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A COMPARISON OF EXPERIMENTAL AND
CALCULATED PROMPT NEUTRON LIFETIMES AND
CENTRAL REACTIVITY COEFFICIENTS IN
ZPR-III ASSEMBLIES AND
THEIR RELATIONSHIP TO OTHER REACTOR PARAMETERS

by

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ABSTRACT

This report is a continuation of a comparison of experimental parameters of ZPR-III fast reactor assemblies with those calculated through use of a cross-section set derived by the author from that of Yiftah, Okrent, and Moldauer,⁽¹⁾ and initially used for a study of the critical sizes of ZPR-III assemblies.⁽²⁾ This cross-section set has also been used to study central fission ratios in ZPR-III assemblies.⁽³⁾ In the present report, the analysis is extended to prompt neutron lifetimes and the relative reactivity effects of the substitution of materials at the reactor centers. The results of the present work are examined in conjunction with those of the critical size and fission ratio studies.

I. INTRODUCTION

The determination of the reactivity effect of the substitution of a material in a reactor is comparatively straightforward and precise, and this technique has been used extensively in investigations with ZPR-III. The value of such data lies both in their usefulness as spectral indices and also in the information regarding cross sections of the sample material. In particular, some of the data are sensitive to the lower-energy ends of the flux and importance spectra, and thus provide information which cannot be given by such indices as fission ratios.

Another parameter which has been measured in most ZPR-III assemblies is the Rossi- α which, together with the effective delayed-neutron fraction, gives the prompt-neutron lifetime in the reactor. The neutron lifetime in a group is inversely proportional to the average neutron velocity in the group, so that the prompt-neutron lifetime is sensitive to the low-energy end of the spectrum and, hence, is an excellent low-energy spectral index.

In earlier work, the author derived a 16-group cross-section set by modification of that (YOM) of Yiftah, Okrent, and Moldauer⁽¹⁾ and has used this set (listed at Argonne National Laboratory as Set 635) in studies of critical size⁽²⁾ and fission ratios⁽³⁾ with ZPR-III assemblies. In the present work, the applicability of this cross-section set is further examined by comparison of experimental and calculated prompt-neutron lifetimes and the central reactivity effects of a selected number of elements, these elements being chosen on the basis of: (a) their importance in reactor construction; (b) reasonable knowledge of their cross sections; and (c) their usefulness as spectral indices.

In this report, summaries of the results of the k calculations and the fission-ratio comparisons are presented, in addition to prompt neutron lifetimes and central reactivity data. These are all considered in an attempt to obtain an overall picture of the comparison of experimental and calculated fast reactor parameters.

II. DERIVATION OF THE CROSS-SECTION SET

The cross-section set⁽²⁾ used in this work consists of the YOM set with: (a) α values of U^{233} , U^{235} , and Pu^{239} modified to agree with the measurements of Diven and Hopkins;⁽⁴⁾ (b) the ν values of U^{235} modified in accordance with recent measurements;⁽⁵⁻⁷⁾ and (c) the σ (transport) and σ (elastic removal) data for steel and aluminum modified in an approximate manner^(8,9) to allow for the effects of strong resonances in these materials. These modified data are given in Appendix I. The other data are taken without modification from the YOM set with the exception of the fission cross sections of U^{234} and U^{236} , which are taken from ANL Set 179 (see Davey⁽³⁾), and the molybdenum data for which some of the published group cross sections were later corrected by Yiftah through personal communication.

III. SUMMARY OF THE RESULTS OF PREVIOUS CALCULATIONS

A. k Calculations

The compositions and calculated values of k for 23 ZPR-III assemblies are presented in Table I. The k values quoted here differ slightly from those given in Davey⁽²⁾ because corrections of small errors in the previous calculations have now been made. The new values replace those previously quoted.

The DSN neutron transport code in the S4 approximation was used to calculate k effective for each assembly, shape factors being used to convert from the experimental cylindrical geometries to idealized spherical geometries. Corrections were applied for the core heterogeneity and some other small effects.

Table I
ZPR-III REACTOR COMPOSITIONS AND COMPUTED VALUES OF k

ZPR-III Assembly No.	Ratio $\frac{U^{238}}{U^{235}}$	Approximate Volume Percentage of Material in Core									Computed k_{eff}	Deviation from Mean k of 1.000	Equivalent Deviation in Critical Mass (%)
		U ²³⁵	U ²³⁸	Steel	Al	Na	C	O	Mo	Zr			
23	0.07	9.3	0.7	9.2	42.8						1.004	+0.004	+1.8
2A	0.07	14.0	1.0	27.8	31.4						1.005	+0.005	+1.7
6F	1.1	14.0	15.9	12.4	31.4						1.003	+0.003	+1.1
5	1.1	14.0	15.9	12.3	31.4						1.005	+0.005	+2.1
29	2.0	5.0	10.0	24.9	24.4			14.5			1.003	+0.003	+1.6
30	1.5	5.9	9.0	24.7	23.4			7.3			1.009	+0.009	+5.1
31	1.6	5.8	9.1	24.8	23.5						1.006	+0.006	+3.4
34	2.2	4.7	10.3	24.8	25.5		9.1				0.997	-0.003	-1.5
9A	3.2	11.7	38.0	14.4	21.5						0.998	-0.002	-0.9
20	3.1	6.1	19.0	14.5	25.1				5.0	4.3	0.992	-0.008	-4.0
41	4.9	6.0	29.1	14.2	17.9						1.002	+0.002	+1.0
10	4.9	11.9	57.9	19.5							0.999	-0.001	-0.5
11	7.5	9.5	71.7	9.2							0.994	-0.006	-4.2
24	9.6	7.6	72.9	9.4							0.995	-0.005	-4.7
25	10.3	7.2	74.2	9.2							0.991	-0.009	-6.5
35	0.07	4.1	0.3	50.0		35.6		4.2			0.994	-0.006	-2.9
36	5.3	9.4	49.5	12.8		18.2					0.994	-0.006	-2.6
33	0.07	9.3	0.7	64.2		18.2					1.014	+0.014	+5.9
32	0.07	9.3	0.7	81.8							1.015	+0.015	+6.3
16	5.3	9.4	50.1	9.2			18.1				0.999	-0.001	-0.5
12	3.8	9.4	35.4	9.2			32.0				0.998	-0.002	-1.2
17	2.2	9.4	20.6	9.2			45.7				0.998	-0.002	-0.9
14	0.07	9.4	0.7	9.2			63.9				0.995	-0.005	-1.9

The results in Table I show that the cross-section set is quite successful in that 14 of the calculated k values lie within $\pm 1/2\%$ of the mean value of 1.000, 7 lie between $1/2$ and 1% , and 2 lie between 1 and $1-1/2\%$. This corresponds to the prediction of the critical masses of 19 of the assemblies to better than 5% accuracy, with none being calculated to worse than $6-1/2\%$ accuracy.

If the nuclear data for a particular nuclide were appreciably in error, then it might be expected there would be a tendency towards an increasingly inaccurately calculated value of k as the fraction of that nuclide increased. But, apart possibly from the tendency for high k values in Assemblies 32 and 33 which have high steel contents, no such trends are apparent. However, the absence of such trends does not necessarily mean that the nuclear data are correct, since it is possible that a number of errors could cancel fortuitously in a k calculation.

B. Fission Ratios

The calculated and experimental fission ratios are compared in Tables II, III, and IV. These are basically the data given in Davey,⁽³⁾ but here the experimental results have been corrected for inelastic scattering in the fission chamber walls. In addition, the ratios are given relative to Pu^{239} in order to facilitate comparison with the central reactivity data which are normalized to Pu^{239} .

Table II

COMPARISON OF EXPERIMENTAL AND CALCULATED U^{235}/Pu^{239} AND
 U^{233}/Pu^{239} CENTRAL FISSION RATIOS IN ZPR-III ASSEMBLIES

ZPR-III Assembly No.	U^{235}/Pu^{239}				U^{233}/Pu^{239}			
	Calculated Ratio (C)	Experimental Ratio ^(a) (E)	(E/C) (Average = 1.06)	Deviation from Average (E/C) (%)	Calculated Ratio (C)	Experimental Ratio ^(a) (E)	(E/C) (Average = 1.00)	Deviation from Average (E/C) (%)
2A	0.775	-			1.213	-		
5	0.776	-			1.218	-		
6F	0.776	-			1.218	-		
23	0.786	0.847	1.08	+2	1.225	1.25	1.02	+2
9A	0.791	-			1.249	-		
10	0.792	0.820	1.04	-2	1.257	1.25	0.99	-1
36	0.800	0.840	1.05	-1	1.267	1.23	0.97	-3
33	0.799	0.826	1.03	-3	1.258	1.25	0.99	-1
11	0.805	0.855	1.06	0	1.278	1.29	1.01	+1
32	0.805	0.833	1.03	-3	1.269	1.26	0.99	-1
31	0.813	0.847	1.04	-2	1.276	1.29	1.01	+1
41	0.814	-			1.283	-		
24	0.816	0.862	1.06	0	1.294	1.24	0.96	-4
25	0.818	0.855	1.05	-1	1.297	-		
20	0.824	0.870	1.06	0	1.294	1.32	1.02	+2
16	0.827	-			1.295	-		
30	0.833	0.893	1.07	+1	1.299	1.33	1.02	+2
12	0.847	0.909	1.07	+1	1.311	1.33	1.01	+1
17	0.865	0.926	1.07	+1	1.325	1.35	1.02	+2
34	0.870	0.935	1.07	+1	1.344	1.36	1.01	+1
29	0.873	0.943	1.08	+2	1.347	1.39	1.03	+3
35	0.879	0.917	1.04	-2	1.355	-		
14	0.896	0.952	1.06	0	1.342	1.38	1.03	+3

^(a)The corrections for wall-thickness effects are zero in these cases.

Average E/C for U^{235}/Pu^{239} = $1.06 \pm 1/2\%$

Average E/C for U^{233}/Pu^{239} = $1.00 \pm 1/2\%$

Table III

COMPARISON OF EXPERIMENTAL AND CALCULATED U^{234}/Pu^{239} AND
 Pu^{240}/Pu^{239} CENTRAL FISSION RATIOS IN ZPR-III ASSEMBLIES

ZPR-III Assembly No.	U^{234}/Pu^{239}				Pu^{240}/Pu^{239}			
	Calculated Ratio (C)	Experimental Ratio ^(a) (E)	(E/C) (Average = 0.93)	Deviation from Average (E/C) (%)	Calculated Ratio (C)	Experimental Ratio ^(a) (E)	(E/C) (Average = 1.00)	Deviation from Average (E/C) (%)
2A	0.388	-			0.408	-		
5	0.379	-			0.397	-		
6F	0.377	-			0.393	-		
23	0.375	0.354	0.94	+1	0.394	-		
9A	0.335	-			0.345	-		
10	0.322	0.282	0.88	-5	0.327	-		
36	0.308	0.273	0.89	-4	0.313	0.294	0.94	-6
33	0.331	0.318	0.96	+3	0.342	0.344	1.01	+1
11	0.295	0.266	0.90	-3	0.296	-		
32	0.321	0.317	0.99	+6	0.330	0.331	1.00	0
31	0.311	0.294	0.95	+2	0.320	0.276	0.86 ^(b)	
41	0.292	-			0.297	-		
24	0.273	0.221	0.81	-12	0.273	-		
25	0.267	0.225	0.84	-9	0.265	-		
20	0.285	0.271	0.95	+2	0.290	0.300	1.03	+3
16	0.291	-			0.297	-		
30	0.292	0.280	0.96	+3	0.302	-		
12	0.290	0.277	0.96	+3	0.300	-		
17	0.296	0.287	0.97	+4	0.310	-		
34	0.261	0.240	0.92	-1	0.269	0.264	0.98	-2
29	0.254	0.254	1.00	+7	0.262	0.283	1.08	+8
35	0.243	0.221	0.91	-2	0.248	0.238	0.96	-4
14	0.309	0.302	0.98	+5	0.329	-		

^(a)These experimental data have been increased by 4% to correct for the effect of inelastic scattering in the fission chamber walls.

^(b)Rejected as more than three standard deviations from the mean.

Average E/C for U^{234}/Pu^{239} = $0.93 \pm 1/2\%$

Average E/C for Pu^{240}/Pu^{239} = $1.00 \pm 2\%$

Table IV

COMPARISON OF EXPERIMENTAL AND CALCULATED U^{236}/Pu^{239} AND
 U^{238}/Pu^{239} CENTRAL FISSION RATIOS IN ZPR-III ASSEMBLIES

ZPR-III Assembly No.	U^{236}/Pu^{239}				U^{238}/Pu^{239}			
	Calculated Ratio (C)	Experimental Ratio ^(a) (E)	(E/C) (Average = 0.98)	Deviation from Average (E/C) (%)	Calculated Ratio (C)	Experimental Ratio ^(a) (E)	(E/C) (Average = 0.98)	Deviation from Average (E/C) (%)
2A	0.140	-			0.0654	-		
5	0.133	-			0.0616	-		
6F	0.132	-			0.0608	-		
23	0.138	-			0.0645	0.0620	0.96	-2
9A	0.105	-			0.0457	-		
10	0.0942	-			0.0395	0.0390	0.99	+1
36	0.0896	0.084	0.94	-4	0.0379	0.0372	0.98	0
33	0.105	0.104	0.99	+1	0.0454	0.0428	0.94	-4
11	0.0805	-			0.0328	0.0328	1.00	+2
32	0.0966	0.097	1.00	+2	0.0406	0.0406	1.00	+2
31	0.0976	0.095	0.97	-1	0.0423	0.0402	0.95	-3
41	0.0855	-			0.0356	-		
24	0.0706	-			0.0280	0.0287	1.03	+5
25	0.0679	-			0.0267	0.0270	1.01	+3
20	0.0857	-			0.0362	0.0358	0.99	+1
16	0.0893	-			0.0384	-		
30	0.0941	0.094	1.00	+2	0.0411	0.0412	1.00	+2
12	0.0957	-			0.0429	0.0436	1.02	+4
17	0.106	-			0.0494	0.0490	0.99	+1
34	0.0820	0.075	0.91 ^(b)		0.0352	0.0342	0.97	-1
29	0.0814	0.082	1.01	+3	0.0358	0.0363	1.01	+3
35	0.0732	-			0.0313	0.0298	0.95	-3
14	0.123	-			0.0599	0.0565	0.94	-4

(a) The U^{236}/Pu^{239} and U^{238}/Pu^{239} experimental fission ratios have been increased by 6% and 8% respectively to correct for the effects of in-elastic scattering in the chamber walls.

(b) Rejected as more than three standard deviations from the mean

Average E/C for U^{236}/Pu^{239} = $0.98 \pm 2\%$

Average E/C for U^{238}/Pu^{239} = $0.98 \pm 2\%$

The data in Tables II, III, and IV show that calculated and measured fission rates for a number of isotopes are in fair agreement. The relative fission rates of Pu^{239} and U^{233} are calculated to about $\pm 1\text{-}1/2\%$ accuracy, and those of Pu^{240} , U^{236} , and U^{238} relative to either Pu^{239} or U^{233} are calculated to about $\pm 5\%$ accuracy. The calculated fission rates of U^{235} and U^{234} , however, are about 6% low and 7% high, respectively, relative to those of the other five isotopes.

The interpretation of these results is entirely dependent upon the accuracy of the fission chamber measurements; for a detailed discussion see Davey.⁽³⁾ However, the basic difficulty lies in the lack of agreement among the data for Pu^{239} , U^{233} , and U^{235} . Their relative fission ratios should be well known, since they are only slightly dependent upon the calculated spectrum. The disagreement could arise if the fission cross-sections of both Pu^{239} and U^{233} relative to that of U^{235} were in error by about 6%, or, more probably, if the calibrations of both the Pu^{239} and the U^{233} chambers were in error relative to the U^{235} chamber. The agreement between experiment and calculation for the five isotopes Pu^{239} , U^{233} , Pu^{240} , U^{236} , and U^{238} is circumstantial evidence for the correctness of these experimental data, and in this event it would appear that the calculated spectra as defined by the threshold fissionable isotopes are reasonably correct. However, if the Pu^{239} and U^{233} data are rejected, then there is disagreement between the calculated and measured fission ratios of Pu^{240} , U^{236} , and U^{238} relative to U^{235} , which would indicate that the calculated spectra are too hard.

Clearly, the question of the accuracy of the intercalibration of the fission chambers, particularly with U^{233} , U^{235} , and Pu^{239} , must be clarified before the fission chamber data can be interpreted. In any event, it should be remembered that the threshold fissile materials only give information regarding the fractions of flux above and below the threshold energies; thus, they cannot define neutron spectra completely. In particular, they cannot give information on the low-energy end of the spectrum.

IV. PROMPT-NEUTRON LIFETIMES

The prompt-neutron lifetime L may be calculated from the following expression:⁽⁹⁾

$$L = \frac{\int_V \left(\sum_i \frac{\phi_i \phi_i^*}{v_i} \right) dV}{\int_V \left(\sum_i N v_i \sigma_{fi} \phi_i \right) \left(\sum_i \chi_i \phi_i^* \right) dV},$$

where the numerator and denominator are integrated over the reactor volume V . The summation is over all energy groups, and v_i , $N \sigma_{fi}$, and χ_i ,

respectively, are the average neutron velocity, the macroscopic fission cross section, and the fraction of the fission spectrum in group i .

Since the fission is principally in U^{235} , and since the adjoint spectrum does not vary greatly from reactor to reactor, the denominator is not very sensitive to neutron spectrum. In the numerator, however, we have a $1/v$ term which strongly emphasizes the low-energy end of the spectrum and which makes the prompt-neutron lifetime a sensitive low-energy spectral index.

The prompt-neutron lifetimes obtained in this report were not obtained from the above equation, but were obtained by means of the DSN neutron transport code of Carlson.⁽¹⁰⁾ The calculations were made in the S4 approximation on the spherical models used for the k calculations.

Two calculations were made for each assembly: (a) the k calculation described in Section III A; and (b) an α -calculation in which the neutron flux is assumed separable with respect to time and the time variation is given by a factor exponential (α_t).

The prompt-neutron lifetime is then given by $(k - 1)/\alpha$.

The values of k and α obtained by solution of the real or the adjoint (importance) equations should be identical, and α and k were obtained by solution of the real and adjoint equations, respectively. This gave the real and adjoint spectra needed for the central reactivity calculations.

The results of the calculations and the experimental prompt-neutron lifetimes⁽¹¹⁾ are given in Table V. It should be noted that the values of $(k - 1)$ quoted in Table V are not, in general, identical with those obtained from the k values given in Table I. This arises because the shape factor used to give the spherical dimensions used in the k and α calculations was in some cases not quite that given by the procedure outlined in Davey.⁽²⁾

In these cases, the k values of Table I were obtained by small corrections applied to the calculated k values to give those appropriate to the correct shape factor. The procedure of using slightly inaccurate shape factors was sometimes needed in order to obtain a value of $(k - 1)$ or α which was reasonably large and hence was accurately known, i.e., was much larger than the convergence criterion applied to the calculation.

Discussion of the Results

The data of Table V show very clearly that the prompt-neutron lifetime calculated with ANL Set 635 is grossly in error for every one of the 21 cases examined, and in every case the calculated lifetime is too small. The assemblies in Table V are listed in order of increasing softness of spectrum as defined by increasing $\sigma(n\alpha)$ for boron-10 (see Table VI), and

there is no indication that the discrepancy varies systematically with the degree of softness of spectrum. Indeed, the discrepancy cannot be associated with any one nuclide and is moderately constant at an average value of 34%, since in 20 of the cases the discrepancy lies between extreme values of 24% and 48%. In one case, Assembly 6F, there is a discrepancy of 64%, but this is suspiciously high, for the core composition of 6F and 5 are identical and the discrepancy in the latter case is only 25%.

Table V

COMPARISON OF EXPERIMENTAL AND CALCULATED PROMPT-NEUTRON LIFETIMES IN ZPR-III ASSEMBLIES

ZPR-III Assembly Number	Calculated $k - 1^{(a)}$ ($\times 10^5$)	Calculated $\rho^{(a)}$ ($\times 10^{-2}$)	Calculated Prompt-neutron Lifetime [$C = (k-1)/\rho$] $\times 10^8 \text{ sec}$	Experimental Prompt-neutron Lifetime (E) ($\times 10^8 \text{ sec}$)	E/C (average = 1.34)	% Deviation from Average (D)
2A	-397	-415	9.57	-	-	-
5	-603	-849	7.10	8.9	1.25	-7
6F	276	614	4.50	7.4	1.64	+22
23	192	198	9.70	12.0	1.24	-7
9A	1294	2275	5.69	8.1	1.42	+6
10	690	1393	4.95	6.9	1.39	+4
36	179	318	5.63	7.9	1.40	+5
33	1274	1288	9.89	12.4	1.25	-7
11	229	448	5.11	7.0	1.37	-2
32	1503	1477	10.2	12.6	1.24	-7
31	980	742	13.2	16.4	1.24	-7
41	150	158	9.49	13.2	1.39	+4
24	232	377	6.15	8.6	1.40	+5
25	187	301	6.21	8.1	1.30	-3
20	-539	-413	13.1	16.2	1.24	-7
16	449	699	6.42	-	-	-
30	1273	907	14.0	17.8	1.27	-5
12	-1223	-1513	8.08	10.7	1.32	-1
17	-571	-548	10.4	13.7	1.32	-1
34	159	94	16.9	25.0	1.48	+10
29	666	380	17.5	23.2	1.33	-1
35A ^(b)	1692	529	32.0	42.0 ^(b)	1.22	-9
35 ^(b)	-114	-29	39.0			
14	-1345	-936	14.4			

(a) The specified convergence for the k and ρ calculations in 10^{-5} in k and 10^2 in ρ , so that the maximum accuracy is unity in the quoted values of $k - 1$ and ρ . The actual convergence may be appreciably less than that specified due to uncertainty in the meaning of the convergence criterion in the DSN code.

(b) The lifetime was measured with a cylindrical assembly ($L/D \sim 1$), with the radial reflector of 35A and axial reflector of 35. The calculated lifetimes are therefore weighted 2 to 1 to give an effective lifetime for the experimental assembly.

The explanation of an approximately constant discrepancy between calculated and measured prompt-neutron lifetimes poses considerable problems. Brunson⁽¹¹⁾ has discussed the experimental data extensively, and although there may be some systematic errors, e.g., in the estimation of the

effective delayed-neutron fraction, it seems certain that the discrepancy is much too large to be attributed to experimental errors. Therefore, it appears that the calculated lifetimes must be erroneous, and hence all the calculated spectra are too hard. Since a large number of diluents are used in these assemblies, e.g., U, steel, Al, Na, O, and C, it would seem possible that there are systematic features in the construction of the cross-section set which give incorrect data for an appreciable number of the materials. Also, if the problem lies in the derivation of down-scattering cross sections, then both inelastic (e.g., U and steel) and elastic (e.g., O and C) scattering data must be affected.

V. CENTRAL REACTIVITIES

The change in reactivity caused by the insertion of a sample in a reactor at a position of zero flux gradient (i.e., the reactor center) is proportional (if perturbation theory is applicable) to the following expression:

$$\begin{bmatrix} \phi_1^* & \dots & \phi_{16}^* \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1,16} \\ A_{21} & A_{22} & \dots & A_{2,16} \\ \vdots & \vdots & \ddots & \vdots \\ A_{16,1} & \dots & \dots & A_{16,16} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \vdots \\ \vdots \\ \phi_{16} \end{bmatrix} = 0,$$

where the perturbation matrix $[A]$ only contains terms constructed from the nuclear data for the sample material. In $[A]$, all terms in the j^{th} column are constructed solely from the nuclear data of group j in the cross-section set. The diagonal term A_{jj} is given by:

$$A_{jj} = -(\text{removal from group } j) + (\text{production in group } j),$$

i.e.,

$$A_{jj} = -[\sigma_{fj} + \sigma(n\gamma)_j + \sigma_{erj} + \sigma_{inj}] + \nu_j \sigma_{fj} \chi_j,$$

where σ_{inj} is the inelastic removal cross section for group j . The non-diagonal terms in column j are neutron-production terms; thus, A_{ij} is the neutron production in group i resulting from fission and scattering in group j , i.e., we have

$$A_{ij} = +(\chi_i \nu_j \sigma_{fj}) + (\sigma \text{ scattering } j \rightarrow i).$$

As we have no upscattering at fast reactor neutron energies, all the terms above the diagonal are of the form $\chi \nu \sigma_F$.

To illustrate the effects of scattering, capture, etc., we will consider some simplified cases.

First consider a pure elastic scatterer, i.e., there is no capture, fission, or inelastic scattering. Carbon comes close to this ideal, as it has no capture or fission, and inelastic scattering only occurs in a few high-energy groups. The carbon perturbation matrix in Appendix II will illustrate the type of matrix obtained. With the group width used here, elastic scattering only transfers neutrons from one group to the one immediately below it.

All the diagonal terms A_{jj} are negative due to scattering out of group j , the only process occurring, and $A_{j+1,j}$ is equal in magnitude to A_{jj} but is of opposite sign. All other terms are zero. The perturbation expression is thus the sum of terms of the form

$$\phi_1^* (A_{11} \phi_1) + \phi_2^* (A_{21} \phi_1 + A_{22} \phi_2) + \dots \quad .$$

Since A_{21} and A_{22} are of opposite sign and almost of the same magnitude (elastic scattering cross sections do not vary rapidly with energy), the magnitude of each term except the first is determined largely by the difference between the fluxes in successive groups. Consequently, the terms in brackets are negative for groups above the flux peak and positive for groups below the peak, and the sign and magnitude of the perturbation effect is fairly sensitive to the relative importance of neutrons above and below the flux peak.

Inelastic scattering effects are similar, of course, to elastic scattering, but since inelastic scattering transfers neutrons over many groups and not just between adjacent ones, there is a less clear correlation between the perturbation and the shape of the neutron spectrum.

The effects of capture (or removal by n, α reactions) are easiest to see for the case in which there is no fission and in which the scattering cross sections are negligible compared with that for capture. Boron-10 in the lower energy groups is a good example of this. In the ideal case, all the diagonal terms are negative and just equal to $\sigma(n\gamma)$ [or $\sigma(n\alpha)$ for B^{10}] for each group, and all the nondiagonal terms are zero. The perturbation is then negative and is the sum over all groups of $-\phi_j^* \sigma(n\gamma)_j \phi_j$. It should be noted that the (n, γ) cross sections of most heavy absorbers and the $B^{10}(n\alpha)$ cross section increase with the decreasing energy; hence, the reactivity effects of these materials are good low-energy spectral indices.

The cases of the nonthreshold fissile materials, U^{233} , U^{235} , and Pu^{239} , are similar to those of the strong absorbers in that the diagonal terms are by far the most important, but here they are of the form $(\nu - 1) \sigma_F - \sigma(n\gamma)$ instead of $\sigma(n\gamma)$.

Threshold fission introduces positive reactivity effects in the high-energy groups, but as threshold fissile materials are also good low-energy absorbers, the net reactivity effect may be either positive or negative.

The relative central reactivities were calculated by means of a Fortran code written for the IBM 1620 computer by Allen L. Hess.

To compare the results obtained in different assemblies, the calculated reactivities were normalized by equating the calculated Pu^{239} reactivity change to the calculated value of $(\nu - 1 - \alpha) \sigma_f$ for Pu^{239} at the

center of the reactor, all results being presented as millibarns per atom. This normalization procedure was adopted because of the insensitivity of $(\nu - 1 - \alpha)\sigma_f$ for Pu^{239} to the neutron spectrum, but it is not completely satisfactory because errors in the Pu^{239} data, both experimental and calculated, are reflected in all the other data.

The unperturbed, central, real and importance spectra obtained from the k and α calculations were used in the reactivity calculations; this procedure is only justified when the sample is such that it does not perturb the reactor flux.

It is possible to examine this self-shielding effect experimentally by determining reactivity changes as a function of sample size and then extrapolating to determine the change for a very small sample, but none of the experimental data given here have been so corrected. It is possible that this effect is not important for some materials, but the very limited amount of data available shows that corrections should be applied for heavily absorbing materials. Hess *et al.*,⁽¹²⁾ have recently made a study of self-shielding effects with samples of U^{238} , tantalum, and B^{10} . Their data indicate that the experimental B^{10} and tantalum reactivities quoted here should be increased by about 5%, but that the effect on the U^{238} data is less than the experimental errors. It must be emphasized that these results are only indicative of the order of magnitude of self-shielding effects, and the data from other assemblies cannot be corrected on this basis. We therefore accept that the experimental data for heavily absorbing samples may be too low by about 5% due to self-shielding.

In comparing the relative reactivities of U^{233} , U^{235} , and Pu^{239} , any self-shielding effects are probably not important, since the samples are of similar size and such effects would cancel. Also, in some cases, such as carbon and boron-10, the discrepancies between results of experiment and calculation are so much larger than any probable corrections for self-shielding that the general conclusions would be unaffected, but the lack of information on this effect is a serious drawback to the interpretation of the experiments.

Spectrum-averaged values of $(\nu - 1 - \alpha)\sigma_f$ for Pu^{239} and of $\sigma(n\alpha)$ for B^{10} are given in Table VI. The results of the central reactivity calculations are given in Tables VII through XX, and the perturbation matrices are given in Appendix II. The central reactivity data are listed in order of increasing $\sigma(n\alpha)$ for B^{10} , i.e., in order of increasing softness of spectrum.

Table VI

CALCULATED AVERAGE VALUES OF $(\nu - 1 - \alpha)\sigma_F$
FOR Pu^{239} AND $\sigma(n\alpha)$ FOR B^{10}

Assembly Number	Average $(\nu - 1 - \alpha)\sigma_F$ for Pu^{239} (mb)	Average $\sigma(n\alpha)$ for B^{10} (mb)	Assembly Number	Average $(\nu - 1 - \alpha)\sigma_F$ for Pu^{239} (mb)	Average $\sigma(n\alpha)$ for B^{10} (mb)
2A	3478	764	24	3314	965
5	3461	768	25	3308	982
6F	3457	768	20	3357	1018
23	3476	826	16	3372	1034
9A	3392	839	30	3388	1072
10	3363	840	12	3410	1147
36	3356	885	17	3458	1249
33	3398	889	34	3391	1277
11	3334	906	29	3388	1296
32	3380	912	35A	3375	1308
31	3386	964	35	3379	1318
41	3352	965	14	3561	1423

Table VII

COMPARISON OF EXPERIMENTAL AND CALCULATED
CENTRAL REACTIVITIES OF U^{235}

Assembly Number	Central Reactivity (mb)		E/C Av = 1.01	Deviation (%)
	Calculated (C)	Experimental (E)±35		
2A	1812	-		
5	1819	-		
6F	1818	-		
23	1818	1817	1.00	-1
9A	1834	-		
10	1845	-		
36	1857	1847	0.99	-2
33	1779	1782	1.00	-1
11	1879	1838	.98	-3
32	1773	1793	1.01	0
31	1820	1837	1.01	0
41	1865	1925	1.03	+2
24	1907	1875	0.98	-3
25	1918	1925	1.00	-1
20	1850	1918	1.04	+3
16	1898	-		
30	1834	1845	1.01	0
12	1910	1921	1.01	0
17	1905	-		
34	1896	1961	1.03	+2
29	1895	2012	1.06	+5
35	1797	1982	1.10	+9
14	1860	-		

Table VIII

COMPARISON OF EXPERIMENTAL AND CALCULATED
CENTRAL REACTIVITIES OF U^{233}

Assembly Number	Central Reactivity (mb)		E/C Av = 1.01	Deviation (%)
	Calculated (C)	Experimental (E) ± 35		
2A	3323	-	0.99	-2
5	3349	-		
6F	3352	-		
23	3343	3311		
9A	3396	-		
10	3417	-	1.01	0
36	3437	3485		
33	3336	3305		
11	3465	3401		
32	3340	3380		
31	3397	3385	1.00	-1
41	3455	3504	1.01	0
24	3502	3360	0.96	-5
25	3514	3480	0.99	-2
20	3443	-	1.03	+2
16	3509	-		
30	3442	3541		
12	3550	3638		
17	3587	-		
34	3576	3744	1.05	+4
29	3574	3741	1.05	+4
35	3516	-	1.02	+1
14	3651	-		

Table IX

COMPARISON OF EXPERIMENTAL AND CALCULATED
CENTRAL REACTIVITIES OF U^{238}

Assembly Number	Central Reactivity (mb)		E/C Av = 1.04	Deviation (%)	Difference E-C (mb) ± 3
	Calculated (C)	Experimental (E) ± 3			
2A	83	-	0.55	-49	-34
5	33	-			
6F	26	-			
23	75	41			
9A	47	-			
10	-70	-	1.23	+19	-18
36	-79	-97			
33	0	-7			
11	-95	-86			
32	-20	-19			
31	-40	-50	1.25	+21	-10
41	-82	-85	1.04	0	-3
24	-106	-92	0.87	-17	+14
25	-109	-98	0.90	-14	+11
20	-72	-83	1.15	+11	-11
16	-89	-	1.31	+27	-16
30	-52	-68			
12	-81	-78			
17	-56	-			
34	-94	-108			
29	-93	-104	1.12	+8	-11
35	-108	-117	1.08	+4	-9
14	19	-	0.96	-8	+3

Table X

COMPARISON OF EXPERIMENTAL AND CALCULATED
CENTRAL REACTIVITIES OF Th^{232}

Assembly Number	Central Reactivity (mb)		E/C Av = 0.88	Deviation (%)
	Calculated (C)	Experimental (E) ± 4		
2A	-132	-	0.84	-5
5	-171	-		
6F	-173	-		
23	-138	-		
9A	-204	-		
10	-204	-	0.77	-9
36	-208	-175		
33	-164	-126		
11	-206	-		
32	-172	-134		
31	-192	-163	0.78	-9
41	-206	-184	0.85	-3
24	-202	-184	0.89	+1
25	-201	-	0.91	+3
20	-202	-	1.02	+16
16	-221	-		
30	-201	-		
12	-231	-		
17	-233	-		
34	-228	-232	1.00	+11
29	-229	-228	1.00	+11
35	-243	-		
14	-209	-		

Table XI

COMPARISON OF EXPERIMENTAL AND CALCULATED
CENTRAL REACTIVITIES OF B^{10}

Assembly Number	Central Reactivity (mb)		E/C Av = 1.31	Deviation (%)
	Calculated (C)	Experimental (E) ± 13		
2A	-808	-	1.21	-8
5	-767	-		
6F	-758	-		
23	-875	-1057		
9A	-753	-		
10	-706	-	1.32	+1
36	-730	-966		
33	-975	-1226		
11	-694	-940 ^(a)		
32	-1006	-1256		
31	-965	-	1.38	+5
41	-817	-1125		
24	-693	-880		
25	-686	-866		
20	-936	-1244		
16	-869	-	1.26	+1
30	-1099	-		
12	-1068	-1399		
17	-1328	-		
34	-1301	-1594		
29	-1315	-1744	1.23	-6
35 ^(b)	-1600	-2585	1.33	-2
14	-1985	-	1.62	-24

^(a) Average of values of -996 and -885 measured in the nominally identical Assemblies 11 and 22.^(b) The central spectra of Assemblies 35 and 35A are essentially identical. See also footnote (b) to Table V.

Table XII

COMPARISON OF EXPERIMENTAL AND CALCULATED
CENTRAL REACTIVITIES OF TANTALUM

Assembly Number	Central Reactivity (mb)		E/C Av = 1.14	Deviation (%)
	Calculated (C)	Experimental (E)±4		
2A	-152	-	1.02	-11
5	-187	-		
6F	-193	-		
23	-169	-173		
9A	-221	-		
10	-224	-	1.11	-3
36	-233	-259		
33	-188	-216		
11	-235	-259	1.10	-4
32	-195	-		
31	-223	-	1.19	+4
41	-240	-285		
24	-238	-275		
25	-239	-261	1.09	-4
20	-240	-269		
16	-264	-	1.12	-2
30	-254	-		
12	-298	-344		
17	-342	-	1.15	+1
34	-320	-406		
29	-323	-386		
35	-375	-	1.20	+6
14	-467	-		

Table XIII

COMPARISON OF EXPERIMENTAL AND CALCULATED
CENTRAL REACTIVITIES OF NIOBIUM

Assembly Number	Central Reactivity (mb)		E/C Av = 1.00	Deviation (%)
	Calculated (C)	Experimental (E)±5		
2A	-42	-		
5	-66	-		
6F	-69	-		
23	-46	-		
9A	-84	-		
10	-88	-	1.09	+9
36	-91	-99		
33	-56	-43		
11	-92	-96	0.77	-23
32	-59	-		
31	-74	-	1.04	+4
41	-89	-95		
24	-92	-91		
25	-92	-83	1.07	+7
20	-83	-		
16	-102	-	0.99	-1
30	-84	-		
12	-113	-120		
17	-127	-	0.90	-10
34	-112	-116		
29	-113	-117		
35	-130	-	1.06	+6
14	-168	-		

Table XIV

COMPARISON OF EXPERIMENTAL AND CALCULATED
CENTRAL REACTIVITIES OF MOLYBDENUM

Assembly Number	Central Reactivity (mb)		E/C Av = 0.91	Deviation (%)
	Calculated (C)	Experimental (E)±2		
2A	-22	-		
5	-50	-		
6F	-53	-		
23	-25	-		
9A	-71	-		
10	-76	-	0.89	-2
36	-79	-70		
33	-28	-23		
11	-82	-68	0.82	-10
32	-29	-22		
31	-51	-	0.83	-9
41	-74	-64		
24	-82	-73		
25	-83	-69	0.83	-9
20	-63	-63		
16	-84	-	1.00	+10
30	-54	-		
12	-82	-83		
17	-75	-	1.01	+11
34	-67	-70		
29	-68	-74		
35	-57	-	1.04	+14
14	-41	-		

Table XV

COMPARISON OF EXPERIMENTAL AND CALCULATED
CENTRAL REACTIVITIES OF ZIRCONIUM

Assembly Number	Central Reactivity (mb)		Difference E-C (mb) ±2
	Calculated (C)	Experimental (E)±2	
2A	1	-	
5	-16	-	
6F	-18	-	
23	2	15	
9A	-28	-	
10	-31	-	+13
36	-32	-24	
33	4	-	
11	-34	-36	+8
32	4	25	
31	-12	-	-2
41	-27	-24	
24	-34	-33	
25	-35	-37	-2
20	-19	-20	
16	-33	-	-1
30	-11	-	
12	-29	-	
17	-20	-	+3
34	-14	-9	
29	-14	-10	
35	1	-	+5
14	19	-	

Table XVI

COMPARISON OF EXPERIMENTAL AND CALCULATED CENTRAL REACTIVITIES OF STAINLESS STEEL TYPE 304

Assembly Number	Central Reactivity (mb)		Difference E-C (mb) ± 2
	Calculated (C)	Experimental (E) ± 2	
2A	1	-	0
5	-12	-	
6F	-13	-	
23	1	1	
9A	-20	-	
10	-23	-	-2
36	-23	-25	
33	3	2	
11	-25	-25	
32	3	4	
31	-8	-6	+2
41	-20	-18	+2
24	-25	-25	0
25	-25	-23	+2
20	-14	-14	0
16	-24	-	0
30	-7	-7	
12	-20	-18	
17	-12	-	
34	-8	-6	
29	-8	-7	+1
35	3	5	+2
14	17	-	

Table XVII

COMPARISON OF EXPERIMENTAL AND CALCULATED CENTRAL REACTIVITIES OF ALUMINUM

Assembly Number	Central Reactivity (mb)		Difference E-C (mb) ± 2
	Calculated (C)	Experimental (E) ± 2	
2A	11	-	-5
5	4	-	
6F	3	-	
23	9	4	
9A	-3	-	
10	-6	-	-5
36	-7	-12	
33	16	25	
11	-11	-14	
32	16	29	
31	5	3	-2
41	-7	-8	-1
24	-15	-17	-2
25	-16	-19	-3
20	-2	-7	-5
16	-8	-	-1
30	4	3	
12	-4	-6	
17	1	-	
34	2	2	
29	2	2	0
35	15	28	13
14	19	-	

Table XVIII

COMPARISON OF EXPERIMENTAL AND CALCULATED CENTRAL REACTIVITIES OF SODIUM

Assembly Number	Central Reactivity (mb)		Difference E-C (mb) ± 6
	Calculated (C)	Experimental (E) ± 6	
2A	21	-	-2
5	15	-	
6F	13	-	
23	18	-	
9A	5	-	
10	1	-	11
36	-1	-3	
33	25	36	
11	-6	-	
32	26	49	
31	14	20	6
41	-2	+1	3
24	-13	-	6
25	-15	-	
20	4	-	
16	-2	-	
30	12	18	
12	4	-	5
17	9	-	
34	8	13	
29	6	27	
35	21	45	
14	28	-	

Table XIX

COMPARISON OF EXPERIMENTAL AND CALCULATED CENTRAL REACTIVITIES OF OXYGEN

Assembly Number	Central Reactivity (mb)		Difference E-C (mb) ± 5
	Calculated (C)	Experimental (E) ± 5	
2A	23	-	-2
5	17	-	
6F	15	-	
23	19	-	
9A	7	-	
10	2	-	16
36	1	-1	
33	29	45	
11	-6	-	
32	30	56	
31	16	23	7
41	-1	-4	-3
24	-14	-20	-6
25	-16	-21	-5
20	4	-	9
16	0	-	
30	14	-	
12	6	-	
17	14	-	
34	11	20	9
29	9	29	20
35	30	54	24
14	34	-	

Table XX

COMPARISON OF EXPERIMENTAL AND CALCULATED
CENTRAL REACTIVITIES OF CARBON

Assembly Number	Central Reactivity (mb)		Difference E-C (mb) ± 2
	Calculated (C)	Experimental (E) ± 2	
2A	32	-	
5	25	-	
6F	23	-	
23	26	40	+14
9A	12	-	
10	6	-	
36	5	0	-5
33	42	66	+24
11	-5	-11	-6
32	43	72	+29
31	23	30	+7
41	3	1	-2
24	-16	-21	-5
25	-18	-25	-7
20	8	5	-3
16	3	-	
30	21	33	+12
12	10	12	+2
17	20	-	
34	17	34	+17
29	16	33	+17
35	43	74	+31
14	45	-	

A. U²³³, U²³⁵, and Pu²³⁹

The data of Tables VII and VIII show that the relative central reactivities of U²³³, U²³⁵, and Pu²³⁹, in general, can be calculated to an accuracy of 5% or better, with the single case of Assembly 35 showing a 9% discrepancy. There is no evidence to show if this 9% discrepancy can be attributed to an unusually large experimental error. The moderately good agreement between results of experiment and calculation indicates that the relative average values of $(\nu - 1 - \alpha)\sigma_f$ for U²³³, U²³⁵, and Pu²³⁹ are correctly calculated (ignoring the small effect of scattering in these materials). Since the values of $(\nu - 1 - \alpha)$ are principally determined by the values of ν , and since the relative ν values for these isotopes should be well known, these reactivity data indicate that the fission cross sections of U²³³, U²³⁵, and Pu²³⁹ used in the calculations are moderately accurate. This leads to the conclusion that the disagreement between the experimental and calculated fission ratios of these isotopes discussed in Section III, B, is most probably attributable to errors in the intercalibration of the fission chambers.

B. Scatterers

The materials we shall consider in this section are carbon, oxygen, sodium, and aluminum. Of these materials only carbon and oxygen have completely negligible capture cross sections; those of sodium and aluminum

are only 1 or 2 mb and are therefore not important. The central reactivity data for these materials are given in Tables XVII through XX. The perturbation matrix for aluminum was obtained by means of resonance-shielded cross sections.

In Tables XVII through XX the data are displayed roughly in the order of increasing softness of the real spectrum, and examination of these tables shows no apparent correlation between spectrum softness and the sign of the coefficients, their magnitudes, or the discrepancies between experimental and calculated data. However, the discussion in the introduction of Section V indicated that the perturbation effects of scatterers are fairly sensitive to the shape of the importance spectrum.

The data are presented in a different way in Table XXI, the assemblies being listed in the order of decreasing U^{238}/U^{235} ratio in the core. Examination of the importance spectra in Appendix III will show that this order indicates a steady diminution of importance at high energies as the fraction of U^{238} fission decreases. It is also interesting to note that the importance spectrum is principally dependent on the U^{238}/U^{235} ratio and varies little with the proportion of other materials in the core (compare Assemblies 36,16; 10,41; 34,17; 2A,23,32,33,35,14).

Table XXI

COMPARISON OF EXPERIMENTAL AND CALCULATED CENTRAL
REACTIVITIES OF SCATTERING MATERIALS AS A FUNCTION OF
THE U^{238}/U^{235} RATIO OF THE CORE COMPOSITION

Ratio U^{238}/U^{235} in Core	Assembly Number	Carbon (± 2 mb)		Oxygen (± 5 mb)		Sodium (± 6 mb)		Aluminum (± 2 mb)	
		Expt (E)	E - C	Expt (E)	E - C	Expt (E)	E - C	Expt (E)	E - C
10.3	25	-25	-7	-21	-5			-19	-3
9.6	24	-21	-5	-20	-6			-17	-2
7.5	11	-11	-6					-14	-3
5.3	36	0	-5	-1	-2	-3	-2	-12	-5
5.3	16								
4.9	10								
4.9	41	1	-2	-4	-3	1	3	-8	-1
3.8	12	12	2					-6	-2
3.2	9A								
3.1	20	5	-3					-7	-5
2.2	34	34	17	20	9	13	5	2	0
2.2	17								
2.0	29	33	17	29	20	27	21	2	0
1.6	31	30	7	23	7	20	6	3	-2
1.5	30	33	12			18	6	3	-1
1.1	5								
1.1	6F								
.07	2A								
.07	23	40	14					4	-5
.07	32	72	29	56	26	49	23	29	13
.07	33	66	24	45	16	36	11	25	9
.07	35	74	31	54	24	45	24	28	13
.07	14								

Examination of Table XXI leads to the following conclusions:

(a) There is a strong correlation between the sign and magnitude of the reactivity coefficient and the softness of the importance spectrum. The coefficients are more positive for softer real spectra (compare those for Assemblies 34 and 29 with their neighbors in Table XXI), but the real spectrum is not apparently as significant as the importance spectrum. Hence, the reactivity coefficients of these scattering materials can be regarded as being principally importance spectrum indices.

(b) The discrepancies $E - C$ are of about the same magnitude as the experimental errors for the harder importance spectra, but there are very large discrepancies when the importance spectra are soft, reaching the same order of magnitude as the coefficients themselves.

(c) The disagreements strongly indicate that the calculated importance spectra are too hard, particularly in the systems with soft importance spectra (low U^{238}/U^{235} ratio). The real spectra may also be in error, but these data are not apparently sensitive to the real spectra and thus cannot give much information on the accuracy of its computation.

C. Weak Absorbers

There are two weak absorbers, steel and zirconium, for which the (n,γ) cross sections are about 10 to 15 mb and, hence, are of the same order of magnitude as the reactivity changes produced by scattering. The experimental and calculated reactivities are given in Tables XV and XVI. As in the case of aluminum, the matrix for steel was obtained from the resonance-shielded cross sections used in the k calculations.

Table XVI shows that the calculated and experimental reactivities of steel are in excellent agreement, the discrepancies being no greater than the experimental uncertainty in any case.

Table XV shows that the calculated and experimental data for zirconium are generally in quite good agreement, but there are considerable discrepancies in both Assemblies 23 and 32, in which there are soft importance spectra, these discrepancies being of the same sign as those occurring with the pure scatterers.

The significance of these results is difficult to assess, as these data do not appear to be consistent with those obtained with the pure scatterers in Section V, B. From Section V, B, we would expect that the contribution of the scattering terms to the reactivity effects of steel and zirconium would cause discrepancies of the same type as the pure scatterers, and, indeed, with zirconium in Assemblies 23 and 32 there is some indication that this is so. The steel data, however, show excellent agreement in every case.

It is difficult to see why steel should be exceptional in this respect, and it appears possible that the good agreement with this material is fortuitous. In this regard it should be noted that the k calculations given in Table I show that the predominantly steel Assemblies 32 and 33 have the greatest deviations, and this might indicate that the steel data could be anomalous.

D. Strong Absorbers

In this section we will consider the central reactivity effects of Mo, Nb, Ta, Th²³², and B¹⁰. The isotope Th²³² is, of course, fissile at high energies, but unlike U²³⁸, which is considered in Section V, E, capture is always the dominant effect in the reactors being examined and Th²³² is most appropriately considered as an absorber.

The discussion in the introduction of Section V indicated that the reactivity effect of a heavy absorber was largely determined by a $\phi\phi^*$ weighting of the absorption cross section, and since the real spectrum varies much more widely than the importance spectrum, it appears preferable to display these reactivities in terms of ϕ . The data given in this section will therefore be listed in order of increasing B¹⁰ $\sigma(n,\alpha)$, i.e., in the order of increasing softness of the real flux spectrum. Before entering into a detailed examination of the data, there are two general points regarding the reactivity measurements which should be noted.

First, the most important cross section for Mo, Nb, Ta, and Th²³² is that for capture, $\sigma(n\gamma)$, and this cross section is difficult to measure accurately for fast neutrons. Consequently, there may be a much greater uncertainty in the perturbation matrices of Mo, Nb, Ta and Th²³² than in, for example, those of Pu²³⁹, B¹⁰, or carbon. An additional feature of the measurements of capture cross sections is that it is probably more difficult to obtain an accurate absolute measurement than it is to obtain the variation with energy accurately, i.e., the shape of the $\sigma(n\gamma)$ curve. Hence, with these four materials we may expect to find discrepancies which could be attributed to inaccurate absolute $\sigma(n\gamma)$ data, i.e., errors of normalization.

Secondly, although capture is the most important process, scattering effects are not negligible, and these effects will be more prominent with the materials with lower capture cross sections.

The central reactivity data are given in Tables X through XIV. A summary of the ratio E/C for these parameters and the deviations from the average E/C is given in Table XXII.

Table XXII

RATIO OF EXPERIMENTAL AND CALCULATED DATA (E/C) AND
PERCENTAGE DEVIATION D FROM AVERAGE E/C FOR THE
HEAVY ABSORBERS

Assembly Number	Mo		Nb		Th ²³²		Ta		B ¹⁰	
	E/C	D (%)	E/C	D (%)	E/C	D (%)	E/C	D (%)	E/C	D (%)
2A										
5										
6F										
23							1.02	-11	1.21	-8
9A										
10										
36	0.89	-1	1.09	+9	0.84	-5	1.11	-3	1.32	+1
33	0.82	-10	0.77	-23	0.77	-9	1.15	+1	1.26	-4
11	0.83	-9	1.04	+4			1.10	-4	1.35	+3
32	0.76	-16			0.78	-9			1.25	-5
31					0.85	-3				
41	0.86	-5	1.07	+7	0.89	+1	1.19	+4	1.38	+5
24	0.89	-2	0.99	-1	0.91	+3	1.16	+2	1.27	-3
25	0.83	-9	0.90	-10			1.09	-4	1.26	-4
20							1.12	-2	1.33	+1
16										
30										
12	1.01	+11	1.06	+6			1.15	+1	1.31	0
17										
34	1.04	+14	1.04	+4	1.02	+16	1.27	+11	1.23	-6
29	1.09	+20	1.04	+4	1.00	+11	1.20	+6	1.33	+2
35									1.62	+24
14										

Average Values of E/C are:

Mo	0.91	Nb	1.00	Th ²³²	0.88
Ta	1.14	B ¹⁰	1.31		

Examination of these tables shows the following:

(a) There are numerous large discrepancies between the experimental and calculated parameters.

(b) The ratio E/C is in general fairly constant for all assemblies for the very heavy absorbers Th²³², tantalum, and B¹⁰. There are a few cases where abnormally high deviations occur.

(c) The ratio E/C is also moderately constant for the less heavy absorbers molybdenum and niobium, but the deviations in these cases are generally appreciably higher than with the other three parameters.

These results very clearly show that few of the five parameters are calculated correctly, and it appears possible that even in the cases where agreement is obtained (e.g., with niobium) that this could be due to fortuitous cancellation of errors.

The B^{10} reactivity is principally determined by the well known $(n\alpha)$ cross section, which is approximately $1/v$ over a considerable energy range. These data strongly indicate that the calculated spectra of the real flux, or importance, or both, are too hard in all the assemblies, the calculated effectiveness of a $1/v$ detector being underestimated by about 30%. It would be expected that the average values of E/C for molybdenum, niobium, Th^{232} and tantalum would also be greater than unity and might approach 1.3; the failure to observe this could be due to errors in the absolute values of the assumed $(n\gamma)$ cross sections as suggested earlier. Also, the greater spread of results obtained with molybdenum and niobium could be attributed to the greater influence of scattering in these cases.

E. U^{238}

The experimental and calculated central reactivities for U^{238} are given in Table IX. The calculated reactivity of this isotope is the result of capture (negative), fission (positive), and scattering (positive or negative); hence, the net reactivity may be positive or negative, and relatively small errors in the nuclear data or cross sections could be magnified into large fractional errors in reactivity. This is seen to be the case in Table IX, as the fractional deviations are large, but the deviations in $E - C$ are not unreasonable.

The complexity of the reactions in U^{238} makes these data difficult to use as spectral indices or as a means of examining the accuracy of the assumed cross sections for U^{238} . The only general observation that will be made here is that in the majority of cases the calculated reactivity is too positive and that, in general, better agreement with experiment would be obtained if the calculated spectra were softened. This would both increase the negative capture term and decrease the positive fission term.

In summary, therefore, we conclude that the central reactivity data for U^{238} do not give clear indications of the causes of discrepancies, but that somewhat better agreement would be obtained if the calculated spectra were assumed to be too hard.

VI. SUMMARY OF CONCLUSIONS

This comparison of experimental and calculated parameters for ZPR-III assemblies has shown that, although there are many discrepancies and much anomalous data, it is possible to identify broad consistent patterns.

The main conclusions are as follows:

- (1) The cross-section set used in this study can predict the critical masses of a large number of the ZPR-III fast reactor assemblies to an accuracy of about 5%. This does not imply that the nuclear data used are free from error.
- (2) The calculated relative fission rates of U^{233} , Pu^{239} , Pu^{240} , U^{236} , and U^{238} are in good agreement with the measured data, but the calculated rates for U^{235} and U^{234} are 6% low and 7% high, respectively, relative to those of the other five isotopes. The disagreement between the calculated and experimental fission rates of U^{235} and those of U^{233} and Pu^{239} is insensitive to errors in the calculated spectrum and may be due to erroneous intercalibration of these chambers. This uncertainty regarding the intercalibration of the fission chambers must be removed before the fission-ratio data can be interpreted.
- (3) In every case examined, the experimental prompt-neutron life-time was larger than the calculated value. On the average, the experimental value was 34% greater than that calculated and was moderately constant for all the assemblies, with extreme values of 24% and 48%. The discrepancy is too large to be attributed to experimental errors, and it appears that all the calculated spectra are too hard.
- (4) The lack of data on the effect of the sample size upon the measured central reactivities is held to be a serious drawback to the interpretation of much of the reactivity data. It is argued that this should not be important in the comparison of the relative reactivity effects of U^{233} , U^{235} , and Pu^{239} , and also should not affect the general conclusions to be drawn from cases such as carbon and boron-10 in which the discrepancies between experiment and calculation are very large.
- (5) The calculated relative central reactivities of U^{233} , U^{235} , and Pu^{239} are in moderately good agreement with experimental data. These data are not sensitive to the calculated spectra. The agreement indicates that the assumed nuclear data for these isotopes are reasonably accurate.
- (6) The reactivity effects of the scatterers carbon, oxygen, aluminum, and sodium are shown to be principally dependent upon the importance spectrum and, hence, can be considered as principally importance-spectrum indices. There are large discrepancies between the experimental and

calculated reactivities, particularly in the systems with soft importance spectra, and it is inferred that the calculated importance spectra are too hard.

(7) The experimental and calculated central reactivities of steel are in excellent agreement, and those for zirconium are in quite good agreement. In view of the evidence of appreciable errors in the calculated real flux and importance spectra, it appears that this agreement might be fortuitous.

(8) There are large discrepancies between the experimental and calculated central reactivities of the heavy absorbers Mo, Nb, Ta, Th²³² and B¹⁰. Of particular note is the experimental reactivity of B¹⁰, 31% greater than calculated. Since the $\sigma(n\alpha)$ for B¹⁰ is roughly $1/v$ in dependence, this is strong corroboration of the prompt-neutron lifetime data for which the discrepancy is 34%. Thus, the B¹⁰ central reactivity data also show that the calculated spectra are too hard.

The data for Mo, Nb, Ta and Th²³² do not generally support the B¹⁰ data, but it is suggested that this may be due to uncertainty in the knowledge of $\sigma(n\gamma)$ for these materials.

(9) The calculation of the reactivity effect of U²³⁸ is shown to be quite sensitive to the assumed cross sections due to competing reactions, and it is not possible to identify errors in U²³⁸ data. The agreement between results of experiment and calculation would be improved if the calculated spectrum were not so hard.

VII. POSSIBLE FUTURE WORK

Clearly, the most important problem is the identification of the reason or reasons that the calculated spectra are too hard, but further improvement in the experimental information is also necessary.

For example, it is essential to establish an accurate intercalibration of the fission chambers; until such data are firmly established, it is possible to select many alternative interpretations of the data.

Another important point is the necessity of allowing for sample size in reactivity measurements so that it is permissible to analyze these results by perturbation theory. This could be achieved by measuring reactivities as a function of sample size and extrapolating to obtain the effect of a very small sample.

A further point is that it is clear that central reactivity coefficients have drawbacks as spectral indices, and it would be of value to obtain alternative indices. For example, an absolute B¹⁰ $(n\alpha)/U^{235} \sigma_f$ ratio would be an

excellent real flux, low-energy spectral index which, if obtained with B^{10} counters and U^{235} fission chambers, could be free of self-shielding effects.

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APPENDIX I

Cross-section Data ANL Set 635

In this appendix are listed the differences between the cross-section set used in these calculations and the YOM data.⁽¹⁾ The derivation of the modified data is given in Davey.⁽²⁾

Group	Pu ²³⁹		U ²³⁵			U ²³³		Steel		Al	
	$\sigma(n\gamma)$	σ_{tr}	$\sigma(n\gamma)$	σ_{tr}	ν	$\sigma(n\gamma)$	σ_{tr}	σ_{er}	σ_{tr}	σ_{er}	σ_{tr}
1	0.002	3.918	0.020	3.880	2.71	0.007	4.180	0.054	1.975	0.118	1.370
2	0.010	4.920	0.035	4.730	2.71	0.013	4.810	0.064	1.938	0.180	1.328
3	0.020	5.064	0.058	4.860	2.60	0.019	4.870	0.099	2.077	0.270	1.900
4	0.045	5.011	0.102	5.067	2.53	0.049	4.882	0.125	1.953	0.339	2.108
5	0.094	6.055	0.167	5.554	2.48	0.113	5.594	0.175	2.276	0.448	2.807
6	0.153	7.241	0.200	6.770	2.46	0.175	7.055	0.152	2.606	0.428	2.980
7	0.187	8.557	0.305	8.549	2.44	0.218	8.851	0.172	2.560	0.450	2.903
8	0.238	10.029	0.397	10.127	2.43	0.259	10.612	0.224	3.096	0.531	3.062
9	0.252	11.218	0.549	11.566	2.43	0.303	12.351	0.263	3.778	0.421	2.841
10	0.290	12.304	0.731	12.707	2.42	0.344	13.638	0.235	4.382	0.209	1.863
11	0.543	13.451	0.954	13.751	2.42	0.399	14.324	0.434	5.176	0.260	1.522
12	0.840	14.356	1.150	14.447	2.42	0.470	15.091	0.205	3.730	0.001	0.533
13	0.980	14.969	1.396	15.247	2.42	0.551	15.918	0.409	7.979	0.121	1.096
14	1.280	16.100	1.732	18.743	2.42	0.637	16.580	0.574	10.960	0.161	1.510
15	1.891	18.000	2.092	22.300	2.42	0.833	18.500	0.165	9.196	0.0507	1.370
16	4.020	13.500	3.825	28.000	2.42	0.990	22.010	0	8.090	0	1.370

APPENDIX II

Perturbation Matrices

P11239

[illegible]

235

[illegible]

U233

[illegible]

U238

[illegible]Th²³²[illegible]

Boron-10

[illegible]

Zirconium

A _{ij}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	-1.751															
2	0.341	-1.3794														
3	0.470	0.5274														
4	0.419	0.32	-0.720													
5	0.274	0.24	0.327	-0.318												
6	0.143	0.15	0.18	0.172	-0.243											
7	0.104	0.09	0.11	0.07	0.236	-0.285										
8		0.05	0.05	0.04		0.277	-0.272									
9			0.03	0.02			0.263	-0.293								
10			0.02	0.01				0.283	-0.276							
11								0.263		-0.284						
12										0.266	-0.319					
13											0.294	-0.288				
14												0.253	-0.284			
15													0.230	-0.336		
16														0.260	-0.1952 0.0852	-0.08

Stainless Steel 304

[illegible]

Aluminum

[illegible]

Na

[illegible]

Oxygen

[illegible]

Carbon

[illegible]

APPENDIX III
Calculated Central Spectra

Real Flux

Ass'y No. Group	2A	5	6F	9A	10	11	12	14	16
1	0.036	0.037	0.036	0.027	0.024	0.020	0.026	0.036	0.024
2	0.072	0.068	0.068	0.049	0.041	0.034	0.048	0.073	0.041
3	0.116	0.104	0.102	0.077	0.066	0.054	0.073	0.111	0.064
4	0.146	0.143	0.142	0.128	0.121	0.109	0.112	0.118	0.111
5	0.177	0.183	0.183	0.193	0.205	0.200	0.156	0.118	0.173
6	0.162	0.171	0.173	0.192	0.206	0.212	0.155	0.107	0.178
7	0.119	0.123	0.125	0.140	0.147	0.156	0.130	0.095	0.143
8	0.078	0.080	0.081	0.093	0.098	0.108	0.102	0.080	0.107
9	0.054	0.051	0.051	0.052	0.045	0.048	0.074	0.072	0.066
10	0.028	0.028	0.028	0.035	0.034	0.042	0.057	0.058	0.051
11	0.007	0.007	0.006	0.008	0.007	0.008	0.030	0.042	0.021
12	0.004	0.004	0.004	0.005	0.005	0.007	0.020	0.032	0.013
13	0.001	0.001	0.001	0.001	0.001	0.002	0.010	0.022	0.005
14	-	-	-	-	-	-	0.004	0.015	0.002
15	-	-	-	-	-	-	0.002	0.012	0.001
16	-	-	-	-	-	-	0.001	0.009	-

Importance

Ass'y No. Group	2A	5	6F	9A	10	11	12	14	16
1	0.052	0.059	0.060	0.071	0.078	0.088	0.072	0.046	0.080
2	0.053	0.059	0.060	0.069	0.075	0.083	0.070	0.048	0.076
3	0.052	0.057	0.057	0.064	0.069	0.077	0.064	0.049	0.070
4	0.050	0.050	0.051	0.053	0.055	0.058	0.054	0.050	0.056
5	0.051	0.051	0.051	0.053	0.054	0.056	0.054	0.053	0.055
6	0.053	0.054	0.054	0.056	0.057	0.059	0.058	0.057	0.058
7	0.056	0.057	0.057	0.058	0.059	0.060	0.060	0.060	0.060
8	0.060	0.061	0.060	0.060	0.060	0.059	0.061	0.063	0.060
9	0.062	0.062	0.062	0.061	0.060	0.058	0.061	0.065	0.060
10	0.065	0.064	0.064	0.062	0.061	0.058	0.063	0.067	0.061
11	0.067	0.066	0.065	0.063	0.061	0.058	0.064	0.069	0.061
12	0.067	0.067	0.067	0.065	0.063	0.059	0.065	0.071	0.062
13	0.075	0.072	0.072	0.068	0.065	0.060	0.066	0.074	0.064
14	0.077	0.074	0.073	0.069	0.066	0.062	0.067	0.075	0.064
15	0.079	0.075	0.075	0.068	0.064	0.059	0.064	0.076	0.061
16	0.081	0.072	0.072	0.060	0.053	0.046	0.057	0.077	0.052

Real Flux

Ass'y No. Group	17	20	23	24	25	29	30	31	32
1	0.030	0.020	0.036	0.017	0.016	0.019	0.021	0.022	0.018
2	0.057	0.038	0.072	0.029	0.028	0.039	0.044	0.044	0.041
3	0.086	0.064	0.113	0.046	0.044	0.067	0.076	0.078	0.080
4	0.114	0.110	0.141	0.100	0.097	0.099	0.114	0.121	0.126
5	0.140	0.168	0.163	0.193	0.191	0.141	0.160	0.175	0.196
6	0.134	0.185	0.157	0.214	0.214	0.146	0.163	0.180	0.187
7	0.117	0.150	0.120	0.163	0.164	0.139	0.141	0.139	0.136
8	0.094	0.107	0.080	0.115	0.118	0.108	0.102	0.095	0.092
9	0.076	0.072	0.062	0.052	0.054	0.086	0.075	0.067	0.061
10	0.059	0.054	0.038	0.050	0.052	0.071	0.056	0.048	0.036
11	0.037	0.017	0.011	0.010	0.011	0.035	0.023	0.016	0.012
12	0.026	0.012	0.006	0.009	0.009	0.029	0.017	0.012	0.010
13	0.015	0.002	0.001	0.002	0.002	0.011	0.005	0.002	0.003
14	0.008	0.001	-	-	-	0.005	0.002	0.001	0.001
15	0.005	-	-	-	-	0.004	0.001	-	0.001
16	0.002	-	-	-	-	0.001	-	-	-

Importance

Ass'y No. Group	17	20	23	24	25	29	30	31	32
1	0.063	0.069	0.053	0.095	0.097	0.062	0.060	0.061	0.051
2	0.062	0.068	0.054	0.088	0.090	0.062	0.060	0.061	0.052
3	0.058	0.064	0.054	0.082	0.084	0.060	0.058	0.058	0.052
4	0.052	0.057	0.052	0.061	0.062	0.056	0.054	0.053	0.051
5	0.054	0.056	0.053	0.059	0.060	0.056	0.054	0.053	0.050
6	0.057	0.059	0.055	0.061	0.061	0.058	0.057	0.056	0.053
7	0.060	0.060	0.058	0.060	0.060	0.059	0.058	0.058	0.055
8	0.062	0.061	0.061	0.058	0.057	0.061	0.060	0.061	0.060
9	0.063	0.061	0.062	0.056	0.056	0.062	0.062	0.062	0.063
10	0.065	0.062	0.063	0.056	0.055	0.063	0.064	0.064	0.067
11	0.066	0.063	0.065	0.055	0.054	0.064	0.065	0.065	0.069
12	0.068	0.064	0.065	0.056	0.055	0.066	0.066	0.066	0.071
13	0.069	0.066	0.072	0.057	0.056	0.069	0.071	0.071	0.076
14	0.069	0.066	0.074	0.059	0.058	0.069	0.071	0.071	0.077
15	0.068	0.066	0.078	0.055	0.054	0.068	0.072	0.072	0.076
16	0.064	0.058	0.081	0.042	0.041	0.065	0.068	0.068	0.077

Real Flux

Ass'y No. Group	33	34	35	35A	36	41
1	0.021	0.018	0.014	0.014	0.023	0.020
2	0.047	0.038	0.033	0.033	0.040	0.038
3	0.088	0.067	0.064	0.064	0.063	0.061
4	0.128	0.106	0.095	0.095	0.115	0.112
5	0.187	0.147	0.142	0.142	0.193	0.182
6	0.185	0.155	0.166	0.166	0.206	0.195
7	0.134	0.132	0.145	0.146	0.151	0.150
8	0.092	0.101	0.112	0.112	0.104	0.105
9	0.060	0.081	0.083	0.083	0.050	0.062
10	0.033	0.069	0.059	0.059	0.038	0.050
11	0.012	0.034	0.030	0.030	0.009	0.013
12	0.009	0.030	0.029	0.029	0.006	0.010
13	0.002	0.012	0.012	0.012	0.002	0.002
14	0.001	0.005	0.006	0.006	-	-
15	0.001	0.004	0.006	0.006	-	-
16	-	0.001	0.004	0.003	-	-

Importance

Ass'y No. Group	33	34	35	35A	36	41
1	0.051	0.063	0.052	0.052	0.080	0.077
2	0.052	0.062	0.052	0.053	0.076	0.074
3	0.052	0.060	0.052	0.053	0.071	0.069
4	0.051	0.055	0.053	0.053	0.056	0.057
5	0.050	0.056	0.053	0.053	0.054	0.056
6	0.053	0.058	0.054	0.055	0.057	0.058
7	0.055	0.060	0.057	0.058	0.059	0.059
8	0.060	0.061	0.061	0.061	0.060	0.060
9	0.063	0.062	0.064	0.064	0.060	0.060
10	0.067	0.063	0.067	0.067	0.061	0.061
11	0.069	0.065	0.069	0.069	0.061	0.061
12	0.071	0.066	0.071	0.070	0.062	0.062
13	0.076	0.069	0.074	0.073	0.064	0.065
14	0.077	0.068	0.073	0.072	0.066	0.065
15	0.075	0.068	0.071	0.070	0.061	0.063
16	0.078	0.064	0.077	0.077	0.052	0.053