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STABILITY OF GAS FLOW DISTRIBUTION
AMONG PARALLEL HEATED CHANNELS*

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ABSTRACT

Although the condition for stability is well known, determination of the minimum stable flow is difficult because it involves transition from turbulent to laminar flow. Measurements of heat transfer and fluid friction with transition were made and the results were applied to the stability problem. Normal hydrogen gas entered a uniformly heated tube at Reynolds numbers from 600 to 13,000 and was heated to temperatures up to 8.9 times the entering temperature. Empirical correlations of the results were used in stability calculations. The use of these local correlations for friction factors generally resulted in the prediction of higher stable exit-to-inlet temperature ratios than previous calculations which assumed fully developed flow. Calculations for a typical channel of a nuclear rocket core with change of area and nonuniform heating show that the entrance impedance of the channel more than compensates for the destabilizing effect of nonuniform heating. At an entrance Reynolds number of 2350, the core configuration should exhibit stable flow distribution for exit-to-inlet temperature ratios less than 10, whereas the limit for an equivalent uniformly heated, unimpeded channel is 7.

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NOMENCLATURE

English Letter Symbols

- A, B, C = constants
- c_p = specific heat at constant pressure, $QM^{-1}\theta^{-1}$
- D = diameter, L
- f = friction factor, nondimensional
- G = mass flow velocity, $ML^{-2}T^{-1}$
- h = heat transfer coefficient, $QL^{-2}T^{-1}\theta^{-1}$
- I = electrical current, amperes
- k = thermal conductivity, $QL^{-1}T^{-1}\theta^{-1}$
- L = length, L
- m = exponent of temperature in $\mu = \mu_o(T/T_o)^m$, nondimensional
- p = pressure, $ML^{-1}T^{-2}$
- Q = heat rate, QT^{-1}
- q' = heat transfer rate per unit length, $QT^{-1}L^{-1}$
- q'' = heat transfer rate per unit area, $QT^{-1}L^{-2}$
- R = resistance, ohms
- T = temperature, θ
- t = time, T
- u = axial velocity component, LT^{-1}
- v = general variable
- $\dot{w}$ = flow rate, MT^{-1}
- x = distance from electrode, L
- x_h = distance from tube inlet, L
- z = axial coordinate, L

Greek Letter Symbols

- α = stability parameter, T^{-1}
- γ = ratio of specific heats, nondimensional
- μ = dynamic viscosity, $ML^{-1}T^{-1}$
- Π = pressure function, $ML^{-1}T^{-2}$

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ρ = density, ML^{-3}

τ = shearing stress, $ML^{-1}T^{-2}$

ϕ = temperature ratio, nondimensional

Similarity Numbers

J = Colburn J factor, $N_{St} N_{Pr}^{2/3}$

N_{Dp} = pressure drop number, $\Delta p D^2 / \mu^2$

N_{Nu} = Nusselt number, hD/k

N_{Pr} = Prandtl number, $\mu c_p / k$

N_{Re} = Reynolds number $4\dot{w} / \pi D \mu$

N_{St} = Stanton number, $h / G c_p$

Y = transition parameter, $N_{Re} (T_b / T_w)^2$

Subscripts

b = bulk condition

$c.p.$ = constant property reference value

e = exit condition

L = lower bound of transition region

l = laminar

m = average over cross section

o = entrance condition

t = turbulent

U = upper bound of transition region

w = wall or surface conditions

INTRODUCTION

The question of the stability of gas flow among the parallel channels of heat exchangers was examined in the mid 1950's by Los Alamos researchers working with a laminar flow concept for a nuclear rocket engine. In recent years the emphasis has shifted to turbulent flow reactor designs, but the requirement that the engine be cooled down and restarted has revived interest in the flow stability problem.

Heat generated by decaying fission products can be removed most efficiently by a continuous flow of propellant which exhausts from the core at the maximum temperature permitted by the structural and corrosion limitations of the engine. As the heat generation and the flow rate diminish, a condition is reached in which the flow is no longer fully turbulent and flow distribution instabilities are possible. At this point some alternate and less efficient cooling program may be required. To determine the minimum propellant requirements for a nuclear rocket mission, it is important to know the minimum steady flow rate for which the flow distribution is stable.

The criterion for stable flow distribution is now well known, but its application to the determination of a minimum stable flow rate in a reactor core with a large gas temperature rise is complicated because the minimum flow is often turbulent in the entrance of the channels, but, due to increasing viscosity, may become laminar in the downstream region. Such a transition from turbulent to laminar flow is seldom encountered, and there is little information in the heat transfer literature on which to base calculations of either wall temperatures or shearing stresses. The nature of the heat transfer and fluid mechanics of turbulent laminar transition flows and their relation to the problem of flow distribution stability was the subject of this investigation.

PREVIOUS INVESTIGATIONS

The first investigator to study flow distribution stability in gas-cooled reactor cores was Longmire.¹ He derived the stability criterion for fully developed laminar flow in long channels and determined that stable distributions exist for $(\partial \Delta p / \partial \dot{w})_Q > 0$. Longmire's results were confirmed experimentally by Guevara et al.² who showed that a region of the constant heat rate, pressure-drop vs. flow curve for helium in a small capillary tube possessed a negative derivative. When the tube was operated in this region with the end pressures fixed, spontaneous flow and temperature excursions were observed.

Calculations for fully developed laminar flow of ideal hydrogen in long tubes with constant heat flux indicate that stable flow distributions are to be expected for ratios of the exit to inlet temperatures of less than 3.6 to 3.7.^{3,4} In a more recent investigation, Harry⁵ describes the results of numerical calculations using the properties of para-hydrogen. These calculations also pertain to tubes of constant area with uniform heat flux and with fully developed flow, but shearing stresses are calculated according to turbulent relations for $N_{Re} > 1000$ and laminar relations for $N_{Re} < 1000$. The results indicate that the minima of the constant heat rate curves occur at temperature ratios from 5 to 15 when the inlet Reynolds numbers are greater than 2100 but the exit Reynolds numbers are less than 1000. Experimental data obtained by Turney⁶ showed reasonable agreement with the analysis described above at low Reynolds numbers and heat flux.

Little heat transfer or fluid friction data are available in the literature for turbulent-laminar transition flows. Only the recent work of Perkins and Worsoe-Schmidt⁷ reports data in this region, and no attempt at correlation is made although a useful classification of the data is made by McEligot et al.⁸ At the low end of the turbulent Reynolds number range, the results of Ref. 7 appear to be the most reliable for large wall-to-bulk temperature ratios. The laminar flow regime has received considerable analytical attention,⁹⁻¹⁶ but experimental data are still sparse. Davenport and Leppert¹⁷ and Kays and Nicoll¹⁸

The usual small perturbation procedure is followed, and separation of the variables is achieved by assumptions of the form

$$v' = v_1(z)e^{\alpha t} \quad (5)$$

where v' represents the perturbation of the variable v about its steady solution $\bar{v}$. The stability problem is thus reduced to an eigenvalue problem for the α 's. If the real parts of the α 's are negative, then perturbations die out in time and the flow is stable. If the real parts are positive, the perturbations grow (at least initially). It is possible to show that the sign of the real part of the α 's is associated with the derivative $(\partial \Delta p / \partial \dot{w})_Q$, and, specifically, that $\text{Re}(\alpha) < 0$ when $(\partial \Delta p / \partial \dot{w})_Q > 0$. The condition for which $\text{Re}(\alpha) = 0$ is called the point of neutral stability and is obtained from $(\partial \Delta p / \partial \dot{w})_Q = 0$. It appears that under the conditions cited the eigenvalues are purely real so that flow excursions rather than oscillation occur, but no proof of this is given.

The conclusions of the mathematical analysis can be illustrated by a physical argument. Figure 1 shows a possible steady flow solution of Equations (1-3). Consider first a point such as A on the portion of the curve for which $(\partial \Delta p / \partial \dot{w})_Q > 0$. If the heat rate to the gas is momentarily increased by a differential amount dQ , examination of Equations (1-3) shows that the flow must decrease, and, since Δp is fixed by assumption, the resultant state is 2. If the heat perturbation is then removed, the flow can return to its original state since the pressure drop is greater than that required at point 3. Likewise, a negative heating perturbation will be restored to the original state.

A positive heating perturbation of the point B for which $(\partial \Delta p / \partial \dot{w})_Q < 0$ will also cause the state point to move to the left along the constant pressure-drop line to the point 2'. Now the situation is quite different: the flow cannot return to its original state when the heating perturbation is removed since to do so would require additional pressure drop. Furthermore, it cannot remain at the state 2' since at this point it is only capable of removing an amount of heat equal to $Q - dQ$ while the boundary wall is delivering an amount Q . The difference represents a perturbation of the new state which will drive the flow

have reported experimental data for moderate wall-to-bulk temperature ratios, but the types of boundary and initial conditions that have been studied by either analytical or experimental methods are quite limited.

THEORY

Predictions of instabilities in the distribution of flow among parallel passages of heat exchangers are based upon the study of the response to small perturbations of the flow in a single channel when the end pressures are held constant. Physically, this condition requires that the flow in a single channel have a negligible effect on the system pressure drop. Thus it is tacitly assumed that the total system flow is stable and steady, and that only the allocation of flow to the individual channels is subject to instabilities.

The mathematical analysis of stability is described in detail in Refs. 1 and 19, and only a summary is given here. The flow in the channel is assumed to be one-dimensional and low speed, and the fluid is assumed to be an ideal gas. Under these assumptions the conservation equations are

$$\frac{\partial p}{\partial t} + \frac{\partial(\rho u)}{\partial z} = 0 \quad (1)$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial}{\partial z} (p + \rho u^2) = \frac{4\tau_w}{D} \quad (2)$$

$$\frac{1}{\gamma - 1} \frac{\partial p}{\partial t} + c_p \frac{\partial}{\partial z} (\rho u T) = \frac{4q''_w}{D} \quad (3)$$

The boundary conditions are

$$p(0, t) = p_0, \quad \rho(0, t) = \rho_0$$

$$\int_0^L q'_w(z) dz = Q \quad (4)$$

$$\int_0^L \frac{\partial}{\partial z} p(z, t) dz = -\Delta p$$

The first three relations state that the entering condition and heat rate to the gas are fixed. The last is a mathematical expression of the condition imposed upon the channel by its neighbors.

still lower to 3'. Hence, it is seen that the result of a positive perturbation is to drive the flow lower and lower until the heat removal mechanism is altered (for example by vaporization of the wall or disassociation of the coolant). A negative perturbation of the heating rate at point B will cause the flow to move horizontally to the right which will further diminish the heat rate so that the excursion continues to point A.

The arguments above are not changed by including the energy equation for the channel wall in the analysis, although it is probable that the heat capacity of the wall will control the rate of growth or decay of perturbations. Furthermore, the physical argument is valid regardless of the exact nature of the fluid or type of flow, so that the stability criteria, $(\partial \Delta p / \partial \dot{w})_Q > 0$ appears to have general validity.

EXPERIMENTAL APPARATUS

The experimental data were obtained for normal hydrogen gas, precooled to the temperature of liquid nitrogen, flowing through long, slender, electrically heated tubes. The general arrangement of the apparatus is shown in Figure 2. High pressure hydrogen was supplied to the liquid nitrogen heat exchanger through two pressure-regulating valves and two independent flow meters. The heat exchanger consisted of a length of copper tubing and a series of flow-reversing traps and filters immersed in a dewar of liquid nitrogen whose level was maintained by a controller. This arrangement supplied dry hydrogen at a temperature of 135°R to the inlet of the test sections. The effluent gas was cooled in a gas-to-water heat exchanger before being discharged to the stack through the flow controlling needle valve. The jacket surrounding the test section was maintained at a pressure of less than 10^{-3} torr, and a separate vacuum pump was provided to purge the system and reactivate the filters when the system was not in use.

The test sections were made from 3/16-inch O.D. by 0.035-inch wall commercial tubing, of an 80 percent nickel and 20 percent chromium alloy. The inside diameters of the tubes were determined from samples cut from each end of the original tubing. The tubes were gold-plated to reduce radiation losses and to

provide a stable surface, and in some of the tests aluminum radiation shields were used. Figure 3 is a schematic drawing of a typical test section installed in the vacuum jacket. The entrance of the tube was cut square and sharp to produce a definite and reproducible entrance condition, and 4 inches of unheated entrance was provided by brazing the tube into a copper sleeve. The inlet gas temperature was measured by a calibrated copper-constantan thermocouple. The exit temperature was measured by a calibrated chromel-alumel thermocouple which was brazed into a perforated copper plug attached to the end of the tube by a ceramic sleeve. A flow-reversing baffle forced the gas to flow back over the sleeve and exit flange to further reduce heat exchange between the end of the tube and the guard heated exhaust chamber. Fourteen, BS 30 gage, chromel-alumel thermocouples were spot welded to the tube at 0.0625, 1.25, 2.5, 5.0, 10.0, 15.0, 20.0, 25.0, 30.0, 35.0, 40.0, 45.0, 49.0, and 50.0 inches from the upstream electrode. The thermocouple leads were wrapped around the tube and secured with copper oxide cement. Piezometer holes, 0.005-inch in diameter, were electric discharge machined through the wall and connected to manometers through 0.032-inch O.D. by 0.004-inch-wall Inconel tubing. The pressure ports were located at -0.1, 12.6, 25.6, and 50.1 inches from the upstream electrode.

Hot wire anemometer probes were installed in some of the test sections and, although they were quite small, they did produce appreciable flow perturbations. All of the data reported here are for test sections without hot wire probes.

PROCEDURE

Each test section was calibrated in place by determining its resistance and temperature coefficient of resistance and its local heat-loss function. Temperatures above ambient were obtained using a small heating current with the tube evacuated; temperatures below ambient were achieved by cooling the tube with gas precooled by dry ice in an alcohol-water solution. Since the resistance of the tubing alloy changes with time at moderate temperatures, the tubes were aged at 500°F until the resistance stabilized.

The test data were collected from a series of tests in which the exit temperature was maintained at a specified value and the heating current was adjusted for each flow rate. This procedure was adopted to minimize departures from equilibrium between data points and because it was simple to implement. It is emphasized that the experimental system was not subject to flow instabilities of the type discussed above because the pressure drop could not be maintained independently.

The total gas temperature was computed by integrating the heat flux to the gas obtained by deducting heat losses from the I^2R term and then adjusting the distribution to bring it into agreement with the measured exit temperature. The adjustment required was generally less than two percent.

The inside wall temperatures were calculated from the wall thermocouple measurements by the methods of Ref. 20, which accounts for variable resistivity and conductivity. The local heat transfer coefficients were then calculated from

$$h = \frac{q''_w}{T_w - T_b} \quad (6)$$

The local pressures were obtained from the measured inlet pressure and pressure drops by a smoothing and interpolating procedure based on the equation

$$\ln \Pi = A + B \ln x_h + C(\ln x_h)^2 \quad (7)$$

where

$$\Pi = (\rho u^2)_m - (\rho_o u_o^2)_m + \left[p_{s_o} - p_s(x_h) \right].$$

The coefficients were determined, by the method of least squares, using the measured pressures at four axial locations weighted by the absolute height of the manometer liquid columns. The shearing stress at the wall is then given by

$$\tau_w = \frac{D}{4} \frac{d}{dx} \left[p_s + (\rho u^2)_m \right] = \frac{D}{4\Pi} \left(B + \frac{2C \ln x_h}{x_h} \right) \quad (8)$$

The enthalpy and thermal conductivity of normal hydrogen were obtained from Ref. 21 and the other properties from Ref. 22.

All of the calculations were carried out using the IBM 7094 digital computer, and detailed listings and graphs of computed dimensional and nondimensional variables were obtained for over two hundred tests.

EXPERIMENTAL RESULTS

Accuracy and Scope

The difficulties of obtaining results of high accuracy and precision in laminar flow heat transfer experiments are well known. In this investigation, detailed analysis of the experimental apparatus and procedures indicated that systematic errors should be negligible for $10 \leq x/D \leq 335$ and $N_{Re} > 600$, but that the effect of random errors on the computed quantities becomes significant as $T_w \rightarrow T_b$. The small random errors in the measurement of experimental quantities result in a rather large scatter in Nusselt numbers at small Reynolds numbers.

This dispersion of the computed results is indicated in Figure 4 which shows the complete set of local Nusselt numbers. This dispersion is reduced in the subsequent figures by eliminating all the points for which the difference between the local gas temperature computed from power measurements and that computed from the exit thermocouple measurement exceeds 5 percent of the difference between the local wall and gas temperatures.

Local heat transfer coefficients and friction factors are reported for Reynolds numbers from 600 to 13,000 with wall-to-bulk temperature ratios to 2.25. Stability limits of the experimental system were determined for exit-to-inlet temperature ratios from 4 to 8.9.

Summary of Results

Measurements of local friction factors for adiabatic hydrogen flow were made for laminar and turbulent flow. The turbulent results were in excellent agreement with the Blasius equation $f = 0.3164/N_{Re}^{0.25}$. The transition from laminar to turbulent flow was found to be quite sharp and to depend on x/D as observed by Rotta.²³ The transition Reynolds number for the average friction factor was 2350. For Reynolds numbers less than 2350, the local friction factors agree closely with values calculated from the velocity profiles obtained by Langhaar.²⁴

The friction factors obtained with heat addition are shown in Figure 5. For low Reynolds numbers, the effect of heating is to increase the friction factor. A statistical analysis of the results for $N_{Re} < 2350$ yielded $f = f_{c.p.} (T_w/T_b)^{1.41 \pm 0.014}$. The constant property friction factor $f_{c.p.}$ is given by

$$f_{c.p.} = \frac{16}{N_{Re}} \gamma(x_h/DN_{Re}) \frac{I_1 \gamma(x_h/DN_{Re})}{I_2 \gamma(x_h/DN_{Re})} \quad (9)$$

where γ is a function given by Langhaar and $I_n(x) = i^{-n} J_n(ix)$.

In turbulent flow, heating increases the friction factors at small x/D while decreasing them slightly for large x/D . This result is in qualitative agreement with the findings of Magee²⁵ and Lel'chuk and Dyadyakin.²⁶

The transition from laminar to turbulent friction factors appears more gradual when there is heat addition.

The local heat transfer results are presented in Figure 6 in the form of Colburn J factors plotted against bulk Reynolds number. The most striking features of this figure are the spread of the J's in the transition region and the fact that the laminar character seems to persist up to Reynolds numbers of 6000 to 7000. It is found on closer examination that the variation of the heat transfer coefficients in the transition region depends strongly on the heating rate and the axial location.

The results for a series of runs at the highest heating rate in which the exit gas temperature was held at 1200°R are shown in Figure 7. For this heating rate, the laminar variation of the J factor is extended well above the adiabatic transition Reynolds number for all axial positions with the transition occurring at progressively larger Reynolds numbers as the entrance is approached.

The dependence of the transition phenomena on the heating rate is clearly indicated in Figure 8 where the results for $x/D = 125.8$ are plotted with the exit gas temperature as a parameter. The results lie along the prolongation of the laminar flow line up to a transition point which depends upon the rate of heating. The J factors then increase rather abruptly to higher and nearly constant values which are characteristic of turbulent flow. The patterns for other

values of x/D are similar, but there is a variation of the Reynolds number of transition and of the slope of the transition line.

The Nusselt numbers for local Reynolds numbers less than 2350 are compared with two constant-property analytical solutions in Figure 9. The lower curve represents the solution of Siegel, et al.¹⁰ for uniform heat flux with a parabolic velocity profile at $x = 0$. The upper curve represents the numerical result of Keys¹¹ which utilizes the velocity profiles of Langhaar and assumes a uniform heat flux and a uniform velocity profile at $x = 0$. The experimental Nusselt numbers are plotted against the parameter

$$x^+ = \frac{2x}{DN_{Re} N_{Pr} (1 + m)} \left[1 + \frac{1 - \frac{N_{Re} N_{Pr}}{N_{Re o} N_{Pr o}}}{\left(\frac{N_{Re} N_{Pr}}{N_{Re o} N_{Pr o}} \right)^{1/m} - 1} \right] \quad (10)$$

This parameter, derived in Ref. 19, was introduced in place of the usual definition ($x^+ = 2x/DN_{Re} N_{Pr}$) in an attempt to account for the large variation of $N_{Re} N_{Pr}$ in the direction of flow.

Most of the experimental results fall between the two curves, as expected, since the isothermal entrance of 33 diameters is not long enough for complete development of a parabolic velocity profile except at the lower Reynolds numbers. The results for entering Reynolds numbers less than 2350 show the same relationship to the analytical solutions.

In Figure 10, the ratio of the experimental Nusselt number to the constant property Nusselt number of Ref. 10 is plotted as a function of the wall-to-bulk temperature ratio. This figure confirms previous findings^{17,18} that the Nusselt numbers are not very sensitive to radial property variations. The ratio is somewhat higher than expected for small x/D , but this is attributed to the entrance configuration rather than to heating effects. Calculations indicated that free convection was negligible for the experimental conditions.

Not enough data were obtained at high Reynolds numbers to allow any meaningful comparisons with previous investigations of turbulent flow.

Empirical Correlations

In the absence of a realistic mathematical description of the turbulent-laminar transition flow, it is unlikely that correlations of general validity can be developed, so that, where possible, the graphical representation should be relied upon. The investigation of flow stability, however, required the calculation of flow pressure-drop relations for a large number of conditions, so that some empirical method of representing the experimental results was required. In developing the empirical equations, the main emphasis was placed on obtaining the proper behavior of the equations outside the transition zone. No attempt was made to reproduce the data within the transition zone with great precision.

In a recent paper, McEligot et al.⁸ discuss the heat transfer results obtained at Stanford by a number of investigators. Some of these results fall into the turbulent-laminar transition zone. They suggested the use of a modified wall Reynolds number in which the density and viscosity are evaluated at the wall temperature. By mapping previously classified points on the plane defined by this modified Reynolds number and the wall-to-bulk temperature ratio, McEligot et al. were able to obtain the boundaries of the laminar, transition, and turbulent zones as curves. In the present investigation, it was found by the same type of approach that the divisions could be represented approximately by the single parameter, $Y = N_{Re\ b} (T_b/T_w)^2$. The value of the parameter was found to depend on x/D , but a reasonable representation was obtained by selection of one set of boundaries for $x/d < 100$ and another for $x/D > 100$. Table 1 indicates the upper and lower transition boundary values of the parameter.

Table 1

Boundaries of Turbulent-Laminar Transition Zone

x/D	Nusselt Numbers		Friction Factors	
	Y_L	Y_U	Y_L	Y_U
0-100	1000	3000	1000	2000
100-350	1500	3000	1500	2000

For values of Y less than the lower bound, the Nusselt number was evaluated from Kays' equation

$$N_{Nu \ell} = 4.36 + \frac{0.036 \left(\frac{N_{Re \ b} N_{Pr \ b}}{x/D} \right)}{1 + 0.0011 \left(\frac{N_{Re \ b} N_{Pr \ b}}{x/D} \right)} \quad (11)$$

and the friction factor was calculated from

$$f_{\ell} = f_{c.p.} (T_w/T_b)^{1.41} \quad (12)$$

where $f_{c.p.}$ is given by Equation (9).

For values of Y larger than the upper bound, the Nusselt numbers were calculated from

$$N_{Nu t} = 0.024 N_{Re \ b}^{0.8} N_{Pr \ b}^{0.4} (T_w/T_b)^{-0.7} \left[1 + (x/D)^{-0.7} (T_w/T_b)^{0.7} \right] \quad (13)$$

and the friction factors for $x/D > 65$ from

$$f_t/f_{c.p. \ w} = \left[\frac{2}{(T_w/T_b)^{\frac{1}{2}} + 1} \right]^2 \quad (14)$$

where $f_{c.p. \ w}$ is the friction factor from the Blasius equation using the modified-wall Reynolds number. These correlations were obtained by Perkins and Worsoe-Schmidt⁷ for low turbulent Reynolds numbers and large wall-to-bulk temperature ratios. For $x/D < 65$, the equation

$$f/f_{c.p.} = (T_w/T_b)^{0.541} - 0.168 \ln(x/D) \quad (15)$$

was used where $f_{c.p.}$ is the friction factor from the Blasius equation using the bulk Reynolds number. This equation was obtained by Magee²⁵ and is in good agreement with the entrance region results of Ref. 7 which were presented graphically.

In the transition zone, the Nusselt number and friction factor were computed from

$$N_{Nu} = \left[1 - \left(\frac{Y - Y_L}{Y_U - Y_L} \right) \right] N_{Nu \ell}(Y) + \frac{Y - Y_L}{Y_U - Y_L} N_{Nu t}(Y) \quad (16)$$

$$f = \left[1 - \left(\frac{Y - Y_L}{Y_U - Y_L} \right) \right] f_l(Y) + \frac{Y - Y_L}{Y_U - Y_L} f_t(Y) \quad (17)$$

where Y is the parameter $N_{Re}(T_b/T_w)^2$, and the subscripts U and L designate its value at the upper and lower boundary of the transition zone.

A comparison of these correlations with the experimental data is shown in Figure 11 for $T_e = 530^\circ R$ and in Figure 12 for $T_e = 1200^\circ R$. Although some systematic trends are apparent in the transition zone, the bulk of the data falls within ± 20 percent of the calculated values.

Stability Limits

The stability limits were determined by graphical analysis of the data plotted on a large log-log graph with the flow rate as the abscissa and the pressure drop squared as the ordinate. On this basis, sets of the data taken at constant exit temperature were represented by nearly linear functions which could be interpolated easily to obtain curves of constant heat rate. These graphs for Test Sections 1 and 3 are shown in Figure 13.

The minima of the constant heat rate curves represent the points of neutral stability according to the stability criterion $(\partial \Delta p / \partial \dot{w})_Q = 0$. The data from both test sections exhibit two minima for some intermediate heat rates. In these cases the absolute minimum is assumed to represent the point of neutral stability.

These data are summarized in Figure 14 in which the exit-to-inlet temperature ratio, ϕ_c , at the point of neutral stability is plotted as a function of the product of the inlet Reynolds number and the length-to-diameter ratio. The discontinuity evident in both sets of data is due to the shift in the absolute minimum as the heat rate increases.

The range of the points was limited by the range of the experimental data and the graphical procedure to temperature ratios between 4 and 8. Within this range the maximum inlet Reynolds number for which a ϕ_c was found was 3000. This confirms that instabilities are possible even when the flow entering the channel is in the turbulent range. For all entering conditions, the critical temperature ratio is seen to be well above the values predicted by the simplified theory for fully developed laminar flow shown by the solid curve.

APPLICATION OF EXPERIMENTAL RESULTS

The application of the experimental results to the study of other systems was made through use of a computer program which performs the numerical integration of the complete one-dimensional conservation equations for fluid mass, momentum, and energy for real compressible gases flowing in channels of constant or slowly varying section with arbitrary heat distribution. The empirical correlations provided the information necessary to evaluate the heating term in the energy equation and the shearing stress term in the momentum equation. The properties of normal hydrogen below 600°K were taken from Ref. 21 and above 600°K, from Ref. 27. Properties of para-hydrogen below 100°K were from Ref. 28, from 100-600°K, from Ref. 22, and above 600°K, from Ref. 26.

Calculations were made for the conditions of the tests in order to check the validity of the numerical results. These results are shown in Figure 15. The abscissa is the entering Reynolds number, and the ordinate is the nondimensional pressure-drop number $N_{Dp} = \Delta p_0 D^2 / \mu_0^2$. Points indicated by squares were obtained with Test Section 3 at an inlet pressure of 16.4 psia, and the points indicated by triangles were obtained with Test Section 1 at an inlet pressure of 26.0 psia. The computed curves are very flat near the minimum, but they do not exhibit the double minima of Figure 13, and, hence, the loci of the computed minima shown in Figure 14 are continuous. The explanation of the double minima may be in the deviations from the empirical correlation or it may be simply a result of applying imprecise graphical techniques to slowly varying functions. The agreement between computed and experimental critical temperature ratios is fairly good above the discontinuity.

A model of a typical reactor core was developed using the dimensions and specifications of the nominal core channel of the LASL Kiwi-301 reactor experiment. The channel was 53.5-inches long with a heated length of 52 inches, the diameter of the channel is 0.096 inch, and the entering flow is restricted by a metering sleeve 0.038-inch in diameter and 0.375-inch long. The normalized heat generation curve shown in Figure 16 was used.

Calculations for this system were performed using the same entrance temperatures, pressures, and flow rate, and total heating rates as were used in the calculations for the experimental system, but with para-hydrogen properties. The results of these calculations are shown in Figure 17.

Constant heat rate curves in this figure have more clearly defined minima and a steeper slope at high Reynolds numbers. The minima occur at lower Reynolds numbers, and, hence, the range of stable exit-to-inlet temperature ratios is greater. It was shown in Ref. 19 that uniform diameter channels with heat flux variations similar to those in Figure 16 have smaller stable temperature ratios than equivalent channels with uniform heating. The length-to-diameter ratio of the core channel is 25 percent greater than that of the experimental system, which also tends to diminish the stability limit. Any increase in stability limit of the core channel over the experimental channel can therefore be attributed to the impedance of the metering sleeve at the core channel entrance. The relative importance of the metering sleeves in stabilizing the flow distribution depends upon the entrance conditions. The entrance conditions used in the present calculation are typical of conditions expected during a cooldown but do not include the constraint imposed upon the flow by a nozzle. They were chosen primarily to facilitate comparisons with the experimental system.

The loci of points of neutral stability for the core channel are also shown in Figure 14. From this figure it may be seen that the critical temperature ratios are appreciably greater for the core channel. For example, at an entrance Reynolds number of 2350, a core channel has a ϕ_c of 10, whereas ϕ_c for an unimpeded, uniformly heated channel is about 7.

CONCLUSIONS

1. The local Nusselt numbers and friction factors in the turbulent to laminar transition region can be represented reasonably well by simple linear interpolation between the turbulent and laminar correlations provided that the limits of the region are properly defined.

2. The conditions for transition are sensitive to the heat flux. The modified Reynolds number $N_{Re}(T_b/T_w)^2$ was found to be useful in describing the limits of the transition region.
3. The critical exit-to-inlet temperature ratios, below which the parallel channel flow distribution is stable, are significantly higher than predicted from equations for fully developed flow for channels with length-to-diameter ratios less than 500.
4. Flow restrictions at the entrance of the channel, such as the metering sleeves used in reactor core channels, increase the critical exit-to-inlet temperature ratio significantly.

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FIG 1. PHYSICAL INTERPRETATION OF FLOW INSTABILITIES
FOR CONSTANT PRESSURE-DROP BOUNDARY CONDITIONS

FIG. 2 GENERAL ARRANGEMENT OF EXPERIMENTAL APPARATUS

**FIG. 3 SCHEMATIC DRAWING OF TEST SECTION
INSTALLED IN VACUUM JACKET**

Figure 4. Local Nusselt number versus bulk Reynolds number.
All data for hydrogen tests in test section 3 are shown.

Figure 5. Local friction factors for heated flow tests. Solid lines represent Hagen-Poiseuille and Blasius equations $\square$ - $x/D = 10.5$, * - $x/D = 21$, + - $x/D = 42$, \$ - $x/D = 84$, I - $x/D = 126$, X - $x/D = 168$ to 411.

Figure 6. Local Colburn J factors. Symbols have the same meaning as in Figure 5.

Figure 7. Local Colburn J factors for a gas exit temperature of 1200° . Symbols have the same meaning as in Figure 5.

Figure 8. Local Colburn J factors at $x/d = 126$ for various gas exit temperatures. - $T_e = 530^\circ\text{R}$, * - $T_e = 600^\circ\text{R}$, X - $T_e = 800^\circ\text{R}$, ϕ - $T_e = 900^\circ\text{R}$, I - $T_e = 1000^\circ\text{R}$, H - $T_e = 1100^\circ\text{R}$, O - $T_e = 1200^\circ\text{R}$.

Figure 9. Local Nusselt numbers for flow with local Reynolds numbers less than 2350. Symbols have the same meaning as in Figure 5.

Figure 10. Effect of heating on local Nusselt number for laminar flow. Local Reynolds numbers less than 2350. Symbols have the same meaning as in Figure 5.

Figure 11. Correlation of Colburn J factors and friction factors for exit temperature of 530°R using boundaries of transition region given in Table 1.

Figure 12. Correlation of Colburn J factors and friction factors for exit temperature of 1200°R using boundaries of transition region given in Table 1.

FIG. 13 DETERMINATION OF STABILITY LIMITS FROM PRESSURE-DROP MEASUREMENTS

FIG.14 EXPERIMENTAL AND CALCULATED STABILITY LIMITS

FIG.15 COMPARISON OF CALCULATED AND MEASURED
PRESSURE DROP FOR CONSTANT HEAT FLUX

FIG. 16 NORMALIZED FISSION HEATING DISTRIBUTION FOR 30I CORE

FIG.17 CALCULATED PRESSURE-DROP FOR CORE CHANNEL