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UCRL - 74530
PREPRINT

CONF-730626--1

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FORMULAS FOR CANISTER AND PIPE DESIGN IN
UNDERGROUND NUCLEAR EMPLACEMENT

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February 1, 1973

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This paper was prepared for presentation at
ASME Meeting, Miami Beach, June 24-28, 1974.

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FORMULAS FOR CANISTER AND PIPE DESIGN IN
UNDERGROUND NUCLEAR EMPLACEMENT*

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ABSTRACT

In this paper, pressure-vessel theory and practice are applied to the analysis and design of long emplacement strings and canister hardware used in a complex environment of underground nuclear tests. Design formulas are given for the collapse pressure of canisters and piping under external soil pressure and for the axial response of gusseted flanges supported by the theoretical and experimental results. The paper is intended primarily as a practical treatment of a complex problem of structural response with the aid of user-oriented, closed form solutions.

1. INTRODUCTION

Experience with underground placement of long pipes and hardware for canisters has prompted some applications of pressure-vessel technology to systems design. The design requirements often range from ample resistance to local buckling of a cylindrical canister, on one hand, to the complete collapse of a pipe by design for containment purposes, on the other.

*Work performed under the auspices of the U.S. Atomic Energy Commission.

A typical structural system consists usually of a large-diameter well casing, grouted to the surrounding soil formations, and a long string of emplacement piping supported at the ground level by a strongback. The total down-hole loads are caused by the system's dead weight and the frictional load of the stemming materials that are deposited for containment between the emplacement pipe and the well casing. These loads are considered to be essentially static.

Although the emplacement system is subjected primarily to dead-weight tensile stresses, the design criteria include considerations of the external, hydrostatic pressure caused by grouting, the head of underground water, and the effects of stemming. The load environment is therefore such that the external hydrostatic pressure controls the sizing of the lower portions of the system while the overall down-hole weight determines the critical dimensions of the upper portion of the string and the supporting hardware.

Under dynamic conditions of a nuclear explosion, the original dead-weight, tensile loading on the system transforms abruptly into axial compression, and considerable radial soil pressures develop. Fortunately, many problems in the emplacement hardware design can be treated as pseudostatic and elastic, so that the existing knowledge of static strength of materials can be used.

In the majority of emplacement situations for underground tests, the pipe string is made up of 40-ft sections with typical outer diameters ranging from 7 to 10.75 in. and with wall thicknesses ranging from 0.23 to 0.60 in. Shorter sections of pipe, however, are used for other components, such as slip-joints, provided to decouple the axial motion of the pipe during the nuclear explosion. Successful operation of a slip-joint system depends on the manufacturing tolerances and the resistance of the cylindrical components to collapse under external pressure due to the ground shock. Other components such as large

canisters, well casing, and underground metal chambers with or without circumferential stiffeners behave as intermediate and short cylinders; some of these may be up to about 10 ft in diameter and may have walls up to 5 in. thick. The designer of emplacement and containment hardware is therefore faced with the problem of selecting a variety of failure modes and working formulas for the analysis.

2. MODES OF STRUCTURAL RESPONSE

It is well known that a short pressure vessel with relatively thick walls fails when the stresses exceed the yield strength of the material. On the other hand, a long, thin-walled vessel fails by instability and buckling at stresses far below the yield point. This is a fundamental statement that applies also to column action. With regard to cylindrical vessels, the essential parameters, influencing the mode of structural behavior under external pressure, include such as $m = r/T$, $k = T/L$ and $\phi = E/S_y$.

The mode of failure and the mathematical expressions suitable for predicting the real pipe behavior can be selected with the aid of the so-called "thinness factor" λ [1,2].* In terms of the parameters defined above, the factor becomes

$$\lambda = 1.19 (m)^{1/4} (k\phi)^{-1/2}. \quad (1)$$

According to the available experimental evidence gathered recently [3], the design formulas based on the theory of elastic stability correlate well with tests when the thinness factor λ exceeds 2.5.

* Numbers in brackets designate References at end of paper.

The modes of structural response and ultimate cylinder collapse are usually initiated by preferential yielding at some region, which in turn leads to unstable geometry and sudden formation of a lobe. In some cases, however, an axisymmetric pattern of deformation develops, which may be described as "waisting" or "nonlobing" response in plastic failure. The experiments indicate [3] that the value of the distinguishing boundary parameter λ between the lobing and nonlobing response may be given as 0.35. It is clear from Eq. (1) that small values of λ can be expected when m is relatively small and the $k\phi$ product is relatively high. These constraints usually imply the features of short, thick cylinders made of low-strength material.

It should be emphasized here that the specific limits of $\lambda = 2.5$, above which elastic stability governs, and $\lambda = 0.35$, below which the stress governs, are intended as a general guide only. The boundaries defined in the above fashion cannot be very sharp, and in everyday terms we often refer to them as "gray areas" requiring a good deal of engineering judgment. Nevertheless, the definition of extreme values of λ is useful in the process of selecting the appropriate design formulas and visualizing the nature of structural response.

In the region where stresses govern the structural integrity of piping, simple hoop-stress formulas can be used for values of m greater than, say, 10. For m values smaller than 10, that is, for heavy-walled pipe with uniform wall thickness, the maximum elastic stress is usually calculated with the aid of Lamé equations [4]. Several industrial empirical formulas are available for calculating the collapse pressures of piping [5]. These are too numerous to bring out in detail.

Our design experience with the emplacement hardware for underground tests to date indicates that the working range of r/T is usually somewhere between

5 and 50. The L/r ratios are seldom less than 1 and seldom more than 20, giving the corresponding limits for k as 0.2 and 0.001, since $k = (1/m)/(L/r)$. For a constant value of ϕ of about 500, corresponding to the typical yield strength of A537 steel, λ values vary in the manner shown in Fig. 1. It is quite apparent from this diagram that a single theoretical formula applicable to all the ranges of λ , k , ϕ , and m would be rather involved.

The parametric chart given in Fig. 1 may be divided into three regions with respect to the major governing criteria. For relatively thick shells for which the parameter λ does not exceed 1.0, a reasonably good prediction of the collapse pressure can be made based on the elementary membrane solution, assuming that the failure takes place when the hoop stress reaches the yield point of the material. It appears that minor initial imperfections in general and out-of-roundness in particular have limited effect in this region, although some additional circumferential bending stress is induced by the presence of ovality or local out-of-circularity.

Since the theoretical predictions based on the hoop stress criteria in this instance are generally conservative, the calculations can be made without due recognition of small initial imperfections. The reader should be cautioned, however, that gross initial imperfections can be detrimental by precipitating the collapse in all regions of cylinder response to external pressure. Unfortunately, the precise definitions of "small" and "gross" imperfections are not generally available.

The intermediate region defined by $2.50 > \lambda > 1.00$ represents a complicated picture of cylinder behavior under external pressure where some combination of the stress and stability considerations must be involved. Usually some local yielding takes place, before the high elastic instability load is reached,

precipitating the onset of ultimate collapse. Because of the simultaneous involvement of the stress and stability criteria, the stresses arising due to imperfections are important. This fact is supported by experimental evidence up to now, although further work in this area is still required.

The material parameter ϕ , representing numerically the inverse of the elastic strain, normally varies between about 100 and 1000 for the great majority of metallic engineering materials. In the actual case of the emplacement hardware, involving the more common range of yield strengths such as, for instance, 60,000 to 150,000 psi, the corresponding ϕ values are 500 and 200, respectively. In this range of values, we may find such steels as A537, HY80, and 4330, which can be specifically recommended for their suitable fracture toughness.

3. COLLAPSE STRENGTH FORMULAS

The design of emplacement and containment hardware is often complicated by the availability of too many design formulas with their restricted ranges of applicability. This situation is not surprising when one considers various possible modes of pipe failure in the elastic or the elastoplastic range, geometrical features, and manufacturing tolerances.

Gradual development of the stress-analysis criteria for statically and pseudostatically loaded deep-hole casing systems [6] and the subsequent experimental studies [7] form the basis for analyzing the collapse strength of canisters, casing, metal rooms, and large piping. Extensive full-scale and model tests provide background material and stress data on the behavior of ring-stiffened cylindrical shells [8,9]. The customary pressure-vessel theory, as outlined in the ASME code, is seldom used, however, because of the specific requirements of one-time, short-duration service and overall weight limitations in underground

tests. Transportation, emplacement techniques, and test conditions tend to indicate the stresses and strains of relatively short duration. Consequently, the design philosophy, engineering practice, and theoretical approaches to the problem of cylinder collapse are under continuous review, with reference to some of the readily available information from this country and elsewhere [3,10,11]. Based on these studies and the immediate requirements for streamlining the design calculation procedures, the following collapse strength formula is applicable:

$$P_u = S_y F(k) F(m) F(\alpha) \quad (2)$$

Equation (2) conforms to the intermediate response region illustrated in Fig. 1.

Here, $F(m)$ depends on r/T ratio, $F(k)$ is a geometrical parameter containing the effect of out-of-roundness and $F(\alpha)$ is a stress-correction parameter involving the effect of superimposing the tensile stresses on the collapse strength of a cylinder carrying external pressure. The design formula expressed by Eq. (2) reduces essentially to a short-cylinder formula when the effects of out-of-roundness and axial tension are neglected. The expressions for the various parameters used in emplacement-hardware calculations are

$$F(k) = K_1 - (K_1^2 - K_2^2)^{1/2} \quad (2a)$$

$$K_1 = (m)^{1/2} + 0.87k\phi(1 + 6n) \quad (2b)$$

$$K_2 = 3.48k\phi(m)^{1/2} \quad (2c)$$

$$F(m) = \frac{\sqrt{2}}{(2m+1)^{3/2}} \quad (2d)$$

$$F(\alpha) = (1 - 0.91\alpha^2)^{1/2} - 0.3\alpha. \quad (2e)$$

The range of applicability of Eq. (2) is determined by the following criteria:

$$(L/r)_{\min} = 0.63(m)^{-1/2} \left(\frac{1.44\phi}{m} + 1 \right) \quad (3)$$

and

$$(L/r)_{\max} = 4.9(m)^{1/2}. \quad (4)$$

When the L/r value, given by the particular design constraints, is found to fall between the values computed from Eqs. (3) and (4), Eq. (2) is deemed to apply. The designer should be cautioned, however, that, under certain circumstances, Eqs. (3) and (4) may indicate that the range of applicability of Eq. (2) is very limited. This is not surprising when one considers the nature of functions and the number of parameters involved. The parameter β , obtained by dividing Eqs. (3) and (4), is illustrated in Fig. 2. It is clear from the diagram that the range of applicability of Eq. (2) decreases substantially with the decrease in r/T ratio and the yield strength of the material. Fortunately, in the field of emplacement hardware, most design cases involve r/T ratios higher than about 20 and ϕ values less than 500.

The correction for out-of-roundness included in Eq. (2) was derived on the basis of initial ellipticity [12].

The out-of-roundness parameter n can be defined in terms of extreme diametral measurements and wall thickness:

$$n = \frac{D_{\max} - D_{\min}}{4t} \quad (5)$$

It should be mentioned here that Eq. (5) applies only to the case where radial deviation can be related to the even number of circumferential lobes of the out-of-roundness pattern. For a conservative estimate based on Eqs. (2) and (5), a purely elliptical mode of cylinder collapse is used in the design of underground test hardware. Unfortunately, the out-of-roundness parameter n is usually unknown before manufacturing. Nevertheless, as the first rational step in design, the extent of the anticipated maximum out-of-roundness can be based on the customary manufacturing tolerances. For instance, when the tolerance on the diameter of a particular cylindrical canister is given, say, as ± 0.05 in., the corresponding value of n can be taken as $n = 0.025/T$.

The effect of out-of-roundness on the collapse strength of vessels under external pressure has been duly recognized by the ASME code [13]. To eliminate any potential gross out-of-roundness, the ASME code recommends that the ratio $e/2r$ not exceed 1% of the nominal diameter. This conservative rule is sometimes used as the upper allowable limit in establishing the design criteria for deep-hole casing systems [6]. Manufacturing and field experience indicates, however, that the 1% rule may often grossly overestimate the extent of radial deviation found by the measurements.

Since the design of large-diameter casings and emplacement piping often presents a unique problem of subjecting the shell simultaneously to lateral

external pressure and axial tension, Eq. (2) includes a simplified correction factor $F_{(\alpha)}$. This combination of loading is known to have adversely affected the collapse strength of cylindrical components under external pressure. The problem has long been recognized by the oil drilling industry [14], which, over the years, expended considerable research effort in developing the required engineering data. The range of parameters covered there included ϕ between 375 and 1000, and the values of m smaller than 11. The ratios of length to diameter were relatively large, indicating that the strength data developed by the oil industry were generally compatible with the theory of relatively long cylinders. The most common range of m values encountered in the design of emplacement hardware for underground nuclear tests was found to be usually between 10 and 40.

The combined effect of stability and stress has been studied previously, leading to inelastic buckling criteria and the concept of a reduced modulus [15,16]. The approximate correction factor $F_{(\alpha)}$ given in Eq. (2) is based on the principle of the maximum strain energy in a biaxial state of stress and a Poisson's ratio of 0.3.

It was found by design experience that, in the majority of cases with underground test hardware, the stress ratio α was relatively small, so that the correction factor $F_{(\alpha)}$ did not significantly differ from unity. In the lower range of m values, applicable to emplacement piping, the extent of reduction in collapse pressure due to axial tension, predicted on the basis of $F_{(\alpha)}$ was found to be in good agreement with the data developed by the oil drilling industry. However, no test data appear to be readily available for higher values of m and smaller length-to-diameter ratios of cylindrical components for correlation of theory and experiment [17].

The effect of parameter n on the collapse pressure can be expected to be more significant in relatively long cylinders. However, since the exact mode of failure depends on at least three important variables such as k , n , and ϕ , no sweeping generalizations should be made. The experimental work, concerning the effect of the parameter n , is still limited, despite the fact that the general concept of out-of-roundness has been known for many years. The main research difficulty here is that the tests have to be performed under uniform control conditions in such areas as the test apparatus, machining tolerances, heat treatment, and measuring techniques of initial imperfections in all experiments. If undertaken, such an experimental program should cover a wide range of the parameter λ , given by Eq. (1).

4. DESIGN OF RING STIFFENERS

The development of the minimum weight vessels, casing, piping, and canisters subjected to external pressure is related directly to the analysis and design of ring stiffeners. The process of weight optimization, however, does not automatically assure the minimum-cost design because of constant changes in material cost, labor practices, and manufacturing techniques.

When the overall size of the vessel and the working pressure become greater, the required thickness of the material for a vessel with no ring stiffeners constitutes a practical limitation, not only because of the weight of the system but also because of the limited capacity of the metal-forming equipment and because of the economic considerations.

Wall thicknesses can be reduced substantially for uses under external pressures, when the proper size of stiffener is selected. The width and depth of the stiffening ring are important, and it is fortunate that the manufacturing

cost of the rings is not too sensitive to the ring size. On that basis, a slightly larger ring can be chosen for the particular application with very little additional cost.

It should be recalled here that a ring-stiffened cylinder subjected to external pressure can fail according to the following two general modes:

- a) panel buckling, with the rings remaining essentially circular, and
- b) overall collapse of the shell and rings.

It is clear, therefore, that in most circumstances designing for panel buckling rather than overall collapse may be important, so oversize stiffeners would be recommended.

Theoretical work on the strength and stability of ring-supported shells under external pressure has been rather extensive [3,18,19]. However, uncertainties due to the boundary conditions, the effect of out-of-roundness, and the residual stress patterns, with particular reference to the overall collapse, decrease the need for extreme accuracy of the calculations.

The proposed ring-sizing formula has been used mainly for the externally stiffened casing, piping, and canisters. It is emphasized, however, that, according to tests, externally stiffened cylinders appear to be somewhat stronger than internally stiffened cylinders. This effect is probably due to the favorable orientation of the axisymmetric distortion in welding, and is likely to get less important as cylinder size increases.

One of the basic questions to be answered in sizing the stiffeners is what portion of the cylindrical wall should be considered as contributing to the second moment of area for the stiffener [20]. The effective length of the shell is often assumed to be equal to $b + 1.57(\pi r)^{1/2}$ multiplied by a design factor

greater than 1. This factor is usually developed through testing experience for a specific task. The design of ring stiffeners for the emplacement hardware was accomplished with the aid of the following nondimensional relation:

$$\frac{PL}{Et} = \frac{0.08\gamma^2(c+1)^4 + 0.13\gamma(m)^{1/2}(c+1)(4c^2 + 2c + 1)}{(0.33m^3 + m^2 + m) [\gamma(c+1) + 1.57(m)^{1/2}]} \quad (6)$$

5. THEORETICAL AND EMPIRICAL ASPECTS OF DESIGN

The theoretical work, directed specifically towards predicting collapse pressures for ring-stiffened cylindrical shells, has evolved pretty much along the three basic directions:

- a) elastic buckling,
- b) yield stress criterion, and
- c) elasto-plastic criterion.

The predictions based on the elastic behavior (a) are usually considered as upper-bound estimates provided a suitable buckling formula can be applied. A good example of this solution is the well-known short-cylinder formula used in predicting the buckling strength midway between the stiffeners. The corresponding mathematical procedure here is to minimize the buckling pressure with respect to the number of lobes, and a simplified version of this formula is sometimes stated as follows:

$$P_c = \frac{SEk}{2(2m+1)1.5} \quad (7)$$

It should be noted that Eq. (7) applied to vessels, the materials of which have Poisson's ratio equal to about 0.3. The theory does not include the effect of the stiffener area on the strength of shell, and it is assumed that the initial out-of-roundness of the shell is zero.

The predictions based on the yield stress criterion (b) follow from the assumption that the shell collapses when the yield stress is reached at the outer surface of the wall midway between stiffeners in the circumferential direction. Local yielding at the stiffeners due to the high longitudinal discontinuity stresses is not expected to precipitate the collapse. The circumferential yield at midbay, however, although quite important, does not provide all the answers. For certain geometries, for instance, the elastic buckling stress can cause the onset of instability before the circumferential yield stress is reached.

In discussing various yield considerations, it can be generally stated that different theoretical formulas used in design predictions fit some definite ranges of the test results available, but none of them appear to account for the differences in the effects between the external and internal stiffeners welded to the shell. The evidence based on the experiments in this area indicates that collapse pressures are about 10% higher when external stiffeners are used in a welded structure.

When the design is such that the stiffeners are close together, the collapse pressure is usually assumed to be independent of the cylinder length, and simple hoop stress is considered to be responsible for the midbay collapse. Then the corresponding design formula is

$$P_y = 0.955 \sigma_y / m. \quad (8)$$

The above formula is based on the original theoretical work of von Sanden-Günter [1], which indicates that the circumferential stress midway between the stiffeners is about 5% greater than the corresponding simple membrane stress. Actually, the difference has been reported as high as 10% with stiffeners of infinite rigidity, but this latter value has never been verified by experience.

The third area of the theoretical endeavor concerns the so-called elasto-plastic or rigid-plastic analysis (c). The rigid-plastic criteria appear to be rather attractive, if we can ignore the initial imperfections, strain-hardening, and stability considerations.

Because of rather substantial theoretical difficulties in the treatment of ring-stiffened cylinders subjected to external pressure, considerable effort has been expended by the pressure-vessel specialists in this country and abroad in gathering well-documented experimental data [3]. A nondimensional plot of the results, representing a lower-bound curve of midbay collapse pressures in cylindrical, ring-stiffened vessels is given in Fig. 3. The plot includes both internally and externally stiffened cylinders, where the parameter $1.05m P_e/S_y$ is shown as a function of $0.92k\phi/m^{1/2}$, in line with the definitions and notations used in this paper. P denotes experimentally determined interstiffener collapse pressures for all published data having m values between 6 and 250, and L/r ratios in the range of 0.04 to 50. In the majority of the test cylinders, the out-of-roundness was much less than 1% of the radius. In some cases, however, the values of out-of-roundness were found to be in the region of 1%, representing relatively poor manufacturing practice. Because of this series of relatively low experimental collapse pressures, a lower-bound curve was selected here for the illustration.

It should be recognized that, although the lower-bound curve represents a wide range of geometrical proportions, the correlation must break down for cylinders with oversized and very closely spaced stiffeners. Naturally, the critical buckling pressures for such structures would become much higher than those that the simple hoop-stress criterion at midbay would tend to indicate.

The plot in Fig. 3 contains the elements of the maximum hoop stress at midpoint between the stiffeners indicated on the vertical axis. It represents the ratio of the critical pressure sought and that expressed by Eq. (8). The parameter on the abscissa is obtained by dividing Eq. (7) by Eq. (8), and by simplifying the expression in the usual range of the geometries involved where the nondimensional ratio $m \gg 1$. For instance, in practical design applications related to the emplacement hardware, the numerical value of ratio m is seldom found lower than 20. Under such circumstances, the error introduced by the above simplification should not exceed 3%.

The advantage of referring to an empirical curve, such as that shown in Fig. 3, is that the results cover a relatively wide range of L/r ratios without drawing any particular distinction between the short, intermediate, or long cylinders. The disadvantages of the empirical curve discussed are 1) lack of the specific details of the out-of-roundness parameter in the chart and 2) the corresponding practical necessity to rely on the lower-bound curve in conservative predictions of the ultimate collapse pressure.

For the empirical data to be of use in designing emplacement hardware, the following approximation was used:

$$P_{CR} = S_y Z_1 (Z_2 - n Z_3) F(\alpha) \quad (9)$$

where

$$Z_1 = e^{\frac{-0.815 m^{1/2}}{k\phi}} \quad (9a)$$

$$Z_2 = \frac{1}{m^{0.95} (k\phi)^{0.10}} \quad (9b)$$

$$Z_3 = \frac{50}{m^{1.95} (k\phi)^{0.10}} - \frac{33}{m^2} \quad (9c)$$

It should be noted that variables such as S_y , k , m , n , α , and ϕ are involved, making a comparison between the various formulas relatively straightforward. The main reason for developing a formula of the type given by Eq. (9) is to have one model applicable to short, intermediate, and long cylinders, eliminating thereby the problems associated with the transitions and grey areas. The formula was originally intended for the following ranges of the parameters:

$$0 \leq n \leq m/100$$

$$10 \leq m \leq 100$$

$$0.001 \leq k \leq 0.200$$

$$100 \leq \phi \leq 1000$$

$$0 \leq \alpha \leq 1.0$$

In order to make a brief comparison between some of the theoretical and published experimental results, a number of geometries and mechanical properties were selected at random from the design records of the emplacement hardware. The examples involve some typical as well as some extreme configurations, for a general comparison. The detailed data and the results

of the correlation for these examples are given in Table 1. In all computations, $\alpha = 1$ was assumed. The last three columns of the table indicate a reasonable agreement between the various estimates, despite the variations in cylinder proportions. The discrepancies appear to be nearly always on the conservative side.

The formula given by Eq. (9) is, at present, undergoing further evaluation, and it is hoped that using it in a practical design will provide additional background for improving the proposed model.

In the region of relatively low values of k and m ratios, the minimum collapse pressure of casing, tubing, or drill pipe is usually estimated on the basis of specifications recommended by the American Petroleum Institute [21]. The formulas suggested by the institute include experimental corrections and apply to yield strengths between 40,000 and 150,000 psi. They are basically of the Lamé [4] and Stewart [1] type. Some of the design considerations include applying these formulas to the development of emplacement hardware for underground tests, where long, thick pipe is required. Correlation of the measured and calculated collapse pressures indicates a definite trend of improvement with decreasing m ratio [22]. The scatter in the area of thick-walled pipe appears to be mainly due to the variation in material properties and initial imperfections. It is less severe for thick-walled tubes, and many test results for thick-walled piping indicate the collapse pressures that exceed the calculated values. While the thick-walled tubes indicate the need for perhaps less restrictive criteria, the results on thin-walled tubes may still require somewhat larger factors of safety such as for instance, those found in the ASME code.

6. DESIGN OF SPECIAL FLANGES

This section is concerned with the design formulas for practical applications of established principles of stress analysis to special flanges with rib stiffeners (Fig. 4). Such flanges are used in line-of-sight experiments and similar engineering applications where relatively high axial pipe loading and low radial pressures are predominant. Insofar as we can determine, there are no established design methods for rib-stiffened flanges.

A review of some of the basic flange formulas and a study of various geometric parameters [11,23-27] established the three basic design criteria for sizing rib-stiffened flanges. These include the maximum hub stress, strength and rigidity of ribs, and local bending stress of the main flange ring. Whenever possible, the compound flange is designed without a backup ring such as that shown in Fig. 4, since the longitudinal stiffening ribs provide sufficient strength and rigidity in the system.

For a simple flange configuration such as that shown in Fig. 5, the maximum hub bending stress was calculated from the following expression:

$$S_b = \frac{W}{T^2} F(k_f, m, n_f) \quad (10a)$$

where

$$F(k_f, m, n_f) = \frac{0.8(k_f + 1)(k_f^2 - 1)(m)^{1/2}}{(1 + 2k_f^2)[2.57(m)^{1/2} + 1.65n_f] + 1.3n_f^3(k_f^2 - 1)} \quad (10b)$$

When Eq. (10a) is applied to a rib-stiffened flange, the dimension h may be replaced by H_o , which for a flange without backup ring is

$$H_0 = \left[\frac{W^2 T (HT + 4h B_T) - 3HT h^2 B_T (2H - h)}{(B_T + T) (HT + h B_T)} \right]^{1/3} \quad (10c)$$

The formula given by Eq. (10a) was used by the author in analyzing compressor casing flanges in jet engines at De Havilland Engine Company in England some 20 yr ago. It is similar to the well-known Waters-Taylor formula [28]. The direct tensile stress in the string should be added to the value obtained from Eq. (10a) to obtain the total hub stress at the point considered. In the case of a rib-stiffened flange, this stress is assumed to exist at the end of the rib. This, no doubt, can be only a rough estimate for the preliminary design [27].

The relative rigidity of the ribs can be estimated on the basis of the theory for a beam on elastic foundation and toroidal deformation of a circular elastic ring. The combined bending and tensile strength of the ribs can then be approximated with the aid of the following equations, which constitute the second design criterion observed in sizing the compound flange. Since this method yields basically the elastic stresses, further correction of the results may be required. The relevant notation is given in Fig. 6.

$$S_{TR} = \frac{6\eta W(R - r)}{(1 + \eta)NT_T(B_T + T)^2} + \frac{W}{NB_T T_T + 2\pi rT} \quad (11)$$

where

$$\eta = \frac{0.16NR_T B_T^3}{(B_T + T)h^3 \psi(\theta, L)} \quad (11a)$$

$$\psi(\theta, L) = \frac{\cosh \theta L \sinh \theta L - \cos \theta L \sin \theta L}{2\theta(\cosh^2 \theta L - \cos^2 \theta L)} \quad (11b)$$

$$\theta = 0.69 \left(\frac{N}{r} \right) \left(\frac{I_m}{I_n} \right)^{1/4} \quad (11c)$$

$$I_m = T^3/12 \quad (11d)$$

$$I_n = T R^3/12. \quad (11e)$$

The analysis of load sharing between the stiffening ribs and the main flange ring indicates that, in the great majority of design situations involving rib-stiffened flanges of the usual proportions, only limited toroidal moment is transferred by the main flange ring [27]. Hence, with only little error, the ribs can be designed on the basis of the total external flange moment.

The third design criterion includes check of the order of magnitude of the local stresses likely to exist in the main flange ring itself under the action of individual bolt loads. Because of symmetry, only one portion of the flange, supported by the two closest ribs and the corresponding portion of the pipe wall, needs to be examined. Utilizing a rectangular plate model with one long edge fixed, one long edge free, and both short edges simply supported, the following bending stress formulas have been derived [27]:

$$S_b = \frac{66.2 V W B^3 N^3 t}{h^2 (2\pi R - \pi T)^4} \quad (12)$$

The bending moment factor V is based on Timoshenko's analysis [29] and may be obtained from Fig. 7, where

$$a_0 = \frac{2\pi R}{N} - T_r . \quad (12a)$$

When the outer flange diameter is large compared with the pipe diameter and $B_r/a_0 > 1$, the following modified equation may be employed in the design:

$$S_b = 0.71 \frac{WB_r^2}{th^2(2\pi R - NT_r)} . \quad (12b)$$

Recently, analytical and experimental studies have been conducted [30] aimed at correlating some of the stresses calculated using the formulas Eqs. (10) through (12) with those obtained by the finite-element and birefringent-coating techniques. The finite-element model, having the advantage of representing the proper geometric relationship of a rib-stiffened flange, predicted local stresses with better accuracy. Both the finite-element model and the above equations, however, agreed reasonably well with the experimental data.

8. CONCLUDING REMARKS

The formulas presented in this paper have been used during the past several years in the analysis and design of the containment and emplacement hardware on many important events involving underground nuclear tests. As far as can be established on the basis of past records covering structural performance, safety, and test results, the designs performed successfully.

The process of improving the mathematical models discussed will be continued at this Laboratory, and the presented account may be considered as the first progress report on the subject matter. Specifically, further effort will be directed towards the verification and simplification of formulas (6), (9), (10),

(11), and (12). The experience indicates so far that the design criteria, represented by the above formulas, are of major influence in developing the minimum-weight systems for underground applications, particularly at greater overburden loads.

9. ACKNOWLEDGMENTS

This work was performed under the auspices of the U.S. Atomic Energy Commission.

The author is indebted to Mr. V. N. Karpenko of the Nuclear Test Engineering Division for his continuous interest and support of the studies that led to the preparation of this paper.

NOMENCLATURE

- a_o = Mean length of flange sector, in.
- b = Width of ring stiffener, in.
- B_r = Depth of flange stiffener, in.
- D_{max} = Maximum external diameter, in.
- D_{min} = Minimum external diameter, in.
- e = Deviation from perfect radius, in.
- E = Modulus of elasticity, psi
- $F(k)$ = Out-of-roundness correction parameter
- $F(k_F, m, n_F)$ = Hub stress factor
- $F(m)$ = Dimensional parameter
- $F(\alpha)$ = Tensile stress correction parameter
- h = Thickness of main flange ring, in.
- h_r = Depth of ring stiffener, in.

H = Total depth of stiffened flange, in.

H_e = Equivalent depth of stiffened flange, in.

I_m = Moment of inertia for a wall element of unit width,
in.⁴/in.

I_n = Moment of inertia of a rib cross-section, in.⁴

$k = T/L$ = Ratio of thickness to length

$k_f = R_o/R_i$ = Flange-ring ratio

K_1, K_2 = Short cylinder parameters

L = Length between ring stiffeners or cylinder heads, in.

L_s = Length of flange stiffener, in.

$m = r/T$ = Ratio of radius to thickness

$n = e/T$ = Deviation to thickness ratio

n_f = Ratio of flange to pipe thickness

N = Number of flange stiffeners

P = External design pressure, psi

P_c = Classical collapse pressure for short cylinder, psi

P_{CR} = Proposed collapse pressure criterion based on theory
and experiment, psi

P_e = Experimental collapse pressure for cylindrical vessels,
psi

P_u = Theoretical collapse pressure for short cylinders corrected
for out-of-roundness and effect of tension, psi

P_y = Pressure at yield, psi

r = Mean radius of pipe wall, in.

R = Bolt circle radius, in.

R_i = Inner radius of pipe, in.

R_o = Outer radius of flange, in.

S_b = Bending stress, psi

S_t = Tensile stress, psi

S_{TR} = Total elastic stress in flange stiffener, psi

S_y = Yield strength of material, psi

t = Distance from pipe wall to bolt circle, in.

T = Thickness of pipe wall, in.

T_r = Thickness of flange stiffener, in.

V = Moment factor in plate analysis

W = Total flange load, lb

Z_1, Z_2, Z_3 = Empirical factors

$\gamma = \frac{b}{T}$ = Ratio of ring width to thickness of shell

$\alpha = S_t/S_y$ = Tensile-stress ratio

$\epsilon = \frac{h_r}{T}$ = Ratio of ring depth to thickness of shell

θ = Elastic foundation parameter, in.⁻¹

λ = Thinness factor

$\phi = E/S_y$ = Inverse strain parameter

$\psi(\theta, L)$ = Auxiliary function for a beam on elastic foundation

η = Load-sharing ratio

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Table 1

Examples of collapsed-pressure calculations for emplacement of typical hardware.

Reference	r, in.	T, in.	L, in.	e, in.	S _y , ksi	E, psi	P _u , psi ^a	P _{CR} , psi ^a	P _e , psi ^b
Experimental canister	35.88	1.125	82.5	0.0625	80	30×10 ⁶	1260	1000	1000
Experimental canister	35.75	0.500	72	0.0625	36	30×10 ⁶	180	152	160
Dummy line-of- sight pipe	8.81	0.375	96	0.0450	35	30×10 ⁶	500	430	470
Well casing	24.38	0.750	96	0.1220	36	30×10 ⁶	410	490	530
Well casing	27.00	0.625	28	0.0400	100	30×10 ⁶	1190	1010	1060
Experimental canister	40.50	1.500	27	0.1000	36	30×10 ⁶	340	510	540
Experimental canister	27.75	0.750	48	0.1425	36	30×10 ⁶	600	570	690
Experimental canister	3.1	0.4	42	0.0050	152	26×10 ⁶	8550	4980	5230
Concrete room	91	27	240	0.4550	12	4×10 ⁶	2920	2460	3120

^aSee Eq. (2).^bSee Fig. 3.

FIGURE LIST

Fig. 1. Thickness factor for pipe and cylindrical canisters under external pressure. $\phi = 500$ for all the curves.

Fig. 2. Parametric chart for intermediate-length cylindrical vessels.

Fig. 3. Lower-bound empirical curve for ring-stiffened cylinders under external pressure.

Fig. 4. Pipe-flange geometry.

Fig. 5. Proposed simplified notation for analysis of straight flanges.

Fig. 6. Notation for a rib-stiffened flange without a backup ring.

Fig. 7. Bending-moment factor for flange ring stresses.

Blake - Fig. 1

Blake - Fig. 2

Blake - Fig. 3

Blake - Fig. 4

Blake - Fig. 5

Blake - Fig. 6

Blake - Fig. 7