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A Study of the Structure of ^{12}B
via Elastic Scattering of Neutrons from ^{11}B

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June 9, 1973

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A STUDY OF THE STRUCTURE OF ^{12}B
VIA ELASTIC SCATTERING OF NEUTRONS FROM ^{11}B

A Dissertation Presented to
The Faculty of the Graduate College of
Ohio University

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

by
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This dissertation has been approved
for the Department of Physics
and the Graduate College by

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Dean of the Graduate College

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I. INTRODUCTION

A detailed experimental study of the energy levels of ^{12}B through neutron scattering from ^{11}B is of particular interest. The neutron binding energy in ^{12}B is only 3.369 MeV, so that a shell model description of these levels is feasible. Furthermore, states above this energy are not readily excited in other reactions. Clarification of the level structure in ^{12}B is useful for the identification of $T = 1$ isobaric analog states in ^{12}C , where the existence of prominent $T = 0$ states makes the analysis of the structure very difficult. Moreover, no systematic differential cross section measurements for $^{11}\text{B} + n$ have been made above a neutron energy of 2.2 MeV.

In the simple shell model, ^{12}B can be treated as a core of ^{12}C with one extra neutron and one proton 'hole'. The lower lying excited states of ^{12}B occur as particle-hole multiplets arising from the $1p_{3/2}^{-1}$ single proton 'hole' configuration coupled (using j-j coupling) to the configuration of the last neutron 'particle'. The splitting within a multiplet is a result of the residual interaction between the proton 'hole' and the last neutron. Calculations of this type, using various residual interactions, are in agreement with experiment, in particular the $^{11}\text{B}(d,p)^{12}\text{B}$ reaction (Mo 71), for the first four levels of ^{12}B (Gu 68, Gi 64). Moreover, recent calculations of this type, in conjunction with a two channel R-matrix analysis of the differential cross section and polarization data for $^{11}\text{B} + n$ in the neutron energy interval $0.075 \text{ MeV} \leq E \leq 2.2 \text{ MeV}$ have led to the assignment of J^π to the

lowermost (3^- , 2^- , 4^-) levels of another multiplet, $1p_{3/2}^{-1}1d_{5/2}$ (La 70). Although there was no evidence of the existence of the 1^- state from this multiplet from any other reaction, including $^{11}\text{B} + n$ total cross section measurements (Bo 51), these simple particle-hole shell model calculations predicted such a state (appearing as a resonance in $^{11}\text{B} + n$) at a laboratory neutron energy of approximately 2.8 MeV. As discussed below, the present experiment and analysis present strong evidence indicating the existence of this state.

The region of excitation in ^{12}B explored in the present experiment is in the vicinity of the giant dipole resonance in ^{12}C ($19.7 \text{ MeV} \leq E_x \leq 22.2 \text{ MeV}$) which has been the subject of much interest for many years (An 72, Gl 72, Re 71, Ya 71, Do 70, Do 68, De 65, Le 64). Details of the $T = 1$ structure at this excitation energy is particularly difficult to obtain because of the large number of $T = 0$ states present. The identification of states in ^{12}B ($T = 1$) from the present experiment will provide valuable information with regard to the $T = 1$ isobaric analog states in ^{12}C .

The region of excitation explored in ^{12}B in the present experiment extends from $E_x \approx 5.3 \text{ MeV}$ to $E_x \approx 7.5 \text{ MeV}$. Neutron scattering is the most practical way of exploring this region (Aj 68). The only previous differential cross section measurements of neutron scattering from ^{11}B in this region have been limited to the measurements of Porter, et al., which have been published as an Aldermaston laboratory report (Po 70). These measurements were limited in scope and did not shed any new light on the structure of ^{12}B . The total cross section measurements of Bockelman, et al. and Fossan, et al. have led to tentative assignments

of J (based on total peak height) or have set only a lower limit to J for the levels of this region (Bo 51, Fo 61). Studies of gamma rays from neutron inelastic scattering from ^{11}B have either been limited in maximum neutron energy available ($E < 3.0$ MeV, Li 61) or have used fission spectrum neutrons (Ni 71). Neutron capture gamma ray measurements have also been limited in maximum neutron energy ($E < 2.5$ MeV, Im 62).

Figure 1-1 shows the total cross section for $^{11}\text{B} + n$ plotted alongside the energy level diagram for ^{12}B (Aj 68). The arrows indicate the energy interval for which the differential cross sections were measured in the present experiment. Pulsed protons from the Ohio University High Current Tandem Van de Graff accelerator incident upon a lithium metal target provided a pulsed (FWHM ≈ 1 nsec) neutron source via the reaction $^7\text{Li}(p,n)^7\text{Be}$. Standard time-of-flight techniques were used throughout this experiment. A companion experiment for neutrons scattered from an enriched ^{10}B sample provided the necessary data to eliminate the contribution from the scattering of neutrons by ^{10}B in the natural boron scattering sample (80.4% ^{11}B , 19.6% ^{10}B) used in the present study.

The data of this experiment have been combined with the differential cross section and polarization data analyzed previously (La 70). This dissertation presents a detailed analysis of the combined data, including predictions of the polarization in the energy interval $2.2 \text{ MeV} \leq E \leq 4.5 \text{ MeV}$, in terms of a two channel, three level R-matrix. The level structure of ^{12}B inferred from this analysis is interpreted in terms of specific shell model particle-hole configurations predicted by previous calculations (La 70). Comparisons are also made to the $T = 1$ isobaric analog states in ^{12}C .

1.600

1

Figure 1-1. The energy level diagram for ^{12}B .

1

9.03	>1
8.49	>1
8.40	
8.23	>1
7.77	>0
7.54	>3
6.76	≥ 0
6.60	>0
5.73	3
5.61	2
5.00	1 ⁺
4.54	4 ⁻
4.37	2 ⁻
4.30	1 ⁻
3.76	2 ⁺
3.39	3 ⁻
2.72	0 ⁺
2.62	1 ⁻
1.67	2 ⁻
0.95	2 ⁺
0.0	1 ⁺

^{12}B

II. EXPERIMENTAL METHOD

The differential cross sections were measured using pulsed beam time-of-flight techniques. The Ohio University High Current Tandem Van de Graff accelerator with pre-acceleration klystron bunching (5 MHz) provided 1 nsec bursts of protons in the 2.0 MeV to 6.8 MeV range. Average beam currents on target varied from 5.0 to 10.0 μ A. Protons from the accelerator, after suitable collimation, struck a thin layer of pure lithium metal and produced neutrons via the ${}^7\text{Li}(p,n){}^7\text{Be}$ reaction which has a threshold of 1.887 MeV. The neutrons emitted at zero degrees were scattered from cylindrical scattering samples, one of natural boron (19.6% ${}^{10}\text{B}$ and 80.4% ${}^{11}\text{B}$) and another enriched to 93% ${}^{10}\text{B}$. The scattering plane was normal to the axis of these cylindrical samples. Angular distributions were measured for neutron energies between 2.2 and 4.5 MeV at seven angles, from 20° to 160° . A separate time-of-flight detector located at zero degrees served as a monitor of the neutron flux. A floor plan of the experimental arrangement is shown in Figure 2-1 (Ne 73).

NEUTRON SOURCE

Neutrons were obtained via the ${}^7\text{Li}(p,n){}^7\text{Be}$ reaction using the wobble target assembly shown in Figure 2-2. Neutrons from the ${}^7\text{Li}(p,n){}^7\text{Be}^*$ reaction, which has a threshold of 2.380 MeV, were completely resolved by the time-of-flight (TOF) system. Thin (100-200 mg/cm^2) layers of pure lithium were evaporated onto a tantalum

Figure 2-1. Floor plan of the experimental arrangement.

Figure 2-2. The Wobble Target Assembly.

The dashed circle on the tantalum 'cup' indicates the path traced by the proton beam as the 'cup' wobbles about the beam axis.

'cup' using an evaporator designed exclusively for this purpose. The cup was transported to the target assembly in a helium atmosphere, where it was mounted and held in place by the vacuum. Most irregularities in the evaporated lithium layer were averaged out by wobbling the cup about the proton beam axis. A simple compressed air jet directed at the cup was found to provide sufficient cooling.

The proton beam was collimated through two sets of completely adjustable slits from which beam current could be measured. One set of slits, immediately preceding the last quadrupole magnet on the beam leg, confined the beam to the axis of the quadrupole. Steering of the beam by this quadrupole was therefore eliminated. The other set of slits was located approximately 66 cm in front of the lithium target cup and was set for an opening of 5 mm by 5 mm. Beam spot sizes on target were kept to less than 6 mm by 6 mm.

The spread in energy of the neutrons, ΔE , was produced chiefly by the proton energy loss in the lithium layer. This spread was determined by measuring the yield of the ${}^7\text{Li}(p,n){}^7\text{Be}$ reaction near threshold (rise curve method), using a long counter positioned at zero degrees. The total energy spread of the neutrons was obtained from the proton energy difference between the threshold of the reaction and the peak in the yield curve. An example of a yield curve is shown in Figure 2-3.

The total neutron cross section of natural boron (80.4% ${}^{11}\text{B}$) exhibits several narrow ($\Gamma \approx 60$ keV), closely spaced resonances in the neutron energy interval of 2.2 to 4.5 MeV (Fo 61). It was essential that the neutron energy spread and, therefore, the lithium target thickness be kept to a minimum if any resonance interference effects

Figure 2-3. Two Yield Curves.

The energy difference between the threshold and the peak gives the total proton energy spread (ΔE_p) at threshold.

were to be noticed. The wobble target assembly provided a convenient means to meet this requirement. First, by developing a standard procedure for evaporating lithium metal, reproducible (± 1 keV) targets could easily be made with neutron energy spreads from several keV up to 60 or 80 keV. Secondly, the lithium targets picked up essentially no contaminants and thus no additional energy spread was introduced during the time it took to pass over a narrow resonance. This was primarily a result of the construction of the wobble target assembly which eliminated the need for a dynamic 'O' ring seal by using a flexible steel bellows. Finally, any changes in neutron energy spread as a result of beam spot movement to another portion of the tantalum 'cup', where the lithium layer might be of a different thickness, were eliminated by the collimation system described earlier.

As a check on the constancy of the neutron energy spread, another rise curve was taken on every target after each run was completed. Despite the fact that some of the runs lasted up to 72 hours with 7 to 10 μ A of average current, in no instance did the target exhibit any appreciable change in thickness. This is illustrated in Figure 2-3, where the two rise curves taken several days apart on the same target indicate only a 2 keV change in target thickness.

BORON SAMPLES

An isotopically pure sample of ^{11}B was not available, thus two scattering samples were used: a high chemical purity natural boron sample and an isotopically enriched sample of ^{10}B . Table 2-1 gives the pertinent physical data of the samples. The boron powder was

TABLE 2-1

	B	^{10}B	NE 102
Chemical Purity (percent by weight)	99.999 [*]	93.6 ^{**}	100.
Atomic Percent ^{11}B	80.40	7.72	...
Atomic Percent ^{10}B	19.60	92.28	...
Atomic Percent H	...	...	52.494
Atomic Percent C	...	...	47.506
Mass (gm)	15.2260	12.6354	10.891
Volume (cm ³)	12.896	12.888	10.553
Density (gm/cm ³)	1.181	.980	1.032

* Supplied by Electronic Space Products, Inc., Los Angeles, California.

** Analysis by Oak Ridge National Laboratory. Impurities consisted of the following (percent by weight in parentheses):

C (0.52)
O (1.30)
H₂O (1.30)

sealed in identical thin-walled (0.25 mm wall thickness) cylindrical aluminum cans whose inside dimensions were $2.540 \pm .013$ cm diameter and $2.540 \pm .013$ cm high. Both samples were carefully packed to approximately $1/2$ crystal density and had neutron transmissions of approximately 70% along a diameter. These samples and an identical empty can were supported at the pivot of the main neutron detectors and were located at zero degrees, 20 cm from the lithium target.

NEUTRON DETECTORS

Each main neutron detector was a 18.24 cm diameter by 5.08 cm length NE 218 liquid scintillator. It was coupled via a sharply tapered lucite light guide to an RCA 4522 12.70 cm diameter fast photomultiplier tube. Both detectors had commercial magnetic shielding and were placed in the center of large water tank shields, 3 meters from the scattering samples. A cutaway view of a water tank shield is shown in Figure 2.4. To reduce background gamma radiation, an annular lead sleeve 5.08 cm thick and 91 cm long surrounded the detector. Sectional polyethylene collimators, each 2 meters in length, collimated the scattered neutrons.

Plexiglass shadow bars, which shadowed the collimator aperture and the detector from direct neutrons of the source reaction, were mounted on the water tank shields. These shadow bars were used for all the measurements except the extreme forward and backward angles. At these angles, the plexiglass shadow bars were removed and specially designed polyethylene shadow bars were used instead, one permanently mounted to the beam leg support (160°) and another rigidly attached to

Figure 2-4. Outaway view of a water tank shield.

POLYETHYLENE
COLLIMATOR

WATER

ANNULAR LEAD
SLEEVE

NEUTRON
DETECTOR

the water tank pivot (20°). Each tank was mounted on air pallets and was attached by a pivot arm to the scattering sample holder. When compressed air was applied to the tank base, the tank could be easily pivoted about the scattering sample by hand, although its total weight was in excess of 4500 lbs.

The lithium target neutron production was monitored at zero degrees by a 2.54 cm diameter by 1.27 cm length NE 102 scintillator coupled directly to the face of an RCA 8575 5.08 cm diameter fast photomultiplier tube, placed 5.2 meters from the wobble target assembly. A long counter placed at 90° served as an additional monitor of the total neutron production of the lithium target.

INSTRUMENTATION

Figure 2-5 shows a block diagram of the neutron detection electronics. The linear (dynode) signal from each main detector was amplified (Ortec 460) and sent to a leading edge discriminator (Ortec 458). This discriminator was set to accept pulses corresponding to neutron energies of approximately 1 MeV and above. The pulses above this level were sent to the pulse shape discriminator (Ortec 458) to identify neutrons and gamma rays. If a neutron was identified, a logic pulse was supplied to the multiple coincidence AND circuit.

The fast zero-crossing (ZC) timing technique was used throughout the experiment to obtain good time resolution at a reasonable cost. The anode signal from each detector was clipped and sent to a ZC discriminator (EG & G T140/N). Since the neutron counting rates were much less than the beam pulse repetition rate (5 MHz), pulses from this

Figure 2-5. Block diagram of the instrumentation.

RRD refers to the repetition rate discriminator.

Delay elements necessary to meet coincidence requirements have been eliminated for simplicity.

discriminator became the start pulses for the time-to-amplitude converters (TAC). Although this procedure results in time being stored in an inverted mode, it greatly reduces the dead time of the TAC (Ortec 437A).

Stop pulses were provided by a simple capacitive pick-off suspended in the proton beam. After fast amplification (Hewlett-Packard 35002A), these pulses were inverted and sent to a ZC discriminator. The output of this discriminator was delayed 400 nsec in RG 213 cable and then regenerated in a fast coincidence unit (LRS 162). These stop pulses then entered the repetition rate discriminator. If the repetition rate of the detected stop pulses was equal to the klystron buncher frequency (to within $\sim 0.1\%$), another logic level was supplied to the multiple coincidence AND unit. This requirement was particularly important during the transmission measurements, to be discussed in a subsequent section. The final logic level was supplied to the AND unit by the beam current integrator.

A Nuclear Data 3300 multichannel analyzer, operating in the digiplex mode, provided two analog-to-digital converters, one for each main detector. When the three coincidence requirements were met, a TAC pulse was accepted by the multichannel analyzer.

The monitor time-of-flight system was identical to the main detector systems except that pulse shape discrimination was not employed. Monitor TOF spectra were stored in a Nuclear Data 1100 multichannel analyzer.

TRANSMISSION MEASUREMENTS

The transmissions (along a diameter) of the natural boron and the enriched ^{10}B samples were required so that the zero degree monitor spectra could be used to normalize the scattering measurements. Transmissions were measured by summing the zero degree monitor spectra over the same TOF peak channels for sample 'in' and sample 'out' runs. Each spectrum was taken for equal amounts of integrated beam, thus the measurements were charge normalized. It was during these measurements that it was important to detect stop pulses at the klystron frequency. If all of the beam pulses from the beam pick-off were not large enough to be counted, as a result of a slight change in accelerator operating conditions for example, a normalization error would result. For equal amounts of integrated charge, there could be substantially different spectra because not all of the start pulses (detected neutrons) would have an associated stop pulse. To avoid this source of possible error, a frequency to voltage converter was constructed (Yo 73). If the output of this device, referred to as the repetition rate discriminator, indicated stop pulses were not being detected at the klystron frequency (to within $\pm 0.1\%$), data acquisition was interrupted.

These transmission measurements were typically taken for 1000 microcoulombs of integrated proton beam and were repeated three times for each sample. The above procedure determined sample transmissions to better than 1%.

SCATTERING MEASUREMENTS

The scattering measurements for this experiment and a concurrent ^{10}B experiment were completed during a five-week period. Very thin lithium targets ($\Delta E \approx 15$ keV) were used for the $^{11}\text{B} + n$ data near resonances, thus the count rate was low. Since an isotopically pure ^{11}B sample was unavailable, data were also taken for the enriched ^{10}B sample to correct for the ^{10}B contribution to the natural boron scattering data. An empty can measurement was taken to measure background neutrons not completely removed by the water tank shields and the plexiglass shadow bars. Thus two background runs were required, one for the empty can and one for the enriched ^{10}B sample. However, since the background counts were only a fraction of the true data counts, it was not necessary to spend an equal amount of time on each of these two less important measurements. Simple statistical considerations led to counting for only one fourth as long for the enriched sample and for the empty can.

Scattering TOF data were taken for seven angles of the main neutron detectors from 20° to 160° . A typical run consisted of 8000 μC of integrated beam current with the natural boron sample in position, 2000 μC of integrated beam for the enriched ^{10}B sample and 2000 μC of integrated beam for the empty can. Off-resonance data were taken for equal amounts of integrated beam for both samples, except for the empty can, because the concurrent ^{10}B experiment required the ^{10}B data.

ABSOLUTE NORMALIZATION

Absolute $^{11}\text{B} + n$ cross sections were calculated relative to the well known $\text{H}(n,n)\text{H}$ differential cross section. This section will discuss how this was accomplished.

Ideally, $\text{H}(n,n)\text{H}$ differential cross sections could be measured for all of the detectors at all the laboratory neutron energies to be encountered in the scattering from ^{11}B . Such a series of measurements would take a large amount of time for the range of neutron energies of the present experiment.

Another method of determining the detector efficiencies involves measuring the efficiency of each detector by comparison to a known source reaction, such as the $^7\text{Li}(p,n)^7\text{Be}$ reaction. This method has several difficulties associated with it. The two primary difficulties are 1) this reaction is known absolutely only to about 7% (Bo 63), and 2) it is difficult to accurately determine the target thickness (i.e. the number of ^7Li nuclei on the target).

A combination of both methods was chosen for the present experiment. The relative efficiencies of all of the detectors were measured to determine the energy dependence of their absolute efficiency. At several energies of the experiment, a differential cross section measurement of $\text{H}(n,n)\text{H}$ was done with the identical equipment, solid angles, etc., as the ^{11}B experiment. These measurements set the absolute scale of the relative efficiencies. In this manner, the need for doing numerous $\text{H}(n,n)\text{H}$ measurements was eliminated while at the same time the entire experiment was normalized to the well known $\text{H}(n,n)\text{H}$ differential cross section.

Basically an iterative procedure was used:

- 1) the relative efficiencies of all the detectors were carefully measured using the zero degree cross section for ${}^7\text{Li}(p,n){}^7\text{Be}$.
- 2) these relative efficiencies were used to calculate the relative yield for the $\text{H}(n,n)\text{H}$ measurements.
- 3) these relative yields, corrected for finite geometry effects, are then proportional to the known $\text{H}(n,n)\text{H}$ differential cross sections.
- 4) hence a conversion factor K was determined from each $\text{H}(n,n)\text{H}$ measurement which converted the ${}^{11}\text{B} + n$ relative yields to absolute cross sections.

This conversion factor was a constant, independent of the energy at which the $\text{H}(n,n)\text{H}$ measurement was made, hence the shape of the relative efficiency curve for each detector was presumed to be correct.

The ${}^{11}\text{B} + n$ measurements at all the laboratory neutron energies were then corrected using the measured relative efficiencies to obtain the relative yields. Multiplication by the conversion constant K converted these relative yields to absolute cross sections.

RELATIVE EFFICIENCY MEASUREMENTS

Relative efficiency measurements were taken to determine the energy dependence of the detection efficiency of each detector. A charge normalized TOF spectrum was taken with each detector at zero degrees and no sample in place using a very thin ($\Delta E \approx 10$ keV) lithium target. The repetition rate discriminator insured correct charge normalization and the analyzer dead time was kept to less than 10% by reducing the beam current on target. Spectra were taken every 50 to 100 keV from the bias of each detector to approximately 4.0 MeV, using the same lithium target for each detector. This insured that any errors in

the target thickness measurement were included systematically for all three detectors. Dead time corrections were made using a digital measure of the Nuclear Data 3300 analyzer busy signals, compared to a standard clock. Accurate 'integrated' dead times could then be calculated. After correcting the TOF yield for this dead time, the zero degree cross section data for ${}^7\text{Li}(p,n){}^7\text{Be}$ of Austin (Au 62) and Borchers (Bo 63) were used to obtain the relative efficiencies of each detector.

H(n,n)H SCATTERING

The well known H(n,n)H differential cross section was used to determine the absolute value of the ${}^{11}\text{B} + n$ cross sections. A cylinder of a hydrocarbon scintillator (NE 102) was used as the hydrogen scatterer. Table 2-1 gives the pertinent physical data of this reference scatterer.

At various times and incident energies throughout this experiment, the boron samples were replaced with this hydrocarbon scatterer. Sample transmissions were determined using the method described previously. A TOF spectrum was taken with each main detector at a laboratory angle of 42° , where the hydrogen and carbon peaks were well resolved. These measurements, corrected for incident flux attenuation, multiple scattering and exit flux attenuation, provided the conversion constant which normalized all of the boron scattering measurements to the H(n,n)H differential cross section.

DATA ACQUISITION SUMMARY

Data were taken in the following sequence:

- 1) the thickness of a freshly evaporated lithium target was ascertained by moving the long counter to zero degrees and measuring a rise curve at threshold.
- 2) the relative efficiency measurements were made for all three detectors.
- 3) the sample transmissions were measured at the laboratory neutron energy of the angular distribution.
- 4) the angular distribution measurements and corresponding zero degree monitor measurements were taken for the three samples:
 - a) the 20° spectrum for each sample. This measurement was done separately because the shadow bar placement for this angle added considerable 'in-scattering' to all the other angles.
 - b) the 42° and 114° spectra, using detector 1 and 2 respectively.
 - c) the 66° and 138° spectra, using detector 1 and 2 respectively.
 - d) the 90° and 160° spectra, using detector 1 and 2 respectively.
- 5) the neutron energy was increased and steps 3) and 4) were repeated.
- 6) at selected energies, the $H(n,n)H$ transmissions and 42° relative yields were measured.
- 7) depending on the requirements of neutron energy spread, a new lithium target was placed on the wobble target assembly. Before removing the previous lithium target, another rise curve was measured.
- 8) the thickness of the freshly evaporated lithium target was ascertained by moving the long counter to zero degrees and measuring a rise curve at threshold.
- 9) the new neutron energy was reached and steps 3) through 6) were repeated.

The experiment was completed in two separate runs, each of approximately 12 days duration, during which time the accelerator operated continuously, except for minor breakdowns. At the beginning of each 12 day run, a linear spectrum from a ^{60}Co source was taken with each detector to check for possible gain shifts. No measurable shifts were observed.

III. DATA REDUCTION

Reduction of the experimental TOF data was carried out in several distinct steps using computer programs designed for the Ohio University Academic Computer Center IBM 360 Model 44 computer. All data taken with the ND 3300 analyzer was 'dumped' onto magnetic tape (Kennedy Model 3112) for direct use by the IBM 360/44. ND 1100 analyzer data was typed out and then punched on cards for computer use.

RELATIVE EFFICIENCY MEASUREMENTS

Figure 3-1 shows a typical TOF spectrum, in histogram form, obtained during the relative efficiency measurements for one of the main neutron detectors. The ground state group from ${}^7\text{Li}(p,n){}^7\text{Be}$ is on the right (time is increasing toward the left). Plots such as this were made for all of the relative efficiency measurements for each detector so that beginning and ending channels could be chosen for the peak integration. Background yields were calculated by integrating a flat portion of the spectrum to the left of the ground state group. After integrating the ground state group TOF peak, and correcting for background yield, the integrated yield was corrected for the analyzer dead time, an accurate measure of which was obtained by the method described previously. The following equation was used to determine the relative efficiency for each detector:

$$Y(E) = Y_{\text{I to } (0^0)} \epsilon d\Omega \quad 3.1$$

Figure 3-1. A Main Detector Relative Efficiency Spectrum.

- where
- $Y(E)$ = the integrated TOF yield, corrected for analyzer dead time and background yield.
 - Y_I = the number of protons incident on the target, obtained from the integrated charge.
 - t = the thickness of the target (number of lithium nuclei/barn) obtained indirectly from the rise curve measurement.
 - $\sigma(0^\circ)$ = the measured zero degree laboratory cross section of ${}^7\text{Li}(p,n){}^7\text{Be}$ in barns/sr (Au 62, Bo 63).
 - ϵ = the relative efficiency of the detector.
 - $d\Omega$ = the solid angle subtended by the detector in sr.

All the relative efficiency measurements were normalized to the integrated proton charge as measured by the BCI. The RRD was used to insure that the repetition rate of the stop pulses was equal to the klystron buncher frequency. The zero degree cross section measurements of the ${}^7\text{Li}(p,n){}^7\text{Be}$ reaction of Austin and Borchers (Au 62, Bo 63) were combined and fit, over selected proton energy regions with a power series using the method of least squares:

$$\sigma(0^\circ, E_p) = \sum_{i=1}^n A_i E_p^i \quad 3.2$$

where E_p is the proton laboratory energy. The values of $\sigma(0^\circ)$ from this fit were used in equation 3.1 to obtain the relative efficiency data points shown in Figures 3-2 and 3-3.

The relative efficiency data points were then fit using the method of least squares to one of the following expressions, depending on the neutron energy:

Figure 3-2. Relative Efficiency Data and Fit for Detector 1.

Figure 3-3. Relative Efficiency Data and Fit for Detector 2.

a) $E \geq 1.5$ MeV

$$\epsilon = k(1 - B/E)(1 - e^{-No(E)x}) \quad 3.3$$

where

E = the neutron energy in the laboratory system.

N = .0511, the number of hydrogen nuclei/barn-cm for NE 218.

x = the detector thickness in cm.

B = the discriminator bias, determined by the least squares fitting.

k = an arbitrary scale factor, determined by the least squares fitting.

$\sigma(E)$ = the total n-p cross section as a function of neutron laboratory energy.

b) $E < 1.5$ MeV

$$\epsilon = \sum_{i=1}^l A_i E^i \quad 3.4$$

The value of N for NE 218 was that determined recently by Masterson (Ma 70). Gammel's expression was used to determine $\sigma(E)$ (Ga 63).

The polynomial fit below $E = 1.5$ MeV was required because the other expression is known to be inaccurate near the bias (Ry 60). Values from the fitted curves were used for all the subsequent data reduction. Figure 3.4 shows the relative efficiency points and the fit for the zero degree monitor. The lowest neutron energy used in this experiment ($E = 2.2$ MeV) was well above the bias of this detector. This enabled the first expression to be used to obtain the fit (using $N = .05233$ for NE 102) for the entire neutron energy range for this detector.

Figure 3-4. Relative Efficiency Data and Fit for the Monitor
Detector.

RELATIONSHIP BETWEEN THE DIFFERENTIAL CROSS SECTION AND THE YIELDS

The microscopic differential cross section (per unit solid angle) can be defined by means of the following relationship:

$$N_S(\theta, E) = N_I(E_0) n \bar{\sigma}(\theta, E_0) \quad 3.5$$

where

- $N_S'(\theta, E)$ = the number of neutrons scattered at angle θ with energy E ($E < E_0$).
- $N_I(E_0)$ = the number of neutrons incident on the sample with energy E_0 .
- n = the total number of nuclei in the sample.
- $\bar{\sigma}(\theta, E_0)$ = the average differential cross section at angle θ for incident neutrons of energy E_0 (uncorrected for any finite sample size effects).

The number of neutrons detected in the laboratory system is related to the number scattered by the following equation:

$$N_S(\theta, E) = \frac{G(\theta, E)}{\epsilon_D(E) d\Omega_D} \quad 3.6$$

where

- $G(\theta, E)$ = the number of neutrons detected at angle θ with energy E .
- $\epsilon_D(E)$ = the detector efficiency for neutrons of energy E .
- $d\Omega_D$ = the solid angle subtended by the detector.

The number of incident neutrons must now be determined. Figure 3-5 illustrates the geometry for a monitor detector located several meters from the sample.

Figure 3-5. The geometry for a monitor detector located at zero degrees several meters from the sample.

For the beam of incident particles defined within the dashes of Figure 3-5:

$$\frac{N}{N_1} = T(E_0) \quad 3.7$$

where

N_1 = the number of neutrons incident within the dashes of energy E_0 .

N = the number of neutrons of energy E_0 that exit within the dashes and strike the detector.

$T(E_0)$ = the measured sample transmission for neutrons of energy E_0 .

Again within the dashes of Figure 3-5:

$$\frac{M_G}{N_1} = T(E_0)\epsilon_M(E_0) \quad 3.8$$

where

M_G = the measured number of counts observed in the monitor detector.

$\epsilon_M(E_0)$ = the monitor detector efficiency for neutrons of energy E_0 .

For the entire sample (assuming an isotropic source):

$$N_1 = N_I(E_0)d\Omega_M \quad 3.9$$

Substituting equation 3.9 into equation 3.8 and solving for the number of neutrons incident on the entire sample, $N_I(E_0)$:

$$N_I(E_0) = \frac{M_G}{T(E_0)\epsilon_M(E_0)d\Omega_M} \quad 3.10$$

Neutron scattering experiments usually require scattering samples which are large (of the order of 2.54 cm high by 2.54 cm diameter cylinders for example) and which, therefore, introduce several finite geometry effects. One such effect is incident flux attenuation and it is convenient to correct for this effect at this time. This effect attenuates the incident flux, $N_I(E_0)$, so that the scattering nuclei farthest from the source are not exposed to the full intensity. To correct for this, the incident flux can be replaced by an average flux, N_A , which illuminates all of the nuclei in the sample. This correction may be performed as follows (Wa 54):

Since the sample is a right circular cylinder of radius a , struck on a lateral side by an isotropic flux $N_I(E_0)$, the average flux in the sample is given by:

$$N_A(E_0) = \frac{N_I(E_0)}{\pi a^2} \int_{-a}^{+a} \int e^{-n_{cc} \sigma_T(E_0) (x + \sqrt{a^2 - y^2})} dx dy$$

3.11

or $N_A(E_0) = f(E_0) N_I(E_0)$

where $f(E_0) = \frac{1}{n_{cc} \sigma_T(E_0) a} \left(I_1(2n_{cc} \sigma_T(E_0) a) - L_1(2n_{cc} \sigma_T(E_0) a) \right)$

3.12

and n_{cc} = the number of nuclei/cm³ in the sample.

$\sigma_T(E_0)$ = the total cross section of the sample in barns.

I_1 = a Modified Bessel Function of the First Kind.

L_1 = a Modified Struve Function of the First Kind.

Figure 3-6 shows the geometry used for this calculation. The above result is an excellent approximation to the true average flux since the ${}^7\text{Li}(p,n){}^7\text{Be}$ differential cross section in the vicinity of zero degrees is essentially isotropic (Au 62) and the samples used in this experiment subtended an angle of only $\pm 3.5^\circ$.

Solving equation 3.11 for $N_I(E_o)$ and substituting this result into equation 3.10 yields

$$N_A(E_o) = \frac{f(E_o)M_G}{T(E_o)\epsilon_M(E_o)d\Omega_M} \quad 3.13$$

The microscopic differential cross section corrected for incident flux attenuation is then given by

$$N_S(\theta, E) = N_A(E_o)n\bar{\sigma}(\theta, E_o) \quad 3.14$$

where $\bar{\sigma}(\theta, E_o) =$ the differential cross section at angle θ for incident neutrons of energy E_o corrected for incident flux attenuation.

Substituting equations 3.6 and 3.13 into equation 3.14 gives

$$\frac{G(\theta, E)}{\epsilon_D(E)d\Omega_D} = \frac{f(E_o)M_G}{T(E_o)\epsilon_M(E_o)d\Omega_M} n\bar{\sigma}(\theta, E_o) \quad 3.15$$

The equation for the absolute yield, $Y(\theta, E)$, is then obtained by setting the absolute yield equal to $n\bar{\sigma}(\theta, E_o)$:

Figure 3-6. The geometry used in the calculation of the incident flux attenuation factor.

$$Y(\theta, E) = \frac{1}{f(E_o)} \left\{ \frac{G(\theta, E) T(E_o)}{M_G} \right\} \frac{\epsilon_M(E_o) d\Omega_M}{\epsilon_D(E) d\Omega_D} \quad 3.16$$

Neutron scattering experiments generally have significant background counts. This term has been omitted thus far for clarity and must now be inserted. For the present experiment an equation for the relative yield is then obtained:

$$Y(\theta, E, S) = \frac{1}{f(E_o, S)} \sum_I \left\{ \frac{G(\theta, E, S, I) T(E_o, S)}{M_G(S)} - \frac{B(\theta, E, S, I)}{M_B(S)} \right\} \frac{\epsilon_M(E_o) d\Omega_M}{\epsilon_D(E(I)) d\Omega_D} \quad 3.17$$

- where
- $G(\theta, E, S, I)$ = the number of counts in the main detector TOF spectrum at angle θ with scattered energy E for sample S 'in' (out) in TOF channel I .
 - $M_G(S)$ = the number of counts in the zero degree monitor detector TOF spectrum, summed over the region corresponding to the ground state neutrons from the ${}^7\text{Li}(p, n){}^7\text{Be}$ reaction, corrected for room background, with sample S 'in' (out).
 - $T(E_o, S)$ = the measured transmission of the sample at energy E_o .
 - $\epsilon_D(E(I))$ = the calculated relative efficiency of the main neutron detector at energy E (in channel I) described earlier.
 - $\epsilon_M(E_o)$ = the calculated relative efficiency of the monitor detector at energy E_o described earlier.
 - $d\Omega_D, d\Omega_M$ = the solid angles of the main neutron detector and the zero degree monitor detector, respectively.
 - $f(E_o, S)$ = the calculated incident flux attenuation factor for sample S at energy E_o .

The incident flux attenuation factor $f(E_0, S)$ was calculated using the sample parameters of Table 2-1 and the total cross sections for B and ^{10}B measured by Bockelman (Bo 51) and Fossan (Fo 61).

In order to determine the beginning and ending channels for the TOF peak integration, background-subtracted TOF histograms of the main detector spectra for each energy and angle for both the natural B and the enriched ^{10}B sample were plotted. An example of one of these histograms is shown in Figure 3-7. Neutrons from $^7\text{Li}(p, n_1)^7\text{Be}$ were completely resolved for all of the spectra. After the beginning and ending channels for all of the TOF peak integrations were chosen, the relative yields were calculated.

Analyzer dead time corrections were made to the main detector counts $G(\theta, E, S, I)$ and $B(\theta, E, S, I)$ using the method described in Chapter II. Dead time could then be measured accurately for spectra that took several hours to accumulate. Main neutron detector analyzer dead times never exceeded 2%. A comparison of the maximum counting rate in the ND 1100 analyzer with its cycle time ($\approx 13 \mu\text{s}$) indicated that its dead time never exceeded 0.1%. Thus no correction was made to the monitor spectra for the analyzer dead time.

DETERMINATION OF THE ABSOLUTE NORMALIZATION CONVERSION FACTOR

The relative yield for the $\text{H}(n, n)\text{H}$ scattering measurements was determined from an expression similar to equation 3.17:

$$Y(\theta, E, H) = \int_I \left(\frac{G(\theta, E, H, I)}{\frac{f_1^M G_1}{T_1 \epsilon_{M1}} + \frac{f_2^M G_2}{T_2 \epsilon_{M2}}} - \frac{B(\theta, E, H, I)}{\frac{M_{B1}}{\epsilon_{M1}} + \frac{M_{B2}}{\epsilon_{M2}}} \right) \frac{d\Omega_M}{\epsilon_D(E(I)) d\Omega_D}$$

3.18

Figure 3-7. A background-subtracted TOF histogram for the natural boron sample at a laboratory angle of 20° .

- where
- f_1, f_2 = the value of the incident flux attenuation factors for the ground and first excited state neutrons, respectively.
- $\frac{M_{G1}}{(M_{B1})}$ = the number of counts in the zero degree monitor detector TOF spectrum summed over the region corresponding to ground state neutrons, corrected for room background, with sample H 'in' (out).
- $\frac{M_{G2}}{(M_{B2})}$ = the number of counts in the zero degree monitor detector TOF spectrum summed over the region corresponding to first excited state neutrons, corrected for room background, with sample H 'in' (out).
- T_1, T_2 = the measured sample transmissions for the ground and first excited state neutrons, respectively.
- $\epsilon_{M1}, \epsilon_{M2}$ = the calculated relative efficiency of the zero degree monitor detector at the energy corresponding to the energy of the ground state and first excited state neutrons, respectively.

All of the other quantities have the same definition as in equation 3.17. The factors involving the first excited state neutrons from the source reaction (${}^7\text{Li}(p, n_1){}^7\text{Be}$) were required because this group was completely unresolved from the ground state group for the hydrogen spectra. An illustration of this is given in Figure 3-8, which shows a histogram of a hydrocarbon spectrum. From right to left it shows the ground state group elastically scattered from carbon, the excited state group elastically scattered from carbon and both groups elastically scattered from hydrogen.

This contribution to the relative hydrogen yield was eliminated as follows (the θ dependence has been dropped since all of the hydrogen spectra were taken at the same laboratory angle, $\theta = 42^\circ$).

Figure 3-8. A background-subtracted TOF histogram for the hydrocarbon sample at a laboratory angle of 42° .

Let
$$Y_x = h\bar{\sigma}_x$$

where
$$Y_x = \text{the relative yield from the } x\text{th group at energy } E_x.$$

$$h = \text{the total number of hydrogen nuclei in the hydrocarbon sample.}$$

$$\bar{\sigma}_x = \text{the average differential cross section for incident neutrons of energy } E_x \text{ corrected for incident flux attenuation.}$$

Then
$$\frac{Y_1}{Y_2} = \frac{\bar{\sigma}_1}{\bar{\sigma}_2}$$

Assuming that the multiple scattering corrections are approximately the same for both groups (this will be shown to be the case in a subsequent section):

$$\frac{Y_1}{Y_2} = \frac{\sigma_1}{\sigma_2}$$

$$Y_2 = \frac{\sigma_2}{\sigma_1} Y_1$$

where
$$\sigma_x = \text{the true hydrogen cross section for neutrons scattered from the } x\text{th group.}$$

Also
$$Y_T = Y_1 + Y_2$$

or
$$Y_1 = \frac{Y_T}{1 + \frac{\sigma_2}{\sigma_1}} \quad 3.19$$

where
$$Y_T = Y(0, E, H), \text{ the measured relative yield from equation 3.18.}$$

Thus Y_1 , the relative hydrogen yield from the first group, was used to determine the absolute normalization. From the definition of the relative yield:

$$\sigma_1(E_o) = \frac{Y_1(E,H)}{h}$$

where $\sigma_1(E_o)$ must still be corrected for multiple scattering and exit flux attenuation. A Monte-Carlo code (Monte-Poly) obtained from A.B. Smith of Argonne National Laboratory was used to do this correction (Sm 72). The code uses as input the known differential cross sections of hydrogen and carbon and the measured physical parameters of the hydrocarbon sample (number of H nuclei/cm³, etc.). The program output is a set of correction factors at various angles:

$$CHI(\theta) = \frac{N_T(\theta)}{N_O(\theta)}$$

where

$N_T(\theta)$ = the true number of single hydrogen scattering events at angle θ .

$N_O(\theta)$ = the actual number of single, double and triple hydrogen scattering events where the neutron escapes the sample at angle θ .

A plot of this correction factor versus incident neutron energy is shown in Figure 3-9. The fact that CHI is a constant at a scattering angle of 42° supports the approximation made previously that the multiple scattering and exit flux attenuation correction is the same for both groups of neutrons from ${}^7\text{Li}(p,n){}^7\text{Be}$.

Figure 3-9. A plot of the correction factor CHI versus the scattered neutron energy for those laboratory neutron energies at which hydrocarbon spectra were taken.

Since $\bar{\sigma}_1(E_0)$ is proportional to $N_O(\theta=42^\circ)$, it follows that

$$\frac{N_T(O)}{N_O(O)} \bar{\sigma}_1(E_0) \propto \left. \frac{d\sigma}{d\Omega} \right|_H, \theta = 42^\circ$$

$$\text{or} \quad K \left[(CHI) \bar{\sigma}_1(E_0) \right] = \left. \frac{d\sigma}{d\Omega} \right|_H \quad 3.21$$

where $\left. \frac{d\sigma}{d\Omega} \right|_H$ = the well known differential cross section for $H(n,n)H$ in barns/sr

K = a conversion factor which converts $\bar{\sigma}_1(E_0)$ ($\bar{\sigma}_1(E_0) = Y_1(\theta = 42^\circ, E, H)/h$, the measured relative yield) to differential cross sections.

A plot of the values of the conversion factor K for each main detector is shown in Figure 3-10. This factor should be a constant, independent of neutron energy, if the calculated shapes of the relative efficiencies are correct. For the six hydrogen spectra taken with each main detector during the experiment, K is a constant to within $\pm 4\%$.

DETERMINATION OF THE ^{11}B DIFFERENTIAL CROSS SECTIONS

It is important to emphasize again what the conversion factor K actually does:

- 1) it normalizes all of the ^{11}B differential cross section data to the well known $H(n,n)H$ differential cross section.
- 2) it converts the measured relative yields directly to differential cross sections

Thus for the two boron samples:

$$\begin{aligned} KY_{EM} &= a\bar{\sigma}_{10} + b\bar{\sigma}_{11} \\ KY_B &= c\bar{\sigma}_{10} + d\bar{\sigma}_{11} \end{aligned} \quad 3.22$$

Figure 3-10. A plot of the conversion factor K versus scattered neutron laboratory energy for each main detector.

where

$$Y_{EM} = \text{the measured relative yield from the enriched sample.}$$

$$Y_B = \text{the measured relative yield from the natural B sample.}$$

$$a, b, c, d = \text{the total number of } ^{10}\text{B} \text{ or } ^{11}\text{B} \text{ nuclei in the enriched or the natural B sample, respectively.}$$

There was a small amount of oxygen and carbon contaminants in the enriched sample. These contaminants contributed to the relative yield of the enriched sample, since the TOF system was incapable of resolving elastically scattered neutrons from ^{16}O or ^{12}C from elastically scattered neutrons from ^{10}B or ^{11}B . Based on a comparison of the differential cross sections and the amount of the contaminant present, the relative yield of the enriched sample was corrected for effects of these contaminants by reducing the measured yield slightly:

$$Y_E = (.968)Y_{EM}$$

The above equations (3.22) in two unknowns can be solved for the ^{11}B cross section to correct for the minor isotope (^{10}B) contribution to Y_B :

$$\sigma_{11} = K \frac{aY_B - cY_E}{ad - bc} \quad 3.23$$

where σ_{11} is the differential cross section (barns/sr) which has been corrected for incident flux attenuation.

These absolutely normalized differential cross sections must be corrected for the effects of multiple scattering and exit flux attenuation. The differential cross sections were corrected using a straight-forward Monte-Carlo code supplied by A.B. Smith of Argonne National Laboratory. The input to this code consists of an energy dependent library of differential cross sections for the sample under study. This library was generated from the results of Lane (La 70, La 71) for the elastic differential cross sections for neutrons scattered from ^{10}B and ^{11}B . A total cross section library for $^{10}\text{B} + n$ and $^{11}\text{B} + n$ is also used to calculate the exit flux attenuation (Bo 51, Fo 61). Finally the code uses the measured uncorrected differential cross section for the sample and the physical parameters of the sample (size, number of nuclei/cm³, etc.). Straight-forward Monte-Carlo random walk techniques are then used to 'simulate' the actual experiment. If the multiple scattering corrections are small (< 30%), one iteration of the code is sufficient to obtain reasonable accurate corrections. The output of the code is a set of correction factors, $KI(\theta)$, which when multiplied by δ , give the true differential cross section:

$$\sigma(\theta, E) = KI(\theta)\delta(\theta, E)$$

The correction never exceeded 25%, thus one iteration of the code was sufficient. If the assumption is made that the multiple scattering corrections of the ^{10}B and ^{11}B nuclei are independent, the multiple scattering correction from the ^{10}B in the natural sample can be ignored. To first order, this is a good approximation since the maximum errors

introduced are small (25% of 20% = 5%). Thus the code was used to obtain the ^{11}B results by ignoring the contribution from the ^{10}B nuclei in the natural sample:

$$\sigma_{11}(\theta, E) = KI(\theta)\hat{\sigma}_{11}(\theta, E)$$

Multiple scattering corrections were made to every angular distribution at considerable expense of computer time. These extensive computations were required because of the resonant nature of the ^{11}B cross section in the laboratory neutron energy region covered by this experiment.

The corrected cross sections were converted to the center-of-mass system and fit to a sum of Legendre Polynomials using the method of least squares:

$$\frac{d\sigma_{11}}{d\Omega} = \sum_{L=0}^5 B_L P_L(\cos \theta)$$

A plot of these coefficients (center-of-mass) versus laboratory neutron energy is shown in Figure 3-11. Those coefficients which were systematically zero have not been plotted. The numerical values of the coefficients and the differential cross sections are given in the appendix.

DETERMINATION OF THE ERRORS

The error bars on the B coefficients represent the standard deviation for each coefficient, unless this deviation was less than the size of the data point, in which case it was not plotted. They are the result of the of the following sources of error, all of which were assumed independent:

Figure 3-11. A plot of the differential cross section expansion coefficients (center-of-mass) versus laboratory neutron energy.

- 1) counting statistics (3-5%).
- 2) $H(n,n)H$ normalization (4%).
- 3) multiple scattering corrections (3%).

The calculated ratio of the relative efficiencies in the relative yield equation (3.17) was assumed to be known to $\pm 2\%$. The calculated value of the hydrogen differential cross section was assumed to have a 1% error. These assumed errors have been included in the values given for the errors in 1) - 3) above. The multiple scattering corrections were estimated to be known to 20% of the correction. The average overall error of the data is approximately 8%.

The present experiment consisted of approximately 50 angular distribution measurements at 7 angles for each of 3 samples (natural B, enriched ^{10}B and empty can), along with 50 transmission measurements for each sample. The final B coefficients and their quoted errors are the result of approximately 1200 'separate' measurements, each with an error associated with it. It thus becomes important to do a statistical check on the final quoted errors to ascertain whether these errors are reasonable estimates of the 'true errors' (Be 69). Such a check might indicate whether a systematic error has been overlooked or simply that somewhere in the data reduction a decimal point was misplaced, etc. A calculation of the reduced χ^2_{ν} for each angular distribution served as this check:

$$\chi^2_{\nu} = \frac{\sum_{\ell=1}^7 \left(\frac{\sigma_{\ell}(0)_C - \sigma_{\ell}(0)_M}{\Delta\sigma(\theta)_M} \right)^2}{\nu}$$

where

- $\sigma(\theta)_C$ = the calculated differential cross section from the least squares fit.
- $\sigma(\theta)_M$ = the measured differential cross section from the experiment.
- $\Delta\sigma(\theta)_M$ = the errors of the differential cross section.
- ν = the number of degrees of freedom in the least squares fit.

The sum extends over the 7 angular measurements made at each energy. The value of the reduced chi-squared should have a value of 1.0, except for statistical fluctuations, if the fitting function is a good approximation to the 'true' function and the errors $\Delta\sigma(\theta)_M$ are good approximations to the 'true errors' (Be 69, Bi 32). There are, however, large statistical fluctuations in the determination of this statistic for the values of ν (1, 2 or 3) of this experiment (Bi 32). Figure 3-12 shows a plot of χ^2_ν for the Legendre Polynomial least square fits used for the determination of the B coefficients, $\nu = 1$. The lack of a systematic trend in this statistic and the fact that the average value is approximately 1.0 are both evident.

It is known that

$$\frac{d\sigma}{d\Omega} = \sum_L B_L P_L(\cos \theta)$$

is an excellent approximation to the parent distribution that describes the data (Bi 52). The average value of 1.0 for χ^2_ν then indicates the lack of any significant systematic error and that the errors $\Delta\sigma(\theta)_M$ were reasonable estimates of the 'true errors' of the experiment. These errors are indicated with the experimental points in Figure 3-11.

Figure 3-12. A plot of the reduced chi-squared versus neutron laboratory energy for the B_5 fit ($\nu = 1$) to the differential cross section data.

IV. DATA INTERPRETATION

The elastic differential cross section data are analyzed in terms of the R-matrix theory of nuclear reactions first expounded by Wigner and Eisenbud (W1 47). Several states in ^{12}B inferred from this analysis are interpreted as states of specific shell model particle-hole configurations predicted by a simple δ -function residual interaction. The data of this experiment are combined with differential cross section and polarization data analyzed previously and a complete R-matrix fit, including predictions of the polarization in the energy interval $2.2 \text{ MeV} \leq E_n \leq 4.5 \text{ MeV}$, is given for $0.2 \text{ MeV} \leq E_n \leq 4.5 \text{ MeV}$. In addition, the states inferred by this analysis are compared with the $T_z = 1$ isobaric analog states in ^{12}C , where the presence of many $T = 0$ states makes the analysis of the ^{12}C structure in the excitation energy range of 19-21 MeV very difficult.

R-MATRIX THEORY

A detailed derivation of the R-matrix theory of nuclear reactions will not be attempted here. Many general treatments have been given elsewhere for the many channel, many level case (La 58, Vo 59, W1 60, Vo 62). A review of the basic principles underlying the theory will be given and a derivation of the R-matrix for the simple case of an s-wave neutron will be done. The results of the many channel, many level case, which is required for the present analysis, will simply be stated.

The R-matrix theory is rigorous (it is not a 'model') and therefore its application is not confined to reactions that proceed through the compound nucleus mechanism (La 58). It is based on the following assumptions:

- 1) the applicability of non-relativistic quantum mechanics.
- 2) the absence or unimportance of all processes in which more than two nuclei are formed.
- 3) the absence or unimportance of all processes of creation or destruction.
- 4) the existence, for any pair of nuclei c , of some finite radial distance of separation a_c beyond which neither nucleus experiences any potential field from the other (other than the Coulomb interaction).

These assumptions introduce the basic concept of R-matrix theory.

The physical space in which the nuclear reaction takes place can be separated by the interaction radii a_c into an internal and an external region.

The calculation of the nuclear cross sections can then be (schematically) divided into two steps. In the first step, the cross section is connected to the asymptotic parts of the wave function by means of the collision matrix (Vo 62). The connection is a general one in that it has nothing to do with the nuclear problem in particular. The second step of the cross section calculation connects the collision matrix to the parameters of the nuclear states. This connection is made through an intermediate quantity - the R-matrix.

Schematically (La 58):

$$\sigma_{cc'}(E) \leftarrow U_{cc'}(E) \begin{matrix} + L, \Omega & \text{- diagonal matrices formed from the 'external' wavefunctions.} \\ + R & \text{- nondiagonal matrix parameterized to represent the 'internal' wavefunction.} \end{matrix}$$

where $\sigma_{cc'}(E)$ = the cross section for the production of the pair of nuclei c' when the two nuclei of the pair c bombard against each other.

$U_{cc'}(E)$ = the amplitude of the outgoing wave of pair c' resulting from unit flux of bombardment with pair c .

Thus $\sigma_{cc'} = |1 - U_{cc'}|^2$. The conservation of probability flux and time-reversibility require that U be unitary and symmetric.

The basic ideas of R-matrix theory are quite simple but their application is so complicated by the details of spin, orbital angular momentum, many channels, etc., that their essential simplicity is obscured. However, the essential part of R-matrix theory can be illustrated by the simple case of s-wave neutrons scattering from a square well, so a brief sketch of this calculation will be given (Vo 62). The results for the general case will then be stated.

The radial part of the Schrodinger equation for the s-wave scattering ($c' = c$) of neutrons from a square well is:

$$-\frac{(\hbar^2)}{(2m)} \frac{d^2 \phi}{dr^2} + V\phi = E\phi \quad 4.1$$

where

$$V = \begin{cases} -V_1 & r < a_c \\ 0 & r > a_c \end{cases}$$

m = the neutron mass

Inside the well, the solutions are:

$$\phi = A \sin Kr \quad r < a_c \quad 4.2$$

where A is an arbitrary constant and K is the wave number inside

the well ($E + V_1 = \frac{\hbar^2 K^2}{2m}$). Outside the well, the wave function is a linear combination of incoming waves I ($\equiv (4\pi v)^{-\frac{1}{2}} e^{-ikr}$) and outgoing waves O ($\equiv (4\pi v)^{-\frac{1}{2}} e^{ikr}$):

$$\phi = I - e^{2i\delta} O \quad r > a_c \quad 4.3$$

where δ is the phase shift. By matching the slope and the value of 4.3 and 4.2 at the boundary $r = a_c$ (the interaction radius), the square well phase shift is obtained:

$$\delta = \tan^{-1} \left(\frac{k}{K} \tan Ka_c \right) - ka_c \quad 4.4$$

The scattering cross section σ is related to the phase shift by:

$$\sigma = \frac{4\pi}{k^2} \sin^2 \delta \quad 4.5$$

In terms of the collision matrix U :

$$U = e^{2i\delta}$$

or
$$\sigma = \frac{\pi}{k^2} |1 - U|^2$$

A neutron wave function at a resonance in the scattering cross section (i.e. unbound in the well) is 'almost' a standing wave ($\phi = A \sin Kr$). The standing waves X_λ which resemble the actual wave functions of the low energy resonances can be constructed as follows:

$$-\frac{(\hbar^2)}{(2m)} \frac{d^2 X_\lambda}{dr^2} + V_\lambda X_\lambda = E_\lambda X_\lambda \quad 4.6$$

with the boundary condition at the square well radius a_c :

$$\left. \frac{r}{a_c} \frac{dX_\lambda}{dr} = b X_\lambda \right|_{r=a_c} \quad 4.7$$

where b is a real number.

The actual physical problem of the square well has different wave functions than the X_λ

The difference arises because of the boundary condition added to the problem...The physical problem is Hermitian in all space ($0 < r < \infty$)... By means of the boundary condition, the problem is made Hermitian in the finite volume ($0 < r < a_c$)... This is done to construct standing waves which can be used to expand the actual wave function...If the boundary condition is properly chosen, then the unbound standing waves bear a close physical resemblance to the wave function at $E = E_\lambda$. However, the bound standing waves should not be connected in a detailed manner with the bound states of the nuclear shell model (Vo 62).

The X_λ form a complete orthonormal set of functions, thus the actual wave function ϕ for $r \leq a_c$ may be expanded in terms of the X_λ :

$$\phi = \sum_\lambda C_\lambda X_\lambda \quad 4.8$$

where

$$C_\lambda = \int_0^{a_c} X_\lambda^* \phi dr \quad 4.9$$

Multiplying 4.1 by X_λ^* and the complex conjugate of 4.6 by ϕ , subtracting and integrating gives:

$$\frac{\hbar^2}{2m} \left(\phi \frac{dX_\lambda}{dr} - X_\lambda^* \frac{d\phi}{dr} \right) \Big|_{r=a_c} = (E - E_\lambda) \int_0^{a_c} \phi X_\lambda^* dr$$

$$\text{or} \quad C_\lambda = \frac{1}{(E_\lambda - E)} \frac{\hbar^2}{2ma_c} X_\lambda^*(a_c) \left(\phi'(a_c) - b \phi(a_c) \right) \quad 4.12$$

$$\text{where} \quad \phi' = \frac{rd\phi}{dr}$$

Substituting 4.12 into 4.10, evaluated at $r = a_c$:

$$\phi'(a_c) = R(\phi'(a_c) - b \phi(a_c)) \quad 4.13$$

$$\text{where} \quad R = \lambda^2 \frac{\gamma_\lambda^2}{(E_\lambda - E)} \quad 4.14$$

$$\gamma_\lambda^2 = \frac{\hbar^2}{2ma_c} |X_\lambda(a_c)|^2 \quad 4.15$$

are the R function (in this case) and the reduced width squared, respectively. From 4.13, the logarithmic derivative $\frac{\phi'}{\phi}$ at $r = a_c$ may be written in terms of R as follows:

$$\frac{\phi'(a_c)}{\phi(a_c)} = \frac{(1 + b R)}{R} \quad 4.16$$

Outside the well, from 4.3,

$$\phi = I - UO$$

The slope and the value (or equivalently the ratio of the slope and the value which is called the logarithmic derivative) of 4.17 and 4.16 are matched at the channel radius and the result is solved for the collision function U :

$$U = O^{-1}(1 - RL)^{-1}(1 - RL^*) \quad 4.18$$

where

$$L = O'O^{-1} \Big|_{r=a_c} - b$$

Approximating R by one term

$$R = \frac{\gamma_\lambda^2}{E_\lambda - E}$$

and inserting U into 4.5a yields the cross section for the s-wave neutron problem (the Breit-Wigner formula):

$$\sigma = \frac{\pi}{k^2} \left| 2(\sin ka_c)e^{ika_c} - \frac{\Gamma_\lambda}{E_\lambda - E + i\Gamma_\lambda/2} \right|^2$$

where

$$\Gamma_\lambda = 2ka_c \gamma_\lambda^2$$

$$= 2P_0 \gamma_\lambda^2$$

$$\Delta_\lambda = b\gamma_\lambda^2$$

and P_0 and Δ_λ are referred to as the penetrability (for s-wave $l = 0$) and level shift, respectively.

In the general case, there can be contributions to σ from states with different values of J^π . In analogy to 4.18 then (La 58):

$$U^{J^\pi} = O^{-1}(1 - R^{J_L})^{-1}(1 - R^{J_L*}) I \quad 4.20$$

where all of the above quantities are matrices and J^π refers to a particular state. All of the matrices in 4.20 are diagonal except for the R-matrix, which can have off-diagonal elements. The individual matrix elements are defined as follows:

$$\frac{1}{O_{cc}} = \frac{1}{i \left(\frac{\pi \rho_\alpha}{2} \right)^{1/2} H_{l+1/2}^1(\rho_\alpha)}, \text{ evaluated at } r = a_c$$

$$I_{cc} = -i \left(\frac{\pi \rho_\alpha}{2} \right)^{1/2} H_{l+1/2}^2(\rho_\alpha), \text{ evaluated at } r = a_c$$

$$R_{cc'}^{J^\pi} = \sum_\lambda \frac{\gamma_{\lambda c} \gamma_{\lambda c'}}{E_\lambda - E}, \text{ where the sum is over levels of the same } J^\pi.$$

$$L_{cc} = S_c - iP_c - b_c$$

and $H_{l+1/2}^1, H_{l+1/2}^2$ = the Hankel functions of the 1st and 2nd kind respectively.

ρ_α = kr (k is the wave number) for the particular channel pair α .

$R_{cc'}$ = the R-matrix element for states in channels c and c' respectively.

S_c, P_c = the level shift and penetrability for channel c .

b_c = the boundary condition for channel c .

The differential cross section for the general case of many channels can be written as follows (B1 52):

$$\frac{d\sigma}{d\Omega} = \sum_L B_L P_L(\cos\theta) \quad 4.21$$

where

$$B_L = \sum_{\alpha} \frac{\lambda_{\alpha}^2 (-1)^{s'-s}}{4(2I+1)(2I+1)} \bar{Z}(\ell_1 J_1 \ell_2 J_2, sL) \\ \times \bar{Z}(\ell'_1 J_1 \ell'_2 J_2, sL) \operatorname{Re} \left\{ \left(\delta_{\alpha'} \delta_{\ell'_1 \ell_1} \delta_{s' s} - U_{\alpha' s' \ell'_1; \alpha s \ell_1}^{J_1 \pi_1} \right) \right. \\ \left. \times \left(\delta_{\alpha'} \delta_{\ell'_2 \ell_2} \delta_{s' s} - U_{\alpha' s' \ell'_2; \alpha s \ell_2}^{J_2 \pi_2} \right) \right\}$$

and λ = the de Broglie wavelength divided by 2π

$\vec{s} = \vec{I} + \vec{i}$, the channel spin. $\vec{I}$ and $\vec{i}$ are the intrinsic spins of the target and incident nucleon.

$\vec{J} = \vec{l} + \vec{s}$, the total angular momentum. $\vec{l}$ and $\vec{s}$ are the orbital angular momentum and channel spin respectively.

$$\bar{Z}(\ell_1 J_1 \ell_2 J_2, sL) = (2\ell_1 + 1)^{1/2} (2\ell_2 + 1)^{1/2} (2J_1 + 1)^{1/2} (2J_2 + 1)^{1/2} \\ \times W(\ell_1 J_1 \ell_2 J_2 sL) C(\ell_1 \ell_2 00 | L0)$$

W is the Racah coefficient (Ra 42) and C is the Clebsch-Gordan coefficient as defined in Condon and Shortley (Co 35). The sum $\sum$ is over all of the following: $J_1, J_2, \ell_1, \ell_2, \ell'_1, \ell'_2, s, s'$. α describes the pair of reaction products so that a particular channel c is completely specified by the quantum numbers α, s, ℓ, J .

A similar equation has been derived to calculate the product of the differential cross section and the polarization.

It is given by the following expression (S1 53):

$$\frac{dP}{d\Omega} = \sum_L C_L P_L^1(\cos \Theta) \quad 4.22$$

$$\frac{dP}{d\Omega} = \text{the differential polarization}$$

$$= \frac{dP}{d\Omega} P(\Theta), \text{ where } P(\Theta) \text{ is the polarization}$$

$$P_L^1(\cos \Theta) = \text{the associated Legendre Polynomial.}$$

The expansion coefficients are defined as follows:

$$C_L = \frac{\lambda_\alpha^2 \sqrt{3}}{4(2I+1)(2I'+1)} \left[\frac{3(2L+1)(2I'+1)}{L(L+1)} \right]^{1/2} \\ (-1)^{I'-I'-s+l_1+J_1-s_1} \\ (2J_1+1)(2J_2+1) \left\{ (2l_1+1)(2l_2+1) \right. \\ \left. (2l_1'+1)(2l_2'+1)(2s_1'+1)(2s_2'+1) \right\}^{1/2} \\ C(l_1 l_2 00 | L0) C(l_1' l_2' 00 | L0) W(I' s_1' I' s_2', I' 1) \\ W(l_1 J_1 l_2 J_2, sL) X(J, l_1' s_1', J_2 l_2' s_2', LL1) \\ \text{Re} \left\{ (i) \left\{ \delta_{\alpha' \alpha} \delta_{l_1' l_1} \delta_{s_1' s_1} - U_{\alpha' l_1' s_1'; \alpha l_1 s_1}^{J_1 \pi_1} \right\}^* \right. \\ \left. \times \left\{ \delta_{\alpha' \alpha} \delta_{l_2' l_2} \delta_{s_2' s_2} - U_{\alpha' l_2' s_2'; \alpha l_2 s_2}^{J_2 \pi_2} \right\} \right\}$$

where X is the Wigner 9-J coefficient. The sum is over all of the quantum numbers and (i) indicates $\sqrt{-1}$, otherwise i is the spin of the incident nucleon.

SHELL MODEL CALCULATIONS

The present experiment was largely motivated by previous shell model calculations (La 70). For completeness, a brief review of these calculations is given below.

The nucleus ^{11}B has 5 protons and 6 neutrons. Its ground state can be treated as a closed neutron shell ($1p_{3/2}$) with a 'hole' in the $1p_{3/2}$ shell. The low lying states of ^{12}B may be interpreted as adding a neutron to the ground state or first excited state of ^{11}B . The states in ^{12}B are the result of the residual interaction between the neutron and the proton 'hole'. Since only the level ordering of the excited states was of interest in the calculation (the absolute magnitude of the energy of the states was unimportant), the existence of the closed shells can be ignored (Sh 63). Table 4-1 shows the various particle and 'hole' states which were considered in the calculation and the resultant J^π of their angular momentum coupling.

For a simple δ -function residual interaction of the form

$$V_{12} = 4\pi V_0 \left(1 + \alpha \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right) \delta(\mathbf{r}_1 - \mathbf{r}_2)$$

it has been shown that the splitting within a particle-particle multiplet is given by (Sh 63):

$$E_J = V_0 \sum_L \ell_1 \ell_2 \begin{pmatrix} \ell_1 & L & \ell_2 \\ 0 & 0 & 0 \end{pmatrix}^2 \left(C_{0L}^2 (W-B) + C_{LL}^2 (W+B) \right) \\ \times \int \frac{1}{r^2} \left(R_{n_1 \ell_1}(r) R_{n_2 \ell_2}(r) \right) dr$$

Table 4-1

Proton-hole plus neutron configurations, the neutron single-particle and hole energies from ^{13}C , the J^π values, and the experimental energies for the multiplets of interest.

Proton-hole + neutron configuration	Single-particle neutron energy from ^{13}C (MeV)	J^π	Experimental energy (MeV)
$1p_{3/2}^{-1}p_{1/2}$	0.0	1^+	0.0
		2^+	0.953
$1p_{3/2}^{-1}2s_{1/2}$	3.09	2^-	1.674
		1^-	2.72
$1p_{3/2}^{-1}1d_{5/2}$	3.85	3^-	3.39
		2^-	4.37
		4^-	4.54
		1^-	...
$1p_{3/2}^{-1}1p_{3/2}^{-1}{}^a$	3.68	1^+	...
		3^+	...
		0^+	...
		2^+	...

^aThis $1p_{3/2}^{-1}$ state represents a neutron configuration of $1p_{3/2}^{-1}(1p_{1/2})^2$ in ^{13}C .

$$\text{where} \quad C_{SL}^2 = (2j_1 + 1)(2j_2 + 1)(2S + 1)(2L + 1) \left\{ \begin{matrix} \frac{1}{2} & l_1 & j_1 \\ \frac{1}{2} & l_2 & j_2 \\ S & L & J \end{matrix} \right\}^2 \quad (66)$$

$$R_{nl}(r) = \text{the harmonic oscillator wave functions.}$$

$$\left\{ \begin{matrix} l_1 & L & l_2 \\ 0 & 0 & 0 \end{matrix} \right\} = \text{the Clebsch-Gordan coefficient, defined as in Condon and Shortley (Co 35).}$$

$$\left\{ \begin{matrix} \frac{1}{2} & l_1 & j_1 \\ \frac{1}{2} & l_2 & j_2 \\ S & L & J \end{matrix} \right\} = \text{the 9-J symbol defined for example by Matsunobu (Ma 55).}$$

$$W + B = 1$$

$$W - B = 1 - 4a$$

The particle-particle to particle-hole transformation of Pandya (Pa 56) was used to give the energy splitting within a particle-hole multiplet:

$$E_{J-}(p-h) = -\sum_J (2J + 1) W(j_1 j_2 j_2 j_1 J J') E_J$$

Figure 4-1 shows the final result of the calculations. The parameters used are given in Table 4-2. A discussion of the parameter variations, including a matrix diagonalization to include the configuration mixing of the 2^- states is given in La 70.

Of particular interest to the present analysis is the $1p_{3/2}^{-1}, 1d_{5/2}$ multiplet. The previous work (La 70) confirmed the existence of the $3^-, 2^-$ and 4^- states by simultaneously fitting the $^{11}\text{B} + n$ elastic differential cross section and polarization data for $.2 \text{ MeV} \leq E_n \leq 2.2 \text{ MeV}$. Although there was no evidence of the existence of a 1^- resonance at

Figure 4-1. Energy level diagram of ^{12}B for the parameters listed in Table 4-2.

Table 4-2

Shell model parameters for Figure 4-1.

Parameters	Values
α	0.10
V_0 (MeV)	40
I_{11}	0.047
I_{00}	0.056
I_{22}	0.045
I_{02}	0.021
$E_{s.p.} (1p_{1/2})$ (MeV)	0.0
$E_{s.p.} (2s_{1/2})$ (MeV)	3.09
$E_{s.p.} (1d_{5/2})$ (MeV)	3.85
$E_{s.h.} (1p_{3/2}^{-1})$ (MeV)	3.68

higher neutron energies from $^{11}\text{B} + n$ total cross section measurements or from any other reaction, these simple particle-hole shell model calculations predicted such a state at a neutron laboratory energy of approximately 2.8 MeV. The present experiment was planned with the hope that this state could be identified.

R-MATRIX CALCULATIONS

The most important open channels in the present experiment are elastic neutron scattering in channel spins $S = 1, 2$ corresponding to the coupling of the neutron spin to the ground state spin ($I^\pi = 3/2^-$) of ^{11}B . The $^{11}\text{B}(n, \gamma)^{12}\text{B}$ reaction has a total cross section of several microbarns and was neglected in the calculations (Im 62). The $^{11}\text{B}(n, n')^{11}\text{B}$ (2.14) reaction has a total cross section < 70 mb in the neutron energy interval $2.2 \text{ MeV} \leq E \leq 4.5 \text{ MeV}$. This is indicated in Figure 4-2 which shows the measurements of Porter *et al.* (Po 70). This particle channel was also neglected in the calculations. The effect of neglecting these channels is to make the R-matrix calculations slightly ($\leq 5\%$) higher than the elastic scattering data.

The consideration of the bound 1^- and 2^- states, as well as one or two 1^- or 2^- scattering states, required the use of a three level R-matrix whose elements are

$$R_{cc'}^{J^\pi} = \left(\sum_{\lambda=1}^3 \frac{\gamma_{\lambda c} \gamma_{\lambda c'}}{E_\lambda - E} + R_{cc'}^0 \right)^{J^\pi}$$

where the $R_{cc'}^0$ represents the contribution from the distant levels of the same J^π . It is assumed that a single ℓ value contributes to a resonance of a given J^π so that c and c' refer to the values of

Figure 4-2. The total inelastic neutron scattering cross section.

the channel spin. A total of 11 different states were included in the calculation.

To calculate the elements of the scattering matrix, values for the reduced-width amplitudes $\gamma_{\lambda c}$ and $\gamma_{\lambda c'}$, for the characteristic energies E_{λ} , and the constant background elements R_{cc}^0 are assumed for each J^{π} . From these, the coefficients $B(L)$ and $C(L)$ are determined at each energy. The above definition of channels is adequate for the positive-parity p states (partial waves for $l \geq 3$ are neglected here) and some of the negative parity states. However 1^{-} and 2^{-} states can be formed via s waves and d waves with appropriate channel spins, and this possibility leads to more than two channels. In such cases only the l value that showed the strongest evidence of participation was considered. All penetrabilities, shift factors and hard-sphere phase shifts necessary for the analysis were based on a radius of 4.5 F.

The boundary conditions for the various states were chosen to be equal to the average value of the shift function in the neutron energy interval $2.2 \text{ MeV} \leq E \leq 4.5 \text{ MeV}$. This choice of boundary conditions makes the background R^0 matrix more meaningful. One should recall that this term is added to account for contributions to the calculation from 'distant' levels of the same J^{π} and has no explicit energy dependence. What has been done is to approximate an energy dependent quantity

$$R_{cc'}^{0J^{\pi}} = \sum_{\lambda=4}^{\infty} \frac{\gamma_{\lambda c} \gamma_{\lambda c'}}{E_{\lambda} - E}$$

with

$$R_{cc'}^{0J^{\pi}} = \text{constant}$$

If the boundary conditions are chosen arbitrarily as $b_c = 0.0$ for all states, considerable level shifts are introduced and $E_\lambda \neq E_{res}$. This is not only inconvenient for fitting purposes. This choice of boundary conditions also makes the R_{cc}^0 term more energy dependent (We 71). Hence approximating this term by a constant becomes questionable. On the other hand, choosing $b_c = -S_l(E_{res})$ is rather arbitrary, particularly when there is more than one level of a single J^π . The choice of the boundary condition as the average value of the shift in the energy region of interest was made in an attempt to minimize the energy dependence of R_{cc}^0 .

The next several sections discuss each state in ^{12}B individually and at least one general statement about the data is useful. For the nucleus ^{12}B , states of positive parity ($l = 1$) most often appear as resonance effects in $B(1)$ and $B(3)$ while states of negative parity ($l = 0, 2$) appear as resonance effects in $B(2)$ and $B(4)$. Of course, states of positive and negative parity appear as resonances in $B(0)$ ($-\frac{\sigma_{elas}}{4\pi}$).

The properties of the ^{12}B states as deduced in this analysis are shown in Table 4-3 and discussed in the following paragraphs. Figure 4-3 shows the best fit to the data and indicates the J^π values for the various resonances as determined by this analysis. The subsequent analysis (in particular the spin value) is based to some extent on the background R_{cc}^0 chosen for each state. In an attempt to decrease the effect of the background on the conclusions about the states, R_{cc}^0 terms were included for every state (except the $l = 0$ states) until a reasonable fit to the non-resonant background was obtained. With this background included, spin values were changed to determine which spin

Table 4-3. R-matrix parameters for levels in ^{12}B used in the best fit calculation

$J^\pi(l)$	E (MeV, lab)	E_x (MeV, c.m.)	E_λ (MeV, c.m.)	$v_{\lambda c}^2$ (MeV, c.m.)		R_{cc}^0		b_c
				S=1	S=2	S=1	S=2	
$1^-(0)$	...	2.62	-0.75	0.680	...	...	...	0.0
	1.02	4.30	0.935	0.0055	...	...	...	0.0
$2^-(0)$	...	1.67	-1.70	...	0.49	...	...	0.0
	...	1.09	0.972	...	0.182	...	...	0.0
$0^+(1)$	...	2.72	...	...	...	-0.4	...	0.0
$1^+(1)$	1.77	5.00	1.613	0.0052	0.0471	0.2	0.2	-0.3
	3.5	6.6	3.191	0.0279	0.0279	...	...	-0.3
	3.8	6.8	3.484	0.0949	0.0949	...	...	-0.3
$2^+(1)$	0.430	3.76	0.348	0.0475	0.0475	0.2	0.2	-0.3
$3^+(1)$	2.45	5.61	2.201	...	0.066	...	0.3	-0.3
$0^-(2)$	...	...	...	...	...	...	...	-1.0
$1^-(2)$	2.70	5.8	2.476	1.158	0.128	0.45	0.05	-1.0
$2^-(2)$	...	...	...	...	...	...	...	-1.0
$3^-(2)$	0.02	3.39	-0.275	0.168	0.253	0.1	0.15	-1.0
	2.58	5.73	2.35	0.019	0.0286	...	...	-1.0
$4^-(2)$	1.27	4.54	0.871	...	0.564	...	0.2	-1.0

Figure 4-3

Figure 4-3. The best R-matrix fit to the data obtained in the present experiment. Also shown are the J^π values for the observed resonances.

gave the best fit. The fact that the various states showed resonance interference effects in the various even and odd coefficients makes it possible to be definitive only about the parity assignments.

The B(5) data points have been excluded from the data since this coefficient cannot be formed with $l \leq 2$, which was the highest l value required to obtain this best fit.

STATE AT $E_x = 5.8$ MeV ($E = 2.7$ MeV)

In the present analysis, the broad resonance in B(2) at $E = 2.7$ MeV could not be reproduced without assuming the existence of a broad negative parity state at $E_x = 5.8$ MeV. An illustration of this is shown in Figure 4-4 where the broad state has been omitted in the dashed line calculation (the solid line indicates the best fit). This resonance was not seen in the earlier natural boron total cross section work of Bockelman, et al. (Bo 51). There is a broad resonance in $^{10}\text{B} + n$ total cross section at a neutron energy of approximately 2.8 MeV. Recent work completed at this laboratory (to be published) indicates a strong B(2) term. Since the effects of the multiple scattering of the ^{10}B in the natural B scattering sample were neglected, this resonance in $^{11}\text{B} + n$, at approximately the same neutron energy, could be attributed to the $^{10}\text{B} + n$ resonance. Evidence that there is a broad state in ^{12}B is illustrated in Figure 4-5. This figure shows the recent measurements of Cance et al. (Ca 71) of the total cross section of ^{11}B obtained using a sample enriched to 95% ^{11}B . The line drawn through the points near a neutron energy of 2.7 MeV clearly indicates a broad resonance, particularly when compared to the dashed line which indicates the R-matrix calculation when this broad state is omitted. Although

Figure 4-4. A comparison of the best fit with the calculation
when the 1^- state is omitted.

Figure 4-5. The total neutron cross section of ^{11}B obtained with a sample enriched to 95% ^{11}B (Ca 71). The dashed lines indicate the R-matrix calculation results when the broad resonances are omitted. Note that $\sigma_{\text{elas}} = 4\pi B(0)$ and that $\sigma_T \approx 4\pi B(0)$ between $E = 2.2$ MeV and $E = 4.5$ MeV.

the amount of contaminant present in Cance's sample is not known, if this contaminant consisted entirely of ^{10}B , it would increase the value of the $^{11}\text{B} + n$ cross section by less than 7%. An increase of such magnitude in the calculation in the vicinity of this resonance would not be noticeable.

Additional evidence for the existence of this state is presented in Figure 4-6. This figure shows the results of the reaction $^{11}\text{B}(n, n'\gamma)^{11}\text{B}$ (2.14) of Lind et al. (Li 61), who measured the total gamma ray yield to the $1/2^-$ first excited state of ^{11}B . The line drawn through the points is meant only to guide the eye. Although the standard errors in the points were approximately 10%, Lind et al. do mention that "...there is some evidence for a broad resonance at about 2.9 MeV (neutron energy)" (Li 61).

This state must have negative parity but it could be formed by $l = 0$ or $l = 2$. Since the resonance is in B(2), the only way it can arise in this coefficient with $l = 0$ neutrons would be through d-s interference with a large d wave background. Calculations assuming $l = 0$ indicated no B(2) interference and a strong interference dip in B(0) from this s wave state interfering destructively with the s wave background. Thus $l = 2$ has been assigned.

For each state it is possible to determine the ratio $\gamma_{s=2}^2/\gamma_{s=1}^2$ from the shell model calculations (La 60). For a 1^- state from the $1p_{3/2}^{-1}, d_{5/2}$ configuration, this ratio equals 1/9. Various ratios were tried in the fitting and this ratio gave the best fit.

Because of the problems mentioned earlier about the determination of the 'true' background, a 2^- state could not be ruled out on the basis of the R-matrix fit. Figure 4-7 shows the 2^- fit in dashes

Figure 4-6. Excitation function for the gamma decay of the first excited state of ^{11}B nucleus as measured in the $^{11}\text{B}(n,n'\gamma)^{11}\text{B}$ reaction (L1 61).

Figure 4-7. A comparison of the best fit, which includes a 1^- broad state with the fit obtained when a 2^- broad state is assigned.

along with the best fit. The channel spin ratio used for the 2^- assignment was that predicted by the shell model for a 2^- state from the configuration $1p_{3/2}^{-1}d_{3/2}$ ($\gamma_{s=2}^2/\gamma_{s=1}^2 = 7/3$), since the 2^- state from the $1p_{3/2}^{-1}d_{5/2}$ configuration has been identified previously (La 70).

Since the 2^- assignment cannot be ruled out on the basis of the R-matrix fits, it is necessary to consider the $T = 1$ analog states in ^{12}C corresponding to the excitation energy of ^{12}B explored in this experiment. Figure 4-8 shows the comparison of these two isobars for the $T = 1$ states in ^{12}C only. The angular momentum coupling schemes indicated on the right identify the particle-hole states in ^{12}C calculated by Gillet et al. (Gi 64). The arrows connecting the states in ^{12}B with those in ^{12}C indicate the positions in ^{12}B of the $1p_{3/2}^{-1}d_{5/2}$ multiplet calculated by Lane et al. (La 70) (Gillet et al. did not consider the 4^- state of this multiplet). Note the position of the lowest state of the $1p_{3/2}^{-1}d_{3/2}$ multiplet, $J^\pi = 2^-$, which is the most likely origin of such a broad negative parity state. It is approximately 2 MeV above the 1^- state of the $1p_{3/2}^{-1}d_{5/2}$ multiplet. Although the zero of energy in particle-hole type calculations is arbitrary to some extent, Gillet's calculations did reproduce the experimental energies of the $1p_{3/2}^{-1}d_{5/2}$ multiplet quite well. It is therefore reasonable to assume that the calculated position of the members of the $1p_{3/2}^{-1}d_{3/2}$ multiplet is correct. This makes the assignment of 2^- unlikely for two reasons. First, this assignment requires Gillet's calculations for the $d_{3/2}$ coupling to be about 2 MeV lower and second, it requires both Gillet's and Lane's calculations for the 1^- member of the $d_{5/2}$ multiplet to be over 2 MeV too low. In other words, if the broad state at $E_x = 5.8$ MeV

Figure 4-8. Comparison of the $T = 1$ isobars in ^{12}B and ^{12}C .
The angular momentum coupling on the various
particle-hole configurations calculated by
Gillet et al. are indicated in brackets.

EXPERIMENT			
[p,d] GILLET et al			
p-h CALCULATIONS			
		0 ⁻	$[p_{3/2}^{-1} d_{3/2}]$
		1 ⁻	$[p_{3/2}^{-1} d_{3/2}]$
		3 ⁻	$[p_{3/2}^{-1} d_{3/2}]$
		23.2	2 ⁻
			$[p_{3/2}^{-1} d_{3/2}]$
		22.5	
(7.66) →	7.77 > 0		
	7.54 > 3		
	6.8 1 ⁺		
	6.6 1 ⁺		
	5.8 1 ⁻	21.4 (1 ⁻)	$[p_{3/2}^{-1} d_{5/2}]$
	5.73 3 ⁻		
	5.61 3 ⁺	20.6 (3 ⁺)	
→	5.0 1 ⁺	20.3 (2 ⁺)	
	4.54 4 ⁻	19.6 (4 ⁻)	
4.37 →	4.3 1 ⁻	19.3 2 ⁻	$[p_{3/2}^{-1} d_{5/2}]$
	3.76 2 ⁺	18.84 19.2 1 ⁻ 2 ⁺	
	3.39 3 ⁻		$[p_{3/2}^{-1} d_{5/2}]$
	2.72 0 ⁺	18.36 3 ⁻	
	2.62 1 ⁻	17.79 0 ⁺	
	1.67 2 ⁻	17.23 1 ⁻	
	0.95 2 ⁺	16.58 2 ⁻	
		16.11 2 ⁺	
	0.0 1 ⁺	15.11 1 ⁺	
	¹² B	¹² C	

has $J^\pi = 2^-$, where is the 1^- state? The data of this experiment, except for the resonance in $B(2)$ at $E = 2.7$ MeV, indicate no other broad negative parity states up to an excitation energy of 7.5 MeV in ^{12}B . This result is particularly disturbing because the energy splitting within a particle-hole multiplet is known to be reproduced quite well with the simple delta function residual interaction (Sh 53), as the calculations of Lane et al. indicate.

Finally a few words should be said concerning the configuration mixing of this 1^- state. Gillet et al. calculated the wave function for this state to be

$$\psi(1^-, T = 1) = .989 \phi_{d_{5/2}} + .106 \phi_{s_{1/2}}$$

If the assumption is made that $\gamma_{s_{1/2}}^2(\text{s.p.}) = \gamma_{d_{5/2}}^2(\text{s.p.})$, a consideration of the $\ell = 0$ and $\ell = 2$ penetrabilities at $E = 2.7$ MeV leads to the conclusion that approximately 70% of the observed width of this state will have $\ell = 2$. This conclusion supports the assumptions of the R-matrix calculation (i.e. only one ℓ -value participates in forming this resonance). When combined with the results obtained when $\ell = 0$ is assumed for this state in the calculations ($B(0)$ destructively interferes and gives an interference dip) it suggests that an enlargement of the code to handle two ℓ values for this resonance would result in a better fit to $B(0)$, since the $\ell = 0$ contribution to the width of this state would most likely decrease $B(0)$.

Thus an assignment of $J^\pi = 1^-$ has been given to a previously unobserved state in ^{12}B at $E_x = 5.8$ MeV corresponding to the 1^- member of the $1p_{3/2}^{-1}d_{5/2}$ particle-hole multiplet. This completes the identification of all the states of this multiplet.

STATES AT $E_x = 5.61$ MeV AND $E_x = 5.73$ MeV ($E = 2.45$ MeV AND $E = 2.58$ MeV)

These states will be treated together because of their interrelation to one another. In the interval $2.2 \text{ MeV} \leq E \leq 3.0 \text{ MeV}$, the previous total cross section measurement was interpreted in terms of only two resonances (Bo 51). The lower of these, a broad ($\Gamma \approx 120 \text{ keV}$) peak at $E = 2.45 \text{ MeV}$ was interpreted as a state with $J = 2$, based on total peak height ($\sigma_T \propto (2J + 1)$). The other, a narrow ($\Gamma \approx 60 \text{ keV}$) peak at $E = 2.58 \text{ MeV}$ was interpreted as a state with $J = 3$ based on similar arguments. Figure 4-3 shows the best R-matrix fit which has 3^+ and 3^- respectively as the assignments for these states. The parity assignments are definitive but some discussion is necessary concerning the spin assignment.

For the resonance at $E = 2.58 \text{ MeV}$, the only spin values which give the required peak height in $B(0)$ are $J = 3$ or 4 . This is primarily a result of the reasonable fit to the off-resonance background near this resonance. Figure 4-9 shows the best fit and a 4^- assignment for this state. Note the severe departure from the data, particularly in $B(2)$. Although fits to this 4^- state assignment were not tried with various $R^0_{4^-}$ background terms, it is very unlikely that a value of $R^0_{4^-}$ can be found that would a) fit the data at $E = 2.58 \text{ MeV}$ without b) destroying the fit at other energies. A spin assignment of 4 cannot be ruled out based on the R-matrix fit, however.

There is other evidence that supports the 3^- assignment. Referring again to the inelastic $(n, n'\gamma)$ data of Figure 4-6, the sharp peak at $E = 2.6 \text{ MeV}$ is the result of this state. This state is seen in this data as a result of inelastic neutron scattering to the first excited state of ^{11}B ($I^\pi = 1/2^-, s' = 0, 1$). To conserve parity in the inelastic

Figure 4-9. A comparison of the best fit with a 4^- assignment for the resonance at $E = 2.58$ MeV.

neutron scattering, $\ell' = \ell_{\text{inel}} = 0, 2, 4$ and to have a spin assignment of 4 requires $\ell' = 4$ ($J = \ell' + s'$). The penetrability for an $\ell = 4$ neutron with $E = .250$ MeV (the approximate energy of the inelastic neutron) is approximately 10^{-6} . This leads, via $\Gamma_{\text{tot}}/2p_{\ell=4} = \sum \gamma^2$, to a reduced width which greatly exceeds the Wigner limit, which is approximately 3.4 MeV for the parameters chosen for the present calculation. A spin value of 3 can conserve parity and have $\ell' = 2$, without exceeding the Wigner limit. Thus this narrow state is assigned $J^\pi = 3^-$.

The broad state at $E = 2.45$ MeV has been assigned $J^\pi = 3^+$. Figure 4-10 shows a comparison of the best fit with a 2^+ assignment for this state. Spins of 1 and 4 are ruled out because of the height of the B(0) peak. Once again, a spin of 2 cannot be ruled out by the R-matrix fit. Referring once more to Figure 4-6, there is no evidence to indicate inelastic scattering from the resonance at $E = 2.45$ MeV. This may be expected as can be seen from the following. The arguments made above for the resonance at $E = 2.58$ MeV indicate that this state decays by inelastic neutrons to the first excited state of ^{11}B with $\ell' = 2$. Thus it appears certain that transitions with $\ell' \leq 2$ could have been seen by Lind et al (Li 61). For a spin assignment of 2 to the resonance at $E = 2.45$ MeV, $\ell' = 1$ is allowed and should have been seen. For a spin of 3, $\ell' = 3$ only, and they would not have seen inelastic neutrons because the penetrability for $\ell' = 3$ for neutrons with $E = .125$ MeV is of the order of 10^{-5} . This penetrability requires that the reduced width greatly exceed the Wigner limit. Thus an assignment of $J^\pi = 3^+$ is made for this state.

Figure 4-10

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Figure 4-10. A comparison of the best fit with a 2^+ assignment for the resonance at $E = 2.45$ MeV.

STATES AT $E_x = 6.6$ and $E_x = 6.8$ MeV ($E = 3.5$ MeV and $E = 3.8$ MeV)

The history of these states is rather curious. Fossan et al. have identified a narrow resonance from a total neutron cross section measurement at $E = 3.5$ MeV with $J > 0$ and a narrow resonance ($\Gamma \approx 65$ keV) with $J \leq 0$ at $E = 3.7$ MeV. Data in the present work was taken in small energy steps with a very thin lithium target. The companion experiment, $^{10}\text{B} + n$, required much less energy resolution and data was taken above $E = 3.7$ MeV with larger energy steps and a thicker lithium target. The data indicate a sharp rise in $B(1)$ at 3.5 MeV and confirm the existence of a resonance at this energy. On the other hand, the data show no evidence of a narrow resonance at $E = 3.7$ MeV. In fact, the data above $E = 3.5$ MeV indicate the existence of a previously unobserved broad resonance at a neutron energy of $E = 3.8$ MeV. If the present experiment had not been systematic, it is doubtful whether this state would have been found, because there was no evidence of it from the total cross section work of Fossan (Fo 61). If this broad state is excluded from the calculations, not only the magnitude but also the shape of $B(1)$ cannot be reproduced. The exclusion of this broad state is shown in Figure 4-5, where the dashed line indicates the calculated $B(0)$ without a broad state included. A single broad state is unable to give the sharp rise in $B(1)$ at $E = 3.5$ MeV.

Since the magnitude of the interference effect is small, this resonance could be due to p-d interference, i.e. a broad negative parity state interfering with the p wave background. When such states were included in the calculations, the shape of $B(1)$ was never reproduced, hence the conclusion is that these states both have positive parity.

Total peak height arguments eliminate combinations of spins 0 or 3 from consideration. Various combinations of narrow and broad state spin values were attempted and the combination 1^+ , 1^+ was chosen as the best fit, primarily because of the magnitude and the shape of the fit to $B(1)$. To avoid the interference normally associated when two states of the same J^π occur so close together, opposite relative phases for the reduced width amplitudes were required for the 1^+ resonance at $E = 3.5$ MeV.

Since there was no other evidence to help determine the spin of these states, another calculation with a 2^+ , 2^+ combination was done with the large $R_0^{2^+}$ background required to fit the magnitude of $B(1)$ on resonance. This fit is compared to the best fit in Figure 4-11, where the dashes again indicate the 2^+ , 2^+ calculation. Aside from the obvious magnitude problems in $B(0)$ and $B(1)$ with this particular 2^+ , 2^+ fit, what is more disturbing is the shape of $B(2)$ and to some extent $B(1)$. The $B(2)$ shape in general and the $B(3)$ shape near the 3.5 MeV resonance do not fit the data. In addition, the narrow 2^+ resonance at $E = 3.5$ MeV gives a much higher peak in $B(0)$ and $B(1)$ than the data indicate. Thus the tentative assignments are $J^\pi = (1)^+$, $(1)^+$ for these states.

This 2^+ , 2^+ calculation indicates several of the difficulties associated with background variations. First, the $R_0^{2^+}$ required to fit $B(1)$ on the broad resonance effectively destroys the $B(2)$ and the $B(0)$ fits. This is not to say that these terms could not be fit with the 2^+ , 2^+ combination and a completely different background. It becomes a matter of varying the background to fit the state requirements and with many different states possible in the calculation, such background variations soon become tedious.

Figure 4-11. A comparison of the best fit with a $2^+, 2^+$ combination
for the resonances at $E = 3.5$ MeV and $E = 3.8$ MeV

COEFFICIENTS (B/SR)

The most unsatisfactory fit to the data is in $B(2)$ above $E = 3.3$ MeV. During all of the various fits to the data, this term was consistently low except when a broad negative parity state was inserted at $E = 3.8$ MeV. This brought $B(2)$ up to the data but did not fit the shape of $B(1)$. The total cross section of $^{11}\text{B} + n$, however, indicates the existence of several broad resonances above $E = 4.5$ MeV. These states are most likely the parents of the $T = 1$ analog states in ^{12}C which arise from the $1p_{3/2}^{-1} d_{3/2}$ particle-hole multiplet. It is clear that an extension of this work to look at the states in this region would lead to a better knowledge of the structure of ^{12}B and possibly remove this discrepancy in the $B(2)$ fit.

COMPARISON WITH $T = 1$ STATES IN ^{12}C

The $T = 1$ states in ^{12}B , ^{12}C and ^{12}N are shown in Figure 4-12. The correspondence of the analogs in ^{12}C with their parents in ^{12}B is reasonably well established up through the 4^{-} state at $E_x = 19.6$ MeV (Re 71, Ya 71). The state tentatively identified as $(1)^{-}$ at $E_x = 31.4$ MeV is a reasonable candidate for the analog of the 3^{-} state identified in the present analysis. In addition, this state is only .3 MeV below the energy predicted by Gillet et al. for the 1^{-} member of the $1p_{3/2}^{-1} d_{5/2}$ multiplet (See Figure 4-8)

Bohne et al. (Bo 70) have identified a 3^{+} state in ^{12}N at $E_x = 5.27$ MeV. More recent work by Yamaguchi et al. (Ya 71) has tentatively identified a 3^{+} state in ^{12}C from electron scattering data. This state is a reasonable candidate for the analog of the 3^{+} state in ^{12}B identified by the present analysis.

Figure 4-12. $T = 1$ energy levels for the $A = 12$ nuclei.

8.12 <u>8.36</u>				8.5
7.55 <u>7.77</u>		23.2		7.7 ($\leq 2^-$)
6.6 <u>6.8</u> 1^+		22.5		7.1
5.8 1^+		21.4 (1^-)		6 (1^-)
5.61 <u>5.73</u> 3^+ 3^-		20.6 3^+		5.27 3^+
5.0 1^+		20.3 2^-		4.6
4.37 <u>4.54</u> 2^- 4^-		19.4 <u>19.6</u> (4^-) 2^+		4.24 ($\leq 2^-$)
3.76 <u>4.3</u> 2^+ 1^-		18.84 <u>19.2</u> 2^+ 1^-	n	3.5
3.39 3^-		18.36 3^-		3.1
2.62 <u>2.72</u> 1^- 0^+		17.79 0^+		2.43
1.674 2^-		17.23 1^-		1.65 <u>2.0</u>
0.953 2^+		16.58 2^-		0.969 <u>1.2</u> 2^+ (2^-)
[15.21] 1^+		16.11 2^+		
^{12}B		15.11 1^+	$\bar{p}$	[14.98] 1^+ $\bar{p}$
		^{12}C		^{12}N

The narrow 3^- and $(1)^+$ states identified in the present analysis would be difficult to see at high excitation in ^{12}C . However, the broad $(1)^+$ state identified at $E_x = 6.8$ MeV should have an analog in ^{13}C which has not yet been identified.

POLARIZATION CALCULATIONS

Figure 4-13 shows the best fit obtained for all of the $^{11}\text{B} + n$ differential cross section and polarization data, including the predicted values of the polarization in the energy interval $2.2 \text{ MeV} \leq E \leq 4.5 \text{ MeV}$. The fit to the data below $E = 2.2$ MeV is not as good as it was in the previous work (La 70). This is not surprising, considering the boundary conditions and the R^0 backgrounds were chosen to give the best fit to the data above $E = 2.2$ MeV. A comparison of the various predicted polarization calculations for the various spin assignments possible for each state is useful as a guide for future experiments. The polarization data can sometimes determine the J value of a state uniquely and hence a measurement of the polarization on resonance may serve to unambiguously determine the spin value.

Figure 4-14 shows the predicted polarization of the best fit and a 2^- resonance at $E = 2.7$ MeV, respectively. Unfortunately, the difference in the calculated polarizations are so slight that it is unlikely that a polarization measurement would uniquely determine which spin assignment is correct.

Figure 4-15 shows the predicted polarization of the best fit and a 2^+ resonance at $E = 2.45$ MeV. Since the 2^+ resonance gives a positive $C(1)$ and the 3^+ resonance gives a negative $C(1)$, a polarization measurement on resonance could uniquely determine the spin of this state.

Figure 4-13. A complete R-matrix fit to all of the $^{11}\text{B} + n$ differential cross section and polarization data, including predicted values of the polarization in the energy interval $2.2 \text{ MeV} \leq E \leq 4.5 \text{ MeV}$.

Figure 4-14. The difference in the predicted polarization for
a 1^- and a 2^- assignment to the resonance at $E = 2.7$ MeV.

Figure 4-15. The difference in the predicted polarization for the best fit and a 2^+ assignment for the resonance at $E = 2.45$ MeV.

Finally, Figure 4-16 shows the predicted polarization of the best fit and the $2^+, 2^+$ calculation with the large $R^0_{2^+}$ background. The difference in the shape of $C(1)$ near these resonances could uniquely determine the spin of one or both of these resonances (the magnitude of $C(1)$ is low because of the large R^0 used in the fit). Although a calculation was done for the $3^-, 4^-$ combination at $E = 2.58$ MeV, the differences in the predicted polarization were negligible.

R^0 VARIATIONS

With no R^0 's included, the calculations were in serious disagreement with the data. The process of determining the set of R^0 's used in the best fit was straightforward. Reasonable values were assumed for the R^0 's of the $1^+, 2^+, 3^+, 1^-, 2^-, 3^-$ and 4^- states and these were inserted in the calculation and the background was fit. Essentially all of the states within and below the energy region of the present experiment were included explicitly, thus all R^0 's included were positive ($E_\lambda > E$), except the $R^0_{0^+}$ which is negative since ^{12}B has a 0^+ bound state. Each R^0 was then varied individually and no strong dependence on a particular R^0 was observed. The ratios of $R^0_{S=1}/R^0_{S=2}$ predicted by the shell model calculations were used for the R^0 's of the $1^-, 2^-$ and 3^- states. However, no significant changes occurred when these ratios were inverted. The $R^0_{2^-}$ had essentially no observable effect and was removed from the subsequent calculations.

Figure 4-16. The difference between the predicted polarization
for the best fit and the 2^+ , 2^+ combination.

CONCLUSIONS

The elastic scattering differential cross sections for $^{11}\text{B} + n$ have been systematically measured using TOF techniques at 45 energies between $E = 2.2$ MeV and $E = 4.5$ MeV. The experimental results have been corrected for all finite sample size effects and have been normalized to an independent neutron scattering measurement from hydrogen. The corrected differential cross sections have been fitted in the center-of-mass system with a sum of Legendre Polynomials. A two channel, three level R-matrix analysis has been completed for this data. This analysis has lead to the discovery of two previously unobserved resonances in $^{11}\text{B} + n$ at $E = 2.7$ MeV and $E = 3.8$ MeV, corresponding to states in ^{12}B at $E_x = 5.8$ MeV and $E_x = 6.8$ MeV. These states have been assigned spin and parity of 1^- and $(1)^+$ respectively. The broad 1^- state is believed to be the 1^- member of the $1p_{3/2}^{-1}1d_{5/2}$ particle-hole multiplet predicted by earlier shell model calculations. Other resonances previously observed in total neutron cross section at $E = 2.45$ MeV, $E = 2.58$ MeV and $E = 3.5$ MeV have been assigned spin and parity values of 3^+ , 3^- and $(1)^+$. Comparisons have been made with the possible $T = 1$ analog states in ^{12}C corresponding to the states observed in this experiment. Calculations of the polarization have been made and suggestions for future work in this energy region in ^{12}B have been made. The existence of several broad resonances above $E = 4.5$ MeV in $^{11}\text{B} + n$ in conjunction with the difficulty in fitting B(2) above $E = 3.3$ MeV suggest that a continuation of the study of the structure of ^{12}B at higher neutron energies might lead to the discovery of the members of the last major particle-hole multiplet, $1p_{3/2}^{-1}1d_{3/2}$, calculated by Gillet et al.

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APPENDIX

Expansion coefficients and differential cross sections (center-of-mass)
and their standard deviations.

EXPANSION COEFFICIENTS IN α/α_0

LAT ($^{\circ}$)	$\alpha(0)$	$\alpha(1)$	$\alpha(2)$	$\alpha(3)$	$\alpha(4)$
	(CENTER OF MASS)				
2.193	0.138	0.020	0.014	0.011	-0.012
	0.002	0.004	0.005	0.006	0.004
2.161	0.143	0.019	0.021	0.016	-0.006
	0.003	0.004	0.006	0.007	0.004
2.130	0.141	0.104	0.060	0.024	-0.009
	0.003	0.006	0.008	0.009	0.011
2.120	0.202	0.149	0.092	0.035	-0.010
	0.004	0.007	0.009	0.010	0.013
2.100	0.192	0.164	0.105	0.017	-0.009
	0.004	0.007	0.008	0.010	0.012
2.080	0.186	0.164	0.120	0.004	0.014
	0.004	0.007	0.009	0.010	0.012
2.060	0.185	0.114	0.132	-0.004	0.025
	0.003	0.006	0.009	0.010	0.010
2.040	0.182	0.091	0.164	-0.003	0.040
	0.003	0.007	0.008	0.010	0.011
2.020	0.214	0.076	0.214	0.037	0.043
	0.004	0.008	0.010	0.011	0.012
2.000	0.210	0.096	0.233	0.063	0.062
	0.004	0.008	0.010	0.012	0.013
2.000	0.190	0.093	0.201	0.073	0.040
	0.003	0.007	0.004	0.011	0.012
2.000	0.173	0.095	0.172	0.059	0.027
	0.003	0.007	0.008	0.010	0.011
2.000	0.163	0.093	0.150	0.053	0.022
	0.003	0.006	0.006	0.009	0.011
2.000	0.142	0.059	0.145	0.036	0.023
	0.003	0.006	0.007	0.008	0.009
2.000	0.144	0.059	0.127	0.017	-0.000
	0.003	0.005	0.007	0.008	0.006
2.000	0.137	0.051	0.126	0.024	0.012
	0.003	0.005	0.006	0.005	0.009
2.000	0.138	0.041	0.127	0.031	0.022
	0.003	0.005	0.006	0.008	0.009
2.000	0.130	0.046	0.113	0.035	0.011
	0.002	0.005	0.006	0.007	0.009
2.000	0.122	0.032	0.017	0.037	0.009
	0.002	0.005	0.006	0.007	0.007
2.000	0.117	0.037	0.092	0.035	0.015
	0.002	0.004	0.005	0.006	0.007
2.000	0.113	0.033	0.089	0.042	0.013
	0.002	0.004	0.005	0.005	0.007
2.000	0.111	0.022	0.087	0.028	0.000
	0.002	0.004	0.005	0.006	0.007

CYPRATION COEFFICIENTS 1" J/SB

LA (IN.)	b(0)	b(1)	b(2)	b(3)	b(4)
(CENTER OF MASS)					
4.219	0.109	0.022	0.016	0.037	0.001
	0.002	0.004	0.005	0.006	0.007
4.303	0.107	0.029	0.093	0.031	0.0
	0.002	0.004	0.003	0.006	0.0
4.400	0.092	0.033	0.083	0.014	0.0
	0.002	0.004	0.005	0.006	0.0
4.440	0.038	0.042	0.086	0.023	0.0
	0.002	0.004	0.005	0.006	0.0
4.456	0.105	0.046	0.034	0.027	0.0
	0.002	0.004	0.005	0.006	0.0
4.470	0.104	0.056	0.097	0.027	0.0
	0.002	0.004	0.005	0.006	0.0
4.500	0.109	0.071	0.041	0.024	0.0
	0.002	0.004	0.006	0.007	0.0
4.540	0.102	0.073	0.042	0.018	0.0
	0.002	0.004	0.005	0.006	0.0
4.560	0.107	0.076	0.190	0.024	0.0
	0.002	0.005	0.006	0.007	0.0
4.600	0.121	0.065	0.110	0.022	0.0
	0.002	0.005	0.006	0.007	0.0
4.637	0.101	0.062	0.101	0.013	0.0
	0.002	0.005	0.006	0.007	0.0
4.700	0.103	0.069	0.103	0.025	0.0
	0.002	0.005	0.006	0.007	0.0
4.710	0.100	0.067	0.104	0.018	0.0
	0.002	0.005	0.005	0.007	0.0
4.731	0.102	0.059	0.106	0.033	0.0
	0.002	0.005	0.006	0.007	0.0
4.817	0.104	0.079	0.124	0.035	0.0
	0.002	0.005	0.006	0.007	0.0
4.830	0.100	0.075	0.109	0.024	0.0
	0.002	0.004	0.005	0.006	0.0
4.930	0.090	0.071	0.109	0.029	0.0
	0.002	0.004	0.005	0.005	0.0
4.983	0.089	0.056	0.110	0.028	0.0
	0.002	0.004	0.006	0.007	0.0
4.100	0.093	0.040	0.126	0.018	0.0
	0.002	0.005	0.006	0.007	0.0
4.240	0.098	0.031	0.112	0.012	0.0
	0.002	0.004	0.006	0.007	0.0
4.271	0.087	0.028	0.117	0.016	0.0
	0.002	0.005	0.006	0.007	0.0
4.341	0.092	0.040	0.121	0.017	0.0
	0.002	0.005	0.006	0.007	0.0
4.391	0.085	0.029	0.124	0.014	0.0
	0.002	0.004	0.006	0.007	0.0

CROSS SECTION IN B/SR

SLAB (MEV) 21.4 45.5 70.8 95.3 118.8 141.5 161.6

(CENTER OF MASS)

2.193	0.166	0.156	0.126	0.126	0.145	0.130	0.108
	0.010	0.007	0.006	0.005	0.005	0.005	0.005
2.391	0.186	0.158	0.134	0.139	0.130	0.149	0.124
	0.010	0.008	0.006	0.006	0.005	0.006	0.006
2.390	0.142	0.257	0.196	0.136	0.138	0.128	0.110
	0.016	0.012	0.009	0.006	0.005	0.006	0.006
2.420	0.423	0.338	0.196	0.153	0.135	0.119	0.112
	0.019	0.015	0.009	0.007	0.005	0.005	0.006
2.450	0.437	0.330	0.206	0.124	0.104	0.109	0.110
	0.019	0.014	0.010	0.006	0.005	0.005	0.006
2.480	0.443	0.323	0.199	0.118	0.086	0.106	0.136
	0.020	0.014	0.009	0.006	0.004	0.005	0.007
2.510	0.388	0.262	0.166	0.096	0.083	0.124	0.188
	0.018	0.012	0.008	0.005	0.004	0.005	0.009
2.542	0.410	0.264	0.167	0.104	0.094	0.182	0.252
	0.019	0.012	0.008	0.005	0.004	0.008	0.010
2.566	0.511	0.290	0.154	0.109	0.142	0.254	0.326
	0.023	0.013	0.007	0.005	0.006	0.010	0.013
2.583	0.552	0.288	0.145	0.114	0.139	0.218	0.303
	0.024	0.013	0.007	0.006	0.006	0.009	0.013
2.595	0.507	0.266	0.132	0.106	0.128	0.190	0.234
	0.022	0.012	0.007	0.005	0.005	0.008	0.010
2.609	0.448	0.252	0.127	0.095	0.121	0.162	0.201
	0.020	0.011	0.006	0.005	0.005	0.007	0.009
2.638	0.406	0.248	0.121	0.097	0.112	0.144	0.181
	0.019	0.011	0.006	0.005	0.005	0.006	0.009
2.701	0.348	0.192	0.106	0.075	0.101	0.154	0.141
	0.016	0.009	0.005	0.004	0.005	0.007	0.009
2.759	0.368	0.215	0.109	0.080	0.107	0.143	0.190
	0.015	0.010	0.005	0.004	0.005	0.006	0.009
2.850	0.305	0.196	0.099	0.079	0.102	0.143	0.190
	0.015	0.009	0.005	0.004	0.004	0.006	0.009
2.900	0.305	0.193	0.094	0.084	0.106	0.150	0.202
	0.015	0.009	0.005	0.004	0.005	0.007	0.009
2.952	0.289	0.178	0.092	0.080	0.106	0.133	0.168
	0.014	0.008	0.004	0.004	0.005	0.006	0.008
3.002	0.255	0.155	0.086	0.079	0.105	0.135	0.151
	0.013	0.007	0.004	0.004	0.004	0.006	0.007
3.056	0.252	0.151	0.084	0.079	0.097	0.119	0.144
	0.013	0.007	0.004	0.004	0.004	0.005	0.007
3.102	0.244	0.146	0.076	0.078	0.098	0.121	0.134
	0.013	0.007	0.004	0.004	0.004	0.005	0.006
3.153	0.215	0.144	0.075	0.072	0.099	0.128	0.146
	0.012	0.007	0.004	0.003	0.004	0.006	0.007

CROSS SECTION IN B/S

25.0 (1.7)	24.3	45.5	70.3	95.3	111.5	141.5	161.4
(CENTER OF MASS)							
3.294	0.221	0.135	0.073	0.073	0.101	0.123	0.137
	0.012	0.007	0.004	0.003	0.004	0.005	0.007
3.303	0.221	0.145	0.072	0.061	0.093	0.126	0.135
	0.012	0.007	0.003	0.003	0.004	0.005	0.007
3.403	0.194	0.132	0.067	0.053	0.069	0.095	0.126
	0.011	0.007	0.003	0.003	0.003	0.004	0.005
3.440	0.220	0.146	0.071	0.054	0.075	0.101	0.119
	0.012	0.007	0.004	0.003	0.003	0.004	0.006
3.456	0.220	0.154	0.083	0.062	0.073	0.110	0.112
	0.112	0.003	0.004	0.003	0.003	0.005	0.006
3.477	0.247	0.163	0.079	0.052	0.079	0.102	0.110
	0.113	0.008	0.004	0.003	0.003	0.004	0.006
3.571	0.250	0.175	0.097	0.063	0.073	0.091	0.087
	0.013	0.003	0.005	0.003	0.003	0.004	0.005
3.583	0.241	0.176	0.083	0.055	0.166	0.091	0.075
	0.013	0.009	0.004	0.003	0.003	0.004	0.005
3.563	0.271	0.180	0.086	0.053	0.073	0.083	0.105
	0.014	0.009	0.004	0.003	0.003	0.004	0.006
3.677	0.266	0.164	0.079	0.042	0.065	0.093	0.110
	0.114	0.009	0.004	0.002	0.003	0.004	0.006
3.687	0.258	0.165	0.074	0.049	0.071	0.098	0.102
	0.014	0.003	0.004	0.003	0.003	0.004	0.006
3.710	0.258	0.173	0.076	0.053	0.064	0.091	0.109
	0.014	0.009	0.004	0.003	0.003	0.004	0.006
3.716	0.250	0.176	0.078	0.044	0.062	0.094	0.107
	0.013	0.009	0.004	0.003	0.003	0.004	0.006
3.731	0.260	0.163	0.073	0.047	0.072	0.102	0.108
	0.114	0.009	0.004	0.003	0.003	0.005	0.006
3.817	0.300	0.173	0.075	0.045	0.059	0.095	0.112
	0.015	0.009	0.004	0.002	0.003	0.004	0.006
3.884	0.274	0.163	0.063	0.041	0.058	0.089	0.102
	0.014	0.003	0.004	0.002	0.003	0.004	0.006
3.901	0.259	0.160	0.065	0.034	0.053	0.078	0.093
	0.013	0.009	0.003	0.002	0.003	0.004	0.006
4.019	0.255	0.133	0.061	0.033	0.052	0.083	0.117
	0.114	0.003	0.004	0.002	0.003	0.004	0.007
4.120	0.243	0.143	0.055	0.033	0.054	0.105	0.151
	0.014	0.004	0.003	0.002	0.003	0.005	0.009
4.240	0.203	0.133	0.054	0.031	0.063	0.112	0.143
	0.013	0.003	0.004	0.002	0.003	0.005	0.008
4.291	0.220	0.127	0.053	0.032	0.059	0.115	0.151
	0.114	0.007	0.004	0.002	0.003	0.005	0.009
4.341	0.240	0.145	0.057	0.030	0.067	0.104	0.152
	0.114	0.009	0.004	0.002	0.003	0.005	0.009
4.391	0.225	0.123	0.049	0.029	0.052	0.113	0.160
	0.114	0.007	0.003	0.002	0.003	0.005	0.009