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METHODS FOR MEASURING AND CHARACTERIZING
TRANSISTOR AND DIODE LARGE SIGNAL PARAMETERS
FOR USE IN AUTOMATIC CIRCUIT ANALYSIS PROGRAMS

APPROVED FOR PUBLIC RELEASE

by

W. H. Sullivan

and

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JULY 1965

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METHODS FOR MEASURING AND CHARACTERIZING TRANSISTOR AND DIODE LARGE SIGNAL PARAMETERS FOR USE IN AUTOMATIC CIRCUIT ANALYSIS PROGRAMS*

Summary

The mathematical bases of the Ebers-Moll, Charge-Control and Linvill models are discussed in order to establish the parameter requirements and the accuracy of these models. A recovery technique for measuring those parameters associated with minority carrier storage in devices is described and typical values are given for several high frequency devices. Other parameters, such as current gain and junction depletion capacitance, do not require special measurement procedures and in these cases the characterization of the bias dependence of the parameter is given primary emphasis. The use of the recovery technique in characterizing permanent radiation (neutron) damage and in studying damage mechanisms in transistors is also discussed.

Introduction

Although the analysis and design of electronic circuits has been greatly simplified by computer programs,^{1,2,3} the choice of component models and the measurement of model parameters must still be made by the engineer. Therefore, the efficient use of these programs requires that the engineer be aware of the approximations inherent in the various device models as well as the techniques for determining the model parameters from laboratory measurements.

This paper considers the modeling problem in three steps. First, the mathematical bases of the Ebers-Moll,⁴ Charge-Control,⁵ and Linvill⁶ models are presented in order to establish the parameter requirements and inherent approximations of these models. Next, a recovery technique for measuring storage parameters, i.e., those parameters associated with the storage of minority carriers in the various regions of a device is presented and storage parameter values are reported for several devices. Other parameters, e.g., current gain and junction depletion capacitance, can be easily measured with standard laboratory equipment and consequently the measurement of these parameters is not discussed in detail. However, the functional rela-

tions between these parameters and the device operating point is important, and this subject is discussed in Appendix I. Finally, the use of storage parameter measurements in the investigation of neutron damage in devices and materials is discussed.

Carrier Storage Models

At high current levels the transient response of diodes and transistors is determined primarily by the effects of minority carrier storage in the various regions of the device. Although these effects can be modeled quite accurately by the continuity and diffusion equations, these equations are not particularly useful in circuit analysis work because of the difficulty in obtaining numerical solutions for arbitrary boundary conditions. Consequently, different approaches for approximating these equations have evolved and form the nucleus for the various models.

Ebers-Moll and Charge-Control Models

The Ebers-Moll and Charge-Control models assume that the device has a one-dimensional geometry and the spatial distribution of minority carriers has a fixed form which is independent of time, voltage, current, etc. Under these assumptions the total minority carrier charge stored in any region is linearly related to the carrier densities at the boundaries of that region. Therefore the excess charge, Q_{AB} , stored in the region between surfaces A and B by minority carrier Z is given by

$$Q_{AB} = K_1 (Z_A - Z_0) + K_2 (Z_B - Z_0) \quad (1)$$

where Z_A is the carrier density at surface A, Z_B is the density at surface B, Z_0 is the equilibrium density, and K_1 and K_2 are proportionality constants. The time variation of Q_{AB} can be determined by integrating the continuity equation

$$\frac{\partial Z}{\partial t} = - (\nabla \cdot J_Z) / q - \frac{Z - Z_0}{\tau}$$

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over the volume bounded by surfaces A and B to obtain

$$\begin{aligned} \frac{dQ_{AB}}{dt} &= \int_{AB} q \frac{\partial Z}{\partial t} dv = - \int_{AB} (\nabla \cdot J_Z) dv - \\ &\quad \int_{AB} \frac{q(Z - Z_0)}{\tau} dv \\ &= I_A + I_R - \frac{Q_{AB}}{\tau} \end{aligned} \quad (2)$$

where I_A is the current entering through surface A, I_B is the current entering through surface B, and τ is the minority carrier lifetime.*

If the region under consideration is the base region of a normally biased transistor, the second term in (1) is essentially zero and $Z_A = Z_{bo} \exp(\theta_e V_{be})$. Therefore, (1) and (2) reduce to

$$\begin{aligned} Q_b &= K_1 Z_{bo} [\exp(\theta_e V_{be}) - 1] \equiv K_1' I_e \\ &= \tau_b (1 - \alpha_N) I_e \end{aligned} \quad (3)$$

$$\frac{dQ_b}{dt} = - \frac{Q_b}{\tau_b} + I_e - I_c = - \frac{Q_b}{\tau_b} + I_b \quad (4)$$

where the value of the constant K_1' may be determined from the base lifetime (τ_b) and the normal common base current gain (α_N), as indicated by Beaufoy and Sparks.⁵ Equation (3) relates total base charge (Q_b) to emitter current (I_e) and equation (4) relates the time rate of change of base charge (dQ_b/dt) to the algebraic sum of the external base current (I_b) and the base recombination current (Q_b/τ_b). These two equations comprise the basic charge control description of carrier storage in the base region of a transistor under normal active region bias conditions. It should be noted that this model uses excess base minority charge as a variable and uses base region lifetime as a parameter. In a complete charge control model, which also includes carrier storage in the collector region, the effective lifetime of the collector region would also be used as a parameter.

*See the Glossary of Terms for the definition of symbols which are not defined in the text.

The Ebers-Moll model describes minority carrier storage in terms of a capacitance effect. The correlation between this and the former model can be derived from (4) by using the relation between Q_b and V_{be} given in (3). Thus, by the chain rule of differentiation,

$$\begin{aligned} \frac{dQ_b}{dt} &= \frac{dQ_b}{dV_{be}} \frac{dV_{be}}{dt} \\ &= K_1 Z_{bo} \theta_e \exp(\theta_e V_{be}) \frac{dV_{be}}{dt} \end{aligned}$$

and

$$\begin{aligned} K_1 Z_{bo} \theta_e \exp(\theta_e V_{be}) \frac{dV_{be}}{dt} &= I_b - \\ &= (1 - \alpha_N) I_e \end{aligned} \quad (5)$$

Therefore, the storage of minority carriers in a transistor base region can be modeled by a junction diffusion capacitance whose value is given by $K_1 Z_{bo} \theta_e \exp(\theta_e V_{be})$. This model includes a parameter, K_1 , which relates the magnitude of diffusion capacitance to the injected emitter current or, in the case of carrier storage in the collector region, collector current. This capacitance is almost linearly related to current and generally dominates the normal junction depletion capacitances at high current levels.

Linville Model

Linville⁶ improved the accuracy of device models by using a lumped parameter network to obtain approximate solutions for the continuity and diffusion equations. This procedure is equivalent to approximating the spatial derivatives in the one-dimensional continuity and diffusion equations by the difference between the minority carrier densities at discrete points in the device regions and results in a set of simultaneous first order differential equations of the form illustrated below.

$$\begin{aligned} \frac{dZ_K}{dt} &= D_Z (Z_{K-1} - 2Z_K + Z_{K+1}) / \Delta x^2 - \frac{Z_K}{\tau_Z} \\ 1 &< K < N + 1 \\ I_K &= qAD_Z (Z_{K+1}) / \Delta x \end{aligned} \quad (6)$$

Here N denotes the number of spatial subdivisions, Δx is the subdivision length, and subscript K denotes the value of the variable at $x = x_K$. In

addition to the lifetime parameter introduced in the previous models, this model is also strongly dependent upon diffusivity, D_Z , as well as the geometrical parameter Δx . The Linvill model therefore requires considerably more knowledge about the device structure than the previous models. However, this added detail does provide for a more accurate model.

Although the above derivations are necessarily brief, they are included here to introduce the parameters contained in the various models and to review the approximations which are implicit in these models. It should be apparent that, although the mathematical forms retained in the Charge-Control and Ebers-Moll models are different, these models result from a common mathematical basis and consequently parameter values for one model can be converted to the parameters of the other. This paper, therefore, deals exclusively with the Ebers-Moll and Linvill models. In addition, the model derivations presented above deal exclusively with the carrier storage problem. A complete device model must also include junction depletion capacitance, bulk resistance, etc. These additions are shown schematically in Figure 1, where the Ebers-Moll model of a diode and transistor are shown. The reader is referred to reference 6 for added detail on the Linvill model.

Storage-Parameter-Measurements

Storage parameters can be evaluated by either small signal reactance and transfer function measurements or by transient response methods. Although the small signal methods are well known, and generally provide greater precision than transient tests, they suffer from one major shortcoming. If the measurements are to adequately describe the devices' large signal behavior, they must be made at reasonably high steady-state power levels. Consequently, junction temperature varies considerably during the measurement sequence and may obscure the relationship between the measured parameter and the bias level. Transient tests have the general advantage that low duty cycle pulses can be used to eliminate the junction heating problem. In addition, relaxation or turnoff measurements can be made with zero external current, thereby eliminating the effects of series bulk resistance and permitting the direct measurement of junction potential.

$$I = I_0 [\exp(\theta V) - 1]$$

$$\theta = \frac{q}{KT}$$

$$C_T = C_s + C_0 / (V_0 - V)^m + K_{dif} I_0 \exp(\theta V)$$

Figure 1a. Diode Equivalent Circuit.

$$I_c = \left\{ \alpha_N I_{e0} [\exp(\theta_e V_e) - 1] - I_{c0} [\exp(\theta_c V_c) - 1] \right\} / (1 - \alpha_N \alpha_I)$$

$$I_e = \left\{ -I_{e0} [\exp(\theta_e V_e) - 1] + \alpha_I I_{c0} [\exp(\theta_c V_c) - 1] \right\} / (1 - \alpha_N \alpha_I)$$

$$\theta_e = \frac{q}{N_e KT}$$

$$\theta_c = \frac{q}{N_c KT}$$

$$CC_T = C_{sc} + \frac{C_{oc}}{(V_{oc} - V_{cb})^{m_c}} + K_{Cdif} I_{c0} \exp(\theta_c V_{cb})$$

$$CE_T = C_{se} + \frac{C_{oe}}{(V_{oe} - V_{eb})^{m_e}} + K_{Edif} I_{e0} \exp(\theta_e V_{eb})$$

Figure 1b. Ebers-Moll Equivalent Circuit for an NPN Transistor

Diode Storage Parameter Measurements

The use of transient tests in the evaluation of model parameters is illustrated by the simple diode recovery technique shown schematically in Figure 2. Switch S_1 is initially closed and a quiescent current, I_{SS} , is established through the diode. Immediately after S_1 opens, the diode voltage decreases abruptly from its quiescent voltage to the actual junction voltage as shown by the two waveforms in Figure 3a. This voltage drop is simply $R_S I_{SS}$ and thus provides a direct measure of R_S , the series diode resistance. The subsequent decay of the junction voltage is primarily dependent upon minority carrier storage and, to a lesser extent, the junction depletion capacitance. Parameters related to these terms can therefore be extracted from this waveform.

Figure 2. Diode Recovery Test Circuit

The capacitance parameters for the diode model (shown in Figure 1a) are obtained in two steps. First, the depletion capacitance of the reverse biased junction can be measured directly using small signal techniques as discussed in Appendix I. These measurements typically produce curves of the form shown in Figure 3c, where the depletion capacitance of a 2N1989 collector-base junction is shown as a function of collector-base voltage. Next, the junction capacitance is measured under forward bias conditions by observing that, for this model, the junction voltage during turnoff is described by

$$(C_{dep} + C_{dif}) \frac{dv}{dt} = I_o [\exp(\theta v) - 1] \equiv I(v) \quad (7)$$

or

$$(C_{dep} + C_{dif}) = I(v) / \frac{dv}{dt} \quad (8)$$

where $I(v)$ is the ideal diode current. Thus, the functional dependence between the total junction capacitance and the junction voltage can be computed from the ratio of the ideal current and the observed slope of the recovery waveform for each value of junction voltage.

The capacitance calculations indicated in (8) are illustrated in Figures 3a and 3b, where recovery waveforms and computed capacitance curves are shown for the collector-base diode of a 2N1989 transistor. In the low voltage portion of Figure 3b, both capacitance curves are coincident and have the same general shape as the theoretical total capacitance function shown by the dotted curve. The theoretical capacitance may be expressed mathematically as the sum of the theoretical depletion* and diffusion capacitance terms as shown in equation (9).

$$C_T = (C_{dep} + C_{dif}) = \left[C_s + C_o / (V_o - V)^m \right] + K_{dif} I_o \exp(\theta v) \quad (9)$$

Figure 3a. Diode Recovery Waveforms

The data of Figure 3b, as well as the reverse bias depletion capacitance data shown in Figure 3c, may be used to obtain a least-squares evaluation of the parameters C_s , C_o , V_o , m and k_{dif} in equation (9).

The observed departure of the capacitance calculations from the form indicated by (9) in the

*See Appendix I.

high voltage portion of Figure 3b is a direct consequence of the approximations inherent in a first order model. The calculated capacitance values shown in this region result from data obtained during the early portion of the turnoff waveform. In this time period, the carrier density distribution in the diode is undergoing rapid change and, as a result, a first order model is not sufficient to describe the experimental data. In other words, a first order model describes the low and mid-frequency behavior of a diode quite accurately. It should not be used to cover the entire frequency spectrum, however.

Figure 3b. Junction Capacitance Versus Voltage

Better correlation between device and model response can be achieved by using higher order models which provide a better approximation to the continuity and diffusion equations. However, these models require material parameters, such as lifetime, which are not given on specification sheets and which must therefore be inferred from electrical measurements.

The theoretical recovery of an ideal diode, i.e., a diode having a one-dimensional geometry, zero conductivity modulation, and approximately unity emitter efficiency (conduction is predominately by base minority carriers), is given by

$$V(t) = V_s \quad (t < t_o)$$

$$V(t) = V_s - R_s I_{ss} + \frac{1}{\theta} \ln \left[\operatorname{erfc} \left(\sqrt{t/\tau_b} \right) \right]$$

$$(t_o < t, V(t) > 0) \quad (10)$$

Figure 3c. Collector Junction Depletion Capacitance Versus Junction Voltage

where τ_b is the base lifetime and erfc is the complementary error function. The lifetime and series resistance of a diode can be estimated by fitting (10) to the measured waveform as illustrated in Figure 4. Although the lifetime values thus obtained are subject to the conditions indicated above and therefore may not be indicative of the actual material constant, this value does describe the overt device performance and is meaningful from a model standpoint.

The increased accuracy afforded by the higher order Linvill models is shown in Figure 5, where an experimental recovery waveform is compared to waveforms predicted by first, second, third, and fifth order models. As illustrated in this figure, the higher order models give a considerable improvement in the predictions, notably in the region immediately following turnoff. This improved high frequency performance is costly because of the added computation required in the more complex models and because the higher order models necessarily introduce short time constants which generally increase the required computer solution time. Therefore, higher order models should only be used when increased high frequency accuracy is necessary to adequately describe the performance of the device and the circuit of which it is a part.

Transistor Storage Parameter Measurement

The emitter diffusion capacitance or base storance of transistors can be determined through a transient test which is very similar to the previously described diode technique. A schematic

representation of the test procedure for transistors is shown in Figure 6a. Initially, switch S_1 is closed and the transistor is biased into the active region. At time t_0 , S_1 opens and the base voltage and collector current decay with typical waveforms as shown in Figure 6b. The slope and magnitude of these waveforms may be used to determine the capacitance or storance parameter values in a manner analogous to the diode case.

Figure 4. Theoretical and Experimental Recovery Waveforms

Figure 5. Computed Recovery Waveforms for Models of Various Order

The total emitter capacitance (CE_T) for the Ebers-Moll model can be computed from the observed transient waveforms and the equation describing the transient response of the model:

$$CE_T = (CE_{dif} + CE_{dep})$$

$$= - \frac{\frac{I_c}{h_{FE}(V_{be}, V_{bc})} - CC_{dep} \left(\frac{dV_{ce}}{dt} - \frac{dV_{be}}{dt} \right)}{\frac{dV_{be}}{dt}} \quad (11)$$

In (11) CE_{dif} denotes emitter diffusion capacitance; CE_{dep} and CC_{dep} are the depletion capacitances of the emitter and collector junctions, respectively; dV_{ce}/dt and dV_{be}/dt are the time derivatives of collector-to-emitter and base-to-emitter voltages; $h_{FE}(V_{bc}, V_{bo})$ is the common emitter current gain (defined in Appendix I) and I_c is total collector current.

Figure 6a. Transistor Test Circuit

Figure 6b. Typical Transistor Recovery Waveforms

As in the diode case, two capacitance curves computed from (11) at two different

initial bias levels will agree for small V_{be} but will not agree for the larger values which are derived from data taken immediately after turn-off. Here, however, the initial transient is not due to an inadequate approximation to the continuity and diffusion equations but rather is thought to be due to a transient change in the injection level over the emitter surface. This comes about because under quiescent conditions base spreading resistance produces a lateral gradient in base-emitter voltage and consequently a gradient in the emitter current density. After the base current terminates, the injection density returns to a uniform distribution. During this transient redistribution, the base voltage decreases more rapidly than predicted because less base voltage is required to maintain a given collector current with a uniform injection density than is required for a nonuniform distribution. All transient base voltage measurements should therefore be made after the initial redistribution transient is complete.

In some devices, it is difficult to determine when the redistribution transient is complete and the base voltage data is meaningful. Diffusion capacitance measurements can be made on these devices by computing an equivalent base-emitter voltage from the measured collector current as indicated in (12) and (13).

$$V_{be} = \frac{1}{\theta} \ln \left[\frac{I_c}{I_{eo} \alpha_N} + 1 \right] \quad (12)$$

$$\frac{dV_{be}}{dt} = \frac{dV_{be}}{dI_c} \frac{dI_c}{dt} = \frac{1}{\theta \alpha_N} \frac{dI_c}{dt} \quad (13)$$

Here it has been assumed that the time variation of the emitter junction voltage is small during one base transit time. Equations (12) and (13) and the previously described recovery measurement techniques were used to measure the total emitter capacitance of four types of transistors. These data were then fitted to an equation of the form shown in (9). The coefficients thus obtained are tabulated in Table I for each of the devices. The devices' alpha cutoff frequencies have also been tabulated to illustrate the approximate inverse relationship between this parameter and the diffusion capacitance coefficient.

Some of the devices tested showed an additional diffusion capacitance characteristic which has not been reported previously. At high current levels, several 2N918 transistors displayed a supra-linear dependence of diffusion capacitance upon collector current. As the current is

increased, the measured capacitance curve departs from the linear relation at approximately the same current level (4 ma.) at which the common emitter current gain reaches a maximum and begins to decrease. Both of these effects are believed to be of a common origin; a shift of the effective collector junction into the collector region. This effect (the Kirk Effect¹⁰) produces an increase in the effective base width and consequently an increase in the diffusion capacitance.

TABLE I
Total Emitter Capacitance Coefficients

Transistor Type		2N917	2N708	2N1051	2N1989
C_s	- Stray Header Capacitance (pf)	0.4	0.4	0.3	0.3
C_o	- Depletion Capacitance Coefficient (pf)	1.0	6.3	8.1	5.8
V_o	- Built-in Diode Potential (volts)	0.8	0.8	0.8	0.8
m	- Junction Grading Constant	0.390	0.381	0.400	0.371
KE_{dif}	- Emitter Diffusion Capacitance Coefficient (pf/ma)	5.2	11.0	40.0	90.0
$f_{\alpha e}$	- Alpha Cutoff Frequency (mc)	800	480	120	60

Charge storage occurs predominately in the base region when a transistor is biased into the active region. However, under saturated conditions (collector junction forward biased) appreciable charge storage may also occur in the collector region and thus give rise to a collector diffusion capacitance. This capacitance can be evaluated in either of the following ways. First, the transistor can be connected in an inverted mode, i.e., with emitter and collector leads interchanged, and the parameters can be measured in the manner previously illustrated. Here it is implied that the appropriate inverted parameters are used in (11). Second, the base-collector junction can be considered as a diode and the parameters can be computed in the manner indicated in the diode section. The latter approach takes advantage of the fact that the collector-base doping ratio of most diffused transistors is such that almost all carrier storage takes place in the collector region and, consequently, emitter bias has little effect upon the measurement.

The transistor measurements presented up to this point have dealt entirely with diffusion capacitance measurements, i.e., the measurement of the storage parameter of a first order model. First order models provide a good description of device behavior so long as the frequency components of base voltage are small compared to the base transit time cutoff frequency. As the input frequencies approach this limit, the inadequacies of the first order model are reflected in reduced computational accuracy

and; therefore, one is forced to resort to higher order models.

High order Linvill models of the base region require values for the base width and base lifetime. These values can be estimated as follows. Under quiescent or low frequency conditions, the minority carrier distribution in the base is nearly linear. Consequently, the base charge and collector or emitter current are approximately given by

$$Q_b = \frac{qW_b AZ_e}{2} \quad (14)$$

and

$$I_e = \frac{qAD_Z Z_e}{W_b} \quad (15)$$

Solving (15) for Z_e and substituting this result and the relationship between I_e and V_{be} into (14) gives

$$Q_b = \frac{W_b^2}{2D_Z} I_e = \frac{W_b^2}{2D_Z} I_{e0} [\exp(\theta_e V_{be}) - 1]$$

or

$$\begin{aligned} CE_{dif} &= \frac{dQ_b}{dV_{be}} = \frac{W_b^2}{2D_Z} I_{e0} \theta_e \exp(\theta_e V_{be}) \\ &= \frac{W_b^2}{2D_Z} \theta_e I_e = KE_{dif} I_e \end{aligned} \quad (16)$$

Therefore,

$$W_b = \sqrt{\frac{2D_Z KE_{dif}}{\theta_e}}$$

The effective base width can thus be computed directly from θ_e , D_Z and KE_{dif} .

The effective base lifetime can be estimated from the DC gain of the device and the base width by using the relation

$$h_{FE} = \frac{2D_Z \tau_b}{W_b^2}$$

or

$$\tau_b = \frac{KE_{dif}}{\theta_e} h_{FE} \quad (17)$$

It should be emphasized that the lifetime determined by (17) may not be the actual material lifetime, since the gain of some devices is controlled by emitter efficiency or collector multiplication, terms which have been neglected in (17). However, the incorporation of these effects into the lifetime term has little effect on the high frequency accuracy of the model and at the same time retains the proper bias dependence of DC current gain.

Permanent Damage Models

When semiconductor materials are exposed to energetic radiation, some of the atoms of the material are permanently displaced. These displacements produce changes in measurable device parameters such as gain, junction capacitance, series resistance, etc. Permanent damage in diodes and transistors can therefore be modeled by predicting or measuring the functional relations between the model parameters and the radiation flux.

Typical neutron induced changes of the Ebers-Moll model parameters are illustrated in Figure 7. In this figure, the average normal and inverted current gains (h_{FEN} , h_{FEI}), emitter and collector diode coefficients (I_{CO} , I_{EO} , θ_C , θ_E), and the emitter and collector diffusion capacitance coefficients of several 2N1989 transistors are shown as a function of neutron flux. Data of the type shown in Figures 7a and 7b have been well documented and, in fact, form the basis for much of the research concerning permanent damage mechanisms in devices. However, capacitance data, such as that shown in Figure 7c, have not been published previously. These data not only

furnish parameter values for use in circuit analysis work but also provide another tool for investigating damage mechanisms.

Figure 7a. Normal and Inverted h_{FE} Versus Neutron Flux

Figure 7b. Junction Leakage Currents and Boltzmann Coefficients Versus Neutron Flux

Collector diffusion capacitance is the result of minority carrier storage occurring primarily in the collector region. In non epitaxial devices, the ratio of this capacitance to collector current is proportional to the square of the collector diffusion length or, in other words, proportional to collector lifetime. The dependence of the collector diffusion capacitance coefficient upon neutron flux, shown in Figure 7c, is therefore a direct

reflection of the neutron induced lifetime degradation in the lightly doped collector region.

Figure 7c. Normalized Collector and Emitter Capacitance Coefficient Versus Neutron Flux

As a further indication of permanent damage mechanisms which might be investigated using data of the type shown in Figure 7, a comparison of Figures 7a and 7c indicates that the neutron damage threshold for inverted gain is approximately a factor of five larger than the collector lifetime damage threshold. These data indicate that inverted gain is not a strong function of collector lifetime, as would be expected if it were predominantly controlled by the emitting efficiency of the collector-base junction, but rather that it is controlled by either the base transport factor or possibly recombination in the space charge region. Figure 7b shows that the collector Boltzmann coefficient is also reduced at higher flux levels, thus indicating that a significant part of the gain reduction may be due to recombination in the depletion region. Although this data is not sufficient to warrant firm conclusions regarding the relative significance of emitter efficiency, base transport factor and space charge recombination in permanent damage effects, it does indicate that data of this type is potentially useful in future investigations of permanent damage mechanisms.

Conclusion

This paper discusses the recovery method of measuring storage parameters in diodes and extends to the measurement of emitter and collector diffusion capacitance in transistors. This

method has been used to characterize storage effects in devices having alpha cutoff frequencies in the range from 50 to 1000 megacycles. The relationship between these parameters and the distributed parameters required by the higher order Linvill models is also considered.

Although emitter diffusion capacitance generally has a strong influence on intrinsic transistor response, many circuit configurations arise where stray wiring capacitance and collector-base depletion capacitance are of equal or greater importance. For example, the Miller capacitance (the collector-base depletion capacitance amplified by the voltage gain of the circuit) of a 2N917 transistor operated in the common emitter mode ($I_C = 1.0$ ma, $V_C = 5$ volts, $R_C = 1000$ ohms) is approximately 40 picofarads. This capacitance is large compared to the emitter diffusion capacitance (≈ 5.0 picofarads at $I_E = 1.0$ ma) and therefore plays a dominate role in determining circuit response. Miller capacitance is also increased by stray wiring capacitance between the collector and base terminals. In the example cited above, as little as 3 picofarads of added wiring capacitance would produce a threefold increase in the Miller capacitance and a corresponding reduction in high frequency performance of the circuit. It should therefore be clear that although the measurement of the detailed bias dependence of model parameters is necessary for the efficient use of computer analysis techniques, additional parameters, such as stray wiring capacitance, are occasionally of equal or greater importance in achieving prediction accuracy and must therefore be taken into account in modeling the complete circuit.

In addition to providing parameter data for circuit analysis work, the measurement of storage parameters in devices also provides a convenient tool for investigating the effects of neutron irradiation on some material parameters and for determining damage mechanisms in devices. The neutron degradation of minority carrier lifetime, for example, can be investigated by making diffusion capacitance measurements on a diode structure. The results of such measurements are illustrated in Figure 7c where the collector diffusion capacitance coefficient of a 2N1989 transistor is plotted versus neutron flux. The presence of such mechanisms as Kirk Effect and the importance of these effects at various neutron exposures can also be investigated through the current dependence of diffusion capacitance.

Although the transient effects of ionizing radiation have not been treated explicitly in this paper, two of the more important parameters required to predict these effects can be determined from the previously described measurements. First, the effective collector lifetime can be obtained by fitting equation (10) to the collector-base diode recovery waveform. Second, the effective base width can be determined from the coefficient relating diffusion capacitance to emitter current, as shown in equation (16).

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APPENDIX I

Common Parameters

Transistor current gain, the DC characteristics of the emitter-base and collector-base junctions and junction depletion capacitance are common to both the Ebers-Moll and Charge-Control models and therefore these parameters will be considered in this appendix without particular reference to a specific device model. The functional relationship between these parameters and device bias levels is described in this appendix. Generally the form of these functional dependencies has been predicted from device theory; however, at extreme bias levels most devices depart from the theoretically predicted behavior. Therefore, a group of empirical formulas are presented which have been found to adequately describe device behavior over large bias ranges and which are useful in modeling transistors and diodes in computer circuit analysis programs.

Junction Characteristics

The DC characteristics of a junction can be represented by

$$I(V) = I_0 \left(\exp\left(\frac{\theta V}{r}\right) - 1 \right) \left(\frac{1}{1 - K\left(\frac{V}{V_B}\right)^n} \right) \quad (I. 1)$$

where I_0 is the ideal leakage current, θ is the Boltzmann term, r is a parameter which modifies the Boltzmann term to include space charge recombination, V_B is the junction breakdown voltage and n and K are empirical constants which describe avalanche multiplication near breakdown. Since the last term in (I. 1) is small for forward bias conditions, I_0 and θ/r may be evaluated from a graph of $\ln(I)$ versus V for positive V . Similarly, constants K and n can be determined from a plot of $\ln(I)$ versus $\ln(|V/V_B|)$ for V approaching $-V_B$.

Equation (I. 1) ignores several effects which can be significant in some situations. First, it is assumed that series bulk resistance is modeled by lumped parameter components appearing elsewhere in the model. Conductivity modulation at high currents must also be taken into account in these lumped components. Second,

the breakdown behavior described in (I. 1) only describes an avalanche type breakdown. Surface breakdown effects are not included in this equation.

Transistor Current Gain Expressions

The large signal current gain of a normally biased transistor is usually stated in terms of the common base (α_N) or common emitter (h_{FEN}) transfer ratios. These parameters (as well as their inverted counterparts α_I and h_{FEI}) are dependent upon bias and consequently the analytical form of this dependence should be included in the device model. Since the inverted parameters have little effect upon model accuracy, they can generally be assumed to be independent of bias and therefore they will not be considered in the following discussion. Furthermore, only the form of h_{FEN} is considered in detail since α_N can be easily computed from h_{FEN} data.

For most diffused transistors operated at low current levels, the current dependence of the current gain (h_{FEN}) is controlled by recombination in the emitter space charge region. Under these conditions, the gain can be approximated by

$$h_{FEN}(V_{be}) = h_{FEO} \exp(\theta_h V_{be}) \quad (I. 2)$$

where θ_h is normally less than q/kT . At intermediate current levels, either recombination in the base region or emitter efficiency may limit the continuing increase of h_{FEN} with current that is predicted by (I. 3). This additional effect can be modeled by modifying (I. 2) by the constant term h_{FE1} , as shown in (I. 3).

$$h_{FEN}(V_{be}) = \frac{h_{FE1} h_{FEO} \exp(\theta_h V_{be})}{h_{FE1} + h_{FEO} \exp(\theta_h V_{be})} \quad (I. 3)$$

At very high current levels, two additional effects tend to decrease the gain. First, if the injection level is sufficiently high, conductivity modulation occurs in the base region and causes a decrease in emitter efficiency and base transport factor. This effect is compounded by the base current generated transverse emitter voltage gradient which increases the injection level at the emitter periphery. Second, the high carrier densities in the base cause the apparent collector junction to shift into the collector region thereby increasing the effective base width and decreasing the

current gain. Relating these last two effects to device structure is very complicated and cannot generally be done with great accuracy. The following modification of (I. 3), which attempts to describe these effects, is entirely empirical and the parameters of this equation are subject to large unit-to-unit variations.

$$h_{FEN}(V_{be}) = \frac{h_{FE1} h_{FEO} \exp(\theta_h V_{be} - \theta_{hl} V_{be}^S)}{h_{FE1} + h_{FEO} \exp(\theta_h V_{be} - \theta_{hl} V_{be}^S)} \quad (I. 4)$$

The variations of h_{FEN} with collector-base voltage is primarily caused by the Early Effect⁸ and avalanche multiplication in the collector depletion region. Although these mechanisms are conceptually different, both can be represented either in terms of a collector multiplication factor, $m_c(V_{bc})$, or in terms of a voltage dependent current gain. This paper considers the former approach and assumes that both effects are described in terms of a leakage current whose magnitude is proportional to $m_c(V_{bc})$ times the collector current. Under this assumption, the current-voltage relations for the emitter and collector junctions may be written as

$$\begin{aligned} I_{cf} &= I_{co} [\exp(\theta_c V_{bc}) - 1] - I_c m_c(V_{bc}) \\ I_{ef} &= I_{eo} [\exp(\theta_e V_{be}) - 1] - I_e m_e(V_{be}) \end{aligned} \quad (I. 5)$$

Under normal bias conditions ($V_{bc} < 0$ and $V_{be} \approx 0.7$), (I. 5) reduces to

$$\begin{aligned} I_{cf} &\approx -I_c m_c(V_{bc}) \\ I_{ef} &\approx I_e \approx I_{eo} [\exp(\theta_e V_{be}) - 1] \end{aligned}$$

since $m_e(V_{be})$ is very small for small voltages and I_{co} is negligible. Furthermore, since I_{cf} is a leakage current under these bias conditions it will always be oriented in opposition to the normal base current; therefore, the total effective base current will be the difference between the external base current, I_b and I_{cf} . Therefore,

$$I_c = -h_{FEN}(V_{be}) [I_b + I_{cf}] = -h_{FEN}(V_{be}) [I_b - I_c m_c(V_{bc})] \quad (I. 6)$$

or, solving for I_c ,

$$\begin{aligned} I_c &= \frac{-h_{FEN}(V_{be}) I_b}{1 - h_{FEN}(V_{be}) m_c(V_{bc})} \\ &= -h_{FEN}(V_{be}, V_{bc}) I_b \end{aligned} \quad (I. 7)$$

Equation (I. 7) shows that the effective current gain becomes infinite whenever $m_c(V_{bc})$ approaches $1/h_{FEN}(V_{be})$. Therefore, the open base breakdown voltage (BV_{CEO}) is always less than the collector junction breakdown potential (BV_{CBO}) since collector breakdown does not occur until $m_c(V_{bc})$ is unity.

The measurement of $h_{FEN}(V_{be})$ and $m_c(V_{bc})$ is facilitated by (I. 6). By varying V_{bc} and I_b such that I_c is constant, the multiplication factor $m_c(V_{bc})$ can be measured directly. In Figure I. 1, a family of I_b versus V_{cb} curves are shown for several collector current levels, each curve having been normalized by dividing by the appropriate collector current. The function $m_c(V_{bc})$ is determined by subtracting any of the curves from the extrapolated ordinate intercept for that curve. The values for $1/h_{FEN}(V_{be})$ are determined by extrapolating each curve to zero collector volts and using the relation

$$V_{be} \approx \frac{1}{\theta_e} \ln(I_c / \alpha_N I_{eo})$$

to determine the corresponding value of V_{be} . These operations yield the $h_{FEN}(V_{be})$ and $m_c(V_{bc})$ curves shown in Figures I.2a and I.2b.

Figure I. 1. Base-Collector Current Ratio Versus Collector Voltage for Fixed Collector Current

Figure I.2a. Current Gain Versus Base-Emitter Voltage.

Figure I.2b. Collector Multiplication Ratio Versus Collector Voltage

Transistor DC Characteristics

The DC characteristics of a transistor are described by two equations⁴ which relate total emitter and collector currents to junction currents and gain coefficients

$$\begin{bmatrix} I_c \\ I_e \end{bmatrix} = \frac{1}{1 - \alpha_N \alpha_I} \begin{bmatrix} 1 & -\alpha_n \\ -\alpha_I & 1 \end{bmatrix} \begin{bmatrix} I_{cf} \\ I_{ef} \end{bmatrix} \quad (I. 8)$$

where α_I is assumed to be constant,

$$\alpha_N = \frac{h_{FEN}(V_{be})}{1 + h_{FEN}(V_{be})} \quad (I. 9)$$

and I_{ef} and I_{cf} are given by (I. 5). Since I_{ef} and I_{cf} are in turn functions of I_e and I_c , (I. 8) is an implicit function of I_e and I_c which can be solved explicitly to obtain

$$\begin{bmatrix} I_c \\ I_e \end{bmatrix} = \begin{bmatrix} 1 - \alpha_N \alpha_I - m_c(V_{bc}) & m_e(V_{be}) \\ m_c(V_{be}) & 1 - \alpha_N \alpha_I - m_e(V_{be}) \end{bmatrix}^{-1} \begin{bmatrix} 1 & -\alpha_N \\ -\alpha_I & 1 \end{bmatrix} \begin{bmatrix} I'_{cf} \\ I'_{ef} \end{bmatrix} \quad (I. 10)$$

where

$$I'_{cf} = I_{eo} \left[\exp(\theta_e V_{bc}) - 1 \right]$$

$$I'_{ef} = I_{co} \left[\exp(\theta_e V_{be}) - 1 \right].$$

Note that all terms in (I. 10) are expressed as functions of the junction voltages V_{be} and V_{bc} -- this feature is required for efficient numerical solution of the models and circuit transient behavior.

Depletion Capacitance

In addition to the DC parameters discussed above, the three models under consideration also include depletion capacitance, i.e., the capacitance associated with the space charge found in junction transition regions. This capacitance can be measured with a standard capacitance bridge as long as the junction is not conducting an appreciable current. Depletion capacitance cannot be measured in the forward conduction region because of the effects of minority carrier storage (diffusion capacitance) and the previously described resistance and temperature effects.

The relationship between the theoretical depletion capacitance and junction voltage is, in general, very complex and cannot be expressed

in closed form. However, an approximate expression, given in (I. 11), has been derived which is reasonably valid for reverse-biased junctions.

$$C_{\text{dep}}(V) = C_s + C_o / (V_o - V)^m \quad (\text{I. 11})$$

This expression has been found to fit abrupt junctions, e. g., the emitter junction of a diffused transistor, quite well ($\sim \pm 1\%$). However, junctions which have a smaller doping gradient, such as the collector junction of a high voltage transistor, do not usually follow this expression and errors of $\pm 15\%$ are frequently encountered. Furthermore, this expression should be used with caution when extrapolating into the forward bias region because of the pole that exists at $V = V_o$. Certainly V_o should never be less than the maximum voltage obtained in the particular application.

The approximate value for the coefficient C_s of (I. 11) can be obtained by direct measurement on a device which has been electrically open-circuited and the remaining coefficients can be obtained from a plot of $\ln(C_{\text{dep}}(V) - C_s)$ versus $\ln(V_o - V)$. Successive plots are made with different V_o values until a straight line is obtained on log-log graph paper. The appropriate V_o value is that value which produces the best straight line. Once the proper V_o value is obtained, the value of n and C_o is given by the slope and intercept, respectively, of the straight-line log-log plot.

Definition of Symbols

A	Junction area
C_{dep}	diode depletion capacitance
C_{dif}	diode diffusion capacitance
C_T	total diode capacitance
CE_{dif}	emitter junction diffusion capacitance
CE_{dep}	emitter junction depletion capacitance
CC_{dep}	collector junction depletion capacitance
CE_T	total emitter junction capacitance
C_s	stray wiring and device header capacitance
C_o, C_1	proportionality constants
D_Z	diffusion constant for charge carrier z
h_{FEI}	Inverted common emitter current gain

h_{FEO}	common emitter gain coefficient
h_{FEI}	common emitter gain coefficient
$h_{\text{FEN}}(V_{\text{be}})$	common emitter DC current gain for $V_{\text{cb}} = 0$
$h_{\text{FEN}}(V_{\text{be}}, V_{\text{bc}})$	common emitter DC current gain as function of V_{be} and V_{bc}
I_{ef}	total emitter diode current
I_{cf}	total collector diode current
I_{eo}	emitter junction saturation current
I_{co}	collector junction saturation current
I_o	diode saturation current
I_c	total device collector current
I_e	total device emitter current
I_b	total device base current
I_{ss}	steady state diode current
I_Z	current carried by carrier Z
I_A	current entering boundary A
I_B	current entering boundary B
J_Z	current density for charge carrier Z
K, K_1, K'_1, K_2	proportionality constants
K_{dif}	diffusion capacitance proportionality constant
KE_{dif}	emitter diffusion capacitance proportionality constant
KC_{dif}	collector diffusion capacitance proportionality constant
$m_c(V_{\text{bc}})$	collector multiplication factor
m	junction grading constant
n	junction breakdown coefficient
N_e	emitter diode Boltzmann modification term
N_c	collector diode Boltzmann modification term
q	electronic charge
Q_b	total minority carrier charge stored in the base region
Q_{AB}	total minority carrier charge stored between surfaces A and B
r	diode Boltzmann term modification parameter
R_s	bulk series resistance of diode
s	gain coefficient
V_o	diode built-in potential

V_s	steady state diode voltage
V_B	junction breakdown voltage
V_{be}	base-emitter voltage
V_{ce}	collector-emitter voltage
W_b	width of base region
Z_A	minority carrier density for arbitrary carrier Z at surface A
Z_B	minority carrier density for arbitrary carrier Z at surface B
Z_K	minority carrier density for arbitrary carrier Z at surface K
Z_o	equilibrium minority carrier density for arbitrary carrier Z
Z_{bo}	equilibrium minority carrier density for arbitrary carrier Z at base-emitter junction
Z_e	emitter junction injection density
α_N	normal DC common base current gain
α_I	inverted DC common base current gain
θ	Boltzmann factor (q/kT)
θ_e	Boltzmann term for emitter junction
θ_c	Boltzmann term for collector junction
θ_h	modified Boltzmann gain coefficient
θ_{hl}	modified Boltzmann gain coefficient
μ_Z	carrier mobility of carrier Z
τ_b	minority carrier lifetime in the base region
τ	minority carrier lifetime

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