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RESONANCE ABSORPTION OF NEUTRONS
IN MIXTURES OF THORIUM-232 AND URANIUM-238:
AN INVESTIGATION OF INTERFERENCE BETWEEN ABSORBERS
BY MEANS OF REACTIVITY MEASUREMENTS IN THE
ADVANCED REACTIVITY MEASUREMENT FACILITIES

Wesley K. Foell

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**PHILLIPS
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ATOMIC ENERGY DIVISION

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ADVANCED REACTIVITY MEASUREMENT FACILITIES***

by

Wesley K. Foell

- * Based on a thesis submitted to the Department of Mechanical Engineering at Stanford University in June, 1964, in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Nuclear Engineering.

**PHILLIPS
PETROLEUM
COMPANY**

Atomic Energy Division

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ABSTRACT

Resonance integrals of single absorber samples of uranium-238 dioxide and thorium-232 dioxide, each in admixture with a lead dioxide diluent, were measured by means of static reactivity techniques in the ARMF-II. The experimental results for the uranium dioxide/lead dioxide samples were in good agreement over a wide range of absorber concentrations with the results predicted with the aid of Nordheim's treatment. There was quantitative disagreement between the experimental results for the thorium dioxide/lead dioxide samples and the corresponding predicted values.

The infinite dilution resonance integral of thorium-232, adjusted to a cutoff energy of 0.5 eV, and including the $1/v$ contribution, was measured as 81.2 ± 4 barns.

The concept of interference between resonances of two absorbers was formulated and defined as the effect on the resonance integral caused by the lack of complete flux recovery between the resonances. Interference was measured in a series of uranium-238/thorium-232 mixtures. The experimental results indicated slightly more than three percent interference in the samples of highest absorber concentration. Interference was also predicted for this series of mixtures by means of multiresonance solutions of the first-flight coupled equations. These calculations predicted an interference of approximately 1.3 percent in the samples of highest absorber concentration. It is believed that the underestimate of interference in the calculations may result from the use of the narrow resonance approximation in the moderator terms of the first-flight coupled equations.

On the basis of the experimental and theoretical results, interference was predicted for uranium-238/thorium-232 metal rods of theoretical density.

Also described are numerous experimental techniques and methods of analysis generally applicable to reactivity and activation measurements in low power reactors.

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TABLE OF CONTENTS

ACKNOWLEDGMENTS	ii
ABSTRACT	iii
I. INTRODUCTION	1
1. HISTORICAL BACKGROUND OF RESONANCE ABSORPTION	1
2. THE CONCEPT OF INTERFERENCE	2
3. THE POSSIBLE EXISTENCE OF INTERFERENCE IN MIXED THORIUM-232/URANIUM-238 SYSTEMS	3
4. A STATEMENT OF THE PROBLEM AND ITS SCOPE	3
5. ORGANIZATION OF THE REPORT	5
II. THEORY AND MEASUREMENT INTERFERENCE	6
1. CURRENT STATE OF THE THEORY OF RESONANCE ABSORPTION	6
2. DEFINITION OF INTERFERENCE	9
3. CALCULATION OF INTERFERENCE BY MEANS OF MULTIRESONANCE SOLUTION OF FIRST-FLIGHT COUPLED EQUATIONS	12
4. EXPERIMENTAL PROCEDURE FOR MEASUREMENT OF INTERFERENCE	14
III. THEORY AND TECHNIQUE OF REACTIVITY MEASUREMENTS ...	19
1. HISTORY OF REACTIVITY MEASUREMENTS	19
2. THE SERVO-CONTROLLED STATIC REACTIVITY MEASUREMENT TECHNIQUE	19
3. DESCRIPTION OF THE ADVANCED REACTIVITY MEASUREMENT FACILITIES	21
3.1 Advanced Reactivity Measurement Facility-I	21
3.2 Advanced Reactivity Measurement Facility-II	21
4. DESCRIPTION OF REACTIVITY MEASUREMENTS IN THE ARMF-II -- SEQUENCE OF MEASUREMENTS: DATA EXTRACTION AND REDUCTION	24
5. PRECISION OF ARMF-II REACTIVITY MEASUREMENTS	29
5.1 Short-Term Precision	29
5.2 Long-Term Precision	30

IV. EXPERIMENTAL AND THEORETICAL CORRECTIONS TO THE OBSERVED REACTIVITY DATA	32
1. EXPERIMENTALLY BASED CORRECTIONS	32
1.1 Aluminum Capsules	32
1.2 Absorption and Scattering in Lead Dioxide; Scattering in the Absorber	32
1.3 Uranium-235 Fission and Capture	33
2. THEORETICALLY BASED CORRECTIONS	36
2.1 Deviation of Real Flux from 1/E Behavior; Energy Dependence of Adjoint Flux	36
2.2 Reactivity Effects of Fast Fissions	50
V. RESONANCE INTEGRALS OF SINGLE ABSORBER SAMPLES OF URANIUM-238 AND THORIUM-232	54
1. CROSS SECTIONS OF GOLD-197, URANIUM-238, AND THORIUM-232	54
1.1 Cross Section of Gold-197	54
1.2 Cross Section of Uranium-238	55
1.3 Cross Section of Thorium-232	58
2. METHODS OF CALCULATION AND RESULTS	60
3. EXPERIMENTAL RESULTS AND COMPARISON WITH THEORETICAL PREDICTIONS	63
VI. INTERFERENCE IN URANIUM-238/THORIUM-232 MIXTURES	69
1. INTERFERENCE CALCULATED BY MULTIRESONANCE SOLUTION OF FIRST-FLIGHT COUPLED EQUATIONS	69
2. MEASURED INTERFERENCE; COMPARISON WITH THEORY	72
VII. DISCUSSION OF RESULTS; CONCLUSIONS	85
VIII. NOMENCLATURE	89
IX. REFERENCES	95
APPENDIX A - CALIBRATION OF ARMF-II REGULATING ROD	99
APPENDIX B - SAMPLE PREPARATION	105
1. MATERIALS	107
1.1 Uranium Dioxide (UO ₂)	107
1.2 Thorium Dioxide (ThO ₂)	107
1.3 Lead Dioxide (PbO ₂)	110
1.4 Gold	110
2. METHODS OF PREPARATION	110

APPENDIX C - MEASUREMENT OF THE GOLD-197 WEIGHTING FACTOR AS A MEANS OF DETERMINING C, THE WEIGHTING FUNCTION SCALE FACTOR	115
1. ACTIVATION EQUIPMENT	117
2. ACTIVATION TECHNIQUES	118
2.1 Foil Weight Determination by Activation Analysis	121
2.2 Activation Measurement of Resonance Integral of Powder Samples Relative to Resonance Integral of Microfoil	121
2.3 Gold Monitor Foils	123
3. METHOD AND RESULTS	124
APPENDIX D - DETERMINATION OF GOLD MICROFOIL WEIGHTS BY ACTIVATION ANALYSIS	129
APPENDIX E - A TRANSPORT THEORY CALCULATION OF THE ARMF-II REAL AND ADJOINT FLUXES	133
1. CYLINDRICAL MODEL OF THE ARMF-II	135
2. GROUP CONSTANTS	136
3. RESULTS	138
APPENDIX F - DERIVATION OF THE FLAT FLUX VERSION OF THE RECIPROCITY THEOREM.	145
APPENDIX G - UNCERTAINTY ANALYSIS	149
1. UNCERTAINTY ANALYSIS OF MEASURED INTERFERENCE . .	151
2. UNCERTAINTY ANALYSIS OF MEASURED INFINITE DILUTION THORIUM RESONANCE INTEGRAL	153

FIGURES

1. Capture cross section of uranium-238 and thorium-232 due to resonances in the range between eighteen and twenty-five eV	4
2. ARMF-I core diagram at midplane: loading 2-62	22
3. ARMF-II core diagram at midplane: loading 12-62	23
4. ARMF-II core diagram at midplane: loading 4-63	24
5. ARMF-II core as viewed from pool surface	24

6. Cutaway view of the ARMF-II core	25
7. Details of cadmium tube with sample capsule positioned	25
8. Lucite positioning plug with capsule attached	26
9. Sample and positioning plug being lowered into ARMF-II cadmium-shielded measurement location	26
10. Low energy flux and weighting function in the ARMF-II cadmium-shielded measurement location as determined from CADOFF-3 calculation	46
11. Average weighting function for uranium dioxide/lead dioxide and thorium dioxide/lead dioxide samples	47
12. Measured and calculated resonance integrals for uranium-238 dioxide/lead dioxide and thorium-232 dioxide/lead dioxide samples .	64
13. S/MUO ₂ dependence of resonance integrals of uranium-238 dioxide/lead dioxide samples	66
14. S/MThO ₂ dependence of resonance integrals of thorium-232 dioxide/lead dioxide samples	67
15. Reactivity worths of dilute thorium-232 dioxide/lead dioxide samples	68
16. Calculated fractional interference among 21.0, 21.84, and 23.48 eV resonances in mixtures with $N_U/N_{Th} = 1.3$	75
17. Calculated fractional interference among 21.0, 21.84, and 23.48 eV resonances in mixtures with $N_U/N_{Th} = 3$	75
18. Calculated fractional interference between 66.3 and 69.2 eV resonances in mixtures with $N_U/N_{Th} = 3$ and $N_U/N_{Th} = 3$	76
19. Calculated fractional interference in the total resonance integral of uranium-238 dioxide/thorium-232 dioxide mixtures	76
20. Calculated energy dependence of flux within sample U/T-135	78
21. Calculated energy dependence of flux within sample U/T-315	78
22. Absorption reactivities of single absorber samples of uranium-238 dioxide/lead dioxide and thorium-232 dioxide/lead dioxide	79
23. Measured interference in mixed uranium-238/thorium-232 samples: $N_U/N_{Th} = 1/3$	83
24. Measured interference in mixed uranium-238/thorium-232 samples: $N_U/N_{Th} = 3$	83

25. Calculated interference in uranium-238/thorium-232 metal rods . .	88
26. Reactivity calibration curve for ARMF-II regulating rod	103
27. Electron photomicrographs of thorium dioxide, uranium dioxide, and lead dioxide particles	109
28. Aluminum capsule for compacted powder samples	110
29. Complete set of powder samples used in resonance integral measurements	110
30. Equipment for compaction of powder samples	112
31. NaI scintillation detector unit	117
32. Source holder assembly	117
33. Multichannel analyzer system	118
34. Decay scheme of gold-198	119
35. Gold-198 gamma ray pulse-height spectrum	120
36. Source geometry for powder-filled disc	122
37. Positions of gold microfoils in infinite dilution samples	125
38. ARMF-II core regions which were homogenized for S-XI transport calculation	135
39. ARMF-II flux spectrum as calculated by the GAM code	139
40. Real and adjoint fluxes in center of ARMF II cadmium tube as determined from S_2 transport calculation	141
41. Weighting function in center of ARMF-II cadmium tube as determined from S_2 transport calculation	142

TABLES

I. Atom Densities of Single Absorber Samples	15
II. Atom Densities of Mixed Uranium-238/Thorium-232 Samples . . .	16
III. Scattering and Absorption Sensitivities of ARMF-I and ARMF-II .	27
IV. Macroscopic Potential Scattering Cross Sections and Reactivities Due to Scattering	34
V. Weights and Concentrations of Samples for Uranium-235 Correction	35

VI. Reactivity Worth of Samples for Uranium-235 Correction	36
VII. Energy and Lethargy Limits of Spectrum Ranges for Uranium-238, Thorium-232, and Gold-197	42
VIII. Calculated Average Weighting Functions for Single Absorber Samples	44
IX. Calculated Average Real Fluxes in Gold Samples	45
X. Multigroup Fluxes and Constants for Calculation of Reactivity Due to Fast Fissions:	52
XI. Summary of Reactivity Data	53
XII. Resonance Parameters of Gold-197	56
XIII. Resonance Parameters of Uranium-238	59
XIV. Resonance Parameters of Thorium-232	61
XV. Calculated Resonance Integrals of Single Absorber Samples	63
XVI. Measured Resonance Integrals of Single Absorber Samples	65
XVII. Calculated Interference in 21.0 eV Resonance of Uranium-238 Due to 21.84 and 23.48 eV Resonances of Thorium-232	71
XVIII. Calculated Interference in 21.84 eV Resonance of Thorium-232 Due to 21.0 eV Resonance of Uranium-238	72
XIX. Calculated Interference in 23.48 eV Resonance of Thorium-232 Due to 21.0 eV Resonance of Uranium-238	73
XX. Calculated Interference in 66.3 eV Resonance of Uranium-238 and 69.2 eV Resonance of Thorium-232	74
XXI. Calculated Fractional Interference in the Total Resonance Integral of Uranium-238/Thorium-232 Mixtures	77
XXII. Weighting Functions, Predicted Reactivities, and Predicted Resonance Integrals of Uranium and Thorium Components of Mixed Uranium-238/Thorium-232 Samples	80
XXIII. Values of the δ Correction and Predicted Resonance Integrals for Mixed Uranium-238/Thorium-232 Samples	81
XXIV. Measured Interference in Mixed Uranium-238/Thorium-232 Samples	82
XXV. Sample Impurities	108
XXVI. Net Weights of Powder Sample Ingredients	113

XXVII. Weights of Gold and Lead Dioxide in Powder-Filled Counting Discs; Gamma-Ray Attenuation Factors	123
XXVIII. Results of Measurement of Weighting Factor, $C \bar{W}_{Au}^{Exp}$	127
XXIX. Results of Gold-197 Activation Analysis in MTR Thermal Column	131
XXX. Size and Composition of Regions in the Cylindrical Model of the ARMF-II	137
XXXI. Group Widths and Boundaries of Spectrum Structure Used in S-XI Calculation	138
XXXII. Calculated Relative Real Flux, Adjoint Flux, and Weighting Function at Center of Cadmium Tube in ARMF-II	143
XXXIII. Uncertainties Associated with Reactivity Measurement of Interference	154

I. INTRODUCTION

1. HISTORICAL BACKGROUND OF RESONANCE ABSORPTION

When Fermi and Szilard made their historic decision in 1940 to explore the possibility of using thermal neutrons as carriers of the chain reaction [1, 2], it was implied that the energetic fission neutrons would have to be slowed down to thermal energy. During this "slowing down" or "moderating" process, the neutrons would have to pass through the energy region in which uranium-238 was known to have strong lines or resonances in which absorption would be likely to occur. Absorption in these lines became known as resonance absorption. Several scientists, realizing that the neutron absorption in these lines would not result in fission, surmised that the chain might be interrupted and brought to a standstill. N. Bohr noted the further possibility that even if the chain reaction could be established while the uranium in the system was cold, the fission heat would increase the temperature of the uranium, thereby increasing its resonance absorption and perhaps halting the chain reaction. Because of the importance of the above and other considerations, the study of resonance absorption proved to be one of the major efforts leading to the first self-sustaining chain reaction in a natural uranium pile.

It is well chronicled history that parasitic resonance absorption did not prevent the achievement of a chain reaction. Fermi and Szilard recognized that the resonance absorption could be diminished by concentrating the uranium in heterogeneous lumps rather than dispersing it uniformly throughout the reactor. Their major problem then was to choose an optimum lattice of uranium lumps and moderator. The pioneering theoretical and experimental work leading to its solution was presented lucidly by Wigner et al [3] in a paper declassified in 1955. After this very successful early work on the resonance absorption problem, interest in the field did not diminish. Both theoretical and experimental investigations continued, and in the years following the declassification of Wigner's almost classic paper, interest was intensified even further and still remains so today.

Throughout the period described above, the concept of the resonance integral as an integrated effective cross section for an energy band, broad in comparison to a resonance width and often encompassing several resonances, has been of great use in the description and prediction of resonance absorption. A general problem in the field today is whether measurements and calculations of resonance integrals can be made to a high accuracy for systems whose composition or configuration makes the resonance absorption process more complex. An affirmative answer to this question requires an evaluation of the applicability of existing techniques, and if necessary, the development of more refined theoretical and experimental methods. In the present work, the author describes an investigation of resonance absorption in a particular system where the conventional definition and treatment of the resonance integral may be inadequate.

2. THE CONCEPT OF INTERFERENCE

Since the early days of the Manhattan Project, when Wigner and his colleagues first developed their theoretical methods for predicting the absorption of neutrons by uranium lumps, resonance absorption has been treated as occurring in widely separated lines or resonances. Because of this wide separation of the resonances, it has been convenient to treat the absorption in each resonance as a separate calculation and then sum their individual contributions to give a total resonance integral. Mathematically this technique may be stated as the following:

$$I_{\infty} = \int \sigma_a(E) \frac{dE}{E} = \sum_i I_{\infty}^i = \sum_i \left[\int \sigma_a^i(E) \frac{dE}{E} \right] \quad (1)$$

where

I_{∞} = total infinite dilution resonance absorption integral of the absorber material

$\sigma_a(E)$ = resonance absorption cross section

$\sigma_a^i(E)$ = resonance absorption cross section due to the ith resonance

I_{∞}^i = resonance integral of the ith resonance.

If the absorber is present in the system in concentration sufficient to cause the flux to deviate from a $1/E$ behavior, the expression analogous to Equation (1) is

$$I = \frac{\int_{\vec{r}} \int_{\vec{\Omega}} \int_E \sigma_a(E) \phi(\vec{r}, \vec{\Omega}, E) d^3\vec{r} d\vec{\Omega} dE}{\phi_{as}} = \sum_i I^i = \sum_i \left[\frac{\int_{\vec{r}} \int_{\vec{\Omega}} \int_E \sigma_a^i(E) \phi^i(\vec{r}, \vec{\Omega}, E) d^3\vec{r} d\vec{\Omega} dE}{\phi_{as}} \right] \quad (2)$$

where

I = total effective resonance absorption integral of the absorber material

$\phi^i(\vec{r}, \vec{\Omega}, E)$ = angular flux which would be present in system if the total resonance absorption cross section is that contributed by the ith resonance

ϕ_{as} = the "asymptotic" flux present in the system in the absence of absorber resonances.

The explicit meaning of Equations (1) and (2) is that absorption can be treated separately in each resonance; that is, regardless of the absorption in higher energy resonances, the asymptotic or $1/E$ flux distribution form is applicable at the high energy side of each resonance without the complication of a disturbance from resonances above. In general, this assumption of "separability" has proved valid due to the fact that for most practical resonance absorbers, the resonances are widely spaced. In particular, certain uranium systems have been studied rather thoroughly by Monte Carlo techniques [4] and the results seem to confirm the assumption. However, some mixture of absorbing isotopes may contain pairs or groups of resonances which are closely spaced, and in these systems, due to the overlapping of the resonance cross sections and/or the absence of complete flux recovery, a separate treatment of each resonance may not be valid. In this work, the failure of the assumption of separability shall be referred to as "interference".

3. THE POSSIBLE EXISTENCE OF INTERFERENCE IN MIXED THORIUM-232/URANIUM-238 SYSTEMS

One system for which separability may not be valid is a mixture of thorium-232 and uranium-238. Previous measurements and calculations of resonance absorption in thorium-232 and uranium-238 have been made only with the individual absorbers. A mixture of these two isotopes gives a system in which resonances are much more closely spaced than in the usual case of a single absorber. Figure 1 illustrates the proximity of three such resonances around the energy of 22 eV. A study of such mixtures, therefore, would provide an opportunity to confirm the adequacy of calculation procedures which assume an absence of interfering effects between resonances.

This particular two-absorber system also provides interest for the reactor designer. Crowther [5] has indicated that a combination of thorium-232/uranium-238 may be attractive in a mixed fuel cycle. Crowther points out that if these fertile isotopes are mixed in the fuel rods of a heterogeneous reactor, the resonance capture of the mixture will be greater than the resonance capture of single rods of either uranium-238 or thorium-232. For a particular system Crowther showed that the resonance capture and, therefore, the conversion ratio of the reactor lattice was maximized with a thorium to uranium ratio of 1.13. In general, mixtures of thorium-232/uranium-238 may be used to vary the resonance capture of a lattice whose geometric and physical properties are determined by heat transfer, economic, or mechanical limitations, and, therefore, the use of the mixed system may be considered to be an additional reactor variable which could provide advantages in both performance and economics.

4. A STATEMENT OF THE PROBLEM AND ITS SCOPE

The work described in this report was an investigation of resonance absorption in systems containing mixtures of uranium-238 and thorium-232. The objective was to determine, by calculation and measurement, the existence and magnitude of interference between close-lying resonances of these two isotopes.

Fig. 1 Capture cross section of uranium-238 and thorium-232 due to resonances in the range between eighteen and twenty-five eV.

The primary purpose of such an investigation was twofold: (a) to check the adequacy of current calculation procedures for a system where interference may exist and (b) to provide information for design of a reactor system containing these mixed resonance absorbers. A further goal was the formulation of a concise definition of the resonance integral of a mixed system, and the development of a precise experimental technique by which this resonance integral may be measured and interpreted in a manner which is consistent with the definition. The reactivity measurement technique was chosen as the means by which the latter could be achieved.

Because of the necessity of confining the experimental program to include a practical number of mixed thorium/uranium systems, the study was performed with a limited number of mixtures, chosen to permit the inference of some general conclusions about the interference phenomenon, while still providing information of interest to a reactor designer. As a by-product of the study of the mixed systems, resonance integrals of single absorber systems were also obtained.

5. ORGANIZATION OF THE REPORT

The plan of the remainder of this report is as follows: Section II opens with the general theory of the calculation of resonance absorption in a heterogeneous system, followed by a precise definition of interference and a method for its calculation. The section closes with an exposition of the general method to be used for measurement of interference by means of reactivity measurements. Section III presents a general description of the reactivity measurement technique, followed by a description of the specific methods developed for the Advanced Reactivity Measurement Facilities. In Section IV, the theoretical and experimental corrections which were applied to the measured data are given in detail. A major portion of this section is devoted to the problem of inferring a resonance integral from a measurement in a system in which the real and adjoint fluxes deviate from their idealized behavior. Section V presents the calculated and experimental results obtained for the single absorber samples of uranium and thorium dioxides. Conclusions are drawn regarding the validity of the experimental technique, and the validity of the resonance parameters used in the calculations. The results of the calculation and measurements of interference are given in Section VI followed by a discussion of results and the conclusions in Section VII.

II. THEORY AND MEASUREMENT OF INTERFERENCE

Because historically uranium-238 was the first isotope for which the phenomenon of resonance absorption was studied and because it has continued to be the most important resonance absorber of interest to designers of nuclear reactor systems, the theory of resonance absorption has been built around the nuclear characteristics of this absorber. The influence of such uranium-238 properties as high atomic number, large level spacing, and the absence of very low-lying resonances can be seen quite clearly in the development of the current theory. Many systems of interest today contain materials which fail to match these characteristics; therefore, new calculational procedures must be developed. Furthermore, to take advantage of the rapid progress in neutron spectrometry which has provided reliable and accurate descriptions of resonance parameters up to several hundred keV for many resonance absorbers of interest, it has been necessary to improve and refine the theory. The past decade has seen the knowledge of resonance absorption pass from a quasi-empirical state to a well-defined science.

1. CURRENT STATE OF THE THEORY OF RESONANCE ABSORPTION

The most successful approach to the problem of calculation of resonance absorption in a relatively complex system is based on a set of equations first suggested by Chernick in 1955 [6]. These equations, which were proposed on an intuitive basis, have since been derived by Corngold [7, 8] as a first approximation to a solution of the Boltzmann equation using a spherical harmonic expansion.

Chernick's method utilizes a coupled set of spatially-independent, energy-dependent equations describing neutron events in moderator and absorber material. The equations are:

$$\begin{aligned}
 F_0(u) = & \left[1 - P_1(u) \right] \sum_i \frac{1}{1-\alpha_i} \int_{u-\ln \frac{1}{\alpha_i}}^u \frac{\sigma_s^i(u') e^{-(u-u')}}{\sigma_{t1}(u')} F_1(u') du' \\
 & + F_0(u) \sum_j \frac{1}{1-\alpha_j} \int_{u-\ln \frac{1}{\alpha_j}}^u \frac{\sigma_s^j(u') e^{-(u-u')}}{\sigma_{t0}(u')} F_0(u') du' \quad (3)
 \end{aligned}$$

and

$$F_1(u) = P_1(u) \sum_i \frac{1}{1-\alpha_i} \int_{u-\ln \frac{1}{\alpha_i}}^u \frac{\sigma_s^i(u') e^{-(u-u')}}{\sigma_{t1}(u')} F_1(u') du'$$

$$+ \left[1 - P_0(u) \right] \sum_j \frac{1}{1-\alpha_j} \int_{u-\ln \frac{1}{\alpha_j}}^u \frac{\sigma_s^j(u') e^{-(u-u')}}{\sigma_{t0}(u')} F_0(u') du'. \quad (4)$$

The subscripts, 0 and 1, refer to the absorber and moderator regions respectively; the sum, i, is over moderator region atoms and the sum, j, is over absorber region atoms. The following nomenclature is used:

$$F(u) = \phi(u) \sigma_t(u) = \text{collision density}$$

$$\sigma_s(u) = \frac{\Sigma_s(u)}{N_a} = \text{scattering cross section per absorber atom}$$

$$\sigma_t(u) = \frac{\Sigma_t(u)}{N_a} = \text{total cross section per absorber atom}$$

$$\alpha_i = \left(\frac{A_i - 1}{A_i + 1} \right)^2$$

$$A_i = \text{atomic weight of the } i\text{th isotope}$$

$$P_k(u) = \text{the probability that a neutron, whose last collision in the } k\text{th region degraded it to lethargy } u, \text{ suffers its next collision also in the } k\text{th region.}$$

Since these equations contain no spatial dependence, the collision densities are space-averaged quantities. The use of flat source P_k 's in (3) and (4) is commonly referred to as the "flat flux" or "flat source" approximation. At this point in the development, the narrow resonance (NR) approximation, originally suggested by Wigner et al [3], is introduced into the moderator terms, ie, into terms within the summation, i. In essence, the use of the NR approximation in the moderator terms is based on the assumption that a single collision of a resonance energy neutron with a moderator nucleus will cause it to hurdle the resonance. The usual requirement for applying this approximation is that

$$E (1 - \alpha_i) \gg \text{width of the resonance} . \quad (5)$$

The left hand quantity above represents the maximum possible energy loss suffered by a neutron in an elastic collision with a moderator nucleus. The quantity on the right is usually considered to be the span between the energies on each wing of the resonance at which the resonance absorption cross section is equal to the potential scattering cross section. The NR approximation in the moderator terms is adequate for most practical systems although it can lead to inaccuracies in the case of absorbers with low-lying resonances [9]. If the NR approximation is valid, the collision density in the moderator can be replaced by its asymptotic value, σ_t . Evaluation of the integral in the moderator terms in (3) and (4) leads to:

$$F_O(u) = \left[1 - P_1(u)\right] \sum_i \sigma_p^i + P_O(u) \sum_j \frac{1}{1-\alpha_j} \int_{u-\ln \frac{1}{\alpha_j}}^u \frac{\sigma_s^j(u') e^{-(u-u')}}{\sigma_{tO}(u')} F_O(u') du' \quad (6)$$

and

$$F_1(u) = P_1(u) \sum_i \sigma_p^i + \left[1 - P_O(u)\right] \sum_j \frac{1}{1-\alpha_j} \int_{u-\ln \frac{1}{\alpha_j}}^u \frac{\sigma_s^j(u') e^{-(u-u')}}{\sigma_{tO}(u')} F_O(u') du' \quad (7)$$

Next, a theorem of transport theory is introduced. The reciprocity theorem, within the confines of the flat flux approximation[a], may be expressed as

$$(1 - P_O) \sigma_{tO} = (1 - P_1) \sigma_{t1} \quad (8)$$

Substitution of the above into Equations (6) and (7) gives

$$F_O(u) = \left[1 - P_O(u)\right] \sigma_{tO}(u) + P_O(u) \sum_j \frac{1}{1-\alpha_j} \int_{u-\ln \frac{1}{\alpha_j}}^u \frac{\sigma_s^j(u') e^{-(u-u')}}{\sigma_{tO}(u')} F_O(u') du' \quad (9)$$

and

$$\begin{aligned} F_1(u) &= \sum_i \sigma_p^i - \left[1 - P_O(u)\right] \sigma_{tO}(u) \\ &+ \left[1 - P_O(u)\right] \sum_j \frac{1}{1-\alpha_j} \int_{u-\ln \frac{1}{\alpha_j}}^u \frac{\sigma_s^j(u') e^{-(u-u')}}{\sigma_{tO}(u')} F_O(u') du' \quad (10) \end{aligned}$$

If a solution for $F_O(u)$ can be obtained from Equation (9), a simple integration gives I as

$$I = \int \sigma_a(u) \phi(u) du = \int F_O(u) \frac{\sigma(u)}{\sigma_{tO}(u)} du \quad (11)$$

[a] A derivation of the flat flux version of the reciprocity theorem is given in Appendix F.

From the introduction of the narrow resonance approximation (NR) or infinitely heavy absorber approximation (IM or NRIA) [a] into the absorber term, in addition to the use of Wigner's rational approximation [a] for (P_0) , it is possible to obtain rather simple but elegant closed-form solutions of the equations. However, in many systems these solutions may suffer from the inaccuracies introduced by the approximations. In an attempt to avoid these approximations, Nordheim and Kuncir [11] noted that Equation (9) is an integral equation of the form,

$$F(u) = A(u) \int_{u-\Delta}^u F(u') R(u') du' + B(u) \quad (12)$$

where A , B , and R are known functions of u and where $F(u)$ is known for $u \leq 0$. Such equations are very amenable to direct machine computation. Although the NR, IM, and rational approximations lead to very simple formulas which are useful for qualitative considerations, the direct numerical solution of Equation (9) now appears to be the most attractive method available for an accurate calculation of resonance absorption. Monte Carlo techniques do provide an alternate and undoubtedly more accurate approach, but at the present time they require long computer runs and seem better suited for the study of certain selected problems rather than routine calculation by reactor designers.

Nordheim and co-workers have developed ZUT [11, 12], a flexible and convenient computer code, which solves Equation (9) and computes a resonance integral from Equation (11). ZUT, which describes each resonance by the Doppler-broadened Breit-Wigner single-level formula, treats only one resonance at a time and then sums up the individual contributions of all levels. It is clear that the separate treatment of each resonance is equivalent to the assumption of complete flux recovery between isolated resonances. The discussion in Section I-2 pointed out certain systems in which this assumption may not be valid. The failure of this assumption, that is, the lack of complete flux recovery between the resonances or the overlapping of the cross sections of closely spaced levels, shall be referred to in this work as "interference". Although this term is also used to describe the interaction between potential and resonance scattering, the two phenomena are ordinarily used in a different context so that no confusion should arise.

2. DEFINITION OF INTERFERENCE

As Nordheim pointed out in Reference 13, the assumption of isolated resonances is not essential for the numerical solution of Equation (9). All that is required is the preparation in equal lethargy intervals of the cross section from all of the appropriate resonances, and the assumption of an asymptotic form of the collision density above a starting energy E_1 . E_1 is necessarily above the highest level included in calculation. If an attempt is made to assess the effect

[a] Those approximations are examined in detail in Reference 10.

of interference between close-lying resonances in a single absorber system, the approach would be rather straightforward. The difference between (a) the total resonance integral with each resonance treated individually and (b) the total resonance integral resulting from a multiresonance calculation, gives the interference in terms of the resulting change in resonance integral.

For a mixture of two absorbing isotopes, the concept of interference is somewhat more complicated. Because the effects of a change in the potential scattering of a system can be calculated by conventional theory, the definition of interference used here shall include only those effects due to a multiresonance treatment of resonance absorption and resonance scattering, and not those due to the potential scattering effects of either isotope. To clarify the need for separating these two types of effects, a system might be considered consisting of an absorbing lump containing (a) a single resonance absorber and (b) a purely scattering diluent. In this system, the lump has a particular resonance integral which can be calculated by means of current theory. If now, a second resonance absorber is substituted for a fraction of the diluent, the resonance integral of the lump would, in general, change even if "interference" did not occur. This change in resonance integral would be caused by any difference between the potential scattering cross section and atomic weight of the diluent and the corresponding parameters of the added resonance absorber. Such an effect is predictable by the methods of Section II-1.

Because of the complexity of this concept of interference in mixtures, the following notation has been developed:

(1) $I_x (N_x; 0; N_m)$ = resonance integral of resonance absorber x in a system containing:

(a) N_x atoms of isotope x.

(b) One or more nonabsorbing materials belonging to the set m. These are often referred to as "moderators" or "scatterers". Each is present in the lump with atom density N_m^i where i refers to a specific isotope or element.

Each isotope present is characterized by a potential scattering cross section, σ_p , and an atomic weight, A.

(2) $I_x (N_x; N_y^r; N_m)$ = resonance integral of resonance absorber x in a system containing:

(a) N_x atoms of isotope x.

(b) N_y atoms of isotope y where y is an isotope having a resonance; the superscript "r" indicates that the effect of this resonance is included in the calculation or measurement of the resonance integral of isotope x.

(c) One or more nonabsorbing materials belonging to the set m, as described in (1) above.

(3) $I_x(N_x; N_y^p; N_m)$ = resonance integral of resonance absorber x in a system containing:

(a) N_x atoms of isotope x.

(b) N_y atoms of isotope y where y is an isotope having a resonance. However, the superscript "p" designates that the effect of this resonance is not included in the calculation or measurement of the resonance integral of isotope x; that is, the resonance is "switched off" so that the only effect of isotope y is its potential scattering. This is a calculational model, of course, and not directly realizable in the laboratory.

(c) One or more nonabsorbing materials belonging to the set m, as described in (1) and (2) above.

$$(4) \quad I_{x,y}^r(N_x; N_y; N_m) = \frac{N_x I_x(N_x; N_y^r; N_m) + N_y I_y(N_y; N_x^r; N_m)}{N_x + N_y}$$

This represents the resonance integral of a mixture containing N_x atoms of resonance isotope, x, N_y atoms of isotope, y, and a set, m, of scattering isotopes. The "r" superscript indicates that each individual resonance integral is defined as in (2) above.

$$(5) \quad I_{x,y}^p(N_x; N_y; N_m) = \frac{N_x I_x(N_x; N_y^p; N_m) + N_y I_y(N_y; N_x^p; N_m)}{N_x + N_y}$$

The above represents the resonance integral of the same mixture as described in (4) except that the individual resonance integrals are defined as in (3).

$$(6) \quad \Delta I_x(N_x; N_y; N_m) = \frac{I_x(N_x; N_y^r; N_m) - I_x(N_x; N_y^p; N_m)}{N_x + N_y}$$

This is the definition of the interference of isotope y on isotope x.

$$(7) \quad \begin{aligned} \Delta I_{x,y}(N_x; N_y; N_m) &= \frac{N_x \Delta I_x(N_x; N_y; N_m) + N_y \Delta I_y(N_y; N_x; N_m)}{N_x + N_y} \\ &= I_{x,y}^r(N_x; N_y; N_m) - I_{x,y}^p(N_x; N_y; N_m) \end{aligned}$$

This represents the total interference in a mixture of two resonance absorbers, x and y. Although the above nomenclature is somewhat complex, it is consistent with a concept of interference which includes only those effects due to the resonances and not those due to a change in effective scattering.

3. CALCULATION OF INTERFERENCE BY MEANS OF MULTIRESONANCE SOLUTION OF FIRST-FLIGHT COUPLED EQUATIONS

Presented below is an example of the calculation of resonance interference. To perform this calculation, the IBM-7090 version of the ZUT code was modified by Brown [14] to handle two Doppler-broadened resonances. Kelber also has written a code [15] which can include up to 300 resonances, but only in the case of a homogeneous system. In the present study, all calculations of interference were done with the modified ZUT described above or a later version [16] modified to treat Equations (19), (20), and (21).

Consider the case where it is desired to calculate the interference of a uranium resonance on a close-lying thorium resonance. The absorber lump consists of a mixture of uranium dioxide, thorium dioxide, and lead dioxide diluent. The equation describing the neutron events in the mixture is essentially the same as Equation (9).

$$\begin{aligned}
 \phi(u)\sigma_t(u) = P_O(u) & \left\{ \frac{1}{1-\alpha_{Th}} \int_{u-\ln\frac{1}{\alpha_{Th}}}^u \frac{\sigma_s^{Th}(u') e^{-(u-u')}}{\sigma_t(u')} F(u') du' \right. \\
 & + \frac{1}{1-\alpha_U} \int_{u-\ln\frac{1}{\alpha_U}}^u \frac{\sigma_s^U(u') e^{-(u-u')}}{\sigma_t(u')} F(u') du' \\
 & + \frac{1}{1-\alpha_{Pb}} \int_{u-\ln\frac{1}{\alpha_{Pb}}}^u \frac{\sigma_s^{Pb}(u') e^{-(u-u')}}{\sigma_t(u')} F(u') du' \\
 & \left. + \frac{1}{1-\alpha_{Ox}} \int_{u-\ln\frac{1}{\alpha_{Ox}}}^u \frac{\sigma_s^{Ox}(u') e^{-(u-u')}}{\sigma_t(u')} F(u') du' \right\} \\
 & + [1 - P_O(u)] \sigma_t(u).
 \end{aligned} \tag{13}$$

As usual, all of the microscopic quantities refer to a cross section per absorber (thorium) atom. In this system, the total cross section is

$$\sigma_t(u) = \sigma_a^{Th}(u) + \sigma_s^{Th} + \sigma_a^U(u) + \sigma_s^U(u) + \sigma_p^{Pb} + \sigma_p^{Ox}. \tag{14}$$

The first flight collision probability is a function of $\sigma_t(u)$; ie,

$$P_O(u) = P_O(\sigma_t(u)) .$$

The subscripts are: "a", absorption; "s", total scattering; "p", potential scattering. The form and nomenclature of Equation (13) are slightly different from those used in the ZUT code. Nordheim's equations in lethargy space were developed from the energy dependent equations using the relationship

$$F(u) = F(E) . \quad (15)$$

Equation (13) is based upon

$$F(u) du = F(E) dE . \quad (16)$$

Nordheim also uses escape probabilities rather than collision probabilities in his equations, and defines α in a different manner. Nevertheless, consistent use of either form produces identical results.

In the nomenclature of Equation (13), the asymptotic collision density and the resonance integral are:

$$F_{as} = \sigma_p^{Th} + \sigma_p^U + \sigma_p^{Pb} + \sigma_p^{Ox} . \quad (17)$$

$$I_{Th} (N_{Th}; N_u^r; N_m) = \int F(u) \frac{\sigma_a^{Th}(u)}{\sigma_t(u)} du . \quad (18)$$

The second half of the interference calculation is performed by solving Equation (13) again, but this time with the resonance "switched off". This is effected by defining the appropriate cross sections as follows:

$$\sigma_a^U(u) = 0 . \quad (19)$$

$$\sigma_s^U(u) = \sigma_p^U . \quad (20)$$

$$\sigma_t(u) = \sigma_a^{Th}(u) + \sigma_s^{Th}(u) + \sigma_p^U + \sigma_p^{Pb} + \sigma_p^{Ox} . \quad (21)$$

A special option [16] was built into the ZUT code to permit the use of Equations (19), (20), and (21) for this second portion of the interference calculation. The resulting resonance integral is

$$I_{Th} (N_{Th}; N_U^p; N_m) = \int F(u) \frac{\sigma_a^{Th}(u)}{\sigma_t(u)} du \quad (22)$$

where $\sigma_t(u)$ refers to the quantity represented in Equation (21).

The calculated interference is then merely the difference between Equations (18) and (22), ie,

$$\Delta I_{Th}(N_{Th}; N_U; N_m) = I_{Th}(N_{Th}; N_U^r; N_m) - I_{Th}(N_{Th}; N_U^p; N_m). \quad (23)$$

An analogous effect caused by the thorium interfering with absorption in the uranium resonance can be calculated in a similar manner.

4. EXPERIMENTAL PROCEDURE FOR MEASUREMENT OF INTERFERENCE

Although the concept of interference appears to be realistic and meaningful, the task of measuring the quantity as it is defined above gives rise to several complicating factors. The elucidation of these complexities and the means by which they were circumvented or surmounted can probably be accomplished most efficiently by first outlining the general experimental approach and then describing in detail the exact methods used to extract the amount of interference as it was defined. The general method follows below.

Two sets of "calibration" samples were prepared [a], each containing compacted powder mixtures of a single absorber and a diluent, lead dioxide. The first set was made up of eight samples of uranium-238 dioxide/lead dioxide mixtures ranging in weight percent of uranium dioxide from 4.6 to 100 percent. The second set consisted of ten samples of thorium-232 dioxide/lead dioxide mixtures varying in weight percent of thorium dioxide from 0.25 to 100 weight percent. The weights of the ingredients of each sample are given in Table XXVI and the corresponding atom densities in Table I. Static reactivity measurements were performed on these samples in the ARMF-II, and from the gross measured reactivity worths, net reactivities due to the absorption alone were obtained according to the methods described in Section IV. In order to deduce effective resonance integrals from these absorption reactivities, it was necessary to compare them to a sample of known absorption cross section. Gold, with its rather accurately known infinite dilution resonance integral, was chosen as the standard. The constant of proportionality relating absorption reactivity and effective resonance integral was determined by successively performing reactivity and activation measurements on a sample of compacted gold powder and then comparing its activity to that of an "infinitely thin" gold foil. This measurement is described in more detail in Appendix C. From the above measurements, "calibration curves", with effective resonance integral as the ordinate and absorber atom density as abscissa, were drawn for both sets of absorber-diluent mixtures. The resonance integral measured for each single absorber sample was interpreted to be (in the nomenclature of Section II-2) $I_U(N_U; 0; N_{m1})$ or $I_{Th}(N_{Th}; 0; N_{m1})$ where N_{m1} refers to the set of scatterers present in the particular calibration sample. Least squares fitting techniques were then used to provide a functional relationship so that resonance integrals could be predicted for other samples within the range covered by the calibration curve.

[a] A description of the materials and sample preparation is given in Appendix B.

TABLE I

ATOM DENSITIES OF SINGLE ABSORBER SAMPLES

Sample	$(10^{21} \frac{N_U}{\text{atom/cm}^3})$	$(10^{21} \frac{N_{Th}}{\text{atom/cm}^3})$	$(10^{21} \frac{N_{Au}}{\text{atom/cm}^3})$	$(10^{21} \frac{N_{Pb}}{\text{atom/cm}^3})$	$(10^{21} \frac{N_{Ox}}{\text{atom/cm}^3})$
ThO ₂ -48	---	0.029866	---	13.40	26.85
-49	---	0.097664	---	13.11	26.42
-50	---	0.39096	---	12.70	26.18
-51	---	0.9926	---	11.56	25.10
-52	---	1.674	---	10.89	25.12
-53	---	2.677	---	9.647	24.64
-54	---	4.373	---	7.796	24.34
-55	---	6.757	---	5.212	23.94
-56	---	9.087	---	2.575	23.32
-57	---	10.97	---	---	21.92
UO ₂ -51	0.5335	---	---	12.35	25.76
-52	1.024	---	---	11.57	25.18
-53	2.102	---	---	10.39	24.98
-54	3.653	---	---	9.007	25.32
-55	5.382	---	---	7.191	25.14
-56	7.571	---	---	4.878	24.90
-57	10.24	---	---	2.707	25.90
-58	12.85	---	---	---	25.70
Au-51	---	---	0.044090	12.99	25.91
-52	---	---	0.21566	12.81	25.62
-53	---	---	0.29021	12.89	25.78
-54	---	---	0.56628	13.08	26.16
PbO ₂ -8	---	---	---	13.25	26.50
-41	---	---	---	13.18	26.36
-50	---	---	---	13.08	26.16

Two sets of "interference" samples were then prepared from mixtures of uranium dioxide, thorium dioxide, and lead dioxide. Because of the desirability of limiting the experimental program to a practical number of absorber compositions and configurations without making impossible the inference of any general conclusions about the interference phenomenon, the range of absorber concentration in these samples was chosen to span the values which might be of interest to a reactor designer. Each set consisted of mixtures containing a fixed ratio of uranium to thorium atom density, with the combined uranium dioxide/thorium dioxide percentage by weight ranging from approximately 15 to 100 percent.

Samples in the first set (samples 131 through 135) each had a uranium to thorium atom density ratio of one to three; in the second set (samples 311 through 315) the ratio was three to one. The atom densities and ingredient weights for each sample are listed in Table II, and Table XXVI, page 113, respectively.

TABLE II

ATOM DENSITIES OF MIXED URANIUM-238/THORIUM-232 SAMPLES

Sample	N_U/N_{Th}	$(10^{21} \frac{N_U}{\text{atom/cm}^3})$	$(10^{21} \frac{N_{Th}}{\text{atom/cm}^3})$	$(10^{21} \frac{N_U + N_{Th}}{\text{atom/cm}^3})$	$(10^{21} \frac{N_{Pb}}{\text{atom/cm}^3})$	$(10^{21} \frac{N_{Ox}}{\text{atom/cm}^3})$
U/T-131	0.3333	0.50210	1.5063	2.0084	10.907	25.83
-132	0.3333	0.99808	2.9944	3.9925	8.5301	25.05
-133	0.3333	1.5106	4.5320	6.0426	6.4404	24.97
-134	0.3333	2.3201	6.9606	9.2807	2.8651	24.29
-135	0.3333	3.00271	9.0082	12.1073	---	24.21
-311	3.000	1.5136	0.50454	2.0181	11.008	26.45
-312	3.000	2.9376	0.97919	3.9168	8.8776	25.59
-313	3.000	4.4426	1.4810	5.9236	6.6801	25.21
-314	3.000	6.5795	2.1931	8.7726	3.4649	24.48
-315	3.000	9.0793	3.0265	12.1058	---	24.21

Reactivity measurements were performed on these samples, and the corresponding resonance integrals of the mixtures were inferred from the results. The resonance integral measured for each sample was interpreted to be

$$I_{U,Th}^r(N_U; N_{Th}; N_{m2}) = \frac{N_U I_U(N_U; N_{Th}^r; N_{m2}) + N_{Th} I_{Th}(N_{Th}; N_U^r; N_{m2})}{N_U + N_{Th}} \quad (24)$$

where N_{m2} refers to the set of scatterers present in the interference sample.

From the two calibration curves constructed from the results of the single absorber measurements, values of $I_U(N_U; 0; N_{m1})$ and $I_{Th}(N_{Th}; 0; N_{m1})$ for each of the interference samples were predicted on the basis of its respective absorber atom densities. It was then possible to calculate for each sample the difference between the measured resonance integral of the mixture as given in Equation (24) and the value predicted from the weighted average of the single absorbers measured at their respective atom densities.

An attempt to deduce from those experiments the amount of interference present in the binary absorber systems immediately brought to light a difficulty common to almost all experimental investigations, namely, that the introduction of the factors necessary to cause the phenomenon of interest also creates additional effects which, although they may be consistent with already established theory and, therefore, to a certain extent predictable, tend to obscure the unknown or new effect. The corrections which must be applied to account for these unwanted effects are often of such magnitude so as to completely mask the

phenomenon under study. The relative success with which these corrections can be applied often depends upon the adequacy of the theory upon which they are based. If the uncertainty of these theoretical corrections exceeds the magnitude of the effect being investigated, then the experimenter must construct a different system in which the magnitudes of the necessary corrections are reduced to a suitable level.

In the specific case of the measurement of interference as described above, the "unwanted" or "extraneous" effect is the change in the resonance integral caused by differences between the scattering properties of the diluent and the added absorber. For a given absorbing lump containing a single absorber and a diluent and having a unique resonance integral, the substitution of a second absorber in place of the diluent would, in general, change the effective resonance integral of the lump even if interference did not occur. The change in resonance absorption would be caused by any difference between the potential scattering cross section and atomic weight of the diluent and the corresponding parameters for the second absorber. Because the current theories describing resonance absorption make it possible to predict the change caused by varying these two parameters, a theoretical correction could be applied to the experimental results. However, it must be remembered that the precisions of the deduced interference then depend on the adequacy of the theory. Fortunately, the degree of accuracy of the theoretical methods described in Section II-1 has been investigated rather thoroughly, allowing the necessary corrections to be made with confidence. Furthermore, lead dioxide was chosen as a diluent because its scattering properties are very similar to those of both uranium dioxide and thorium dioxide and, therefore, the corrections were relatively small.

For each interference sample, therefore, it was necessary to calculate a resonance integral correction, referred to as " δ ", to compensate for the differences between the scattering properties of that sample and the appropriate calibration samples. The value of δ was calculated for each sample as

$$\delta_{Th} = I_{Th}(N_{Th}; 0; N_{m1}) - I_{Th}(N_{Th}; N_U^P; N_{m2}) \quad (25)$$

$$\delta_U = I_U(N_U; 0; N_{m1}) - I_U(N_U; N_{Th}^P; N_{m2}) \quad (26)$$

where N_{m1} is the scatterer set present in the appropriate single absorber calibration sample, and N_{m2} is the scatterer set present in the interference sample. Using these values of δ , it was then possible to predict from the calibration samples the resonance integral that would be realized for each interference sample if no interference occurred in the mixture; ie,

$$\begin{aligned} I_{U,Th}^P(N_U; N_{Th}; N_{m2}) &= \frac{N_U [I_U(N_U; 0; N_{m2}) - \delta_U] + N_{Th} [I_{Th}(N_{Th}; 0; N_{m2}) - \delta_{Th}]}{N_U + N_{Th}} \\ &= \frac{N_U I_U(N_U; N_{Th}^P; N_{m2}) + N_{Th} I_{Th}(N_{Th}; N_U^P; N_{m2})}{N_U + N_{Th}} \end{aligned} \quad (27)$$

The difference between the value obtained from (27) and the measured resonance integral described in (24) was considered to be the measured interference for each binary absorber mixture; ie,

$$\Delta I_{U,Th} (N_U; N_{Th}; N_{m2}) = I_{U,Th}^r (N_U; N_{Th}; N_{m2}) - I_{U,Th}^p (N_U; N_{Th}; N_{m2}). \quad (28)$$

III. THEORY AND TECHNIQUE OF REACTIVITY MEASUREMENTS

1. HISTORY OF REACTIVITY MEASUREMENTS

The use of a thermal reactor as an instrument for measuring neutron cross sections dates to the first reactor itself. The first chain reacting pile -- known as CP-1 in its original version, assembled under the grandstands at the University of Chicago -- was equipped with a number of control and measuring devices which made possible the development of the static reactivity measurement technique. It is now a well-chronicled piece of history that the builders of this first pile, realizing that the most direct reliable measure of the effect of an impurity would be obtained by inserting a sample of the material into a pile and observing its effect on the multiplication of the neutrons sustaining the chain reaction, labeled this technique "the pile danger method" [a]. Foremost in their minds was the study of "dangerous" impurities which might prevent operation of a reactor. Since that time the danger method has been modified and improved to the point where it is used for a much wider variety of experiments. Already during the years 1945 and 1946 the dynamic method of measurement suggested by Wigner and Fermi, known as the pile-oscillator technique, increased the precision of the reactivity measurements by an order of magnitude [17].

During those early years the reactor was used as a tool to determine relative purities of various samples of such materials as uranium, graphite, aluminum, and others to be used in reactor construction. In such measurements, the cross section obtained did not have to be analyzed further to determine if the measured value was due entirely to absorption since the measured effect was the one pertinent for reactor construction purposes. Because of the development of analytical methods to provide rather accurate descriptions of the neutron population in a reactor, in addition to the availability of more sophisticated control and detection devices, it is now easier to separate the composite measured reactivity effects into their component parts as caused by absorption, scattering, fission, and so forth. These developments also now make it possible to use smaller quantities of materials in a wide variety of experimental conditions.

2. THE SERVO-CONTROLLED STATIC RE- ACTIVITY MEASUREMENT TECHNIQUE

Historically, reactivity measurement techniques have been divided into three major classifications according to the method by which the change in the reactor multiplication is determined. They are:

- (1) Period measurements
- (2) Static reactivity measurements using a calibrated control mechanism

[a] A lucid and comprehensive description of the development of this technique and the associated equipment is given by A. Langsdorf in Reference 17.

- (3) Oscillator measurements (including pile-oscillator and servo-oscillator techniques).

The method used in the ARMF-II falls into the second category. The reactor power is kept essentially constant by means of a servo-driven control rod which is actuated by the output of a compensated ion chamber located at the edge of the core (see Figure 4, page 24). Any reactivity change due to the insertion of a sample into the reactor will be exactly compensated by a motion of the control rod. If the rod is calibrated so as to give a correlation between reactivity and rod position, the net reactivity worth of the inserted sample can be determined directly from the rod position reading before and after insertion.

Some experimenters have resorted to the pile-oscillator^[17] or servo-oscillator^[18] reactivity measurement techniques, primarily because they are not so sensitive to the presence of reactivity effects of spurious or exterior origin. The reason for this can be seen more clearly by examining the sources of errors in reactivity measurements. The errors can actually be divided into two distinct types:

- (1) Those arising from "pile noise" due to fluctuations in the finite neutron population
- (2) Those due to noise of exterior origin, eg, a spurious drift caused by the effects of a gradual temperature change in the core.

Because there usually exists a practical upper bound on the reactor power at which convenient measurements can be performed, errors of type (1) place a limit on the precision of reactivity measurements even in the case where all exterior sources of error have been removed. However, the fact that the complete removal of spurious noise is generally not possible, and furthermore, that the behavior of this noise is often not well known, usually prevents the experimenter from reaching the ultimate limit defined by the pile noise. To reduce the drift noise the pile-oscillator technique is used. Here, the critical assembly is modulated by the sample of interest. By limitation of the band pass of the measurement system to a narrow area enclosing the frequency of modulation, much of the external noise (which usually has a preponderance of very low frequencies) is eliminated. A modification of the pile-oscillation technique is the servo-oscillator method in which a sensitive servo-driven control rod is utilized to maintain a fixed flux level in response to the periodic reactivity change caused by the sample. The pertinent difference between the pile-oscillator and the servo-oscillator is feedback; the former uses the open loop response of the reactor whereas the latter operates in a closed loop.

Due to the ability of each to limit the exterior noise effects on the measuring system, the dynamic measurement techniques will, in general, yield errors much closer to the precision limits dictated by the pile noise than will the static reactivity technique. The ARMF reactors are an exception. Because one of the prime goals in design and operation has been to minimize the noise level from all external sources, the actual measured reactivity fluctuations of the ARMF are close to those predicted from the inherent pile noise of the finite neutron population. The achievement of this precision is due primarily to (a) stabilization of the pool water temperature, (b) mechanical stability of the core structure, sample positioning devices, and all control equipment, and (c) isolation of the primary power supply to reduce interference from line noise.

3. DESCRIPTION OF THE ADVANCED REACTIVITY MEASUREMENT FACILITIES[a]

The Advanced Reactivity Measurement Facilities, ARMF-I and ARMF-II, are low-power reactors designed to perform precise static reactivity measurements. The facilities are used as tools in a wide range of experiments which include investigations in reactor physics as well as the measurement of reactor parameters and cross sections. Although the preponderance of the experimental work described in this report was performed in the ARMF-II, some of the preliminary orienting experiments were carried out in the ARMF-I. In fact, the initial core arrangement of the former was conceived primarily as a result of the insight gained through resonance integral measurements in the ARMF-I. In order to elucidate the advantages and disadvantages of the respective facilities, a brief description of both reactors follows with primary emphasis given to the achieved operating characteristics of the ARMF-II and their effect on the experimental results.

3.1 Advanced Reactivity Measurement Facility-I

The ARMF-I is a low-power, pool-type reactor situated in a building adjacent to the Materials Testing Reactor (MTR). Its design is similar to that of the old RMF [19], dismantled in April 1962, after seven years of operation in the MTR canal. The core consists of 24 plate-type elements and four partially loaded elements. The 28 elements are arranged around a six-inch-square, water-filled experimental hole as shown in Figure 2. Located at the corners of the water hole are the four partial elements, constructed so as to permit direct insertion of samples or hollow air-filled tubes into which samples may be inserted. Also shown in Figure 2 are the locations of the detectors and the safety and control elements. Fine control is accomplished by the regulating rod which is a rotating cylinder with a cadmium strip covering 126 degrees of its surface. Control instrumentation relates the servo-ionization chamber output to a reference voltage and, thereby, maintains the reactor at a constant power level by rotating the regulating rod in the appropriate manner to compensate for any perturbation introduced into the reactor. Experimental positions for samples are located throughout the water hole and also in the partial elements where the spectrum is considerably harder. Because many of the experiments performed in the ARMF-I require the ability to distinguish between thermal neutron absorption and fast neutron production in the same sample, the dimensions of the water hole were optimized to give markedly different weighting functions for fast and thermal neutrons in its center and corner areas.

3.2 Advanced Reactivity Measurement Facility-II

The original design of the ARMF-II provided for a virtual duplicate of the ARMF-I. However, because of the increasing interest in resonance absorption of neutrons, it was decided to load a core which would have high sensitivity to absorption rather than maximum separability of thermal and fast statistical weights. In this context, sensitivity refers to the net reactivity effect due to the introduction of one unit of absorption cross section into the system.

[a] A more complete description of these facilities can be found in IDO-17005, E. E. Burdick et al, The Advanced Reactivity Measurement Facilities; A Description and Performance Evaluation (October 1964).

Fig. 2 ARMF-I core diagram at midplane: loading 2-62.

A midplane section of the ARMF-II core loading^[a] and a view from the pool surface are shown in Figures 4 and 5. There are substantial differences between this loading and that described above for the ARMF-I. The large central water-filled experimental position has been replaced by an element containing a partial fuel loading and an air-filled hollow cadmium tube. A cutaway of the ARMF-II core is shown pictorially in Figure 6. Figure 7 shows the details of the cadmium tube containing one of the resonance integral capsules positioned for a measurement. Because the top portion of the sample location is shielded by a cadmium disc fixed to the end of the lucite plug, the capsule is completely surrounded by a cadmium filter. The purpose of the lucite plug, in addition to positioning and holding the capsule, is to simulate water above the cadmium portion of the tube. The lucite plug with a capsule attached is shown in Figure 8. Samples were lowered into position through a tube extending from the surface of the pool as in Figure 9.

The choice of this loading for the ARMF-II was based on the following considerations.

3.21 High Sensitivity to Resonance Absorption. Because the lower critical mass of this assembly causes the neutron balance to be more affected by the removal of a neutron, the absorption sensitivity of this system is increased over that of the ARMF-I. The attractiveness of a centrally located sample position is

[a] A more detailed description of the dimensions and nuclear characteristics of the core can be found in Appendix E.

enhanced by the fact that both the adjoint and real flux peak at this position, giving it maximum statistical weight. The partially loaded element was fabricated with curved fuel plates which left only a small space between the cadmium-shielded sample position and the fuel while still conserving the overall metal-water ratio. The design goal was to maximize the resonance flux in the sample position.

3.22 Low Sensitivity to Scattering. In general, a flux gradient exists at all points in a reactor core except at its center. Since there is a net flow of neutrons in the direction of the gradient, any scattering material placed in this gradient will obstruct the flow of neutrons and decrease the net flow. Ordinarily this decrease in leakage will increase the reactivity of the pile and, unless very accurate corrections are applied to the experimental results, a large scattering effect may completely obscure the measurement of an absorption cross section. Since this scattering effect is proportional to the product of the real and adjoint flux gradients, a sample position at the center of a symmetrical core loading should provide the minimum sensitivity to scattering. The ARMF-II core provides a high degree of symmetry which results in a high ratio of absorption to scattering sensitivity.

A series of experiments was performed to determine the optimum ARMF-II core loading on the basis of 3.21 and 3.22 in the foregoing. In addition to the experiments carried out in the loading of Figure 4, which was chosen as the best arrangement, measurements were performed in another loading (designated as loading 12-62) shown in Figure 3. The configuration was similar to that of Figure 4 except that the cadmium tube was surrounded by water instead of a partial fuel assembly. The results of these preliminary experiments are

Fig. 3 ARMF-II core diagram at midplane: loading 12-62.

Fig. 4 ARMF-II core diagram at midplane: loading 4-63.

given in Table III along with the corresponding results for the ARMF-I. Loading 4-63 was chosen as optimum on the basis of its high sensitivity to absorption, even though its sensitivity to aluminum scattering was higher than in loading 12-62. The aluminum mass in the sample or sample holder is usually so low that its reactivity worth is a small fraction of the gross effect of the sample.

4. DESCRIPTION OF REACTIVITY MEASUREMENTS IN THE ARMF-II -- SEQUENCE OF MEASUREMENTS: DATA EXTRACTION AND REDUCTION

The net reactivity worth of a given sample is arbitrarily defined as the reactivity increment which corresponds to the net change in reactivity rod position due to the insertion of the sample into the "empty" critical reactor. The correspondence between reactivity and regulating rod position is given by the rod calibration curve. For measurements described here, the term "empty"

Fig. 5 ARMF-II core as viewed from pool surface.

(abbreviated MT) refers to the reactor with no sample in the cadmium-shielded experimental position, and "sample-in" refers to the condition with the sample inserted. In literature dealing with reactor perturbation theory the empty reactor is more commonly denoted as the "reference reactor", but empty is probably more descriptive in the present work since, if the change in rod position is neglected, insertion of the sample into the empty experimental hole is the only perturbation introduced.

Fig. 6 Cutaway view of the ARMF-II core.

Fig. 7 Details of cadmium tube with sample capsule positioned.

Fig. 8 Lucite positioning plug with capsule attached.

Fig. 9 Sample and positioning plug being lowered into ARMF-II cadmium-shielded measurement location.

TABLE III

SCATTERING AND ABSORPTION SENSITIVITIES OF ARMF-I AND ARMF-II [a]

Material	ARMF-I	ARMF-II	
	Reactivity ($\mu\text{k/g}$)	Loading 12-62 Reactivity ($\mu\text{k/g}$)	Loading 4-63 Reactivity ($\mu\text{k/g}$)
Aluminum	0.211	0.099	0.478
Lead	0.064	0.036	0.085
Gold in Lead Matrix [b]	-22	-38	-90

[a] This table lists the sensitivities to scatterers and resonance absorbers for various loadings of the ARMF-I and ARMF-II. All measurements were made inside a hollow cadmium tube which had a wall thickness of 0.020 inch. The samples were hollow cylinders of 5-inch length, 0.860-inch diameter, and 0.030-inch wall thickness. All measurements were made with the shim rod in approximately the same position and in the same region of the regulating rod.

[b] The absorption cross section of the gold in the particular samples used was appreciably self-shielded. A higher sensitivity would be obtained with more dilute samples.

Because the "critical position" of the regulating rod may drift slightly due to gradual changes in parameters affecting the reactor's neutron population, sample-in readings are always sandwiched between empty readings so as to permit corrections for drift. A sample's net reactivity worth is then calculated on the basis of a linearly interpolated value of the empty reactor rod position at the midpoint time of the sample-in reading.

The ARMF-II is equipped with a digital averager which provides a means of extracting regulating rod position data in digital form. For a given reactor condition, eg, "sample-in", it accumulates a number of samples of rod position, derives an average, and prepares a single punch card record of several of these averages along with other data significant of the conditions under which the averages were taken. A more complete description of this data reduction technique can be found in IDO-17026, D. H. Gipson and R. A. Peereboom, DUZIT - ARMF Data Analysis Program for the IBM 7040 (October 1964).

On each data card there exist eight octal regulating rod position averages. After the octal averages from a card are converted to their corresponding decimal numbers r_i ($i = 1, 2, \dots, 8$), the average $\bar{r}$ and a standard deviation, s_r , are calculated.

$$\bar{r} = \sum_{i=1}^8 r_i / 8 \quad (29)$$

$$s_r = \sqrt{\frac{\sum_{i=1}^8 (r_i - \bar{r})^2}{7}} \quad (30)$$

$\bar{r}$ is then considered to represent the regulating rod position for the sample on that data card. For every data card, a reactivity ρ is calculated at the average rod position $\bar{r}$, in addition to t_m , the midpoint time for that data sampling [a]. Using the following notation:

$\bar{r}_{k_1}$ = ith MT rod position

$\bar{r}_{k_2}$ = (i + 1)th MT rod position

t_{k_1} = midpoint time of ith MT sampling

t_{k_2} = midpoint time of (i + 1)th MT sampling

and the following definitions:

$$B = \frac{(\bar{r}_{k_2} - \bar{r}_{k_1})}{(t_{k_2} - t_{k_1})}$$

$$A = \bar{r}_{k_1}$$

then the interpolation can be written

$$\bar{r}_k = A + B t_x \quad (31)$$

where

$\bar{r}_k$ = MT rod position at time t_x

t_x = midpoint time of sample-in reading for sample x

$t_{k_1} \leq t_x \leq t_{k_2}$

The net reactivity worth of sample x is then

$$\rho_x = \rho(\bar{r}_x) - \rho(\bar{r}_k) \quad (32)$$

where

$\rho(\bar{r}_x)$ = reactivity evaluated from the rod calibration function at

[a] A discussion of the technique used to calibrate the regulating rod is presented in Appendix A.

$r = \bar{r}_x$, the sample-in rod position for sample x
 $\rho(\bar{r}_k)$ = reactivity evaluated from the rod calibration function at
 $r = \bar{r}_k$.

Usually in the course of a series of measurements, a given sample is measured several times. The average net worth for that sample is calculated by

$$\bar{\rho}_x = \frac{\sum_{j=1}^J \rho_j}{J} \quad (33)$$

where J = number of measurements.

The standard deviation is given by

$$s_{\rho} = \sqrt{\frac{\sum_{j=1}^J (\rho_j - \bar{\rho})^2}{J - 1}} \quad (34)$$

In practice, the reactor drift was usually very slight, making it possible to sandwich at least three sample-in readings between each pair of empties without increasing the standard deviation. A typical sequence for three different samples A, B, and C, was

MT, A, B, C, MT, A, B, C, MT, A, B, C, MT, A, B, C, MT, A, B, C, MT.

Five was chosen as the number of repeats for each sample on the basis of a compromise between the time required for each additional measurement and the increased precision contributed by each repeat.

5. PRECISION OF ARMF-II REACTIVITY MEASUREMENTS

A description of the precision obtained in the ARMF-II static measurements is divided here into two categories: (a) Short-time reproducibility (5.1). Short-term refers to a measurement period lasting a day or less and usually involving less than ten repeats with the same sample. (b) Long-term reproducibility (5.2). Long-term refers to a measurement period of several days, weeks, or even months, and involves the comparison of net reactivity worths of a given sample determined at various intervals throughout this period. The two categories are discussed below.

5.1 Short-Term Precision

The following three quantities were used to evaluate the short-term precision of the ARMF-II:

(1) The statistic s_ρ , which represents the standard deviation of a particular sampling of reactivity data.

(2) The statistic $\bar{\rho}$ which represents the mean reactivity value obtained from the finite data sampling.

(3) σ_ρ , the true standard deviation of the universe being sampled.

$\bar{\rho}$ and s_ρ are defined in Equations (33) and (34). If it is assumed that σ_ρ , the true standard deviation for the universe of reactivity measurements on a particular class of sample configurations in a particular core configuration, is constant, then σ_ρ may be calculated from a sufficiently large number of values of s_ρ , each obtained for a particular sample. That is

$$\sigma_\rho = \sqrt{\frac{\sum_{i=1}^N (J_i - 1) s_{\rho_i}^2}{\sum_{i=1}^N J_i}} \quad (35)$$

where

s_{ρ_i} = standard deviation for J_i measurements of the i th sample

N = number of different samples measured.

Data were collected from all of the reactivity measurements performed in this work. The total number of measurements included, ie, $\sum_{i=1}^N J_i$, was approximately 660. The resulting standard deviation was

$$\sigma_\rho = 0.24 \mu k$$

The value of s_ρ obtained from each series of measurements on a given sample was tested according to the Dixon Outlier Test [20] to see if any outlying values should be rejected. Only a few data were rejected on this basis. Although mechanical tolerances have been reduced as far as possible, the major contribution to short-term errors probably arises from slight changes in the location of the sample or sample positioning plug during successive measurements.

5.2 Long-Term Precision

Long-term precision was checked by periodic measurements on a "standard" sample, ThO₂-57, over the duration of the experimental program. The gross reactivities and corresponding dates of measurements follow.

<u>Date</u>	<u>Gross Reactivity (μk)</u>
8/19/63	-264.51 $\pm$ 0.18
9/26	-263.81 $\pm$ 0.18
10/2	-263.67 $\pm$ 0.16
12/17	-263.01 $\pm$ 0.22
12/17-18	-262.89 $\pm$ 0.28
12/18	-263.00 $\pm$ 0.29
12/19	-262.99 $\pm$ 0.11
12/18	-263.00 $\pm$ 0.48
12/20	-262.92 $\pm$ 0.30
12/23	-263.01 $\pm$ 0.29
12/31	-263.14 $\pm$ 0.01

Between the measurements of August 19 and December 17, there occurred a downward drift in relative gross worth of approximately six-tenths of a percent. Although this drift was extremely small, the majority of the measurements performed before the latter date were repeated as a precautionary measure against the introduction of any systematic, slightly lower values. The old data were rejected. No detectable drift occurred between December 17 and the conclusion of the experimental program.

IV. EXPERIMENTAL AND THEORETICAL CORRECTIONS TO THE OBSERVED REACTIVITY DATA

Energy dependent perturbation theory [21, 22] states that the gross reactivity effect caused by the introduction of a sample into a reactor is the composite result of many diverse effects. By a judicious choice of sample materials and configurations, in addition to a careful selection of the reactor environment in which the measurement is performed, the experimenter can usually minimize the effects other than those he wishes to measure. However, to extract or deduce from observations the reactivity component due to one particular phenomenon, corrections based either on theory or supplementary experiments must usually be applied.

In order to deduce the net reactivity effect due to resonance absorption in the uranium and thorium samples, it was necessary to make several corrections to the measured data. Experimentally based corrections were made to account for the effects of the aluminum capsule, scattering of the absorber, absorption and scattering of lead dioxide, and uranium-235 fission and capture. Theoretical corrections were made to account for deviation of the ARMF flux from a $1/E$ spectrum throughout the resonance range, energy dependence of the adjoint flux, and the reactivity effects of fast fissions. In the interpretation of all reactivity measurements, the principle of "additivity" was assumed to be valid; that is, the gross measured reactivity was considered to be a superposition of all the individual effects. Each of the effects, and the appropriate correction, is discussed in detail below. A summary of the measured gross reactivity values, the reactivity corrections, and the inferred net absorption reactivities for each sample, is given in Table XI, page 53.

1. EXPERIMENTALLY BASED CORRECTIONS

1.1 Aluminum Capsules

The reactivity worth of each aluminum capsule was measured before the capsules were loaded with powder. Although aluminum has a very small positive reactivity effect inside the cadmium, the capsules were fabricated with as little mass as possible so as to minimize their effect. This effect, approximately $0.2 \mu\text{k/g-Al}$, was between 7 and $8 \mu\text{k}$ for the majority of the capsules.

1.2 Absorption and Scattering in Lead Dioxide; Scattering in the Absorber

Lead dioxide was chosen as a diluent for two reasons: (a) low absorption cross section and (b) scattering properties similar to those of uranium dioxide and thorium dioxide. The total effect of absorption and scattering was determined by a reactivity measurement on a pure lead dioxide sample. The absorption reactivity was calculated to be

$$\left[\rho_a \right]_{\text{Pb}} = C \bar{w}_{\text{Pb}} N_{\text{Pb}} V I_{\infty}^{\text{Pb}} \quad (36)$$

where

C = weighting function scale factor [a]

$\bar{W}_{Pb}$ = absorption weighting function for lead

N_{Pb} = atom density of lead in sample, 10^{24} atoms/cm³

V = sample volume = 59.06 cm³

I_{∞}^{Pb} = infinite dilution resonance integral of lead = 0.1 barns [b].

Evaluation of Equation (36) gives

$$\left[\rho_a \right]_{Pb} = - 116.9 N_{Pb} \mu k \quad . \quad (37)$$

The scattering reactivity of the pure lead dioxide sample was determined by subtracting the calculated absorption effect from the measured sum of scattering and absorption. The average scattering effect, based on measurements with three different lead dioxide samples, was

$$\frac{\rho_s}{\Sigma_s} = 52.80 \mu k - cm \quad . \quad (38)$$

The calculated scattering reactivities for uranium dioxide and thorium dioxide were based on the lead dioxide results above. Because the elastic slowing down kernels of these three materials are very similar, their scattering effects should be approximately the same. It was realized that the inelastic scattering cross sections of lead, thorium, and uranium may not be identical. Because this effect is probably very small, and since any theoretical correction would be of questionable validity, no corrections were made for differences in inelastic scattering. The macroscopic potential scattering cross sections of each sample and its corresponding reactivity due to scattering are given in Table IV.

1.3 Uranium-235 Fission and Capture

Because the uranium dioxide used in all measurements was depleted to 350 ppm uranium-235, this correction was relatively small. To determine its magnitude, reactivity measurements were made on two depleted uranium dioxide samples which were identical in all respects except that one of them was spiked with a small amount of highly enriched uranium-235 dioxide. Any difference in the reactivity worths of these two samples was then due to the uranium-235 additive. The concentrations and reactivity worths of these samples are tabulated in Tables V and VI. The net reactivity effect of the uranium-235 is

$$\left(\frac{\rho}{M} \right)_{U^{235}} = 19.70 \frac{\mu k}{g} \quad (39)$$

[a] C and $\bar{W}$ are discussed in detail in Section IV-2.

[b] This value obtained from Reference 23 agrees well with the resonance integral deduced from the 2200 meter/sec cross section of lead.

TABLE IV
MACROSCOPIC POTENTIAL SCATTERING CROSS
SECTIONS AND REACTIVITIES DUE TO SCATTERING

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Sample	Σ_p^{Ox}	Σ_P^U	Σ_p^{Th}	Σ_p^{Pb}	Σ_p^{Au}	Σ_p^{Total}	ρ_s
ThO ₂ -48	0.10205	---	0.00036	0.15139	---	0.25380	13.40
49	0.10040	---	0.00117	0.14818	---	0.24975	13.19
50	0.09948	---	0.00469	0.14350	---	0.24767	13.08
51	0.09538	---	0.01191	0.13063	---	0.23792	12.56
52	0.09546	---	0.02009	0.12306	---	0.23861	12.60
53	0.09363	---	0.03212	0.10901	---	0.23476	12.40
54	0.09249	---	0.05248	0.08810	---	0.23307	12.31
55	0.09097	---	0.08108	0.05890	---	0.23095	12.19
56	0.08862	---	0.10904	0.02910	---	0.22676	11.97
57	0.08337	---	0.13164	---	---	0.21501	11.35
UO ₂ - 51	0.09789	0.00507	---	0.13956	---	0.24252	12.81
52	0.09568	0.00973	---	0.13074	---	0.23615	12.47
53	0.09492	0.01997	---	0.11741	---	0.23230	12.27
54	0.09622	0.03470	---	0.10178	---	0.23270	12.29
55	0.09553	0.05112	---	0.08126	---	0.22791	12.03
56	0.09462	0.07192	---	0.05512	---	0.22166	11.70
57	0.09842	0.09728	---	0.03059	---	0.22629	11.95
58	0.09766	0.11208	---	---	---	0.21974	11.60
U/T-131	0.09825	0.00477	0.01808	0.12325	---	0.24435	12.90
132	0.09517	0.00948	0.03593	0.09639	---	0.23697	12.51
133	0.09487	0.01435	0.05438	0.07278	---	0.23638	12.48
134	0.09231	0.02204	0.08352	0.03238	---	0.23025	12.16
135	0.09201	0.02876	0.10896	---	---	0.22973	12.13
311	0.10052	0.01438	0.00605	0.12665	---	0.24760	13.07
312	0.09723	0.02791	0.01175	0.10032	---	0.23721	12.52
313	0.09579	0.04220	0.01777	0.07549	---	0.23125	12.21
314	0.09301	0.06251	0.02632	0.03915	---	0.22099	11.67
315	0.09201	0.08625	0.03632	---	---	0.21458	11.33

TABLE IV (Cont.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Sample	Σ_p^{Ox}	Σ_p^U	Σ_p^{Th}	Σ_p^{Pb}	Σ_p^{Au}	Σ_p^{Total}	ρ_s
Au - 51	0.09847	---	---	0.14641	0.00036	0.24524	12.95
52	0.09737	---	---	0.14478	0.00201	0.24416	12.89
53	0.09796	---	---	0.14565	0.00270	0.24631	13.00
54	0.09938	---	---	0.14776	0.00527	0.25241	13.33
PbO ₂ - 8	0.09283	---	---	0.13804	---	0.23087	12.19
- 41	0.10015	---	---	0.14891	---	0.24906	13.26
- 50	0.09941	---	---	0.14780	---	0.24721	13.99
U ²³⁵ 1	0.12862	0.16078	---	---	---	0.28940	15.28
2	0.12911	0.16139	---	---	---	0.29050	15.34

Note: Σ_p^{Total} = total macroscopic potential scattering cross section obtained from the sum of columns 2 through 6.

Macroscopic cross sections were calculated using the atom densities in Tables I and II, and the following microscopic potential scattering cross sections.

$$\sigma_p^{Ox} = 3.8 \text{ b}$$

$$\sigma_p^{Pb} = 11.3 \text{ b}$$

$$\sigma_p^U = 9.5 \text{ b}$$

$$\sigma_p^{Au} = 9.3 \text{ b}$$

$$\sigma_p^{Th} = 12.0 \text{ b}$$

Note: Columns 2 - 7 have units of cm^{-1} .
Column 8 has units of μk .

TABLE V

WEIGHTS AND CONCENTRATIONS OF SAMPLES FOR URANIUM-235 CORRECTION

Sample	Mass of Depleted UO ₂ (0.22 w/o U-235) (grams)	Mass of Enriched UO ₂ (93.2 w/o U-235) (grams)	Total Mass of U-235 (grams)	N_{U-235} (10^{21} atom/cm ³)
U-235-1	448.07	0.0	0.8689	0.037706
U-235-2	448.12	1.6295	2.2055	0.09570

TABLE VI

REACTIVITY WORTH OF SAMPLES FOR URANIUM-235 CORRECTION

Sample	Capsule	ρ_g (μk)	ρ_c (μk)	ρ_s (μk)	ρ_{ff} (μk)	ρ_a (μk)	ρ_{U-235} (μk)
U-235-1	46	-293.14	7.95	15.28	167.61	-501.10	17.12
U-235-2	48	-264.88	9.59	15.34	168.04	-501.10	43.45

Note: ρ_g = gross reactivity of sample
 ρ_c = reactivity of aluminum capsule
 ρ_s = scattering reactivity of sample
 ρ_{ff} = reactivity due to fast fissions in fertile material
 ρ_a = reactivity due to absorption
 ρ_{U-235} = reactivity due to uranium-235

or

$$\left(\frac{\rho}{N}\right)_{U235} = 7.687 \times 10^3 \frac{\mu k}{10^{24} \text{ atoms}} \quad (40)$$

2. THEORETICALLY BASED CORRECTIONS

2.1 Deviation of Real Flux From 1/E Behavior; Energy Dependence of Adjoint Flux.

Because the energy distribution of the capture of neutrons is a function of the composition and configuration of each particular sample which is being measured, it is difficult to relate the absorption reactivity worth of a sample to its effective absorption cross section. To do this properly, information describing both the real adjoint flux in the sample position must be available. If these flux distributions are well known, it is possible to relate the results of reactivity measurements to the effective resonance integral of the sample.

2.11 Theory. From energy dependent perturbation theory, the relationship between the properties of a purely absorbing sample and its absorption reactivity effect is

$$\rho_a = CNV \int \int \int_{\vec{r} \vec{\Omega} u} \delta \sigma_a(u) \phi'(\vec{r}, \vec{\Omega}, u) \phi^+(\vec{r}, \vec{\Omega}, u) d^3\vec{r} d\vec{\Omega} du \quad (41)$$

where

C = a constant related to the fission source of the particular reactor under consideration

N = atom density of absorber atoms in sample, atoms/cm³

V = volume of absorber lump, cm³

$\delta \sigma_a(u)$ = lethargy dependent cross section representing the difference between the reference sample cross section and the cross section of the sample of interest. In most practical cases the reference sample is merely air, having essentially zero absorption cross section

$\phi'(\bar{r}, \bar{\Omega}, E)$ = perturbed angular flux in the sample

$\phi^+(\bar{r}, \bar{\Omega}, E)$ = angular importance function (adjoint flux) in the reference sample. In general, the adjoint or importance function is the function obtained by solving the reactor equation arising from application of the appropriate adjoint operator [22].

From ρ_a , the absorption reactivity worth of the sample, the effective resonance integral of the absorber may be deduced. The effective resonance integral may be expressed as

$$I_{\text{eff}} = \frac{\int \int \int \sigma_a(u) \phi'(\bar{r}, \bar{\Omega}, u) d^3\bar{r} d\bar{\Omega} du}{\phi_{as}} \quad (42)$$

where ϕ_{as} is the flux present in the absence of the absorber resonance. It is implicitly assumed in formulating the definition of the resonance integral that an asymptotic "1/E" flux does exist in the absence of absorption. In reality, however, the flux in the sample position may differ markedly from a 1/E behavior. It is clear that one must have a description of this deviation in order to obtain a resonance integral from the experimental results.

Consider the reactivity worth of the sample in a reactor in which the following conditions are satisfied for the unperturbed real and adjoint fluxes.

$$\phi_I(u) = \text{constant} = \phi_I \quad (43)$$

$$\phi_I^+(u) = \text{constant} = \phi_I^+ \quad (44)$$

where the subscript "I" indicates the "ideal fluxes". The "ideal reactivity" may be expressed as

$$\rho_I = CNV \int \phi_I^+ \phi_I(u) \sigma_a(u) du \quad (45)$$

These fluxes are considered to be averaged over the spatial and angular coordinates. One can further write

$$\rho_I = \text{CNV } \phi_I^+ \phi_I \int_u \frac{\phi'(u) \sigma_a(u) du}{\phi_I} = \text{CNV } \phi_I^+ \phi_I I_{\text{eff}} \quad (46)$$

In a real reactor, the unperturbed fluxes deviate from this ideal behavior. The actual measured reactivity will be

$$\rho_M = \text{CNV} \int_u \phi^+(u) \left[\frac{\phi(u)}{\phi_I} \right] \phi'(u) \sigma_a(u) du \quad (47)$$

Here it is assumed that the perturbed real flux may be expressed as the product of a factor $\frac{\phi(u)}{\phi_I}$ indicating the deviation of the unperturbed flux from the ideal behavior and a factor ϕ' which is the perturbed flux of the ideal system.

Rearranging,

$$\rho_M = \text{CNV } \phi_I^+ \phi_I \int_u \left[\frac{\phi^+(u) \phi(u)}{\phi_I^+ \phi_I} \right] \frac{\phi'(u) \sigma_a(u) du}{\phi_I} \quad (48)$$

Combining the above expression with Equation (46) gives

$$\rho_I = \rho_M \frac{\int_u \frac{\phi'(u) \sigma_a(u) du}{\phi_I}}{\int_u \left[\frac{\phi^+(u) \phi(u)}{\phi_I^+ \phi_I} \right] \frac{\phi'(u) \sigma_a(u) du}{\phi_I}} \quad (49)$$

More directly, the effective resonance integral may be deduced from the measured reactivity,

$$I_{\text{eff}} = \int_u \frac{\phi'(u) \sigma_a(u) du}{\phi_I} = \frac{\rho_M}{\text{CNV}} \frac{\int_u \frac{\phi'(u) \sigma_a(u) du}{\phi_I}}{\int_u \left[\frac{\phi^+(u) \phi(u)}{\phi_I^+ \phi_I} \right] \frac{\phi'(u) \sigma_a(u) du}{\phi_I}} \quad (50)$$

If the weighting function is defined as

$$W(u) = \phi^+(u) \phi(u) \quad (51)$$

then

$$\bar{W} = \frac{\int_u W(u) \phi'(u) \sigma_a(u) du}{\int_u \phi'(u) \sigma_a(u) du} \quad (52)$$

where $\bar{W}$ is the absorption weighted average of the weighting function. For a sample designated as x, the resonance integral can then be written

$$I_x = \frac{\rho_x^x M}{C \bar{W}_x N_x V_x} \quad (53)$$

Clearly, in order to calculate an approximate value of $\bar{W}_x$, it is necessary to know the behavior of $\phi^+(u)$, $\phi(u)$, $\sigma_a(u)$, and $\phi'(u)$. These must be determined from additional calculations.

Similar equations may be written for an activation experiment in the ideal and actual reactor. The count rates corresponding to the saturated activities of the sample may be expressed as

$$A_I = \epsilon NV \int_u \phi'(u) \sigma_a(u) du = \epsilon NV \phi_I \int_u \frac{\phi'(u) \sigma_a(u) du}{\phi_I} = \epsilon NV \phi_I I_{\text{eff}} \quad (54)$$

$$A_M = \epsilon NV \int_u \left[\frac{\phi(u)}{\phi_I} \right] \phi'(u) \sigma_a(u) du \quad (55)$$

where the source term and counter efficiencies are included in ϵ .

$$A_I = A_M \frac{\int_u \frac{\phi'(u) \sigma_a(u) du}{\phi_I}}{\int_u \left[\frac{\phi(u)}{\phi_I} \right] \frac{\phi'(u) \sigma_a(u) du}{\phi_I}} = \frac{\phi_I}{\bar{\phi}} A_M \quad (56)$$

The average flux, $\bar{\phi}$, is defined as

$$\bar{\phi} = \frac{\int_u \phi(u) \phi'(u) \sigma_a(u) du}{\int_u \phi'(u) \sigma_a(u) du} \quad (57)$$

Then it follows that, for a sample of material x,

$$I_x = \frac{A_M^x}{\epsilon \bar{\phi}_x N_x V_x} \quad (58)$$

A further measurement may be performed in the same reactor on an infinitely dilute sample of the same material.

$$A_M^\infty = \epsilon N_\infty V_\infty \bar{\phi}_\infty I_\infty \quad (59)$$

where

$$\bar{\phi}_\infty = \frac{\int_u \phi(u) \sigma_a(u) du}{\int_u \sigma_a(u) du} \quad (60)$$

and

$$I_\infty = \int_u \sigma_a(u) du \quad (61)$$

Equations (58) and (59) can be combined to give

$$I_x = \frac{A_M^x}{A_M^\infty} \frac{N_\infty V_\infty}{N_x V_x} \frac{\bar{\phi}_\infty}{\bar{\phi}_x} I_\infty \quad (62)$$

Equating the reactivity and activation values of I_x leads to

$$\frac{\rho_M^x}{\epsilon N_x V_x \bar{W}_x} = \left[\frac{A_M^x}{A_M^\infty} \frac{N_\infty V_\infty}{N_x V_x} \right] \frac{\bar{\phi}_\infty}{\bar{\phi}_x} I_\infty \quad (63)$$

$$C \bar{W}_x = \rho_M^x \frac{\bar{\phi}_x}{\bar{\phi}_\infty} \frac{A_M^\infty}{A_M^x} \frac{1}{N_\infty V_\infty I_\infty} \quad (64)$$

If $\phi(u)$ is known, then for an appropriate calibration material such as gold, $\bar{\phi}_x$ and $\bar{\phi}_\infty$ may be calculated for a thick gold sample x and an infinitely dilute sample, respectively. These quantities, in combination with the corresponding measured values of ρ_M^x , A_M^x , and A_M^∞ , make possible the determination of $C \bar{W}_x$ from Equation (64) for a particular sample, x . This experimentally determined quantity shall be referred to as $C \bar{W}_x^{\text{Exp}}$. A corresponding value, $\bar{W}_x^{\text{Calc}}$, can be

calculated from Equation (52). On the assumption that $\bar{W}_x^{\text{Calc}}$ and $\bar{W}_x^{\text{Exp}}$ differ only by a constant scale factor, this factor C can be determined from

$$C = \frac{C \bar{W}_x^{\text{Exp}}}{\bar{W}_x^{\text{Calc}}} \quad (65)$$

This value of C is then assumed to be representative of the particular reactor configuration and valid for the interpretation of similar reactivity measurements on materials other than the calibration material. For example, the resonance integral of a sample y would be deduced from reactivity measurements as

$$I_y = \frac{\rho_M^y}{N_y V_y C \bar{W}_y^{\text{Calc}}} \quad (66)$$

where $\bar{W}_y^{\text{Calc}}$ is determined in the same manner as $\bar{W}_x^{\text{Calc}}$. (For convenience, the superscript "Calc" will be omitted in future sections.) The calculation of $\bar{\phi}$ and $\bar{W}$ from the results of an S_n multigroup transport calculation is discussed later in this section. The measurement of C is described in Appendix C.

The above method provides a means of interpreting reactivity measurements which is superior to the more usual "boron calibration". Although boron does have the advantage that its well-known $1/v$ absorption cross section brings about the absorption of neutrons over a relatively broad range of energies and, therefore, for a sample of given reactivity worth, will exhibit less self-shielding than a resonance absorber, it does have one overwhelming disadvantage. Because such a large fraction of its resonance integral comes from the energy range just above "cadmium cutoff", the epicadmium effective resonance integral of boron is strongly dependent upon the cadmium cutoff energy, E_{Cd} . More explicitly,

$$I_{\text{Boron}} = 2 \sigma_{2200} \sqrt{\frac{E_{2200}}{E_{\text{Cd}}}} = \frac{K}{(E_{\text{Cd}})^{\frac{1}{2}}} \quad (67)$$

Although methods are available for calculating E_{Cd} , any errors inherent in these techniques would seem to be most serious in the case of a $1/v$ absorber.

2.12 Calculation of the Average Weighting Function $\bar{W}_x$ and the Average Flux $\bar{\phi}_x$. As shown in Equation (66), the average weighting factor $\bar{W}_x$ must be calculated for a sample x before an effective resonance integral can be deduced from its measured absorption reactivity, ρ_x . In addition, Equation (57) states that the calculated average flux, $\bar{\phi}_x$, is required before a resonance integral can be inferred from the observed count rate of an activated sample. For each of the thorium, uranium, and gold resonance integral samples described in Section II, the appropriate quantities $\bar{W}$ and/or $\bar{\phi}$ were calculated according to Equations (52) and (57).

In obtaining these averages, contributions to the measured resonance integral were regarded as coming from four ranges of the spectrum as follows:

- (1) $E_4 \leq E \leq E_3$: low energy absorption occurring between an arbitrary energy point on the low energy wing of the first resonance and the energy at which $\phi(u)$ and $\phi(u) \phi^+(u)$ become negligibly small.
- (2) $E_3 \leq E \leq E_2$: absorption in the resolved region.
- (3) $E_2 \leq E \leq E_1$: absorption in the unresolved region below 32 keV.
- (4) $E_1 \leq E \leq \infty$: absorption in the region above 32 keV.

The energy limits of each range depend, of course, on the particular absorber being considered. The energy limits (and the corresponding lethargy values) for uranium, thorium, and gold are in Table VII.

If the effective resonance cross sections are known, a value of I_1 can be calculated for each of the four ranges for every sample. If, in addition, the energy

TABLE VII
ENERGY AND LETHARGY LIMITS OF SPECTRUM
RANGES FOR URANIUM-238, THORIUM-232, AND GOLD-197

Energy and Lethargy Limits [a]	Uranium-238	Thorium-232	Gold-197
u_1	5.74	5.74	5.74
E_1	32.0 keV	32.0 keV	32.0 keV
u_2	9.13	9.13	9.26
E_2	1000.0 eV	1000.0 eV	940.0 eV
u_3	14.856	13.142	15.000
E_3	3.533 eV	19.623 eV	3.06 eV
u_{cd}	16.812	16.812	16.812
E_{cd}	0.5 eV	0.5 eV	0.5 eV
u_4	17.50	17.50	17.50
E_4	0.251 eV	0.251 eV	0.251 eV

[a] Each range and its limits are discussed above.

dependence of the unperturbed real and adjoint fluxes is known, $\bar{W}_z$ and $\bar{\phi}_z$ can be calculated for each of the ranges in accordance with Equations (52) and (57). The calculation of the effective cross sections of uranium, thorium, and gold is described in Section V. The fluxes were obtained from the S-XI multigroup transport study described in Appendix E. The values of $\bar{W}$ and $\bar{\phi}$ for each of the four ranges were combined to give an average weighting function for the entire spectrum, ie,

$$\bar{W}_{\text{Avg}} = \frac{\sum_{z=1}^4 \bar{W}_z I_z}{\sum_{z=1}^4 I_z} \quad (68)$$

$$\bar{\phi}_{\text{Avg}} = \frac{\sum_{z=1}^4 \bar{\phi}_z I_z}{\sum_{z=1}^4 I_z} \quad (69)$$

where $\bar{\phi}_z$ and $\bar{W}_z$ are the average flux and average weighting functions for the z th range, and I_z is the effective resonance integral for that range. Only $\bar{W}$ was needed for the uranium and thorium samples; both $\bar{\phi}$ and $\bar{W}$ were required to interpret the activation and reactivity measurements on the gold samples. The final values for $\bar{W}_{\text{Avg}}$ and $\bar{\phi}_{\text{Avg}}$ are given in Tables VIII and IX. A smooth curve drawn through the average values of $\bar{W}$ for each of the single absorber samples is shown in Figure 11, page 47. The decrease in $\bar{W}$ for high thorium concentration is caused by the increasing fractions of the total resonance integral contributed by low energy absorption.

The values of I_z are presented in Table XV, page 63, in Section V. Each of the individual ranges is discussed below.

(a) Low Energy Range. Because of the rapid increase of the cadmium cross section in the energy range below a few electron-volts, both the real and the adjoint flux fall off sharply from their more or less constant behavior in the resonance portion of the spectrum. The dropoff of the flux $\phi(u)$, is clearly attributable to the attenuation by cadmium of the neutron flux passing through the cadmium tube into the sample position. A further reason for this non-constant flux behavior is the decrease of neutron current impinging on the outer cadmium surface; that is, fewer neutrons are able to pass through the tube, be reflected from the surrounding medium, and reenter the sample region. The results of the S-XI calculation (Figure 40, page 141) show this flux diminution quite vividly down to an energy of 0.414 eV which was the upper limit of the thermal group. The flux below this energy was calculated by means of the CADOFF-3 code of Brown and Connolly [24]. The flux obtained from CADOFF-3, shown in Figure 10, page 46, was scaled so as to equal to the S-XI flux at an energy of 0.470 eV.

TABLE VIII

CALCULATED AVERAGE WEIGHTING FUNCTIONS
FOR SINGLE ABSORBER SAMPLES [a]

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Sample	$\bar{W}$ Low Energy Range ($E_{Cd} = 0.251$ eV)	$\bar{W}$ [b] Resolved Range	$\bar{W}$ [b] 4.91 eV Resonance of Gold-197	$\bar{W}$ Unresolved Range	$\bar{W}$ Fast Capture Range	Average $\bar{W}$ for all Ranges
Thorium-232						
Infinite Dilution	0.4636	1.0003	---	1.098	2.107	1.0002
ThO ₂ -48	0.4636	1.0010	---	1.099	2.107	1.0007
-49	0.4636	1.0018	---	1.099	2.107	1.0014
-50	0.4636	1.0044	---	1.099	2.107	1.0036
-51	0.4636	1.0075	---	1.100	2.107	1.0052
-52	0.4636	1.0093	---	1.103	2.107	1.0064
-53	0.4636	1.0108	---	1.103	2.107	1.0062
-54	0.4636	1.0119	---	1.107	2.107	1.0062
-55	0.4636	1.0123	---	1.107	2.107	1.0036
-56	0.4636	1.0128	---	1.112	2.107	1.0025
-57	0.4636	1.0129	---	1.112	2.107	1.0011
Uranium-238						
Infinite Dilution	0.5122	0.97073	---	1.096	2.178	0.9733
UO ₂ -51	0.5122	0.97682	---	1.096	2.178	0.9817
-52	0.5122	0.97890	---	1.100	2.178	0.9885
-53	0.5122	0.98085	---	1.100	2.178	0.9936
-54	0.5122	0.98193	---	1.105	2.178	0.9977
-55	0.5122	0.98243	---	1.105	2.178	1.0003
-56	0.5122	0.98265	---	1.110	2.178	1.0027
-57	0.5122	0.98288	---	1.110	2.178	1.0053
-58	0.5122	0.98293	---	1.113	2.178	1.0068
Gold-197						
Infinite Dilution	0.5650	1.0307	0.9277	1.0703	0.0	0.9163
Au-51	0.5650	1.0307	0.9277	1.0703	0.0	0.9080
-52	0.5650	1.0307	0.9277	1.0703	0.0	0.8896
-53	0.5650	1.0307	0.9277	1.0703	0.0	0.8842
-54	0.5650	1.0307	0.9277	1.0703	0.0	0.8690

[a] The energy limits of each range are given in Table VII.

[b] The large 4.91 eV resonance of gold-197 was treated separately in column (4).

TABLE IX
CALCULATED AVERAGE REAL FLUXES IN GOLD SAMPLES [a]

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Sample	$\bar{\phi}$ Low Energy Range ($E_{Cd} = 0.251$ eV)	ϕ [b] Resolved Range	$\bar{\phi}$ [b] 4.91 eV Resonance of Gold-197	$\bar{\phi}$ Unresolved Range	$\bar{\phi}$ Fast Capture Range	Average $\bar{\phi}$ for all Ranges
Gold-197 Infinite Dilution	0.7140	1.0278	0.9865	1.0428	0.0	0.9761
Au-51	0.7140	1.0278	0.9865	1.0428	0.0	0.9688
-52	0.7140	1.0278	0.9865	1.0428	0.0	0.9534
-53	0.7140	1.0278	0.9865	1.0428	0.0	0.9494
-54	0.7140	1.0278	0.9865	1.0428	0.0	0.9370

[a] The energy limits of each range are given in Table VII.

[b] The large 4.91 eV resonance of gold-197 was treated separately in column 4.

The dropoff of the adjoint flux, $\phi^+(u)$, is even more substantial. This can be partially attributed to the fact that a neutron in the tube, even if not absorbed in the sample, runs the risk of being captured during its outward passage through the cadmium wall. Furthermore, the importance of resonance neutrons at energies above the upper wing of the cadmium resonance can be decreased because some of these neutrons may escape from the tube, undergo moderation in the surrounding fuel-water medium, and then be reflected back into the cadmium where they are finally captured. In general then, a correction must be made to account for the dropoff of both $\phi^+(u)$ and/or $\phi(u)$. In activation measurements, only $\phi(u)$ is involved, while for reactivity measurements, the appropriate weighting factor is $W(u) = \phi(u) \phi^+(u)$. It might be said that in reactivity measurements the neutrons suffer "double jeopardy"; that is, not only is their ability to enter the sample by passage through the cadmium a factor of interest, but also their chance of escaping back out through the cadmium and contributing further to the neutron multiplication.

The use of the absorption-weighted average flux, $\bar{\phi}$, or the absorption-weighted average weighting function, $\bar{W}$, replaces the effective cadmium cutoff energy as a means of correcting measured resonance integrals for deviation of $\phi(u)$ and $\phi^+(u)$ from a nonconstant behavior at low energies. If $\bar{\phi}$ and $\bar{W}$ are calculated over the entire range of the spectrum in which there is a contribution to the measured activity or reactivity, the corrected value of the resonance integral corresponds to that range also. On the basis of the results of a CADOFF calculation for the case of a cadmium cylindrical filter with 20-mil wall thickness, 0.251 eV was chosen as the energy E_4 at which $\phi(u)$ and $W(u)$ are negligibly small. Because none of the absorbers studied in this investigation

Fig. 10 Low energy flux and weighting function in the ARMF-II cadmium-shielded measurement location as determined from CADOFF-3 calculation.

Fig. 11 Average weighting function for uranium dioxide/lead dioxide and thorium dioxide/lead dioxide samples.

have extremely low-lying resonances, the choice of the correct value of E_4 is not a critical one. All of the experimental resonance integrals in this work were further corrected to an arbitrary cutoff of 0.5 eV by subtraction of the resonance integral contribution between 0.5 and 0.251 eV.

The quantities $\bar{\phi}$ and $\bar{W}$ are calculated on the assumption that no self-shielding occurs in this low energy range; that is, the sample does not perturb the flux $\phi(u) = \phi'(u)$. On the basis of this assumption

$$\bar{\phi} = \frac{\int_{u_3}^{u_4} \phi(u) \sigma_a(u) du}{\int_{u_3}^{u_4} \sigma_a(u) du} \quad (70)$$

$$\bar{W} = \frac{\int_{u_3}^{u_4} W(u) \sigma_a(u) du}{\int_{u_2}^{u_4} \sigma_a(u) du} \quad (71)$$

The expressions used for the absorption cross sections are those given in Section V and the boundaries of the low energy range are those of Table VII. A special code INTEGR [25] was programmed to perform the necessary integration in Equations (70) and (71) using the multigroup S-XI fluxes and multigroup fluxes constructed from the very low energy results of CADOFF-3. A gaussian integration scheme was used to calculate $\int \sigma_a(u) du$, after which $\bar{\phi}$ and $\bar{W}$ were calculated from

$$\bar{\phi} = \frac{\sum_{i=g_3}^{g_4} \phi_i \int_{u_3}^{u_4} \sigma_a(u) du}{\int_{u_3}^{u_4} \sigma_a(u) du} \quad (72)$$

$$\bar{W} = \frac{\sum_{i=g_3}^{g_4} \phi_i \phi_i^+ \int_{u_3}^{u_4} \sigma_a(u) du}{\int_{u_3}^{u_4} \sigma_a(u) du} \quad (73)$$

where:

i = multigroup subscript designating the i th group

g_3 = subscript of the group containing lethargy u_3

g_4 = subscript of the group containing lethargy u_4 .

The denominator in Equations (70) and (71) is the resonance integral between u_3 and u_4 . The calculated values of $\bar{\phi}$ and $\bar{W}$ are given in Tables VIII and IX, pages 44 and 45, respectively.

(b) Resolved Resonance Range. In the resolved resonance range between u_2 and u_3 , the real flux and the weighting factor are slowly varying functions of lethargy. Both drop off gradually toward higher lethargies, primarily due to the effects of absorption in the fuel region and in the cadmium tube. To simplify the calculation of $\bar{W}$ and $\bar{\phi}$, a least squares fitting technique was used to fit smooth curves through the multigroup results for ϕ_i and ϕ_i^+ (as shown in Figures 40 and 41, pages 141 and 142, respectively). The following fourth order polynomials gave excellent fits between $u = 9$ and $u = 15.5$.

$$W(u) = -5.0204 + 2.2360 u - 0.30097 u^2 + 0.0176618 u^3 - 0.0003853 u^4 \quad (74)$$

$$\phi(u) = -1.3586 + 0.87566 u - 0.11722 u^2 + 0.0068491 u^3 - 0.00014895 u^4 \quad (75)$$

$\bar{\phi}$ and $\bar{W}$ could then be evaluated for a particular sample, x , from

$$\bar{\phi}_x = \frac{\sum_{j=1}^R \phi(u_j) I_x^j}{\sum_{j=1}^R I_x^j} \quad (76)$$

$$\bar{W}_x = \frac{\sum_{j=1}^R \phi(u_j) \phi^+(u_j) I_x^j}{\sum_{j=1}^R I_x^j} \quad (77)$$

where

I_x^j = effective resonance integral in sample x of the jth resonance whose peak occurs at lethargy u_j . I_x^j was calculated using the ZUT code

R = total number of resolved resonances of the material present in sample x

$\phi(u_j)$ = Equation (74) evaluated at $u = u_j$

$\phi(u_j)\phi^+(u_j)$ = Equation (75) evaluated at $u = u_j$.

$\bar{W}$ for all samples containing uranium, thorium, and gold was obtained with the appropriate contribution from all resolved resonances of the respective absorbers and the appropriate p-wave contribution which shall be discussed in Section V-2. The results are presented in Tables VIII and IX.

(c) Unresolved Resonance Range. The flux and weighting function are relatively constant in this range except at the high end where the influence of the fission spectrum has a slight effect. The evaluation of $\bar{\phi}$ and $\bar{W}$ was accomplished in the same manner as in the resolved range. $\phi(u)_j$ and $W(u)_j$ were obtained from the S-XI results, and the values of I_x^j were group values of the resonance integral due to the unresolved resonances as calculated by the TUZ [11] code. The results are given in Tables VIII and IX.

(d) High Energy Range Above 32 keV. Because the energy group structure of the multigroup calculation was rather coarse in the high energy range, the fine group real fluxes obtained from a GAM infinite medium calculation [a] were used for the weighting factor calculation instead of the S-XI fluxes. However, the S-XI adjoint fluxes were used because their almost negligible dependences on lethargy in this portion of the spectrum made the group structure relatively unimportant. The infinite medium spectrum obtained from the GAM calculation is shown in Figure 39, page 139. It was necessary to normalize the GAM fluxes to values that were appropriate for use with the S-XI values. This was done by

[a] These fluxes are discussed in more detail on page 52.

multiplying the GAM fluxes by the appropriate normalizing factor so that the total GAM real flux above 32 keV was equal to the total S-XI flux in the same range. Capture cross sections were obtained from BNL-325 [26]. The resonance integral and weighting function due to fast capture were then calculated as

$$I_{fc}^x = \int_0^{5.75} \sigma_c(u) du = \sum_{k=1}^{23} \sigma_{c_k} \Delta u_k \quad (78)$$

$$\bar{W}_{fc}^x = \frac{\sum_{k=1}^{23} \phi_k \sigma_{c_k} \phi_k^+ \Delta u_k}{I_{fc}^x} \quad (79)$$

where:

σ_{c_k} = capture cross section for the kth group of the fine group structure

Δu_k = lethargy width of kth fine group = 0.25

ϕ_k = normalized GAM flux for kth group

ϕ_k^+ = S-XI adjoint flux for kth group.

The results are given in Tables VIII and IX.

2.2 Reactivity Effects of Fast Fissions

Because of the hard spectrum in the ARMF-II cadmium-shielded sample position, fast fissions in the fertile isotopes may result in an appreciable reactivity effect. To each of the measured gross reactivities of the fertile samples it was necessary, therefore, to apply a theoretical reactivity correction to account for fast fission. This correction differs in principle from the usual fast effect calculation because the majority of the fast neutrons causing fissions in the fertile material originate outside of the absorbing lump. The basis of the correction is given below.

Energy dependent perturbation theory gives the change in reactivity due to fast fission as

$$\rho_{ff} = C N V \left[\int_u \int_{u'} \phi(u) \sigma_f(u) \nu(u) \chi(u') du' du - \int_u \phi(u) \sigma_f(u) \phi^+(u) du \right] \quad (80)$$

where

C = a constant related to the fission source of the reactor

N = atom density of fertile atoms in sample, $\frac{\text{atoms}}{\text{cm}^3}$

V = volume of absorber lump, cm^3

$\sigma_f(u)$ = microscopic fission cross section of fertile material, cm^2

$\nu(u)$ = number of fission neutrons born per fission

$\chi(u')$ = fraction of fission neutrons born in a unit lethargy interval about lethargy u'

$\chi(u')$ is normalized so that

$$\int \chi(u') du' = 1$$

$\phi(u)$ = real flux in the sample

$\phi^+(u)$ = adjoint flux in the sample.

It has been assumed that the fluxes in the sample position are flat and that the sample causes no significant flux perturbations at the energies where fast fission is appreciable.

By defining an average fission importance as

$$\bar{\phi}_\chi^+ = \frac{\int_{u'} \phi^+(u') \chi(u') du'}{\int_{u'} \chi(u') du'} = \int_{u'} \phi^+(u') \chi(u') du'. \quad (81)$$

Equation (80) can be written as

$$\rho_{ff} = C N V \left[\bar{\phi}_\chi^+ \int_u \phi(u) \nu(u) \sigma_f(u) du - \int_u \phi(u) \sigma_f(u) \phi^+(u) du \right]. \quad (82)$$

This calculated reactivity can be scaled to an actual predicted value in the ARMF if C is given a numerical value equal to the weighting function scale factor discussed in the previous section. Because $\phi(u)$ and $\phi^+(u)$ were calculated by means of a multigroup transport code, it is convenient to replace the integrals in Equations (81) and (82) by discrete summations.

$$\bar{\phi}_\chi^+ = \sum_i \phi_i^+ \chi_i \Delta u_i. \quad (83)$$

$$\rho_{ff} = C N V \left[\bar{\phi}_X^+ \sum_i \phi_i (\nu \sigma_f)_i \Delta u_i - \sum_i \phi_i \sigma_{f_i} \phi_i^+ \Delta u_i \right] . \quad (84)$$

The multigroup transport calculation utilized only three "broad groups" ($\Delta u_1 = 1.25$, $\Delta u_2 = 1.75$, $\Delta u_3 = 4.25$), in the fast fission energy region of the spectrum. However, the GAM [a] infinite medium calculation, performed to obtain broad group constants for the multigroup study, contained 29 "fine groups" in this region of the spectrum, each with a width $\Delta u = 0.25$. At high energies the infinite medium spectrum and the spectrum at the center of the cadmium tube as given by the multigroup transport calculation should be very similar. Therefore, since GAM produces "broad group" fission cross sections averaged over the "fine group" spectrum, the products of the broad group fluxes and cross sections should give the same reaction rate as the fine group products. The broad group constants and fluxes are given in Table X. If the constants and fluxes from Table X are inserted into Equations (83) and (84), the following expressions are obtained:

$$(\rho_{ff})_U = 9.9259 \times 10^3 N_U \quad (85)$$

$$(\rho_{ff})_{Th} = 2.6909 \times 10^3 N_{Th} \quad (86)$$

TABLE X
MULTIGROUP FLUXES AND CONSTANTS FOR
CALCULATION OF REACTIVITY DUE TO FAST FISSIONS

Group	Δu_i	ϕ_i	ϕ_i^+			
1	1.25	1.8024	0.68562			
2	1.75	4.7439	0.96476			
3	4.25	1.7000	1.03401			
<hr/>						
<div style="display: flex; justify-content: space-around;"><div><u>Thorium-232</u></div><div><u>Uranium-238</u></div></div>						
Group	σ_{f_i} (barns)	σ_{f_i}	χ_i	σ_{f_i} (barns)	$\nu \sigma_{f_i}$	χ_i
1	0.17238	0.54712	0.18207	0.60746	1.8435	0.18207
2	0.05238	0.146684	0.36617	0.23395	0.62642	0.36617
3	0.0	0.0	0.03096	0.0	0.0	0.03096

[a] Appendix E describes the GAM calculation and gives both fine and broad group structure in energy and lethargy coordinates.

The reactivity corrections are tabulated for each sample in Columns 8 and 9 of Table XI.

TABLE XI

SUMMARY OF REACTIVITY DATA

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Sample	Capsule	ρ_g	ρ_c	ρ_s	$[\rho_a]_{Pb}$	ρ_{U-235}	$[\rho_{ff}]_{Th}$	$[\rho_{ff}]_U$	ρ_a
ThO ₂ -48	52	16.68	8.90	13.40	-1.57	----	0.08	----	- 4.13
-49	54	7.64	8.60	13.19	-1.53	----	0.26	----	- 12.88
-50	55	- 18.73	8.98	13.08	-1.48	----	1.05	----	- 40.36
-51	1	- 54.77	7.79	12.56	-1.35	----	2.67	----	- 76.44
-52	2	- 90.67	8.09	12.60	-1.27	----	4.50	----	-104.59
-53	3	-110.17	7.79	12.40	-1.13	----	7.20	----	-136.43
-54	4	-151.56	7.92	12.31	-0.91	----	11.77	----	-182.65
-55	5	-197.07	7.66	12.19	-0.61	----	18.18	----	-234.49
-56	6	-233.92	7.66	11.97	-0.30	----	24.45	----	-277.70
-57	7	-263.00	7.83	11.35	----	----	29.52	----	-311.70
UO ₂ -51	31	- 61.37	9.20	12.81	-1.44	0.09	----	5.30	- 87.33
-52	32	- 88.26	9.28	12.47	-1.35	0.18	----	10.16	-119.00
-53	33	-129.07	9.52	12.27	-1.21	0.36	----	20.86	-170.87
-54	34	-170.42	8.18	12.29	-1.05	0.63	----	36.26	-226.73
-55	35	-204.24	7.87	12.03	-0.84	0.93	----	53.42	-277.65
-56	36	-233.65	9.72	11.70	-0.57	1.30	----	75.15	-330.95
-57	37	-271.22	7.70	11.95	-0.32	1.76	----	101.64	-393.95
-58	14	-303.50	7.56	11.60	----	2.21	----	127.55	-452.42
U/T-131	131	-151.17	7.33	12.90	-1.28	0.09	4.05	4.98	-179.64
-132	132	-219.63	9.23	12.51	-1.00	0.17	8.06	9.91	-258.51
-133	133	-273.27	8.84	12.48	-0.75	0.26	12.20	14.99	-321.29
-134	134	-340.84	8.65	12.16	-0.33	0.40	18.73	23.03	-403.48
-135	135	-396.96	8.27	12.13	----	0.52	24.43	30.05	-472.36
-311	311	-155.28	7.07	13.07	-1.31	0.26	1.36	15.02	-190.75
-312	312	-218.09	8.97	12.52	-1.04	0.51	2.63	29.16	-270.84
-313	313	-268.86	8.88	12.21	-0.78	0.76	3.99	44.10	-338.02
-314	314	-322.24	8.61	11.67	-0.41	1.13	5.90	65.31	-414.45
-315	315	-377.29	7.56	11.33	----	1.56	8.14	90.12	-496.00
Au-51	9	- 45.23	7.68	12.95	-1.51	----	----	----	- 64.35
-52	10	-165.31	7.36	12.89	-1.50	----	----	----	-184.06
-53	11	-200.44	7.37	13.00	-1.51	----	----	----	-219.30
-54	47	-299.08	9.27	13.33	-1.53	----	----	----	-320.15
PbO ₂ -8	8	18.30	7.66	12.19	-1.55	----	----	----	----
-41	41	21.22	9.50	13.26	-1.54	----	----	----	----
-50	50	22.12	9.66	13.99	-1.53	----	----	----	----
U-235-1	46	-293.14	7.95	15.28	----	17.12	----	167.61	-501.10
-2	48	-264.68	9.59	15.34	----	43.45	----	168.04	-501.10

$$1 \mu k = 10^{-6} \frac{\delta k}{k}$$

ρ_g = gross reactivity (This quantity represents the average of five or more measurements.)

ρ_c = reactivity of aluminum capsule

ρ_s = scattering reactivity of sample

$[\rho_a]_{Pb}$ = reactivity due to absorption in lead

ρ_{U-235} = reactivity due to uranium-235

ρ_{ff} = reactivity due to fast fissions in fertile material

ρ_a = net reactivity due to absorber of interest

$$\rho_a = \rho_g - \rho_c - \rho_s - [\rho_a]_{Pb} - \rho_{U-235} - \rho_{ff}$$

V. RESONANCE INTEGRALS OF SINGLE ABSORBER SAMPLES OF URANIUM-238 AND THORIUM-232

Section V-1 describes the gold-197, uranium-238, and thorium-232 cross sections used in the calibration of the resonance integrals of these three isotopes. Because a description of the resonance calculations is included in the discussion of the weighting function calculations in Section IV-2, only a synopsis is given in this section. It is presented in Section V-2 along with the resulting calculated values. Section V-3 is comprised of experimental results for the single absorber samples and a comparison with theory.

1. CROSS SECTIONS OF GOLD-197, URANIUM-238, AND THORIUM-232

1.1 Cross Section of Gold-197

Because the absolute values of all of the resonance integrals measured in this work were based upon the gold-197 infinite dilution resonance integral as a calibration standard, it was important to determine the best available value for this quantity. Desjardins et al [27] at Columbia have resolved the gold resonances up to an energy of 940 eV and have measured the energies at which additional levels lie between 940 and 2470 eV. These data are consistent with the levels reported previously in BNL-325, but because of the improvement in resolution at high energies, the resolved range has been extended considerably. The Columbia data were, therefore, adopted as the best available; the parameters are listed in Table XII. The additional resonance energies above 940 eV are also listed in the table.

At energies below 3.06 eV, where the gold cross section is dominated by the 4.91 eV resonance, the capture cross section was considered to be the sum of the contribution from this dominant resonance and a residual $1/v$ component arising from higher energy resonances. This gives rise to the following expression:

$$\sigma_a = \sigma_o \sqrt{\frac{E_o}{E}} \frac{1}{\left(\frac{E - E_o}{\Gamma/2}\right)^2 + 1} + \frac{C_o}{\sqrt{E}} \quad (87)$$

where

$$E_o = 4.91 \text{ eV}$$

$$\sigma_o = 32850 \text{ b}$$

$$\Gamma = 0.1396 \text{ eV}$$

C_o was evaluated by setting the 2200 meter/sec value of σ_a equal to 98.8 barns. This gave for $0 \leq E \leq 3.06 \text{ eV}$,

$$\sigma_a(E) = 32850 \sqrt{\frac{4.91}{E}} \frac{1}{\left(\frac{E - 4.91}{0.0698}\right)^2 + 1} + \frac{0.8591}{\sqrt{E}}. \quad (88)$$

In the unresolved energy region above 940 eV, the contribution to the resonance integral was treated on a statistical basis using the following averages of the parameters in Table XII:

$$\bar{D} = 21.1 \text{ eV}$$

$$\bar{\Gamma}_\gamma = 0.196$$

$$\bar{\Gamma}_n^0 = 0.0051 \text{ eV}$$

$$\bar{g} = 1/2.$$

Because they contribute less than a few barns to the resonance integrals, the effect of partial waves of higher angular momentum has been neglected.

The above data gave the infinite dilution resonance integral, based on a 0.5 eV cutoff, as

$$I_\infty^{\text{Au}} = \int_{0.5}^{\infty} \sigma_a(E) \frac{dE}{E} = 1579.0 \text{ barns}. \quad (89)$$

The 1579.0 barn value includes the so-called "1/v contribution". Subtraction of the 1/v portion gives

$$\int_{0.5}^{\infty} \left(\sigma_a(E) - 98.8 \sqrt{\frac{0.0253}{E}} \right) \frac{dE}{E} = 1534.6 \text{ barns}. \quad (90)$$

Jirlow and Johansson [28] measured the same integral by means of cadmium ratio measurements of thin foils in a neutron beam and obtained 1490 ± 40 barns, excluding 1/v capture and using a cutoff of 0.5 eV. Because of the excellent agreement between the new Columbia data and the old BNL parameters, the calculated value of 1579 ± 45 barns was chosen as a standard for the present work.

1.2 Cross Section of Uranium-238

Resonance parameter data for uranium-238 are available from several sources [29, 30, 31]. The agreement among those sets of parameters is excellent and leads one to use the data with a high degree of confidence. In addition, the wealth of experimental investigations of systems containing this important resonance absorber has somewhat substantiated the accuracy of these parameters. The set used in this work was taken from the 1960 Brookhaven National Laboratory Compilation [32]. The resolved parameters are shown in Table XIII. The data at the bottom of this table were used to treat the unresolved energy range on a statistical basis.

TABLE XII
 RESONANCE PARAMETERS OF GOLD-197

E_0 (eV)	g	Γ_n (10^{-3} eV)	Γ_n^0 (10^{-3} eV)	Γ_γ (10^{-3} eV)
4.91	5/8	15.6	7.0	124
46.7	---	0.11	0.016	---
58.2	---	4.7	0.62	143
60.3	5/8	76.0	9.8	120
78.7	---	13.0	1.46	145
107.3	---	9.3	0.9	165
122.6	---	0.8	0.07	---
144.8	---	6.5	0.54	190
151.8	5/8	23.2	1.9	179
163.6	---	40.0	3.1	---
165.6	---	12.0	0.9	---
190.8	3/8	36.0	2.5	189
209.6	---	0.72	0.05	---
241.0	5/8	68.0	4.4	185
256.2	---	0.6	0.37	---
263.4	3/8	150.7	9.3	136
275.0	---	5.0	0.3	127
294.5	---	400.0	23.0	250
330.5	---	200	11.0	---
332.5	---	60	3.2	---
357.0	5/8	40.8	2.2	177
373.0	---	115	5.9	---
377.0	---	12	0.6	---
384.0	---	90	4.6	---
403.0	---	26	1.3	590
442.0	3/8	306.7	15.0	280
453.0	3/8	120	5.6	250
479.0	---	300.8	13.6	180
492.0	---	45	2.0	---
496.0	---	15	0.7	---

TABLE XII (cont.)

E_0	g	Γ_n	Γ_n^0	Γ_γ
(ev)		(10^{-3} eV)	(10^{-3} eV)	(10^{-3} eV)
538.0	---	39	1.7	280
551.0	---	36	1.5	282
565.0	---	3.3	0.14	---
583.0	---	370	15.0	---
590.0	---	30	1.2	---
606.0	5/8	198.4	8.0	140
621.0	3/8	110.7	4.4	160
628.0	---	60	2.4	---
632.0	---	35	1.4	---
664.0	3/8	6.2	0.24	---
691.0	---	10	0.38	---
702.0	---	400	15.0	---
706.0	---	240	9.0	---
721.0	5/8	86.4	3.2	210
743.0	---	11	0.4	---
766.0	3/8	426.7	15.3	154
781.0	---	148	5.3	---
791.0	---	125	4.4	170
803.0	3/8	285.3	10.0	217
819.0	---	40	1.4	---
825.0	---	150	5	---
831.0	---	150	5	---
871.0	---	16	0.54	150
886.0	---	65	2.2	125
940.0	5/8	434.4	14.2	110

ADDITIONAL E_0 VALUES OBSERVED FOR GOLD-197
(In eV)

968	1137	1297	1465	1660	1905	2220
993	1140	1323	1485	1712	1935	2250
998	1194	1340	1505	1725	1965	2270

TABLE XII (cont.)

1002	1217	1347	1514	1740	1995	2315
1047	1228	1367	1545	1752	2055	2360
1072	1232	1375	1566	1775	2105	2395
1086	1255	1408	1595	1825	2140	2440
1100	1265	1438	1610	1840	2155	2470
1130	1293	1445	1630	1870	2175	

Note: Data taken from Reference 27.

On the assumption that the 2.71 barn 2200 meter/sec cross section is due to the contribution from the 6.68 eV resonance and a residual $1/v$ component arising from higher energy resonances, the low energy cross section for $0 < E \leq 3.533$ eV is

$$\sigma_a(E) = \frac{1}{\sqrt{E}} \left[0.2243 + \frac{9.1540}{(E - 6.68)^2} \right]. \quad (91)$$

The total infinite dilution resonance integral, based on a 0.5 eV cutoff, was calculated as

$$I_{\infty}^{U^{238}} = 270.0 \text{ barns.} \quad (92)$$

Subtraction of the $1/v$ contribution gives

$$\int_{0.5}^{\infty} \left(\sigma_a(E) - 2.71 \sqrt{\frac{0.0253}{E}} \right) \frac{dE}{E} = 268.8 \text{ barns.} \quad (93)$$

1.3 Cross Section of Thorium-232

The knowledge of the thorium-232 resonance parameters is somewhat more uncertain than for gold and uranium-238. Data obtained from several different sources [27, 33, 34, 35] give rise to inconsistencies in Γ_γ , the measured gamma widths. Early values reported by Levin and Hughes [33] resulted in a Γ_γ value of 0.030 eV. The Columbia data for thorium-232 resonances above 100 eV, reported in 1960 [27], gave an average gamma width of 0.040 eV. Very recent measurements by Uttley and Jones [34] have produced an average width of 0.0214 eV; this value has been somewhat reinforced by the new Columbia data reported by Garg et al [35]. Values of the neutrons widths for the important lower two resonances also differ considerably among the above sets of data. Infinite dilution resonance integrals calculated from these sets range from 76 to 97 barns.

TABLE XIII

RESONANCE PARAMETERS OF URANIUM-238

E_o	$\Gamma_n(\text{eV})$	$\Gamma_\gamma(\text{eV})$	$E_o(\text{eV})$	$\Gamma_n(\text{eV})$	$\Gamma_\gamma(\text{eV})$
6.7	1.52×10^{-3}	2.46×10^{-2}	536.0	5.40×10^{-2}	2.46×10^{-2}
21.0	8.90×10^{-3}	2.46×10^{-2}	557.0	10.00×10^{-4}	2.46×10^{-2}
36.9	3.25×10^{-2}	2.46×10^{-2}	581.0	4.20×10^{-2}	2.46×10^{-2}
66.3	2.50×10^{-2}	2.46×10^{-2}	596.0	6.60×10^{-2}	2.46×10^{-2}
81.3	2.10×10^{-3}	2.46×10^{-2}	605.0	6.00×10^{-4}	2.46×10^{-2}
90.0	9.00×10^{-5}	2.46×10^{-2}	621.0	3.90×10^{-2}	2.46×10^{-2}
102.5	6.50×10^{-2}	2.46×10^{-2}	629.0	9.00×10^{-3}	2.46×10^{-2}
116.5	1.50×10^{-2}	2.46×10^{-2}	662.0	1.25×10^{-1}	2.46×10^{-2}
145.6	8.00×10^{-4}	2.46×10^{-2}	680.0	1.30×10^{-3}	2.46×10^{-2}
165.2	3.50×10^{-3}	2.46×10^{-2}	695.0	5.30×10^{-2}	2.46×10^{-2}
189.6	1.35×10^{-1}	2.46×10^{-2}	710.0	1.70×10^{-2}	2.46×10^{-2}
208.5	5.50×10^{-2}	2.46×10^{-2}	723.0	1.47×10^{-2}	2.46×10^{-2}
237.5	3.20×10^{-2}	2.46×10^{-2}	732.0	4.25×10^{-3}	2.46×10^{-2}
264.5	2.30×10^{-4}	2.46×10^{-2}	766.0	9.00×10^{-3}	2.46×10^{-2}
274.0	2.70×10^{-2}	2.46×10^{-2}	782.0	3.00×10^{-3}	2.46×10^{-2}
291.0	1.90×10^{-2}	2.46×10^{-2}	792.0	1.10×10^{-2}	2.46×10^{-2}
311.5	10.00×10^{-4}	2.46×10^{-2}	825.0	6.00×10^{-2}	2.46×10^{-2}
348.0	4.50×10^{-2}	2.46×10^{-2}	855.0	1.30×10^{-1}	2.46×10^{-2}
377.0	1.50×10^{-3}	2.46×10^{-2}	859.0	6.00×10^{-2}	2.46×10^{-2}
398.5	10.00×10^{-3}	2.46×10^{-2}	867.0	2.20×10^{-3}	2.46×10^{-2}
411.0	1.70×10^{-2}	2.46×10^{-2}	896.0	1.30×10^{-3}	2.46×10^{-2}
435.0	1.40×10^{-2}	2.46×10^{-2}	909.0	9.00×10^{-2}	2.46×10^{-2}
455.0	7.00×10^{-4}	2.46×10^{-2}	928.0	3.70×10^{-2}	2.46×10^{-2}
464.0	7.00×10^{-3}	2.46×10^{-2}	940.0	1.95×10^{-1}	2.46×10^{-2}
479.0	4.50×10^{-3}	2.46×10^{-2}	960.0	1.90×10^{-1}	2.46×10^{-2}
490.0	10.00×10^{-4}	2.46×10^{-2}	983.0	10.00×10^{-4}	2.46×10^{-2}
519.0	3.80×10^{-2}	2.46×10^{-2}	995.0	4.00×10^{-1}	2.46×10^{-2}

Unresolved Resonances

$$\Gamma_\gamma = 2.46 \times 10^{-2} \text{ eV} \quad D = 18.5 \text{ eV}$$

$$(\Gamma_n^o) = 1.74 \times 10^{-3} \text{ eV} \quad E_C = 1005 \text{ eV}$$

Note: Data taken from Reference 32.

The thorium data used for calculation in the present investigation are those obtained from the Neutron Cross Section Evaluation Center of Brookhaven in 1962. They consist of the 1960 Columbia data, modified to give $\Gamma_\gamma = 0.030$ eV for the four lower resonances. The parameters are listed in Table XIV. The average values derived from the data of Table XIV are as follows:

$$\bar{D} = 20.1 \text{ eV}$$

$$\bar{\Gamma}_\gamma = 40 \times 10^{-3} \text{ eV}$$

$$\bar{\Gamma}_n^0 = 1.27 \times 10^{-3} \text{ eV}.$$

In the unresolved region between 1000 eV and 32 keV, the resonance integral was treated on a statistical basis using the above average parameters.

In the low energy range from 0.251 to 19,623 eV, it was necessary to take into account the deviation of the absorption cross section from a $1/v$ dependence. This deviation has been interpreted by Cooper et al [36] as arising from a virtual level lying at a negative energy of several electron volts. They reason that the residual $1/v$ components from distant positive energy levels contribute about 0.6 barns to the 2200 meter/sec cross section and that a similar contribution can be expected from the distant virtual levels. On this basis, six barns of the 2200 meter/sec cross section are unaccounted for and thereby attributed to a nearby virtual level. They have accounted for this additional contribution with the following set of parameters for the negative level:

$$E_0 = -4.3 \text{ eV}$$

$$\bar{\Gamma}_n^0 = 0.703 \times 10^{-3} \text{ eV}$$

$$\bar{\Gamma}_\gamma = 40 \times 10^{-3} \text{ eV}.$$

The resulting low energy cross section is

$$\sigma_a(E) = \frac{1}{\sqrt{E}} \left[0.159 + \frac{18.28}{(E + 4.3)^2} + \frac{9.4}{(E - 21.84)^2} + \frac{15.1}{(E - 23.48)^2} \right]. \quad (94)$$

The first term within the brackets represents the residual effect due to distant virtual and real resonances, and the second term is the contribution of the first virtual level. The total infinite dilution resonance integral, based on a 0.5 eV cutoff and including the $1/v$ contribution, was calculated as

$$I_\infty^{\text{Th}^{232}} = 96.7 \text{ barns.} \quad (95)$$

2. METHODS OF CALCULATION AND RESULTS

Contributions to the resonance integral of each absorber were regarded as coming from four ranges of the energy spectrum. The ranges and the cor-

TABLE XIV

RESONANCE PARAMETERS OF THORIUM-232

E_o	$\Gamma_n(\text{eV})$	$\Gamma_\gamma(\text{eV})$	$E_o(\text{eV})$	$\Gamma_n(\text{eV})$	$\Gamma_\gamma(\text{eV})$
21.84	2.40×10^{-3}	3.00×10^{-3}	456.40	2.82×10^{-2}	4.10×10^{-2}
23.48	4.00×10^{-3}	3.00×10^{-3}	465.00	4.50×10^{-2}	4.10×10^{-2}
59.55	4.60×10^{-3}	3.00×10^{-3}	491.00	5.00×10^{-2}	4.10×10^{-2}
69.20	3.90×10^{-2}	3.00×10^{-3}	513.00	3.50×10^{-3}	4.10×10^{-2}
113.15	1.10×10^{-2}	4.20×10^{-2}	531.50	1.50×10^{-2}	4.10×10^{-2}
121.00	1.85×10^{-2}	4.10×10^{-2}	543.00	10.00×10^{-4}	4.10×10^{-2}
128.50	10.00×10^{-5}	4.10×10^{-2}	573.00	2.50×10^{-2}	4.10×10^{-2}
129.40	3.10×10^{-3}	4.10×10^{-2}	581.00	1.60×10^{-3}	4.10×10^{-2}
146.20	10.00×10^{-5}	4.10×10^{-2}	601.50	1.10×10^{-2}	4.10×10^{-2}
154.60	2.20×10^{-4}	4.10×10^{-2}	621.50	5.00×10^{-3}	4.10×10^{-2}
170.80	5.80×10^{-2}	4.80×10^{-2}	660.50	3.20×10^{-2}	4.10×10^{-2}
193.00	1.40×10^{-2}	5.00×10^{-2}	669.00	1.70×10^{-2}	4.10×10^{-2}
196.80	1.30×10^{-4}	4.10×10^{-2}	679.50	1.25×10^{-1}	4.10×10^{-2}
199.80	9.00×10^{-3}	4.10×10^{-2}	692.00	3.80×10^{-2}	4.10×10^{-2}
203.00	1.88×10^{-2}	4.10×10^{-2}	705.00	1.80×10^{-2}	4.10×10^{-2}
221.80	2.30×10^{-2}	4.60×10^{-2}	717.50	2.50×10^{-2}	4.10×10^{-2}
252.30	2.40×10^{-2}	4.20×10^{-2}	746.00	1.60×10^{-1}	4.10×10^{-2}
264.00	1.50×10^{-2}	4.10×10^{-2}	783.00	10.00×10^{-3}	4.10×10^{-2}
286.80	2.10×10^{-2}	5.00×10^{-2}	810.00	1.25×10^{-1}	4.10×10^{-2}
306.50	2.00×10^{-2}	4.10×10^{-2}	843.00	2.30×10^{-2}	4.10×10^{-2}
330.40	5.50×10^{-2}	5.30×10^{-2}	872.00	10.00×10^{-3}	4.10×10^{-2}
343.30	3.00×10^{-2}	4.10×10^{-2}	896.00	3.00×10^{-2}	4.10×10^{-2}
366.70	3.50×10^{-2}	4.10×10^{-2}	950.00	4.00×10^{-2}	4.10×10^{-2}
371.00	2.40×10^{-2}	4.10×10^{-2}	970.00	7.50×10^{-3}	4.10×10^{-2}
402.80	9.00×10^{-3}	4.10×10^{-2}	990.00	2.40×10^{-2}	4.10×10^{-2}
413.40	2.68×10^{-2}	4.10×10^{-2}	998.00	7.00×10^{-2}	4.10×10^{-2}

Unresolved Resonances

$$\Gamma_\gamma = 4.0 \times 10^{-2} \text{ eV}$$

$$D = 20.1 \text{ eV}$$

$$\langle \Gamma_n^0 \rangle = 1.27 \times 10^{-3} \text{ eV}$$

$$E_C = 1008 \text{ eV}$$

Note: Data taken from Reference 27 and modified as stated on page 60.

responding nomenclatures are described in detail, beginning on page 42 in Section IV. On page 42, these ranges were denoted by the following:

- (1) Low energy absorption
- (2) Absorption in the resolved region
- (3) Absorption in the unresolved region below 32 keV
- (4) Absorption in the region above 32 keV.

The energy limits of each range for gold, uranium, and thorium are given in Table VII, page 42. The calculations of the resonance integrals for each range, using the cross sections given in Section V-1 of this chapter, are described briefly below.

Because the absorption cross sections are small in the low energy range, self-shielding is negligible and resonance integral was calculated directly from

$$I_{\text{low}} = \int_{U_{\text{Cd}}}^{U_3} \sigma_a(u) du. \quad (96)$$

The integration was performed by means of a gaussian integration scheme in INTEGR [25], the IBM 7040 code programmed for calculation of the average weighting functions. The cadmium cutoff energy used was 0.5 eV.

The resolved resonance contribution for the single absorber samples were calculated from Equations (9) and (11) using the single-resonance ZUT code as described in Section II. An additional contribution due to p-wave resonances was estimated from the work of Vernon [37]. For uranium, this gave 0.83 barn, and in the case of thorium, a contribution of 1.0 barn. Because the p-wave resonances are relatively unshielded, the same resonance integral contribution was added for all concentrations of a given absorber. The contribution of the unresolved resonances was calculated on a statistical basis by means of Nordheim's TUZ code [11, 12]. In summary, this code makes use of both the Wigner rational approximation and the narrow resonance approximation, and uses an average over the Thomas-Porter distribution of neutron widths. In all of the single absorber samples considered, the contribution of the unresolved samples amounted to less than ten percent of the total resonance integral. The resonance integral in the high energy range was calculated by numerical integration of the capture cross sections taken directly from BNL-325.

The contribution from each range and the total resonance integral for each single absorber sample and the gold calibration samples are given in Table XV. The results for the uranium-238 dioxide/lead dioxide and thorium-232 dioxide/lead dioxide samples are also presented graphically in Figure 12.

TABLE XV

CALCULATED RESONANCE INTEGRALS OF SINGLE ABSORBER SAMPLES

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Sample	Low Energy Range ($E_{Cd} = 0.251$) (barns)	Resolved Range (barns)	Unresolved Range (barns)	Fast Capture Range (barns)	p-wave Capture (barns)	Total Resonance Integral ($E_{Cd} = 0.251$) (barns)	Total Resonance Integral ($E_{Cd} = 0.5$) (barns)
Thorium-232							
Infinite Dilution	2.99	89.32	3.50	1.13	1.0	97.94	96.72
ThO ₂ -48	2.99	83.92	3.47	1.13	1.0	92.51	91.29
-49	2.99	75.67	3.45	1.13	1.0	84.24	83.02
-50	2.99	55.62	3.35	1.13	1.0	64.09	62.87
-51	2.99	39.02	3.16	1.13	1.0	47.30	46.08
-52	2.99	30.72	3.00	1.13	1.0	38.84	37.62
-53	2.99	24.25	2.82	1.13	1.0	32.19	30.97
-54	2.99	18.69	2.58	1.13	1.0	26.39	25.17
-55	2.99	14.73	2.35	1.13	1.0	22.20	20.98
-56	2.99	12.48	2.17	1.13	1.0	19.77	18.55
-57	2.99	11.18	2.05	1.13	1.0	18.35	17.13
Uranium-238							
Infinite Dilution	1.54	264.35	2.92	0.85	0.83	270.49	269.96
UO ₂ -51	1.54	81.80	2.73	0.85	0.83	87.75	87.22
-52	1.54	57.94	2.59	0.85	0.83	63.75	63.22
-53	1.54	39.64	2.38	0.85	0.83	45.24	44.71
-54	1.54	29.80	2.16	0.85	0.83	35.18	34.65
-55	1.54	24.42	1.99	0.85	0.83	29.63	29.10
-56	1.54	20.52	1.82	0.85	0.83	22.56	22.03
-57	1.54	17.70	1.68	0.85	0.83	22.60	22.07
-58	1.54	15.79	1.56	0.85	0.83	20.57	20.04
Gold							
Infinite Dilution	73.74	1519.94	6.55	0.0	0.0	1599.83	1578.97
Au-51	73.74	874.36	6.51	0.0	0.0	954.01	933.35
-52	73.74	445.28	6.37	0.0	0.0	524.99	504.13
-53	73.74	387.15	6.32	0.0	0.0	466.81	445.95
-54	73.74	279.86	6.15	0.0	0.0	359.35	338.49

3. EXPERIMENTAL RESULTS AND COMPARISON WITH THEORETICAL PREDICTIONS

Measured resonance integrals of the single absorber samples were deduced from the net absorption reactivities by means of Equation (66), using the average weighting functions given in Table VIII. The measured resonance integrals and their corresponding values after adjustment to a cutoff energy of 0.5 eV are given in Columns (2) and (3) of Table XVI. The corresponding calculated values are given in Column (4). These results are presented graphically in Figure 12.

The calculated and measured values for the uranium dioxide/lead dioxide samples are in good agreement over the wide range of absorber concentrations.

Fig. 12 Measured and calculated resonance integrals for uranium-238 dioxide/lead dioxide and thorium-232 dioxide/lead dioxide samples.

TABLE XVI
MEASURED RESONANCE INTEGRALS OF SINGLE ABSORBER SAMPLES

(1)	(2)	(3)	(4)
Sample	Measured Resonance Integral ($E_{Cd} = 0.251$)[a] (barns)	Measured Resonance Integral (Adjusted to $E_{Cd} = 0.5$) (barns)	Calculated [b] Resonance Integral (Adjusted to $E_{Cd} = 0.5$) (barns)
Thorium-232			
Infinite Dilution	82.46	81.24	96.72
ThO ₂ -48	79.40	78.18	91.29
-49	75.67	74.45	83.02
-50	59.10	57.88	62.87
-51	44.02	42.80	46.08
-52	35.67	34.45	37.62
-53	29.10	27.88	30.97
-54	23.85	22.63	25.17
-55	19.87	18.65	20.98
-56	17.52	16.30	18.55
-57	16.31	15.09	17.13
UO ₂ -51	95.81	95.28	87.22
-52	67.55	67.02	63.22
-53	47.01	46.48	44.71
-54	35.74	35.21	34.65
-55	29.63	29.10	29.10
-56	25.05	24.52	25.03
-57	21.99	21.46	22.07
-58	20.09	19.56	20.04

[a] The use of this value of E_{Cd} for the measured resonance integrals is discussed on page 45, in Section IV-2.

[b] Taken from column (8) of Table XV, page 63.

Because the validity of the resonance parameters used in the uranium calculations is rather firmly established, the substantial agreement between theory and measurement is gratifying and tends to support the basis of the experimental techniques. Furthermore, most previous measurements of the resonance integrals of uranium were made on samples containing the pure absorber. The effective resonance integrals of these materials were usually varied by using samples of different size, each having the same absorber density. In the present work, the lump configuration was kept fixed while the concentrations of the absorbers were varied over a wide range by use of the lead dioxide diluent. This technique provided a further test for the adequacy of resonance absorption theory. Because the

single absorber samples contained a lead dioxide diluent, the results could not be directly correlated with the work of other experimenters who have performed measurements on pure uranium dioxide. However, in the more concentrated samples, the lead dioxide scattering cross section per absorber atom is small, and the resonance integrals should be comparable to that of a pure uranium dioxide sample having the same S/M_{UO_2} (the ratio of surface of the absorber lump to mass of uranium dioxide in the lump) value. Figure 13 shows a comparison of the calculated and measured results with the widely accepted correlation obtained in 1957 by Hellstrand [38]. It is seen that the present work extends much further into the range of low absorber concentrations, but does not cover the low S/M values obtained with samples of high density or small diameter. The present results are consistent with those of Hellstrand; ie, the additional scattering of the lead dioxide increases the resonance integral at high S/M values, while at the lower end point (a pure uranium dioxide sample) the agreement is excellent.

Fig. 13 S/M_{UO_2} dependence of resonance integrals of uranium-238 dioxide/lead dioxide samples.

The values of the thorium dioxide/lead dioxide resonance integrals calculated with the Columbia data are in quantitative disagreement with the experimental results. The theoretical predictions are between 10 and 15 percent higher than the measured values throughout the entire range of measurements. The experimental and theoretical results are compared in Figure 14 with the separate work of Pettus [39] and Weitman [40] on pure thorium dioxide samples. The present work appears to be consistent with the S/M correlation obtained by Weitman; good agreement is again obtained at the low end of the range. The results of Pettus are 20 to 30 percent higher than the measured values obtained here. However, Pettus' own recent calculations [41] indicate that his weighting function corrections were incorrect and may have given results that were 10 to 20 percent too high [a].

[a] Pettus is repeating his resonance integral measurements on thorium metal and thorium dioxide [42], but the results are not yet available.

Fig. 14 S/M_{ThO_2} dependence of resonance integrals of thorium-232 dioxide/lead dioxide samples.

The infinite dilution resonance integral of thorium was determined by extrapolating the reactivity worths to yield a value of reactivity per absorber atom at zero absorber concentration in the sample. The absorption reactivities for the most dilute samples are shown in Figure 15. The solid curve represents a fit to the data to the expression

$$\rho_a = A(1 - e^{-BN}). \quad (97)$$

The fit is seen to be excellent. The slope of this curve at $N = 0$ is

$$\left(\frac{d\rho_a}{dN}\right)_{N=0} = AB. \quad (98)$$

Equation (66) also gives the slope at zero concentration as follows:

$$\left(\frac{d\rho_a}{dN}\right)_{N=0} = \bar{C}_\infty^{\text{Th}} V I_\infty^{\text{Th}}. \quad (99)$$

The combination of (98) and (99) gives

$$I_\infty^{\text{Th}} = \frac{AB}{\bar{C}_\infty^{\text{Th}} V}. \quad (100)$$

The resulting infinite dilution resonance integral, adjusted to a cutoff energy of 0.5 eV and including the $1/v$ contribution, is

$$I_\infty^{\text{Th}232} = 81.2 \pm 4 \text{ barns}. \quad (101)$$

Fig. 15 Reactivity worths of dilute thorium-232 dioxide/lead dioxide samples.

The value calculated was 96.7 barns. This result and the comparison shown in Figures 12 and 14 would seem to indicate that the Columbia parameters used in the calculation are incorrect. Somewhat lower values of Γ_n and/or Γ_γ would result in much better agreement between experiment and theory. The results of recent measurements [34, 35], received just before this work went to press, tend to substantiate the fact that Γ_γ is approximately 0.020 eV [a].

[a] It was also learned by oral communication with the Cross Section Evaluation Center at Brookhaven [35] that J. Harvey of Oak Ridge is just completing a critical analysis of all the available thorium resonance parameter data. He has derived a "best" set of values, which, when compared to the values used in Table XIV, contains slightly lower neutron widths for the low energy resonances and gives a much lower average gamma width.

VI. INTERFERENCE IN URANIUM-238/THORIUM-232 MIXTURES

1. INTERFERENCE CALCULATED BY MULTIRESONANCE SOLUTION OF FIRST-FLIGHT COUPLED EQUATIONS

The basic principles of calculation of interference by means of a multi-resonance solution of the coupled first flight equations were presented in Section II. The method of solution described there permits a treatment of interference between two close-lying Doppler broadened resonances. Because of the large number of resolved resonances in both uranium and thorium [a] it was impractical to perform an interference calculation for each possible pair of resonances for each of the ten interference samples, even though a complete treatment of the interference problem would require this. For this reason, only selected uranium-thorium resonance pairs were studied. In particular, pairs of resonances were studied which were not only close-lying, but also contributed an appreciable fraction to the total resonance integral of each respective absorber. The latter criterion was important since a high fractional interference, $\Delta I/I$, in a relatively small resonance, might contribute very little to the change in the total resonance integral (ie, the measured resonance integral) of the isotope. On this basis, the following pairs of resonances listed below were considered. Also given is I_{∞}^i , the infinite dilution resonance integral of each resonance, and I_{∞}^i/I_{∞} , the fractional contribution of that resonance integral of the isotope.

Isotope	E_0 (eV)	I_{∞} (barns)	$\frac{I_{\infty}^i}{I_{\infty}}$
Uranium-238	21.0	61.03	0.226
Thorium-232	21.84	19.03	0.197
Uranium-238	21.0	61.03	0.226
Thorium-232	23.48	26.15	0.270
Uranium-238	66.3	11.05	0.041
Thorium-232	69.2	14.46	0.150

There are other pairs which might interact to a small degree, eg, 116.5 eV in uranium-238 and 113.15 eV in thorium-232, but because of computer time considerations, the calculations were limited to the resonances listed above.

Because the 21.0, 21.84, and 23.48 eV resonances are so closely spaced, (Figure 1, page 5) a thorough study would require the simultaneous treatment of all three resonances. The calculation of interference among these resonances was, therefore, performed with groups of three resonances at a time rather than with pairs. In each case, however, the third resonance was not Doppler

[a] Fifty-four for uranium and fifty-two for thorium in the data used in this work.

broadened, but merely represented by the nautral line shape. The addition of this third resonance required a further modification to the ZUT code [16].

The calculation of interference in each resonance in each sample required two calculations: One with the interfering resonance or resonances "turned on", and a second with them "switched off". The resonance integrals calculated in this way, the resulting interference, and the fractional interference, are presented for each sample in Tables XVII through XX on pages 71, 72, 73, and 74. The results are shown graphically in Figures 16, 17, and 18, on pages 75 and 76. Curves have been drawn through the calculated values. Substantial interference does occur within the groups of resonances, particularly in the 21.0 and 21.84 eV resonances where the fractional interference is as high as eight percent in the samples of highest absorber concentration. The 23.48 eV resonance is affected to a substantially lesser extent. In the 66.3 - 69.2 eV pair, the maximum fractional effect was less than six percent. Particularly interesting is the fact that the effect is much greater in the lower energy resonance, apparently indicating that the depression of the slowing down flux in the lump by the high resonance is important in this particular case. For each of the resonances, the fractional interference was approximately a linear function of the combined absorber atom density.

The calculated neutron flux energy dependence within sample U/T-135 resulting from the 21.0, 21.84, and 23.48 eV resonances is shown in Figure 20. The lack of flux recovery between the lower two resonances is quite apparent. Figure 21 shows the calculated flux in sample U/T-315 in the range of the 66.3 and 69.2 eV resonances. The lack of recovery is not so apparent, but particularly interesting is the flux hump on the lower side of each resonance, apparently due to interference between potential and resonance scattering.

The results in Tables XVII through XX were combined to give the fractional change in the total resonance integral of each interference sample. The total fractional interference is expressed as follows:

$$\frac{\Delta I}{I} = \frac{N_U \Delta I_U (N_U; N_{Th}; N_{m_2}) + N_{Th} \Delta I_{Th} (N_{Th}; N_U; N_{m_2})}{N_U I_U (N_U; N_{Th}^P; N_{m_2}) + N_{Th} I_{Th} (N_{Th}; N_U^P; N_{m_2})} \quad (102)$$

The values of I_U and I_{Th} were obtained from the calculated curve in Figure 12. The total fractional interference is presented in Table XXI and Figure 19. The values are substantially reduced from those obtained with each resonance pair because the individual resonances represent only a small fraction of the resonance integral. The maximum ΔI value obtained amounted to less than four-tenths of a barn, corresponding to a percentage change of approximately 1.4 percent. It is true that there may be additional interference contributions from other resonances, but these should not be as great as the interference in the important low energy resonances considered in these calculations.

TABLE XVII

CALCULATED INTERFERENCE IN 21.0 eV RESONANCE OF
URANIUM-238 DUE TO 21.84 AND 23.48 eV RESONANCES OF THORIUM-232

Sample	$I_{21.0}^p$ (barns)	$I_{21.0}^r$ (barns)	$\Delta I_{21.0}$ (barns)	$\frac{\Delta I_{21.0}}{I_{21.0}^p}$
U/T-131	17.5038	17.1465	-0.3573	-0.02041
-132	11.6164	11.1942	-0.4222	-0.03634
-133	9.08534	8.62463	-0.46071	-0.05071
-134	7.05871	6.56401	-0.49470	-0.07008
-135	6.06803	5.55351	-0.51452	-0.08479
-311	9.14502	9.01370	-0.13132	-0.01436
-312	6.21083	6.03836	-0.17247	-0.02776
-313	4.92666	4.72942	-0.19724	-0.04004
-314	3.99475	3.75761	-0.21714	-0.05463
-315	3.35212	3.12186	-0.23026	-0.06869

Note: 21.0 and 21.84 eV resonances are Doppler broadened; 23.48 eV resonance is represented by natural line shape.

In this Table and also in Tables XVIII, XIX, and XX the abbreviated column headings designate the following:

$$I_{E_0}^p = I_{E_0} (N_x; N_y^p; N_{m_2})$$

$$I_{E_0}^r = I_{E_0} (N_x; N_y^r; N_{m_2})$$

$$\Delta I_{E_0} = I_{E_0}^r - I_{E_0}^p$$

where E_0 = energy at peak of resonance in which interference occurs

N_x = atom density of isotope in which interference occurs

N_y = atom density of interfering isotope.

TABLE XVIII

CALCULATED INTERFERENCE IN 21.84 eV RESONANCE OF
THORIUM-232 DUE TO 21.0 eV RESONANCE OF URANIUM-238

Sample	$I_{21.84}^P$ (barns)	$I_{21.84}^r$ (barns)	$\Delta I_{21.84}$ (barns)	$\frac{\Delta I_{21.84}}{I_{21.84}^P}$
U/T-131	5.96553	5.87003	-0.09550	-0.01601
-132	3.91045	3.79048	-0.11997	-0.03068
-133	3.02472	2.88796	-0.13676	-0.04521
-134	2.31989	2.16405	-0.15584	-0.06718
-135	1.97882	1.8107	-0.16812	-0.08496
-311	10.5329	10.2927	-0.2402	-0.02280
-312	7.58591	7.31517	-0.27074	-0.03569
-313	5.97657	5.6921	-0.28447	-0.04760
-314	4.68761	4.3889	-0.29871	-0.06372
-315	3.82635	3.51458	-0.31077	-0.08124

Note: 21.0 and 21.84 eV resonances are Doppler broadened;
23.84 eV resonance is represented by natural line shape.

2. MEASURED INTERFERENCE; COMPARISON WITH THEORY

The general method for the measurement of interference in the uranium dioxide/thorium dioxide mixtures was described in Section II. The detailed results and a comparison with theory are presented below.

The net absorption reactivity worths (absolute values were used) of the single absorber samples were fitted by nonlinear least squares fitting techniques [43] to a functional relationship of the form

$$\rho_a = A + B N + C N^2 + D (1 - e^{-\lambda N}) \quad (103)$$

where A, B, C, D, and λ are parameters determined by the best fit. The data were fitted only in the range of absorber concentrations used in the interference samples. Therefore, it was not necessary that $\rho \rightarrow 0$ as $N \rightarrow 0$, ie, $A \neq 0$. For two primary reasons this particular functional form of $\rho(N)$ was chosen over several others which were fitted to the data:

- (1) It satisfied the requirements that

TABLE XIX

CALCULATED INTERFERENCE IN 23.48 eV RESONANCE OF
THORIUM-232 DUE TO 21.0 eV RESONANCE OF URANIUM-238

Sample	$I_{23.48}^p$ (barns)	$I_{23.48}^r$ (barns)	$\Delta I_{23.48}$ (barns)	$\frac{\Delta I_{23.48}}{I_{23.48}^p}$
U/T-131	6.56992	6.54806	-0.02186	-0.00333
-132	4.29939	4.26856	-0.03083	-0.00717
-133	3.34458	3.30606	-0.03852	-0.01152
-134	2.58872	2.54009	-0.04863	-0.01878
-135	2.22200	2.16614	-0.05586	-0.02514
-311	12.1641	12.1366	-0.0275	-0.00226
-312	8.44799	8.42058	-0.02741	-0.00324
-313	6.58136	6.55049	-0.03087	-0.00469
-314	5.14522	5.10853	-0.03669	-0.00725
-315	4.20895	4.16366	-0.04529	-0.01076

Note: 21.0 and 23.48 eV resonances are Doppler broadened;
21.84 eV resonance is represented by natural line shape.

$$(a) \rho \geq 0$$

$$(b) \frac{d\rho}{dN} > 0$$

$$(c) \frac{d^2\rho}{dN^2} \leq 0$$

These three requirements are based on the theoretically predicted and experimentally verified behavior of the reactivity worth of an absorbing sample in which the concentration of absorber atoms is continually increasing. That is, each additional absorber atom must increase the net worth of the sample in a smooth, monotonic manner.

(2) At each data point, it gave residuals that did not greatly exceed the experimental precision. High order polynomials gave rise to residuals of the same order of magnitude, but those functions had "bumps" and did not meet requirements in (1) above. The resulting parameters for the two sets of samples are as follows:

TABLE XX

CALCULATED INTERFERENCE IN 66.3 eV RESONANCE OF
URANIUM-238 and 69.2 eV RESONANCE OF THORIUM-232

<u>Sample</u>	$I_{66.3}^p$ (barns)	$I_{66.3}^r$ (barns)	$\Delta I_{66.3}$ (barns)	$\frac{\Delta I_{66.3}}{I_{66.3}^p}$
U/T-131	4.84254	4.80145	-0.04109	-0.008485
-132	3.32675	3.26229	-0.06446	-0.019376
-133	2.61802	2.53965	-0.07837	-0.029935
-134	2.03286	1.94006	-0.09220	-0.045335
-135	1.74482	1.64471	-0.10011	-0.057376
-311	2.63689	2.63038	-0.00651	-0.002469
-312	1.78893	1.77923	-0.00970	-0.005422
-313	1.41623	1.40462	-0.01161	-0.008198
-314	1.14261	1.12917	-0.01344	-0.011762
-315	0.96709	0.95224	-0.01485	-0.015362

<u>Sample</u>	$I_{69.2}^p$ (barns)	$I_{69.2}^r$ (barns)	$\Delta I_{69.2}$ (barns)	$\frac{\Delta I_{69.2}}{I_{69.2}^p}$
U/T-131	3.03120	3.02573	-0.00547	-0.001804
-132	2.11282	2.10430	-0.00852	-0.004032
-133	1.72206	1.71117	-0.01089	-0.006324
-134	1.40350	1.38962	-0.01388	-0.009990
-135	1.24404	1.22800	-0.01604	-0.012893
-311	5.43696	5.42977	-0.00719	-0.001322
-312	3.80365	3.79573	-0.00792	-0.002082
-313	3.03598	3.02561	-0.01037	-0.003416
-314	2.45566	2.44079	-0.01487	-0.006055
-315	2.07725	2.05681	-0.02044	-0.009840

Note: Both resonances are Doppler broadened.

Fig. 16 Calculated fractional interference among 21.0, 21.84, and 23.48 eV resonances in mixtures with $N_U/N_{Th} = 1/3$.

Fig. 17 Calculated fractional interference among 21.0, 21.84, and 23.48 eV resonances in mixtures with $N_U/N_{Th} = 3$.

Fig. 18 Calculated fractional interference between 66.3 and 69.2 eV resonances in mixtures with $N_U/N_{Th} = 1/3$ and $N_U/N_{Th} = 3$.

Fig. 19 Calculated fractional interference in the total resonance integral of uranium-238 dioxide/thorium-232 dioxide mixtures.

TABLE XXI

CALCULATED FRACTIONAL INTERFERENCE IN THE TOTAL
RESONANCE INTEGRAL OF URANIUM-238/THORIUM-232 MIXTURES

Sample	I (barns)	ΔI (barns)	$\frac{\Delta I}{I}$
U/T-131	49.41	-0.192	-0.00388
-132	38.82	-0.241	-0.00621
-133	32.72	-0.279	-0.00852
-134	27.25	-0.310	-0.01139
-135	24.28	-0.334	-0.01374
-311	54.43	-0.172	-0.00316
-312	40.71	-0.206	-0.00505
-313	34.05	-0.238	-0.00699
-314	28.92	-0.261	-0.00901
-315	25.46	-0.278	-0.0109

Note:

$$\Delta I = \frac{N_U \Delta I_U (N_U; N_{Th}; N_{m2}) + N_{Th} \Delta I_{Th} (N_{Th}; N_U; N_{m2})}{N_U + N_{Th}}$$

$$I = \frac{N_U I_U (N_U; N_{Th}^P; N_{m2}) + N_{Th} I_{Th} (N_{Th}; N_U^P; N_{m2})}{N_U + N_{Th}}$$

$$UO_2 - PbO_2: \quad A = 47.5932$$

$$C = -0.10360 \quad B = 24.7506$$

$$\lambda = 0.56165 \quad D = 103.9689$$

$$ThO_2 - PbO_2: \quad A = 10.2110$$

$$C = -0.51482 \quad B = 27.4045$$

$$\lambda = 0.97997 \quad D = 62.2342$$

The data points and the fitted curves are shown in Figure 22. The fits are excellent, giving at each data point a residual of less than one μk .

From the analytical expressions for the single absorber calibration curves, reactivities were predicted for both the uranium and thorium components of the

Fig. 20 Calculated energy dependence of flux within sample U/T-135.

Fig. 21 Calculated energy dependence of flux within sample U/T-315.

Fig. 22 Absorption reactivities of single absorber samples of uranium-238 dioxide/lead dioxide and thorium-232 dioxide/lead dioxide.

interference samples on the basis of the atom densities of the respective absorbers in the mixture. Resonance integrals for each of these components were obtained from Equation (66) by application of the appropriate weighting functions. The weighting functions were taken from Figure 11 at the respective

atom densities of each absorber. The weighting functions, predicted reactivities, and predicted resonance integrals are given in Table XXII.

TABLE XXII

WEIGHTING FUNCTIONS, PREDICTED REACTIVITIES, AND
PREDICTED RESONANCE INTEGRALS OF URANIUM AND THORIUM
COMPONENTS OF MIXED URANIUM-238/THORIUM-232 SAMPLES

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Sample	$(\rho_a)_{Th}$ (μk)	w_{Th}	$I_{Th}(N_{Th}; 0; N_{m1})$ (barns)	$(\rho_a)_U$ (μk)	w_U	$I_U(N_U; 0; N_{m1})$ (barns)
U/T-131	- 98.34	1.0061	37.282	- 85.54	0.9816	99.721
-132	-146.50	1.0065	27.943	-116.81	0.9876	68.090
-133	-185.34	1.0060	23.356	-144.21	0.9909	55.353
-134	-238.19	1.0035	19.592	-180.18	0.9943	44.076
-135	-278.83	1.0019	17.609	-206.55	0.9965	39.343
-311	- 48.18	1.0039	54.653	-144.35	0.9909	55.300
-312	- 74.95	1.0053	43.746	-203.41	0.9962	39.937
-313	- 97.32	1.0061	37.526	-250.90	0.9994	32.468
-314	-122.82	1.0066	31.965	-307.34	1.0023	26.777
-315	-147.46	1.0065	27.814	-367.11	1.0045	23.128

Note: N_{m1} refers to the set of scatters present in the calibration samples.

In order to predict the exact resonance integral that would be realized for each interference sample if no interference occurred in the mixture, it was necessary to correct for any difference between the scattering properties of the calibration and interference samples as described in Equations (25) and (26) in Section II-4. The values of N_{m1} , the lead dioxide scatterer set which would be present in a calibration sample having the same single absorber atom density as the corresponding interference sample, were determined by interpolation using the lead dioxide atom densities in the actual calibration samples. The δ corrections were calculated for only the first eight resonances of both uranium and thorium, and from these results the total δ correction for all resonances was estimated. The values of δ and the resulting corrected resonance integrals are listed in Table XXIII. For the majority of the samples, the correction was extremely small, totaling less than a few tenths of a barn. Also given in Table XXIII is the corrected resonance integral, $I_{U,Th}^p(N_U; N_{Th}; N_{m2})$, predicted for the interference sample on the assumption of no interference.

Measured resonance integrals of the interference samples were inferred from the absorption reactivities of these samples by means of Equation (66) using $\bar{w}_{U,Th}$, the appropriate weighting function, for the mixtures. The correct value of $\bar{w}_{U,Th}$ is

TABLE XXIII

VALUES OF THE δ CORRECTION AND PREDICTED
RESONANCE INTEGRALS FOR MIXED URANIUM-238/THORIUM-232 SAMPLES

(1)	(2)	(3)	(4)	(5)	(6)
Sample	δ_{Th} (barns)	$I_{Th}(N_{Th}; N_U^p; N_{m_2})$ (barns)	δ_U (barns)	$I_U(N_U; N_{Th}^p; N_{m_2})$ (barns)	$I_{U,Th}^p(N_U; N_{Th}; N_{m_2})$ (barns)
U/T-131	-0.043	37.325	-0.207	99.928	52.975
-132	-0.002	27.945	0.027	68.063	37.974
-133	-0.019	23.375	0.027	55.326	31.363
-134	-0.013	19.605	0.095	44.781	25.900
-135	-0.028	17.637	0.074	39.269	23.045
-311	-0.067	54.720	-0.214	55.514	55.316
-312	0.050	43.696	-0.039	39.076	40.906
-313	0.095	37.431	0.003	32.465	33.706
-314	0.157	31.808	0.050	26.727	27.997
-315	0.165	27.649	0.054	23.074	24.218

Note: N_{m_2} refers to the set of scatterers present in the interference sample.

$$\bar{w}_{U,Th} = \frac{(\rho_a)_{U,Th}}{\frac{(\rho_a)_U}{W_U} + \frac{(\rho_a)_{Th}}{W_{Th}}} \quad (104)$$

where $(\rho_a)_U$, $(\rho_a)_{Th}$, W_U , and W_{Th} are the single absorber quantities listed in Table XXII. The measured resonance integral was then determined from

$$I_{U,Th}(N_U^r; N_{Th}; N_{m_2}) = \frac{(\rho_a)_{U,Th}}{C V \bar{w}_{U,Th} (N_U + N_{Th})} \quad (105)$$

where $(\rho_a)_{U,Th}$ is the absorption reactivity of the interference sample. The measured resonance integral and the resulting measured interference, ΔI , as defined in (28), determined from the difference between the measured values and the predicted values in Column 6, Table XXIII, are presented in Table XXIV. Also given is the measured fractional interference $\Delta I/I$. The calculation of the uncertainties in ΔI and $\Delta I/I$ is described in Appendix G. The measured fractional interference is displayed graphically in Figures 23 and 24 along with the results of the multiresonance calculations.

The measured interference is small, as expected. The maximum decrease in resonance integral is less than 1.4 barns and the maximum fractional interference is approximately three and a half percent. The measured interference

TABLE XXIV

MEASURED INTERFERENCE IN MIXED URANIUM-238/THORIUM-232 SAMPLES

Sample	$I_{U,Th}^p$ (barns)	$I_{U,Th}^r$ (barns)	$\Delta I_{U,Th}$ (barns)	$\frac{\Delta I_{U,Th}}{I_{U,Th}^p}$
U/T-131	52.975	51.672	-1.303 ± 0.498	-0.0245 ± 0.0094
- 132	37.974	37.276	-0.698 ± 0.238	-0.0183 ± 0.0063
- 133	31.363	30.565	-0.798 ± 0.158	-0.0254 ± 0.0050
- 134	25.900	24.992	-0.908 ± 0.105	-0.0350 ± 0.0041
- 135	23.045	22.426	-0.619 ± 0.076	-0.0269 ± 0.0033
- 311	55.316	54.630	-0.686 ± 0.493	-0.0124 ± 0.0089
- 312	40.906	39.784	-1.122 ± 0.245	-0.0274 ± 0.0060
- 313	33.706	32.744	-0.962 ± 0.159	-0.0285 ± 0.0047
- 314	27.997	27.050	-0.947 ± 0.103	-0.0338 ± 0.0037
- 315	24.218	23.422	-0.796 ± 0.081	-0.0329 ± 0.0033

Note:

$$I_{U,Th}^p = I_{U,Th}^p (N_U; N_{Th}; N_{m_2})$$

$$I_{U,Th}^r = I_{U,Th}^r (N_U; N_{Th}; N_{m_2})$$

$$\Delta I_{U,Th} = I_{U,Th}^r - I_{U,Th}^p$$

The calculation of the uncertainties in Column 4 and 5 is described in Appendix G.

is of the same order for both sets of samples. The scatter of the data would appear to reflect the problem of attempting to measure the extremely small difference (a fraction of a barn for most samples) between two large values. In general, the results for samples U/T 311-315, with the higher ratio of uranium to thorium content, exhibit a smoother variation. This probably arises in part from the slightly better analytical fit which was obtained for the reactivity data of the uranium dioxide/lead dioxide samples. The uncertainties ascribed to the experimental values are believed to be realistic ones. They

Fig. 23 Measured interference in mixed uranium-238/thorium-232 samples: $N_U/N_{Th} = 1/3$.

Fig. 24 Measured interference in mixed uranium-238/thorium-232 samples: $N_U/N_{Th} = 3$.

amount to only a few tenths of a barn because of the use of the difference method which tends to cancel the effects of many possible systematic errors.

The measured interference is consistently higher than the values calculated by the methods described in Section VI-1. The theoretical predictions are deficient in certain respects, however. In particular, the calculated results included only a few selected pairs of interfering resonances; if all slightly interfering resonances were taken into account, the calculated interference would increase by a small amount. Furthermore, the use of the narrow resonance approximation in the moderator terms [Equations (5), (6), (7) on pages 7 and 8] tends to underestimate the interference effect by neglecting the flux depression in the moderator caused by the absorption in the resonance. The results are discussed in more detail in the following chapter, along with the conclusions drawn from them.

VII. DISCUSSION OF RESULTS; CONCLUSIONS

The results obtained from this investigation and the conclusions drawn from them are discussed below.

(1) Resonance integrals of single absorber samples of uranium-238 dioxide and thorium-232 dioxide, each in admixture with a lead dioxide diluent, were measured by means of static reactivity techniques in the ARMF-II. Calibration was effected by a combination of activation and reactivity measurements on gold using a calculated gold infinite dilution resonance integral as a standard. It is believed that this method of calibration provides an improvement over both the usual boron calibration method [39] and the techniques utilizing an extrapolation to infinite dilution. The experimental results for the uranium dioxide/lead dioxide samples were in good agreement with the results predicted with the aid of Nordheim's treatment. Because the resonance parameters of uranium-238 are well established, this agreement was considered to substantiate the adequacy of the experimental techniques. The present work was consistent with the widely accepted results of Hellstrand at low S/MUO_2 (the ratio of surface of the absorber lump to mass of uranium dioxide in the lump) values where the lead dioxide scattering was negligible.

Resonance integrals of thorium dioxide/lead dioxide samples, calculated with the Columbia data, were in quantitative disagreement with the experimental results. The theoretical predictions were between 10 and 15 percent higher than the measured values throughout a very wide range of absorber concentrations. However, the experimental work is consistent with S/M correlation obtained by Weitman. Because there have been inconsistencies in the thorium resonance parameters reported from several independent sources, the present results are significant in that they lend weight to the sets having the lower values of Γ_n and Γ_γ .

(2) The infinite dilution resonance integral of thorium-232, adjusted to a cutoff energy of 0.5 eV, and including the $1/v$ contribution, was measured as 81.2 ± 4 barns. This value is significant in that it provides a further test for the validity of the various sets of thorium resonance parameters. It also tends to support the lower values of neutron and gamma widths.

(3) The concept of interference between resonances of two absorbers was formulated and defined as the effect on the resonance integral caused by the lack of complete flux recovery between the resonances. The definition included only those effects due to the presence in a system of an additional resonance or resonances; effects predictable by conventional theory were not considered to be interference. This concept is important with regard to future reactor design problems, in that many contemplated systems contain pairs or groups of resonances which are closely spaced; in these systems, due to the overlapping of resonance cross sections and/or the absence of complete flux recovery, a separate treatment of each resonance may not be valid. Mixed resonance absorbers in control devices and mixed fertile isotopes are two such systems of particular significance.

(4) Interference was measured in a series of uranium-238/thorium-232 mixtures. The experimental results indicated slightly more than three percent interference in the samples of highest absorber concentration. Interference was

also predicted for this series of mixtures by means of multiresonance solutions of the first-flight coupled equations. As many as three close-lying resonances were treated simultaneously with a modification of the Nordheim treatment. The results, based on an interference calculation for several of the most prominent resonances, predicted an interference of approximately 1.3 percent in the samples of highest absorber concentration.

From the experimental and theoretical results, it can be concluded that interference in the particular mixtures investigated was rather small. For this reason it is very difficult to confirm quantitatively the adequacy of the multiresonance calculation techniques which were used. However, the comparison of theory and experiment shown in Figures 23 and 24 does indicate a discrepancy between the calculated and measured values. The theoretical predictions of interference are systematically lower than the measured value throughout the entire range of concentrations, and in each case they fall outside the limits of uncertainty assigned to the experiment. Since these limits of uncertainty have been assigned on a rather firm basis (Appendix G), and since a meticulous review of the experimental procedures has not revealed any systematic errors, it must be concluded that the calculations underestimated the amount of interference.

One deficiency of the calculation, of course, is that it was practical to include only a few selected pairs of interfering resonances; if all slightly interfering resonances were taken into account, the calculated interference would increase. However, because the most prominent resonances of uranium and thorium were included in the calculation, it is felt that the inclusion of all of the resolved resonances could increase the total fractional interference by no more than 10 to 20 percent. It is realized also that accidental overlap may occur in the unresolved range. Assuming that these two possibilities would, together, increase the fractional interference by approximately twenty-five percent, the calculations would then appear to underestimate the interference by slightly more than a factor of two.

A second and possibly more serious cause for the underestimate of the interference may lie in the use of the narrow resonance approximation in the moderator terms of the first-flight coupled equations. This approximation, which is inherent in the present formulation of the Nordheim treatment, is based on the assumption that a single collision with a moderator nucleus will cause it to hurdle the energy range of interest. The mathematical formulation of the approximation is presented on pages 7 and 8. If the approximation is valid, the moderator can be considered to be a black medium for resonance neutrons. However, the total practical width of a pair or group of resonances such as those encountered in the present work is quite large. For this reason, a collision with a moderator nucleus may not remove a neutron from the energy range of interest; hence, absorption in a higher energy resonance may actually cause a deficiency in the number of neutrons entering the fuel lump at the energy of the lower resonance. The NR approximation neglects this deficiency. At the present time, it is not clear whether the so-called "narrow resonance failure" can entirely account for the underestimate of interference. It would be worthwhile to investigate the effect further by means of analytical or experimental methods. An appropriate starting point for future work, attempting to estimate or elucidate the effect of the narrow resonance failure, might be based on the work of Iijima [9]. Selected Monte Carlo problems might also provide some insight,

although in many instances, it is difficult to arrive at generalizations from the results of a few Monte Carlo studies.

For experimental reasons, the uranium-238/thorium-232 samples studied had relatively low absorber densities (less than six g/cm³ in the samples of highest concentration). In a reactor the density of both fertile materials would be somewhat higher, and, therefore, the interference would be increased. Calculations, identical to those performed for the experimental samples, were carried out for cylindrical samples containing uranium-238 and thorium-232 metals of approximately theoretical density. The results are shown in Figure 25. The fractional interference, presented as a function of the cylinder radius, approaches three percent for rods of large radius. If these values were then to be corrected, on the basis of the experimental work performed with the samples of lower density, it would be necessary to multiply the calculated results by a factor of approximately two and a half. The interference in a typical uranium-238/thorium-232 system might, therefore, be significant. In a metal rod of one centimeter radius, for example, the effect could amount to five percent or more. Because of their lower densities, the effect should be less in a mixture of the oxides. Because such interference could compromise the advantages to be gained from a mixed system, this phenomenon should definitely be considered in future work on binary systems.

Fig. 25 Calculated interference in uranium-238/thorium-232 metal rods.

VIII. NOMENCLATURE^[a]

ENGLISH LETTER SYMBOLS

Lower Case Letters

- f_A = correction factor applied to measured count rate to account for gamma-ray self-attenuation in source
- f_R = correction factor applied to measured count rate to account for effect of source-detector distance
- g = spin factor in the single-level Breit-Wigner cross section formula
- g_3 = subscript of the group (in the multigroup structure of Table XXXI) containing lethargy u_3
- g_4 = subscript of the group (in the multigroup structure of Table XXXI) containing lethargy u_4
- i = multigroup subscript designating the i th group
- i_o = unattenuated count rate due to a slab gamma-ray source of thickness dx , counts/sec
- j = subscript or superscript; refers to a particular resonance absorber
- k = GAM fine group subscript designating the k th fine group
- p = subscript designating potential scattering
- p = superscript designating that the interfering resonance has been switched off
- r = regulating rod position
- $\bar{r}$ = average regulating rod position
- r_1^j = regulating rod position with "sample out"
- r_2^j = regulating rod position with "sample in"
- $\bar{r}_k$ = MT regulating rod position at time t_x
- $\bar{r}_{k1}$ = regulating rod position of i th MT
- $\bar{r}_{k2}$ = regulating rod position of $(i + 1)$ th MT

[a] Some symbols are used more than once with different meanings; the correct meaning should be obvious from the context in which it is used. Many symbols with obvious meanings have not been included in this list.

$\bar{r}_x$ = "sample in" regulating rod position for sample x

s = subscript representing "scattering"

s_r = standard deviation of a given regulating rod position average

s_ρ = standard deviation for a particular sampling of reactivity data

$s_{\rho i}$ = standard deviation for J_i measurements of the ith sample

t = thickness, cm

t_c = elapsed time during counting, sec

t_{k1} = midpoint time of ith MT sampling

t_{k2} = midpoint time of (i + 1)th MT sampling

t_w = elapsed time between discharge and commencement of counting

t_x = midpoint time of "sample in" reading for sample x

w_i = weight fraction of element i in the mixture

Capital Letters

A = count rate at discharge from reactor, counts/sec

A' = measured count rate before correction for decay, counts/sec

A_i = atomic weight of isotope i

A_I = activity of a sample due to absorption in idealized constant flux, counts/sec

A_M = activity of a sample due to absorption in flux varying with lethargy, counts/sec

A_∞ = average specific count rate of microfoils, counts/sec

A_∞^m = average specific count rate of the monitor foils for the infinite dilution sample, counts/sec

B = buckling, cm^{-1}

C = weighting function scale factor

$\bar{D}$ = average spacing between resonances, eV

E_1 = upper limit of unresolved resonance range, ≈ 32 keV

E_2 = upper limit of resolved resonance range, eV

- E_3 = upper limit of "1/v" region or low energy range, eV
 E_4 = energy at which $\phi(u)$ and $\phi(u) \phi^+$ in cadmium covered sample become negligibly small, eV
 E_{Cd} = cadmium cutoff energy, eV
 E_0 = energy at which peak resonance cross section occurs, eV
 F_{as} = asymptotic collision density in the lump in the absence of absorber resonances
 $F(E)$ = energy dependent collision density
 $F(u)$ = lethargy dependent collision density
 I = effective resonance integral, barns
 I_{eff} = effective resonance integral, barns
 I^i = resonance integral of the ith resonance
 I_0 = unattenuated count rate due to activity uniformly distributed throughout a slab gamma-ray source, counts/sec
 I_∞ = infinite dilution resonance integral, barns
 I_x = resonance integral of resonance absorber x
 I_x^j = effective resonance integral in sample x of the jth resonance, barns

The following symbols are used to describe interference; they are precisely defined on pages 10 and 11.

$$I_x (N_x; 0; N_m)$$

$$I_x (N_x; N_y^r; N_m)$$

$$I_x (N_x; N_y^p; N_m)$$

$$I_{x,y}^r (N_x; N_y; N_m)$$

$$I_{x,y}^p (N_x; N_y; N_m)$$

$$\Delta I_x (N_x; N_y; N_m)$$

$$\Delta I_{x,y} (N_x; N_y; N_m)$$

J = number of measurements for each individual sample
 MT = designates condition of empty reactor ("sample out")
 N = atom density of absorber atoms in sample, atoms/cm³
 N = number of different samples measured
 N_m = a set of scatterers or moderators present in a sample
 N_{m1} = scatterer set present in the single absorber calibration sample
 N_{m2} = scatterer set present in the interference sample
 N_x = atom density of isotope x, atoms/cm³
 $P_k(u)$ = probability that a neutron whose last collision in the k th region degraded it to lethargy u , suffers its next collision also in the k th region
 R = total number of resolved resonances of the material present in sample x
 V = volume of absorber lump, cm³
 $W = \phi \phi^+ =$ weighting function
 $\bar{W}_x$ = average weighting function for sample x

GREEK LETTER SYMBOLS

Lower Case Letters

$$\alpha_i = \left(\frac{A_i - 1}{A_i + 1} \right)^2$$

δ = a correction to compensate for the change in resonance integral due to the difference between the scattering properties of the calibration sample and the interference sample, barns

δ = reactor savings, cm

$\delta \sigma_a(u)$ = lethargy dependent cross section representing the difference between the reference sample cross section and the cross section of the sample of interest, cm²

ϵ = detector efficiency

μ = linear gamma-ray attenuation coefficient, cm⁻¹

$$\mu k = 10^{-6} \frac{\delta k}{k}$$

ν = number of fission neutrons born per fission

ρ = mean reactivity value obtained from the infinite data sampling, μk

ρ_a = reactivity due to absorption, μk

ρ_c = reactivity of aluminum capsule, μk

ρ_{ff} = reactivity due to fast fissions in fertile material, μk

ρ_g = gross reactivity of sample, μk

ρ_I = reactivity due to absorption in idealized constant flux, μk

ρ_M = reactivity due to absorption in flux varying with lethargy, μk

$\rho(\bar{r}_k)$ = reactivity evaluated from the rod calibration function at $r = \bar{r}_k$, μk

ρ_s = scattering reactivity of sample, μk

ρ_{U-235} = reactivity due to uranium-235, μk

$\sigma_a(E)$ = resonance absorption cross section, cm^2

$\sigma_a^i(E)$ = resonance absorption cross section due to the i th resonance, cm^2

σ_{c_k} = capture cross section for the k th group of the fine group structure, cm^2

σ_f = microscopic fission cross section of fertile material, cm^2

σ_o = peak resonance cross section at $E = E_o$, cm^2

σ_p = potential scattering cross section per absorber atom, $cm^2/atom$

σ_ρ = true standard deviation of the universe being sampled, μk

σ_s = scattering cross section per absorber atom, $cm^2/atom$

σ_t = total cross section per absorber atom, $cm^2/atom$

σ_{2200} = 2200 meter/sec cross section, cm^2

$\chi(u')$ = fraction of fission neutrons born in a unit lethargy interval about lethargy u'

Capital Letters

Γ = total width of resonance, eV

Γ_γ = gamma width of resonance, eV

Γ_n = neutron width of resonance, eV

$\bar{\Gamma}_n^0$ = reduced neutron width of resonance, eV

ΔI = difference between the effective resonance integrals in the absence and in the presence of interference, barns

Δu_k = lethargy width of kth fine group = 0.25

Σ_p = macroscopic potential scattering cross section, cm^{-1}

Σ_s = macroscopic scattering cross section of sample, cm^{-1}

ϕ_{as} = the asymptotic flux present in the system in the absence of absorber resonances

ϕ^i = flux which would be present in system if the total resonance absorption cross section is that contributed by the ith resonance

ϕ_I = constant real flux

ϕ_I^+ = constant adjoint flux

ϕ_k = normalized GAM flux for kth group

ϕ_k^+ = S-XI adjoint flux for kth group

$\bar{\phi}_x$ = absorption weighted average flux for sample x

$\bar{\phi}_x^+$ = adjoint flux averaged over the fission spectrum

ϕ' = perturbed flux

ϕ^+ = importance function (adjoint flux)

$\bar{\phi}_\infty$ = absorption weighted average flux for infinite dilution samples

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APPENDIX A

CALIBRATION OF ARMF-II REGULATING ROD

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APPENDIX A

CALIBRATION OF ARMF-II REGULATING ROD

A least squares fit technique was used to obtain the functional relationship between reactivity and the regulating rod position [43]. The method is outlined below.

Assuming the true function to be $\rho = f(r)$, where r is the position of the regulating rod and ρ is the reactivity, an approximating function, $\rho_n = F_n(r)$ is to be found such that the functional difference $f(r) - F_n(r)$ is as small as possible for all values of r in some specified range of the regulating rod. A poison sample of relatively small reactivity worth was used to obtain paired readings of regulating rod position. It was assumed that this sample had a constant relative reactivity over the entire range of the regulating rod. The paired readings are the regulating rod position with the sample in and out of the reactor core, respectively, at different points over the range of travel of the regulating rod. The increment between each of these points is the reactivity necessary to compensate for a given insertion or withdrawal of the shim blade. By means of successive movements of the shim, interspersed with pairs of regulating rod readings, the desired range of the rod was calibrated.

Let $\Delta\rho$ = relative reactivity worth of the sample

r_2^j = regulating rod position with sample in

r_1^j = regulating rod position with sample out.

Then

$$\Delta\rho = f(r_2^j) - f(r_1^j) \quad (106)$$

for m pairs of readings, $j = 1, 2, \dots, m$

From the approximating function,

$$\rho_n = F_n(r) = a_0 + a_1 G_1(r) + a_2 G_2(r) + \dots + a_n G_n(r), \quad (107)$$

the values of the coefficients a_i , ($i = 1, 2, \dots, n$), are to be found such that the function

$$\Delta\rho_n^j = F_n(r_2^j) - F_n(r_1^j) \quad (108)$$

is a good fit to the experimental values (r_2^j, r_1^j) and $\Delta\rho$. The method of least squares was used to evaluate the coefficients where the function to be minimized was

$$S(a_1, a_2, \dots, a_n) = \sum_{j=1}^m (\Delta\rho_n^j - \Delta\rho)^2 \quad (109)$$

The value a_0 was determined by setting ρ_n equal to zero at the arbitrary regulating rod position $r = 1600$, which is the upper limit of the regulating rod motion.

An arbitrary value, $\Delta\rho$, was assigned to the calibration sample to produce an unnormalized calibration curve. A period measurement, which measured the increment of reactivity between two regulating rod positions, was then used to

scale the curve with true reactivity values. Because the reactivity worths of samples of unknown cross sections are always compared to the worth of some standard sample, only relative and not absolute reactivity worths are required. Therefore, the shape of the calibration curve is all that is necessary.

The calibration curve was normalized by a set of period measurements which measured the reactivity increment between rod positions 650 and 950. The average value obtained for this interval was 264.76 μ k. The delayed neutron fraction used was 0.00755.

The following trigonometric function gave the best fit.

$$\rho(r) = a_0 + \sum_{k=1}^3 a_k \sin k\theta + \sum_{k=1}^3 b_k \cos k\theta \quad (110)$$

where

$$\theta = \frac{\pi r}{1600}$$

for $450 \leq r \leq 1500$

$$a_0 = 360.35108$$

$$a_1 = 117.46679$$

$$a_2 = 26.287097$$

$$a_3 = -3.2379034$$

$$b_1 = 398.32743$$

$$b_2 = 35.195464$$

$$b_3 = -2.7778766$$

The resulting calibration curve is shown in Figure 26.

Fig. 26 Reactivity calibration curve for ARMF-II regulating rod.

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APPENDIX B

SAMPLE PREPARATION

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APPENDIX B

SAMPLE PREPARATION

Although sample procurement and fabrication were without a doubt the least interesting and least inspiring aspects of this work, they were among the most time consuming. Undoubtedly, many others performing work in experimental reactor physics will agree with such an observation. Much of the fabrication work could have been sublet to another laboratory, but because of the loss of close control of the sample, with the attendant loss in confidence in the specifications, this expedient was avoided whenever possible.

1. MATERIALS

1.1 Uranium Dioxide (UO₂)

The uranium dioxide used in these experiments was obtained on loan from the Isotopes Department of Oak Ridge National Laboratory. The Oak Ridge assay gave its uranium-235 content as 330 ppm; this figure was verified by mass spectrometer measurements at NRTS. Table XXV gives the results of a spectrographic analysis to determine the maximum detected contaminants. None of the impurities were present in quantities sufficient to be detectable even in the reactivity measurements. Because the urania was received in the form of a coarse grained powder (approximately one-hundred mesh), it was necessary to reduce the particles to a size in which self-shielding of resonance neutrons would not be significant. The reduction was accomplished by repeated shaking of the powder and a tungsten carbide ball in a tungsten carbide mixer-grinder mill. The resulting particles were then sized in a "Roller Particle Size Analyzer". This analyzer is a single-stage air elutriator which performs separations of dry, finely divided powders according to the particle diameter. From a single operation of this instrument, the powder was divided into two fractions, one of them containing particles smaller than the required diameter. The other fraction was then processed again in the tungsten carbide grinding mill. By several repetitions of this operation, the entire batch of uranium dioxide was reduced to a suitable particle size. An electron photomicrograph of the resulting uranium dioxide particles is pictured in Figure 27. The average particle size appears to be less than a micron. Photomicrographs of thorium dioxide and uranium dioxide are also given.

1.2 Thorium Dioxide (ThO₂)

The thoria was a high purity nuclear grade obtained from the Davison Chemical Division of W. R. Grace and Co. An isotopic analysis by means of mass spectrometer measurements showed the isotopic content to be 100 percent thorium-232 [a]. A spectrographic analysis is given in Table XXV.

[a] A small percentage of Th-230 has been found in some thorium batches, coming from ores containing both uranium and thorium. Thorium is a member of the uranium-238 decay scheme.

TABLE XXV

SAMPLE IMPURITIES

	Element	ppm	Element	ppm
Uranium Dioxide ^[a]	Be	< 0.01	B	2.0
	Cr	2	Cd	< 0.1
	Li	< 0.2	Fe	80
	Ni	8	Mn	1
	Al	6	V	75
	Ca	< 10	Co	< 1
	Cu	6	Na	40
	Mg	6	Si	30
	P	< 100		
Thorium Dioxide ^[b]	Fe	43	Ca	15
	Ni	10	Mg	6
	Cr	15	Zn	30
	Mn	2	Si	6
	Cu	2	Al	28
	Na	< 75	B	< 0.20
	K	< 10	Cd	0.10
	Al	20	Fe	80
Lead Dioxide ^[c]	B	< 20	Mg	2
	Pb	1000	Mn	30
	Ca	3	Si	300
	Cr	5		
	Cu	100		
	Ag	< 10	Mg	< 7
	B	< 20	Ni	< 7
Gold ^[d]	Cu	< 10	Pb	< 160
	Fe	< 100	Si	< 50

[a] Spectrographic analysis received from Oak Ridge National Laboratory.

[b] Spectrographic analysis received from W. R. Grace and Co.

[c] Spectrographic analysis performed by CPP Analytical Section at National Reactor Testing Station.

[d] Spectrographic analysis performed by CPP Analytical at National Reactor Testing Station.

THORIUM DIOXIDE

URANIUM DIOXIDE

LEAD DIOXIDE

Fig. 27 Electron photomicrographs of thorium dioxide, uranium dioxide, and lead dioxide particles.

1.3 Lead Dioxide (PbO₂)

The lead dioxide was a reagent grade powder obtained from J. T. Baker Chemical. Its average particle size was approximately 0.36 micron. A spectrographic analysis is given in Table XXV.

1.4 Gold

The gold obtained from Western Gold and Platinum of Belmont, California, in the form of finely precipitated powder, was sized as 325 mesh or less. As pointed out in Section 3 of Appendix C, particle self-shielding in the gold will not affect the final results of the gold calibration procedure.

2. METHODS OF PREPARATION

When the design of the experimental samples was first conceived, it was envisioned that the absorber powders would be compacted into thin pellets or wafers which would be stacked into thin-walled hollow aluminum cylinders. It was later discovered, however, that a compact of the oxides of uranium, thorium, and lead would not hold together unless they were sintered. Since the development of an adequate sintering process could be rather lengthy and time consuming, the process of direct compaction in the cylinders was chosen instead. The aluminum capsule was, therefore, designed to withstand the compaction pressures, and yet be fabricated with as little material as possible in order to minimize its own reactivity worth. The capsules were five inches long with an inside diameter of 0.958 inch. A picture of the capsule is shown in Figure 28. Figure 29 shows a set of capsules containing compacted powders. A tiny O-ring seal closed the sample tightly so that none of the alpha-emitting powders could leak during handling [a].

Fig. 28 Aluminum capsule for compacted powder samples.

Fig. 29 Complete set of powder samples used in resonance integral measurements.

Because both uranium and thorium are long-lived alpha emitters, precautions were taken to ensure that no contamination was introduced into the laboratories during sample preparation. The majority of the required weighing was performed in a glove box or with carefully decontaminated weighing bottles. After the appropriate amounts of each powder were weighed, the mixtures were

[a] A series of reactivity measurements was performed to ensure that the O-ring caused no significant moderation inside the cadmium tube.

formed and thoroughly homogenized by tumbling them in a glass container for one hour and shaking them for fifteen minutes in a wrist-action shaker-grinder mill. A chemical analysis on several small aliquots extracted from each sample verified that the resulting distributions of the ingredients were uniform throughout the mixtures.

A small laboratory hand press was used to compact the powders in the capsules at a pressure of approximately 3500 pounds per square inch. To avoid an overall density gradient along the length of the powder-filled cylinder, the powders were compacted in thin layers, each about one-quarter inch thick. The volume of powder compacted into each layer was rigidly controlled by the loading and leveling devices pictured in Figure 30. The press and jig to hold the capsules are also shown. The ingredient weights of each compacted powder sample are given in Table XXVI.

Fig. 30 Equipment for compaction of powder samples.

TABLE XXVI

NET WEIGHTS OF POWDER SAMPLE INGREDIENTS

Sample	Net Weight (g)	UO ₂ Mass (g)	ThO ₂ Mass (g)	PbO ₂ Mass (g)	Gold Mass (g)	Mass of Enriched UO ₂ (g)
UO ₂ - 51	304.01	14.12	---	289.8	---	---
- 52	298.43	27.10	---	271.3	---	---
- 53	299.31	55.64	---	243.6	---	---
- 54	307.98	96.71	---	211.3	---	---
- 55	311.21	142.5	---	168.7	---	---
- 56	314.98	200.5	---	114.4	---	---
- 57	334.55	271.0	---	63.50	---	---
- 58	340.31	340.3	---	0.0	---	---
ThO ₂ - 48	315.02	---	0.77314	314.25	---	---
- 49	310.09	---	2.5283	307.59	---	---
- 50	308.00	---	10.121	297.88	---	---
- 51	296.93	---	25.70	271.2	---	---
- 52	298.85	---	43.33	255.5	---	---
- 53	295.62	---	69.29	226.3	---	---
- 54	296.08	---	113.2	182.9	---	---
- 55	297.11	---	174.9	122.2	---	---
- 56	295.60	---	235.2	60.40	---	---
- 57	283.92	---	283.9	0.0	---	---
U/Th-131	308.12	13.293	38.994	255.83	---	---
-132	304.03	26.425	77.517	200.09	---	---
-133	308.38	39.995	117.32	151.07	---	---
-134	308.82	61.427	180.19	67.205	---	---
-135	315.20	80.144	235.06	0.0	---	---
U/Th-311	316.05	40.073	13.061	262.91	---	---
-312	311.36	77.774	25.349	208.24	---	---
-313	312.65	117.62	38.340	156.69	---	---
-314	312.25	174.20	56.775	81.274	---	---
-315	318.73	240.38	78.347	0.0	---	---
Au - 51	304.76	---	---	303.9	0.8517	---
- 52	304.72	---	---	300.5	4.166	---
- 53	307.91	---	---	302.3	5.606	---
- 54	317.66	---	---	306.7	10.939	---
PbO ₂ - 8	310.72	---	---	310.72	---	---
- 41	309.11	---	---	309.11	---	---
- 50	306.80	---	---	306.80	---	---
U-235- 1	448.07	448.07	---	---	---	---
- 2	449.75	448.12	---	---	---	1.6295

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APPENDIX C

MEASUREMENT OF THE GOLD-197 WEIGHTING FACTOR AS
A MEANS OF DETERMINING C, THE WEIGHTING FUNCTION SCALE FACTOR

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APPENDIX C

MEASUREMENT OF THE GOLD-197 WEIGHTING FACTOR AS A MEANS OF DETERMINING C, THE WEIGHTING FUNCTION SCALE FACTOR

Section IV-2 describes a correction to measured reactivities which accounts for the variation with lethargy of the real and adjoint fluxes. In the idealized system, the fluxes are lethargy-independent. The details of the measurement of the gold-197 weighting function, $C \bar{W}_{Au}^{Exp}$, which is required for the correction, are given below.

1. ACTIVATION EQUIPMENT

All of the activation experiments in this investigation utilized a gamma-ray scintillation spectrometer for the purpose of measuring the neutron induced activities of the absorber materials. A brief description of the spectrometer equipment and the data-reduction techniques is given below.

A cylindrical 3- x 3-inch NaI scintillation crystal was used in all of the counting work. The particular unit used was a Harshaw Integral Line Unit, No. 12 S-13, which contained a DuMont 6363 photomultiplier tube. A drawing of this unit is shown in Figure 31. For the purpose of establishing accurate source distances it was necessary to measure the distance from the top of the crystal to the top of the can. A contact radiograph with a Tm-170 radioisotope camera showed this distance to be 0.125 inch [44].

The source holder arrangement is shown in Figure 32. Adjustable shelves are located at an appropriate distance from the detector. The source distance

Fig. 31 NaI scintillation detector unit.

Fig. 32 Source holder assembly.

chosen for each particular foil depended upon its disintegration rate. Since the error in quantitative results generally increases as the source-detector distance is reduced, the distances were kept as great as was practical. Most of the gold monitor foils were counted at a distance of thirty centimeters. The thin gold micro-foils were counted at three or ten centimeters, and the gold powders were counted at ten centimeters. Each foil counted was mounted in a 3.25- x 2.50-inch paper card with a 1.0-inch diameter hold in the center. Mylar tape was used to mount the foil. Metal source holders were avoided so as to minimize Compton scattering from the source holder. In order to prevent the entry of beta particles into the detector, a beta absorber was placed on top of the detector. The absorber chosen was 1.024 g/cm² of plastic. Reference 45 discusses the effect of this absorber on the gamma spectrum. The detector was located in a "graded shield" to minimize the effect of both photoelectric interaction and background from external sources.

Differential counting was accomplished with a 512 channel Nuclear Data-130 multichannel analyzer. A view of the system is shown in Figure 33. Shown in addition to the analyzer are an A-8 amplifier, a perforated tape data storage system, a high voltage power supply, a total-count scaler for monitoring the discriminator output of the amplifier, and an automatic typewriter.

Fig. 33 Multichannel analyzer system.

2. ACTIVATION TECHNIQUES

As described in Section II, gold was used as the primary standard for all of the uranium and thorium reactivity measurements. Since gold-197 was the only material studied by means of activation analysis, all of the data analysis and reduction techniques described in this section apply specifically to that particular isotope. In addition to the neutron-absorption characteristics which make gold-197 an attractive standard for cross section measurements, its decay scheme is well suited for activation analysis by means of gamma-ray spectroscopy. As shown in Figure 34, its primary mode of decay gives rise to a 411 keV gamma ray. From the almost identical results of several experimenters [46], a value of 2.70 days has been chosen as the half-life.

A channel width of 10 keV was chosen for all of the spectrometer measurements. The counts under the 411 keV photopeak, summed between the energies of 330 and 480 keV (Channels 33 through 47) were chosen as a measure of the activity of the gold-197 isotope. An example of the total gamma spectrum obtained is shown in Figure 35. The small peak at 69 keV is caused by the mercury X-ray, while the peak at 820 keV can be attributed to "random summing" effects. The measured spectrum is in excellent agreement with the results of Heath [45]. Before each counting operation, the energy scale of the analyzer was established with a cesium-137 standard source. For the 10 keV channel scale used, this meant adjusting the gain of the system so that the

Fig. 34: Decay scheme of gold-198.

cesium photopeak fell in channel location 66.162. Any shifts in the photomultiplier resulting from use of the fine-gain vernier were avoided. Periodically, during the months in which these counting experiments were performed, the analyzer system was checked by calibration procedures and performance tests such as those described by Heath in Reference 47.

Although the total number of counts measured under the peak usually exceeded 30,000, a minimum of 10,000 counts was obtained from each respective foil in order to obtain adequately small variances in count rates. The total count rate at the discriminator output of the amplifier was kept below 5000 cps by careful adjustment of waiting time or source-detector distance. Because a severe atmospheric inversion occasionally causes the MTR stack effluent to create a fluctuating radiation background, extreme care was taken to run frequent checks on the background count. Counting was temporarily discontinued during the few instances of high background. Even for the lowest activity foils the background never exceeded ten percent of the gold activity, and in the majority of the cases, the background counts amounted to less than one percent of the foil activity. Foil count rate at time of discharge from the reactor was obtained by applying the appropriate decay correction. This discharge count rate is

$$A = A' \frac{\lambda e^{t_w}}{(1 - e^{-\lambda t_c})} \quad (111)$$

where

A = count rate at discharge

A' = measured count rate

λ = decay constant of gold-197

t_w = elapsed waiting time between discharge and commencement of counting

t_c = elapsed time during counting.

Fig. 35 Gold-198 gamma ray pulse-height spectrum.

Because no counting times were greater than 1000 seconds, Equation (111) was approximated by

$$A = \frac{A'}{t_c} e^{-\lambda t} \quad (112)$$

Various foil geometries were used in the gold activation measurements described in this appendix and in Appendix D. Since each of these required slightly different corrections to the measured count rates, the counting techniques for each type of measurements are discussed separately below.

2.1 Foil Weight Determination by Activation Analysis

The weight of a gold microfoil was determined by comparing its activity to that of a weighable 0.1-mil thick gold foil irradiated simultaneously in the same highly thermalized flux. Since both foils have the same diameter and were similarly sandwiched between 2-mil layers of aluminum foil, their geometries were essentially identical. For this reason it was possible to obtain reasonably high count rates without introducing any errors due to source geometry by counting with a three centimeter source-detector distance. In addition to background corrections, a slight correction was made for the activity of the 2-mil aluminum. This was effected by measuring the activity of a blank 2-mil aluminum sandwich which was irradiated with the gold foil of interest. After a waiting time of two days between irradiation and counting, the activity of the aluminum was usually negligible.

2.2 Activation Measurement of Resonance Integral of Powder Samples Relative To Resonance Integral of Microfoil

Relative resonance integrals were determined by measuring relative activities of gold powders and gold microfoils. In order to minimize the effects of geometry differences between the microfoils and the powder-filled discs, they were counted at a source-detector distance of ten centimeters. A correction to account for the slightly different source distance of the disc and foil was made as follows:

$\bar{r}$ = average source-detector distance

$$\bar{r}_1 = (\bar{r})_{\text{microfoil}} = 10.005 \text{ cm}$$

$$\bar{r}_2 = (\bar{r})_{\text{disc}} = 10.073 \text{ cm}$$

$$\Delta \bar{r} = \bar{r}_2 - \bar{r}_1 = 0.068 \text{ cm.}$$

From the detector efficiency curves of Heath ^[45], the differential fractional efficiency of a 3- x 3-inch NaI crystal at a source distance of ten centimeters is

$$\frac{\partial \epsilon}{\partial \bar{r}} = 0.18/\text{cm} \quad (113)$$

(The foils are assumed to act as point sources.)

Then

$$\Delta \epsilon = \frac{\partial \epsilon}{\partial \bar{r}} \Delta \bar{r} = 0.0122 \quad (114)$$

A multiplicative factor to correct the measured powder activity can be expressed as follows:

$$f_R = \frac{1}{1 - \Delta\epsilon} = 1.0122 . \quad (115)$$

A second correction factor was calculated to correct for attenuation of the 411 keV gamma ray by the lead dioxide/gold powder. The dimensions of the source and detector are shown in Figure 36. Because the source-detector distance is large compared to the detector dimensions, it is a good approximation to treat the powder-filled disc as a thin slab from which all radiation is emitted perpendicular to its face. If i_0 is the "unattenuated" count rate of the detector due to any increment of slab thickness, dx , the attenuated count rate due to the slab thickness, t , is

$$I = \int_0^t i_0 e^{-\mu x} dx = \frac{i_0}{\mu} (1 - e^{-\mu t}) = \frac{i_0}{\mu t} (1 - e^{-\mu t}) \quad (116)$$

where

μ = linear attenuation constant of disc

$I_0 = i_0 t$, unattenuated count rate due to uniformly distributed activity throughout the slab.

The multiplicative correction factor may then be written as follows:

$$f_A = \frac{I_0}{I} = \frac{\mu t}{(1 - e^{-\mu t})} . \quad (117)$$

The attenuation constant of the gold/lead dioxide mixture is computed from the weighted sum of the individual mass attenuation constants.

$$\frac{\mu}{\rho} = \sum_i \left(\frac{\mu}{\rho} \right)_i w_i \quad (118)$$

w_i = weight fraction of element i in the mixture.

Because the gold constitutes such a small weight fraction of the powder, the attenuation constant was calculated for pure lead dioxide. Attenuation in the

Fig. 36. Source geometry for powder-filled disc.

aluminum disc was also taken into account. The following mass attenuation constants were obtained from Reference 48.

<u>Material</u>	$\frac{\mu}{\rho} \left(\frac{\text{cm}}{\text{g}} \right)$
Al	0.091
O	0.0944
Pb	0.201
PbO ₂ (Calculated)	0.187

Table XXVII lists the masses of gold and lead dioxide in each powder disc and the resulting attenuation correction factor $(f_A)_{\text{PbO}_2}$. Column 4 gives the correction factor $(f_A)_{\text{Al}}$ for the aluminum. The overall correction is $(f_A)_{\text{PbO}_2} \times (f_A)_{\text{Al}}$. Attenuation in the lower layer of the 2-mil aluminum surrounding the gold microfoils was negligible.

TABLE XXVII
WEIGHTS OF GOLD AND LEAD DIOXIDE IN
POWDER-FILLED COUNTING DISCS: GAMMA-RAY ATTENUATION FACTORS

<u>Sample</u>	<u>Disc. No.</u>	M_{PbO_2} (mg)	M_{Au} (mg)	$(f_A)_{\text{PbO}_2}^{[a]}$	$(f_A)_{\text{Al}}^{[a]}$	$(f_A)_{\text{PbO}_2} \times (f_A)_{\text{Al}}$
Au-51	9	336.35	0.9427	1.0210	1.0069	1.0280
	10	315.02	0.8828	1.0197	1.0069	1.0267
	11	360.19	1.0094	1.0225	1.0069	1.0296
	12	349.12	0.9784	1.0218	1.0069	1.0289
	13	327.28	0.9172	1.0204	1.0069	1.0274
Au-52	4	337.23	4.6745	1.0218	1.0069	1.0289
	5	453.41	6.2050	1.0287	1.0069	1.0358
	6	498.39	6.9085	1.0316	1.0069	1.0387
	7	402.72	5.5823	1.0255	1.0069	1.0326
	8	460.02	6.3767	1.0291	1.0069	1.0362
Au-54	1	239.94	8.558	1.0155	1.0069	1.0225
	2	281.27	10.032	1.0181	1.0069	1.0251
	3	314.19	11.206	1.0203	1.0069	1.0273
	4	279.82	9.9810	1.0181	1.0069	1.0251

[a] $(f_A)_{\text{PbO}_2}$ and $(f_A)_{\text{Al}}$ described on pages 122 and 123.

2.3 Gold Monitor Foils

These foils, 5 mil thick and 0.152 inch in diameter, were used as monitors in both the MTR activation analysis and the ARMF irradiations. Because of

their relatively large gold mass, resulting in high activities, these MTR monitors of ARMF monitors were counted at distances of 20 cm and at 30 cm.

3. METHOD AND RESULTS

The resonance integral samples used in these measurements consisted of four different mixtures of finely precipitated gold and a lead dioxide diluent. Homogenization and compaction of these mixtures were achieved in a manner similar to that used for the uranium and thorium samples (Appendix B).

Because there was neutron self-shielding even in the finest gold powder available, it was necessary to use a different form of absorber for the infinite dilution measurements. "Microfoils" of gold were prepared by a vacuum evaporation of metallic gold onto a 0.375-inch diameter, 2-mil thick aluminum foil. The evaporation was accomplished by mounting the appropriate amount of thin sheet gold upon a tungsten electrode which was located approximately five inches from the blank 2-mil aluminum foils. After the system was enclosed under a vacuum jar and evacuated to approximately 3×10^{-5} mm Hg, the electrodes were electrically heated to 1100°C or slightly above the melting point of gold. Because the evaporating gold approximated a point source equidistant from every point on each aluminum foil, the resulting thin layer of gold was assumed to be of uniform thickness. Subsequent activation analysis showed the gold to be of the desired thickness -- approximately 10^{-6} cm, equivalent to 25 $\mu\text{g}/\text{cm}^2$. After the evaporation, each foil was covered and cemented to a similar 2-mil aluminum blank to ensure against loss of the thin gold. The microfoils, with lead dioxide as a diluent, were used for the infinite dilution samples. Five of them were positioned approximately 1.25 inch apart throughout the length of one of the aluminum capsules. A blank aluminum foil was also included for the purpose of determining the proper correction for the aluminum backing. Two separate runs were made. The positions of the foils for each of these runs are shown in Figure 37. The same microfoils were used in both measurements.

Because the time-integrated flux to which the sample is exposed is determined by the activation requirements of each particular sample, gold monitor foils were used to normalize all measurements to the same flux exposure. These foils were 0.152 inch in diameter and 5 mils thick.

The quantity $C \bar{W}_{\text{Au}}^{\text{Exp}}$ was measured three times: once each in the powder samples Au-51 and Au-52, with microfoils 41 through 46; the third time with powder sample Au-54 and microfoils 41 through 46 again. The weights and concentrations of all of the gold samples used in the calibration measurements are given in Tables XXVI and I. It should be mentioned at this point that the very small amount of particle self-shielding in the -325 mesh gold powder will not affect the measurement of the weighting factor since the effective resonance integral of the sample is affected identically in both the activation and reactivity measurement.

In the first set of measurements, static reactivity measurements were performed on samples Au-51 and Au-52, and the observed data were reduced in

Fig. 37 Positions of gold microfoils in infinite dilution samples.

accordance with the methods of Section IV in order to give net absorption reactivities. Both of these samples and the infinite dilution sample containing the microfoils were then individually irradiated in the same location where the reactivity measurements were performed. During each irradiation, four gold monitor foils were located above the cadmium-shielded sample position on the teflon plug which ordinarily serves to position the sample in the reactor (Figure 7, page 25). This location for the monitor foils was chosen primarily for the following three reasons;

- (1) The samples and the monitors enter and leave the core simultaneously.
- (2) Because great pains have been taken to ensure reproducibility of the sample and plug position for reactivity measurements, the position of the monitor foils can also be accurately reproduced for each activation measurement.
- (3) The monitor foils are sufficiently isolated from the samples so as to be unaffected by any local flux perturbation due to the sample.

After the powder samples had been taken from the core the powders were removed from the capsule and again tumbled for one hour and violently shaken for fifteen minutes to ensure a homogeneous activity. After this mixing, a minimum of five small portions of the powder in each sample was taken and enclosed in the small aluminum counting discs shown in Figure 34, page 118. These powders, the infinite dilution foils, and the monitor foils were counted as described in Appendix C-2.

If the activities of the monitor foils are introduced into Equation (62), it may be written as

$$I_x = \frac{A_x}{A_\infty} \frac{A_\infty^m}{A_x^m} \frac{\bar{\phi}_\infty}{\bar{\phi}_x} I_\infty \quad (119)$$

where

A_x = average specific count rate of the powder sample

A_∞ = average specific count rate of microfoils [a]

A_∞^m = average specific count rate of the monitor foils for the infinite dilution sample

A_x^m = average specific count rate of the monitor foils for the powder sample

$\bar{\phi}_x$ = absorption weighted average flux for sample x as calculated in Section IV-2.13

$\bar{\phi}_\infty$ = absorption weighted average flux for infinite dilution sample as calculated in Section IV-2.13.

The effective weighting factor was then determined from the relationship

$$C \bar{W}_{Au}^{Exp} = \frac{\rho_x}{N_x V_x I_x} \quad (120)$$

The entire procedure of reactivity and activation measurements was repeated with powder sample Au-54 and microfoils 41 through 46. The results for both sets of measurements are presented in Table XXVIII. C was obtained in each set by equating the calculated and experimental values of $\bar{W}$; ie,

$$C = \frac{C \bar{W}_x^{Exp}}{\bar{W}_x^{Calc}} \quad (130)$$

The final value of C obtained by averaging the results from the three samples is $C = 29.469$.

[a] The microfoils were "weighed" by activation analysis in the MTR thermal column; a description of the procedure is presented in Appendix D.

TABLE XXVIII

RESULTS OF MEASUREMENT OF WEIGHTING FACTOR, $C \bar{W}_{\text{Au}}^{\text{Exp}}$

Sample	Au-51	Au-52	Au-54
A_x (Counts per sec per mg)	943.93	105.10	178.98
A_{∞} (Counts per sec per mg)	1625.4	1625.4	3890.9
A_x^m (Counts per sec per mg)	157.20	28.680	74.746
A_{∞}^m (Counts per sec per mg)	153.95	153.95	355.86
$\bar{\phi}_{\infty}$ (n/cm ² /sec)	0.9761	0.9761	0.9761
$\bar{\phi}_x$ (n/cm ² /sec)	0.9688	0.9534	0.9370
I_x (barns)[a]	916.70	568.48	364.96
ρ_x (μk)	64.35	184.06	320.15
$N_x V$ (10^{24} atoms)	2.6040×10^{-3}	12.740×10^{-3}	33.444×10^{-3}
$C \bar{W}_{\text{Au}}^{\text{Exp}}$	26.928	25.414	26.229
$\bar{W}_{\text{Au}}^{\text{Calc}}$	0.9080	0.8896	0.8690
C	29.656	28.568	30.183
$(C)_{\text{Avg}} = 29.469$			

[a] Referred to a low energy cutoff of 0.251 eV

$$\int_{0.251}^{\infty} \sigma_a \frac{dE}{E} = 1599.8 \text{ barns used as value for } I_{\infty}$$

This corresponds to a value of 1579.0 barns for a cutoff of 0.5 eV.

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APPENDIX D

DETERMINATION OF GOLD MICROFOIL WEIGHTS BY ACTIVATION ANALYSIS

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APPENDIX D

DETERMINATION OF GOLD MICROFOIL WEIGHTS BY ACTIVATION ANALYSIS

Because the gold microfoils could not be weighed accurately on a microbalance, it was necessary to "weigh" them by a neutron activation technique. Each foil was activated in the MTR thermal column along with a 0.1-mil thick gold foil of known weight. The ratio of their masses was then determined from their relative activities. The thickness of the weighable gold foil was chosen on the basis of a compromise between its thermal neutron self-shielding and the accuracy with which its weight could be determined on a microbalance. One-tenth mil is the optimum thickness since it has less than 0.3 percent self-shielding for 0.025 eV neutrons and can be weighed to a relative accuracy of better than 0.25 percent.

The irradiations were performed in a cylindrical hole located in a graphite stringer in position T2-H3 of the MTR thermal column. The microfoils and 0.1-mil foils were sandwiched in pairs in slots cut into a lucite rod which could easily be inserted into and removed from the stringer while the reactor was at power. Activation of bare and cadmium covered 0.1-mil gold foils located in the stringer at a distance of two feet from the outer face of the reactor shield gave a cadmium ratio of 4340. Since all other foils were irradiated at distances greater than two feet, the effects of resonance flux were assumed to be negligible. The results are given in Table XXIX.

TABLE XXIX

RESULTS OF GOLD-197 ACTIVATION ANALYSIS IN MTR THERMAL COLUMN

Microfoil No.	Weight of 0.1 Mil Foil (mg)	Microfoil Count Rate (counts/sec)	0.1 Mil Foil Count Rate (counts/sec)	Weight of Microfoil (μ g)
41	2.001	60.420	9306.4	12.99
42	1.934	53.561	8410.4	12.32
43	2.065	52.351	8635.8	12.52
44	2.008	48.477	7977.1	12.20
45	2.048	46.323	7566.0	12.54
46	2.089	44.432	7461.5	12.44

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APPENDIX E

A TRANSPORT THEORY CALCULATION OF THE ARMF-II REAL AND ADJOINT FLUXES

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APPENDIX E

A TRANSPORT THEORY CALCULATION OF THE ARMF-II REAL AND ADJOINT FLUXES

The correct interpretation of reactivity measurements in the ARMF requires a knowledge of the energy dependence of both the real and adjoint fluxes in the sample location. To obtain this information, a cylindrical representation of the ARMF-II was studied by means of the S-XI code [49]. S-XI, written for the IBM 7090, is a one dimensional code based on the discrete S_n approximation as a method of solving the Boltzmann equation [50]. It can accommodate a large number of energy groups and mesh points and provides great versatility with its many options. A particularly significant feature for this study was the ability of the code to provide space and energy dependent scalar fluxes, adjoints, and weighting functions.

1. CYLINDRICAL MODEL OF THE ARMF-II

The ARMF-II core configuration (shown in Figure 4, page 24) of a centrally located, cadmium shielded, sample position surrounded by a symmetrical array of fuel elements, lends itself rather well to a one dimensional representation composed of a group of concentric annular regions. The radius of each annulus was chosen so that its area was equivalent to the corresponding region that it described in the actual reactor. Figure 38 shows the reactor regions described. A six-inch water region surrounding the core was used to describe the reflector.

Fig. 38 ARMF-II core regions which were homogenized for S-XI transport calculation.

Table XXX. presents the size, composition, and atom density of each annular region. The atom densities are the result of regional homogenizations. The fuel regions were homogenized using the following fuel element characteristics:

Fuel Plate Thickness	0.065 in.
Fuel Plate Width	2.887 in.
Side Plate Thickness	0.1875 in.
Side Plate Width	3.262 in.
Metal Water Volume Ratio	0.60

Region 1 represents a lead dioxide sample located inside the cadmium tube.

Initial attempts to select proper mesh spacing resulted in several difficulties. Within a region, this spacing is determined by the material cross sections. At or near an interface, a gradual transition of spacing must occur from one region to the other. If the transition is not sufficiently gradual, eigenvalue oscillation or negative fluxes may occur. Mesh spacing also appears to significantly influence the rate of convergence. The final calculation utilized 86 mesh points. The S_2 approximation was used since, with a reasonable number of mesh points, all higher order approximations overloaded the computer memory.

2. GROUP CONSTANTS

The energy spectrum was divided into 17 fast groups and one thermal group. The group widths, which are multiples of 0.25 lethargy units, were chosen to provide the highest resolution in the resonance region and at the lower end of the energy spectrum. The boundaries and width of each group are presented in Table XXXI.

The GAM-I code ^[51] was used to calculate the constants for the 17 groups. GAM is a P_1 multigroup code for the calculation of fast neutron spectra. In this code the P_1 equations are integrated over quarter lethargy units from 10 MeV to 0.414 eV, yielding a total of 68 "fine groups". The 17 "broad group" constants were then calculated by averaging over the 68 group spectrum. The ARMF-II flux spectrum as calculated by this zero-dimensional code is shown in Figure 39.

The SOFOCATE code ^[52] was used to provide the thermal, or eighteenth group cross sections. SOFOCATE calculates thermal constants by averaging energy dependent cross sections over either a Wigner-Wilkins or a Maxwellian spectrum. In this particular case, the Wigner-Wilkins spectrum was used.

Previous studies of the ARMF-I [53] have indicated that the reflector savings, δ , is approximately 7.5 cm. Thus, the equivalent reactor height, H , follows:

$$H = 2\delta + H_o = 76 \text{ cm} \quad (131)$$

where

$$H_o = \text{active fuel length} = 24 \text{ in.} = 61 \text{ cm.}$$

TABLE XXX

SIZE AND COMPOSITION OF REGIONS IN THE CYLINDRICAL MODEL OF THE ARMF-II

Region	Inner and Outer Radii (cm)	Region Composition	Atom Densities (10^{24} atom/cm ³)
1	0 - 1.4732	PbO ₂	$N_{\text{Pb}} = 0.010$ $N_{\text{Ox}} = 0.020$
2	1.4732 - 1.5240	Cadmium	$N_{\text{Cd}} = 0.0464$
3	1.5240 - 4.7244	131 gram fuel element in water	$N_{\text{Al}} = 0.022593$ $N_{\text{U-235}} = 0.0000794$ $N_{\text{Ox}} = 0.020857$ $N_{\text{H}} = 0.041714$
4	4.7244 - 14.1224	179 gram fuel element in water	$N_{\text{Al}} = 0.022593$ $N_{\text{U-235}} = 0.0001085$ $N_{\text{Ox}} = 0.020857$ $N_{\text{H}} = 0.041714$
5	14.1224 - 22.5552	(a) 57.14% 179 & 25 gram fuel elements (b) 42.86% water	$N_{\text{Al}} = 0.012910$ $N_{\text{U-235}} = 0.000115$ $N_{\text{Ox}} = 0.026259$ $N_{\text{H}} = 0.052518$
6	22.5552 - 37.7952	Water	$N_{\text{Ox}} = 0.033460$ $N_{\text{H}} = 0.066922$

TABLE XXXI

GROUP WIDTHS AND BOUNDARIES OF
SPECTRUM STRUCTURE USED IN S-XI CALCULATION

Group	Upper Limit (eV)	Upper Limit (Lethargy)	Group Width (Lethargy)
1	1.0×10^7	0.0	1.25
2	2.87×10^6	1.25	1.75
3	4.98×10^5	3.00	4.25
4	7.10×10^3	7.25	1.75
5	1.20×10^3	9.00	1.25
6	3.54×10^2	10.25	1.25
7	1.01×10^2	11.50	0.75
8	47.9	12.25	0.50
9	29.0	12.75	0.75
10	13.7	13.50	0.75
11	6.48	14.25	0.75
12	3.06	15.00	0.50
13	1.86	15.50	0.50
14	1.125	16.00	0.25
15	0.876	16.25	0.25
16	0.683	16.50	0.25
17	0.532	16.75	0.25
18	0.414	17.00	

The vertical buckling used was

$$B_z^2 = \left(\frac{\pi}{H} \right)^2 = 0.00171 \text{ cm}^{-2}. \quad (132)$$

The axial neutron leakage was included by means of a fictitious absorption cross section equal in magnitude to DB_z^2 where D is the diffusion coefficient and B_z is the geometric buckling.

3. RESULTS

The S_2 approximation was used in the calculation and gave the following output quantities:

$$\text{Scalar real flux} = \phi(r, u) \Delta u = \int_{\Omega} \phi(r, u, \Omega) d\Omega \Delta u. \quad (133)$$

Fig. 39 ARMF-II flux spectrum as calculated by the GAM code.

$$\text{Scalar adjoint flux} = \phi^+(r, u) = \int_{\Omega} \phi^+(r, u, \Omega) d\Omega \quad (134)$$

$$\begin{aligned} \text{Scalar statistical weight} &= \phi(r, u) \phi^+(r, u) \Delta u \\ &= \int_{\Omega} \phi(r, u, \Omega) \phi^+(r, u, \Omega) d\Omega \Delta u. \end{aligned} \quad (135)$$

Distribution functions of the real flux and the statistical weight were obtained by dividing the corresponding output quantities by the appropriate group width, Δu . Analysis of the results revealed that both $\phi(r, u)$ and $\phi^+(r, u)$ are spatially flat inside the cadmium tube; the quantities $\phi(u)$ and $\phi^+(u)$ are therefore adequate for representation of these fluxes. In addition, it was noted that

$$\int_{\Omega} \phi(r, u, \Omega) d\Omega \int_{\Omega} \phi^+(r, u, \Omega) d\Omega = \int_{\Omega} \phi(r, u, \Omega) \phi^+(r, u, \Omega) d\Omega. \quad (136)$$

The product of the integrals rather than the integral of the products was therefore used as the statistical weight.

Distribution functions for $\phi(u)$, $\phi(u)^+$, and $\phi(u) \phi^+(u)$ are presented in lethargy space in Figures 40 and 41, and a corresponding tabulation is given in Table XXXII. Since a $1/E$ flux distribution corresponds to a constant flux in lethargy space, it is apparent that the real flux deviates somewhat from a $1/E$ behavior. The flux diminution at low energies is apparently due primarily to the influence of the cadmium tube. The adjoint flux also shows the same rapid decrease at energies of a few electron volts. It is apparent that the proper interpretation of reactivity and activation measurements performed inside the cadmium tube must include some corrections to account for both the non- $1/E$ behavior of the real flux and the lack of constancy of the adjoint flux.

Fig. 40 Real and adjoint fluxes in center of ARMF-II cadmium tube as determined from S_2 transport calculation.

Fig. 41 Weighting function in center of ARMF-II cadmium tube as determined from S_2 transport calculation.

TABLE XXXII

CALCULATED RELATIVE REAL FLUX, ADJOINT FLUX, AND
WEIGHTING FUNCTION AT CENTER OF CADMIUM TUBE IN ARMF-II

<u>Group</u>	<u>$\phi(u)$</u>	<u>$\phi^+(u)$</u>	<u>$\phi(u)\phi^+(u)$</u>
1	1.8024	0.6856	1.2358
2	4.7439	0.9647	4.5767
3	1.7000	1.0340	1.7578
4	1.0429	1.0286	1.0728
5	1.0383	1.0206	1.0597
6	1.0310	1.0051	1.0363
7	1.0052	0.9740	0.97914
8	1.0250	0.9864	1.0111
9	1.0062	0.9725	0.9786
10	0.9950	0.9657	0.9609
11	0.9862	0.9352	0.9223
12	0.9726	0.9099	0.8851
13	0.9475	0.8787	0.8326
14	0.9027	0.8318	0.7509
15	0.8350	0.7661	0.6397
16	0.7264	0.6559	0.4765
17	0.5226	0.4380	0.2290

Note: Fluxes obtained from S-XI calculation.

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APPENDIX F

DERIVATION OF THE FLAT FLUX VERSION OF THE RECIPROCITY THEOREM

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APPENDIX F

DERIVATION OF THE FLAT FLUX VERSION OF THE RECIPROCITY THEOREM^[a]

Consider the case of a spatially uniform isotropic source of neutrons, $S(r)$, in volume V_0 . The macroscopic total cross section of this region is Σ_{t0} . Exterior to V_0 is a second region of volume V_1 also containing a uniform isotropic source and having cross section Σ_{t1} .

The probability of a source neutron from V_0 suffering its first collision in V_1 is $1 - P_0$. P_0 is the first flight collision probability of region 0 as defined in Section II-1.

$$(1 - P_0) = \frac{\int_{V_0} S(r') d^3r' \int_{V_1} \Sigma_{t1} \phi(r, r') d^3r}{\int_{V_0} S(r) d^3r} \quad (137)$$

where $\phi(r, r')$ is the uncollided flux at r , due to a unit isotropic source at r' . Equation (137) can be simplified to read

$$\begin{aligned} (1 - P_0) &= \frac{S_0 \Sigma_{t1} \int_{V_0} d^3r' \int_{V_1} \phi(r, r') d^3r}{S_0 \int_{V_0} d^3r} \\ &= \frac{\Sigma_{t1}}{V_0} \int_{V_0} d^3r' \int_{V_1} \phi(r, r') d^3r \end{aligned} \quad (138)$$

where S_0 is the isotropic source strength. The probability of a source neutron from V_1 suffering its first collision in V_0 is similarly

$$(1 - P_1) = \frac{\Sigma_{t0}}{V_1} \int_{V_0} d^3r \int_{V_1} \phi(r', r) d^3r' \quad (139)$$

where P_1 is the first flight collision probability of region 1. Here the reciprocity theorem of monoenergetic transport theory is introduced^[54].

It states

$$\phi(r, r') = \phi(r', r). \quad (140)$$

When Equation (140) is substituted into (139), the following can be obtained.

$$(1 - P_1) = \frac{\Sigma_{t0}}{V_1} \int_{V_0} d^3r \int_{V_1} \phi(r, r') d^3r' \quad (141)$$

[a] The general procedure in this derivation is similar to that in Reference 10.

The interchanging of r and r' in (141) gives

$$(1 - P_1) = \frac{\Sigma_{t_0}}{V_1} \int_{V_0} d^3r' \int_{V_1} \phi(r, r') d^3r. \quad (142)$$

Comparison of (138) and (142) then leads to

$$(1 - P_0) V_0 \Sigma_{t_0} = (1 - P_1) V_1 \Sigma_{t_1}. \quad (143)$$

Equation (143) can be written in the nomenclature of Section II as follows:

$$(1 - P_0) \sigma_{t_0} = (1 - P_1) \sigma_{t_1} \quad (144)$$

where

$$\sigma_{t_0} = \frac{\Sigma_{t_0}}{N_0}$$

$$\sigma_{t_1} = \frac{V_1 \Sigma_{t_1}}{V_0 N_0}$$

N_0 = atom density of a particular absorber in region 0.

Equation (144) is precisely the relationship used in Section II.

APPENDIX G

UNCERTAINTY ANALYSIS

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APPENDIX G

UNCERTAINTY ANALYSIS

1. UNCERTAINTY ANALYSIS OF MEASURED INTERFERENCE

This analysis is best performed by examining the uncertainties in the appropriate reactivity values and then, from these, deducing the uncertainties in the corresponding resonance absorption. Because the average weighting function, $\bar{W}_X$, is approximately the same for all the uranium and thorium samples, the measured interference for a sample containing a binary mixture is written approximately as follows:

$$\Delta I_{U,Th} = \frac{\Delta \rho_a}{C\bar{W} V(N_U + N_{Th})} + \delta = \frac{(\rho_a)_{U,Th} - [(\rho_a)_U + (\rho_a)_{Th}]}{C\bar{W} V(N_U + N_{Th})} + \delta \quad (145)$$

where $(\rho_a)_{U, Th}$ = absorption reactivity of binary mixture

$(\rho_a)_U$ and $(\rho_a)_{Th}$ = absorption reactivities of the corresponding single absorber samples of the appropriate atom densities

δ = the small resonance integral correction described on page 17.

The uncertainty in $\Delta I_{U,Th}$ can then be determined from the uncertainties in the right hand side of (145).

The δ term contribution to ΔI was relatively small for most of the samples. Because it was estimated that this small term could be calculated to within an accuracy of ten percent or better, its contribution to the uncertainty in ΔI was negligible. Weighing errors (contributing to an uncertainty in $N_U + N_{Th}$) and the uncertainty in the value of $C\bar{W}$ are also unimportant when compared to the uncertainty resulting from the small differences between the two large numbers in the numerator. The analysis for each interference sample is, therefore, focused on the uncertainties in the quantity

$$\Delta \rho_a = (\rho_a)_{U,Th} - [(\rho_a)_U + (\rho_a)_{Th}] \quad (146)$$

The quantity $(\rho_a)_{U,Th}$ is deduced from measurements using the expressions at the bottom of page 52; i.e.,

$$\rho_a = \rho_g - \rho_c - \rho_s - (\rho_a)_{Pb} - \rho_{U-235} - \rho_{ff} \quad (147)$$

It contains (a) a "random error" arising from the directly measured quantities ρ_g and ρ_c , and (b) a "systematic error" due to any possible inaccuracies in the calculated or indirectly measured quantities ρ_s , $(\rho_a)_{Pb}$, ρ_{U-235} , and ρ_{ff} . The quantities $(\rho_a)_U$ and $(\rho_a)_{Th}$ were obtained from the analytical expressions resulting from the least squares fitting procedures described on pages 72-79. Each reactivity value obtained from one of these curves contains (a) a systematic error analogous to that described above and (b) an "estimation error" due

to the uncertainty associated with the coefficients A, B, C, D, and λ obtained from the fit. Fortunately, the systematic errors associated with the uranium dioxide and thorium dioxide cancel because of the difference term in (146). This can be easily verified by substituting the appropriately subscripted form of (147) into each of the three terms on the right hand side of (146). The cancellation is based, of course, on the assumption that, for a given atom density of each of the absorbers, the quantities ρ_g , ρ_{U-235} and ρ_{ff} are the same in both the calibration sample and the interference sample. After this cancellation, $\Delta\rho_a$ may be written as follows:

$$\Delta\rho_a = [\rho_g - \rho_c]_{U,Th} - [\rho_g - \rho_c]_U - [\rho_g - \rho_c]_{Th} +$$

$$[(\rho_a)_{Pb} + (\rho_s)_{Pb}]_U + [(\rho_a)_{Pb} + (\rho_s)_{Pb}]_{Th} - [(\rho_a)_{Pb} + (\rho_s)_{Pb}]_{U,Th}. \quad (148)$$

As described on page 32, the total effect of absorption and scattering by lead (lead dioxide) was determined by reactivity measurements on pure lead dioxide samples. The effect was assumed to be proportional to the number of lead dioxide molecules present in the sample. Equation (148) can, therefore, be rewritten as

$$\begin{array}{cccc} (1) & (2) & (3) & (4) \\ \Delta\rho_a = [\rho_g - \rho_c]_{U,Th} - [\rho_g - \rho_c]_U - [\rho_g - \rho_c]_{Th} + [K\Delta N_{Pb}] & & & (149) \end{array}$$

where K = reactivity per molecule of lead dioxide. K was measured for several different lead dioxide samples so that an uncertainty can be ascribed to it.

$$\Delta N_{Pb} = (N_{Pb})_U + (N_{Pb})_{Th} - (N_{Pb})_{U,Th}$$

The uncertainty in each term in (149) is discussed below.

The uncertainty (random error) in the first term should be approximately the same as σ^2 , the "variance of the fit" obtained from the fits described on page 72. These two fits gave variances of $0.739 \mu k^2$ and $1.348 \mu k^2$, respectively; the variance of term (1) in (149) was taken as the root-mean-square value of these, ie, $1.278 \mu k^2$.

The second and third terms each contain uncertainties (estimation errors) due to uncertainties associated with the coefficients of the fit. The resulting uncertainties associated with the reactivity value obtained from the fitted function can be obtained from the concept of "propagation of error" [55, 56]. In summary, this concept states that the variance of the reactivity function at a given atom density, N , can be estimated as follows:

$$\sigma_p^2(N) = \sigma^2 \left[\sum_{j=1}^5 \sum_{k=1}^5 \left(\frac{\partial \rho}{\partial \alpha_j} \right) \left(\frac{\partial \rho}{\partial \alpha_k} \right) C_{jk} \right] \quad (150)$$

where σ^2 = the variance of fit

α_j or α_k = the parameters of the fit, A, B, etc

C_{jk} = kth element of the jth row of the inverse of the matrix of the normal equations.

The uncertainty in the fourth term was calculated directly from the uncertainty ascribed to K.

The variances arising in each of the four terms are tabulated in columns 1-4 of Table XXXIII, along with their combined variance, σ_{tot}^2 , in Column 5 and the standard deviation, σ_{tot} , in Column 6. The standard deviation in ΔI was then calculated from

$$\sigma_{\Delta I} = \frac{\sigma_{tot}}{C\bar{W} V(N_U + N_{Th})} \quad (151)$$

The values of $\pm \sigma_{\Delta I}$ and $\pm \sigma_{\Delta I}/I$ are given in Columns 4 and 5 of Table XXIV. It can be seen that $\sigma_{\Delta I}$ ranges from one-half barn to less than one-tenth of a barn.

2. UNCERTAINTY ANALYSIS OF MEASURED INFINITE DILUTION THORIUM RESONANCE INTEGRAL

The infinite dilution resonance integral of Th-232 was calculated from Equation (100) as follows:

$$I_{\infty}^{Th} = \frac{A B}{C\bar{W}_{\infty}^{Th} V} \quad (100)$$

Because an excellent fit to the reactivity data was obtained from Equation (97), the error contributed by uncertainties in A and B was negligible. The uncertainty in $C\bar{W}$ was estimated to be 4.1 percent, including errors in the calculated spectra and in the measurement of C. An additional uncertainty of three percent was contributed by the infinite dilution gold standard. The final result and uncertainty are then

$$I_{\infty}^{Th} = 81.2 \pm 4 \text{ barns.}$$

TABLE XXXIII

UNCERTAINTIES ASSOCIATED WITH REACTIVITY MEASUREMENT OF INTERFERENCE

	(1)	(2)	(3)	(4)	(5)	(6)
	σ^2_U	σ^2_{Th}	$\sigma^2_{U,Th}$	σ^2_{Pb}	σ^2_{tot}	σ_{tot}
Sample	$(\mu k)^2$	$(\mu k)^2$	$(\mu k)^2$	$(\mu k)^2$	$(\mu k)^2$	(μk)
131	0.7283	0.9077	1.2779	0.0994	3.0133	1.736
132	0.3374	1.0169	1.2779	0.1021	2.7343	1.653
133	0.4749	0.9116	1.2779	0.0990	2.7634	1.662
134	0.4109	1.0614	1.2779	0.1039	2.8541	1.689
135	0.3340	0.8793	1.2779	0.0939	2.5851	1.607
311	0.4753	1.1243	1.2779	0.0963	2.9738	1.725
312	0.3377	1.0839	1.2779	0.1024	2.8019	1.674
313	0.3893	0.9229	1.2779	0.1045	2.6946	1.642
314	0.3183	0.7511	1.2779	0.1104	2.4577	1.568
315	0.4741	1.0254	1.2779	0.1108	2.8882	1.699

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