$

Here λ_D is the Debye radius. So $\omega_i = \frac{1}{2} (ne^2/\epsilon_0 m_i)^{1/2} = \frac{1}{2} \omega_p^2$ as before.

We thus see that the maximum growth rate is exactly the same as before.

Therefore the growth rate in the unstable case would give the same density threshold as before, except for the saturated value.

We next compute the saturated value for the instability. It is well-known that these instabilities saturate (cease to grow) when the maximum potential energy fluctuation in the wave, $e\Delta V$ becomes comparable to kT . Thus, the maximum potential fluctuation in the beam is $\approx kT/e$. Since the maximum growth rate corresponds to a very short wavelength with $k \approx \frac{\Delta v}{\omega}$, the maximum E-field is on the order of several

volts/wavelength or about 1 kV/cm as before.

If the kinetic energy in the relative motion of the particles is higher than the potential energy of the wave, separation will continue at the saturated value of the instability, i.e., $\frac{1}{2} m (\Delta v)^2 > e\Delta v \approx KT$. Hence, $(\Delta v)^2 > (\frac{2KT}{m})$ which is about the value for the threshold for the instability anyway.

Waves at Angles to the Direction of Propagation

These waves have a dispersion relation given by

$$\left(\frac{m_i}{KT} \right) = \frac{1}{2(\omega/k - \cos \theta \Delta v)^2} + \frac{1}{2(\omega/k + \cos \theta \Delta v)^2}$$

Here $\cos \theta$ is the angle between k and $\vec{\Delta v}$. Hence these waves are excited for higher values of Δv than for those waves along the $\vec{v}$ direction. However, for a long narrow beam the waves are lost on the sides of the beam.⁶ Thus, these diagonally-propagating waves should be less serious than those along the $\vec{v}$ direction. Also, the saturated potential is still on the order of magnitude of $e\Delta v \approx KT$.

SUMMARY OF RESULTS

From considerations of wave growth and saturated amplitude, we discover that the instability-caused throughput limit in isotrons (time-of-flight isotope separators) in effect does not exist if $(\Delta v)^2 > (\frac{KT}{m})$, where $2\Delta v$ is the velocity difference between two ion species and $(\frac{KT}{m})^{1/2}$ is the ion-acoustic velocity (K is Boltzmann's constant, T is the electron temperature, and m is the ion mass.) Thus, the fundamental limit should be the granular nature of the beam and this possibly

could be much less restrictive than the instability-induced limit—possibly by several orders of magnitude. This last statement is an educated guess on my part based on an optimistic view of the granularity limit.

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