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**ESTIMATION OF COUNTING RATES FROM PERIODIC
OBSERVATIONS OF A SMALL SET
OF BINARY SCALERS**

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ABSTRACT

The counts received by a radiation detector located on a space satellite were stored in a small set of binary scalers. The number of counts stored in these scalers was read out once per second for eventual transmission to earth via telemetry. This information was then used to determine counting rates as a function of location in space.

The number of scalers used was too small to contain all the counts which could be received during the time interval between successive samplings of the scaler contents. This posed the problem of estimating the probability that the scalers had been filled, had automatically reset to zero, and had begun storing counts again during the interval between samplings.

Because of telemetry limitations it was not possible to sample the condition of all the binary scalers. Only the information stored in the last few scalers is known, and hence there is the problem of estimating how many counts are stored in the preceding scalers.

A technique for analysis of the data is presented, together with flow charts for a FORTRAN code using this technique.

INTRODUCTION

In many experiments measuring radiation flux in space, the counts received by a radiation detector are stored in a set of binary scalers located on a space satellite. A voltage pulse whose magnitude is proportional to the number of counts stored in the scalers is sampled once per second. This

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signal is stored on magnetic tape and later relayed to earth. Its magnitude is usually restricted to a few discrete values, say four. Thus zero volts would correspond to zero counts, one volt to one count, etc., up to a maximum of three counts. Because of this restriction on the magnitude of the output pulse, the total number of counts stored in the complete set of scalars cannot be read out; instead only the counts stored in a segment of the scalar chain is known. A typical case is illustrated in Fig. 1. In this paper the scalars preceding those read out will be called the prescalars. In general the scalars are not cleared after each sampling; however, whenever the total counts received is just equal to the storage capacity of the total chain of scalars, the scalars automatically reset to zero. The total number of binary scalars in the chain must be limited, and an analysis is needed to select the optimum configuration.

The problem is to determine the actual counting rate by observing how the number of counts stored in the scalars changes with time, taking into account two complications. The first complication is that we cannot tell how many counts entered the prescalars during the time interval between readouts. The second complication arises because we must limit the number of prescalars, and hence the storage capacity of the complete set of scalars. Therefore it is possible that the counts received during the time interval between readouts overflowed the scalars and started a new cycle (beginning with zero counts).

THEORY

Let us now seek a solution to the following problem. Counts are fed into a system of $n + m$ binary scalars; of these, the first n form a prescaling unit which we cannot see, and the remaining m scalars are sampled once per second. Given the number of counts stored in the m scalars at many successive samplings, what is the average counting rate?

We define

$$A = 2^n,$$

$$B = 2^m.$$

Suppose we have made a series of samplings of the counts stored in the last scalars. Let N_0 be the number of times that a difference of 0 counts is

Fig. 1. (a) Typical scaler configuration. (b) Probability of the observed counts as a function of counting rate.

observed between successive samplings, and in general let N_i be the number of times a difference of i counts is observed between successive samplings. The maximum value i can have is $B - 1$, the minimum value is 0.

We seek the probability of observing the set of differences $N_0, \dots, N_{B-1}$, assuming an average counting rate R .

If J counts are observed in the m scalers, the actual number of counts stored in the $n + m$ scalers is at least AJ and at most $AJ + (A - 1)$. The term $A - 1$ arises from the unseen counts stored in the n prescalers. All these cases result in the same observation. Hence when we see J counts in the last m scalers we actually have $AJ + \ell$ counts stored in the $n + m$ scalers, with ℓ some integer between 0 and $A - 1$.

Suppose J_1 and J_2 counts are observed in two successive samplings. Then initially we have $AJ_1 + \ell_1$ counts stored, finally we have $AJ_2 + \ell_2$ counts stored. Aside from possible overflow and recycling, the difference in stored counts equals the counts received during the time interval between samplings. Hence the number of counts received during this time interval is

$$A(J_2 - J_1) + (\ell_2 - \ell_1) + ABK \quad (1)$$

where K is any integer and ℓ_2 and ℓ_1 are any integers between 0 and $A - 1$. We observe only J_2 and J_1 ; K , ℓ_1 , ℓ_2 are not known to us.

Now we define

$$J = J_2 - J_1,$$

$$\ell = \ell_2 - \ell_1.$$

Our expression for the counts received is

$$AJ + \ell + ABK \quad (2)$$

where ℓ is an integer.

Table 1 illustrates what happens when ℓ_1 counts are initially stored in the prescalers. As additional counts are received, the prescalers fill up and start over again from zero. One sees that $\ell = \ell_2 - \ell_1$ runs from a minimum of $-\ell_1$ to a maximum of $A - 1 - \ell_1$. Now ℓ_1 can have any value from 0 to $A - 1$. For this range of values of ℓ_1 , ℓ will range from a minimum of $-(A - 1)$ to a maximum of $A - 1$. The probability of any given value of ℓ_1 depends on the past history of the scalers and the counting rate. For simplicity each value of ℓ_1 will be assumed to be equally probable; comparison

of theory and experiment will show if this approximation is reasonable. Note that each value of ℓ arises $A - |\ell|$ ways. As an example, $\ell = A - 2$ can arise in two ways, namely $\ell_1 = 0, \ell_2 = A - 2$, or $\ell_1 = 1, \ell_2 = A - 1$.

Table 1

Counts received	$\ell_1 + \text{counts received}$	ℓ_2 (counts in prescalers)	ℓ
0	ℓ_1	ℓ_1	0
1	$\ell_1 + 1$	$\ell_1 + 1$	1
2	$\ell_1 + 2$	$\ell_1 + 2$	2
3	$\ell_1 + 3$	$\ell_1 + 3$	3
.	.	.	.
.	.	.	.
.	.	.	.
$A - 1 - \ell_1$	$A - 1$	$A - 1$	$A - 1 - \ell_1$
$A - \ell_1$	A	0	$-\ell_1$
$A + 1 - \ell_1$	$A + 1$	1	$1 - \ell_1$
.	.	.	.
.	.	.	.
.	.	.	.
A	$A + \ell_1$	ℓ_1	0

We can now write an expression giving the probability $P_{AB}(J, R)$ of observing a positive difference J in the m scalers sensed, assuming an actual average counting rate of R counts per second:

$$P_{AB}(J, R) = \sum_{k=0}^{\infty} \sum_{\ell=-A+1}^{A-1} \frac{(A - |\ell|)}{A} P(AJ + \ell + ABK, R) \quad (3)$$

where $P(n, R)$, which is the Poisson probability function giving the probability of observing n counts when R is the average number of counts observed, is given by

$$P(n, R) = \frac{R^n e^{-R}}{n!} \quad \text{if } n > 0,$$

$$P(n, R) = 0 \quad \text{if } n < 0.$$

In the limiting case in which all counts can be observed (no prescalers) we expect to get the usual Poisson function, and this is easily verified to be the case:

$$\lim_{B \rightarrow \infty} P_{1B}(J, R) = \frac{e^{-R} R^J}{J!}.$$

Recall that N_J is the number of times a difference $J = J_2 - J_1$ is observed. The probability of observing the set $N_0, N_1, \dots, N_{B-1}$ when the average number of counts per second is R may be written as

$$P(R) = \frac{N!}{N_0! N_1! \dots N_{B-1}!} [P_{AB}(0, R)]^{N_0} [P_{AB}(1, R)]^{N_1} \dots [P_{AB}(B-1, R)]^{N_{B-1}} \quad (4)$$

where

$$N = \sum_{J=0}^{B-1} N_J.$$

For very small counting rates R , we expect a difference of 0 to be observed most often:

$$\lim_{R \rightarrow 0} P_{AB}(0, R) = 1,$$

$$\lim_{R \rightarrow 0} P_{AB}(J, R) = 0, \quad J \neq 0.$$

For very large counting rates R we expect all differences J to be observed with equal frequency:

$$\lim_{R \rightarrow \infty} P_{AB}(J, R) = \frac{1}{B}.$$

For any value of R we must have

$$\sum_{J=0}^{B-1} P_{AB}(J, R) = 1.$$

Hence we expect $P(R)$ to be very small for small R (assuming at least one of the N_J for $J \neq 0$ is nonzero) and to approach a larger constant for large R . For intermediate values of R , $P(R)$ will oscillate and have many minima and maxima as seen in Fig. 1(b). The existence of many maxima in $P(R)$ makes sense physically as corresponding to recycling of the scalers.

DATA ANALYSIS

The data available to us consists of the number of counts stored in a few binary scalers at successive instants of time 1 second apart. The first step in the analysis of this data is to find the number of times a difference of J counts is observed during some time interval over which we wish to average. We then have a set of numbers $N_0, N_1, N_2, \dots, N_{B-1}$; N_J is the number of times a difference of J counts is observed. But the theoretical frequency with which a difference of J counts should be observed is $P_{AB}(J, R)$, assuming an average counting rate R . Hence the expected number of times a difference of J counts is observed is N times this frequency. We can measure how well the observed $N_0, N_1, \dots, N_{B-1}$ agree with the theoretical value by forming the usual chi-squared function:

$$\chi^2(R) = \sum_{J=0}^{B-1} \left[\frac{(N_J - NP_{AB}(J, R))^2}{NP_{AB}(J, R)} \right] \quad (5)$$

χ^2 is a function of the assumed average counting rate R . When χ^2 is sufficiently small, then observation and theory agree and the assumed counting rate R is a possible value of the true counting rate. Tables of χ^2 exist and enable us to choose this figure of merit such that we can have any desired probability of being wrong when we say a given rate R is a possible rate; the smaller χ^2 must be to be acceptable, the smaller the probability that we are wrong in accepting some value of R as being possible. However, we cannot make the maximum value of the accepted χ^2 too small or we may make an error of another kind and reject a rate R when it actually is the correct one. An analysis of this kind frequently yields several possible values of the prob-

able counting rate — this simply means that the data available are not sufficient to yield a single value for the probable counting rate. When such multiple possible counting rates are found, they usually differ approximately by kAB , corresponding to recycling the scalers.

The analysis can be set up in such a way as to guard against the possibility that the scalers were flooded with so many counts that a determination of the counting rate is not possible. The way this is done is to require that at very high assumed rates χ^2 must be large enough so we can say with reasonable probability that this was not the case. Again we must choose a compromise value of χ^2 . If we set the required value too low we will lose cases that could have been correctly analyzed; if we set the required value too high we will analyze cases which are beyond the capacity of the scalers.

Corresponding to an observed set $N_0, N_1, \dots, N_{B-1}$ there is a minimum possible counting rate R_0 given by

$$R_0 = \frac{\sum_{J=0}^{B-1} AJ N_J}{\sum_{J=0}^{B-1} N_J} \quad (6)$$

This expression is easily derived by noting that $N = \sum_{J=0}^{B-1} N_J$ is the number of

seconds during which the counts have been sampled. To get the minimum possible count rate we assume the minimum possible counts were added to the scalers. This would occur if there were no overflow of the scalers, and no counts were unseen in the prescalers. This corresponds to setting $\ell = k = 0$ in eq. (1); so an observed difference J means only AJ counts were received.

RESULTS

An experiment was performed to determine how reliable the computations are. A radioactive source was placed in front of a scintillation detector. The pulses from the photomultiplier tube behind the detector were recorded in a large group of binary scalers. The counts stored in the scalers were observed at 1-second intervals for approximately 70 seconds. Then the counting rate was changed and the observations repeated. From the data taken,

the actual counting rate was determined, and the various configurations of differences $N_0, N_1, \dots, N_{B-1}$ for different values of A and B were found. These values were fed into the computer program for determination of the probable counting rate.

Table 2 compares the measured and computed values of the counting rate. Only those values of R for which χ^2 was less than 6.0 were taken as possible rates and are shown in the table. Furthermore it was required that $\chi^2(R)$ be larger than 7.0 for very large values of R, thus providing positive indication that the scalers were not flooded. If $\chi^2(R)$ was not larger than 7.0 for very large values of R, the scalers are taken to be flooded, and a value of ∞ is indicated in the table. In some cases no value of R resulted in a χ^2 less than 6.0; in such cases a dash appears in the table. In some cases $\chi^2(R)$ is below 6.0 for all values of R greater than some minimum value; this indicates that only a lower bound can be placed on the counting rate.

Looking over the table one sees that for rates less than AB (the storage capacity of the scalers), computed and experimental values agree except in an occasional case where the disagreement is probably due to unusual fluctuations.

Table 2

A = 2, B = 4		A = 4, B = 4		A = 8, B = 4	
True Rate	Computed Rate	True Rate	Computed Rate	True Rate	Computed Rate
2.74	2.7	2.74	-	2.74	2.7
3.43	10.9	3.43	3.1	3.43	3.08
8.78	> 7.1	5.89	6.1	5.89	-
9.09	10.1	8.9	8.8	8.9	8.8
11.05	∞	11.2	11.5, 27.1	13.5	13.5
12.32	> 5.4	13.5	13.7	20.6	52.1
12.45	> 9.1	14.5	∞	23.2	22.8
14.46	∞	15.7	15.7	24.7	24.1
14.98	> 5.5	20.35	20.0	28.7	28.8, 59.7
15.70	5.0, > 11	20.6	18.0, > 28.0	33.8	34.0
16.19	> 7.1	23.2	21.4	35.8	35.8, 68.0
20.35	> 5.7	24.7	21.3, > 31.0	36.6	22.2
21.86	∞	33.82	19.2, > 31.0	41.2	40.7, 71.8
23.33	> 7.6	37.4	20.1	47.8	49.3, 80.0
24.21	-	41.2	20.1, > 33.0	51.8	-
26.99	5.3, > 11.4	47.8	16.9, > 28.0	59.75	61.0, 88.0
28.66	> 8.0	61.05	28.0, > 42.0	61.05	28.0, 60.0
49.2	> 6.8	62.27	> 27.0	68.0	37.9
59.75	-	68.	> 23.0	74.45	76.1, 107.0, 141.0
61.05	> 5.0	74.45	> 22.0	98.17	63.0, 96.0, 125.0

For true rates between AB and $2AB$, corresponding to one recycling of the scalers, it is sometimes not possible to tell if a recycling has occurred. For example, for $AB = 32$, a rate of 61.05 counts per minute led to a calculated rate of either 28 or 60.

As the true rate is increased still further it becomes more difficult to calculate the true counting rate. The result of a calculation is usually a series of values differing by approximately a multiple of AB , or else it is only a lower bound on the possible counting rate. Unless great care is taken, it is easily possible to get incorrect values of the counting rate, because it is difficult to tell how many times the scalers have filled and started again during the time interval between samplings.

The greatest usefulness of this analysis is in the design of experiments of this kind when the counting rate to be measured is approximately known.

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APPENDIX: THE COMPUTER CALCULATIONS

In Fig. 2 we see the flow chart of the code used to analyze the data. B was taken as 4, but A was a variable which formed part of the input. The heights of the output pulse form the input data, and are designated V(I) in the flow chart. It was assumed that perturbations in pulse height due to noise fluctuations were not large enough to raise or lower the pulse heights to the next level (which would change the count by 1); however, pulses less than -0.03 V or more than 5.2 V were considered unreliable and were discarded. The time interval being considered is N seconds long.

The first step was to convert the pulse heights into the number of counts stored in the scalers, count (I). Then the difference in the number of stored counts in successive samplings is calculated (disregarding those counts made unreliable by noise by setting count (I) equal to 10). Negative differences were converted into positive differences by adding B to their value. The number of times a difference of 0, 1, 2, 3 = B - 1 occurred is then calculated. Because a zero index is not permitted by FORTRAN, N_J is designated by $N(J + 1)$ in the flow chart. At this point A, B, $N_0, N_1, \dots, N_3$ are known. The minimum rate R_0 to which the set of differences $N_0, N_1, \dots, N_{B-1}$ can correspond is calculated. Then subroutine CAROL computes $P_{AB}(J, R)$ and $P(R)$ for values of R starting at R_0 and increasing in steps of 2.5% to $R_0 + 100$. This maximum value of R was arbitrarily chosen and represents a counting rate considerably beyond the scaler's capacity. For each value of R the code then calculates χ^2 . All values of $\chi^2(R)$ are searched and those values of R for which χ^2 is less than 6.0 are printed out.

The input to subroutine CAROL (see Fig. 3) is R and $N_0, N_1, \dots, N_{B-1}$. This subroutine calculates $P_{AB}(J, R)$, starting with $J = 0$ and going to $J = B - 1$. For each value of J the calculation begins by setting k in the sum in eq. (2) equal to zero, and summing over ℓ . The result is denoted by PP(1). Then k is increased to 1, and the summation over ℓ is again carried out; the result is called PP(2). The subroutine then tests to see if the sum for $k = 1$ has changed $P_{AB}(J, R)$ by at least 0.1%; if it has, then k is increased by one and another summation over ℓ is performed for this new value of k. This is continued until a value of k is reached for which the sum over ℓ adds less than 0.1% to $P_{AB}(J, R)$. Then J is increased by one, k is set equal to zero, and the computation of $P_{AB}(J + 1, R)$ begins. This continues until $J = B$. Then $P(R)$ is computed.

Fig. 2. Flow chart of code used to analyze data.

Fig. 2. Continued.

Fig. 3. Flow chart of subroutine computing probability of observations.

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