A Comment on the Possible Néel Temperature of Solid ^3He *

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A calculation of the effect of the conjectured three-body exchange on the possible Néel temperature of solid ^3He is made. A value of 2.24 mK is obtained from $P_V(T)$ data, whereas susceptibility data indicate 2.9 mK. These calculations indicate that three-body exchange may lead to the occurrence of the transition above 2.0 mK, the previously expected value.

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The nuclear spin system of crystalline ^3He has been extensively studied for many years. On the basis of the assumption that only nearest-neighbor exchange was important, values of J (the exchange integral) were deduced from NMR¹ and dP/dT^2 measurements. Then use of the high-temperature series expansion treatment of the Heisenberg antiferromagnet led to an expected value for the Neel temperature of about 2.0 mK. Recent high-magnetic-field measurements of Kirk and Adams³ have shown that the series expansion when only nearest neighbor exchange is included is not correct for bcc ^3He . In fact, their results (i.e., that the pressure is less than predicted) suggest that there might be some "ferromagnetic" exchange in the system. On an intuitive basis, such exchange might be expected to raise the Neel temperature.

Zane^{4,5} has offered an explanation for the results of Kirk and Adams in terms of three-body exchange, which leads to an effective next-nearest neighbor ferromagnetic exchange interaction. Using the results of Zane's theory it is possible to estimate the effect of three-body exchange on T_N . In addition to the usual two-body nearest-neighbor exchange, $J_2(\text{nn})$, Zane considers several "higher-order" exchange processes, the most important being three-body exchange J_3 . Two-body next-nearest-neighbor exchange, $J_2(\text{nnn})$, is of considerably less importance. The exchange Hamiltonian can then be written in the form

$$H_{\text{ex}} = - 2J(\text{nn}) \sum_{i < j}^{\text{nn}} \underline{I}_i \cdot \underline{I}_j - 2J(\text{nnn}) \sum_{i < j}^{\text{nnn}} \underline{I}_i \cdot \underline{I}_j, \quad (1)$$

where $J(\text{nn})$ and $J(\text{nnn})$ are the effective nearest-neighbor and next-nearest-neighbor exchange, respectively, given by

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$$J(nn) \cong J_2(nn) + 6J_3 \quad (2)$$

and

$$J(nnn) \cong J_2(nnn) + 4J_3 \quad (3)$$

The mathematical problem of determining the transition temperature resulting from the interaction in Eq. (1) is a difficult one for which there is presently no satisfactory solution. The molecular field model⁶ gives for the bcc lattice, in the case $J(nn) < 0$ and $J(nnn) > 0$,

$$k_B T_N = -4J(nn) + 3J(nnn) \quad (4)$$

However, we know that the coefficients of $J(nn)$ and $J(nnn)$ are not correct. If only nearest-neighbor exchange is considered, high-temperature series expansions⁷ give $T_N = -2.75 J(nn)/k_B$. Therefore, it is intuitively reasonable to scale (4) by the factor 2.75/4 to obtain

$$k_B T_N \cong -2.75 J(nn) + 2.06 J(nnn) \quad (5)$$

Equation (5) is, of course, not strictly correct, but it should give a reasonable indication of the Neel temperature to be expected.

The theory of Zane gives expressions for $J(nn)$ and $J(nnn)$ in terms of various experimental results. The ratio $-J_3/J_2(nn) \equiv b$ (called a in Ref. 4) is taken as an adjustable parameter which is given the value 0.05 in order to obtain a fit of the theoretically calculated high-field pressure, $P_V(T,H)$ to the experimentally observed behavior.³ A second parameter which occurs in the expression for $P_V(T,H)$ is

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$$\frac{d \ln |J(nnn)|}{d \ln V} \bigg/ \frac{d \ln |J(nn)|}{d \ln V} \equiv B$$

which Zane, by plausibility arguments, found to be 1.9. With $b = 0.05$, Eqs. (2) and (3) give $J(nnn)/J(nn) \equiv A \cong -0.22$. Upon substitution of this expression for $J(nnn)$ into Eq. (5) we have

$$k_B T_N = -3.2 J(nn) . \quad (6)$$

$J(nn)$ will be evaluated using results from $P_V(T)$ and susceptibility experiments. The experiments have previously been analyzed in terms of the two-body nearest-neighbor exchange assuming that there are no other exchange processes, $J_2^0(x)$, where x labels the experiment. In the case of $P_V(T)$, $J(nn)$ is related to $J_2^0(P)$ by⁵

$$J(nn) = J_2^0(P) \left(1 + \frac{3}{4} A^2 B\right)^{-1/2} . \quad (7)$$

Using the value of $J_2^0(P)/k_B = -0.72$ mK appropriate to the melting solid near $T = 0^2$, and the values of A and B previously quoted, one finds $J(nn)/k_B = -0.70$ mK. Substituting this value of $J(nn)$ into Eq. (6) gives $T_N = 2.24$ mK, compared to 2.0 mK when only two-body nn exchange was considered. The uncertainty quoted² for $J_2^0(P)$ was 3%; however, the uncertainty in $T_N = 2.24$ mK is considerably greater because of the approximations made in arriving at Eq. (5) and the uncertainty in the parameters A and B .

From susceptibility data, T_N may be found using the relation⁵

$$J(\text{nn}) = \frac{\Theta k_B}{4} (1 + \frac{3}{4} A)^{-1} \quad (8)$$

where Θ is the measured Curie-Weiss constant. Several groups have measured Θ ; we use for the melting solid near $T = 0$ the value 3.0 mK.^{1,2} The corresponding value of $J(\text{nn})/k_B$ from Eq. (8) is 0.89 mK and of T_N from Eq. (6) is 2.9 mK. (It will be noted that the previous excellent agreement between $J_2^0(x)$ derived from $P_v(T)$ and measurements has deteriorated when three-body exchange is included. However, the good agreement between $P_v(T)$ and other NMR determinations is preserved.) The best quoted precision for Θ was $\pm 10\%$, but again the resulting T_N is less precise. Given the uncertainties in these calculations, the values for T_N as calculated from $P_v(T)$ and χ data are in reasonable agreement.

Recently Osheroff, Richardson and Lee⁸ have studied the melting pressure P_m of ^3He during Pomeranchuk compression cooling. Because of the non-equilibrium conditions of their experiment, P_m was studied versus time as the compression and decompression took place. A discontinuity in dP_m/dT was observed at 2.65 mK which has been interpreted as indicating a transition in either the liquid or solid phase. If it is shown in further experiments that the transition is in the solid, our calculations offers a possible explanation for its occurrence at a higher temperature than previously expected. In any case, the high-field $P_v(T, H)$ results call into question the previously expected value of $T_N = 2.0$ mK.

References

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