EFFECTS OF CORE EXCITATIONS IN THE VIOLATION OF THE NUCLEAR PARTICLE-PARTICLE TO PARTICLE-HOLE TRANSFORMATION

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Abstract: A core excited shell model for the mass-40 region is developed to account for the discrepancies between particle-particle (pp) and particle-hole (ph) nuclear partners. Explicitly, second order perturbative-type calculations are complete failures when used in an attempt to simultaneously account for the spectral discrepancies between $^{40}\text{K}(f_{7/2}^v - d_{3/2}^\pi)^{-1}$ and the Racah transform of $^{38}\text{Cl}(f_{7/2}^v - d_{3/2}^\pi)$ and those between $^{48}\text{Sc}(f_{7/2}^\pi - f_{7/2}^v)^{-1}$ and the Racah transform of $^{42}\text{Sc}(f_{7/2}^\pi - f_{7/2}^v)$. However, an (fd)-shell model, which includes core excitations

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from the $0d_{3/2}$ -shell to the $0f_{7/2}$ -shell, yields a simultaneous explanation for both sets of spectral discrepancies. The usefulness of the model in the investigation of the positioning of core excited states in ^{40}Ca (like $8p-8h$) and in the generation of effective operators is also explored.

1. Introduction

There are a host of largely unexplained experimental results in the mass-40 region. One class of these results pertains to the states which are believed to be primarily spherical in nature. Some of these discrepancies in the so called spherical states are nearly as old as the nuclear shell model itself.

Pandya¹⁾, Goldstein and Talmi²⁾ showed that the simplest shell model states of a particle-hole nucleus was related to the simplest shell model states of a particle-particle nucleus by an unitary transformation. Explicitly they showed that the energies of the lowest lying $J^\pi = 2^-, 3^-, 4^-$ and 5^- states of ^{40}K ($f_{7/2}^\pi - d_{3/2}^{v-1}$ in the simplest shell model) and the corresponding J^π states in ^{38}Cl ($f_{7/2}^\pi - d_{3/2}^{v-1}$ in the simplest shell model) were closely related by the Racah transformation

$$E_J(^{40}\text{K}) = -\sum_I [I] \begin{Bmatrix} 7 & 3 & I \\ 2 & 2 & J \end{Bmatrix} E_I(^{38}\text{Cl}) \equiv T_J^I(E_I(^{38}\text{Cl})),$$

where the E 's are interaction energies as discussed in Sec. 4. If the simplest shell model was correct and the nuclear interaction were two-body, then this relationship would be exact.

However, neither condition seems to be satisfied and the relationship is not exact. In 1957 Pandya and French³⁾ showed that including configuration mixing only made matters worse.

Pandya and French³⁾ made an effort to account for the residual discrepancies between ^{38}Cl and ^{40}K . They introduced the simplest configuration mixing terms to the $f_{7/2}-d_{3/2}$ multiplets and found that the correction terms went in the wrong direction. This is tantamount to a failure of second order perturbation theory to explain the discrepancies between ^{40}K and ^{38}Cl .

Although second order perturbation theory fails quite badly for ^{38}Cl and ^{40}K , West and Koltun⁴⁾ have shown that it works quite well for ^{42}Sc and ^{48}Sc . ^{42}Sc and ^{48}Sc are particle-particle (pp) and particle-hole (ph) partners in the $f_{7/2}$ -shell for which the residual discrepancies are considerably larger than those between ^{38}Cl and ^{40}K . In this success of second order perturbation theory we see an example of what is a pattern for the results in the mass-40 region. Namely, second order perturbation theory works in one case, but makes matters worse when applied to what seems to be a similar situation.

There are similar difficulties associated with the ground state magnetic moments of nuclei in the mass-40 region. In particular, if one does a second order perturbation calculation for the magnetic moments of ^{41}Ca and ^{39}K , the results are, roughly speaking,

$$\mu_{g.s.} (^{41}\text{Ca}) = -1.91 + \Delta \text{ n.m.}$$

and

$$\mu_{g.s.} (^{39}\text{K}) = 0.13 + \Delta \text{ n.m.}$$

Experimentally, $\mu_{g.s.} (^{41}\text{Ca}) = -1.60 \text{ n.m.}$ and $\mu_{g.s.} (^{39}\text{K}) = 0.39 \text{ n.m.}^5$). We see that any choice of Δ which will yield an improved magnetic moment (over the Schmidt value) for one nucleus will lead to a worsened magnetic moment for the other nucleus⁶). Thus, we see again the haphazard success and failure of second order perturbation theory.

Recently M1 transitions have been measured in $^{38}\text{Cl}^7$) and $^{40}\text{K}^8$) so that it is possible to discuss the (pp)-to (ph)- transformation of the transition operator. When we assume the M1 operator is one body in nature, a particular transition rate $(J_i \rightarrow J_f)$ is identical in ^{38}Cl and ^{40}K , in the limit of no configuration mixing. When simple configurations are allowed to mix (whether in second order perturbation theory or a matrix in which the second order configurations are included - no multi-particle, multi-hole excitations), it is not surprising that M1 results are satisfactory for some transitions and inadequate for others.

It is the primary purpose of this paper to examine the role of simple excitations vis-a-vis multi-particle, multi-hole excitations treated in a core excited shell model. That is to say we investigate the relative importance of "simple" and "complicated" configurations in treating the properties of

states which are thought to be primarily pure configurations.

2. Model

We want to investigate the effects of the core-excited states on the simple shell model states. The most straightforward approach would be to diagonalize matrices which include multi-particle and multi-hole excitation over several single j -shells. Unfortunately to reduce the dimensionalities of the shell model matrices involved to a reasonable size it is usually necessary to truncate to very few single- j shells. Since we are interested in the mass-40 region we truncate to a closed ^{32}S core and take the $d_{3/2}$ - and $f_{7/2}$ -shells as the valence space. There have been other calculations which have started with similar notions. Perhaps the most successful core calculation using a core excited basis is that of Zuker, Buck and McGrory⁹) in which they treated light (sd)-shell nuclei using a ^{12}C core and allowing all possible excitations between the $0p_{1/2}, 0d_{5/2}$ and $1s_{1/2}$ -shells. A primary consideration in these previous calculations was to properly position these core-excited states.

The present calculation is more conservative in its expectations of the model. Firstly, we calculate the valence effective interaction appropriate to our model space, rather than choosing it in some ad hoc fashion. Secondly, we are not too concerned with properly positioning the core excited states. This is due to our interest in the effect of the core excited

states on the spherical ones. Thus, if the core excited states are a little too high or a little too low in energy it is of secondary importance to the spherical states which are perturbed by these core excited ones. Thirdly, insofar as the spectra are concerned we are only interested in energy differences, in which case many of the effects which we do not treat exactly right would appear both in the (pp)- and (ph)-nucleus, thus tending to cancel in a calculation of differences.

3. Calculation of the Effective Interaction

In our model space there are 24 two body matrix elements corresponding to $(d^2|v|d^2)$, $(d^2|v|f^2)$, $(df|v|df)$, and $(f^2|v|f^2)$ where we have taken $0d_{3/2} = d$ and $0f_{7/2} = f$. There is one single-particle energy, $\epsilon_f - \epsilon_d$, which we take to be 2.94 MeV. This choice is made because we are using a closed ^{32}S core and 2.94 MeV is the separation of the $3/2^+$ ground state and first excited $7/2^-$ state in ^{33}S .

The procedure for renormalizing the two body matrix elements so that they are appropriate for use in an (f-d)-shell model is straightforward, in principle. By starting with bare G-matrix elements, we hope to include effects more than $2\hbar\omega$ in energy away from our model space. Therefore we use the bare Kuo Brown¹⁰⁾ matrix elements for a mass-40 core because we are interested in the mass-40 region. We then include the effects which are $2\hbar\omega$ in energy outside the valence space. We do this by renormalizing

in second order perturbation theory - actually this calculation was done by Kuo and Brown (RKB - renormalized Kuo Brown matrix elements) for all the (fd)-shell matrix elements¹⁰) except $(d^2|v|d^2)$. To the extent that the G-matrix is good and second order perturbation theory is adequate, we have thus eliminated effects which are two or more major shells away from our two single j-shells. To account for effects which are within two major shells but still outside our desired valence space, we renormalize in a matrix because the energy denominators involved are so small. Explicitly we calculate two particle correlations in a matrix (using the single-particle energies of ref.¹¹) and then we allow one particle (lp) to excite out of the ^{32}S core (using experimental single-particle energies) and mix with the relevant matrix element. The eigenvalue of the matrix corresponding to the matrix element is the final value of the (fd)-model interaction for that two body matrix element.

To illustrate the calculation of the effective interaction, we will discuss it in detail for $(f^2|v|f^2)$. We start with RKB matrix elements (see fig. 1a) which include the effects of particle-hole excitations in which the particle and hole are separated by two major shells. The next step is to include 0h ω ladders (2p correlations), that is we solve the two particle problem, $(0f_{7/2}-1p_{3/2}-0f_{5/2}-1p_{1/2})^2$, by diagonalization. The energy of the state with the most f^2 strength (about 90% or more in most cases) is associated with the perturbed RKB matrix element. We do this diagonalization because our (fd)-model does

not include two particle correlations in the valence space and therefore these correlations must be put in the valence effective interaction. Typical contributions of this ladder type term to the renormalization are represented in fig. 1b. Next we must include the renormalization due to excitations from the ^{32}S core. That is, since we are eliminating the $1s_{1/2}$ and $0d_{5/2}$ -shells from our valence space, we must include their contribution in the effective interaction. We estimate their contribution by calculating, in a matrix, the lp renormalization by mixing the f^2 states with states in which lp is excited out of the ^{32}S core ($1s_{1/2}$ - and $0d_{5/2}$ -shells) into the $0d_{3/2}$ and (fp)-shells. Typical terms are shown in fig. 1c. The same association made for the ladder term is made here between the eigenenergies and the renormalized $(f^2|v|f^2)$ term. The eigenenergy for each (JT) matrix would be our final effective interaction except that the matrix will automatically include single-particle energy modifying terms which must be eliminated. A typical term of this sort is shown in fig. 1d. These terms are directly calculated in a mass -33 matrix, then subtracted from mass-34 as was described in ref.⁶). This being done we have included the effects outside the (fd)-model space and excluded those that are inside it, therefore, we have our desired $(f^2|v|f^2)$ valence effective interaction. We remark that care was taken in the lp renormalization to exclude f^2 to d^2 ladder terms in the matrix since these ladders would be generated in our (fd)-shell model calculation.

For the matrix elements $(d^2|v|d^2)$ the calculation is a bit simpler but somewhat more ambiguous. The particle-particle ladders into the lower (sd)-shell is blocked by the ^{32}S core and the ladders to the (fp)-shell are already partly included in the bare G-matrix elements (so we do not include them again in a matrix calculation like that for f^2 matrix elements). This means that some double counting will occur in the fd model because the d^2 matrix elements already have some f^2 ladders in them. However, we believe that these double counting effects are fairly small because of the large gap between the occupied and unoccupied levels in the G-matrix calculation of Kuo and Brown¹⁰). The renormalization with respect to the ^{32}S core proceeds in the same manner as that for $(f^2|v|f^2)$.

The RKB matrix elements for $(df|v|df)$ and $(d^2|v|f^2)$ have 1hw and 2hw renormalizations built in. We, therefore, renormalize $(df|v|df)$ only with respect to the ^{32}S core. Since $(d^2|v|f^2)$ is an off-diagonal matrix element, therefore we cannot use our matrix approach to renormalize with respect to a ^{32}S core and we use these RKB numbers as they stand.

We then have our (fd)-model interaction in which effects outside the model space have been approximately folded into the effective interaction. The calculated matrix elements are listed in Table 1. The renormalizations were done in order of largest terms first and smallest terms last.

The calculated two body matrix elements we use cannot be too accurate. Therefore, we calculated them several different

ways (some of them quite unreasonable, for example, adding a constant to each matrix element). We found that the position of the core excited states was fairly sensitive to the choice of interaction. On the other hand, the energies of interest, when treating differences were fairly insensitive to the variation. Furthermore, the trends of the differences were quite insensitive. For example, if we took f^2 from ^{42}Sc (this is pure double counting, because we include in the interaction terms like $f^4 d^{-2}$) we get the same trends in the spectral differences between ^{38}Cl and the Racah transform of ^{40}K as we get using the calculated interaction in Table 1. We further remark that even the deformed state positions are fairly insensitive to a 10% change in the two body matrix elements. We also found out results to be fairly insensitive to the order in which the renormalizations were performed.

There is another type of difficulty which is associated with our severe space truncation. That is, we have excluded from our valence space configurations which surely lie "fairly" low. This is a problem that all such models have, and the effects of the truncation will surely become important if we try to extend the model too far or expect too much of the model.

4. The Calculation

In the spectral calculations we will be interested in interaction energies of states. That is, in ^{38}Cl the interaction

energy of a particular I state is

$$E_I(1d) = E_I(^{38}\text{Cl}) + E_{g.s.}(^{36}\text{S}) - E_{g.s.}(^{37}\text{S}) - E_{g.s.}(^{38}\text{S})_{(4.1)}$$

and similarly in ^{40}K for a particular J state

$$E_J(1d) = E_J(^{40}\text{K}) + E_{g.s.}(^{38}\text{Ca}) - E_{g.s.}(^{39}\text{Ca}) - E_{g.s.}(^{40}\text{Ca}). \quad (4.2)$$

If the simplest shell model were exactly valid (no configuration mixing),

$$\Delta E_J = E_J(1d) - T_J^I(E_I(1d)) = 0,$$

where T_J^I is the Racah transformation and the repeated index, I, is summed. Alternatively, we can re-write the interaction energy in its multipole form in which

$$\alpha_\lambda(1d) = - \sqrt{\frac{[1]}{32}} \sum_I [I] (-)^I \left\{ \begin{matrix} \frac{7}{2} & \frac{3}{2} & I \\ \frac{3}{2} & \frac{7}{2} & \lambda \end{matrix} \right\} E_I(1d)$$

and

$$\alpha_\lambda(1d^{-1}) = - \sqrt{\frac{[1]}{32}} \sum_J [J] (-)^J \left\{ \begin{matrix} \frac{7}{2} & \frac{3}{2} & J \\ \frac{3}{2} & \frac{7}{2} & \lambda \end{matrix} \right\} E_J(1d^{-1}).$$

In the simple shell model limit,

$$\Delta \alpha_\lambda = \alpha_\lambda(1d^{-1}) - (-)^{\lambda+1} \alpha_\lambda(1d) = 0.$$

Analysis in terms of the multipole discrepancies is more convenient. For example, α_0 , the monopole gives the centroid, $\alpha_0 = \frac{\sum_J E_J}{\sum_J [J]}$, of the levels. Experimentally, for the $^{38}\text{Cl}/^{40}\text{K}$ system almost the entire violation, $\Delta \alpha_\lambda$, is in the monopole. That is, the ΔE_J 's are essentially J independent

(see Table 2).

It is clear that in the limit of no core excitation the (fd)-shell model would predict that $\Delta\alpha_\lambda = 0$. Thus, we already see a fault of our severe space truncation, the second order perturbation effects of Pandya and French³⁾ will be absent. To get around this we do an ordinary shell model calculation (OSM) using the bare Kuo-Brown interaction¹⁰⁾, for the eight nuclei listed in equations (4.1) and (4.2) to modify the valence effective interaction for $(df|v|df)$. By ordinary shell model we mean the simplest configurations without massive core excitations. Explicitly for the nuclei in (4.1) we start with a ^{36}S core and allow up to two protons to excite from the $0d_{5/2}$ and $1s_{1/2}$ -shells into the $0d_{3/2}$ -shell as well as any nucleon in the $(f_{7/2})$ -shell (one at most) is allowed anywhere in the $f_{7/2}$ - $p_{3/2}$ - $f_{5/2}$ single j -shells. For the four nuclei in (4.2) having a ^{40}Ca core, only ^{40}K is not a pure configuration, for ^{40}K the valence neutron particle is allowed to roam over the (sd)-shell. The single-particle energies used for these cores are listed in Table 3. Thus, where we calculate differences we will be able to see the relative roles of the OSM and the (fd)-model in contributing to the discrepancies, $\Delta\alpha_\lambda$ and ΔE_J , between the (pp)- and (ph)-partners. These OSM calculations are used to modify the (fd)-model effective interaction, so that the (fd)-model does not yield $\Delta\alpha_\lambda = \Delta E_J = 0$ in the limit of no core excitation. Rather, in this limit the model will yield the OSM results. Explicitly, the particle-particle interaction for ^{38}Cl is no longer the

same as that for ^{40}K because the matrix elements

$$\left(d_{7/2}^{\pi} f_{7/2}^{\nu} \mid v \mid d_{7/2}^{\pi} f_{7/2}^{\nu} \right)$$

have been modified to account for OSM effects (see Table IV). A similar though much larger, effect occurs for the

$$\left(f_{7/2}^{\pi} f_{7/2}^{\nu} \mid v \mid f_{7/2}^{\pi} f_{7/2}^{\nu} \right)$$

interaction between ^{42}Sc and ^{48}Sc . The larger difference between the effective interaction elements in $f_{7/2}$ -shell is reflective of the larger role of OSM effects in this shell.

In the (fd)-model, we are interested in the effects of the core excited states on the spherical ones. It turns out in difference calculations, the core excited states which have more than two more holes than the spherical state, have a negligible effect on the spherical states of interest. This simplification is not too surprising because a two body operator will not directly connect to states with higher particle-hole rank. We therefore make this approximation in our spectral calculations, unless we are specifically interested in the core excited states.

The (fd)-model is also used to generate effective operators to be used in transition calculations in the OSM. Explicitly magnetic moments are calculated in the model for the "single-particle" nuclei, and put in as effective moments for the "two-particle" nuclei, ^{38}Cl and ^{40}K . The model is also used to discuss the position of some core excited states, although we believe these results to be at the limit of the model.

5. The Results

5.1. $^{38}\text{Cl}/^{40}\text{K}$ Spectral Differences

In this section we will concentrate on a discussion of the various $^{38}\text{Cl}/^{40}\text{K}$ spectral differences calculations. The OSM calculation, which is an update of the Pandya and French³⁾ calculation, yields completely unsatisfactory results for the difference quantities ΔE_J and $\Delta \alpha_\lambda$ (see Table V). Explicitly all the ΔE_J 's have the wrong sign when compared to their experimental counterparts. There is some resemblance between the OSM experimental $\Delta \alpha_\lambda$'s for $\lambda > 0$, however, the experimental $\Delta \alpha_\lambda$'s for $\lambda > 0$ are quite small compared to $\Delta \alpha_\lambda = 0$. The fact that most of the experimental $\Delta \alpha_\lambda$ is concentrated in the $\lambda = 0$ term is seen in the fact that the experimental ΔE_J 's are almost constant. That is, the monopole is the (J)-weighted average of the energy levels; therefore, if the ΔE_J 's are constant,

$$\Delta E_J = a$$

then there is only a monopole discrepancy,

$$\Delta \alpha_\lambda = \delta_{\lambda,0} a$$

The unsatisfactory results for the OSM are essentially independent of choice of single-particle energies and two-body matrix elements. Rather, the results are reflective of the unsatisfactory nature of a second order perturbation theory approach to the problem.

To further clarify the distinction between an OSM type calculation and an (fd)-model calculation and to emphasize the

need of the latter, we will consider the contribution of second order perturbation theory results to ΔE_J and $\Delta \alpha_\lambda$. West and Koltun⁴⁾ have shown that the primary contribution, in second order perturbation theory, to the monopole discrepancies can be obtained from a calculation of the graphs shown in figure 3. We will confine our discussion to the monopole because in the $^{38}\text{Cl}/^{40}\text{K}$ and $^{42}\text{Sc}/^{48}\text{Sc}$ systems it is by far the largest discrepancy. We remark that the role of the excited single-nucleon state shown in figure 2 increased the closer it lies, energetically, to the model space, thus simplifying the calculations of West and Koltun⁴⁾.

The expression for the energy of the term represented by Figure 3a is

$$E_I(jk) = \sum_{\alpha} \frac{(jkI|r|\alpha kI)^2}{-(E_{\alpha} - E_j)}$$

and that for figure 2b is

$$\bar{E}_J(jk^{-1}) = \sum_{\alpha} \frac{(jk^{-1}J|r|\alpha k^{-1}J)^2}{-(E_{\alpha} - E_j)}$$

We add that

$$\Delta E_J = E_J(jk^{-1}) - T_J^{-1}(E_I(jk))$$

For the sake of simplicity, we take

$$(jkI|r|\alpha kI) = a \quad \text{and} \quad (jk^{-1}J|r|\alpha k^{-1}J) = b$$

By this restriction, we fix ΔE_J to be a constant (J-independent) and therefore $\Delta \alpha_\lambda = 0$ if $\lambda \neq 0$. Therefore, we cannot discuss the discrepancies of higher multipolarity within this framework. However, we emphasize that it is quite adequate for a discussion of the monopole discrepancy.

If there are N single particle states each with the same single-particle energy, ϵ_α , then

$$E_I(J, \theta) = \frac{Na^2}{-(\epsilon_\alpha - \epsilon_J)} \quad \text{and} \quad T_J^I(E_I(J, \theta)) = \frac{Na^2}{(\epsilon_\alpha - \epsilon_J)}$$

Furthermore,

$$\Delta \mathcal{A}_0 = \Delta E_J = -\frac{Nb^2}{(\epsilon_\alpha - \epsilon_J)} + \frac{Na^2}{-(\epsilon_\alpha - \epsilon_J)}$$

which is less than zero for all cases. It is clear that this result is independent of our choice of single-particle energies or two-body matrix elements (which appear quadratically). For the $^{42}\text{Sc}/^{48}\text{Sc}$ conjugate pair this result is in agreement with experiment, however, for $^{38}\text{Cl}/^{40}\text{K}$ the result is contrary to experiment. This trend would not change even if we treated these type configurations in a matrix (OSM) instead of perturbation theory. Rather we are outside the limit of the validity of the OSM or second order perturbation theory for the $^{38}\text{Cl}/^{40}\text{K}$ case. We have neglected the contribution of core excited configurations. This neglect is usually attributed to the fact that their unperturbed energy is so large, i.e., a large energy denominator, hence a small contribution. Even though the unperturbed energies may be large, the perturbed energies of many

of these configurations is small and, thus, their effect is not negligible. Therefore, an OSM calculation is inadequate for $^{38}\text{Cl}/^{40}\text{K}$ because it rests on the assumption that the perturbation expansion of the energy for the model space interaction converges much more rapidly than it really does. It is the effect of core excited configurations that we are attempting to account for with our (fd)-shell model.

Firstly, in the (fd)-model we take a ^{32}S core and obtain two body matrix elements, v_{eff} , which obey the Racah transform (i.e., same $(df|v|df)$ particle-particle matrix elements for ^{38}Cl and ^{40}K . The results of the calculations are presented in Table VI. It is seen that the calculated ΔE_J 's and $\Delta\alpha_\lambda = 0$ have the correct signs when compared to experiment although they are too large. To summarize, we have factored the terms which give rise to the violations into two parts - OSM and core excitations. To estimate their relative contributions we can just add ΔE_J^{OSM} to ΔE_J^{fd} and $\Delta\alpha_\lambda^{\text{OSM}}$ to $\Delta\alpha_\lambda^{\text{fd}}$. This is done in Table VII. It is seen that the combined results are slightly better than the (fd) results. However, the appropriate way to combine these terms is to use one calculation to generate a modified effective interaction for the other. Since the OSM results are more than 90% pure configurations, we use them to account for the difference in cores between ^{38}Cl (a ^{36}S core) and ^{40}K (a ^{40}Ca core). This difference is manifest in different particle-particle interactions for ^{38}Cl and ^{40}K for

$$(d_{\frac{1}{2}}^\pi \frac{1}{2}^\nu I | v | d_{\frac{1}{2}}^\pi \frac{1}{2}^\nu I).$$

See Table IV.

With v_{eff} no longer obeying the Racah transform we recalculate ^{38}Cl and ^{40}K and the appropriate reference energies. We remark that the changes in the interaction matrix elements do little to the individual spectra, but the changes are quite important in the differences. The results of the new (fd)-model calculation are presented in Table VIII. We see that the agreement between theory and experiment is quite astonishing. We note also that although $\Delta\alpha_0$ is insensitive to a matrix treatment, the higher multipolarities are fairly sensitive.

It is worth noting that agreement of the calculated individual spectra when compared to experiment is not outstanding, even though the differences are well described. This is due to the fact that many terms which contribute to individual spectra are absent. However, West and Koltun¹⁾ have shown that a good portion of these terms obey the Racah transform so that they make no contribution to the differences. For example, many diagrams with high-lying intermediate states will transform in spite of different intermediate states. For this reason it is easier to obtain a calculated spectral of differences than to calculate spectra.

5.2. Results, Particle Rank of the Interaction

In 1957, Pandya and French³⁾ saw the value of using a higher particle rank interaction in the calculation of the spectral dis-

crepancies between ^{40}K and the Racah transform of ^{38}Cl . In particular, they did an OSM type calculation in addition to which they allowed a three body force to admix in the pure model space (no configuration mixing space). They showed that their residual three body interaction yielded a cancellation of the effects of their OSM calculation so that their resultant calculated discrepancies ($\Delta\alpha_\lambda$) were about the size of the experimental discrepancies for $\lambda > 0$. Unfortunately, their three body force was repulsive which is contradiction to what is required by experiment to yield the proper sign for $\Delta\alpha_0$. As will be seen, the experimental three body force in this case is attractive.

The point of this section is to show how our (fd)- model results can be presented in terms of an expansion in the particle rank of the interaction, thereby enabling us to understand the successes of a Pandya and French-type calculation. At the same time we will see how the effects of space truncation can be simulated in the particle rank of an effective interaction. Thus, the interaction energy of ^{38}Cl as calculated in the (fd)- model will yield a monopole which we can view as the effective monopole in the pure shell model (no configuration mixing). Similarly, the (fd)-model calculation of the ^{40}K (with respect to a ^{36}S core) monopole can be viewed in the pure shell model space as resulting from a d-f interaction which is 2 plus 3 plus 4 body in nature. Therefore we are simulating the effect of

core excitation by including a linear combination higher particle rank component of the interaction in the pure shell model space. Due to the limited experimental information available and the size of the monopole discrepancies, we will restrict the discussion in this section to monopoles.

There are a new class of discrepancies of the difference type that are associated with the interaction energies $d_{\pi}f_v$, $d_{\pi}^2f_v$, $d_{\pi}^3f_v$ and $d_{\pi}^4f_v$ which are represented in figure 4. The monopole discrepancy is associated with the fact that the monopoles which are obtained from figure 4 are not linear in the number of d-particles. This linearity would apply if the interaction were only two body in nature and there were only pure configurations.

Since the calculations are to be done in the (fd)-model and the results are to be re-expanded in terms of various particle ranks for the $d_{\pi}^n f_v$ (particle rank is ≥ 2 and $\leq n + 1$) interaction, we must explicitly account OSM effects. That is, it appears that we should calculate a separate G-matrix for each set represented in figures 4a-4d. Fortunately, we are saved this task because the OSM calculation yields less than a 10 keV effect on the monopole going from ^{38}Cl to ^{40}K (remember, we now have only particle-particle terms - compare figure 2 with figure 4). Since OSM results need not be included in v_{eff} , the OSM is clearly an insufficient model to be used to account for the discrepancies in which we are interested in this section.

Experimentally, the interaction monopole for ^{39}Ar is

unknown, however,

$$\text{for } ^{38}\text{Cl}: \quad v_2^e = -0.925 \text{ MeV}$$

$$\text{and for } ^{40}\text{K}: \quad 1/3 (3v_2^e + 3v_3^e + v_4^e) = -0.980 \text{ MeV}$$

$$\text{and for } ^{41}\text{Ca}: \quad 1/4 (4v_2^e + 6v_3^e + 4v_4^e + v_5^e) = -0.985 \text{ MeV},$$

where the subscript on v^e denotes the particle rank of the interaction. If we assume there is only a two-body force and no configuration mixing, then $v_2^e (^{38}\text{Cl})$ should equal $v_2^e (^{40}\text{K})$ in the pure shell model limit. However, $\Delta v^e (^{38}\text{Cl}/^{40}\text{K}) = -0.980 + 0.925 = -0.055 \text{ MeV}$, which has about the same magnitude as the monopole violation in the pp- to ph-transformation of ^{38}Cl to ^{40}K .

To see how the (fd)-model fares, we calculate the terms represented in figure 4 to obtain

$$\text{for } ^{38}\text{Cl}: \quad v_2^{\text{fd}} = -0.75 \text{ MeV}$$

$$\text{and for } ^{39}\text{Ar}: \quad 1/2 (2v_2^{\text{fd}} + v_3^{\text{fd}}) = -0.78 \text{ MeV}$$

$$\text{and for } ^{40}\text{K}: \quad 1/3 (3v_2^{\text{fd}} + 3v_3^{\text{fd}} + v_4^{\text{fd}}) = -0.80 \text{ MeV}$$

$$\text{and for } ^{41}\text{Ca}: \quad 1/4 (4v_2^{\text{fd}} + 6v_3^{\text{fd}} + 4v_4^{\text{fd}} + v_5^{\text{fd}}) = -0.81 \text{ MeV}.$$

When we solve the equations we find that the monopoles are

$$v_2^{\text{fd}} = -0.75 \text{ MeV}$$

$$v_3^{\text{fd}} = -0.06 \text{ MeV}$$

$$v_4^{\text{fd}} = 0.03 \text{ MeV}$$

$$v_5^{\text{fd}} = 0.00 \text{ MeV}.$$

We remark that there is some accuracy (round off) problem associated with the calculation of the higher particle rank monopoles. Nonetheless, the effective three-body monopole term is more than ten times smaller than the corresponding two body term.

When comparing ^{38}Cl and ^{40}K in the pure model space we see the origin of the discrepancy between their monopoles can be attributed to an effective three plus four body interaction.

We can deduce V_3^e and V_4^e , if we assume that V_5^e is zero which is consistent with the (fd)-model calculations. Doing this we obtain

$$\begin{aligned} V_2^e &= -0.925 \text{ MeV}, \\ V_3^e &= -0.070 \text{ MeV}, \\ \text{and } V_4^e &= 0.045 \text{ MeV}. \end{aligned}$$

We see that the comparison between calculated and "experimental" monopoles is quite good. We add that the trends for the "experimental" monopoles remain the same for different choices of V_5^e as long as it remains smaller in magnitude than V_4^e .

To make a further comparison between experiment and theory, we recall that

$$\Delta V^e (^{38}\text{Cl}/^{40}\text{K}) = -0.055 \text{ MeV}$$

alternatively, using the "experimental" values for V_3^e and V_4^e we obtain

$$\Delta V^e (^{38}\text{Cl}/^{40}\text{K}) = \frac{1}{3} (3 V_3^e + V_4^e) = -0.055 \text{ MeV}$$

Further, we see that we can account for most of the experimental discrepancy by using our calculated values for V_3^{fd} and V_4^{fd} in which case

$$\Delta V^{fd} (^{38}\text{Cl}/^{40}\text{K}) = \frac{1}{3} (3 V_3^{fd} + V_4^{fd}) = -0.05 \text{ MeV}.$$

We remark that in this case ΔV^{OSM} is about the size of the residual discrepancy between experiment and theory. In addition, by re-expanding our (fd)- results for the interaction energies of ^{38}Cl , ^{39}Ar , ^{40}K and ^{41}Ca , we obtain these monopole variations in terms of the particle rank of the force in the pure model space. We emphasize that we have shown in a particular example how some effects of a two-body interaction in a large space can be simulated by a higher particle rank interaction in a pure shell model valence space.

5.4. Results - $^{42}\text{Sc}/^{48}\text{Sc}$

In this section we present the results of an (fd)-model difference calculation for $^{42}\text{Sc}/^{48}\text{Sc}$. We do this to determine whether or not the $^{42}\text{Sc}/^{48}\text{Sc}$ pair reveal an embarrassment to the (fd)-model in the same way that the success of an OSM calculation for $^{42}\text{Sc}/^{48}\text{Sc}$ is tarnished by the failure of the same type of calculation to account for the differences of the $^{38}\text{Cl}/^{40}\text{K}$ system. That is, we have seen that a second order perturbation theory treatment fails badly when applied to the discrepancies between ^{40}K and the Racah transform of ^{38}Cl , in spite of the fact the treatment is satisfactory when applied to the discrepancies between ^{48}Sc and the Racah transform of ^{42}Sc).

We have seen that the use of the (fd)-model reveals that core excitations overcancel the effects of an OSM calculation for the $^{38}\text{Cl}/^{40}\text{K}$ discrepancies - yielding satisfactory agreement between calculation and experiment. How does this result apply to the $^{42}\text{Sc}/^{48}\text{Sc}$ system differences? We will see that

there is no overcancellation and that the (fd)-model calculation yields slightly improved results over the OSM difference calculation of Goode, Koltun and West¹¹⁾. In their (fp)-shell model calculation, they allowed at most two nucleons to excite from the $0f_{7/2}$ -shell to the $1p_{3/2}$ - $0f_{7/2}$ - $1p_{1/2}$ single-j shells. They used renormalized Kuo-Brown matrix elements¹⁰⁾ and the (fp)-shell single particle energies listed in Table III.

As in the case of $^{38}\text{Cl}/^{40}\text{K}$, we must use a different effective interaction for each nucleus. In the present case it is the $(f^2|v|f^2)$ two body matrix elements which differ between ^{42}Sc and ^{48}Sc . The effective (fd)-model elements for the respective nuclei are taken from the results of the OSM calculation of Goode, Koltun, and West¹¹⁾. These numbers appear in Table IX. We do not include the renormalization effect of the ^{32}S core (see Table I) because so much of it is blocked at ^{48}Sc . The results of the $^{42}\text{Sc}/^{48}\text{Sc}$ difference calculations are about the same whether or not these ^{32}S core renormalization terms are excluded. We emphasize that the J dependent energy difference, ΔE_J , between the effective interaction for ^{48}Sc and the Racah transform of that for ^{42}Sc is identical to the corresponding OSM spectral difference as calculated by Goode, Koltun, and West¹¹⁾. Therefore, as in the $^{38}\text{Cl}/^{40}\text{K}$ case, the (fd)-model results for $^{42}\text{Sc}/^{48}\text{Sc}$ include the OSM results as well as the effects of core excitations.

The results of the (fd)-model difference calculation for

the $^{42}\text{Sc}/^{48}\text{Sc}$ conjugate pair appear in Table X. It is clear that a good part of the $^{42}\text{Sc}/^{48}\text{Sc}$ discrepancy is accounted for. In Table XI the calculated discrepancies for the (fd)-model and the OSM calculation are compared with experiment¹²). The results of both calculations compare favorably with experiment. To compare the two calculations with experiment, we use a measure introduced by West and Koltun⁴). We consider the $\Delta\alpha_\lambda$'s to be the basis vectors in an orthogonal eight dimensional space, they then compute the cosine of the experimental and theoretical vector,

$$X = \frac{\sum_{\lambda} \Delta\alpha_{\lambda}^{\text{calculated}} \Delta\alpha_{\lambda}^{\text{experiment}}}{\sqrt{\left(\sum_{\lambda} \Delta\alpha_{\lambda}^{\text{calculated}}\right)^2 \left(\sum_{\lambda} \Delta\alpha_{\lambda}^{\text{experiment}}\right)^2}}$$

If $X = 1$, the experimental and theoretical violations are identical. For the OSM calculation of Goode, Koltun, and West¹¹), $X = 0.95$, which is about the same as the 0.97 that we obtain for our (fd)-model calculation. If we leave the monopole out of the space then $X = 0.79$ for the OSM calculation and 0.90 for the (fd)-model. Thus we see that the violation of the higher multipoles is better described on the (fd)-model. Most importantly, we see that the (fd)-model yields a consistent explanation of the $^{42}\text{Sc}/^{48}\text{Sc}$ and $^{38}\text{Cl}/^{40}\text{K}$ violations.

The improvement the (fd)-model yields over OSM is not so startling in $^{42}\text{Sc}/^{48}\text{Sc}$ as it was in $^{38}\text{Cl}/^{40}\text{K}$ where

$$X_{\text{osm}} = -0.59$$

but

$$X_{\text{fd}} = 0.89.$$

Still the $^{42}\text{Sc}/^{48}\text{Sc}$ results on the model go in the direction of improved agreement with experiment. The (fd)-model for ^{42}Sc and ^{48}Sc has its most profound effect in improving the individual spectra compared to experiment. For example, the interaction energy of the ground state of ^{42}Sc is -3.20 MeV, the OSM yields -2.60 MeV for this state whereas the (fd)-model yields -3.34 MeV.

5.5. Results: ^{40}Ca

With the success of the difference calculations, we calculate the spectrum of ^{40}Ca including 0p-0h, 2p-2h, 4p-4h, 6p-6h, and 8p-8h configurations. Although we have previously emphasized that we do not demand that the (fd)-model do well in positioning the core excited states, it is worthwhile to do this calculation to make a connection with the results of those who have used an (fd)-model to calculate the energies of core-excited states.

Federman and Pittel¹³⁾ have calculated the 0^+ spectrum of ^{40}Ca in an (fd)-model neglecting 6p-6h and 8p-8h configurations. In their calculation the interaction will lead to double counting. For example, they take $\langle f^2 | v | f^2 \rangle$ from experiment (spectrum of ^{42}Sc), therefore core excited effects are present in the interaction and will be duplicated in the calculations. To account for this they reduce off-diagonal matrix elements like $\langle (2p-2h) | v | (0p-0h) \rangle$ and $\langle (4p-4h) | v | (2p-2h) \rangle$. Zuker¹⁴⁾ has also calculated the 0^+ spectrum of ^{40}Ca arising from the same con-

figurations we use, but with a fitted interaction.

In Table XII we compare the calculated energies of the first three 0^+ states with experiment. The calculated energies of the 0^+ states is in fair agreement with experiment and the aforementioned calculations. As we have discussed previously, the location of the core-excited state is sensitive to the choice of residual interaction. This point is also made by Zucker¹⁴⁾ in comparing his results to those of Federman and Pittel¹³⁾. We add that, like Zuker¹⁴⁾, we see no sizeable 6p-6h or 8p-8h strength lying low (below 15 MeV). This latter fact is in contradiction to the calculations of Gerace and Green¹⁵⁾ in which they attribute an 8p-8h nature to the 5.20 MeV 0^+ state in ^{40}Ca . Because of the sensitivity of the calculation for the position of core-excited states to the choice of residual interaction, it is not at all unlikely that we could get a more compressed spectrum (and better agreement with experiment) with a different interaction. Even in that event, however, it is difficult indeed to imagine the 8p-8h strength coming low because it is so high (above 15 MeV) to begin with.

5.6. Results: Transition Rates

With the success of the (fd)-model in describing the differences in the $^{38}\text{Cl}/^{40}\text{K}$ and the $^{42}\text{Sc}/^{48}\text{Sc}$ partners, we now use the model to calculate transition rates in the mass-40 region. We point out that we are no longer using the model in spectral or difference calculations, and we therefore expect more difficult-

ies than previously encountered. Still, it is worthwhile to ascertain the value of the model in the calculation of effective transition operators.

In an OSM calculation of the M1 transition rates of ^{40}K , it is appropriate to use effective magnetic moments for the one body terms representing $f_{7/2}^{\nu}$ and $d_{3/2}^{\pi-1}$, which are the primary OSM configurations for ^{40}K and therefore must be treated with utmost care. Since $f_{7/2}^{\nu} - d_{3/2}^{\pi-1}$ is the principle configuration, it is not so important to include effective moments for the other single-particle states. We therefore confine our discussion (and calculations) of the one body terms exclusively to the ground state moments of ^{41}Ca and ^{39}K . An OSM calculation of these two terms (OSM, as we have discussed it, allows no configuration mixing for these two nuclei) yields the Schmidt moments,

$$\mu_{f_{7/2}}^{\nu} = -1.91 \text{ n.m.}$$

and

$$\mu_{d_{3/2}}^{\pi-1} = 0.13 \text{ n.m.}$$

whereas, the experimental moments are

$$\mu_{f_{7/2}}^{\nu} = -1.60 \text{ n.m.}$$

and

$$\mu_{d_{3/2}}^{\pi-1} = 0.39 \text{ n.m.}$$

These ground state moments are labelled $f_{7/2}$ and $d_{3/2}^{-1}$ on the basis of the simple, pure shell model configuration assignment. Mavromatis and Zamick⁶) have shown that a second order perturbation theory calculation in which all excitations within $2\hbar\omega$ in energy of the ground state are included, yields inconsistent

results for the ground state moments of ^{39}K and ^{41}Ca . In particular, they obtain

$$\mu_{\frac{3}{2}}^{\nu} = -1.91 + |\delta_1|$$

and

$$\mu_{d_{3/2}}^{\pi} = 0.13 - |\delta_2|$$

where $|\delta_1| \approx |\delta_2| \approx 0.30$ n.m. for the Kallio-Kolltveit potential. Therefore, although they obtain a reasonable ground state moment for ^{41}Ca , their value for the ground state moment for ^{39}K is quite unsatisfactory. We add that the second order perturbation theory calculation of Mavromatis and Zamick⁶⁾ is more inclusive than the ones we have discussed. That is, in our OSM calculations we include Ohw excitations only (do not allow excitations to cross a major shell), therefore our OSM would give the Schmidt values for the ground state moment of ^{41}Ca and ^{39}K .

However, in the (fd)-model we include core excitations but only of a $d_{3/2}$ to $f_{7/2}$ nature. Still, we obtain

$$\mu_{\frac{3}{2}}^{\nu} = -1.47 \text{ n.m. for } ^{41}\text{Ca} \text{ and } \mu_{d_{3/2}}^{\pi} = 0.12 \text{ n.m.}$$

for ^{39}K , which is not in excellent agreement with experiment, but is a significant improvement over the second order perturbation theory results. In particular, an improvement over the Schmidt value for one nucleus (^{41}Ca) does not imply a poorer result for the other nucleus (^{39}K). Chemtob¹⁶⁾ has shown that two-body mesonic currents make important contributions to the ground state moments of ^{41}Ca and ^{39}K . Since we have ignored mesonic effects we estimate their effects in the (fd)-model

moments by adding Chemtob's correction terms to our calculated moments. We obtain

$$\mu_{f_{7/2}}^{\nu} = -1.66 \text{ n.m. for } ^{41}\text{Ca} \text{ and } \mu_{d_{3/2}}^{\pi-1} = 0.31 \text{ n.m. for } ^{39}\text{K},$$

both of which are in good agreement with experiment. With this in mind, we would expect mesonic contributions to be important for the calculation of transition rates in ^{38}Cl and ^{40}K . On the other hand, we do not claim to have explained the ground state moments of ^{41}Ca and ^{39}K , because there are terms we have left out (like $2p-1h$). The role of these terms has been discussed in perturbation theory by Mavromatis and Zamick⁶) and Dieperink¹⁷).

It turns out that the calculated OSM transition rates are fairly sensitive to the choice of one body moment so that it is fairly difficult to use the (fd)-model to generate effective transition operators for OSM. As discussed previously, the OSM states of interest are primarily $f_{7/2}^{\nu}-d_{3/2}^{\pi-1}$ states in nature so that we choose effective moments for the ground states of ^{41}Ca and ^{39}K . The calculated OSM transition rates are compared with the known transition rates for ^{40}K in Table XIII. Three different values are used for $\mu_{f_{7/2}}^{\nu}$ and $\mu_{d_{3/2}}^{\pi-1}$ - the Schmidt values, the (fd)-model calculated values (without mesonic corrections), and the experimental values. It is clear from Table XIII that the transition rates are quite sensitive to the choice of one body moment and only the experimental one body moments lead to a satisfactory comparison with experiment for ^{40}K . Unfortunately, the picture for ^{38}Cl is

cloudier still.

For ^{38}Cl , the appropriate one body terms are the ground state moments of ^{37}Cl and ^{37}S (which is unknown). In ^{37}Cl the ground state moment is

$$\mu_{d_{3/2}}^{\pi} = 0.68 \text{ n.m.}$$

which is quite a bit larger than the Schmidt value of 0.13 nm.

In an OSM calculation for this term, we get a moment of 0.42 nm with less than 0.5% configuration mixing (recall that there is no OSM configuration mixing for ^{39}K or ^{41}Ca). This is reflective of the additional difficulties in the calculation of the transitions in ^{38}Cl . The ^{38}Cl transitions are sensitive to the choice of one body moments and the one body moments are themselves quite sensitive to configuration mixing. Furthermore, the ground state moment of ^{37}S is unknown.

In Table XIV, we present a comparison between the calculated and experimental transition rates in ^{38}Cl between the states which are primarily $f_{7/2}^{\nu}-d_{3/2}^{\pi}$ in nature. We see that an OSM calculation with effective moments from the (fd)-model yields, as in the case of ^{40}K , smaller BML's than an OSM calculation with Schmidt moments. Still we see that no one case gives satisfactory agreement with experiment. Furthermore, the OSM calculated BML's are somewhat sensitive to our choice of single-particle energies. Owing to the many sensitivities we remark that we probably could find some combination of input which would lead to satisfactory transition rates in ^{38}Cl . However, such an approach is not within the spirit of this work and

therefore, we will not pursue this point further.

6. Discussion

We have introduced an (fd)-shell model which we use to account for the effects of core excited states on spherical states in difference calculations. In particular, we have seen that ordinary perturbation theory or OSM calculations are unable to simultaneously account for ΔE_J between $E_J(^{40}\text{K})$ and $T_J^I(E_I(^{38}\text{Cl}))$ and ΔE_J between $E_J(^{48}\text{Sc})$ and $T_J^I(E_I(^{42}\text{Sc}))$. However, we have shown that our core excited model, in which the residual interaction is independently calculated, can be used to give a simultaneous explanation for both sets of discrepancies. In determining the various interaction energies, E_J , different effective interactions are used for each case because there is no effective interaction appropriate to both members of a pair (e.g. in determining the interaction energy of ^{38}Cl and that of ^{40}K we use different effective interactions for each). This is due to the fact that there are effects outside the valence space which do not exactly obey the Racah transformation.

We have shown that the effects of a two body force in large vector space can be simulated by a higher particle-rank force in a smaller space. This accounts for the success of Pandya and French in accounting for some of the discrepancies between ^{40}K and ^{38}Cl by appealing to a three-body force for ^{40}K . Still, it may be disheartening to some that seemingly simple states

cannot be completely described without including the effects of core excitations.

We emphasize, however, that difference calculations are quite appropriate within the framework of the (fd)-model. For example, if a core excited state is poorly treated in the model calculation, this effect will appear consistently and thus tend to cancel in the difference calculation. This is reflective of the fact that individual calculated spectra need not compare very well to experiment in spite of the fact that the discrepancies in the differences are very well accounted for. We see that we approach the limit of the model when we try to calculate the position of the core excited states or various transition rates.

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Table Captions

- Table I. The two body matrix elements for the (fd)-model are listed in MeV. For each matrix element the far right hand column represents the value of that two body matrix element which is used in the (fd)-model calculation. RKB means renormalized Kuo Brown matrix elements, pp means $0h\omega$ ladders, and Δ means the renormalization with respect to a ^{32}S core.
- Table II. The experimental results for ^{38}Cl and ^{40}K in terms of energies and multipoles.
- Table III. The single-particle energies used in the OSM calculations.
- Table IV. The effective interactions for ^{38}Cl and ^{40}K which do not obey the Racah transform compared to the (fd)-model v_{eff} which obeys the transformation.
- Table V. The OSM results for ^{38}Cl and ^{40}K in terms of energies and multipoles.
- Table VI. The (fd)-model results for ^{38}Cl and ^{40}K in terms of energies and multipoles for the case in which v_{eff} obeys the Racah transformation.
- Table VII. The sum of the results of Table V and Table VI.
- Table VIII. The (fd)-model results for ^{38}Cl and ^{40}K in terms of energies and multipoles for the case in which v_{eff} includes OSM effects

- Table IX. The v_{eff} for ^{42}Sc and ^{48}Sc from the OSM calculation of Goode, Koltun, and West¹¹⁾.
- Table X. The (fd)-model results for ^{42}Sc and ^{48}Sc in terms of energies and multipoles for the case in which v_{eff} includes OSM effects.
- Table XI. The multipole discrepancies are compared. $\Delta\alpha_{\lambda}^{\text{fd}}$ is from Table X and $\Delta\alpha_{\lambda}^{\text{exp}}$ and $\Delta\alpha_{\text{OSM}}^{\lambda}$ are from ref.¹¹⁾.
- Table XII. The 0^+ spectrum of ^{40}Ca calculated in the (fd)-model including 0p-0h, 2p-2h, 4p-4h, 6p-6h, and 8p-8h effects and excluding OSM effects.
- Table XIII. The transition rates in ^{40}K . Experiment is compared to ordinary OSM and OSM with effective moments for $f_{7/2}$ and $d_{3/2}$. The two sets of effective moments are from the (fd)-model (without OSM effects) and experiment.
- Table XIV. The transition rates in ^{38}Cl . Experimental rates are compared to ordinary OSM, OSM with effective moments from the (fd)-model (without OSM effects), and the rates calculated in the (fd)-model.

Figure Captions

- Figure 1. Diagram (a) represents the 2hw renormalization of Kuo and Brown. Diagram (b) represents the 0hw ladder terms which have 2hw effects included at each vertex. Diagram (c) represents the renormalization with respect to a ^{32}S core and 0hw and 2hw effects are included at each vertex. Diagram (d) represents a typical double counting term generated in the renormalization with respect to a ^{32}S core. These terms are eliminated because the single-particle energies are taken from experiment.
- Figure 2. Diagram (a) is a representation of the interaction energy of ^{38}Cl with respect to a ^{36}S core. Diagram (b) is a representation of the interaction energy of ^{40}K with respect to a ^{40}Ca core.
- Figure 3. Diagram (a) yields the (pp)-nucleus' largest (uncancelled) second order contribution to the violation of the (pp)- to (ph)-Racah transformation. Diagram (b) yields the (ph)-nucleus' largest (uncancelled) second order contribution to the violation of the (pp)- to (ph)-Racah transformation.
- Figure 4. Diagram (a) represents the interaction energy of ^{38}Cl with respect to a ^{36}S core. Diagram (b) represents the interaction energy of ^{39}Ar with respect to a ^{36}S core. Diagram (c) represents the interaction energy of ^{40}K with respect to a ^{36}S core. Diagram (d) represents the interaction energy of ^{41}Ca with respect to a ^{36}S core.

Table I.

	Bare	RKB	RKB+Δ
$(d^2_{01} v d^2_{01})$	-0.43	-1.03	-1.06
$(d^2_{10} v d^2_{10})$	-0.29	-0.18	-0.38
$(d^2_{21} v d^2_{21})$	-0.27	0.26	0.39
$(d^2_{30} v d^2_{30})$	0.17	-1.84	-1.73

	Bare	RKB	RKB+pp	RKB+pp+Δ
$(f^2_{01} v f^2_{01})$	-0.87	-1.81	-2.60	-2.79
$(f^2_{10} v f^2_{10})$	-0.23	-0.53	-1.82	-2.08
$(f^2_{21} v f^2_{21})$	-0.66	-0.79	-1.05	-1.00
$(f^2_{30} v f^2_{30})$	-0.21	-0.21	-1.07	-1.22
$(f^2_{41} v f^2_{41})$	-0.30	-0.09	-0.24	-0.16
$(f^2_{50} v f^2_{50})$	-0.60	-0.50	-1.25	-1.24
$(f^2_{61} v f^2_{61})$	-0.12	0.23	0.13	0.22
$(f^2_{70} v f^2_{70})$	-2.18	-2.20	-2.20	-2.08

	Bare	RKB	RKB+Δ
$(df_{20} v df_{20})$	-3.62	-3.11	-2.93
$(df_{21} v df_{21})$	-0.35	0.28	0.19
$(df_{30} v df_{30})$	-1.90	-1.63	-1.40
$(df_{31} v df_{31})$	-0.32	0.07	0.08
$(df_{40} v df_{40})$	-0.87	-0.88	-0.86
$(df_{41} v df_{41})$	-0.22	0.34	0.49
$(df_{50} v df_{50})$	-2.22	-1.30	-1.37
$(df_{51} v df_{51})$	-1.11	-0.59	-0.53

	Bare	RKB
$(d^2_{01} v f^2_{01})$	1.92	2.62
$(d^2_{10} v f^2_{10})$	-1.27	-0.45
$(d^2_{21} v f^2_{21})$	0.36	0.45
$(d^2_{30} v f^2_{30})$	-0.30	-0.24

Table II. Experimental $^{38}\text{Cl}/^{40}\text{K}$ Differences.

(a) Energies

J	$E_J(^{40}\text{K})$	$-E_J(^{38}\text{Cl})$	$-T_J^J(E_I(^{38}\text{Cl}))$	ΔE_J^{exp}
2	1.36	1.69	-1.30	0.06
3	0.59	0.93	-0.54	0.05
4	0.56	0.38	-0.47	0.09
5	1.45	1.01	1.36	0.09

(b) Multipoles

λ	$a_\lambda(^{40}\text{K})$	$-(-)^{\lambda+1}a_\lambda(^{38}\text{Cl})$	$\Delta a_\lambda^{\text{exp}}$
0	1.00	-0.92	0.08
1	0.17	-0.15	0.02
2	0.39	-0.39	0.00
3	-0.05	0.04	-0.01

Table III. Single Particle Energies for OSM.

	^{40}K	^{38}Cl
$\epsilon_{p_{1/2}}$	4.0	2.0
$\epsilon_{f_{5/2}}$	6.0	3.0
$\epsilon_{p_{3/2}}$	2.0	0.5
$\epsilon_{f_{7/2}}$	0.0	0.0
$\epsilon_{d_{3/2}}$	0.0	0.0
$\epsilon_{s_{1/2}}$	-2.5	-4.8
$\epsilon_{d_{5/2}}$	-5.4	-7.8

Table IV. v_{eff} for ^{38}Cl and ^{40}K

	(fd)-model		
	v_{eff} with no OSM effects	v_{eff} for mass-40	v_{eff} for mass-38
$(f_{7/2}^{\nu} d_{3/2}^{\pi} J=2 v_{\text{eff}} f_{7/2}^{\nu} d_{3/2}^{\pi} J=2)$	-1.37	-1.26	-1.32
$(f_{7/2}^{\nu} d_{3/2}^{\pi} J=3 v_{\text{eff}} f_{7/2}^{\nu} d_{3/2}^{\pi} J=3)$	-0.66	-0.68	-0.68
$(f_{7/2}^{\nu} d_{3/2}^{\pi} J=4 v_{\text{eff}} f_{7/2}^{\nu} d_{3/2}^{\pi} J=4)$	-0.19	-0.07	-0.14
$(f_{7/2}^{\nu} d_{3/2}^{\pi} J=5 v_{\text{eff}} f_{7/2}^{\nu} d_{3/2}^{\pi} J=5)$	-0.95	-0.66	-0.77

Table V. OSM Results for $^{38}\text{Cl}/^{40}\text{K}$ Differences.

(a) Energies				
J	$E_J(^{40}\text{K})$	$-E_J(^{38}\text{Cl})$	$-T(E_J(^{38}\text{Cl}))$	ΔE_J^{OSM}
2	1.79	1.96	-1.96	-0.17
3	0.66	+1.11	-0.82	-0.16
4	0.61	-0.51	-0.64	-0.03
5	1.56	+1.51	-1.60	-0.04

(b) Multipoles

λ	$\alpha_\lambda(^{40}\text{K})$	$-(-)^{\lambda+1}\alpha_\lambda(^{38}\text{Cl})$	$\Delta\alpha_\lambda^{\text{OSM}}$
0	1.13	-1.21	-.08
1	.10	-.05	.05
2	.49	-.50	-.01
3	-.09	.06	-.03

Table VI. (fd)-Model Results for $^{38}\text{Cl}/^{40}\text{K}$ Differences
for the Case v_{eff} Obeys the Racah Transform.

(a) Energies

J	$E_J(^{40}\text{K})$	$-E_J(^{38}\text{Cl})$	$-T(E_J(^{38}\text{Cl}))$	ΔE_J^{fd}
2	1.12	1.21	-0.96	0.16
3	0.67	0.73	-0.54	0.13
4	0.60	0.44	-0.47	0.13
5	1.10	0.80	-1.01	0.09

(b) Multipoles

λ	$\alpha_\lambda(^{40}\text{K})$	$-(-)^{\lambda+1}\alpha_\lambda(^{38}\text{Cl})$	$\Delta\alpha_\lambda^{\text{fd}}$
0	0.87	-0.75	0.12
1	0.07	-0.09	-0.02
2	0.23	-0.23	0.00
3	-0.02	0.01	-0.01

Table VII. Combined OSM and (fd)-Model Differences.

(a) Energies

J	ΔE_J^{OSM}	ΔE_J^{fd}	$\Delta E_J^{\text{OSM}} + \Delta E_J^{\text{fd}}$	ΔE_J^{exp}
2	-0.14	0.16	0.02	0.06
3	-0.16	0.13	-0.03	0.05
4	-0.03	0.13	0.10	0.09
5	-0.04	0.09	0.05	0.09

(b) Multipoles

λ	$\Delta \alpha_\lambda^{\text{OSM}}$	$\Delta \alpha_\lambda^{\text{fd}}$	$\Delta \alpha_\lambda^{\text{OSM}} + \Delta \alpha_\lambda^{\text{fd}}$	$\Delta \alpha_\lambda^{\text{exp}}$
0	-0.08	0.12	0.04	0.08
1	0.05	-0.02	0.03	0.02
2	-0.01	0.00	-0.01	0.00
3	-0.04	-0.01	-0.05	-0.01

Table VIII. (fd)-Model Results for ^{38}Cl and ^{40}K
 Differences for the Case v_{eff} Does Not
 Obey the Racah Transform.

(a) Energies

J	$E(^{40}\text{K})$	$-E_J(^{38}\text{Cl})$	$-T(E_J(^{38}\text{Cl}))$	ΔE_J
2	0.85	1.19	-0.78	0.07
3	0.45	0.75	-0.41	0.04
4	0.54	0.39	-0.46	0.08
5	1.03	0.65	-1.00	0.03

(b) Multipoles

λ	$\alpha_\lambda(^{40}\text{K})$	$-(-)^{\lambda+1}\alpha_\lambda(^{38}\text{Cl})$	$\Delta\alpha_\lambda$
0	0.74	-0.69	0.05
1	0.14	-0.15	-0.01
2	0.20	-0.21	-0.01
3	-0.04	0.03	-0.01

Table IX.

	v_{eff} for mass-42	v_{eff} for mass-48
$(f_{7/2}^2 \ J=0 \ T=1 v f_{7/2}^2 \ J=0 \ T=1)$	-2.60	-2.25
$(f_{7/2}^2 \ J=1 \ T=0 v f_{7/2}^2 \ J=1 \ T=0)$	-1.82	-1.37
$(f_{7/2}^2 \ J=2 \ T=1 v f_{7/2}^2 \ J=2 \ T=1)$	-1.05	-0.75
$(f_{7/2}^2 \ J=3 \ T=0 v f_{7/2}^2 \ J=3 \ T=0)$	-1.07	-0.50
$(f_{7/2}^2 \ J=4 \ T=1 v f_{7/2}^2 \ J=4 \ T=1)$	-0.24	0.05
$(f_{7/2}^2 \ J=5 \ T=0 v f_{7/2}^2 \ J=5 \ T=0)$	-1.25	-0.54
$(f_{7/2}^2 \ J=6 \ T=1 v f_{7/2}^2 \ J=6 \ T=1)$	0.13	0.27
$(f_{7/2}^2 \ J=7 \ T=0 v f_{7/2}^2 \ J=7 \ T=0)$	-2.20	-1.94

Table X.

(a) Energies				
J	$E_J(^{48}\text{Sc})$	$-E_J(^{42}\text{Sc})$	$-T_J^J(E_I(^{42}\text{Sc}))$	ΔE_J
0	5.01	3.32	-5.47	-0.46
1	2.69	1.56	-2.88	-0.17
2	0.85	1.36	-0.59	0.26
3	0.89	1.13	-1.04	-0.15
4	0.25	0.28	-0.58	-0.33
5	0.44	1.23	-0.73	-0.29
6	-0.05	0.08	-0.50	-0.55
7	1.13	1.95	-1.53	-0.40

(b) Multipoles			
λ	$\alpha_\lambda(^{48}\text{Sc})$	$-(-)^{\lambda+1}\alpha_\lambda(^{42}\text{Sc})$	$\Delta\alpha_\lambda^{fd}$
0	+0.73	-1.05	-0.32
1	-0.22	0.07	-0.14
2	0.65	-0.59	0.06
3	-0.12	0.18	0.06
4	0.36	0.38	-0.02
5	-0.11	0.22	0.11
6	0.22	-0.25	-0.03
7	0.03	-0.07	-0.04

Table XI.

λ	$\Delta\alpha_{\lambda}^{fd}$	$\Delta\alpha_{\lambda}^{OSM}$	$\Delta\alpha_{\lambda}^{exp}$
0	-0.32	-0.37	-0.54
1	-0.14	-0.08	-0.16
2	0.06	0.05	0.03
3	0.06	0.06	0.02
4	-0.02	-0.10	-0.08
5	0.11	0.09	0.10
6	-0.03	-0.09	-0.07
7	-0.04	0.04	-0.06

Table XII.

	Experimental Energy (in MeV)	Present (fd)-model Energy (in MeV)
0_1^+	0.00	0.00
0_2^+	3.35	4.04
0_3^+	5.20	6.89

Table XIII. Transition Rates in ^{40}K .

	Experiment	OSM	OSM with $\mu_{f7/2}^{\nu}$ and $\mu_{d3/2}^{\pi-1}$ from fd model	OSM with $\mu_{f7/2}^{\nu}$ and $\mu_{d3/2}^{\pi-1}$ from experiment
BM1($2^{-} \rightarrow 3^{-}$)	$0.21 \pm 0.04 \text{ nm}^2$	0.17	0.10	0.23
BM1($3^{-} \rightarrow 4^{-}$)	$0.27 \pm 0.01 \text{ nm}^2$	0.22	0.15	0.26
BM1($4^{-} \rightarrow 5^{-}$)	$0.06 \pm 0.01 \text{ nm}^2$	0.04	0.01	0.06
$\mu(4^{-})$	-1.29 nm	-1.73	-1.35	-1.31

Table XIV. Transition Rates in ^{38}Cl .

	Experiment	OSM	OSM with $\mu_{f7/2}^{\nu}$ and $\mu_{d3/2}^{\pi}$ from OSM	(fd)-Model
BM1($2^{-} \rightarrow 3^{-}$)	0.35 ± 0.10	0.19	0.11	0.14
BM1($3^{-} \rightarrow 4^{-}$)	$0.08^{+0.07}_{-0.03}$	0.32	0.08	0.27
BM1($4^{-} \rightarrow 5^{-}$)	$0.16^{+0.14}_{-0.06}$	0.18	0.05	0.15

Fig. I.

Fig. 2.

(a.)

(b.)

Fig. 3.