ANL-7110

Reactor Technology (TID-4500)

AEC Research and

Development Report.

Argonne National Laboratory

9700 South Cass Avenue

Argonne, Illinois 60440

REACTOR PHYSICS DIVISION ANNUAL REPORT

July 1, 1964 to June 30, 1965

Robert Avery, Division Director

December, 1965

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

RELEASED FOR ANNOUNCEMENT
IN NUCLEAR SCIENCE ABSTRACTS

Operated by The University of Chicago
under

Contract W-31-109-eng-38

with the

U.S. Atomic Energy Commission

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Foreword

This, the second annual report published by the Reactor Physics Division, is similar to the first report, ANL-7010 (January 1965), in its format and objectives. It delineates in a succinct fashion the analytical and experimental work performed by members of the Reactor Physics Division during the year July 1, 1964 to June 30, 1965. Some of the efforts described are continuations of studies reported in the previous annual report; some are new efforts. Some will be continued while others have been completed or, for one reason or another, terminated.

Again, the report is pointed toward descriptions of the technical accomplishments of individuals rather than toward the programmatic phases of work within the Division. References within each paper are included to assist readers who may wish to delve more deeply into a given topic. A list of open-literature and report publications by Reactor Physics staff during the reporting period are also appended to further aid those who would like to have additional information.

Table of Contents

SECTION I

FISSION PROPERTIES AND CROSS SECTION DATA

I-1.	Fast Neutron Scattering Program	3
	S. G. BUCCINO, J. CHIEN ¹ , S. A. COX, D. LISTER ² , A. B. SMITH and W. G. VONACH	
I-2.	Scattering From Heavy Elements	3
	W. G. VONACH and A. B. SMITH	
I-3.	Elastic Scattering From Intermediate Weight Elements	5
	S. A. COX	
I-4.	Fast Neutron Scattering From Na, Al and Be	8
	J. P. CHIEN ¹ and A. B. SMITH	
I-5.	Fast Neutron Scattering From Ag, In and Cd	10
	A. B. SMITH	
I-6.	Experimental Interpretation Using a Surface Absorption Optical Potential for Low Energy Neutron Scattering	11
	A. B. SMITH	
I-7.	High Resolution Total Neutron Cross Sections	13
	J. F. WHALEN, J. W. MEADOWS and A. B. SMITH	
I-8.	Total Neutron Cross Sections at High Energies	14
	D. LISTER ² and A. B. SMITH	
I-9.	Precision Determinations of Fast Neutron Total Cross Sections in the Heavy Elements	15
	J. F. WHALEN, A. B. SMITH and J. W. MEADOWS	
I-10.	Measurement of the Li ⁶ (n, absorption) and B ¹⁰ (n, absorption) Cross Sections by the Shell Transmission Method	16
	S. A. COX	
I-11.	Gamma Rays from Inelastic Neutron Scattering	18
	J. F. BARRY ³ and D. STUPEGIA ⁴	
I-12.	Energy Dependence of $\bar{\nu}_p$ for U-235 from 0.039 to 1.0 MeV	20
	J. MEADOWS and J. WHALEN	
I-13.	Mass-Energy Distributions for 6.0 MeV Neutron Fission of U-235	21
	J. W. MEADOWS and J. F. WHALEN	
I-14.	Spontaneous Fission Half-Lives of Am-241, Cm-242, and Cm-244	23
	R. J. ARMANI and R. GOLD	
I-15.	(d, n) Reactions in the Medium Weight Nuclei; Fe-54(d, n)Co-55 and Ni-58 (d, n) Cu-59	25
	D. S. GEMMELL ⁵ , L. L. LEE ⁵ , J. P. SCHIFFER ⁵ and A. B. SMITH	
I-16.	Stripping Reactions in Light Nuclei	26
	S. G. BUCCINO and A. B. SMITH	

SECTION II

THERMAL REACTORS

II-1.	Linear Relationship Between Cadmium Ratio and Reactor Core Moderating Ratio . .	31
	G. S. STANFORD, K. E. PLUMLEE and W. R. ROBINSON	
II-2.	Slow Neutron Spectrum Studies by Neutron Activation	34
	P. H. KIER	

¹ Graduate Student, University of Michigan, Ann Arbor, Michigan; Faculty member, National Tsing Hau University, Taiwan, China

² Post-Doctor, Columbia University, New York, N. Y.

³ Atomic Weapons Research Establishment, Aldermaston, England

⁴ Chemical Engineering Division, Argonne National Laboratory

⁵ Physics Division, Argonne National Laboratory

II-3.	Small Reactivity Measurements in the ATSR.....	36
	R. C. DOERNER and J. HAUGSNES	
II-4.	Argonne Advanced Research Reactor Calculations—Preface.....	42
	D. H. SHAFTMAN, C. N. KELBER, B. N. KRISTIANSON ¹ , H. F. REED and R. P. SAVIO ²	
II-5.	Argonne Advanced Research Reactor Calculations—Radiation Doses Beyond the Reactor Containment Building Following a Postulated Maximum Credible Accident..	43
	H. F. REED, C. N. KELBER and D. H. SHAFTMAN	
II-6.	Argonne Advanced Research Reactor Calculations—Preliminary Heating Rates for Water, Beryllium, and Aluminum in the Internal Thermal Column, and for the Beryllium Reflector.....	47
	H. F. REED	
II-7.	Argonne Advanced Research Reactor Calculations—Multiplication of Photoneutrons in the Subcritical AARR; Effects on Reactor Power and Location of Startup Detector.....	50
	D. H. SHAFTMAN and H. F. REED	
II-8.	Argonne Advanced Research Reactor Calculations—Analysis of Criticality-Facility Loadings.....	51
	D. H. SHAFTMAN, C. N. KELBER, B. N. KRISTIANSON ¹ and R. P. SAVIO ²	
II-9.	Argonne Advanced Research Reactor Calculations—Comparison of Analog and Digital Transient Temperature Calculations.....	60
	W. P. STEPHANY	
II-10.	Critical Experiments for AARR Design.....	61
	J. DAUGHTRY, T. JOHNSON, F. KODRICK, K. PLUMLEE, W. ROBINSON and G. STANFORD	
II-11.	Measurement of Gamma Heating in the Argonne Advanced Research Reactor Critical Assembly by Thermoluminescent Dosimetry.....	73
	G. S. STANFORD, T. W. JOHNSON and W. R. ROBINSON	
II-12.	Onset of Oscillations in a Natural Circulation Boiling System.....	76
	D. MILLER, K. C. JAIN ³ , M. PETRICK ¹ , and S. G. BANKOFF ³	
II-13.	An Analytic Study of the Space Dependent Dynamics of Boiling Reactors.....	77
	C. K. SANATHANAN	
II-14.	High Conversion Critical Experiment.....	82
	A. R. BOYNTON, K. E. PLUMLEE, W. C. REDMAN, W. R. ROBINSON and G. S. STANFORD	
II-15.	Hi-C Uniform Lattice Calculations.....	84
	E. M. PENNINGTON	
II-16.	Thermal Disadvantage Factors in the Hi-C Cores.....	90
	A. R. BOYNTON	
II-17.	The EBWR Plutonium Recycle Experimental Program.....	92
	P. H. KIER	
II-18.	Consistent Heavy-Atom Cross Section Evaluation for Plutonium Recycle Calculation.....	93
	D. P. MOON	
II-19.	Determination of Capture-to-Fission Ratio for Pu-239 and Pu-241.....	94
	H. P. ISKENDERIAN	

SECTION III

FAST REACTORS

III-1.	Physics Parameters and Sodium Void Coefficients in a 950-Liter Pancake Core....	101
	F. H. HELM, W. Y. KATO, G. MAIN and G. K. RUSCH	
III-2.	Zoned Core Simulating a 2600 Liter Carbide Fast Reactor.....	114
	R. A. KARAM, A. ANCARANI ⁴ , F. HELM, W. Y. KATO and G. K. RUSCH	
III-3.	The Neutron Spectrum in ZPR-6 Assembly No. 4Z.....	116
	E. F. BENNETT	

¹ Reactor Engineering Division, Argonne National Laboratory

² Reactor Operations Division, Argonne National Laboratory

³ Northwestern University, Evanston, Illinois

⁴ Euratom, Brussels, Belgium

III-4.	Doppler Effect Measurements in Plutonium-Fueled Fast Power Breeder Reactor Spectra.....	120
	G. J. FISCHER, D. A. MENELEY, R. N. HWANG, E. F. GROH and C. E. TILL	
III-5.	A Check Calculation of Doppler Coefficient in the ZPR-3 Mockup of SEFOR.....	124
	R. N. HWANG	
III-6.	Doppler Effect Calculations with Interference Corrections.....	127
	R. N. HWANG	
III-7.	Fast Reactor Physics Parameters from a Pulsed Source.....	127
	W. Y. KATO, H. H. MEISTER ¹ , G. W. MAIN, J. L. RUSSELL, JR. ² and K. L. CROSBIE ²	
III-8.	Evaluated Fast Cross Sections.....	139
	D. M. O'SHEA, H. H. HUMMEL, W. B. LOEWENSTEIN and D. OKRENT ³	
III-9.	Preparation of Cross Sections and Parametric Studies of the Reactivity Coefficients for Large U-233-Th Fueled Fast Reactors.....	141
	R. S. SINGH ⁴ and H. H. HUMMEL	
III-10.	Evaluation of Fast Multigroup Cross Sections for Light Element Reflectors.....	145
	D. MENEGHETTI	
III-11.	Theoretical Studies of Thin Slab Heterogeneity Effects upon the Spectral Reactivity Component of Sodium Voiding.....	148
	D. MENEGHETTI	
III-12.	EBR-II Physics.....	151
	W. B. LOEWENSTEIN, F. KIRN ⁵ and R. R. SMITH ⁵	
III-13.	The Experimental Breeder Reactor II (EBR-II) as a Fast Flux Irradiation Facility.....	152
	W. B. LOEWENSTEIN	
III-14.	Physics Analysis of TREAT with an EBR-II Subassembly.....	154
	J. T. MADELL	
III-15.	Transient Effects in Meltdown of Irradiated EBR-II Mark I Fuel Samples.....	157
	C. E. DICKERMAN, L. E. ROBINSON and C. AUGUST	
III-16.	Fast Reactor Safety Experiments on Thorium-Uranium EBR-II-Type Fuel Samples in the Absence of Coolant.....	159
	C. E. DICKERMAN, L. E. ROBINSON and R. STEWART	
III-17.	Initial TREAT Meltdown Experiments in the Integral Sodium Loop.....	161
	L. E. ROBINSON, C. DICKERMAN, W. STEPHANY and F. L. WILLIS	
III-18.	Behavior of US Fuel Samples in High Temperature Transient TREAT Experiments.....	163
	L. E. ROBINSON, C. E. DICKERMAN, R. CARLANDER ⁶ , C. AUGUST and C. MUELLER ⁷	
III-19.	Correlation of Results from In-Pile Transient Pressure Measurements.....	164
	C. E. DICKERMAN and W. P. STEPHANY	
III-20.	FARET Core I—Preliminary Fuels Irradiation Program and Design.....	165
	P. J. PERSIANI, W. R. SIMMONS ⁸ and T. R. BUMP ⁸	
III-21.	FARET Critical Experiments.....	169
	A. HESS ⁹ , J. K. LONG ⁵ , G. BRUNSON ⁵ and P. J. PERSIANI	
III-22.	Preliminary Study of FARET Radial Shield Configurations.....	170
	A. E. MCARTHY and P. J. PERSIANI	
III-23.	Note on Rossi-Alpha in Reflected Fast Reactors.....	172
	R. A. KARAM	
III-24.	Neutronics Calculations for a 1,000 MWe Plutonium-Uranium Metal Fueled, Fast Breeder Reactor Design Study.....	173
	D. K. BUTLER, W. B. LOEWENSTEIN, D. MENEGHETTI and K. E. PHILLIPS	
III-25.	Sodium Void Reactivity Effects Associated with Evaluation of Four 1,000 MWe Reactor Design Studies: Effect of Beryllium Oxide Addition.....	180
	H. H. HUMMEL and K. E. PHILLIPS	
III-26.	Spectral and Capture Components of Sodium Void Effect for Four 1,000 MWe Fast Breeder Studies.....	182
	H. H. HUMMEL and J. WHITE ⁹	

¹ Kernforschungszentrum, Karlsruhe, West Germany

² J. J. Hopkins Laboratory, General Atomic, San Diego, California

³ Laboratory Director's Office, Argonne National Laboratory

⁴ Atomic Energy Establishment, Trombay, India

⁵ Idaho Division, Argonne National Laboratory

⁶ Metallurgy Division, Argonne National Laboratory

⁷ Student, Northwestern University, Evanston, Illinois

⁸ Reactor Engineering Division, Argonne National Laboratory

⁹ Student, University of Illinois, Urbana, Illinois

III-27.	Evaluation of Breeders from a Conservation Standpoint	184
	P. F. GAST	
III-28.	Quick, Approximate Evaluation of the Economics of Mixed Plutonium Fueled Breeder Designs	185
	P. F. GAST	
III-29.	Mixed Fuel Cycle Fast Breeder Reactors	187
	W. B. LOEWENSTEIN	
III-30.	Additional Physics Studies of Molten-Salt-Fueled Fast Reactor Systems	189
	D. K. BUTLER and D. MENEGHETTI	
III-31.	A Simplified Model for Calculating Criticality Trends in Fast Reactors	191
	B. I. SPINRAD	
III-32.	An Estimation of the Temperature-Dependent Cross Sections for Tungsten Isotopes in the Doppler Region	207
	R. N. HWANG	
III-33.	Reactivity Coefficients of Fuel Expansion into Moderating and Marginally Mod- erating Reflectors	208
	W. B. LOEWENSTEIN, K. K. ALMENAS, W. G. KNAPP and R. C. DOERNER	
III-34.	Recent Developments in the Physics Design of Some Nuclear Rockets	215
	W. B. LOEWENSTEIN, K. K. ALMENAS, D. BUTLER, R. C. DOERNER and W. G. KNAPP	
III-35.	Experimental Physics Studies in Fast Reactor Nuclear Rocket Engine Cores	215
	R. C. DOERNER, W. G. KNAPP, K. K. ALMENAS, C. COHN, R. A. KARAM, W. Y. KATO and W. B. LOEWENSTEIN	
III-36.	Small Core Reactors for Space Propulsion	221
	D. K. BUTLER and H. L. DRYER ¹	
III-37.	A Study of Small, Fast, W-UO ₂ Rocket Engines for Advanced Scientific Missions	223
	B. I. SPINRAD, E. J. CROKE ¹ , M. STEPHENSON ¹ and A. J. ULRICH	
III-38.	Study of Reflector Based Control of Fast Nuclear Rocket Reactors	224
	K. K. ALMENAS	
III-39.	Further Study of the Rocket Fuel Test Reactor Concept	227
	J. T. MADELL	

SECTION IV

EXPERIMENTAL TECHNIQUES AND FACILITIES

IV-1.	Calculation of Pile-Up Distortion	233
	R. GOLD	
IV-2.	Numerical Evaluation of Fourier Integrals	233
	R. GOLD, C. COHN and I. OLSON	
IV-3.	Proportional Counter Multiplication Process	235
	R. GOLD and E. F. BENNETT	
IV-4.	The Effect of Deadtime on Counting Errors	239
	C. E. COHN	
IV-5.	Theory and Methods of Correction for Resolution Losses of Counting Equipment	242
	K. E. PLUMLEE	
IV-6.	Computer Techniques for the Investigation of Counting System Deadtime	247
	G. S. STANFORD	
IV-7.	A FORTRAN Code for Normalizing and Combining Large Amounts of Foil Counting Data	252
	G. S. STANFORD	
IV-8.	The Interpretation of Rod Drops with Finite Drop Times	254
	C. E. COHN	
IV-9.	Preparations for the Measurement of the Low Energy Component of the Fission Neutron Spectrum	254
	K. G. PORGES and A. DEVOLPI	
IV-10.	On the Precision of Measurements of Exponential Growth and Decay Constants	255
	C. E. COHN	
IV-11.	Obtaining Space-Dependent Zero-Power Transfer Functions Using Statics Tech- niques	259
	C. E. COHN	

¹ Reactor Engineering Division, Argonne National Laboratory

IV-12.	Direct Measurement of Effective Capture-to-Fission Ratio at Low Flux.....	261
	W. C. REDMAN and M. M. BRETSCHER ¹	
IV-13.	Oscillation Measurement Procedure in Critical Experiments.....	266
	K. E. PLUMLEE and C. E. COHN	
IV-14.	Modification of the TREAT Integral Sodium Loops.....	268
	L. E. ROBINSON, R. T. PURVIANCE and F. L. WILLIS	
IV-15.	Loading and Remote Handling of Integral TREAT Sodium Loops.....	274
	F. L. WILLIS, R. J. SCHILTZ ² and L. E. ROBINSON	
IV-16.	Pressure Transducers for TREAT Sodium Loops.....	279
	L. E. ROBINSON and R. T. PURVIANCE	
IV-17.	Remote Encapsulation of Pre-Irradiated Fuel Elements for Exposure in TREAT..	280
	F. L. WILLIS, R. J. SCHILTZ ² and J. H. MONAWECK ²	
IV-18.	Fast Neutron Transient Detection Hodoscope for Reactor Fuel Meltdown Studies..	285
	A. DEVOLPI and C. H. FREESE ³	
IV-19.	Remotely-Controlled Camera Stage for Hot-Cell Work.....	290
	C. AUGUST	
IV-20.	Improvements in Absolute Neutron Detection Techniques.....	291
	A. DEVOLPI and K. G. PORGES	
IV-21.	Mn-55 and Co-59 Resonance Detectors.....	293
	R. A. KARAM	
IV-22.	Design Improvements and Sensitivity Tests of the FERD System for EBR-II.....	294
	K. G. PORGES	
IV-23.	Standardization of Radiochemical Methods for Measurement of Uranium Fission Rate and U-238 Neutron Capture Rate.....	295
	R. J. ARMANI and D. M. SMITH	
IV-24.	Multichannel Analyzers.....	297
	G. S. STANFORD	
IV-25.	Incorporation of Non-Standard Input-Output Devices into a FORTRAN System..	298
	C. E. COHN	
IV-26.	Calibration of Small Fission Chambers by the Coincidence Method.....	299
	K. G. PORGES and A. DEVOLPI	
IV-27.	Solid-State Neutron Spectrometer.....	300
	R. A. KARAM	
IV-28.	Note on the Reduction of Rossi-Alpha Data.....	304
	C. E. COHN	
IV-29.	Serial Klystron-Magnetic Bunching.....	306
	A. B. SMITH	
IV-30.	Multi-angle Time-of-Flight System.....	306
	C. H. NELSON and A. B. SMITH	
IV-31.	Accelerator Digital System.....	307
	J. F. WHALEN, R. ROGE, J. W. MEADOWS and A. B. SMITH	

SECTION V

MISCELLANEOUS

V-1.	Resonance Parameter Analysis.....	313
	E. M. PENNINGTON	
V-2.	Resonance Integrals in Slab Lattices Using Various Escape Probabilities.....	316
	E. M. PENNINGTON	
V-3.	Tables of Resonance Integral Functions, $L'(Z', \xi)$	318
	E. M. PENNINGTON	
V-4.	Equivalence Between Homogeneous and Heterogeneous Resonance Integrals in Cylindrical Geometry.....	319
	H. H. HUMMEL	
V-5.	Calculation of Resonance Absorption Using Non-Asymptotic Neutron Sources in Two Region Cells.....	321
	P. H. KIER	

¹ Valparaiso University, Valparaiso, Indiana² Reactor Engineering Division, Argonne National Laboratory³ Electronics Division, Argonne National Laboratory

V-6.	Moderator Scattering and Flux Depression Near Resonances.....	324
	P. F. GAST	
V-7.	A Simple Estimate of the Effects of Resonance Interference.....	324
	C. N. KELBER	
V-8.	Improved Rational Escape Probability in Lumped Absorbers.....	325
	C. N. KELBER	
V-9.	Effects of Chemical Binding on the Doppler Broadened Cross Section of Uranium in a UO_2 Lattice.....	327
	C. R. ADKINS ¹	
V-10.	Spatial Dependence of Prompt-Neutron Chains.....	330
	R. A. KARAM	
V-11.	On the Synthesis of Space Dependent Transfer Functions.....	332
	C. K. SANATHANAN	
V-12.	The Neutron Flux Distribution in a Subcritical Multiplying Medium with a Constant External Source.....	335
	D. H. SHAFTMAN	
V-13.	The Americium-Alpha-Lithium Neutron Spectrum.....	337
	E. F. BENNETT	
V-14.	Reactor Computation Development.....	339
	B. J. TOPPEL	
V-15.	The Argonne Revised THERMOS Code.....	340
	B. J. TOPPEL	
V-16.	Multigroup Constants Code MC ²	341
	D. M. O'SHEA, B. J. TOPPEL and A. L. RAGO ²	
V-17.	Aids for Maintaining and Checking an Evaluated Nuclear Data File.....	342
	D. M. O'SHEA, J. J. ZENOR and (MRS.) P. L. WALKER	
V-18.	MACH 1, A One-Dimensional Diffusion Theory Calculation System.....	344
	D. A. MENELEY, L. C. KVITEK and D. M. O'SHEA	
V-19.	Numerical Calculation of Laplace Transforms.....	345
	R. A. MEWALDT ³ and H. C. THACHER, JR.	
V-20.	Evaluation of Mathematical Functions.....	351
	H. C. THACHER, JR.	
V-21.	Stochastic Techniques for Reactor Analysis.....	357
	L. MILLER ⁴	
V-22.	Further Evaluation of a Rotating Power Generating System.....	359
	A. J. ULRICH, R. O. BRITTAN, R. G. POST ⁵ , B. J. HENDERSON ⁶ and J. C. CARTER	
V-23.	A Parametric Design Study of a Reactor System to Produce High Temperature Gaseous Lithium Hydride.....	361
	M. B. RODIN and J. C. CARTER	
V-24.	Efficiency of Direct Conversion of Nuclear Energy to Coherent Laser Light.....	362
	J. A. DESHONG, JR.	
V-25.	The Amplification of Coherent Gamma Photons.....	366
	J. C. CARTER	
V-26.	Thermionic Energy Conversion with a Plasma Thermocouple.....	372
	H. K. RICHARDS	
V-27.	Reactor Physics Constants Center—Information Storage and Retrieval.....	378
	L. TEMPLIN, H. THACHER and (MRS.) P. L. WALKER	
V-28.	Reactor Physics Constants Center—Resonance Integral and Age Compilation....	379
	P. J. PERSIANI and A. E. MCARTHUR	
V-29.	Vaporization of Sodium in Reactor Coolant Channels.....	380
	D. R. MACFARLANE	
V-30.	Parametric Studies Using a Coolant Expulsion Model.....	385
	C. DICKERMAN, W. STEPHANY and A. WRIGHT ⁷	
V-31.	The Effects of the Thermal Characteristics of UO_2 Upon the Dynamics of High Performance Fast Reactors.....	390
	C. K. SANATHANAN, J. C. CARTER and R. O. BRITTAN	
V-32.	Fuel Element Thermal Displacements and Stresses for Reactor Feedback and Structural Integrity Analyses.....	397
	C. K. YOUNGDAHL	

¹ Student, Carnegie Institute of Technology, Pittsburgh, Pennsylvania

² Applied Mathematics Division, Argonne National Laboratory

³ Student, Lawrence University, Appleton, Wisconsin

⁴ Student, University of Illinois, Urbana, Illinois

⁵ University of Arizona, Tucson, Arizona

⁶ Student, University of Georgia, Athens, Georgia

⁷ Student, Northwestern University, Evanston, Illinois

V-33.	A Graphical Method for Determining the Radial Distribution of Thermal Stress in Fuel Rods Possessing Temperature-Dependent Material Properties.....	397
	C. K. YOUNGDAHL	
V-34.	Properties of Materials Near Their Thermodynamic Critical Point.....	398
	D. MILLER	
V-35.	Thermophysical Properties of Reactor Materials at High Temperatures.....	400
	D. MILLER and R. D. JOHNSON ¹	

SECTION VI
PUBLICATIONS
REACTOR PHYSICS DIVISION
(April 1, 1964 to June 30, 1965)

Open Literature.....	403
Reports.....	404
Abstracts.....	405

¹ Student, Northwestern University, Evanston, Illinois

Section I

Fission Properties and Cross Section Data

A program in experimental and theoretical studies of microscopic neutron interactions with nuclei utilized in reactor systems is being pursued. The motivation is to provide the basic information requisite to efficient and optimal reactor physics design. The major portion of this effort is devoted to studies of fast neutron induced processes, particularly those which are important in fast reactors. Attention is given to: neutron elastic and inelastic scattering, neutron induced reactions including capture, and to the characteristics of the fission processes. Of necessity, these activities require creative development of both experimental and theoretical methods in order to obtain the desired objectives.

1-28

I-1. Fast Neutron Scattering Program

S. G. BUCCINO, J. CHIEN*, S. A. COX, D. LISTER†, A. B. SMITH and W. G. VONACH

It is the intent in the continuing comprehensive program of fast neutron scattering studies to provide the extensive and sound foundation of experimental cross sections required for heuristic design of reactor systems. The experimental program now extends over fifty elements ranging from $A = 9$ to $A = 238$ and includes both inelastic and elastic phenomena. The associated incident neutron energy interval is 0.3 to 1.5 MeV which is defined by existing equipment. The experimental investigations are of two types: a) fast time-of-flight studies of both elastic and inelastic neutron scattering and b) detailed studies of elastic scattering with exceptional resolution. The time-of-flight studies provide an incident neutron resolution of 5–30 keV and a scattered neutron resolution of $\gtrsim 15$ keV. They are most applicable to complex scattering processes where both elastic and inelastic events are contributing factors; however, they are slow in execution and somewhat limited in resolution. Studies of elastic scattering in light to medium nuclei, where inelastic scattering is not a contributing factor, non-time dependent techniques utilizing pulse shape-sensitive detectors to discriminate against undesired background provide high sensitivity and good resolution ($\Delta E_n \gtrsim 2$ keV) are employed. The latter method is particularly

useful for studying microscopic and intermediate resonance structure.

Results obtained thus far have been extensively described in the literature. Therefore, only representative examples of the work are given in the following several papers on neutron scattering (see Papers I-2 through I-5). The reader may examine the accompanying references for a more extensive discussion of the experimental program and for results obtained.

REFERENCES

1. W. G. Vonach and A. B. Smith, *A Compendium of Fast Neutron Scattering Studies*, Proc. International Conference on Nuclear Physics, Paris, France, July 2, 1964.
2. W. G. Vonach, A. B. Smith and P. A. Moldauer, *Shell Model Effects on Neutron Optical Model Absorption*, Phys. Letters **11**, 331 (1964).
3. A. B. Smith and P. T. Guenther, *Some Elastic Neutron Angular Distributions*, Trans. Am. Nucl. Soc. **1**, No. 2, 267 (1964).
4. R. Avery et. al., *Physics of Fast Reactors*, Proc., 1964 Geneva Conference (to be published).
5. J. A. M. deVilliers, C. A. Engelbrecht, W. G. Vonach and A. B. Smith, *Neutron Scattering from Au, Hg, and Tl*, Z. Physik **183**, 323 (1965).
6. W. G. Vonach and A. B. Smith, *Scattering of Fast Neutrons from Ag, In and Cd*, Bull. Am. Phys. Soc. **10**, 498 (1965).
7. J. P. Chien and A. B. Smith, *Fast Neutron Cross Sections of Na and Al*, Bull. Am. Phys. Soc. **10**, 576 (1965).
8. S. A. Cox, *Elastic Scattering from Intermediate Weight Nuclei*, Bull. Am. Phys. Soc. **10**, 576 (1965).

* Graduate Student, University of Michigan and National Tsing Hau University, Taiwan, China.

† Post-Doctor, Columbia University.

I-2. Scattering From Heavy Elements

W. G. VONACH and A. B. SMITH

Fast neutron scattering from heavy elements is usually characterized by complex inelastic processes due to the profusion of low lying excited states resulting from the rotational and vibrational nature of the target nuclei. Due to this complexity the experimental observation of inelastic scattering in the heavy element region is often difficult. Despite this, extensive experimental studies have recently been made for a number of heavy elements including Re, W-186, W-182, Au, Tl,

Hg, Sm, Gd, Yb, and Pt. The results of the determination of the inelastic scattering cross sections of Au, Hg, and Tl are indicated in Figs. I-2-1 through I-2-3 and are representative of those obtained for heavy nuclei of an essentially spherical form. The solid curves shown on the figures represent the results of optical model and Hauser-Feshbach calculations and do give qualitative agreement with the experimental results (see Paper I-6). The dotted curves indicate calculations including

FIG. I-2-1. The Inelastic Excitation Cross Sections of Au-197. (The data points represent the measured values, the solid curves indicate Hauser-Feshbach calculations, and the dotted curves Hauser-Feshbach results corrected for resonance width fluctuations). ANL Neg. No. 112-3901 Rev. 2.

width fluctuation corrections. There appears little justification for such correction terms from a phenomenological basis in these cases. Application of the "cor-

FIG. I-2-2. The Inelastic Excitation Cross Sections of Hg. (Data points denote experimental results. The dotted and solid curves are defined as for Fig. I-2-1. However, in the lower two sections of this figure, they represent the sum of the noted excitation functions and should be compared with the solid points). ANL Neg. No. 112-3897 Rev. 2.

rected" theory in these instances can be questioned. In the more violently deformed nuclei, such as Yb and W, the experimental results cannot be described in terms of spherical model. Calculations using a deformed potential are now in progress and will, hopefully, provide a quantitative description of the experimental elastic and inelastic scattering results. Proper application of such nuclear models not only leads to a better understanding of the basic phenomena but also provides a realistic method for extrapolating the available experimental information into regions of most importance to applied reactor design.

FIG. I-2-3. The Inelastic Excitation Cross Sections of Tl. (Data points represent experimental results. The solid and dotted curves have the same meaning as in Figs. I-2-1 and I-2-2. In the two lower sections of the figure, the calculated curves refer to the indicated sums of two excitation functions and should be compared to the solid experimental points). ANL Neg. No. 112-3902 Rev. 2.

I-3. Elastic Scattering From Intermediate Weight Elements

S. A. Cox

A multiple detector system utilizing pulse shape discrimination was used to measure the angular distribution of neutrons elastically scattered from cylindrical samples of Cr, Mn, Fe, Co, Ni, Cu, Zn, Ge, Se, Zr, and Nb.¹

Differential cross sections were measured at 8 angles between 20 deg. and 160 deg. Angular distribution measurements were taken every 8 keV over the incident neutron energy range from 680 keV to 1250 keV. The neutron source was the Li(p, n) reaction and the target

thickness was approximately 20 keV as determined by the rise method. All data were corrected for multiple scattering effects, flux attenuation in the scatterer and for finite solid angle of the detectors. Absolute cross sections were obtained by making all measurements relative to scattering from carbon and utilizing the differential scattering cross section for carbon as reported by R. Lane, et al.²

Each angular distribution was fitted by the method of least squares with a fourth order Legendre poly-

FIG. I-3-1. Total Elastic Scattering Cross Sections for Even Z Elements. ANL Neg. No. 112-5150.

FIG. I-3-2. The b_2 Coefficient in the Legendre Polynomial Expansion of the Differential Elastic Scattering Cross Section for Even Z Elements. The b_2 coefficient represents a large part of the resonance scattering. ANL Neg. No. 112-5152.

FIG. I-3-3. Comparison Between the Experimental Results and Calculations from the Optical Model and Hauser-Feshbach Theory. ANL Neg. No. 112-5604.

nomial series. The results are given in terms of the total elastic scattering cross section and the 4 coefficients of the Legendre polynomials according to the following expression.

$$\frac{d\sigma_{el}}{d\Omega} = \sum_1^4 b_L P_L(\cos \theta)$$

Some results for the even Z elements are given in Figs. I-3-1 and I-3-2. Prominent fluctuations are evident in both the elastic scattering cross section and the b_2 polynomial coefficient. Similar fluctuations were observed in the other polynomial coefficients for the even Z elements and in the scattering cross sections and polynomial coefficients for the odd Z elements Mn and Co. The data for Cu and Nb did not exhibit any significant fluctuations. The data shown in Figs. I-3-1 and I-3-2 were averaged over 50 keV intervals in order to increase the statistical accuracy and to smooth out some of the fine structure present in the data for Cr, Fe, and Ni.

Cross section fluctuations have been observed by many investigators in a number of charged particle induced and neutron induced reactions.³⁻⁶ Three mechanisms have been proposed to explain the observed effects. These may be classified as Ericson fluctuations⁷, random fluctuations in the nuclear level density³, and doorway state phenomena.⁸⁻¹⁰ The interpretation of the present data in terms of one or more of the above mentioned processes is uncertain; however, three aspects of the data are clear. The Ericson theory predicts a fluctuation spacing at least an order of magnitude smaller than the observed spacing. The observed fluctuations are only weakly dependent on mass number whereas the nuclear level density shows a strong dependence. The character of the fluctuations is qualitatively consistent with the predictions of the doorway state model.

Figure I-3-3 compares the present results and predictions of the statistical model. For the purpose of a meaningful comparison the data were averaged over a 200 keV interval in order to reduce effects due to the above mentioned gross structure. The optical model parameters used in the calculations were those proposed by P. Moldauer¹¹ and were the same for all of the elements studied. Contributions due to resonance scattering were taken into account in a Hauser-Feshbach calculation.¹² The agreement between theory and experiment is rather good, considering that no attempt was made to fit individual elements by adjusting the optical model parameters.

One interesting point in the comparison is that even though the optical model parameters were fixed, the calculations closely follow the point by point fluctuations in the data.

REFERENCES

1. S. A. Cox, *Elastic Neutron Scattering from Fe, Ni, and Cu*, Bull. Am. Phys. Soc. **8**, 6 (1963).
2. R. O. Lane, A. S. Langsdorf, Jr., J. E. Monahan and A. J. Elwyn, *The Angular Distributions of Neutrons Scattered from Various Nuclei*, Ann. of Phys. **12**, No. 2, 135 (1961).
3. J. Konijn and A. Lauber, *Cross Section Measurements of the Ni-58 (n,p) and Si-29 (n, α)Mg-26 Reactions in the Energy Range 2.2 to 3.8 MeV*, Nucl. Phys. **48**, 191 (1963).
4. A. J. Elwyn, J. E. Monahan, R. O. Lane and A. Langsdorf, Jr., *Neutron Scattering from Nuclei Near $A = 20$ at Energies below 2.2 MeV*, Nucl. Phys. **59**, 13 (1964).
5. G. Pappalardo, *Statistical Fluctuations and Intermediate Structures in Nuclear Reaction Cross Sections*, Phys. Letters **13**, 320 (1964).
6. A. Agodi and G. Pappalardo, *Statistical Fluctuations of Compound Nucleus Level Density and Nuclear Cross Sections*, Nucl. Phys. **47**, 129 (1963).
7. T. Ericson, *Fluctuations of Nuclear Cross Sections in the 'Continuum' Region*, Phys. Rev. Letters **5**, 430 (1960).
8. B. Block and H. Feshbach, *The Neutron Strength Function and the Shell Model*, Ann. Phys. **23**, 47 (1963).
9. A. Lande and B. Block, *Multiparticle Excitations in the*

Intermediate Compound State, Phys. Rev. Letters **12**, 334 (1964).

10. A. K. Kerman, L. S. Rodberg and J. E. Young, *Intermediate Structure in the Energy Dependence of Nuclear Cross Sections*, Phys. Rev. Letters **11**, 422 (1963).

11. P. A. Moldauer, *Optical Model for Low Energy Neutron Interactions with Spherical Nuclei*, Nucl. Phys. **47**, 65 (1963).

12. W. Hauser and H. Feshbach, *The Inelastic Scattering of Neutrons*, Phys. Rev. **87**, 366 (1952).

I-4. Fast Neutron Scattering From Na, Al and Be

J. P. CHIEN* and A. B. SMITH

Time-of-flight techniques were employed to determine the differential elastic and inelastic scattering cross sections of Na and Al and the differential elastic scattering cross sections of Be. The elastic scattering measurements were made at $\lesssim 50$ keV intervals using resolutions of $E_n \sim 20$ keV. The results were expressed as a series expansion in Legendre polynomials. The values obtained for Al are shown in Fig. I-4-1. The measured quantities vary sharply with energy due to the resonance nature of the processes. This is clear from the total cross sections also indicated in Fig. I-4-1. The indicated total cross sections were the result of very good resolution measurements (see Paper I-7) and agree well with the measured elastic cross section when

proper cognizance is taken of the respective resolutions (20 keV compared with $\lesssim 2$ keV). Results similar to those shown for Al in Fig. I-4-1 were obtained for elastic scattering from Na. The only qualitative difference is a somewhat wider resonance spacing in Na than Al. Beryllium is very light with only two resonances in the region of measurement (520 keV, 3^- ; 560 keV, 2^+). Thus, the observed angular distributions were primarily due to shape elastic scattering.

The experimental energy range permitted inelastic excitation of a single state in Na (0.438 MeV, $5/2^+$) and in two states in Al (0.842, $3/2^+$ and 1.013, $5/2^+$ MeV). The differential cross section for the inelastic excitation of each of these states was determined from within a few hundred kilovolts of the respective thresh-

* Graduate student, University of Michigan.

FIG. I-4-1. The Measured Total Elastic Scattering Cross Sections and the Coefficients of Legendre Expansion for Aluminum are Indicated by Circular Points. The Measured Total Neutron Cross Section is Indicated by the Dotted Line and the Solid Curve Denotes the Results of Optical Model Calculations. ANL Neg. No. 112-4866.

FIG. I-4-2. Sodium Total Inelastic Scattering Cross Sections, 0.438 MeV Level. ANL Neg. No. 112-5072 Rev. 1.

FIG. I-4-3. The Optical Model Calculations for Aluminum. ANL Neg. No. 112-5091.

olds to 1.5 MeV. All observed inelastic scattering in Al was essentially isotropic and the cross sections were small ($\lesssim 10$ mb/sr). The single inelastic excitation cross section in Na was relatively large and displayed a resonance behavior as shown in Fig. I-4-2. The results

of these measurements agree well with the indirect determinations of the same cross section using the $(n, n'\gamma)$ measurements reported in the literature.

The optical model and average compound nucleus theories were employed in the interpretation of the experimental results. Initial calculations utilized a surface absorption model and the results were unsatisfactory. However, parameter adjustments were made, including the use of a volume absorption term, to achieve good agreement with experimental results, as indicated in Fig. I-4-3 in which the calculated and measured differential cross sections of Al are shown averaged over an energy interval sufficient to smooth the resonance structure. Similar agreement between calculated and experimental values was obtained for Na. Starting with such a good optical model description of elastic scattering, the inelastic cross sections were calculated using Hauser-Feshbach theory. A representative result is indicated in Fig. I-4-2 by the line noted as "H-F." The "conventional" theory was then corrected for width fluctuation effects with the result indicated by the "W. F." Curve of Fig. I-4-2. The Hauser-Feshbach result does give average qualitative agreement with experiment. The corrections for width fluctuation did not improve the correlation between theory and experiment.

The startling facet of the experimental interpretation is the remarkable description of the experimental results provided by a model requiring statistical averaging of structures in the compound nucleus. This fundamental precept is not strictly adhered to in these light nuclei. Despite this fact, the model does appear successful.

I-5. Fast Neutron Scattering From Ag, In and Cd

A. B. SMITH

Representative of the scattering studies in medium weight nuclei is the determination of the cross sections for the elastic and the inelastic scattering of fast neutrons from Ag, In, and Cd. The cross sections were determined for incident neutron energies of 0.3 to 1.5 MeV in increments of 50 keV or less. The available time-of-flight resolution was sufficient to resolve the elastic scattering component and most of the complex of inelastic groups. The measured differential elastic scattering cross sections were expressed as expansions in Legendre polynomials. An example pertaining to the differential elastic scattering cross sections of Cd is graphically shown in Fig. I-5-1.

Neither Ag nor Cd are mono-isotopic. Thus, the observed differential inelastic scattering cross sections were the composites of contributions from several nuclei. Despite this complexity inelastic excitation cross sections were determined for states in Ag at 0.09, 0.33, 0.42, 0.71, 0.78, 0.95, and 1.15 MeV and in Cd at 0.31 and 0.62 MeV. The inelastic excitation cross sections

of states in essentially mono-isotopic In at 0.33, 0.58, 0.83, 0.96, and 1.10 MeV were also experimentally measured. None of the observed differential inelastic cross sections differed significantly from isotropy. Measurements were made from within several hundred kilovolts of the respective inelastic reaction thresholds to 1.5 MeV. The results obtained for Ag are shown in Fig. I-5-2.

The experimental results were interpreted in terms of the optical model and Hauser-Feshbach theory. The thesis of the optical calculations was the application of a model having wide validity rather than the detailed adjustment of the model parameters to the particular experimental cases in hand.¹ The success of this interpretation is indicated by the solid curves of Fig. I-5-1 which represent the results of optical model calculations (see Paper I-6). Hauser-Feshbach theory was employed to determine the contributions to compound elastic scattering and the various inelastic cross sections. Good qualitative agreement with experiment was achieved

FIG. I-5-1. The Differential Elastic Scattering Cross Section of Cd Expressed in its Form $d\sigma/dr = (\sigma/4\pi) (1 + \sum \omega_i P_i)$. ANL Neg. No. 112-4755.

FIG. I-5-2. Inelastic Excitation Cross Sections of Ag. ANL Neg. No. 112-4958 Rev.

as indicated by the solid curves of Fig. I-5-2. Subsequent adjustment of the Hauser-Feshbach results in accord with the theoretical calculations of P. Moldauer gave the results indicated by the dotted curves in Fig. 2, Ref. 2. The application of this "corrected" theory can be questioned particularly in view of the generality required of the optical potential, a tenet that may not lead to a sufficiently detailed agreement between theory

and experiment to permit a rigorous test of the validity of the correction factors.

REFERENCES

1. P. A. Moldauer, *Average Compound Nucleus Cross Sections*, Rev. Nucl. Phys. **36**, 1079 (1965).
2. P. A. Moldauer, *Optical Model of Low Energy Neutron Interactions with Spherical Nuclei*, Nucl. Phys. **47**, 65 (1963).

I-6. Experimental Interpretation Using a Surface Absorption Optical Potential for Low Energy Neutron Scattering

A. B. SMITH

A number of successful and specific applications of the optical model to the interpretation of neutron phenomena have been described in the literature. These optical model evaluations tend to be confined to energies where the compound elastic scattering process is negligible (above several MeV), or to cases where inelastic scattering is not a contributing factor. In either of these circumstances the interpretation of the

reaction cross section is simplified. Successful as these interpretations may be they are not particularly useful to the reactor designer as the incident neutron energies are too high and/or the important inelastic process is not considered. For reactor application the energy region near and below 1.0 MeV is most critical and since the reactor system often contains materials having large inelastic cross sections, the effect of both com-

FIG. I-6-1. Experimental and Calculated Values of the Cross Sections for Both Elastic and Inelastic Scattering of 1.0 MeV Neutrons from a Wide Range of Nuclei. (The Solid curve represents the result obtained with a surface absorption optical potential and a 14 MeV imaginary potential. Other values of the imaginary potential are also indicated.) ANL Neg. No. 112-5625.

pound elastic scattering and inelastic scattering must be included in any useful calculation. The calculation of the relative contributions of the elastic and inelastic scattering processes depends upon a knowledge of the detailed structure of the target nucleus and upon significant experimental information dealing with both types of scattering. The requisite experimental scattering quantities have resulted from the extensive studies described in Papers I-1 through I-5. Thus it is now

possible to meaningfully apply a model in the region of most interest.

The basic model employed in this work is the surface absorption optical potential of P. Moldauer¹ and the compound nucleus theory of W. Hauser and H. Feshbach.² With this foundation the imaginary portion of the optical potential, W , was varied in size and position so as to obtain the best overall agreement with the measured experimental results. The intent was to

obtain a satisfactory description of the entire mass ($A = 23$ to 238) and energy ($E = 0.3$ to 1.5 MeV) intervals rather than to carry out localized parameter adjustments that would destroy the general applicability of the model. It was realized that such local adjustments would improve the calculated fit to experiment in isolated cases and might be necessary to interpret fine details of reaction theories. With this philosophy a number of calculations were carried out including all of the nuclei measured in the scattering program. A representative comparison of results pertaining to a neutron energy of 1.0 MeV is given in Fig. I-6-1. The upper three sections of the figure show the elastic scattering cross section and the first two coefficients of the Legendre expansion of the elastic differential distributions as a function of target mass number. The calculated result, obtained with an imaginary potential of 14 MeV, is indicated by the solid line which follows the measured experimental points with considerable definition. The results obtained with $W = 2, 8$ MeV are also indicated but neither provides a generally satisfactory agreement with experiment. The lower portion of the figure compares the measured inelastic cross sections (data points) with the result calculated using $W = 14$ MeV. Again the agreement with experiment is good, except in the region of deformed nuclei where the spherical model employed is

not strictly applicable. The effect of nuclear shell structure is clearly evident in the figure and it is due principally to the shell dependence of the density of low lying excited states. Beyond this there is a more subtle effect upon the optical potential itself, since near the doubly closed shell at $A = 208$ it is necessary to reduce the imaginary potential to 5 or 2 MeV to achieve a good agreement with experiment. A similar but less pronounced reduction is required in the region of the closed neutron shell $N = 50$ at $A = 90$. The model illustrated here, in the context of simple scattering, is also capable of qualitatively interpreting other neutron induced phenomena, including polarization. The model can also be extended to cover a wide range of incident neutron energies by introducing an energy dependence of the parameters in a logical manner.

With a model such as that described above it is not clear, a priori, that a desired cross section for scattering should be obtained experimentally. Indeed, proper extrapolation of experimental values using the model may lead to a more accurate cross section than can be obtained experimentally.

REFERENCES

1. P. A. Moldauer, *Optical Model for Low Energy Neutron Interaction With Spherical Nuclei*, Nucl. Phys. **47**, 65 (1963).
2. W. Hauser and H. Feshbach, *The Inelastic Scattering of Neutrons*, Phys. Rev. **87**, 366 (1952).

I-7. High Resolution Total Neutron Cross Sections

J. F. WHALEN, J. W. MEADOWS and A. B. SMITH

The previously described nanosecond time-of-flight technique for determining fast neutron total cross sections has been employed to study Fe, Al, Pb-208, Pb-206, and Na in appreciable detail.^{1, 2} Resolutions were less than ~ 1 keV at 0.5 MeV, providing a knowledge of resonance structure not easily obtainable in other ways. Improvements in the method have included the use of on-line computers to directly determine the cross sections in barns as the data is being taken. An example of the results obtained for Fe is shown in Fig. I-7-1. Some of the resonances shown have measured widths of less than 1 keV. This fine resolution makes possible a reasonable multi-level fit to the experimental results and has led to the specific identification of a number of excited states in Fe-27 and Al-28.

The effectiveness of the experimental resolution obtained by the time-of-flight method is demonstrated by comparing part A and part B of Fig. I-7-1. The former set of data was obtained using a conventional mono-energetic source technique giving a resolution of ~ 2.5 keV. Clearly, much of the detailed resonance structure is lost.

REFERENCES

1. J. F. Whalen, R. Roge and A. B. Smith, *Fast Neutron Total Cross Sections with an Automated Facility*, Bull. Am. Phys. Soc. **9**, 651 (1964).
2. W. G. Vonach, J. F. Whalen and A. B. Smith, *Fast Neutron Total Cross Sections*, Reactor Physics Division Annual Report July 1, 1963 to June 30, 1964, ANL-7010 (January 1965) p. 8.

FIG. I-7-1. Total Neutron Cross Section of Fe. (Part A was obtained using mono-energetic source techniques providing a resolution of ~ 2.5 keV. Part B was obtained with the time-of-flight method and a resolution of ~ 0.1 nsec/M). ANL Neg. No. 112-5325.

I-8. Total Neutron Cross Sections at High Energies

D. LISTER and A. B. SMITH

A number of recent theoretical and experimental studies have indicated that the total neutron cross sections of light and medium weight nuclei display a gross structure at energies ≥ 4.0 MeV.¹ Unfortunately,

measurements in this high energy region are difficult due to the lack of a suitable mono-energetic neutron source. Recent detailed studies of the reaction $\text{Be-9}(d,n)\text{B-10}$ using fast time-of-flight techniques at

FIG. I-8-1. Total Neutron Cross Section of Al. ANL Neg. No. 112-5613.

both the ANL tandem accelerator and the 3.0 MeV pulsed Van de Graaff make it attractive to use this (d,n) reaction as a source for fast neutron total cross section measurements. The observed neutron groups are very well defined, the neutron energies available extend over a wide range (1.0 to 12.5 MeV) and are well known, and the yield is sufficient to permit reasonably accurate measurements. Initial studies have been made at the tandem generator and have resulted in total cross sections for Co, Mg, Fe and Al in the energy interval 4–12 MeV. A representative result pertaining to Al is given in Fig. I-8-1. The energy coverage is not

continuous as yet and the results display too great a spread in cross sections to permit a detailed knowledge of any small structure effects. However, the feasibility of the method has been amply demonstrated and measurements soon to be undertaken with the intense pulsed beam of the 3.0 MeV machine should provide definitive results. The development of sources suitable for this and other work has been the motivation for the (d,n) studies described in Paper I-15.

REFERENCES

1. *Neutron Cross Sections*, Edited by J. Stehn, BNL-325, 2nd Supplement to Second Edition, 1965.

I-9. Precision Determinations of Fast Neutron Total Cross Sections in the Heavy Elements

J. F. WHALEN, A. B. SMITH and J. W. MEADOWS

FIG. I-9-1. Measured Total Cross Sections of U-235 and Re. *ANL Neg. No. 112-5383*.

In view of the recent observation of sizable discrepancies in such important cross sections as σ_f (U-235) it is of interest to carefully reexamine certain of the fast neutron total cross sections important to reactor design concepts. These include the total cross sections of U-nat, U-235, Re, and W. The closed-loop, fully automated facility described in Paper IV-31 below is admirably suited to such investigations since the on-line computer systems can calculate the cross sections, assay their precisions, and make such control adjustments as are warranted to obtain the best possible information. Furthermore, the automated system does free the scientific personnel from much of the

laborious data reduction usually associated with the determination of total neutron cross sections.

Using the above system a number of important materials have been studied. As an example, results obtained for U-235 and Re are shown in Fig. I-9-1. These and many other measurements are being used to formulate an extensive library of total cross sections. The library is available for those interested in establishing multi-group cross section sets and it has proven valuable as a reference for checking other measurements of microscopic partial cross sections as, for example, the elastic and inelastic scattering cross sections.

I-10. Measurement of the Li-6 (n, absorption) and B-10 (n, absorption) Cross Sections by the Shell Transmission Method

S. A. Cox

INTRODUCTION

A knowledge of the absolute neutron flux is usually required in the measurement of absolute neutron cross sections. The reactions most commonly utilized in neutron flux monitors are: B-10 (n, α), Li-6 (n, α) and U-235 (n,f). In the region below 150 keV incident neutron energy little was known concerning the U-235 fission cross section until the recent measurements of P. H. White et al.¹ While these precision measurements provide several excellent calibration points, still more measurements would be useful.

EXPERIMENTAL METHOD

The shell transmission method consists of placing a neutron detector some distance from a neutron source, measuring the neutron intensity of the source alone, then repeating the measurement with a spherical shell of some material surrounding the source. The ratio of the two measured intensities, i.e. shell on/shell off, is defined as the sphere transmission. Since there is a definite relationship between the sphere transmission and the absorption cross section of the shell material², the measurement is independent of any knowledge of the absolute neutron flux.

The sphere transmission method is well suited to the measurement of the lithium and boron absorption cross sections. Since the (n, α) cross section below 100 keV is a relatively large fraction of the total cross section for both nuclei, the elastic scattering cross section need not be known to high accuracy. Indeed for B-10 (n, α) at $E_n = 10$ keV a calculation using the method of H.

Bethe et al² shows that a 50% error in the elastic scattering cross section introduces less than a 4% error in the absorption cross section. Also, the somewhat uncertain resonance self protection correction³ which must be considered for heavy nuclei does not apply to nucleides as light as Li-6 and B-10 where the resonances are broad and widely spaced.

In the present work, measurements were made with the neutron source inside and with the neutron source outside the sphere. Since the corrections were quite different for the two cases, it was believed that the chance of any systematic error due to the various corrections would be reduced if both methods were used. The Li-7 (p,n) reaction was utilized as the neutron source outside the sphere and the V-51 (p,n) reaction was used for the neutron source inside the sphere.

MEASUREMENT WITH THE Li-7 (p,n) SOURCE

The lithium targets were in the form of vacuum deposited films of either Li₂F or Li₃N and were 5 keV to 20 keV thick to 1.88 MeV protons. The films were deposited on silver backings to insure adequate cooling during proton bombardment. The protons were obtained from a 3 MeV Van de Graaff accelerator.

The neutron detector was a cadmium covered spherical proportional counter 4 cm in diameter filled to one atmosphere with BF₃. The counter was developed at AWRE for use as a hydrogen recoil counter and is described in detail elsewhere.⁴

Measurements were made with the neutron detector either at 0 or 120 deg with respect to the incident proton

direction. Use of both of these angles was necessary in order to cover the neutron energy range from 9 keV to 300 keV and at the same time insure that the incident neutrons were monoenergetic. Below 150 keV, measurements were made at 120 deg and above 150 keV at 0 deg.

Since the cadmium covered BF_3 counter used in the measurements was 20 to 30 cm from the source a large fraction of the total counting rate was due to epi-cadmium neutron background. The fraction varied with experimental conditions but was typically 20% to 40%. Also because both Li-6 and B-10 have high absorption cross sections in the epi-cadmium region, the background was smaller with a sphere around the detector than with the bare detector. In order to separate the epi-cadmium counts from the counts due to neutrons coming directly from the source, it was necessary to pulse the proton beam. The repetition rate was 10 kilocycles per second and the pulse was rectangular with a width of 12 micro-seconds. The essentially time-independent background due to epi-cadmium neutrons was easily distinguished from the time-dependent counts due to neutrons coming directly from the target

MEASUREMENTS WITH THE V-51(p,n) SOURCE

Since the vanadium target was inside the sphere all neutrons emitted from 0 to 180 deg passed through the shell. Seven paraffin wax moderated BF_3 counters were used to integrate over the neutron angular distributions. The seven identical counters were placed at the following angles: (30, 50 deg), 90 deg, (70, 110 deg), (130, 150 deg). Six of the counters were connected in pairs as indicated by the parenthesis. The total counts accumulated by each of the "groups" of counters were recorded separately in order to measure the degree of anisotropy of the angular distributions. The measured forward/backward anisotropy varied from ~ 2 to nearly isotropic.

The boron shell consisted of 89.4 a/o B-10 powder compacted between two thin spherical shells of copper. The outside diameter of the sphere was 6.043 cm and the shell thickness was 0.925 cm.

Two different size shells of Li-6 were used. Both were machined from cast lithium metal which consisted of 95.4 a/o Li-6. The outside diameter and shell thickness were 10.207 cm and 5.138 cm respectively for the large sphere and 7.695 cm and 2.626 cm respectively for the small sphere.

RESULTS AND DISCUSSION

The data were fully corrected for multiple scattering and finite distance effects. The experimental errors represent the combined effect of uncertainties in count-

FIG. I-10-1. Experimental Results for the B-10 Neutron Absorption Cross Section. [The dashed line represents a $1/\sqrt{E_n}$ extrapolation from the thermal neutron absorption cross section (3480 barns)]. ANL Neg. No. 112-5606.

FIG. I-10-2. Experimental Results for the Li-6 Neutron Absorption Cross Section. [The dashed line represents a $1/\sqrt{E_n}$ extrapolation from the thermal neutron absorption cross section (945 barns)]. ANL Neg. No. 112-5605.

ing, background determination and neutron energy determination, and also include an estimated uncertainty in the knowledge of the elastic scattering cross sections used in the multiple scattering correction.

The results are given in Figs. I-10-1 and I-10-2. The dashed line in each figure represents a $1/\sqrt{E_n}$ extrapolation from the thermal neutron absorption cross section.

It is evident from the figure that for B-10 the experimental results agree very well with the $1/\sqrt{E_n}$ line while the Li-6 data lies much higher than the $1/\sqrt{E_n}$

line. The present B-10 results, which are in very good agreement with the recent results of F. Mooring,⁵ indicate that the B-10 neutron absorption follows a $1/\sqrt{E_n}$ cross section behavior up to at least 200 keV.

The Li-6 results are in good agreement with the measurements of F. Gabbard, et al.,⁶ G. Gorlov, et al., S. Bame and R. Cubbit,⁸ and H. Bichsel and T. Bonner.⁹ They are in disagreement with the results of S. Schwarz, et al.¹⁰ and A. Bergman, et al.¹¹ at low neutron energy. At the higher energies near 100 keV, all the measurements are in fair agreement.

The fact that the results of the measurements made with two different lithium spheres and with two different neutron sources were in good agreement lends support to the present results.

REFERENCES

1. P. H. White, Atomic Weapons Research Establishment, Aldermaston, England (Private Communication).
2. H. A. Bethe, J. R. Beyster and R. E. Carter, *Inelastic Cross Sections for Fission Spectrum Neutrons-I*, J. Nucl. Energy **3**, 207 (1956); Part II, J. Nucl. Energy **3**, 273 (1956); Part III, J. Nucl. Energy **4**, 3 (1957); and Part IV, J. Nucl. Energy **4**, 147 (1957).
3. H. W. Schmitt and C. W. Cook, *Absolute Neutron Absorption Cross Sections for Sb-Be Neutrons*, Nucl. Physics **20**, 202 (1960).
4. R. W. Benjamin, C. D. Kemshall and J. Redfearn, *A High Resolution Spherical Proportional Counter*, AWRE Report NR 1/64.
5. F. P. Mooring, Physics Division (Private Communication).
6. F. Gabbard, R. H. Davis and T. W. Bonner, *Study of the Neutron Reaction Li-6(n, α)H-3, F-19(n, γ)F-20, and I-127(n, γ)I-128*, Phys. Rev. **114**, 201 (1959).
7. G. V. Gorlov, et al., *Measurements of the Li-6(n,t)He-4 Cross Section for Neutrons with Energies from 9 to 700 keV*, Soviet Phys. Doklady, **1**, 705 (1956).
8. S. J. Bame and R. L. Cubbit, *Li-6(n,t)He-4 Reaction at Intermediate Energies*, Phys. Rev. **114**, 1580 (1959).
9. H. Bichsel and T. W. Bonner, *Reactions Li-7(α ,n)B-10, Li-7(α , α'), and B-10(n, α)Li-7*, Phys. Rev. **108**, 1025 (1957).
10. S. Schwarz, L. G. Stromberg and A. Bergstrom, *A Relative Measurement of the Li-6(n, α)H-3 Cross Section*, Research Institute for National Defense, Stockholm, Sweden (Private Communication).
11. A. A. Bergman and F. L. Shapiro, *Measurements with a Slowing Down Spectrometer Employing Lead*, Soviet Physics JETP **13**, 895 (1961).

I-11. Gamma Rays from Inelastic Neutron Scattering

J. F. BARRY* and D. STUPEGIA†

INTRODUCTION

Measurements of neutron inelastic scattering cross sections are commonly made by detecting either the scattered neutrons or the gamma rays which follow the neutron emission. The former method has undergone intensive development and is now a more precise technique in most of the energy regions of interest. However, it is limited at energies immediately above the threshold for excitation of a state due to the difficulty of detecting the emitted low energy neutrons. The gamma ray detection method does not have this limitation. It is particularly worthwhile to study inelastic reactions near threshold energies since this region is critical in theoretical interpretations in terms of a nuclear model and the level structure of the nucleus involved.

This discussion describes a system that has been set up to measure inelastic γ -ray production cross sections and their differential distributions. A sodium iodide scintillation spectrometer with an anticoincidence

Compton shield is used to obtain γ -ray spectra and, further, a fast time-of-flight selection system is employed to reduce backgrounds. The nucleus Nb-93 has been investigated and inelastic excitation cross sections at 90 deg determined for levels at 0.741, 0.809, 0.956, and 1.080 MeV for incident neutron energies up to 1.44 MeV. In addition, angular distribution measurements were made at 1.06 and 1.41 MeV. Absolute values of the cross sections were obtained by calibration of the γ -ray detector with standardized sources and by calibration of the neutron monitors in terms of the fission cross section of U-235.

EXPERIMENTAL METHOD

The principle problem in determining useful γ -ray spectra from neutron scattering is the suppression of the intense background due to both neutrons and γ -rays from the neutron source. A heavy shield was necessary to reduce direct and room-scattered radiation, and time-of-flight selection was employed to reduce the remaining background and eliminate pulses from unwanted radiation scattered by the sample. The neutron flux during irradiations was monitored with

* Atomic Weapons Research Establishment, Aldermaston, England.

† Chemical Engineering Division.

long counters which, in turn, were calibrated in a separate measurement using a Np-237 fission counter.

The determination of the over-all efficiency of the counting arrangement was made with the aid of several gamma emitting sources of known disintegration rates. Sources of Cr-51 (0.32 MeV), Be-7 (0.478 MeV), Cs-137 (0.661 MeV), and Mn-54 (0.840 MeV) were prepared from solutions whose disintegration rates had been measured with a calibrated scintillation counter.

RESULTS

INELASTIC EXCITATION CROSS SECTIONS OF Nb-93

In the present work the yield of inelastic γ -rays from levels in Nb at 0.741, 0.809, 0.958, and 1.080 MeV was measured in steps of approximately 50 keV from thresh-

FIG. I-11-1. Cross Section for the Production of 0.741 MeV State of Nb-93 by Neutron Inelastic Scattering. ANL Neg. No. 112-5623.

old up to 1.44 MeV and the respective cross sections calculated assuming isotropic γ -ray emissions. A typical result pertaining to the 0.741 state is given in Fig. I-11-1. The standard deviation of the results is estimated to be 10% (7% relative) and is principally due to: uncertainties in the neutron flux measurement (7%), spectrum stripping procedure (4%), and the γ -ray efficiency determination (3%). The absolute error in the measured excitation of the 809 keV level is subjectively estimated to be 20% due to the difficulty in analyzing the respective photopeak in the presence of the strong 741 keV photopeak.

Excitation of the level at 1.080 MeV appears mainly through the production of a 335 keV γ -ray which is first produced at a neutron energy of 1.14 MeV. Cascade decay of the 1.08 level through the 741 keV level was therefore taking place and a correction was applied for this effect in obtaining the cross section for the excitation of the 741 keV level. At the highest neutron energy measured an additional gamma ray of 540 keV energy was observed and attributed to excitation of the level at 1.28 MeV cascading through the level at 741 KeV.

Comparison of the results with various other measurements shows that there is good agreement with the excitation function determined by D. Reitmann, et al.¹ for the 741 KeV level but only fair agreement of the excitation of 809 and 958 KeV levels. In all cases, the data of N. Nath, et al.² seems high.

ANGULAR DISTRIBUTION MEASUREMENTS

The cross sections given above were based on the assumption that the angular distributions of each of the γ -rays is isotropic. This is not necessarily the case and

FIG. I-11-2. Cross Section for Production of 0.741 MeV State of Nb-93 by Neutron Inelastic Scattering (Angular Distribution). ANL Neg. No. 112-5622.

measurements have been made to determine the distribution. The procedure was identical to that followed in making the 90 deg excitation functions.

Measurements were made at angles of 38, 60, 90, 120 and 142 deg for neutron energies of 1.060 MeV and 1.41 MeV. The angular distributions were determined for the 741 keV + 809 keV gamma rays, combined, and for the 958 keV gamma ray. The former result is shown in Fig. I-11-2. The gamma rays are isotropic within

$\pm 15\%$ but the spread in the results is outside the 7% errors expected. The reason for this is not apparent and it is proposed to repeat these measurements and to extend the work to a number of other nuclei.

REFERENCES

1. D. Reitmann, C. A. Engelbrecht and A. B. Smith, *Inelastic Neutron Scattering in Nb-93*, Nucl. Phys. **48**, 593 (1963).
2. N. Nath, et al., *Scattering of Fast Neutrons from Natural Zirconium and Niobium*, Nucl. Phys. **14**, 78 (1959).

I-12. Energy Dependence of $\bar{\nu}_p$ for U-235 from 0.039 to 1.0 MeV

J. MEADOWS and J. WHALEN

INTRODUCTION

Recent measurements^{1, 2} of $\bar{\nu}_p$ (the average number of prompt neutrons emitted per fission) for fast-neutron induced fission of U-235 indicated that at about 0.3 MeV there may be a departure from an approximately linear increase with energy. In an attempt to confirm this phenomenon, a series of measurements has been made over the energy range from 0.039 to 1.0 MeV.

EXPERIMENTAL METHOD

The experimental method has been previously described in detail.³ Minor modifications were made in order to adapt the equipment to the specific measurements. These included the installation of two concentric rings of BF₃ counters (twelve counters each) replacing the original single ring and providing considerably increased sensitivity. All measurements were made relative to $\bar{\nu}_p$ for Cf-252. In order to avoid errors due to a change in the background or in the over-all neutron detection efficiency, measurements of $\bar{\nu}_p$ for U-235, for Cf-252, and the background determinations were made simultaneously. Basically the apparatus consisted of two fission chambers, one containing Cf-252 and the other containing U-235, and a neutron detector. Fission neutrons were counted for a 170 μ sec interval after each respective fission event while the background was counted for a 340 μ sec interval beginning 900 μ sec after the fission. The ratio of $\bar{\nu}_p$ for U-235 to $\bar{\nu}_p$ for Cf-252 is given by

$$\bar{\nu}_p(\text{U-235})/\bar{\nu}_p(\text{Cf-252}) = C \left[\frac{N_U - B_U}{F_U} \right] \left[\frac{N_{\text{Cf}} - B_{\text{Cf}}}{F_{\text{Cf}}} \right]^{-1}$$

where N_i is the neutron counting rate, B_i is the background counting rate, F_i is the number of fissions, and C is a correction factor. Subscripts denote the U-235 or Cf-252 fission events. The correction factor, C , in-

TABLE I-12-I. PROMPT $\bar{\nu}$ VALUES FOR U-235

E_n , MeV	$\bar{\nu}_p$
0.039	2.401 $\pm$ 0.017
0.046	2.403 $\pm$ 0.016
0.150	2.449 $\pm$ 0.018
0.225	2.466 $\pm$ 0.018
0.265	2.455 $\pm$ 0.022
0.298	2.458 $\pm$ 0.022
0.325	2.496 $\pm$ 0.018
0.358	2.422 $\pm$ 0.018
0.375	2.467 $\pm$ 0.022
0.405	2.454 $\pm$ 0.022
0.425	2.519 $\pm$ 0.017
0.476	2.499 $\pm$ 0.019
0.548	2.487 $\pm$ 0.017
0.675	2.501 $\pm$ 0.017
0.785	2.518 $\pm$ 0.014
1.000	2.540 $\pm$ 0.016

cludes effects due to: (1) dead time, (2) geometric asymmetry of the neutron detector, (3) fission by neutrons of lower energy, and (4) energy and angular dependence of the neutron detector efficiency. The first three corrections were made as described in Ref. 3. The fourth correction was made by employing a Monte Carlo calculation. It was not possible to compare the calculated angle and energy dependence of the detector efficiency with experimental values. However, the computed total neutron detector efficiency and the relative efficiency of the two counter rings were comparable with the experimental values. For Cf-252 spontaneous fission neutrons the respective values are

	Calculated	Observed
Total Efficiency, %	10.8 $\pm$ 0.3	9.7 $\pm$ 0.1
Relative Efficiency—Inner to Outer Ring, %	2.1 $\pm$ 0.1	1.93 $\pm$ 0.04

FIG. I-12-1. $\bar{\nu}_p$ for U-235. (The horizontal bars indicate the spread in the incident neutron energy. The vertical bars indicate the statistical error). ANL Neg. No. 112-5603.

RESULTS

The results are shown in Table I-12-I and Fig. I-12-1. All absolute values are based on $\bar{\nu}_p$ of Cf-252 = 3.782. The errors quoted are statistical errors due to counting only. However these do account for the major part of the error in the relative values. While the general appearance of the data gives some indication of fluctuations in the region around 0.4 MeV, in no case do any of the data points depart from a linear fit by more than ~ 2.5 standard deviations. Although most of the larger fluctuations in the data are concentrated in this region the statistical accuracy is not yet sufficient to

permit a definitive conclusion regarding structure in $\bar{\nu}_p$ (U-235) in the energy region 0.3–1.0 MeV.

REFERENCES

1. D. S. Mather, P. Fieldhouse and A. Moat, *Prompt Average Number of Prompt Neutrons from U-235 Fission Induced by Neutrons from Thermal to 8 MeV*, Phys. Rev. **133**, B1403 (1964).
2. Yu A. Blyumkin, et al., *Channel Effects in the Energy Dependence of the Number of Prompt Neutrons and the Kinetic Energy of Fragments in the Fission of U-235 and U-233 by Neutrons*, Nucl. Phys. **52**, 648–672 (1964).
3. J. W. Meadows and J. F. Whalen, *Energy Dependence of Prompt $\bar{\nu}$ for Neutron Induced Fission of U-235*, Phys. Rev. **126**, 197 (1962).

I-13. Mass-Energy Distributions for 6.0 MeV Neutron Fission of U-235

J. W. MEADOWS and J. F. WHALEN

INTRODUCTION

In many experimental studies of the fission process the identity and excitation energy of the fissioning nucleus is uncertain due to the possibility of neutron emission prior to fission. However, in neutron induced fission below 6 MeV both of these quantities are known. The results of a comparison between thermal and 6.0 MeV neutron induced fission of U-235 are reported here.

EXPERIMENTAL METHOD

The basic experimental method has been reported previously¹ and thus is only summarized here. Two-parameter pulse height measurements were made with solid state detectors. The pulse height distributions were

converted into energy distributions and corrected for: (1) energy losses in the source, support foil, and detector windows, (2) a mass dependent pulse height defect, and (3) the center of mass motion. The energy calibrations were based on the time-of-flight studies of U-235 thermal fission carried out by J. Milton and J. Fraser.² The energy distributions were converted into mass-total kinetic energy distributions and corrected for neutron emission.

Neutrons of 6.0 MeV energy were obtained from the $D(d,n)T$ reaction using 2.75 MeV deuterons incident on a deuterium gas target. Thermal neutrons were obtained by encasing the fission chamber in a paraffin block. Measurements were made on fission fragments

TABLE I-13-I. SUMMARY OF RESULTS OBTAINED FROM TWO-PARAMETER MEASUREMENTS OF U^{235} FISSION INDUCED BY THERMAL AND 6.0 MeV NEUTRONS

Neutron Energy	Thermal	6 MeV		
Detector Angle, deg	—	0	100	All
Average mass of light peak $\bar{M}_L$, amu	96.4	98.1	96.7	97.6
Variance of light peak σ^2_L , amu ²	34.1	41.7	48.2	44.9
Average mass of heavy peak $\bar{M}_H$, amu	139.2	139.3	138.4	138.9
Variance of heavy peak σ^2_H , amu ²	33.3	45.4	43.2	44.7
Average total kinetic energy $\bar{E}_T$, MeV	166.5	165.7	165.9	165.8
Average total kinetic energy corrected for neutron emission $\bar{E}_{TC}$, MeV	168.3	168.1	168.3	168.2
Peak-to-valley ratio	~470	24.4 ± 2	22.7 ± 2	23.4 ± 1.4
Number of fissions	132,123	19,420	13,908	33,328

FIG. I-13-1. Percent Mass Distributions of Thermal and 6.0 MeV Neutron Induced Fission of U^{235} . ANL Neg. No. 112-5600.

FIG. I-13-2. The Distribution of the Average Total Kinetic Energy, $\bar{E}_{TC}$ as a Function of the Mass of the Heavy Fragment. These energy values have been corrected for neutron emission. ANL Neg. No. 112-5601.

FIG. I-13-3. The Variance of the Kinetic Energy, $\sigma^2(E_K)$, as a Function of the Mass of the Heavy Fragment. ANL Neg. No. 112-5602

emitted at 0 and 100 deg to the incident neutron direction. Angular resolutions were 20 deg. Thermal measurements were made at frequent intervals and these were used to correct for the slow decrease in pulse height which was apparently caused by radiation damage to the detectors.

RESULTS

The results are summarized in Figs. I-13-1 through I-13-3 and Table I-13-I. The most prominent difference at the two energies is the 20-fold increase in the fission yield at symmetry for 6.0 MeV fission, Fig. I-13-1. There is also a general broadening of the mass distribution as can be seen from the variances of the mass peaks in Table I-13-I. This is a continuation of the trend indicated by a previously observed value¹ of 38.5 for the mass variance in 1.0 MeV fission of U-235. The distribution of the variance of the kinetic energy in Fig.

I-13-3 indicates a broadening in the energy direction, particularly in the vicinity of $M_H = 127$.

Little, if any, of the incident neutron energy appears in the average kinetic energy of the fragments (see values of E_{TC} in Table I-13-I). The statistical accuracy of the measurements is not sufficient to show changes of the order of a few tenths of an MeV. However, inspection of Fig. I-13-2 indicates that the average kinetic energy for 6.0 MeV fission may differ from that of thermal fission in a mass-dependent manner. Thus the agreement in the values of E_{TC} at the two energies may be coincidental.

REFERENCES

1. J. W. Meadows and J. F. Whalen, *Mass and Kinetic Energy Distributions in MeV Neutron Induced Fission*, Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010 (January 1965).
2. J. C. D. Milton and J. S. Fraser, *Time-of-flight Fission Studies on U-233, U-235, and Pu-239*, Can. J. Phys. **40**, 1626-1663 (1962).

I-14. Spontaneous Fission Half-Lives of Am-241, Cm-242, and Cm-244

R. J. ARMANI and R. GOLD

The standard method used to determine spontaneous fission half-lives consists of fission fragment counting and alpha counting, which respectively determine fission rate and the number of atoms of the isotope present. In the present measurement alpha decay was

measured, but instead of counting fission fragments, neutrons emitted in the fission process were counted. This method eliminates the problems of self absorption of the fission fragments and alpha pulse pileup but it does introduce limitations of its own. In particular, un-

certainties in neutron detection efficiency and $\bar{\nu}$ are introduced.

The neutron detector was a lithium-iodide crystal, enriched to 95.6% in Li-6, mounted on a photomultiplier tube. Samples were electroplated on platinum planchets 1 in. in diameter by 0.005 in. thick. Uniform deposits were obtained over an area of 0.625 in. diameter. To prevent the spread of alpha contamination, the planchets were sealed in lucite holders. The detector assembly was modified to accept these holders as shown in Fig. I-14-1. In order to increase the sensitivity of the neutron detector, and to make the detector response less sensitive to the fission neutron spectrum, the detector assembly was centered in a D_2O neutron moderating system. A stainless steel tank 2 ft in diameter and 2 ft tall was used. Figure I-14-2 shows the arrangement of the detector in the D_2O moderating system.

A sample of Cf-252, whose fission rate was known to $\pm 5\%$, was prepared and counted in this configuration for calibration purposes. The nominal sample activities used were 10 mC of Am-241, 1 mC of Cm-242, 10 mC of Cm-244, and 1000 fissions/sec of Cf-252. The purity of each sample was determined by alpha pulse analysis. More precise alpha activities of the samples were determined by alpha counting. The absolute counting was done in a special low geometry alpha counter.¹ The

FIG. I-14-1. Neutron Detector and Sample Holder Assembly, ANL Neg. No. 112-5456.

FIG. I-14-2. Neutron Moderating System. ANL Neg. No. 112-5457 Rev.

particular low geometry counter employed for these measurements used a solid-state detector. Utilizing this system, accuracy obtained for absolute alpha activity depends mainly on the certainty with which the solid angle is known. In the worst possible case, the solid angle uncertainty was no more than 0.5% relative error.

PRELIMINARY RESULTS

The spontaneous fission half-life, T_f , was calculated from the equation

$$T_f = \frac{A_\alpha T_\alpha \bar{\nu} \epsilon}{R}, \quad (1)$$

where A_α is the alpha decay rate, T_α is the alpha half-life, $\bar{\nu}$ is the average number of neutrons emitted per fission, ϵ is the detector efficiency, and R is the neutron counting rate. As previously stated, ϵ was determined by counting a standard Cf-252 neutron source. Hence ϵ is simply

$$\epsilon = m/\bar{\nu}S, \quad (2)$$

where m is the neutron counting rate due to a Cf-252 sample of fission rate S and $\bar{\nu}$ is taken as 3.78 ± 0.04 .⁽²⁾

The preliminary results for Cm-242, Cm-244, and Am-241 are given in Table I-14-I. The relative error associated with the spontaneous fission half-life, $(\Delta T_f/T_f)$, can be calculated from Eq. (1). One finds

TABLE I-14-I. PRELIMINARY RESULTS

Isotope	T_α	$\bar{\nu}$	T_f Present Work, yr	T_f Previous Work, yr
Am-241	458.1 $\pm$ 0.5 yr ⁽⁴⁾	2.34 ^a	(3.80 $\pm$ 0.39) 10 ¹⁰	(2.3 $\pm$ 0.08) 10 ¹⁴ ⁽⁷⁾
Cm-242	162.5 $\pm$ 2 days ⁽⁶⁾	2.65 $\pm$ 0.09 ⁽³⁾	(6.47 $\pm$ 0.27) 10 ⁶	(7.2 $\pm$ 0.2) 10 ⁶ ⁽⁷⁾
Cm-244	18.11 $\pm$ 0.07 yr ⁽⁶⁾	2.810 $\pm$ 0.059 ⁽³⁾	(1.23 $\pm$ 0.03) 10 ⁷	(1.346 $\pm$ 0.006) 10 ⁷ ⁽⁶⁾

^a Estimated from the general behavior of $\bar{\nu}$ as a function of Z^2/A .

$$\left(\frac{\Delta T_f}{T_f}\right)^2 = \left(\frac{\Delta T_\alpha}{T_\alpha}\right)^2 + \left(\frac{\Delta \bar{\nu}}{\bar{\nu}}\right)^2 + \left(\frac{\Delta R}{R}\right)^2 + \left(\frac{\Delta A_\alpha}{A_\alpha}\right)^2 + \left(\frac{\Delta \epsilon}{\epsilon}\right)^2. \quad (3)$$

There exist correction factors which must be applied to these data. These correction factors are presently under experimental evaluation. For example, the present data have not been corrected for the energy dependence of the spontaneous fission neutron spectrum upon $\bar{\nu}$.⁽³⁾ Consequently it must be emphasized that the present results given in Table I-14-I can only be regarded as preliminary.

REFERENCES

1. M. L. Curtis, et al., *Absolute Alpha Counting*, Nucleonics **13**, No. 5, 38 (1955).

2. P. R. Fields and H. Diamond, *Californium-252, A Primary Standard for Neutrons*, Proc. Symp. Neutron Detection, Dosimetry and Standardization, IAEA, Harwell, England, December 10-14, 1963, Vol. II, pp. 581-593.
3. H. Soodak, *Reactor Handbook*, (Interscience Publishers, Div. of John Wiley & Sons, New York, 1962), Vol. III, Part A.
4. R. G. Hall and T. L. Markin, *The Alpha Half-Life of Am-241*, J. Inorg. Nucl. Chem. **4**, 137 (1957).
5. G. C. Hanna, B. G. Harvey and N. Moss, *The Half-Life of Cm-242*, Phys. Rev. **78**, 617 (1950).
6. D. Metta, et al., *Nuclear Constants of Nine Transplutonium Nuclides*, J. Inorg. Nucl. Chem. **27**, 33 (1965).
7. V. A. Druin and G. N. Flerov, *Spontaneous Nuclear Fission*, Physics of Nuclear Fission, Edited by N. A. Perfilov and V. P. Eismont (Israeli Program for Scientific Translations, Jerusalem, 1964).
8. G. C. Hanna, et al., *Spontaneous Fission in Cm-242*, Phys. Rev. **81**, 466 (1951).

I-15. (d, n) Reactions in the Medium Weight Nuclei; Fe-54 (d, n) Co-55 and Ni-58 (d, n) Cu-59⁽¹⁾

D. S. GEMMELL,* L. L. LEE,* J. P. SCHIFFER,* and A. B. SMITH

The fast time-of-flight system at the Argonne National Laboratory tandem Van de Graaff generator has been employed to study the reactions Fe-54(d,n) and Ni-58(d,n). Time-of-flight techniques were utilized to resolve the emitted neutron groups. Angular distributions were determined at incident neutron energies of

7.0 MeV and D.W.B.A. calculations employed to determine the respective ℓ_p values of the various transitions. The results are pertinent to the structure of the nucleus in this intermediate region.

REFERENCES

1. D. S. Gemmell, L. L. Lee, J. P. Schiffer and A. B. Smith, *The Fe-54(d,n) Co-55 Reaction*, Bull. Am. Phys. Soc. II, **9**, 676 (1964).

* Physics Division.

I-16. Stripping Reactions In Light Nuclei

S. G. BUCCINO and A. B. SMITH

THE Be-9(d,n)B-10 REACTION¹

Measurements of the angular distributions of 13 neutron groups from the Be-9(d,n)B-10 reaction have been completed at deuteron energies of 2.6, 3.0, 3.2 and 7.0 MeV. The initial phases of this work are reported in the literature.² The 7.0 MeV data taken with the pulsed-beam facility at the tandem Van de Graaff was reduced to absolute differential cross sections. These cross sections were compared with the results of distorted wave Born approximation (D.W.B.A.)³ calculations yielding ℓ_p -values and spectroscopic factors, S_ℓ . Figure I-16-1 shows the angular distributions and theoretical fits to the ground and first three excited states. The results of the measurements and comparisons to previous data are summarized in Tables I-16-I and I-16-II.

Preliminary work using this reaction as a neutron source for transmission studies in the 6-11 MeV region are underway (see Paper I-8).

TABLE I-16-I. MEASURED STATES IN B-10
(Note: References are designated by numerical superscripts)

This Experiment		Previous Work	NDS (Ref. 6)	
E_x^a , MeV	ℓ_p	ℓ_p	E_x , MeV	J^π
g.s.	1	1 ^(2, 4)	g.s.	3 ⁺
0.72 ^b	1	1 ⁽²⁾	0.72	1 ⁺
1.74 ^b	1	1 ⁽²⁾	1.74	0 ⁺
2.15 ^b	1	1 ⁽²⁾	2.15	1 ⁺
3.59 ^b	1	1 ^(2, 4)	3.59	2 ⁺
4.77 ^{b, c}	(2)	2 ⁽⁴⁾	4.77	(2 ⁺)
5.11 ± .02	0	0 ^(3, 4)	5.11	(2 ⁻)
5.16 ± .02	1	1 ⁽³⁾	5.16	(2 ⁺)
5.90 ± .08 ^d	(1)		5.92	2 ⁺
6.10 ± .08 ^{e, d}	(2)		6.04	4 ⁺
6.35 ± .05 ^e	(0)		6.12, 6.40	
6.50 ± .05	1		6.57	
6.95 ± .03	0		6.88, 6.97	1 ⁻
7.50 ± .05			7.48	2 ⁺ , 2 ⁻
7.60 ± .05			7.56	0 ⁺
(7.85 ± .05)			7.78	2
(8.07 ± .05)				
(8.12 ± .05)			8.89	3 ⁻

^a Assuming Be-9 ground state Q-value is 4.362 MeV.

^b The excitation energies of these levels were taken from the Nuclear Data Sheets.

^c Angular distribution anomalous at all incident energies.

^d Contaminated by O-16(d, n)F-17 reaction.

^e Contaminated by O-16(d, n)F-17* reaction.

FIG. I-16-1. Differential Cross Sections at $E_d = 7.0$ MeV for the Ground and First Three Excited States of B-10. Solid Curves are D.W.B.A. Calculations, Dotted Line is Plane-wave (Butler) Calculation. ANL Neg. No. 112-5135.

FIG. I-16-2. J -dependence Effects for the $5/2^+$ and $3/2^+$ Transitions in the $\text{Mg-24} (d,n) \text{Al-25}$ Reaction. (Curves are meant to show the trend of the data). ANL Neg. No. 112-4990.

FIG. I-16-3. Time-of-flight Spectrum for the $\text{Al-27} (d,n) \text{Si-28}$ Reaction at $E_d = 7.0 \text{ MeV}$, $\theta = 16 \text{ deg}$. ANL Neg. No. 112-5469.

TABLE I-16-II. SPECTROSCOPIC FACTORS FOR THE Be-9(d, n) REACTION

Level, MeV	Absolute ^a Spectroscopic Factors	Relative ^a Spectroscopic Factors	Relative Spectroscopic Factors, Ref. 16	Relative ^b Spectroscopic Factors, Ref. 2
g.s.	0.49	1.0	1.0	1.0
0.72	1.12	2.3	2.1	2.24
1.74	0.50	1.0	3.9	0.85
2.15	0.20	0.41	0.87	0.60
3.59	0.22	0.45	—	0.32
4.77	0.21	0.43		
5.11 ^c	0.64	1.3		
5.92	0.66	1.3		
6.35 ^d	(0.42), 0.25	(0.86), 0.51		
6.95	0.90	1.8		

^a Obtained at $E_d = 7.0$ MeV.^b Obtained at $E_d = 2.82$ MeV.^c Contains small (<5%) contribution from the 5.16 MeV level.^d Values in parenthesis assume $J = 1$, others are results assuming $J = 2$.THE Si-28(d,n)P-29 AND Mg-24(d,n)Al-25 REACTIONS⁴

These reactions have been studied at incident deuteron energies of 7.0, 9.0, and 10.0 MeV. A number of transitions were observed and absolute differential cross sections at $E_d = 9.0$ MeV were obtained for the ground state and 0.45, 0.95, 1.81, 2.50, 2.69, and 3.08 MeV levels in Al-25. Relative cross sections were obtained for the ground state and 1.36, 1.95, 3.46, and 4.34 MeV levels in P-29. These results were compared with D.W.B.A. calculations. For the $\ell = 2$ transitions J -dependence effects to those observed in (d,p) reactions⁵ were seen. This is shown in Fig. I-16-2 for the $5/2^+$ ground state and 0.94 MeV $3/2^+$ transitions in Mg-24(d,n)Al-25. It should be noted that the $3/2^+$ transition shows a minimum at ~ 55 deg while the minimum for the $5/2^+$ transition is displaced to larger angles. These effects were present for the $\ell_p = 2$ transitions in both reactions for the three energies studied. This is believed to be the first reported observation of this effect in (d,n) reactions.

THE Al-27(d,n)Si-28 REACTION

Preliminary data at an incident deuteron energy of 7.00 MeV indicates the presence of a large number of transitions (~ 25) some of which lead to previously unreported levels. The data are being reduced to obtain absolute differential cross sections. Preliminary analysis

TABLE I-16-III. ENERGY LEVELS IN Si-28 RESULTING FROM THE REACTION Al-27(d, n)Si-28

Group No.	Present Work	NDS (Ref. 6)
	E_x , MeV	E_x , MeV
0	0	0
1	1.72 ± 0.16	1.78
1'	(2.12 ± 0.30)	—
2	4.81 ± 0.17	4.61
3	5.04 ± 0.13	4.98
4	6.28 ± 0.03	6.27
5	6.87 ± 0.04	6.88, 6.89
6	7.44 ± 0.06	7.38, 7.42
7	7.91 ± 0.06	7.80, 7.93
9	8.34 ± 0.04	8.26, 8.33, 8.41
10	8.63 ± 0.04	8.59
11	8.97 ± 0.07	8.94
13	9.40 ± 0.08	9.31, 9.38
13'	9.48 ± 0.08	9.49
14	9.82 ± 0.05	9.76, 9.93
15	10.23 ± 0.06	10.31
16	10.43 ± 0.06	—
17	10.72 ± 0.05	10.71
18	10.98 ± 0.06	—
19	11.13 ± 0.05	—
20	11.49 ± 0.07	—
21	11.63 ± 0.05	—
	—	11.79
21'	11.86 ± 0.06	11.86, 11.89
22	12.00 ± 0.07	11.97, 12.07
23	12.27 ± 0.04	—

^a Average excitation energies obtained with 15 measurements at 13 angles and for $E_d = 7.0$ MeV.

indicates the levels listed in Table I-16-III. Figure I-16-3 shows a time-of-flight spectrum at a reaction angle of 16 deg.

REFERENCES

1. S. G. Buccino and A. B. Smith, *Levels in B-10 Excited by the Be-9(d,n) Reaction*, submitted to Phys. Letters.
2. S. G. Buccino and A. B. Smith, *The Be-9(d,n)B-10 Reaction*, Bull. Am. Phys. Soc. **9**, 628 (1964).
3. R. H. Bassel, R. M. Drisko and G. R. Satchler, *Distorted-wave Theory of Direct Nuclear Reactions*, ORNL-3240 (1962).
4. S. G. Buccino, D. S. Gemmel, L. L. Lee, Jr., J. P. Schiffer and A. B. Smith, *The Si-28(d,n)P-29 and Mg-24(d,n)Al-25 Reactions*, Bull. Am. Phys. Soc., **10**, 511 (1964).
5. L. L. Lee, Jr. and J. P. Schiffer, *Dependence of the Angular Distribution of the (d,p) Reaction on the Total Angular Momentum Transfer*, Phys. Rev. **B126**, 405 (1964).
6. Nuclear Data Sheets, National Academy of Sciences, National Research Council, Washington, D. C., (May 1962).

Section II

Thermal Reactors

The work reported in this section pertains to reactor systems in which a substantial portion (greater than twenty percent) of the fissions are caused by neutrons of thermal energy; nevertheless, for some cases reported here the term "Thermal" is a misnomer. For both the Argonne Advanced Research Reactor and the Hi-C experiments the term "epi-thermal" or "intermediate" is more nearly applicable.

The first three papers relate to a variety of experimental techniques, including: a new correlation for cadmium ratios; the response of radioactive materials as spectral indices in lattices; and precise reactivity measurements for integral data measurements. Seven papers refer to the Argonne Advanced Research Reactor. The first five papers represent highly technical theoretical contributions to the project design effort and safety analysis. The next is a summary of work on the critical experiment—also highly technical and oriented to produce design data. The last AARR paper is on the use of a relatively new experimental technique—Thermo-Luminescent Dosimetry to measure gamma doses in critical experiments.

Two papers follow on oscillations in natural circulation boiling water reactors; one short paper describes work largely reported elsewhere and a longer paper summarizes an analytical study of space dependent dynamics of boiling water reactors.

The Hi-C (High Conversion) experiments are represented by three papers; one on the experiments themselves, one on improved theoretical correlations with experiment, and one devoted specifically to thermal disadvantage factors where an anomalous result may have been found.

This chapter closes with three papers on the EBWR-Pu Recycle experiment; the experimental program is summarized; the method of developing consistent heavy atom cross sections is summarized, and a critique made of various methods for determining the capture-to-fission ratio in EBWR lattices.

II-1. Linear Relationship Between Cadmium Ratio and Reactor Core Moderating Ratio

G. S. STANFORD, K. E. PLUMLEE and W. R. ROBINSON

Cadmium ratios for several materials have been measured in a number of different thermal critical assemblies, covering a wide range of fuel-to-moderator ratios, with emphasis on undermoderated systems. It has been found, for the cores and detector materials investigated, that the cadmium ratio for a given material is primarily a function of the moderating ratio of the core, with only secondary dependence on the actual materials or configuration of the lattice.

THEORY

The moderating ratio is defined by the equation

$$MR \equiv \frac{\sum \xi N \sigma_s}{\sum N \sigma_a},$$

where the summations are for the different core materials, weighted according to their concentration in the lattice. The numerator is the slowing-down power, and the denominator is the thermal absorption. The values of MR were calculated assuming homogenized cores, using 20.4 b and 0.45 b as the values of $\xi \sigma_s$ for hydrogen and oxygen, respectively. That the cadmium ratio should depend, to a first approximation, only on the moderating ratio is indicated by the following two-group considerations.

Using the superscripts *sub* and *epi* to indicate the subcadmium and epicadmium groups, and indicating the foil material by the subscript x , one can write

$$CdR - 1 = \frac{\text{subcadmium activation}}{\text{epicadmium activation}} = \frac{\sigma_{ax}^{sub} \phi_x^{sub}}{\sigma_{ax}^{epi} \phi_x^{epi}}, \quad (1)$$

where the absorption cross sections σ_a^{sub} and σ_a^{epi} are averaged over the subcadmium and epicadmium spectra respectively, and the flux ϕ_x is averaged over the foil.

For internal lattice positions, neglecting leakage, a thermal neutron balance equation

$$\sum_i N_i \sigma_{ai}^{sub} \phi_i^{sub} = c \sum_i \xi_i N_i \sigma_{si}^{epi} \phi_i^{epi} \quad (2)$$

can be written. Here N_i is the average number of atoms of core component i per unit cell volume, and c is an approximate constant of proportionality. Multiplying Eq. (1) by Eq. (2), rearranging, and considering that in the cores investigated the right-hand side of Eq. (2) was dominated by the hydrogen, with a small

contribution from the oxygen, one gets

$$CdR - 1 = c \cdot \frac{\sigma_{ax}^{sub}}{\sigma_{ax}^{epi}} \cdot \frac{(n \xi \sigma_s^{epi})_H (\phi_H^{epi} / \phi_x^{epi}) + (n \xi \sigma_s^{epi})_O (\phi_O^{epi} / \phi_x^{epi}) + \dots}{(n \sigma_a^{sub})_f (\phi_f^{sub} / \phi_x^{sub}) + (n \sigma_a^{sub})_H (\phi_H^{sub} / \phi_x^{sub}) + \dots}, \quad (3)$$

where H refers to hydrogen, O to oxygen, f to fuel and x to foil. Experimentally the foils were located between fuel pellets, so that $\phi_f^{sub} / \phi_x^{sub} \approx 1$. It happens that the other flux ratios in Eq. (3) can also be set equal to unity without introducing much error, since the epicadmium flux is typically quite flat, and since the denominator of Eq. (3) is dominated by the first term. Now we can write

$$CdR - 1 \approx c \frac{\sigma_{ax}^{sub}}{\sigma_{ax}^{epi}} \frac{(n \xi \sigma_s^{epi})_H + (n \xi \sigma_s^{epi})_O}{(n \sigma_a^{sub})_f + (n \sigma_a^{sub})_H} \approx \text{constant} \cdot MR. \quad (4)$$

Thus it is in fact predicted that the cadmium ratio in the fuel should be approximately a linear function of the moderating ratio if the latter is calculated on the basis of a homogenized core.

MEASUREMENTS

The lattices investigated were assembled in the course of the High Conversion (Hi-C) and the Argonne Advanced Research Reactor (AARR) critical-assembly programs. The Hi-C cores were described in the preceding annual report,¹ which included a discussion of how the cadmium ratios were determined and also gave some of the results.

The AARR lattices are described elsewhere in this annual report (see Paper II-10). The foils and the aluminum and cadmium capsules used were similar to those used in the Hi-C experiments. For the cadmium ratio measurements, the capsules containing the foils were taped in holes cut in 40-mil thick by $\frac{1}{2}$ -in. wide aluminum strips. Each strip contained two foils, one aluminum covered and one cadmium covered, which, when the strip was inserted into one of the 40-mil water channels between the fuel-plate positions, were equidistant from the core midplane. The cadmium ratios reported below were measured at the locations indicated by the circles on Fig. II-1-1, or at equivalent positions in other parts of the core.

FIG. II-1-1. Core Layout—AARR Critical Assembly (The small circles indicate the positions at which the cadmium ratios were measured.) ANL Neg. No. 112-5177.

FIG. II-1-2. Cadmium Ratio for Dysprosium as a Function of Lattice Moderating Ratio. ANL Neg. No. 112-5178.

FIG. II-1-3. Cadmium Ratio for 1.2-mil Indium as a Function of Lattice Moderating Ratio. ANL Neg. No. 112-5175.

FIG. II-1-4. Cadmium Ratio for 1.1-mil Gold as a Function of Lattice Moderating Ratio. ANL Neg. No. 112-5172.

Results of the cadmium ratio measurements to date are plotted in Figs. II-1-2 through II-1-8. In these graphs the solid lines are least-square linear fits to all of the data points except those for the AARR cores. The lengths of the error bars are determined by the reproducibility of the determinations, or by the scatter of the measurements for several core locations which should give similar results. The cadmium ratios as plotted in these figures have not been corrected for flux depression, self shielding in the foil, or perturbations due to the cadmium covers. As mentioned above, the moderating ratios were calculated using the 2200 m/sec absorption cross sections, with no corrections for lattice inhomogeneities.

FIG. II-1-5. Cadmium Ratio for Manganese as a Function of Lattice Moderating Ratio. ANL Neg. No. 112-5174.

FIG. II-1-6. Cadmium Ratio for Copper as a Function of Lattice Moderating Ratio. ANL Neg. No. 112-5176.

Figure II-1-8 shows the results of some careful measurements made in a particular type of lattice—namely, those lattices in the Hi-C series for which the fuel was contained in stainless steel tubing. (For many of the points in the other figures, the fuel was in aluminum tubes.) These particular measurements were made using the “dysprosium substitution” method,² which is particularly accurate when the cadmium ratio to be determined is very close to unity.

DISCUSSION

It can be seen from Figs. II-1-2 through II-1-7 that the moderating ratio is the most important factor in determining the cadmium ratio, with the details of the

FIG. II-1-7. Uranium-235 Fission Cadmium Ratio as a Function of Lattice Moderating Ratio. ANL Neg. No. 112-5465.

FIG. II-1-8. Uranium-238 Capture Cadmium Ratio as a Function of Lattice Moderating Ratio, for Hi-C cores with Stainless Steel-Clad Fuel. ANL Neg. No. 112-5173.

core composition playing a strictly secondary role. This is especially true for dysprosium, gold, and indium. For the other materials the results for the various kinds of Hi-C and BORAX-V cores are in reasonable agreement, but the cadmium ratios for the AARR lattices are appreciably higher. For a partial explanation of this, one can consider the very heavy stainless steel content of the AARR cores. Stainless steel contains some manganese, and a reduction in the neutron flux at energies near the 337-eV manganese resonance by this manganese would increase the cadmium ratio measured with manganese foils.

There is also an additional consideration. The two-group treatment outlined above shows that the effects of lattice inhomogeneities tend to cancel when the foils

are exposed to a representative sample of the flux in the fuel, as was the case with the Hi-C and BORAX-V cores. However, with the foils outside the fuel, as in AARR, the thermal flux in the foil will be greater than that in the fuel, so that a higher cadmium ratio would be expected. Moreover the effect should be greater in the loadings with the least fuel, which are therefore more inhomogeneous. The measurements reported here are consistent, in general, with these observations.

The presence of copper in the AARR cores cannot be invoked to help explain the copper results, but the epithermal activation of the copper foils is undoubtedly depressed somewhat by the manganese and other resonances.

In those cases where the data from the AARR cores diverge from the other curves, the AARR points nevertheless seem to fall fairly well on a straight line of their own, so that one can predict with some confidence the cadmium ratios that would be observed for other fuel densities in the AARR configuration.

The fact that the extrapolated curves do not pass closer to the origin is perhaps due to spectral changes from core to core, since the "constant" of Eq. (4) contains effective foil cross sections which are somewhat dependent on the spectral shape.

CONCLUSIONS

Using the moderating ratio as the independent

variable (abscissa), instead of the more common H:U-238 atom ratio, makes it possible to plot data from diverse reactor cores on the same basis. The data in this report come from two radically different lattice types, AARR and Hi-C, covering in each case a range of moderating ratios. Cores of the Hi-C variety included lattices with two different cladding materials (aluminum and stainless steel), two fuel diameters (0.87 cm and 0.935 cm), and two fuel compositions (3 and 5 w/o enriched UO_2).

The measurements reported here make possible the prediction, within limits, of the cadmium ratios that would be observed in reactor cores having a wide variety of lattice configurations, it being only necessary to make a simple calculation of the moderating ratio for the given lattice. If a few measurements have been made in somewhat similar lattices, the predictions can be made with more confidence.

REFERENCES

1. A. R. Boynton, Q. L. Baird, J. M. Christenson, K. E. Plumlee, W. C. Redman, W. R. Robinson and G. S. Stanford, "High Conversion Critical Experiment", Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010 (January 1965), p. 33.
2. R. H. Lewis, T. C. Engelder, D. M. Roberts and D. B. Wehmer, "Thermal Activation Method for ρ_{28} Measurements in Slightly Enriched Uranium Oxide Lattices", BAW-1268 (May 1963).

II-2. Slow Neutron Spectrum Studies by Neutron Activation

P. H. KIER

As part of the EBWR Plutonium Recycle Experiment, it is planned to irradiate materials whose activation will yield information on the slow neutron spectrum and the effect of fuel composition and/or temperature on this spectrum. Our primary interest in these irradiations is to detect the depressions in the flux caused by absorption in the 0.3 eV resonance of Pu-239 and the 1.03 eV resonance of Pu-240 rather than in detecting the gross effects of the temperature and density of the moderator on the spectrum. It has been proposed that the materials, contained in spheres, be irradiated within Inconel tubes located in the moderator equidistant from four fuel pins. To determine if the required information can be obtained from such irradiations or whether foils should be placed at the surface of the cladding, a series of THERMOS¹ problems were calculated and the ratios of the activities of various nuclides were recorded

for either the energy range from zero to 0.532 eV or from 0.532 eV to 1.56 eV. Materials considered for the lower interval were Pu-239, U-235, Eu-151 and Lu-176 while those considered for the upper interval were Ir-191 and Ir-193.²

The problems were run for a four-region, cylindrical cell that simulates the pin cell of the plutonium zone. The central region, which had a radius of 0.4725 cm, contained the $\text{PuO}_2\text{-UO}_2$ fuel having a molecular density of $0.022 \times 10^{24}/\text{cm}^3$. The fuel cladding was zirconium with a wall thickness of 0.0635 cm, and the water region had an outer radius of 0.8154 cm. The outer region contained three mean free paths of a fictitious purely scattering, nonmoderating material. Use of this extra region has been found to reduce errors introduced by the Wigner-Seitz cell approximation in closely packed lattices.³

TABLE II-2-I. RESONANCE PARAMETERS FOR MATERIALS USED IN THERMOS CALCULATIONS

Material	Energy, eV	Γ_n , MV	Γ_γ , MV	Γ_f , MV	g
U-235	-1.40	2.82	33	194	0.5
	-0.02	0.0003	33	86	0.5
	0.273	0.0029	29	99	0.5
	0.420	0.0012	27	213	0.5
	0.910	0.0137	30	330	0.5
	1.140	0.0172	44	125	0.5
	2.035	0.077	35	12	0.5
Pu-239	-0.26	0.085	40	160	0.25
	0.297	0.126	39	62	0.5
	7.84	1.31	38	42	0.75
	10.93	2.73	32	147	0.75
	11.90	1.75	41	22	0.75
Pu-240	1.055	2.4	32	0.007	1.0
Eu-151	0.327	0.065	70		0.5
	0.461	0.756	93		0.5
	1.055	0.235	90		0.5
	1.830	0.035	90		0.5
	2.712	0.230	92		0.5
Lu-176 ^a	3.361	1.85	92		0.5
	3.71	0.74	88		0.5
	0.143	0.092	61		0.5
	1.574	0.46	55		0.5
Ir-191	0.654	0.5	73.5		0.5
	5.360	6.6	84.5		0.5
Ir-193 ^b	1.303	0.91	86.5		0.5
	43.400	50	102		0.5
Sm-149	0.0976	0.5	63		0.5625
	0.873	0.82	60		0.5
	4.98	2.26	67		0.5
	6.48	0.8	62		0.5
	9.00	9.0	62		0.5

^a Add 1/v component with σ_{act} (2200 m/sec) = 1.9×10^3 b.^b Add 1/v component with σ_{act} (2200 m/sec) = 40 b.

For resonance absorbers, cross sections were computed for each temperature from resonance parameters. These resonance parameters, given in Table II-2-I, yielded cross sections in good agreement with the curves in BNL-325. Calculations were made for uniform cell temperatures of 68°F, 325°F and 489°F for the fuel composition at zero burnup. The density of water was taken as that of the saturated liquid except for one problem, constructed to yield the effect of voids on the activation ratios, in which the cell temperature was 68°F but the water density was that for 325°F. The gas kernel using H. Honeck's¹ parameters was used for H₂O.

To determine the effect of burnup on the activation ratios, problems were run for 0.6 a/o fuel burnup and a moderator temperature of 68°F. At zero burnup, the composition of the fuel was 98.5 a/o UO₂ (99.8 a/o U-238 and 0.2 a/o U-235) and 1.5 a/o PuO₂ (92 a/o Pu-239 and 8 a/o Pu-240). At 0.6 a/o burnup, the composition was taken as: 97.9 a/o U-238, 0.952 a/o Pu-239, 0.30 a/o Pu-240, 0.085 a/o Pu-241, 0.16 a/o U-235,

TABLE II-2-II. ACTIVATION RATIOS FOR THE ENERGY INTERVAL 0-0.532 eV

Radius, cm	Ratio	Zero Burnup				0.6% Burn-up
		68°F	325°F	489°F	68°F 9% void	68°F
0	Fission, Pu-239/U-235	1.0	1.159	1.292	1.016	1.146
0.4295		1.027	1.203	1.356	1.045	1.124
0.5534		1.046	1.235	1.402	1.065	1.103
0.7281		1.042	1.235	1.408	1.061	1.076
0	Absorption, Eu-151/(1/v)	1.0	1.190	1.391	1.045	1.167
0.4295		0.989	1.184	1.387	1.032	1.092
0.5534		0.979	1.179	1.384	1.022	1.032
0.7281		0.954	1.155	1.360	0.996	0.976
0	Absorption, Lu-176/(1/v)	1.0	1.278	1.417	1.009	1.084
0.4295		0.985	1.261	1.398	0.994	1.047
0.5534		0.972	1.245	1.381	0.980	1.015
0.7281		0.955	1.227	1.364	0.942	0.984
0	Lu-176 absorption Pu-239 fission	1.0	1.127	1.131	0.995	0.953
0.4295		0.957	1.068	1.060	0.951	0.934
0.5534		0.924	1.025	1.010	0.917	0.918
0.7281		0.909	1.007	0.991	0.902	0.909
0	Pu-239 fission (1/v) absorption	1.0	1.134	1.253	1.013	1.138
0.4295		1.030	1.180	1.319	1.046	1.121
0.5534		1.052	1.215	1.368	1.069	1.105
0.7281		1.050	1.218	1.377	1.069	1.082

TABLE II-2-III. ACTIVATION RATIOS FOR THE ENERGY INTERVAL 0.532 eV-1.56 eV

Radius	Absorption Ratio	Zero Burnup			0.6% Burnup
		68°F	325°F	489°F	68°F
0	Ir-191/Ir-193	1.0	0.920	0.810	1.873
0.4295		1.010	0.930	0.819	1.627
0.5534		1.020	0.940	0.830	1.273
0.7281		1.025	0.944	0.817	1.017
0	Ir-191/(1/v)	1.0	0.966	0.913	1.272
0.4295		1.023	0.987	0.934	1.189
0.5534		1.045	1.010	0.967	1.055
0.7281		1.057	1.020	0.973	0.943
0	Ir-193/(1/v)	1.0	1.049	1.128	0.679
0.4295		1.012	1.062	1.141	0.730
0.5534		1.024	1.074	1.165	0.829
0.7281		1.031	1.080	1.191	0.931

0.000353 a/o Sm-149 and 0.60 a/o fission products, which were assumed to be 1/v absorbers with a 2200 m/sec cross section of 50 barns.

The results of the calculations for the energy interval

from zero to 0.532 eV are given in Table II-2-II and those for the energy interval from 0.532 eV to 1.56 eV are given in Table II-2-III. The activation ratios, normalized to unity at the center of the cell for 68°F and zero burnup, are given at the following four positions in the cell: at the center of the fuel, in the fuel near the fuel-cladding interface, in the moderator near the moderator-cladding interface, and in the moderator near the edge of the cell.

From Tables II-2-II and II-2-III it is noted that the activation ratios at each position change significantly with temperature, indicating that irradiations in an Inconel tube can detect changes in the slow neutron spectrum arising from uniform temperature changes. From Table II-2-II, it is observed that the activation ratio changes negligibly when the density of the moderator is reduced by 9%, showing that these activation ratios cannot be used to investigate the effects of voids on the neutron spectrum.

Another question of interest is whether the irradiations can detect changes in the spectrum arising from changes in the composition of the fuel with burnup. From the tables, the difference between the activation ratios at zero burnup and 0.6 a/o burnup is dependent upon position in the cell. The activation ratios are sensitive to the magnitude of the flux depression caused

by absorption due to a resonance of one of the plutonium isotopes. The magnitude of a depression is dependent upon concentration of the resonance absorber and position in the cell. Near the edge of the cell the flux depressions have nearly completely disappeared so that the spectrum is characteristic of the moderator. Thus irradiations in the Inconel tubes cannot yield information on the details of the spectrum near the low energy resonances of plutonium and foils should be used for this purpose. The results of the computation indicate that the fission ratio of Pu-239 to U-235 will yield data on the spectrum below the cadmium cutoff and that the activation ratio of Ir-191 to Ir-193 will yield data on the spectrum near the 1.03 eV resonance of Pu-240, and that foils containing these materials will have to be placed near the surface of the cladding of a fuel pin.

REFERENCES

1. H. C. Honeck, "A Thermalization Transport Theory Code for Reactor Lattice Calculations", BNL-5826 (1961).
2. D. Jakeman and E. J. Maunders, "Use of Iridium Foils for the Measurement of the Pu-240 Resonance Escape Probability in Plutonium-Fueled Reactor Assemblies", J. Nucl. Energy A/B **18**, 173-199 (1964).
3. Y. Fukai, "Zur Randbedingung mit isotroper Reflexion bei der Berechnung des zylindrischen Zellenrandes", Nukleonik **7**, 144-152 (1965).

II-3. Small Reactivity Measurements in the ATSR

R. C. DOERNER and J. HAUGSNES

INTRODUCTION

Small reactivity measurements ($< 1.5 \times 10^{-5} \Delta k/k$) are being made at the Argonne Thermal Source Reactor (ATSR). Samples are pneumatically driven into the center of the reactor core. The resulting reactivity change is just compensated by the motion of a fine control rod (servo rod) which is automatically positioned so that the signal current from a compensated ion chamber near the edge of the core is maintained constant. The performance characteristics of this system have been analyzed by E. Bennett and R. Long.¹ The change in the position of the servo rod is a measure of the reactivity worth of the sample. The reactivity worth per unit change in the servo rod position is directly determined from period measurements. The measured worth of a sample can be converted into an integral reactor parameter, such as a resonance integral, by comparison with the corresponding re-

activity worth of an appropriate standard sample and by the application of several correction factors. A description of experimental techniques and some selected results of recent reactivity measurements are given here.

EXPERIMENTAL METHOD

The ATSR is a fully enriched (93 w/o U-235), water-moderated reactor which is described in Ref. 2. The core is divided into a square array of 25 vertical compartments, each approximately 7.85 cm x 8.65 cm in cross section; the fueled length of the core is 60.0 cm. The sample moves in a voided aluminum tube mounted on the axis of the central compartment and running the full length of the core. The rest of the central compartment is filled with water. The four corner compartments are filled with graphite and the remaining twenty compartments contain fuel assem-

blies. The total core volume occupied by one fuel assembly is 3.81 liters. Of this, 2.71 liters is water and 1.10 liters is metal (157.1 g of U-235 plus aluminum). The H:U-235 atomic ratio for the core is about 450:1.

Three sides and the bottom of the core are reflected by 15 cm to 20 cm of water followed by 15 cm of lead shielding followed by more water which serves as shielding. The top of the core is reflected and shielded by about 90 cm of water. The leakage face of the core is now reflected and shielded by 1.9 cm of aluminum followed by 10 cm of lead followed by a large stack of graphite. At critical, the two safety rods are fully removed from the core and the two control rods are four-fifths of the way out of the core. The latter are located between the core and the reflector on opposite sides of the core.

Resonance integrals are of considerable interest in reactor technology. They are defined in terms of a $1/E$ slowing-down spectrum. For the present measurements, the desired spectrum was approximated over the sample volume by wrapping the full core length of the central voided aluminum tube (2.86 cm o.d.) with a sheet of cadmium 0.102 cm thick. A second aluminum tube (3.81 cm o.d. and voided) running the full length of the core contains the inner aluminum tube and its cadmium wrapping. Both tubes are filled with carbon dioxide because it has a negligible reactivity effect as it drives the sample in and out of the core. The vertical position of the sample is fixed by an adjustable platform extending from the bottom of the inner tube.

Foil activation measurements in the central core compartment, both with and without the cadmium wrapped around the inner tube, indicate that the non-symmetric reflector in ATSR does not produce a tilting of the flux at the center of the core. The safety rods have no effect and the control rods have a negligible effect on the flux gradient at the core center.

R. Long³ has made activation measurements with foils of gold, indium, manganese, cobalt, and molybdenum to determine the gross features of the flux spectrum in the central region around the sample. In these measurements it was assumed that each of the foil materials has a single well-defined resonance. Self-shielding effects were eliminated by extrapolating cadmium ratio measurements for detecting foils of different thicknesses to equivalent zero foil thickness conditions. Results of a least-squares fit to these data are shown in Fig. II-3-1 for a reactor power of 67 ± 7 W. Micro-structure effects due principally to resonance absorption in the fuel and structural materials cannot be determined by this method but these are expected to be fairly well eliminated by the relatively large water channel surrounding the sample.

A wedge-shaped stainless steel servo-controlled rod

FIG. II-3-1. Flux Spectrum in the Center of ATSR. ANL Neg. No. 112-5738.

located near the edge of the core provides an approximate linear reactivity change with position. The magnitude of the reactivity control associated with this rod is fixed by its location. Small adjustments in its integral worth can be made by changing the fuel loading in the area immediately surrounding the rod. For the data reported here, this rod had an integral worth of $1.5 \times 10^{-5} \Delta k/k$ and its total motion was limited to 3.18 cm (divided into 10^4 units).

Limitations on the precision with which sample worths can be determined are essentially determined by the servo rod performance limitations. These, in turn, depend on the reactor and instrument noise. Measurements on the ARMF-II,⁴ a facility very similar to the ATSR, indicate the limiting precision for this type of measurement ("90% confidence interval") is about $5 \times 10^{-8} \Delta k/k$.⁵ Motion of the servo rod has been calibrated in terms of both the absolute reactivity controlled by the rod and by its response to a standard gold sample so that the position of this rod is the primary source of sample worth data.

By use of shim heaters which hold the water temperature to ± 0.3 degree, residual drift was held to about 20% of full-scale rod deflection per hour, depending upon the initial temperature balance. Operating power was sufficient to cause the rod to exhibit a pile noise fluctuation of about 10%. Higher powers caused increased core heating and made it difficult to achieve the

correct moderator-environment temperature balance. A reactor power of about 70 W has been found to provide optimum equilibrium between the rate of heat generated by fission and rate of heat loss from the core to its surroundings.

In a typical measurement, the sample is alternately driven into and out of the core center in a square wave manner. The reactivity changes due to sample motion are compensated by reactivity changes due to servo rod motion. A random fluctuation of the servo rod position (about 10% of full rod travel at 70 W) in response to reactor and instrument noise is superimposed upon the rod motion in response to the sample. The signal is separated from the noise by a precision analog integrator which electronically integrates the rod position signal over 90-sec time intervals. The integrator output is converted to digital form and printed on paper tape. During steady-state operation, a succession of such measurements produces a series of mean rod position values, P_k , with a Gaussian distribution about the series mean value.

For alternating sample-in and sample-out operation, the changes, D_k , in the mean servo rod position are calculated by grouping the mean rod position, P_k , according to

$$D_k = (-1)^{k+1} [P_k - \frac{1}{2}(P_{k-1} + P_{k+1})],$$

where k ranges from 1 to $N - 1$, N being the observed

number of rod position integrations, P_k , starting with the zeroth integration for the sample-out case.

Linear variation in reactivity with time will not appear in the D_k . Moreover, the sum of any two adjacent D_k 's will exactly cancel any reactivity drift of second order in time.

The mean change in servo rod position for a set of oscillations of a sample is defined by

$$\bar{D} \equiv N^{-1} \sum_1^N D_k.$$

Define the root mean square deviation of the values, D_k , by

$$\sigma \equiv (N - 1)^{-1/2} \left[\left(\sum_1^N D_k^2 \right) - N \bar{D}^2 \right]^{1/2}.$$

Because the D_k 's are not completely independent of each other, the above definition does not correspond exactly to the standard definition. However, if one starts with the definition of the D_k and propagates the errors in the independent measured quantities P_k , it can be shown that the root mean square error in the mean servo rod position change $\bar{D}$ is given by

$$E = \sigma [8/(3N)]^{1/2}.$$

For the ATSR, the root mean squared deviation of the D_k 's about the mean value has been measured to be

FIG. II-3-2. Typical Chart Record of Servo-Rod Position. ANL Neg. No. 112-5703A.

FIG. II-3-3. Distribution of Change in Servo-Rod Position for Repeated Measurements on Two Samples. ANL Neg. No. 112-5742.

$\sigma = 7.3 \pm 2 \times 10^{-8} \Delta k/k$ at a reactor power of 70 W,¹ very close to the value predicted for the ARMF-II by L. Cloud.⁵

A typical trace of servo rod position is shown in Fig. II-3-2. The small variations in rod position about the mean represent its response to the reactor plus instrument noise. Starting from the right side of Fig. II-3-2, the sample is introduced into the core center at the time marked "start". Ten seconds are allowed for the system to come to equilibrium. During the next ninety seconds, the rod position signal is integrated. Then the sample is removed, the rod comes to its new position, and again after a 10-sec wait and a 90-sec integrating period, the sample is reintroduced. The 90-sec period provides sufficient time to integrate over a number of cycles of the lowest frequency component in the noise spectrum and short enough so that long-term drifts do not become a problem.

RECENT MEASUREMENTS

Typical results are shown in Fig. II-3-3 for a strong and a weak absorber. The latter, shown to the left of Fig. II-3-3, consists of 301 separate determinations of the worth of the empty aluminum sample holder. The large number of determinations illustrated in Fig. II-3-3 were spread over a period of several months. Similar results for 226 determinations of the worth of the more strongly absorbing sample are shown to the right of Fig. II-3-3. This particular sample was an alloy of gold and copper that is used to check the day-to-day variations in the mean value and the dispersion of the mean, and normally requires between 15 and 20 sample-in, sample-out differences to determine the mean to better than $\pm 2\%$.

In reactivity measurements, the effective cut-off energy (E_c) of the cadmium tube is defined as that energy below which reactions in the sample produce zero reactivity change. For a given cadmium tube thickness, H. Brown and T. Connolly⁶ have shown that the effective

cut-off energy in terms of zero reactivity change may be considerably higher than that for activation measurements. As an example, they report that for an infinitely dilute gold sample in a 40-mil thick cadmium tube $E_c = 1.634$ eV in a reactivity measurement compared to 0.971 eV in an activation measurement. In terms of correcting the gold resonance integral measurement for the $1/v$ component below the resonance, this amounts to 16 b in an activation measurement and 28 b in a reactivity measurement. The same foil in a 60-mil cadmium tube has $E_c = 2.428$ eV and a correction to the resonance integral of 37b when measured by reactivity techniques. For the corresponding activation measurement, E_c is about half this value (1.293 eV) and the resonance integral correction is 22b.

Experimentally, E_c can be determined from the ratio of the reactivity response for a resonance absorber to that for a pure $1/v$ absorber, such as boron. In absorbers whose resonance structure can be adequately described in the dilute form as $(\pi/2)(\sigma_R \Gamma/E_R)$, the reactivity ratio is

$$\begin{aligned} \frac{\rho_{\text{sample}}}{\rho_B} &= \frac{K[\sigma_0^s(1/v) + \sigma^s(\text{res})]}{K\sigma_0^B(1/v)} \\ &= \frac{\int_{E_c}^{\infty} \sigma_0^s \sqrt{\frac{0.025}{E}} \frac{dE}{E} + \sum_i \frac{\pi}{2} \left(\frac{\sigma_R \Gamma}{E_R} \right)_i}{\int_{E_c}^{\infty} \sigma_0^B \sqrt{\frac{0.025}{E}} \frac{dE}{E}}. \end{aligned}$$

In this expression, the superscripts s and B designate the sample and boron, respectively; the subscript 0 refers to the 2200 m/s cross sections; and the resonance terms, σ_R , Γ , and E_R are the peak cross section, the width and energy, respectively of the i th resonance.

For a gold sample (using BNL-325 values for σ_0 and the resonance parameters), this equation can be solved for the lower limit of the integral:

$$E_c = \left[0.158 \left(\frac{\rho_{Au}}{\rho_B} - 0.1309 \right) \right]^2$$

on a per-atom basis; or

$$E_c = \left[2.25 \left(\frac{\rho_{Au}}{\rho_B} - 0.00918 \right) \right]^2$$

on a reactivity-per-milligram basis.

Measurements⁷ made in the ATSR of the central worth of B₄C samples ranging in weights from 1.5 to 54 mg indicate that the infinitely dilute reactivity is $(10.2 \pm 0.8) \%$ of full rod travel per mg of boron ($1.51 \pm 0.12 \times 10^{-6} \Delta k/k/\text{mg}$). Similar results for gold samples between 5 and 500 mg indicate an infinitely dilute reactivity of $(4.0 \pm 0.5) \%$ of full rod travel per mg. From these, the estimated cut-off energy for the 40-mil cadmium tube in the ATSR is $E_c = 0.75 \pm 0.21$ eV, somewhat lower than predicted by Brown and Connolly.

If the calibration of the servo rod is made in terms of some standard resonance absorber such as gold as is the case in the ATSR measurements, a correction must also be made for the change of importance of capture at energies other than those of the calibrating standard. A simple but useful measure of the average adjoint flux above E_c is the ratio of reactivity to activation of a given sample:

$$\frac{\rho}{A} \approx \frac{\int \phi \Sigma_a \phi^* dE}{\int \phi \Sigma_a dE} = \bar{\phi}^*.$$

Normalization of the numerator and denominator of this approximation is best done by making measurements both at the center of the reactor and a point outside the reflector where the reactivity is small enough to be neglected and the importance, ϕ^* , goes to zero. Relative to the importance of capture in gold normalized to

FIG. II-3-4. Reactivity Measurements for Gold Samples. ANL Neg. No. 112-5743.

TABLE II-3-I. ABSORPTION INTEGRALS EXTRAPOLATED TO INFINITE DILUTION

Sample	% Rod Change per Gram of Sample		Infinite Dilution Absorption Integral, b	Form of Sample ^a
	For lightest sample measured	Extrapolated to zero sample mass		
Gold	740 ± 180 (3.5 mg)	820 ± 70	1580 ^b	Machined pellets of Au/Cu and Au/Pb alloy; pressed powder with Al diluent.
Indium	424 ± 7 (91.4 mg)	1500 ± 300	2840 ± 618	Machined In/Pb alloy pellets.
Silver	447 ± 32 (23.41 mg)	600 ± 140	1136 ± 282	Pressed powder with Al diluent.
Cobalt	414 ± 43 (14.12 mg)	800 ± 300	1517 ± 583	Pressed powder with Al diluent.
Gold	310 ± 150 (1.77 mg)	320 ± 50	1580 ^b	Au foil, 1.016-cm dia.
U-238	10.47 ± 0.23 (2089 mg)	55 ± 20	267 ± 106	U foil, hollow cylinder.

^a Pellet samples were 1.016-cm diameter and from 0.45 to 1.27-cm long.

^b Known value used to calibrate the servo rod.

TABLE II-3-II. EFFECTIVE ABSORPTION INTEGRALS FOR VARIOUS MATERIALS

Sample	Mass, mg	% Rod Change per Gram of Sample	Effective Absorption Integral, barns	Form of Sample ^a
LiF	9.17	1535 ±67	2905 ±127	Pressed and sintered powder pellet; Al diluent.
	32.72	1052 ±23	1991 ±44	
	70.80	771 ±11	1459 ±21	
B ₄ C	4.47	2405 ±134	4552 ±254	Pressed and sintered powder pellet; Al diluent.
	5.81	2350 ±141	4448 ±267	
	15.30	1402 ±61	2653 ±115	
	38.23	1300 ±28	2460 ±53	Machined solid pellet.
C	1859.7	-2.00 ±0.20	-3.79 ±0.40	
MgO	1083	3.73 ±0.51	7.06 ±0.97	
Al	26148.71	0.056 ±0.005	0.10 ±0.16	The sample holder
Al ₂ O ₃	~800	1.46 ±0.26	2.76 ±0.51	
CaO	1716	6.69 ±0.13	12.66 ±0.28	
TiO ₂	1024	9.73 ±0.87	18.41 ±1.65	
V ₂ O ₅	1320	3.94 ±1.49	7.46 ±2.82	
MnO ₂	3033	6.74 ±0.45	12.76 ±0.85	
Fe	3256	1.75 ±0.19	3.31 ±0.37	Machined solid metal pellet.
Co	1714	27.97 ±0.28	52.94 ±0.53	
Cu	7293.18	1.84 ±0.05	3.48 ±0.16	

unity, measurements with cobalt and indium indicate an importance weighting of 0.815 and 1.02, respectively.

Gold has been the most extensively studied material in the ATSR measurements, both in the amount of data taken and in the variety of sample composition and geometry. Lead was found to be the least objectionable diluent material (with regard to self-shielding) but only a few materials of interest alloy well with lead. Powdered metallurgical techniques using 300-mesh aluminum powder as the diluent material proved unsatisfactory because of the large amount of Al₂O₃ apparently present in the aluminum. Chemical analysis at different times indicated varying and unpredictable amounts of water in the aluminum powder. The presence of even known quantities of oxygen and hydrogen introduces moderately large and statistically uncertain corrections to the data.

TABLE II-3-II—Continued

Sample	Mass, mg	% Rod Change per Gram of Sample	Effective Absorption Integral, barns	Form of Sample ^a
CuO	2837	2.82 ±0.24	5.34 ±0.46	Loose powder.
Zn	3804.45	1.56 ±0.09	2.95 ±0.20	Metal granules (30-mesh) in Al capsule.
ZnO	1226.70	1.69 ±0.23	3.20 ±0.46	Oxide powder in Al capsule.
Se	3346	5.57 ±0.24	10.54 ±0.46	Loose powder.
ZrO ₂	1936	3.39 ±0.40	6.42 ±0.76	Loose powder.
Cd	2939.96	12.59 ±0.12	23.83 ±0.26	Rolled 20-mil sheet.
Sn	4889.34	2.07 ±0.04	3.92 ±0.16	Metal granules (30-mesh) in Al capsule.
BaO	6482	3.12 ±0.19	5.90 ±0.37	Loose powder.
TaO ₂	991	60.71 ±0.69	114.90 ±1.31	Loose powder.
Pb	9296.40	-0.089 ±0.015	-0.17 ±0.16	Machined solid metal pellet.
Lucite	228.11	119.9 ±2.2	227 ±4	Machined bushing.

^a All samples were contained in the sample holder and corrections have been made for the worth of the holder.

The reactivity measurements on the gold samples are shown in Fig. II-3-4 which gives the logarithm of the per cent change in rod position per gram of gold as a function of the mass of the gold sample. These data can be extrapolated to the case of zero sample mass to get $(820 \pm 70)\%$ rod change per gram for dilute pellet samples and $(320 \pm 50)\%$ rod change per gram for thin foil samples. The calibration of the servo control rod is then found to be 1.893 ± 0.162 b per percent change in rod position per gram for dilute pellet samples and 4.850 ± 0.758 b per percent change in rod position per gram for foil samples. This calibration is based upon a value of 1580 b for the infinite dilution absorption integral of gold from which 28 b has been subtracted to account for the higher effective cadmium cut-off energy in reactivity measurements.

The servo rod calibration can be used to convert reactivity measurements on samples into effective absorption integrals provided the rod position varies linearly with reactivity (this has been checked from time to time) and provided reactivity varies directly with the effective absorption integral of the sample (if not, an importance weighting correction should be applied).

For the samples listed in Table II-3-1, enough data were taken for different sample dilutions so that the crude extrapolations could be made to the case of in-

finitely dilute samples. Tentative infinite dilution absorption integrals, listed in Table II-3-I, have not been corrected to a standard value of cadmium cut-off energy nor has any correction been made for importance weighting which may differ from that of the gold. The large spread in the infinite dilution values results from the uncertainties in extrapolating to zero sample mass which can be appreciated by comparing columns 2 and 3 of Table II-3-I. The uncertainties can be reduced by taking data for more dilute samples but this rapidly becomes more time consuming if one wants to get reasonable statistical accuracy in the measurements.

A survey was made of the reactivity effects of a number of samples in the 1 to 5-g weight range and these measurements are listed in Table II-3-II. Column 4 lists tentative values of the effective absorption integrals for these measurements based upon the calibration (for dilute pellets) of the servo rod in terms of gold. No corrections have been made for the effective cadmium cut-off energy or differences in importance weighting. Table

II-3-II also lists measurements made on light samples of the $1/v$ absorbers boron and lithium.

REFERENCES

1. E. F. Bennett and R. L. Long, "Precision Limitations in the Measurement of Small Reactivity Changes," Nucl. Sci. Eng. **17**, 425 (1963).
2. R. J. Armani, "The Argonne Thermal Source Reactor," ANL-6440 (1962).
3. R. L. Long, "An Automatic Control Rod Method for Resonance Integral Measurements," Thesis, Purdue University (June 1962).
4. E. E. Burdick, E. Fast and D. W. Knight, "The Advanced Reactivity Measurements Facility, Description and Performance Evaluation," IDO-17005 (1964).
5. L. G. Cloud, "Theoretical Development of Statistical Precision Limitations for Differential Reactivity Measurements Made with a Servo Controlled Regulating Rod," Trans. Am. Nucl. Soc. **8**, No. 1, 259 (1965).
6. H. L. Brown, Jr. and T. J. Connolly, "Cadmium Cut-Off Energies for Resonance Absorbers in Activation and Reactivity Measurements," Trans. Am. Nucl. Soc. **6**, 236 (1963).
7. M. V. Davis, University of Arizona (Private Communication).

II-4. Argonne Advanced Research Reactor Calculations—Preface

D. H. SHAFTMAN, C. N. KELBER, B. N. KRISTIANSON,* H. F. REED and R. P. SAVIO†

The various contributions to this section of the Annual Report have been categorized into two principal subsections:

1. Topics concerned directly with the theoretical reactor physics analysis of AARR conceptual designs;
- and
2. Analysis of the AARR Criticality-Facility loadings, and particularly of those parameters which have been measured and which are directly pertinent to theoretical reactor physics evaluations of AARR conceptual designs.

At the time of writing of the previous Annual Report,¹ the criticality experiments had not yet begun. However, in one of the papers,² a summary was given of the experiments planned and of the hazards analysis for the criticals. In the present report, emphasis has been placed upon theoretical reactor physics studies of actual Criticality-Facility systems for which measured data are available (see Paper II-10). Comparisons of measurement and of calculation are presented for the available excess reactivity, the reactivity coefficients for

partial voiding of water in the core and in the Internal Thermal Column (ITC), and the spatial distributions of activity of various neutron detectors (see Paper II-8). Other theoretical results are presented for which measured data are not yet available (see Paper II-8).

The engineering design of the reactor core has reached a stage of development wherein the loading of fissionable material is more closely defined. In the work reported in the previous Annual Report,³ and in the bulk of the theoretical reactor physics studies since then, the U-235 loading was taken to be approximately 1 kg per liter of core. The present reference engineering core design defines a total U-235 loading of approximately 0.8 kg per liter of core volume, based on a fuel-plate matrix of stainless steel and a maximum 37 w/o of UO₂ highly enriched in U-235. The density of the UO₂ has been reduced somewhat to improve the resistance of the cermet to damaging effects of fuel burnup. Also, the requirements for the width of the non-fueled framing of the fuel plates indicate that roughly 10% of the width of the plate must be allocated to this framing. The effects of this engineering design on the reactor physics parameters of the system are under study. The results are too preliminary to permit a detailed reporting at this time.

* Reactor Engineering Division.

† Reactor Operations Division.

Alternative core designs are being analyzed, involving the possible augmentation of the fuel loading by using a higher fraction of UO_2 in the cermet or by increasing the thickness of the cermet in the bulk of the reactor core. The time schedule for the AARR core design requires that the first AARR core be specified in the very near future, and it is expected that the maximum UO_2 content of the cermet will be set at approximately 37 w/o for this core.

There are certain topics which are relatively insensitive to the loading of fuel in the core. Three of these topics are discussed in separate papers under Category No. 1, namely: (1) the calculation of radiation doses to persons outside the AARR complex, for a hypothesized Maximum Credible Accident (MCA) in the AARR; (2) preliminary calculations of heating rates of mate-

rials in the ITC and in the reflector, for two cases: (a) equilibrium power operation, and (b) a freshly shut-down reactor; and (3) calculations pertaining to the effect of the photoneutron production, in the outer beryllium reflector, on the startup of a reactor which has been shut down after some operation at power (see Papers II-5, II-6, and II-7).

REFERENCES

1. Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010 (January 1965).
2. *ibid*, ANL-7010, C. N. Kelber, "Argonne Advanced Research Reactor—Critical Experiment Planning and Hazards Analysis", pp. 63–66.
3. *ibid*, ANL-7010, B. N. Kristianson and C. N. Kelber, "Argonne Advanced Research Reactor—Physics Calculations", pp. 57–62.

II-5. Argonne Advanced Research Reactor Calculations—Radiation Doses Beyond the Reactor Containment Building Following a Postulated Maximum Credible Accident

H. F. REED, C. N. KELBER and D. H. SHAFTMAN

INTRODUCTION

To provide data for the preliminary safety evaluation for the AARR, radiation doses have been computed for a hypothesized Maximum Credible Accident (MCA). The details of the supposed incident which has led to the MCA are not of concern here. The assumptions regarding the fractional core melt and the subsequent dispersal of fission-product solids and gases are described in detail, below, under the heading "General Methods and Assumptions."

It is assumed that the incident leading to the MCA fission-product release also causes an over-pressure in the Reactor Containment Building (RCB), e.g., as a result of a massive spill of hot reactor water in that building. It is assumed also that the reactor has been operating at 240 MWt, and that reactor shutdown is achieved at the time of occurrence of the MCA.

[Note: All of these analyses have been made for the present concept of the AARR Reactor Containment Building, which provides single containment.]

Two separate cases have been studied. For Case 1, it is assumed that the RCB has been sealed, except for leakage due to a net over-pressure in the building. Conservatively, the leakage is assumed to result in a single (point) source, at ground level. The leakage rate is as-

sumed to be a step-function, stepping at the time of the incident to a level of 0.1%/day of the air volume ($1.2 \times 10^6 \text{ ft}^3$) in the RCB. The Case-1 release is of the type recommended for evaluation of reactor siting.¹ The results are presented in Tables II-5-I and II-5-II.

For Case 2, it is assumed that the contaminated air in the RCB is exhausted at a fixed rate. This air is filtered and then it leaves the RCB at the top of a 250-ft-high stack. Two sub-cases have been studied: (a) a ventilation rate of 600 cfm; and (b) a rate of 6000 cfm. The results are presented in Table II-5-III.

For the MCA hypothesized, at the low exhaust rate there would be very little reduction in the overpressure developing in the building, and also little decontamination of the RCB in the first 2 hours. At the high exhaust rate, the ground-level-leakage calculation (Case 1) would be affected, but the magnitude of this effect has not been calculated. Of course, for sufficiently high ventilation rates, a net inward air flow would develop at other openings in the RCB, and therefore there would be zero uncontrolled leakage from the building.

Note that, for the low ventilation rate (600 cfm), the direct photon dose is essentially common to Case 1 and Case 2. The other dose contributions, from both cases, should be summed, and combined with the direct

TABLE II-5-I. DOSES FROM GROUND-LEVEL LEAKAGE OF 0.1%/DAY OF THE REACTOR CONTAINMENT BUILDING VOLUME

Period	Dis- tance, m	Whole-Body Doses, rem							Bone Doses, rem			Thyroid Doses, rem	Total ^b
		Direct photon	Plume beta-ray	Plume photon	Radionuclide			Total	Radionuclide		Total ^a		
					Sr-89	Sr-90	Iodines		Sr-89	Sr-90			
2 hours	100	77.	15.	0.64	0.46	0.80	7.2	101.	16.	13.	159.	3,990.	4,070.
	200	12.	4.4	0.40	0.13	0.22	2.0	19.	4.6	3.8	36.	1,120.	1,140.
	500	0.59	0.85	0.17	0.025	0.043	0.39	2.1	0.88	0.72	5.2	215.	216.
	1,000	0.015	0.25	0.094	0.0072	0.012	0.11	0.50	0.26	0.21	1.4	63.	63.
	2,000	—	0.073	0.050	0.0020	0.0034	0.031	0.16	0.071	0.58	0.40	17.	17.
	5,000	—	0.011	0.011	0.0002	0.0004	0.0038	0.026	0.0085	0.0070	0.053	2.1	2.1
30 days ^c	100	657.	91.	4.1	21.	45.	97.	914.	757.	751.	4,080.	53,800.	54,500.
	200	98.	25.	2.1	5.5	11.	25.	168.	195.	193.	979.	13,900.	14,000.
	500	4.3	5.0	0.92	1.1	2.3	4.9	19.	38.	38.	178.	2,740.	2,750.
	1,000	0.095	1.6	0.55	0.34	0.71	1.6	4.8	12.	12.	55.	867.	869.
	2,000	0.0002	0.52	0.28	0.11	0.24	0.52	1.7	4.0	4.0	18.	288.	288.
	5,000	—	0.13	0.13	0.029	0.060	0.13	0.48	1.0	1.0	4.8	75.	75.
	10,000	—	0.050	0.066	0.011	0.023	0.052	0.20	0.40	0.40	1.9	29.	29.

^a These values include other bone-seeker doses and whole-body iodine and photon doses.

^b These values include whole-body strontium and photon doses.

^c The inhalation period for the iodines is assumed to be infinite.

TABLE II-5-II. FIFTEEN-MINUTE DOSES AT 100 METERS FOR THE CASE OF GROUND-LEVEL LEAKAGE OF 0.1%/DAY OF THE REACTOR CONTAINMENT BUILDING VOLUME

Receptor	Fission-Product Physical State	Exposure Dose, rem				
		Direct photon	Plume		Inhalation	Total
			Beta-ray	Photon		
Whole body	Rare gases	7.1	1.6	0.091	—	20.
	Halogens	7.7	0.97	0.049	1.1	
	Solids	0.5	0.32	0.0065	0.16	
		15.3 (total)	2.9 (total)	0.15 (total)	1.3 (total)	
Bone	Rare gases	7.1	—	0.091	—	26.
	Halogens	7.7	—	0.049	1.1	
	Solids	0.5	—	0.0065	9.3	
		15.3 (total)		0.15 (total)	10.4 (total)	
Thyroid	Rare gases	7.1	—	0.091	—	614.
	Halogens	7.7	—	0.049	598.	
	Solids	0.5	—	0.0065	0.2	
		15.3 (total)		0.15 (total)	598. (total)	

photon dose, in estimating the total dose—at least for the 2-hour dose.

GENERAL METHODS AND ASSUMPTIONS

It is assumed that the MCA occurs after a total energy output of 9000 MWD, with the reactor operating at 240 MWt. It is assumed further that 100 % of the core melts, and that the fission products escape instantaneously and contaminate the RCB. The postulated

escape fractions are: 1.0 for the noble gases; 0.5 for the halogens; and 0.01 for the solids.

The published data of J. Perkins and R. King² have been modified and updated³ on the basis of papers by A. Avery, et al.,⁴ and by G. O'Kelley, et al.⁵ These revised values have been used to determine the beta ray and gamma ray activity subsequent to the time of reactor shutdown from 240 MWt.

To calculate the direct photon radiation from the

TABLE II-5-III. DOSES FROM FORCED VENTILATION THROUGH THE STACK

Ventilation Rate, ft ³ /min.	Dose Period	Distance, m	Whole-body Doses, rem				Thyroid Doses, rem	Total
			Direct photon	Plume beta-ray	Plume photon	Total		
600	2 hours	100	62.	—	6.0	68.	—	68.
		200	9.7	—	5.9	16.	—	16.
		500	0.47	—	5.7	6.2	—	6.2
		1,000	0.012	0.00004	6.0	6.0	0.00016	6.0
		2,000	0.00004	0.079	7.2	7.3	0.28	7.5
		5,000	—	0.47	3.5	4.0	1.3	4.8
600	30 days (∞ for Iodines)	100	278.	—	19.	297.	—	297.
		200	43.	—	19.	62.	—	62.
		500	1.9	—	19.	21.	—	21.
		1,000	0.034	0.0002	21.	21.	0.0014	21.
		2,000	0.0001	0.53	28.	29.	3.1	31.
		5,000	—	5.7	24.	30.	33.	57.
		10,000	—	6.1	17.	23.	34.	51.
		20,000	—	4.6	8.4	13.	25.	33.
6000	2 hours	100	51.	—	50.	101.	—	101.
		200	8.0	—	50.	58.	—	58.
		500	0.39	—	49.	49.	—	49.
		1,000	0.010	0.0004	51.	51.	0.0013	51.
		2,000	0.00003	0.69	63.	63.	2.4	65.
		5,000	—	4.5	33.	37.	12.	45.
6000	30 days (∞ for Iodines)	100	188.	—	72.	260.	—	260.
		200	29.	—	72.	101.	—	101.
		500	1.3	—	72.	74.	—	74.
		1,000	0.030	0.0006	79.	79.	0.0024	79.
		2,000	0.00007	1.3	106.	108.	5.1	111.
		5,000	—	14.	91.	105.	49.	140.
		10,000	—	15.	66.	81.	47.	113.
		20,000	—	12.	32.	43.	34.	66.

contaminated RCB, the RCB was treated at a point source. The source strength was assumed to be effectively 30 % less because part of the RCB is below grade. It is considered that a concrete wall, at least 18 in. thick, will be used for shielding beyond the perimeter of the RCB. With an 18 in. thickness of ordinary concrete, the direct photon dose is attenuated by factors of from 50 to 120, depending upon the time duration and the distance of the receptor from that wall. In these computations of dose rates, the varying effectiveness of the wall has been included.

The external doses and the internal doses due to fission products leaking or being exhausted from the building were calculated with the Gaussian Plume model for a point source.^{6, 7} The results apply for a small, stationary, tissue-equivalent receptor located at ground level. For Case 1, it is assumed that the receptor is on the axial centerline of the plume; for Case 2, the receptor is taken to be directly below the axial centerline.

In calculating inhalation-dose rates, the postulated breathing rates were 30m³/day for the first 2-hour period, and 20m³/day for subsequent times.

SPECIAL ASSUMPTIONS FOR CASE 1 (GROUND-LEVEL LEAKAGE)

1. Fifty percent of the halogens which escape from the molten fuel deposit on equipment within the RCB or on the inside surfaces of the building wall. The remaining fission products uniformly contaminate the building air.

2. For the first 2 hours, atmospheric conditions are moderately stable (see Category F, Ref. 7), with a wind speed of 1 m/sec. Subsequently, conditions are slightly stable (see Category E, Ref. 7), with a wind speed of 2 m/sec.

3. There is no decay of fission products enroute to the receptor. Except for the external photon dose, exposure begins when the plume front reaches the receptor.

SPECIAL ASSUMPTIONS FOR CASE 2 (FORCED VENTILATION THROUGH THE STACK)

1. Fifty percent of the halogens and of the solids which enter the building air deposit on building internals. Half of the deposited material is assumed to be

transferred to a shielded location, e.g., by a water-spray system.

2. Atmospheric conditions are moderately stable, with a wind speed of 1 m/sec, for all time after the MCA is initiated.

3. Except for the iodines, there is no decay of fission products enroute to the receptor. Except for the external photon dose, exposure begins when the plume front reaches the receptor.

4. The exhausted air is replaced by uncontaminated air entering the RCB. This air mixes uniformly with the rest of the air in the building.

5. The systems of filters are only 99 % efficient in trapping the halogens and the solids.

SUMMARY AND CONCLUSIONS: COMPUTED DOSES FROM A POSTULATED MCA

CASE 1 (GROUND-LEVEL LEAKAGE ONLY)

On the basis of the conservative assumptions of these MCA analyses, both the inhalation (thyroid) dose and the whole-body dose exceed the 2-hour limits for persons at short distances from the AARR, e.g., at 100 meters. The computed 15-minute whole-body dose is within limits, but the inhalation dose (~ 600 rem) still is too high.

[Note: For the present siting of AARR, the nearest approach to CP-5 is less than 100 meters.]

CASE 2 (FORCED VENTILATION THROUGH FILTERS AND OUT A 250-FT-HIGH STACK)

For the assumed circumstances of forced ventilation, ventilation at 600 cfm would not relieve the hazards of ground-level leakage, described above, during the initial 15-minute period.

The overwhelming contributors to the plume whole-body dose are the noble gases. Therefore, for a given ventilation rate, the efficiency of filtration of halogens and solids, within the range 95 % to 99.9 %, would little affect the whole-body dose. *The whole-body dose is controlling, for the case of forced ventilation.*

At a ventilation rate of 6000 cfm, the rate of development of overpressure within the RCB would be reduced by ~ 0.08 psi/min, which would be a significant reduc-

tion. Nevertheless, there might be sufficient uncontrolled leakage of fission products to create a hazard to persons evacuating CP-5, under certain atmospheric conditions. Even if the RCB subsequently were sealed (forced ventilation discontinued) after the evacuation of CP-5 and of the various AARR buildings, say 30 minutes after the beginning of the MCA, at large distances from AARR the 30-day whole-body dose would be substantial. At 5000 meters, the resulting 30-day whole-body dose would total approximately 10 rem, in comparison with 0.48 rem if there had been no forced ventilation.

Several different means are being explored to reduce the dose rates in the event of a catastrophe of the postulated magnitude of the MCA. One concept involves an approximation to double containment, wherein the RCB is sealed and leakage products are diluted in an enclosed volume and exhausted through filters and then out the stack. This should eliminate the problem of an uncontrolled leakage of fission products which could form a dangerous plume. To reduce the direct photon dose to personnel outside the RCB, the thickness of the shielding wall at the periphery of that building might be greatly increased. These and other ideas now are under study.

REFERENCES

1. J. J. DiNunno, et al., "Calculation of Distance Factors for Power and Test Reactor Sites", TID-14844, 2nd Printing (March 23, 1962).
2. J. F. Perkins and R. W. King, "Energy Release from the Decay of Fission Products", Nucl. Sci. Eng. **3**, 726-746 (1958).
3. K. Santhanam, Argonne National Laboratory (Private Communication).
4. A. F. Avery, et al., "Methods of Calculation for Use in the Design of Shields for Power Reactors", AERE-R-3216 (May 1960).
5. G. D. O'Kelley, E. Eichler and N. R. Johnson, "Studies of Short-Lived Fission Products and Their Importance to Reactor Technology", Proc. 1958 Geneva Conference, **15**, Paper No. 672, pp. 469-474.
6. F. A. Gifford, Jr., "Atmospheric Dispersion Calculations Using the Generalized Gaussian Plume Model", Nuclear Safety, **2**, 56-59 (December 1960).
7. F. A. Gifford, Jr., "Use of Routine Meteorological Observations for Estimating Atmospheric Dispersion", Nuclear Safety, **2**, No. 4, 47-51 (June 1961).

II-6. Argonne Advanced Research Reactor Calculations—Preliminary Heating Rates for Water, Beryllium, and Aluminum in the Internal Thermal Column, and for the Beryllium Reflector

H. F. REED

INTRODUCTION

Heating rates have been estimated conservatively for various materials in the Internal Thermal Column (ITC), and for the outer radial beryllium reflector of the AARR. All heating rates were calculated for an equilibrium power level of 240 MWt near the beginning of the core cycle, or for shutdown to 7 % of this power level.

The reactor was approximated as a set of five, nested, cylindrical annuli with circular boundaries (see Table II-6-I). The fuel zone is 45.7 cm high, and the volume of this active fuel zone is 77 liters.

HEATING RATES FOR MATERIALS IN THE ITC AT 240 MWt

Heating rates in the ITC were calculated for 240 MWt operation, for small volumes of beryllium, aluminum, and water positioned at various radii on a plane bisecting the core height. The results are summarized in Table II-6-II.

There are three principal contributors to heating of materials in the ITC: (1) capture of photons originating in the core; (2) capture of photons produced in the ITC from (n, γ) events; and (3) energy absorption during processes of neutron moderation in the ITC. These contributing events are described in more detail below.

CAPTURE OF PHOTONS PRODUCED IN THE CORE

The contributions to the heating rates from photons produced in the reactor core were calculated by determining the probability, P_a , that, in its first flight, the photon would leak from the core and be absorbed in the sample volume in the ITC. For these calculations, use was made of a program that yields collision probabilities for multi-region cylindrical cells of infinite height.¹

In one case studied, the sample volume was a 2-cm-radius cylinder with vertical axis coinciding with that of the ITC. Assuming a uniform source of photons in the core annulus, $P_a = 0.000975$ with pure water in this sample volume. Based upon other calculations using this program, it is estimated conservatively that P_a is higher by 20 % because of the actual non-uniform radial distribution of photons in the core. For a small sample located at the mid-height of the ITC, an additional factor of ~ 1.2 is applied to account for the finite height of the actual reactor. Assuming 21 MeV/fission for the average photon-energy production rate, the net

contribution of core photons to the heating rate of water at the radial and axial center of the ITC is estimated conservatively to be 63 Watts/gram. For small volumes of materials other than water, this contribution was assumed to be proportional to the energy-absorption coefficient for 2-MeV photons.

A similar procedure was used to estimate heating rates in small samples located at the ITC-core interface, at the mid-height of the active fuel zone. The results are summarized in Table II-6-II.

For these calculations, the removal cross sections,

TABLE II-6-I. PHYSICAL PARAMETERS OF THE FIVE
NESTED CYLINDRICAL ANNULI REPRESENTING
THE REACTOR

Region	Materials	Volume Fraction	Outer Radius, cm
ITC	H ₂ O	1.	7.
Core	H ₂ O	0.503	24.2
	Stainless steel	0.384	
	UO ₂	0.113	
Followers of peripheral control blades	Zr	1.	24.7
Inner beryllium reflector	Be	0.93	30.5
	H ₂ O	0.07	
Permanent beryllium reflector	Be	0.81	65.5
	H ₂ O	0.03	
	Zr	0.06	
	Void	0.1	

μ_i , were assigned to yield overestimates of the heating rates, due to 2-MeV photons. In the water sample, $\mu \equiv \mu_{\text{absorption}} = 0.0264 \text{ cm}^{-1}$; in the core, $\mu \equiv \mu_{\text{absorption}} = 0.121 \text{ cm}^{-1}$; and, in the remaining water of the ITC, $\mu \equiv \mu_{\text{total}} = 0.0493 \text{ cm}^{-1}$.

CAPTURE OF PHOTONS PRODUCED BY (n, γ) EVENTS IN THE ITC

Radiative capture of neutrons by ITC materials produces photons, some of which are captured in the ITC in their first flight. The collision-probability method was used to estimate this contribution to the heating rate in materials in the ITC.

For these computations, the ITC was partitioned

TABLE II-6-II. HEATING RATES IN WATER AND IN SMALL VOLUMES OF BERYLLIUM, AND ALUMINUM IN THE ITC AT THE MID-HEIGHT OF THE ACTIVE FUEL ZONE (240 MWt OPERATION)

	Heating Rate Contributions, W/g					
	Axis of ITC			ITC-Core Interface		
	H ₂ O	Be	Al	H ₂ O	Be	Al
Photons from the Core	63.	50.	56.	78.	62.	68.
Photons from (n, γ) events in the ITC						
Source in ITC water	10.	6.4	6.9	5.1	4.1	4.5
Self-absorption of photons	—	0.24	5.8	—	0.06	1.4
Neutron slowing down	64.	21.	3.8	87.	29.	5.1
	138.	78.	72.	170.	95.	79.

into 3 nested cylindrical annuli. Average thermal neutron fluxes were assigned to these regions, on the basis of multigroup diffusion theory calculations: 1.0×10^{16} n-cm/cm³-sec, in region 1 (outer radius: 1 cm); 0.72×10^{16} in region 2 (outer radius: 6 cm); and 0.24×10^{16} in region 3 (outer radius: 7 cm, the core-ITC interface). An average photon energy of 2.2 MeV was postulated per neutron capture.

With the associated step-function distributions of source photons in the ITC, the heating rates due to photon capture were calculated, both at the center of the ITC and at the core-reflector interface (see Table II-6-II).

HEATING DURING MODERATION OF NEUTRONS

In Table II-6-II are listed heating rates contributed by neutron slowing down in small samples of various materials in the ITC. The top 6 neutron energy groups, covering the range from 17 keV to 10 MeV, were assumed to be the dominant contributors; the spatial distributions of the neutron fluxes were obtained from 16-group calculations of the reactor.² The average neutron energy loss, in scattering from one group to another, was calculated as the (positive) difference of the mean energies of these groups.³ In-group scattering was neglected.

HEATING RATES IN THE BERYLLIUM REFLECTOR AT 240 MWt

The contributions of heating rates in the beryllium reflector, which are due to capture of photons generated in the reactor core, were calculated by a contractor. These calculations did not include heating by photons of energies less than 1 MeV. A net correction factor of 2.3 has been derived by correcting for these low-energy

TABLE II-6-III. AXIALLY AVERAGED HEATING RATES IN THE BERYLLIUM REFLECTOR DURING AND IMMEDIATELY AFTER REACTOR OPERATION AT 240 MWt

Distance From Center of ITC, cm	Heating Rates, W/g of Be					
	During operation			At shutdown		
	Photon	Neutron	Total	Fission-product photons	Delayed-fission photons and neutrons	Total
24.7	27.	17.	44.	8.9	4.0	13.
28.	14.	7.0	21.	5.0	1.9	6.9
32.	8.8	2.9	12.	2.8	1.1	3.9
40.	4.2	0.67	4.9	1.1	0.44	1.5
48.	2.1	0.29	2.4	0.49	0.22	0.71
63.5	0.54	0.094	0.63	0.12	0.057	0.18

TABLE II-6-IV. EFFECTIVE^a PHOTON-ENERGY SOURCE INTENSITY IN ANNULAR CORE FOR FISSION-PRODUCT PHOTONS

Photon Energy Range, MeV	Assigned Photon Energy, MeV	Effective Photon Source, MeV/fission
0.0-0.5	0.5	0.8
0.5-1.5	1.0	3.28
1.5-2.5	2.0	2.47
2.5-5.0	3.0	1.51
		<u>8.06</u>

^a The average photon source intensity, at the mid-height of the active core zone, has been increased by 15% to account for the radial variation of power density.

photons and by conservatively estimating an average photon-energy source of 21 MeV/fission in the core. The results in Table II-6-III were determined by applying this factor of 2.3 to the spatial distribution of heating by photons which was calculated by the contractor.

Axially-averaged heating rates due to capture of these photons by the beryllium, and due to slowing down of neutrons, are shown in Table II-6-III.

HEATING RATES IN THE BERYLLIUM REFLECTOR AFTER REACTOR SHUTDOWN FROM 240 MWt

In these calculations, it was assumed that the reactor is shut down rapidly, from equilibrium operation at 240 MWt to a state of subcriticality by \$10. Immediately after shutdown, the thermal power level due to fissions by the reduced neutron flux is 9% of the 240 MWt level. Subsequently, the neutron flux level falls, and 100 seconds after shutdown the heating in the beryllium due to fissions in the core may be ignored. The values listed in Table II-6-III correspond to 9% of the heating rates at 240 MWt.

TABLE II-6-V. AXIALLY AVERAGED PHOTON-ENERGY FLUXES RESULTING FROM FISSION-PRODUCT DECAY PROMPTLY AFTER SHUTDOWN

Reactor Radius, cm	Photon Energy, MeV	0.5	1.0	2.0	3.0
	Photon energy flux, 10^{14} MeV-cm/cm ² -sec				
24.2 (core-reflector interface)		1.4	9.3	8.5	5.0
30.5		0.41	2.9	3.0	2.1
43.6		0.074	0.59	0.71	0.52
63.5		0.011	0.098	0.13	0.10

TABLE II-6-VI. EQUIVALENT REGION COMPOSITIONS USED TO DETERMINE BUILDUP FACTORS AT VARIOUS RADII

Photon Energy, MeV	Equivalent Composition of Region		
	24.2 cm- 30.5 cm	30.5 cm- 43.6 cm	>43.6 cm
0.5 and 1.0	Tin	Iron	Aluminum
2.0 and 3.0	Iron	Aluminum	Water

Immediately after shutdown, the accumulated fission products in the core continue to produce beta rays and gamma rays, with a total decay power equal to 7 % of the equilibrium, pre-shutdown level. Half of this energy stems from the delayed, fission-product gammas (photons). Only these photons are important contributors to heating in the beryllium, for the betas have a very short range of penetration. In the calculations of reflector heating due to capture of fission-product gammas from the core, the effective photon source level was assumed to be 15 % higher to account for the power peaking at the outer perimeter of the reactor core near the beginning of core life.

The core photon source was divided into 4 energy groups (see Table II-6-IV). The photon energy-absorption fluxes in the beryllium reflector are listed in Table II-6-V for 4 different radial locations. In determining these fluxes, the following models were used: at the core-reflector interface (24.2 cm), the equation for a solid-sphere source⁵ was applied; in the beryllium reflector, the line-source equation was applied for a solid cylinder with a height of 45 centimeters.

The photon cross sections of Ref. 6 were used. The energy-absorption buildup factors were approximated by exponentials, and the coefficients in the exponent were obtained from Table IV of Ref. 7. The effective, or "equivalent", compositions assumed in selecting the buildup factors are given in Table II-6-VI.

The numerical results for the heating rates are summarized in Table II-6-III, under the heading "Fission-Product Photons". These are axially-averaged heating rates in the beryllium zone bounded by planes at the top and at the bottom of the 45.7-cm-high active core zone.

The total energy-absorption rates in the entire radial beryllium reflector, in this 45.7-cm-high zone, were obtained from numerical integrations. During operation, this beryllium absorbs 2.3 % of the thermal power of the system, or 5.5 MWt. With the effect of delayed fissions included, immediately after shutdown the beryllium absorbs 4.5 % of the thermal power, or 1.7 MWt. After the heating from fission events becomes negligible, the beryllium absorbs 7.3 % of the power level after shutdown.

These percentages indicate that the cooling requirements of the beryllium reflector during shutdown determine the assignments of coolant flow in the reactor.

REFERENCES

1. E. M. Pennington, "Cylindrical Lattice Collision Probability Codes, B 692/RP", ANL-6836 (February 1964).
2. B. N. Kristianson and C. N. Kelber, "Argonne Advanced Research Reactor—Physics Calculations", Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010, 57-62 (January 1965).
3. G. E. Hansen and W. H. Roach, "Six and Sixteen Group Cross Sections for Fast and Intermediate Critical Assemblies", LAMS-2543 (December 1961).
4. M. Roos, "Source of Gamma Radiation in a Reactor Core", J. Nuc. Energy (Pergamon Press, 1959), Part B: Reactor Technology, Vol. I, pp. 98-104.
5. T. Rockwell, III, *Reactor Shielding Design Manual*, TID-7004 (March 1956).
6. *Reactor Physics Constants*, ANL-5800, 2nd Edition (1963), pp. 652-655.
7. S. Buscaglione and R. Manzini, "Buildup Factors: Coefficients of the J. J. Taylor Equation", ORNL-TR-80 (June 1964).

II-7. Argonne Advanced Research Reactor Calculations—Multiplication of Photoneutrons in the Subcritical AARR; Effects on Reactor Power and Location of Startup Detector

D. H. SHAFTMAN and H. F. REED

INTRODUCTION

During power operation of an AARR core, fission products accumulate which continue to provide a source of photons when the reactor has been shut down. Most of these photons are captured in the core. Some of the photons which leak into the radial beryllium reflector are captured in (γ, n) reactions with beryllium. The resulting photoneutrons diffuse through the reactor and provide a background of thermal neutron flux. This neutron flux is increased by the multiplication of the diffusing photoneutrons in the subcritical reactor. Two questions to be answered are:

- (1) Will a thermal-neutron startup detector, positioned at or radially beyond the beryllium reflector, provide interpretable changes in count rate during control rod withdrawal with the reactor safely subcritical?

and

- (2) Does the subcritical multiplication of these photoneutrons increase the reactor thermal power significantly?

SUMMARY AND CONCLUSIONS

The answers to the two questions are:

- (1) A startup detector of thermal neutrons, positioned at the outer surface of the beryllium reflector, at or near the mid-height of the active core, would provide interpretable changes, in response to control rod withdrawal, when the reactor is safely subcritical initially. For a system which is 10 dollars subcritical, the ratio of the thermal neutron fluxes (multiplied flux vs unmultiplied flux) is ~ 2.2 at this radial location. At a radial distance of 14 cm beyond the beryllium, the ratio decreases to 1.6. If the reactor were subcritical by only 1 dollar, the ratios would be ~ 12 and ~ 6.5 , respectively. The computed unmultiplied thermal neutron flux, at the distance of 14 cm from the beryllium, is more than 3×10^9 when the reactor is subcritical by 10 dollars.
- (2) Assume that the reactor has been operating to "equilibrium" at 240 MWt, and that the reactor then is shut down. Promptly after shutdown, the thermal power due to fission-product decay is approximately 7 % of the equilibrium level, or ~ 17 MWt. If the reactor is subcritical by 10 dollars,

the multiplication of the photoneutrons results in a contribution of ~ 0.2 MWt; at 1 dollar subcriticality, the contribution is ~ 2.2 MWt.

THE COMPUTATIONAL PROCEDURE; RESULTS

In outline, the computational procedure was:

- (1) To determine the spatial and energy distribu-

FIG. II-7-1. Distribution of Unmultiplied Photoneutron Sources in the Reflector Due to Fission-Product Gammas After Shutdown of AARR from 240 MWt Operation. ANL Neg. No. 112-5488.

tions of photoneutron sources due to leakage of photons from the shut-down reactor; and then

- (2) To use the "one-shot" option of the multigroup-diffusion-theory program RP-122 (REX) to calculate the thermal neutron flux resulting from the cascade processes of subcritical multiplication

FIG. II-7-2. Contributions to Group-16 Thermal Neutron Flux From Successive-Generation Multiplication of Photoneutron Sources in the Be Reflector After Shutdown of AARR to \$10. Subcriticality (Group-16: 0-0.1 eV). ANL Neg. No. 112-5487.

of these source neutrons. Sixteen energy groups were used for the computations. The mathematical formulation of this procedure is discussed in Paper V-12.

The threshold for the (γ, n) reaction in beryllium is 1.67 MeV. Therefore, only the photons in the so-called 2-MeV and 3-MeV groups contribute to the production of photoneutrons in the beryllium. The average energies of the photoneutrons generated are approximately 0.3 MeV and 1.3 MeV, respectively. For an idealized beryllium zone, without voids, water, or other metals, the macroscopic cross sections for the (γ, n) reaction are approximately $1 \times 10^{-4} \text{ cm}^{-1}$ (2-MeV photons), and $7 \times 10^{-5} \text{ cm}^{-1}$ (3-MeV photons).

The 0.3-MeV photoneutrons comprise roughly $\frac{3}{4}$ of the total source throughout the beryllium. Therefore, the photoneutron spectrum was taken to be 0.25 in group No. 3 (0.9 MeV to 1.4 MeV) of the 16-group multigroup partition of the total energy range, and 0.75 in group No. 5 (0.1 MeV to 0.4 MeV). Figure II-7-1 shows the calculated radial distribution of the photoneutron sources, at the vertical position corresponding to the center of the active core zone. The amplitude is based upon an equilibrium pre-shutdown thermal power level of 240 MWt, with a prompt shutdown to 7% of this power level; half of this 7% is attributable to fission-product photons. Figure II-7-2 shows the unmultiplied thermal neutron flux distribution, ψ_0 , in energy group 16 (0 eV to 0.1 eV).

Also shown in Fig. II-7-2 are the shapes of the group-16 fluxes contributed by successive-generation sources to the multiplied group-16 flux. For example, ψ_1 is the renormalized group-16 flux generated by the first-generation fissions caused by the diffusion and moderation of unmultiplied photoneutrons. On this figure, each of the successive-generation neutron flux contributions has been renormalized to a fixed total fission-neutron production rate in the core. The unmultiplied group-16 flux, ψ_0 , is not so renormalized, of course; the correct relationship is that $\psi_0/\psi_1 = 6.8$ at the outer surface of the beryllium zone.

Additional discussion of the results shown in Fig. II-7-2 is presented in the reference Paper V-12 of this report.

II-8. Argonne Advanced Research Reactor Calculations—Analysis of Criticality-Facility Loadings

D. H. SHAFTMAN, C. N. KELBER, B. N. KRISTIANSON* and R. P. SAVIO†

INTRODUCTION

The AARR Criticality Facility has been in operation

* Reactor Engineering Division.

† Reactor Operations Division.

since the first loadings of fuel assemblies early in October, 1964. Experiments performed with that facility are described in Paper II-10 of this report, which also includes summaries of interpreted measured data.

It should be noted that the thicknesses, both of the Internal Thermal Column (ITC) and of the reactor fuel zone, in the Criticality Facility are smaller than the values used in analyses of reference AARR systems. Moreover, the uranium is in the form of uranium metal, not UO_2 , and none of the three fuel loadings for which measurements are reported contained as much U-235 as the present reference design system. Experiments are being planned for a modified Criticality Facility, with a larger ITC and a correspondingly smaller fuel zone; the modification will be easily reversible, however, so as to provide continuity in measurements with the future, heavier, fuel loadings. The next loading of fuel foils is to contain a total of ~ 62 kg of U-235 in 1215 fuel foils of highly enriched uranium; this is approximately equal to the total U-235 loading in the present reference design reactor.

There are three principal purposes of the program of experiments. One purpose is to measure integral parameters (e.g., available excess reactivity) and differential parameters (e.g., void coefficients of reactivity) with a variety of fuel loadings with small annular cores. These measurements provide data with which results of theoretical analysis may be compared, for check points of the calculation methods. They also provide data of direct use in designing the AARR and in evaluating safety characteristics of the AARR.

A second principal purpose is to measure spatial distributions of neutron flux and of power, in a detail which is extremely difficult, or not feasible, to determine by theoretical analysis. This information is needed for heat-transfer and thermal-stress analyses of proposed reactor cores, from which the first operating-core design is to be selected. With simulated beam tubes, measurements of neutron fluxes and gamma fluxes are being made in an attempt to provide guidance to the reactor design layout of beam tubes and through tubes. Also, experiments are being conducted in the ITC to measure reactivity effects and flux-perturbation effects of materials which might be introduced into that flux-trap zone. These experiments represent a significant effort of the carefully planned Criticality Facility program.

A third principal purpose of the experiments is to provide approximate answers for problems as they arise in the course of design of the AARR. An example of these experiments is the set of measurements of the effectiveness of neutron "windows", placed within the pressure vessel, in raising the level of the thermal neutron flux just beyond the vessel, in the vicinity of neutron detectors of the nuclear instrumentation system.

In early planning of the program of experiments in the Criticality Facility, it was intended that little time

would be spent on measurements in systems where the total U-235 content was not much more than half that of the reference system. One principal purpose of experiments with smaller loadings was to permit a gradual, safe progress to loadings with larger total masses of U-235, and to determine the corresponding requirements of boron for reactivity compensation in these loadings. The other principal purpose of these experiments was to provide information about reactivity effects in annular-core reactors with high fuel contents, to permit comparison with calculations in a range of cores for which little or no experimental data were available.

Because of delays in the delivery of fuel foils to the Laboratory, there has been time for additional experimentation with each loading. Some of the additional experiments have yielded data of basic interest, e.g., relative reactivity-control effectiveness of various neutron absorbers in low-epithermal reactors. Other experiments have been directed to specific design evaluations, such as preliminary estimates of the effects of "water" (Plexiglas) in the beryllium reflector on reactivity and on the magnitude of the peak flux in the beryllium per unit reactor power.

Theoretical analyses have been conducted on a number of the experiments with reduced loadings of U-235, but, as yet, not with a very detailed treatment of the substantial spatial heterogeneities of these loadings. Nevertheless, many of the important reactor parameters computed are in fair agreement with measured data. These analyses and comparisons have been helpful both in improving calculations and in improving experiments. It is expected that the effects of these heterogeneities on reactivity will be much smaller for loadings of direct interest to the AARR design.

The computations discussed below may be divided into two major categories: (1) differential and integral reactivity effects of various loadings of fuel and boron; and (2) radial distributions of neutron activation and of power. Where measured data are available, comparisons are supplied in tabular or in graphical form.

For the most part, methods of multigroup diffusion theory have been applied. The geometric complexities and asymmetries of the reactors have been treated only approximately. Primarily, the computations have been for one-dimensional cylindrical geometry, with approximate corrections for the effects of axial neutron leakage.

It is clear that an understanding of a number of the reactor characteristics will be obtained only by a greater detail and a greater sophistication of calculation in future studies. This will involve the use of more accurate approximations to the multigroup solution of the Boltzmann equation for neutron transport, both in one dimension and in two dimensions. The number of

neutron energy groups encompassing the thermal energy range will be increased above the two groups used in the analyses reported here. Some important spatial fine structures of power and of neutron flux will remain unresolved, computationally, except in crude approximations. This is due to the considerable geometric complexities of the arrangements of materials in the reactor.

REMARKS ON THE GEOMETRY APPROXIMATIONS OF THE COMPUTATIONS

Geometrically, the reactor has been approximated as a nested sequence of cylindrical annuli. The area of each annulus was set equal to the actual area of the zone in question. The beryllium reflector, however, was approximated as an annulus of thickness 30.5 cm (12 inches). In actual fact, the outside surface of this zone is rather far from circular, as may be seen in Fig. II-10-1 of Paper II-10. This difficulty of simulating geometry makes it quite difficult to compare adequately theory and experiment for neutron activation mappings in the vicinity of the interfaces of the Internal Thermal Column (ITC) and the core, the core and the beryllium reflector, and the beryllium reflector and the water beyond it.

Each region was assumed to be uniform in composition. The heterogeneous core arrays of coolant (H_2O), fuel foils, and strips of stainless steel and boron-stainless steel were volume-averaged to provide approximate core compositions. For those experiments with layers of Plexiglas in the reflector (see Paper II-10), the beryllium and Plexiglas were volume-averaged to obtain the atom densities of the various materials. The equivalent radii of the nested cylindrical annuli are shown in Table II-8-I.

Three basic core loadings have been studied experimentally. They will be referred to by indicating the number of fuel foils and then the number of poison strips of borated stainless steel, namely, 315/0 (or 315), 615/420, and 810/810. This same notation will be applied in summarizing results of calculations for other systems, including 615/0, 615/210, 810/0, 810/405, and loadings of 1215 fuel foils and various numbers of poison strips.

The fuel foils were distributed essentially uniformly in each loading, with fuel foils at the inner and outer edges of the reactor core. Thus, in the 315-fuel-foil minimum-critical loading, each fuel foil in an assembly was separated from the next foil by three fuel-plate cells with zero uranium content. It is to be expected, therefore, that both the measured reactivity data and the data from calculations of volume-averaged cores will exhibit unusual behavior. For example, the measured uniform-void coefficient of reactivity in the 315-fuel-foil core was very much smaller in magnitude than the cal-

TABLE II-8-I. RADII OF EQUIVALENT CYLINDRICAL ANNULI^a

Composition	Outer Radius, cm
ITC water	5.61
ITC liner (SS)	5.77
ITC water	5.88
Fuel zone	23.30
Aluminum control-blade follower plus water	23.96
Beryllium reflector	54.44
Water ^b	84.44

^a The effective bare height of the reactor was taken to be 60 centimeters.

^b For the reactivity calculations, the termination of the water zone at 30 cm is inconsequential. There is some effect on the flux distributions at the outer edge of the beryllium, but, for purposes of this report, the effect is very small.

TABLE II-8-II. MULTIGROUP STRUCTURE

Group Number	Neutron-Energy Range, eV	GAM-I Neutron-Energy Range
1	$3. \times 10^6$ – $10. \times 10^6$	2.865×10^6 – $10. \times 10^6$
2	1.4×10^6 – 3.0×10^6	1.353×10^6 – 2.865×10^6
3	0.9×10^6 – 1.4×10^6	0.821×10^6 – 1.353×10^6
4	0.4×10^6 – 0.9×10^6	0.388×10^6 – 0.821×10^6
5	0.1×10^6 – 0.4×10^6	0.111×10^6 – 0.388×10^6
6	$17. \times 10^3$ – $100. \times 10^3$	15.0×10^3 – $111. \times 10^3$
7	$3. \times 10^3$ – $17. \times 10^3$	3.35×10^3 – 15.0×10^3
8	0.55×10^3 – 3×10^3	0.583×10^3 – 3.35×10^3
9	100. –550.	101. –583.
10	30. –100.	29.0 –101.
11	10. –30.	10.7 –29.0
12	3. –10.	3.06 –10.7
13	1. –3.	1.13 –3.06
14	0.4–1.	0.414–1.13
15	0.1–0.4	— —
16	0 –0.1	— —

culated coefficient, and, also, it was much smaller in magnitude than the coefficients measured in the other cores. In general, agreement between theory and experiment is much improved for the heavier loadings of fuel.

CONSTRUCTION OF CROSS SECTION SETS FOR SIXTEEN-GROUP DIFFUSION THEORY CALCULATIONS

For each of four basic core loadings (315/0, 615/420, 810/810, and 1215/2000) a cross section set has been selected for a 16-group partition of the neutron energy range (see Table II-8-II). For the first 14 groups, extending above 0.4 eV, the cross sections of the Hansen-and-Roach library¹ were used. For materials not in that library, cross sections were obtained from GAM-I⁽²⁾ computations, using a coarse-group energy partition which corresponded fairly closely to the Hansen-and-

Roach partition (see Table II-8-II). Except for U-235 and U-238, these cross sections were assumed to be independent of the core loading. For U-235 and U-238, the equivalence formula of P. Bakshi³ was applied to estimate the effective excess potential scattering per atom for an equivalent homogeneous mixture of the uranium isotopes with the other constituents of the fuel zone. Then, Figs. 10 and 11 of Ref. 1 were applied to determine group-average fission and capture cross sections of U-235 in the energy range of resolved resonances (groups 10-13, 1 eV to 100 eV). Similarly, for U-238, Fig. 16 of Ref. 1 was used to obtain effective group-average capture cross sections for groups 8-12 (3 eV to 3 keV). The U-235 and U-238 cross sections were assumed to be independent of core loading in all other groups above 0.4 eV.

Two extreme cases were considered for each loading, in applying the Bakshi formula: (1) isolated foil; and (2) black foil. Multigroup (16-group) calculations with the resulting spread of cross section values indicated negligibly small reactivity uncertainty for all loadings but the 1215-fuel-foil loadings. For the 1215 loading, a preliminary maximum-error estimate of $\sim 0.4\%$ was inferred, which is the change in reactivity from the black-foil approximation to the isolated-foil approximation. In subsequent computations, the isolated-foil approximation was used.

The group-average transport cross sections for hydrogen were taken from the Hansen-and-Roach set (groups 1-14, Table XV)⁽¹⁾. These values of σ_{tr}^H were calculated for use in S_n computations of homogeneous epithermal systems, and their use for computations in the ITC is open to question. It is reassuring, therefore, that rather good agreement was obtained between measured and calculated radial distributions of activations of neutron detectors in the ITC and in the bulk of the fuel zone (see Figs. II-8-1 through II-8-4). Since there are other uncertainties in estimating power distributions at the edge of the reactor core (e.g., the use of only two thermal energy groups) there is not enough information, at present, to determine whether substantial changes in the hydrogen group transport cross sections are needed at the core-ITC interface.

Group 15 (0.1 eV-0.4 eV) and group 16 (0-0.1 eV) cover what might be designated the "thermal energy range." It is a good approximation to assign zero upscattering to groups above 0.4 eV, but there is substantial upscattering of neutrons from group 16 to group 15. In the 16-group computations, it was assumed that there is zero upscattering. Therefore, cross sections for *net* downscattering from group 15 to group 16 were calculated. (This is discussed in greater detail in the next subsection.) The average microscopic capture, fission, and elastic-transport cross sections for the various

FIG. II-8-1. Radial Distribution of Relative Activity of Manganese Foils (315-Fuel-Foil Loading; 10% Plexiglas in Beryllium Reflector). Refer to Fig. II-10-7, Paper No. II-10. ANL Neg. No. 112-5885.

materials were calculated for groups 15 and 16 with the program THERMOS⁴ for thermal spectra. Because the ITC and the core have such different thermal spectra, cross sections were calculated for each of these two regions. The ITC-average cross sections were assumed to apply to the outer-reflector zone as well; this assumption is reasonable, as was shown by subsequent calculations with the THERMOS program. As a result, the cross section library includes "ITC-hydrogen" and "core-hydrogen", and "ITC-oxygen" and "core-oxygen", with corresponding cross sections assumed to differ only in groups 15 and 16.

REMARKS ON THE COMPUTATIONS OF NET-DOWN-SCATTER CROSS SECTIONS FOR NEUTRON ENERGY GROUP 15 (0.1 eV - 0.4 eV)

In the 16-group model of the diffusion theory analyses, it has been assumed that there is no upscattering

FIG. II-8-2. Radial Distribution of Relative Activity of Manganese Foils (615/420 Core Loading; No Plexiglas in Beryllium Reflector). ANL Neg. No. 112-5884.

of neutrons from groups 15 and 16 to energies above 0.4 eV (groups 1-14). Further, it has been assumed that the neutron exchange between groups 15 and 16, due to downscattering and upscattering, may be approximated by computing a net-downscattering cross section $\sigma_{s_{\text{net}}}^i(15 \rightarrow 16)$ for material i .

Symbolically,

$$L_{15}\phi_{15} = \Sigma_s(16 \rightarrow 15)\phi_{16} + S_{15} \quad (1)$$

$$L_{16}\phi_{16} = \Sigma_s(15 \rightarrow 16)\phi_{15} + S_{16},$$

where, for $j = 15, 16$:

$$L_j \equiv \Sigma_{a_j} + \sum_{k \neq j} \Sigma_s(j \rightarrow k) - \nabla \cdot D_j \nabla$$

$$S_j \equiv \sum_{k=1}^{14} \Sigma_s(k \rightarrow j)\phi_k.$$

In the approximation that $\Sigma_s(15 \rightarrow k) = 0$ except for $k = 16$, and $\Sigma_s(16 \rightarrow k) = 0$ except for $k = 15$, Eqs. (1)

FIG. II-8-3. Radial Distribution of U-235-Fission Activity of Bare Foils (615/420 Core Loading; No Plexiglas in Beryllium Reflector). ANL Neg. No. 112-5887.

may be rewritten:

$$\begin{aligned} -\nabla \cdot D_{15} \nabla \phi_{15} + \Sigma_{a_{15}} \phi_{15} + \Sigma_{s_{\text{net}}}(15 \rightarrow 16)\phi_{15} &= S_{15} \\ -\nabla \cdot D_{16} \nabla \phi_{16} + \Sigma_{a_{16}} \phi_{16} &= S_{16} + \Sigma_{s_{\text{net}}}(15 \rightarrow 16)\phi_{15}, \end{aligned} \quad (2)$$

where

$$\Sigma_{s_{\text{net}}}(15 \rightarrow 16) \equiv \frac{\Sigma_s(15 \rightarrow 16)\phi_{15} - \Sigma_s(16 \rightarrow 15)\phi_{16}}{\phi_{15}}$$

is a function of position ($\phi_{15} \neq 0$). The leakage terms in Eqs. (2) include *all* leakage, radial and axial.

It would be desirable to use a space-independent $\Sigma_{s_{\text{net}}}(15 \rightarrow 16)$ within a given region, rather than a value which varies from point to point, in keeping with the usual philosophy of multiregion, multigroup calculations. The usual questions arise as to the adequacy of this approximation throughout the given region. To attempt to answer these questions, the integral-transport program THERMOS⁴ was applied to calculations of the reactor, assuming a step-function source of neutrons into the range (0-1.4 eV) of the THERMOS computation; the source was made proportional to the atom density of hydrogen in the region. The results showed

FIG. II-8-4. Radial Distribution of U-235-Fission Activity of Bare Foils (810/810 Core Loading; No Plexiglas in Beryllium Reflector). ANL Neg. No. 112-5886.

that the ratio of the group-16 flux (the integrated flux from 0 to 0.1 eV) to the group-15 flux was almost independent of position in the vicinity of the radial center of the ITC. However, it was not feasible to determine accurately the radial leakage so as to solve Eq. (2) for $\Sigma_{s_{\text{net}}}(15 \rightarrow 16)$. Subsequently, THERMOS was applied to the calculation of uniform, one-region slab cells of water and of beryllium to obtain data with which to solve Eq. (2). In the reactor core, however, no such difficulty was experienced, and it was found that $\Sigma_{s_{\text{net}}}(15 \rightarrow 16)$ rapidly approached asymptotic values in the various cores. The leakage was a small part of the total "loss" of neutrons in each group, in the core, and the model of Eqs. (2) appears to be satisfactory with a constant value of $\Sigma_{s_{\text{net}}}(15 \rightarrow 16)$, at least for purposes of calculating reactivity effects, if not for flux calculations.

The source term S_{16} was calculated as:

$$S_{16} = \int_{0.1}^{1.4} \phi(E) \Sigma_s^H(E \rightarrow \text{group 16}) dE + \int_{1.4}^{\infty} \phi(1.4) \frac{1.4}{E} \Sigma_s^H(E \rightarrow \text{group 16}) dE, \quad (3)$$

TABLE II-8-III. MICROSCOPIC NET-DOWNSCATTER CROSS SECTIONS

Element	σ_s, b	$\sigma_{s_{\text{net}}}(15 \rightarrow 16)$				
		$\sigma_{s_{\text{net}}}(15 \rightarrow 16)$ in the Medium, b/atom				
		ITC or reflector	315/0 $\bar{\Sigma}_{s_{16}} = 0.397$ cm ⁻¹	615/420, $\bar{\Sigma}_{s_{16}} = 0.736$ cm ⁻¹	810/810, $\bar{\Sigma}_{s_{16}} = 0.973$ cm ⁻¹	1215/2000, $\bar{\Sigma}_{s_{16}} = 1.528$ cm ⁻¹
Oxygen	4.2	1.233	0.385	0.299	0.266	0.218
Aluminum	1.4	—	0.079	0.062	0.054	0.043
Chromium	3.0	—	0.089	0.066	0.060	0.047
Iron	11.0	—	0.300	0.223	0.204	0.158
Nickel	17.5	—	0.453	0.337	0.308	0.239
Hydrogen	—	0.834	5.749	5.685	5.539	5.371
Beryllium	—	0.065	—	—	—	—

where $\phi(E)$ was taken from the output of THERMOS, with appropriate corrections to convert from flux per unit velocity to flux per unit energy. The second term on the right-hand side of Eq. (3) stems from the assumption, in THERMOS, that the flux is $1/E$ above the upper boundary (in this case ~ 1.4 eV). Knowing all the other terms in the balance equation for group 16 [Eq. (2)], $\Sigma_{s_{\text{net}}}(15 \rightarrow 16)\phi_{15}$ is determined, and hence $\Sigma_{s_{\text{net}}}(15 \rightarrow 16)$.

The flux-spectrum output of THERMOS was applied also to the calculation of $\sigma_s(15 \rightarrow 16)$ for each element but hydrogen, as the total slowing down from group 15 to group 16 (per atom of the material) divided by the total flux in group 15. Then, by subtraction, the net slowing-down cross section of hydrogen was obtained. The results are shown in Table II-8-III. The uranium isotopes have very small $\sigma_s(15 \rightarrow 16)$, and they have been ignored in these computations.

Note (Table II-8-III) that the oxygen cross section is very much larger than the Hansen-and-Roach value (Ref. 1, p. 620: 0.331 barns) in the ITC, and it is smaller than 0.33 barns in the cores for loadings of 615 suel foils and more. In the very weakly absorbing ITC, the dominant contribution to the group-15 flux is from the "Maxwellian" component, in the vicinity of 0.1 eV. It is only in the range from 0.1 eV to ~ 0.13 eV that oxygen can scatter neutrons into group 16, and the result is that the cross section for downscatter by oxygen is large in the ITC. In the core, the flux above 0.1 eV drops less rapidly than $1/E$ with increasing energy, for loadings of 615/420 or more. Consequently, those values of $\sigma_s^O(15 \rightarrow 16)$ are lower than the $1/E$ -averaged value, which is 0.31 barns.

In the case of the weakly-absorbing, moderating regions (ITC and beryllium reflector), the thermal neu-

tron flux spectrum is very close to Maxwellian. A defining principle of the Maxwellian distribution is that the rate of upscattering from one energy interval to another is equal to the rate of down scattering from the second interval to the first. This is the Equilibrium Principle. It is not at all surprising, therefore, that the cross section for net downscattering from group 15 to group 16 is very small in the ITC and in the beryllium. However, this value of $\sigma_{\text{net}}^{Be}(15 \rightarrow 16)$ in the reflector is very sensitive to the rate of neutron leakage (radial, as well as axial), and, therefore, it is poorly determined for AARR conditions. There is discussion of this uncertainty, in the subsection titled *Radial Distributions of Neutron Activation*, below.

REACTIVITY AND REACTIVITY COEFFICIENT

The construction of the cross section libraries for the various fuel loadings are described above. In calculating effects of perturbations of a given system, the cross-section library was unchanged. The adequacy of this approximation was demonstrated for fairly sizeable perturbations by detailed and laborious changes of the cross section set. For massive changes, such as the removal of the boron from the 1215/2000 core, the approximation is less convincing, but it has not been checked yet.

Table II-8-IV summarizes the results of calculations of the available excess reactivity of various loadings of fuel and poison (boron). Where measured data are available they are listed for comparison. In Table II-8-V are the calculated and measured coefficients of reactivity for small, uniform voidings of the core and of the ITC.

It may be seen that there is no apparent trend or bias in the discrepancies between calculated and measured

available excess reactivity. Credit has been taken in Table II-8-IV for part of the reactivity effects of the wide separations of fuel foils in the 315-fuel-foil loading; from S_n computations of fuel-plate cells and of the volume-averaged medium, it was found that 3.0% reactivity was lost due to this heterogeneity. No calculations have been made, however, of the local effects of the heterogeneities at the inner and outer edges of the core. It is likely, for example, that the effective radius of the ITC is larger and the effective thickness of the core is smaller than the values used in the calculations—both effects reduce reactivity.

The calculated void coefficient in the ITC is almost independent of fuel loading, as is the measured coefficient. The calculated value is much smaller than the measured value. Assuming that the measured data are more accurate, it is not known whether this discrepancy arises from: inadequacy of the hydrogen cross sections in the ITC; the use of diffusion theory; the omission of the positive reactivity effect of an increase in axial reflector savings with increasing void; or other effects not mentioned here.

Except in the case of the 315-fuel-foil loading, the calculated coefficient of reactivity for small, uniform voidings in the reactor core is in good agreement with measurement. The measured datum for the 315 system simply may indicate that the gross heterogeneities in the reactor core contribute large positive-reactivity components, thereby reducing the magnitude of the negative void coefficient. This effect was studied with S_n computations of fuel-plate cells and of the homogenized infinite medium. The result was that the reactivity coefficient of voiding some of the moderator in an infinite lattice of fuel-plate cells was only $\sim 10\%$ of the measured value in the finite system. It would

TABLE II-8-IV. AVAILABLE EXCESS REACTIVITY OF AARR CRITICALITY-FACILITY SYSTEMS

Number of Fuel Foils	Approximate Loading of U-235, kg	Number of Poison Strips ^a	Approximate Loading of B, g	Plexiglas Content of Reflector, %	Calculated Excess Reactivity, %	Measured Excess Reactivity, ^b %
315	16.1	0	0	10	+5.0 ^c	+0.4
315	16.1	0	0	0	+6.1 ^c	—
615	31.5	420	220.5	10	+5.0	+3.6
615	31.5	420	220.5	0	+6.0	+4.8
615	31.5	210	110.3	0	+11.2	—
615	31.5	0	0	0	+16.5	—
810	41.4	810	425.	0	+4.4	+2.7
810	41.4	405	212.5	0	+11.0	—
810	41.4	0	0	0	+18.9	—
1215	62.1	2000	1050.	0	+0.9	—
1215	62.1	1000	525	0	+10.5	—
1215	62.1	0	0	0	+21.4	—

^a The poison strips are borated stainless steel, with a nominal mass of 0.525 g natural boron per strip.

^b See Paper II-10.

^c Of this, it is calculated that 3.0% is due to the error in k_∞ in not calculating the heterogeneous medium of fuel-plate cells.

TABLE II-8-V. VOID COEFFICIENTS OF REACTIVITY IN AARR CRITICALITY-FACILITY SYSTEMS

Number of Fuel Foils	Number of Poison Strips	Plexiglas Content of Reflector, %	Calculated Void Coefficient, ^a %/‰ void in water	Measured Void Coefficient, ^b %/‰ void in water
315	0	10	Core, -0.28 ITC, +0.037	Core, -0.16 —
615	420	10	— —	Core, -0.23 ITC, +0.047
615	420	0	Core, -0.24 ITC, +0.032	— —
810	810	0	Core, -0.22 ITC, +0.033	Core, -0.19 ITC, +0.047
1215	2000	0	Core, -0.17 ITC, +0.032	— —

^a In the computation, 10% of the water in the region was removed.

^b See Table II-10-III, Paper II-10.

appear, therefore, that primarily it is the leakage of neutrons from the annular core that determines the void coefficient in the core.

RADIAL DISTRIBUTIONS OF NEUTRON ACTIVATION

Using multigroup fluxes determined from the 16-group, one-dimensional, diffusion theory calculations, radial distributions of neutron-activation-rates have been calculated for various neutron detectors, including Mn and U-235 (fission). The group cross sections for the manganese were obtained from GAM-I computations. The U-235 microscopic fission cross sections were taken directly from the corresponding 16-group library employed for the multigroup calculation, except for the two thermal groups. These average fission cross sections were computed with respect to a flux spectrum characteristic of the medium. In the calculation, no correction was made for self-shielding within the detector or for flux-depression effects.

Figure II-8-1 shows the relative radial distributions of activation of manganese detectors in the 315-fuel-foil system, normalized to a bare-detector activity of 100 (arbitrary units) at the center of the ITC. The calculated "epicadmium" distribution is simply the (normalized) activity by captures of neutrons of energy greater than 0.4 eV (Groups 1-14). On this figure are plotted the measured activation distributions, normalized to a bare-foil activity of 100 at the center of the ITC. The calculated distributions are shown as solid lines; the measured data are the points.

The dashed line, shown in the range from 24 cm to 45 cm is the result of a calculation wherein the net-down-

FIG. II-8-5. Calculated Radial Distribution of U-235-Fission Activity of Bare Foils (1215/2000 Core Loading; No Plexiglas in Beryllium Reflector). ANL Neg. No. 112-5883.

scatter cross section, $\sigma_{s_{\text{net}}}^{\text{Be}} (15 \rightarrow 16)$, was increased to three times the value shown in Table II-8-II. It is believed that this larger value for $\sigma_{s_{\text{net}}}^{\text{Be}} (15 \rightarrow 16)$ is within the uncertainties of the calculation of this cross section. Note that much of the discrepancy between theory and measurement in the reflector is removed by this change. However, Figs. II-8-2 through II-8-5 are based upon the smaller value for the beryllium cross section.

It may be seen (see Figs. II-8-1 through II-8-3) that, in the 315 and 615/420 systems, the calculated relative activity in the reactor core is consistently lower than the measured values. This might be due, in part, to the heterogeneity of the fuel loading, with detector foils located in the water near fuel foils. Agreement between measurement and calculation is good, lending assurance that the computational model and the cross sections should not lead to large errors in estimating the magnitudes of the peak "fluxes" in the ITC and in the reflector per unit reactor power, in actual design systems.

In Fig. II-8-2 similar comparisons of calculated and measured activation distributions are shown for

manganese foils in the 615/420 system with no Plexiglas in the reflector. In Fig. II-8-3, U-235-fission distributions are shown for this system (bare foils).

In Fig. II-8-4 bare-foil activities are displayed for U-235 fission in the 810/810 system. There was no Plexiglas in the reflector.

In Fig. II-8-5 the calculated bare-foil activity for U-235 fission in the 1215/2000 system is presented, with no Plexiglas in the reflector. As yet, no experiments have been performed with a loading of 1215 fuel foils.

REMARKS ON THE ACTIVATION DISTRIBUTIONS

It should be noted that what have been compared are detector activities, not neutron fluxes. The test, therefore, is a very practical one for the ITC since this will be a medium for neutron captures. The calculated ratio of group-16 flux to group-15 flux is ~ 4 , at the center of the ITC, on the basis of REX calculations. THERMOS calculations for a thin slab cell (i.e. infinite medium) of water yield a ratio of 6.5. If this difference is real, as it appears to be, the thermal neutron flux spectrum at the center of the ITC may be substantially harder than a Maxwellian. It would be misleading, therefore, to take the sum of the group-15 and group-16 fluxes as representing the "2200 m/s neutron flux" in the ITC.

The corresponding situation in the beryllium reflector has not been investigated yet. In that region, the beam-tube fluxes and through-tube fluxes are of primary importance. The over-all exit spectrum at the end of the tube will be a function of the angular distribution of scattered neutrons in a small cone around the longitudinal axis of the tube.

It is worth repeating here that there are substantial differences between the systems for which these measurements and calculations have been performed and the present reference AARR design system. Three important differences are:

1. The reference AARR includes a *graded* loading of U-235 at the edges of the reactor core; the criticals have not, in the loadings to date. With a reduced loading of U-235 near the ITC, the peak flux per unit power is decreased. There is additional decrease in this ratio as the core is operated at power.

2. In the reference AARR, the fuel is included as a cermet of UO_2 and stainless steel. In the criticals, the fuel is high-density uranium metal. Therefore, there is more steel, and less oxygen, per fuel plate in the criticals.

3. The reference AARR includes an ITC with a larger radius, so as to reduce the flux-depression effects of samples in the ITC.

Modifications of the Criticality Facility are being

planned to permit experimentation with a larger ITC. The core thickness will be reduced, correspondingly.

CALCULATED REACTIVITY EFFECTS OF VOID IN ITC'S CONTAINING MIXTURES OF BERYLLIUM AND WATER

Calculations were made of the reactivity affect of partial voidings of ITC's which contain mixtures of beryllium and water. The purpose of these computations was to determine whether replacement of some of the ITC water with beryllium would greatly reduce the positive reactivity affect of massive voidings of ITC water without a large reduction in the magnitude of the absolute peak flux in the ITC.

From detailed analyses with 810/810 and 1215/2000 cores, it was found that the reactivity effects of partial voidings of ITC water were insensitive to the fuel loading in these "black" cores. Also, whereas the magnitude of this positive void coefficient is reduced as the volume fraction of beryllium in the ITC is increased, the calculated coefficient was insensitive to the radial distribution of the beryllium in the ITC, at least up to a beryllium volume fraction of 0.3. This was shown by calculations with: (1) rings of beryllium, displacing ITC water at the core-ITC interface; (2) homogeneous mixtures of beryllium and water in the ITC; and (3) cylinders of beryllium displacing water at the axis of the ITC.

The maximum reactivity gain due to uniform, partial voidings of the ITC water was calculated to be $\sim 1.0\%$, in the existing Criticality Facility, with no beryllium in the ITC. Additional computations were made, for a modified system, with an ITC of larger radius (7.3 cm versus 5.9 cm); the maximum reactivity gain increased to 1.8%. On the basis of measurements in the vicinity of zero ITC void, in the Criticality Facility, it is believed that these values may be gross underestimates of the actual reactivity gains of voidings of water in the ITC.

An experiment is being planned for the existing ITC, to measure the reactivity effect of massive, uniform voidings in the ITC, ranging up to $\sim 70\%$ of the ITC volume. Concurrently, much more detailed analysis will be made for this reactivity coefficient, including two-dimensional computations and/or S_n -type computations with linearly anisotropic scattering by hydrogen.

These calculations and experiments are exploratory in nature; no design change from a 100%-water ITC (with samples) is planned at this time. The preliminary analyses described above indicate that the use of $\sim 35\%$ beryllium in the larger ITC caused a reduction in the calculated maximum reactivity effect of voiding of ITC water from the 1.8% down to 1.1%, at the ex-

pense of a 7% reduction in the magnitude of the peak flux in the ITC. In these calculations, a 1.4-cm thick ring of beryllium displaced ITC water at the periphery of the 7.3-cm-radius ITC.

REFERENCES

1. G. E. Hansen and W. H. Roach, "Six and Sixteen Group Cross Sections for Fast and Intermediate Critical Assemblies," LAMS-2543 (December 1961).
2. G. D. Joanou and J. S. Dudek, "GAM-I: A Consistent P_1 Multigroup Code for the Calculation of Fast Neutron Spectra and Multigroup Constants," GA-1850 (June 1961).
3. P. Bakshi, "Equivalence Principles and Resonance Integrals," BNL-4381 (September 1959).
4. H. C. Honeck, "THERMOS, A Thermalization Transport Theory Code for Reactor Lattice Calculations" BNL-5826 (September 1961).

II-9. Argonne Advanced Research Reactor Calculations—Comparison of Analog and Digital Transient Temperature Calculations

W. P. STEPHANY

INTRODUCTION AND DEFINITION OF THE PROBLEM

To satisfy the requirements of the Preliminary Safety Analysis Report of the AARR project, an analog computer was used to study the dynamic thermal responses of the proposed AARR core to various perturbing influences. Preference was given to an analog rather than to a digital code because of apparent advantages in programming effort and output presentation. However, the present analog facility imposed a limit on the number of spatial node points to be specified for the problem. Because of this restriction, the ARGUS digital code, which is capable of providing a detailed spatial mesh for the thermal model, was used to check the results of the analog calculations.¹

The analog calculations were designed to determine whether certain hypothesized power excursions initiated from the 100 MW steady state level could be effectively controlled by the reference safety system before significant core damage occurred. Comparison of analog results with ARGUS results indicated that the differences are within expected bounds.

SIGNIFICANT DIFFERENCES BETWEEN THE ANALOG AND ARGUS MODELS

GEOMETRY AND SPATIAL MESH

A typical thermal cell was defined for the AARR flat-plate core matrix, consisting of a central fuel zone, cladding, and an outer coolant zone. The thickness of each zone was chosen such that the results of this single calculation would represent similar sections throughout the core. Thus, the fuel zone thickness was specified as one-half the actual fuel thickness, the cladding zone identical to the actual cladding and the coolant zone as one-half the actual coolant channel. This typical flat-plate cell was used for the analog model but for the ARGUS

model the cell was represented by a series of concentric annuli with a central dummy cylinder having a radius of 5 cm, large enough to simulate the flat-plate cell in the circumferential annuli.

The analog model divided the typical cell into twelve rectangular sections, four axially and three radially. The axial division was made to accommodate the axial power profiles, the radial division gave representation of the fuel, cladding, and coolant zones. One node point gave representative temperatures for each section. In addition, there was one node point at the boundary of each axial section in the coolant zone including inlet and outlet positions. Hence, there were 17 node points.

The ARGUS model divided the typical cell into thirty annular sections, ten axially and three radially. The program requires that axial sections be of equal length. Radially, the division was made to represent the three distinct materials, as in the analog model. There was a radial row of node points at the boundary of each axial section, as well as at each extremity of the cell. Each radial row consisted of nine node points: five equally spaced in the fuel zone, three equally spaced in the cladding zone, and one in the coolant representing bulk coolant temperature. Thus, the model used an 11×9 nodal matrix.

POWER GENERATING FUNCTION

For a specified initial rate of reactivity increase, the analog model used space independent neutron kinetics equations with six delayed neutron groups to calculate the changes in the power generating function based on the net reactivity worth including temperature changes in the core. ARGUS cannot do this, since it uses only a prespecified power generating function without a feedback loop to correct for the variation in reactivity with reactor power. Therefore, the time-dependent power

curves obtained as analog output were used for ARGUS input.

AXIAL POWER DISTRIBUTION

Two axial power profiles were used. A "chopped cosine" form represented the axial profile in those sections of the core where the flux is not influenced by the control blades. A step form was used to approximate the skewing effect due to the control blades, specifying a power level 147% higher in the lower three-tenths of the core than in the upper seven-tenths.

ARGUS evaluates the $\partial T/\partial Z$ term of the heat conduction equation for the coolant by a linear finite difference approximation, using the coolant temperatures at the extremities of two adjacent axial sections to calculate the coolant temperature at the intermediate boundary. This technique works well for smooth axial profiles. However, the sharp discontinuity of the step form caused error. For the equilibrium case at the 100 MW level, ARGUS results were compared with an analytic solution. For the transient case, an energy balance was made between coolant temperature rise and the heat flux from the cladding surface. These comparisons showed the error to be insignificant.

FUEL TEMPERATURE DISTRIBUTION AND HEAT FLUX

The more detailed fuel temperature results of the ARGUS calculations pointed out that the analog used the centerline fuel temperature as the single temperature associated with an axial fuel section. The clad temperature, used by the analog to calculate the heat flux from the clad to the coolant, was based on the heat flux from the fuel to the clad. The fuel temperature distribution calculated by ARGUS showed a steeper gradient at the outer edge of the fuel compared with that

determined by the analog, the latter being based on the linear difference between the temperature of the fuel and cladding. Thus, use of the fuel centerline temperature in the analog better approximated the heat flux than use of an average temperature for each fuel section. ARGUS, however, gave higher values for the heat flux from the clad surface due to generally higher clad surface temperatures caused by the steeper gradient near the outer edge of the fuel.

GAP CONDUCTIVITY

The analog model assumed an infinite gap conductivity at the fuel-clad interface. ARGUS cannot employ this assumption, since inaccuracies due to round-off errors will occur for a gap conductivity greater than 56 W/cm²·°C. Although the ARGUS problems used 56 W/cm²·°C for the gap conductivity, the analog's assumption was effectively duplicated by subtracting the interface temperature drops from the values of fuel temperature.

CONCLUSIONS

The generally good agreement between the results of the two approaches showed the analog model was capable of the accuracy required for the preliminary safety calculations. Also, this application illustrated the capability of the ARGUS code to describe a system ostensibly beyond the range for which it was designed.

REFERENCES

1. D. F. Schoeberle, J. Heestand and L. B. Miller, *A Method of Calculating Transient Temperatures in a Multiregion, Axisymmetric, Cylindrical Configuration. The ARGUS Program, 1089/RE248, written in FORTRAN II, ANL-6654* (1963).
2. Op. cit., p. 36.

II-10. Critical Experiments for AARR Design

J. DAUGHTRY, T. JOHNSON, F. KODRICK, K. PLUMLEE, W. ROBINSON and G. STANFORD

DESCRIPTION OF ASSEMBLIES 1, 2a, 2b, AND 3

The Argonne Advanced Research Reactor (AARR) is intended to provide a small central irradiation facility operating at approximately 10^{16} thermal flux intensity, and several additional peripheral thimbles for beam extraction, sample irradiation, and high flux experiments. The critical experiment now in progress in the ZPR-5 facility provides experimental measurements of the performance obtainable from precise full-scale assem-

blies which duplicate many of the design features being studied. Numerous adjustments and variations are evaluated to assist in optimization of location, size, and composition of the components. These include fuel density, control-safety blades, reflector composition, beam tubes, and nuclear instruments for reactor operation.

The reactor includes a central hexagonal-shaped H₂O-filled internal thermal column (ITC), as shown in Fig. II-10-1. The critical experiment is limited to operating

I DRY THIMBLES FOR FLUX DETECTORS
(ROMAN NUMERAL INDICATES
CHANNEL NUMBER)

FIG. II-10-1. Top View of AARR Critical Core. ANL Neg. No. 112-5435.

levels less than a few hundred watts because of the lack of shielding or cooling, and it produces a measured flux of up to 10^{10} thermal neutrons/cm²-sec in the ITC. Proportionality indicates that a power level of 220–300 MW would be sufficient to produce a flux level of 10^{16} thermal neutrons/cm²-sec.

The fuel region is also hexagonal in shape. Its inner boundary is formed by the 0.16 cm thick by 6 cm wide sides of the ITC liner. The outer fuel boundary is the 0.64 cm thick by 25 cm wide slots for the six safety blades as shown in Fig. II-10-1. Fuel height is 46 cm. There are 1,215 fuel positions having exchangeable con-

tents. Normally the metal-to-water ratio is unity, with 0.1-cm thick metal plate assemblies being separated by 0.1-cm thick coolant channels. A plate assembly may include 0.1-cm thickness of stainless steel, or alternatively one or more 0.025-cm thick stainless steel-jacketed fuel foils, boron-stainless steel foils (1% natural boron), and 0.025, 0.050, or 0.075-cm thickness of stainless steel. Twenty-seven parallel plates are present in each subassembly and three subassemblies provide a uniformly spaced 81-plate array extending from a face of the ITC to the periphery of the core, for example.

The basic fuel foil is $5.7 \times 45.7 \times 0.01$ cm thick

highly enriched uranium metal, weighing about 54.8 g. Thinner pieces are provided for graduated loadings in regions having large gradients of neutron flux intensity. This permits experimental power flattening and simulation of non-uniform burnout. The AARR operating level will probably be limited by hot spots which may be eased by power flattening. Each fuel foil has an envelope of 0.007-cm (or thicker) stainless steel, with over-all thickness of approximately 0.025 cm. One edge of each envelope was formed by folding and the other three were sealed by electron beam welding.

There is little experimental information available for cores of the high fuel loading density involved in AARR, consequently the first loading (Assembly No. 1) used 17.2 kg of highly enriched uranium (315 fuel foils, one in each fourth available position). Assemblies Nos. 2a and 2b included 33.7 kg of fuel plus one atom of B-10 per 32 atoms of U-235 (615 fuel foils and 420 boron-stainless steel foils). The third assembly included 44.4 kg of uranium and had an atom ratio of B-10/U-235 of 1/21.9 (810 foils each of fuel and boron-stainless steel). The stepwise increase in loading helped to determine fuel requirements, excess reactivity, poison, and control blade worth.

Excess reactivity of the first assembly was 0.4 %. That of Assembly No. 2a was 3.6 %; however, there was a subsequent removal of all Plexiglas from the radial reflector (90 % beryllium and 10 % Plexiglas in alternating layers, 10 cm and 1.1 cm thick, respectively). Excess reactivity was increased to 4.8 % by the removal of Plexiglas and the assembly was designated as Assembly No. 2b thereafter. For the third assembly the excess reactivity was 2.7 % initially (with a tightly packed 100 % beryllium reflector).

Subsequent installation of a 10 × 10 cm(o.d.) empty aluminum through-tube and two radial beam tubes, (Fig. II-10-2), which resemble typical design penetrations of the beryllium reflector, reduced reactivity by 1.3 %. An additional loss of 0.2 % resulted from flooding all three with water; however, individual measurements showed that no loss accompanied flooding of the No. 2 radial tube. Installation of beryllium fillers to replace as much as was mechanically feasible of the beryllium displaced by the beam tubes raised the excess reactivity to 2.5 %. Four-fifths of the beryllium was replaced and five-sixths of the reactivity loss was recovered. Reactivity of the aluminum present in the tube walls will be measured by varying the aluminum content since that

FIG. II-10-2. Position of Beam Tubes and Be Fillers. ANL Neg. No. 112-5748.

effect was too small to isolate accurately from the preceding measurements. Such measurements are needed as a basis for estimating reactivity losses from design penetrations into the AARR reflector, assessment of operating variables, conjectured mishaps, etc., which could affect the reactor.

Twelve 0.39-cm thick hafnium control/safety blades (attached to 0.012-cm thickness of stainless steel) are in use. The positions are indicated in Fig. II-10-1. Six cut off the beryllium reflector from the fuel region and six operate inside the fuel region. Worths of the blades varied somewhat with orientation of the adjacent fuel foils (parallel or edge-on), and with fuel loading density. Comparison of the second and the first assemblies indicated a fractional reduction of approximately 0.1 in each blade worth. A further reduction of 0.05–0.10 was noted with the third core assembly. For that assembly the integral worths ranged from 1.25% to 1.45% in reactivity for unshadowed blades. Blade worth increased significantly with reactor temperature and tended to reinforce the negative temperature coefficient.

FIG. II-10-3. Manganese Radial Activation Traverses. ANL Neg. No. 112-5718.

FIG. II-10-4. Manganese Radial Activation Traverse (Assembly No. 2a). ANL Neg. No. 112-5719.

Because of anticipated rapid attrition of absorber material at the neutron flux levels of AARR, comparisons of several materials were made for evaluation as control elements or burnable poison. This was necessary because little prior experimental information was available for the epithermal neutron energy fluxes present in AARR.

ACTIVATION DISTRIBUTIONS, POWER, AND CADMIUM RATIOS IN AARR CRITICAL ASSEMBLIES

RADIAL ACTIVATION DISTRIBUTIONS

Figures II-10-3 through II-10-6 show the radial activation distributions for manganese foils in the three core assemblies. Radial traverses were taken at a level 3.2 cm below the core midplane. Foils were located between fuel plates at the locations shown in Fig. II-10-7. Foils in the internal thermal column (ITC) and in the beryllium reflector were supported by Plexiglas holders. The reactor was controlled by adjusting the water level with all control blades fully withdrawn. With Assembly No. 3 vertical traverses at positions shown in Fig. II-10-7 were made using manganese foils. Reactor operation was by water level control at approximately 10 cm below the top of the fuel. These traverses indicated that

the peak activity occurred at the same level of approximately 3 cm below the core midplane for all four traverse positions.

As the loading density increased the principal changes were:

(1) A noticeably lessened thermal-neutron activation of foils in the fuel region relative to those in the ITC and in the radial reflector.

(2) A decrease of the cadmium ratio in the fuel region. Ratios in the ITC and in the radial reflector remained unchanged.

(3) A readily apparent dip in the distribution of the manganese activity in the core near the edge of the fuel region.

During the experiments with Assembly No. 2, the original radial reflector of 90% beryllium and 10% Plexiglas by volume was modified to give an all-beryllium reflector. The change was made to indicate the effects of prospective changes in coolant content for the design reflector. Mechanical considerations made it necessary to shift the traverse positions about an inch laterally because of the modification of the reflector.

FIG. II-10-5. Manganese Radial Activation Traverses—Assembly No. 2b. (100% Beryllium Reflector). ANL Neg. No. 112-5720.

TABLE II-10-I. PEAK-TO-AVERAGE MIDPLANE ACTIVATION RATIOS

Assembly No.	Detector	Ratio of Peak Flux to Flux at Radius of 15 cm
1	Mn	18
2a	Mn	28
2b	Mn	27
3	Mn	34
2b	U-Al	39
3	U-Al	55

New foil holders were made of teflon to avoid the hydrogen content of the original Plexiglas holders. Comparison of traverses from Assembly No. 2b (Fig. II-10-5) with Assembly No. 2a (Fig. II-10-4) shows increased activity in the outer portion of the core. In the 100% beryllium reflector, the peak relative activity was lowered by approximately 10%, the region of peak activity was broadened, and the activity dropped off much less rapidly with increasing radius. Flux at the outer perimeter of the beryllium was roughly doubled, and the over-all average was increased.

Uranium-aluminum radial activation (U-235 fission) traverses were also taken in the second and third loadings. Table II-10-I summarizes the ITC-to-fuel midplane activation ratios for the various loadings.

VERTICAL ACTIVATION DISTRIBUTIONS

Figures II-10-7 and II-10-8 show the positions and relative activities of vertical uranium-aluminum (U-235 fission) traverses in a typical loading. The foils were supported by Plexiglas in the ITC, by aluminum strips inserted between fuel plates in the fuel region, and by a dummy aluminum safety blade on the core periphery. During irradiations the reactor was controlled by the blades farthest removed from the traverse locations; a full top water reflector (approximately 37 cm) was present. In Assembly No. 3 a manganese *radial* traverse was irradiated with the above operating conditions. Results indicated that while the activations were depressed near the controlling blades, no perturbation occurred in the region of the normal vertical traverses.

POWER CALIBRATION

In the first assembly, a determination of the absolute thermal flux in the fuel region was made by irradiating two gold foils at a fairly high operating power for the experimental loading (approximately 250 W). From absolute gamma counting of the irradiated foils, corrected for self shielding and flux depression, it was deduced that the midplane thermal flux at a distance of approximately 15 cm from the center of the ITC was approximately 3.5×10^8 n/cm²-sec. By proportion with the subcadmium activation levels of Fig. II-10-3, the

FIG. II-10-6. Manganese Radial Activation Traverses—Assembly No. 3 (100% Beryllium Reflector). ANL Neg. No. 112-5747.

thermal flux in the ITC was estimated to be approximately 10^{10} .

In the third assembly, a run was made with fully enriched U-235 metal foils in the fuel region and in the ITC, plus two gold foils in the ITC. The fission rates in the uranium foils were determined radiochemically by Mo-99 separation. The absolute activations of the gold foils were determined by two independent methods: by counting in a calibrated system, and by comparison with a gold foil irradiated in a standard pile; the two resulting activations agreed to within 1.5%. This experiment gave the following results:

Thermal flux at the center of the ITC:

$$2.0 \times 10^9 n/cm^2\text{-sec} \pm 6\%$$

Total power in the core: $53 \text{ W} \pm 10\%$

The total power was obtained by numerical integration, using data from radial and vertical U-235 fission-product traverses. The assigned errors represent estimates of the over-all precision of the measurements.

CADMIUM RATIOS

Cadmium ratios were measured with foils of U-235 (fission), dysprosium, indium, gold, manganese, and copper. For irradiation, the foils were contained in 20-mil thick capsules of aluminum or cadmium. The capsules containing the foils were taped in holes cut in 40-mil thick by $\frac{1}{2}$ -in. wide aluminum strips. Each strip

FIG. II-10-7. Typical Foil Placement for Radial and Vertical Activation Traverses. ANL Neg. No. 112-5724.

contained two foils, one aluminum-covered and one cadmium-covered, which, when the strip was inserted into one of the 40-mil water channels between the fuel-plate positions, were equidistant from the core midplane. To preserve vertical symmetry about the core midplane, a full top reflector was used and the reactor was controlled with blades far removed from the foils.

The data obtained so far are summarized in Table II-10-II and are plotted in Figs. II-1-1 through II-1-7 of Paper II-1. The errors shown are indicative of the scatter observed for several typical measurements in the fuel zone.

TEMPERATURE COEFFICIENTS AND VOID WORTH OF H₂O MODERATOR

Temperature reactivity coefficients and H₂O void worths are of interest in design studies and in safety analyses of AARR. Assessment of the experimental measurements indicates that the uniformly heated reactor loses less reactivity than would be expected from

water expansion alone as the temperature rises; however the temperature coefficient is negative nevertheless. The measurements were made in Assemblies Nos. 2a and 3.

The coefficient was measured over a range of approximately 21°C to 76°C. Two methods were utilized: (1) the change in the critical position of a calibrated control blade versus the change in moderator temperature with moderator level well above the core; and (2) the change in critical water level versus the change in moderator temperature with all control blades withdrawn. For the second method the differential water worth was obtained from positive period measurements.

Preliminary results obtained by the two methods with the second loading did not agree. Later measurements with the third loading showed that this disagreement could be resolved within the limits of experimental accuracy by considering the change in the control blade calibration with temperature. Furthermore, in comparing the results obtained by the two methods, the difference in core configuration should be noted. In the

FIG. II-10-8. U-235 Fission Vertical Activation Traverses in the 810 Fuel Foil System. ANL Neg. No. 112-5749.

TABLE II-10-II. CADMIUM RATIOS IN AARR CORES

Assembly No.	U-235 (fission)	Dy	In (1.2 mil)	Au (1.1 mil)	Mn	Cu
1	3.36 ± 0.10	10.8 ± 0.6	1.30 ± 0.02	1.24 ± 0.02	2.38 ± 0.06	2.11 ± 0.02
2	2.00 ± 0.04	5.87 ± 0.20	1.185 ± 0.020	1.131 ± 0.006	1.59 ± 0.02	1.54 ± 0.02
3		4.36 ± 0.15			1.37 ± 0.01	1.29 ± 0.03

first method the water level was approximately 37 cm above the fuel region, one or two control blades were fully inserted and another blade was partially inserted. In the second method, all control blades were fully withdrawn and the water level was approximately 10 cm below the top of the fuel.

A control blade located in the fuel region was calibrated for the full length of its travel at the extremities of the temperature range with Assembly No. 3. At the higher temperature the blade worth was approximately 3.3 % greater, relatively, than at room temperature. The differential water worth did not appear to change with temperature and had a value of approximately $(0.14 \pm$

$0.02) \% (\Delta k/k)/\text{cm}$ of water in both assemblies. The temperature coefficient of reactivity was found to be $(-0.0020 \pm 0.0005) \% (\Delta k/k)/^\circ\text{C}$ for both cases.

For all void worth measurements the voids were simulated by the insertion of teflon to displace water. This procedure was justified on the basis of a study of the reactivity equivalence of teflon and air made during the experimental program with the AHFR Critical Assembly.¹

In the fuel region, 0.081-cm thick by 1.26-cm wide teflon strips were inserted between fuel plates. The strips extended at least 2 cm beyond the fuel at the top and bottom of the core. The void fraction was calculated

on the basis of the cross-sectional area of the inserted teflon and the cross-sectional area of the water in the core. The total amounted to about one-half per cent of the water volume in the fuel region. The teflon strips were distributed in such a manner that the resulting void worth measurement would represent an approximate average over the fuel region. With the reactor critical the teflon strips were withdrawn and the rising periods were recorded.

In the third core the void worth was measured at four radial positions within the fuel region, using the methods described above.

For the measurements in the ITC five teflon rods, 1.27-cm in diameter and 60 cm long, were supported by Plexiglas brackets in the positions shown in Fig. II-10-9. The worth of each rod was determined by withdrawing the rod from the ITC and observing the change in the critical position of a calibrated control blade. Here again the void fraction was calculated on the basis of relative cross-sectional areas. The void worth in the ITC was

FIG. II-10-9. Positions of Teflon Rods in ITC. ANL Neg. No. 112-5721.

TABLE II-10-III. AVERAGE VOID WORTH IN AARR CRITICAL ASSEMBLIES^a

As- sembly No.	No. of Fuel Foils	Void Worth, $\%(\Delta k/k)/\% \text{ Void}$		
		Fuel region	ITC	Total
1	315	-0.159 ± 0.010	—	—
2	615	-0.228 ± 0.010	$+0.045 \pm 0.001$	-0.18 ± 0.01
3	810	-0.190 ± 0.010	$+0.045 \pm 0.001$	-0.14 ± 0.01

^a Cross-sectional areas used in the calculation of void fractions: area of ITC = 100 cm², area of water in the fuel region = 749 cm²; and area of one teflon rod = 1.27 cm².

TABLE II-10-IV. RADIAL DEPENDENCE OF VOID WORTH IN AARR CRITICAL ASSEMBLY—ASSEMBLY 3

	Radius, cm	Void Worth, $\%(\Delta k/k)/\% \text{ Void}$
ITC	0.00	0.40 ± 0.01
	1.59	0.40 ± 0.01
	3.18	0.41 ± 0.01
	4.58	0.41 ± 0.01
	5.33	0.26 ± 0.01
Fuel Region	6.67	-0.16 ± 0.01
	10.24	-0.24 ± 0.01
	14.85	-0.25 ± 0.01
	20.32	-0.16 ± 0.01

the same for Assembly No. 2 and Assembly No. 3 within the accuracy of the measurements.

Table II-10-III summarizes the results obtained for the average void worth in the AARR critical assemblies. In recording the void worth the basis for computing the void fraction should be clearly defined. In Table II-10-III, for the void worth given for the fuel region, the void fraction was computed in terms of the total volume of water in the fuel region and this indicates the loss which should result from boiling. For the ITC the void fraction was computed in terms of the total volume of water in the ITC. Thus, the two results may be added directly to give a value for the void worth that is representative of the total assembly. This has been given in the last column of Table II-10-III. The loss of H₂O from density decrease during temperature coefficient tests indicated larger negative coefficients than were measured at elevated temperatures.

The measured value of the void worth in the fuel region is less in Assembly No. 3 than in Assembly No. 2a. This result was not expected, but may be related to the absence of the 10% Plexiglas component from the beryllium reflector after the Assembly No. 2a experiments.

Table II-10-IV gives the void worth in Assembly No. 3 as a function of the radial distance from the center of the ITC. These data have been plotted in Fig. II-10-10. The void fractions in this case were computed in terms of the total water volume, ITC plus fuel region. Figure II-10-10 shows that the void worth is positive and uniform throughout most of the ITC. This affects the design since failure of sample coolant flow during AARR operation might result in voiding the ITC sufficiently, if improperly designed, to increase reactivity by a dangerous amount. The positive coefficient falls off somewhat near the edge of the ITC and becomes negative near the inner fuel region boundary. It is fairly uniform

FIG. II-10-10. Radial Dependence of Void Worth-Assembly 3. ANL Neg. No. 112-5727.

in the fuel region and becomes less negative near the beryllium reflector.

The height of the boxes in Fig. II-10-10 represents the estimated uncertainty in the measurements. The width of the boxes represents the radial extent of the teflon in each measurement.

All void worth measurements were made with full top water reflector and a water temperature of about 23°C.

REACTIVITY COEFFICIENTS OF MATERIALS IN AARR CRITICAL LOADINGS

Measurements have been made to determine the reactivity worth of the following materials in the fuel region of each of the first three AARR loadings:

Boral (B_4C -Al)	hafnium
boron-stainless steel	stainless steel
cadmium	U-235-aluminum
europium oxide-stainless steel	water
gadolinium oxide-stainless steel	

The stainless steel, U-235, and water data are of interest since these materials are major constituents in the design of the AARR core and in the critical assembly. Europium oxide is under consideration as a possible control blade material, although the critical assembly

control blades are hafnium. The boron-stainless steel is being used in the critical assembly to control the amount of excess reactivity of the core. Boral, cadmium, and gadolinium have been included for comparison as control materials and poisons.

Measurements were made for samples of different thicknesses and with the samples at different radial locations within the fuel region. However, all measurements were made with the samples at the vertical midplane of the core.

The samples were inserted into the core by means of a dummy control blade-sample holder in one of the three radial control blade slots. The sample holder had the same dimensions as the AARR radial control blades and was made from 2S aluminum. The maximum sample thickness was limited to the thickness of the sample holder, 0.51 cm. The sample positions were in the lower portion of the sample holder so that during reactor startup the samples were below the core. They could then be raised to the midplane of the core with the standard control blade drive mechanism. Three sample holes, 2.18×2.18 cm, were in a horizontal row across the width of the sample holder. The centers of these sample positions were at radii of 9.5 cm, 15.7 cm, and 21.9 cm. A later modification of the sample holder provided larger sample holes for the purpose of investigat-

TABLE II-10-V. COMPARISON OF REACTIVITY WORTHS IN THE FUEL REGION OF AARR CRITICAL ASSEMBLY CORES

Sample Description	Sample Thickness, in.	Sample Weight, g	Sample Worth, $10^{-4} \Delta k/k$		
			Assembly 1	Assembly 2a	Assembly 3
Boral (B_4C -aluminum) (22.2% B by wt)	0.200	17.859	-12.07	-10.15	-8.45
Boron-stainless steel (1.01% B by wt)	0.200	53.955	-5.71	—	-3.72
	0.100	26.978	—	-2.88	-1.91
	0.050	13.489	—	-1.46	-1.15
Cadmium	0.200	62.703	-7.53	-5.70	-3.89
	0.100	31.352	-6.18	-4.47	-3.15
	0.050	15.676	-5.49	-3.56	-2.78
	0.020	6.270	-4.71	-2.88	—
Europium oxide-stainless steel (31% Eu by wt)	0.156	43.299	-12.18	-10.65	-8.63
Gadolinium oxide-stainless steel (0.7% Gd by wt)	0.200	56.052	-3.58	-2.34	-1.54
Hafnium-zirconium (97.5% Hf by wt)	0.200	92.274	—	-9.97	-8.00
	0.119	55.132	-8.78	-7.89	-6.37
	0.100	46.137	—	-7.15	-5.83
Stainless steel (Type 304)	0.200	56.868	-0.76	-0.53	-0.46
Uranium-aluminum (17.44% U by wt, 93.17% U-235)	0.182	19.914	+1.46	—	+0.42
	0.078	8.535	—	+0.36	+0.23
Water	0.200	21.657	+0.76	+0.68	+0.51

ing edge effects. The larger holes were at the same radial distances from the center of the reactor. Sample holes that were not in use were always filled with aluminum blanks. When a sample thinner than 0.51 cm was used, the remaining volume of the sample hole was filled with an aluminum spacer to bring the total thickness to 0.51 cm.

The worth of each sample was obtained by observing either a rising period or a change in position of a calibrated control blade when the sample was removed from the midplane of the core to a position some distance below the core. This movement of the sample holder was equivalent to replacing the sample by 0.51-cm thick aluminum. Thus the measured reactivity values were relative to a 0.51-cm thick aluminum sample in the same position.

Table II-10-V is a comparison of results obtained in

the three assemblies. The measurements for Assembly No. 2a were made prior to the removal of the Plexiglas from the reflector. The tabulated reactivity values were obtained with identical samples in each of the three sample positions. Additional measurements have shown that the reactivity worth with more than one sample in the core is equal to the sum of the worths of the individual samples. This indicates that the samples are spaced sufficiently far apart so that there is very little interaction. The estimated error in the tabulated values is $0.1 \times 10^{-4}(\Delta k/k)$; however, this does not include any error that may be present in the conversion from period to reactivity.

Figure II-10-11 shows the relative worth of cadmium as a function of sample thickness for the three assemblies. The data have been plotted in terms of worth per

FIG. II-10-11. Cadmium Worth Versus Sample Thickness. ANL Neg. No. 112-5714.

FIG. II-10-12. Hafnium Worth Versus Sample Thickness. ANL Neg. No. 112-5715.

FIG. II-10-13. Rossi-Alpha Data for AARR Critical Assembly, Assembly 1. ANL Neg. No. 112-5713.

TABLE II-10-VI. ROSSI-ALPHA AND PROMPT NEUTRON LIFETIME IN AARR CRITICAL EXPERIMENTS

Assembly No.	Rossi-Alpha, sec^{-1}	ℓ , μsec
1	187.1 ± 4.2	38.8
2a	273.7 ± 7.3	26.5
3	247.3 ± 3.5	29.4

unit surface area. Figure II-10-12 is a similar presentation of the hafnium data.

PROMPT NEUTRON LIFETIME OF CRITICAL LOADINGS BY ROSSI-ALPHA MEASUREMENTS

Prompt neutron lifetimes in each loading of the AARR critical assembly have been determined by the Rossi-alpha method. The theory of this method^{2,3} shows that for a multiplying system near critical, the prompt neutron time behavior is proportional to $e^{-\alpha t}$, where $\alpha = (1 - k_p)/\ell$; k_p is the prompt multiplication factor and ℓ is the prompt neutron lifetime. At delayed critical $1 - k_p = \beta$ and α is then equal to β/ℓ . The decay constant, α , is obtained by observing the time distribution of pulses from a neutron detector. The exponential behavior of the neutron counting rate after an initial neutron pulse is a result of the time correlation of

events in the same neutron chain. Neutrons from separate chains or from extraneous sources are uncorrelated and produce a constant background on which the exponential decay is superimposed.

Neutrons were detected by a single fission chamber placed above the internal thermal column and fuel zone of the AARR Critical Assembly. The chamber and its preamplifier were sealed in a polyethylene jacket and secured to the top spacer plate, since they were below the water level during reactor operation. The pulses from the detector were transmitted to an ANL model TA-26 time analyzer and a 400-channel pulse height analyzer. The time analyzer channel width is variable over a wide range; however, for these measurements 40 μsec and 80 μsec channel widths were used.

The reactor was operated at a very low power level. A neutron counting rate of about 2000 cpm was found to be optimum. At higher counting rates the ratio of uncorrelated to correlated counts was too great. At lower operating levels reactor control became increasingly difficult, and the time required to obtain satisfactory statistics was prohibitive. Data were normally accumulated for three to five hours.

Data obtained in the runs were corrected for deadtime losses and then analyzed to obtain the exponential decay constants. Figure II-10-13 shows the results of a typical run. Each experimental point represents the sum of four

time analyzer channels. The solid curve is the result of a weighted least-squares fit of the data to the expression

$$R(t) = A_0 + A_1 e^{-\alpha_1 t} + A_2 e^{-\alpha_2 t}.$$

The counting rate, R , is made up of a background, A_0 , of uncorrelated counts, a slowly decaying fundamental mode, and a rapidly decaying portion due to higher order transients.⁴ The fundamental mode decay constant, designated here as α_1 , is the Rossi-alpha. The least-squares analysis was done using the variable metric method for minimization.⁵

Values obtained for the Rossi-alpha and for the prompt neutron lifetime are presented in Table II-10-VI. The errors shown are standard deviations obtained from the least-squares analysis. To obtain the prompt neutron lifetime, ℓ , a value of 0.00726 was used for β .⁶

The fact that the measured value of the prompt

neutron lifetime in Assembly No. 3 is greater than that in Assembly No. 2a was unexpected and as yet is unexplained.

REFERENCES

1. J. W. L. De Villiers (Ed.), *Critical Experiment for the Preliminary Design of the Argonne High Flux Reactor*, ANL-6357, 41 (1961).
2. F. de Hoffman, *Statistical Aspects of Pile Theory*, The Science and Engineering of Nuclear Power (Addison-Wesley Press, Inc., Cambridge, 1949), Vol. II, pp. 116-119.
3. J. D. Orndoff, *Prompt Neutron Periods of Metal Critical Assemblies*, Nucl. Sci. Eng. **2**, 450-454 (1957).
4. R. A. Karam, *Spatial Dependence of the Decay Rates of Prompt-Neutron Chains in Reflected Reactors*, Trans. Am. Nucl. Soc. **8**, No. 1, 224 (1965).
5. W. C. Davidon, *Variable Metric Method for Minimization*, ANL-5990 (Rev.), (1959).
6. D. H. Shaftman, Argonne National Laboratory (Private Communication).

II-11. Measurement of Gamma Heating in the Argonne Advanced Research Reactor Critical Assembly by Thermoluminescent Dosimetry

G. S. STANFORD, T. W. JOHNSON and W. R. ROBINSON

SUMMARY

The use of CaF_2 thermoluminescent dosimetry for determining gamma-ray heating in the reflector of a critical assembly is being investigated, and preliminary results indicate that it is basically a simple and reliable method. Results good to about $\pm 10\%$ have been obtained, and it is expected that improvements in technique can decrease the error limits. An additional uncertainty related to possible sensitivity to fast neutrons remains to be investigated.

INTRODUCTION

To predict accurately the required capacity of the cooling system for the reflector of a high-power reactor, it is necessary to know the amount of heat that must be removed from the various parts of the reflector at operating power. The reflector is heated partly by energy lost by neutrons in the course of slowing down, and partly by the slowing-down of electrons which have absorbed energy from the gamma-ray flux. Most of the gamma rays originate in the core, but there is a small component due to neutron capture in the reflector.

The cooling requirements for experimental equipment in the reflector, such as beam tubes and contents, must also be determined. In some cases—examples being copper, aluminum, and Hf-180—absorption of a

neutron leads to emission of a beta particle, providing an additional mechanism for the local generation of heat in the presence of a large neutron flux.

To determine the cooling requirements for the beryllium reflector of AARR, measurements of the gamma-ray heating in the critical assembly are being made. Since the critical assembly cannot operate much above 50 W, meaningful calorimetric measurements would be difficult. After considering various possible indirect methods for getting the desired information, it was concluded that thermoluminescent dosimetry (TLD) offered the most promise.*

TLD has several advantages. It can be used at critical assembly power levels, and it requires no temperature stabilization of the reactor. The measurement is almost as easy and flexible as a foil irradiation, although the subsequent readout tends to be more time-consuming. The phosphor used (CaF_2 powder, manganese-activated) is insensitive to thermal neutrons, and therefore can be used for accurate gamma-ray field measurements in the presence of a high thermal neutron flux. However, the thermoluminescent sensitivity to fast neutrons remains to be investigated.

* H. F. Reed, Reactor Physics Division, initially suggested this possibility.

GAMMA-RAY HEATING

Gamma rays transfer their energy to a medium by imparting kinetic energy to electrons, which in turn produce ionization and atomic and molecular excitation. The energy eventually appears as heat. Thus in principle the rate of gamma-ray heating could be determined either by measuring the temperature rise in an insulated portion of the medium, or by measurement of the rate of energy loss by the gamma-ray field. For the first method, some form of calorimetry is required. For the second, a detector can be used which responds to the presence of ionizing radiation, such as an ionization chamber, photographic emulsion, photoluminescent or thermoluminescent material.

Any of this latter group of detectors can be used so as to take advantage of the Bragg-Gray principle,^{1,4} which states, in brief, that the response of a detector is proportional to the rate of energy loss in the medium surrounding the detector, provided that the detector is thin enough that an ionizing electron will lose only a small portion of its initial energy in the detector. Thus the rate of gamma heating in any medium can be determined merely by surrounding the sensitive volume of the detector with the material being investigated. The wall thickness should exceed the range of the primary electrons. If the mass stopping power of the detecting material matches that of the medium, the detector need not be thin.

If the detector is an ionization chamber, an absolute determination of the energy being imparted by the gamma field can be found from a knowledge of the chamber current, volume, and pressure, and the average energy required to produce an ion pair in the filling gas. Detectors of the other types must be calibrated, usually by comparison with the response of an ionization chamber in a convenient flux (or by exposing in a flux previously standardized with an ionization chamber).

The Bragg-Gray consideration gives TLD an advantage over photoluminescence. Since a TLD phosphor is in the form of a powder, it can be exposed in thinner layers than the photoluminescent materials. The latter come usually as small glass rods, typically $1 \times 1 \times 6$ mm.

THERMOLUMINESCENT DOSIMETRY

When an ionizing particle passes through a thermoluminescent phosphor, some of the electrons of the phosphor become trapped in metastable excited states. When the phosphor is heated, the trapped electrons can rise to the conduction band, from which they fall to the ground state with the emission of light. A detailed discussion of thermoluminescent (and also photoluminescent) detectors, including an extensive bibliography, will be found in Ref. 5.

FIG. II-11-1. Thermoluminescent Glow Curve for 5-mg Sample of CaF_2 (Mn). Exposure: ~ 60 rads of Gamma Rays. ANL Neg. No. 112-5382.

The most commonly used TLD detectors are suitably activated LiF or CaF_2 , in powder form. LiF has some advantages over CaF_2 , but for the present application an unwanted neutron response seemed likely, even with LiF highly depleted in Li-6 . Consequently, it was decided to use CaF_2 exclusively. An investigation of its possible neutron sensitivity, due to the presence of manganese activant, will be discussed below.

The useful exposure range of the TLD materials is large. Doses ranging from the mrad region to 10^6 rad or more have been measured, and the response of both LiF and CaF_2 has been found to be linear over a large part of this range.

EXPERIMENTAL METHOD

For irradiation, sample sizes of 50–100 mg were used. Since aliquots of only about 5 mg were required for readout, multiple readouts were possible. Readout was accomplished with an apparatus⁶ assembled by J. Kastner.† The light intensity from the sample as it was being heated was plotted as a function of time by a strip-chart recorder, producing a “glow curve” like that shown in Fig. II-11-1.

The thermal emission from the heater components and sample holder approximates the broken background line drawn in Fig. II-11-1. The areas of the glow curves minus background were determined with a planimeter. The duration of heating was typically about 35 sec. The sample region was flushed with dry nitrogen for about one minute prior to heating, since with CaF_2 the presence of oxygen causes spurious thermoluminescence.⁵ The nitrogen flow was shut off just prior to heating, because it was found that the

† Radiological Physics Division.

cooling of the powder by the flowing gas broadened the glow curve.

At least two aliquots were read out from each sample. For different aliquots from the same irradiation sample, the reproducibility in the area per mg was about $\pm 7\%$. Efforts to improve the readout consistency are under way.

Calibration was accomplished by exposing samples of the phosphor, at about the same time that the reactor irradiation was made, in the calibrated Co-60 irradiation facility in the ANL Biology Division. Since the known quantity in this facility is rad/sec in a tissue-equivalent sample, the phosphor was enclosed in thick lucite. The calibration exposures were in the range of the reactor exposures, and the calibration samples were read out along with the reactor samples.

MEASUREMENTS IN THE REFLECTOR OF THE AARR CRITICAL ASSEMBLY

The AARR critical assembly is described elsewhere in this report (see Paper II-10). TLD measurements have been made with the first and third cores. Since the primary object of these measurements was to develop a technique for use in the fully loaded core, no effort was made to surround the phosphor probes with particular materials.

A "probe" consisted of about 50 mg of phosphor. For these irradiations the probes for a traverse through the beryllium reflector were contained in wells milled in lucite (first loading) or teflon (third loading) strips

FIG. II-11-2. Dose Rate in the Be Reflector of the AARR Critical Assembly. ANL Neg. No. 112-5363.

about 1 in. wide by $\frac{3}{16}$ in. thick. The strips fitted in slots near the midplane of the beryllium reflector. The arrangement was essentially the same as the one used for the neutron activation traverses, described elsewhere in this annual report (see Paper II-10).

An irradiation time of about 15 min at a reactor power of 20 W gave ample thermoluminescent response. Absolute power measurements, described elsewhere, were made at the same time so that the gamma-ray field in rad/sec could be determined per watt of reactor power. Results for the two loadings are shown in Fig. II-11-2.

INVESTIGATION OF NEUTRON SENSITIVITY

THERMAL NEUTRONS

To provide electron trapping suitable for TLD, CaF_2 must be doped with a small percentage of manganese. Manganese absorbs thermal neutrons with a cross section of some 13 b/atom, and subsequently emits a beta particle with energy ranging up to 2.9 MeV. These beta particles will cause some thermoluminescent activation of the phosphor, although if the layer of powder is thin, most of their energy will be lost, not in the powder, but in the material surrounding it. Some experiments are under way to determine whether this source of activation is appreciable in the neutron fluxes present in the AARR reflector.

It is intended that some of the phosphor will be adulterated by the addition of a known additional amount of manganese, in the form of a white manganese-containing powder (manganous tartrate). The relative response of adulterated and unadulterated phosphors to a pure gamma field and to gamma-rays plus neutrons will then be determined, permitting the response due to the original manganese to be calculated. A preliminary experiment of this nature has been performed, using manganous sulphate instead of the tartrate. Because of uncertainty as to the exact amount of water with the sulphate, and because the presence of water tends to cause spurious thermoluminescence, accurate quantitative results were not obtained. The indications are, however, that the response to neutrons is very small, if not negligible, for a thin layer of powder in a field where the ratio of neutron to gamma-ray flux is as high as 3×10^8 n/cm per rad of gamma rays (i.e. in the beryllium reflector of the AARR critical assembly).

Thus the gamma-ray heating results reported in this paper are subject to a possible small correction for thermal neutron sensitivity of the phosphor.

FAST NEUTRONS

In a collision with a fast neutron, a light nucleus can receive appreciable kinetic energy. In a head-on colli-

sion, for example, a 1-MeV neutron will transfer 190 keV to an F-19 nucleus. This process can be expected to produce a thermoluminescent response, and thus represents an important area of uncertainty in interpreting the measurements plotted in Fig. II-11-2.

An investigation of the magnitude of this effect is being started, using an approach suggested by R. Gold.⁷ The energy deposited in a medium by a gamma field of given intensity is a function of the atomic number of the medium. Using a pure gamma source, the response of a thin layer of phosphor surrounded by, say, carbon, can be determined relative to the response when the medium is lead. Then the same comparison in the presence of fast neutrons will give an indication of the effect of the neutrons, since the latter component of the thermoluminescent activation will not depend greatly on the nature of the medium immediately surrounding the phosphor.

As of this writing, the magnitude of this effect is not

known, so that the reported results are further subject to an unknown correction.

REFERENCES

1. G. S. Hurst, *Fast Neutron Physics, Part I*, (J. B. Merion and J. L. Fowler, Eds., Interscience Publishers, Inc., New York: 1960), Chapter IVG: *Fast Neutron Dosimetry*.
2. L. V. Spencer and F. H. Attix, *A Theory of Cavity Ionization*, *Radiation Res.* **3**, 239-254 (1955).
3. O. Glasser and J. Victoreen, *The Thimble Ionization Chamber*, *Radiology* **29**, 341-345 (1937).
4. H. Fricke and O. Glasser, *A Theoretical and Experimental Study of the Small Ionization Chamber*, *Am. J. Roentgenol. Radiation Therapy Nucl. Med.* **13**, 453-461 (1925).
5. F. H. Attix, *Present Status of Dosimetry by Radiophotoluminescence and Thermoluminescence Methods*, NRL Report 6145 (September 18, 1964).
6. J. Kastner and R. Hukkoo, *LiF Thermoluminescent Dosimetry for Beta Rays*, Abstract 85, *Radiation Res.* **25**, No. 1, 203 (1965). (Paper to be published.)
7. R. Gold, Argonne National Laboratory (Private Communication).

II-12. Onset of Oscillations in a Natural Circulation Boiling System

D. MILLER, K. C. JAIN,* M. PETRICK* and S. G. BANKOFF†

Natural-circulation boiling systems have considerable technological importance in such diverse applications as desalinization evaporators and the safety of nuclear reactor systems. As the power input is increased, these systems tend to show oscillatory behavior of flow, pressure, voids and temperature of increasing amplitude. The oscillations are almost periodic in nature. The prediction of their frequency and amplitude is a severe test of the description of two-phase flow.

The effects of geometry, subcooling and pressure on the inception and development of oscillatory behavior in a natural circulation boiling water loop were studied experimentally. The following range of parameters was covered by the study:

Heat-flux, Btu/hr-ft ² ,	0.2-3.1 × 10 ⁵
Pressure, psia,	200-1500
Inlet subcooling, °F,	0-62
Test section, length, in.,	48, 72, 96
i.d., in.,	0.364, 0.625, 0.8125
Riser length, in.	48, 60
i.d., in.	0.3125, 0.625, 1.049

The critical power density at which the amplitude of

the oscillations becomes large was determined by an arbitrary criterion, since the oscillation amplitude increases continuously with power. For large oscillations, periodic inlet flow reversals were found, which were large compared to the steady inlet flow velocities. These phenomena are emphasized by lower system pressure and by smaller diameters of the test section and riser. Other parameters did not affect these phenomena in a distinct fashion. The frequencies of the oscillations varied from 0.24 to 0.76 cycles per second and, in general, increased with increasing power as did the flow amplitudes.

Two theoretical models were investigated; one linear and one nonlinear. A linear analysis in the frequency domain, due to A. Jones,¹ showed good ability to predict the oscillation threshold and the frequency associated with the initial oscillations. The predictions of the threshold power were correct within ±5 % for over 60 % and within ±10 % for over 80 % of the tested cases.

The nonlinear model of S. Jahnberg² had to be modified to provide a satisfactory prediction of the steady inlet flow velocities. However, this modification and the original model failed to give reasonable predictions of the oscillation thresholds and frequencies. Neither

* Reactor Engineering Division.

† Northwestern University, Evanston, Illinois.

form of the model provided a satisfactory prediction of the amplitudes. It is thought that the one-dimensional formulation fails to take into account periodic vaporization of superheated liquid near the leading edge of the two-phase region, which acts as a periodic forcing function. The introduction of such a forcing function allows the computation of periodic bounded oscillations similar to those observed experimentally, and also explains qualitatively the observed trends with parameter variation.

The results of this investigation are reported in significantly greater detail elsewhere.³

REFERENCES

1. A. B. Jones, *Hydrodynamic Stability of a Boiling Channel*, KAPL-2170, (October 1961).
2. S. Jahnberg, *A One-Dimensional Model for Calculation of Non-Steady Two-Phase Flow*, EAES Symposium, Studsvik, Sweden, October 1963.
3. K. C. Jain, *Self-Sustained Hydrodynamic Oscillations in a Natural Circulation Two-Phase Flow Boiling Loop*, ANL-7073 (1965).

II-13. An Analytic Study of the Space Dependent Dynamics of Boiling Reactors

C. K. SANATHANAN

INTRODUCTION

The present trend in power plant design, emphasizing large boiling nuclear reactors with high power density, makes it necessary for the designer to have a more sophisticated understanding of the dynamics of the reactor.

The main difficulty in the analysis of a transient in boiling reactors is that the coolant density, and consequently the nuclear parameters, depend on both space and time. The space-time dependence of the coolant density is obtained by solving the conservation equations of mass, momentum and energy pertaining to the coolant flow through the heated channels in the reactor core. This implies that the space-time dependent conservation equations of coolant flow must be solved simultaneously with the space dependent neutron kinetics equations. The solution of these nonlinear partial differential equations has been the object of many investigations in the past.¹⁻⁸ Prior to this time the approach to theoretical solutions has been to use such simplifying assumptions as the separability of space-time functions, the homogeneity of coolant in two-phase flow, etc.

Recently, because of elaborate numerical methods,^{1, 2} the simplifying assumptions have been kept to a minimum, and there has been good agreement between predicted and experimentally observed results. Numerical solution is probably the only choice when one considers transients in which the reactor power undergoes large variations.

However, for the purposes of understanding the stability and the transient behavior of the system in the neighborhood of a known steady operating condi-

tion, one may resort to the small perturbation techniques yielding the system transfer function. Such an analysis carried out at several power levels would yield significant information about the system performance and stability. This information is of interest to the designer. The simplicity and straightforwardness of the transfer function method is particularly useful at the initial stages of design. It is emphasized that numerical methods are more suitable if one is interested in a detailed investigation and evaluation of a specific design.

In boiling reactors the only significant spatial dynamic effects are those associated with the coolant along the axial dimension of the reactor. Hence the partial differential equations are written with only two independent variables, distance z along the core axis and time t .

THE MODEL AND THE ASSUMPTIONS

Figure II-13-1 illustrates the model under consideration. It consists of a vertical reactor core with several coolant channels immersed in water:

The spatial variation of pressure is considered negligible in comparison with the operating pressure of the system and hence the fluid properties such as enthalpy, density, etc. are evaluated at the operating pressure. Further, during small transients the system pressure does not change in time.

The nuclear parameters are assumed to depend only upon the coolant density (or the void fraction) during small transients. That is, the variation of the nuclear properties of the core materials for small temperature variations in them is considered negligible.

FIG. II-13-1. Model of a Natural Circulation Boiling Water Reactor. ANL Neg. No. 112-4687.

The two phase flow in the coolant channels is assumed to be separated. The axial variation of the slip ratio is taken into account. However, it is assumed that the slip ratio does not vary with time during transients.⁹ In other words,

$$V_g^*(z, t) = S(z)V_f^*(z, t), \quad (3)$$

where $S(z)$ is the slip ratio; V_g^* and V_f^* are the velocities of the gaseous and the liquid phases of the coolant respectively.

Flow of heat from the wall surface to the coolant is assumed normal to the wall surface. The dynamics of the heat flow through the fuel is represented by just one time constant. More sophisticated representations may sometimes be necessary.

Fission is assumed to be due only to thermal neutrons.

Since all perturbations are zero for $t \leq 0$, the independent variable t and the corresponding Laplace transform variable s are freely interchanged.

Only one independent variable in space, z , (along the vertical axis) is considered. This is justified by the fact that the spatial dynamic effects are mainly associated with the axial flow of the coolant. z varies from 0 to L , where L is the length of the channel.

SPACE-TIME REPRESENTATION OF THE TRANSIENT VOIDS IN THE HEATED CHANNELS

Since the rate of fission reaction in the reactor depends on the thermal neutron flux, any change $\Delta\psi$ in

the thermal neutron flux is followed by a corresponding change $\Delta\phi$ in the heat flux at the channel wall surface. If the dynamics of heat conduction in the fuel is represented by one time constant, one may write

$$\Delta\phi(z, s) = \frac{F(z)}{1 + s\tau} \Delta\psi(z, s), \quad (4)$$

where s is the Laplace transform variable for t , $F(z)$ is a conversion factor that converts thermal neutron flux to the equivalent fission heat flux, and τ is the time delay associated with the heat conduction in the fuel. More complicated expressions may sometimes be necessary for the representation of the dynamics of heat conduction.

The perturbation in the heat flux gives rise to a corresponding perturbation in the void fraction along the channel. The conservation equations of the two-phase flow through the channel must be solved to obtain the space-time dependent perturbation of the void fraction. For completeness of this paper, the contents of Ref. 9 along with the necessary modifications are presented in the following.

The conservation equations of mass, energy and momentum are written, assuming separated flow. The dependent variables are $\alpha^*(z, t)$, the void fraction; $V_f^*(z, t)$, the velocity of the liquid phase; and $V_g^*(z, t)$, the velocity of the gaseous phase. $\phi^*(z, t)$ represents the heat flux along the channel. These quantities are expressed as a sum of their steady state and transient parts. For example,

$$\alpha^*(z, t) = \alpha(z) + \Delta\alpha(z, t). \quad (5)$$

Next, the conservation equations are written in their linearized perturbed form. In other words, the unknowns are made to be the perturbations $\Delta\alpha$, ΔV_f and ΔV_g for a corresponding $\Delta\phi$. ΔV_g is eliminated by writing

$$\Delta V_g = S(z)\Delta V_f. \quad (6)$$

Each of the resulting two unknown quantities, $\Delta\alpha$ and ΔV_f , are expressed as a series of Legendre polynomials, characterizing the coefficient of each term of the series as an unknown function of time. To facilitate the application of the orthogonality properties of the Legendre polynomials, a new space variable x is defined in place of z , where x is obtained by the transformation,

$$x = \frac{2z}{L} - 1. \quad (7)$$

Note that $0 \leq z \leq L$ implies $-1 \leq x \leq +1$. Thus,

$$\begin{aligned} \Delta\alpha(x, t) = & a_0(t)P_0(x) + b_1(t)P_1(x) \\ & + b_2(t)P_2(x) + \dots \end{aligned} \quad (8)$$

and

$$\Delta V_f(x, t) = a_0(t)P_0(x) + a_1(t)P_1(x) + a_2(t)P_2(x) + \dots \quad (9)$$

The coefficients b_0, b_1, b_2 , etc. are obtained by substituting the above series expansions into the linearized form of the conservation equations and applying the orthogonality and the boundary conditions. The above series approximation is shown to converge quite rapidly; and as a result of this, the space-time dependence of $\Delta\alpha$ is explicitly expressed.

In Ref. 9 it is assumed that $\Delta\phi$ is expressed as a product of a function of space and a function of time, i.e.,

$$\Delta\phi = f(x)\theta(t). \quad (10)$$

Keeping $\theta(t)$ arbitrary, the transfer functions $T_0 = [b_0(s)]/[\theta(s)]$, $T_1 = [b_1(s)]/[\theta(s)]$, etc. are first derived for a given $f(x)$. T_0, T_1 , etc. are used to obtain $b_0(s), b_1(s)$, etc. for any given $\theta(s)$.

In the present analysis,

$$\Delta\phi(x, s) = \frac{F(x)}{1 + s\tau} \Delta\psi(x, s). \quad (11)$$

Substituting the series expansion for $\Delta\psi$, Eq. (2),

$$\Delta\phi(x, s) = \frac{F(x)}{1 + s\tau} [\theta_0(s)P_0(x) + \theta_1(s)P_1(x) + \theta_2(s)P_2(x) + \dots]. \quad (12)$$

From Eq. (12) it is clear that $\Delta\phi$ in the present case is not just one term expressed as a product of a function of space and a function of time (as done in Ref. 9), but is a sum of many such terms.

Consider the i th term of $\Delta\phi$:

$$\Delta\phi_i(x, s) = \frac{F(x)}{1 + s\tau} \theta_i(s)P_i(x). \quad (13)$$

Let the corresponding perturbation in the void fraction be written as

$$\Delta\alpha_i = b_0^i(s)P_0(x) + b_1^i(s)P_1(x) + b_2^i(s)P_2(x) + \dots \quad (14)$$

Now, using the procedure discussed in Ref. 9, the transfer functions

$$T_j^i = \frac{b_j^i(s)}{\theta_i(s)}, \quad j = 0, 1, 2, \dots \quad (15)$$

are obtained.

Since the conservation equations are used in their linearized form, the total perturbation in the void fraction due to all of $\Delta\phi$ may be written as

$$\Delta\alpha = \sum_i \Delta\alpha_i. \quad (16)$$

Using Eqs. (14) and (16)

$$\begin{aligned} \Delta\alpha = & \theta_0[T_0^0P_0(x) + T_1^0P_1(x) + T_2^0P_2(x) + \dots] \\ & + \theta_1[T_0^1P_0(x) + T_1^1P_1(x) + T_2^1P_2(x) + \dots] \\ & + \theta_2[T_0^2P_0(x) + T_1^2P_1(x) + T_2^2P_2(x) + \dots] \\ & + \dots \end{aligned} \quad (17)$$

Rearranging,

$$\begin{aligned} \Delta\alpha = & [\theta_0T_0^0 + \theta_1T_0^1 + \theta_2T_0^2 + \dots] \\ & + P_1(x)[\theta_0T_1^0 + \theta_1T_1^1 + \theta_2T_1^2 + \dots] \\ & + P_2(x)[\theta_0T_2^0 + \theta_1T_2^1 + \theta_2T_2^2 + \dots] \\ & + \dots \end{aligned} \quad (18)$$

Equation (18) shows how the coefficients of expansion of the neutron flux perturbation, θ_j , are related to the corresponding space-time dependent perturbation in the void fraction.

VARIATION OF THE NUCLEAR PARAMETERS DURING TRANSIENTS

The nuclear parameters, such as the diffusion length, absorption cross section, etc., of a boiling reactor vary as a function of the local void fraction. It is shown in the previous section that for any change in the neutron flux level of the reactor, there is a corresponding change in the void fraction. Consequently the nuclear parameters themselves undergo certain changes in space and time during transients. This phenomenon is called the "internal feedback" of a boiling reactor. It is the purpose of this paper to take a simultaneous account of the space-time variation of the neutron flux and the corresponding space-time variation of the nuclear parameters.

It is assumed that the perturbations in the nuclear parameters are linearly related to the perturbation in the void fraction.

For example, the perturbation in the diffusion coefficient ΔD is given by,

$$\Delta D(z, t) = \frac{dD}{d\alpha}(z)\Delta\alpha(z, t) \quad (19)$$

$$= g(z)\Delta\alpha(z, t) \quad (20)$$

where $g = dD/d\alpha$ is obtained from steady state information.

The perturbation in the absorption cross section has two parts:

- (1) *Internal*: due to the change in local void fraction, and
- (2) *External*: due to the addition or withdrawal of poison.

Hence, the perturbation in the absorption cross section $\Delta\Sigma$ is written as

$$\Delta\Sigma = \Delta\Sigma_{int} + \Delta\Sigma_{ext}, \quad (21)$$

where

$$\begin{aligned} \Delta\Sigma_{int}(z, t) &= \frac{d\Sigma}{d\alpha}(z)\Delta\alpha(z, t) \\ &= h(z)\Delta\alpha(z, t). \end{aligned} \quad (22)$$

$\Delta\Sigma_{ext}$ is a known function of z and t .

In the present analysis, $\Delta\Sigma_{ext}(z, t)$ is considered as the "input" or the forcing function and $\Delta\psi(z, t)$ as the "output" of the system. In what follows, the input is considered arbitrary. This enables one to derive the space dependent transfer function directly from the linearized dynamic equations of the system. The transient behavior of the system for any specific input (for small perturbations) is then studied by utilizing the transfer function.

Further, it is convenient to use the transfer function to investigate the system's stability.

The above ideas are illustrated in the following sections.

A REACTOR WITH ONE ENERGY GROUP OF NEUTRONS

A simple model of a nuclear reactor is considered in this section to clearly illustrate the present method of dynamic analysis. The analysis is extended to a more realistic model in the next section.

A boiling water reactor with only one energy group of neutrons is studied in the following. The delayed neutrons are assumed to be absent.

The diffusion equation for the neutron flux in one dimensional space may be written as follows:

$$\begin{aligned} D^*(z, t) \frac{\partial^2 \psi^*}{\partial z^2} - \Sigma^*(z, t) \psi^*(z, t) \\ + K(z) \psi^*(z, t) = \frac{1}{v} \frac{\partial \psi^*}{\partial t}(z, t) \end{aligned} \quad (23)$$

where ψ^* is the neutron flux, D^* the diffusion coefficient, Σ the macroscopic absorption cross section, and v the neutron velocity. $K(z)\psi^*$ is the source term for thermal neutrons. In the case of a boiling water reactor, $K(z)$ is independent of void fraction for the first order of approximation.¹⁰ $K(z)$ is determined by the fuel composition, and since the latter is assumed not to change with time, $K(z)$ is independent of t . The macroscopic absorption cross section Σ^* depends both on the local void fraction α^* and the external addition or withdrawal of poison (for control purposes). D^* is a function of α^* . The term $(\partial D^*/\partial z)(\partial \psi^*/\partial z)$ is neglected in comparison with $D^*(\partial^2 \psi^*/\partial z^2)$ in writing Eq. (23). Let $\psi^* = \psi(z)$

represent the steady state of the reactor, when $D^* = D(z)$ and $\Sigma^* = \Sigma(z)$. Suppose, at time $t = 0$, a "small" quantity of poison is introduced or withdrawn from the reactor. The problem is to obtain ψ^* as a function of z and t , given the space-time dependence of the poison introduced or withdrawn from the reactor. This is accomplished by the small perturbation technique.

ψ^* , Σ^* and D^* are expressed as a sum of their steady state and transient (perturbed) parts,

$$\psi^*(z, t) = \psi(z) + \Delta\psi(z, t) \quad (24)$$

$$\Sigma^*(z, t) = \Sigma(z) + \Delta\Sigma(z, t) \quad (25)$$

and

$$D^*(z, t) = D(z) + \Delta D(z, t), \quad (26)$$

where $\psi(z)$, $\Sigma(z)$, and $D(z)$ are known functions of z , and Δ denotes the perturbation. Substituting Eqs. (24) to (26) into Eq. (23), removing the steady state part, and retaining only the terms that contain the first power of the perturbations, one may write

$$\begin{aligned} \Delta D(z, t) \frac{d^2 \psi(z)}{dz^2} + D(z) \frac{\partial^2 \Delta\psi(z, t)}{\partial z^2} - \Delta\Sigma(z, t) \psi(z) \\ - [\Sigma(z) - K(z)] \Delta\psi(z, t) = \frac{1}{v} \frac{\partial \Delta\psi(z, t)}{\partial t}. \end{aligned} \quad (27)$$

Equation (27) is the *linearized perturbed form* of Eq. (23). Substituting Eqs. (20), (21), (22) into Eq. (27), and performing the Laplace transformation for t , and the transformation z to x given by Eq. (7),

$$\begin{aligned} \left(\frac{2}{L}\right)^2 g(x) \frac{d^2 \psi(x)}{dx^2} \Delta\alpha(x, s) + \left(\frac{2}{L}\right)^2 D(x) \frac{\partial^2 \Delta\psi(x, s)}{\partial x^2} \\ - h(x) \psi(x) \Delta\alpha(x, s) - [\Sigma(x) - K(x)] \Delta\psi(x, s) \\ - \frac{1}{v} s \Delta\psi(x, s) = \psi(x) \Delta\Sigma_{ext}(x, s). \end{aligned} \quad (28)$$

$\Delta\psi$ in Eq. (28) is to be solved as a function of x and s .

Let $\Delta\psi(x, s)$ be expressed as follows:

$$\begin{aligned} \Delta\psi(x, s) = \theta_0(s) P_0(x) + \theta_1(s) P_1(x) \\ + \theta_2(s) P_2(x) + \dots \end{aligned} \quad (29)$$

where P_0 , P_1 , $P_2 \dots$ are Legendre polynomials and θ_0 , θ_1 , $\theta_2 \dots$ are unknown functions of s .

$\Delta\alpha(x, s)$ is related to $\Delta\psi(x, s)$ through the equation of heat conduction in the fuel elements and the conservation equations of the coolant flow along the channels. Equations (4) and (18) express this relationship. Hence, if Eqs. (4) and (18) are substituted into Eq. (28), the only unknowns are the coefficients θ_0 , θ_1 , $\theta_2 \dots$ etc. They are obtained by utilizing the orthogonality and the boundary conditions.

The boundary conditions on $\Delta\psi$ are that it vanishes at $x = \pm 1$. Therefore,

$$\theta_0 + \theta_1 + \theta_2 + \cdots = 0 \quad (30)$$

and

$$\theta_0 - \theta_1 - \theta_2 - \cdots = 0. \quad (31)$$

Let $\Delta\psi$ be expanded up to n terms. Consequently, there would be n unknown coefficients and $(n - 2)$ more equations are necessary to solve for the n coefficients. They are obtained by multiplying Eq. (28) by $P_0, P_1, P_2 \cdots P_{n-3}$ respectively and integrating over x between -1 and $+1$ (orthogonality conditions).

It is found¹¹ that the series approximation of $\Delta\psi$ converges quite rapidly, and hence, the present method is useful in many practical cases. A numerical example illustrating the present method in detail may be found in Ref. 11.

A REACTOR WITH TWO ENERGY GROUPS OF NEUTRONS AND SIX GROUPS OF DELAYED NEUTRONS

The purpose of this chapter is to present a more realistic formulation of the problem. Two energy groups of neutrons are considered, along with six groups of delayed neutrons. Fission is assumed to be caused only by the slow group.

The linearized form of the neutron kinetics equations are as follows

$$D^{(2)} \frac{\partial^2 \Delta\psi^{(2)}}{\partial z^2} + \frac{d^2 \psi^{(2)}}{dz^2} \Delta D^{(2)} - \Sigma_a^{(2)} \Delta\psi^{(2)} - \psi^{(2)} \Delta \Sigma_a^{(2)} + \Sigma_R \Delta\psi^{(1)} + \psi^{(1)} \Delta \Sigma_R = \frac{1}{v_2} \frac{\partial \Delta\psi^{(2)}}{\partial t}, \quad (50)$$

$$D^{(1)} \frac{\partial^2 \Delta\psi^{(1)}}{\partial z^2} + \frac{d^2 \psi^{(1)}}{dz^2} \Delta D^{(1)} + (1 - \beta) \nu \Sigma_f \Delta\psi^{(2)} - (\Sigma_R + \Sigma_a^{(1)}) \Delta\psi^{(1)} - \psi^{(1)} (\Delta \Sigma_R + \Delta \Sigma_a^{(1)}) + \sum_{i=1}^6 \lambda_i \Delta c_i = \frac{1}{v_1} \frac{\partial \Delta\psi^{(1)}}{\partial t}, \quad (51)$$

$$\frac{\partial \Delta c_i}{\partial t} = \beta_i \nu \Sigma_f \Delta\psi^{(2)} - \lambda_i \Delta c_i \quad i = 1, 2, \cdots 6. \quad (52-57)$$

The superscripts (1) and (2) indicate the quantity pertaining to the fast and the thermal groups respectively.

Δ denotes perturbation

$\psi \equiv$ neutron flux intensity

$c_i \equiv$ i th delayed neutron emitter concentration

$\Sigma_a \equiv$ absorption cross section

$D \equiv$ diffusion coefficient

$\Sigma_R \equiv$ fast removal cross section

$\Sigma_f \equiv$ fission cross section (does not change with time)

$\nu \equiv$ average number of neutrons per fission

$\beta_i \equiv$ fraction of delayed fission neutrons in i th group

$\beta \equiv$ sum of the β_i values

$\lambda_i \equiv$ decay constant of the i th delayed-neutron-emitter

$v_1, v_2 \equiv$ velocities of the fast and slow groups of neutrons

Equations (52) to (57) are Laplace transformed and $\Delta c_i(z, s)$ is expressed in terms of $\Delta\psi^{(2)}(z, s)$. Thus

$$\Delta c_i(z, s) = \beta_i \nu \Sigma_f \frac{1}{s + \lambda_i} \Delta\psi^{(2)}(z, s) \quad (58-63)$$

$$i = 1, 2, \cdots 6.$$

Laplace transforming Eqs. (50) and (51), substituting Eqs. (58-63) into Eq. (51) and transforming z to x , using Eq. (7), one obtains the following equations:

$$\left(\frac{2}{L}\right)^2 D^{(2)} \frac{\partial^2 \Delta\psi^{(2)}}{\partial x^2} + \left(\frac{2}{L}\right)^2 \frac{d^2 \psi^{(2)}}{dx^2} \Delta D^{(2)} - \Sigma_a^{(2)} \Delta\psi^{(2)} - \psi^{(2)} \Delta \Sigma_a^{(2)} + \Sigma_R \Delta\psi^{(1)} + \psi^{(1)} \Delta \Sigma_R = \frac{s}{v_2} \Delta\psi^{(2)}, \quad (64)$$

$$\left(\frac{2}{L}\right)^2 D^{(1)} \frac{\partial^2 \Delta\psi^{(1)}}{\partial x^2} + \left(\frac{2}{L}\right)^2 \frac{d^2 \psi^{(1)}}{dx^2} \Delta D^{(1)} + (1 - \beta) \nu \Sigma_f \Delta\psi^{(2)} - (\Sigma_R + \Sigma_a^{(1)}) \Delta\psi^{(1)} - \psi^{(1)} (\Delta \Sigma_R + \Delta \Sigma_a^{(1)}) + \sum_{i=1}^6 \beta_i \nu \Sigma_f \frac{\lambda_i}{s + \lambda_i} \Delta\psi^{(2)} = \frac{s}{v_1} \Delta\psi^{(1)}. \quad (65)$$

$\Delta D^{(1)}, \Delta D^{(2)}, \Delta \Sigma_a^{(1)}, \Delta \Sigma_a^{(2)}$ and $\Delta \Sigma_R$ are assumed to depend on the first power of $\Delta\alpha$ as was done in the previous section [see Eqs. (20) and (22) for instance]. Further, $\Delta\alpha$ is expressed as a function of $\Delta\psi^{(2)}$ [see Eq. (18)]. Consequently, the only unknowns are $\Delta\psi^{(1)}$ and $\Delta\psi^{(2)}$ in Eqs. (64) and (65). Each is expressed in a series of Legendre polynomials with unknown coefficients. These coefficients are solved by using the boundary and orthogonality conditions. From this point on, the method of analysis is the same as in the previous chapter and hence not discussed further.

DISCUSSION

The present study has resulted in a straightforward method of obtaining the space dependent transfer function of a boiling reactor. Valuable information on the stability and transient behavior of the system are readily obtained from the transfer function. Such information is expected to be particularly useful in design.

The transfer function of the system is obtained by

linearizing the dynamic equations. Hence, the result of the present dynamic analysis is expected to be valid only for small deviations of the dependent variables about a steady operating point.

It is noted, however, that the linearized dynamic equations are solved by making no restriction on the space or time dependence of the perturbations. Previous studies^{3, 9, 12} have shown that such restrictions could introduce serious errors in the space-dependent transfer function.

Although a natural circulation system has been considered in presenting this method, the analysis is equally applicable to a forced circulation system if a few minor modifications are made in the treatment of the coolant dynamics. This is considered in detail in Ref. (9).

The method given in Ref. (9) may not be adequate to solve the dynamic equations of the coolant flow, if the coolant entering the core has a large amount of subcooling. In this case, the method given in Ref. 13 is used. The analysis in Ref. 13 gives the space-time dependence of the perturbation of coolant density, instead of the void fraction. In this case, the variation of the nuclear parameters is taken into account by either of the following two methods:

1. The nuclear parameters are expressed as a function of the coolant density, instead of the void fraction.
2. The perturbation in density is converted to the corresponding perturbation in void fraction using the information about the distribution of the fluid enthalpy along the channel, thus reducing the problem to the one considered here.

REFERENCES

1. A. N. Nahavandi and R. F. Von Hollen, *A Digital Computer Solution for Space Dependent Neutron Kinetics Equations*, Nucl. Sci. Eng. **18**, 335-350 (1964).
2. A. N. Nahavandi and R. F. Von Hollen, *A Space Dependent Dynamic Analysis of Boiling Water Reactor Systems*, Nucl. Sci. Eng. **20**, 392-413 (1964).
3. J. E. Meyer and E. A. Reinhard, *A Small Perturbation Approach to the Study of Parallel Channel Boiling Flow Oscillations*, WAPD-TM-342 (January 1963).
4. J. A. Fleck, Jr., *The Dynamic Behavior of Boiling Water Reactors*, J. Nucl. Energy, Part A, Reactor Science, **11**, (1960).
5. A. Z. Akcasu, *Theoretical Feedback Analysis in Boiling Water Reactors*, ANL-6221 (1960).
6. H. L. Garabedian and C. B. Leffert, *A Time Dependent Analysis of Spatial Flux Distributions*, Nucl. Sci. Eng., **6**, 26 (1959).
7. H. L. Garabedian and R. E. Lynch, *Nonlinear Reactor Kinetics Analysis*, Nucl. Sci. Eng., **21**, 550-564 (1965).
8. E. Toops, *Reactor Kinetics*, CENRD-163, Combustion Engineering, Inc. (1961).
9. C. K. Sanathanan, *Dynamic Analysis of Coolant Circulation in Boiling Water Nuclear Reactors*, ANL-6847 (1964).
10. J. A. Thie, et al., *Void Measurements in a Boiling Reactor*, Nucl. Sci. Eng., **11**, 1-6 (1961).
11. C. K. Sanathanan, *An Analytic Study of the Space Dependent Dynamics of Reactors*, Chem. Eng. Symposium Series, AICHE Annual Meeting, December 1965, Philadelphia (to be published).
12. E. R. Quandt, *Analysis and Measurement of Flow Oscillations*, Chem. Eng. Progress Symposium Series, Heat Transfer, Buffalo, **57**, No. 32.
13. C. K. Sanathanan, *Further Developments in the Dynamic Analysis of Coolant Circulation in Boiling Water Nuclear Reactors*, ANS Transactions **7**, No. 2 (November 1964).

II-14. High Conversion Critical Experiment

A. R. BOYNTON, K. E. PLUMLEE, W. C. REDMAN, W. R. ROBINSON and G. S. STANFORD

INTRODUCTION

The High Conversion Critical Experiment Program (Hi-C) was undertaken to extend the range of investigations with light water-moderated, slightly enriched, uranium oxide cores into the very undermoderated region of small H to U-238 atom ratios. The results of critical and micro-parameter measurements in assemblies having H:28 atom ratios from 5 down to 1 have been reported.¹⁻³

After microparameters had been obtained in the 1.127-cm triangular stainless steel-clad zone core (H:28 atom ratio of 0.98), it seemed desirable to make meas-

urements in an even drier core. Such a core was constructed as a central zone in the 1.127-cm triangular lattice. The zone was loaded into an aluminum box of inside dimensions 19.44 × 20.31 cm and 0.64-cm thickness, which was positioned in the center of the 1.127-cm triangular grid. The box was filled with 388 stainless steel-clad Hi-C fuel pins in a triangular pattern. The dimensions of the box and the number of pins which it held indicated an average gap of 0.012 cm between fuel pins. This spacing corresponds to a 1.069-cm triangular lattice pitch with an H:28 atom ratio of 0.50. These are the values used to describe the loading; however, local variations could have been considerable since

l variations in the gap between fuel pins result in large local variations in H:28 atom ratio in this very tight lattice.

Criticality was obtained with about 3400 aluminum-clad Hi-C fuel pins loaded in the 1.127-cm triangular lattice surrounding the tight zone. From the outer edge of the box (about 10-cm equivalent radius), aluminum-clad fuel pins were loaded uniformly to a radius of approximately 32 cm. A final outer zone in which some of the fuel pin positions were left to be filled with water, extended to almost a 40-cm radius.

The Hi-C fuel was 3.04 w/o enriched uranium oxide pellets, 0.935-cm in diameter loaded to 122-cm length, in cladding tubes of either aluminum or stainless steel. The aluminum tubes were 1.058 cm o.d. and 0.048 cm thick while the stainless steel tubes were 1.057 cm o.d. and 0.0496 cm thick.

ACTIVATION TRAVERSES OF CORE

Foil activation traverses were taken at the horizontal midplane of the core to discover any region of asymptotic spectrum. Cadmium-covered and bare gold and manganese and bare 0.21 w/o uranium metal foils were used. Typical of the results is the bare gold traverse shown in Fig. II-14-1. Local irregularities in loading along the sides of the box containing the tight zone is responsible for the activation peak just outside the box.

The tight zone was slightly larger than 10-cm equivalent radius and the micro-parameter measurements were obtained at radii ranging from 3.4 to 4.8 cm. Judging by other studies of two-region cores in which comparisons were made with uniformly loaded critical cores,⁴ these measurements are expected to be within a few percent of results which would have been obtained in a region of asymptotic spectrum. The extent of the region in which the spectrum may have been asymptotic is indicated in Fig. II-14-2 which presents plots of

FIG. II-14-1. Bare Gold Activation Traverse in 1.069 cm Triangular Zoned Core. ANL Neg. No. 112-5674.

FIG. II-14-2. Activation Traverses in 1.069 cm Triangular Zoned Core. ANL Neg. No. 112-5673.

TABLE II-14-I. INITIAL CONVERSION RATIOS IN STAINLESS STEEL CLAD BORAX-V AND HI-C CORES

Lattice	H:28	α^{25}	ICR
1.27-cm square BORAX-V	4.65	0.234	0.297 ± 0.015
1.27-cm triangular BORAX-V	3.53	0.251	0.330 ± 0.016
1.24-cm square Hi-C	2.92	0.244	0.500 ± 0.005
1.27-cm triangular Hi-C	2.29	0.257	0.581 ± 0.017
1.166-cm triangular Hi-C	1.33	0.296	0.835 ± 0.005
1.127-cm triangular Hi-C	0.98	0.317	0.913 ± 0.019
1.069-cm triangular Hi-C	0.50	0.362	1.485 ± 0.034

cadmium-covered gold and bare depleted uranium activation, both of which have been normalized to the bare gold traverse. Figure II-14-2 was constructed from smoothed plots rather than from the data points.

MICRO-PARAMETER VALUES

The micro-parameters given below were determined in the central zone by methods previously described.¹⁻³

INITIAL CONVERSION RATIO

Data from which the ICR (defined as the ratio of the Pu-239 production rate to the U-235 destruction rate in a fresh core) was computed were taken at a distance of $4\frac{1}{2}$ cm from the core center. Nine independent determinations gave

$$\text{ICR} = 1.485 \pm 0.034.$$

A value of 0.362 for α^{25} was used. Improved values of epicadmium α 's were obtained from GAM-I calculations⁵ and the revised ICR values for previously reported Hi-C and BORAX-V cores are given in Table II-14-I.

U-238 CAPTURE CADMIUM RATIO (C_{28})

Three irradiations were performed at an average distance of $4\frac{1}{2}$ cm from the core center. The thermal

activation method was used to give a value of

$$C_{28} = 1.035 \pm 0.001.$$

U-238 TO U-235 FISSION RATIO, δ^{28}

All measurements were made at a distance of $3\frac{1}{2}$ cm from the core center. Twenty independent determinations gave a value of

$$\delta^{28} = 0.2622 \pm 0.0036.$$

U-235 FISSION CADMIUM RATIOS AND THERMAL DISADVANTAGE FACTOR, ζ

All measurements were made at an average distance of $4\frac{1}{2}$ cm from the core center. The U-235 fission cadmium ratios obtained in the fuel and in the moderator, respectively, were

$$(C_f^{25})_{fuel} = 1.59 \pm 0.03,$$

and

$$(C_f^{25})_{moderator} = 1.69 \pm 0.09.$$

The bare moderator foil to bare fuel foil fission product activity was

$$(A_{25}^b)_{mod}/(A_{25}^b)_{fuel} = 1.081 \pm 0.041.$$

The thermal disadvantage factor, computed from the above, is

$$\zeta_{25} = 1.189 \pm 0.109.$$

CADMIUM RATIOS

The cadmium ratio (0.051-cm cadmium covers) was obtained for dysprosium, indium, gold, manganese, and copper. The measurements were made at an average distance of 4 cm from the core center and were made in the fuel pins. The results were:

dysprosium	4.31 ± 0.10
indium	1.135 ± 0.008
gold	1.099 ± 0.008

manganese	1.322 ± 0.030
copper	1.210 ± 0.030

As was done with the micro-parameter results in previous cores,³ comparison of experimental results with the theoretical results of J. Bengston and R. Hellens⁶ may be performed. Their computations (for unclad oxide rods) were made utilizing MUFT-IV and SOFO-CATE to generate constants for three fast groups and one thermal group. The calculated micro-parameters, such as the initial conversion ratio, appear to be in fair agreement with experiments even at this very low H:28 atom ratio, although the trend is one of decreasing agreement with decreasing H:28 atom ratio. More recent calculations⁵ utilizing THERMOS and GAM-I for generating thermal and fast constants also indicate fair agreement with experiment. Once again, the trend is one of increasing disagreement with decreasing lattice pitch.

REFERENCES

1. Q. L. Baird and A. R. Boynton, "Bucklings, Disadvantage Factors and δ^{28} Measurements in Some Undermoderated Slightly Enriched Cores," Trans. Am. Nucl. Soc. 6, No. 2, 248 (1963).
2. Q. L. Baird, A. R. Boynton, W. R. Robinson and J. M. Christenson, "Initial Conversion Ratio and Epicadmium to Subcadmium Uranium-238 Capture Rate Measurements in Very Undermoderated Slightly Enriched Oxide Cores," Trans. Am. Nucl. Soc. 7, No. 1, 80 (1964).
3. A. R. Boynton, Q. L. Baird, J. M. Christenson, K. E. Plumlee, W. C. Redman, W. R. Robinson and G. S. Stanford, "High Conversion Critical Experiment," Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010, (1965), pp. 33-43.
4. J. L. Volpe, et al., "Two Region Studies in Slightly Enriched Water-Moderated Uranium and Uranium Dioxide Lattices," Nucl. Sci. Eng. 5, 360-370 (1959).
5. E. M. Pennington, Argonne National Laboratory (Private Communication).
6. J. Bengston and R. L. Hellens, "A Survey of the Beginning of Life Characteristics of Uranium-Fueled, Water-Moderated Lattices," CEND-137 (1961).

II-15. Hi-C Uniform Lattice Calculations

E. M. PENNINGTON

INTRODUCTION

Calculations of four-group constants, material bucklings, reflector savings and various lattice parameters were performed for the light water moderated lattices studied during the High Conversion (Hi-C) experimental program in the ZPR-VII critical assembly.

Constants for three fast groups with lower energy boundaries at 1.35 MeV, 1.23 keV and 0.414 eV were obtained with the GAM-1⁽¹⁾ code, while those for the thermal group were found using the THERMOS² code. Preliminary work in this area was reported in Ref. 3. The preliminary material bucklings were considerably

higher than the critical geometric bucklings of the experiments. The cross sections of U-235 and U-238 in the original GAM-1 library were therefore replaced by versions obtained from Hanford which were based on the data in Ref. 4. The new U-238 cross sections differ from the original in the values of σ_c , σ_f , ν and resonance parameters; transfer cross sections remained unchanged.

TABLE II-15-I. HI-C AND BORAX-V CORE DESCRIPTIONS

Core Code	Lattice Pitch, cm	Fuel	Clad	H ₂ O/ UO ₂ Volume Ratio	H/ U-238 Atom Ratio
B 1.27 □S	1.27 square	BORAX-V	SS	1.504	4.654
B 1.27 △S	1.27 triangular	BORAX-V	SS	1.141	3.533
H 1.24 □A	1.24 square	Hi-C	Al	0.9596	2.911
H 1.24 □S	1.24 square	Hi-C	SS	0.9620	2.918
H 1.27 △A	1.27 triangular	Hi-C	Al	0.7544	2.289
H 1.27 △S	1.27 triangular	Hi-C	SS	0.7568	2.296
H 1.166 △A	1.166 triangular	Hi-C	Al	0.4347	1.319
H 1.166 △S	1.166 triangular	Hi-C	SS	0.4371	1.326
H 1.127 △A	1.127 triangular	Hi-C	Al	0.3218	0.9760
H 1.127 △S	1.127 triangular	Hi-C	SS	0.3242	0.9833
H 1.069 △S	1.069 triangular	Hi-C	SS	0.1635	0.4959

In the new version all U-235 cross sections are changed from the original, and furthermore, resonance parameters are included so that resonance integrals are no longer calculated in the infinite dilution limit. Use of the new cross sections led to bucklings which were in much better agreement with experimental values. Reference 5 reports similar results for calculated bucklings upon changing the U-238 cross sections to compare calculated and experimental reactivities for the Yankee critical experiments.

Table II-15-I briefly describes the lattices involved in the study. Note that the fuel rods are closely spaced, as is inferred from the low volume-ratios of water to fuel. The Hi-C fuel is 3.04 w/o enriched uranium in UO₂ pellets loaded into either stainless steel or aluminum cladding tubes, while the BORAX-V fuel is 4.95 w/o enriched uranium in UO₂ pellets in stainless steel cladding tubes.

CRITICAL BUCKLINGS

Table II-15-II presents four-group diffusion theory values of critical bucklings for all the lattices obtained by fundamental mode calculations. The B-1 and P-1 options refer to the GAM-I code. The main differences between B-1 and P-1 constants lie in the diffusion coefficients. In all cases THERMOS problems with the Nelkin kernel for hydrogen in H₂O were used for the thermal group. Both the calculated critical bucklings, B_m^2 , and the experimental critical bucklings, B_g^2 , are given in Table II-15-II, along with $k_{eff}(B_g^2)$. For five of the eleven lattices no experimental values are given, either because B_g^2 is negative, or because not enough fuel was available to build a critical lattice for the smaller positive values of B_g^2 . It is seen that all of the calculated reactivities are higher than the experimental reactivities, with the average k_{eff} for the experimentally critical lattices being 1.0118 for the B-1 option and 1.0062 for the P-1 option. This represents much better agreement with experiment than was the case for preliminary values.³

TABLE II-15-II. CALCULATED AND EXPERIMENTAL BUCKLINGS

Core Code	Option	B_m^2, cm^{-2}	B_g^2, cm^{-2}	$k_{eff}(B_g^2)$	Core Code	Option	B_m^2, cm^{-2}	B_g^2, cm^{-2}	$k_{eff}(B_g^2)$
B 1.27 □S	B-1	0.011270	0.010734	1.0143	H 1.27 △S	B-1	0.003807	—	—
	P-1	0.010873		1.0038		P-1	0.003764		
B 1.27 △S	B-1	0.009589	0.009147	1.0127	H 1.166 △A	B-1	0.002674	0.002436	1.0117
	P-1	0.009301		1.0045		P-1	0.002649		1.0105
H 1.24 □A	B-1	0.007302	0.007076	1.0079	H 1.166 △S	B-1	0.000756	—	—
	P-1	0.007111		1.0013		P-1	0.000752		
H 1.24 □S	B-1	0.005052	0.004747	1.0104	H 1.127 △A	B-1	0.001386	—	—
	P-1	0.004968		1.0076		P-1	0.001380		
H 1.27 △A	B-1	0.005899	0.005538	1.0140	H 1.127 △A	P-1	-0.000480	—	—
	P-1	0.005777		1.0094					
					H 1.069 △S	P-1	-0.002773	—	—

TABLE II-15-III. TWO GROUP PARAMETERS

Core Code	B1.27 □S	B1.27 △S	H1.24 □A	H1.24 □S	H1.27 △A	H1.27 △S	H1.166 △A	H1.166 △S	H1.127 △A	H1.127 △S	H1.069 △S
p_1	0.62246	0.55482	0.57211	0.56485	0.51405	0.50566	0.37130	0.35721	0.30155	0.28620	0.14425
$(\eta f)_1$	0.86548	0.87682	0.77482	0.73432	0.78359	0.73886	0.78282	0.73204	0.78684	0.72781	0.70016
$(\eta f)_2$	1.7019	1.7217	1.7239	1.5549	1.7396	1.5675	1.7628	1.5857	1.7692	1.5916	1.5983
τ , cm ²	34.266	36.026	42.785	38.311	45.706	40.469	53.729	47.396	58.787	50.932	61.316
L^2 , cm ²	1.1763	1.1646	1.9489	1.6767	2.0701	1.7381	2.4387	1.9740	2.7223	2.1092	2.4734
λ_1^a	0.2380	0.2923	0.2542	0.2684	0.3012	0.3169	0.4308	0.4541	0.5084	0.5326	0.7207
λ_2^a	0.7620	0.7077	0.7458	0.7316	0.6988	0.6831	0.5692	0.5459	0.4916	0.4674	0.2793

^a λ_1 and λ_2 are the first and second terms of the left member of Eq. (1), representing fast and thermal fissions, respectively.

TABLE II-15-IV. RADIAL REFLECTOR SAVINGS, cm

Core Code	Four-Group		Two-Group		Experimental
	B-1	P-1	B-1	P-1	
B 1.27 □S	6.95	7.21	7.04	7.30	7.35 ± 0.27
B 1.27 △S	7.35	7.57	7.40	7.63	7.55 ± 0.10
H 1.24 □A	8.25	8.45	8.25	8.46	8.13 ± 0.16
H 1.24 □S	7.64	7.76	7.58	7.69	7.69 ± 0.08
H 1.27 △A	8.85	9.02	8.76	8.94	8.77 ± 0.17
H 1.166 △A	10.72	10.86	10.23	10.32	10.73 ± 0.14

TWO GROUP PARAMETERS

Parameters involved in the two-group criticality equation used in the analysis of Brookhaven light water moderated lattices⁷ are given in Table II-15-III. This equation is

$$\frac{(\eta f)_1(1 - p_1)}{1 + \tau B^2} + \frac{(\eta f)_2 p_1}{(1 + \tau B^2)(1 + L^2 B^2)} = 1 \quad (1)$$

where the subscripts 1 and 2 designate fast and thermal groups, respectively, and

$$(\eta f)_1 = \frac{\nu_1 \Sigma_{f1}}{\Sigma_{a1}}, \quad (\eta f)_2 = \frac{\nu_2 \Sigma_{f2}}{\Sigma_{a2}}, \quad p_1 = \frac{\Sigma_{12}}{\Sigma_{a1} + \Sigma_{12}},$$

$$\Sigma'_{a1} = \Sigma_{a1} - \Sigma_{11}^{n,2n}, \quad \tau = \frac{D_1}{\Sigma'_{a1} + \Sigma_{12}} \text{ and } L^2 = \frac{D_2}{\Sigma_{a2}}.$$

The fast group constants were determined from the one group edit of the GAM-I P-1 problems. Table II-15-III shows clearly the increasing importance of non-thermal events as the water to fuel ratio is decreased, with the parameters p_1 and λ_1 being especially significant.

REFLECTOR SAVINGS

Radial reflector savings calculations were done for the six lattices which went critical in the experiments. The multigroup one-dimensional diffusion theory code RP-122 (REX) was used. Both four-group and two-group computations were carried out using both B-1 and P-1 constants from GAM-I for the non-thermal

groups and THERMOS constants for the thermal group. GAM-I and THERMOS constants were also used for the radial H₂O reflector. Calculated and experimental⁶ results are presented in Table II-15-IV. To determine criticality the REX code iterates on the axial buckling B_z^2 . Then the critical B_z^2 is subtracted from the known B_m^2 of Table II-15-II to obtain the radial buckling B_R^2 , and hence the reflector savings, λ_R . Calculations of radial reflector savings are in fairly good agreement with experiment. No axial calculations were done because of the complicated structure of the bottom axial reflector. Also, λ_R is much more important than λ_z in determining the experimental B_0^2 since the experimental B_R^2 are considerably larger than B_z^2 .

U-238-TO-U-235 FISSION RATIO, (δ^{28})

The ratio of the number of fissions in U-238 to those in U-235 can be calculated from GAM-I and THERMOS constants as

$$\delta^{28} = \left(N^{28} \sum_{i=1}^4 \sigma_{fi}^{28} \phi_i \right) / \left(N^{25} \sum_{i=1}^4 \sigma_{fi}^{25} \phi_i \right) \quad (2)$$

In addition to the critical bucklings given in Table II-15-II, the fluxes in Eq. (2) were calculated using a Fortran code, BUCKLE, written for the CDC 160-A computer. Values of δ^{28} determined from Eq. (2), approximate values computed by methods described below, and experimental values^{6,8} where available, are presented in Table II-15-V.

The diffusion equation for the first energy group, which ranges from 10 MeV to 1.35 MeV in the work reported here, may be written as

$$D_1 \nabla^2 \phi_1 - \Sigma_{R1} \phi_1 + \Sigma_{11}^{n,2n} \phi_1 + \chi_1 \nu_1 \Sigma_{f1} \phi_1 + \chi_1 \sum_{i=2}^4 \nu_i \Sigma_{fi} \phi_i = 0 \quad (3)$$

which yields

$$\phi_1 = \left(\chi_1 \sum_{i=2}^4 \nu_i \Sigma_{fi} \phi_i \right) / \left(\Sigma_{R1} - \Sigma_{11}^{n,2n} - \chi_1 \nu_1 \Sigma_{f1} + D_1 B^2 \right). \quad (4)$$

TABLE II-15-V. U-238-TO-U-235 FISSION RATIO

Core Code	B1.27□S	B1.27△S	H1.24□A	H1.24□S	H1.27△A	H1.27△S
4 group GAM-I (P-1 homogeneous)	0.05424	0.06674	0.07991	0.08121	0.09440	0.09513
4 group GAM-I (P-1 heterogeneous) ^a	0.05686	0.06908	0.08230	0.08379	0.09649	0.09742
1 group GAM-I (P-1 homogeneous)	0.05132	0.06309	0.07535	0.07644	0.08892	0.08943
1 group GAM-I (fission spectrum homogeneous)	0.05373	0.06630	0.07909	0.08040	0.09363	0.09447
1 group GAM-I (fission spectrum heterogeneous)	0.05633	0.06862	0.08146	0.08295	0.09570	0.09674
1 group MUFT-4 (fission spectrum homogeneous)	0.05564	0.06914	0.08323	0.08486	0.09942	0.1006
1 group MUFT-4 (fission spectrum heterogeneous)	0.05837	0.07159	0.08579	0.08767	0.1017	0.1031
experimental	0.0650 ±0.006	0.1190 ±0.010	—	0.1014 ±0.0007	—	0.1160 ±0.0016

Core Code	H1.166△A	H1.166△S	H1.127△A	H1.127△S	H1.069△S
4 group GAM-I (P-1 homogeneous)	0.1334	0.1320	0.1555	0.1524	0.1971
4 group GAM-I (P-1 heterogeneous) ^a	0.1349	0.1337	0.1566	0.1537	0.1979
1 group GAM-I (P-1 homogeneous)	0.1253	0.1234	0.1459	0.1422	0.1827
1 group GAM-I (fission spectrum homogeneous)	0.1327	0.1317	0.1551	0.1525	0.1977
1 group GAM-I (fission spectrum heterogeneous)	0.1342	0.1334	0.1562	0.1538	0.1985
1 group MUFT-4 (fission spectrum homogeneous)	0.1448	0.1437	0.1717	0.1685	0.2255
1 group MUFT-4 (fission spectrum heterogeneous)	0.1465	0.1456	0.1731	0.1701	0.2266
experimental	—	0.1539 ±0.0037	—	0.1877 ±0.0020	0.2622 ±0.0036

^a Approximate 4-group heterogeneous values were calculated by multiplying the 4-group homogeneous values by the ratio of the δ^{28} values for the 1-group GAM-I fission spectrum heterogeneous case and the corresponding homogeneous case.

Now make the following approximations:

- (1) All U-238 fissions occur in group 1
- (2) All U-235 fissions occur below group 1, except that U-235 fissions in group 1 are included in the $\chi_1 \nu_1 \Sigma_{f1}$ term in the denominator of Eq. (4).
- (3) For U-235 fission, $\nu_2 = \nu_3 = \nu_4 = \nu_t = 2.43$.

The above approximations lead to the following approximate formula for δ^{28} :

$$\delta^{28} = \nu_i \chi_1 \Sigma_{fi}^{28} / (\Sigma_{R1} - \Sigma_{11}^{n,2n} - \chi_1 \nu_1 \Sigma_{f1} + D_1 B^2) \quad (5)$$

Equation (5) has the advantage of involving only group 1 constants. The first of the three approximations is the poorest, causing the values of δ^{28} given by Eq. (5) to be a few percent too low. Values of δ^{28} were calculated according to Eq. (5) using cross sections from the GAM-I output. Also Eq. (5) was used to make calculations with cross sections obtained by averaging the first eight fine groups, having lethargy widths of $\Delta u = 0.25$, of the GAM-I⁽⁹⁾ and MUFT-4⁽¹⁰⁾ libraries over a U-235 fission spectrum.

The calculations outlined above were done in the homogeneous approximation; that is, no allowance was made for the spatial variation of the fast flux in the unit

cell. This is reasonably well justified in the energy region of importance for fast fission since the cross sections are relatively small, and so the mean free paths are long. In order to study heterogeneous effects, average fluxes were calculated for the three regions (fuel, clad and moderator) of the unit cell with the RP 325 X collision probability code^{11, 12}. Both GAM-I and MUFT-4 fission spectrum averaged cross sections were used. The average regional fluxes were then combined with the cross sections for the individual cell regions to yield homogenized constants for Eq. (5).

Table II-15-VI presents the average fluxes in group 1 for the stainless steel clad cores, calculated using the GAM-I fission spectrum constants. The subscripts 1, 2 and 3 on ϕ denote fuel, clad and water regions respectively. The flux peaking in the fuel is not great even for the BORAX-V square lattice, and becomes quite small for the most tightly packed Hi-C lattice.

Inspection of the δ^{28} values in Table II-15-V shows that heterogeneous effects on δ^{28} are small and become smaller as the lattice pitch decreases, as would be expected from the results of Table II-15-VI. One-group values of δ^{28} are smaller than four-group values, as

TABLE II-15-VI. RATIOS OF AVERAGE GROUP
1 REGIONAL FLUXES

	Core Code						
	B1.27 □S	B1.27 △S	H1.24 □S	H1.27 △S	H1.166 △S	H1.127 △S	H1.069 △S
$\bar{\phi}_2/\bar{\phi}_1$	0.95910	0.96655	0.96671	0.97185	0.98047	0.98376	0.98883
$\bar{\phi}_3/\bar{\phi}_1$	0.93577	0.94961	0.95150	0.96009	0.97372	0.97869	0.98609

anticipated above, because of the neglect of U-238 fissions below 1.35 MeV. Averaging GAM-I cross sections over a fission spectrum increases δ^{28} somewhat above the value obtained using cross sections averaged over the spectrum generated by GAM-I. MUFT-4 values of δ^{28} are higher than the corresponding GAM-I values.

Since all experimental values are considerably higher than the corresponding theoretical values, it is apparent that agreement between theory and experiment on δ^{28} is not good. The explanation for the discrepancy is not known. It may be caused by inadequacies in the cross sections used in the calculations or by some systematic experimental error.

Computations were made to determine the effects on reactivity of heterogeneous calculations in group 1, and also of using MUFT-4 instead of GAM-I cross sections. Fast constants obtained using both GAM-I and MUFT-4 fission spectrum cross sections in both homogeneous and heterogeneous calculations were substituted for the original GAM-I P-1 constants in group 1 of the four-group BUCKLE problems. Results for the three lattices which were studied are presented in Table II-15-VII in the form of reactivity differences. Table II-15-VII shows that the use of heterogeneous instead of homogeneous cross sections increases reactivity by little more than 0.1% even for the loosest lattice. Use of MUFT-4 cross sections in place of GAM-I cross sections decreases reactivity for most lattices, but increases it for the 1.069 cm Hi-C triangular lattice with stainless steel clad.

RESONANCE ABSORPTION IN U-238 AND U-235

Resonance absorption in the isotopes U-238 and U-235 in the GAM-I code is calculated according to the methods of F. Adler, G. Hinman and L. Nordheim.¹³ The escape probability for the fuel rod is computed by the formula for an isolated rod with the mean chord length, $\bar{l}$, being replaced by $\bar{l}/(1-C)$ where C is the Dancoff factor. Thus Dancoff factors must be calculated for the GAM-I input. These were determined from the tables¹⁴ calculated by I. Carlvik. The clad and H₂O cross sections were homogenized to yield the moderator cross section used in looking up the Dancoff

TABLE II-15-VII REACTIVITY CHANGES UPON
SUBSTITUTION OF GROUP 1 CROSS SECTIONS
INTO FOUR-GROUP "BUCKLE" PROBLEMS, %

Core Code	B1.27□S	H1.127△A	H1.069△S
GAM - I homogeneous →	+0.134	+0.059	+0.045
GAM-I heterogeneous			
MUFT - 4 homogeneous →	+0.104	+0.063	+0.052
MUFT-4 heterogeneous			
GAM - I homogeneous →	-1.579	-0.749	+0.702
MUFT-4 homogeneous			

TABLE II-15-VIII. DANCOFF FACTORS

Core Code	Ref. 14	RP 325X 2-Region	RP 325X 3-Region
B 1.27 □S	0.250	0.22934	0.23418
B 1.27 △S	0.307	0.29903	0.30413
H 1.24 □A	0.365	0.35370	0.36910
H 1.24 □S	0.331	0.31674	0.32405
H 1.27 △A	0.425	0.42608	0.44094
H 1.27 △S	0.383	0.37908	0.38610
H 1.166 △A	0.576	0.58655	0.59809
H 1.166 △S	0.513	0.51512	0.52049
H 1.127 △A	0.652	0.66358	0.67280
H 1.127 △S	0.577	0.57939	0.58365
H 1.069 △S	0.690	0.69195	0.69407

factors in the table. Since the tables involve two parallel rods only, an approximate method was used for determining the effect of shielding by rods intervening between pairs of rods. The contribution for two rods partly shielded by intervening rods was determined by multiplying the C value for the two unshielded rods by d_1/d . Here d is the rod diameter and d_1 is the perpendicular distance between two lines drawn in a plane perpendicular to the rods, parallel to the line joining the axes of the rods, and tangent to the intervening rods. The Dancoff factors so calculated are listed in Table II-15-VIII along with values computed by revised versions of the RP 325X collision probability codes^{11, 12}. The two-region version evaluates C from Eq. (17) of Ref. 12, while the three-region version uses the three-region extension of Eq. (17).

Three-region values of C from the RP 325X code are a little larger than two-region ones. Differences between three- and two-region values are greater for Al clad fuel than for SS clad fuel, since the cross section of aluminum differs from that of water by more than the SS cross section differs. Comparison of the values based on Carlvik's tables and the RP 325X values shows that the Carlvik values tend to be a little lower than those from RP 325X for tight lattices, and a little higher for looser lattices. At any rate, the use of a Dancoff factor as in the GAM-I code implies that the

TABLE II-15-IX. RESONANCE INTEGRALS
FROM GAM-I OUTPUT, barns

Core Code	U-238 Absorption	U-235 Absorption	U-235 Fission
B 1.27 □S	19.104	271.48	168.96
B 1.27 △S	18.253	269.27	167.68
H 1.24 □A	16.686	279.65	173.80
H 1.24 □S	17.187	280.81	174.46
H 1.27 △A	15.793	277.42	172.52
H 1.27 △S	16.418	279.01	173.43
H 1.166 △A	14.660	270.85	168.79
H 1.166 △S	15.645	274.14	170.65
H 1.127 △A	13.297	265.99	166.04
H 1.127 △S	14.644	270.79	168.75
H 1.069 △S	12.730	263.10	164.40
Infinite Dilution	273.80	319.70	197.22
Smooth	5.167	92.85	78.33

TABLE II-15-X. INITIAL CONVERSION RATIOS
AND α^{25}

Core Code	α^{25} Calcu- lated	α^{25} Experi- mental	ICR Calculated	ICR Experimental
B 1.27 □S	0.2326	0.234	0.3418	0.297 ± 0.015
B 1.27 △S	0.2472	0.251	0.3958	0.330 ± 0.016
H 1.24 □A	0.2326	—	0.5045	—
H 1.24 □S	0.2376	0.244	0.5397	0.500 ± 0.005
H 1.27 △A	0.2451	—	0.5676	—
H 1.27 △S	0.2510	0.257	0.6100	0.581 ± 0.017
H 1.166 △A	0.2806	—	0.7645	—
H 1.166 △S	0.2897	0.296	0.8357	0.835 ± 0.005
H 1.127 △A	0.3031	—	0.8726	—
H 1.127 △S	0.3134	0.317	0.9635	0.913 ± 0.019
H 1.069 △S	0.3697	0.362	1.339	1.485 ± 0.034

clad is treated in the narrow resonance approximation. This may not be very well justified for broad low-lying resonances, especially for stainless steel clad.

Resonance integrals from the GAM-I output for U-238 absorption and U-235 absorption and fission are given in Table II-15-IX. These values include only contributions from resolved and unresolved resonances, and not the contribution from the smoothly varying part of the capture and fission cross sections. The decrease in the resonance integrals with decreasing lattice pitch is apparent in Table II-15-IX, along with the fact that resonance integrals for SS clad rods are higher than for Al clad rods, especially for the tighter lattices. All U-235 resonance integrals are greater than 80 % of the corresponding infinite dilution values, so that U-235 is not strongly self-shielded even in the tightest lattices.

INITIAL CONVERSION RATIO AND α^{25}

The initial conversion ratio is defined as the number of captures in U-238 per absorption in U-235 in a virgin

core. Thus it is given by

$$ICR = \left(N^{28} \sum_{i=1}^4 \sigma_{ci}^{28} \phi_i \right) / \left(N^{25} \sum_{i=1}^4 \sigma_{ai}^{25} \phi_i \right). \quad (6)$$

The capture-to-fission ratio in U-235 is given by:

$$\alpha^{25} = \left(\sum_{i=1}^4 \sigma_{ci}^{25} \phi_i \right) / \left(\sum_{i=1}^4 \sigma_{fi}^{25} \phi_i \right). \quad (7)$$

Values of ICR and α^{25} calculated using GAM-I P-1 and THERMOS constants are listed in Table II-15-X along with experimental values⁸ which are available only for stainless steel clad fuel. The α^{25} values agree reasonably well with experiment; but calculated initial conversion ratios are generally somewhat high, especially for the looser lattices.

DISCUSSION

Use of the Hanford versions of U-238 and U-235 in the GAM-I library greatly improved the agreement between theoretical and experimental bucklings. Agreement between theoretical and experimental values of various parameters ranges from good to rather poor. There are improvements which could be made in the theoretical treatment, especially in the resonance region. A treatment of resonance absorption involving neither the extreme NR nor NRIA approximations, but using numerical integration of the transport theory integral equations would be an improvement. Such a treatment could also allow for interference effects between resonances lying close together. The Breit-Wigner single-level formula is not adequate for fissionable isotopes, although parameters for multilevel formulae are available only over small energy ranges. This makes accurate treatment of resonance absorption in U-235 difficult. As mentioned above, the treatment of scattering by the clad in the narrow resonance approximation, which is implicit in the use of the Dancoff factor, is not always well justified. In determining the source in the THERMOS problems, the flux is assumed to have a $1/E$ energy dependence above the thermal cutoff. Such an assumption is not good for the tight lattices, although it is difficult to estimate the errors caused by it.

Some of the defects of the theoretical treatment have been mentioned above. However, it should be emphasized that, even with very good theoretical methods, the theoretical study of tightly packed light water moderated lattices is difficult. This is because non-thermal events are of considerable importance, and inaccuracies in cross section data produce uncertainties in the calculation. The large changes in reactivity obtained by substituting the Hanford U-238 and U-235 cross sections for the original ones in the GAM-I library illustrate this well.

REFERENCES

1. G. D. Joanou and J. S. Dudek, *GAM-I: A Consistent P-1 Multigroup Code for the Calculation of Fast Neutron Spectra and Multigroup Constants*, GA-1850 (June 1961).
2. H. C. Honeck, *THERMOS, A Thermalization Transport Code for Reactor Lattice Calculations*, BNL-5826 (September 1961).
3. E. M. Pennington, *Hi-C Uniform Lattice Calculations*, Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010 (January 1965), p. 44.
4. R. C. Liikala, *Updated RBU Basic Library*, HW-75716, Vol. 1, 2, 3 (May 20, 1963).
5. D. P. Moon, *Consistent Heavy-Atom Cross Section Evaluation for Plutonium Recycle Calculations*, ANL-6972 (December 1964).
6. A. R. Boynton, Q. L. Baird, J. M. Christenson, K. E. Plumlee, W. C. Redman, W. R. Robinson and G. S. Stanford, *High Conversion Critical Experiment*, Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010 (January 1965), p. 33.
7. R. L. Hellens and H. C. Honeck, *A Summary and Preliminary Analysis of the BNL Slightly Enriched Uranium, Water Moderated Lattice Measurements*, Technical Reports, Series No. 12: Light Water Lattices; International Atomic Energy Agency, Vienna (1962).
8. A. R. Boynton, Argonne National Laboratory (Private Communication).
9. G. D. Joanou, et al., *Nuclear Data for GAM-I Data Tape*, GA-2451, Vol. I, II, III (August 1961).
10. A. F. Henry, *54 Group Library for P-1 Programs*, WAPD-TM-224 (April 1960).
11. E. M. Pennington, *Cylindrical Lattice Collision Probability Codes, B692/RP*, ANL-6836 (February 1964).
12. E. M. Pennington, *Collision Probabilities in Cylindrical Lattices*, Nucl. Sci. Eng. **19**, 215, (June 1964).
13. F. T. Adler, G. W. Hinman and L. W. Nordheim, *The Quantitative Evaluation of Resonance Integrals*, Proc., 1958 Geneva Conference, **16**, Paper No. 1988, pp. 155-171.
14. Reactor Physics Constants, ANL-5800 2nd Edition, (July 1963), Table 4-25.

II-16. Thermal Disadvantage Factors in the Hi-C Cores¹

A. R. BOYNTON

As part of the High Conversion Critical Experiment (Hi-C), the thermal disadvantage factor (ξ) has been measured in five lattices which had water-to-uranium oxide volume ratios ranging from 0.96 down to 0.16. The data obtained in these cores indicate that ξ does not decrease monotonically with decreasing water-to-uranium oxide volume ratio but passes through a minimum and increases as the lattice pitch decreases beyond a certain value.

The fuel in these cores was 3.04 w/o enriched uranium in the form of 10.17 g/cm³ UO₂ pellets of 0.935-cm diameter loaded to 122-cm length in 1.057-cm o.d. clad tubes of 0.0496-cm thick stainless steel. Both square and triangular lattice pitches were used.

The disadvantage factor was obtained by the so-called integral technique. Foils of uranium-aluminum alloy of 17.5 w/o uranium, enriched to 92.75 w/o U-235, were shaped to measure the U-235 fission rate in representative portions of the fuel and moderator volumes of a unit cell. Cadmium ratios in both fuel and moderator sections were determined with 0.051-cm cadmium covers. Various foil thicknesses (0.0051 to 0.066 cm) were used. The foil activities were corrected for thermal self shielding. In the cadmium-covered irradiations, various masses of cadmium were used and the cadmium ratios corrected to zero cadmium mass.

The results of these measurements are given in

Table II-16-I. Also given are the results of two computations, the first being 27-energy group Nelkin kernel THERMOS results for an upper energy of 0.415 eV. The unit cells used in the calculations had an extra scattering region to approximate an isotropic return at the boundary. Both neutron density and U-235 fission ξ 's are given. The disadvantage factor was also calculated using a collision probability technique² and the flat-flux approximation. The collision probabilities were computed by the B692/RP code³ which considered a three-region cell (fuel, clad, and moderator). The cross sections used for this one-group calculation were averaged values obtained from the THERMOS outputs. This computation results in flux disadvantage factors.

Figure II-16-1 presents the calculated and experimental disadvantage factors versus water-to-oxide volume ratios. Two disagreements are noted in the comparison of the calculations with experimental data. First, except for the tightest lattice, THERMOS values are larger than experimental disadvantage factors. This was observed by H. Honeck⁴ when he found that theoretically predicted disadvantage factors were 3-5% higher than measured values, the 3% corresponding to the largest W/U lattices (about 4) and the 5% to the smallest W/U lattice (about 1.2) for which he had experimental data. This trend appears to continue through

TABLE II-16-I. THERMAL DISADVANTAGE FACTORS AND U-235 FISSION CADMIUM RATIOS IN HI-C LATTICES

Lattice		Experimental Results				Calculated ζ		
Pitch, cm	Water-to-oxide volume ratio	Fuel cadmium ratio	Moderator cadmium ratio	Bare moderator to bare fuel ratios	ζ	THERMOS number density	THERMOS U-235 fission	B692/RP flux
1.24 (square)	0.962	4.49 ± 0.11	5.19 ± 0.15	1.118 ± 0.009	1.160 ± 0.015	1.238	1.248	1.163
1.27 (triangular)	0.757	3.75 ± 0.08	4.14 ± 0.10	1.075 ± 0.009	1.112 ± 0.015	1.225	1.234	1.154
1.166 (triangular)	0.437	2.55 ± 0.06	2.72 ± 0.08	1.054 ± 0.004	1.096 ± 0.026	1.190	1.198	1.140
1.127 (triangular)	0.324	2.15 ± 0.05	2.33 ± 0.08	1.050 ± 0.008	1.120 ± 0.038	1.186	1.193	1.136
1.069 (triangular)	0.164	1.59 ± 0.03	1.69 ± 0.09	1.081 ± 0.041	1.189 ± 0.109	1.157	1.162	1.133

FIG. II-16-1. Thermal Disadvantage Factors in Hi-C Cores. ANL Neg. No. 112-4782.

the loosest Hi-C lattices. For these, the theory predicts disadvantage factors from 6 to 10 % larger than measured.

The second difficulty is that while the experimental values increase at values of water-to-oxide volume ratios lower than about 0.5, the calculations show the disadvantage factor to be monotonically decreasing with smaller lattice pitch. Some investigators have calculated a minimum for the disadvantage factor⁵⁻⁷ but their results seem to depend to a great extent upon the use of unhardened cross sections. This may suggest that the average cross sections obtained from THERMOS may be overly hardened. There may also exist difficulties associated with the reduction of the unit cell to

cylindrical geometry for the purpose of computation. On the other hand, some of these experimental results were obtained in central zones which possessed negative buckling,⁸ since critical assemblies of stainless steel-clad Hi-C fuel pins cannot be assembled at water-to-oxide volume ratios of less than about 0.5.

The small cell problem is a severe test of calculation techniques; it is an equally severe test of experimental techniques. At this point it is not clear whether the discrepancy between experiment and theory results from errors of experimental or computational origin.

REFERENCES

1. A. R. Boynton, *Experimental Thermal Disadvantage Factors in Some Very Undermoderated, 3% Enriched, Water-Uranium Oxide Lattices*, Trans. Am. Nucl. Soc. **8**, No. 1, 249 (1965).
2. E. M. Pennington, *Collision Probabilities in Cylindrical Lattices*, Nucl. Sci. Eng. **19**, 215 (1964).
3. E. M. Pennington, *Cylindrical Lattice Collision Probability Codes, B692/RP*, ANL-6836 (1964).
4. H. C. Honeck, *The Calculation of the Thermal Utilization and Disadvantage Factor in Uranium/Water Lattices*, Nucl. Sci. Eng. **18**, 49 (1964).
5. J. A. Thie, *Failure of Neutron Transport Approximations in Small Cells in Cylindrical Geometry*, Nucl. Sci. Eng. **9**, 286 (1961).
6. J. Hardy, Jr., J. J. Volpe, D. Klein, E. Schmidt and E. Gelbard, *Thermal Neutron Spectral and Spatial Distributions in Light Water Moderated Uranium Lattices*, WAPD-T-1594 (1963).
7. Z. Weiss and R. Stamm'ler, *Calculation of Disadvantage Factors for Small Cells*, Nucl. Sci. Eng. **19**, 374 (1964).
8. Q. L. Baird and A. R. Boynton, *Bucklings, Disadvantage Factors, and δ^{28} Measurements in Some Undermoderated Slightly Enriched Cores*, Trans. Am. Nucl. Soc. **6**, No. 2, 258 (1963).

II-17. The EBWR Plutonium Recycle Experimental Program

P. H. KIER

The EBWR Plutonium Recycle experimental program consists of two categories. The first category contains those experiments which will yield information required for safe operation of the reactor. These experiments, which will be performed with the full fuel loading, will measure control rod worths, boric acid worth, the temperature coefficient of reactivity, shutdown margins and will insure that the eight rod and the nine rod shutdown criteria are satisfied at all times. Also transfer function measurements will be made during the approach to power to determine the highest operating power for which the system is dynamically stable.

The fully loaded EBWR core consists of three types of fuel assemblies. The central region of 36 assemblies contains the mixed UO_2 - PuO_2 fuel. The composition of the mixed oxide fuel is nominally 98.5 a/o UO_2 (depleted to 0.2 a/o in U-235) and 1.5 a/o PuO_2 (enriched to 8 a/o in Pu-240). Surrounding the plutonium zone are two driver or shim zones of 60 assemblies initially containing UO_2 (enriched to 6 a/o in U-235) fuel pins. The outer 48 assemblies contain natural UO_2 fuel pins.

The second category contains those experiments which will yield information on the physics of assemblies whose fuel contains plutonium with a significant Pu-240 content. These experiments, which will be performed in a critical system composed of the plutonium zone and a water reflector, will be used to test and refine our calculational models. Experiments of this type which have been planned are: determination of unpoisoned, unrodded critical mass; critical mass as a function of boric acid concentration, boric acid worths, temperature coefficient of reactivity and foil activations. Special foils will be irradiated at the surface of the cladding of fuel pins to study the slow neutron spectrum, with emphasis on energies near the 0.3 eV resonance of Pu-239 and the 1.05 eV resonance of Pu-240.

Irradiation of the fuel during power operation will allow an investigation of the buildup of higher isotopes in plutonium initially containing 8 a/o Pu-240. Periodically, every 2000-3000 MWD/Tonne, the reactor will be shutdown and the composition of selected fuel pins and special pins will be sampled. While the reactor is shutdown, the natural uranium and shim zones will

be removed and experiments will be performed in the plutonium zone critical.

The program of experiments is given in the outline that follows.

Second Category Experiments:

I. Initial Plutonium Zone Critical

- A. Unpoisoned, unrodded, cold critical mass
- B. Critical mass vs boric acid concentration
- C. Full plutonium critical
 1. Eight and nine rod shutdown
 - a. Control rod worths
 - b. Critical bank height vs H_3BO_3 concentration
 2. Temperature coefficient of reactivity
 3. Foil activations

First Category Experiments:

II. Full Loading

- A. Loading of the shim and the natural uranium zones
 1. Eight and nine rod shutdown vs loading
 2. Rod worths vs loading
 3. Shutdown margin vs loading
- B. Temperature coefficient of reactivity
- C. Flux traverses

III. Approach to Power

- A. Training of reactor operators
- B. Transfer function measurements
 1. Dynamic stability vs power
 2. Effect of special riser on stability
- C. Power coefficient of reactivity

IV. Power Operation

- A. Irradiation of fuel to specified burnup
- B. Shutdown
 1. Check of shutdown margin
 2. Unload natural and shim zones

Second Category Experiments:

V. Plutonium Zone Critical

- A. Reactivity check (add shim elements if subcritical)
- B. Repeat I-C
- C. Sampling of plutonium pins
 1. Replace specified pins with fresh
 2. Effect of fresh pins on reactivity

Repeat II through V until specified burnup is attained.

II-18. Consistent Heavy-Atom Cross Section Evaluation for Plutonium Recycle Calculation

D. P. Moon

A calculational model employing the GAM,¹ DSN,² and SOFOCATE³ codes to obtain effective cross sections is being used for the physics evaluation of the EBWR Plutonium Recycle Experiment.⁴ Resonance absorption in U-238, U-235, and Pu-240 is calculated using resonance parameters in GAM. The resonance self shielding factor for Pu-239 is calculated using the IITRI resonance integral code.⁵

The GAM-I cross section library⁶ has been employed except for U-238, U-235, and Pu-240. The U-238 cross sections have been obtained from the Hanford RBU library.⁷ The U-235 and Pu-240 cross sections in the GAM library have been modified to include resonance parameters. The SOFOCATE thermal group calculation makes use of the hydrogen free gas moderator option with 2200 m/s cross section data for Pu-239 as shown in Table II-18-I. The values of $\sigma_f(E)$ for Pu-239 were obtained from BNL-325 (Supplement 1) as was also the shape of the α curve for Pu-239.

The above prescription has been used to analyze recent light-water-moderated PuO₂-UO₂ critical experiments performed at Hanford,⁸ Hanford subcritical experiments on Pu-Al alloy rods in H₂O,^{9, 10} and YANKEE critical experiments on 2.7 w/o enriched UO₂ in H₂O.^{11, 12}

The Hanford criticals on PuO₂-UO₂ were performed with fuel pins almost identical to those which will be used in the EBWR Plutonium Recycle Experiment. Four lattices were examined with H₂O/UO₂ ratios varying from 1 to 5. The fuel was arranged in a triangular pattern in pins of 0.472 cm radius, 123 cm height, and 0.0685 cm thick zirconium cladding. The isotopic enrichments were 1.366 a/o Pu²³⁹, 0.117 a/o Pu²⁴⁰, 0.011 a/o Pu²⁴¹, and 0.219 a/o U²³⁵.

The Hanford Laboratory's subcritical experiments with plutonium-aluminum fuel elements were performed over a wide range of H/Pu ratios and for two different plutonium enrichments (5 and 1.82 w/o). The experiments with the lower enrichment are the more accurate ($\pm 0.1\%$ uncertainty in k_{eff}) and, hence, extensive calculations have been carried out only for the 1.82 w/o plutonium-aluminum lattices. The fuel was arranged in a triangular pattern using pins of 0.635 cm radius, 111.3 cm height, and 0.0762 cm thick zirconium cladding. The plutonium by weight consisted of 93.97% U-239, 5.58% Pu-240, and 0.44% Pu-241. Five lattices were examined with H/Pu atom ratios varying from 630 to 1418.

TABLE II-18-I. CROSS SECTION DATA FOR Pu-239 (2200 m/s)

σ_a, b	σ_f, b	ν	η	α
1026	742	2.89	2.09	0.383

TABLE II-18-II. ERRORS IN CALCULATED MULTIPLICATION CONSTANTS FOR LIGHT-WATER PuO₂ AND UO₂ CRITICAL EXPERIMENTS

Lattice	H ₂ O/UO ₂	Error
PuO ₂ -UO ₂	1.10	-0.0035
	2.71	-0.0070
	3.79	-0.0050
	5.14	+0.0005
YANKEE (UO ₂)	1.05	+0.0070
	1.41	+0.0040
	1.86	+0.0025
	3.38	+0.0045

TABLE II-18-III. ERRORS IN CALCULATED MULTIPLICATION CONSTANTS FOR LIGHT-WATER Pu-Al SUBCRITICAL EXPERIMENTS

Lattice	Pu/H	Error	Experimental Uncertainty
1.82 w/o Pu-Al	632	+0.0070	± 0.0010
	1004	+0.0000	± 0.0010
	1423	-0.0030	± 0.0010

The YANKEE criticals consisted of six lattices with H₂O/UO₂ ratios varying from 1 to 5. For the purpose of this comparison the four lattices with the lowest H₂O/UO₂ ratios were chosen. The fuel was arranged in pins of 0.381 cm radius, 122 cm in height, and 0.0407 cm thick stainless steel cladding.

The multiplication constants of the lattices were determined using four-group, one-dimensional diffusion theory calculations.¹³

The calculational results are in good agreement with experiment for all lattices. Errors in the calculated multiplication constants for the various lattices are summarized in Tables II-18-II and II-18-III. Details of the calculation together with graphical comparisons of the experimental and calculational results are given in ANL-6972.¹⁴ Further details of the physics calculational model for the EBWR plutonium experiment are given in ANL-7019.¹⁵

REFERENCES

1. G. D. Joanou and J. S. Dudek, *GAM-1: A Consistent P₁ Multigroup Code for the Calculation of Fast Neutron Spectra and Multigroup Constants*, GA-1850 (June 1961).
2. B. Carlson, C. Lee and J. Worlton, *The DSN and TDC Neutron Transport Codes*, LAMS-2346 (February 1960).
3. H. Amster, *The Calculation of Thermal Constants Averaged Over a Wigner Wilkens Flux Spectrum: Description of the SOFOCATE Code*, WAPD-TM-39, (January 1957).
4. B. J. Toppel, *EBWR Plutonium Recycle Experiment*, Reactor Physics Annual Report July 1, 1963 to June 30, 1964, ANL-7010 (January 1965) pp. 69-73.
5. D. A. Klopp and W. E. Zagotta, *Resonance Integral Determination*, ARF Progress Report A-1918-8 (March 1962).
6. G. D. Joanou, et al., *Nuclear Data for GAM-1 Data Tape*, GA-2451 (Vol. 1 and 2) (August 1961).
7. R. C. Liikala, *Updated RBU Basic Library*, Vol. 1, 2, 3, HW-75717 (1963).
8. L. E. Schmid, et al., *Experiments PuO₂-UO₂ Fuel Elements in Light Water*, HW-81659 (April 1964).
9. V. I. Neeley, R. C. Lloyd and E. D. Clayton, *Neutron Multiplication Measurement with Pu-Al Alloy Rods in Light Water*, HW-70944 (August 1961).
10. W. P. Stinson and L. C. Schmid, *Subcritical Measurements with 1.8 wt % Pu-Al Rods in Light Water*, HW-74190 (July 1962).
11. P. W. Davison, et al., *YANKEE Critical Experiments—Measurements on Lattices of Stainless Steel Clad Slightly Enriched Uranium Dioxide Fuel Rods in Light Water*, YAE-94 (April 1959).
12. V. E. Grob, et al., *Multi-Region Reactor Lattice Studies—Results of Critical Experiments in Loose Lattices of UO₂ Rods in H₂O*, WCAP-1412 (March 30, 1960).
13. M. K. Butler and J. M. Cook, *UNIVAC Programs for the Solution of One-Dimensional Multigroup Reactor Equations*, ANL-5437 (October, 1956).
14. D. P. Moon, *Consistent Heavy-Atom Cross-Section Evaluation for Plutonium Recycle Calculations*, ANL-6972 (December, 1964).
15. B. J. Toppel and R. Avery, *Some Physics Aspects of the Plutonium Experiment in EBWR*, ANL-7019 (April, 1965).

II-19. Determination of Capture-to-Fission Ratio for Pu-239 and Pu-241

H. P. ISKENDERIAN

In connection with the Plutonium Recycle Program, it is proposed to carry out some physics irradiation experiments in EBWR for the determination of α^{49} and α^{41} , especially in the epithermal range. α is the ratio of capture-to-fission cross sections; 49 and 41 designate Pu-239 and Pu-241. In this memorandum some equations are derived relating α to quantities measurable in irradiation experiments. Methods of experimentation are described^{1, 2} and their estimated accuracies are given in what follows.

Evaluations are made of the differences in the values of α^{49} corresponding to irradiation of Pu-foils placed on different types of fuel elements (i.e., 1.5 a/o Pu-239 enriched in U-238, 6 a/o U-235 in U-238, and natural uranium elements). Differences in the values of α^{49} are also evaluated corresponding to irradiation of Pu-239 foils placed in the moderator-coolant between the fuel rods, on the surface of the fuel rod, and homogenized in the fuel rod.

Our discussion refers specifically to Pu-239; it may, however, readily be extended to apply to Pu-241.

EQUATIONS FOR DETERMINATION OF α^{49}

The isotopic differential equations for Pu-239 and Pu-240 are:

$$\frac{dN^{49}}{d\tau} = -N^{49}\sigma_a^{49} \quad (1)$$

$$\frac{dN^{40}}{d\tau} = N^{49}\sigma_c^{49} - N^{40}\sigma_a^{40}, \quad (2)$$

where the superscript 40 designates Pu-240 and

$\tau \equiv$ neutron flux $\times$ time

$\sigma_a \equiv$ total absorption cross section

$\sigma_c \equiv$ capture cross section

$N \equiv N(\tau) \equiv$ atomic density

The solutions of Eqs. (1) and (2) are

$$N^{49} = N_0^{49} \exp(-\sigma_a^{49}\tau) \quad (3)$$

and

$$N^{40} = N_0^{40} \exp(-\sigma_a^{40}\tau) + N_0^{49}\sigma_c^{49} \frac{\exp(-\sigma_a^{49}\tau) - \exp(-\sigma_a^{40}\tau)}{\sigma_a^{40} - \sigma_a^{49}}, \quad (4)$$

where

$$N_0 \equiv N(0).$$

For small to moderate burnups, Eq. (3) may be put into the form:

$$N_0^{49} - N^{49} \approx \frac{N_0^{49} + N^{49}}{2} (\sigma_a^{49}\tau). \quad (5)$$

For burnups as high as 25%, Eq. (5) is in agreement with Eq. (3) within 0.4%.

For small burnups, Eq. (4) may be put into the form

$$N^{40} - N_0^{40} = N_0^{49} \sigma_c^{49} \tau (1 - \sigma' \tau + \sigma'' \tau^2), \quad (6)$$

here

$$\sigma' \equiv \frac{\sigma_a^{40} + \sigma_a^{49}}{2!}; \quad \sigma'' \equiv \frac{(\sigma_a^{40})^2 + \sigma_a^{40} \sigma_a^{49} + (\sigma_a^{49})^2}{3!}. \quad (7)$$

METHODS OF DETERMINING α^{49}

Methods have been referred to in current literature^{1, 2} for the determination of α^{25} in a thermal reactor and α^{25} , α^{33} and α^{49} in a fast reactor. The values of $\alpha_{\text{epicadmium}}$ are our main interest and the evaluations refer to very thin foils or capsules of Pu-239. The methods may readily be applied to Pu-241.

DETERMINATION OF α^{49} FROM SMALL EXPOSURES OF FOILS IN TERMS OF $(\Delta N^{40}/N^{49})$, N^{137} , N^{49} and Y^{137} :

$$\begin{aligned} N_0^{49} - N_\tau^{49} &\equiv \Delta N^{49} = \Delta N_{\text{capture}}^{49} + \Delta N_{\text{fission}}^{49} \\ &= \Delta N_{\tau m}^{40} + N_{\tau m}^{137}/Y^{137} \end{aligned} \quad (8)$$

FIG. II-19-1. Accuracy of Mass Spectrometric Measurements (shown by shaded area) ANL Neg. No. 112-5608.

where the subscript τm refers to exposure " τ " but without accounting for losses of the elements by absorption. N^{137} and Y^{137} refer to concentration and yield of Cs^{137} , a fission product of Pu-239.

With sufficiently low burnup of Pu-239 ($\sigma' \tau \ll 1$), Eq. (6) yields,

$$N^{40} - N_0^{40} \equiv \Delta N^{40} \approx N_0^{49} \sigma_c^{49} \tau \text{ and } \Delta N^{40} \approx \Delta N_{\tau m}^{40}. \quad (9)$$

Since σ^{137} is also very small, it follows from Eq. (8) that

$$\alpha^{49} = \frac{\Delta N^{40}/(1 - \sigma' \tau)}{N^{137}/Y^{137}} \quad (10)$$

where $\sigma' \tau$ is now a small correction term.

Equation (10) may also be written as

$$\alpha^{49} = \frac{(\Delta N^{40}/N^{49}) N^{49} (1 - \sigma' \tau)}{N^{137}/Y^{137}}. \quad (11)$$

Determination of α^{49} may be obtained by

- Mass-spectrometric measurement of the isotopic ratio $\Delta N^{40}/N^{49}$
- Radiochemical measurement of the concentration N^{137}
- Chemical analysis of N^{49} .

Accuracy of α^{49} given by Eq. (11) may be obtained from the expression

$$\left| \frac{\delta \alpha^{49}}{\alpha^{49}} \right|_{\max} = \left| \frac{\delta \Delta N^{40}}{\Delta N^{40}} \right| + \left| \frac{\delta N^{137}}{N^{137}} \right| + \left| \frac{\delta N^{49}}{N^{49}} \right| + \left| \frac{\delta Y^{137}}{Y^{137}} \right|, \quad (12)$$

where δx = error in the quantity x .

For the determination of α_{epi}^{49} , which is the principle interest in the measurements, consider an irradiation exposure of 30 days in the plutonium and shim zone of EBWR with a plutonium loading. From the constants of Tables II-19-Ia and II-19-Ib and Eq. (9), irradiation in the plutonium zone of the core is given by

$$\frac{\Delta N^{40}}{N_0^{49}} = \sigma_c^{49} \tau|_{epi} \approx (5.88 \times 10^{-24})$$

$$\cdot (2.26 \times 10^{14} \times 8.64 \times 10^4 \times 30) = 0.00344.$$

TABLE II-19-Ia. CROSS SECTION AND NEUTRON FLUX DATA OBTAINED FOR PLUTONIUM LOADED CORE OF EBWR AT OPERATING POWER OF 40 MW

	Group: 1	2	3	4	5	6	7 (Thermal)
E_{low}	0.183 MeV	5.53 keV	275. eV	6.48 eV	1.4400 eV	0.532 eV	0
σ_c^{49}, b	0.1157	0.74353	2.90694	35.231	3.4037	23.165	315.60
σ_f^{49}, b	1.8336	1.99177	4.5048	37.7458	11.7939	51.2881	523.6
$\phi_{Pu \text{ zone}}, \frac{n \cdot 10^{-13}}{\text{cm}^2 \text{ sec}}$	9.245	3.830	2.343	2.493	0.892	0.522	3.4867
$\phi_{shim \text{ zone}}, \frac{n \cdot 10^{-13}}{\text{cm}^2 \text{ sec}}$	4.763	1.941	1.61	1.12	0.39	0.23	1.085
σ_a^{40}, b	1.11757	0.4455	2.7994	42.139	61.913	3968.	199.4

The above values of neutron flux must be multiplied by the factor 1.17 to correspond to flux values at 40 MW power level.

TABLE II-19-Ib. AVERAGED CROSS SECTION DATA IN THE EPICADMIUM ENERGY RANGE FOR PLUTONIUM LOADED CORE OF EBWR AT OPERATING POWER OF 40 MW

	Pu Zone	Shim Zone	Natural U Zone
$\sigma_c^{49} = \frac{\sum_{i=1}^6 \phi_{epi} \sigma_{a epi}^{49}}{\sum_{i=1}^6 \phi_{epi}}, b$	5.8825	5.370	6.313
$\sigma_f^{49} epi, b$	8.617	7.972	9.214
$\sigma_a^{40} epi, b$	119.5	103.4	136.5
α_{epi}^{49}, b	0.6826	0.6736	0.6852
$\phi_{th}(40 \text{ MW}), \frac{n}{\text{cm}^2 \text{ sec}}$	4.086×10^{13}	1.272×10^{13}	4.345×10^{13}
$\sum_{i=1}^6 \phi_{epi}, \frac{n}{\text{cm}^2 \text{ sec}}$	2.26×10^{14}	1.13×10^{14}	0.63×10^{14}

^a $\phi_{th}(\text{reactor}) = 3 \times 10^{13} \frac{n}{\text{cm}^2 \text{ sec}}$ at 40 MW power level.

TABLE II-19-II. ACCURACY STATEMENT OF TERMS APPEARING IN EQUATIONS FOR α^{49}

- 1) Mass Spectrometric Analysis (see Fig. II-19-1)
(By sample dilution and addition of known amount of Pu-242)
Accuracy of measurements = 0.5%^a
- 2) Radiochemical Analysis $\delta N^{137}/N^{137} = 1$ to 2%
Count measurements—statistical error of $\sim 1\%$
error of standard $\sim 1\%$
- 3) Chemical Analysis of N^{49} (unirradiated) $\pm 0.1\%$

^a By taking special precautions this may be reduced to 0.2%.

The magnitude of each term in Eq. (12) may now be determined from the accuracy statements of Table II-19-II.

$$\frac{\delta \alpha_{epi}^{49}}{\alpha_{epi}^{49}} \simeq 0.005 + 0.02 + 0.001 + 0.07 = 0.096$$

The largest contribution to this maximum estimated error is that due to uncertainty (7.%) in the fission yield term, Y^{137} . It is assumed that the active isotope Cs^{136} , which is produced from the stable fission product Cs^{135} by (n, γ) reaction, has duly decayed.

DETERMINATION OF α^{49} FROM SMALL EXPOSURES OF FOILS IN TERMS OF $(\Delta N^{40}/N^{49})$, N^{137} , N^{49} AND α_{th}^{49}

α_{epi}^{49} may be obtained by excluding Y^{137} in Eqs. (11) and (12). From Eq. (11), irradiation of two sets of foils with and without cadmium covers yields

$$\alpha_{epi}^{49} = \frac{N_{epi}^{40}}{N_{epi}^{137}} Y_{epi}^{137};$$

$$\alpha_{th}^{49} = \frac{N_{th}^{40}}{N_{th}^{137}} Y_{th}^{137} = \frac{N^{40} - N_{epi}^{40}}{N^{137} - N_{epi}^{137}} Y_{th}^{137},$$

where N^{40} and N^{137} refer to foils with no cadmium covers. Assuming $Y_{epi}^{137} \approx Y_{th}^{137}$,

$$\alpha_{epi}^{49} = \left[\frac{(N^{137}/N_{epi}^{137}) - 1}{(N^{40}/N_{epi}^{40}) - 1} \right] \alpha_{th}^{49} \quad (13)$$

or

$$\alpha_{epi}^{49} = \left[\frac{(N^{137}/N_{epi}^{137}) - 1}{\left\{ \frac{N^{40}}{N^{49}} / \left(\frac{N_{epi}^{40}}{N_{epi}^{49}} \right) \left(\frac{N_{epi}^{49}}{N^{49}} \right) \right\} - 1} \right] \alpha_{th}^{49}. \quad (14)$$

The accuracy of α_{epi}^{49} may be obtained from:

$$\left| \frac{\Delta \alpha_{epi}^{49}}{\alpha_{epi}^{49}} \right|_{\max} = \left| \frac{\delta N^{40}}{N^{40}} \right| + \left| \frac{\delta N_{epi}^{40}}{N_{epi}^{40}} \right|$$

$$\left(1 - \frac{N_{epi}^{40}}{N^{40}} \right)$$

$$+ \left| \frac{\delta N^{137}}{N^{137}} \right| + \left| \frac{\delta N_{epi}^{137}}{N_{epi}^{137}} \right| + \left| \frac{\delta \alpha_{th}^{49}}{\alpha_{th}^{49}} \right|$$

$$\left(1 - \frac{N_{epi}^{137}}{N^{137}} \right)$$

Estimated accuracy may be obtained from Eq. (15), Tables II-19-Ia, II-19-Ib and accuracy statements of Table II-19-II:

$$\left| \frac{\Delta \alpha_{epi}^{49}}{\alpha_{epi}^{49}} \right|_{\max} = \left| \frac{0.005 \times 2}{1 - 0.0935} \right| + \left| \frac{0.02 \times 2}{1 - 0.0836} \right|$$

$$+ \frac{\delta \alpha_{th}^{49}}{\alpha_{th}^{49}} = 0.055 + \frac{\delta \alpha_{th}^{49}}{\alpha_{th}^{49}}.$$

A more probable value of this estimated maximum error should be:

$$\left| \frac{\Delta \alpha_{epi}^{49}}{\alpha_{epi}^{49}} \right|_{\text{probable}} = \frac{0.005 \times \sqrt{2}}{1 - 0.0935} + \frac{0.02 \times \sqrt{2}}{1 - 0.0836}$$

$$+ \frac{\delta \alpha_{th}^{49}}{\alpha_{th}^{49}} = 0.039 + \frac{\delta \alpha_{th}^{49}}{\alpha_{th}^{49}}.$$

Adequate decay of the active isotope Cs^{136} is again assumed to have taken place, as explained under the preceding method of measurement.

DETERMINATION OF α^{49} FROM IRRADIATION OF FOILS WITH SHORT AND LONG EXPOSURES

Determination of α^{49} may be made by mass-spectrometric analysis of two sets of foils with short and long exposures, respectively.

For a foil with long exposure, we obtain from Eq. (5):

$$\sigma_a^{49} \tau \approx 2 \frac{N_0^{49} - N^{49}}{(N_0^{49} + N^{49})}. \quad (16)$$

For a second foil with short exposure, we obtain from Eq. (6):

$$\sigma_a^{49} \tau' = \Delta N_{\tau'}^{40}/N_0^{49}, \quad (17)$$

here

$$\Delta N_{\tau'} \equiv N(\tau') - N(0). \quad (18)$$

From Eqs. (16) and (17), we obtain:

$$\alpha^{49} = \frac{c_0 \Delta N_{\tau'}^{40}/N_0^{49}}{(\Delta N_{\tau'}^{49}/N_{av}^{49}) - c_0(\Delta N_{\tau'}^{40}/N_0^{49})}, \quad (19)$$

where

$$c_0 = \tau/\tau'$$

and

$$N_{av}^{49} = (N_0^{49} + N^{49})/2.$$

From Eq. (19), we obtain:

$$\left| \frac{\delta \alpha^{49}}{\alpha^{49}} \right| = \frac{\left| \frac{\delta(\Delta N_{\tau'}^{49}/N_{av}^{49})}{\Delta N_{\tau'}^{49}/N_{av}^{49}} \right| + \left| \frac{\delta[\Delta N_{\tau'}^{40}/N_0^{49}]}{(\Delta N_{\tau'}^{40}/N_0^{49})} \right| + \frac{\delta c_0}{c_0}}{1 - c_0 \frac{\Delta N_{\tau'}^{40}/N_0^{49}}{\Delta N_{\tau'}^{49}/N_{av}^{49}}}. \quad (20)$$

The estimated accuracy may be obtained from Eq. (20), Tables II-19-Ia, II-19-Ib and accuracy statements of Table II-19-II. Length of irradiation times may be obtained from data of Table II-19-Ib and Eq. (5). For epicadmium α_{epi}^{49} :

$$\left| \frac{\delta \alpha_{epi}^{49}}{\alpha_{epi}^{49}} \right|_{\max} = \frac{0.05 + 0.007 + \delta c_0/c_0}{1 - 15 \times \frac{0.0035}{0.10}} = 0.12 + 2.1 \frac{\delta c_0}{c_0}$$

for 10 % burnup at an irradiation time of 336 days;

$$\left| \frac{\delta \alpha_{epi}^{49}}{\alpha_{epi}^{49}} \right|_{\max} = \frac{0.033 + 0.007 + \delta c_0/c_0}{1 - 15 \times \frac{0.0035}{0.15}} = 0.062 + 1.5 \frac{\delta c_0}{c_0}$$

for 15 % burnup at an irradiation time of 493 days;

$$\left| \frac{\delta \alpha_{epi}^{49}}{\alpha_{epi}^{49}} \right|_{\max} = \frac{0.025 + 0.007 + \delta c_0/c_0}{1 - 15 \times \frac{0.0035}{0.20}} = 0.043 + 1.4 \frac{\delta c_0}{c_0}$$

for 20 % burnup at an irradiation time of 644 days.

These calculations do not allow for changes in the neutron spectrum which occurs with such long irradiation times.

α^{49} IN FUEL ELEMENT COMPARED WITH α^{49} IN COOLANT MODERATOR

Whether there would be appreciable differences in the results of measurements if test samples of foils were placed on the surface of fuel elements or were placed in the coolant between fuel rods (such as a U-tube) is an important consideration. Placement in the U-tube allows easy insertion and removal. It may, however, smooth out important effects of the resonance structure of the fuel elements.

TABLE II-19-III. CALCULATED VALUES OF α_{epi}^{49} (BY RIFF RAFF CODE) DETERMINED IN COOLANT OR ON FUEL ELEMENTS INDICATED

Type Fuel Element	In Coolant-Moderator	On Surface of Fuel Rod	Average in the Fuel Rod
1.5% Pu	0.7819	0.8695	0.8630
6% cur. U-235	0.7819	0.8967	0.8966
Natural U	0.7819	0.8852	0.8850

The above results are obtained by multigroup calculations in the neutron energy range of 0.532 to 90 eV. Included are the 22 resonance lines of Pu-239 (0.296 to 85.5 eV), 7 resonance lines of U-238 (6.68 to 90 eV), and 58 resonance lines of U-235 (-1.40 to 47.0 eV).

TABLE II-19-IV. COMPOSITION OF PLUTONIUM ISOTOPES AVAILABLE AT ORNL (IN SMALL AMOUNTS) FOR RESEARCH PURPOSES

Pu-239	Pu-241	Pu-242	
		Batch Pu-242-114B	103A
Pu-239 99.97%	Pu-239 0.81%	0.017%	1.96%
Pu-240 0.03%	Pu-240 2.89%	0.039%	3.96%
Pu-241 50 ppm	Pu-241 95.46%	0.064%	2.61%
Pu-242 —	Pu-242 0.84%	99.88%	91.47%

To estimate these differences, the α_{epi}^{49} in the coolant was evaluated from resonance integral data ranging from 0.296 eV to 85.5 eV, which included all of the 22 resolved resonances in Pu-239. The contribution due to the 0.296 eV line was limited to that above 0.532 eV, assuming a 1/E flux.

α^{49} was calculated as a function of position in the lattice of the test foil, by use of the RIFF RAFF code.³ Interference due to U-238 and U-235 resonance lines was also included. The cut off of energy was 0.532 eV. Calculations of α_{epi}^{49} were made for the three types of elements to be used in the plutonium recycle core, that is for 1.5 a/o enriched plutonium, 6 a/o enriched U-235, and natural U elements. The results of our calculations are listed in Table II-19-III.

It will be noted from the data of Table II-19-III that:

- 1) α_{epi}^{49} calculated from irradiation of foils in the coolant-moderator is 11 % and 15 % lower than the corresponding values of α_{epi}^{49} on the surface of a U-238 rod enriched with 1.5 a/o Pu-239 and with 6 a/o U-235, respectively.
- 2) α_{epi}^{49} calculated on the surface of a U-238 rod enriched with 6 a/o U-235 is 3.1 % greater than the corresponding value for a U-238 rod enriched with a 1.5 a/o Pu-239.
- 3) The effect of the plutonium resonance self-shielding varies with position in the lattice. As

the data of Table II-19-III show, the α_{epi}^{49} detected by a foil on the surface of a 1.5 a/o enriched Pu element is 7.5% greater than the average value in the rod. The U-238 resonances also perturb the flux as the data in Table II-19-II show; the α_{epi}^{49} detected with a foil in the moderator (actually in a U-tube) would be about 0.1 lower than that measured at a fuel rod location.

CONCLUSIONS

It appears from analysis of the three methods of measurement of α^{49} that irradiation of foils with and without cadmium covers for a moderate length of time (30–60 days), would be most suitable for determining α^{49} . Irradiation of the foils yields α^{49} in terms of Y^{137} or α_{th}^{49} , in addition to other terms.

Both methods involve mass spectrometric measurements of the isotopic ratio $\Delta N^{40}/N^{49}$, which can be obtained with a relatively good accuracy ($\pm 0.5\%$ or better) for the burnups considered. Both methods involve a radiochemical analysis of Cs^{137} , a fission product currently used in such studies. Both methods involve a waiting period for the decay of the active isotope Cs^{136} ($T_{1/2} = 15$ days) as a consequence of the $Cs^{135}(n,\gamma)Cs^{136}$ reaction. Cs^{135} is a stable fission product of Pu-239.

Long term irradiation would be desirable to reduce the effect of Cs^{136} . High burnups would, however, increase the magnitude of correction terms [see Eq. (6)]. Irradiation times of 30–60 days appear to be more suitable.

This analysis should apply at least equally as well for Pu-241, since the daughter Pu-242 has a much lower absorption cross-section than Pu-240.

This method of analysis for long and short term ir-

radiation of foils does not account for spectral changes resulting from long term irradiation. To account for such changes corrections must be made, based on additional activation analysis.

The calculations indicate that measurements of α^{49} from activation data for foils placed in a coolant-moderator will be appreciably different from those for foils placed on the surfaces of fuel rods. This is probably due to significant dips in resonance flux at the rod surfaces. Calculations also indicate that α^{49} on the surface of a U-238 rod enriched with 1.5 a/o Pu-239 is 3.1% lower than that on a U-238 rod enriched with 6 a/o U-235.

In order to obtain accurate data for the determination of α^{49} and α^{41} and to check our calculations, especially in the epithermal range, it is proposed to carry out a set of irradiation experiments with very thin foils or capsules. The foils will be placed on the surfaces of fuel elements enriched with 1.5 a/o Pu-239 or with 6 a/o U-235. Additionally, irradiation of foils in the moderator-coolant between the fuel rods will be made.

Composition of the foil materials is given in Table II-19-IV.

REFERENCES

1. C. E. Crouthamel, D. C. Stuepegia and C. M. Stevens, *Capture to Fission Ratios of U-233, U-235, U-238 and PU-239 in EBR-I, Mark III*, Nucl. Sci. Eng., **21**, 179–185 (1965).
2. D. E. Conway, H. D. Cook and S. B. Gunst, *Measurement of the Epi-Cadmium Capture-to-Fission Cross Section Ratio in U-235*, Nucl. Sci. Eng., **22**, 20–23 (1965).
3. P. H. Kier, *RIFR RAFF, A Program for Computation of Resonance Integrals in a Two-Region Cell*, ANL-7033 (to be published).

Section III

Fast Reactors

This chapter summarizes analytical and experimental work related to fast reactor development. While most of the studies treat fast power breeder reactors, some work is directed toward the development of a fast spectrum nuclear rocket. Many neutronic problems are common in these two areas of application.

Individual papers fall into a number of related topical categories: General Fast Reactor Research and Development, Support of Project Activities, Fast Reactor Safety, and General Studies of Fast Reactor Concepts with related Evaluations.

III-1. Physics Parameters and Sodium Void Coefficients In a 950-Liter Pancake Core

F. H. HELM, W. Y. KATO, G. MAIN and G. K. RUSCH

INTRODUCTION

In the design of large fast sodium-cooled power reactors it is important for safety reasons to maintain a negative sodium void coefficient. Depending on core shape and neutron spectrum, this coefficient may either be positive or negative. The sodium void effect has been measured and calculated for several previous assemblies of Argonne's ZPR-3 and ZPR-6⁽¹⁻³⁾ and extensive studies have been performed by a number of industrial laboratories (see for example Ref. 4). In the work reported here, the effect was measured on Assembly No. 3 of ZPR-6, a pancake-shaped core with a large axial leakage. Some of the results are compared with identical measurements in Assembly No. 2, which had the same composition but a length-to-diameter ratio of approximately 1:1.

GENERAL REACTOR PARAMETERS

CORE SIZE AND COMPOSITION

Assembly No. 3 of ZPR-6 was a 950-liter uranium carbide core with an axial length of 50.8 cm and a diameter of 155 cm. It had an axial and radial U-238 blanket of 30 cm thickness. The outline of the assembly is shown in Fig. III-1-1; typical drawer loadings are shown in Fig. III-1-2.

A shortage in sodium cans made it necessary to use aluminum in place of sodium near the axial blanket in about 870 of the 1250 core drawers. In order to simulate the characteristics of sodium as closely as possible, perforated aluminum plates were used which had about 45% of the aluminum's natural density. The change in critical mass due to this replacement was calculated to be smaller than 0.1%. In the sodium void experiments, only drawers with full length sodium columns were used.

Core and blanket composition are shown in Table III-1-I. The values given for sodium were calculated under the assumption that all sodium columns extend through the whole length of the core, including the space which was actually occupied by aluminum.

CRITICAL RADIUS AND MASS

Calculated and measured values of the critical parameters are given in Table III-1-II.

In all calculations, the composition was assumed to be fixed and a search was made for the critical radius. When a one-dimensional code was used, this had to be done in three steps. First, the critical radial buckling for the axially reflected core was determined in a slab calculation. Then the critical length for an axially bare core was calculated for this radial buckling. These first two steps yield the axial reflector savings which are used in a cylindrical calculation of the critical radius.

Two-dimensional calculations were performed with the four-group code PDQ and with the multigroup code CANDID. When PDQ was used, the 22-group cross section set 224 was collapsed into four groups. This was done in a one-dimensional calculation using MACH 1.

The differences in the results for the critical radius and mass were enhanced by the very low reactivity worth of the core material at the radial edge of the core. In terms of k_{eff} , the agreement between the different calculations is quite good, as shown in the last column of Table III-1-II.

Comparing the calculated values with the results of the measurements, one has to take in account that the calculations do not include the effects of the heterogeneous material arrangement, the irregular boundary, and the central gap which, when they are combined, are estimated to cause a decrease in critical radius of 2 to 3 cm. The experimental values given in Table III-1-II are the uncorrected measured results.

CONTROL AND SAFETY RODS

Assembly No. 3 had ten fuel loaded dual purpose (control-safety) rods and twelve B-10 safety blades. In order to achieve sufficient shut-down reactivity, the dual purpose rods were each loaded with five fuel columns during the initial start-up of the assembly. After criticality was achieved, this number was reduced to four, as measurements showed that this could be done with safety. To reduce spectral distortion, the two drawers horizontally adjoining each dual purpose rod were loaded with only one column of fuel.

Rods were calibrated by positive period, digital-rod-travel and rod-drop techniques. Total rod worths are shown in Table III-1-III. For the calculations, the withdrawal of a rod was simulated by reducing the density of the core composition in a cylindrical shell.

TABLE III-1-I. COMPOSITION FOR ASSEMBLIES 2 AND 3 OF ZPR-6

	Core			Blanket		
	Atom density, atoms cm ⁻³ × 10 ²⁴	Avg. density, g cm ⁻³	YOM volume fraction ^a	Atom density atoms cm ⁻³ × 10 ²⁴	Avg. density, g cm ⁻³	YOM volume fraction
U-235	0.002285	0.8915	0.04760	8.104 × 10 ⁻⁵	0.03162	0.001688
U-238	0.009853	3.892	0.2052	0.03993	15.777	0.8318
C	0.01290	0.2573	0.1540			
Na	0.007910	0.3019	0.3595			
Fe	0.01038	0.9621	0.1226	0.004223	0.3916	0.04986
Cr	0.002983	0.2575	0.03724	0.001214	0.1048	0.01516
Ni	0.001390	0.1355	0.01522	0.000566	0.0552	0.0062
SS-304	0.01475	1.355	0.17506	0.006003	0.552	0.07122

^a Corrected volume fractions for use with the macroscopic cross sections by Yiftah, Okrent and Moldauer.²

FIG. III-1-1. Outline of Assembly 3 of ZPR-6. ANL Neg. No. 112-5712.

GAP WORTH

The reactivity worth of small gaps between the reactor halves was measured by establishing critical rod positions for different table separations.

There was some difficulty in defining the gap width as the distance between the reactor surfaces varied across the interface by about 0.2 mm. An effective gap width was defined as the average separation between the halves at the four corners of the matrix. The separation was measured by linear potentiometers which were mounted on the matrix support structure. Their readings were normalized to zero when the halves were fully closed. Measurements were made for air gaps and for gaps filled with stainless steel. Results are shown in Fig. III-1-3.

For large gaps, the worth was determined by the subcritical source multiplication technique. In this method, the subcriticality of the system is determined from the counting rates of neutron detectors in or near the reactor. In the limit for small table separations when the effect of higher harmonics in the flux distribution is negligible, the inverse counting rate is proportional to $1 - k_{eff}$. The normalization factor was found in a reference run in which the subcriticality was known from the position of calibrated control rods.

Figure III-1-4 shows the normalized inverse counting rate versus the table separation for a fission counter in the core center and a BF₃ counter outside of the blanket. For each gap size two different neutron source distributions were used. One distribution consisted only of the spontaneous fission neutrons from the uranium in the reactor, the other was generated by introducing the start-up source to a position near the edge of the core. The fission counter inside the core showed an approximately linear relationship between the inverse counting rate and the gap width and no significant influence of the source distribution. The curves taken with the BF₃ counters outside the blanket, however, flatten out for larger gap widths. The disagreement between the results for different counter positions and source distributions is probably due to the presence of higher harmonics and to the increase in the counting rate of the outside counter caused by neutrons leaking from the gap. Both of these effects were largely elimi-

FIG. III-1-2. Typical Loading of the Core-Section of Drawers Without (Top) and With (Bottom) Replacement of Sodium-Cans by Aluminum and Stainless Steel. (Figure shows loading for stationary half. In the movable half, the sequence of materials across the drawer is reversed). ANL Neg. No. 112-5702.

TABLE III-1-II. CRITICAL PARAMETERS

Code	Cross Sec- tion Set	Critical Radius, cm	Critical Mass, kg	Calculated k_{eff} for $R = 77.86$ cm
RE 122 ⁽⁶⁾	635 ⁽⁶⁾	77.6	855	—
MACH 1 ⁽⁷⁾	199 ^(8, 9)	75.6	813	1.0047
MACH 1	224 ⁽¹⁰⁾	79.8	906	0.9962
PDQ ⁽¹¹⁾	224	73.5	769	1.0087
CANDID ⁽¹²⁾	224	78.8	887	0.998
Measured Values		77.5	850	—

TABLE III-1-III. TOTAL ROD WORTH

Rod	Method	Worth, Δk
Dual Purpose Rod ^a No. 5	Period Measurement	77.8 ± 0.5
Dual Purpose Rod ^a No. 6	Period Measurement	71.3 ± 0.5
	Calculated ^b	68.5
B-10 Safety Rod No. 1	Rod Drop	46.6 ± 2
B-10 Safety Rod No. 5	Rod Drop	52.5 ± 4

^a Rod loaded with four fuel columns.

^b Calculated with the code RE 122 and cross section set 635.

nated in the measurement with the counter in the core center and with the spontaneous fission source only. This measurement therefore should give the most reliable result for the subcriticality.

ROSSI-ALPHA AND PROMPT NEUTRON LIFETIME

The prompt neutron decay constant was determined by the Rossi-alpha technique and was found to be $3.14 \times 10^4 \text{ sec}^{-1}$. With the calculated β_{eff} of 0.0072, this yields a prompt neutron lifetime of $\ell = \beta/\alpha =$

$0.23 \mu\text{sec}$. The calculated prompt neutron lifetime was $\ell = 0.186 \mu\text{sec}$. The calculation of β_{eff} and ℓ was performed with the subroutine BAILIFF of MACH 1 and cross section set 224.

SODIUM VOID MEASUREMENTS

SCOPE OF THE EXPERIMENTS

Measurements of the reactivity changes caused by the removal of sodium comprised the main part of the

FIG. III-1-3. Gap Worth for Small Gaps. ANL Neg. No. 112-5696.

experimental program on Assembly No. 3. The measurements can be divided into three categories:

First, the dependence of the sodium void coefficient on the heterogeneous material arrangement in the core drawers was studied. Measurements were made in a central region for a number of different loading patterns. The results will be useful in interpreting the other sodium void measurements in Assembly No. 3 and measurements in other assemblies having a similar loading pattern.

The second category includes measurements in central regions of different sizes, the mapping of the sodium void coefficient along the core axis and radius and a measurement in a sector comprising $\frac{1}{8}$ of the total core. For these measurements the unmodified drawer loading (Fig. III-1-2) was used throughout.

Finally, in order to study the influence of the blanket composition, zones of light blanket (which had the composition of the core except for the fuel) were introduced in each half in the center of the axial reflector. The sodium void coefficient then was remeasured along the core axis.

EXPERIMENTAL METHOD

A sodium-void measurement was generally performed by three reactor runs in each of which the critical position of the control rods was established at the same power.

1. A reference run, establishing critical rod positions for the normal core configuration.

2. A run with the sodium voided from the region of interest. The sodium cans were replaced by stainless steel frames which contained, with an accuracy of about 10%, the same amount of stainless steel as the cans.

3. A run for the measurement of the stainless steel worth. For this run, the stainless steel frames were replaced by solid stainless steel plates in a representative fraction of the void. The measurements of the stainless steel worth were necessary in order to be able to correct

FIG. III-1-4. Gap Worth for Large Gaps. ANL Neg. No. 112-5695.

for the difference in stainless steel weight between the sodium cans and the frames.

Some of the sodium-void measurements were repeated in order to check the reproducibility of the results. In general, the results were reproducible within about 0.5 lh. In isolated cases discrepancies up to 2.5 lh were found.

THE HETEROGENEITY EFFECT

Experiments to study the heterogeneity effect were done in the center of the core in a (~ 28 cm diameter) test region which contained 21 drawers in each half of the reactor (radial outline as in Region B in Fig. III-1-6). Axially, the voids extended either 30.5 cm (centered at the interface) or 50.8 cm (through the whole core). Measurements were made with:

1. Different materials placed next to the sodium.
2. Different width of the sodium columns.
3. Partial voids in a 1 in. wide sodium column.
4. Discontinuous or "staggered" voids in the normal drawer loading.

Results are given in Table III-1-IV.

The first measurements were made with the normal drawer loading and with two rearranged configurations in which the sodium cans were predominantly contiguous either to U-235 and carbon or to U-238 (Fig. III-1-5 a-c). The axial length of the voids was 30.5 cm. The measured effect varied with the material arrangement but the results were difficult to interpret because they may have been influenced by changes in several different parameters. Both modified drawer loadings, for example, have a sodium can next to each wall. Therefore, in these loadings the cans at two adjoining drawer walls were very close together and may have enhanced streaming effects.

The drawer loadings shown in Fig. III-1-5 d-f were designed for studying the effect of voiding sodium next to different materials without changing the arrangement of fuel and diluents. In this loading pattern two of the sodium columns were surrounded completely by U-238, while the other two were surrounded mainly by graphite. Measurements were performed voiding the sodium next to the U-238 only, next to the graphite only, and voiding all of the sodium. Voids of 30.5 and 50.8 cm length were used. In all cases the sodium void coefficient was found to be more negative next to the graphite than next to the U-238.

To study the effect of the width of the sodium columns, the materials in the drawers in the test region were rearranged so that the sodium columns were $\frac{1}{4}$, $\frac{1}{2}$, 1, or 2 in. wide while the remaining core materials for each case formed approximately a homogeneous mixture (Fig. III-1-5 g-l). The sodium-void coefficient became more negative as the width of the sodium

columns increased. For the 1 and 2 in. columns measurements were made with the columns aligned and non-aligned in two opposing drawers at the interface of the halves. The void coefficient for non-aligned columns was larger (more negative) than for aligned columns.

It was thought at first that the increased sodium-void coefficient for wider sodium columns is due to void streaming. This explanation, however, becomes questionable if one considers the results of the experiment in which the width of the sodium columns was kept at a constant 1 in. while the effect of partial voids which had widths of $\frac{1}{4}$, $\frac{1}{2}$, and 1 in. was being measured (Fig. III-1-5 m-q). All of these measurements yielded essentially the result which had been found for the voiding of the whole width of a 1 in. column using the loading pattern of Fig. III-1-5 h. In other words, as long as the overall width of the sodium columns was kept constant, the sodium-void coefficient did not depend on the width of the void.

A measurement in which most of the streaming was eliminated was made by voiding from the normal drawer loading every other sodium can as shown in Fig. III-1-5 r. Vertically adjoining drawers had the opposite voiding pattern so that in no direction were uninterrupted streaming channels formed. In this configuration, which extended through the whole axial length of the core, the sodium-void coefficient was about 7% less than for total voiding of the sodium columns.

The measured heterogeneity effects are combinations of a spectral component and a non-isotropic leakage effect which are difficult to calculate. However, some qualitative understanding has been achieved and a detailed analysis of the data is being pursued.¹³

REGIONWISE MEASUREMENTS

Using the normal drawer loading, the sodium void coefficient was measured for a number of regions which are shown in Fig. III-1-6. These measurements have been interpreted assuming a homogeneous mixture of the materials. The heterogeneity effect for the normal drawer loading is probably quite small because the positive effect due to the large amount of U-238 next to the sodium tends to compensate the negative effect due to the finite width of the sodium columns. Nevertheless it is possible that the discrete material arrangement causes errors in the order of 10% of the total effect.

The results of the measurements are shown in Table III-1-V for five central regions and in Table III-1-VI for four regions along the core axis and for four rings of different radii situated next to the core midplane. The measurements show that the sodium void coefficient is

FIG. III-1-5. Drawer Loadings for Measurements of the Heterogeneity Effect (See Table III-1-IV). ANL Neg. No. 112-5701.

FIG. III-1-6. Regions for Sodium Void Measurements. ANL Neg. No. 112-5745, Revised.

negative in all outlying regions but becomes approximately zero in the core center.

In order to gain information about the effect of

TABLE III-1-IV. MEASUREMENTS OF THE HETEROGENEITY EFFECT

Material Arrangement ^a	Axial Length of Void	Fig. III-1-5	Ih	Ih/kg
Different materials next to sodium:				
Normal drawer loading	30.5	a	-7.4	-1.31
Sodium predominantly next to U-235 and C	30.5	b	-8.4	-1.48
Sodium predominantly next to U-238	30.5	c	-7.1	-1.25
Modified drawer loading where two sodium columns are surrounded by U-238 and two mostly by C:				
Sodium voided next to U-238	30.5	d	-3.4	-1.20
	50.8		-14.5	-3.10
Sodium voided next to C	30.5	e	-5.6	-1.97
	50.8		-19.0	-4.06
All sodium voided	30.5	f	-7.6	-1.34
	50.8		-33.6	-3.59
Sodium columns of different width:				
$\frac{1}{4}$ in. sodium columns	30.5	a	-7.1	-1.31
$\frac{1}{2}$ in. sodium columns	30.5	g	-8.6	-1.51
1 in. sodium columns	30.5	h	-11.1	-1.96
1 in. sodium columns (not aligned at interface)	30.5	i	-12.0	-2.12
2 in. sodium columns ^b	30.5	k	-12.8	-2.64
2 in. sodium columns (not aligned at interface)	30.5	l	-14.5	-2.99
1-in. wide sodium column with partial voids of different width:				
$\frac{1}{4}$ in. voids	30.5	m	-5.5	-1.94
$\frac{1}{4}$ in. voids (not aligned at interface)	30.5	n	-5.4	-1.89
$\frac{1}{2}$ in. voids	30.5	o	-5.7	-2.00
$\frac{1}{2}$ in. voids (not aligned at interface)	30.5	p	-5.4	-1.90
1 in. voids	30.5	q	-10.5	-1.85
Staggered voids:				
Every other sodium can removed from each column	50.8	r	-15.0	-3.21
All sodium voided	50.8		-32.5	-3.48

^a If not otherwise noted the drawers in the movable and in the stationary halves were loaded symmetrically, so that all material columns were aligned at the interface.

^b For this experiment the drawers were loaded alternately with all sodium and with a mixture of the other core materials; 18 sodium-containing drawers were voided.

totally voiding the sodium from the reactor, a sector-shaped region which comprised about $\frac{1}{8}$ of the core was voided (Fig. III-1-6E). Table III-1-VII shows the result and its extrapolation to the whole core.

MEASUREMENTS WITH A LOW DENSITY BLANKET ZONE

For this group of measurements, the axial U-238 blanket was replaced by a low density (or light) blanket in a zone of ~ 41 cm diameter in the center of each half. The composition of the light blanket was the same as that of the core with the U-235 being replaced by U-238. The atomic densities were (in units of 10^{24} atoms/cm³) U-238, 0.01232; U-235, 0.000025; stainless steel, 0.01475; carbon, 0.0129; sodium, 0.00791. The

TABLE III-1-V. MEASUREMENTS IN CENTRAL REGIONS

Region ^a	Length, cm	Diameter, cm	Measured Effect, Ih	Sodium Void Coefficient, Ih/kg		
				Experimental	Calculated	
					Set 199	Set 224
A	15.2	18.7	0	0	-1.11	-0.58
B	30.5	28.5	-7.4	-1.31	-2.09	-1.49
C	40.6	41.3	-37.0	-2.29	-2.82	-2.26
D	50.8	49.8	-94.2	-3.23	-3.49	-2.99
E	50.8	18.7	-14.4	-3.54	-3.60	-3.09
F	50.8	28.5	-32.5	-3.48	-3.57	-3.05

^a Region designations are given as used in Fig. III-1-6.

^b Diameter of a circle enclosing an equal area as the outline of the region.

TABLE III-1-VI. MEASUREMENTS ALONG CORE AXIS AND RADIUS

Region ^a	Dimensions, cm		Mea- sured Effect, Ih	Sodium Void Coefficient, Ih/kg			
				Experi- mental	Calculated		
	Inner ^b axial boundary	Outer ^b axial boundary	Axial regions (diam 28.5 cm)				
					Set 199	Set 224	
G	0	7.62	-0.5	-0.18	-1.15	-0.62	
H	7.62	15.24	-6.6	-2.33	-2.80	-2.34	
I	15.24	20.32	-9.2	-5.0	-4.77	-4.41	
K	20.32	25.4	-10.8	-5.8	-6.02	-5.9	
	Inner radius	Outer radius	Rings				
L	25.9	36.5	-2.5	-1.06	-1.63	-1.19	
M	46.3	58.2	-9.6	-2.16	-2.16	-1.84	
N	65.1	70.0	-5.0	-2.25	-2.23	-2.07	
O	72.7	77.9	-4.6	-2.13	-2.12	-1.98	

^a Region designations are given as used in Fig. III-1-6.

^b Distance from interface.

TABLE III-1-VII. THE SODIUM VOID COEFFICIENT FOR A SECTOR AND FOR THE WHOLE CORE

Region	Ih	Ih/kg
Sector	-108.5	-2.83
Whole core (extrapolated from sector measurement)	-847	-2.83
Whole core (calculated) Set 199	-778	-2.70
Set 224	-669	-2.43

FIG. III-1-7. Light Blanket Zones with Regions for Sodium Void Measurements. ANL Neg. No. 112-5729.

introduction of the light blanket zone produced a reactivity increase of 13 ± 3 Ih compared to the core with the unmodified high density blanket.

The sodium-void coefficient was measured in a number of regions on the core axis all of which had a diameter of about 28 cm. The outlines of the modified blanket zones and of the sodium-void test regions are shown in Fig. III-1-7.

Some of the sodium-void measurements were repeated with the sodium being removed from the light blanket for both the reference runs and the void runs within the circumference of the test region. The results of the measurements (Table III-1-VIII) show that the sodium-void coefficients in general become slightly more negative as the U-238 blanket is replaced by a light blanket, and that this effect, in the core, becomes more significant when the sodium is removed from the light blanket.

CALCULATIONS

Most calculations for the sodium-void coefficient analysis in Assembly No. 3 were performed using the one-dimensional code MACH 1.⁽⁷⁾ Since spherical calculations were not adequate because of the pancake shape of the core, the one-dimensional calculations were

done in two steps, one in slab geometry and one in cylindrical geometry.

First, k_{eff} with and without sodium was calculated in infinite slab geometry. The voiding was simulated by introducing a region without sodium which had the axial boundaries of the actual void. As the radial boundaries are not taken into account by this calculation, the resulting sodium-void effect was multiplied by the fraction of the flux-importance integral which corresponded to the radial dimensions of the void. The sodium-void effect caused by absorption, spectral changes and axial leakage should be approximately proportional to this fraction. The effect of the change in radial leakage (which is not proportional to the flux-importance integral) was excluded from this part of the calculation by introducing the radial leakage as a fictitious material whose concentration was left unchanged when the sodium was voided. (Usually the radial leakage is taken into account by introducing a radial buckling, B^2 , from which the code forms an absorptive term DB^2 ; this term, however, would change upon voiding the sodium.)

The effect of the change in radial leakage was found by a calculation in (infinite) cylindrical geometry. Only the leakage effect was of interest in this part of the calculation; therefore, the removal of sodium was simulated by adding a negative concentration of a fictitious material which had the transport cross sections of sodium and zero values for all other cross sections. The axial dimensions of the void were taken into account by multiplying the result with the proper fraction of the axial flux-importance integral. For the same reasons as above the axial leakage was not introduced as a buckling but as a fictitious material.

TABLE III-1-VIII. SODIUM-VOID COEFFICIENTS FOR DIFFERENT BLANKET COMPOSITIONS

Regions		Measured Values (Ih/kg)		
		Heavy U-238 blanket	Light blanket	
Inner ^a boundary	Outer ^a boundary		With Na	Without Na ^b
0	7.62	-0.18	-0.3	-0.6
15.24	25.4	(-5.9) ^c	-6.38	-6.82
0	25.4	-3.48	-3.57	-3.92
				Blanket
25.4	33.0	—	-4.54	—
25.4	55.8	—	-2.90	—

^a Distance from midplane, in cm.

^b For all measurements under this heading the sodium in the blanket was removed permanently while the sodium-void coefficient in the core was measured.

^c This value was estimated from the results for two smaller regions, extending from 15.24 to 20.32 cm, and from 20.32 to 25.4 cm.

The total sodium-void coefficient was found by adding the result of the two partial calculations.

These calculations were performed with the 18-group cross section set 199^(8, 9) and with the new 22-group cross section set 224.⁽¹⁰⁾ The cross sections for sodium, carbon, iron, chromium and nickel, which were taken from set 224, were ELMOE¹⁴ corrected for a spectrum typical for large carbide cores.

The one-dimensional method discussed above does not take into account the radial variation of the real and adjoint spectrum in the calculation of the spectral component of the sodium void coefficient. This variation, however, was very small for radii smaller than 50 cm, and for regions near the radial edge the spectral effect is small compared to the radial leakage term. The method of solving two-dimensional problems by two one-dimensional calculations is a very economical tool for the analysis of the sodium-void effect; caution must be used where the spectrum is strongly space-dependent.

For some regions two-dimensional calculations have been performed using the codes PDQ¹¹ and CANDID¹² with cross section set 224. The collapsing of the cross sections into four energy groups, which is necessary when the code PDQ is used, was done separately for each region and for the compositions with and without sodium. In the CANDID calculations, some inaccuracy may have been introduced by the use of a very wide mesh (only 13 points in each dimension) in order to minimize use of computer time.

Table III-1-IX compares the results for different methods of calculation. The MACH 1 and CANDID calculations agree quite well, but the void coefficients calculated by PDQ are considerably more positive. This deviation is very probably due to the loss of the large negative contribution from the down scattering within the energy bounds of the second of the four coarse groups. For further outlying regions the significance of the spectral effect is smaller, which results in smaller deviations for the PDQ calculations.

A comparison between calculations and measurements shows the largest discrepancies in the core center and good agreement near the core edge. The discrepancies are mainly caused by the high sensitivity of the

spectral component which is predominant in the core center to the real and adjoint spectrum, and by uncertainties in the elastic removal cross sections of the sodium.

FISSION RATIO MEASUREMENTS

Information on the fast neutron spectrum in Assembly No. 3 was obtained by making absolute fission ratio measurements with gas flow counters for several materials in the core center and by taking foil traverses with U-235 and U-238 foils.

The gas flow counters were similar to Kirn counters.¹⁵ They had a diameter of about 5 cm and an axial thickness of about 2.5 cm. Their wall thickness was small in comparison with that of the surrounding stainless steel matrix. The amount of fissionable material in the counters was determined to better than 2%, so that they could be considered absolute fission counters.

The central fission ratios were measured for the normal core loading and for the core with a central void of about 50 cm length and diameter, from which all sodium had been removed. The U-238/U-235 fission ratio for the normal (sodium-filled) core loading was also taken with the 41 cm diameter light blanket zones in place.

The results which are given in Table III-1-X show that the removal of the sodium causes a considerable spectral hardening, while the modification in the blanket

TABLE III-1-IX. COMPARISON OF ONE- AND TWO-DIMENSIONAL CALCULATIONS

Sodium Void Coefficient (1h/kg)				
Region	MACH 1	PDQ	CANDID	Measured values
Total core	-2.43	-2.06	-2.50	-2.72
Central region ^a (G)	-0.62	+1.02	-0.63	-0.18
Region next to axial blanket ^a (K)	-5.9	-5.54	-6.07	-5.8
Ring next to radial blanket ^a (O)	-1.98	-1.83	-1.66	-2.13

^a The letters in parenthesis refer to the designation of the region in Fig. III-1-6.

TABLE III-1-X. MEASURED AND CALCULATED FISSION RATIOS

		U-233 to U-235	U-234 to U-235	U-236 to U-235	U-238 to U-235	Np-237 to U-235	Pu-239 to U-235	Pu-240 to U-235	Am-241 to U-235
Na	Flow counter	1.453	0.222	0.0690	0.0331 ^a	0.242	1.019	0.244	—
	Calculated	1.535	0.253	0.0802	0.0356	0.237	1.099	0.262	0.141
Void	Flow counter	1.461	0.250	0.0716	0.0380	0.269	1.038	0.259	—
	Calculated	1.544	0.282	0.0886	0.0389	0.264	1.125	0.292	0.156

^a The same U-238/U-235 fission ratio of 0.0331 was measured when the light blanket zones were in place.

FIG. III-1-8a. Radial Traverses of U-238/U-235 Fission Ratio. ANL Neg. No. 112-5739.

FIG. III-1-8b. Axial Traverses of U-238/U-235 Fission Ratio. ANL Neg. No. 112-5740.

does not bring an observable spectral change in the core center.

Calculations of the central fission ratios were made with the MAIM VI code (an earlier version of MACH 1) and the cross section set 224. The calculated values are also given in Table III-1-X. The largest discrepancies between measured and calculated values are about 15 % but in most cases the agreement is much better.

The following fission traverses were taken with U-235 and U-238 foils.

For the normal coreloading and for the core with the 50 cm central void:

Axial traverse, radial traverse and fine traverse through a central drawer.

For the normal core loading with the light blanket zones in place:

Axial and radial traverse.

For the core with the sector-shaped void:

A radial traverse extending through the center of the sector and through the opposite (sodium-filled) side of the core (dotted line in Fig. III-1-6E).

Results are shown in Figs. III-1-8 through III-1-11. The fission ratios were, when possible, normalized to the central measurements with the absolute fission counters. For the sector void no such measurement was made; therefore, the fission ratio was arbitrarily normalized to unity in the core center.

Figures III-1-8 and III-1-9 clearly show the spectral hardening in the regions from which the sodium had been voided. The fine structure traverses (Fig. III-1-10)

FIG. III-1-9. Radial Fission Traverse with the Voided Sector. ANL Neg. No. 112-5733.

FIG. III-1-10. Measured Fine Structure of U-238/U-235 Fission Ratio Across Drawers With and Without Sodium. ANL Neg. No. 112-5737.

FIG. III-1-11a. Axial Traverses of the U-235 and U-238 Fission Rates with Normal Blanket and with Light Blanket Zone. ANL Neg. No. 112-5735.

FIG. III-1-11b. Axial Traverses of U-238/U-235 Fission Ratio with Normal Blanket and with Light Blanket Zone. ANL Neg. No. 112-5741.

TABLE III-1-XI. CENTRAL REACTIVITY COEFFICIENTS (Ih/kg)

Element	With Na	Without Na
U-235	+75.1	+64.5
Pu-239	+119.9	+105.9
U-233	+142.2	+116.8
U-238	-4.12	-3.95
SS	-2.24	-2.24
C	+10.4	+7.84
Fe	-2.27	-2.12
Al	-0.43	-1.47
Be	+29.1	+24.4
Zr	-1.98	-2.01
W	-6.89	-5.92

TABLE III-1-XII. CORE PARAMETERS OF ASSEMBLIES No. 2 AND 3

	Assembly No. 2	Assembly No. 3
Length	91.4	50.8
Diameter	92.1	155.0
Volume	610	958
kg U-235 in core	544	850
Central U-238/U-235 fission ratio	0.0358 ± 0.0005	0.0346 ± 0.0005
Rossi-alpha	3.49×10^4	3.14×10^4

TABLE III-1-XIII. COMPARISON OF SODIUM VOID COEFFICIENTS IN ASSEMBLIES No. 2 AND 3

	Ih/kg		Ih/% of Core Volume	
	Assembly No. 2	Assembly No. 3	Assembly No. 2	Assembly No. 3
Total core	-5.14 ^a	-2.72 ^b	-9.66 ^a	-7.83 ^b
Core center ^c				
16 cm diameter region	-1.6	0.0	-3.01	0.0
30 cm diameter region	-2.07	-1.31	-3.89	-3.77
40 cm diameter region	-2.99	-2.29	-5.62	-6.59
Near axial blanket boundary, on core axis	-7.5	-5.8	-14.1	-16.7
Near radial blanket boundary, in core midplane	-7.26	-2.13	-13.6	-6.13

^a Extrapolated from measurements in cylindrical rings.

^b Extrapolated from measurements in a sector.

^c The central regions are cylindrical and have approximately equal length and diameter.

show the increase of the fission ratio near the fuel columns. This increase is due solely to the variation in the U-238 fission rate. There is not much variation of the U-235 fission rate across the drawer. The fine struc-

ire curves were normalized by setting their average values equal to the fission ratio measured with the absolute gas flow counters.

Figure III-1-11 shows axial traverses taken with the normal loading with and without the light blanket zones in place. The fission ratios show little dependence on the blanket composition but the total rates of fissions show that the flux in the light blanket is significantly higher than in the U-238 blanket. This probably caused the increase in reactivity when the light blanket zones were introduced. The axial traverses shown in Fig. III-1-11 were also calculated and the agreement with the measurements was very good.

MEASUREMENTS OF CENTRAL REACTIVITY COEFFICIENTS

The central reactivity coefficients of several materials were measured using an automatic sample changer. The size of the samples was 5 x 5 x 2.5 cm. The measurements were made in the center of the core, once when the sodium was voided from a central region of 50 cm diameter and length, and again when the normal amount of sodium was put back into place. The results are tabulated in Table III-1-XI.

COMPARISON WITH ASSEMBLY No. 2

Assembly No. 2 and Assembly No. 3 of ZPR-6 had the same core and blanket composition and the same thickness of the axial and radial blanket. The length-to-diameter ratio of the core, however, was about 1:1 for Assembly No. 2 and 1:3 for Assembly No. 3. A comparison of the results from both assemblies shows the influence of the core shape on critical size, neutron spectrum and sodium void coefficients. Table III-1-XII gives a survey over the core parameters.

The transition from equal length and diameter to a pancake core brought an almost 50% increase in critical volume but very little change in the central spectrum. The slight spectral softening which is indicated by the results of the fission ratio and Rossi-alpha measurements is due to the proximity of the axial blanket to all parts of the core in Assembly No. 3.

Measured sodium void coefficients for both assemblies are shown in Table III-1-XIII. In order to eliminate the influence of the different core sizes, the results were not only given in Ih/kg but also in Ih/percent of core volume (1 percent of the core volume contained 1.88 kg sodium in Assembly No. 2 and 2.88 kg sodium in Assembly No. 3). In terms of Ih/kg the sodium void coefficient is less negative in the pancake-shaped

Assembly No. 3 in all regions which have been compared. The large difference for the total core is at least in part due to the fact that in Assembly No. 3 the regions of high axial flux gradient are to a great extent eliminated by the axial reflector savings, while in Assembly No. 2 this effect is of less importance. In terms of $\text{Ih/\%}$ of core volume, the differences were smaller; and for regions containing a strong axial flux gradient, the values are more negative for Assembly No. 3 than for Assembly No. 2.

REFERENCES

1. P. I. Amundson, et al., *Critical Studies of Uranium-Steel and Uranium Steel-Sodium Fast Reactor Cores (ZPR-3 Assemblies 32 and 33)*, ANL-6690 (1963).
2. J. M. Gasidlo, et al., *Doppler Effect Measurements, Sodium Void Coefficients, and Critical Studies of a 5000-Liter Fast Power Breeder Reactor by the Two-Zone Method (ZPR-3 Assembly 43)*, ANL-6838 (1964).
3. G. K. Rusch, E. R. Böhme, L. R. Dates, S. Grifoni, W. Y. Kato, G. W. Main, H. H. Meister, M. Nozawa and R. L. Stover, *Investigations of a 600-Liter Uranium Carbide Core (ZPR-6 Assembly No. 2)*, Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010, 91 (January 1965).
4. Reactor Engineering Division, *An Evaluation of Four Design Studies of a 1000 MWe Ceramic Fueled Fast Breeder Reactor*, COO-270 (December 1964).
5. Report RE 122, ANL-AMD Program Library, Argonne National Laboratory.
6. W. G. Davey, *K Calculations for 22 ZPR-3 Fast Reactor Assemblies, Using ANL Cross Section Set 635*, ANL-6570 (1962).
7. D. A. Meneley and L. C. Kvitek, Argonne National Laboratory (Private Communication).
8. M. G. Bhide and H. H. Hummel, *Calculations of the Doppler Coefficient of Large Ceramic Fueled Fast Reactors*, ANL-6601 (1962).
9. P. Greebler, et al., *Calculation of Doppler Coefficient and Other Safety Parameters for a Large Fast Oxide Reactor*, GEAP-3646 (March 1961).
10. D. M. O'Shea, H. H. Hummel, W. B. Loewenstein, D. Okrent and C. N. Kelber, *26-Group Cross Sections*, ANL-6858 (to be published).
11. G. G. Bilodeau, et al., *PDQ—An IBM-704 Code to Solve the Two-Dimensional Few-Group Neutron-Diffusion Equations*, WAPD-TM-70 (August 1957).
12. Report R 152, ANL-AMD Program Library, Argonne National Laboratory.
13. D. Meneghetti, *Calculational Studies of Effects of Thin Slab Heterogeneities Upon Spectral Reactivity Component of Sodium Voiding*, Trans. Am. Nucl. Soc. **8**, No. 2 (1965).
14. A. L. Rago and H. H. Hummel, *ELMOE: An IBM-704 Program Treating Elastic Scattering Resonances in Fast Reactors*, ANL-6805 (1964).
15. F. S. Kirm, *An Absolute Fission Counter*, Trans. Am. Nucl. Soc., 2nd Winter Meeting, 61 (October 1957).

III-2. Zoned Core Simulating a 2600 Liter Carbide Fast Reactor

R. A. KARAM, A. ANCARANI,* F. HELM, W. Y. KATO and G. K. RUSCH

Assembly 4Z went critical during the first week of May 1965. The assembly is a zoned reactor having four radial regions: central, buffer, driver, and reflector. The purpose of the zoned assembly is to test, in conjunction

TABLE III-2-I. VOLUME FRACTIONS IN ASSEMBLY 4Z AND THE REFERENCE CORE

Material	Volume Fractions in Reference Assembly ^a	Volume Fractions in Assembly No. 4Z ^b			Volume Fractions in Blanket (Common to Both Assemblies)
		Region			
	Core	Central ^c	Buffer ^d	Driver ^e	
U-235	0.0319	0.0319	0.00095	0.1428	0.00169
U-238	0.2213	0.2213	0.4539	0.1119	0.8319
Na	0.4208	0.4208	0.00	0.4208	—
C	0.1539	0.1539	0.3601	0.1539	—
SS	0.1483	0.1483	0.0708	0.1483	0.0708

^a Reference core volume = 2554 liters; critical mass = 1525 kg U-235.

^b Zoned core volume = 1100 liters.

^c Central region radius = 41.43 cm.

^d Buffer region outer radius = 46.96 cm.

^e Driver region outer radius = 53.88 cm.

TABLE III-2-II. COMPARISON OF CALCULATED AND MEASURED CRITICAL MASS (kg) OF ASSEMBLY No. 4Z (ZPR-6)

	Code and Cross Section Set			
	MAIM-6 (Set 224—22 groups)	RE-122 (Set 635—16 groups)	RE-122 (Set 149—4 groups)	SNARG-S4 (Set 224—22 groups)
Calculated ^a	1082	1103	1152	1074
Measured	1101			

^a Calculations were done in cylindrical geometry.

with the reference core (full size), the extent to which physics parameters of the reference assembly may be predicted on the basis of measurements performed on the zoned system. The fundamental advantage of zoned cores is that they require a smaller fuel inventory than cores which are not zoned.

COMPOSITION AND CRITICAL MASS

Assembly No. 4Z consists of four regions: (1) the central region extends from the center of the core to

* Euratom, Brussels, Belgium.

41.43 cm in the radial direction and has the same composition as the reference core; (2) the buffer region extends from 41.43 to 46.96 cm; (3) the driver region extends from 46.96 to 53.88 cm; and (4) the blanket is 30 cm of depleted uranium. Assembly No. 4Z is cylindrical in form with axial length 122 cm. The axial blanket is 30 cm of depleted uranium. Table III-2-I lists the concentrations of materials in Assembly No. 4Z and the reference core.

TABLE III-2-III. COMPARISON OF FLUXES AND ADJOINT FLUXES IN THE REFERENCE AND ZONED CORES^a

Group	E_L , ^b keV	ϕ_{Center}		ϕ_{Center}^*		$\phi\phi_{\text{Ref}}^* - \phi\phi_{4Z}^*$
		Ref. Assembly	Assembly No. 4Z	Ref. Assembly	Assembly No. 4Z	
1	3680	1.462	1.475	5.411	5.425	-0.091
2	2230	2.837	2.835	5.130	5.137	-0.009
3	1350	4.774	4.763	4.676	4.678	+0.042
4	821	7.301	7.282	4.054	4.051	+0.099
5	498	10.479	10.459	4.029	4.024	+0.133
6	302	12.995	12.981	4.070	4.061	+0.174
7	183	13.197	13.193	4.064	4.052	+0.175
8	111	11.445	11.448	4.046	4.034	+0.205
9	67.4	9.823	9.832	4.034	4.024	+0.062
10	40.9	7.801	7.814	4.037	4.029	+0.010
11	24.8	5.952	5.565	4.063	4.059	-0.029
12	15.0	4.742	4.756	4.137	4.135	-0.048
13	9.11	2.977	2.986	4.252	4.254	-0.044
14	4.31	2.330	2.336	4.445	4.448	-0.034
15	2.61	0.251	0.251	4.621	4.625	-0.001
16	2.03	0.518	0.519	4.637	4.641	-0.007
17	1.23	0.488	0.489	4.630	4.634	-0.007
18	0.961	0.256	0.256	4.635	4.640	-0.001
19	0.583	0.243	0.243	5.025	5.029	-0.001
20	0.275	0.082	0.082	5.284	5.289	-0.001
21	0.101	0.0255	0.0255	5.393	5.397	-0.000
22	0.029	0.0016	0.0016	5.323	5.327	-0.000

^a Calculations were done in spherical geometry (MAIM-6, cross section set 224).

^b E_L is the lower boundary of energy group.

Table III-2-II shows a comparison of the calculated and measured critical mass of Assembly No. 4Z. The measured value does not include any corrections for heterogeneity effects.

SPECTRAL MATCHING

The fluxes and adjoint fluxes at the center of the zoned and reference assemblies are compared in Table

TABLE III-2-IV. COMPARISON OF FLUXES AND ADJOINTS AT 30 CM FROM THE CENTER OF THE REFERENCE AND ZONED CORES^a

Group	ϕ_{30} cm		ϕ_{30}^* cm		$\phi\phi_{Ref.}^* - \phi\phi_{4Z}^*$
	Ref. Assembly	Assembly No. 4Z	Ref. Assembly	Assembly No. 4Z	
1	1.476	1.470	5.411	5.53	-0.142
2	2.838	2.800	5.130	5.219	-0.054
3	4.775	4.681	4.676	4.737	+0.154
4	7.362	7.242	4.054	4.077	+0.320
5	10.478	10.490	4.028	4.048	-0.254
6	12.993	13.042	4.069	4.089	+0.540
7	13.194	13.240	4.064	4.068	-0.240
8	11.443	11.480	4.046	4.033	-0.001
9	9.822	9.874	4.035	4.008	+0.057
10	7.801	7.850	4.037	4.000	+0.093
11	5.953	5.985	4.064	4.019	+0.139
12	4.744	4.741	4.137	4.086	+0.254
13	2.979	2.963	4.252	4.203	+0.214
14	2.332	2.319	4.445	4.409	+0.142
15	0.251	0.250	4.621	4.600	+0.010
16	0.518	0.515	4.637	4.611	+0.027
17	0.489	0.481	4.630	4.607	+0.048
18	0.257	0.249	4.636	4.616	+0.042
19	0.244	0.232	5.025	5.010	+0.064
20	0.0819	0.0775	5.284	5.280	+0.024
21	0.0256	0.0243	5.392	5.400	+0.007
22	0.00165	0.00158	5.323	5.340	+0.003

^a Calculations were done in spherical geometry using MAIM-6 and cross section set 224.

III-2-III. The fluxes and their adjoints are normalized to 100 n/cm² sec at the core center. Although the values reported in Table III-2-III were calculated for spherical geometry, a similar agreement between the fluxes and adjoint fluxes was obtained in cylindrical geometry. Table III-2-IV compares the fluxes and adjoint fluxes in the two assemblies at 30 cm from the center of each core. The last columns of Tables III-2-III and III-2-IV indicate the "goodness" of spectral matching.

FATMAN EFFECT

The fatman effect was calculated for the reference core by means of a one-dimensional diffusion code. The fatman consisted of a slab of Benelex placed adjacent to the face of one-half of the reactor. Two thicknesses for the slab, 20 cm and 50 cm, were used. The calculated k_{eff} and the measured k_{eff} of Assembly No. 4Z are shown in Fig. III-2-1. Although the calculation and measurement of k_{eff} are associated with two different assemblies, the k_{eff} of the bare half of each assembly was the same. The uncertainty in the measurement is perhaps

FIG. III-2-1. Dependence of k_{eff} of One-half of the Assembly on the Thickness of Benelex Placed Adjacent to the Midsection of the Core. ANL Neg. No. 112-5463.

5%. The discrepancy between measurement and calculation is perhaps due to the fact that the half was far subcritical.

MEASUREMENT OF THE SPECTRUM AND DOPPLER EFFECT

The neutron spectrum, at various locations in Assembly No. 4Z, was measured with a proton recoil counter. The data are reported in Ref. 1.

Experiments for measuring Doppler coefficients are now underway.

CONTROL ROD CALIBRATION

The control rods were calibrated by the period method. The worths of the safety rods were determined by the rod-drop method in relation to a calibrated rod. The total shutdown margin was found to be 2.7% $\Delta k/k$.

REFERENCES

1. E. F. Bennett, *The Neutron Spectrum Measurement in a Fast Critical Assembly*, (to be published).

III-3. The Neutron Spectrum in ZPR-6 Assembly No. 4Z

E. F. BENNETT

This report is an extension and an application of in-core neutron spectrum experimentation described previously.¹

The core ZPR-6, No. 4Z (see Paper III-2) used for the measurements described herein was constructed primarily for studies of the Doppler reactivity effect. This effect is sensitive to the low energy region of the spectrum and there is appreciable interest in a direct measurement of the spectrum extended to the lowest attainable energy response (approximately 1 keV) of existing experimental techniques.

The approach used was that of 4π -recoil proportional counting.² The detector arrangement is shown in Fig. III-3-1. The counter body was constructed of stainless steel (1.2-mm wall thickness). Ceramic seals were placed at each end and hypodermic needles inserted to define the active counting volume. The anode was an 0.025-mm-diameter stainless steel wire. Two different counters were used to make each spectrum measurement. The first was 2.5-cm in diameter and had an effective length of 6.5 cm. It was filled to a pressure of 3 atm of hydrogen gas including 20 cm Hg of methane for quenching and 10 cm Hg of nitrogen for calibration. The second counter was 3.8-cm in diameter with an effective length of 9.4 cm. This counter was filled with methane to a pressure of 3 atm and contained 10 cm Hg of nitrogen for calibration. The hydrogen-filled counter was used for measurements in the energy interval from 1 keV to 100 keV while the methane-filled counter, because of its increased size and stopping power for protons, was used in the interval from 100 keV to 1 MeV.

The counters, along with the first tube of a preamplifier, were placed within a 5- by 5-cm drawer (stainless steel) of the same type containing the various pieces of core material used to fabricate the reactor. Only one counter could be used at a time. Some lead (a sleeve 1-cm thick for the hydrogen counter and 0.6-cm thick for the methane counter) was placed around the sensitive volume. The lead served both to support the counters in the drawer and to provide some additional shielding from gamma radiation existing in the core. That part of the drawer not needed for the detector or preamplifier was filled with representative core material except for U-235.

The center of the counter was displaced axially about 11 cm from the center of the core; the axial dimension of the core was 122 cm. The core was of the "zoned"

variety with the central region having the composition listed in Table III-3-I.

EXPERIMENTAL TECHNIQUE

The elimination of gamma-ray background from the ionization spectrum of a proportional counter by means of pulse shape discrimination has been described previously.² For measurements reported here an improved electronics system was used^{3, 4} in which each pulse from the detector is analyzed by separate amplifiers. The first produces a pulse proportional to the rate-of-rise of the incident pulse, and the second is a conventional delay-line-clipped amplifier with a peak output pulse proportional to the initial ionization. The ratio of pulses is taken by an analog pulse height computer; the ratio is proportional to the specific ionization of the event and is large for a recoil-proton pulse and considerably smaller, on the average, for an electron-induced pulse of gamma-ray origin. The "ionization" pulse and the "specific ionization" pulse are simultaneously recorded using a two-parameter pulse height analyzer. Figure III-3-2 is a two-parameter spectrum of a mixed gamma-neutron field. Ionization is recorded along the ordinate (ranging from 0 to 10 keV) and specific ionization along the abscissa. Only those memory channels with a content in excess of a pre-set amount intensify. For any given ionization level, events resolve into two distinct regions. The region at low values of specific ionization (abscissa) corresponds to gamma ray events and is quite broad, while the region at high values of specific ionization corresponds to recoil-proton events and is narrower.

Measurements in the mixed field of the reactor were compared with those from a pure gamma source and in this way a correction for the overlap of events of gamma origin with that part of the specific ionization spectrum associated with proton-recoil events could be made. The gamma background was rather low in the reactor at the time these measurements were made and the correction for the overlap between gamma channels and neutron channels did not exceed a few percent except for the lowest values of ionization where preamplifier noise broadened the distributions and corrections increased to 5% or so.

A 1024-channel, two-parameter distribution (64 ionization channels by 16 specific ionization channels) was accumulated at each value of gas multiplication. The spectrum was examined (by computer) and compared

pure gamma spectra. An integrated recoil-proton number, corrected for background, was determined for each value of ionization. The corrected recoil-proton pulse height distributions were used to evaluate the neutron spectrum. The well-known equation

$$\phi(E) = \frac{1}{NT} \frac{E^2}{\sigma(E)} \frac{dD(E)}{dE} \quad (1)$$

relates neutron flux (per unit lethargy), $\phi(E)$, to the measured distribution of recoil protons (per unit energy), $D(E)$; $\sigma(E)$ is the n,p scattering cross section and NT is the product of the number of hydrogen atoms in the sensitive region of the detector by the live counting time.

Calibration of instrumental pulse amplitudes versus recoil-proton energies was done using the $N^{14}(n,p)C^{14}$ reaction in the manner described in Ref. 2. In order to relate energy and ionization, knowledge of W , the energy expended by protons per ion pair formed in the counter gas, is necessary. For the hydrogen-filled counter (1 keV to 100 keV) W was assumed to be infinite below 290 eV and constant for protons in excess of this value,² whereas for the methane-filled counter (100 keV to 1 MeV) tungsten was assumed to be infinite below 8 keV and to be constant above.

Differentiation of $D(E)$ consisted of a determination (by computer) of a slope (and associated error) to sections of the recoil-proton distribution of approximately fixed fractional width. The slope-taking interval was chosen relative to the intrinsic line width of the counters; it varied between $\pm 7\%$ and $\pm 10\%$ depending on the counter used and the energy region under study. The energy resolution that can be assigned to a neutron

spectrum derived by Eq. (1) is very nearly the larger of either the intrinsic width (full width at half maximum response to mono-ionizing events) or the slope-taking interval. Resolution of measured neutron spectra was about 20% from 100 keV to 1 MeV, about 15% from 10 keV to 100 keV and about 25% below 10 keV.

RESULTS

Neutron flux levels in the assembly were adjusted to provide the desired counting rates, which were less than about 3000/sec, by a combination of external sources and sub-criticality. The facility was kept between 1 and 2% subcritical and the flux maintained by a "start-up" source placed near the driver region but diametrically opposed to the region where counting was done. The extent of the departure of the spectrum from that at critical is not known but is probably small since multiplication was in excess of about 50.

A spectrum of neutrons near the center of the core is shown in Fig. III-3-3. Statistical accuracy and resolution are included on the figure; total running time was about six hours.

TABLE III-3-I. VOLUME FRACTIONS FOR ZPR-6
ASSEMBLY NO. 4Z (CENTRAL REGION)

Material	Volume Fraction
U-235	0.0319
U-238	0.2213
Sodium	0.4208
Carbon	0.1539
Stainless Steel	0.1483

FIG. III-3-1. Proportional Counter Probe. ANL Neg. No. 112-4409-A.

FIG. III-3-2. Two-Parameter Spectrum of a Mixed Gamma and Neutron Field. *ANL Neg. No. 143-1909-Rev. 2.*

At 45 keV the spectral structure was finer than the instrumental resolving power. This structure was not reproducible; it appeared at only two of the four radial positions in the core where spectral measurements were taken. The core was constructed of vertically stacked plates of various thicknesses of the materials listed in Table III-3-I. The local environment of the detector was not, therefore, uniform and it seems likely that an observable local spatial dependence of the spectrum exists. This question is complicated by the finite size of the detectors. The spectrum shape exhibits a broad maximum between 80 and 250 keV; it then tends to flatten below about 6 keV with a distinct peaking in the 2 to 3 keV region, probably attributable to the presence

of sodium in the core. Neutron flux diminishes rapidly below 6 keV.

Figure III-3-4 contains a calculated and an observed histogram for the center position spectrum. The experimental histogram is an average of the data over the appropriate lethargy interval. The group calculation was done with MAIM-6, using homogenized cross sections (cross section set 801) which were averaged over the actual material concentrations using the program ELMOE⁵. There is general agreement between calculated and measured values except at lower energies, although agreement cannot be considered as at all striking, regardless of the way one chooses to normalize the results.

FIG. III-3-3. Neutron Spectrum at the Core Center. *ANL Neg. No. 112-5169.* FIG. III-3-4. Comparison of Central Spectrum with a Group Calculation. *ANL Neg. No. 112-5161.*

REFERENCES

1. E. F. Bennett and J. Haugsnes, *In-Core Fast Neutron Spectroscopy*, Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010, (January 1965) pp. 225-227.
2. E. F. Bennett, *A Study of the 1/E Slowing-Down Neutron Spectrum Using 4 π -Recoil Proportional Counters*, ANL-6897 (1964).
3. M. Strauss and R. Brenner, Argonne National Laboratory (Private Communication).
4. R. N. Larsen, Argonne National Laboratory (Private Communication).
5. C. E. Till, Argonne National Laboratory (Private Communication).

III-4. Doppler Effect Measurements in Plutonium-Fueled Fast Power Breeder Reactor Spectra

G. J. FISCHER, D. A. MENELEY, R. N. HWANG, E. F. GROH and C. E. TILL

INTRODUCTION

The Doppler effect is of considerable importance to the large fast power breeder reactors which are currently being evaluated and designed. The lower effective neutron energy spectrum which results from a high ratio (frequently about seven to one) of fertile to fissile isotopes, the use of ceramic fuel, and high coolant fractions is the source of a significant reactivity shutdown potential by means of the Doppler effect in case of a nuclear incident. That the effect represents an instantaneous shutdown response is an important feature. Finally, the existence of possible gains of reactivity in these large reactors upon loss of sodium increases the importance of a careful measurement to verify the magnitude of the Doppler effect in the spectrum of the large plutonium-fueled fast reactor.

The present set of measurements was performed at the ZPR-3 fast critical assembly at the NRTS in Idaho. It is a continuation, with new apparatus, of a program of Doppler measurements begun with ZPR-3 Assembly No. 43. Relatively small Doppler effect measuring samples, one-half inch in diameter and 70 % of theoretical density, were used in order to produce a minimal distortion of the neutronic environment in which the samples were placed.

The procedure of the experiment consisted of determining the change in the worth of an isotope or isotopes, in samples having the form just described and sealed into a heating jacket to make a "Doppler element," with respect to a reference material as the temperature of the Doppler element was varied. Since the principal source of experimental uncertainty was reactor drift, produced largely by the decay heat of the plutonium fuel, considerable pains were taken to control this problem. A second source of uncertainty—arising from expansion of the Doppler elements upon heating, especially for the fissile isotopes—was partially controlled by design of the apparatus. The remainder of the ex-

pansion effect was accounted for theoretically by use of two-dimensional transport computations.

REACTOR LOADING

The geometry of ZPR-3 Assembly 45A is shown in Fig. III-4-1. The central zone mocked up a large Pu-239-U-238 monocarbide fast power breeder reactor in its central zone. It had a cylindrical height of 91.4 cm, representative of a breeder reactor, and was reflected by a mockup of a typical, clean, low-density axial blanket.

Careful choice of materials for the driver and filter regions produced a satisfactory neutron energy spectrum a few centimeters within the boundary of the central zone. By being able to produce Doppler meas-

FIG. III-4-1. Cross Sectional View of ZPR-3 Assemblies 45 and 45A. (A special radial reflector was employed next to the driver for Assembly 45 only). ANL Neg. No. 112-5401.

ements using a central zone of relatively restricted radius, the problem of spontaneous neutron source from the plutonium was reduced. Also, the decay heat of the plutonium, a serious source of reactor drift, was minimized.

Assembly 45 had essentially the same geometry as Assembly 45A except that a light element reflector was

used outside of the driver in order to better preserve the lower energy neutrons of this loading.

EQUIPMENT AND PROCEDURE

Figure III-4-2 is a schematic diagram of the Doppler effect apparatus¹ used in ZPR-3. The drive unit moves the Doppler equipment back and forth and exchanges the hot Doppler elements with the reference material at the center of the assembly. The reactor is maintained at constant power throughout the exchanges and the experiment by motions of a fine control rod. If there are no other interfering events occurring at the same time, the change in the reactivity worth of the Doppler elements upon heating is ascribed to the Doppler effect changes in reaction rates within the Doppler elements.

The construction of the basic Doppler element is shown in Fig. III-4-3. Pellets of isotopes of interest, in the form of oxides at 0.70 theoretical density, were pressed into cylinders of 0.5003 in. diameter and 0.500 in. length, and fired. Twelve of these pellets were used to produce an element six inches long.

Two Doppler elements were mounted in line, sym-

Fig. III-4-2. Schematic Diagram Showing Arrangement of Doppler Element Drive System in ZPR-3 Reactor (Proportions have been distorted). ANL Neg. No. 112-4834-Rev.

Fig. III-4-3. Doppler Element-Assembled and Opened to Show Centered UO_2 Pellets. ANL Neg. No. 112-5704.

metrically about the axial center of the reactor in which they were to be used, in the manner shown in Fig. III-4-2. One end of each Doppler element was fixed in position in such manner that both elements expanded in the same direction upon heating. This caused the material of one element to move in the direction of a slightly increasing flux gradient and the material of the other element to move into a correspondingly decreasing flux gradient, resulting in a first-order cancellation of flux gradient expansion effects. Two-dimensional transport theory calculations were used to evaluate the magnitude of the remaining uncompensated expansion effects. These were significant for the fissile isotopes only.

RESULTS AND CONCLUSIONS

The experimental results obtained using Doppler elements made up of U-238 and of U-238 mixed with Pu-239 in a ratio of seven to one are shown in Table III-4-I. Results of theoretical interpretation of these measurements are also included in the table.

It can be seen that there is relatively satisfactory agreement between experiment and theory for the cases of Table III-4-I. For these cases there is much less theoretical uncertainty than for fissile isotopes. The uncertainties which do exist are due to uncertainties in basic nuclear data. The results for U-238 indicate a significant reactivity loss of $-16 \times 10^{-6} \Delta k/k$ for the temperature range used in the power breeder spectrum of Assembly 45A. For the mixed 1:7 isotopes, the reactivity loss of $-13.6 \times 10^{-6} \Delta k/k$ per kg is within 3% of being proportional to the relative density of U-238 in this Doppler element compared with that of the U-238 element.

The consequence of spectral softening can be seen by comparing the U-238 result for Assembly 45A to that for Assembly 45. A factor of almost two in Doppler response occurs.

The theoretical estimations of the Doppler reactivity changes were based upon perturbation theory calculations using multigroup cross sections appropriate for the experiment conditions. Radial values of the real and adjoint multigroup fluxes were obtained by means of cylindrical one-dimensional diffusion theory calculations. The axial dependence of these fluxes was estimated from results of nine-group two-dimensional transport calculations, while the normalization of the perturbation integral was taken from a two-dimensional transport calculation of plutonium worth at the core center. This worth calculation checked well against the experimentally measured worth of plutonium at the same position in the reactor, giving -185 lh/kg theoretically versus -176 ± 4 lh/kg experimentally. The diffusion theory radial flux dependence also compared well with that of a similar one-dimensional transport calculation for the case of a plutonium Doppler element at the center of Assembly 45A. A two-dimensional nine-group transport calculation of one of the Doppler reactivity changes agreed within a few percent with the perturbation calculation of the same case.

Measurement with the empty or dummy "Doppler elements" indicates a lack of serious instrumental or other non-nuclear reactivity effects.

Table III-4-II contains the results of measurements made with Pu-239 and contains in parentheses, for completeness, experimental Doppler results for U-235 obtained by J. Gasidlo and P. Amundson in the same

TABLE III-4-I. DOPPLER EFFECT REACTIVITY CHANGES FOR U-238 AND U-238 MIXED WITH PU-239 IN 7:1 RATIO
($\Delta T = 500$ to $1100^\circ K$)

Measurements ^a	Assembly	Experiment			Theory ($\Delta k/k \times 10^{-6}$)		
		$\Delta\rho$, lh ($\pm \sim 0.020$)	$\Delta k/k$ per kg $\times 10^{-6}$	$\Delta k/k \times 10^{-6}$	Columbia Data	Firk, et al. data	Exact calculation
U-238 $\frac{1}{2}$ in. D.E.	45	-0.327	-27.7	-6.39	-6.19	—	—
U-238 1.0 in. D.E.	45	-1.38	-23.3	-26.8	-26.11	-23.8	-26.4
Dummy U-238	45	-0.006	—	-0.112	—	—	—
$\frac{1}{2}$ in. D.E.	45A	-0.184	-16.0	-3.68	-2.66	—	—
U-238, Pu-239 7:1 Mixture $\frac{1}{2}$ in. D.E.	45A	-0.156	-13.6	-3.12	-1.99	—	—

^a D.E. = Doppler element.

TABLE III-4-II. DOPPLER EFFECT REACTIVITY CHANGES FOR PU-239 AND U-235
($\Delta T = 500$ to $1100^\circ K$)

Measurement	Assembly	Experiment					Theory
		$\Delta\rho$, Inhours ($\pm \sim 0.020$)	$\Delta k/k \times 10^{+6}$	$\Delta k/k$ per kg $\times 10^{+6}$	Expansion ^a $\Delta k/k \times 10^{+6}$	Corrected $\Delta k/k \times 10^{+6}$	$\Delta k/k \times 10^{+6}$
Pu-239	45A	-0.060	-1.21	-5.36	—	—	—
Pu-239—3 weeks later	45A	-0.049	-0.99	-4.39	—	—	—
Pu-239 Average	45A	-0.055	-1.10	-4.88	+1.35	+0.25	+5.06
Pu-239 Homogeneous	45A	-0.034	-0.68	-3.02	+1.35	+0.67	—
Pu-239 in B ₄ C	45A	+0.042	+0.83	+3.70	+0.90	+1.73	+2.33
Pu-239 1 Element	45A	-0.044	—	—	—	—	—
Pu-239	45	-0.024	-0.56	-2.49	+1.63	+1.07	+12.95
(U-235; JG, PA)	45A	+0.054	+1.1	+4.64	+1.05	+2.15	+2.90

^a Calculated correction to experiment.

assembly.² These fissile isotope results lead to some very interesting observations which must be tested by further experimental work. The expansion effect correction has been evaluated with some care, so that the experimental results, as corrected for expansion effects, will be compared with the theoretical results.

The experimental results for Pu-239 give surprisingly small reactivity responses. This holds true even for the much softer spectrum of Assembly 45, where there should be many more neutrons in the region of the Doppler effect.

It can be seen from Table III-4-II that the experimental Pu-239 Doppler response becomes more positive when B₄(nat)C is used in the scattering zone in place of stainless steel and graphite, and that agreement with calculation immediately improves. It can also be seen that agreement is worse when the spectrum is softened, going to Assembly 45. These results suggest that much of the disagreement might be due to poor Pu-239 cross section information in the lower energy portion of the Doppler reaction. The basic cross section information of U-235, in particular its α -value,³ is relatively better known than is that of Pu-239. That the U-235 Doppler reactivity change is reasonably well calculated in the same spectrum tends to indicate that the flux calculated for the region of the Doppler element probably is not in serious error.

The case labeled "Homogeneous" represents the experimental difference resulting from replacing the "scattering zone" and "fuel zone" by a completely uniform core region. One sees that the Dancoff correction is not large.

The U-235 measurement employed the same apparatus except for the U-235 Doppler elements, which were fabricated in the same manner as the plutonium elements. The procedure of the measurement was also the same.

These results indicate that for a large Pu-239-U-238 fast power breeder using ceramic fuel the positive

Doppler reactivity contribution of plutonium is small in magnitude compared with the strong negative Doppler effect of the U-238 in the same reactor. It should be remembered that these results are for a "clean" fast reactor. The presence of fission products which occurs after some degree of burnup and the presence of Pu-240 which occurs with fuel recycle, would be expected to reduce the magnitude of the Doppler effect.

EXPERIMENTAL AND THEORETICAL UNCERTAINTIES

The principal source of experimental uncertainty was reactor drift during the experimental data-taking operation, largely due to heat production by the plutonium fuel. The heat was removed by drawing air through the zone and this procedure led to a drift sensitivity related to the temperature of the reactor cell air as it entered the matrix. Control of these parameters improved continuously during the course of the experiments and has been worked on in anticipation of future measurements.

The corrections for expansion effects employed in this report were calculated using two dimensional transport theory. Both 5- and 9-group cross sections were produced for the 2-D code for each reactor region by a flux weighting routine working from a 25-group cross section set related to the set 224 reported elsewhere.⁵ Only a very slight reactivity difference was noted when the expansion effect was calculated using 9 groups instead of 5 groups and when the convergence criterion was tightened by a factor of two.

A relatively large and probably quite conservative overall experimental uncertainty of $\pm 0.40 \times 10^{-6} \Delta k/k$ has been assigned to all data of Tables III-4-I and III-4-II. Improved data-taking procedures currently being used for Doppler measurements on ZPR-VI(CET) are producing results with uncertainties approximately an order of magnitude smaller than this.

A continuous problem of all fast reactor calculations is whether the calculational procedure produces the correct flux magnitude in the tail of the fast reactor neutron energy spectrum where the entire Doppler effect occurs.⁶ For the calculations reported here, most regional cross sections were produced using the ELMOE code⁷ with radial leakage buckling corrections being added. Other cross sections were prepared from the new ANL code MC² which produces multigroup cross sections from a basic library containing very detailed nuclear data.

REFERENCES

1. D. A. Meneley, G. J. Fischer and E. F. Groh, *Design and Use of Doppler Effect Equipment for Zero Power Fast Reactor Experiments*, ANL-7037 (1965).
2. J. Gasidlo and P. Amundson, Argonne National Laboratory (Private Communication).
3. G. deSaussure, L. W. Weston, R. Gwin, J. E. Russell and R. W. Hockenbury, *Measurement of α , the Ratio of the Neutron Capture Cross Section to the Fission Cross Section, for U-235 in the Energy Region from 3.25 eV to 1.8 keV*, ORNL-3738 (1965).
4. H. H. Hummel and A. L. Rago, *Effect of Parameter Variation in Doppler Effect Calculations*, Proc. of the Conf. on Breeding, Economics, and Safety in Large Fast Power Reactors, ANL-6792 (1963) p. 747.
5. D. M. O'Shea, et al., *Twenty-Six Group Cross Sections*, ANL-6858 (to be published).
6. D. Meneghetti and J. R. White, *Calculations of ZPR-III Fast Assemblies Using a Twenty-Six-Group Cross Section Set*, Trans. Am. Nucl. Soc. 7, No. 2, 237 (1964).
7. A. L. Rago and H. H. Hummel, *ELMOE: An IBM-704 Program Treating Elastic Scattering Resonances in Fast Reactors*, ANL-6805 (1964).

III-5. A Check Calculation of Doppler Coefficient in the ZPR-3 Mockup of SEFOR

R. N. HWANG

A check calculation on the Doppler coefficient of reactivity for the ZPR-3 mockup of the Southwest Experimental Fast Oxide Reactor (SEFOR) has been carried out. This coefficient is quite important with respect to the safety of performing the mockup. The problem was investigated previously by A. Reynolds and S. Stewart.¹

The mockup will be assembled on ZPR-3 using two types of fuel plates. The central or inner core region will be fueled with U-238-Pu-Mo plates. The outer core region will be fueled with pure Pu plates. The geometrical configuration is given in Fig. III-5-1.

The overall Doppler coefficient of the system under consideration depends almost entirely on the relative magnitude of the negative coefficient attributed to U-Pu-Mo alloy plates in the inner core region with fixed dimension (due to limited U-Pu-Mo alloy inventory) compared with the positive coefficient attributed to pure Pu-plates in the outer core region. Therefore the actual critical dimension of the whole system must be determined quite accurately. To illustrate the effect of the outer region critical dimension on the overall Doppler coefficient of the mockup, calculations have assumed k_{eff} for the system to equal 1.0, 1.05, and 1.152, respectively, in order to determine various critical dimensions of the outer region (region II). The investigation by Reynolds and Stewart was based on $k_{eff} = 1.152$. The critical dimension obtained by using

$k_{eff} = 1.152$ is believed to be too pessimistic from a practical point of view since such pessimism of predicted reactivity will undoubtedly require the outer core region to be significantly larger in the calculation than will prevail in the experiment. This approach tends to overestimate the relative contribution of the positive component of the Doppler coefficient. Conversely, the Doppler coefficient obtained by using $k_{eff} = 1$ may be too optimistic for a safety evaluation if the critical dimension of the outer core region is underestimated. The intermediate value of $k_{eff} = 1.05$ is believed to reasonably account for predictive uncertainties.

The Doppler coefficient in this investigation is specially defined. An increase in power level causes the temperature of the outer core fuel to rise more than that in the inner core. For safety analyses, the pure Pu fuel plates in the outer core, but near the inner core boundary, are at the highest temperature and would melt first during a hypothetical excursion. Thus the overall system reactivity change due to a change in power level and resulting temperature changes, is referred to that point of maximum steady state temperature.

METHOD OF CALCULATION

The method of calculation used here is similar to that given by Reynolds and Stewart.¹ The Doppler coefficient in terms of the change in temperature at the

FIG. III-5-1. Schematic Diagram of Proposed ZPR-3 Mockup of SEFOR. ANL Neg. No. 112-5614.

interface can be written as:

$$\frac{dk}{dT_{\text{Pu(max)}}} = \left(\frac{dk}{dT} \right)_1 \frac{dT_1}{dT_{\text{Pu(max)}}} + \left(\frac{dk}{dT} \right)_{2\text{U}} \frac{dT_{2\text{U}}}{dT_{\text{Pu(max)}}} + \left(\frac{dk}{dT} \right)_{2\text{Pu}} \frac{dT_{2\text{Pu}}}{dT_{\text{Pu(max)}}} \quad (1)$$

where the type of fuel plates in the inner and outer region are designated by subscripts 1 and 2 respectively. $T_{\text{Pu(max)}}$ is the maximum temperature in the pure Pu-plates at the interface of the inner and outer cores. In region II, the outer core, there are both plutonium (Pu) and depleted uranium (U) plates, hence the designation 2Pu and 2U, respectively.

DETERMINATION OF CRITICAL RADIUS

The dimension of the inner core region which contains the Pu-U-Mo fuel is essentially fixed ($r_1 = 32.811$ cm; height = 71.12 cm). These dimensions are fixed because of the limited inventory of this type of fuel. The radius of the outer region is to be determined by the criticality of the system. The reliability of the overall Doppler coefficient is extremely sensitive to the relative dimensions of the inner and outer core regions.

The uncertainties in determining the critical size may either overestimate or underestimate the negative Doppler coefficient of the actual critical system. Hence, it is more desirable for a safety analysis to estimate what the overall Doppler coefficient will be for various critical dimensions. These various critical dimensions have been determined by using multigroup diffusion theory (MAIM-6; see Paper V-16) with k_{eff} corresponding to various outer region dimensions.

Two types of cylindrical analyses are used:

1. The axial buckling is assumed to be group-independent. B_z^2 , the axial buckling, is determined by the reflector savings. The latter is obtained from two one-dimensional slab calculations for the system with and without axial reflectors. The results are given in Table III-5-I.

2. The axial buckling is assumed to be group-dependent. B_z^2 is determined by the axial leakage in each energy group from a slab calculation. For $k_{\text{eff}} = 1.152$, the axial bucklings are larger than those given by Reynolds and Stewart.¹ The corresponding critical radius is also larger than that obtained in Ref. 1. Results are given in Table III-5-II.

The discrepancy between the two types of analyses appears to be negligible for all cases considered here.

DETERMINATIONS OF $(dk/d\bar{T})_x$

$(dk/d\bar{T})_x$ is the Doppler coefficient of the fuel type x per unit change in the average temperature of that fuel type. This is related to the calculated Doppler reactivity change $(\Delta k_{\text{eff}}/k_{\text{eff}})_x$ by the following equation:

$$(dk/d\bar{T})_{x,T_0} \approx \frac{(\Delta k_{\text{eff}}/k_{\text{eff}})_x}{T_0 \ln [(\Delta \bar{T} + T_0)/T_0]}, \quad (2)$$

TABLE III-5-I. DOPPLER COEFFICIENTS OF REACTIVITY $\times 10^6$ (BASED ON CONSTANT AXIAL BUCKLING)
(Note: Per °C at room temperature; axial reflector saving = 9 cm, radial reflector saving = 12 cm)

Constituent Materials	$(dk/d\bar{T})_x (d\bar{T}_x/dT_{\text{Pu(max)}})$	
	$k_{\text{eff}} = 1.152$	$k_{\text{eff}} = 1.0$
Region I Pu-U-Mo Alloy	-5.3045	-9.8307
Region II Pu-plates	+4.3025	+0.7197
U-plates	-0.1813	-0.02806
Net Effect		
$dk/dT_{\text{Pu(max)}}$	-1.1833	-9.139
Critical Radius, cm		
r_1	32.811	32.811
r_2	53.4	34.2

TABLE III-5-II. DOPPLER COEFFICIENTS OF REACTIVITY
 $\times 10^6$ (BASED ON GROUP DEPENDENT AXIAL
 BUCKLING)
 (Note: Per °C at room temperature)

Constituent Materials	$(dk/d\bar{T}_x) (d\bar{T}/dT_{P_{i(\max)}})$		
	$k_{eff} = 1.152$	$k_{eff} = 1.05$	$k_{eff} = 1.00$
Region I Pu-U-Mo Alloy	-5.6432	-8.7323	-1.0412
Region II Pu-plates	+4.0134	+2.1642	+0.6915
U-plates	-0.1812	-0.1018	-0.03707
Net Effect			
$dk/dT_{Pu(\max)}$	-1.8111	-6.67	-9.757
Critical Radius, cm			
r_1	32.811	32.811	32.811
r_2	52.9	39.0	34.5

where the initial temperature T_0 and the increment ΔT are taken to be 300°K and 400°K, respectively. It is assumed that $(dk/d\bar{T})_x \sim T^{-1}$.

$(\Delta k_{eff}/k_{eff})_x$ for a particular k_{eff} value can be obtained quite readily by using multigroup diffusion theory. Δk_{eff} is just the difference in k_{eff} between two critical calculations using "hot" and "cold" cross sections for the fuel type x , respectively. All temperature-dependent effective cross sections used in the calculations were corrected for the overlap effect due to the neighboring resonances of the same, as well as different, nuclides in the mixture. The flux weighting factors used in determining the effective cross sections were also corrected for the depression due to resonance structures of the heavy nuclides. The inclusion of the overlap effect accounts for the higher negative Doppler coefficient in the inner core region with alloy plates for the $k_{eff} = 1.152$ case than that obtained in Ref. 1 for the same k_{eff} .

DETERMINATION OF $d\bar{T}_x/dT_{Pu(\max)}$

The heat produced in the fuel per unit volume is proportional to the corresponding number of fissions per unit volume in x . Assuming no heat removal, the relative rates of temperature rise are given by:

$$\frac{d\bar{T}_x}{dT_{Pu(\max)}} = \frac{\sum_i \int_E \Sigma_f^i(E) \bar{\phi}(E) dE}{\int_E (\Sigma_f^{Pu^{239}}(E) + \Sigma_f^{Pu^{240}}(E)) \phi(E, r_b) dE} \quad (3)$$

$$\frac{C_p^{Pu} \rho_{Pu}}{C_p^x \rho_x}$$

where

- $\Sigma_f^i(E) = N\sigma_f^i(E)$, the macroscopic fission cross section of isotope i in x
 r_b = radial distance (cm) of inner-outer core boundary
 ρ_x = atom density of fuel type x
 C_p^x = specific heat of fuel type x
 $\bar{\phi}(E)$ = spatial average of neutron flux in region where fuel type x is located
 i = fissionable isotope in x

All of the calculations used the specific heats given in Ref. 1. These are reproduced in Table III-5-III as are other parameters related to this study.

TABLE III-5-III. PHYSICAL PROPERTIES OF THE MOCKUP
 Atomic Densities and Dimensions of Core Regions I and II

Material	Core Region I	Core Region II
Be	7.2821×10^{21}	7.2821×10^{21}
O	2.6929×10^{22}	2.6929×10^{22}
Na	4.6926×10^{21}	4.6926×10^{21}
Cr	4.3891×10^{21}	4.3891×10^{21}
Fe	1.3897×10^{22}	1.3897×10^{22}
Ni	2.3337×10^{21}	2.3337×10^{21}
Mo	5.8941×10^{20}	—
U-238	7.8588×10^{21}	7.9457×10^{21}
Pu-239	1.8026×10^{21}	1.8026×10^{21}
Pu-240	1.6209×10^{20}	7.5149×10^{19}

$r_1 = 32.811$ cm; r_2 to be determined
 inner core height = 71.12 cm; total core height = 86.14 cm

Volume Fractions and Dimensions of Non-Core Regions

Inner radial sodium and steel reflector	Outer radial reflector	Inner axial reflector	Outer axial Na and SS reflector
V(Na) 0.57	V(Ni) = 0.82	V(Ni) = 0.45	V(Na) = 0.67
V(SS) 0.26	V(SS) = 0.04	V(SS) = 0.35	V(SS) = 0.33
V(Void) 0.17	V(Void) = 0.14	V(Na) = 0.20	
Thickness, cm: 12	16	10	8

Specific Heats	
Material	C_p^x cal/mole/°C
Pu	9.0
U	6.6
U-Pu-Mo	7.1

CONCLUSION

The results in Tables III-5-I and III-5-II clearly indicate that the positive component of the overall Doppler coefficient depends strongly upon the actual critical dimension of the outer core region. It is always desirable to have a relatively large negative component in the event of power excursion. Based on the calculated values, it is reasonable to believe that the system has a significant negative Doppler coefficient of reactivity as long as the actual critical core radius does

not exceed 39 cm. For a system with $r_2 \leq 39$ cm, the predicted negative component of the overall Doppler coefficient is at least four times larger than the corresponding positive component in the event of power excursion.

REFERENCES

1. A. B. Reynolds and S. L. Stewart, *Calculation of Doppler Coefficient in the ZPR-3 Mockup of SEFOR*, General Electric Co., (Private Communication to F. W. Thalgott, Argonne National Laboratory).

III-6. Doppler Effect Calculations with Interference Corrections¹

R. N. HWANG

Because of great concern about the effect of resonance interactions on the Doppler-effect calculations, extensive studies have been made for various dilute systems. A simplified method which allows the simultaneous occurrence of more than one type of interference has been developed to calculate the temperature-dependent effective cross sections. Consequently, the numerical work involved is simplified considerably.

Calculations have been made for typical Pu-239-

U-238, U-235-U-238, and Pu-239-U-238-Pu-240 systems. The results of these calculations seem to indicate that the resonance interference will substantially increase the negative Doppler effect of the system as compared to the case where resonance interferences are ignored.

REFERENCES

1. R. N. Hwang, *Doppler Effect Calculations with Interference Corrections*, Nucl. Sci. Eng. **21**, 523-535 (1965).

III-7. Fast Reactor Physics Parameters from a Pulsed Source

W. Y. KATO, H. H. MEISTER,* G. W. MAIN, J. L. RUSSELL, JR.† and K. L. CROSBIE†

INTRODUCTION

All of the important parameters which characterize a fast reactor system are calculated using the neutron spectrum. Experimental determination of the neutron spectrum is important because it provides a means of verifying calculational techniques and averaged cross section sets. With the gradual shift in emphasis in fast reactor research toward large (approximately 3000-5000-liter core) dilute fast reactors which have softer spectra than those currently operating, there is greater interest in the intermediate energy neutron distribution from a few hundred electron volts to a few hundred

kiloelectron volts. Many different methods have been used in an attempt to determine core spectra, including such techniques as cloud chambers,¹ nuclear emulsions,² proton recoil counters,³ helium-3 detectors, and lithium detectors with solid state devices.⁴ In thermal reactors, the Maxwellian distribution has been measured with time-of-flight techniques using a pulsing source or chopper.^{5, 6, 7} With fast reactors whose spectra range from a few hundred electron volts upward, it becomes difficult to measure the spectra by time-of-flight techniques using a chopper to interrupt the neutron beam because of the lack of good shutter materials and the difficulty of designing and constructing a shutter which acts fast enough to obtain good resolution.

With the availability of high-intensity short-duration pulsed neutron sources, it becomes possible to pulse a

* Kernforschungszentrum, Karlsruhe, West Germany.

† J. J. Hopkins Laboratory, General Atomic, San Diego, California.

fast system and measure the emerging burst of neutrons using time-of-flight techniques. In order to determine the feasibility of measuring fast spectra using time-of-flight techniques and of measuring other reactor physics parameters with a pulsed source, a large depleted uranium assembly and auxiliary equipment was assembled at the General Atomic LINAC facilities. The study was conducted jointly by personnel of Argonne National Laboratory and General Atomic. This paper presents the results of time-of-flight spectra, spectral indices, fission rates and U-238 capture measurements from various parts of the depleted uranium assembly.

EXPERIMENTAL ARRANGEMENT

Approximately 1700 bricks (10 metric tonnes) of depleted uranium (0.21% U-235) were stacked in sixteen layers on a movable pedestal as shown in Fig. III-7-1. The uranium was stacked on a movable table so that one face of the 10-tonne assembly could be brought up to the electron beam tube of the LINAC.

The bricks in the ninth layer were arranged, as shown in Fig. III-7-2, so that 5-cm high by 5-cm wide traverse

channels could be made in five different positions across the layer. The bricks of uranium were so arranged that it was possible to remove 5- × 5-cm columns of depleted uranium for the insertion of counters or foils, or for the emergence of a neutron beam. The five transverse traverse channels were located at distances of 5.1, 20.3, 35.6, 50.8 and 66.0 cm (2, 8, 14, 20, 26 in.) from the target.

The over-all dimensions of the block were 81-cm long, 76-cm wide, and 81-cm high (32 × 30 × 32 in.). The entire uranium assembly was covered with a 1-mm thick cadmium sheet and a 7-mm thick boron-aluminum (Boral) sheet to minimize the effect of room scattered neutrons.

A beam tube from the LINAC was arranged so that the 30-MeV electron beam would impinge upon a nickel coated 2- × 2- × 2-in. brick of depleted uranium whose center is located about 8 cm (3 in.) from the outside face along the center line in the ninth layer of the large block of uranium. The center of the target was located approximately 43.2 cm from the table top. The target was air cooled with a small stream of air and its

CUTAWAY VIEW

FIG. III-7-1. Uranium Block on Pedestal. ANL Neg. No. 112-4779A.

FIG. III-7-2. Cross Section of 9th Layer. ANL Neg. No. 112-4863.

temperature was monitored with a thermocouple. During the experiment the beam current of the LINAC was usually adjusted so that the surface temperature of the target was no greater than about 500°C.

The 30 MeV electron beam from the LINAC incident on the target is converted into Bremsstrahlung which, in turn, produces photo-fission neutrons in the U-238 target and surrounding material. The arrangement of the experiment with respect to the beam from the LINAC and the flight path is shown in Fig. III-7-3. A detailed description of the LINAC characteristics, its facilities, and 50-m flight path may be found in Ref. 8.

Experimental measurements of spectra or spectral indices were made from one of the five transverse channels, shown in Fig. III-7-2. In order to pin-point the positions where an experiment was conducted, a rectangular coordinate system is used to describe the horizontal plane locations in the ninth layer where all of the measurements were made. The center of the target block is considered to be (0, 0), the center of the (x, y) coordinate system. All of the channels, except the one where measurements were being conducted, were normally filled with depleted uranium bricks in order to

avoid perturbations of the flux distribution. Depleted uranium bricks were also used to fill the channel to the location where measurements were being conducted.

NEUTRON DETECTORS

Three different types of neutron detectors were used for the fast spectrum time-of-flight measurements: a cluster of BF_3 counters, a liquid scintillator, and a B_4C block in combination with the liquid scintillator. The BF_3 counter cluster consisted of thirty-two 5-cm (2-in.) diameter thin walled 0.08 cm (0.30 in.) thickness B^{10}F_3 filled (one atmosphere pressure) gas counters with an active length of about 40 cm.⁹ For measuring neutrons with energies greater than about 200 keV, a 13-cm (5-in.) diameter liquid organic scintillator¹⁰ was used. The third type of detector used for time-of-flight measurements was a boron-carbide and water mixture¹¹ and the 5-in. liquid scintillator. An aluminum box of approximately $51 \times 25 \times 10$ cm ($20 \times 10 \times 4$ in.) was filled with about 14 kg (30 lb) of commercial grade boron carbide and 6 kg (14 lb) of water, and the 0.48 MeV gammas from the (n, α) reaction were observed with the liquid scintillator. In addition to these detectors for

FIG. III-7-3. Experimental Arrangement of Neutron Flight Path. ANL Neg. No. 112-4864.

time-of-flight neutrons, a conventional BF_3 gas-filled counter was placed in the same room where the uranium assembly was located to monitor the over-all neutron flux.

SPECTRA MEASUREMENTS

Neutron spectrum measurements were made at the following locations in the assembly: (20.3, 0), (20.3, 12.7), (20.3, 25.4), (35.6, 0) and (50.8, 0). Measurements of the neutron spectrum at (20.3, 0) were made with the three types of detectors at the 50-m flight path station. At the other locations the principal detector was the B_4C detector. Details of the collimator and shielding around the cell where the uranium assembly was located may be found in a paper by J. Beyster, et al.⁸ The detector pulses were analyzed using a TMC-CN1024 Analyzer with a TMC-211 or TMC-201 logic plug-in unit for the time-of-flight measurements. With the B_4C detector, a channel width of 125 nsec was used while a 500-nsec width was used for the BF_3 counters

and 31.25 nsec for the 5-in. scintillator. The data stored in the time analyzer was processed using ECTOPLASM, a computer code developed by Y. Naliboff¹² for reducing time-of-flight spectral information on the IBM-7044 and subsequently adapted to the CDC-3600 computer. This code corrects the data for background, either directly measured or calculated, time analyzer deadtime losses, mean neutron emission time in the assembly, and source strength variations from run to run based on a monitor counting rate. The program converts the time dependent neutron distribution into an energy dependent neutron spectrum. In these measurements, since there was some uncertainty about how the background should be measured, the background correction was calculated by taking the average of the last 100 analyzer channels.

Spectra obtained from the time-of-flight measurements of neutrons originating at the (20.3, 0) location in the uranium assembly are shown in Fig. III-7-4. Curves A and B are curves obtained from the same run.

FIG. III-7-4. Spectra at (20.3, 0). ANL Neg. No. 112-4924.

Curve A shows the effect of approximately 0.54 cm (0.214 in.) of aluminum in the flight path. The aluminum resonances at 35.04 ± 0.09 keV, 89 ± 1 keV, and about 150 keV are clearly observed in Curve A which

has not been corrected for the aluminum in the flight path. When the neutron distribution is corrected for the aluminum resonances using the total cross sections tabulated by R. Howerton,¹³ the resulting neutron dis-

tribution is given by Curve B of Fig. III-7-4. Curve C shows the data obtained with the BF_3 counters uncorrected for material in the flight path. Since the BF_3 detector efficiency drops off rapidly at high energies, the region above 100 keV shows the effects of poor statistics. In addition, the region between 10 and 100 keV shows the effects of the aluminum resonances. Curves D and E were obtained using transport theory analysis described later.

A measurement of the spectrum at (20.3, 0) with the 5-in. organic scintillator resulted in spectral information from about 300 keV to 10 MeV. The shape of the curve in the 300 keV to 1 MeV region is almost identical with the shape of the curve obtained with the B_4C detector in that location.

Measurement of the spectrum at (35.6, 0) is shown in Fig. III-7-5. Only the data from the B_4C detector is shown compared with a calculated curve. The experimental data are shown uncorrected for the aluminum in the flight path. Spectrum measurements attempted at the (50.8, 0) position were not successful because of the limitations on neutron intensity due to the lack of a properly cooled target.

It should be noted that the neutrons which were observed in the time-of-flight measurements emerged either from the $y = 0$ plane or a plane parallel to the $y = 0$ plane. For the case of the measurements from the $y = 0$ plane, the neutrons emerged essentially at a 90° angle from the incident electron beam. In order to determine if an angular dependence of the spectra exists, measurements were obtained from (20.3, 0), (20.3, 12.7) and (20.3, 25.4) locations. These locations were chosen since the neutrons would be from positions having different angles with respect to the incident electron beam.

Experimental spectra obtained at (20.3, 0), (20.3, 12.7) and (20.3, 25.4) are compared in Fig. III-7-6. The data shown in this figure, however, do not show strong angular dependence effects. A summary of the LINAC conditions, analyzer channel widths, and time to obtain the aforementioned data are given in Table III-7-I.

COMPARISON WITH THEORY

One purpose of the uranium spectrum measurements was to provide a means of evaluating methods used to calculate fast and intermediate neutron spectra. Al-

FIG. III-7-5. Spectra at (35.6, 0). ANL Neg. No. 112-4925.

FIG. III-7-6. Experimental Spectra at Different Locations. ANL Neg. No. 112-4922.

ough the geometry is not one-dimensional, a one-dimensional calculation is a reasonable approximation for comparison with the measured spectra near the source. The first efforts to calculate the spectra ob-

tained in various parts of the assembly were made with one-dimensional, multigroup transport-theory computer codes. Two codes, the GAPLSN¹⁴ and DSN (ANL-RE 209)¹⁵ were used with GAM II⁽¹⁶⁾ to obtain

TABLE III-7-I. TIME-OF-FLIGHT PARAMETERS

Run No.	Curve	Position (x, y)	Run Time, min	Detector	LINAC		Analyzer	
					Flt. Path, m	Burst Width, nsec	Channel Width, nsec	Total Counts
6	Fig. III-7-5	(35.6, 0)	142	B ₄ C + 5" Scintillator	50	100	125	—
10	Curve A & B, Fig. III-7-4	(20.3, 0)	59	B ₄ C + 5" Scintillator	50	20	125	300,400
11		(20.3, 0)	63	5" Scintillator	50	20	31½	302,400
13	Curve C, Fig. III-7-4	(20.3, 0)	157	BF ₃ Counters	50	100	500	293,000
15	Curve B, Fig. III-7-6	(20.3, 12.7)	44	B ₄ C + 5" Scintillator	50	50	125	305,000
16	Curve C, Fig. III-7-6	(20.3, 25.4)	33	B ₄ C + 5" Scintillator	50	100	125	145,000

the cross section set for the GAPLSN code; cross section set 199,⁽¹⁷⁾ a modified YOM¹⁸ set, was used in the DSN code to calculate the spectrum in the depleted uranium assembly.

Results of the GAPLSN calculation are shown by Curve D in Fig. III-7-4. Since the experimental spectrum (Curves A, B, and C) were obtained at an angle of 90 degrees to the source, the calculated curve is obtained by averaging angular groups 5 and 6.

The DSN calculation using a modified 18-group YOM cross section set was done assuming a fission spectrum source, isotropic scattering, and an S-8 approximation. The calculated results which represent the total flux at (20.3, 0) are shown in Curve E in Fig. III-7-4. There is reasonable agreement between shapes of the theoretical curves (D & E) above about 30 keV and a wide discrepancy below this value. The difference between the two theoretical curves is due to differences in such factors as resonance self-shielding, assumed shape of the fission neutron spectrum, shape of the source neutron distribution, and the number of down scattering groups.

Comparing the experimental (Curve B) and the theoretical spectra (Curves D & E), there appears to be qualitative agreement in the shapes of the spectra above about 30 keV. The experimental curve lies between the two theoretical curves below 30 keV. In addition, if one compares closely the shapes of the experimental and theoretical curves, the peak of the experimental spectrum appears to be shifted slightly toward the lower energy region.

Another comparison between experimental data and theoretical calculations is shown in Fig. III-7-5 for the spectrum at (35.6, 0). The theoretical curve was that obtained using the DSN code. If one compares the experimental and theoretical curves in Figs. III-7-4 and III-7-5, it is found that there is qualitative agreement in the shapes of the curves above about 30 keV. The theoretical spectrum of Fig. III-7-5 seems to indicate a

slightly harder spectrum than that obtained from experimental measurements. There are large discrepancies between experiment and theory in the lower energy regions which indicates that further effort, both experimentally and theoretically, is needed.

FISSION DETECTORS

In order to obtain spectral indices to compare with those obtained from the time-of-flight spectra data or theoretical spectrum calculations, two types of special fission detectors were used: small gas-flow-type fission chambers and solid state devices.

The small, absolute gas-flow-type fission chambers used to measure effective fission cross section ratios were of parallel plate design similar to those described by F. Kirn.¹⁹ The body of the chambers (5-cm diameter, 1.9-cm long) was made with thin aluminum (0.05-cm thick) walls in order to minimize spectral distortion caused by inelastic scattering.

The solid state counters which were used to measure the effective fission ratios were constructed using surface barrier detectors. The counters were 1.1-cm in diameter, 1.2 cm long, with a wall thickness of 0.05 cm. Each counter contained a 0.012-cm thick stainless steel disc, located 0.06 cm from the sensitive face of the detector, on which was electroplated a coating of fissionable material. The thickness of the coatings ranged from 50 to 100 $\mu\text{g}/\text{cm}^2$.

Plates coated with U-233, U-234, U-235, U-238, Np-237, Pu-239, and Pu-240 were available for fission rate measurements. Since the detectors could take only one plate at a time, it was necessary to interchange plates in order to obtain fission rates for the different fissile isotopes.

FISSION RATE MEASUREMENTS

Two types of fission rate measurements using the solid state and gas-flow detectors were of interest in the assembly: the observation of the die-away of the neutron spectrum, and the time averaged fission rate.

During the initial testing of the fission chambers in the accelerator beam, an additional background was observed in the gas-flow chambers which appeared about 10 μ sec after each accelerator burst. These background pulses were caused partially by the intense gamma radiation produced in the target and partially by pickup of the radio frequency voltage of the accelerator through the preamplifier input and power supply. It was found that the amplitude of these background pulses was proportional to the average beam current; thus, by running the accelerator at a reduced beam intensity, it was found possible to keep these pulses below the threshold of the discriminator so that fission rate measurements could be made. Generally, when fission rate measurements were made using the gas-flow counter, a beam current of the order of 10 % of that used for the time-of-flight measurements was used. It was found that this background was rather high at the (20.3, 0) position, so that it was possible to make die-away measurements only at the (35.6, 0) position with the gas-flow counters.

The solid state detectors, which had a better shielded preamplifier, showed a much lower amplitude for the background signal. Because of the long rise time (>1 μ sec) on the low noise preamplifier (Tennelec Model 100-A) used with the solid state detectors only the time averaged fission rate were measured at the (20.3, 0) position. Three types of corrections were introduced to the counting rate in order to obtain correct fission ratios: (1) corrections for counting losses, (2) normalization for variations of beam intensity, and (3) corrections for the fission contributions of impurity isotopes. Corrections for counting losses were necessary even at relatively low average counting rates because of the pulsed operation of the neutron source and the short decay time of the neutron spectrum. The experimentally-determined, time-averaged fission ratios at (20.3, 0) are given in Table III-7-II. For comparison, fission ratios obtained from the DSN calculation are also given in the same table.

Two types of fission rate measurements were made at the (35.6, 0) position with gas-flow fission chambers: a time-averaged fission rate determination using a conventional 1 Mc scaler and fast amplifier, and a time-dependent fission rate with the TMC 1024 channel time analyzer and the TMC 201 logic plug-in unit.

The time-dependent fission rates after the injection of the neutron burst were measured for the following isotopes: U-233, U-235, Np-237, Pu-239, and Pu-240.

Typical die-away or burst decay curves are shown in Figs. III-7-7 and III-7-8 for U-235 and Pu-239, respectively. A channel width of 62.5 μ sec was used with the analyzer to obtain these decay curves.

It is expected that higher spatial and energy modes of

TABLE III-7-II. TIME-AVERAGED FISSION RATIOS AT (20.3, 0)

	Experimental	Calculated (DSN-Spectra)
U-233/U-235	1.678 $\pm$ 0.020	1.569
U-234/U-235	0.133 $\pm$ 0.003	0.171
U-238/U-235	0.0091 $\pm$ 0.0003	0.00643
Np-237/U-235	0.182 $\pm$ 0.003	0.148
Pu-239/U-235	—	1.114
Pu-240/U-235	—	0.157

FIG. III-7-7. Time Dependence of U-235 Fission Rate at (35.6, 0). ANL Neg. No. 112-4918.

the flux distribution will prevail in the first part of the decay curves; however, the data for the U-233 and U-235 isotopes indicate that the data can be approximated by a single exponential function e^{-at} over most of the curve. An exponential fit for times greater than 500 nsec yield values of $1/\alpha$ for five different isotopes at (35.6, 0) as follows: U-233, 261 ± 15 nsec; U-235, 284 ± 15 nsec; Np-237, 274 ± 30 nsec; Pu-239, 263 ± 15 ; and Pu-240, 274 ± 25 . It was not possible to obtain a decay curve with the U-238 detector because of limited intensities. These results indicate that after a transition time of about 500 nsec, a time-asymptotic spectrum is established which decays exponentially.

FIG. III-7-8. Time Dependence of Pu-239 Fission Rate at (35.6, 0). *ANL Neg. No. 112-4919*.

The variation of the neutron spectrum as a function of time after the injection of the neutron burst was studied by plotting the fission ratios, σ_f^z/σ_f^{25} , as a function of time. The curves shown in Fig. III-7-9 show the dependence of the fission ratios with time at the (35.6, 0) location. With the exception of that of U-233/U-235 and Pu-239/U-235, which are not too sensitive to spectrum changes, the fission ratios indicate a decreasing value or a softening of the neutron spectrum with time. After about 500 nsec, the fission ratios within experimental error arrive at a constant value which is characteristic of the establishment of a time-asymptotic spectrum. The asymptotic values of σ_f^z/σ_f^{25} which have been obtained by adding all of the channels with $t > 500$ nsec are given in Table III-7-III. The time-averaged fission ratios obtained using a conventional scaler are also listed in Table III-7-III. The limits of error in Table III-7-III are based on statistical counting uncertainties only. They do not include uncertainties such as that in the quantity of material coated on the plates. It is expected that uncertainties due to the coating material are less than 1% for the U-235 and U-238 detectors, and of the order of 3% for U-233, U-234, Pu-239, and Pu-240. For comparison purposes fission

FIG. III-7-9. Fission Ratios at (35.6, 0). *ANL Neg. No. 112-4921*.

TABLE III-7-III. FISSION RATIOS AT (35.6, 0)

	Experimental (Time Averaged)	Calculated (DSN-Spectra)	Ex- perimental- Asymptotic
U-233/U-235	1.599 ± 0.024	1.548	1.59
U-234/U-235	0.0909 ± 0.0015	0.107	—
U-238/U-235	0.0031 ± 0.0004	0.00252	—
Np-237/U-235	0.0836 ± 0.0018	0.0877	0.0246
Pu-239/U-235	1.012 ± 0.016	1.060	0.819
Pu-240/U-235	0.1011 ± 0.002	0.0925	0.0463

ratios obtained from the DSN calculation are also given in Tables III-7-II and III-7-III for the (20.3, 0) and (35.6, 0) locations.

FOIL ACTIVATION MEASUREMENTS

The U-235 and U-238 fission and the U-238 capture distributions were measured in the assembly by irradiating 0.013-cm thick foils of 93% enriched uranium, weighing about 0.3 g each and by depleted uranium foils (0.2 w/o U-235) of about 1 g each. The irradiation was carried out at the same time that time-of-flight spectrum measurements were in progress from the (20.3, 0) position. The foils were analyzed for Mo-99 to determine fission rates and Np-239 was separated

radiochemically and analyzed using techniques of Refs. 20 and 21 to determine U-238 capture rates.

The U-235 fission distribution in the assembly, as measured by this activation technique, is shown in Fig. III-7-10. The axial (x axis) fission distributions appear to approach an exponential $e^{-\alpha x}$ decay. The U-235 and U-238 fission rates and U-238 capture rates are tabulated in Table III-7-IV. The time-averaged U-238/U-235 fission ratio using the activation techniques was obtainable only at $x = 20.3$ and 35.6 because of limited neutron intensity. The U-238 capture to U-235 fission ratios and the U-238/U-235 fission ratios are given in Table III-7-V. There are considerable uncertainties in these U-238/U-235 fission ratios because of the low Mo-99 activity in the U-238 foils. Theoretically determined U-238 capture/U-235 fission calculated from the DSN calculations are also given in the table.

Since an asymptotic spectrum is not established for points less than about 50 cm from the source,²² it is possible to obtain only a rough estimate of the diffusion length in depleted uranium from the decay curves given

FIG. III-7-10. U-235 Fission Distributions. ANL Neg. No. 112-4923-T.

TABLE III-7-IV. U-238 CAPTURE AND U-235 FISSION DISTRIBUTION

(Note: Capture and fission rates normalized to source intensity during Run No. 13 in the time-of-flight spectra measurements).

Position (x, y)	Captures/sec/g-U-238	Fissions/sec/g-U-235	Fissions/sec/g-U-238
(20.3, 0)	—	—	—
(35.6, 0)	2.62×10^5	2.02×10^6	1.14×10^4
(50.8, 0)	3.75×10^4	2.69×10^5	—
(66, 0)	4.58×10^3	3.39×10^4	—
(20.3, -12.7)	1.06×10^6	7.97×10^6	9.17×10^4
(35.6, -12.7)	1.92×10^5	1.43×10^6	8.43×10^3
(50.8, -12.7)	3.02×10^4	2.08×10^5	—
(66, -12.7)	4.08×10^3	2.85×10^4	—
(20.3, -25.4)	3.03×10^5	2.44×10^6	1.38×10^4
(35.6, -25.4)	8.28×10^4	5.59×10^5	6.16×10^3
(50.8, -25.4)	1.51×10^4	1.06×10^5	—
(66, -25.4)	2.58×10^3	3.97×10^4	—

LINAC (Run No. 13)
Rep. rate 360/sec
Burst width 100 nsec
BF₃ monitor 2.3×10^5 c/m

TABLE III-7-V. U-238 CAPTURE AND U-235 FISSION RATIO DISTRIBUTIONS

Position (x, y)	$\sigma_c^{U-238} / \sigma_f^{U-235}$		$\sigma_f^{U-238} / \sigma_f^{U-235}$
	Experimental	Calculated	
(20.3, 0)	—	0.160	—
(35.6, 0)	0.131	0.141	0.0038
(50.8, 0)	0.141	0.143	—
(66, 0)	0.137	0.146	—
(20.3, -12.7)	0.135	—	0.0097
(35.6, -12.7)	0.136	—	0.0040
(50.8, -12.7)	0.146	—	—
(66, -12.7)	0.145	—	—
(20.3, -25.4)	0.127	—	0.0038
(35.6, -25.4)	0.150	—	0.0093
(50.8, -25.4)	0.143	—	—
(66, -25.4)	0.066	—	—

in Fig. III-7-10. These curves give a value of 7.5 ± 0.4 cm for the axial decay length. Assuming a value of 2.5 cm for the radial extrapolation length and the values of 81 cm and 76 cm for the height and width, a value of $L = 8.2$ cm for the diffusion length is obtained; this is considerably smaller than the value of 9.17 ± 0.18 obtained by C. Chezem.²²

CONCLUSIONS

These experiments have shown that a LINAC provides a high-intensity, short-duration neutron burst for neutron spectra measurements from a fast assembly using time-of-flight techniques. Spectra from a fast

reactor assembly may be measured with good resolution from a few kiloelectron volts to about 10 MeV. The B_4C detector has been shown useful up to about 2 MeV. Its basic limitation is due to the uncertainty in its response function above 2 MeV. For higher energies the liquid scintillator would be the more appropriate detector to be used. The measured gross neutron spectrum is in qualitative agreement above 30 keV with the theoretically calculated spectrum. Below 30 keV there are large discrepancies between experiment and theory which require further effort for clarification. The time-of-flight technique with a LINAC could produce high energy resolution spectra to obtain the fine structure in the spectrum. This might be a means to check the methods for obtaining averaged cross section sets useful in reactor analysis. The experiments also gave a survey of the time-space dependence of fission rates and spectral indices, although statistical accuracy was, in some cases, limited by low neutron intensities.

The neutron intensity with the air-cooled U-238 target was sufficient to measure the U-235 fission and U-238 capture rates throughout the assembly, using radiochemical separation techniques. Measurements of fission ratios with solid state detectors and gas-flow-type absolute counters was possible at the $x = 20.3$ and 35.6 positions. The gamma sensitivity of the gas-flow counters and radio frequency noise pickup made it necessary to operate the LINAC at reduced intensities so that it was not possible to obtain spectral index measurements at distances greater than about 35 cm from the target where an asymptotic spectrum might be expected.

Fission rates were measured at (35.6, 0) as a function of the time after the injection of the neutron burst. The die-away curves for U-233 and U-235 can be approximated by a single exponential with a decay time of about 270 nsec. The variation of the fission ratios with time indicates a softening of the neutron spectrum during a transition time of about 500 nsec, after which an exponentially decaying asymptotic spectrum is established.

ACKNOWLEDGMENT

The authors express their appreciation to L. R. Dates who designed the components for the uranium pedestal, to J. R. Brown† and W. Mowry† who assisted in assembling and dismantling the experimental equipment and J. Trimble† and R. Hackney† who assisted in the data reduction and theoretical calculations. Appreciation is expressed to R. Armani and his group who analyzed the activity of uranium foils using radiochemical techniques. Acknowledgment is also extended to D.

Meneghetti for helpful discussions on the theoretical analysis of spectra from an uranium assembly.

REFERENCES

1. H. V. Lichtenberger, F. W. Thalgot, W. Y. Kato and M. Novick, *Operating Experience and Experimental Results Obtained from a NaK Cooled Fast Reactor*, Proc. 1955 Geneva Conference **3**, Paper No. 813, p. 245.
2. W. C. Redman and J. H. Roberts, *Some Current Techniques of Fast Neutron Spectrum Measurements*, Proc. 1958 Geneva Conference **12**, Paper No. 597, p. 72.
3. E. F. Bennett, *Fast Neutron Spectroscopy by 4π Recoil Proportional Counting*, Trans. Am. Nucl. Soc. **7**, No. 2, 369 (1964).
4. R. J. Huber, *In-Core Experiments with a Lithium-6 'Sandwich' Fast Neutron Spectrometer*, Trans. Am. Nucl. Soc. **7**, No. 2, 368 (1964).
5. G. R. Keepin, *Pulsed Neutron Research in Neutron and Reactor Physics*, At. Energy Rev. **2**, No. 2, IAEA, Vienna (1964).
6. K. H. Beckurts, *Reactor Physics Research With Pulsed Neutron Sources*, Nucl. Instr. Methods **11**, 144-168 (1961).
7. R. S. Stone and R. Slovacek, *Neutron Spectra Measurements*, Nucl. Sci. Eng. **6**, 466-474 (1959).
8. General Atomic Linear Accelerator Facility, USAEC Report, GA-1855 (rev.).
9. J. R. Beyster, et al., *Measurements of Neutron Spectra in Water, Polyethylene, and Zirconium Hydride*, Nucl. Sci. Eng. **9**, 168-184 (1961).
10. J. R. Beyster, et al., *Integral Neutron Thermalization*, USAEC Report, GA-4659 (January 1964).
11. J. L. Russell, et al., *Neutron Spectra in Bulk Media by Time-of-Flight*, Proc. IAEA Symp. Pulsed Neutron Research, SM 62/61 (1965).
12. Y. D. Naliboff, *FORTAN ECTOPLASM Program*, USAEC Report, GAMD-2887 (January 1962).
13. R. J. Howerton, *Tabulated Neutron Cross Sections, Part I: 0.001-14.5 MeV*, USAEC Report, UCRL-5226 (May 1958).
14. J. H. Alexander, G. W. Hinman and J. R. Triplett, *GAPLSN, A Modified DSN Program for the Solution of the One-Dimensional Anisotropic Transport Equation*, USAEC Report, GA-4972 (March 1964).
15. B. Carlson, C. Lee and J. Worlton, *The DSN and TDC Neutron Transport Codes*, USAEC Report, LAMS-2346 (February 1960).
16. G. D. Joanou and J. S. Dudek, *GAM-II, A B_3 Code for the Calculation of Fast-Neutron Spectra and Associated Multigroup Constants*, USAEC Report, GA-4265 (September 1963).
17. M. G. Bhide and H. H. Hummel, *Calculations of the Doppler Coefficients for Large Ceramic Fueled Fast Reactor*, ANL-6601 (1962).
18. S. Yiftah, D. Okrent and P. A. Moldauer, *Fast Reactor Cross Sections*, (Pergamon Press, New York, 1960).
19. F. S. Kirn, *Neutron Detection with an Absolute Fission Counter*, Proc. IAEA Symp. Neutron Dosimetry, IAEA, Vienna, **2** (1963).
20. W. Y. Kato, F. S. Kirn and W. C. Redman, *Measurement of the Conversion Ratio for a Fast Breeder Reactor*, Reactor Operational Problems, **2**, D. J. Hughes, et al., eds., (Pergamon Press, New York, 1957).
21. R. R. Smith, et al., *The Breeding Ratio of a Plutonium Loading in EBR-I*, USAEC Report, ANL-6789 (1964).
22. C. G. Chezem, *A Uranium-Metal Exponential Experiment*, Nucl. Sci. Eng. **8**, 652-669 (1960).

† General Atomic, San Diego, California.

III-8. Evaluated Fast Cross Sections

D. M. O'SHEA, H. H. HUMMEL, W. B. LOEWENSTEIN and D. OKRENT*

INELASTIC SCATTERING

Inelastic scattering distributions which are calculated from cross section data for individual levels have been compared with those which are calculated from the statistical model of inelastic scattering. Fig. III-8-1 shows such a comparison for U-238. Here cross sections for individual level excitation have been measured to 1.5 MeV.¹ Measurements of nuclear temperature have been made at higher energies. The measured nuclear temperature has been extrapolated to the lower energies and has been used in the statistical model. In the figure the distribution represents the scattering cross section from the initial energy E to the final energy E' , versus E' . An average has been taken over an energy interval from 1.1 to 1.4 MeV and the cross section is shown as distributed over a number of final energy intervals. It may be noticed that a considerable portion of the cross section of the resolved levels is due to the scattering from the lower lying levels, much more than would be predicted by the statistical model. The dashed line in the upper right hand corner indicates that there is still about 0.8 barns scattering from the lowest lying levels. This in turn forces the remaining portion of the distribution to have fewer low energy neutrons than indicated by the nuclear temperature model. A smooth transition from the level model to the nuclear temperature model should improve at energies where the scattering cross section for the lower lying levels does not form the major portion of the total inelastic cross section.

A similar comparison for iron is shown in Fig. III-8-2. Here the inelastic distribution from the resolved levels in iron is computed from cross sections measured at Los Alamos,² for levels up to 5 MeV. Again there is still appreciable scattering from the lowest lying levels. The transition from the level model to the nuclear temperature model in this case is not particularly good.

Similar comparisons have been carried out for Mo, Nb and W. In the first two (Mo and Nb) the transition from the level model to the nuclear temperature model is quite smooth. Tungsten exhibited a sharp discontinuity on the basis of the known data.

Another alternative to using the nuclear temperature model at energies which are comparatively low is to make estimates for the cross section for scattering from groups of levels. This procedure was used in YOM³ where levels were still unresolved at relatively low energies. In the case of U-235, P. Moldauer had made a

FIG. III-8-1. Inelastic Scattering Distributions for U-238. ANL Neg. No. 112-5778.

FIG. III-8-2. Inelastic Scattering Distributions for Fe. ANL Neg. No. 112-5779.

theoretical calculation of the individual level excitation cross section for the first twelve levels of U-235. These were subsequently grouped in YOM into four levels up to 500 keV.

Figure III-8-3 shows a comparison of the inelastic distributions from the statistical model, the twelve levels, and the four groups of levels. Here the distribution from the resolved levels shows appreciable scattering from the lower lying levels. This is not evident in the distribution from the statistical model. The distribution obtained from the grouped levels predicts an energy loss which is too small. There is no cross section for scattering in the low energy tail of the distribution.

* Laboratory Director's Office.

FIG. III-8-3. Inelastic Scattering Distributions for U-235. ANL Neg. No. 112-5780.

FIG. III-8-4. U-238 Capture Cross Sections. ANL Neg. No. 112-5732.

The difficulty arises in trying to break up the grouped levels in a reasonable way and to assign an appropriate energy loss to each segment.

CAPTURE CROSS SECTIONS

Fig. III-8-4 is a summary of recent measurements of capture in U-238, along with measurements reported in the second edition of BNL-325. The disagreement between these is quite large. The British⁴ and Russian⁵ measurements lie below the Oak Ridge⁶ measurements by a considerable amount. They also lie below the measurements reported in BNL-325 in the vicinity of 100 keV. Above 100 keV measurements made at Aldermaston⁷ are in general agreement with the data in BNL-325. The solid curve adopted for reactor analysis

FIG. III-8-5. Th Capture Cross Sections. ANL Neg. No. 112-5734.

FIG. III-8-6. U-235 Fission Cross Sections. ANL Neg. No. 112-5736.

in the figure lies slightly below the curve in BNL-325 from 50 keV to 1 MeV.

This discrepancy is not unique to U-238. Figure III-8-5 contains a similar summary on the capture in thorium. The Oak Ridge measurements agree with the Russian measurements below 50 keV. The British measurements lie below these and also below the old measurements in BNL-325. The solid curve adopted for reactor analysis is slightly lower than that in BNL-325, except from 300 keV to 1 MeV where it is slightly higher.

FISSION CROSS SECTIONS

Several significant changes have occurred in the status of fission cross sections since Supplement No. I of BNL-325 was issued. Recent measurements of the U-235 fission cross section are summarized in Fig. III-

FIG. III-8-7. Pu-240 Fission Cross Sections. ANL Neg. No. 112-5744.

8-6, along with the measurements reported in BNL-325. The measurements from Aldermaston^{8, 9} are consistently about 5-8% lower than the curve given in BNL-325. There are some Russian measurements^{10, 11} which are also lower; however, the experimental errors in the Russian results are consistent with the BNL-325 curve. The upper curve in Fig. III-8-6 is the curve from BNL-325. The dashed line below this was used in generating the first version of the twenty-six group set.¹² The lower end of the curve at 10 keV is anchored to a Russian point just below 10 keV and to a British measurement⁴ of the average fission cross section between 9 and 10 keV.

Some interesting data are also available in the case of Pu-240 fission. Figure III-8-7 shows some recent Russian measurements¹³ for Pu-240 fission along with the values in BNL-325. The agreement is good at high energies, but the Russian measurements show the existence of a sizable fission cross section below what was considered to be the Pu-240 fission threshold at 200 keV. There is apparently a sizable cross section at energies as low as 10 keV. These data should make a

significant difference in the calculation of large dilute fast reactors with high Pu-240 content.

REFERENCES

1. A. B. Smith, *Recent Changes in Heavy Element Cross Sections*, Proc. Conf. on Breeding, Economics and Safety in Large Fast Power Reactors, ANL-6792, 29-47 (1963).
2. J. C. Hopkins and M. G. Silbert, *Inelastic Scattering of 2 to 5 MeV Neutrons by Iron*, Nucl. Sci. Eng. **19**, No. 4, 431 (1964).
3. S. Yiftah, D. Okrent and P. A. Moldauer, *Fast Reactor Cross Sections*, (Pergamon Press, New York, 1960).
4. E. R. Rae, et al., *Neutron Interactions with Reactor Materials*, Proc. 1964 Geneva Conference (to be published).
5. V. A. Tolstikov, L. E. Sherman and Yu Ya Stavinskii, *Measurement of the Capture Cross Section of 5 to 200 keV Neutrons for Uranium-238 and Thorium-232*, Atomnaia Energiia **15**, No. 11, 414 (1963).
6. R. L. Macklin and J. H. Gibbons, Oak Ridge National Laboratory (Private Communication).
7. J. F. Barry, J. Bunce and P. H. White, *Cross Section for the Reaction U-238 (n,γ) in the Energy Range 0.12-7.6 MeV*, Reactor Sci. Technol. (J. Nucl. Energy Parts A/B) **18**, No. 9, 481 (1964).
8. P. H. White, *Measurements of the U-235 Neutron Fission Cross Section in the Energy Range 0.04-14 MeV*, Reactor Sci. Technol. (J. Nucl. Energy Parts A/B) **19**, No. 5, 325 (1965).
9. J. L. Perkins, et al., *The Fission Cross Sections of U-233, U-234, U-235, U-236, NP-237, Pu-239, Pu-240, and Pu-241 for 24 keV Neutrons*, Reactor Sci. Technol. (J. Nucl. Energy Parts A/B) **19**, No. 6, 423 (1965).
10. G. A. Dorofeev and Y. P. Dobrynin, *Effective Fission Cross Sections of U-233, U-235, Pu-239, and Pu-240 in the Neutron Energy Range 30 keV-5 MeV*, J. Nucl. Energy **5**, No. 2, 217 (1957).
11. G. V. Gorlov, et al., *The Fission Cross Sections of U-233 and U-235 for Neutrons Having Energies Between 3 and 800 keV*, Reactor Sci. Technol. (J. Nucl. Energy Parts A/B) **12**, No. 1/2, 79 (1960).
12. D. M. O'Shea, H. H. Hummel, W. B. Loewenstein, D. Okrent and C. N. Kelber, *Twenty-Six Group Cross Sections*, ANL-6858 (to be published).
13. V. G. Nesterov and G. N. Smirenkin, *The Fission Cross Section of Pu-240 for 0.04-4.0 MeV Neutrons*, Reactor Sci. Technol. (J. Nucl. Energy Parts A/B) **16**, No. 1, 51 (1962).

III-9. Preparation of Cross Sections and Parametric Studies of the Reactivity Coefficients for Large U-233-Th Fueled Fast Reactors

R. S. SINGH* and H. H. HUMMEL

ABSTRACT

The feasibility of using U-233-Th as a fuel in a large t breeder reactor has been studied from the point of

view of the Doppler and sodium void coefficients. The composition and temperature dependent cross sections for thorium have been evaluated using the latest resonance parameters available, taking into account the s-wave and p-wave contributions. A set of 22-group

* Atomic Energy Establishment, Trombay, India.

cross sections has been prepared for U-233, U-234, and fission products. The sodium void coefficient has been studied in detail over a range of compositions and sizes of core for metal and oxide fuels and the effect of the addition of U-234 and fission products on the coefficient is also investigated. The reactivity coefficients due to the Doppler effect are calculated for metal and oxide fueled systems of interest. The results indicate that the reactivity coefficients due to sodium loss and the Doppler effect are encouraging enough to go to larger sizes of fast breeder cores if U-233 and thorium are used as the fuel.

INTRODUCTION

The selection of a suitable fuel for a very large fast breeder reactor of the type mentioned¹ in connection with the desalination of water may depend very much on the nuclear properties of the fuel and the inherent reactivity coefficients. It has been observed^{2, 3} that the sodium void coefficient in Pu-239-U-238 fueled fast reactors becomes positive as one goes to larger core volumes. This problem may, to a large extent, be minimized with U-233-Th fuel because of the nuclear properties of these materials. The fission cross section of U-233 varies more rapidly with energy than does that of Pu-239. Thorium has a much smaller fission cross section and a higher threshold energy for fission than U-238. As a consequence, the reactivity gain due to the spectral effects is much smaller with U-233-Th fuel than with Pu-239-U-238 fuel. This study investigated details of the reactivity coefficients due to sodium loss and Doppler effect in different sizes and compositions of U-233-Th fueled cores. As a first step in the problem, efforts were made to prepare cross section sets with the latest experimental data available.

CROSS SECTION PREPARATION

A 22-group cross section set⁴ was prepared by using the first 13 groups of the YOM set⁵ and adding 9 groups for the lower energies. Below group 11 (25 keV), the capture and fission cross sections of heavy elements are composition dependent because of the resonance self-shielding effects.

Resolved resonance parameters⁶ for Th-232 obtained up to 3 keV at Columbia University were used to evaluate the effective resonance integrals in the narrow resonance approximation with the RP-270 code. Temperature-dependent effective cross sections were obtained for the lower energy groups (below group 11) covering both the unresolved and resolved resonance regions for a value of the potential scattering cross section, σ_p , per absorber (Th) atom of 40 barns. The p -wave contribution to the capture cross sections was also evaluated separately in the unresolved resonance

TABLE III-9-I. TEMPERATURE DEPENDENT CAPTURE CROSS SECTIONS OF THORIUM, barns

Unresolved Resonance Parameters	s -waves: $\Gamma_n^0 = 1.27 \times 10^{-3}$ eV; $D = 20.1$ eV; $\Gamma_\gamma = 0.04$ eV.				
	p -waves: $\frac{\langle \Gamma_n^0 \rangle}{D} = 1.0 \times 10^{-4}$; $D = 20.1$ for $J = \frac{1}{2}$; $D = 10.0$ for $J = \frac{3}{2}$; $\Gamma_\gamma = 0.03$ eV				
	$\sigma_p = 40$ barns				
Energy Group, j	E_L , ^a keV	300°K	750°K	1500°K	2500°K
12	15.0	0.515	0.538	0.552	0.561
13	9.1	0.614	0.731	0.757	0.773
14	4.0	0.730	0.986	1.033	1.063
15	2.8	0.855	1.230	1.311	1.366
16	2.0	0.995	1.144	1.253	1.328
17	1.4	1.348	1.577	1.759	1.894
18	1.0	1.07	1.254	1.395	1.498
19	0.55	1.088	1.323	1.528	1.692
20	0.30	1.03	1.288	1.54	1.758
21	0.10	0.895 ^b	1.119 ^b	1.359 ^b	1.576 ^b
22	0.03	0.723	0.829	0.958	1.087

^a E_L = lower energy of the group.

^b These numbers were subsequently found to be in error. Correct values are 1.28, 1.60, 1.93, and 2.28 barns at 300, 750, 1500, and 2500°K, respectively. It is not expected that these errors would affect the results here significantly.

TABLE III-9-II. FISSION AND CAPTURE CROSS SECTIONS OF U-233 BELOW 9.1 keV, barns

Energy Group, j	E_L , ^a keV	σ_f	$\sigma_{n\gamma}$
13	9.1	5.02	0.82
14	4.0	6.7	1.4
15	2.8	7.5	1.6
16	2.0	7.7	1.8
17	1.4	8.4	2.0
18	1.0	8.8	2.1
19	0.55	11.0	3.4
20	0.30	15.0	6.2
21	0.10	25.0	10.0
22	0.03	33.0	13.0

^a E_L = lower energy of the group.

region and included in the capture cross sections. The s -wave and p -wave resonance parameters in the unresolved resonance region and the temperature dependent capture cross sections for thorium are given in Table III-9-I. Capture cross sections at higher energies were as given in YOM⁵ except that in groups 9, 10, and 11, values of 0.32, 0.37, and 0.515 barns, respectively, were used.

The experimental values of the capture and fission cross sections for U-233 are available only in a limited range of energies. The 22-group cross sections were prepared using the same values as the YOM set³ for the first 13 groups and making some adjustments in the

TABLE III-9-III. COMPONENTS OF SODIUM VOID EFFECT IN SPHERICAL CORES FOR UNIFORM REMOVAL OF SODIUM FROM CORE

Fuel Type ^a	% Na	Volume, liters	Th-232 + U-234 U-233 (approx.)	% δk : Full - Half			% δk : Half - Nil		
				Total	Spectral and capture	Leakage	Total	Spectral and capture	Leakage
Metal (No U-234) $T = 300^\circ\text{K}$	70	6,000	6.6	—	-2.1	—	—	-3.2	—
		8,000	7.1	—	-1.7	—	—	-2.9	—
		15,000	8.9	-3.7	-0.6	-3.1	-5.9	-1.4	-4.5
Metal (U-234) $T = 300^\circ\text{K}$	70	8,000	7.0	—	-0.8	—	—	-1.5	—
		15,000	9.0	-2.4	+0.5	-2.9	-4.4	+0.2	-4.6
Oxide (No U-234) $T = 300^\circ\text{K}$	70	6,000	5.5	-6.8	-3.0	-3.8	-8.3	-3.5	-4.8
		15,000	7.5	-6.0	-2.9	-3.1	—	—	—
Metal (U-234 + F.P.) $T = 300^\circ\text{K}$	70	—	8.0	—	+0.5	—	—	+0.2	—
Oxide (U-234 + F.P.) $T = 300^\circ\text{K}$	70	—	8.0	—	0.0	—	—	+0.3	—
Metal (No U-234) $T = 300^\circ\text{K}$	50	15,000	11.8	-0.6	+1.1	-1.7	-0.9	+1.3	-2.2
Metal (U-234 + F.P.) $T = 1500^\circ\text{K}$	70	15,000	8.1	-1.3	+1.5	-2.8	-4.5	+0.8	-5.3
Oxide (U-235 + F.P.) $T = 1500^\circ\text{K}$	70	—	8.0	—	+1.5	—	—	—	—
Metal (U-234 + F.P.) $T = 1500^\circ\text{K}$	50	15,000	11.7	+0.7	+1.8	-1.1	—	—	—

^a See text for composition; F.P. = fission products.

capture and fission cross sections to obtain reasonable values of the ratio of parasitic capture to fission cross sections in the lower energy groups. The values tabulated in the Russian cross section sets⁷ were also consulted in preparing the cross sections for U-233. Detailed investigations to obtain shielded cross sections for U-233 were not made in view of the absence of reliable resonance parameters for this element because of the problem of multilevel effects, and also because its contribution to the Doppler effect is probably negligible. The fission and capture cross sections thus prepared are given in Table III-9-II for U-233.

As the buildup of the heavier isotopes and fission products increases with the burnup of fuel, their importance on the reactivity coefficients has to be considered. Therefore, the cross sections for U-234 and fission products were also prepared. The capture cross sections of fission products were evaluated for U-233 fission products making use of the data provided by J. Garrison and B. Roos.⁸

All of these cross sections were put into the standard

22-group set No. 215 to be used in the subsequent calculations.

SODIUM VOID COEFFICIENT

To study the reactivity coefficients in large core volumes, three cases were considered corresponding to the volumes 6,000, 8,000, and 15,000 liters. Both metal and oxide fuels were studied with volume fractions of 70 % sodium, 15 % stainless steel, and 15 % fuel. To compare the effect of the coolant volume fraction on the sodium void coefficient, two cases with volume fractions 50 % sodium, 20 % stainless steel, and 30 % fuel were studied. The effect of adding U-234 and fission products to the sodium coefficient was also investigated. The methods employed to calculate the spectral, the capture, and the leakage components of the sodium coefficient are the same as were used in Ref. 9. The results of the calculations are given in Table III-9-III.

It is noted from Table III-9-III that the spectral plus capture component becomes less negative with decreasing enrichment as has been observed earlier for

TABLE III-9-IV. DOPPLER REACTIVITY EFFECTS BY DIRECT K CALCULATION

Fuel Type	Th/U-233	Sodium	300-750°K		750-1500°K		1500-2500°K	
			$-\% \Delta k$	$-\frac{\Delta k}{\Delta T} \times 10^5$	$-\% \Delta k$	$-\frac{\Delta k}{\Delta T} \times 10^5$	$-\% \Delta k$	$-\frac{\Delta k}{\Delta T} \times 10^5$
Metal (No U-234) 70 v/o Na	6.0	Yes	0.55	1.22	0.38	0.51	0.25	0.25
		No	0.04	0.09	0.02	0.03	0.01	0.01
	6.8	Yes	0.70	1.56	0.48	0.78	0.32	0.32
		No	0.04	0.09	0.03	0.04	0.02	0.02
	8.0	Yes	0.92	2.04	0.63	0.84	0.42	0.42
		No	0.07	0.16	0.04	0.05	0.02	0.02
Oxide (No U-234) 70 v/o Na	4.4	Yes	0.98	2.18	0.73	0.98	0.52	0.52
		No	0.24	0.53	0.16	0.21	0.10	0.10
	5.5	Yes	1.33	2.96	0.98	1.31	0.71	0.71
		No	0.36	0.80	0.23	0.31	0.15	0.15
	6.5	Yes	1.72	3.82	1.29	1.72	0.92	0.92
		No	0.52	1.16	0.38	0.51	0.15	0.15

Pu-U-238 systems.⁸ Also, the addition of U-234 [$N(U-234) = 0.4N(U-233)$] and fission products (5 a/o as pairs of the fuel material) to metal as well as oxide fueled systems changes the sign of the spectral component. The total sodium void coefficient is observed to be negative even in the case of a 15000 liter core at 70 v/o sodium and 1500°K, and only slightly positive for 50 v/o sodium. The expected decrease of leakage component with increasing size and the increase in leakage component with sodium content also are observed. Nonlinearities are noted in both the spectral and leakage components when the sodium is lost from full to half, and half to nil. The spectral plus capture component becomes more positive when evaluated at higher temperatures, as can be noted in Table III-9-III. The spatial dependence of the spectral and capture components is not considered here and this has to be investigated in future studies of a particular system. The results of sodium void coefficient thus obtained are encouraging enough to consider the feasibility of large fast breeder reactors based on U-233-Th fuel.

DOPPLER EFFECT

The importance of the Doppler coefficient, which may be the only reliable prompt-acting negative reactivity feedback, with regard to the safety of a fast reactor, has been recognized by many people. The Doppler effect occurs in isotopes which show a resonance structure in their cross sections and it is due mostly to neutrons with energies below about 10 keV. Therefore, its effect is most pronounced in the systems with softer spectra. The spectra in the U-233-Th metal fueled cores are more degraded than in corresponding Pu-U-238 systems. The low density fuel and high sodium volume fraction (70%) in these studies cause the spectral softening.

The ELMOE¹⁰ code was used for these calculations. The problems were first run with the room temperature (300°K) cross sections of thorium to obtain a critical value of the buckling, B^2 . They were then rerun with the cross sections at higher temperatures using the same buckling to obtain k_{eff} . The change in reactivity, δk , in going from one temperature to another was thus determined. These δk values were obtained for both metal-fueled and oxide-fueled cores with the total amount of sodium present and, alternately, without sodium in the core. The results are given in Table III-9-IV. It can be seen from Table III-9-IV that the values of $\Delta k/\Delta T$ for the metal fueled systems are large and comparable in magnitude with those that can be obtained in Pu-239-U-238 oxide fueled systems. It is interesting to note that the large Doppler coefficient in metal fueled systems with sodium in the core drops to almost negligible values (by a factor of 15) when the sodium is removed from the core. This can be attributed to the hardening of the spectrum in the core when the sodium is expelled. The latter effect is most pronounced in a metal fueled system with large sodium volume fraction in the core. The disadvantage can be overcome by adding a small amount of moderating material like BeO. Doppler coefficients for the oxide-fueled cores are found to be quite large; they increase with decreasing enrichment. The reduction in the Doppler coefficient when the sodium is expelled (from an oxide system) is only a factor of 5, as has been observed earlier³ in cores with 70 v/o sodium.

It can be concluded from the present study that there seems to be little problem from the standpoint of reactivity coefficients in designing large fast reactors suitable for desalination of sea water using U-233-Th fuel.

REFERENCES

1. R. P. Hammond, *Large Reactors May Distill Sea Water Economically*, *Nucleonics* **20**, No. 12, 45 (1962).
2. J. B. Nims and P. F. Zweifel, *Preliminary Report on Sodium Temperature Coefficients in Large Fast Reactors*, APDA-135 (1959).
3. S. Yiftah and D. Okrent, *Some Physics Calculations on the Performance of Large Fast Breeder Power Reactors*, ANL-6212 (1960).
4. H. H. Hummel and A. L. Rago, *Effect of Parametric Variation in Doppler Effect Calculations*, ANL-6792, 747 (1963).
5. S. Yiftah, D. Okrent and P. Moldauer, *Fast Reactor Cross Sections*, (Pergamon Press, New York, 1960).
6. J. Garg, J. Rainwater and W. W. Havens, Jr., AEC Nuclear Cross Sections Advisory Group Meeting, WASH-1044 (August, 1963).
7. L. P. Abagyan, et al., *Group Constants of Fast and Intermediate Neutrons for the Designing of Nuclear Reactors*, Physical Power Engineering Inst., State Committee for Use of Atomic Energy, U.S.S.R. (1963).
8. J. D. Garrison and B. W. Ross, *Fission Product Capture Cross Sections*, *Nucl. Sci. Eng.* **12**, 115 (1962).
9. H. H. Hummel, K. Phillips, and A. L. Rago, *Calculations of Sodium Void Reactivity Effect for Large Fast Oxide Reactors*, ANL-6792, 65 (1963).
10. A. L. Rago and H. H. Hummel, *ELMOE, An IBM-704 Program Treating Elastic Scattering Resonances in Fast Reactors*, ANL-6805 (1964).

III-10. Evaluation of Fast Multigroup Cross Sections for Light Element Reflectors

D. MENEGHETTI

INTRODUCTION

The ELMOE^{1, 2} code is often used^{3, 4, 5} to determine homogenized multigroup transport and elastic transfer cross sections for fast reactor cores containing light materials such as aluminum, steel, nickel, sodium, graphite, etc. For a given composition the ELMOE code determines the bare core neutron flux spectrum (normal mode) using many hundreds of very fine energy groups. It produces fine-flux-weighted coarse group cross sections. These are then used in the core region of a subsequent coarse, multigroup, reflected criticality calculation.

The possible presence of light element resonance scattering materials in reflectors introduces the question of how one determines coarse group cross sections for reflector regions in an analogous manner. In addition, the possibility of adapting the ELMOE code to enable suitable cross section averages to be calculated for such non-normal mode regions is of significant interest.

METHOD

Multigroup cross sections which include considerations of group and regional dependent leakage into and out of reflector regions were evaluated.⁶ These were obtained by iterations between coarse group multi-region criticality calculations and modified fine-group ELMOE calculations. In the latter, the ELMOE material-bucklings were set at zero and, instead, the leakage effects in the ELMOE calculations were simulated by insertion of energy dependent positive and/or negative fictional capture cross sections. These were

determined from the regional group leakages derived from the criticality calculations.

Three definitions were used for the fine group leakage absorptions, Σ_j^L , derived from the coarse group leakage absorptions, Σ_J^L :

$$A. \quad \Sigma_j^L \equiv \Sigma_J^L = \frac{(\text{Net Leakage})_J^R}{(\text{Integral Flux})_J^R}$$

$$B. \quad \Sigma_j^L = \frac{B_J^2}{3\Sigma_J^{Tr}} \equiv \frac{\Sigma_J^L \Sigma_J^{Tr}}{\Sigma_J^{Tr}}$$

$$C. \quad \Sigma_j^L = \frac{[B_J^2(\text{Smoothed})]_j}{3\Sigma_J^{Tr}}$$

where R , j , J , and Σ^{Tr} denote the region, fine group, coarse group, and transport cross section respectively. In "A" the fine-group leakage-absorptions within a given coarse group are assumed constant and equal to the coarse group value. "B" allows for fine energy variation in the leakage absorption caused by fine structure variations of the transport cross section. The latter are obtained from the ELMOE library. "B" still retains constant fine-group bucklings equal to the coarse group bucklings. "C" is similar to "B" except that the coarse group bucklings have been smoothed (by a parabolic smoothing in each coarse group). This eliminates the abrupt steps in buckling values at coarse group boundaries. Comparison of results obtained with the three definitions then indicates whether these detailed considerations are important in any particular case. For cross sections other than the transport and elastic transfer of aluminum and nickel, the cross

sections of YOM⁷ with the conservative ν values for U-235 were used.

CROSS SECTION AVERAGING RESULTS

The spherical geometry fast systems considered are listed in Table III-10-I. The second system has a more degraded core neutron spectrum than the first system. The first and third systems have identical core compositions. The first system is similar, though not identical, to the ZPR-9 Assembly No. 1 experiment.⁸ The other two systems have no direct experimental analogue. For analytical purposes the reflectors were considered to be two identical composition regions of 6 cm and 24 cm thickness, respectively. The criticality-ELMOE iterations were carried out separately but simultaneously for these two regions to ascertain the importance of considering spatial leakage effects upon the cross section averaging.

In general the need for the leakage-iterative method will largely depend upon the ratio of fine-group leakage absorption to fine-group removal absorption. The latter includes parasitic capture as well as energy degradation. Since the ELMOE groups are very narrow, the fine group removal absorption cross sections are essentially equal to the fine group transport cross sections. The need for the use of the leakage—iteration methods tends to increase as the absolute value of the ratio of leakage-absorptions to transport cross sections increases. Leakage absorptions may be both positive and negative.

Comparisons of converged, leakage-iterated values of $3\Sigma_{Tf}^{A_i}$ for the first 6 cm thick reflector region of the first system are listed in Table III-10-II. The group structure is that of the YOM⁷ set. Column A contains the cross section values determined by assuming the coarse absorption-leakage option. Columns B/A and C/A give the ratios of the coarse buckling-fine transport and smoothed buckling-fine transport options, respectively, relative to the coarse leakage values,

TABLE III-10-I. COMPOSITIONS OF FAST SYSTEMS WITH LIGHT ELEMENT RESONANCE REFLECTORS

System	Core ^a Composition (v/o)				Reflector ^b	
	U-235	U-238	C	SS ^c	Material	v/o
1	9.4	70.1	0	9.3	Al	94
2	9.4	0	74.5	9.3	Al	94
3	9.4	70.0	0	9.3	Ni	100

^a Core radii = 30–35 cm.

^b Inner and outer reflector regions are 6 and 24 cm thick, respectively.

^c Stainless steel.

TABLE III-10-II. RATIOS OF CONVERGED $3\Sigma_{Tf}^{A_i}$ FOR AN AL REFLECTOR (U-238 Diluted Core, System No. 1)^a

Energy Group J	E_L^b , MeV	S_4 Calc.: Inner 6 cm			$A^{DT}/A^{S_4 c}$	S_4 A^{outer}/A^{inner}
		A	B/A	C/A		
1	3.67	(0.233)	(1.00)	(1.00)	(1.00)	(1.00)
2	2.23	0.235	0.99	0.98	1.00	1.00
3	1.35	0.323	1.00	1.00	1.00	1.00
4	0.825	0.359	1.00	1.00	1.00	1.00
5	0.50	0.469	1.00	1.00	1.00	1.00
6	0.30	0.492	1.00	1.00	1.00	1.00
7	0.18	0.448	1.01	1.00	1.00	1.01
8	0.11	0.416	1.01	1.01	1.01	0.99
9	0.067	0.398	1.02	1.01	1.01	0.96
10	0.041	0.322	1.00	1.01	0.99	1.01
11	0.025	0.132	1.05	1.02	1.11	0.95
12	0.015	0.110	1.01	1.01	0.95	1.00
13	0.0091	0.183	1.01	1.01	1.03	0.99
14	0.0055	0.246	1.00	1.00	0.99	1.00
15	0.0021	0.235	1.00	1.00	1.00	1.00
16	0.0005	(0.233)	(1.00)	(1.00)	(1.00)	(1.00)

^a Values in parentheses were not averaged.

^b Lower boundary of energy group.

^c DT = Diffusion theory.

Column A. In general, the three options give very similar values. Even the values for group eleven which contains not only a prominent scattering resonance but also a very sharp decrease of the fine flux envelope, do not differ much. This effect of a rapidly decreasing flux envelope combined with a prominent scattering resonance shows up again in the fifth column. Here the values obtained by diffusion theory iterated with the modified ELMOE code, relative to S_4 iterated with the modified ELMOE code, are compared. The less rapidly decreasing flux spectrum envelope obtained in group eleven by S_4 calculations relative to diffusion theory calculation, results in an eleven percent greater cross section value. The last column indicates the spatial leakage effects upon the transport cross sections. In general, then, even for this relatively leaky reflector the choice of options is not too important with respect to averaging the transport cross section. Although not demonstrated here, the simple assumption of zero leakage absorption may suffice for some applications. This is not the case in averaging of the elastic transfer cross sections. Two to three leakage iterations are necessary to obtain suitably averaged elastic transfer cross sections.

Table III-10-III lists the corresponding values and ratios for the elastic transfer cross sections in the same format as Table III-10-II. For many groups the B/A and C/A ratios differ considerably from unity. Further-

TABLE III-10-III. RATIOS OF CONVERGED $\Sigma_J^{ER(a)}$ FOR
AN AL REFLECTOR
(U-238 Diluted Core, System No. 1)^b

Energy Group J	Inner 6 cm S_4			ADT/AS_4	S_4 A_{outer}/A_{inner}
	A	B/A	C/A		
1	(0.0067)	(1.00)	(1.00)	(1.00)	(1.00)
2	0.0155	0.86	0.79	0.99	1.03
3	0.0188	0.99	0.99	0.99	1.03
4	0.0244	0.98	1.00	1.00	1.01
5	0.0327	1.00	0.97	1.00	0.96
6	0.0257	1.00	0.97	0.99	1.12
7	0.0241	1.02	1.01	1.00	1.10
8	0.0293	1.00	0.98	0.99	1.03
9	0.0239	0.95	0.97	0.97	1.10
10	0.0147	1.05	1.07	0.96	1.04
11	0.0075	0.88	0.93	0.81	1.09
12	0.0022	1.18	1.27	0.50	0.95
13	0.0064	1.03	1.00	1.48	0.89
14	0.0066	1.05	1.02	0.88	1.14
15	0.0017	1.00	1.00	1.00	1.24

^a ER = Elastic removal.

^b Values in parentheses were not averaged.

more, similarity of corresponding B/A and C/A values indicate that whereas use of fine transport variations in the leakages can be important, the presence of the buckling steps at the coarse group boundaries are not too significant. (If cases arise for which a particular choice of coarse group boundaries, coupled with a particular composition and geometry, lead to significant differences in some groups between the B/A and C/A ratios it is suggested that such coarse groups be subdivided.)

Quite large deviations from unity are noted in the leakage-iterated results based on the diffusion theory spectrum relative to the S_4 spectrum for this system. (These, of course, do not indicate errors in the leakage iteration method. Diffusion theory is simply a poor choice for obtaining the coarse group spectrum for this core-reflector combination.)

Greater differences occur in the elastic transfer values for the outer aluminum reflector region relative to the inner reflector constants than are determined for the corresponding transport cross sections.

CRITICAL VOLUME RESULTS

Comparative results of calculated critical radii using YOM cross sections⁷, non-leakage ELMOE derived cross-sections, and the cross sections obtained from the leakage-iterative options are shown in Table III-IV for the three systems described in Table III-10-I. As might be expected from the relative insensitivity

of the averaged transport cross section to the leakage options, the critical volumes based on zero-leakage reflector cross sections differ by less than 1 % from the leakage-iterated results. The direct application of the YOM values result in critical volumes about 12 % to 18 % smaller than the leakage-iterated values.

FLUX SPECTRA RESULTS

The previously shown sensitivity of the elastic transfer cross sections to reflector leakage suggests that the neutron spectra are sensitive to the use of the cross sections derived from the leakage-iterated method. Table III-10-V lists the ratios of the group fluxes obtained by using the various methods and options for the inner reflector region of the first system. Multigroup

TABLE III-10-IV. CRITICAL CORE RADII (R_{LA}) AND
RATIOS OF CRITICAL CORE RADII (R_x/R_{LA})^a

Sys- tem	Re- flector	Principal Core Diluent	$\frac{R_Y}{R_{LA}}$	$\frac{R_0}{R_{LA}}$	R_{LA} , cm	$\frac{R_{LB}}{R_{LA}}$	$\frac{R_{LC}}{R_{LA}}$
1	Al	U-238	0.954	1.002	35.21	1.000	1.000
2	Al	C	0.934	1.003	31.55	1.000	1.000
3	Ni	U-238	0.959	0.999	29.53	1.000	1.000

^a $x = Y \equiv$ YOM⁷ constants

$x = 0 \equiv$ non-leakage ELMOE, $B^2 = 0$

$x = LA \equiv$ leakage option A

$x = LB \equiv$ leakage option B

$x = LC \equiv$ leakage option C

All criticality calculations are based on multigroup diffusion theory.

TABLE III-10-V. COMPARISON OF NEUTRON SPECTRA
IN INNER REGION OF AL REFLECTOR, $\phi_J(x)$ ^a
(U-238 Diluted Core, System No. 1)

Energy Group J	$\frac{\phi_J(Y)}{\phi_J(LA)}$	$\frac{\phi_J(0)}{\phi_J(LA)}$	$\phi_J(LA)$	$\frac{\phi_J(LB)}{\phi_J(LA)}$	$\frac{\phi_J(LC)}{\phi_J(LA)}$
1	0.92	1.01	0.0125	1.00	1.00
2	1.01	1.02	0.0219	1.02	1.03
3	0.98	1.01	0.0415	0.99	0.98
4	0.98	1.05	0.0848	1.01	1.00
5	1.01	1.08	0.148	0.99	1.01
6	0.89	0.99	0.194	1.00	1.01
7	0.86	0.97	0.172	0.99	0.99
8	0.92	0.99	0.120	1.01	1.01
9	0.92	0.94	0.0893	1.02	1.01
10	1.28	0.96	0.0794	0.97	0.96
11	1.25	0.89	0.0290	1.11	1.09
12	5.56	1.36	0.0063	0.95	0.95
13	6.04	2.16	0.0009	1.08	1.12
14	4.03	1.86	0.0003	1.00	1.00
15	4.69	2.22	0.0001	1.03	1.00
16	9.56	8.97	0	1.00	0.93

^a Various options are identified in Table III-10-IV.

diffusion theory was used for the coarse group criticality calculation.

In general, the larger spectral discrepancies, due to use of the non-leakage ELMOE cross sections, occur in the lower energy portion of the reflector spectrum where the inelastic scattering sources are negligible and where the fine flux envelope is rapidly decreasing with decreasing energy. The close similarity of the values in the columns $\phi_j(LB)/\phi_j(LA)$ and $\phi_j(LC)/\phi_j(LA)$ show that the smoothing of the coarse energy dependent buckling is not significant. The deviations of both $\phi_j(LB)/\phi_j(LA)$ and $\phi_j(LC)/\phi_j(LA)$ from unity, however, indicate that considerations of fine structure variations of transport cross sections occurring in the leakage terms are important in those coarse groups where the flux spectrum envelope is rapidly decreasing with decreasing energy and which also contain scattering resonances.

For the system having a 30 cm thick nickel reflector the need for using the leakage-iterated elastic transfer cross sections is not as pronounced as for the aluminum reflector. This is true since, in the lower energy groups, the transport cross sections of nickel are considerably larger than those of aluminum. If the nickel reflector thickness were thinner then the leakage-iteration would also become more necessary for this case.

REFERENCES

1. H. H. Hummel and A. L. Rago, *An Accurate Treatment of Resonance Scattering in Light Elements in Fast Reactors*, Proc. of Seminar on Physics of Fast and Intermediate Reactors, IAEA, Vienna, 1, Paper No. SM-18/45 (1962), pp. 231-250.
2. A. L. Rago and H. H. Hummel, *ELMOE: An IBM-704 Program Treating Elastic Scattering Resonances in Fast Reactors*, ANL-6805 (January 1964).
3. M. G. Bhide and H. H. Hummel, *Reactivity Coefficients of Sodium in Some Large Fast Reactors*, Proc. of Seminar on Physics of Fast and Intermediate Reactors, IAEA, Vienna, 2, Paper No. SM-18/72 (1962), pp. 177-186.
4. D. Meneghetti, *Recent Advances and Problems in Theoretical Analyses of ZPR-III Fast Critical Assemblies*, Proc. of Seminar on Physics of Fast and Intermediate Reactors, IAEA, Vienna, 2, Paper No. SM-18/37 (1962), pp. 457-487.
5. R. Avery, H. H. Hummel, R. N. Hwang, D. Meneghetti, P. A. Moldauer and J. B. Nims, *Physics of Fast Reactors*, Proc. 1964 Geneva Conference, Paper No. 259 (to be published).
6. D. Meneghetti, *Evaluation of Group Cross Sections for Light Reflectors of Fast Reactors*, Trans. Am. Nucl. Soc. 8, No. 1, 247 (1965).
7. S. Yiftah, D. Okrent and P. A. Moldauer, *Fast Reactor Cross Sections*, (Pergamon Press, New York, 1960).
8. R. C. Doerner, W. G. Knapp and K. K. Almenas, *Physics Measurements in Tungsten-based Aluminum-reflected Fast Reactors*, Trans. Am. Nucl. Soc. 7, No. 2, 236 (1964).

III-11. Theoretical Studies of Thin Slab Heterogeneity Effects upon the Spectral Reactivity Component of Sodium Voiding

D. MENEGHETTI

INTRODUCTION

Results of few groups DSN (Discrete S_N) calculations of critical mass corrections due to thin slab-cell heterogeneities in ZPR-III fast critical assemblies have been previously reported.¹ The heterogeneities are caused by the various configurations of fuel and diluent plates in the core drawers of the honeycomb arrays which form the two halves of the critical assembly machine. Recently, measurements of variations in the values of the sodium void reactivity effects with different intradrawer loading configurations have been reported.² The present calculations were directed toward understanding the effects of such heterogeneities upon the spectral reactivity component due to sodium voiding.

THEORETICAL CONSIDERATIONS

Let Δk_{in}^{het} and Δk_{out}^{het} be the respective heterogeneity corrections relative to the corresponding homogeneous

"Na in" and "Na out" situations. For fast critical assemblies Δk_{in}^{het} and Δk_{out}^{het} generally are positive. Reactivity effects resulting from removal of sodium from heterogeneous slab cells will then be more positive or more negative than for the homogeneous cell depending upon whether or not Δk_{out}^{het} is greater or less than Δk_{in}^{het} , respectively.

Whether Δk_{out}^{het} is greater or less than Δk_{in}^{het} depends primarily upon the changes in the values of the $\bar{\phi}_{fuel j}/\bar{\phi}_{diluent j}$ ratios for each energy group (j). ($\bar{\phi}_{fuel j}$ and $\bar{\phi}_{diluent j}$ are spatial averages of the cell neutron flux over fuel and diluent regions, respectively.) These changes are due to the sodium removal and its resulting overall spectral shift as well as upon the importance reweighting of the ratios for the groups relative to each other because of the overall spectral shift. For repetitive binary cells the values of $\bar{\phi}_{fuel j}/\bar{\phi}_{diluent j}$ tend to be closer to unity primarily because of the composite

cts of diluent density reduction, which simulates reduction in diluent thickness, and because of the changes in the relative values of regional sources (fission and scattering) within each group. $\bar{\phi}_{\text{fuel } j} / \bar{\phi}_{\text{diluent } j}$ values close to unity imply a decrease in heterogeneity, i.e., tend to make $\Delta k_{\text{out}}^{\text{het}} < \Delta k_{\text{in}}^{\text{het}}$. Importance reweighting of the flux ratios relative to each other generally tends to increase the heterogeneity, i.e., tends to make $\Delta k_{\text{out}}^{\text{het}} > \Delta k_{\text{in}}^{\text{het}}$. The overall spectral shift in the cell, due to sodium reduction, shifts the spectrum toward higher energies where $\bar{\phi}_{\text{fuel } j} / \bar{\phi}_{\text{diluent } j}$ values are greater than unity and away from the lower energies where the ratios are less than unity. The relative opposing effects of the changes in $\bar{\phi}_{\text{fuel } j} / \bar{\phi}_{\text{diluent } j}$ and of the importance reweighting determine whether the spectral component of the heterogeneous sodium void reactivity value is more negative or more positive than the corresponding homogeneous value.

RESULTS AND DISCUSSION

Twenty-two group³ calculations of assumed cell compositions and configurations have been made using the SNARG (Argonne Discrete S_N) code utilizing 12 or 14 discrete angles in an MSG⁴ (modified single Gaussian) quadrature. For each energy group the overall leakage out of the cell was simulated by a group dependent absorption equal to $D_j B^2$, where B^2 is the material buckling of the "Na-in" homogenized cell composition and D_j is the "Na-in" homogenized cell value of the group diffusion constant. The identical group leakage absorption values were also used in the "Na-out" cell calculations. The resulting calculated

homogeneous values of changes in multiplication constants due to sodium removal therefore correspond to the spectral components of the sodium void reactivities for the case of bare cores. Furthermore, these studies have assumed that heterogeneity effects upon the leakage reactivity component of the sodium void, if any, do not affect the evaluation of the heterogeneous spectral component.

Results of the analyses for two assumed core compositions are given in Tables III-11-I and III-11-II respectively. The results given in Table III-11-I, (Nos. 1 and 3), show that placement of the small U-238 quantity present either uniformly in the sodium diluent or at the center of the sodium diluent give more negative heterogeneous values relative to homogeneous values. The more positive correction increment values of configurations 2 and 4 indicate that the spectral shift importance reweighting of the various group contributions over-ride or equal, respectively, the advantage factor changes upon removal of sodium. For a given cell size this tends to occur more frequently for configurations having relatively larger values of $\Delta k_{\text{in}}^{\text{het}}$. The latter is the result of placing the light diluent materials predominantly together and midway between regions of clusters or alloys of fissile and fertile materials.

The results of configurations 1, 2, and 3 for the second core composition (Table III-11-II) indicate that if the same overall cell size is retained and if only a small portion of the total fertile material is alloyed or clustered with the fissile material, the heterogeneity reactivity correction is not significantly altered.

Comparison of configurations 4 and 5 indicate that

TABLE III-11-I. EFFECT OF SODIUM REMOVAL ON HETEROGENEOUS REACTIVITY VALUES FOR BARE CORE
(Pu-239, U-238, Na Core)

No.	Regional Volume Fractions and Half Cell Dimensions, cm	$\Delta k_{\text{in}}^{\text{het}}, \%$	$\Delta k_{1/2\text{out}}^{\text{het}}, \%$	$\Delta \equiv \Delta k_{1/2\text{out}}^{\text{het}} - \Delta k_{\text{in}}^{\text{het}}, \%$	$\frac{\Delta}{k_{1/2\text{out}}^{\text{homo}} - k_{\text{in}}^{\text{homo}}}, \%$ ^a
1	$\left \begin{array}{c} \text{Pu} \\ 0.16 \end{array} \right \left \begin{array}{c} \frac{1}{15}\text{U} + \frac{14}{15}\text{Na} \\ 2.40 \end{array} \right $	3.45	3.17	-0.28	+37
2	$\left \begin{array}{c} \text{Pu} \\ 0.16 \end{array} \right \left \begin{array}{c} \text{U} \\ 0.16 \end{array} \right \left \begin{array}{c} \text{Na} \\ 2.24 \end{array} \right $	4.38	4.54	+0.16	-21
3	$\left \begin{array}{c} \text{Pu} \\ 0.16 \end{array} \right \left \begin{array}{c} \text{Na} \\ 2.24 \end{array} \right \left \begin{array}{c} \text{U} \\ 0.16 \end{array} \right $	3.09	2.80	-0.29	+38
4	$\left \begin{array}{c} \frac{1}{2}\text{Pu} + \frac{1}{2}\text{U} \\ 0.32 \end{array} \right \left \begin{array}{c} \text{Na} \\ 2.24 \end{array} \right $	3.01	3.01	0.00	0
5 ($\frac{1}{2}$ size cell)	$\left \begin{array}{c} \frac{1}{2}\text{Pu} + \frac{1}{2}\text{U} \\ 0.16 \end{array} \right \left \begin{array}{c} \text{Na} \\ 1.12 \end{array} \right $	1.53	1.51	-0.02	+3

^a Percentage by which heterogeneous sodium removal reactivity value differs from the homogeneous(homo) sodium removal value $k_{1/2\text{out}}^{\text{homo}} - k_{\text{in}}^{\text{homo}} = -0.766\%k$.

TABLE III-11-II. EFFECT OF SODIUM REMOVAL ON HETEROGENEOUS SPECTRAL COMPONENT REACTIVITY VALUES FOR BARE CORE
(Pu-239, U-238, C, Na Core)

No.	Regional Volume Fractions and Half Cell Dimensions, cm	$\Delta k_{in}^{het}, \%$	$\Delta k_{1/2out}^{het}, \%$	$\Delta \equiv \Delta k_{1/2out}^{het} - \Delta k_{in}^{het}, \%$	$\frac{\Delta}{k_{1/2out}^{homo} - k_{in}^{homo}}, \%$ ^a
1	$\left \begin{array}{c} \text{Pu} \\ 0.16 \end{array} \right \left \begin{array}{c} \frac{1}{3}\text{U} + \frac{1}{3}\text{C} + \frac{1}{3}\text{Na} \\ 2.40 \end{array} \right $	3.33	3.27	-0.06	-40
2	$\left \begin{array}{c} \frac{1}{2}\text{Pu} + \frac{1}{2}\text{U} \\ 0.32 \end{array} \right \left \begin{array}{c} \frac{1}{14}\text{U} + \frac{5}{14}\text{C} + \frac{5}{14}\text{Na} \\ 2.24 \end{array} \right $	2.62	2.56	-0.06	-40
3	$\left \begin{array}{c} \text{Pu} \\ 0.16 \end{array} \right \left \begin{array}{c} \text{U} \\ 0.16 \end{array} \right \left \begin{array}{c} \frac{1}{14}\text{U} + \frac{5}{14}\text{C} + \frac{5}{14}\text{Na} \\ 2.24 \end{array} \right $	3.64	3.58	-0.06	-40
4 ($\frac{1}{2}$ cell size)	$\left \begin{array}{c} \frac{1}{4}\text{Pu} + \frac{3}{4}\text{U} \\ 0.32 \end{array} \right \left \begin{array}{c} \frac{2}{12}\text{U} + \frac{5}{12}\text{C} + \frac{5}{12}\text{Na} \\ 0.96 \end{array} \right $	0.94	0.91	-0.03	-20
5 ($\frac{1}{3}$ cell size)	$\left \begin{array}{c} \frac{1}{4}\text{Pu} + \frac{3}{4}\text{U} \\ 0.16 \end{array} \right \left \begin{array}{c} \frac{2}{12}\text{U} + \frac{5}{12}\text{C} + \frac{5}{12}\text{Na} \\ 0.48 \end{array} \right $	0.47	0.455	-0.015	-9
6 ($\frac{1}{2}$ cell size)	$\left \begin{array}{c} \frac{1}{4}\text{Pu} + \frac{3}{4}\text{U} \\ 0.32 \end{array} \right \left \begin{array}{c} \text{U} \\ 0.16 \end{array} \right \left \begin{array}{c} \text{C} \\ 0.40 \end{array} \right \left \begin{array}{c} \text{Na} \\ 0.40 \end{array} \right $	1.17	1.22	+0.05	+30
7 ($\frac{1}{2}$ cell size)	$\left \begin{array}{c} \frac{1}{4}\text{Pu} + \frac{3}{4}\text{U} \\ 0.32 \end{array} \right \left \begin{array}{c} \text{U} \\ 0.16 \end{array} \right \left \begin{array}{c} \text{Na} \\ 0.40 \end{array} \right \left \begin{array}{c} \text{C} \\ 0.40 \end{array} \right $	1.08	1.06	-0.02	-10
8 ($\frac{1}{2}$ cell size)	$\left \begin{array}{c} \frac{1}{4}\text{Pu} + \frac{3}{4}\text{U} \\ 0.32 \end{array} \right \left \begin{array}{c} \text{U} \\ 0.16 \end{array} \right \left \begin{array}{c} \frac{1}{2}\text{C} + \frac{1}{2}\text{Na} \\ 0.80 \end{array} \right $	1.11	1.10	-0.01	-6
9 ($\frac{1}{2}$ cell size)	$\left \begin{array}{c} \frac{1}{6}\text{Pu} + \frac{5}{6}\text{U} \\ 0.48 \end{array} \right \left \begin{array}{c} \frac{1}{2}\text{C} + \frac{1}{2}\text{Na} \\ 0.80 \end{array} \right $	0.73	0.70	-0.03	-20

^a Percentage by which heterogeneous sodium removal reactivity value differs from the homogeneous sodium removal value $k_{1/2out}^{homo} - k_{in}^{homo} = +0.16\%$.

decreasing the overall cell (and regional) dimensions reduces the correction. Comparison of configurations 6, 7, and 8 show that even though the fissile and fertile materials are alloyed and clustered the locations of the light diluent materials relative to each other can themselves cause substantial differences in the correction increment. Furthermore, those configurations which tend to cluster the sodium toward the middle of the diluent region have the larger k_{in}^{het} values and correspondingly positive or relatively less negative correction increments, i.e., $\Delta \equiv (\Delta k_{1/2out}^{het} - \Delta k_{in}^{het})$.

Comparison of configurations 4 and 9 show that simply alloying all of the fissile and fertile material

with retention of the identical overall cell size does not insure a smaller correction increment.

REFERENCES

1. D. Meneghetti, *Recent Advances and Problems in Theoretical Analyses of ZPR-III Fast Critical Assemblies*, Proc. of Seminar on Physics of Fast and Intermediate Reactors, IAEA, Vienna, 2, 457 (1962).
2. G. K. Rusch and F. H. Helm, *The Influence of Heterogeneity on the Measured Sodium Void Coefficient in a 950-Liter Pancake Fast Core*, Trans. Am. Nucl. Soc., 8, 1, 246 (1965).
3. D. M. O'Shea, H. H. Hummel, W. B. Loewenstein, D. Okrent and B. J. Toppel, *Twenty-Six Group Cross Sections*, ANL-6858 (to be published).
4. D. Meneghetti, *Discrete Ordinate Quadratures for Thin Slab Cells*, Nucl. Sci. Eng. 14, 295 (1962).

III-12. EBR-II Physics

W. B. LOEWENSTEIN, F. KIRN* and R. R. SMITH*

The Experimental Breeder Reactor II (EBR-II)^{1, 2, 3} is a fast neutron system designed to produce 62.5 MWt with a maximum power density exceeding 1000 kW/liter. The core is fueled with ~ 370 kg of uranium in a 5 w/o fission² alloy. The uranium is about 48% enriched in the U-235 isotope. The core is surrounded on all sides by a thick depleted uranium blanket. Most of the conversion ($\sim 90\%$) of fertile (U-238) to fissile (Pu-239) atoms takes place in the blanket.

The Dry Critical Experiment (without coolant sodium)⁴ was conducted in 1961. The Wet Critical and Low Power Experiments (with coolant sodium)⁵ were conducted during the latter half of 1963 and early 1964. The approach to power was initiated during July 1964. The power level was raised to 37.5 MW during October 1964 and raised to 45 MW during March 1965. Since then the reactor has operated principally at 45 MW with periodic investigations at lower and zero powers. Maximum burnup of 1.15% in the fuel alloy has been achieved (August 1965). This report is primarily concerned with measurements and observations on the reactor since the previous summary⁶ on the low power experiments.

The initial stages of increasing the power level and reactor temperature to those consistent with design objectives were designed to continuously monitor system kinetics and reactivity to assure that the system was operating as predicted. Significant deviations from predictions were the basis for ancillary investigations to assure understanding before proceeding further.

In general, the approach to power did not produce any data that differed significantly from expectations. The reactor was found to be extremely stable at all power levels of operation. Interpretation of these data suggests system stability for all contemplated modes of operation. The steady state power coefficient of reactivity was measured to be essentially constant (for constant flow) and in substantial agreement with expectations.⁶ There is no evidence for significant non-linear effects; this finding gives reasonable assurance that fuel subassembly or control rod bowing is not manifesting itself in any significant way.

Reactivity losses due to fuel alloy burnup are in good agreement with expectations.¹ Measured reactivity changes due to plutonium buildup during zero power operation following an extended power run are especially as predicted. There is no evidence that the

power distribution is significantly different from predictions.

Since many nuclear parameters related to power operation were obtained through indirect correlation with critical experiments on ZPR-III it is concluded that such correlation has been generally successful.

One unexpected observation pertains to a very small amplitude oscillation at 10 cps. This was completely unexpected and cannot be observed with normal plant nuclear monitoring equipment. Highly sensitive, high speed recorders are required to even observe the phenomenon. Many explanations have been postulated but none seem adequate for complete explanation. It is felt that the basic phenomenon is somehow mechanical as opposed to thermal and manifests itself on reactivity through some motions as opposed to thermal expansion. This effect is and will be carefully monitored as a function of time, power level and multiplicity of irradiation experiments.

The reactivity oscillator is an important tool for determining system kinetics. Unfortunately, the EBR-II oscillator rod failed mechanically after 37.5 MW power had been achieved. Since that failure other methods of kinetic monitoring were developed. These include the development of a small reactivity "control rod" which could be used for meaningful rod drop experiments. Another method involves the programmed insertion, holding and removing of small amounts of reactivity with a normal control rod at some power level. The reactivity signal is essentially trapezoidal with the holding time at peak reactivity amplitude (both positive and negative) being closely related to the period of an oscillator rod.

The neutronics of a rotating reactivity oscillator had been experimentally studied on ZPR-III.⁷ These, as well as analytical data have been used to specify the parameters of a new oscillator mechanism. The neutronics are flexible in that it will be possible to have reactivity frequencies that are double the mechanical frequencies. However, such high frequencies are accompanied by a loss of reactivity amplitude. It is not clear at this time that such reduced reactivity amplitude oscillations are adequate for stability investigations at the high frequencies where the corresponding power level perturbation is least.

REFERENCES

1. W. B. Loewenstein, *The Physics Design of the EBR-II*, ANL-6383(1961); also Proc. of Seminar on Physics of

* Idaho Division.

- Fast and Intermediate Reactors, IAEA, Vienna, **3**, p. 263 (1962).
2. L. J. Koch, W. B. Loewenstein and H. O. Monson, *Addendum to Hazard Summary Report—Experimental Breeder Reactor II*, ANL-5719 Addendum (1962).
 3. L. J. Koch, H. O. Monson, D. Okrent, M. Levenson, W. R. Simmons, J. R. Humphreys, J. Haugsnes, V. Z. Jankus and W. B. Loewenstein, *Hazard Summary Report—Experimental Breeder Reactor II*, ANL-5719 (1957).
 4. R. L. McVean, G. S. Brunson, B. C. Cerutti, W. B. Loewenstein, F. W. Thalgott and G. K. Whitham, *EBR-II Dry Critical Experiments—Experimental Results*, ANL-6462 (1962).
 5. F. Kirn and W. B. Loewenstein, *EBR-II Wet Critical Experimental Results*, Trans. Am. Nucl. Soc. **7**, No. 1, (1964); also *Results of the Wet Critical Experiments*, ANL-6864 (1964).
 6. W. B. Loewenstein, *EBR-II Physics*, Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010, 141 (January 1965).
 7. W. P. Keeney and J. K. Long, *Argonne National Laboratory Idaho Division Summary Report for July, August, September, 1960*, ANL-6301.

III-13. The Experimental Breeder Reactor II (EBR-II) as a Fast Flux Irradiation Facility

W. B. LOEWENSTEIN

After its primary neutronic and engineering objectives have been met, EBR-II will be operated primarily as a fast-flux irradiation test facility. Even before that time, however, every practicable enhancement of the facility for irradiation purposes will be effected short of compromising completion of the primary objectives. One possible improvement is to increase the maximum system reactivity to enable accommodation of a larger number of irradiation subassemblies.

Reactivity may be enhanced in two obvious ways. First, the enrichment of the fuel alloy may be increased slightly. Second, the reflector effectiveness may be increased by using nickel, steel and beryllium—separately or in judicious mixtures. The advantage of the latter approach is that the magnitude of neutron flux for a given maximum power density is maximized.

Altering the enrichment of the fuel alloy can be effected without significantly altering the reactor and its characteristics. A sacrifice is made in the magnitude of neutron flux for given maximum power density. This sacrifice is small in terms of the large irradiation test capacity that becomes available. The ability to accommodate a substantially larger number of irradiation experiments, even though experiment residence times may be increased by five or ten percent to achieve a given burnup, represents greatly increased utilization of the reactor as an irradiation test facility.

A number of rather simple calculations were carried out to demonstrate the nature of the reactivity problem. When an experiment is placed in the reactor core, it generally holds less reactivity than the driver fuel subassembly it replaces. Therefore, one or more driver fuel subassemblies must be added in the 6th row (by replac-

ing an inner blanket subassembly) to bring system reactivity back to where the reactor is operable. The reactivity worth of the experiment can range from essentially zero for structural materials to about 30 % of the driver fuel subassembly for a very heavily loaded irradiation experiment in a nineteen capsule irradiation subassembly. (A driver fuel subassembly contains 91 fuel pins.) With such replacements, the total power in the system can also be expected to change, assuming that the maximum power density is held fixed. To understand these phenomena a detailed replacement is outlined:

- 1) A driver subassembly (*D*) is removed at r_1
- 2) An experiment (*E*) is inserted at r_1
- 3) One or more driver subassemblies (*D*) are added at r_2 (where $r_2 > r_1$) to maintain reactivity.
- 4) The reactivity worth of the experiment is:
 - a) zero or
 - b) 30 % of the driver worth at the same position.
- 5) The power in the experiment is:
 - a) zero or
 - b) 30 % of the driver power at the same position.

If $\rho_x(r)$ is the reactivity worth for the replacement x , the reactivity balance equation must read:

$$[\rho_E(r_1) - \rho_D(r_1)] \Delta V_1 + \rho_D(r_2) \Delta V_2 = 0,$$

where ρ represents the worth of a single subassembly. ΔV_1 represents the in-core space that is perturbed for an experiment. ΔV_2 represents the core boundary space to be filled up with driver subassemblies to maintain reactivity.

Similarly, there is a fractional change in total power (assuming fixed maximum power density) as a result of such substitutions. Generally such replacement causes

an increase in total power, which can become problematical. The fractional change in power ($\Delta P/P$) may be expressed as:

$$\frac{\Delta P}{P} = \frac{P_D(r_2) \Delta V_2 - [P_D(r_1) - P_E(r_1)] \Delta V_1}{\bar{P}_{\text{core}} V_{\text{core}}}$$

where $P_x(r)$ is the power distribution for driver ($x=D$) and experiment ($x=E$).

The important parameters in these considerations are $\Delta V_2/\Delta V_1$ (the ratio of the number of driver subassemblies added to accommodate a given experiment) and $(\Delta P/P)/(\Delta V_1/V_{\text{core}})$, the fractional change in power to accommodate a given experiment. To obtain some insight into numerical values the following assumptions are made:

$$\rho_x(r) = B_x \left(1 - q \frac{r^2}{b^2}\right)^2$$

and

$$P_x(r) = A_x \left(1 - q \frac{r^2}{b^2}\right).$$

A_x and B_x are constants (with A_D being essentially proportional to the maximum power density), b is the core radius while q is a core power shaping constant. With $q \approx 0.5$, the reactivity and power distributions of Fig. 24 of Ref. 1, and Fig. 41 of Ref. 2, respectively, may be quite well approximated.

With such assumptions on power and reactivity distribution, the fractional change in total power may be expressed as

$$\begin{aligned} \frac{\Delta P}{P} &\approx \frac{(1-q)}{\left(1 - \frac{q}{2}\right)} \left[1 - \frac{1-q}{1 - q(r_1/b)^2}\right] \frac{\Delta V_2}{V_{\text{core}}} \\ &\approx \frac{2}{3} \left[1 - \frac{1}{2\{1 - \frac{1}{2}(r_1/b)^2\}}\right] \frac{\Delta V_2}{V_{\text{core}}}. \end{aligned}$$

Similarly, the reactivity balance equation gives

$$\frac{\Delta V_2}{\Delta V_1} \approx \frac{2}{3} \frac{[1 - q(r_1/b)^2]^2}{[1 - q]^2} \approx \frac{8}{3} \left[1 - \frac{1}{2} (r_1/b)^2\right]^2$$

if the experiment has $\frac{1}{3}$ the worth of the driver subassembly at r_1 ; or

$$\frac{\Delta V_2}{\Delta V_1} \approx 4 \left[1 - \frac{1}{2} \left(\frac{r_1}{b}\right)^2\right]^2$$

if the experiment has zero worth relative to the driver subassembly at r_1 . Some numerical results for small perturbations are given in Table III-13-I.

From Table III-13-I it can be seen that if 10 to 15 % the core volume is occupied by irradiation experiments and if the maximum power density is to be maintained, the system must be able to accept 10 to 15 %

TABLE III-13-I. MAKEUP REACTIVITY AND CORE POWER PERTURBATION FOR TWO TYPES OF IRRADIATION EXPERIMENTS

$\frac{r_1^a}{b}$	$\Delta V_2/\Delta V_1^b$		$(\Delta P/P)/(\Delta V_2/V)^d$	$(\Delta P/P)/(\Delta V_1/V)^e$	
	Zero Worth ^c	1/3 Worth ^c		Zero Worth ^c	1/3 Worth ^c
0	4.0	2.7	+0.334	+1.34	+0.90
0.25	3.8	2.5	+0.322	+1.22	+0.81
0.50	3.1	2.1	+0.286	+0.89	+0.60
0.75	2.1	1.4	+0.202	+0.42	+0.28
1.0	1.0	0.67	0	—	—

^a Normalized radius.

^b $(\Delta V_2/\Delta V_1) \approx$ Number of driver fuel subassemblies required at core boundary to accommodate one experiment at (r_1/b) .

^c Refers to reactivity worth of experiment relative to driver at (r_1/b) .

^d Fractional change in core power divided by fractional change in core volume. (Includes volume of irradiation experiments).

^e Fractional change in core power divided by fraction of core volume that represents irradiation experiments.

TABLE III-13-II. ESTIMATED REQUIRED FUEL ALLOY ENRICHMENT FOR SEVERAL REACTOR CONFIGURATIONS

Length of Fuel Pin, in.	Unperturbed Core Size ^a	Approximate Fuel Alloy Enrichment ^e (% U-235 in U)	Approximate Perturbed Core Size ^f	Number of Possible Experiments in Row 6 ^g	Total Number of Experiments ^h (Row 4 and Row 6)
14.22	70	48.4	91	none	10
13.5	74-75	48.4	96 ⁱ	none	10 ⁱ
13.5	70	~49.6	91	none	10
13.5	67	~51.1	87	4	14
13.5	61	~52.5	80	10	20
13.5	55 ^b	~55.7	69-70	21	31
13.5	55 ^c	~57.2	65-66	25	35
13.5	58 ^d	~55.0	71	20	30

^a Number of subassemblies in unperturbed core including control and safety subassemblies. (Does not include reactivity perturbation due to oscillator rod.)

^b Six dummy subassemblies in Row 5; these could be zero worth irradiation experiments.

^c Six dummy subassemblies in Row 4; these could be zero worth irradiation experiments.

^d Three dummy subassemblies in Row 4; these could be zero worth irradiation experiments.

^e These are estimates. Any effort to achieve such unperturbed core sizes is subject to careful analysis and evaluation.

^f Number of subassemblies (driver, control, safety and experiment) for 10 experiments in Row 4; worth of experiment $\approx 1/5$ driver worth.

^g Assuming zero reactivity worth.

^h If a reasonable number of experiments are placed in Row 5, the number possible will be between 10 and the number in this column.

ⁱ Or ~8 experiments in Row 4 with 91 subassembly core.

more core power. Note that core size is not explicitly factored into the results of Table III-13-I.

The numbers in Table III-13-I are approximate; nevertheless, they may be used as a guide to determine the irradiation potential, with respect to multiplicity of experiments, of the EBR-II. The majority of the irradiation experiments will be located such that $0.6 < r_1/b < 0.7$; hence, for every irradiation experiment about two driver subassemblies will be required in Row 6. Sixteen available spaces for drivers in Row 6 are consistent with about eight experiments in Row 4. However, if the unperturbed (by experiments) total power for a 75 subassembly core is 62.5 MW, the perturbed core power would have to increase to about 66.7 MW which is beyond the full design power of the facility.

If the unperturbed core were significantly smaller, e.g. 55-61 core subassemblies, the actual changes in power to accommodate an additional in-core irradiation experiment would be larger; however, the unperturbed power would initially be less, assuming that maximum power density is held constant.

The core power of an unperturbed 61 subassembly core is less than that of an unperturbed 74-75 subassembly core if the maximum power density is held constant. If the core power shape constant is roughly constant ($q \approx 0.5$), the power of the 61 subassembly core is about 20 % less than that of the larger core. This 20 % can be quite important toward providing fractional changes

in power to accommodate irradiation experiments. The smaller core also provides a significant number of irradiation positions in Row 6 which would probably not be a reactivity drain on the reactor. Table III-13-II summarizes some estimated enrichments consistent with achieving certain unperturbed core sizes. It is to be noted that these are estimates and if one of these objectives is desired, the corresponding enrichment must be arrived at on the basis of careful analysis and correlation with integral data.

The analysis of 13.5 in. core height as opposed to the 14.221 in. height reflects a contemplated change in fuel element design to enhance burnup potential of the driver fuel.

Other neutronic aspects related to enhancing the use of EBR-II as a fast flux irradiation facility have been³ and are being studied. The latter include overall effects of altering the radial reflector adjacent to the core; altering the ratio of core height to diameter as well as system adaptability to a variety of fuel types.

REFERENCES

1. F. S. Kirn and W. B. Loewenstein, *EBR-II Wet Critical Experiments*, ANL-6864 (1964).
2. L. J. Koch, W. B. Loewenstein and H. O. Monson, *Addendum to Hazard Summary Report—Experimental Breeder Reactor II (EBR-II)*, ANL-5719 Addendum (1962).
3. W. B. Loewenstein, *EBR-II Physics*, Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010 (1965).

III-14. Physics Analysis of TREAT with an EBR-II Subassembly

J. T. MADELL

The Transient Reactor Test Facility (TREAT) has been employed to test single and small groups of EBR-II fuel pins without sodium coolant. The presence of flowing sodium and the interaction of the fuel pins within a subassembly are important factors in the transient behavior of fast fuel elements. A test facility is being designed to insert a complete EBR-II subassembly into the center of TREAT through which sodium is pumped. Since such a test facility has not previously been employed, an analysis of its affect on the operation and safety of TREAT was required. The objectives of the physics analysis were to determine the reactivity effect of the subassembly and its components, the power distribution in the subassembly and in

TREAT, and the kinetic parameters of TREAT with the subassembly present.

DESCRIPTION OF TEST FACILITY

The cylindrical approximation of the large test facility is shown in Table III-14-I. The EBR-II subassembly can contains 91 fuel pins in a hexagonal array. The array is represented by a center pin and five concentric rings containing 6, 12, 18, 24 and 30 pins. Sodium flows between the pins in the subassembly can. Outside of the subassembly are two concentric pipes with sodium flowing inside. A void region exists between the outer sodium pipe and the liner of the test facility. The facility is placed in TREAT by removing six center fuel elements and it extends the entire axial

TABLE III-14-I. DESCRIPTION OF THE REGIONS IN THE CYLINDRICAL APPROXIMATION OF THE LARGE SODIUM LOOP IN THE TREAT CORE

Radical Location From Center of Facility	Region (Material)	Region Outer Radius, cm
1	EBR-II Subassembly	2.68
2	Flowing sodium	3.41
3	Steel pipe	3.65
4	Flowing pipe	4.86
5	Steel pipe	5.72
6	Void	11.09
7	Zirconium liner	11.29
8	TREAT core	81.87
9	Zircaloy clad reflector	91.0
10	Aluminum clad reflector	108.9
11	Permanent reflector	184.7

length of TREAT, including the top and bottom reflectors.

REACTIVITY EFFECT

A sixteen group cross section set (Set #201) prepared by Hansen and Roach¹ was selected for the study. Set #201 permitted an accurate description of nuclear reactions in the highly thermal TREAT core and reflector as well as in the fast reactor subassembly. In the first series of calculations the results of the one-dimensional diffusion theory code REX were compared with experiments and previous calculations described in Ref. 3. Experimentally, TREAT achieved cold, clean criticality with 141 elements. Previous calculations, using a three group cross section set and a spherical representation of TREAT, yielded $k = 1.025$. With the 16 group set and the same geometrical representation a k of 1.040 was obtained. TREAT was then represented as a cylinder in a one-dimensional calculation using calculated values of its equivalent height. The calculation in cylindrical geometry gave $k = 1.08$ for the 141 element assembly. Attempts to determine the cause of discrepancy were unsuccessful. It was felt that the reflector was of much poorer grade graphite than the calculations represented. For this reason additional boron was added to the reflector material so that the calculated values would agree with the experimental results. The one-dimensional model of the 141 element TREAT with the boron poisoned reflectors was used as a basis for comparison in subsequent calculations.

A series of calculations determined the geometrical representation of TREAT with the test section inserted. The test section was represented by 15 regions with 2 intervals each. The rest of TREAT was divided into five regions: the core, three reflectors, and an aluminum partition wall.

It was also shown that further reduction in the number of regions or intervals in the test section results in insignificant changes in the calculated values of reactivity and power distribution.

Three areas which may introduce some error into the calculation were investigated; self-shielding in the EBR-II fuel pins, the method of treating the void regions and the method of treating the axial reflectors in the test section.

SELF-SHIELDING IN TEST SECTION FUEL

In the initial calculation, corrections for self-shielding in the fuel pins of the test section were not included. Self-shielding factors were later calculated for U-238 using the figure in Ref. 1 and the calculated self-shielding factors were used to generate a new set of cross sections for U-238 in the EBR-II fuel pins. Using the new cross sections produced a negligible change in the calculated value for the reactivity and power distribution.

AXIAL REFLECTORS IN THE TEST SECTION

The TREAT reactor and the test section have axial reflectors. In the one-dimensional cylindrical geometry calculations of TREAT the axial dimension was represented by a bare core whose height yields an equivalent axial buckling.

The test section was represented in the same manner but the value of the equivalent height could only be approximated. The importance of the axial reflectors of the test section to the reactivity was determined by considering two extreme cases: the full length of the test section being composed of fuel assembly (no reflector) in one case, and of reflector (no fuel) in the second case. The value of k for the two cases differed by 0.002. It was seen that the one-dimensional representation of the axial reflectors in the test section does not greatly affect the value of the reactivity of the system.

VOID REGION

A void region (see Table III-14-I), exists between the liner and the outer sodium pipe. The one-dimensional diffusion theory cannot accurately describe a void region in a two-dimension geometry and therefore a method of adjusting the calculation to agree with experiment was devised. The change in reactivity upon voiding a known fuel element position in TREAT was measured to be $-0.97\% \Delta k$.⁽³⁾ A series of calculations was performed to determine the value of the axial buckling or equivalent height of the center void which yields a -0.97% change in the reactivity. The void region was mocked up by 1% density aluminum. The equivalent height was found to be 596 cm. In subsequent

calculations the void region in the test section was represented by 1% density aluminum with an equivalent height of 596 cm.

The one-dimension representation of a TREAT Test Facility with improved treatment of self-shielding and void region in the test section, was used in calculating the reactivity effects and power distribution in the test section.

REACTIVITY EFFECTS

In developing a model to represent the void region, it was recognized that significant reactivity ($\sim 12\% \Delta k$) was lost due to neutron streaming in the void region. The purpose of the void region is to act as a thermal insulator around the outer sodium pipe. An arrangement of graphite and air gaps was designed to reduce the neutron leakage and also to meet the requirement of thermal insulation. The arrangement was represented by 50% density graphite with an equivalent height of 177.17 cm in the calculations and the addition of the graphite yielded an 8% Δk increase in reactivity.

The reactivity changes upon substituting various materials was investigated. Exchanging zirconium for iron in the liner and outer sodium pipe resulted in increases of 2 and 3% Δk , respectively. The substitution of a zirconium pipe for the iron pipe was found to be

FIG. III-14-1. Power Density Distribution in Test Section for Various Enrichments of Fuel Pins. ANL Neg. No. 112-5611 Rev. 1.

FIG. III-14-2. Product of Real and Adjoint Fluxes for Treat With and Without the Test Section. ANL Neg. No. 112-5612.

TABLE III-14-II. CALCULATED VALUES OF REACTIVITY USING THE DDF AND REX CODES FOR THREE ARRANGEMENTS OF TREAT

System	Calculated k	
	DDF-2	REX
141 element TREAT	1.003	1.00
Fully loaded TREAT	1.210	1.186
Fully loaded TREAT with EBR-II Subassembly	1.140	1.114

impractical and therefore the increase in reactivity could not be realized.

The effect of changing the material composition of the fuel pins was also investigated. Five concentric rings surrounded a center pin. Starting from the center pin and proceeding outward radially the fuel enrichment was 6, 6, 6, 4, 4, and 3%. The substitution of graphite, iron and zirconium decreases the reactivity by 0.007 Δk or less. The effect of fuel enrichment upon the

activity and power distribution was studied in the fully loaded TREAT core.

CALCULATIONS WITH FULL TREAT CORE

In the remaining one-dimensional calculations the removable reflector elements were replaced with TREAT fuel elements to expand the core from 141 elements to 361 elements. For the following calculations the test section liner was made of zirconium, and the void region was filled with 50% density graphite. Three sets of fuel enrichments were studied: (a) 6, 6, 6, 4, 4, and 3%; (b) 12, 12, 12, 8, 8, and 6%; and (c) uniform 6%. The reactivity calculated for each of these three cases was essentially the same. The power distribution in the test section is shown in Fig. III-14-1. The change of reactivity caused by the removal of the fuel pins and sodium coolant was calculated to be $-0.002 \Delta k$. The removal of the pins and sodium corresponds to a serious accident during a transient experiment.

TWO-DIMENSIONAL CALCULATIONS

Two-dimensional transport calculations were used to verify the treatment of the axial reflectors and void regions in the one-dimensional diffusion calculations of the TREAT facility. The same 16 group cross section set was used in the DDF transport code.⁴ Three cases were considered using the DDF code; the 141 element TREAT, the 361 element TREAT, and the 361 element TREAT with the test section. The values of k_{eff} for three cases are given in Table III-14-II for the DDF and one-dimensional REX calculations. It is pointed out that the convergence criterion was less stringent in the DDF problems (5×10^{-3}) than in the REX problems (1×10^{-4}) because of practical limits in the running time of the DDF code. The values of k_{eff} calculated by DDF would be about 1 to 2% lower if the same convergence criterion were used in both cases. The figures in Table III-14-II indicate that the

one-dimensional model is a sufficiently accurate representation of TREAT.

KINETIC PARAMETER

The real and adjoint fluxes from the REX calculations of the 361 element TREAT with the test section inserted were used in the RE 1188 code to calculate some kinetics parameters. The value of the neutron lifetime in the fully loaded TREAT with the test section inserted is 4.735×10^{-4} sec. For TREAT with 141 elements and no test section, the lifetime is 5.38×10^{-4} sec. The plot of the product of the real and adjoint fluxes in Fig. III-14-2 shows that the control rods are in a region of greater neutron importance when the test section is present.

The new temperature coefficient for TREAT was not calculated. It is reasoned that the temperature coefficient should be more negative. The negative temperature coefficient in TREAT is due primarily to increased leakage from the core and reflector. Since the neutron density is peaking closer to the reflector when the subassembly is inserted, more neutrons have a greater probability of leaking with an increase in temperature than in the smaller TREAT core with no test section. In addition, the test section serves as a sink for neutrons and an increase in the TREAT temperature will allow more neutrons to diffuse into this sink.

REFERENCES

1. G. E. Hansen and W. H. Roach, *Six and Sixteen Group Cross Sections for Fast and Intermediate Critical Assemblies*, LAMS-2543 (December, 1961).
2. Reactor Physics Constants, ANL-5800, 2nd edition, p. 761.
3. D. Okrent, C. E. Dickerman, J. Gasidlo, D. M. O'Shea and D. F. Schoeberle, *The Reactor Kinetics of the Transient Reactor Test Facility (TREAT)*, ANL-6174, 22-25 (September, 1960).
4. M. Shapiro, United Nuclear Corporation (Private Communication to G. Fischer, Argonne National Laboratory).
5. L. C. Kvitek, *Multigroup Calculations of Effective Delayed Neutron Fractions, Prompt Neutron Lifetime and Related Kinetics Parameters*, ANL-6511 (March, 1962).

III-15. Transient Effects in Meltdown of Irradiated EBR-II Mark I Fuel Samples

C. E. DICKERMAN, L. E. ROBINSON and C. AUGUST

Previously, experiments in which irradiated metallic fuel pins have been subjected to transient nuclear heating in the Transient Reactor Test Facility (TREAT) were carried out to study the failure thresh-

olds, mechanisms of failure, and density changes upon meltdown of EBR-II Mark I and Fermi A specimens containing appreciable inventories of fission gases.^{1, 2} As an extension of those investigations, further experi-

ments were conducted in TREAT, using standard transparent meltdown capsules for irradiated samples,³ to measure the transient expansion of unrestrained irradiated U-5 w/o fission fuel alloy, and to observe the failure mechanisms of an irradiated EBR-II fuel pin subjected to an axial power profile shaped to approxi-

FIG. III-15-1. Comparison of Experimental and Calculated Linear Expansion of 9.1 cm Sample, Using Data for Unirradiated Alloy. ANL Neg. No. 112-5687.

FIG. III-15-2. Comparison of Experimental and Calculated Linear Expansion of 14.6 cm Sample, Using Data for Unirradiated Alloy. ANL Neg. No. 112-5688.

mate a typical fast reactor core "chopped cosine" power distribution.

Both expansion samples were taken from a single pin irradiated to approximately 1.3 a/o burnup in the Materials Testing Reactor at a calculated maximum centerline temperature of 445°C. After irradiation it was annealed in a furnace for four hours at 550°C, and then disassembled remotely to provide bare specimens. One sample, about 9.1 cm long was given a total energy

input of approximately 600 W-sec/g during a power pulse with a half width ~ 0.5 sec. The other sample was about 14.1 cm long, and was exposed to a "flattened" power pulse of about 24 sec duration at 38 W/g. Each specimen was instrumented with a linear motion transducer to measure transient axial expansion. The complete pin used for the shaped axial power test had been irradiated to ~ 0.6 a/o burnup in MTR at the same temperature and annealed the same as the samples. Tantalum neutron absorbers were incorporated in the meltdown capsule to shape the axial power profile. Chromel-alumel fast-response thermocouples were spot-welded to the surface of the clad pin. Attempts to weld similar thermocouples to the surfaces of the bare pins were unsuccessful.*

The nominal heating rate for the 9.1 cm long pin was 1000°C/sec; that for the longer fuel cylinder was 80°C/sec. Sample temperatures, as functions of time, were inferred from reactor transient instrument data reactor power/sample power ratios, and fuel alloy enthalpy data.⁴ Calculated temperatures were corrected for heat lost from the samples during the times of interest (the maximum energy correction was 5%).

Expansion data are shown in Figs. III-15-1 and III-15-2 for the 9.1 cm and 14.1 cm samples, respectively. Also given for comparison are the expansions computed from the temperature calculations, using published data on the expansion during the initial heating of un-

* Although procedures were established for welding thermocouple wires directly to bare, unirradiated U-5 w/o fission alloy, the remote adaptation of the technique was not successful with the irradiated material.

radiated U-5 w/o fissium specimens which had previously been annealed for 11 days at 500°C and water quenched.⁵ Because of uncertainties in calibrations, a band (rather than a curve) is shown for the calculated expansions. Each band corresponds to a power calibration uncertainty of $\pm 10\%$.

The data from both experiments indicate that the transient expansion of this material is similar to that of the unirradiated alloy up to about 700°C. (This temperature is the approximate limit of data on the unirradiated fuel.⁵) At higher temperatures, the expansion is appreciably higher than expected from extrapolations of the data on unirradiated fuel. For equal temperatures, expansion appears to be $\sim 20\%$ lower for the slow heating experiment than for the high-heating rate transient. However, this difference is within possible experimental errors, estimated at $\pm 15\%$.

Both samples melted during the course of the experimental runs. The sharp drops in expansion shown at about 3.9 and 27 sec in Figs. III-15-1 and III-15-2, respectively, are consistent with melting. Calculated temperatures, at these times, are indicated on the figures. The final residue had a nominal density about one-half the original value for the longer sample and about three quarters for the shorter sample subjected to the higher heating rate. A control portion of the pin

indicated $\sim 6\%$ change in fuel density from the MTR irradiation.

Behavior of the complete pin run with shaped axial power was similar to that of unirradiated samples run under the same conditions. Failure occurred in the hottest central region, within a few mm of the fuel center. Fuel was found to be ejected from the failed cladding. Although fuel movement occurred from the cooler ends to the center, only about half the fuel originally inside the cladding was ejected. Fuel remaining in the area of the cladding break had density decreases as much as 20% from the original, while retaining a cylindrical geometry. Several other areas of the cladding showed signs of small penetrations by dissolution.

REFERENCES

1. J. H. Monaweck, C. E. Dickerman and E. S. Sowa, *Behavior of Irradiated Metallic Fuel Elements Exposed to Nuclear Excursions in TREAT*, Trans. Am. Nucl. Soc. **6**, No. 2, 374 (1963).
2. C. E. Dickerman and L. E. Robinson, *Photographic Experiments on Meltdown of Irradiated Metallic Fuel Pins*, Trans. Am. Nucl. Soc. **7**, No. 1, 137 (1964).
3. L. E. Robinson and C. August, *Transparent Facility for TREAT Meltdown Experiments on Pre-Irradiated Samples*, ANL (to be published).
4. H. Savage and D. Seibel, *Heat Capacity Studies of Uranium and Uranium-Fissium Alloys*, ANL-6702 (1963).
5. S. T. Zegler and M. V. Nevitt, *Structures and Properties of Uranium-Fissium Alloys*, ANL-6116 (1961).

III-16. Fast Reactor Safety Experiments on Thorium-Uranium EBR-II-Type Fuel Samples in the Absence of Coolant¹

C. E. DICKERMAN, L. E. ROBINSON and R. STEWART

Thorium-uranium metallic fuel is of potential interest for use in fast breeder reactors, for reasons arising both from nuclear and metallurgical properties. Physics calculations show that sodium-cooled fast reactor cores operating on the thorium-uranium cycle tend to have significantly more negative sodium density coefficients of reactivity than those on the uranium-plutonium cycle.² Steady state irradiation tests indicate superior stability of thorium-uranium alloys under conditions of high temperature and burnup.³ Because of the promise of this fuel, a series of safety-oriented experiments were conducted on steel-clad, sodium-bonded thorium-20 w/o uranium fuel samples contained in an inert gas atmosphere, in order to survey the threshold of failure under transient nuclear heating and to study the mechanisms of fuel movement.

Six specimens of the Experimental Breeder Reactor-II Mark I-type were tested. Each consisted of a fuel cylinder approximately 12 cm long and 0.356 cm in diameter, sodium-bonded to a steel jacket of 0.442 cm outer diameter and 0.023 cm wall. They were exposed in standard transparent meltdown capsules in the Transient Reactor Test Facility (TREAT). Normally, low-enrichment fuel is used in such experiments. Because the uranium in this experiment was fully enriched, the capsules were lined with cadmium to reduce the ratio of sample energy to reactor energy in order to provide experimental flexibility in specifying reactor transient conditions. Two fast-response chromel-alumel thermocouples were spot-welded to the cladding surface of each specimen. Irradiation conditions are summarized in Table III-16-I. Sample energy inputs listed are those

TABLE III-16-I. Th-U SAMPLE IRRADIATION CONDITIONS

Cap- sule	Initial Reactor Period, sec	Power Burst Half-Width, sec	Sample Fuel Energy Input, cal/g	Maximum Cladding Thermocouple Readings	
				TC 1 ^a , °C	TC 2 ^b , °C
1	0.146	0.3	30.1	350	505
1	0.217	0.7	46.1	605	710
1	0.125	0.5	80.6	1120	1200
2	0.112	0.43	91.0	1360	1330
3	0.099	0.35	106.0	1400	1350
4	0.50	8.9 ^c	121.7	1220	1300
5	0.076	0.26	107.5	>1400 ^d	>1400 ^d
6	0.080	0.31	119.8	>1400 ^d	>1400 ^d

^a One cm above fuel center.^b One cm below fuel center.^c "Flat-top" power transient.^d Readings erratic at or near maximum.

obtained using a ratio of sample energy to reactor energy established by radiochemical analysis for the fission yield of Ce-144 and Cs-137 from the first sample⁴ (the only one not suffering physical damage to the fuel). This ratio is about 11 % below the preliminary value used in planning the series of experiments, and about 6 % above an approximate energy ratio based on the metallurgical condition of fuel from samples Nos. 3 and 5.

Capsule No. 1 was given a succession of three bursts with monotonically-increasing integrated power in order to obtain check points on the calculated relationship between sample temperature and reactor energy release. Appearance of the first fuel cylinder after irradiation was unchanged from the pretransient condition. Neither sample No. 1 nor No. 2 failed; however, the second specimen showed evidence of selective melting and flow of a molten phase which solidified against the inside surface of the cladding without penetrating. Sample No. 3 did fail, indicating that the threshold for failure in a short burst occurs at a cladding temperature of $\sim 1360^{\circ}\text{C}$, in reasonable agreement with a bursting pressure estimate of $\sim 1312^{\circ}\text{C}$ made for an isothermal sample model.

Sample No. 4 was subjected to much slower heating: its rate of temperature rise in the range $1000\text{--}1100^{\circ}\text{C}$ was $\sim 180^{\circ}\text{C}/\text{sec}$. Despite the somewhat more vigorous attack of cladding for this specimen, no evidence was found for vigorous fuel movement, and no fuel was expelled from the cladding.

Both sample Nos. 5 and 6, which were run to extensive failure, also produced fuel movement dominated by gravity.

High-speed color motion pictures were taken of the six capsules during the experiments. Each of the four cases of failure showed high velocity release of a sodium

cloud, with maximum measured cloud front velocities as high as ~ 25.8 meters/sec. There was no indication that any significant portion of the momentum of escaping sodium was transferred to fuel, even in the case of sample Nos. 5 and 6. This is in direct contrast with the vigorous fuel expulsion found to be the case for similar samples containing uranium-fissium fuel cylinders which have a melting range of about $1000\text{--}1080^{\circ}\text{C}$.⁵ When extensive failure occurred in the thorium-uranium case, the cladding was much closer to its melting point than in the uranium-base alloy case;⁵ thus, the steel jacket was not effective in allowing the internal pressure to build up and cause rapid expulsion of fuel from the point of cladding failure.

Appreciable attack of the steel cladding by the fuel appears—for the time scale of these experiments, at least—to occur within about 150°C of the melting point, beginning at about 1350°C . This is more than 200°C higher than the threshold for rapid attack of steel or iron by uranium.⁶ Rate data for the thorium alloy-iron dissolution comparable to those for the uranium-iron system are not available; however, a thorium-iron eutectic has been reported at 860°C ,⁷ 145°C above the lowest uranium-iron eutectic. This difference in eutectic temperatures is qualitatively in agreement with the difference in penetration observed for the specimens of this experiment and for uranium-based fuel. Direct comparison with the uranium-based alloy results, however, requires actual rate data, since rates depend not only on temperature but on the compounds involved.

These results indicate that one mechanism for producing relatively rapid motion of fuel following fuel failure apparently is not operative for the steel-clad, sodium-bonded thorium-uranium metallic fuel type. This behavior might be modified significantly by such conditions as greatly different heating rates, presence of sodium coolant, and effects of internal inventories of fission product gases, none of which was studied in this series of experiments.

REFERENCES

1. C. E. Dickerman, L. E. Robinson and R. Stewart, *Fast-Reactor Safety Experiments on EBR-II-Type Thorium-Uranium Fuel*, Trans. Am. Nuc. Soc. **8**, No. 1, 309 (1965).
2. G. A. Sofer, A. H. Kazi and J. Bobker, *Economics and Safety Aspects of Large Ceramic U-Th Fast Breeder Reactors*, Proc. Conf. on Breeding, Economics and Safety in Large Fast Power Reactors, ANL-6792, 435 (1963).
3. J. H. Kittel, J. A. Horak, W. F. Murphy and S. H. Paine, *Effects of Irradiation on Thorium and Thorium-Uranium Alloys*, ANL-5674 (1963).
4. R. Armani and D. M. Smith, Argonne National Laboratory (Private Communication).

- C. E. Dickerman, D. Okrent and E. Sowa, *The Fast Reactor Safety Programme In TREAT*, Physics of Fast and Intermediate Reactors, IAEA, Vienna **III**, 171, (1962).
6. C. M. Walter and L. R. Kelman, *Penetration Rate Studies*

- of Stainless Steel by Molten Uranium and Uranium-Fissium Alloy*, J. Nuc. Mat., **6**, No. 3, 281 (1962).
7. F. A. Rough and A. A. Bauer, *Constitutional Diagrams of Uranium and Thorium Alloys*, (Addison-Wesley, Reading, Massachusetts, 1958), pp. 74-76.

III-17. Initial TREAT Meltdown Experiments in the Integral Sodium Loop

L. E. ROBINSON, C. DICKERMAN, W. STEPHANY and L. WILLIS

Analysis of hypothetical fast reactor meltdown accidents requires information on the fuel failure thresholds, fuel movement mechanisms, effects of a sodium environment on fuel behavior, pressure surges, and coolant movement.¹ As a continuation of earlier experiments on EBR-II Mark I-type fuel pins, a series of transient heating experiments was carried out in the Transient Reactor Test Facility (TREAT) using a sample fuel pin contained in an integral sodium loop.

Earlier meltdown experiments on this fuel type were performed to study transient behavior in an inert gas environment,¹ kinetics of cladding penetration,² effects of exposure of pins inside a stagnant sodium annulus,³ and phenomena associated with appreciable sample inventories of fission products.^{4, 5} The loop experiments reported here were the first TREAT tests in which fast reactor fuel pins have been run in a geometrically realistic mockup of a core flow passage formed by a fuel pin cluster, with sodium flow capability. The tests yielded results on the performance of the loop with its associated instrumentation when used in meltdown experimentation, sample central temperature data for comparison with calculated temperatures, and data on the threshold of failure resulting from dissolution of cladding by molten fuel.

The sample pin was of the standard EBR-II design, consisting of a 4.1%-enriched U-5 w/o fissium alloy cylinder, 0.365 cm diameter by 36 cm long, bonded by Na to a 304 stainless steel jacket with 0.396 cm inner diameter and 0.023 cm wall thickness.

Surrounding the fuel sample was a ring of six empty cladding tubes arranged on the EBR-II cluster pitch, forming a seven-element cluster. A Ta-Mo thermocouple, sheathed in an open 0.11 cm diameter Ta tube was cast into the fuel sample to measure transient fuel center temperatures.⁶ Miniature, fast-response strain gauge pressure transducers mounted on short NaK-filled stand offs were located at the test section inlet and outlet (See Paper IV-16). The sodium loop was of the integral type,⁷ designed to fit completely within the volume of a TREAT fuel element (approximately 10

cm square by 240 cm high), and with maximum sodium flow capability of about 4.5 meters/sec. (See Paper IV-14).

Six transients were run—three with 3.8 meters/sec flow and three with no flow. Maximum sample temperatures occurred in the last transient, a 144 MW-sec excursion with no flow which carried the sample to a point just short of the failure threshold. Reactor power burst durations were of the order of a half-second. Sample power was essentially uniform along the sample axis.

Experimental central fuel temperatures were compared with values calculated using reactor power instrument traces, sodium flow values, a radiochemical power calibration,⁸ basic material properties of fuel,⁹ sodium, and steel, and the ARGUS transient heat transfer code for cylindrical geometry.¹⁰ Agreement was satisfactory for analyzing the experiments. Direct comparison of maximum central fuel temperatures for the four highest energy transients indicated that the effective power calibration was about 4.4 % low. This discrepancy is within the range of experimental error arising from uncertainties in power instrument inter-calibrations, radiochemistry, property data and thermocouple measurements.

The maximum central fuel temperature and the maximum fuel-cladding interface temperature were 1200°C and 950°C, respectively, for the most severe transient. Post-mortem inspection of the sample showed that the cladding had not failed. However, the fuel displayed surface cracks filled with solidified fuel-cladding eutectic, apparently produced by high temperature molten fuel from the pin interior which penetrated the fuel surface through the cracks and reacted with the cladding. This result confirms a mechanism suggested earlier³ to describe how significant alloying of fuel and cladding could occur, although heat transfer calculations based on the geometry before melting indicated maximum interface temperatures below the threshold for rapid formation of eutectic.² Figure III-17-1 con-

FIG. III-17-1. Closeup Views of Fuel Cylinder Carried to Failure Threshold in First Integral Sodium Loop Meltdown Experiment. ANL Neg. No. 112-4640-A.

tains four close-ups of the uranium fuel cylinder after removal of cladding and bond sodium. Segment A shows solidified fuel-cladding eutectic which flowed downward inside the annulus between fuel and cladding, and froze against the cladding without penetration.

Close-ups B, C, and D show the eutectic-filled surface cracks.

REFERENCES

1. C. E. Dickerman, D. Okrent and E. S. Sowa, *The Fast Reactor Safety Programme in TREAT*, Physics of Fast

- and Intermediate Reactors, IAEA, Vienna, **III**, 171 (1962).
2. C. M. Walter and C. E. Dickerman, *TREAT Study of the Penetration of Uranium and U-5 w/o Fs Alloy Through Type 304 Stainless Steel*, Nucl. Sci. and Eng. **18**, 518 (1964).
 3. C. E. Dickerman, E. S. Sowa, J. H. Monaweck and A. Barsell, *In-Pile Experiments on Meltdown of EBR-II Mark I Fuel Elements in Stagnant Sodium*, Nucl. Sci. and Eng. **18**, 319 (1964).
 4. J. H. Monaweck, C. E. Dickerman and E. S. Sowa, *Behavior of Irradiated Metallic Fuel Elements Exposed to Nuclear Excursions in TREAT*, Trans. Am. Nuc. Soc. **6**, No. 2, 374 (1963).
 5. C. E. Dickerman and L. E. Robinson, *Photographic Experiments on Meltdown of Irradiated Metallic Uranium Fuel Pins*, Trans. Am. Nuc. Soc. **7**, No. 1, 137 (1964).
 6. D. L. Rall and S. W. Bovarney, *Development of Temperature Instrumentation for EBR-II Fuel Pins*, ATL-188 (1959).
 7. E. S. Sowa, C. R. Breden and L. W. Fromm, Jr., *Equipment Used in the Performance of Experiments in Nuclear Reactors at the Argonne National Laboratory*, Nucl. Sci. and Eng. **15**, 415 (1963).
 8. R. J. Armani, Argonne National Laboratory, (Private Communication).
 9. H. Savage and R. D. Seibel, *Heat Capacity Studies of Uranium and Uranium-Fissium Alloys*, ANL-6702 (1963).
 10. J. Heestand, D. F. Schoeberle and L. B. Miller, *A Method of Calculating Transient Temperatures in a Multiregion, Axisymmetric, Cylindrical Configuration, The ARGUS Program, 1089/RE-248, Written in FORTRAN II*, ANL-6654 (1963).

III-18. Behavior of US Fuel Samples in High Temperature Transient TREAT Experiments¹

L. E. ROBINSON, C. E. DICKERMAN, R. CARLANDER,* C. AUGUST and C. MUELLER†

Transient heating experiments carried out in the TREAT reactor on Ta-clad EBR-II-type uranium monosulfide elements in the absence of coolant² demonstrated that this fuel has stable behavior for heating times of the order of a half-second and up to the melting point of US (2463°C).³ In order to complete the survey of transient US fast reactor type fuel performance, additional transient tests were carried out on two similar specimens, clad with W-26 w/o Re tubing to study effects of gross fuel melting. Fuel consisted of pellets of approximately 96% theoretical density, containing approximately 2.5 w/o UO₂ and 3.5 w/o UOS. Pellet diameter was 0.381 cm, and the total pellet stack height for each sample was about 19.0 cm. A blanket gas of one atmosphere of argon, at room temperature, was contained in each sample. Samples were exposed to reactor power pulses of about a half-second duration in a helium atmosphere inside standard TREAT transparent meltdown capsules. Fast-response W-5% Re versus W-26% Re thermocouples were spot-welded to the outer surfaces of the cladding tubes. Experimental conditions are summarized in Table III-18-I.

Both specimens were found to be cast into their tubing. Neither sample failed. Motion pictures taken during the experimental runs showed shifting zones of light and dark appearance on the cladding, indicating flow of hot fuel inside the cladding. No significant

variation in fuel structure was found between samples; however, both displayed a change in structure with radial position. Grains at the extreme edge were almost entirely composed of large amounts of UOS and possibly some UO₂ in a matrix of US. At a distance from the edge of about one-quarter of the cylinder radius, UOS was found to be migrating from the grain boundaries into the US matrix. At a distance from the

TABLE III-18-I. EXPERIMENTAL CONDITIONS

Sample No.	Experiment	Estimated US Energy Release, W sec/g	Max. Cladding Surface Temperature, °C	Est. Max. Central Fuel Temperature, °C
1	1	812	1115	1850-2100
1	2	1111	1310	2300-2462
1	3	1667	1965	melted
2	4	2069	2160	melted
2	5	2077	2180	melted

edge of one-half to three-quarters of the cylinder radius, the UOS was found both at the grain boundaries and dispersed within the matrix. At the center, the UOS was distributed as a fine dispersion in the US grains.

Some evidence of a possible reaction between fuel and cladding was observed. The tungsten alloy was cracked intergranularly at the fuel interface, and an alien phase was seen at the metal grain boundaries. This phase, which could have been either a reaction product

* Metallurgy Division.

† Student, Northwestern University, Evanston, Illinois.

or merely fuel which had penetrated the cracks while molten, was not identified.

Analyses were made of the effective fuel-cladding interface thermal conductance by means of comparing experimental data and theoretical calculations of the delay between reactor power pulse and peak cladding surface temperature, initial rates of cladding surface temperature rise, and the general shapes of cladding surface temperature curves. Interface conductance values deduced varied from 0.1–0.2 W/cm²C for the lowest temperature excursion to the relatively wide range of 0.3–0.7, typical of the three highest temperature runs.

In general, these higher temperature exposures

showed the same resistance to mechanical damage displayed in the earlier tests. No evidence was found either for massive cracking, or for a central zone of "loosely bonded" ceramic particles.

REFERENCES

1. L. E. Robinson, C. E. Dickerman, R. Carlander, C. August and C. Mueller, *Behavior of Uranium Sulfide Fast-Reactor-Type Fuel Specimens Under Transient Heating in TREAT*, Nuclear App. 1, 168 (1965).
2. L. E. Robinson, C. E. Dickerman and C. August, *Behavior of Uranium Sulfide Fast Reactor Type Fuel Specimens Under Transient Heating in TREAT*, Trans. Am. Nucl. Soc. 7, No. 1, 105 (1964).
3. E. D. Cater, *The Vaporization, Thermodynamics and Phase Behavior of Uranium Monosulfide*, ANL-6140 (1960).

III-19. Correlation of Results from In-Pile Transient Pressure Measurements¹

C. E. DICKERMAN and W. P. STEPHANY

Analytical studies have been made on the effects of pressure generated by vaporizing coolant in heated channels (see Paper V-30). As a test of the theoretical work, a generalized approximate analytical description developed in the course of the investigations has been used to analyze results reported for experiments on transient void formation in water, performed in the KEWB reactor by L. Wentz, et al.² Because of the lack of applicable transient heat transfer data, the model was used for correlation of results, rather than quantitative calculations.

The approximate analytical model predicts that

$$\Delta P = K_1 \sqrt[3]{q_1 D_1^2 z m_0 / A}, \quad (1)$$

where ΔP is pressure rise, K_1 is numerical factor, q_1 is a suitably-averaged value of power, D_1 is a suitably-averaged value of the derivative of pressure with respect to energy, z is the distance of coolant expulsion, m_0 is the mass to be accelerated, and A is the cross sectional area across which the pressure is exerted. Similarly,

$$\dot{z} = K_2 \sqrt[3]{A q_2 D_2^2 z^2 / m_0}, \quad (2)$$

where $\dot{z}$ is the time derivative of z .

In applying Eqs. (1) and (2) to correlate the experimental data, it was assumed that there exists a reasonably well-defined distance of expulsion, z , beyond which the power no longer is effective in producing pressure, and that z did not vary "greatly" over the range of the experiments (ΔP is proportional only to $z^{1/3}$; $\dot{z}$ shows a

greater dependence, being proportional to $z^{2/3}$). It must also be assumed that there is no hidden dependence upon reactor period and m_0 through q and D , which implicitly contain the heat transfer component of the model. It is assumed that q is proportional to reactor power.

Wentz, et al. have described their experimental arrangement in detail.³ The range of reactor periods for which significant pressure pulses occurred was 1 to 10 msec. A variety of "after pulses" was observed; this analysis is restricted to the initial pressure pulses.

From the published correlations of experimental data, the ratio of ΔP for a mass loading of 225 grams to that for 30 grams is approximately 2.17. Equation (1) predicts a ratio of 1.96.

Available data² on maximum velocity give a value of 5.11 ± 1.13 cm/sec for a 30 gram loading over all reactor periods; 3.72 ± 2.38 cm/sec for a 124 gram loading; and 2.75 ± 1.18 cm/sec for 225 grams. The ratio of velocity for 30 grams to that at 225 grams is 1.86; the ratio for velocity at 124 grams to that at 225 grams is 1.35. Equation (2) predicts a value of 1.96 for the first ratio and a value of 1.22 for the second. Although agreement appears to be satisfactory, it may be fortuitous, in view of the experimental uncertainties quoted.

Since significant pressure rises do not occur while the reactor power is rising on a constant exponential, period dependence can be estimated, within the limits of the model, by replacing the actual power pulse by an

quivalent square wave pulse with the proper ratio of amplitude to maximum power and sufficient duration to yield the actual integrated power. Experimentally, the maximum reactor power is approximately proportional to the period raised to the -1.63 power (see Fig. 6 of Ref. 4). Equation (1) predicts that the ratio of pressure rise at 1 millisecond to that at 10 milliseconds is 12.3. The experimental correlation² gives a value of 13.8.

Dependence of maximum velocity upon period can be estimated using Eq. (2), which yields a ratio of velocity for 1 millisecond to that at 10 milliseconds of 3.47. Because of scatter in the available data, this ratio cannot be compared directly; it is, however, in qualitative

agreement with the fitted curves,² which yield ratios of 3.42 and 2.03 for 225 grams and 30 grams, respectively.

REFERENCES

1. C. E. Dickerman and W. Stephany, *Correlation of In-Pile Transient Pressure Measurements*, Nucl. Structural Eng. **1**, No. 2, 342 (1965).
2. L. B. Wentz, S. M. Zivi and D. B. Langmuir, *Quarterly Progress Report for Period Ending June 30, 1964, Kinetic Studies of Heterogeneous Water Reactors*, STL-372-6 (1964).
3. L. B. Wentz, R. W. Wright and S. M. Zivi, *Annual Summary Report 1963, Kinetic Studies of Heterogeneous Water Reactors*, STL-6312 (1963).
4. M. S. Dunenfeld and R. K. Stitt, *Summary Review of the Kinetics Experiments on Water Boilers*, NAA-SR-7087 (1963).

III-20. FARET Core I—Preliminary Fuels Irradiation Program and Design¹

P. J. PERSIANI, W. R. SIMMONS* and T. R. BUMP*

PURPOSE

The primary objective of the initial experiment in the Fast Reactor Test Facility (FARET) is the investigation of the performance characteristics of fuel systems being considered in the AEC's Fast Reactor Development Program.

The FARET Core I is designed to simultaneously test several different fuel element designs, the sampling size of each design to be such as to provide good statistical data. The performance of these elements is to be investigated under a variety of neutronic and thermal conditions.

Oxide, carbide, and metallic fuels are all being considered for test in Core I under operating conditions appropriate to the prototype reactors in the AEC's program. By selection of location in the reactor and/or enrichment of the test element, performance ratings will be achieved which "bracket" the desired conditions and which will permit assessment of fuel operating capability.

PRELIMINARY CORE I DESCRIPTION

The above objectives will be achieved with a core consisting of about 61 subassemblies and having a height of 51 cm (20 in.). A preliminary configuration of such a core has been chosen for the purpose of initial physics and heat transfer studies. The first, second, fourth, and one-half of the fifth subassembly rows are assumed to be loaded with oxide fuel, for a total of 37

subassemblies. The third row is loaded with 12 carbide fuel subassemblies, and the other one-half of the fifth row with 12 metal fuel subassemblies. This division, in amounts of fuel, is tentative and may be changed as more becomes known about the relative availability and/or potential of the various fuels.

The representative loading pattern is given in Fig. III-20-1 and the related power parameters are listed in Table III-20-I. Table III-20-II summarizes the nuclear and control parameters for Core I. Tables III-20-III and III-20-IV provide design information on some preliminary subassembly concepts. Table III-20-IV gives pertinent data on two oxide-, two carbide-, and one metal-fueled subassemblies.

Specific subassemblies for fuel test in the first core of FARET are now under study. The final specifications will depend on information gained from irradiation tests, compatibility and corrosion experiments, manufacturing developments, and further large reactor studies in the near future.

It is intended to install six or more instrumented subassemblies, one or two for each kind of fuel. These subassemblies should remain in place for the core life of that particular fuel element type, providing continuous indications of fuel and coolant temperatures, and possibly of coolant flow rates, neutron flux, and fission gas release. The allowable number of permanent instrumented subassemblies is limited since the presence of their instrument leads interferes with some aspects of fuel handling. When details of optimum fuel handling are combined with fuel irradiation requirements, the

* Reactor Engineering Division.

FIG. III-20-1. FARET Core I (Loading Pattern). ANL Neg. No. 112-5728.

TABLE III-20-I. PERFORMANCE PARAMETERS OF FARET CORE I PRELIMINARY DESIGN;^a REACTOR POWER 50 MW(t)

Assembly		Fuel Type	Approx. U-235 Enrichment	Elements Per Assembly		Avg. Power Per Assembly, MW	Avg. Linear Power, kW/ft	Range of Linear Power, kW/ft	Power Density, MW/liter	% Heavy Atom Burnup (180 days full power)
No.	Row			No.	Diam					
1	1	(Pu, U)O ₂	0.400	37	0.29	0.88	13.6	—	0.58	3.3
6	2	(Pu, U)O ₂	0.400	37	0.29	0.88	13.6	12.5-13.6	0.58	3.3
12	3	(Pu, U)C	0.489	37	0.29	1.17	18.0	17-20	0.76	3.3
18	4	(Pu, U)O ₂	0.400	37	0.29	0.71	11	10-13	0.46	2.7
24	5	(Pu, U)Ti	0.114	61	0.22	0.72	7	6.5-8.2	0.47	1.8

^a Volume 93.2 liters, height 50.8 cm (20 in.). Core Composition (v/o): Fuel 36/Coolant 44/Structure 20. Uranium/Plutonium (Atom Ratio): Ceramic Fuel, 6/1; Metal Alloy Fuel (30 a/o Ti), 4/1.

best patterns of fuel loading and of instrumentation will be chosen. Some additional instrumented subassemblies, chosen deliberately for early examination, may also be desirable. Additional development work is still necessary to accomplish all of these objectives, but in the extensive work done to date in this area, no insurmountable obstacles have been encountered.

It is planned to irradiate the Core I fuel to about 10

percent burnup of heavy atoms, or until fuel element failure seems imminent, whichever occurs first. The ability to predict failure will be gained by removing for inspection subassemblies containing each type of fuel (probably more than six assemblies in all) each time a carbide fuel subassembly undergoes about 1.5 to 2 percent burnup. Each subassembly removed will be replaced by a new unit of the same type and enrichment.

The fuel handling would occur about every 3 to 4 months depending on a 50 to 80 percent plant factor respectively. Since fuel with burnup is continuously being replaced with fresh fuel, the loss in reactivity over the entire 10 percent burnup range is reduced from

TABLE III-20-II. CALCULATED NUCLEAR AND CONTROL PARAMETERS FOR FARET IRRADIATION CORE I
(Preliminary Design)

<i>Nuclear Data</i>		
Neutron flux at center of core, n/cm ² -sec		
above 1.35 MeV		0.7×10^{15}
below 1.35 MeV		2.3×10^{15}
total		3×10^{15}
Prompt neutron lifetime, μ sec		0.26
Effective delayed neutron fraction		0.0055
<i>Reactor Control; average reactivity coefficients, $\Delta k/\Delta T^\circ\text{C}$</i>		
Overall Doppler @750°K	Uranium	-0.63×10^{-6}
	Pluto-nium	$< +0.1 \times 10^{-6}$
Radial core expansion		-9.5×10^{-6}
Axial fuel expansion (Row)		
oxide fuel 1 & 2		-0.55×10^{-6}
carbide fuel 3		-1.63×10^{-6}
oxide fuel 4		-1.11×10^{-6}
metal fuel 5		-1.80×10^{-6}
Sodium expansion (Row)		
oxide region 1 & 2		-1.1×10^{-6}
carbide 3		-2.5×10^{-6}
oxide 4		-4.1×10^{-6}
metal 5		-1.6×10^{-6}
Total core		-9×10^{-6}
Total core and reflector		-14×10^{-6}
<i>Estimated Range of Total Worth of Control Rods</i>		
Composition (v/o): B ₄ C 55/coolant 25/structure 20		
No. of Rods—11		
60% B ¹⁰ enrichment		0.08
80% B ¹⁰ enrichment		0.095
<i>Excess and Shutdown Reactivity</i>		
Heavy atom burnup		0.015–0.04 ^a
Operational temperature		
Sodium $\Delta T_{av} = 500^\circ\text{C}$ core and reflector	0.0070	
Radial core expansion $\Delta T_{av} = 500^\circ\text{C}$	0.0048	
Axial fuel expansion $\Delta T_{av} = 1500^\circ\text{C}$	0.0025	
oxide		
$\Delta T_{av} = 1100^\circ\text{C}$		
carbide		0.0018
$\Delta T_{av} = 500^\circ\text{C}$	0.0009	
metal		
Total operational temperature	0.017	
Excess reactivity	0.032–0.057	
Shutdown reactivity	0.020	
Design worth of control system	0.052–0.077	

^a Depending on the manner of compensating for burnup.

TABLE III-20-III. PRELIMINARY CORE I DATA

<i>Entire Core</i>	
equivalent diameter, in.	19
height, in.	20
volume, liters	93
<i>Subassemblies</i>	
mixed oxide	37
mixed carbide	12
Pu, U alloy	12
control rods	8–12
radial reflector	228
total	301
configuration	Hexagonal
dimension across flats, in.	2.290
lattice spacing (pitch), in.	2.320
tube wall thickness, in.	0.040
<i>Upper Reflector</i>	
height, in.	12
equivalent diameter, in.	19
<i>Radial Reflector</i>	
height, in.	80
equivalent outside diam, in.	42.3
radial thickness, in.	11.6
<i>Control Rod Composition, v/o</i>	
boron carbide	55
stainless steel	20
sodium	25
<i>Upper Reflector Composition, v/o</i>	
stainless steel	48
sodium	52
<i>Radial Reflector Composition, v/o</i>	
stainless steel	82
sodium	18

FIG. III-20-2. Methods of Providing Burnup Reactivity in FARET. ANL Neg. No. 112-5730.

TABLE III-20-IV. PRELIMINARY FUEL DATA

Fuel Elements	Mixed Oxide		Mixed Carbide		Pu, U Alloy
	Design no. 1	Design no. 2	Design no. 1	Design no. 2 ^a	
Elements per subassembly	37	37	37	37	61
Fuel height, in.	20	20	20	20	20
Fuel outside diam, in.	0.255	0.245	0.245	0.185	0.170
Fuel inside diam, in.	0	0.100	0	0	0
Clad outside diam, in.	0.290	0.290	0.290	0.220	0.220
Clad thickness, in.	0.015	0.020	0.020	0.015	0.015
Sodium bond gap, in.	—	—	—	—	0.010
Spacer wire diam, in.	0.057	0.057	0.057	0.117	0.052
Axial blanket height (each end), in.	6	6	6	6	6
Gas reservoir, in.	25	25	25	25	25
Element height, in.	57	57	57	57	57
Fuel U/Pu (atom ratio) ^b	6/1	6/1	6/1	6/1	4/1
Fuel fabrication ^c	Pressed	Pressed	Pressed	Pressed	Cast
Fuel pellet density, %	92	92	92	92	95
"Effective" fuel density, %	88	74	88	88	77
Bond material ^d	Helium	Helium	Helium	Helium	Sodium
<i>Core Composition, v/o</i>					
Fuel	0.40	0.31	0.37	0.21	0.30
Structure	0.19	0.22	0.22	0.19	0.22
Sodium	0.40	0.40	0.40	0.59	0.48
Void	0.01	0.07	0.01	0.01	0.00

^a This design is intended to produce a high burnup rate without excessive fuel temperatures.

^b U/Pu ratio is optional. Uranium enrichment is adjusted to provide criticality and desired heat generation rate.

^c Other cladding materials and methods of fabrication are also under consideration.

^d Sodium bonding for ceramic fuels is also under study.

about 8 percent to about 5 percent $\Delta k/k$ (see Fig. III-20-2).

This loss in reactivity can be made up by several techniques. The entire 10 percent burnup could be accommodated by initial fuel enrichment with the excess reactivity controlled by the boron carbide control rods. Considering the additional reactivity to be added for operational temperature and shutdown margin the total worth of the control system could amount to about 9 to 10 percent.

The total amount of reactivity required in each of the control rods could be reduced by adopting a loading procedure which adds fueled subassemblies in the reflector region. A fueled assembly placed in the immediate reflector region is worth 0.3 to 0.5 percent. In Fig. III-20-2 the effect of adding two fueled assemblies is indicated by the uppermost curve. It appears that excess reactivity requirements in the control system for burnup could be held to about 0.015 $\Delta k/k$. The long term reactivity requirements in FARET Core I will probably be supplied by a combination of control rods and fuel subassembly additions in the reflector region.

Table III-20-II summarizes the predicted reactivity coefficients for the Core I loading. The reactivity effects due to the expansion of coolant and fuel for the anticipated operating temperature changes are also given in

Table III-20-II. Adding the temperature effect, a shutdown margin of not less than 2% $\Delta k/k$ and the heavy atom burnup excess reactivity requirements of 1.5% $\Delta k/k$, and then assuming that fueled subassemblies are added in the reflector region, the total worth of control rods must be about 0.060 $\Delta k/k$.

It is anticipated that higher burnup rates may be desired in a few subassemblies. To achieve this, some subassemblies may be fitted with pins of smaller diameter and higher enrichment to keep the linear heat fluxes and fissionable material constant in the subassembly.

It is planned to have three in-pile loops eventually operating in the reactor, with the expectation that one is installed in the reflector prior to startup. The first use of this loop could be to test fuel elements under more severe conditions than in the core. For example, ceramic fuels may be operated partially molten. If advisable, reference fuel elements for the prototype reactor will be installed in this loop for testing under more extreme conditions than permissible when directly exposed in the core.

Later cores will test improved fuel elements which have evolved from experience gained with the earlier cores. It is likely that these later cores will be smaller, to emphasize high rates of burnup, high flux, and high

pecific performance parameters. Special fuel design features to enhance safety, such as controlled expansion elements may also be tested. In the later stages of fuel development, it is anticipated that fuel elements may be vented directly to the reactor coolant, to gain experience with the problems involved in operating a large system in that manner. Safety experiments are

expected to be interspersed between various irradiation runs.

REFERENCES

1. W. R. Simmons, P. J. Persiani and T. R. Bump, *Fast Reactor Test Facility (FARET) and Initial Experimental Program*, Am. Nucl. Soc. Fast Reactor Technology, National Topical Meeting, Detroit, Michigan; April 26-28, 1965. ANS-100.

III-21. Faret Critical Experiments

A. HESS,* J. K. LONG,* G. BRUNSON* and P. J. PERSIANI

A series of FARET critical experiments was initiated on the ZPR-III critical facility during January 1965. The main objective was to confirm the physics analysis of multi-fueled, non-uniform core loadings. A primary interest was to experimentally determine the predicted control rod reactivity worths of FARET core systems.¹ Of equal importance was the substantiation of the neutronic behavior of a mixed core, with simulated subassemblies containing different types of fuels being interchanged or replaced. Other measurements included in the program were the distributed worths of

volume fraction composition of 82/18, respectively. The configuration and composition details, along with the preliminary results, have been published in Ref. 2.

Some analytical results for the reactivity worths of control rods in the 84 liter mixed oxide core are compared with the experimental values in Table III-21-I. The reactivity effects were studied for two B-10 enrich-

TABLE III-21-I. CONTROL ROD WORTHS FOR FARET CRITICAL EXPERIMENTS WITH MIXED OXIDE LOADING

B ¹⁰ Enrichment	30%		70%	
	Inhours	Δk	Inhours	Δk
Experiment	-2544	0.051	-3897	0.077
Calculation	-2450	0.049	-3800	0.076

fuel, coolant, and structural materials, reaction rate traverses and Rossi-alpha measurements.

The three core configurations that have been studied are: a mixed carbide core having fuel/sodium/stainless steel volume fractions of about 36/39/18 respectively; a mixed oxide loading having approximately the same volume fractions; and a zoned configuration involving carbide, oxide and metal fueled assemblies. The mixed carbide loading was approximately 68 liters in volume with a core height of 61 cm. The latter two configurations had core heights of 51 cm and volumes of about 34 and 93 liters, respectively. The reflector used in all of the core loadings was a stainless steel and sodium mockup of the FARET system, having a percent

* Idaho Division.

TABLE III-21-II. SUBASSEMBLY REPLACEMENTS IN THE FARET CRITICAL EXPERIMENT

A. Oxide Core (84 liters); 503 Ih = 1%k			
Replacement (1.5 liter assembly)	Row	Reactivity change, Ih	
		Experiment	Calculations
Carbide for oxide	1	+258	+297
Carbide for oxide	3	+204	+239
Carbide for oxide	5	+110	+122
Carbide for reflector	6	+254	+242
Carbide for reflector	5/6	+348	+335
Oxide for reflector	6	+180	+166
B. Carbide Core (68 liters); 545 Ih = 1%k			
Oxide for carbide	1	-173	-156
	2	-161	-147
	3	-128	-112
	4	-81	-76
5 Oxide assemblies in Row 3	3	-618	-583

ments of the boron-carbide control rods. Agreement between the computations and experiments established the range of control that may be obtained in FARET core systems.

Referring to Table III-21-II, the experimental data or the interchange of differently fueled assemblies in the oxide and carbide core loadings can be predicted to within 10 %. In the oxide core, the row indicated by 5/6 means that the fueled assembly location was better

represented by placing the assembly partly in the 5th and 6th row of the matrix in the critical facility.

Pulsed neutron measurements made on the carbide and oxide core, were found to be in reasonable agreement with the computed Rossi-alpha (ratio of delayed neutron fraction to neutron lifetime) value of $2.0 \times 10^4 \text{ sec}^{-1}$ and $2.7 \times 10^4 \text{ sec}^{-1}$ for the two cores, respectively.

The agreement between the experimental data on the preliminary analysis thus far indicates that the be-

havior of a non-uniform core loading is understood as well as that of a uniform core.

REFERENCES

1. W. R. Simmons, P. J. Persiani and T. R. Bump, *Fast Reactor Test Facility and Initial Experimental Program*, Am. Nucl. Soc., Fast Reactor Technology National Topical Meeting, Detroit, Michigan; (April 26-28, 1965) ANS-100.
2. Argonne National Laboratory Reactor Development Program Progress Reports; ANL-7017, (March 1965); ANL-7028, (April 1965); ANL-7045, (May 1965); ANL-7046, (June 1965).

III-22. Preliminary Study of FARET Radial Shield Configurations

A. E. McARTHUR and P. J. PERSIANI

Several radial shielding configurations for FARET have been examined to study the relative heating in the shield regions and the flux changes in the concrete. The basic objective of the study was to obtain a configuration which would not require cooling of the concrete shield. A further consideration was that the level of the thermal neutron flux in the concrete be of the order of 5×10^{10} to $10^{11} \text{ n/cm}^2\text{-sec}$. This would allow startup instruments to be located in a low temperature region. Configuration IV in Table III-22-I appears to satisfy these objectives.

Detailed core gamma-ray and capture gamma-ray calculations were made using all possible sources. Capture gamma-ray sources were determined by first finding the neutron capture distribution in each region, using multigroup neutron fluxes and cross sections. Neutron fluxes were calculated by obtaining a source distribution of first collisions from a removal theory formula assuming a uniform source in the core. This source distribution then was used as an inhomogeneous source for 14 coupled diffusion groups in an RP-122⁽¹⁾ diffusion code problem to get the neutron flux distributions resulting from this source. The lower energy fluxes at the edge of the core, obtained from a diffusion solution, were used to fix the values of the fluxes in the 14 diffusion groups at the inner boundary. Four of the configurations studied are listed in Table III-22-I.

Gamma-ray and thermal neutron flux distributions through the outer regions of configurations III and IV are shown in Figs. III-22-1, III-22-2, and III-22-3. The fluxes correspond to a total power of 50 MW (t).

Boron (n, α) heating was compared to the total gamma-ray radiation heating in the borated graphite

regions. It was found that the (n, α) heating at the inner face of the borated graphite regions was about 5 times the heating from gamma-ray absorption.

Placing boral next to the concrete instead of in front of the steel liner was much more effective in reducing the capture gamma-ray sources in the steel; these sources were found to be a major contributor to the

TABLE III-22-I. FARET RADIAL SHIELD CONFIGURATIONS

Region	Region Outer Radius, cm			
	Configuration			
	I	II	III	IV
Core	76.0	76.0	76.0	76.0
Reflector	83.9	83.9	83.9	83.9
Sodium	153.8	130.9	130.9	130.9
Borated graphite	none	153.8	153.8	153.8
Vessel	167.8	167.8	167.8	167.8
Insulation	228.7	198.3	198.3	198.3
Borated graphite	213.5	none	221.1	none
Steel	231.3	199.5	222.4	199.5
Boral	none	none	none	200.8
Ord. concrete (10 feet)	536.1	504.3	527.2	505.6

concrete heating. The boral prevents a large fraction of the low energy neutrons produced in the concrete from reaching the steel.

The current design concept is to place the borated graphite section in the reactor vessel adjacent to the core support barrel. This allows the heat generated to be removed by the sodium coolant and furthermore, the graphite region is more accessible.

REFERENCE

M. K. Butler and J. M. Cook, *Univac Programs for the Solution of One-Dimensional Multigroup Reactor Equations*, ANL-5437, (1956).

FIG. III-22-1. Gamma-Ray Flux in Outer Regions of Configurations III and IV. ANL Neg. No. 112-5147.

FIG. III-22-2. Thermal Neutron Flux Configuration IV. (Regions outside insulation). ANL Neg. No. 112-5146.

FIG. III-22-3. Thermal Neutron Flux Configuration III. (Regions outside insulation). ANL Neg. No. 112-5145.

III-23. Note on Rossi-Alpha in Reflected Fast Reactors

R. A. KARAM

Rossi-alpha is defined as $\alpha = \beta_{eff}/\ell$ for a delayed critical system, where β_{eff} is the effective delayed neutron fraction and ℓ is the effective lifetime of the prompt neutrons. The measured values of α in reflected fast reactors have been about 30% lower than the calculated values.¹⁻⁴ Better agreement between measurements and calculations have been reported by G. Hansen⁵ on small bare fast reactors. In most cases, the discrepancies between calculations and measurements on the small bare cores were about 10% according to Hansen. Errors in β_{eff} have been estimated by G. Brunson, et al² to be about 5%. It should be pointed out that the calculation of β_{eff} is not sensitive to the cross section set that one uses in multigroup diffusion theory calculation, and 5% uncertainty in β_{eff} is perhaps an upper limit. The uncertainty in the measurement of α is perhaps another 5%, although it can be lower with good statistics. By using the theorem of propagation of errors, one finds the inherent uncertainty in ℓ to be less than 10%. This uncertainty approaches the discrepancies found between measurements and calculations for the small bare reactors; however, for reflected reactors, the discrepancies are outside any statistics associated with either α or β_{eff} . This note discusses the incompatibility of the measurement and the calculation.

Consider the eigenvalue problem

$$\frac{\partial \Phi}{\partial t} = H\Phi, \quad (1)$$

where Φ is the flux of neutrons and H is the appropriate operator. It should be noted that either the diffusion equation or the time-dependent Boltzmann equation can be represented by Eq. (1).

Now let

$$\Phi = \sum_i a_i \phi_i \quad (2)$$

where each ϕ_i is a well behaved eigenfunction. Similarly define an adjoint function

$$\Phi^* = \sum_i a_i^* \phi_i^*. \quad (3)$$

Require that these functions be orthonormal; i.e.,

$$\int_V \Phi^* \Phi dV = 1, \quad (4)$$

where V is the core volume. Operating on Φ by H ,

multiplying by Φ^* and integrating over all space,

$$\int_V \Phi^* H \Phi dV = \int_V \sum_i a_i^* \phi_i^* H \sum_i a_i \phi_i dV \quad (5)$$

or

$$\bar{W} = \sum_i a_i^* a_i W_i, \quad (6)$$

where $\bar{W}$ is the eigenvalue of the system and W_i is the eigenvalue of ϕ_i .

From the condition set in Eq. (4), another equation emerges,

$$\sum_i a_i^* a_i = 1. \quad (7)$$

Equation (6) shows that if there is more than one eigenstate in the system, then the eigenvalue of the whole system is an averaged sum of all the eigenvalues. Alpha is the slope of the asymptotic decay of the prompt neutrons and, therefore, it is equal to the smallest W_i . In the calculation of alpha, the following steps are usually carried out.

1. The steady state condition of Eq. (1); i.e., $H\Phi = 0$, is solved for the fluxes and adjoints usually without the delayed neutrons.

2. The fluxes and adjoints are used in the perturbation approximation to calculate the effective lifetime of the neutron, ℓ . An alternate method for calculating ℓ is to arbitrarily allow the neutrons in a reactor to grow or decay with some constant λ and calculate k_{eff} . Then ℓ is equated to $(1 - k_{eff})/\lambda$. This alternate method is available with transport codes. A third method is based on the use of $1/v$ poisoning. In this method, a fictitious material with a macroscopic capture cross section of c/v is inserted uniformly throughout the reactor and k_{eff} is calculated. (With this technique, ℓ is obtained through the relation $\ell = -\delta k/c$ where δk is the difference in the k_{eff} 's of the system with and without the $1/v$ poison.)

3. Based on the results of Step 1, β_{eff} is calculated by appropriate weighting of all the delayed neutrons.

4. The calculated alpha is equated to $-\beta_{eff}/\ell$ when the system is just critical.

In all of these methods for calculating ℓ , neutrons in all modes are taken into consideration. It is obvious that the measurements that have been reported are based on the asymptotic mode; hence, the incompatibility.

Another physically unrealizable situation is that

When Eq. (1) is solved in the steady condition without incorporation of the delayed neutrons, the eigenvalue of such a system is zero; i.e., $\bar{W}$ of Eq. (6) is zero. This is possible only if all the W_i of Eq. (6) are zero. Since there are delayed neutrons, the W_i 's in Eq. (6) for the prompt-neutron chains cannot be all zeros. The error in ℓ caused by the omission of the delayed neutrons is estimated to be less than 2%.

It is possible that in certain reactors the asymptotic mode predominates. In such cases there should be better agreement between measurement and calculation of alpha. Bare reactors may fall into such a category. Experimentally, reactors with light reflectors exhibit fast decaying modes besides the asymptotic mode (see Paper V-10). There is preliminary evidence (see Paper V-10) that the agreement between the measured and calculated α improves considerably when all of these modes are averaged in the same manner in which the delayed neutron fractions are averaged into one group. The study of the two-region reactor kinetics model^{3,6} showed that the decay rates of the fast modes depend on the worth of the reflector. The decay rates of the fast modes increase with an increase in the worth of the reflector. This conclusion is substantiated when the findings of Brunson, et al.² on Assembly No. 40 of ZPR 3, where the reflector worth was about 5% $\Delta k/k$, are compared with the results reported in Paper V-10 on Assembly No. 6 of ZPR 9 where the reflector worth was 12% $\Delta k/k$. The degree to which the fast decaying modes influence the calculated α depends on the system. A general method for making the calculation of α compatible with measurement, regardless of the system, is the solution of the time-dependent diffusion or transport equations when the reactor is made \$1.00

subcritical by decreasing the fission neutron distribution by β_{eff} ; i.e., $\sum_i \chi_i = 1 - \beta_{eff}$. The slope of the asymptotic decay of the prompt neutrons is the alpha. With the recent advent of time-dependent codes,^{7,8} this procedure is now possible. With this method a meaningful comparison of the measured and calculated alpha can be made. Furthermore, this method can provide a valuable check on the accuracy of cross section sets and calculational techniques.

REFERENCES

1. W. G. Davey, *An Analysis of 23 ZPR 3 Fast Reactor Critical Experiments*, Nucl. Sci. Eng. **19**, 259-273 (1964).
2. G. S. Brunson, R. N. Curran, J. M. Gasidlo and R. J. Huber, *A Survey of Prompt-Neutron Lifetimes in Fast Critical Systems*, ANL-6681 (1963).
3. R. C. Doerner, K. K. Almenas, R. A. Karam, W. Y. Kato and W. B. Loewenstein, *Physics Measurements in Tungsten-Based Aluminum-Reflected Fast Reactors*, Reactor Physics Annual Report, July 1, 1963 to June 30, 1964, ANL-7010, (1964) pp. 116-123.
4. W. G. Knapp, R. C. Doerner, K. K. Almenas, C. E. Cohn and R. A. Karam, *Physics Measurements of Modified Tungsten-Based Aluminum-Reflected Fast Reactors*, Trans. Am. Nucl. Soc. **8**, No. 1, 241 (1965).
5. G. E. Hansen, *Properties of Elementary Fast-Neutron Critical Assemblies*, Proc. 1958 Geneva Conference, **12**, Paper No. 592, 84-88 (1958).
6. G. Kistner, *Rossi-Alpha Theory for Assemblies with Two Prompt-Neutron Groups*, Nukleonik **7**, No. 2, 106-109 (1965).
7. J. B. Yasinsky and A. F. Henry, *Some Numerical Experiments Concerning Space-Time Reactor Kinetics*, Nucl. Sci. Eng. **22**, 171-181 (1965).
8. A. Brickstock, A. R. Davies and I. C. Smith, *A Time-Dependent Multigroup Diffusion Code with Automatic Resonance Self-Shielding of Cross Sections*, Intern. Conf. on the Application of Computing Methods to Reactor Problems, ANL-7050 (to be published).

III-24. Neutronics Calculations for a 1,000 MWe Plutonium-Uranium Metal-Fueled, Fast Breeder Reactor Design Study

D. K. BUTLER, W. B. LOEWENSTEIN, D. MENEGHETTI and K. E. PHILLIPS

INTRODUCTION

Physics calculations have been carried out in support of a design study for a 1,000 MWe plutonium-uranium, metal-fueled fast breeder reactor.¹ The reactor design concept is a ring of six core modules separated and surrounded by a fertile blanket. Each module has a core volume of about 800 liters which is loosely coupled to the neighboring core modules. A simplified schematic of the system is shown in Fig. III-24-1.

The modular design reduces the positive Na-coefficient of reactivity relative to that of configurations which would have greater breeding ratios. To improve the negative Doppler coefficient of reactivity and to flatten the radial power distribution, a beryllium region has been placed between each core module and its surrounding radial blanket. The design also uses two core zones having different fuel alloy enrichments to further flatten the radial power distribution.

FIG. III-24-1. Simplified Schematic of Horizontal Section of Reactor at Horizontal Midplane of Core Modules. *ANL Neg. No. 112-5746*.

TABLE III-24-I. SINGLE FUEL ENRICHMENT MODULE (DIMENSIONS FOR ANALYSIS)

Core	840 liters, 92 cm height, 53.9 cm radius
Axial blanket	33 cm thick
Axial reflector	55 cm thick
Radial Be annulus	5.3 cm thick
Radial blankets { Reg. 1	6.1 cm thick
Reg. 2	Range of thicknesses: 17 to 38 cm
Radial reflector	37 cm thick

The inner and outer peripheries of the fertile blankets of the composite ring of the six modules are surrounded by beryllium reflector regions. External to these are steel inner and outer radial reflectors. The external beryllium regions reduce fertile inventory and improve blanket breeding.

The neutronic breeding penalty caused by an assumed presence of fission in the metallic fuel alloy has been estimated. The improvements in breeding and Na-void reactivity which accompany the use of a neutronically inert fuel alloy additive, such as titanium or zirconium rather than fission, have been calculated. The reference design uses the neutronically inert fuel alloy additive.

Analyses pertaining to fuel cycle reactivity and overall system kinetics were also carried out in support of the reactor design effort.

COMPOSITIONS AND DIMENSIONS OF SINGLE FUEL ENRICHMENT MODULE

The regional compositions and dimensions in the neutronics analyses are those of a single module core and its associated blankets and reflectors. The effective radial blanket thickness of a module is dependent upon the radial direction. For example, in a direction between modules the neighboring core modules share radial blanket material and no radial reflectors are used. In a radial direction perpendicular to the ring of the six core modules, the radial blanket thicknesses differ and there are also peripheral reflectors. In intermediate radial directions, the effective blanket thicknesses are again different.

These preliminary studies have been directed primarily toward understanding the neutronics of a given 840 liter core module with a range of radial blanket thickness. Outer radial boundary conditions of zero current with no radial reflector and zero flux with radial reflector have been studied.

The dimensions of the single fuel zone core module, associated blanket, and reflectors used in these analyses are given in Table III-24-I.

The core is constituted as follows: 45 v/o fuel element, 40 v/o sodium, 9 v/o steel structure, and 6 v/o control mechanism. The fuel element composition is 30 v/o Ti-V (20 w/o Ti, 80 w/o V) clad and 70 v/o (U + Pu + 4 w/o fission + fission products). Using

heavy atom % burnup in the core as a basis, the fission product pairs in the analyses represent an average of 2.75% heavy atom burnup, i.e. 2.75% of the (U + Pu) atoms are fission product pairs. The "expanded" fuel density (U + Pu + Fz. + F.P.) has been assumed to be 12 g/cc. This is consistent with providing ~30% void for irradiation growth of fresh fuel. The atomic density ratios of Pu isotopes to total Pu atomic density have been assumed to be: 0.751, 0.210, 0.0295, and 0.0085 for Pu-239, Pu-240, Pu-241, and Pu-242,

TABLE III-24-II. Be-REGIONS AND BLANKET THICKNESS EFFECTS (ISOLATED, REFLECTED MODULES)

Be Region	Reactor Parameter	Radial Blanket Thickness	
		29.1 cm	44.1 cm
No Be regions	Crit.	0.190	0.190
	Conc. ^a		
	C.C.R. ^b	0.60	0.60
	T.B.R. ^c	1.46 ^d	1.58 ^e
	Na ^f	-0.06%	-0.07%
		$\Delta k/k$	$\Delta k/k$
Be at core-blanket interface	Dopp. ^g	-0.16	-0.16 (-0.11)
	Crit.	0.189	0.189
	Conc. ^a		
	C.C.R. ^b	0.63	0.63
	T.B.R. ^c	1.40 ^h	1.43 ^h
	Na ^f	-0.17%	-0.17%
Be at core-blanket and blanket-reflector interfaces		$\Delta k/k$	$\Delta k/k$
	Dopp. ^g	-0.36 (-0.33)	-0.35 (-0.32)
	Crit.	0.189	
	Conc. ^a		
	C.C.R. ^b	0.63	
	T.B.R. ^c	1.42 ^h	
	Na ^f	-0.18%	
		$\Delta k/k$	
	Dopp. ^g	-0.35 (-0.32)	

$$^a \text{ Crit. Conc.} = \frac{(\text{total Pu})_{\text{in core}}}{(\text{total Pu} + \text{U-238})_{\text{in core}}}$$

$$^b \text{ C.C.R.} = \frac{(\text{captures in U-238, Pu-240 and Pu-242})_{\text{in core}}}{(\text{absorptions in Pu-239 and Pu-241})_{\text{in core}} + (\text{captures in U-238, Pu-240 and Pu-242})_{\text{in core and blanket.}}}$$

$$^c \text{ T.B.R.} = \frac{\text{Pu-242}_{\text{in core and blanket.}}}{(\text{absorptions in Pu-239 and Pu-241})_{\text{in core and blanket.}}}$$

^d Includes 0.33 axial contributions.

^e Includes 0.38 axial contributions.

^f Reactivity resulting from a 40% reduction of core sodium.

^g Based on reactivity resulting from U-238 Doppler effect in core for temperature change 750°K to 1500°K with constant Na composition and with 40% less Na in core. The Δk 's have been converted to $(T \partial k / \partial T)_{\text{Doppler}} \times 10^2$, assuming this quantity independent of temperature.

^h Includes 0.31 axial contributions.

respectively. The 6 v/o of control material was replaced by 4 v/o sodium and 2 v/o steel for most of the analyses. This corresponds to no control poison in the core.

The axial blanket densities correspond to those in the core; 45 v/o fertile material and clad, 40 v/o sodium coolant, 9 v/o steel structural material, and 6 v/o for control mechanisms. The latter consists of 4 v/o sodium and 2 v/o steel. The 45 v/o breeder element consists of about 70 v/o U-238 and 30 v/o of the titanium-vanadium cladding alloy. One w/o of the uranium fuel is assumed to be Pu-239 which represents the average fissile concentration at core midlife. The expanded fuel density, i.e. (U-238 + Pu-239), is 15 g/cc in the breeding blankets.

The axial reflector composition above and below the axial blankets is 60 v/o sodium, 26 v/o steel, and 14 v/o titanium-vanadium cladding alloy.

The radial beryllium zone surrounding the module consists of 5 v/o sodium, 12 v/o steel, and 83 v/o beryllium.

The inner radial blanket contains 47 v/o fertile material and clad, 10 v/o steel structure, and 43 v/o sodium. The 47 v/o breeder element consists of about 70 v/o U-238 and 30 v/o titanium-vanadium cladding alloy. 2.4 weight percent of the uranium fuel is Pu-239; this is the average fissile concentration at midlife.

The outer radial blanket region contains 68 v/o fertile material and clad, 7 v/o steel structure, and 25 v/o sodium. The 68 v/o breeder element consists of about 74 v/o U-238 and 26 v/o titanium-vanadium alloy. The average fissile concentration at midlife is about 1 w/o Pu-239 in the uranium fuel.

The steel reflector contains 95 v/o steel and 5 v/o sodium. The outer beryllium region composition is the same as that of the core-blanket Be interface region. All compositions and dimensions correspond to the system design parameters.¹ The corresponding considerations for the two-fuel enrichment zone module are listed in a subsequent section. The calculations are all based on multigroup diffusion theory. A modification of the Argonne 26-group cross section set² was used. Calculated Na-void reactivities include consideration of altered axial reflector savings upon core Na removal.

RADIAL BERYLLIUM REGION AND RADIAL BLANKET THICKNESS STUDIES

The effects of the radial beryllium regions at the core-blanket and/or blanket-reflector interfaces upon critical composition, breeding ratio, core conversion ratio, sodium void reactivity and Doppler reactivity of isolated modules are summarized in Table III-24-II.

The beryllium region at the blanket-reflector interface had the same composition as that of the core-

blanket interface region but it was 15 cm thick rather than 5.3 cm.

The beryllium at the core blanket interface was found to augment the negative core Doppler reactivity by more than a factor of two over that of the system without such a thin light element annulus. Calculations show that the beryllium annulus also tends to augment the negative reactivity due to uniform voiding of core sodium.

The addition of an outer beryllium region around the thinner (~ 29 cm) radial blanket results in a breeding ratio of 1.42 which may be compared with 1.43 corresponding to a thicker blanket (~ 44 cm) surrounded by a non-moderating reflector.

The beryllium annulus at the radial core blanket interface provides some radial power flattening. This is evident in Fig. III-24-2 where the radial source distributions with and without the beryllium annulus are compared. The beryllium annulus causes preferential degradation of the neutron spectrum near the radial core boundary. This is advantageous in that centrally located Na voiding of the core does not diminish the peripherally-weighted Doppler effect.

TITANIUM-ALLOY FUEL SUBSTITUTED FOR FIZZIUM-ALLOY FUEL

The neutronics of the fizzium core alloy have been compared with the neutronics of a titanium core alloy. An equal number of titanium atoms were substituted for the fizzium atoms. The calculations are summarized in Table III-24-III. The analyses utilized cylindrical geometry with an axial extrapolated bare core. The systems have a radial blanket of about 29 cm thickness, surrounded by a 15 cm Be annulus which, in turn, is surrounded by a 37 cm reflector.

FIG. III-24-2. Radial Source Distribution (Effect of Be Annulus). ANL Neg. No. 112-5716.

TABLE III-24-III. COMPARISON OF TITANIUM AND FIZZIUM ALLOYED FUELS

Be Region	Reactor Parameter	With Titanium	With Fizzium
No Be at core-blanket interface; Be at blanket-reflector interface	Crit. Conc. ^a	0.182	0.190
	C.C.R. ^b	0.64	0.60
	T.B.R. ^c	1.62 ^d	1.56 ^d
	Na ^e	-0.28% $\Delta k/k$	-0.08% $\Delta k/k$
	Dopp. ^f	-0.23 (-0.17)	-0.16 (-0.12)
Be at core blanket interface and at blanket-reflector interface	Crit. Conc. ^a	0.179	0.189
	C.C.R. ^b	0.68	0.63
	T.B.R. ^c	1.49 ^e	1.42 ^e
	Na ^e	-0.32% $\Delta k/k$	-0.18% $\Delta k/k$
	Dopp. ^f	-0.52 (-0.47)	-0.35 (-0.32)

$$^a \text{ Crit. Conc.} = \frac{(\text{total Pu})_{\text{in core}}}{(\text{total Pu} + \text{U-238})_{\text{in core}}}$$

$$^b \text{ C.C.R.} = \frac{(\text{captures in U-238, Pu-240 and Pu-242})_{\text{in core}}}{(\text{absorptions in Pu-239 and in Pu-241})_{\text{in core}}}$$

$$^c \text{ T.B.R.} = \frac{(\text{captures in U-238, Pu-240 and Pu-242})_{\text{in core and blanket}}}{(\text{absorptions in Pu-239 and in Pu-241})_{\text{in core and blanket}}}$$

^d Includes 0.38 axial contributions.

^e Reactivity resulting from a 40% reduction of core sodium.

^f Based on reactivity resulting from U-238 Doppler effect in core for temperature change 750°K to 1500°K with constant Na composition and with 40% less Na in core. The Δk have been converted to $T(\partial k/\partial T)_{\text{Doppler}} \times 10^2$ assuming this quantity independent of temperature.

^g Includes 0.31 axial contributions.

The use of titanium rather than fizzium alloy enhances the gain of breeding by about +0.06.

Comparison of the Na-void reactivity calculations given in Tables III-24-II and III-24-III shows that the replacement of fizzium by titanium gives a substantial improvement in the magnitude of the overall negative Na void effect. It is also noted, however, that the core-blanket interface Be annulus does not affect the Na reactivity as significantly with the use of titanium alloy fuel as with the fizzium alloy fuel.

TWO FUEL ENRICHMENT ZONED CORE

Two fuel enrichment zoned core modules together with a Be annulus between the core and the radial blanket have been considered to improve radial power flattening. In the zoned core, the module dimensions and compositions differ from those listed in Table III-24-I. These dimensions are given in Table III-24-IV.

e overall core volume is about 784 liters. The analyses were based on one-dimensional cylindrical geometry with zero flux outer boundary conditions. The same axial reflector savings and variation of axial reflector savings used in the previous single fuel enrichment cores with Ti fuel alloy were used.

The atom densities for the inner and outer core zones correspond to 45.79 and 49.69 v/o fuel element, 39.42 and 42.40 Na, and 8.80 and 7.91 steel structure, respectively. The inner core zone also contains 6 v/o control which consists of 58 v/o Na, 35 v/o steel, and about 7 v/o $B_4^{nat}C$ to represent control present during operation. The fuel element composition is 31.3 v/o TiV (80 w/o V and 20 w/o Ti) clad and 68.7 v/o of (U + Pu + Ti) alloy plus fission product pairs. It was assumed that the fuel could remain in the reactor until about 6% of the heavy atoms are fissioned. Therefore, the analyses utilized fission product pairs consistent with a time average value of 3% in the burnup cycle;

TABLE III-24-IV. TWO FUEL ENRICHMENT ZONE-CORE MODULE (DIMENSIONS FOR ANALYSIS)

Core (Reg. 1)	511.2 liters, 92 cm height, 42.2 cm radius
Core (Reg. 2)	272.4 liters, 92 cm height, 10.1 cm thick
Axial blanket	33 cm thick
Axial reflector	55 cm thick
Inner radial Be region	4.2 cm thick
Radial blankets	Reg. 1 9.7 cm thick
	Reg. 2 24.0 cm thick
Outer Be region	12.0 cm thick
Radial reflector	36.0 cm thick

i.e. 3% of the U + Pu atoms are fission product pairs. Expanded (U + Pu + Ti + F.P.) density has been retained at 12 g/cc. In these analyses the fuel alloy has contained about 8.9 a/o Ti.

One-dimensional cylindrical calculations for the two-zoned core with and without the core blanket Be annulus are given in Table III-24-V for three outer enrichment ratios, $[Pu/(Pu + U)]_{\text{core reg. 2}} = 0.20, 0.21,$ and 0.22. Criticality was obtained by variation of $[Pu/(Pu + U)]_{\text{core reg. 1}}$.

Comparison of these two-zoned results with those of the analogous one-zoned Ti alloy fueled cases in Table III-24-III shows that the breeding ratios are effectively unchanged even though B_4C is in the central zone for the analyses summarized in Table III-24-V. Doppler effects are not significantly altered by using the two-zone enrichment. The somewhat lesser Doppler reactivity in the two-zoned case with a Be annulus is largely due to the smaller effective thickness of the annulus in these analyses. Zoned loading does, however, significantly alter negative Na void reactivity of the single enrichment core. This becomes more evident with the increase in outer core enrichment. The overall effect using a two fuel-zone core loading with a moderate size Be annulus does, however, considerably flatten the radial power distribution. The radial source distribution for one of the two fuel-zone cases with the Be annulus is shown in Fig. III-24-3.

EQUILIBRIUM FUEL CYCLE AND REACTIVITY LOSS FROM BURNUP

Two important properties of the system associated with long term operation are: the equilibrium fuel

TABLE III-24-V. TWO FUEL ENRICHMENT ZONE CORE MODULES (CYLINDRICAL CALCULATIONS)

Be Region	Atomic Concentrations				C.C.R.	T.B.R.	Na Re-activity, $\% \Delta k/k$	Doppler ^b		Source Peak-to-Average at Cylindrical Midplane
	$\left(\frac{\text{Pu}}{\text{Pu} + \text{U}}\right)_{\text{core}}^{\text{a}}$		$\left(\frac{\text{Fissile Pu}}{\text{Pu} + \text{U}}\right)_{\text{core}}$					Na in	Na out ^e	
	Reg. 1	Reg. 2	Reg. 1	Reg. 2						
No Be at core-blanket interface; but Be at blanket-reflector in- terface.	0.178	0.200	0.139	0.156	0.63	1.63 ^d	+0.02	-0.21	-0.16	1.35
	0.176	0.210	0.137	0.164	0.63	1.63 ^d	+0.06	-0.21	-0.16	1.32
	0.173	0.220	0.135	0.172	0.62	1.63 ^d	+0.10	-0.21	-0.16	1.29
Be at core-blanket interface; and Be at blanket-reflector in- terface.	0.176	0.200	0.137	0.156	0.66	1.50 ^e	-0.05	-0.41	-0.36	1.25
	0.173	0.210	0.135	0.164	0.65	1.50 ^e	0.00	-0.41	-0.36	1.20
	0.170	0.220	0.132	0.172	0.65	1.50 ^e	+0.04	-0.41	-0.36	1.16

$$^a \text{ Crit. Conc.} = \frac{(\text{total Pu})_{\text{in core}}}{(\text{total Pu} + \text{U-238})_{\text{in core}}}$$

$$^b 10^2 \left(T \frac{\partial k}{\partial T} \right)_{\text{Doppler}}$$

^c For 40% less Na in core.

^d Includes 0.38 axial contribution.

^e Includes 0.31 axial contribution.

FIG. III-24-3. Radial Source Distribution. (Two fuel zones).
ANL Neg. No. 112-5723.

composition and the change in reactivity between refuelings. These were studied using the CYCLE code.³ CYCLE is a one-dimensional code that calculates the equilibrium fuel composition for a specified burnup and refueling scheme. It also calculates the isotopic changes and changes in reactivity which must be made to maintain criticality over the cycle. Because a single dimension is assumed in cylindrical geometry, the code cannot account for changes in the axial reflector savings that would occur in the actual system. On the other hand, if spherical geometry is used, the code assumes a uniform blanket. It is therefore necessary to estimate the characteristics of the actual system on the basis of a series of problems with appropriately varied compositions and geometries.

In calculating the equilibrium composition an additional restriction requires that the core loading be uniform. Therefore, the properties of the reference design with the two fuel enrichment zones were estimated by assuming a single enrichment with the same volume fraction of fuel, clad, structure, and coolant as the two-zoned system. It was also necessary to use a "single batch" scheme where the whole core is reprocessed at one time. To approximate the actual processing scheme it was assumed that $\frac{1}{3}$ of the fission

products and 2% of the heavy atoms corresponding to reprocessing losses were removed from the core fuel. The remainder was returned, combined with sufficient fresh U-238, Pu-239, and boron to restore the core to its initial uniform composition. The code adjusts the feed enrichment so that the system is critical at mid-cycle, but allows the reactivity to vary during the cycle as the composition changes with burnup.

The equilibrium composition was obtained in spherical geometry with both the axial and radial blankets of the reference design. The equilibrium atom densities obtained were essentially those used in the reference design.

The code also computes the change in isotopic composition of the blanket over the cycle. These were found to be in satisfactory agreement with those estimated for the reference design.

The reactivity change due to burnup during a cycle is determined in a separate calculation. A desired burnup, the fraction of fissions of the heavy atoms, during the cycle is specified. The burnup consistent with the refueling scheme is 2% in the core. The reactivity change for the actual two-dimensional cylinder was estimated from a series of one-dimensional calculations. The reactivity change for the reference systems with two fuel enrichment zones in cylindrical geometry was determined to be $-2.6\% \Delta k/k$. In the calculation, it was assumed that the axial reflector saving was constant throughout the cycle. A correction for axial effects was estimated by two subsidiary calculations. First, the reactivity change for a single enrichment zone core was calculated to be $-2.8\% \Delta k/k$. The reactivity changes less with burnup in the two-zone core than in the single zone core. The equivalent single zone core was next used in spherical geometry with the radial blanket all around. The reactivity change was $-2.5\% \Delta k/k$.

The burnup calculations support the original assumptions on which the reference design was based both as to equilibrium composition and the amount of reactivity which will be lost by burnup during each cycle. The latter quantity also supports the estimate of the average amount of boron which must be provided to compensate for the excess reactivity that must be provided at the beginning of a burnup cycle.

REACTIVITY COEFFICIENTS AND DYNAMIC BEHAVIOR OF A MODULE

The dynamic behavior of a single module of the 1000 MW system has been explored for two kinds of transient conditions important to reactor safety. These transient conditions are ramp insertions of reactivity and loss of coolant flow. Calculations were performed using a modified version (RP351X) of the FORE⁴ pro-

In all these studies it was assumed that the modules are relatively uncoupled. Detailed study of the interaction of a malfunction in one module and its influence upon a neighboring module is a problem requiring additional study.

TABLE III-24-VI. SINGLE MODULE DYNAMIC BEHAVIOR

No.	Type of Problem	$\left(\frac{dk}{dT}\right)_{\text{expansion}}$	$\left(T\frac{dk}{dT}\right)_{\text{Dopp.}}$	Fuel Melts, sec		Coolant Boils, sec		Feedback Reactivity, ^a \$
				Peak channel	Average channel	Peak channel	Average channel	
1	Ramp insertion \$2/sec; large Doppler coef.		-0.0047	0.44	0.52	>2.0 ^b	>2.0 ^b	-0.18
2	Ramp insertion \$2/sec; small Doppler coef.		-0.0012	0.36	0.45	>0.57 ^b	>0.57 ^b	-0.04
3	Ramp insertion \$2/sec; small Doppler coef. and axial fuel expansion	$-8.1 \times 10^{-6}/^{\circ}\text{F}$	-0.0012	0.48	0.74	1.46	1.56	-0.40
4	Ramp insertion \$.20/sec; small Doppler coef.		-0.0012	2.18	2.39	2.49	2.57	-0.10
5	Loss of coolant flow ^c ; large Doppler coef.		-0.0047	6.25	7.40	7.60	8.16	-0.17
6	Loss of coolant flow; large Doppler coef. and radial expansion	$-3.7 \times 10^{-6}/^{\circ}\text{F}$	-0.0047	9.20	9.70	>10.0 ^b	>10.0 ^b	-0.58
7	Loss of coolant flow; small Doppler coef. and radial expansion	$-3.7 \times 10^{-6}/^{\circ}\text{F}$	-0.0012	8.30	8.80	9.47	9.47	-0.42

^c Flow decays exponentially from 22 ft/sec for 13.2 sec to a final velocity of 0.8 ft/sec.

REFERENCES

1. L. E. Link, J. E. Ayer, D. K. Butler, K. A. Hub, W. B. Loewenstein, W. J. Mechem, D. Meneghetti, D. H. Thompson, V. G. Trice Jr. and J. K. Weills, *1000 MWe Metal Fueled Fast Breeder Reactor*, ANL-7001 (to be published).
2. D. M. O'Shea, H. H. Hummel, W. B. Loewenstein, D.

- Okrent and B. J. Toppel, *Twenty-Six Group Cross Sections*, ANL-6858 (to be published).
3. B. J. Toppel, R. Avery and G. J. Fischer, *CYCLE and COST, Codes for Fast Reactor Fuel Cycle Analysis and Related Cost Evaluation*, Trans. Am. Nucl. Soc. 5, No. 1, 92 (1962).
4. P. Greebler, D. B. Sherer and N. H. Walton, *FORE, A Computational Program for the Analysis of Fast Reactor Excursions*, GEAP-4090 (October 1962).

III-25. Sodium Void Reactivity Effects Associated with Evaluation of Four 1000 MWe Reactor Design Studies: Effect of Beryllium Oxide Addition

H. H. HUMMEL and K. E. PHILLIPS

A series of calculations of sodium void effect was made (Table III-25-I) with reactor geometry and composition similar to that in the General Electric 1000 MWe Fast Reactor Design Study,¹ and corresponding to cases A and B described in that study. Cores were 12 ft in diameter, with the axial blanket 1.25 ft thick. The calculations were made in one-dimensional finite slab geometry. Cores were of uniform composition, while the blanket in all cases had the case A composition of 33 v/o UO₂, 17 v/o steel, and 50 v/o Na.

The purpose of this series of calculations was to study the effect of the introduction of BeO into the core on the sodium void effect. This was done first by replacing

fuel with BeO, in case 2, and also by displacing sodium with BeO, as in case 5. The effect of simply reducing the amount of fuel without inserting any BeO was studied in case 3, while the effect of filling the resulting void with Na was found in case 4.

Cross sections used in these calculations were taken from Set 224,⁽²⁾ as given in Table III-25-II. Material 16 was used for U-238 and Material 2 for Pu-239; even though the latter was evaluated at 300°K it was the only Pu-239 cross section available in the library at the time the calculations were done. It has subsequently been found that because of an improper interpolation procedure, fission cross sections for this material in the energy range 1.0 to 2.8 KeV are probably much too

TABLE III-25-I. EFFECT IN % Δk FOR UNIFORM REMOVAL OF 50% OF SODIUM FROM CORE

Case	Core Composition	Fertile Fissile	Na	Axial Leakage	Radial Leakage	Elastic Transfer	Inelastic Transfer	Capture	Total	Total Full-Half by k Calculation
1	33 v/o Fuel 17 v/o Steel, 50 v/o Na (GEA)	6.69	Full	-0.67	-0.24	0.86	0.58	0.14	0.67	0.68
			Half	-0.59	-0.24	0.95	0.65	0.14	0.91	
			No	-0.50	-0.25	0.97	0.74	0.16	1.12	
2	25 v/o Fuel, 8 v/o BeO, 17 v/o Steel, 50 v/o Na (GEB)	5.29	Full	-0.64	-0.20	0.41	0.49	0.19	0.25	0.24
			Half	-0.57	-0.21	0.43	0.56	0.21	0.42	
			No	-0.49	-0.22	0.26	0.64	0.28	0.47	
3	25 v/o Fuel, 17 v/o Steel, 50 v/o Na, 8 v/o Void	5.24	Full	-0.74	-0.28	0.61	0.51	0.17	0.27	0.27
			Half	-0.64	-0.29	0.69	0.58	0.17	0.51	
			No	-0.52	-0.30	0.66	0.67	0.21	0.72	
4	25 v/o Fuel, 15 v/o Steel, 60 v/o Na	5.35	Full	-0.93	-0.41	0.81	0.65	0.19	0.31	0.28
			Half	-0.75	-0.43	0.93	0.76	0.18	0.69	
			No	-0.55	-0.45	0.94	0.92	0.21	1.07	
5	33 v/o Fuel, 8 v/o BeO, 17 v/o Steel, 42 v/o Na	6.57	Full	-0.47	-0.13	0.46	0.46	0.14	0.46	0.47
			Half	-0.43	-0.13	0.48	0.50	0.15	0.57	
			No	-0.40	-0.13	0.36	0.55	0.20	0.58	

TABLE III-25-II. MATERIALS FROM SET 224
USED IN CALCULATIONS
(Ref. 2)

Material No. (REX)	Isotope	Temp., °K	σ_p , barns	Comments
2	Pu-239	300	300	See text
4	Pu-241	—	—	
8	Fission product pairs for Pu-239			
16	U-238	1500	40	BNL-325 parameters: $\frac{\Gamma_n^0}{D} = 1 \times 10^{14}$
27	Pu-240	1500	600	
43	Na	These materials had an approximate ELMOE correction.	40	Firk, Lynn, Moxon Parameters Contained U-238-Pu- 239 overlap correc- tion
44	O			
46	Fe			
47	Ni			
48	Cr			
61	U-238	1500	40	
63	Pu-239	1500	200	
65	Pu-239	1500	300	

high. It is believed, subject to further checking now in progress, that this may have resulted in sodium void effects for uniform removal of 50% of core sodium of the order of 0.5% Δk too negative for this series of calculations which should be accepted on a relative basis only.

In this series of calculations materials 43–48 were used throughout for the listed elements. They had an approximate ELMOE³ correction obtained for a representative composition. It is not believed that the use of these approximate cross sections instead of exact ELMOE results would make a significant difference.

The results given in Table III-25-I are the reactivity changes corresponding to uniform removal of 50% of the sodium from the core. The entries for “full,” “half,” and “no” are results of perturbation calculations using the Del code⁴ based on fluxes and adjoints for full, half, and no sodium present. There is seen to be some non-linearity in the individual components, particularly at a high ratio of sodium to fuel, which causes an even stronger non-linearity in the total reactivity change. Given for comparison is the total reactivity change from full to half sodium by direct k calculation. It would be expected that this would approximately equal an average of the “full” and “half” sodium values. The fact that it is about equal to the “full” values points to some inconsistency.

The radial leakage term was obtained by an equivalent bare radius calculation. The magnitude of this term is reduced about 10% by an assumed increase in reflector savings upon removal of sodium.

The effect of various factors on the elastic and in-

elastic transfer components of the sodium void effect is readily understood in terms of the energy variation of the neutron importance function for group j , ϕ_j^+ . It will be recalled that these components are proportional to

$$\sum_j \sum_k \phi_j (\phi_k^+ - \phi_j^+) \Sigma_{j \rightarrow k} \quad (1)$$

where ϕ_j is the flux for group j and $\Sigma_{j \rightarrow k}$ is an elastic or inelastic transfer cross section from group j to group k .

The importance function itself for a fundamental mode with buckling B^2 may be written as

$$\phi_j^+ = \frac{\eta_j + \sum_{k>j} \phi_k^+ \Sigma_{j \rightarrow k}}{1 + \frac{D_j B^2}{\Sigma_{aj}} + \frac{\Sigma_{remj}}{\Sigma_{aj}}} \quad (2)$$

where η_j is the number of fission neutrons produced, per neutron absorbed, Σ_{aj} is the macroscopic total fission and radiative capture cross section, and Σ_{remj} the total elastic plus inelastic removal cross section, all for group j . $\Sigma_{j \rightarrow k}$ in this case corresponds to the sum of elastic and inelastic transfers.

It is seen from Eq. (2) that variations in η_j tend to produce variations in ϕ_j^+ , particularly if elastic and inelastic transfer is small. As these transfers become large, the effect of variations in η_j becomes damped out.

It is observed that in going from case 1 to case 2 there is a substantial decrease in elastic transfer component. This is caused partly by the fact that the moderation provided by the BeO lessens the energy variation of the neutron importance function, and partly by the fact that the fertile-to-fissile ratio is less. This increases the leakage which, occurring as it does preferentially at higher energies, depresses the high energy portion of the neutron importance function, again decreasing its energy variation.

A relative increase in fissile material control also lessens the energy variation of η . The results in cases 3 and 5 give these effects separately. The fact that the separate effects do not add up to the total change in elastic transfer component in going from case 1 to case 2 may be due to a differing effect of the BeO moderation at different enrichment ratios.

The use of BeO does seem to produce a modest improvement in the sodium void effect. For the case in which BeO is substituted for fuel, the change is little more than would be obtained simply by removing the same amount of fuel. For a fuel with less moderation already present, such as a metal fuel, considerably stronger effects could be expected.

REFERENCES

1. *Liquid Metal Fast Breeder Reactor Design Study*, GEAP-4418 (January, 1964), pp. 3–96.

2. D. M. O'Shea, H. H. Hummel, W. B. Loewenstein, D. Okrent and B. J. Toppel, *Twenty-Six Group Cross Sections*, ANL-6858 (to be published).
3. A. L. Rago and H. H. Hummel, *ELMOE: An IBM-704 Program*

Treating Elastic Scattering Resonances in Fast Reactors, ANL-6805 (1964).

4. L. Kvitek, *Del, A Perturbation Program Written in Fortran II for the IBM-704*, ANL-7052, (to be published).

III-26. Spectral and Capture Components of Sodium Void Effect for Four 1000 MWe Fast Breeder Studies

H. H. HUMMEL and J. WHITE*

Fundamental mode calculations have been made of the spectra and capture components of the sodium void effect for core compositions of the four 1000 MWe fast breeder studies sponsored by the AEC.^{1, 2, 3, 4} In the case of the General Electric Company study, the hard spectrum alternate with no BeO was used. The effect of variation in fuel composition was also studied in some cases. The purpose of this set of calculations was to study the influence of differences in core composition among the various studies on the spectral and capture components of the sodium void effect. Set 224 cross sections⁵ were used in this work, with ELMOE⁶ being employed directly for light element cross sections.

Materials 61 and 63 were used for U-238 and Pu-239 in this case. Material 63 did not contain the error in Pu-239 fission cross section mentioned in Paper No. III-25 for Material 2.

The spectral component of sodium void effect, defined here as the sum of elastic and inelastic transfer components, depends on core composition in several different ways. The most important variable is the ratio of fertile to fissile material. An increase in this ratio makes the spectral component more positive for the reasons stated in Paper No. III-25. The isotopic composition of the fuel is also important. An increase in the ratio of Pu-240-to-Pu-239 tends to make the spectral component more positive, while an increase of Pu-241 relative to Pu-239 makes this component more negative, because of the more rapid energy variation of the Pu-241 fission cross section. These ratios varied somewhat from one study to another (Table III-26-I), but no attempt has been made to eliminate this inconsistency here. The addition of fission products causes a more rapid energy variation of the importance function and therefore makes the spectral component more positive. The studies were quite consistent in their assumptions in this regard.

An increase in moderation usually acts to decrease the spectral effect, as explained in Paper No. III-25. There are differences in moderation among the cases considered because the fuel is in the oxide form for the General Electric and Allis-Chalmers cases and in the carbide form for the Westinghouse and Combustion Engineering cases, and also because of the differences in the ratios of sodium-to-steel and of fuel-to-steel (Table III-26-I).

Another effect introduced by these last named differences in composition is that the diffusion coefficient will vary differently with energy. This will cause the leakage spectrum to vary differently with energy; so that, although the over-all ratio of leakage-to-absorption may be the same from one case to another, differences will be produced in the energy variation of the importance function and therefore in the spectral component. No evaluation of this effect has been made here, but it is probably small.

A final factor that must be taken into consideration is that the sodium void effect varies directly with the ratio of sodium atoms to fuel atoms. This has been compensated for by renormalizing all of the results to the General Electric result by multiplying by the ratio of sodium content for the General Electric case to that for the other case in question and dividing by the corresponding fuel atom ratio (Table III-26-I).

Results of the calculations, normalized as described, are given in Table III-26-I. In this table, "% F.P." denotes the fission product pairs as percent of the fuel heavy atoms (counting a pair as a heavy atom). The "Na present" column gives amount of original sodium present for the calculation of the flux and adjoint used in the perturbation calculation. It is seen that there are no strong non-linearities in Δk as a function of sodium content.

The total spectral component has been plotted as a function of fertile-to-fissile atom ratio in Fig. III-26-1 for full sodium flux. In this plot, "clean" denotes the cases in Table III-26-I in which there are no fission products

* Student, University of Illinois, Urbana, Illinois.

TABLE III-26-I. EFFECT ON $\% \Delta k/k$ FOR UNIFORM REMOVAL OF 50% OF SODIUM FROM CORE ($T = 1500^\circ\text{K}$)
BY PERTURBATION THEORY

	v/o Fuel	v/o Steel	v/o Na	$\%$ F.P.	Atom Ratios			Na Present	Normalized Values of $\% \Delta k/k$				Comments
					Pu-240 Pu-239	Pu-241 Pu-239	Fertile Fissile		Elastic Re- moval	In- elastic Re- moval	Cap- ture	Total Spectral Plus Capture	
Westinghouse, Carbide Fuel ($N = 0.00856$) ^a	29.4	15.5	55.1	0	0	0	4.00	Full Half No	0.04 -0.07 -0.37	0.30 0.31 0.35	0.11 0.11 0.11	0.45 0.35 0.10	
Atom Ratio, Steel/ Fuel = 1.54							5.54	Full Half No	0.59 0.57 0.27	0.52 0.56 0.60	0.16 0.15 0.16	1.27 1.28 1.03	
Atom Ratio, $\frac{1}{2}$ Na/ Fuel = 0.706 $N = 0.00605$ for $\frac{1}{2}$ Na Norm Factor 1.23							9.00	Full Half No	1.90 2.14 2.02	0.97 1.06 1.19	0.23 0.23 0.26	3.10 3.43 3.47	
				0	0.48	0.069	5.54	Full Half No	0.73 0.70 0.51	0.62 0.70 0.79	0.14 0.12 0.12	1.49 1.52 1.42	
				5.5	0.48	0.069	5.54	Full Half No	1.12 1.07 0.89	0.68 0.77 0.85	0.14 0.12 0.12	1.94 1.96 1.86	1000 MWe Reference Design
Combustion Engi- neering, Carbide Fuel ($N = 0.00829$) ^a	25.6	7.9	66.5	5.7	0.38	0.061	7.00	Full Half No	1.58 1.64 1.40	0.94 1.05 1.21	0.17 0.16 0.16	2.69 2.85 2.77	Average of inner and outer zone com- positions of 1000 MWe Reference Design
Atom Ratio, Steel/ Fuel = 0.81							8.48	Full Half No	2.18 2.34 2.24	1.12 1.28 1.46	0.20 0.18 0.18	3.50 3.80 3.88	Inner zone composi- tion
Atom Ratio, $\frac{1}{2}$ Na/ Fuel = 0.851 $N = 0.007$ for $\frac{1}{2}$ Na Norm Factor 1.02													
General Electric, Oxide Fuel ($N = 0.00626$) ^a	29.0	19.0	47.0	5.8	0.56	0.127	6.25	Full Half No	1.26 1.34 1.28	0.59 0.66 0.74	0.17 0.17 0.19	2.02 2.17 2.21	Hard Spectrum Al- ternate, 1000 MWe Design Study
Atom Ratio, Steel/ Fuel = 2.59				0	0.56	0.127	7.70	Full Half No	1.18 1.35 1.34	0.56 0.63 0.71	0.20 0.20 0.24	1.94 2.18 2.29	
Atom Ratio, $\frac{1}{2}$ Na/ Fuel = 0.868 $N = 0.00544$ for $\frac{1}{2}$ Na Norm Factor 1.00				5.8	0.56	0.127	7.70	Full Half No	1.80 1.80 2.09	0.74 0.83 0.94	0.20 0.20 0.24	2.74 2.83 3.27	
Allis-Chalmers, Oxide Fuel ($N = 0.0055$) ^a	23.5	30.6	40.0	6.3	0.57	0.096	4.38	Full Half No	0.79 0.81 0.74	0.36 0.39 0.41	0.14 0.14 0.17	1.29 1.34 1.32	1000 MWe Reference Design
Atom Ratio, Steel/ Fuel = 4.75 n Ratio, $\frac{1}{2}$ Na/ iel = 0.770 0.00424 for $\frac{1}{2}$ Na m Factor 1.12													

^a N denotes average atoms/cc in core $\times 10^{-24}$.

FIG. III-26-1. Total Spectral Component for Removal of Half of Sodium (Full Sodium flux, $T = 1500^\circ\text{K}$). ANL Neg. No. 112-5489 Rev.

and no higher Pu isotopes present. It is seen that the carbide compositions give lower results than the oxide as was expected. The scatter of points for cases with fuel containing higher isotopes and fission products is not extremely large. This indicates that differences in

steel content and Pu isotopic content among the compositions considered here are not of great importance.

No comparison of the results of fundamental mode and space dependent calculations has been made for the cross sections used here. A comparison made with the cross sections used in Paper No. III-25 indicates that the fundamental mode results for the spectral component are too negative by 0.1 to 0.3 % Δk for removal of 50 % of sodium. This appeared to be due to the effect of the blanket region on the energy variation of the adjoint function. Since that variation is much sharper with the present cross sections because of the mentioned difference in low energy Pu-239 fission cross section, the same result may not apply here.

REFERENCES

1. *Liquid Metal Fast Breeder Reactor Design Study*, GEAP-4418 (January, 1964).
2. *Large Fast Reactor Design Study. Final Report*, ACNP-64503 (January, 1964).
3. *Liquid Metal Fast Breeder Reactor Design Study*, CEND-200 (January, 1964).
4. *Liquid Metal Fast Breeder Reactor Design Study*, WCAP-3251-1 (January, 1964).
5. D. M. O'Shea, H. H. Hummel, W. B. Loewenstein, D. Okrent and B. J. Toppel, *Twenty-Six Group Cross Sections*, ANL-6858 (to be published).
6. A. L. Rago and H. H. Hummel, *ELMOE: An IBM-704 Program Treating Elastic Scattering Resonances in Fast Reactors*, ANL-6805 (1964).

III-27. Evaluation of Breeders from a Conservation Standpoint

P. F. GAST

In a rapidly expanding national power system the surplus plutonium from breeder reactors may initially be insufficient to supply the inventory required for new reactors, and the deficit must be made up by fissile material supplied ultimately from mining operations.¹ With a typical growth curve of system capacity, e.g., the AEC Report to the President,² the material supplied from mining operations reaches a finite maximum.

Designs of breeder reactors may be compared from a conservation standpoint by computing this maximum requirement if the entire system were to consist of reactors of the specified type. For this purpose, designs are basically characterized by two parameters: specific power, kW/kg fissile plutonium including ex-reactor inventory, and breeding gain; or alternatively by specific power and doubling time.

The time at which the system becomes self-sufficient is determined from the condition:

$$\frac{1}{C} \frac{dC}{dt} = \frac{(BR - 1)APS}{(1 + F)(1 + I)}$$

where

$C(t)$ = capacity growth curve, installed kW

t = the time in years

S = specific power, kW/kg fissile plutonium

BR = breeding ratio

F = fertile fissions/fissile plutonium fission

I = external inventory/internal inventory

P = plant load factor

E = power plant efficiency

A = kg fissile plutonium burned/kW year $\approx 365 \times 10^{-6} (1 + \alpha)$.

For typical growth curves, which increase less than exponentially, the left hand side of the expression decreases with time and the condition is eventually satisfied. For the special case of continued exponential growth, it can be satisfied only by matching growth rate and reactor characteristics.

The net outside supply required is:

$$D = \frac{1 + I}{S} \frac{C}{E} - \frac{(BR - 1)AP}{(1 + F)} \int \frac{C}{E} dt.$$

The growth curve used can be taken as linear after the year¹ 2000:

$$C(t) = a + bt$$

in which case D_{max} can be written in the alternative form

$$D_{max} = \frac{\text{Surplus Pu, kg/yr}}{MW(th)} \left[\frac{b}{2} T^2 + \frac{a^2}{2b} - c \right]$$

where T = doubling time of the system,

$$c = \int_{1964}^{2000} \frac{C}{E} dt; \text{ i.e., the contribution from the initial non-linear, growth period.}$$

D_{max} and the time at which it occurs have been computed for four designs of ceramic fueled breeders supplying 1000 MWe whose design parameters had been reduced to a common basis.³ Table III-27-I presents the results together with some of the reactor design characteristics.

It is clear that the order of rank of the designs from a conservation standpoint is different from that obtained on the basis of fuel cycle cost when computed for near future conditions. In fact, two of the designs

TABLE III-27-I. PLUTONIUM REQUIREMENTS FOR FOUR 1000 MWe DESIGN CONCEPTS

Design Organization	Combustion Engineering	Allis-Chalmers	Westinghouse	General Electric
Fuel form	Carbide	Oxide	Carbide	Oxide
Surplus Pu, kg/yr	275	245	400	240
Doubling time, yrs	6.3	17.3	11.2	12.2
Fuel cycle cost, mills/kWh	0.43	0.70	0.42	0.57
Peak outside supply metric tons, Pu	146	2900	1710	1270
Time of peak, AD	2004	2015	2009	2010
Fuel cycle cost, zero Pu credit	0.82	1.05	0.99	0.91

with the same fuel cost differ by a factor of more than 10 with respect to outside supply requirements.

The peaks of the requirement curves come at a time when uranium prices may be rising sharply.¹ Reactors starting up in this period may have to pay high prices for initial inventory but be faced with a declining market for surplus plutonium in the years following. Quantitative estimation of the effect upon fuel cycle economics is difficult. However, if plutonium credits were entirely wiped out, the ranking order of these designs is the same as that obtained from the conservation viewpoint as shown in the last line of Table III-27-I.

REFERENCES

1. J. R. Dietrich, *Efficient Utilization of Nuclear Fuels*, Power Reactor Tech., **6**, No. 4, 1 (1963).
2. Civilian Nuclear Power, A Report to the President—1962, U. S. Atomic Energy Commission (November 20, 1962).
3. L. E. Link, G. J. Fischer and E. L. Zebroski, *Fuel Cycle Economics of Fast Reactors*, Proc. 1964 Geneva Conference, Paper No. 248 (to be published).

III-28. Quick, Approximate Evaluation of the Economics of Mixed Plutonium Fueled Breeder Designs

P. F. GAST

Detailed economic evaluations of breeder designs are required in the final stages of comparing their merits with those of other reactors or other methods of power production. However, these detailed analyses tend to obscure, in many cases, the contributions made to the final result by the various parameters of the design. An approximate method has been developed for making

such analyses, at the same time making clearer the relative importance of various economic factors. The method may be used for quick evaluations of individual designs, for estimating the improvement required in one parameter to offset the decline of another, or for estimating various combinations of parameter values which will meet a given economic requirement.

TABLE III-28-I. *S* AND *G* VALUES FOR FOUR
1000 MWe PLUTONIUM BREEDER DESIGN
CONCEPTS

Reactor	<i>S</i> (kg/MWe)	<i>G</i>
General Electric (G.E.), oxide fuel	3.40	0.17
Westinghouse Electric (W), carbide fuel	4.95	0.37
Argonne National Laboratory (ANL), metal fuel	4.07	0.42
Combustion Engineering (C.E.), carbide fuel	3.00	0.35

In this method, total power costs are broken down into the following categories: fuel processing costs, working capital costs, returns from sale of excess plutonium, and construction capital costs.

Fuel processing costs include the cost of fabricating fuel and blanket elements, shipping, and chemical reprocessing. The sum of these costs has the form:

$$\text{mills/kWh} = A/B$$

where *B* is the fuel burnup at discharge, expressed in MWD(th)/kg and the parameter *A* can be obtained from

$$A = \frac{\$/\text{kg}}{24E}$$

where *E* is the power plant thermal to net electrical conversion efficiency. For sodium cooled breeder designs, $24E \approx 9.6$. The \$/kg is to be understood as the total cost for both core and blanket processing per kg of core fuel.

Values of *A* do not differ widely for various fuel designs; and when different types of fuels are not being compared, *A* may be taken as 50. When it is desired to differentiate between fuels, the following representative values may be used.¹

Oxide	55
Carbide	51
Metal	40 (close coupled pyrometallurgical processing)

The fuel processing costs will be a major item in the cost of power from breeder reactors; and improved fuel designs or processing methods which can result in significantly lowered values of *A*, should be the objective of research and development effort. The small variation in values for different fuels may reflect the present lack of detailed information on what costs will actually be.

The major component of working capital will be that tied up in plutonium inventory both in the reactor and in the reprocessing and fuel fabrication pipe line. Capital tied up in fuel fabrication charges represents a smaller component. Other minor amounts would be re-

quired in the operation of the plant but these have been neglected in view of the uncertainties in the amount of ex-reactor inventory required.

Working capital costs in mills/kWh are given approximately by:

$$F_w S \left[\frac{C}{7} + 0.15 \right],$$

where

F_w is the annual fixed charge rate.

S is the total kg inventory of Pu(239 + 241)/MWe.

C is the price of Pu(239 + 241), \$/gm.

7 arises from the 7000 hours of operating time per year (80% plant factor).

0.15 represents in an approximate way the fuel fabrication inventory value.

The value of the excess plutonium produced is given by

$$0.11 C G \text{ mills/kWh,}$$

in which *G* is written for

$$(BR - 1) \frac{(1 + \bar{\alpha})}{(1 + F)},$$

where

BR is the breeding ratio.

$\bar{\alpha}$ is the mean value of α for Pu-239 and Pu-241 in the reactor.

$1 + F$ is the ratio of total fissions to fissions of Pu-239 plus Pu-241.

Information usually given about breeder designs may be used to compute values of *G* as follows:

$$G = 0.95 \frac{\text{Net gms Pu (239 + 241)}}{MWd(th)}$$

or

$$G = 22,800 \frac{\text{Net gms Pu (239 + 241)}}{KWh(e)}$$

or

$$G = \frac{3.26}{MW(th)} [\text{Excess Pu (239 + 241), kg/yr}].$$

Representative values of *S* and *G* for several designs are given in Table III-28-I.

The difference between the inventory cost and the return from excess plutonium produced might be termed the "net plutonium cost," although the inclusion of the fabrication inventory term makes this not quite correct. For handy reference, a plot of equal plutonium cost lines in the *S*-*G* plane may be made. Figure III-28-1 is an example of such a plot made for a plutonium price

FIG. III-28-1. Net Plutonium Cost, $C = \$10/\text{g}$, $F_w = 10\%$. ANL Neg. No. 112-5333.

of \$10/gm and a fixed charge rate of 10%/yr, representative of private utility financing.

Estimates of construction costs for fast breeders are not abundant. However, for comparative purposes, an

estimated \$25/kW more for a sodium cooled breeder than for a boiling water reactor may be adopted at the 1000 MWe level. A corresponding estimate at other levels would be 25% of the cost per kilowatt. The \$25/kW amounts to 0.5 mills/kWh with private utility financing.

As an example of the use of the method here presented, consider the evaluation of the competitive position of breeders vis-a-vis boiling water reactors. With an obtainable exposure of 20,000 MWd/T in boiling water reactors, the fuel cycle cost is estimated from the Oyster Creek analysis³ to be:

$$\begin{aligned} \text{mills/kWh} &= 1.26 + 4.0 F_w, \text{ or} \\ &1.66 \text{ mills/kWh for an } F_w \text{ of } 10\%. \end{aligned}$$

In a ceramic fueled breeder with an exposure of 100 MWd/kg fuel, fuel processing costs would be 0.53 mills/kWh using a mean of the values of A given above. If 0.50 mills/kWh is allowed for higher construction costs, then competitive breeder design must lie below the 0.63 mills/kWh line in Fig. III-28-1 (and be capable of reaching 100 MWd/kg fuel exposure). The three ceramic fueled designs listed in Table III-28-I are plotted on Fig. III-28-1 and all are seen to meet the requirement postulated.

REFERENCES

1. K. A. Hub, Argonne National Laboratory (private communication).
2. *An Evaluation of Four Design Studies of a 1000 MWe Ceramic Fueled Fast Breeder Reactor*, COO-279, Reactor Engineering Division, Chicago Operations Office, USAEC.
3. *Report on Economic Analysis for Oyster Creek Nuclear Electric Generating Station*, Jersey Central Power and Light Co., Nuclear News, **1**, No. 4, Supplement (April 1964).

III-29. Mixed Fuel Cycle Fast Breeder Reactors

W. B. LOEWENSTEIN

Much of the emphasis in the recent 1000 MWe fast breeder reactor studies^{1, 2} has focused upon reactor safety. Those aspects related to the sodium coefficient of reactivity and loss of core sodium appeared to have a profound influence upon the various reactor configurations. To obtain a small core sodium reactivity coefficient requires some significant compromises on nuclear performance. In particular the core conversion ratio tends to be lower, the fissile loading in the fuel is higher and the specific power may be lower than for the

corresponding system with optimum nuclear performance having a considerably larger core-sodium reactivity coefficient.

The relatively low core conversion ratio in such systems requires a significant amount of built in excess reactivity at the start of core life to provide an uninterrupted burnup cycle. This excess reactivity must be controlled, thus, probably increasing demands on the multiplicity of control mechanisms. Furthermore, for metallic fueled systems, the choice of a modular array

TABLE III-29-I. A PRELIMINARY COMPARISON OF MIXED FUEL CYCLE WITH SINGLE FUEL CYCLE FAST BREEDER REACTORS (3000 Liter Core Volume)

Core Volume, liters		Reactor Fuel Type	Kg ^U /Kg Th (In Central Core)	Kg ^{Pu} /Kg ^U (In Outer Core)	Na Void Effect, % $\Delta k/k$		Fertile Fissions, %	Total Core Peak to Average Power Density	Core Conversion Ratio	Total Breeding Ratio	Comments ^b
Central	Outer or total				Central core	Outer core					
0	3000	M	—	0.14	—	+0.79	22.0	2.03	1.02	1.67	$N^{233} = 0.75 N^{Pu}$
600	2400	M	0.110	0.157	+0.14	+0.11	17.1	1.62	0.89	1.50	
800	2200	M	0.113	0.161	+0.11	-0.01	15.8	1.64	0.86	1.47	
1200	1800	M	0.117	0.167	+0.001	-0.15	13.3	1.71	0.81	1.39	$N^{233} = 0.75 N^{Pu}$
0	3000	C	—	0.151	—	+0.92	19.1	2.05	0.97	1.49	$N^{233} = 0.75 N^{Pu}$
800	2200	C	0.131	0.155	+0.002	+0.013	13.5	1.79	0.83	1.33	
800	2200	C	0.144	0.173	+0.21	+0.12	15.3	1.40	0.84	1.38	
0	3000	O	—	0.26	—	+0.55	17.4	2.02	0.80	1.29	$N^{233} = 0.667 N^{Pu}$
800	2200	O	0.16	0.25	-0.27	-0.26	11.3	2.14	0.70	1.14	
800	2200	O	0.19	0.29	-0.03	-0.13	12.6	1.88	0.69	1.13	
800	2200	O	0.20	0.27	-0.12	-0.17	12.0	2.18	0.70	1.12	$N^{233+234} = 0.75 N^{Pu}$

^a For 40% removal of core sodium.^b N^{234} pessimistically assumed $\approx 0.25 N^{233}$ where indicated. Higher Pu isotopes are consistent with those used in 1000 MWe studies.

with relatively higher plutonium content in the fuel alloy as compared with the optimum geometric configuration, directly affects the maximum possible operating temperature.

Quite apart from these considerations, it has been reported³ that unmoderated U-233-Th fueled systems generally have more negative sodium reactivity coefficients than corresponding Pu-U-238 fueled systems. Conceptually it is then possible to replace part of a Pu-U-238 fuel system with U-233-Th fuel with some advantages to the sodium reactivity coefficient. If this replacement is carried out on a system with reasonably optimum nuclear performance and geometry, some performance advantages might be realized relative to the Pu-U-238 systems that have been specifically designed to counteract a possible deleterious positive sodium reactivity coefficient.

The sodium coefficient of reactivity generally varies rapidly as a function of position, being positive near the center of the core and becoming negative near the core boundary. The crossover point, from positive to negative, is a sensitive function of core volume, composition and geometry. Nevertheless, it seems reasonable to introduce the U-233-Th fuel in those regions of the core where the presence of Pu-U-238 fuel gives the more positive contributions to the sodium reactivity coefficients.

It is desirable to have the over-all system operating on the Pu-U-238 cycle. Therefore, the breeding blankets are assumed to contain natural or depleted uranium. The core will be constituted of two types of fuel sub-assemblies. One will be the standard type of sub-assembly which contains Pu-U-238 fuel elements with axial segments of fertile material. The other sub-

assembly will be similar to the standard subassembly, having the same fuel element or pin height. However, the central region of unspecified height is constituted of U-233-Th fuel while the axial core extremities consist of Pu-U-238 fuel.

The core that is assembled in this manner will have a central region of U-233-Th fuel which is surrounded on all sides by a larger region of Pu-U-238 fuel. The core is reflected by a blanket of natural or depleted uranium with the necessary engineering compromises. These usually imply a lower atom density of uranium in the axial blanket than is acceptable in the radial reflector.

This over-all core assembly principle can be applied to any of those major fuel types (e.g. oxide, carbide and metal) as well as others that are not as well developed (e.g. sulfide and phosphide). The simplest application assumes that whatever type of fuel is used for the Pu-U-238 material is also used in the U-233-Th region. However, the rather good irradiation stability⁴ of U-233-Th metal does suggest that this material may be used in conjunction with a carbide or oxide fuel bearing Pu-U-238.

Table III-29-I summarizes the results of a number of spherical multigroup diffusion theory calculations using 22 groups of the ANL-26 group set.⁵ The light element materials in this set were obtained in the same manner as described in Ref. 3. A few comparative calculations with more detailed light element cross section generation procedures exhibited about the same small uncertainties as indicated in Ref. 3. The 3000 liter core volume was chosen as typical and in the range of interest. It is not intended to imply any particular heat removal capability.

Detailed comparisons of data in Table III-29-I are difficult since fission distributions are not always continuous across the inner-outer core boundary. However, the cited peak to average core power densities give some indication of the extent of effective "power flattening" that has been factored into the analyses. Furthermore, the ceramic and metallic systems were assumed to contain fission products for an average of 5% and 2.75% burnup, respectively, consistent with the 1000 MWe^{1,2} studies. The metallic systems all utilize the "high void" fuel elements² to accommodate irradiation growth.

In general, it can be seen from these preliminary studies that the potentially deleterious core sodium reactivity may be significantly altered in a favorable direction by utilizing the U-233-Th cycle over a small region near the center of the core. The size of the region will depend upon basic core volume and higher isotope content in both plutonium and fissile uranium. Current uncertainties in the Pu-240 and U-234 cross-sections could have significant bearing upon the ultimate configurations. The effect of the 27 day Pa-233 half life relative to the shorter Np-239 half life has not been factored into these considerations; this will present some unique control problems.

Since the conversion ratio of the U-233-Th inner

core is less than unity, makeup U-233 could be obtained by placing fertile thorium in the blankets, presumably away from the core boundary so that the low Th-232 fission cross-section does not significantly alter the fertile fission fraction.

Finally, metallurgical considerations may suggest configurations where the U-233-Th fuel material is different from the Pu-U-238 fuel material. For example, there is no reason why a U-233-Th-Oxide fuel may not be used with a Pu-U-238 metal fuel when in a stainless steel clad. This particular combination is favorable for enhancing the Doppler coefficient of reactivity. Furthermore, such axial segmentation of the fuel may afford a basis for increasing the average fuel burnup in a subassembly if the fuel in the central U-Th section has a higher burnup capability than the Pu-U-238 fuel.

REFERENCES

1. *An Evaluation of Four Design Studies of a 1000 MWe Ceramic Fueled Fast Breeder Reactor* COO-279 (December 1964).
2. L. E. Link, et al., *1000 MWe Metal Fueled Fast Breeder Reactor*, ANL-7001 (to be published).
3. D. Okrent, K. Cohen and W. Loewenstein, *Some Nuclear and Safety Considerations in the Design of Large Fast Power Reactors*, (presented at 1964 Geneva Conference).
4. J. H. Kittel, J. A. Horak, W. F. Murphy and S. H. Paine, *Effects of Irradiation on Th and Th-Uranium Alloys*, ANL-5674 (1963).

III-30. Additional Physics Studies of Molten-Salt-Fueled Fast Reactor Systems

D. K. BUTLER and D. MENEGHETTI

The study previously reported¹ has been extended with two new objectives: 1) to evaluate systems with updated design parameters and 2) to compare the previous results with those using recently developed chlorine cross sections.

The updated parameters were those for systems analogous to types A, B, and C of Ref. 1. The systems included compositions based on a more detailed study of physical characteristics of the molten salts. In addition, fission products in the salt, as well as a simulated reactor vessel, were included in the analyses.² To facilitate the comparison, core volumes of 10,000 liters were analyzed as in the previous study. The reactor vessel was assumed to be $\frac{3}{4}$ in. thick high nickel content stainless steel alloy. An additional change was made in the reactor whose core consisted of a homogeneous salt mixture (Type A). This system was reflected by a molten salt blanket. The systems B and C retained

their uranium metal blankets (45 cm of 60 v/o U, 20 v/o Na, 20 v/o Fe).

The new chlorine cross sections were assembled by modifying the original Hansen-Roach cross sections for chlorine³ according to the more recent tabulation of M. Kalos and J. Ray.⁴ Capture and inelastic scattering cross sections from Kalos and Ray were used for groups 1 through 10. In addition, the Hansen-Roach elastic removal cross section for groups 2 through 11 were adjusted to correspond to weighting by a flat spectrum. The flat spectrum is closer to that actually present in the molten salt systems than is the original Hansen-Roach spectrum.

The nuclear performance with these changes is summarized in Table III-30-I. The breeding ratio for the homogeneous system (Type A) is reduced from that obtained in Ref. 1 by 0.23. The detailed neutron balance shows that this results primarily from increased capture

TABLE III-30-I. CHLORIDE FUELED REACTORS

	Homo- geneous	Heterogeneous	
	Type A	Type B	Type C
Core Characteristics			
Volume, liters	10,000	10,000	10,000
Fuel Composition, mole/o ($\text{Pu}_x\text{U}_{1-x}\text{Cl}_3$)	30 ^a	50 ^a	18 ^a
Core Composition, v/o Fuel, ($\text{Pu}_x\text{U}_{1-x}\text{Cl}_3$)	100	45	30
Solid Metallic U-238- 1% Pu	0	0	15
Structural Material of 60% Ni, 30% Fe, 10% Mo	0	11	11
Sodium Coolant	0	44	44
Blanket Characteristics			
Thickness, cm	121.9	45	45
Composition, v/o	100 salt ^b	60U, 20Na, 20Fe	°
Power Generation, % of Total	14	—	—
Neutron Absorptions in Whole Reactor			
Structure and Vessels, %	4.7	13.3	8.1
Chlorine, %	2.9	2.8	1.1
U-238 Captures, %	49.8	41.9	45.0
Pu-239 Captures and Fissions, %	33.7	37.2	33.0
Breeding Ratio	1.48	1.13	1.37 ^d
% Δk for Voiding 40% of Sodium Coolant from Core	—	+0.23	+1.88
% Δk for Fuel Expansion for $\Delta T = 100^\circ\text{C}$ (Approx.)	-1.5	-1.5	-1.5

^a Remainder is chlorides such as NaCl, KCl, MgCl_2 , and fission product chlorides.

^b Salt composition: 29.4 mole/o UCl, 0.6 mole/o PuCl_3 , remainder chlorides, such as NaCl, KCl, MgCl_2 , and fission product chlorides.

^c Two-thirds of core surface surrounded by blanket of 60 v/o U, 20 v/o Na, 20 v/o Fe; one-third surrounded by reflector of 44 v/o Na, 56 v/o Fe.

^d Based on breeding ratio of 1.48 for a radial blanket and 1.07 for an axial reflector; see footnote c.

in the reactor vessel. Increased capture in chlorine also contributes to the reduced breeding ratio. Part of this reduction is caused by the softening of the neutron spectrum near the core boundary because of the molten salt blanket. The leakage out of the blanket is not significantly altered. However, the molten salt blanket is considerably thicker than the corresponding metal blanket.

The performance for the system with coolant pipes

containing molten metallic sodium (Type B) is similar to that obtained before.¹

The neutronics of the system when metallic U-238 elements are added to the internally cooled core (Type C) are also similar to those of the previous analysis.¹ However, the present calculation allows for an axial reflector which would be present in the actual reactor system.

Loss of sodium coolant from the core by boiling remains an important safety problem for Types B and C. The Δk for voiding 40 % of the sodium coolant in Type B is slightly more positive than with the previous parameters and chlorine cross sections. With the uranium metal present in the core in Type C, the new calculations show a significantly more positive sodium void effect than the previous analysis. However, the reactivity decrease from thermal expansion of the molten fuel is large enough so that an actual design may be possible in which fuel expansion counteracts any reactivity gain from coolant boiling.

Results based on the new chlorine cross sections were compared with those obtained with the original Hansen-Roach chlorine cross sections. Comparisons show insignificant changes in the over-all system parameters. The neutron losses due to (n,p), (n, α), and (n, γ) reactions increase by 1 to 2 % relative to the original calculations. Increasing the inelastic scattering cross sections decreases the very high energy flux; however, the change in the inelastic removals due to the altered inelastic spectrum tends to decrease the fluxes in the lowest energy groups. The two combined effects largely offset each other in their over-all effect on criticality and neutron inventory.

It is possible that the modified cross sections still do not accurately represent the chlorine cross sections. Experimental neutron data for this material are limited. It is therefore not obvious that separated chlorine isotopes will not be required for good nuclear performances even though the present study tends to indicate that natural chlorine may be used.

REFERENCES

1. D. Butler and D. Meneghetti, *Physics Studies of Some Molten-Salt-Fueled Fast Reactor Systems*, Reactor Physics Division Annual Report, ANL-7010 (January 1965), p. 156.
2. P. A. Nelson, D. K. Butler, M. G. Chasanov and D. Meneghetti, *Fuel Properties and Nuclear Performance of Fast Reactors Fueled with Molten Chlorides*, Trans. Am. Nucl. Soc. 8, No. 1, 153 (1965).
3. G. E. Hansen and W. H. Roach, *Six and Sixteen Group Cross Sections for Fast and Intermediate Critical Assemblies*, LAMS-2543 (November, 1961).
4. M. H. Kalos and J. H. Ray, *Neutron Cross Sections of Natural Chlorine*, UNC-5067 (September 1963).

III-31. A Simplified Model for Calculating Criticality Trends in Fast Reactors

B. I. SPINRAD

INTRODUCTION

Survey calculations require that a large number of criticality calculations be performed. Because small errors in criticality are tolerable in survey calculations, the detailed and highly precise codes which are available do not have to be used. Instead, a simplified method is appropriate.

This paper describes such a simplified calculation system, which has been used to obtain critical mass estimates for parametric calculations of tungsten-UO₂, beryllium-reflected fast rocket reactors. The method is reasonable for any system for which a reasonably clear distinction can be made between "fundamental mode" neutrons (i.e., neutrons with the unreflected critical core spectrum) and reflector-moderated neutrons.

GENERAL DESCRIPTION

The method cascades a number of approximations. The starting point is that, even for small reactors, the major contribution to criticality is from neutrons which obey the fundamental mode equation in space and energy. An equivalent statement is that the contribution of the reflector to reactivity consists of only two components: a source, feeding the fundamental mode in the core and consisting of neutrons whose energy does not conform to the fundamental mode and which cause fission when they enter the core; and a direct reflection of the fundamental mode without energy change.

The next step is to calculate the fundamental mode for systems consisting only of core material. Examination of the spectra is required at this point: if the fundamental modes for all core compositions of interest have spectra which show a rapid low-energy fall-off below the energy at which the highest flux is found, it is reasonable to equate the fundamental mode with a single fast energy group. This is the next step in the approximation.

Core constants for this group (fission and absorption cross-sections and diffusion coefficients) are then obtained by spectrum-averaging the fission and absorption cross-sections over the entire fundamental-mode spectrum, and adjusting diffusion coefficients to obtain the correct critical one-group buckling.

Reflector constants for the fundamental mode are obtained as follows:

Diffusion coefficient, by flux averaging the reflector, energy-dependent diffusion coefficient over fundamental mode spectrum.

Removal cross section, by taking the ratio $D(E_i, E_L)/\tau(E_i, E_L)$ (where $D(E_i, E_L)$ is the average diffusion cross section for an infinite material with source energy well above E_i and for an energy group between E_i and E_L ; and $\tau(E_i, E_L)$ is the neutron age between E_i and E_L) and considering this as a cross-section for neutrons of energy E_i to be slowed down to an energy below E_L . This is standard in two-group theory. E_L is taken as the cutoff energy of the fundamental mode. The removal cross section as a function of E_i is flux-weighted over the fundamental mode, but only considering groups above E_L .

Below the fundamental mode cutoff energy, a few-group theory is employed. It is assumed that, for these groups, the core does not contribute to neutron energy change. Therefore, the flux spectrum in the reflector and the reentrant low-energy-neutron current spectrum to the core are taken as the infinite region reflector slowing-down spectrum. Core absorption and fission cross-sections and diffusion coefficient, and reflector diffusion coefficient and group age are calculated from this spectrum by spectral averaging.

To compute criticality, we start with one fundamental-mode neutron leaking from core to reflector. This results in some neutron slowing down in the reflector, and the return of a definite number of neutrons to the core at lower energies. Some of these neutrons, in turn, cause fissions. Neutrons from these fissions represent a source to the fundamental mode in the core, which is a subcritical multiplying system. This system will have a buckling value at which enough fundamental-mode fissions occur to leak one neutron to the reflector. This buckling value is the critical value. It is achieved by a geometric change over the entire system (core and reflector), which geometric change alters the reflector leakage pattern, and therefore generates an iteration. We have used only one-dimensional geometric changes (radius or height of right circular cylinders), and have achieved rapid convergence.

To synthesize two-dimensional criticality, we make use of the reflector-savings concept. We compute the core as though it is bare in the transverse dimension, and assume that the actual core transverse dimension is reduced by the reflector savings in that dimension.

The neutrons which are reflector-moderated only affect criticality by producing a core-edge fission spike which is a source to the fundamental mode. The importance of such fissions is not very rapidly varying

TABLE III-31-I. FORTRAN LISTING OF SIGMA PROCESSOR-2

```

* SIGMA PROCESSOR = 2
  DIMENSION SIG(9),STORA(200),STORB(200),STORC(200)
  READ 100, J, K, M
100 FORMAT (3I6)
  DO 1 L=1,200
    STORA(L)=0.
    STORB(L)=0.
  1 STORC(L)=0.
  DO 2 L=1,M
    READ 101, DA, DB, DC
101 FORMAT (3F15.7)
    NEXT=0
    DO 2 LL=1,22
      IF(LL-16) 5,5,6
  5 NUM=9
    GO TO 7
  6 NUM=25-LL
  7 READ 102, (SIG(N),N=1,NUM)
102 FORMAT (6E12.6)
    IF(LL-J) 3,4,2
  3 IF(J-LL-K) 13,14,14
13 LAST=J-LL+3
    GO TO 15
14 LAST=K+3
15 SUM=0.
    DO 11 N=LAST,NUM
11 SUM=SUM+SIG(N)
    SIG(LAST)=SUM
    GO TO 17
  4 LAST=3
17 DO 12 N=1,LAST
    NN=NEXT+N
    STORA(NN)=STORA(NN)+DA*SIG(N)
    STORB(NN)=STORB(NN)+DB*SIG(N)
12 STORC(NN)=STORC(NN)+DC*SIG(N)
  2 NEXT=NEXT+9
  PRINT 200
200 FORMAT (7H10UTPUT///)
  DO 18 N=1,3
    NEXT=0
    PRINT 103
    PUNCH 103
103 FORMAT (80X)
    DO 18 L=1,J
      IF(J=L-K) 23,23,24
23 LAST=J-L+3
    GO TO 25
24 LAST=K+3
25 DO 26 I=1,LAST
    II=I*NEXT
    GO TO (27,28,29) N
27 SIG(I)=STORA(II)
    GO TO 26

```


TABLE III-31-I—Continued

```

28 SIG(I)=STORB(I)
   GO TO 26
29 SIG(I)=STORC(I)
26 CONTINUE
   PRINT 102, (SIG(I),I=1, LAST)
   PUNCH 102, (SIG(I),I=1, LAST)
18 NEXT=NEXT+9
   END

```

near the core edge, so that only a crude estimate of the shape of the spike is needed. This estimate requires only a few-group description of the reflector-moderation; the groups are, in fact, identified with energy ranges over which the neutron fission/absorption ratio of the core is crudely constant.

The last energy group is atypical. Since no neutrons of lower energy are considered, any slowing down from that group leads to neutron loss. The slowing-down cross section may therefore be adjusted between 0 (which means that the neutron economy is no longer affected by moderation) and a maximum value of the true slowing-down cross section. This permits a compromise which may be equated to interface losses, such as core vessel absorption.

FUNDAMENTAL MODE COMPUTATIONS

MICROSCOPIC CROSS SECTIONS

Any appropriate cross section set can be used. The set most easily available at Argonne is the 22 group ANL-224⁽¹⁾ set, which has been evaluated as valid for fast reactor calculations. A fission spectrum for this breakdown is also available.

The cross sections were prepared and used without taking into account resonance self-shielding, which is of considerable importance below about 10 keV. It may be neglected if neutrons at these lower energies do not contribute importantly to the fundamental mode, which is the case in the application reported here.

The data furnished were repunched on working cards in a more compact format. For any group i , the first card contains: $3\sigma_{tr}$, σ_a , σ_f , $\sigma_{r,i+1}$, $\sigma_{r,i+2}$, $\sigma_{r,i+3}$; the second card contains $\sigma_{r,i+4}$, $\sigma_{r,i+5}$, and $\sigma_{r,i+6}$. Here, σ_{tr} is (microscopic) transport cross-section in barns, σ_a absorption cross-section, σ_f fission cross-section, and $\sigma_{r,i+j}$ cross-section for transfer of neutrons from group i to group $i+j$. Elastic transfer is included in $\sigma_{r,i+1}$. For $i > 16$, $\sigma_{r,i+j}$ is simply omitted when $i+j > 22$, and for $i \geq 19$, there is no second card. The card format is 6E12.6. Set 224 only lists six groups of inelastic

scattering, but the preprocessing routine used will accept up to 27 inelastic scattering groups and reduce them to 6 by piling up all the rest of the scattering into group $i+6$.

MACROSCOPIC CROSS SECTIONS

Macroscopic cross-sections for a given composition were computed in a separate program, which was needed so as not to burden the storage capacity of the CDC-160A computer used in this work. The program, "Sigma Processor-2" is reproduced as Table III-31-I.

The program accepts as inputs, first a single card containing three integers, J , K , M (316 Format). These are: J , the number of groups to be outputted; K , the maximum number of inelastic transfer groups; and M , the number of constituents (nuclear species) in the mixture. We require $J \leq 20$, $K \leq 6$, and M is unlimited.

TABLE III-31-II. GROUP STRUCTURE OF THE GROUPS USED IN FUNDAMENTAL MODE CALCULATIONS, AND ASSOCIATED FISSION SPECTRUM

Group	Upper Energy, keV	Lower Energy, keV	Δu (Lethargy Units)	Fraction of Fission Neutrons Born in Group
1	10000.0	3679.0	1.0	0.130350
2	3679.0	2231.0	0.5	0.210890
3	2231.0	1353.0	0.5	0.232230
4	1353.0	820.8	0.5	0.179410
5	820.8	497.9	0.5	0.114870
6	497.9	302.0	0.5	0.064840
7	302.0	183.2	0.5	0.034097
8	183.2	111.1	0.5	0.017125
9	111.1	67.38	0.5	0.008396
10	67.38	40.87	0.5	0.004070
11	40.87	24.79	0.5	0.001953
12	24.79	15.03	0.5	0.000933
13	15.03	9.119	0.5	0.000440
14	9.119	4.307	0.75	0.000267
15	4.307	2.613	0.5	0.000068
16	2.613	2.035	0.25	0.000022
17	2.035	1.234	0.5	0.000022
18	1.234	0.961	0.25	0.000006
19	0.961	0.583	0.5	0.000007
20	0.583	0.275	0.75	0.000004

The next card contains three atom densities (DA , DB , DC) in 3F15.7 Format. These are the atom densities of the first component of the mixtures, three of which are computed at a time. The processed microscopic cross-sections previously prepared for that component, which have the designation $SIG(N)$ (described in the immediately preceding section) are then read in and number-weighted by DA , DB , and DC . Another atom density set is then read in for the next component, the microscopic cross-sections are read in, number-weighted, and cumulated; until finally, the M materials have been processed.

The output consists of a set of cards containing macroscopic cross-sections (in cm^{-1}) for each of three mixtures. The order and format are the same as for the inputs.

In reducing the number of groups, we simply ignore the groups curtailed, so that the group reduction merely means that we do not allow slowing down below group J . In curtailing K , which is possible, and is necessary for the last few groups when J has been curtailed, removal cross-sections which are forbidden are displaced and cumulated in the last group into which scattering is allowed.

The output used, uniformly, for the studies reported here was a twenty-group, six inelastic group system. The group structures and the associated fission spectrum are listed in Table III-31-II.

FUNDAMENTAL MODE CRITICAL BUCKLING CALCULATION

The fundamental mode computation is performed on another 160A program entitled "Subelmoe." It owes its name to the fact that its initial version was pieced together as a subset of the ELMOE² program. Blocks of cards from the ELMOE deck were extracted and used. Although the program has since been drastically changed, the name has remained.

The Fortran printout of Subelmoe is displayed as Table III-31-III, and the program explained subsequently.

Subelmoe accepts input as cross sections for J groups ($J \leq 20$) and K group transfers ($K \leq 6$). It also accepts a fission spectrum input as B .

For a new problem, or a problem which incorporates a group or inelastic structure or fission spectrum, entry is at order 70. J and K (in 216 Format) are read in, followed by $B(I)$, the fission spectrum. B is in 6E12.6 Format—that is, there are six fission-neutron yields per card, in group order, until the groups are all described.

For subsequent problems using a common group and inelastic structure and fission spectrum, entry is at order 1. Two alphanumeric cards are read in and read out to describe the problem. Both cards must be furnished as input; only the first 60 columns of each

card are used. Since J and K are not included in problem printouts, it is recommended that this "title" include that information.

The following steps, down through order 126, read in the cross section information, which is input to this program and is the output of "Sigma Processor-2" (q.v.). As a precaution for later operations, all cross section storage is initially cleared. The cross section symbols are:

$$\begin{aligned} TSTR &= 3N\sigma_{tr} \\ SA &= N\sigma_a \\ ENUFS &= N\sigma_f \\ SIS(I, J) &= N\sigma_{in}^{i \rightarrow i+j} \end{aligned}$$

Transport, absorption, and fission cross sections are subscripted according to group. The two indices on $N\sigma_{in}$ describe group transfer from group " i " to group " $i+j$ " as indicated.

Down to this point, PHI , which obviously symbolizes the group flux vector, has not been needed, and has therefore been used as temporary storage for both the title and for the cross section inputs.

At order 4, we have an entry point for repeating a problem with an altered value of " ν ", neutrons/fission. This may be desirable for perturbation problems. GNU is, of course, the symbol for ν ; read in with it is an integer, $NEXT$, which controls the entry point for the next problem. GNU is in F12.6 and $NEXT$ in I2 Format; thus, column 14 of the card controls the next problem, which will enter at order 1 if $NEXT = 0$, at order 4 if $NEXT = 1$, and at order 70 if $NEXT = 2$ (or 3–9).

Orders 4 through 20 constitute a fivefold iteration loop to determine critical buckling. Buckling is first set to zero, and group fluxes computed by a source-sink balance:

$$\phi(I) = \frac{1}{N\sigma_{in}(I) + N\sigma_a(I) + B^2 D(I)} \cdot \left[B(I) + \sum_{j=1}^{I-1} N\sigma_{in}^{j \rightarrow I} \phi(j) \right]. \quad (1)$$

In Eq. (1), B^2 is the buckling (BB is its Fortran symbol) and B is the fission source to the group. The Fortran symbol for $\Phi(I)$ is $PHI(I)$.

Also cumulated are:

$$\left. \begin{aligned} FEE &= \sum_{L=1}^J \phi(L), \\ SIGA &= \sum_{L=1}^J N\sigma_a(L)\rho(L), \\ SIGF &= \sum_{L=1}^J N\sigma_f(L)\rho(L), \\ D &= \sum_{L=1}^J \phi(L)/3N\sigma_{tr}(L). \end{aligned} \right\} \quad (2)$$

TABLE III-31-III. FORTRAN LISTING OF SUBELMOE

* SUBELMOE

```

C MAKIMUM GROUPS = 20
  DIMENSION B(20),TSTR(20),SA(20),ENUFS(20),SIS(20,6),PHI(20)
100 FORMAT(10A6)
101 FORMAT ( F12.6,I2)
103 FORMAT (1H1)
104 FORMAT (3I6)
105 FORMAT(6E12.6)
106 FORMAT (E12.6)
108 FORMAT (//4H NU=,F6.3)
122 FORMAT(//5X8HBUCKLING5X11HK EFFECTIVE//)
124 FORMAT (//7X,4HFLUX//)
141 FORMAT (///4X,4HSABS,7X,5HSFISS,11X,1HD/3E12.6)

C
C ENTRY, INITIAL OR NEXT=2. NEW J,K,FSSION SPECTRUM
C
  70 READ 104, J,K
  READ 105,(B(I),I=1,J)

C
C ENTRY FOR NEXT =0. NEW PROBLEMS IN SET
C
  1 READ 100, (PHI(I),I=1,20)
  PRINT 103
  PRINT100, (PHI(I),I=1,20)

C
C PHI IS TITLE TO HERE. PUT J,K IN TITLE
C
  DO 113 I=1,J
  TSTR(I)=0.
  SA(I)=0.
  ENUFS(I)=0.
  DO 113 IM=1,6
113 SIS(I,IM)=0.
  DO 126 IK=1,J
  DO 114 I=1,20
114 PHI(I)=0.
  NOW=3+(J=IK+K-ABSF(J-IK=K))/2
  READ 105, (PHI(I),I=1,NOW)
  TSTR(IK)=PHI(1)
  SA(IK)=PHI(2)
  ENUFS(IK)=PHI(3)
  DO 126 I=4,9
  IKI=I-3
126 SIS(IK,IKI)=PHI(I)

C
C ENTRY POINT FOR NEXT=1
C
  4 READ 101,GNU,NEXT
  PRINT 108, GNU
  PRINT 122

```

I

TABLE III-31-III—Continued

	BB=0.
	DO 20 IM=1,5
	SIGA=0.
	SIGF=0.
	D=0.
	FEE=0.
	DO 12 IK=1,J
	SIN=0.
	DO 21 I=1,6
21	SIN=SIN+SIS(IK,I)
	DENOM=SA(IK)*BB/TSTR(IK)*SIN
	SIN=B(IK)
	IF (IK-1) 60,60,62
62	LAST=IK-1
	IN=(1+IK-K+ABSF(1-1K+K))/2
	DO 13 I=IN, LAST
	IKI=IK-I
13	SIN=SIN+PHI(I)*SIS(I,IKI)
60	PHI(IK)= SIN/DENOM
	SIGA=SIGA+SA(IK)*PHI(IK)
	SIGF=SIGF+ENUFS(IK)*PHI(IK)
	D=D+PHI(IK)/TSTR(IK)
12	FEE=FEE+PHI(IK)
	SIN=GNU*SIGF
	PRINT 105,BB,SIN
20	BB=BB+(SIN-1.)/D
	PRINT 124
	PRINT 106, (PHI(I),I=1,J)
	SIGA=SIGA/FEE
	SIGF=SIGF/FEE
	D=D/FEE
	PRINT 141, SIGA, SIGF, D
	IF (NEXT-1) 1,4,70
	END

The iteration algorithm is:

$$B_{n+1}^2 = B_n^2 + (\nu^*SIGF-1)/D. \quad (3)$$

This will obviously converge when the source-sink balance is achieved for the problem: $\nu^*SIGF = 1$. In practice, it has never failed to converge so far in five iterations.

In the sequence between orders 21 and 60, there is some "bookkeeping" calculation to insure that the proper inelastic sources and sinks are used. The total inelastic cross section for a group is obtained as though there were always six exit groups, which is why these cross sections were initially cleared. *IN* is an index which is either 1 or *IK-K* (*IK* being current group number), depending on whether $IK \leq K+1$; it establishes the first group which can feed group *IK* with inelastic transfers.

After five iterations, for each of which ν^*SIGF and *BB* are printed out, the problem is assumed converged, and terminates with a printout of $\phi(I)$ —group flux in order—and of *SIGA*, *SIGF*, and *D*, which, when normalized, are spectrum averaged cross sections and diffusion coefficient.

CRITICALITY OF REFLECTED SYSTEMS

The result of the fundamental mode computation is used to determine criticality of reflected systems in simple geometries. An analytical version of multigroup theory is employed.

FUNDAMENTAL MODE GROUP

Group 1 is a representation of the fundamental mode. It is characterized by core group constants—absorption and fission cross sections and diffusion

coefficient—which are the output of the Subelmoe calculations.

For the purpose of determining reflector group-constants for group 1, it is necessary to examine the Subelmoe spectrum. If, as is the case with most heavily loaded systems, this spectrum falls off rapidly on the low energy side of the peak, an effective cut-off energy may be estimated. This cut-off energy is used to distinguish reflector neutrons which are to be considered as “fundamental mode” from those which have been moderated into energy regions where few fundamental-mode neutrons exist.

(If essentially non-moderating reflectors, such as W or U-238 are employed, an estimate of reflector savings may be made in a one group formalism. Subelmoe spectral averages of absorption, fission, and reciprocal transport cross sections serve to define one-group reflector constants. In the following treatment, we assume that the reflector is a moderator.)

With this cut-off, which is, at the time of writing, subjectively estimated, group 1 constants may now be defined for the reflector. Let:

$$\phi(E) dE = \text{Subelmoe flux in } (E, E + dE)^*$$

$$E_c = \text{cut-off energy}$$

$$\tau(E, E_c) = \text{age from } E \text{ to } E_c$$

$$D(E) = \frac{1}{3N_{\sigma_{tr}}(E)} = \text{diffusion coefficient at energy } E$$

$$N_{\sigma_{SD}} = \frac{\int_{E_c}^{\infty} D(E)\phi(E) dE}{\int_{E_L}^{\infty} \tau(E)\phi(E) dE} = \begin{array}{l} \text{group 1 cross} \\ \text{section for remo-} \\ \text{val of neutrons} \end{array} \quad (4)$$

$$D = \frac{\int_{E_L}^{\infty} D(E)\phi(E) dE}{\int_{E_L}^{\infty} \phi(E) dE} = \begin{array}{l} \text{group 1 diffu-} \\ \text{tion coefficient.} \end{array} \quad (5)$$

If the reflector is absorbing, its absorption cross section is determined by spectral averaging over $\phi(E)$.

Equations (4) and (5) are identical with formulae used to determine high energy group constants in few-group theory, except that the Subelmoe spectrum above E_c is used instead of the fission spectrum.

OTHER GROUPS—CORE

The core is considered non-moderating at energies below E_c . It is therefore necessary to define only

* Presented in these sections are idealized formulae for continuous functions of energy. The Subelmoe flux and many cross-section sets exist only as bar diagrams and fine-group constants. Appropriate numerical approximations must be used; those employed in the sample calculations will be reported with the calculations.

N_{σ_a} , N_{σ_f} , and D for as many groups as are needed to describe the (energy-variable) neutron economy in the core and, crudely, the varying core blackness.

Within any given group, the spectrum is assumed to be that of a constant cross section, infinite medium moderator: dE/E . For each group, N_{σ_f} , N_{σ_a} , and D are determined by averaging over this spectrum.

OTHER GROUPS—REFLECTOR

Standard multigroup theory for an infinite moderating region is used:

$$N_{\sigma_{SD}} = 1 \left/ \int_{E_L}^{E_U} \frac{dE}{\xi N_{\sigma_S} E} \right., \quad (6)$$

$$D = \int_{E_L}^{E_U} \frac{D(E) dE}{\xi N_{\sigma_S} E} \left/ \int_{E_L}^{E_U} \frac{dE}{\xi N_{\sigma_S} E} \right. \quad (7)$$

In Eqs. (6) and (7), E_L and E_U are the lower and upper energy bounds, respectively, for a given group.

As previously mentioned, the slowing down cross sections for the last group is anomalous, if the lower energy is not zero. Neutrons moderated from that group vanish! If their contribution to criticality is to be accounted, $N_{\sigma_{SD}}$ for that group must be equated to zero; in which case, neutrons in the last group in the reflector can only choose between core return or external leakage. Adjustment of the slowing down cross section between zero and its physical value may be used to control the fate of these low energy neutrons, simulating the effects of low energy absorbing barriers, poorer core neutron economy, and so forth.

SOCRATES

Method

Socrates is a one-dimensional diffusion code for two-region problems. The core is assumed to be devoid of moderation: the system is assumed reflector-moderated. The reflector is assumed to be non-absorbing. Fission may occur in all core groups, but is a source only to group (1).

The group equations are, for N groups:

Core, Group 1

$$D_{(1)} \nabla^2 \phi(1) + D_{(1)} B^2 \phi(1) + \sum_{i=2}^N \nu N_{\sigma_f}(i) \phi(i) = 0, \quad (8)$$

Core, Groups 2-N

$$D(j) \nabla^2 \phi(j) - N_{\sigma_a}(j) \phi(j) = 0; \quad (9)$$

Reflector, Group 1

$$D(1) \nabla^2 \phi(1) - N_{\sigma_{SD}}(1) \phi(1) = 0, \quad (10)$$

Reflector, Groups 2-N

$$D(j)\nabla^2\phi(j) - N\sigma_{SD}(j)\phi(j) + N\sigma_{SD}(j-1)\phi(j-1) = 0. \quad (11)$$

In each group, flux and normal current are continuous at core reflector interface. Flux is assumed to vanish at the outer reflector boundary.

Let us normalize the net outward current at the core-reflector interface to unity. Equation (10) is a homogeneous equation in $\phi(1)$ which, for regular geometries, may be solved in closed form as the sum of two functions multiplied by arbitrary constants. One relation between the two constants is set by the exterior boundary condition. (In what follows, since this is a linear relation, this reduction is always assumed, and the homogeneous part of the diffusion equation subject to exterior boundary conditions will be assumed solvable as a single closed function with a single arbitrary constant.) The net leakage normalization of $D\nabla\phi = -1$ at the core interface then specifies completely the group 1 flux in the reflector.

Knowing group (1), Eq. (11) can be solved for $\phi(2)$ exactly, analytically, to within an arbitrary normalizer to the solution of the homogeneous part of Eq. (11). $D\nabla\phi(2)$ is, of course, also determined to within the same constant (multiplying a different function), and so $\phi(2)/D\nabla\phi(2)$, evaluated at the core interface, is a known function of this constant.

Since ϕ and $D\nabla\phi$ are both continuous, the value of its ratio must also be continuous.

But Eq. (9) is homogeneous; and if a homogeneous boundary condition exists, such as symmetry or flux vanishing at a point (which is true in all well-defined problems) then its solution has the property that $\phi/D\nabla\phi$ in the core has no undetermined constants. Its value may be evaluated at the reflector interface and equated with $\phi/D\nabla\phi$ on the reflector side; which permits the arbitrary constant in the reflector flux to be determined.

The reflector flux in group (2) is now completely determined, and a repetition of the preceding argument permits reflector fluxes in all succeeding groups to be determined.

Knowing reflector flux values at the core interface permits us to evaluate core fluxes in groups (2) et. seq. The solution of Eq. (9) may be evaluated to within an arbitrary constant, which is determined by flux continuity.

We thus know the source term for Eq. (8). Under

these conditions, and with the internal core boundary conditions and the known flux-to-current ratio at the reflector interface (known, because Eq. (10) is homogeneous), Eq. (8) may be solved for leakage current.

In general, the leakage current will be unity (our starting value) only under some critical conditions. We have chosen as the variable parameter the buckling in the two dimensions perpendicular to the dimension of the problem. This buckling is physically realized by altering the transverse dimension of the system. This alteration modifies slightly the solutions of the reflector equations: so an iteration is generated, which, with caution exercised, can be made to converge to a definite transverse dimension for which the system is critical.

The entire set of group fluxes are then known, as well as critical dimensions, and appropriate properties of the system may then be printed out.

Cylinders, Finite and Bare Axially, Reflected Radially

The method just summarized can be illustrated by reference to Table III-31-IV, which is a Fortran listing of the cylindrical version of Socrates, suitable for running on the CDC-160A computer. This program is described below. Table III-31-V should be consulted for the meaning and scale of the symbols.

Problem entry is at order 66, with the reading and subsequent printing of a title card which may be punched in any fashion in columns 1-72. This title card is the first data card. A card containing NOGPS, A, B, EPS in I12, 3E15.8 Format and another card in 3F12.6 Format containing three parameters on which the group cross-sections may depend are read in next.

Next the group constants are calculated via a subroutine SIGMA (not shown). SIGMA may be changed from problem to problem, and be either a set of Fortran value statements or a computed set depending on the parameters CO, POC, and POR. The group constants are printed out.

Next, axial buckling (BZ) is set equal to zero to start. The various functions which are the solutions of the homogeneous group equations are computed and combinations which are needed are computed and stored.

These combinations are (in conventional notation):

$$\left. \begin{aligned} \text{(Core: Let } \kappa_i^2 &= B_z^2 + \Sigma_{ai}/D_i), \\ \text{BIGK(I)} &= \kappa_i \frac{I_1}{I_0}(\kappa_i A); \end{aligned} \right\} \quad (12)$$

$$\left. \begin{aligned} \text{(Reflector: Let } \mu_i^2 &= B_z^2 + \Sigma_{SDi}/D_i), \\ \text{BIGMU(I)} &= \mu_i \frac{K_1(\mu_i A)I_0(\mu_i[A+B]) + I_1(\mu_i A)K_0(\mu_i[A+B])}{K_0(\mu_i A)I_0(\mu_i[A+B]) - I_0(\mu_i A)K_0(\mu_i[A+B])} \end{aligned} \right\} \quad (13)$$

TABLE III-31-IV. FOETRAN LISTING OF SOCRATES, A FEW-GROUP CRITICALITY PROGRAM FOR REACTIONS WITH HARD-SPECTRUM FAST CORES AND MODERATING REFLECTORS

* SOCRATES - CYLINDER

```

0  DIMENSION TITLE(12),BIGMU(6),BIGK(6),C(6),SIGSD(6),SIGA(6),
1  DR(6),DC(6),SIGF(6),S(6),AC(6)
101 FORMAT(/15H IN GROUP ORDER/7X,8HFISSIONS,6X,9HEDGE RATE,10X,
1  5HKAPPA/(3F15.8))
102 FORMAT(/31H SUBCRITICAL AT INFINITE HEIGHT)
103 FORMAT (/17H CRITICAL HEIGHT=,F7.2,2HCM)
104 FORMAT (/9H SAVINGS=,F7.2,2HCM)
105 FORMAT (1H1)
106 FORMAT (12A6)
107 FORMAT (6F12.6)
108 FORMAT(I12,3E15.8)
109 FORMAT (////)
110 FORMAT (/8X,4HSABS,7X,5HSFISS,7X,5HDCORE,7X,5HSIGSD,8X,4HDREF)
PRINT 105
66 READ 106, (TITLE(I),I=1,12)
PRINT 109
PRINT 106, (TITLE(I),I=1,12)
READ 108 ,NOGPS,A,B,EPS
READ 107, CO,POC,POR
PRINT 110
DO 23 I=1,NOGPS
CALL SIGMA(I,CO,POC,POR,SIGSD(I),SIGA(I),SIGF(I),DC(I),DR(I))
23 PRINT 107,SIGA(I),SIGF(I),DC(I),SIGSD(I),DR(I)
BZ=0.
19 DO 11 I=1,NOGPS
BIGMU(I)=SQRTF(BZ+SIGSD(I)/DR(I))
BIGK(I)=SQRTF(BZ+SIGA(I)/DC(I))
X=A*BIGMU(I)
Y=B*BIGMU(I)+X
IF (X-1.) 1,10,10
1 IF (Y-1.) 12,13,13
10 Y=EXPF(2,*(X-Y))*(1.+4,*(Y-X)/(16,*(X*Y+Y/X+X/Y))
OK=(X+1.5+.375/X)/(1.+X)
CALL DIVI(X,OI)
ZEK=1.
ZI=1.
GO TO 15
12 CALL BESSEL(Y,ZI,ZEK,OI,OK)
Y=ZEK/ZI
GO TO 14
13 Y=3.1416*EXPF(-2.*Y)*(Y+.1)/(Y+.15)
14 CALL BESSEL(X,ZI,ZEK,OI,OK)
15 QFG=(OK+Y*OI)/(ZEK-Y*ZI)
BIGMU(I)=BIGMU(I)*QFG
X=A*BIGK(I)
CALL DIVI(X,QUOTI)
11 BIGK(I)=BIGK(I)*QUOTI
C(1)=1./(DR(1)*BIGMU(1))
AC(1)=C(1)
DO 2 I=2,NOGPS
C(I)=0.

```

TABLE III-31-IV—Continued

```

AC(I)=0.
M=I-1
DO 3 J=1,M
C(J)=C(J)*SIGSD(M)/(SIGSD(I)-SIGSD(J)*DR(I)/DR(J))
AC(I)=AC(I)+C(J)
3 C(I)=C(I)-C(J)*(DR(I)*BIGMU(J)+DC(I)*BIGK(I))
C(I)=C(I)/(DR(I)*BIGMU(I)+DC(I)*BIGK(I))
2 AC(I)=AC(I)+C(I)
ARG=((2.43*SIGF(1)-SIGA(1))/DC(1)-BZ)*A**2
CALL DIVJ (ARG,QUOTJ)
SUM=0.
DO 8 K=2,NOGPS
ZK=SIGA(K)/DC(K)+BZ
8 SUM=SUM+AC(K)*SIGF(K)*(QUOTJ/A+BIGK(K))/(ARG/A**2+ZK)
X=DR(1)*A*BIGMU(1)*(1.-2.43*SUM)/DC(1)
IF (X) 20,20,7
20 BZ=2*(BZ+.000001)
GO TO 19
7 ARG=ARG+2.*ARG*(X-QUOTJ)/(ARG+QUOTJ**2)
CALL DIVJ (ARG,QUOTJ)
IF (ABSF(QUOTJ/X-1.)-.0001) 16,16,7
16 X=-ARG/A**2+(2.43*SIGF(1)-SIGA(1))/DC(1)
IF (BZ) 24,24,18
24 IF (X) 17,18,18
17 PRINT 102
GO TO 66
18 IF (ABSF(BZ-X)-EPS) 21,21,22
22 IF (ABSF(BZ-X)-10.*EPS) 5,5,6
5 BZ=X
GO TO 19
6 BZ=(BZ+X)/2.
GO TO 19
21 H=3.1416/SQRTF(BZ+.000001)
PRINT 103,H
C(1)=1.
X=0.
DO 9 K=2,NOGPS
AC(K)=AC(K)*SIGF(K)
C(K)=AC(K)*BIGK(K)/(SIGA(K)/DC(K)+BZ)
X=X+C(K)
9 C(1)=C(1)-2.43*C(K)
C(1)=C(1)/(2.43-(BZ*DC(1)+SIGA(1))/SIGF(1))
X=X+C(1)
AC(1)=SIGF(1)/(DR(1)*BIGMU(1))
DO 4 I=1,NOGPS
AC(I)=AC(I)*A/(2.*X)
C(I)=C(I)/X
4 S(I)=SQRTF(BZ+SIGA(I)/DC(I))
PRINT 101, (C(I),AC(I),S(I),I=1,NOGPS)
X=2.404826/SQRTF((2.43*SIGF(1)-SIGA(1))/DC(1)-BZ)=A
PRINT 104,X
GO TO 66
END

```

TABLE III-31-IV—Continued

SUBROUTINE BESSEL (X,ZI,ZEK,QI,OK)

 $ZI = 1. + .25 * (1. + .0625 * X^{**2}) * X^{**2}$
 $ZEK = ZI * \text{LOGF}(2./X) - .577 + (.106 + .014 * X^{**2}) * X^{**2}$
 $QI = .5 * X * (1. + (.125 + .005 * X^{**2}) * X^{**2})$

 $OK = (1./X - ZEK * QI) / ZI$

RETURN

END

C
C

SUBROUTINE DIVI(A,Q)

 $Z = .6366 * \text{ATANF}(A)$
 $Q = Z * (.785 + Z * (.040 - Z * (.022 - Z * (1.335 - 1.147 * Z))))$

RETURN

END

C
C

SUBROUTINE DIVJ (ARG,QUOTJ)

IF (ARG-5.76) 1,2,2

2 ARG=5.76

 $10 \text{ QUOTJ} = \text{ARG} * (1. + \text{ARG} * (.125 + \text{ARG} * (1./48. + \text{ARG} * (11./3072. + \text{ARG} * (19./30720. + \text{ARG}/3840.)))) / 2.$

RETURN

END

TABLE III-31-V. SOCRATES SYMBOLS (CYLINDER)

NOGPS	== number of groups
A	== core radius (cm)
B	== reflector thickness (cm)
EPS	== convergence precision for buckling (cm ⁻²)
CO,POC,POR	== parameters in cross-section formulation
SIGA(I)	== core macroscopic absorption cross sections for group I (cm ⁻¹)
SIGF(I)	== core macroscopic fission cross sections for group I (cm ⁻¹)
DC(I)	== core diffusion coefficient for group I (cm)
SIGSD(I)	== reflector slowing-down cross section for group I (cm ⁻¹)
DR(I)	== reflector diffusion coefficient for group I (cm)
BZ	== axial buckling (cm ⁻²)
BIGK(I)	== flux relaxation function in core
BIGMU(I)	== flux relaxation function in reflector
AC(I)	== flux coefficient for group I at core edge (in final printout, fission rate at core edge)
H	== critical system height (cm)

Their calculation, which occurs between orders 19 and 11, is carried out to a precision of 1%, using approximation formulae, some of which are contained in the main body of the program, and others in the subroutines "BESSEL" and "DIVI".†

The calculation of flux values at the core edge is

† The author is moderately proud of DIVI, which computes $\varrho = I_1(A)/I_0(A)$ to one percent precision over the entire range $(0, \infty)$ of A . It was derived by a fitting procedure.

performed between orders 11 and 2. The numbers produced, denoted by AC(I), are actually $\phi_i(A)/I_0(\kappa_i a)$.

The determination of critical transverse buckling is performed between orders 2 and 21. First, $B_r^2 A^2$, the radial buckling multiplied by the square of the core radius is computed, and the function $B_r(J_1/J_0)(B_r A)$ obtained with the help of subroutine DIVJ. Note that this function (times A) is a function of $B_r^2 A^2$, rather than its square root.

Next, the value which this function is supposed to have, in order that one neutron leak to the reflector, is estimated. The value of the function just procured enters this estimate weakly.

If, according to the estimate, the function should be negative or zero, there is far too much leakage: the system is highly supercritical. The transverse buckling is therefore doubled, and solution attempted anew from the point where BIGMU and BIGK are to be calculated (since these values depend on B_z^2). If, however, the function is positive, then the value of B_r^2 is found which lets $B_r(J_1/J_0)(B_r A)$ equal its estimate. This value is obtained by Newton's method of finding roots. From this B_r^2 and the known materials buckling of group (1) given by

$$B_m^2 = (\nu \Sigma_f - \Sigma_a) / D, \quad (14)$$

a new value of B_z^2 can be computed. It is denoted by X in this region of the problem (order 16 et. seq.).

If the assumed axial buckling is zero (first-go-around)

and the iterated value is negative, the system is subcritical at infinite height. This fact is printed out and a new problem called. If either the working axial buckling or the iterate is positive, a convergence test on the iteration is made.

If convergence succeeds (working value close enough to iterate) we proceed to order 21. If it fails, a further test is made to see whether convergence is close; if it is not, working value and iterate are averaged to provide a new working value; if it is, the iterate is the new working value. In any case, a failed convergence returns to the start of the problem.

When B_z^2 has converged, we proceed. Critical system height is computed and printed. Then, for each group, the fraction of the fissions occurring in that group, the edge fission rate (fission rate at the core edge from neutrons in the group, normalized to unity for the average total fission rate in the core) and κ are computed and printed.

Finally reflector savings are computed as

$$S = \frac{2.404826}{B_r} - A \quad (15)$$

and printed out.

A more complete exposition of Socrates may be found in Ref. 3.

Other Geometries

A version of Socrates using the same logic, exists to calculate one-sided reflector savings for cylinders, bare on the radius and at one end, and reflected at the other. The transformation of this version to the cylinder reflected symmetrically at each end, can be made by substituting a few cards in the Fortran deck. Persons wishing to use these programs may get them from the author.

SOME SAMPLE RESULTS

Calculations have been made for a large number of W- UO_2 cases, reflected with beryllium. Some of these are reported here to furnish examples of the output and to provide data for comparison with other methods.

SUBELMOE RESULTS

ANL-224 cross section values were processed as described¹ and used to establish fundamental mode properties. The systems reported here were mixtures of W and U-235 O_2 at densities corresponding to about 1800°C values. The parameter "C" is the volume fraction of UO_2 in the mixture. Atom densities used are (full density for each constituent):

$$\begin{aligned} \text{W: } & 0.06120 \times 10^{24} \text{ atoms W/cc} \\ \text{U-235O}_2: & 0.02295 \times 10^{24} \text{ atoms U-235/cc} \\ & 0.04590 \times 10^{24} \text{ atoms O/cc.} \end{aligned}$$

50 VOLUME PERCENT URANIA IN TUNGSTEN ZERO POROSITY

$$\text{NU} = 2.430$$

BUCKLING	K EFFECTIVE
.000000E 00	.169525E 01
.161868E-01	.977445E 00
.153472E-01	.100091E 01
.153805E-01	.999963E 00
.153791E-01	.100000E 01

FLUX

.799911E 00
.147805E 01
.182465E 01
.287659E 01
.462970E 01
.428538E 01
.400873E 01
.292448E 01
.181303E 01
.848698E 00
.347269E 00
.121566E 00
.410035E-01
.168751E-01
.213373E-02
.386719E-03
.238038E-03
.472483E-04
.432597E-04
.201128E-04

SABS	SFISS	D
.222417E-01	.158163E-01	.105284E 01

FIG. III-31.1. Sample Output—50% Volume Percent Urania in Tungsten, Zero Porosity.

Figure III-31-1 is a sample output. The buckling- K_{eff} listing is not only useful as a check that convergence is achieved, but also permits estimating K_∞ (value of K_{eff} at zero buckling) and an effective one group migration area from the slope of the K_{eff} - B^2 curve.

The flux listing is in arbitrary units, in order of group number: the first number is the flux in ANL-224 group 1, and so forth.

$SABS$, $SFISS$, and D are the spectrum averages of $N\sigma_a$, $N\sigma_f$, and D , respectively, with macroscopic cross sections in cm^{-1} and D in cm .

FIG. III-31-2. SUBELMOE Spectra (at zero porosity). ANL Neg. No. 112-5838.

TABLE III-31-VI. VALUES OF FUNDAMENTAL-MODE-GROUP CONSTANTS FOR W-UO₂ CORES AT ZERO POROSITY

$C, v/o$	$N\sigma_{a0}, \text{cm}^{-1}$	$N\sigma_{f0}, \text{cm}^{-1}$	D_0, cm
0.60	0.02464	0.01871	1.106
0.50	0.02224	0.01582	1.053
0.45	0.02105	0.01436	1.027
0.40	0.01986	0.01289	1.001
0.30	0.01754	0.00995	0.948

Fitting Formulae: $D_0 = 0.790 + 0.527 C \text{ (cm)}$
 $N\sigma_{f0} = 0.00120 + 0.02921 C \text{ (cm}^{-1}\text{)}$
 $N\sigma_{a0} = 0.01042 + 0.02367 C \text{ (cm}^{-1}\text{)}$

Figure III-31-2 presents the spectra computed for zero porosity at three different urania concentrations. Note that the height of the bar for each group is $\phi/\delta u$ for that group. The bar diagrams are the true results; the curves are drawn in by eye simply as a graphic aid.

From Fig. III-31-2, it is clear that, as urania concentration decreases, the spectrum (as expected) becomes softer. Even at 30% urania concentration, there are very few neutrons below group 12.

Table III-31-VI gives the values of $N\sigma_a$, $N\sigma_f$, and D for core compositions of variable urania concentration ($C \equiv v/o \text{ UO}_2$ in fuel meat) at zero porosity. Below are fitting formulae for these quantities, which are scripted with zero to indicate the zero porosity. The fitting formulae are perfect to the number of significant figures retained; notwithstanding this, they are merely

TABLE III-31-VII. FEW GROUP CROSS SECTIONS OF CORE MATERIALS
(Entries are cross sections in barns)

Group	Type of Cross Section (Element-Type)					
	W_a	W_{tr}	O_{tr}	U_a^{235}	U_f^{235}	U_{tr}^{235}
2	0.46	11.44	3.39	4.88	3.48	14.77
3	1.15	15.17	3.62	10.26	6.89	20.79
4	2.92	18.74	3.62	24.91	16.87	42.08

fitting formulae, as evidence of which, observe that $N\sigma_{f0}$ has a positive value when c goes to zero. Thus, the formulae may only be applied within the range of $0.3 \leq c \leq 0.6$.

SOCRATES RESULTS

Presented here are calculations of the reflector savings in cylindrical geometry, of Be reflectors of various thicknesses (and full density) with cores of three U-235O₂ concentrations in W (and full density).

A four group structure is used. Group 1 is the fundamental mode in the core. It was taken as cutting off at 41 keV, which, from inspection of Fig. III-31-2, omits a relatively small part of the fundamental mode. Forty-one keV is also the lower bound of group 10 in the ANL-224 set.

Group 2 of these calculations is taken as groups 11, 12, 13, 14 of ANL-224; Group 3, as groups 15, 16, 17, 18, 19 of ANL-224; Group 4, as groups 20, 21, 22 of ANL-224.

Cross Sections: Core

Core constants for Group 1 are taken directly from the fitting formulae of Table III-31-VI.

For groups 2, 3, and 4, the ANL-224 cross sections for fission and absorption and the reciprocal of the transport cross section were lethargy averaged for each of the core materials: U-235, O, and W. Results are listed in Table III-31-VII.

The cross section for oxygen absorption is taken as zero.

The cross sections for both U-235 and W are "unshielded" for resonance self-protection. This makes a major difference only if the shielding leads to a major change in U-235 fission/total absorption, which is unlikely.

Beryllium Cross Sections

For group 1, use was made of a cross section set in ANL-224 format.⁴ This set has negative absorption in groups 1 and 2, corresponding to $(n, 2n)$ reactions; and positive absorption in group 3 because of (n, α) reaction dominance. Thus, within each ANL-224 group, there is a value of: σ_a ; σ_{tr} ; and a set of $\sigma_{i \rightarrow i+k}$. Absorption was eliminated in each group by setting σ_a to zero, writing

TABLE III-31-VIII. Be CROSS SECTIONS IN SOCRATES
Group 1 (σ in barns)

ANL-224 Group	σ_{tr}	$\sigma_{i \rightarrow i+1}$	$\sigma_{i \rightarrow i+2}$	$\sigma_{i \rightarrow i+3}$	$\sigma_{i \rightarrow i+4}$	$\sigma_{i \rightarrow i+5}$	$\sigma_{i \rightarrow i+6}$	T_i
1	0.805	0.124	0.112	0.117	0.312	0.020	0	3.478
2	1.371	0.475	0	0.32	0.06	0.09	0.11	2.598
3	1.104	0.433	0				0	3.195
4	2.494	1.025						1.103
5	3.536	1.455						0.712
6	3.467	1.428						0.518
7	4.024	3.500						0.316
8	4.821	2.293						0.245
9	5.371	2.355						0.155
10	5.417	2.434	0				0	0.076

$\sigma_{tr} = \sigma_{tr} - \sigma_a$, and renormalizing $\sigma_{i \rightarrow i+k}$ so that their sum equals the input value of $\sum_k \sigma_{i \rightarrow i+k}$. This admittedly crude treatment of Be multiplication and absorption is, it is hoped, made more permissible by the fact that all reactions in Be occur within Socrates group 1.

The resulting cross sections were then used to calculate ages to the bottom of group 10 from the formulae:

$$\tau_{10} = \frac{1}{3N^2\sigma_{tr}(10)\sigma_{10 \rightarrow 11}} \quad (16)$$

$$\tau_i = \left[\frac{1}{3N^2\sigma_{tr}(i)} + \left(\sum_k \sigma_{i \rightarrow i+k} \tau_k / \sum_k \sigma_{i \rightarrow i+k} \right) \right] \quad (17)$$

Values of the adjusted cross section¹ set, and of $T_i \equiv 3N^2\tau_i$, are given in Table III-31-VIII.

A peculiar feature of this data set is that the age from group 2 is less than that from group 3, due to the fact that the $(n, 2n)$ reaction is an excellent energy reducer. Although this rationalization is correct, the result is not reasonable in detail, and requires further improvement.

Equations (4) and (5) were evaluated, using Subelmoe fluxes for ϕ and replacing the integrals by sums over ANL-224 groups 1 to 10.

The Subelmoe fluxes for several U-235 concentrations were used, and the spectrum dependence on concentration is responsible for a dependence of the Be group 1

TABLE III-31-IX. FORTRAN LISTING OF ZERO POROSITY "SIGMA" SUBROUTINE

Under Each Socrates Group Number in the left hand margin are listed in order for that group: DC—Core Diffusion Coefficient; F—Macroscopic Fission Cross Section in Core; ABS—Macroscopic Absorption Cross Section in Core; DR—Reflector (Be) Diffusion Coefficient; SD—Reflector Group Transfer Cross Section.

```

SUBROUTINE SIGMA (I,C,PC,PR,SD,ABS,F,DC,DR)
GO TO (1,2,3,4), I
1 DC=.79+.527*C
F=.0012+.02921*C
ABS=.01042+.02367*C
DR=.755/(1+.5*C)
SD=.0625+.02454*C
RETURN
2 DC=1./(2.1+.616*C)
F=.07987*C
ABS=.02813+.08385*C
DR=.495
SD=.0666
RETURN
3 DC=1./(2.785+.855*C)
F=.15813*C
ABS=.07038+.16509*C
DR=.486
SD=.0764
RETURN
4 DC=1./(3.441+.045*C)
F=.38717*C
ABS=.17870+.39298*C
DR=.486
SD=.00001
RETURN
END

```

ZERO POROSITY = .45,20,10

SABS	SFISS	DCORE	SIGSD	DREF
.021071	.014344	1.027150	.051457	.974193
.065862	.035941	.548606	.066600	.495000
.144670	.071158	.416623	.076400	.486000
.355541	.174226	.292333	.000010	.486000

CRITICAL HEIGHT= 39.59CM

IN GROUP ORDER		
FISSIONS	EDGE RATE	KAPPA
.85934141	.42410956	.16373685
.05960931	.22746576	.35545660
.02910543	.18020206	.59459281
.05194385	.58807970	1.10567273

SAVINGS= 8.48CM

FIG. III-31-3. Sample Output—Zero-Porosity Core, Zero-Porosity Reflector.

crosssections on C . The fitted formula for thick Be are:

$$\begin{aligned}\sigma_{tr}(1) &= 3.58 - 1.79C \text{ barns} \\ \sigma_{SD}(1) &= 0.507 - 0.189C \text{ barns} \\ D(I) &= 0.755/(1 - 0.5C) \text{ cm} \\ N_{\sigma SD}(1) &= 0.0625 - 0.02454C \text{ cm}^{-1}\end{aligned}$$

For groups 2, 3, and 4 the cross section curve of BNL-325⁵ and Eqs. (6) and (7) were used. For group 4, σ_{SD} was set equal to 0.00001, a number which is essentially zero, but which avoids mathematical complexities in Socrates.

Results of the Be computations are summarized, along with the core constants in Table III-31-IX, which is the printout of the SIGMA subroutine used for these calculations.

Results

Results are reported here for a variety of zero-porosity core, zero-porosity reflector calculations. Figure III-31-3 is a sample output of a single calculation.

REFLECTOR SAVINGS

The mathematical precision of Socrates is about 0.2 cm in savings for most of the problems reported here.

A number of problems were run at a constant reflector thickness of 10 cm. Concentration was varied between $C = 0.3$ and 0.6 , and core radius between 20 and 30 cm. No dependence of savings on concentration was observed, nor directly, any dependence on core radius. average value was obtained from 16 problems:

$$S_{10} = 8.3 \pm 0.2 \text{ cm}, \quad (18)$$

TABLE III-31-X. REFLECTOR SAVINGS VERSUS Be THICKNESS

t , cm	S/t (computed)
0	1.025
2.5	0.82 ± 0.15
5	0.97 ± 0.08
7.5	0.89 ± 0.05
10	0.83 ± 0.02
12.5	0.70 ± 0.02

where S_{10} is the savings due to 10 cm of Be. The highest value was 8.54 and the lowest 7.93. The 7.93 cm savings value was for one of the shortest cores (critical at 29 cm), and other, still shorter cores, gave values of about 8.0 cm; so that a very slight trend toward decreasing savings with short cores (and therefore short reflectors) is discernible. This is to be expected, since the small height means a large axial buckling which has the same effect as increasing reflector absorption.

For a core radius of 20 cm and $C = 0.6$, many calculations were performed with varying reflector thickness, from $t = 2.5$ to 12.5 cm. At zero thickness, $S/t = D_{\text{Core}}/D_{\text{Ref}} = 1.025$. Table III-31-X presents the values of S/t computed. They all seem reasonable except for the 2.5 cm value, which would seem to require adjustment to the top of its estimated error range.

FLUX PEAKS

The same set of problems used to generate savings data also can be examined for the effect of reflector

TABLE III-31-XI. EDGE/AVERAGE FISSION RATES

A. Variable Reflector Thickness ($C = 0.6$, Core radius = 20 cm)

Reflector Thickness, cm	Total Edge/Average	Edge (Group 4)/Average (all groups)
2.5	0.36	0.02
5	0.69	0.11
7.5	1.02	0.29
10	1.27	0.47
12.5	1.44	0.61

B. Variable Core Concentration (Reflector thickness = 10 cm)

1. Core Radius = 20 cm

C , v/o UO_2	Total Edge/Average	Edge (Group 4)/Average (all groups)
35	1.49	0.64
40	1.48	0.63
45	1.42	0.59
50	1.37	0.55
55	1.32	0.51
60	1.27	0.47

2. Core Radius = 25 cm

30	1.40	0.62
35	1.39	0.61
40	1.35	0.57
45	1.29	0.52
50	1.24	0.48
55	1.17	0.44
60	1.13	0.41

thickness on edge peaking of the flux. Table III-31-XI gives edge peaking rates (edge fission rate/radial average fission rate) and the contribution of group 4—low energy neutrons—to it.

The trends in part B of Table III-31-XI are readily explained: as core radius increases, or as core concentration increases, critical system height decreases. This increases axial leakage in the reflector, which leakage competes with the moderation process, slightly decreasing reflector savings (as previously noted), and significantly decreasing the soft component of reentrant current.

As a last result, the fraction of all fissions in the

various groups may be examined. In the worst case ($C = 35$ v/o core radius 20 cm, reflector thickness 11 cm) 84% of all fissions occurred in the fundamental mode, and only 6% in group 4. Thus, it is apparent that, although the reflector is an excellent moderator, its major effectiveness is due to good reflection of the fundamental mode. It is therefore expected that reflectors thicker than 12.5 cm will add only marginally to reactivity, while continuing to aggravate the core edge peak (unless it is destroyed by an absorber); that the use of absorbing control in the reflector is only likely to be effective if high energy neutrons are absorbed; and that the core edge peak may be tailored, or used to provide first-pass preheating, without major effect on criticality. Finally, it may be concluded that Be is as superior a reflector for this type of system as it is for water-moderated thermal reactors or for small fast assemblies.

SUMMARY AND CONCLUSIONS

A set of programs is presented for calculating criticality in systems with hard core spectra and moderating reflectors. The programs may be run quickly on the CDC-160A computer, or linked into more comprehensive survey programs on a computer such as the CDC-3600.

Some sample results for systems of interest for nuclear heat-transfer rockets are presented. These results may be used to compare with more detailed theoretical methods. Although the physical description underlying these programs is admittedly crude, the results of the calculations are intuitively reasonable.

REFERENCES

1. D. M. O'Shea, H. H. Hummel, W. B. Loewenstein, D. Okrent and C. N. Kelber, *26-Group Cross Sections*, ANL-6858 (to be published).
2. A. L. Rago and H. H. Hummel, *ELMOE: An IBM-704 Program Treating Elastic Scattering Resonances in Fast Reactors*, ANL-6805 (1964).
3. B. I. Spinrad, *A Simplified Computational System for Fast Reactors with Moderating Reflectors*, ANL-7141 (to be published).
4. P. Greebler, General Electric Co. to D. O'Shea, Argonne National Laboratory (Private Communication).
5. *Neutron Cross Sections*, BNL-325, Second Edition (1958).

I-32. An Estimation of the Temperature-Dependent Cross Sections for Tungsten Isotopes in the Doppler Region

R. N. HWANG

A preliminary study has been made to evaluate the temperature-dependent cross sections of tungsten in the energy regions of interest in fast-reactor studies.

The most recent measurements of the resonance parameters for tungsten¹ have provided data to about 500 eV. Since the measured values seem to show that the level densities of individual tungsten isotopes are rather low, with the exception of perhaps W-183, it is extremely difficult to obtain a reliable set of average parameters to be used in the statistical region. However, Table III-32-I gives average parameters for individual isotopes that have been obtained by using known parameters at low energies and theoretical predictions as summarized by J. Harvey.²

Since the resonance self-shielding and Doppler effect rely heavily on the $l = 0$ neutron, the S -wave capture cross sections have been carefully examined. In order to check the accuracy of these S -wave parameters, calculations for the average infinite diluent cross sections above 500 eV were made. The Porter-Thomas distribution for the neutron width was used in the statistical averaging. The results seem to show excellent agreement with the measured values reported by S. Kaphigasheve and Y. Popov³ below about 7 keV where the S -wave is believed to be the major contributor. The P -wave contribution above 7 keV was also estimated and the results are given in Fig. III-32-1 together with the S -wave contribution.

Calculations of the capture cross sections for the case $\Sigma_p = 30$ b/W-atom, taking into account the resonance self-shielding, were also made for various temperatures. Table III-32-II gives the results for some typical energy groups consistent with the ANL-26-group structure.

Currently there is significant interest in highly enriched compact reactors containing high concen-

FIG. III-32-1. Calculated (σ_r) for Natural Tungsten (Infinite dilution). ANL Neg. No. 112-5466.

TABLE III-32-II. TUNGSTEN MULTIGROUP CAPTURE CROSS SECTIONS (BARNs)
($\sigma_p = 30b$)

Energy Group	E_L , ^a keV	$\tilde{\sigma}_\gamma$ (S-wave)				$\tilde{\sigma}_\gamma^b$ (P-wave)
		300°K	750°K	1500°K	2000°K	
9	67.0	0.04358	0.04399	0.04430	0.04453	0.08269
10	40.7	0.06773	0.06870	0.06941	0.06992	0.09564
11	25.0	0.1034	0.1056	0.1072	0.1083	0.1046
12	15.0	0.1517	0.1598	0.1632	0.1656	0.1079
13	9.1	0.2309	0.2397	0.2467	0.2518	0.1002
14	4.0	0.3580	0.3773	0.3927	0.4041	0.08523
15	2.6	0.5424	0.5818	0.6141	0.6387	—
16	2.0	0.6873	0.7456	0.7946	0.8326	—
17	1.4	0.7968	0.8707	0.9341	0.9839	—
18	1.0	0.9442	1.0398	1.1243	1.1922	—
19	0.55	1.1444	1.2689	1.3842	1.4791	—

^a Lower boundary of the group; upper boundary of Group 9 = 110 keV

^b P -wave $\tilde{\sigma}_\gamma$ is taken to be temperature independent

trations of natural tungsten surrounded by a beryllium reflector. The reflector returns Doppler neutrons in the low keV region to the core. This raises the question of accurate neutronic description of the returned neutrons. It is not likely that the resonance absorption of the

TABLE III-32-I. AVERAGE S -WAVE PARAMETERS FOR TUNGSTEN ISOTOPES

(Note: P -wave strength functions were taken to be 0.3×10^{-4} for all tungsten isotopes)

Isotope	Average Spacing $\langle s \rangle$ eV	Strength Function $\langle \Gamma_n^0 \rangle / \langle s \rangle$	Γ_γ , mv
W-182	60	2.5×10^{-4}	60
W-183	58.5 ($J = 0$), 19.6 ($J = 1$)	2.5×10^{-4}	100
W-184	105	3.0×10^{-4}	60
W-186	105	3.0×10^{-4}	60

system can be described in all detail by the conventional method based on the narrow resonance approximation and the equivalence relation. Much theoretical and experimental work is needed in this area.

REFERENCES

1. A. B. Smith, ed., Reports to the AEC Nuclear Cross Sections Advisory Group, Wash.-1046 and 1048, (1964).

2. J. A. Harvey, *General Survey on Time-of-Flight Measurements Applied to Nuclear Physics*, Proc. Symp. on Neutron Time-of-Flight Methods, Brussels, September 1961, pp. 23-56.
3. S. V. Kapchigasheve and Yu P. Popov, *Capture Cross Sections in Construction Materials for Neutrons with Energies up to 50 KeV*, Atomnaya Energiya, **15**, No. 2, 120-126 (1963).

III-33. Reactivity Coefficients of Fuel Expansion into Moderating and Marginally Moderating Reflectors

W. B. LOEWENSTEIN, K. K. ALMENAS, W. G. KNAPP and R. C. DOERNER

The reactivity change produced by the expansion of fuel plates into the axial reflectors of critical assemblies such as the ZPR-3, ZPR-6 and ZPR-9 is a very important safety parameter. It is this "expansion coefficient of reactivity" which provides the negative temperature coefficient of reactivity required for the safe operation of the assemblies. The dependence of the sign and magnitude of the expansion coefficient on the reflector material is of particular importance to the ZPR-9 experimental program. An integral part of the program is the construction of critical assemblies reflected by a variety of reflector materials. Accordingly, an experimental apparatus was constructed to measure reactivity effects related to core expansion and the problem was analyzed theoretically. To date, measurements of fuel expansion into Al and Al_2O_3 reflectors have been made. This report describes the experiments, the experimental results and the calculations of fuel expansion coefficients of reactivity into marginally moderating (Al and Al_2O_3) reflectors and moderating (Be) reflectors.

EXPERIMENTAL APPARATUS AND EXPERIMENTAL RESULTS

A schematic representation of the experimental apparatus is given in Fig. III-33-1. The mechanism fits into a ZPR-9 drawer and consists of two special fuel plates extending from the core center to the core edge. Each plate is split along a diagonal and is thus composed of two identical triangular pieces. The lower triangular piece is fixed in the drawer and the upper piece is attached to a rod which extends through the axial reflector to an activating mechanism located outside of the assembly. By means of this rod, the upper piece can be moved along the core axis over short predetermined distances without disturbing any other core parameter. The displacement of the upper section closely resembles the localized spatial transfer of fuel which occurs during an expansion caused by a temperature rise. As in the case of expansion caused by temperature rise, a small fraction of the fuel is removed uniformly from the whole length of the plate and transferred a short dis-

FIG. III-33-1. Schematic Representation of Mechanism for Reactivity Measurement (Clearance between plates and between plates and reflector are exaggerated). ANL Neg. No. 112-5636.

ce into a region adjoining the reflector. Specially located reflector blocks allow the transfer to occur directly into the reflector.

In an experiment, the expansion mechanism is usually located in the central drawer in order to maximize the reactivity effect of plate motion. The plate motion itself is activated by remote control when the assembly is in a critical or near critical condition. Two methods have been employed to measure the resulting reactivity change:

A. MEASUREMENT OF RESULTING POSITIVE OR NEGATIVE PERIOD

After a time lapse allowing the transients to die out, the power rise is fed into a 400 channel analyzer. The analyzer data is fitted by a least squares method to a single exponential which then represents the period produced by the plate motion.

B. THE REACTIVITY SHIFT PRODUCED BY THE PLATE MOTION

The reactivity shift produced by the plate motion is balanced by a calibrated sensitive control rod.

The apparatus has been used to measure the effect of expanding a fuel plate into an Al and a partial (9 drawers) Al_2O_3 reflector. The upper plate was moved by 0.5 cm. For a 12 in. long fuel section, a 0.5 cm expansion is equivalent to the expansion caused by a 1200°C temperature rise. Experimental results are given in Table III-33-I.

ANALYTICAL ASPECTS

The qualitative behavior of fuel plate expansion may be most easily interpreted with diffusion theory perturbation theory. The important terms determining the reactivity change accompanying fuel expansion are:

$$\begin{aligned} & \int_V N(\mathbf{r}) \left\{ \int_{E'} \phi^*(E', \mathbf{r}) \chi(E') dE' \int_E \nu(E) \sigma_f(E) \phi(E, \mathbf{r}) dE \right. \\ & - \int_E \phi^*(E, \mathbf{r}) [\sigma_f(E) + q\sigma_{n,\gamma}(E)] \phi(E, \mathbf{r}) dE \\ & - \int_E \nabla \phi^*(E, \mathbf{r}) D(E, \mathbf{r}) \nabla \phi(E, \mathbf{r}) dE \\ & \left. + \int_{E'} \int_E [\phi^*(E', \mathbf{r}) - \phi^*(E, \mathbf{r})] \right. \\ & \quad \left. \cdot \sigma(E \rightarrow E') \phi(E, \mathbf{r}) dE dE' \right\} d\mathbf{r}, \end{aligned}$$

where

$$\begin{aligned} V & \equiv \text{unperturbed core volume} \\ N(\mathbf{r}) & \equiv \text{number of U-235 atoms/cc} \\ \phi^*(E, \mathbf{r}) & \equiv \text{neutron importance function} \\ \phi(E, \mathbf{r}) & \equiv \text{neutron flux} \end{aligned}$$

TABLE III-33-I. EXPERIMENTAL RESULTS FOR SIMULATED FUEL PLATE EXPANSION

	Reactivity Change Measured by Method A, 1h/0.5 cm	Reactivity Change Measured by Method B, 1h/0.5 cm
Expansion into full Al axial reflector	-1.25	-1.24
Expansion into cluster of 9 Al_2O_3 axial reflector drawers	-1.18	-1.29

$$\begin{aligned} \chi(E) & \equiv \text{fission spectrum} \\ \nu(E) & \equiv \text{neutron yield/fission of U-235} \\ \sigma_f(E) & \equiv \text{fission cross section for U-235} \\ q & \equiv \text{constant to allow for homogenization of U-235 with other core materials} \\ \sigma_{n,\gamma}(E) & \equiv \text{parasitic capture cross section for U-235 and other materials that move} \\ D(E, \mathbf{r}) & \equiv \text{diffusion coefficient for U-235 and other materials that move, normalized by } N(\mathbf{r}) \\ \nabla \phi^*(E, \mathbf{r}) & \equiv \text{gradient of neutron importance} \\ \nabla \phi(E, \mathbf{r}) & \equiv \text{gradient of neutron flux} \\ \sigma(E \rightarrow E') & \equiv \text{inelastic and elastic transfer cross sections for all materials with } E' < E. \end{aligned}$$

A thermal expansion will cause $N(\mathbf{r})$ to decrease. However, the integral will have to be evaluated over the larger expanded volume. Assuming only axial expansion the reactivity change (Δk) in a one group formalism, including scattering, is given by

$$\begin{aligned} \Delta k & = k_{\text{final}} - k_{\text{initial}} \\ & = \frac{\Delta V}{F} \left\{ N \left[\frac{P_b - P_c}{1 + (\Delta V/V)} \right] - \frac{1}{N} \left[R_c + \left(1 + \frac{\Delta V}{V} \right) R_b \right] \right\}, \end{aligned}$$

where

$$\begin{aligned} \Delta V & \equiv \text{increased "core" volume due to expansion of fuel (positive)} \\ V & \equiv \text{unperturbed core volume} \\ F & \equiv \text{reactivity normalization term (positive)} \\ N & \equiv \text{number of U-235 atoms/cc} \\ P_c & = \frac{\phi^* \sigma \phi^c}{\phi^* \sigma \phi^b} \equiv \text{average over core} \\ P_b & = \frac{\phi^* \sigma \phi^b}{\phi^* \sigma \phi^c} \equiv \text{value at core boundary} \\ R_c & = \frac{\nabla \phi^* (1/\sigma_t) \nabla \phi^c}{\phi^* \sigma \phi^c} \equiv \text{average over core} \\ R_b & = \frac{\nabla \phi^* (1/\sigma_t) \nabla \phi^b}{\phi^* \sigma \phi^b} \equiv \text{value at core boundary} \\ \sigma & \equiv \nu \sigma_f - \sigma_f - \sigma_{n,\gamma} \\ \sigma_t & \equiv 3 \times \text{transport cross section.} \end{aligned}$$

If Δk is to be negative (as $\Delta V \rightarrow 0$), the following

FIG. III-33-2. Fission Distribution and Reactivity Worth of U-235: a. Along Axial Centerline of ZPR-9, Assembly No. 6; b. Along Central Axis of ZPR-9, Assembly No. 7; c. Along Axial Centerline of a ZPR-9 Assembly No. 6 Type Core Reflected by Be. *ANL Neg. No. 112-5647.*

dition must be satisfied:

$$N(P_b - P_c) < \frac{1}{N} (R_c + R_b).$$

If scattering effects are negligible, as they usually are for fissionable materials, the required condition is simply

$$\overline{\phi^* \sigma \phi^b} < \overline{\phi^* \sigma \phi^c}.$$

The most direct method of gaining insight into the fuel expansion coefficient consists of measuring and calculating the spatial distribution of the fuel worth along the axis of a given reactor. A more complex but very flexible second method involves the prediction of k_{eff} for expanded and unexpanded cores. Because of the small expansion regions, these calculations are usually performed with transport theory (e.g., S_N methods). Both types of calculations have been used to determine the fuel expansion coefficient for a range of probable ZPR-9 assemblies.

FUEL PLATE EXPANSION INTO marginally MODERATING REFLECTORS

Reflector materials like Al, especially Al_2O_3 , have neutron spectra significantly softer than the spectrum of the fast reactor core they may surround. Therefore, these materials have been termed "marginally moderating." In many neutronic aspects significant qualitative differences exist between the marginally moderating reflectors (e.g., Al and Al_2O_3) on the one hand, and good moderating reflectors (e.g., Be) on the other. This difference is quite pronounced in the effect these two types of reflectors have upon the reactivity coefficient of core fuel plate expansion. Even infinitely thick reflectors of Al or Al_2O_3 cannot change the sign of the core fuel-plate expansion coefficient of reactivity from negative to positive. Moderately thick Be reflectors can cause the sign to change. The extent of this neutronic difference is illustrated in Fig. III-33-2 a-c. The spatial distributions along the core axis of the U-235 worth and the U-235 fission rate for identical cores which are axially reflected by three different materials are shown. The cores of Fig. III-33-2 a and b are reflected by 30 cm of Al and Al_2O_3 respectively. The core of Fig. III-33-2c is reflected by 20 cm of Be.

These figures may be used to estimate the fuel plate expansion coefficient of reactivity. The results are given in Table III-33-II along with corresponding experimental data from Table III-33-I.

The estimates in Table III-33-II are made for the expansion of two $\frac{1}{8}$ in. fuel plates located along the centerline of the core. This analytical evaluation affords a direct comparison with the experimental data.

TABLE III-33-II. FUEL PLATE EXPANSION DATA

Expansion Into	Estimated from Fig. III-33-2a-c 1h/0.5 cm	Experimental Data from Table III-33-I 1h/0.5 cm
Al reflector	-1.65	-1.29
Al_2O_3 reflector	-1.36	-1.24
Be reflector	-3.6	—

The large difference between the power and the fuel worth at the core edge for these two classes of reflectors (e.g., Al and Al_2O_3 vs Be) is primarily due to thermal and near-thermal neutrons. Fifty percent of the neutrons producing the core edge power spike of the reactor with a 20 cm Be reflector have energies below 0.1 eV. The corresponding neutron fraction for the 30 cm Al_2O_3 reflector is only 4 %.

FUEL PLATE EXPANSION INTO A MODERATING REFLECTOR

The expansion of a fuel plate into a 20 cm thick Be reflector can cause a reactivity gain. In general, this is an undesirable condition for operating a critical facility and it stimulated investigations toward providing an acceptable negative fuel plate expansion coefficient, even with an axial Be reflector. Possible critical assembly modifications which singly or in combination can produce a negative expansion coefficient include:

1. Decreasing the reflector thickness.
2. Decreasing the moderator density in the reflector.
3. Using only one axial moderating reflector—the other axial reflector being Al_2O_3 , U, etc.
4. Inserting an absorbing "sheet" between the core and axial moderating reflector.
5. Selectively "staggering" the fuel columns.

Calculations to quantitatively evaluate the effect of such modifications are summarized in Table III-33-III.

The sign of the fuel plate expansion effect may be qualitatively evaluated by inspection of Fig. III-33-3. The spatial distribution of the energy integrated $\phi^* \sigma \phi$ is given as a function of radius for 300 and 1500 liter core volume reactors. Each core volume was studied with both a 10 and 20 cm Be reflector. It can be seen that for a 10 cm reflector, the ratio of $\phi^* \sigma \phi^b / \phi^* \sigma \phi^c$ is less than unity and for a 20 cm reflector it is greater than unity. The radial distribution of $\phi^* \phi$ is also given simply to demonstrate that the effective $\sigma = (\nu - 1 - \alpha) \sigma_f^{U-235}$ increases with increasing radius near the core boundary.

Figure III-33-3 is a graphic demonstration that the axial "fuel importance" must be carefully considered in planning a critical experiment with an axial moderating reflector. The reduction of the "fuel importance" peak

TABLE III-33-III. TYPICAL CORE EXPANSION COEFFICIENTS OF REACTIVITY (PRINCIPALLY LIGHT ELEMENT REFLECTORS)

Core Volume, liters	Description	Reflector		$(\Delta k/k)/(\Delta L/L)^a$	$(\Delta k/k)/^\circ\text{C}^b, \times 10^{-6}$
		Material	Thickness, cm		
300	Core uniformly expands to fill 0.63 cm gap between core and reflector.	Be	20	-0.12	-1.7
300			10	-0.17	-2.4
800			20	-0.10	-1.4
800			10	-0.16	-2.2
300	Core uniformly expands 0.63 cm; reflector is pushed outward 0.63 cm; reflector density conserved.	Be	20	-0.16	-2.2
800				-0.15	-2.1
300	Core uniformly expands 0.63 cm; reflector is pushed outward 0.63 cm; reflector mass conserved.	Be	20	-0.23	-3.2
800				-0.22	-3.1
300	U-235 uniformly expands to fill 0.63 cm gap between core and reflector	Be	20	+0.33	+4.6
300			10	-0.23	-3.2
800			20	+0.38	+5.3
800			10	-0.23	-3.2
300	U-235 "staggered"; long fuel columns expand into region of low fuel worth; reflector unperturbed.	Be	20	-0.18	-2.6
800			20	-0.20	-2.8
300	U-235 uniformly expands 0.63 cm; reflector is pushed outward 0.63 cm; reflector density conserved.	Be	20	+0.28	+3.9
300			10	~ -0.24	~ -3.4
800			20	+0.32	+4.5
800			10	~ -0.22	~ -3.1
300	U-235 uniformly expands 0.63 cm; reflector is pushed outward 0.63 cm; reflector mass conserved.	Be	20	+0.19	+2.7
300			10	~ -0.30	~ -4.2
800			20	+0.24	+3.4
800			10	~ -0.27	~ -3.8
300	U-235 uniformly expands 0.63 cm; reflector is pushed outward 0.63 cm; reflector mass conserved.	50 v/o Al 50 v/o Be	20	-0.11	-1.5
800				-0.18	-2.5
300	U-235 uniformly expands 0.63 cm; reflector is pushed outward 0.63 cm; reflector mass conserved.	0.5 cm W ^c + Be	20	-0.18	-2.5
300	U-235 uniformly expands 0.63 cm; reflector is pushed outward 0.63 cm; reflector mass conserved.	U	20	-0.43	-6.0
800		U	20	-0.45	-6.3

^a Reactivity change $(\Delta k/k)$ per fractional change in axial length $(\Delta L/L)$ of all fuel plates or core composition (as described).

^b Assuming $(\Delta L/L)/^\circ\text{C} \approx 14 \times 10^{-6}/^\circ\text{C}$.

^c 0.5 cm W shell between core and Be reflector.

in Fig. III-33-3, accomplished by halving the reflector thickness, has over-all effects similar to some of those summarized in Table III-33-III. There the "fuel importance" was reduced by lowering the Be moderator density in the reflector or by inserting a strongly absorbing region between the core and the axial reflector.

Figure III-33-3 may also be qualitatively used to estimate how the fuel plates should be selectively "staggered" to provide reactivity reduction upon heating fuel plates. Any axial expansion that effectively moves fuel from positions of greater to lesser $\phi^*\sigma\phi$ will tend toward reactivity reduction. With reference to Fig. III-33-3, a single fuel column ~ 35 cm long when

the half core height is ~ 41 cm is appropriate for the 300 liter core. The illustrated 1500 liter core single fuel column could be ~ 63 cm long when the half core height is ~ 71 cm.

In addition to causing reactivity reduction due to fuel expansion of the above cited "long" fuel column, there will be a tendency for reactivity addition due to expansion of the "short" staggered fuel column near the core boundary. This smaller reactivity addition can be minimized if the "long" fuel column pushes strongly absorbing material (e.g., W or U-238) against the moderating reflector. In addition, it is then most desirable if the absorbing material pushes a significant

CURVE	CORE VOLUME, liter	REFLECTOR THICKNESS, cm
PARAMETER: $\phi^*(\nu-1-\alpha)\sigma_f$ (U-235) ϕ		
A	1500	10
A-1	300	10
B	1500	20
B-1	300	20
PARAMETER: $\phi^*\phi$		
C	1500	10
C-1	300	10

FIG. III-33-3. Spatial Distribution of U-235 Importance (300 and 1500 liter cores). ANL Neg. No. 112-2986.

portion of the reflector as opposed to a single column. The effect of fuel pushing an absorber against the reflector is demonstrated in Fig. III-33-4.)

The core expansion studies cited above are based on multienergy group spherical analyses. For non-moder-

ating reflector systems, spherical expansion calculations have been found to represent fairly well the neutronics of a cylindrical system core expansion. (These systems usually have a ratio of core height to diameter, such that $0.7 < L/D < 1.2$.) The neutronics of the reflector

FIG. III-33-4. Reactivity Effect for Axial Fuel Expansion as a Function of the Amount of Accompanying Absorber Motion. *ANL Neg. No. 112-2985 Rev.*

moderated system are somewhat more complex. A number of questions regarding the geometry of the analytical model, uncertainty of nuclear constants and the effect of varying the core composition or density must be pursued for complete understanding.

The effect of perturbing the core composition was studied in spherical geometry for a 300 liter core that is well-moderator reflected. The core void was decreased from 45% to 30%. The corresponding critical enrichments [volume fraction of U-235/volume fraction

of (U-235 + W)] were lowered from 45% to 29%. The magnitude of the fuel plate expansion coefficient of reactivity was decreased by $\sim 30\%$ (from $+3.9$ to $+2.6 \times 10^{-6} (\Delta k/k)/^{\circ}\text{C}$). In these particular analyses the core fuel was assumed to expand into a gap between the core and the reflector. This decrease becomes significantly less pronounced if it is assumed that fuel plate expansion causes the axial reflector to move. The decrease is primarily due to the lesser penetration of the "spike" into the higher density core.

III-34. Recent Developments in the Physics Design of Some Nuclear Rockets¹

W. B. LOEWENSTEIN, K. K. ALMENAS, D. BUTLER, R. C. DOERNER and W. G. KNAPP

The physics design of typical unmoderated core nuclear rockets was reviewed. The emphasis focused on those design criteria that have been the most difficult in terms of practical rocket reactor design. These include allowable fuel loading in the core, power distribution, reactivity for control, and some important reactivity coefficients. The physics design of a nuclear rocket having a fast spectrum, refractory metal core is realistic in that many pertinent experimental data have recently been generated on the ANL-ZPR-9 facility. Typical experimental data were described along with their implication in analytical techniques. Included is a variety of detailed experiments with tungsten, rhenium, hydrogen, and boron. The design problem of using a light metal reflector has been subjected to both experimental and analytical study. The radial power distribution

and the related reflector control drums represent important areas of nuclear analysis. The use of preheaters and reflectors other than beryllium were considered. Methods of enhancing the worth of control mechanisms by mixing moderator with poison was demonstrated to have significant advantages under some reflector conditions. The status of pertinent cross-section data, their interpretation, corresponding significant uncertainties, and their implications in rocket design were considered.

REFERENCES

1. W. B. Loewenstein, K. K. Almenas, D. Butler, R. C. Doerner and W. G. Knapp, *Recent Developments in the Physics Design of Some Nuclear Rockets*, AIAA Bulletin 2, 249 (May 1965).

III-35. Experimental Physics Studies in Tungsten-Based Fast Reactors

R. C. DOERNER, W. G. KNAPP, K. K. ALMENAS, C. COHN, R. A. KARAM,
W. Y. KATO and W. B. LOEWENSTEIN

Studies of the fast tungsten reactor previously reported¹ have been extended during the past year to include measurements of the control margin available in reflector oriented control elements and the effects of coolant voiding in the reflector. In the earlier report, efforts were directed toward studying the effect of replacing the depleted uranium diluent material in the aluminum reflected fast core with varying quantities of tungsten. The primary object of those measurements was to provide checkpoints for both the cross section and the calculational model used to provide predictions of the critical mass and the central material replacement worths. The present report summarizes the studies made on tungsten-based fast cores diluted with rhenium (Assembly No. 4B), graphite (Assembly No. 5), and void in the form of 45% density aluminum (Assembly No. 6). Also included are a number of idealized measurements of the propellant void coefficient and the boron control margin in the reflector.

In order to best illustrate the general trends in the data with respect to the diluent material, two of the

assemblies already reported are included in the present report. These include the all tungsten-diluted assembly (No. 4) and the part tungsten-part depleted uranium diluted assembly (No. 3).

All of the assemblies discussed here have certain features in common. They were constructed on the ZPR-9, a split table type of facility, in which both the matrix and the fuel drawers are aluminum. Within the limits of available material sizes and configurations all of the cores were roughly cylindrical and were reflected axially and radially with 30 to 40 cm of full density aluminum. The fuel was 93.2% enriched U-235 in $\frac{1}{16}$ in. thick plates. All assemblies contained at least 36.4 v/o tungsten. In the all-tungsten and the tungsten-rhenium diluted assemblies (Nos. 4 and 4B), a lack of sufficient diluent materials made it necessary to increase the fuel concentration by 50%. The physical properties of the various assemblies are shown in Table III-35-I.

Calculated and measured critical masses are given in Table III-35-II. The calculations were performed in

TABLE III-35-I. MEASURED AND CALCULATED PARAMETERS FOR TUNGSTEN-BASED ALUMINUM-REFLECTED ZPR-9 ASSEMBLIES

	Assembly No. 4 (Ref.)		Assembly No. 4B		Assembly No. 3		Assembly No. 5		Assembly No. 6	
	Exp	Calc ^a	Exp	Calc ^a	Exp	Calc ^a	Exp	Calc ^a	Exp	Calc ^a
Critical Mass, ^b kg U-235	285.2		339.6		493.0		357.0		453.5	
ESHCM, ^c kg U-235	268.8	304.6	325.0	415.5	480.9	562.0	339.2	340.9	428.6	444.6
Volume ^b liters	107.9		128.8		282.2		204.7		258.1	
ESHCV, ^c liters	101.7	116.1	131.2	158.3	275.2	321.5	192.7	192.0	243.9	253.2
L/D ^b	1.129		1.039		1.006		0.825		0.836	
Atom Density ($\times 10^{24}$ atoms/cm ³) ^d										
U-235	0.00675		0.00674		0.00448		0.00451		0.00450	
U-238	0.00049		0.00049		0.0168		0.00033		0.00033	
W	0.0396		0.0364		0.0214		0.0213		0.0213	
Al	0.00687		0.00749		0.00681		0.00681		0.01597	
Re	0		0.00247		0		0		0	
C	0		0		0		0.02951		0	

^a All calculations performed by means of one-dimensional diffusion code RP-122, 16-group cross-section set 201 with W(37) and Re, per Ref. 2.

^b Experimental, cylindrical only.

^c Equivalent spherical homogeneous critical mass and volume. Calculated values may use slightly different atom densities.

^d Core. (Reflector: 0.05639×10^{24} atoms/cm³ aluminum.)

TABLE III-35-II. CRITICAL MASSES OF ZPR-9 ASSEMBLIES

	Assembly No.				
	4 (Ref) (all W)	4B (W-Re)	3 (W-U-238)	5 (W-C)	6 (Al-Void)
Exp. Crit. Mass (M), kg U-235	285.2	339.6	493.0	357.0	453.5
Fuel Edge Worth, lh/kg	31.87	23.9	12.11	19.1	17.6
$\frac{\Delta M}{M} / \frac{\Delta k}{k}$	4.67	5.5	7.01	6.2	5.4
Heterog. Corr., kg U-235	+4.05	+5.39	+33.51	+9.00	+16.12
Central Gap ^a , kg U-235	-0.42	-0.56	-1.14	-0.70	-0.76
Edge Effects ^b , kg U-235	-0.27	-0.35	-0.74	-0.45	-0.48
EHCCM ^c , kg U-235	288.6	344.1	524.6	364.9	468.4
Shape Factor	0.931	0.944	0.916	0.929	0.915
EHSC ^d {	Mass, kg U-235	268.7	325.0	480.5	339.2
	Radius, cm	28.95	31.52	40.35	35.83
	Volume, liters	101.7	131.2	275.2	192.7
Calculated {	Crit. Mass, kg U-235	304.6	415.5	562.0	340.9
	Volume, liters	116.1	158.3	321.5	192.0
Calc - Exp Calc	11.8%	21.8%	14.4%	0.5%	3.6%

^a Taken as $-0.031\% \Delta k/k$ for each assembly.

^b Taken as $-0.02\% \Delta k/k$ for each assembly.

^c Equivalent homogeneous cylindrical critical mass.

^d Equivalent homogeneous spherical critical.

one-dimensional spherical geometry by diffusion theory using slightly modified Hansen-Roach 16-group cross section sets supplemented with specially generated tungsten and rhenium cross section sets.² With regard to the magnitude of the errors between the calculated and the measured values for the critical mass, it should be noted that once a particular cross section set had been adopted for these studies it was used throughout

in order that the trends in the computed values would not be influenced by changes in the cross section sets. It is also important to note that there are three options available in the particular code used for these calculations. The one used here was a search for a critical radius with a fixed fuel composition and for a k_{eff} equal to 1. For a fixed radius, calculations of either the fuel concentration at critical or of k_{eff} for a given fuel con-

ntration would have produced errors on the order of $\pm 3\%$ rather than the 10 to 20% errors indicated in Table III-35-II. This is a consequence of the r^3 dependence of critical mass and the lesser worth of peripheral fuel. DSN (transport code) calculations are known to give relatively small errors but, because of the excessive computer time required, were used only to check errors in diffusion code calculations and to perform calculations which could not be adequately handled by the diffusion codes.

Critical masses measured in cylindrical geometry have been corrected for the excess reactivity and for edge irregularities according to the methods outlined in Ref. 2. A fuel heterogeneity correction was determined by a fuel bunching experiment in one or two octants of the core. The reactivity effects associated with bunching were extrapolated to full core and to infinite dilution. This type of measurement has been presented in Ref. 2 for Assembly No. 4 and is the basis for the correction applied for Assemblies No. 4 and 4B. A similar measurement on Assembly No. 6 indicates that a correction on the order of $0.66\% \Delta k/k$ is needed to account for the heterogeneity effect in the fuel plates in that assembly. Measurements were not made on Assembly No. 5, but, because of its overall softer spectrum, a small amount of self-shielding might be expected to cancel part of the true heterogeneity effect. Assuming that the magnitude of this correction is proportional to the number of fuel plates in the reactor, a heterogeneity correction of $0.4\% \Delta k/k$ has been used for Assembly No. 5.

In addition to the thickness of the aluminum drawer fronts, there exists an air gap at the core midplane estimated to average 0.076 cm. The worth of the central gap for varying gap thicknesses was experimentally determined by calibrated control rods for the different assemblies. Extrapolating these data to the existing gap yields the corrections indicated in Table III-35-II.

In these studies a two-step empirical shape factor presented by W. Davey³ is used to convert the experimental homogeneous critical mass determined in cylindrical geometry to an equivalent spherical mass. These shape factors consist of the product of a volume-

dependent term and a term dependent on the cylindrical core length-to-diameter ratio. These are also listed in Table III-35-II.

It should be noted that in Assembly No. 4B the presence of a small amount of rhenium with its high resonance absorption cross section has greatly increased the calculated versus experimental difference. Later comparisons between the measured and calculated values for the central worth of rhenium indicate that the assumed absorption cross section is 25 to 40% too high.

The general experience with the large number of assemblies in ZPR-3, -6, and -9 has shown that the Hansen-Roach cross section, in general, predicts a harder spectrum in the core than is usually inferred from the measurements. In particular, differences of from 25 to 40% have been observed in many of the calculated prompt neutron lifetimes relative to those inferred from Rossi-alpha measurements. Using some of the very recently generated cross sections, this discrepancy has been reduced to about 10% for a large number of ZPR-3 assemblies.⁴ In addition to the cross section studies, an attempt to fit the measured data to a two-region statistical model is now in progress. It is tentatively concluded that there are two separate and distinct decaying components in the measured Rossi-alpha data. As one might expect from a two-region model, one of these components is associated with the core, the other with the reflector. These studies are not yet sufficiently advanced to make any definitive conclusions from them. For this reason, the conventional definition of alpha (is equal to β_{eff}/ℓ) is used in these studies. Kinetic parameters from the various assemblies are summarized in Table III-35-III.

Central material replacement reactivity worths have been measured and calculated for a number of materials. These are summarized in Table III-35-IV. In the measurements the various materials were loaded into cans which fit into an automatic remotely operated sample changer. Use of the changer did not require the separation of the table halves in order to change samples. For this reason, the experimental error associated

TABLE III-35-III. KINETIC PARAMETERS OF ZPR-9 ASSEMBLIES

	Assembly No.				
	4	4B	3	5	6
Inhours per percent reactivity (calc)	430	432.3	446.0	424.8	430.6
α (calc)	0.00664	0.00666	0.00708	0.00674	0.00664
ssi- α , in units of 10^4 sec^{-1}	5.2	7.1	6.6	3.9	4.3
mpt Neutron Lifetime, nsec	128	93	107	172	154
	95.0	68.1	81.7	121	122
	(Calc - Meas)/Calc	-34%	-36%	-42%	-26%

TABLE III-35-IV. MEASURED AND CALCULATED CENTRAL REACTIVITY WORTHS, Ih/kg

	Assembly No. 4		Assembly No. 4B		Assembly No. 3		Assembly No. 5		Assembly No. 6	
	Exp	Calc	Exp	Calc	Exp	Calc	Exp	Calc	Exp	Calc
U-235	270.0	285.9	218.7	223.1	162.4	165.4	197.1	221.4	148.2	141.0
U-238	-5.5	-7.5	-5.2	-6.1	-6.3	-8.4	-8.2	-9.6	-3.4	-3.2
B-10	-3660.0	-3835.0	-3064.0	-2914.0	-2110.0	-2162.0	-3330.0	-4108.0	-2065	-1920.0
W	-19.6	-22.3	-17.0	-18.0	-14.8	-15.6	-16.3	-21.0	-10.6	-11.0
Re	-71.7	-109.5	-60.8	-81.9	-44.3	-59.0	-65.6	-106.8	-39.1	-54.7
Al	-6.6	8.0	-6.0	7.8	—	-5.4	0.3	3.6	-3.0	4.2
C	-2.8	59.6	-18.0	21.3	-21.1	-9.9	20.6	23.0	8.6	31.4
Au	-39.3	—	-31.9	—	-23.4	—	-31.6	—	-20.1	—
H	3350	3490	-2910	-396	—	-2720	3880	600	3510	2500

with any single measurement is estimated to be $\pm 0.2 \text{ Ih}$.

The calculated values were determined from a standard perturbation code in which the normalization is an integral of the importance over the core volume. Thus the differences between the calculated values of the given material for the various assemblies is a direct measure of the core volume as well as the unperturbed central real and adjoint spectra. It is of interest to note that in Assembly No. 4 and 4B, the sign of the carbon worth is incorrectly predicted while in all of these assemblies at least the sign of the hydrogen worth is correctly predicted. As was mentioned earlier, the calculations for rhenium overpredict the absorption in that material by a considerable amount. Excepting for Assembly No. 4, the predicted worths of hydrogen are generally badly undercalculated. This is probably due to the fact that the cross section library does not allow downscattering into more than five groups. Since the calculated spectra are generally harder than that inferred from the experimental measurements, the adjoint spectra probably has the same deficiencies. This would result in a general tendency to underestimate the absorbing effect and overestimate the effects of inelastic downscattering for any of the samples.

In addition to the materials listed in Table III-35-IV, a number of central worths were determined for varying size samples of lucite and polyethylene. Values for the central hydrogen worth given in Table III-35-IV correspond to extrapolation of the measured samples to zero sample size. In all cases, a small correction was made for the carbon content. Results of these measurements are summarized in Fig. III-35-1. It can be seen that size effects are negligible in Assemblies No. 4, 4B, and 6 while in Assemblies No. 3 and 5, they are significant.

The central hydrogen worth studies were extended to investigate the enhancement effect that hydrogen might have on the central worth of boron. For this purpose, samples were fabricated from lucite powder inti-

FIG. III-35-1. Central Hydrogen Worths as a Function of Sample Size *ANL Neg. No. 112-5144*.

mately mixed with amorphous boron powder in varying proportions. Samples were prepared to reflect effects of size and weight as well as composition. Several of the samples were molded in a metallurgical press to give highly compacted pellets while others were loose, low density powder mixtures. There was no observable effect due to size or preparation technique. Enhancement of the boron worth is defined as the difference between the total mixed sample worth and the sum of the worths of the component parts divided by the worth of pure boron. This corresponds to the enhancement in the absorption of boron resulting from the addition of the moderating material. Results of these types of measurements for the various assemblies are shown in Fig. III-35-2. It can be seen that the results for the softer spectrum (Assembly No. 5) and the hardest spectrum (Assembly No. 4B) form a band within which results for the other assemblies lie.

FIG. III-35-2. Boron Enhancement Effect in B-H Mixtures. ANL Neg. No. 112-5722

Only a limited number of studies have been made of the interrelated problems of control margin and propellant void. Reactivity effects of hydrogen homogeneously distributed through the reflector of Assembly No. 6 were studied by adding polyethylene sheets (20 mils thick). As the hydrogen was added, a few drawers at a time, small changes in the reactivity were measured by a carefully calibrated control rod. When it became necessary to remove large amounts of reactivity, fuel was removed from the central core drawers. In this way, the radial leakage was kept constant and the measured reactivity effects did not have to be corrected for changes in total core leakage. Thirty mils of polyethylene per drawer corresponded to a homogeneous hydrogen concentration of 1.412 g of hydrogen/liter and 8.462 g of carbon/liter. No correction was applied to the measured data for the effects of carbon. A completely hydrogenated reflector containing 1.143 kg of hydrogen added 0.44% $\Delta k/k$ in reactivity.

tails of this experiment are outlined in Table III-V. After polyethylene had been added to all of the reflector drawers, the assembly was designated as Assembly No. 6A.

TABLE III-35-V. REACTIVITY ASSOCIATED WITH HYDROGEN ADDED TO THE REFLECTOR, ZPR-9 ASSEMBLY No. 6

Polyethylene Weight, kg	Total No. of Drawers Loaded	Reactivity Added, $\Delta k/k$
0.148	8	1.82
0.206	10	2.73
0.593	32	16.64
1.168	66	25.50
1.755	100	35.84
4.031	202	102.70
5.900	324	151.43
7.936	434	193.06

In order to gain some insight into the amount of control which is available in the reflector, a series of worth measurements were made for boron columns inserted into the reflector. A single boron column consists of 5 cans placed end to end to form a slab 10 in. long, 2 in. wide and $\frac{1}{8}$ in. thick and having a total B-10 content of 20.3 ± 0.55 g. The quoted error includes the RMS dispersion in empty can weights, boron enrichments, non-boron content and packing density within the cans.

Columns of boron were loaded into the reflector in positions symmetrically located around the core, a few drawers at a time. A typical column had a mean radius of 42 cm and subtended an azimuthal angle of 6 deg. Variations of as much as 1 cm in mean radius and 1 deg in subtended angle were experienced because of the necessity of loading the columns in some drawers in a vertical position and others in a horizontal position. Figure III-35-3 shows the total reactivity held up by the boron at each stage of the loading plotted against the total subtended angle loaded. When the loading was complete (440 cans) the reactivity held up was 0.672% $\Delta k/k$ at a total subtended angle of 268 deg. To estimate the effect of extending the 10 in. long boron columns to the full core length (12 in.) an extra $2 \times 2 \times \frac{1}{8}$ in. can was added to 24 boron columns removing 8.8 Ih. Adding 4 times this to the previous total and extrapolating this to a full 360 deg ring gives a total reactivity of 1.01% $\Delta k/k$ and a total B-10 weight of 2.878 kg. These extrapolations were necessary because the limited supply of B-10 cans did not allow more extensive actual loadings. In this type of measurement, it was necessary to separate the halves of the reactor for each increment of boron addition. Experience has shown that this produces uncertainties in the reactivity measurements of ~ 0.5 Ih. Scaling this error for individual measurements up to the total worth of the ring indicates an expected error on the order of about 5 Ih in the measured worths. This is quite consistent with the spread in the points shown in Fig. III-35-3.

The same experiment was repeated with the same boron and following the same procedures in Assembly No. 6A after the hydrogen had been added to the reflector. In this case, the complete loading of 440 cans had a measured worth of 1.30% $\Delta k/k$ and the effect of extending 26 of the 88 10-in. long boron columns to

FIG. III-35-3. Boron Ring Experimental Data for Assemblies 6 and 6A. ANL Neg. No. 112-5690.

LOCATIONS OF RING SEGMENTS

FIG. III-35-4. Worth of B-10 in the Reflector. ANL Neg. No. 112-5672.

12 in. removed 17.7 Ih of reactivity. Extrapolating the extra 2 in. over the remaining part of the core and then extrapolating over a 360 deg ring yields 1.43% $\Delta k/k$ as the reactivity tied up by a complete boron ring in the hydrogenated reflector. Results of these measurements are also shown in Fig. III-35-3.

An additional study of boron in the reflector region was made in Assembly No. 5. A 43 deg ring segment of B-10 was placed in the reflector with the radius being changed from measurement to measurement. Additional boron was added as necessary to maintain ap-

roximately a 43 deg azimuthal subtended angle. Both the geometry of the experiment and the spatial dependence of B-10 worth in the reflector of this assembly are shown in Fig. III-35-4.

REFERENCES

1. R. C. Doerner, K. K. Almenas, R. A. Karam, W. Y. Kato, W. G. Knapp and W. B. Loewenstein, *Physics Measure-*

- ments in Tungsten-Based Aluminum-Reflected Fast Reactors*, Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010, p. 116 (1965).
2. R. C. Doerner, W. G. Knapp, K. K. Almenas and R. A. Karam, *Physics Measurements in Tungsten-Based Aluminum-Reflected Fast Reactors* ANL-7007, (in preparation).
3. W. G. Davey, *An Analysis of 23 ZPR-3 Fast Reactor Critical Experiments*, Nucl. Sci. Eng. **19**, 259 (July 1964).
4. R. Avery, et al., *Physics of Fast Reactors*, Proc. 1964 Geneva Conference (to be published).

III-36. Small Core Reactors for Space Propulsion

D. K. BUTLER and H. L. DYER*

TABLE III-36-I. CRITICAL PROPERTIES OF Al_2O_3 AND Be REFLECTED SPHERES AND OF Be REFLECTED CYLINDERS (NO PREHEATERS)

Geometry and Fuel	Reflector		Core Porosity, %	Core Radius, cm	Core Height, ^a cm	Core Volume, liters	Reflector Savings, cm		
	Thickness, cm	Material							
<i>Spheres</i> U-235	10.16	Al ₂ O ₃	25	23.4	—	53.6	5.3		
			40	36.4	—	202.1	7.6		
			55	146.3	—	1311.9	14.0		
	20.32	Al ₂ O ₃	25	21.3	—	40.4	7.4		
			40	32.8	—	148.0	11.2		
			55	139.0	—	1125.4	21.3		
	10.16	Be	25	22.1	—	45.1	6.4		
			30	25.2	—	67.3	7.1		
			40	35.1	—	179.7	8.7		
	20.32	Be	55	147.9	—	1354.5	12.4		
			25	19.1	—	29.2	9.5		
			30	21.9	—	43.9	10.5		
	 U-233	10.16	Be	40	30.3	—	114.4	13.6	
				55	138.2	—	1106.1	22.1	
				25	14.5	—	12.7	5.6	
		20.32	Be	30	16.1	—	17.6	6.3	
				40	21.2	—	39.9	8.2	
				25	12.4	—	8.0	7.7	
		 <i>Cylinders</i> U-235	10.16	Be	30	13.7	—	10.7	8.7
					40	17.6	—	22.9	11.8
					25	17.4	62.5	59.4	6.4
			20.32	Be	30	20.0	69.7	87.3	8.1
					40	28.5	93.5	238.7	8.9
					25	14.7	59.4	40.2	8.9
 U-233			10.16	Be	30	16.9	66.3	59.2	10.1
					40	24.0	88.8	161.0	13.2
					25	11.1	43.9	16.9	5.5
			20.32	Be	30	12.4	47.9	23.2	6.5
					40	16.7	60.0	52.7	8.0
					25	8.3	41.8	9.0	7.3
			10.16	Be	30	10.7	42.0	15.0	7.9
					40	13.4	56.4	32.4	11.4
					25	17.4	62.5	59.4	6.4
			20.32	Be	30	20.0	69.7	87.3	8.1
					40	28.5	93.5	238.7	8.9
					25	14.7	59.4	40.2	8.9
	10.16		Be	30	16.9	66.3	59.2	10.1	
				40	24.0	88.8	161.0	13.2	
				25	11.1	43.9	16.9	5.5	
	20.32		Be	30	12.4	47.9	23.2	6.5	
				40	16.7	60.0	52.7	8.0	
				25	8.3	41.8	9.0	7.3	
	10.16	Be	30	10.7	42.0	15.0	7.9		
			40	13.4	56.4	32.4	11.4		
			25	17.4	62.5	59.4	6.4		
	20.32	Be	30	20.0	69.7	87.3	8.1		
			40	28.5	93.5	238.7	8.9		
			25	14.7	59.4	40.2	8.9		
10.16	Be	30	16.9	66.3	59.2	10.1			
		40	24.0	88.8	161.0	13.2			
		25	11.1	43.9	16.9	5.5			
20.32	Be	30	12.4	47.9	23.2	6.5			
		40	16.7	60.0	52.7	8.0			
		25	8.3	41.8	9.0	7.3			
10.16	Be	30	10.7	42.0	15.0	7.9			
		40	13.4	56.4	32.4	11.4			
		25	17.4	62.5	59.4	6.4			
20.32	Be	30	20.0	69.7	87.3	8.1			
		40	28.5	93.5	238.7	8.9			
		25	14.7	59.4	40.2	8.9			
10.16	Be	30	16.9	66.3	59.2	10.1			
		40	24.0	88.8	161.0	13.2			
		25	11.1	43.9	16.9	5.5			
20.32	Be	30	12.4	47.9	23.2	6.5			
		40	16.7	60.0	52.7	8.0			
		25	8.3	41.8	9.0	7.3			
10.16	Be	30	10.7	42.0	15.0	7.9			
		40	13.4	56.4	32.4	11.4			
		25	17.4	62.5	59.4	6.4			
20.32	Be	30	20.0	69.7	87.3	8.1			
		40	28.5	93.5	238.7	8.9			
		25	14.7	59.4	40.2	8.9			
10.16	Be	30	16.9	66.3	59.2	10.1			
		40	24.0	88.8	161.0	13.2			
		25	11.1	43.9	16.9	5.5			
20.32	Be	30	12.4	47.9	23.2	6.5			
		40	16.7	60.0	52.7	8.0			
		25	8.3	41.8	9.0	7.3			
10.16	Be	30	10.7	42.0	15.0	7.9			
		40	13.4	56.4	32.4	11.4			
		25	17.4	62.5	59.4	6.4			
20.32	Be	30	20.0	69.7	87.3	8.1			
		40	28.5	93.5	238.7	8.9			
		25	14.7	59.4	40.2	8.9			
10.16	Be	30	16.9	66.3	59.2	10.1			
		40	24.0	88.8	161.0	13.2			
		25	11.1	43.9	16.9	5.5			
20.32	Be	30	12.4	47.9	23.2	6.5			
		40	16.7	60.0	52.7	8.0			
		25	8.3	41.8	9.0	7.3			
10.16	Be	30	10.7	42.0	15.0	7.9			
		40	13.4	56.4	32.4	11.4			
		25	17.4	62.5	59.4	6.4			
20.32	Be	30	20.0	69.7	87.3	8.1			
		40	28.5	93.5	238.7	8.9			
		25	14.7	59.4	40.2	8.9			
10.16	Be	30	16.9	66.3	59.2	10.1			
		40	24.0	88.8	161.0	13.2			
		25	11.1	43.9	16.9	5.5			
20.32	Be	30	12.4	47.9	23.2	6.5			
		40	16.7	60.0	52.7	8.0			
		25	8.3	41.8	9.0	7.3			
10.16	Be	30	10.7	42.0	15.0	7.9			
		40	13.4	56.4	32.4	11.4			
		25	17.4	62.5	59.4	6.4			
20.32	Be	30	20.0	69.7	87.3	8.1			
		40	28.5	93.5	238.7	8.9			
		25	14.7	59.4	40.2	8.9			
10.16	Be	30	16.9	66.3	59.2	10.1			
		40	24.0	88.8	161.0	13.2			
		25	11.1	43.9	16.9	5.5			
20.32	Be	30	12.4	47.9	23.2	6.5			
		40	16.7	60.0	52.7	8.0			
		25	8.3	41.8	9.0	7.3			
10.16	Be	30	10.7	42.0	15.0	7.9			
		40	13.4	56.4	32.4	11.4			
		25	17.4	62.5	59.4	6.4			
20.32	Be	30	20.0	69.7	87.3	8.1			
		40	28.5	93.5	238.7	8.9			
		25	14.7	59.4	40.2	8.9			
10.16	Be	30	16.9	66.3	59.2	10.1			
		40	24.0	88.8	161.0	13.2			
		25	11.1	43.9	16.9	5.5			
20.32	Be	30	12.4	47.9	23.2	6.5			
		40	16.7	60.0	52.7	8.0			
		25	8.3	41.8	9.0	7.3			
10.16	Be	30	10.7	42.0	15.0	7.9			
		40	13.4	56.4	32.4	11.4			
		25	17.4	62.5	59.4	6.4			
20.32	Be	30	20.0	69.7	87.3	8.1			
		40	28.5	93.5	238.7	8.9			
		25	14.7	59.4	40.2	8.9			
10.16	Be	30	16.9	66.3	59.2	10.1			
		40	24.0	88.8	161.0	13.2			
		25	11.1	43.9	16.9	5.5			
20.32	Be	30	12.4	47.9	23.2	6.5			
		40	16.7	60.0	52.7	8.0			
		25	8.3	41.8	9.0	7.3			
10.16	Be	30	10.7	42.0	15.0	7.9			
		40	13.4	56.4	32.4	11.4			
		25	17.4	62.5	59.4	6.4			
20.32	Be	30	20.0	69.7	87.3	8.1			
		40	28.5	93.5	238.7	8.9			
		25	14.7	59.4	40.2	8.9			
10.16	Be	30	16.9	66.3	59.2	10.1			
		40	24.0	88.8	161.0	13.2			
		25	11.1	43.9	16.9	5.5			
20.32	Be	30	12.4	47.9	23.2	6.5			
		40	16.7	60.0	52.7	8.0			
		25	8.3	41.8	9.0	7.3			
10.16	Be	30	10.7	42.0	15.0	7.9			
		40	13.4	56.4	32.4	11.4			
		25	17.4	62.5	59.4	6.4			
20.32	Be	30	20.0	69.7	87.3	8.1			
		40	28.5	93.5	238.7	8.9			
		25	14.7	59.4	40.2	8.9			
10.16	Be	30	16.9	66.3	59.2	10.1			
		40	24.0	88.8	161.0	13.2			
		25	11.1	43.9	16.9	5.5			
20.32	Be	30	12.4	47.9	23.2	6.5			
		40	16.7	60.0	52.7	8.0			
		25	8.3	41.8	9.0	7.3			
10.16	Be	30	10.7	42.0	15.0	7.9			
		40	13.4	56.4	32.4	11.4			
		25	17.4	62.5	59.4	6.4			
20.32	Be	30	20.0	69.7	87.3	8.1			
		40	28.5	93.5	238.7	8.9			
		25	14.7	59.4	40.2	8.9			
10.16	Be	30	16.9	66.3	59.2	10.1			
		40	24.0	88.8	161.0	13.2			
		25	11.1	43.9	16.9	5.5			
20.32	Be	30	12.4	47.9	23.2	6.5			
		40	16.7	60.0	52.7	8.0			
		25	8.3	41.8	9.0	7.3			
10.16	Be	30	10.7	42.0	15.0	7.9			
		40	13.4	56.4	32.4	11.4			
		25	17.4	62.5	59.4	6.4			
20.32	Be	30	20.0	69.7	87.3	8.1			
		40	28.5	93.5	238.7	8.9			
		25	14.7	59.4	40.2	8.9			
10.16	Be	30	16.9	66.3	59.2	10.1			
		40	24.0	88.8	161.0	13.2			
		25	11.1	43.9	16.9	5.5			
20.32	Be	30	12.4	47.9	23.2	6.5			
		40	16.7	60.0	52.7	8.0			
		25	8.3	41.8	9.0	7.3			
10.16	Be	30	10.7	42.0	15.0	7.9			
		40	13.4	56.4	32.4	11.4			
		25	17.4	62.5	59.4	6.4			
20.32	Be	30	20.0	69.7	87.3	8.1			
		40	28.5	93.5	238.7	8.9			
		25	14.7	59.4	40.2	8.9			
10.16	Be	30	16.9	66.3	59.2	10.1			
		40	24.0	88.8	161.0	13.2			
		25	11.1	43.9	16.9	5.5			
20.32	Be	30	12.4	47.9	23.2	6.5			
		40	16.7	60.0	52.7	8.0			
		25	8.3	41.8	9.0	7.3			
10.16	Be	30	10.7	42.0	15.0	7.9			
		40	13.4	56.4	32.4	11.4			
		25	17.4	62.5	59.4	6.4			
20.32	Be	30	20.0	69.7	87.3	8.1			
		40	28.5	93.5	238.7	8.9			
		25	14.7	59.4	40.2	8.9			
10.16	Be	30	16.9	66.3	59.2	10.1			
		40	24.0	88.8	161.0	13.2			
		25	11.1	43.9	16.9	5.5			
20.32	Be	30	12.4	47.9	23.2	6.5			
		40	16.7	60.0	52.7	8.0			
		25	8.3	41.8	9.0	7.3			
10.16	Be	30	10.7	42.0	15.0	7.9			
		40	13.4	56.4	32.4	11.4			
		25	17.4	62.5	59.4	6.4			
20.32	Be	30	20.0	69.7	87.3	8.1			
		40	28.5	93.5	238.7	8.9			
		25	14.7	59.4	40.2	8.9			
10.16	Be	30	16.9	66.3	59.2	10.1			
		40	24.0	88.8	161.0	13.2			
		25	11.1	43.9	16.9	5.5			
20.32	Be	30	12.4	47.9	23.2	6.5			
		40	16.7	60.0	52.7	8.0			
		25	8.3	41.8	9.0	7.3			
10.16	Be	30	10.7	42.0	15.0	7.9			
		40	13.4	56.4	32.4	11.4			
		25	17.4	62.5	59.4	6.4			
20.32	Be	30	20.0	69.7	87.3	8.1			
		40	28.5	93.5	238.7	8.9			
		25	14.7	59.4	40.2				

^a Assumes total axial reflector savings equal to the spherical reflector savings.

* Reactor Engineering Division.

TABLE III-36-II. CRITICAL PROPERTIES OF BERYLLIUM REFLECTED SPHERES AND CYLINDERS WITH 3.81 cm PREHEATER

Geometry and Fuel	Reflector Thickness, cm	Core Porosity, %	Core Radius, cm	Core Height, ^a cm	Core Volume, liters	Reflector Savings, cm
<i>Spheres</i>	U-235	25	22.5	—	47.7	5.0
		30	25.5	—	70.3	5.7
		40	35.1	—	181.8	7.6
		25	20.6	—	36.8	7.0
		30	23.4	—	53.8	7.8
		40	32.4	—	142.7	10.4
	U-233	25	15.0	—	14.3	4.1
		30	16.7	—	19.4	4.7
		40	21.9	—	43.8	6.3
		25	13.7	—	10.7	5.5
		30	15.2	—	14.7	6.2
		40	19.7	—	32.0	8.6
<i>Cylinders</i>	U-235	25	16.9	62.5	56.3	5.6
		30	20.3	69.7	90.5	6.5
		40	28.3	93.5	235.0	7.4
		25	16.5	59.4	50.9	6.2
		30	18.7	66.3	77.9	6.5
		40	26.0	88.8	188.0	9.7
	U-233	25	11.9	43.9	19.7	3.7
		30	13.3	47.9	26.4	4.2
		40	17.5	60.0	58.0	5.9
		25	11.1	41.8	16.1	4.6
		30	12.7	42.0	21.3	5.1
		40	15.8	56.4	44.2	7.6

^a Assumes total axial reflector savings equal to the spherical reflector savings.

The neutronic characteristics of some small fast reactors that would be suitable for low thrust space applications have been studied. The study provides part of the information necessary to determine whether a sufficiently light weight and efficient nuclear engine can be built to serve in those space applications where low thrusts are desirable.

The design concept assumed hydrogen as propellant, heated in a UO₂ fueled cylindrical core, with tungsten and rhenium as the core refractory materials. The concept considered both U-235 and U-233 as fuels, the latter being appropriate for achieving the smallest critical volumes. Either Al₂O₃ or Be reflector surrounded the cores radially and in one axial direction. Some of the systems had preheater sections between the core and the radial reflector. These were fueled with the

same uranium isotopes and compounds as the corresponding cores. In the preheater the compounds were assumed to be in a stainless steel cermet clad with stainless steel.

The primary objective of the study was to determine the critical volume as a function of the shape and porosity of heavily loaded cores. The radial power distribution was also studied. The calculations were obtained with the one-dimensional multigroup discrete ordinate code (SNARG) in spherical and cylindrical geometry using the S4 angular approximation.

In order to determine the optimum reactor in terms of total weight and thermal performance, it was necessary to obtain information about cores with L/D ratios ranging from 0.2 to 4.0. The approximation formula described by G. Hansen¹ was used. This formula is based on diffusion theory. It is most accurate for large, unreflected cores, less so for smaller ones. It was used for these calculations because it provided an analytic relationship between L/D and critical volume that is suitable for inclusion in computing machine programs to determine optimum configurations. Final determination of the optimum system will require more accurate techniques to verify the results of the approximate formula. It was assumed that a sufficiently accurate reflector saving required for the formula could be determined from one-dimensional spherical and cylindrical calculations.

The range of the study covered core porosities from 25% to 50%, and two reflector thicknesses (10.16 cm and 20.32 cm). In those systems which included preheaters, the fuel loading of the preheater was adjusted so that the one-dimensional cylindrical calculation yields about 7.5% of the fission source in the preheater.

The results for systems without preheaters are shown in Table III-36-I and for systems with preheaters in Table III-36-II. The spherical calculations with preheaters were performed to obtain parameters for the analytical formula.

The radial distribution of sources for the smallest U-233 fueled system with and without preheater shows severe thermal spikes adjacent to the blanket. These can be expected to present a problem in the practical design of a small system with beryllium reflector.

REFERENCES

1. G. E. Hansen, *Properties of Elementary Fast-Neutron Critical Assemblies*, Proc. 1958 Geneva Conference, **12**, Paper No. 592, p. 84.

III-37. A Study of Small, Fast, W-UO₂ Rocket Engines for Advanced Scientific Missions

B. I. SPINRAD, E. J. CROKE*, M. STEPHENSON* and A. J. ULRICH

A number of quite advanced, unmanned scientific missions are, upon casual examination, within the potential capabilities of a Saturn IB launch system with a nuclear third stage. Among these are: radiometric mapping of the solar system from Mercury to Jupiter orbits (11 km/sec, 1.5 tonnes); asteroid mapping and particle sampling (7 km/sec, 2.5 tonnes); and permanent Martian orbiter, photographer, and signaller (7 km/sec, 2.5 tonnes).†

Using the published figure of 34,000 lb, or 15.4 tonnes, as the useful mass which Saturn IB will be able to place in low earth orbit, it is readily determined that, with an engine system having a mass of 3 tonnes, the desired (δv , payload) are marginal. Studies were therefore undertaken to determine:

1. Whether the startup of a nuclear third stage before Saturn IB has reached orbit could permit a significant addition to the mass injected.

2. Whether a sufficiently lightweight, operable nuclear engine could be projected.

With regard to the first point, trajectory analysis of the Saturn IB boost indicates that: if the Saturn IV stage is fired at an angle whose cosine is 12/13 with respect to the horizon; if it is fired for 66 seconds less than the time needed for orbital injection; and if, subsequently, a nuclear third stage of 300 MW jet power and specific impulse of 750 sec is fired at 45° to horizontal; then 20.3 tonnes (including the nuclear engine) would be injected into low orbit. This number is not yet optimized; however, the extra mass injected should not be less than 1.7 tonnes/100 MW of nuclear jet power, and should be relatively insensitive to specific impulse.

Reactors of 300 MW—or less—jet power have relatively low-thrust. Nevertheless, the incentive to achieve low power exists in the potential saving of

engine mass. In the class of systems with 50–300 MW of power and 15–20 tonnes of mass, the low thrust would lead to heavy gravity losses near the earth unless perigee propulsion could be used. By this latter means, between two and ten restarts are needed to achieve low losses.

The type of nuclear engine system which we believe to be most easily miniaturized, and to have the greatest potential for durability is the ANL W-UO₂ fast reactor.¹ Accordingly, design studies were undertaken to estimate mass-performance features of this engine type in the 50–300 MW range.

The following preliminary results of the studies are reported:

1. Critical reactors of this type can be constructed in the porosity range 0.10–0.25 which, with reflector, weigh less than one tonne.

2. The smallest critical reactors which we are able to construct could accommodate coolant flows capable of removing up to several hundred megawatts of heat.

3. At about 200 MW the pressure shells were reduced to the minimum thickness dictated by good practice.

4. Engine systems in the 150–200 MW range can be designed in the 1.5–2.0 tonne mass range.

5. There is a tradeoff of mass versus exit temperature, achieved by high heat transfer performance which favors cores slightly longer than those with minimum system masses.

6. A reactor of 150–300 MW in power, with specific impulse >800 sec, can be designed with a mass of less than 2 tonnes, excluding the shield.

7. Therefore, for further mission studies of this type, we may consider a reactor engine system with jet power of about 200 MW, a mass of about 4 tonnes (including the shield), and a specific impulse of about 800 sec.

REFERENCES

1. J. F. Marchaterre, J. J. Dickson, B. M. Hoglund and V. M. Kolba, *Refractory Metal Fast Reactor for Nuclear Rocket Propulsion*, AIAA Bulletin **2**, 207 (May 1965).

* Reactor Engineering Division.

† Numbers in parentheses are crude estimates of δv from low earth orbit, and of payload weight (additional to engine) required.

III-38. Study of Reflector Based Control of Fast Nuclear Rocket Reactors

K. K. ALMENAS

Extremely high temperatures exist in nuclear rocket reactor cores. Therefore, these reactors may best be controlled by moving mechanisms located in the reflector where environmental conditions can be kept tolerable for a wide range of control materials. Transfer of control mechanisms from the core into the reflector seriously affects their neutronic effectiveness. The ability to control such a reactor thus becomes an important and, in some cases, even a limiting design parameter.

A study has been conducted which investigated many aspects of this control problem. Part of the study has been reported in Ref. 1 and a summary of all the work is being published.² One important aspect of these studies is presented here; namely, a demonstration that optima exist in some of the design parameters of a reflector-located control mechanism.

Figure III-38-1 is a schematic of a control drum and control vane in a reflector. Since most of the indicated dimensions are not optimum with respect to maximizing reactivity for control, the design values of these dimensions have to be imposed by some other criteria. The dimensions include the control drum diameter, the reflector thickness, and the very important distance between the control drum and the core-reflector interface. At least two design parameters of the control vane itself, the control vane thickness (T) and its composition, usually will have well-defined optima with respect to the achievable control span.

The reasons for the existence of these optima can be explained qualitatively. The control span is defined as the reactivity change produced by transferring the control vane poison from the drums IN to the drums OUT position. The poison is not removed but merely transferred to a region of lesser importance. The reactivity held by the poison while the control vane is in the OUT position will directly reduce the total available control span. Further, as the control vane thickness (T) is increased, the average distance over which the poison is moved between the drums IN and OUT positions is reduced. The effect of these geometric properties on the control span are amplified by the self-shielding of the poison.

The loss of control span due to incomplete removal of the poison when the control drums are in the OUT position is shown in Fig. III-38-2. The dependence of control span loss on vane thickness for vanes located in three different reflectors is demonstrated. The aluminum

oxide reflector curve exhibits both the self-shielding effect of the boron poison and the increase in reactivity loss (not available for control) which results when the vane poison is moved closer to the core by increased control vane thickness. The thin vane region of the curve departs from linearity as the effectiveness of the added boron is reduced by self-shielding. This effect is reversed for the larger control vane thicknesses for which the reactivity effect of moving the control poison closer to the core becomes dominant.

The curves for the aluminum reflector show one of the mechanisms which determine the composition dependence for optimum control. The substitution of hydrogen for boron reduces the control drum OUT position reactivity loss for all control vane thicknesses. For certain hydrogen-to-boron composition ratios no corresponding reactivity decrease occurs while the poison is at the drum's IN position. The lower energy neutrons which are preferentially absorbed by the

- θ = ANGLE SPANNED BY CONTROL VANE
- T = CONTROL VANE THICKNESS
- R = CONTROL DRUM RADIUS
- TT = DISTANCE OF DRUM FROM CORE-REFLECTOR INTERFACE
- TR = THICKNESS OF REFLECTOR

FIG. III-38-1. Schematic of Control Drum and Vane in Reflector. ANL Neg. No. 112-5534.

FIG. III-38-2. Reactivity Loss Produced by Control Vane at the OUT Position. ANL Neg. No. 112-5535.

FIG. III-38-3. Control Span Versus Control Vane Thickness for Al Reflected Reactor. ANL Neg. No. 112-5537.

FIG. III-38-4. Control Span Versus Control Vane Thickness for Al_2O_3 Reflected Reactor. ANL Neg. No. 112-5536.

boron and hydrogen mixture have a relatively high importance close to the core but a very low importance further away from it.

The effect that the self-shielding of poison has on the optimum control vane thickness is shown in Figs. III-38-3, III-38-4, and III-38-5. The figures demonstrate the relationship between the achievable control span and the control vane thickness for vanes located in three neutronically different reflectors. The optimum

vane thicknesses tend toward smaller values for the reflector materials which have the more degraded spectra where self-shielding becomes more important.

In the aluminum reflector (Fig. III-38-3), which has the hardest equilibrium neutron spectrum, the optimum vane thickness is about half of the control drum radius (the control drum diameter for the calculations is 20 cm). The optimum thickness is smaller for the softer

FIG. III-38-5. Control Span Versus Control Vane Thickness for Be Reflected Reactor. ANL Neg. No. 112-5538.

FIG. III-38-6. Relative Real and Adjoint Fluxes 4.2 cm from the Core-Reflector Interface in Al and Al₂O₃ Reflectors. ANL Neg. No. 112-5541.

spectrum aluminum oxide reflector (Fig. III-38-4) and becomes quite thin for the beryllium reflector (Fig. III-38-5).

Figures III-38-3 and III-38-4 also show the effect on optimum thickness produced by the composition of the control vane. The optimum is seen to move toward larger thicknesses for vanes containing a mixture of hydrogen and boron. This is due to the lower importance of the neutrons absorbed in the mixture vanes when the drums are at the OUT position as noted above. The significant increase in control span that can be obtained with a mixed composition vane has significant practical importance. There is a strong incentive to optimize control vane composition to enhance the reactor control span.

Composition optimization does not imply mere enhancement of the poison worth. The volume available for poison material is at a premium. If a poison additive is to be considered, it must increase the effectiveness of the control vane while actually displacing poison

material. As shown in Fig. III-38-3 and Fig. III-38-4, a homogeneous moderator has this effect when displacing natural boron or even B-10. The poison volume for all curves of Figs. III-38-3 and III-38-4 is constant and represents 40% of the vane volume both for the pure boron and for the boron and hydrogen mixture control vanes.

Initially the composition optimization focused on a search of the most effective neutron absorber. It was determined that the choice of the absorber is not very crucial since practical absorber concentrations are fairly black to thermal and resonance neutrons. Some of the rare earth resonance absorbers have resonance peaks which exceed the B-10 cross section. However,

FIG. III-38-7. Control Span Versus Control Vane Composition for Al Reflected Reactor. ANL Neg. No. 112-5539.

FIG. III-38-8. Control Span Versus Control Vane Composition for Ni Reflected Reactor. ANL Neg. No. 112-5540.

these resonance peaks did not contribute significantly to the total absorption of the vane because of extreme self-shielding. The most important absorber requirement is a high volumetric specific absorption of fast neutrons with sufficient resonance and thermal absorption to render the vane black for those neutrons. Because of its high atomic density, natural boron can compete in this respect with all of the possible absorbers in most applications. The B-10 isotope is a significantly better absorber per unit volume. For this reason, boron and B-10 were used in the comparative evaluations.

The variable in the composition optimization is the ratio of hydrogen-to-boron in a fixed control vane volume. As demonstrated in Fig. III-38-3 and Fig. III-38-4, H/B volume ratios of up to 5 can produce significant control span gains in the reflectors with the more energetic neutron spectra. An important mechanism contributing to this effect is demonstrated in Fig. III-38-6. There, the relative energy distribution of the real and adjoint fluxes 4 cm into the reflector is shown. This position is at the location that the control poison occupies when the drums are at the IN position. The pertinent feature is the decrease of the adjoint flux with decreasing energy in the region of maximum real flux density (0.4 to 0.04 keV). This decrease means

that the moderation of neutrons will result in a reactivity loss which is quite independent of parasitic capture in the drums. The fluxes of Fig. III-38-6 were calculated for the poison-free case. Any addition of boron would only increase the negative adjoint flux gradient and thus increase the negative reactivity effect produced by neutron moderation.

The extent to which boron can be replaced by a hydrogenous moderator is illustrated in Figs. III-38-7 and III-38-8. The curve for nickel (Fig. III-38-7) shows that the removal of $\frac{4}{5}$ of the natural B and its replacement by TiH_2 can produce an almost doubled control span. The increase is not so drastic for the case where the absorber is B-10. However, the substitution of TiH_2 can be advantageous because it reduces the B-10 isotope inventory by a factor of 5 with no loss in the resultant control span.

REFERENCES

1. K. K. Almenas, *The Neutronics of Reactor Control with Reflector Materials*, Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010 (January 1965), 131-134.
2. K. K. Almenas, *Study of Reflector Based Control of Fast Nuclear Rocket Reactors*, ANL-7099 (to be published).

III-39. Further Study of the Rocket Fuel Test Reactor Concept

J. T. MADELL

This work is a continuation of work reported in the 1963-64 Annual Report.¹ The purpose of the Rocket Fuel Test Reactor (RFTR) is to evaluate the performance of rocket fuel elements in an environment similar to that of a nuclear rocket. The two types of test reactors studied last year, $\text{UO}_2\text{-BeO}$ fueled-sodium cooled and U-ZrH_2 fueled-sodium cooled, were characterized by large size and high power output.

Two new approaches were investigated to achieve the high performance requirements in the test section without excessive size and power output in the test reactor. One is a bootstrap approach. The rocket fuel elements in the test section are surrounded by less heavily fueled rocket elements in the test reactor. The power density and temperatures of the fuel and coolant in the test reactor will be 50 to 60% of the values in the test section. It is then possible to operate the test reactor within existing operating limitations while improved designs are evaluated in the test section at power densities and temperatures 2 or 2.5 times greater

than previously obtained. The improvements arising from each set of tests are incorporated into the fuel elements of the test reactor. Each set of tests thus allows an increase in the power density and temperatures in both the test sections and test reactor. The desired performance requirements for the rocket fuel is obtained after a series of bootstrap operations.

A series of one dimensional problems using the REX² code were performed to determine the size and power level of various test facilities. As was the case in the previous study, the rocket fuel elements in the test section were composed of a $\text{UO}_2\text{-W}$ cermet. The surrounding rocket fuel elements in the test reactor were composed of either a $\text{UO}_2\text{-W}$ or $\text{UO}_2\text{-Mo}$ cermet. The lower temperature in the test section permits the use of molybdenum initially. The volumetric concentration of U-235 in the test reactor is varied radially to flatten the power density. Previous studies have shown that the size and power output are reduced by varying the fuel concentration in such a manner. A

description of test facilities investigated and the results of the investigation are presented in Tables III-39-I and III-39-II. Problems 457 and 458 are studies of designs where the power is flattened by varying the loading of 95% enriched uranium in the fuel elements. In the remaining problems the enrichment of the UO_2 is varied to flatten the power but the UO_2 content is fixed. The values of the power output given in Table III-39-II are obtained by assuming that the maximum power density in the test section is 10 MW/liter.

The lowest value of the power output is obtained from designs studied in Problems 449 and 450. The ratio of the maximum power density in the test section to that in the test reactor for each of the two problems is 2.0 and 2.5, respectively. The size and power output of the test reactors in the bootstrap approach are within a practical limit and they also have the advantage of an open heat cycle which reduces the capital cost of the facility. It appears both practical and

TABLE III-39-I. VOLUMETRIC COMPOSITION OF VARIOUS TEST REACTORS IN THE "BOOTSTRAP" APPROACH

Problem No.	UO_2	Mo/W	Void, %
447 & 448	27% (varying enrichment)	33% Mo	40
449 & 450	16% (varying enrichment)	37% Mo	47
451 & 452	24% (varying enrichment)	26% W	50
453 & 454	24% (varying enrichment)	26% Mo	50
457	14.5-15.6% (varying loading)	35.5-34.4% Mo	50
458	11.1-14.2% (varying loading)	38.9-35.9% Mo	50

TABLE III-39-II. PERFORMANCE OF VARIOUS TEST REACTORS IN THE "BOOTSTRAP" APPROACH

Prob. No.	Test Reactor				Max. Power Density Ratio
	Core	Reflector	Volume	Power	
447	$\text{UO}_2\text{-Mo}$ (v.e.) ^a ↓	BeO ↓	215	670	2.0
448			285	935	2.5
449			190	595	2.0
450			315	770	2.5
451	$\text{UO}_2\text{-W}$ (v.e.) ^a		355	1070	2.0
452	$\text{UO}_2\text{-W}$ (v.e.) ^a		970	2125	2.5
453	$\text{UO}_2\text{-Mo}$ (v.e.) ^a		210	645	2.0
454	$\text{UO}_2\text{-Mo}$ (v.e.) ^a		345	840	2.5
457	$\text{UO}_2\text{-Mo}$ (v.l.) ^b	↓	220	660	2.0
458	$\text{UO}_2\text{-Mo}$ (v.l.) ^b		340	825	2.5

^a v.e. = variable enrichment

^b v.l. = variable loading

TABLE III-39-III. COMPARISON OF PERFORMANCE OF VARIOUS TEST REACTORS WITH TEST SECTION FUELED WITH U-235 AND U-233

Prob. No.	Test Section			Test Reactor		Max. Power Density Ratio
	Fuel	Loading, kg	Radius, cm	Core volume, liters	Power, MW	
(a) U-ZrH ₂ Fueled, Sodium Cooled Test Reactor with Nickel Reflector						
463	U-235	107	13	620	1240	3.0
474	U-233	107	13	110	110	6.7
476	U-233	40	8	200	380	3.5
(b) "Bootstrap" Test Reactors Fueled with UO ₂ -W and UO ₂ -Mo ^b						
451	U-235	107	13	355	1070	2.0
478	U-233	40	8	190	430	3.0
447	U-235	107	13	215	670	2.0
477	U-233	40	8	130	285	3.0
473	U-233	124	14.03 ^a	Nickel Reflector Only		—

^a Calculated critical radius.

^b Refer to Problem No. 451 and 477, respectively, in Table II-39-I for compositions.

possible to develop a bootstrap operation in which the power density and temperatures can be increased by factors of 2 or 2.5 at each step.

A second approach which was investigated is the use of U-233 instead of U-235 in the fuel elements of the test section. Uranium-233 with its higher values of $\nu\sigma_f$ is more reactive than U-235. Increasing the reactivity of the test section by using U-233 reduces the size and fuel loadings required in the surrounding test reactor. The substitution of U-233 would still allow the experiments to be interpreted in terms of rocket fuel elements loaded with U-235, and at the same time would result in greatly improved performance of the test facility. Problems were run to compare the performance of different test facilities upon the substitution of U-233. The description and results of the problems are given in Table III-39-III.

A U-ZrH_2 fueled, sodium cooled test reactor has an eleven fold decrease in power and a 2.2 fold increase in the maximum power density ratio upon substituting U-233 for U-235 in the test section. Reducing the test section radius from 13 cm to 8 cm still results in an attractive performance.

Significant improvements in the performance of the "bootstrap" test reactors are also seen in Table III-39-II. Problems 447 and 451 are used as a basis of comparison.

Since the test section fueled with U-233 and reflected

by nickel is critical at a radius of 14.03 cm (see Problem 473), the radius of the test section was reduced to 8 cm for more meaningful results. Even with a smaller test section, the size and power output of the test reactors are greatly reduced. Also, the power density ratio increased from two to three upon the substitution of U-233; thus, fewer sets of tests in a bootstrap operation are needed to attain the desired power density and temperature. One limitation of this approach is the amount of U-233 available for testing rocket fuel elements. U-233 is being produced in kilogram quantities but the demand still appears to be greater than the supply.

The objection of excessive size and power output for

a Rocket Fuel Test Reactor is greatly reduced or removed by the use of the "bootstrap" approach or the substitution of U-233 in the test section (or a combination of both). Although there still exist technical difficulties which have not been investigated, an RFTR appears to be practical as well as technically feasible.

REFERENCES

1. J. T. Madell and D. R. MacFarlane, *Investigation of a Rocket Fuel Test Reactor*, Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010 (January, 1965) p. 138.
2. Reactor Physics Constants, ANL-5800, 2nd edition, (July, 1963) p. 761.

Section IV

Experimental Techniques and Facilities

Measured integral reactor physics parameters are necessary for either direct introduction into reactor design calculations or for comparison with values obtained analytically using basic microscopic cross section data. In the latter case the objective is to verify either the cross section data or the calculational techniques. Reactor integral parameters are obtained using special measurement techniques in exponential or critical facilities. The papers presented in this section deal largely with the development of such special techniques, facilities, and radiation detection devices. Included also are papers which describe specially developed equipment useful in obtaining microscopic nuclear data. In addition, there are papers which discuss methods for analyzing and interpreting experimentally determined quantities, and others which discuss the development of special equipment used in measurements relating to power reactor fuel element failures.

231 -310

IV-1. Calculation of Pile-Up Distortion

R. GOLD

In many experiments there may exist a high background of undesirable radiation which is detected, when the detection of relatively rare events, or another type of radiation, is actually desired. In such circumstances, reasonably accurate values of the pile-up count rate may be required. For many such measurements in nuclear spectroscopy, a knowledge of pile-up distortion would prove helpful in both the preparation and analysis of experiments.

It is convenient to introduce accepted experimental terminology in discussing this effect. Thus, the time dependence of the detector response will be referred to as the pulse shape of the detector. The distribution of the maxima in response amplitude (or the distribution of peak values) will be called the pulse height distribution of the detector.

A distinction must be made between two different types of pulse height distributions that have been considered; those which contain pile-up distortion and those which do not. These distributions are designated as actual and *ideal* pulse height distributions, respectively. An actual pulse height distribution is one that

arises from the output of a detector and therefore inherently contains pile-up distortion. In contrast, an *ideal* pulse height distribution is purely conceptual; that is, it cannot occur in practice. It may be regarded as that pulse height distribution which is obtained from a detection system in the limit of vanishing count rate.

A general solution has been obtained for the actual pulse height distribution, $D(\nu)$.¹ It may be expressed as an integral representation of the form

$$D(\nu) = \int_0^\infty B(\nu')P(\nu - \nu') d\nu' \quad (1)$$

where $P(\nu)$ is the *ideal* pulse height distribution and the function $B(\nu')$ depends upon the *ideal* pulse height distribution, the pulse shape (i.e. the time-dependent response of the detector), and the average count rate.

Some important special cases, which are of interest in experimental applications, have been studied. Table IV-1-I depicts the *ideal* pulse height distributions and pulse shapes corresponding to the special cases that have been considered. Computer programs have been developed to evaluate the integral representations which arise for these special cases. Analysis of numerical results obtained for Cases Ib, Ic, and IIa has been published.²

REFERENCES

1. R. Gold, *Theoretical Estimates of Pile-Up Distortion*, ANL-6949, (March 1965)
2. R. Gold, *Calculation of Spectral Distortion Due to Pile-Up Effect*, Rev. Sci. Instr. **36**, No. 6, 784-794 (June 1965).

TABLE IV-1-I. SPECIAL CASE DESIGNATIONS

Ideal Pulse Height Distribution	Pulse Shape	
	I. Square	II. Exponential
a. Delta function	Ia	IIa
b. Exponential	Ib	IIb
c. Gaussian	Ic	IIc

IV-2. Numerical Evaluation of Fourier Integrals

R. GOLD, C. COHN and I. OLSON

INTRODUCTION

In many applications, one must evaluate Fourier transforms of the form

$$f(\omega) = \int_0^\infty f(t) \cos(\omega t) dt. \quad (1)$$

In Eq. (1) the function $f(t)$ is either known analytically or the result of an experimental measurement. It should be noted that customary quadrature techniques, such as the Laguerre-Gauss or the Hermite-Gauss method,¹ cannot be applied to Fourier integrals. Due to the oscillatory term in the integrand [i.e. $\cos(\omega t)$],

the interpolation formulas used in these methods do not provide a close fit for the behavior of the integrand. Consequently, different techniques must be employed for the efficient evaluation of Fourier integrals. The present method may be compared with other techniques that have been introduced to treat this problem.^{2, 3}

METHOD

In the present approach, the range of integration is partitioned into two regions $0 \leq t \leq t_m$ and $t_m \leq t < \infty$. The region $t_m \leq t < \infty$ is designated as the asymptotic region. In terms of this partitioning, the Fourier integral in Eq. (1) reduces to

$$f(\omega) = \int_0^{t_m} f(t) \cos(\omega t) dt + J(\omega, t_m) \quad (2)$$

where the expression $J(\omega, t_m)$ represents the asymptotic contribution. The appropriate range of values for t_m depends only upon the behavior of $f(t)$. Consequently different techniques need be employed in the asymptotic region, depending upon whether $f(t)$ is known analytically or measured experimentally.

CASE (a): $f(t)$ IS A KNOWN ANALYTICAL FUNCTION

In this event, t_m is chosen large enough so that a simple asymptotic form of $f(t)$ is valid for all $t > t_m$. Thus

$$f(t) \sim f_a(t), \quad t \geq t_m. \quad (3)$$

Consequently, the approximate asymptotic contribution reduces to

$$J_a(\omega, t_m) = \int_{t_m}^{\infty} f_a(t) \cos(\omega t) dt. \quad (4)$$

The expression given above may also be evaluated in an asymptotic series, since t_m has been chosen sufficiently large. Hence, the contribution used in Eq. (2) is the asymptotic form of the integral given in Eq. (4). This expansion can usually be found utilizing integration by parts. One can retain as many terms as is desired.

CASE (b): $f(t)$ DETERMINED EXPERIMENTALLY

If $f(t)$ results from a measurement, then it can only be determined over a finite region. In this event, the interval $0 \leq t \leq t_m$ represents the domain of the experimental data. Consequently $f(t)$ can be determined in the region $t \geq t_m$ only by means of some extrapolation or continuation method. For example, a least-squares fit of the experimental points in the neighborhood of $t = t_m$ can be used in the region $t \geq t_m$. In fact, for many experiments the asymptotic behavior of $f(t)$ is closely approximated by a decreasing exponential

function. For such cases

$$f(t) \approx f_a(t) = Ce^{-\alpha t}, \quad t \geq t_m \quad (5)$$

where the constants C and $\alpha(>0)$ are determined from the least-squares fit to the experimental data. Hence in these circumstances, the asymptotic contribution becomes

$$J_a(\omega, t_m) = Ce^{-\alpha t_m} \left[\frac{\omega \sin(\omega t_m) - \alpha \cos(\omega t_m)}{\alpha^2 + \omega^2} \right]. \quad (6)$$

There remains only the evaluation of the integral in the region $0 \leq t \leq t_m$ in Eq. (2). This term can be determined accurately if $f(t)$ is a slowly varying function. Let $\{t_i\}$, $i = 1, 2 \dots m$, be a proper net (subdivision) on the interval $(0, t_m)$; then the contribution to the integral which arises between t_i and t_{i+1} is

$$I_i = \int_{t_i}^{t_{i+1}} f(t) \cos(\omega t) dt. \quad (7)$$

Applying the mean value theorem to Eq. (7),

$$\begin{aligned} I_i &= f(\bar{t}) \int_{t_i}^{t_{i+1}} \cos(\omega t) dt \\ &= \frac{f(\bar{t})}{\omega} [\sin(\omega t_{i+1}) - \sin(\omega t_i)] \end{aligned} \quad (8)$$

where $\bar{t}$ is some value of t in the interval $t_i \leq t \leq t_{i+1}$. To ensure the applicability of the mean value theorem the function $\cos(\omega t)$ must not change sign in the interval $t \leq \bar{t} \leq t_{i+1}$. This can be accomplished by including the zeros of $\cos(\omega t)$ as a subset of the set of net points. Moreover, for slowly varying $f(t)$, one can use

$$f(\bar{t}) \approx \frac{f(t_{i+1}) + f(t_i)}{2}, \quad (9)$$

whence

$$I_i = \left[\frac{f(t_{i+1}) + f(t_i)}{2\omega} \right] [\sin(\omega t_{i+1}) - \sin(\omega t_i)]. \quad (10)$$

Since

$$\int_0^{t_m} f(t) \cos \omega t dt = \sum_{i=1}^{m-1} I_i \quad (11)$$

the Fourier transform becomes

$$\begin{aligned} f(\omega) &= \frac{1}{2\omega} \sum_{i=1}^{m-1} [f(t_{i+1}) + f(t_i)] \\ &\quad \cdot [\sin(\omega t_{i+1}) - \sin(\omega t_i)] + J_a(\omega, t_m). \end{aligned} \quad (12)$$

This form is easily adapted for computer evaluation.

The accuracy of this method can be estimated by utilizing the fact that the Fourier transform $f(\omega)$ should be independent of the choice of t_m (provided t_m is large enough to ensure that the asymptotic expansions are valid). By employing a number of different values of

FIG. IV-2-1. The Fourier Transform of $f(t)$ Given in Eq. (14) for $\alpha = 0.01$ and $\beta = 0.06$ in the Region $\omega \geq 1.5$. ANL Neg. No. 112-5455.

t_m and examining the change which occurs in $f(\omega)$ the accuracy of this method can be ascertained for any specific case of interest.

EXAMPLE

The method has been employed to evaluate the following Fourier integral, which arises for a special

case of pileup:⁴

$$f(\omega) = \frac{2e^{-\alpha}}{\pi} \int_0^{\infty} \cos[(\omega - 1)t] e^{-\beta^2 t^2} \cdot \exp[\alpha e^{-\beta^2 t^2} \cos t] \cdot \cos[\alpha e^{-\beta^2 t^2} \sin t] dt. \quad (13)$$

Here, one has

$$f(t) = \frac{2e^{-\alpha}}{\pi} e^{-\beta^2 t^2} \exp[\alpha e^{-\beta^2 t^2} \cos t] \cdot \cos[\alpha e^{-\beta^2 t^2} \sin t], \quad (14)$$

where α and β are parameters.

The asymptotic contribution is, to third order:

$$J_a(\omega, t_m) = \frac{2e^{-\alpha}}{\pi} e^{-\beta^2 t_m^2} \left\{ \frac{\cos[(\omega - 1)t_m]}{2\beta^2 t_m} - \frac{(\omega - 1) \sin[(\omega - 1)t_m]}{4\beta^4 t_m^3} - \frac{\cos[(\omega - 1)t_m]}{4\beta^4 t_m^3} \right\}. \quad (15)$$

This can be reduced to

$$J_a(\omega, t_m) = - \frac{(-1)^n (2e^{-\alpha}/\pi) (\omega - 1) e^{-\beta^2 t_m^2}}{4\beta^4 t_m^3} \quad (16a)$$

when t_m is chosen such that

$$(\omega - 1)t_m = (n + \frac{1}{2})\pi \quad (16b)$$

where n is an integer.

The accuracy obtained for this Fourier transform was better than 1-2% of $f(\omega)$ for values of $f(\omega)$ as low as 10^{-8} . A graph of the Fourier transform, $f(\omega)$, for the parameters $\alpha = 0.01$ and $\beta = 0.06$, is given in Fig. IV-2-1.

REFERENCES

1. F. B. Hildebrand, *Introduction to Numerical Analysis*, (McGraw-Hill Book Co., Inc., New York 1956).
2. L. N. G. Filon, *On a Quadrature Formula for Trigonometric Integrals*, Proc. Royal Soc. Edinburgh **49**, 38-47 (1928).
3. H. Hurwitz, Jr. and P. F. Zweifel, *Numerical Quadrature of Fourier Transform Integrals*, Math Tables and Other Aids to Computation **10**, 140-149 (1956).
4. R. Gold, *Theoretical Estimates of Pile-Up Distortion*, ANL-6949 (March 1965).

IV-3. Proportional Counter Multiplication Process

R. GOLD and E. F. BENNETT

INTRODUCTION

Employing pulse-shape rejection techniques to imitate gamma-ray induced events, 4π -recoil proportional counters have been utilized to measure fast

neutron spectra from 1 MeV to below 1 keV.¹ At the low energy end of this region, the proportional counter multiplication process can significantly broaden experimental measurements. Hence, the low-energy limit of

FIG. IV-3-1. Experimental Arrangement of Proportional Counter, Window Assembly, and Ultraviolet Source. ANL Neg. No. 112-5301.

4π -neutron spectroscopy with proportional counters must rest upon a knowledge of this multiplication process. Consequently, an experimental examination of the electron multiplication process in these proportional counters has been undertaken.

EXPERIMENTAL METHOD

Figure IV-3-1 displays the experimental arrangement, which is similar to the original experiment of S. Curran, A. Cockroft, and J. Angus.² With such an arrangement, the response of the proportional counter to single electrons can be determined. Ultraviolet quanta enter the counter through the quartz window, traverse the active volume of the counter gas, and

TABLE IV-3-I. PROPORTIONAL COUNTER GAS MIXTURE^a

	Partial Pressure		
	Methane	Argon	Nitrogen
Counter I	10	76	4
Counter II	20	76	4
Counter III	76	0	4

^a The partial pressures are given in units of centimeters of mercury.

FIG. IV-3-2. The Diethorn Representation of Gas Amplification A as a Function of Anode Voltage V for Counters I and II. ANL Neg. No. 112-5307.

FIG. IV-3-3. The Diethorn Representation of Gas Amplification A as a Function of Anode Voltage V for Counter III. *ANL Neg. No. 112-5306*.

create photoelectrons at the opposite counter wall. Once created, a photoelectron will drift in toward the anode wire of the counter where an electron avalanche is subsequently generated and detected. Measurement of the response distribution which arises from these single electron events permits a study of the statistics of the multiplication process.

Different counter gas fillings of argon and methane have been investigated. These are designated in Table IV-3-I as Counters I, II, and III, respectively. In each of these fillings, a small amount of nitrogen has been added so that the $N-14(n,p)C-14$ reaction could be utilized to evaluate counter operation. The counter was placed in the center of a small moderating assembly, which contained a plutonium-beryllium neutron source. In this environment, monoenergetic protons from the exothermic $N-14(n,p)C-14$ reaction give rise to a well-defined response peak. This peak affords measurement of the gas gain of the counter as a function of anode voltage. In the region of higher gas gain, the average value of the single electron response distribution can be used to define the gas amplification.

The formula of W. Diethorn³ has been used to treat the experimental gas gain data. In this description, the gas amplification A and the anode voltage V are given by

$$\frac{\log A}{V} = C_1 \log V + C_2 \quad (1)$$

where C_1 and C_2 are constants characteristic of the

proportional counter construction and gas filling. The behavior of gas gain has been utilized as a criterion for defining proper proportional counter operation. Any deviation in gas gain from a smooth monotonically increasing function, such as that suggested in Eq. (1), has been utilized to imply the onset of phenomena which are not truly representative of a pure electron multiplication process.

The gas amplification data obtained with Counters I, II, and III is presented in Figs. IV-3-2 and IV-3-3. For Counters I and II, the gas gain rapidly departs from the expected extrapolation at higher values of gas gain. This behavior implies that more methane is necessary to quench ultraviolet instabilities which arise in Counters I and II in the region of large A .

The data of Fig. IV-3-3 obtained with the methane filling (Counter III) indicates that although this difficulty has been overcome, a distinctly different phenomenon has arisen. For Counter III, the departure from predicted behavior, at the highest values of A , now falls below the extrapolation. This effect is attributed to saturation in the electron avalanche due to space charge limitation in the neighborhood of the anode wire. In spite of this effect, which occurs at the highest values of A , in the region $\log V \leq 8.1$, the gas gain behaves as a smooth monotonically increasing function of V . Hence the response distribution obtained in this region should be representative of proper proportional counter operation.

FIG. IV-3-4. The Single Electron Spectra Obtained with Counter III for Anode Voltages of (a) 2700 and 3050 V; (b) 2800 and 3200 V; and (c) 2900 and 3350 V. ANL Neg. No. 112-5326.

Some single electron spectra obtained in this region are displayed in Fig. IV-3-4. These figures present the measured distribution $P(\nu)$ in normalized form where

$$\nu = n/\bar{n} \quad (2)$$

with n and $\bar{n}$ the final number and average final number

of electrons in the avalanche, respectively. It is evident that the single electron response distribution is not constant, but changes with anode voltage. Consequently one can conclude that the electron branching process in proportional counters is not invariant with respect to the average final number of electrons.⁴

REFERENCES

1. E. F. Bennett, *A Study of the 1/E Slowing-Down Neutron Spectrum Using 4 π -Recoil Proportional Counters*, ANL-6897 (September 1964).
2. S. C. Curran, A. L. Cockcroft and J. Angus, *Investigation of Soft Radiation by Proportional Counters—V. Use As a De-*

tector of Ultra-violet Quanta and Analysis of the Gas Multiplication Process, Phil. Mag. **40**, 929 (1949).

3. W. Diethorn, *A Methane Proportional Counter System for Natural Radiocarbon Measurements*, NYO-6628 (1956).
4. R. Gold and E. F. Bennett, *Electron Multiplication in Proportional Counters* (submitted for publication).

IV-4. The Effect of Deadtime on Counting Errors

C. E. COHN

It is well known that counts taken at high rates must be corrected for losses due to the deadtime of the counting system. However, it is not commonly realized that the estimated standard deviations of these counts must also be corrected. This paper will consider such corrections for two simple cases, and will consider the question of how high in count rate it is worthwhile to go in seeking maximum precision.

To evaluate these corrections, one first finds the average, $\bar{t}$, and its variance, σ_t^2 , of the time between successive recorded counts. Then, one considers the total number of counts N recorded in a time T , and one applies the asymptotic formulas¹

$$C \equiv \frac{N}{T} = \frac{1}{\bar{t}} \quad (1)$$

and

$$\frac{\sigma_N^2}{T} = \frac{\sigma_t^2}{\bar{t}^3}, \quad (2)$$

where C is the observed count rate and σ_N^2 is the variance of N .

The first case to be considered is that of the non-paralyzable counter. When such a counter records a pulse, it is incapable of recording another pulse until a time interval τ , the "deadtime", has elapsed. Pulses arriving during this interval have no effect. The probability $P(t) dt$ that the time interval between two recorded pulses will be between t and $t + dt$ will then be the probability that no pulse arrives from τ (assuming that the first pulse occurred at $t = 0$) to t , times the probability that one pulse arrives between t and $t + dt$. For a Poisson-distributed source of pulses with count rate n , this is

$$P(t) dt = ne^{-n(t-\tau)} dt, \quad (3)$$

from which may be calculated

$$\bar{t} = \tau + \frac{1}{n} \quad (4)$$

and

$$\sigma_t^2 = \frac{1}{n^2}. \quad (5)$$

Equation (1) then yields the familiar formula

$$N = \frac{nT}{1 + n\tau}, \quad (6)$$

which may be solved for an estimate of n , given an observed N .

The function $C\tau = N\tau/T$ plotted on Fig. IV-4-1 shows how C approaches $1/\tau$ as $n\tau$ becomes very large. (The line labeled "no deadtime loss" is merely included for comparison, showing C increasing linearly with n . Note that τ is merely an arbitrary number, of no significance for that case.)

Equation (2) yields

$$\sigma_N^2 = \frac{nT}{(1 + n\tau)^3}. \quad (7)$$

This yields the variance of the estimated n through the law of propagation of error,

$$\sigma_n^2 = \left(\frac{dn}{dN}\right)^2 \sigma_N^2. \quad (8)$$

Differentiating Eq. (6) implicitly with respect to n yields

$$\frac{dn}{dN} = \frac{1}{T} (1 + n\tau)^2. \quad (9)$$

Eqs. (7), (8) and (9) yield

$$\sigma_n = \sqrt{\frac{n}{T} (1 + n\tau)} \quad (10)$$

for the standard deviation of n , and

$$\frac{\sigma_n}{n} = \sqrt{\frac{\tau}{T}} \sqrt{\frac{1 + n\tau}{n\tau}} \quad (11)$$

for the relative standard deviation. Here, the second

FIG. IV-4-1. Observed Count Rate as Function of Mean Input Pulse Rate. ANL Neg. No. 112-5490.

radical is a dimensionless function of a dimensionless argument and is plotted in Fig. IV-4-2. (The line marked "no deadtime loss" is again included for comparison purposes, showing $(\sigma_n/n)\sqrt{T}$ decreasing as the square root of n .)

Note that the relative standard deviation with deadtime losses is greater than that without such losses, for a given n and T . Equation (7) shows that the variance of the observed count rate is smaller with deadtime losses than without. In fact, the variance approaches zero for very large n , since here a count will be recorded right after the dead interval for the previous count, and the recorded counts will form a regular train. However, Eqs. (6) and (9) show that with increased deadtime losses the observed count rate yields less information about the source count rate. The superposition of these two effects leads, in Eq. (11), to a relative standard deviation which approaches $\sqrt{\tau/T}$ as n becomes very large. This asymptotic behavior can be readily seen in Fig. IV-4-2.

In many experimental situations, one is interested in relative rather than absolute information about radiation intensities; e.g., in various reactor kinetic studies. In addition, one often is free to vary the detection efficiency of the counter so that one may choose the value of n at which he will work. In such cases, it is of interest to know how high in n one may go and continue to get worthwhile improvements in precision. It is then evident from Fig. IV-4-2 that little is gained in going beyond values of 1 or 2 for $n\tau$ for the nonparalyzable counter. Even these rates are far higher than customary, and the deadtime corrections are of such magnitude that the rates are obviously not prac-

tical unless the counter deadtime is well known and unless the characteristics of the counter conform closely to the mathematical model. (Perhaps, in the design of counting systems, it would be worthwhile to make some effort to attain well-behaved resolution characteristics.)

When an estimate for n has been obtained from a formula such as Eq. (6), an estimate of standard deviation must also be made. It should be evident from this discussion that the result of the usual formula $\sigma_n = \sqrt{n/T}$ must be corrected for deadtime effects. For the nonparalyzable counter, the corrections may be obtained from Eq. (11). The correction factor by which the result of the usual formula must be multiplied is plotted in Fig. IV-4-3 for both cases considered here.

The second case considered is that of the paralyzable counter. Here, as before, a recorded pulse initiates a dead interval of length τ seconds, during which no pulses can be recorded. However, a pulse arriving during the dead interval, although not recorded, causes the dead interval to start again from the beginning. Thus, the counter is always dead for τ seconds after the last received pulse, whether recorded or unrecorded.

For this case, it has been shown² that

$$\bar{t} = \frac{1}{n} e^{n\tau} \quad (12)$$

and

$$\sigma_t^2 = \frac{e^{2n\tau}}{n^2} - \frac{2\tau e^{n\tau}}{n} \quad (13)$$

Equation (1) then yields

$$N = nTe^{-n\tau} \quad (14)$$

FIG. IV-4-2. Relative Error of Estimated n as Function of Mean Input Pulse Rate. ANL Neg. No. 112-5492.

FIG. IV-4-3. Correction Factors for Error of Estimated n (σ_n = error correction factor times $\sqrt{n/T}$) ANL Neg. No. 112-5491.

Since this is a transcendental equation for n , with the solution not expressible in closed form, obtaining an estimate for n is not as straightforward as previously. Series approximations are commonly used³ for small values of $n\tau$, although with a digital computer it would be quite practical to solve the equation by iteration.

Figure IV-4-1 shows how the observed count rate for a paralyzable counter is affected by deadtime. Unlike the former case, the observed count rate does not increase monotonically with the input pulse rate. Rather, it has a maximum at $n\tau = 1$ and decreases thereafter.

Eqs. (2) and (13) yield

$$\sigma_N^2 = nTe^{-n\tau} - 2n^2T\tau e^{-2n\tau}. \quad (15)$$

Again, σ_n is to be estimated using Eq. (8). Differentiating Eq. (14) implicitly with respect to N yields

$$\frac{dn}{dN} = \frac{e^{n\tau}}{T(1 - n\tau)}. \quad (16)$$

Note that the aforementioned maximum in count rate leads to a singularity in this equation at $n\tau = 1$.

Substituting Eqs. (15) and (16) into Eq. (8) yields

$$\sigma_n = \sqrt{\frac{n(e^{n\tau} - 2n\tau)}{T(1 - n\tau)^2}} \quad (17)$$

for the standard deviation of n , and

$$\frac{\sigma_n}{n} = \sqrt{\frac{\tau}{T}} \sqrt{\frac{e^{n\tau} - 2n\tau}{n\tau(1 - n\tau)^2}} \quad (18)$$

for the relative standard deviation. Note that the aforementioned singularity leads to an infinite error at $n\tau = 1$ (although the law of propagation of error undoubtedly breaks down here).

The second radical in Eq. (18) is also plotted in Fig. IV-4-2. Here, the relative error has a minimum at $n\tau$ around 0.38, and significant reductions in error are not obtained beyond about 0.2. These rates, although less than the worthwhile range for the nonparalyzable counter, are still somewhat greater than commonly used.

A convenient way to check the behavior of a given counting system at high counting rates might be to use a nuclear reactor on a rising period. After the transients have decayed, and before power feedbacks begin to affect the reactivity, the reactor flux should be very accurately exponential in time. A curve of count rate versus time, plotted on semi-log paper, will then look like the curves in Fig. IV-4-1. Fitting a function, such as Eq. (6) or (14), to this curve will yield an estimate of τ and will give an indication of the validity of the assumed model.

It is apparent from the figures that the behavior is not particularly sensitive to the nature of the model for $n\tau \leq 0.1$, so that use of counting rates below this does not require such information.

REFERENCES

1. Emanuel Parzen, *Stochastic Processes* (Holden-Day, Inc., San Francisco, 1962), p. 180.
2. *Ibid.*, pp. 166-168.
3. K. E. Plumlee, *Theory and Methods of Correction for Resolution Losses of Counting Equipment*, Trans. Am. Nucl. Soc. 7, No. 2, 341 (November 1964).

IV-5. Theory and Methods of Correction for Resolution Losses of Counting Equipment

K. E. PLUMLEE

METHODS OF CALIBRATING EQUIPMENT

The use of small irradiated foils permits measurement of reaction rates and power distributions in experimental facilities. Study of foil activation counting led to the development of improved theory and methods of measurement and correction for resolution losses in radiation counting equipment. This requires calibration with several source intensities covering the range of operation of the equipment.

If the absolute activities and counting efficiencies are available for the sources used in calibration, a calibration curve may be plotted or an interpolation formula fitted to the calibration points. An interpolation formula permits accurate correction of counting data within the range of calibration, and eliminates reading errors. Moreover, the use of an appropriate interpolation formula which can be analyzed by theoretical models permits evaluation of higher ordered equipment characteristics. Older methods are shown to accommodate no more than a first-order measured correction; however, they do include various arbitrary higher-ordered terms.

By Eq. (1) the relation of a source term N is given as a function of the associated count rate n . The term N is also a function of radiation source activity S and counter efficiency E , i.e., $N = SE$. Equation (1) is a slightly modified power series interpolation formula. It also resembles a theoretical formula proposed by T. Kohman¹ for resolution loss correction. Several other correction formulas are readily related to equations derived from Eq. (1).

$$N = n(C_0 + C_1n + C_2n^2 + C_3n^3 \dots). \quad (1)$$

The coefficients $C_0, C_1, C_2, \dots$ may be solved by determinants by providing a series of measurements of the ratio (N/n) using different source intensities. The number of coefficients obtainable is known to be one less than the number of measurements made, because of mathematical restrictions. However, C_0 is seen to be unity.

The equation may be rewritten for the case involving only a knowledge of the relative source intensities $R_2, R_3, \dots, R_j = N_j/N_1$, and the measured count rates $n_1, n_2, n_3, \dots, n_j$. The use of aliquots of liquid solutions is a standard method by which sources of known relative intensities may be prepared. Other methods include graduated irradiation levels or times in pre-

paring sources, or use of sources having convenient decay constants which can be counted at known relative activations during decay. Equation (1) is rewritten with the following change of coefficients, to obtain Eq. (2):

$$C_i = C'_i N_1 / n_1 = C'_i / C'_0, \quad (\text{note } C_i \geq C'_i),$$

$$\frac{n_1}{n_j} R_j = C'_0 + C'_1 n_j + C'_2 n_j^2 + C'_3 n_j^3 + \dots \quad (2)$$

If even the relative source activities are unknown, R_j may be eliminated from Eq. (2) but this requires a second set of counts $n_2^*, n_3^*, \dots, n_j^*$ involving a reference source paired with each other calibration source in turn. Counting efficiency must be uniform during counting of single and paired sources. Noting that $R_j^* = (R_j + 1) = (N_j + N_r)/N_r$, Eq. (2) involving R_j is subtracted from the equation involving R_j^* to obtain

$$\frac{n_r}{n_j^* - n_j} = C'_0 + \frac{n_j^{*2} - n_j^2}{n_j^* - n_j} C'_1 + \frac{n_j^{*3} - n_j^3}{n_j^* - n_j} C'_2$$

$$+ \dots + \frac{n_j^{*i+1} - n_j^{i+1}}{n_j^* - n_j} C'_i. \quad (3)$$

An approximate solution for C'_1 and for resolution time $\tau = C_1$ is readily obtained by rearranging Eq. (3) to obtain Eq. (4). As shown in Table IV-5-I, this approximation overestimates τ by less than 0.3% if the product $(n_j^* \tau)$ is no greater than 0.002, or 6% if $n_j^* \tau < 0.02$.

$$C'_1 = \frac{-C'_0}{n_j^* + n_j} - \frac{n_r}{n_j^{*2} - n_j^2}$$

$$- \left[\frac{n_j^{*3} - n_j^3}{n_j^* - n_j} C'_2 + \frac{n_j^{*4} - n_j^4}{n_j^* - n_j} C'_3 + \dots \right],$$

$$\tau = C_1 = C'_1 N_r / n_r = \frac{-1}{n_j^* + n_j} + \frac{N_r}{n_j^{*2} - n_j^2}$$

$$- \frac{N_r}{n_r} [\text{bracketed quantity above}] \quad (4)$$

$$\tau = C_1 \leq \frac{N_r + n_j - n_j^*}{n_j^{*2} - n_j^2}.$$

A slightly different approximation was used by H. Kirby² for paired source measurements, following P. Moon's³ suggestion. Although the errors involved are about 50% larger than those resulting from Eq. (4), the greater availability of n_r compared with N_r makes

TABLE IV-5-I. ESTIMATED RATIOS OF MEASURED TO ACTUAL τ BY EQS. (4) AND (5) USING PAIRED EQUAL SOURCES

$n\tau$ (Single Source)	Formula A			Formula B			Formula C		
	n^*/n	τ_4/τ	τ_5/τ	n^*/n	τ_4/τ	τ_5/τ	n^*/n	τ_4/τ	τ_5/τ
10^{-4}	1.9998	1.0003	1.0004	1.9998	1.0003	1.0004	1.9998	1.0003	1.0004
2×10^{-4}	1.9996	1.0005	1.0008	1.9996	1.0005	1.0008	1.9996	1.0005	1.0008
10^{-3}	1.9980	1.003	1.004	1.9980	1.003	1.004	1.9980	1.003	1.004
2×10^{-3}	1.9960	1.003	1.004	1.9960	1.003	1.004	1.9960	1.003	1.004
10^{-2}	1.9798	1.037	1.052	1.9792	1.062	1.083	1.9795	1.050	1.068
2×10^{-2}	1.9591	1.080	1.112	1.9564	1.139	1.193	1.9577	1.110	1.153
5×10^{-2}	1.8947	1.219	1.325	1.873	1.453	1.679	1.885	1.321	1.475
10^{-1}	1.7778	1.543	1.915	0.623	neg.	neg.	1.726	1.984	2.805

Eq. (5) the more convenient approximation. A generalized derivation of Kirby's approximation is obtained by using Eq. (1) to eliminate N_r , N_j , and N_j^* , providing alternatives to Eqs. (3) and (4):

$$0 = N_r + N_j - N_j^* = (n_r + n_j - n_j^*)C_0 \\ + (n_r^2 + n_j^2 - n_j^{*2})C_1 + \dots, \\ \tau = C_1 = \frac{n_r + n_j - n_j^*}{n_j^{*2} - n_r^2 - n_j^2} C_0 \\ + \left(\frac{n_r^2 + n_j^2 - n_j^{*2}}{n_j^{*2} - n_r^2 - n_j^2} C_2 + \dots \right),$$

and

$$\tau = C_1 \leq \frac{n_r + n_j - n_j^*}{n_j^{*2} - n_r^2 - n_j^2}. \quad (5)$$

Application of Eq. (3), (4), or (5) requires accuracy of four or five significant figures in input data. In order to average statistical fluctuations of counting sufficiently to obtain four or five figure significance, cumulative counts must range upwards from 10^6 , with 10^8 being preferred. If $(n_j^*/\tau) \leq 0.02$ or $(n_j^*/n_j) \geq 1.98$ for an equal paired source measurement, the shortest counting time to obtain 10^6 or 10^8 counts is $10^8\tau$ or $10^{10}\tau$ respectively, while counting a single source.

The range of equipment characteristics usually encountered does not greatly affect the application of Eq. (4) or (5), as indicated in Table IV-5-I for somewhat extreme as well as for typical cases. It is assumed that equipment obeys correction Formula (A),⁴ (B),⁵ or (C)⁶ given below. Formula (A) indicates very good equipment characteristics; Formula (B) indicates rather poor characteristics; and Formula (C) is typical of good equipment.

$$N_A = \frac{n}{1 - n\tau}, \quad (\text{Ref. 4}) \quad (A)$$

$$N_B = \frac{n}{1 - N_B\tau}, \quad (\text{Ref. 5}) \quad (B)$$

$$N_C = n \exp(N_C\tau), \quad (\text{Ref. 6}) \quad (C)$$

TABLE IV-5-II. COMPARISON OF ARBITRARILY CORRECTED COUNT RATES

Input Data $n\tau$	Resulting Correction Factors			Range of N , %
	N_A/n	N_B/n	N_C/n	
10^{-4}	1.0001	1.0001	1.0001	$\ll 0.01$
2×10^{-4}	1.0002	1.0002	1.0002	$\ll 0.01$
10^{-3}	1.0010	1.0010	1.0010	< 0.01
2×10^{-3}	1.0020	1.0020	1.0020	< 0.01
10^{-2}	1.0101	1.0102	1.01015	0.01
2×10^{-2}	1.0204	1.0208	1.0206	0.04
5×10^{-2}	1.0526	1.0556	1.0541	0.3
10^{-1}	1.1111	1.1270	1.1182	1.4
2×10^{-1}	1.250	3.618	1.296	289.

Equations (4) and (5) are linear approximations determined by fitting to two data points. The resulting estimates of τ are not entirely consistent, but the discrepancy is less with Formula (A) than with Formula (B) or (C) which depart more drastically from linearity as indicated by the comparison under "Formulas Showing Equipment Characteristics" in this paper. As shown in Table IV-5-II, very little error appears in the corrected results at low count rates regardless of use of Formula (A), (B), or (C). Use of Formulas (B) and (C) compounds any overestimate of τ ; consequently Formula (A), or others of Table IV-5-III, often provides better corrections than Formula (B) or (C) even though Formula (B) or (C) would be more appropriate if the correct value of τ were used.

Equation (1) is conveniently rewritten with $C_0 = 1$ and $C_1 = \tau$, the resolving time, and the coefficients are redefined as $c_i = C_i/\tau^i$, yielding Eq. (6):

$$N = n[1 + c_1n\tau + c_2(n\tau)^2 + c_3(n\tau)^3 + \dots]. \quad (6)$$

Equation (6) serves as an interpolation formula between calibration points obtained with calibration sources counted in fixed geometry. All coefficients may be determined analytically via Eq. (1), (2), or (3) as well. Interestingly, the established formulas for re-

TABLE IV-5-III. POWER SERIES COEFFICIENTS OF RESOLVING LOSS FORMULAS

Formula	c_1	c_2	c_3	c_4
(A) $N = \frac{n}{1 - n\tau}$	1	1	1	1
(B) $N = \frac{n}{1 - N\tau}$	1	2	5	14
(C) $N = n \exp(N\tau)$	1	$\frac{3}{2}$	$\frac{3^2}{2!}$	$\frac{1 \cdot 2 \cdot 5}{2 \cdot 4}$
(D) $N = n \exp(n\tau)$	1	$\frac{1}{2}$	$\frac{1}{3!}$	$\frac{1}{4!}$
(E) $\int_t^\infty N dt = \int_t^\infty n \exp(N\tau) dt$	$\frac{\lambda}{2}$	$\frac{\lambda^2}{3}$	$\frac{\lambda^3}{4}$	$\frac{\lambda^4}{5}$

solving loss correction all yield the same series expansion as Eq. (6), but with differing values of the coefficients c_i . These were obtained by simple methods, including iteration and reversion of series, with results shown in Table IV-5-III.

Study of the formulas in Table IV-5-III shows that Formula (A)⁴ applies a fixed resolving interval τ for each event scaled, but Formula (B)⁵ applies an interval τ for every event occurring whether resolved or not. Formula (C),⁶ the Poisson approximation, includes statistical adjustment to account for overlapping of intervals since an unresolved event commences during the interval τ accompanying the preceding event. Formula (D)⁷ is a rough approximation of Formula (C). Formula (E) applies Formula (C) to a practical case where the activity decays rapidly during counting. These coefficients correct the observed count rather than the count rate, and are seen to be functions of the activity decay constant λ . This illustrates a useful point, that the c_i , including even c_1 , are functions of both counting interval and activity decay constant.

The values of c_2 , c_3 , and c_4 are seen to be arbitrary in the formulas of Table IV-5-III, thus affecting all terms beyond the first-order correction. Measurement of these coefficients avoids arbitrary assumptions and extends the range in which accurate data corrections may be made, as well as providing a basis for evaluation of equipment characteristics.

Reasonable perspective is essential in calibration of counting equipment. Interpolation formulas pass through the calibration points, and consequently do not improve or compensate for errors therein. However, several calibration points may be used profitably. Older methods generally involved a paired source measurement although H. Kirby,² D. Harris,⁸ and J. Thomas⁹ considered many measurements. If the curve fitting process is to compensate for errors in calibration it becomes elaborate and laborious, and reportedly is unprofitable.² There are points of diminishing returns for effort expended in three directions—accuracy of calibration, number of calibration points, and data

fitting. Figure IV-5-1 indicates the actual values of τ obtained by various truncated and standard correction formulas. The highest-ordered coefficient fitted appears to vary noticeably with count rate, consequently it appears necessary to solve for one more than is to be used in data correction, if a highly accurate result is desired. Data were taken with equipment described in Ref. 10. The following equations were obtained as truncation of the fitted series took place:

$$N = n[1 + n\tau + (5.4 \pm 0.2)n^2\tau^2 + (7 \pm 1)n^3\tau^3],$$

$$N = n[1 + (1.01 \pm 0.01)n\tau + (6.4 \pm 0.4)n^2\tau^2],$$

$$N = n[1 + (\text{variable})n\tau].$$

FORMULAS SHOWING EQUIPMENT CHARACTERISTICS

The fitted values of the c_i terms may be related to equipment characteristics by comparison with like coefficients of a suitable derived formula. The following formulas apply to a particular type of counting device and include explicit terms for certain characteristics as described in the derivation. A typical counting device scales at a rate n ; however, some fraction F_1 of the observed counts consists of single resolved events which occur at a rate $P_1 = nF_1$, and other fractions F_2 , F_3 , F_4 , ... consist of unresolved pairs of events (F_2), and longer sequences. For such equipment the following equations apply:

$$n = \sum_{i=1}^{\infty} P_i = n \sum_{i=1}^{\infty} F_i,$$

$$N = \sum_{i=1}^{\infty} iP_i = n \sum_{i=1}^{\infty} iF_i,$$

FIG. IV-5-1. Variation of Measured T With Series Selection and Count Rate. ANL Neg. No. 112-5717.

$$N = n + \sum_{i=2}^{\infty} (i-1)P_i \quad (7)$$

$$= n(1 + F_2 + 2F_3 + 3F_4 + \dots).$$

If a scaled event is accompanied by an average interval τ during which resolution cannot occur, and events occur randomly at an average rate N , then the probability that a scaled event will be followed by one or more unresolved events is approximately

$$N\tau \approx F_2 + F_3 + F_4 + \dots = \sum_{i=2}^{\infty} F_i.$$

Similar estimates may be written for progressive sums of the series. If the i th event of an unresolved sequence of events is followed by an average resolution interval τ_i , and if we define $B_i\tau^i \equiv \tau_1\tau_2\tau_3 \dots \tau_i$, we may write and sum as follows:

$$\begin{aligned} N\tau_1 &\approx F_2 + F_3 + F_4 + \dots \\ N^2\tau_1\tau_2 &\approx F_3 + F_4 + F_5 + \dots \\ N^3\tau_1\tau_2\tau_3 &\approx F_4 + F_5 + F_6 + \dots \\ N\tau + B_2N^2\tau^2 + \dots B_iN^i\tau^i + \dots \\ &\approx \sum_{i=2}^{\infty} (i-1)F_i = (N/n) - 1. \end{aligned}$$

The preceding argument should be refined to remove a systematic overestimate in each term of the series in $N\tau$. Since each unresolved event ($i+1$)st occurs before the expiration of the interval τ_i following the preceding event, it initiates an interval τ_{i+1} which overlaps τ_i . At low count rate there is approximately an equal chance of early or late occurrence and the overlap averages approximately $\tau_i/2$ (but increases as $n^{-1} \rightarrow \tau$). Moreover the number of events which might occur during a single interval τ_i is unbounded; consequently, there are further corrections, $\tau_i/4$, $\tau_i/8$, etc., to be applied to longer sequences. The improved series in $N\tau$ is as follows, using the identity $\tau_i = B_i\tau/B_{i-1}$ as previously defined:

$$\begin{aligned} \sum_{i=2}^{\infty} (i-1)F_i &\approx N\tau - \left(\frac{1}{2}\right)N^2\tau^2 - \left(\frac{1}{4}\right)N^3\tau^3 - \left(\frac{1}{8}\right)N^4\tau^4 - \dots \\ &\quad + B_2N^2\tau^2 - \left(\frac{1}{2}\right)B_2^2N^3\tau^3 - \left(\frac{1}{4}\right)B_2^3N^4\tau^4 - \dots \\ &\quad + B_3N^3\tau^3 - \left(\frac{1}{2}\right)B_3^2N^4\tau^4 - \dots \\ &\quad + B_4N^4\tau^4 - \dots, \\ \sum_{i=2}^{\infty} (i-1)F_i &\approx N\tau + A_2N^2\tau^2 + A_3N^3\tau^3 + A^4N^4\tau^4 + \dots \\ A_i &\leq B_i - \left(\frac{1}{2}\right)B_{i-1}^2 - \left(\frac{1}{4}\right)B_{i-2}^3 \\ &\quad - \dots - \left(\frac{1}{2}\right)^{i-1}. \end{aligned}$$

Reversion of this equation by iteration produces Eq. (8) having like coefficients with Eq. (6).

$$\begin{aligned} N &= n[1 + n\tau + (A_2 + 1)n^2\tau^2 \\ &\quad + (A_3 + 3A_2 + 1)n^3\tau^3 \\ &\quad + (A_4 + 4A_3 + 2A_2^2 + 6A_2 + 1)n^4\tau^4 \\ &\quad + (A_5 + 5A_4 + 10A_3 + 5A_3A_2 + 10A_2^2 \\ &\quad + 10A_2 + 1)n^5\tau^5 + \dots]. \end{aligned} \quad (8)$$

It is immediately seen that Eq. (8) corresponds with Formulas (A) and (B), the first having the property that $0 = A_2 = A_3 = \dots$, which is characteristic of a non-paralyzable counter, and the second that $1 = A_2 = A_3 = \dots$.

If $1 = B_1 = B_2 = \dots$, then $A_2 \approx \frac{1}{2}$, $A_3 \approx \frac{1}{4}$, $A_4 \approx \frac{1}{8}$, $\dots$, and the higher terms of the A_i sequence are inconsequential in Eq. (8). The appearance of smaller values than these indicates heavy dependence on scaled events and little dependence on unresolved events. The appearance of a sequence where $A_i \approx A_{i-1}/2$ indicates heavy dependence on the terminal event regardless of the length of an unresolved sequence (i.e. systems with relatively short memories). The c_i terms obtained for such systems should be nearly identical to those shown for the Poisson approximation in Table IV-5-I. Larger A_i values indicate increased dependence on length of unresolved sequences, i.e. systems having relatively long memories.

FORMULA FOR MEASUREMENT OF RAPIDLY DECAYING ACTIVITIES

COUNTING INTERVALS OF SEVERAL HALF-LIVES DURATION

The preceding formulas apply to steady count rates. In counting rapidly decaying activities the count rate may vary significantly during the counting interval, thus affecting the coefficients c_i .

The corrected count S during a counting interval T and the measured count s were related by the following derivation which utilizes the Poisson approximation to relate N to n :

$$\begin{aligned} S &= \int_0^t N dt = \frac{-1}{\lambda} (N_t - N_0) \\ &= \frac{N_0}{\lambda} [1 - \exp(-\lambda t)] \end{aligned}$$

and

$$\begin{aligned} s &= \int_0^t n dt = \int_0^t N \exp(-N\tau) dt \\ &= \frac{1}{\lambda\tau} [\exp(-N_t\tau) - \exp(-N_0\tau)]. \end{aligned}$$

Note that, as t becomes infinite, N decays to zero,

and the terms S_∞ and s_∞ for long counting intervals are easily related by the common factor N_0 , yielding Eq. (9), or Formula (E) of Table IV-5-III.

$$S_\infty = \frac{N_0}{\lambda} = s_\infty \left[1 + \frac{1}{2} (s_\infty \lambda \tau) + \frac{1}{3} (s_\infty \lambda \tau)^2 + \cdots + \frac{1}{j} (s_\infty \lambda \tau)^{j-1} \right]. \quad (9)$$

COUNTING INTERVALS OF HALF-LIFE OR LONGER DURATION

Polynomial solutions may be derived for variable counting intervals; however an exact series, Eq. (10), is not convenient to use and approximations are limited to restricted ranges of λt . Derivation of Eq. (10) is similar to that of Eq. (9):

$$\begin{aligned} s\lambda\tau &= 1 - N_t\tau + \frac{1}{2!} N_t^2 \tau^2 - \frac{1}{3!} N_t^3 \tau^3 + \cdots \\ &\quad - 1 + N_0\tau - \frac{1}{2!} N_0^2 \tau^2 + \frac{1}{3!} N_0^3 \tau^3 - \cdots \\ s\lambda\tau &= (N_0 - N_t)\tau - \frac{1}{2!} (N_0^2 - N_t^2)\tau^2 \\ &\quad + \frac{1}{3!} (N_0^3 - N_t^3)\tau^3 - \cdots + \frac{(-1)^{j-1}}{j!} (N_0^j - N_t^j)\tau^j. \end{aligned}$$

Define

$$\begin{aligned} F_j &\equiv \frac{(-1)^{j-1}}{j!} \frac{[1 - \exp(-j\lambda t)]}{[1 - \exp(-\lambda t)]^j} \\ &= \frac{(-1)^{j-1}}{j!} \frac{\left[(j\lambda t) + \frac{1}{2!} (j\lambda t)^2 + \cdots \right]}{\left[(\lambda t) + \frac{1}{2!} (\lambda t)^2 + \cdots \right]^j}, \end{aligned}$$

$$F_j \approx \frac{(-1)^{j-1}}{j!}, \quad \text{for large values of } (\lambda t);$$

$$s\lambda\tau = S\lambda\tau + F_2(S\lambda\tau)^2 + F_3(S\lambda\tau)^3 + \cdots + F_j(S\lambda\tau)^j.$$

Reversion of the series produces a few terms of Eq. (10):

$$\begin{aligned} S &= s[1 - F_2(s\lambda\tau) + (2F_2^2 - F_3)(s\lambda\tau)^2 \\ &\quad - (5F_2^3 - 5F_2F_3 + F_4)(s\lambda\tau)^3 \\ &\quad + (14F_2^4 - 21F_2^2F_3 + 3F_3^2 + 6F_2F_4 - F_5) \\ &\quad \cdot (s\lambda\tau)^4 + \cdots]. \end{aligned} \quad (10)$$

COUNTING INTERVALS OF HALF-LIFE OR SHORTER

Another series was derived for use with small values of $(\lambda\tau)$. Defining the following quantities is helpful:

$$\begin{aligned} \bar{N} &\equiv \frac{1}{2} (N_0 + N_t) = N_0 - \frac{\Delta N}{2} \\ &= N_t + \frac{\Delta N}{2} = \frac{S\lambda G}{2} \xrightarrow{t \rightarrow 0} \frac{S}{t}, \end{aligned}$$

$$\begin{aligned} G &\equiv \frac{1 + \exp(-\lambda t)}{1 - \exp(-\lambda t)} \\ &= \frac{2 - \lambda t + \frac{1}{2} \lambda^2 t^2 - \frac{1}{3!} \lambda^3 t^3 + \cdots}{\lambda t - \frac{1}{2} \lambda^2 t^2 + \frac{1}{3!} \lambda^3 t^3 - \cdots} \xrightarrow{\lambda t \rightarrow 0} \left(\frac{2}{\lambda t} - 1 \right), \end{aligned}$$

$$\frac{\Delta N}{\bar{N}} \equiv \frac{2}{G} \xrightarrow{t \rightarrow 0} \lambda t,$$

$$\begin{aligned} s\lambda\tau &= (N_0 - N_t)\tau - \frac{1}{2!} (N_0^2 - N_t^2)\tau^2 \\ &\quad + \frac{1}{3!} (N_0^3 - N_t^3)\tau^3 - \cdots, \end{aligned}$$

$$\begin{aligned} s\lambda\tau &= S\lambda\tau \left\{ 1 - \bar{N}\tau + \frac{1}{2!} \bar{N}^2 \tau^2 \left[1 + \frac{1}{12} \left(\frac{\Delta N}{\bar{N}} \right)^2 \right] \right. \\ &\quad \left. - \frac{1}{3!} \bar{N}^3 \tau^3 \left[1 + \frac{1}{4} \left(\frac{\Delta N}{\bar{N}} \right)^2 \right] \right. \\ &\quad \left. + \frac{1}{4!} \bar{N}^4 \tau^4 \left[1 + \left(\frac{\Delta N}{\bar{N}} \right)^2 + \frac{1}{80} \left(\frac{\Delta N}{\bar{N}} \right)^4 \right] - \cdots \right\}, \end{aligned}$$

$$\begin{aligned} s\lambda\tau &= S\lambda\tau \left\{ 1 - S\lambda\tau(G/2) + \frac{1}{2!} (S\lambda\tau)^2 \left[(G/2)^2 + \frac{1}{12} \right] \right. \\ &\quad \left. - \frac{1}{3!} (S\lambda\tau)^3 \left[(G/2)^3 + \frac{1}{4} (G/2) \right] + \cdots \right\}. \end{aligned}$$

Reversion is necessary to obtain Eq. (11):

$$\begin{aligned} S &= s \{ 1 + S\lambda\tau(G/2) + \frac{3}{2} (s\lambda\tau)^2 [(G/2)^2 - \frac{1}{96}] \\ &\quad + \frac{8}{3} (s\lambda\tau)^3 [(G/2)^3 - \frac{1}{16} (G/2)] + \cdots \}. \end{aligned} \quad (11)$$

Since

$$s\lambda\tau(G/2) \rightarrow (s/t)\tau = n\tau, \quad \text{as } (t \rightarrow 0),$$

this series corresponds almost identically term-by-term with those of the Poisson approximation, for short counting intervals.

Although some discretion may be required in applying these results to other equations, it appears that the substitution of the (F) and (G) terms should apply reasonably accurate corrections into series having coefficients similar to those of Table IV-5-III.

EFFECT OF CORRECTION FOR LENGTH OF COUNTING INTERVAL

As may be seen by comparison of Eqs. (9) and (11), the first-order correction may vary radically, if counting intervals vary between long and short. Since the correction depends on the second power of (λt) , the correction is usually negligible with short counting intervals. When calibrating equipment these terms may become significant because of the long counting intervals required. The following approximations indicate the

ure of this correction:

$$s\lambda(G/2) = \frac{s}{t} \left(1 + \frac{\lambda^2 t^2}{12} - \frac{\lambda^4 t^4}{720} + \frac{\lambda^6 t^6}{30,240} + \dots \right),$$

$$S \approx s \left\{ 1 + \left(\frac{s\tau}{t} \right) \left(1 + \frac{\lambda^2 t^2}{12} \right) + \frac{3}{2} \left(\frac{s\tau}{t} \right)^2 \left(\frac{95}{96} \right) \left(1 + \frac{\lambda^2 t^2}{12} \right)^2 + \dots \right\}.$$

LIMITATIONS OF RESOLVING LOSS CORRECTION FORMULAS

POISSON APPROXIMATION—FORMULA (C)

The Poisson approximation indicates a maximum at $n\tau \approx 0.37$, or $N\tau = 1.0$, since $dn/dN = 0$. Obviously a small variation of n near that region, such as is inherent in counting statistics, results in a variation proportional to dN/dn in the estimate of N . This magnifies the counting errors. Series approximations converge slowly, and iteration requires essentially an equal number of operations, if $n\tau > 0.25$. Even though the Poisson approximation is double-valued, and is satisfied by either a small or a large estimate of N for any value of $n\tau < \exp(-1)$, the series approximations given herein are single-valued, because of iteration or reversion to fit only the lower value.

INTERPOLATION FORMULA

Although a set of calibration data may be fitted by an interpolation formula, the comments relative to the

Poisson formula apply in the vicinity of any maximum in n . The other possibility is that the equipment may saturate, so that dn/dN approaches zero asymptotically as N increases. The region where dn/dN is small (or dN/dn is large) may require least-squares fit if extension into this region is considered necessary.

REFERENCES

1. T. P. Kohman, *A General Method for Determining Coincidence Corrections of Counting Instruments*, Paper No. 22.50, The Transuranium Elements; G. T. Seaborg, J. J. Katz and W. M. Manning, Eds. (McGraw-Hill Book Co., Inc., N.Y., 1949); National Nuclear Energy Series Div. IV, First Ed., Vol. 14B, p. 1655.
2. H. W. Kirby, *Determination of Coincidence Corrections in a Proportional Counter; I. Double Source Methods*, MLM-1197 (1964).
3. P. B. Moon, *Recent Developments in Geiger-Muller Counters*, J. Sci. Instr. XIV, No. 6, 6 (1937).
4. S. A. Korff, *Electron and Nuclear Counters*, (D. Van Nostrand Co., Inc., Princeton, N.J., 1946).
5. K. E. Plumlee, *Resolving Loss Formulation for Scaler Data*, USAEC Report TID-15297 (1962).
6. H. Volz, *Über die Trennschärfe von Zählansordnungen*, Z. Physik **93**, 539 (1935).
7. D. E. Hull and H. Seelig, *The Half-Life of Iodine (128)*, Phys. Rev. **60**, 553 (1941).
8. D. R. Harris, *Large Counting Losses in Neutron Detection Channels*, WAPD-TM-92 (1957).
9. J. Thomas, *Determination of Count-Rate Dependent Corrections by the Substitution Method*, Danish AEC Risø Report No. 57.
10. K. E. Plumlee and M. T. Wiggins, *Automatic Foil Activity Counting Facility and Data Reduction Program*, ANL-6628 (1963).

IV-6. Computer Techniques for the Investigation of Counting System Deadtime

G. S. STANFORD

SUMMARY

An investigation is being conducted of the deadtime characteristics of the counting equipment used in the ANL Reactor Physics Laboratory. Two computer codes have been written for the purpose. One of them determines the deadtime, plus some higher series coefficients, by taking advantage of the change in counting rate as the source decays. The other one processes the data from a conventional paired-source measurement, calculating the deadtime for user-specified values of the few series coefficients. For the counting systems being investigated, it is proving difficult to determine the effective deadtime for a given set of counting conditions to better than $\pm 1 \mu\text{sec}$, although the reproducibility of a particular determination is considerably better than that.

bility of a particular determination is considerably better than that.

INTRODUCTION

If the relative activation of a set of irradiated foils covers a wide range, the accurate determination of this relative activation frequently depends upon, among other things, an accurate knowledge of the resolving-time losses in the counting equipment. To take a quantitative example, suppose that background and counting-time limitations make it desirable that the weakest foil gives a counting rate of at least 100/sec, and suppose that the strongest foil is a factor of 100 more active. A two-microsecond uncertainty in the deadtime

(regardless of the magnitude of the deadtime) will then result in a 2 % uncertainty in the activation ratio under the optimum counting-rate conditions—conditions which cannot be maintained unless the sample has a relatively long half-life. It is not unusual for the resolving-loss errors to amount to three per cent or more, unless special precautions are taken.

The actual mechanism by which resolving-time losses occur varies from one counting system to another.¹ In the particular case that each pulse counted causes a fixed deadtime τ , with no extension of that interval if an unresolved pulse occurs, the true counting rate N (excluding background) is given exactly by

$$N = \frac{n}{1 - n\tau} - b, \quad (1)$$

where n is the observed counting rate and b is the background rate. In another possible case, each resolved or unresolved pulse has associated with it a deadtime τ which starts when the pulse occurs. In this case the true counting rate is given by

$$N = ne^{(N+b)\tau} - b, \quad (2)$$

which cannot be solved analytically for N .

There are also other possibilities. In general, N can be expressed by the series

$$N + b = n[1 + n\tau + B(n\tau)^2 + C(n\tau)^3 + D(n\tau)^4 + \dots], \quad (3)$$

where the values of the coefficients $B, C, D, \dots$ depend on the type of mechanism operating. For the series corresponding to Eq. (1) all coefficients are unity, whereas for Eq. (2) the values for B, C , and D are respectively $3/2, 8/3$, and $125/24$.⁽²⁾

For a given counting system it is possible, at least in principle, to determine reasonable values for the first few coefficients, either from a knowledge of how the system works, or by actual measurement. Usually not more than the first few coefficients are important, since there is an upper limit to the tolerable counting rate (typically $\sim 10^4$ count/sec), determined by system stability. The counting data obtained in the automatic counting facility³ are usually processed by means of the computer code RP-202,⁽³⁾ a code which permits the user to specify estimated values of τ, B , and C to be used in making the deadtime corrections. The higher coefficients are neglected.

Until recently it was our practice to determine a value of τ with a double-pulse generator. However, in working with data from foils that were counted over several half-lives it was observed that the input deadtimes apparently were often in error.

The two computer programs described here were developed to permit a detailed examination of the dead-

time characteristics of the counting systems. Both codes were written in FORTRAN for the CDC-3600 computer. The first (DEADTIME) uses a series of counts from a decaying foil to determine the deadtime, plus, where statistics permit, approximate values of some of the coefficients. It is assumed that the decay constant of the material is accurately known. The second program (TWØSØRCE) processes the data for double-source deadtime measurements. The program gives the deadtime according to an exact solution of Eq. (1), and it also gives a value of τ for input values of B, C , and D selected by the experimenter, the latter value of τ being determined by an iterative solution of Eq. (3). The operation of the two codes will be described in more detail below.

The deadtime investigation is still in progress, but one of the observations made so far is that the effective deadtime for the equipment used, even for a given counting channel, has been a variable quantity. In particular, it depends to such an extent on the discriminator level that for very accurate work it appears necessary to measure it each time a set of foils is counted. Circuit changes to alleviate this problem are being investigated.

PROGRAM DEADTIME

Program DEADTIME, RP-362X, requires as input the punched specific activation cards which can be produced by the standard ANL foil data processing code, RP-202.⁽³⁾ It also requires a set of cards with appropriate input parameters, and the original counting-data cards which were used with RP-202 to produce the specific activation cards.

For successful operation of the code, a foil must have been counted several times, over a period of one or more half-lives, and the counting rate at the beginning must be high enough that the deadtime losses are appreciable. Thus half-lives greater than several hours or so are unsuitable, good materials to use being, for example, indium (54 min) or manganese (2.6 h). The counting data are first processed with RP-202, using an approximate deadtime, and requesting that the specific-activation cards be punched out. These cards contain, for each foil and each scaler, the specific saturated activation after corrections for background, decay, irradiation time, and deadtime.

Let the subscript i refer to a particular count for the foil, and let S'_i be the saturated activation as calculated for that count by RP-202, using the approximate deadtime τ' and the coefficients B' and C' . Then the calculation performed by RP-202 can be represented by

$$S'_i = k_i n_i (1 + n_i \tau' + B' n_i^2 \tau'^2 + C' n_i^3 \tau'^3), \quad (4)$$

where k_i contains the decay and other non-deadtime

corrections, and where background has been assumed negligible in comparison with the high counting rates involved. If the input deadtime and coefficients are incorrect, the S'_i will not in general be the same for different i . The DEADTIME program calculates the values for τ and the desired higher coefficients B , C , D , $\dots$ which yield identical values of S'_i for a set of counts on a decaying foil. Designating this value of S'_i by S , we have

$$S = k_i n_i (1 + n_i \tau + B n_i^2 \tau^2 + C n_i^3 \tau^3 + \dots). \quad (5)$$

Let V_i be the value of the parenthetical expression in Eq. (4). Since the n_i are measured quantities and the k_i do not include the deadtime corrections, it follows that the quantity S'_i/V_i is independent of the input values of τ' , B' , and C' . Thus if it is intended that the DEADTIME program will be used for a particular set of data, only approximate values of τ' , B' , and C' are needed for the first RP-202 processing.

Eliminating k_i from Eqs. (4) and (5), one obtains

$$(V_i/S'_i)S - n_i \tau - n_i^2 B \tau^2 - n_i^3 C \tau^3 - \dots = 1. \quad (6)$$

This can be regarded as a set of linear equations, the unknowns being S , τ , $B\tau^2$, $C\tau^3$, $\dots$. Thus in principle, two counts of a foil are needed to determine τ , plus one more count for each additional coefficient desired. The resulting set of equations is solved by Program DEADTIME by matrix inversion.

Because of the statistical precision required for accurate determination of the higher coefficients, it has turned out in practice that even the results of calculating only the second coefficient (B) by the above method show considerable scatter. Nevertheless it is sometimes possible to determine approximate values of B and C in that way.*

Even though it is difficult to determine accurately the optimum values of the higher coefficients, and even though this difficulty indicates that, for reasonable counting rates, it is not important to optimize the coefficients with high precision, it is at the same time desirable to use values at least of B and C appropriate to the counting system.† Consequently a subroutine (BGIVEN) was added to Program DEADTIME, permitting the calculation of τ for specified input values of B and C . Provision was also made for calculating C , D , $\dots$ as before, for specified values of B .

* The capability of solving for more than two coefficients was included in the code largely because no additional programming effort was required.

† It should perhaps be emphasized that, although the values of B and C are not critical, the best value of τ is rather strongly dependent on the values selected for B and C . It will be shown below, in the discussion of Table IV-6-I, that use of unsuitable values for B and C leads to a somewhat indeterminate dead time.

It can be seen from Eq. (6) that if B is specified, the number of unknowns is reduced by one, but the equation is no longer linear since τ and τ^2 are not independent. Equation (6), however, can be rewritten as follows:

$$(V_i/S'_i)S - n_i(1 + n_i B \tau + n_i^2 C_a \tau^2) \tau - n_i^3 C_b \tau^3 - n_i^4 D \tau^4 - \dots = 1, \quad (7)$$

which is now linear in the unknowns S , τ , $C_b \tau^3$, $D \tau^4$, $\dots$. To provide the option of either specifying or calculating the coefficient C , C has been separated into the known and unknown components C_a and C_b ; the latter is set to zero when C is not to be calculated. The set of equations represented by Eq. (7) can be solved by iteration—by determining a value of τ_t such that the value of τ as determined by matrix inversion satisfies $|\tau - \tau_t| < \delta$, where δ is specified.

Table IV-6-I contains an example of the output from the BGIVEN subroutine for fixed B and C . The first column contains the number of iterations needed to meet the convergence criterion, and the fourth column shows the calculated value of S . In the case shown there were 39 counts on a single foil, permitting 19 independent deadtime determinations with B and C fixed. A variation with the passage of time is evident both in the calculated specific activity and in the calculated deadtime. These trends change as B and C change, both trends tending to disappear at the same pair of values of B and C . It has also been found that the optimum values of the coefficients vary markedly from one counting channel to another, even when the circuits are supposedly similar.

PROGRAM TWOSOURCE

GENERAL PRINCIPLES

The paired-source technique, widely used for the determination of counting-system deadtime, is based on the fact that the observed counting rate when two sources are counted together is less than the sum of their individual counting rates, the difference being due to increased deadtime losses.

Consider first the case where Eq. (1) applies; i.e. the deadtime is non-extendable. Let the subscripts 1, 2, and 12 refer respectively to source 1, source 2, and both together. Then from the three measurements of a paired-source experiment one gets three equations like Eq. (1). Remembering that $N_{12} = N_1 + N_2$, the equations can be solved to give τ in terms of the observed counting rates:‡

$$\tau = \frac{1}{n_{12}} - \sqrt{\left(\frac{1}{n_1} - \frac{1}{n_{12}}\right)\left(\frac{1}{n_2} - \frac{1}{n_{12}}\right) + \frac{b}{n_1 n_2 n_{12}}} \quad (9)$$

‡ The following approximate formula is sometimes used in

TABLE IV-6-I. PORTION OF OUTPUT FROM PROGRAM DEADTIME, SUBROUTINE BGIVEN
(Copied from computer output)

MAY 5 1965 DEADTIME INVESTIGATION INDIUM								
SCALER 1 CYCLES	FOIL	1ST CARD	SPEC. ACT.	DEAD TIME		B	C	D
6	0000	3402	5.923427+009	7.392-006	7.392	1.000+000	2.667+000	
6	0000	3403	5.927882+009	7.402-006	7.402	1.000+000	2.667+000	
6	0000	3404	5.921435+009	7.356-006	7.356	1.000+000	2.667+000	
5	0000	3405	5.913866+009	7.347-006	7.347	1.000+000	2.667+000	
5	0000	3406	5.926766+009	7.368-006	7.368	1.000+000	2.667+000	
5	0000	3407	5.942241+009	7.413-006	7.413	1.000+000	2.667+000	
5	0000	3408	5.934240+009	7.413-006	7.413	1.000+000	2.667+000	
5	0000	3409	5.952597+009	7.477-006	7.477	1.000+000	2.667+000	
5	0000	3410	5.968183+009	7.557-006	7.557	1.000+000	2.667+000	
5	0000	3411	5.958593+009	7.517-006	7.517	1.000+000	2.667+000	
4	0000	3412	5.971644+009	7.564-006	7.564	1.000+000	2.667+000	
4	0000	3413	6.083867+009	8.041-006	8.041	1.000+000	2.667+000	
3	0000	3414	6.025206+009	7.842-006	7.842	1.000+000	2.667+000	
3	0000	3415	6.006833+009	7.755-006	7.755	1.000+000	2.667+000	
4	0000	3416	6.009747+009	7.765-006	7.765	1.000+000	2.667+000	
4	0000	3417	6.012266+009	7.769-006	7.769	1.000+000	2.667+000	
4	0000	3418	6.015356+009	7.852-006	7.852	1.000+000	2.667+000	
4	0000	3419	6.035821+009	7.958-006	7.958	1.000+000	2.667+000	
4	0000	3420	6.029782+009	7.961-006	7.961	1.000+000	2.667+000	

AVERAGE DEAD TIME = 7.618-006 SECONDS (7.618 MICROSECONDS)

STD DEVIATION = 5.415-008 SECONDS (0.054 MICROSECONDS) (ERROR IN THE AVERAGE)

RMS DEVIATION = 2.297-007 SECONDS (0.230 MICROSECONDS) (INDIVIDUAL DEAD TIMES)

It has been assumed that $bn_{12}\tau$ and similar terms are negligible with respect to b , which is reasonable since the background correction itself is normally small.

Turning now to the series expansion formulation, which includes Eqs. (1) and (2) as special cases, one gets from the three measurements of a paired-source determination three equations like Eq. (3) (and one also has $N_1 + N_2 = N_{12}$). From these equations one can eliminate N_1 , N_2 , and N_{12} , to get

$$(n_{12} - n_1 - n_2 + b) + (n_{12}^2 - n_1^2 - n_2^2) \\ \cdot \tau + (n_{12}^3 - n_1^3 - n_2^3) \cdot B\tau^2 + (n_{12}^4 - n_1^4 - n_2^4) \quad (10) \\ \cdot C\tau^3 + (n_{12}^5 - n_1^5 - n_2^5) \cdot D\tau^4 = 0,$$

where only the first four coefficients have been kept. For given values for B , C , and D , Eq. (10) can be solved for τ by iteration.

By terminating Eq. (3) after the first few terms, one of course makes the implicit assumption that the place of Eq. (9):

$$\tau = \frac{n_1 + n_2 - n_{12}}{n_{12}^2 - n_1^2 - n_2^2} = \frac{n_1 + n_2 - n_{12}}{2n_1n_2} \quad (9a)$$

where the second expression comes from letting $n_{12} = n_1 + n_2$ in the denominator. Equation (9a) can be obtained from Eq. (3) by dropping the quadratic and higher terms. As $n\tau$ increases to moderately high values, the first denominator in Eq. (9a) can actually pass through zero and become negative.

higher terms are negligibly small—a condition which is easily violated for large counting rates or large dead-times. Thus, for accurate use of the series representation, it is important that the product $n\tau$ be kept small enough that the higher terms are unimportant.

OPERATION OF PROGRAM TWØSØRCE

The input for Program TWØSØRCE consists of a title card; a card with the decay constant of the material counted; a card with additional control constants, including values for the coefficients B , C , and D ; and the deck of counting data cards, as produced by the automatic foil counting equipment.³ The first of the counting data cards is a background card, followed by three cards containing respectively the count of foil 1, foil 2, and both together. This last set of three cards can be repeated up to fifty times, and the program will average the results.

Using the times and durations of the counts (punched automatically on the counting data cards) and the input decay constant, the program calculates the individual counting rates that would have been observed at the time when the combined count was taken, and then calculates τ using Eq. (9). With this value of τ as a starting point, an iterative solution of Eq. (10) is found, using the input values of B , C , and D . Both values of τ are printed out. When all counting data have

TABLE IV-6-II. SELECTED PORTIONS OF OUTPUT FROM PROGRAM TWOSORCE
(Copied from computer output)

6/17/65 DEAD TIME FOR MANGANESE RADIAL TRAVERSE IN AARR. BIAS UNDER PEAK
SCALER 3. SOUTH COUNTING ROOM

CLOCK3	RATE A	RATE B	RATE C	DEAD TIME		DEAD TIME USING INPUT COEFFICIENTS
216.1	10429.2	12127.5	19329.7	1.4899-005	14.899	12.957
233.1	10486.0	12112.2	19345.0	1.4971-005	14.971	13.018
252.1	10448.3	12101.5	19361.2	1.4696-005	14.696	12.786
451.7	10396.1	11935.0	19302.9	1.4124-005	14.124	12.309
469.4	10391.2	12002.3	19311.8	1.4335-005	14.335	12.485
486.5	10375.9	11929.9	19297.5	1.4053-005	14.053	12.250

AVERAGE DEAD TIME FOR SCALER 3 = 14.5131 MICROSECONDS
STANDARD DEVIATION IN AVERAGE = 0.1619 MICROSECONDS
RMS DEV. IN INDIVIDUAL DEAD TIMES = 0.3620 MICROSECONDS
AVERAGE USING THE INPUT COEFFICIENTS 1.5000, 2.6667, -0.0000, FOR B, C, AND D
AVERAGE DEAD TIME FOR SCALER 3 = 12.6345 MICROSECONDS
STANDARD DEVIATION IN AVERAGE = 0.1354 MICROSECONDS
RMS DEV. IN INDIVIDUAL DEAD TIMES = 0.3028 MICROSECONDS

been processed, the average τ is computed for each of the two methods, along with the standard deviation in each average and rms deviation in the individual values of τ .

The above calculations are made for each scaler used. Portions of the output for a typical problem are shown in Table IV-6-II.

DISCUSSION

In processing the counting data from irradiated foils, using RP-202 or a similar program, the resulting saturated activation obviously should not be a noticeable function of the time at which the foil was counted. If on initial processing it is observed that this condition is violated, it is possible to use the DEADTIME program for an after-the-fact determination of an appropriate deadtime for reprocessing. In practice, satisfactory results have been obtained in this way, for cases where foils have been repetitively counted for one or more half-lives. An advantage of this method is that some counting-rate-dependent gain shifts, if present, will tend to be compensated artificially along with the actual deadtime correction.

For long half-life activations, or other cases where the foil has not decayed appreciably between initial and final counting, the DEADTIME program cannot be used. However, with Program TWOSORCE it is possible to use two of the more active foils to make an expeditious before-the-fact determination of the deadtime prior to the start of counting, at the actual gain voltage settings to be used. This procedure is now followed as part of the counting routine.

It must be admitted, however, that at present there is no confidence that the deadtimes are known, in any

given case, to better than $\pm 1 \mu\text{sec}$, the lower errors indicated by Tables IV-6-I and IV-6-II notwithstanding. The reason for the current uncertainty lies not so much in the reproducibility of a given experiment, as in unpredictable and apparently inconsistent changes in measured deadtime as parameters, such as counting rate or bias level, are changed.

It is felt that part of the solution could lie in determining the optimum values of the higher coefficients on a routine basis, but such a determination is time-consuming. It has been found that the optimum values vary widely from one counting channel to another, as was mentioned above. In general it seems that the coefficients cannot be expected to match one of the clear-cut cases like Eq. (1) or (2), especially for those systems where pile-up rejection circuitry rejects pulses which normally would be counted, but which are superimposed on the tail of other pulses which were too small to be counted at the bias setting used. In this latter case, even the introduction of an artificially fixed deadtime following each counted pulse cannot be expected to lead to an effective deadtime which is independent of the bias setting.

The investigation is continuing.

REFERENCES

1. D. Halliday, *Introductory Nuclear Physics*, (John Wiley & Sons, Inc., New York, 1950), p. 208ff.
2. K. E. Plumlee, *Theory and Methods of Corrections for Resolution Losses of Counting Equipment*, Trans. Am. Nucl. Soc. 7, No. 2, 341 (1964).
3. K. E. Plumlee and M. T. Wiggins, *Automatic Foil Activity Counting Facility and Data-Reduction Program*, ANL-6628 (1962).

IV-7. A FØRTRAN Code for Normalizing and Combining Large Amounts of Foil Counting Data

G. S. STANFORD

INTRODUCTION

In making detailed activation traverses in experimental reactor cores, large numbers of foils—50 to 100 or even more—are often irradiated simultaneously. All the foils must be counted before the activity has decayed, and it is frequently necessary or desirable to use two or more counting systems simultaneously, or to divide the set of foils into groups for counting at different times. It is then necessary to normalize the results in some manner, and this can be done by having in each group one or more foils which also have been counted in another group. An appropriate normalizing factor is then the average ratio of the counting rates for the common foils.

In the ANL Reactor Physics Laboratory, activated foils are usually gamma counted in an automatic foil changing system having a dual crystal arrangement with one detector above the foil and the other below. Two complete systems are in operation, each having two automatic changers feeding a bank of amplifiers and single-channel analyzers. The foils in the group being counted are repeatedly cycled, and the data are punched out on IBM cards. Corrections for decay, dead time, background, and reactor running time can be made by means of the foil data processing code RP-202. The maximum number of foils that can conveniently be counted in one group is about 40. A detailed discussion of the code and the counting facilities can be found in Ref. 1, although various additions and improvements have been made since that report was written.

Recent practice has been to feed the output from each detector to a linear amplifier, and thence in parallel to a set of two or three single-channel analyzers, each with its own scaler. Each detector provides an independent count of each foil, and the redundancy of having more than one analyzer-scaler for each detector provides additional insurance against some of the possible malfunctions that might otherwise cause the loss of large amounts of data. In fact, it has happened more than once or twice that erratic behavior has developed somewhere in a counting channel, a situation that tends not to be discovered until the data are analyzed. The redundancy in collected data has made it easier both to diagnose the problem and to identify and reject bad data.

Thus, it frequently happens that counting data are on

hand for several groups of foils, with counts from up to six scalers for each foil. Even given the output from RP-202 (which provides for each scaler the weight-corrected average of the corrected successive counts for each foil), the complete job of normalizing and averaging the results for all the scalers, and then determining and applying the normalizing factors for combining the data from the different counting groups, is a formidable task without further computer assistance. It is for these reasons that Program FØILNØRM was developed.

It is relative, not absolute, activation rates that are being considered here—for instance, the neutron flux in one part of the reactor core is desired relative to the flux somewhere else. Placing these measurements on an absolute basis, where required, is done with a different type of calibration experiment.

OPERATION OF THE PROGRAM

FØILNØRM (RP-367X) is a program written in FØRTRAN for the CDC-3600 computer. The principal input consists of the weighted-average specific saturated activation (i.e., activation per unit weight) for each foil and each scaler, as punched out on IBM cards by RP-202.

Processing proceeds in two stages. First, for each group of foils the data for all the scalers used are normalized and combined, with the result stored for subsequent use and also printed out. In the second stage the program searches for the common foils, determines the factors for normalizing one group to the next, combines everything into one large group, and prints the result.

For the first stage of processing, input information is required giving for each group the desired weighting factor to be used in averaging the results for the scalers used. In determining these factors, the experimenter takes into account any feelings he may have regarding the relative reliability of the different channels, plus some additional considerations. It has been found, for instance, with the counting equipment used in the Reactor Physics Laboratory, that although the counting rate in each of the gamma detectors is sensitive to small vertical displacements of the foil, the average of the results from the two detectors, weighted inversely according to their relative efficiencies, is insensitive to such displacements. Thus, it has been usual in using FØILNØRM, to weight the scalers so that the bottom

d top detectors get equal emphasis. Secondly, it might sometimes happen that the same set of foils has been counted at different times or on different systems, and it is desired to combine all the data collected. This can be accomplished by assigning a scaler number and suitable weighting factor to each set of data.

The operation of Program FØILNØRM will now be discussed in somewhat greater detail.

Let N and I be the number of foils and the number of scalers, respectively, and let A_{ni} be the activity of the n th foil as determined by the i th scaler. Using the first scaler as an arbitrary reference, the program calculates for each scaler after the first (i.e., for $i \geq 2$) an average ratio R_i :

$$R_i = \frac{1}{N} \sum_{n=1}^N \frac{A_{ni}}{A_{n1}}. \quad (1)$$

The standard deviation σ_i in the individual ratios is then determined, and a new value for R_i is found, this time rejecting any ratios that happen to differ from the average by more than $j\sigma_i$, where j is 3 unless otherwise specified in the input. The process is repeated until σ_i is unchanged, and a printed message identifies the foils for which the ratios were finally rejected. This feature has been found useful in pinpointing bad counts which might otherwise have been missed.

The average activation A_n for each foil is then calculated:

$$A_n = \left(\sum_{i=1}^I w_i R_i A_{ni} \right) / \left(\sum_{i=1}^I w_i \right), \quad (2)$$

where R_1 is identically unity and the w_i are the scaler weights assigned by the experimenter. The results are printed out for each group, and processing proceeds to the second stage, where the groups are combined.

The second-stage processing is done by a subroutine called NØRMGRUP. It starts with the first two groups of foils. The foils common to both groups are located, and the normalizing ratio is determined according to

$$R_k = \frac{1}{N} \sum_{n=1}^N \frac{A_{n1}}{A_{nk}} \quad (3)$$

where N is now the number of common foils, and the subscripts 1 and k refer, respectively to the first and second groups. A message is printed out which identifies the common foils, their ratios, and the standard deviation in the ratios. The activities of the foils in group k are multiplied by R_k and combined with the foils from the first group into a new group 1. The process is then repeated with $k = 3$, then 4, and so on. (The

de is dimensioned to handle up to 10 groups, with up to 100 foils in any group, of which 50 or less may be common to other groups.)

When two groups of foils are merged, the activity

assigned to the common foils is computed according to

$$A_n = \frac{u_n A_{n1} + R_k A_{nk}}{u_n + 1}, \quad (4)$$

where u_n is an integer which is equal to the number of times the n th foil has appeared as a common foil. This ensures that each count of a common foil has the same weight in determining its final relative activation.

The final printout includes a column containing the foil names (as assigned originally in the input to RP-202) and two columns labeled "Normalized Activity". Of this latter pair of columns, the first contains the activities as multiples of the activity of the weakest foil, and the second contains the activities as fractions of the activity of the strongest foil.

INTERCALIBRATED SETS OF FOILS

In working with foils made from certain alloys, notably Dy-Al and U-Al, the concentration of the material of interest is so variable that ordinary weight corrections cannot be made to the activations. In these cases, it is necessary to intercalibrate sets of foils by some process such as simultaneous irradiation on a rotating wheel. The FØILNØRM program can be used to facilitate the assembly of such sets of intercalibrated foils. In particular, a feature has been provided which makes it possible to request in the input that the program punch out a set of intercalibrated "weight" cards in a format compatible with the input requirements of RP-202.

SUMMARY

A FØRTRAN routine, Program FØILNØRM (RP-367X), has been written for the CDC-3600 computer, for use in conjunction with the standard ANL foil-data-processing code RP-202. The program is divided into two parts. The first part provides weighted averages when the same group of foils has been counted on more than one scaler, in more than one system, or at several different times. The second part is for use in internormalizing the data from an irradiated set of foils which has been separated into two or more groups—for counting on different channels, in different systems, or at different times. It is necessary that each group include one or more foils which are common to some other group.

For preparing intercalibrated sets of foils, the program can be requested to punch out weight cards for use as input to RP-202.

REFERENCES

1. K. E. Plumlee and M. T. Wiggins, *Automatic Foil Activity Counting Facility and Data Reduction Program*, ANL-6628 (October 1962).

IV-8. The Interpretation of Rod Drops with Finite Drop Times

C. E. COHN

Rod drops are frequently used for reactivity measurements, as well as for the determination of delayed-neutron parameters. In the analysis, it is commonly assumed that the drop occurs in zero time, although this is never actually the case. It was the purpose of this study to examine the errors incurred by this assumption.

The study has been limited to cases in which the negative Δk increases linearly during the drop time. A version of the RE-129 code¹ was run from critical with equilibrium precursor concentrations at time zero, and with Δk linearly varying at a rate $-k'$ per sec. At various subsequent points in time, values of flux and precursor concentrations are read out. Now, such values read out at a time T correspond to the conditions existing at the end of the drop time T for a rod drop with total $\Delta k = k'T$.

Since Δk remains constant after the drop is completed, the subsequent behavior of the flux can be expressed as

$$\phi(t) = \sum_{j=1}^n A_j e^{s_j(t-T)}.$$

Here the s_j are the roots of the inhour equation and are found by a modified version of the RE-126 code,² while the A_j are calculated from the aforementioned values of flux and precursor concentrations at time T .

If the linear drop is interpreted as if it were instantaneous, the resulting errors will depend upon which point in time is assumed as the origin of the instantaneous drop. For example, consider a linear drop with a total Δk of -1% and a drop time of 1 sec in a reactor with a prompt neutron lifetime $\ell^* = 10^{-3}$ sec. If this is interpreted as if it were an instantaneous drop occurring at the beginning of the linear fall, then the actual flux would be 10% high 1 sec after the assumed instantaneous drop, and 2% high 10 sec after. If, on the other hand, the instantaneous drop is assumed to occur at the end of the linear fall, then the actual flux is 8% low 1 sec after the instantaneous drop, and 3% low 10 sec after.

This reversal suggests that the error could be minimized if the instantaneous drop were assumed to occur at some time T' between the beginning and end of the linear fall. Furthermore, a long time after the drop the detailed history of the drop should have little bearing on the reactor behavior so that an equivalent instantaneous drop should provide an accurate description. The time T' is chosen such that

$$A_1 e^{s_1 t} = A'_1 e^{s_1(t+T-T')},$$

where s_1 is the smallest of the s_j and the A'_j are the coefficients appropriate to an instantaneous drop. This assures that the fluxes from the linear drop and the equivalent instantaneous drop will coincide for large t .

The ratio T'/T is always less than 0.5, and approaches that value as T and $|\Delta k|$ decrease. This approach becomes slower as the prompt neutron lifetime decreases. The flux predicted by the equivalent instantaneous drop is always less than that of the actual drop.

For the particular case previously considered, the ratio T'/T is 0.382. The discrepancy between the fluxes resulting from the instantaneous and linear drops is 0.15% at 1 sec after the end of the linear fall and 0.017% at 10 sec after.

REFERENCES

1. R. O. Brittan, *Some Problems in the Safety of Fast Reactors*, ANL-5577 (1956).
2. C. E. Cohn and B. J. Toppel, *IBM-704 Codes for Reactivity Step Calculations (RE-126 and RE-135)*, ANL-6134 (1960).

IV-9. Preparations for the Measurement of the Low Energy Component of the Fission Neutron Spectrum

K. G. PORGES and A. DE VOLPI

The fission neutron energy spectrum for thermal or spontaneous fission has been measured with considerable precision in the 1 to 15-MeV region.¹ Less information, however, is available on the component of

this spectrum below 1 MeV; in particular, the region below 100 keV, in which neutron detection by recoil detectors becomes difficult, is almost entirely unexplored.

This information, in view of its potential technological importance,² is being sought at Argonne in two independent experiments, of which the work reported here aims especially at the region between 5 and 100 keV. Over this energy region, advantage may be taken of the well-known energy dependence of the B-10-(n, α)Li-7 cross section by using a boron coating as the primary neutron detecting element. In the experiment described here, such a boron coating is viewed at close distance by a number of solid-state alpha particle detectors, whose signals are processed in a fast electronics channel including anti-walk discrimination. The time of the arrival of a detected neutron can thus be measured to approximately 1 nsec. Time of departure is fixed through detection of the fission events, which take place in a small weightless deposit of Cf-252 on the negative electrode of a fission chamber. This order of time resolution allows one to keep the flight path short; hence, the solid angle subtended by the neutron detection system may in turn be made rela-

tively large, compensating to some extent for the rather poor detection efficiency of the boron coating and solid-state detector combination.

For the first measurements the boron coating is put on a hemisphere of 6-cm radius; the whole experiment is arranged under an evacuated bell jar to prevent air absorption of alpha particles and to minimize scattering of neutrons. Extensive tests of the electronics, particularly with regard to mixing the signals from 16 solid-state detectors without undue increase in noise or rise-time, have been essentially completed. The fission channel electronics and time-of-flight system are ready for use. For the initial run, a count rate of the order of 10^3 – 10^4 counts/day is anticipated in the spectrum region of interest.

REFERENCES

1. L. Cranberg, G. Frye, N. Nereson and L. Rosen, *Fission Neutron Spectrum of U-235*, Phys. Rev. **103**, 662–670 (1956).
2. G. C. Hanna, *Fission Data and Nuclear Technology*, AFCL-2168 (February 1965).

IV-10. On the Precision of Measurements of Exponential Growth and Decay Constants

C. E. COHN

Formulas have been derived for predicting the error due to counting statistics in measurements of exponential growth and decay constants. It is assumed that the measurement is made by subdividing the measuring time into a number of equal time intervals, accumulating Poisson-distributed counts from a detector during each interval, and fitting the results to an exponential. It is further assumed that there are no background or transient terms. The following conclusions have been reached.

1. The fractional standard deviation of the observed period is proportional to the inverse of the square root of the total number of counts, C_T , accumulated during the run.
2. For an infinite number of time intervals, N , this proportionality factor is close to unity for a total measuring time, T_0 , greater than 3.7 periods, and is inversely proportional to T_0 for smaller values.
3. For finite N , the fractional error increases slightly, for example by less than a factor of 1.3 for $N \geq 2$ and $|T_0| \leq 4$.
4. If a significant amount of time is required for data recording after each counting interval, an increase in N will lead to a net loss in precision owing to the decrease in C_T . The only justification for N

greater than 2 is to allow a check for transient components, improperly corrected deadtime losses, etc., by testing the goodness of fit.

Other authors^{1, 2} have considered this problem, but the above conclusions have not specifically been stated.

Assume that a run has a total duration T which is divided into N intervals. The accumulated counts, y , from the N intervals are to be fitted to an exponential function

$$y = Ae^{Bt}.$$

(Note that when B is positive, it is referred to as a "growth constant" and its reciprocal is known as an "e-folding time" or "positive period," while a negative B is referred to as a "decay constant" and its reciprocal is known as a "mean life" or "negative period.") The most convenient way of doing this fitting is to write $z_i \equiv \ln y_i$, giving a least-squares sum

$$M = \sum_{i=1}^N w_i (z_i - A' - Bt_i)^2, \quad (1)$$

where $t_i = iT/N$ and $A' = \ln A$. The normal equations are then

$$A' \sum_{i=1}^N w_i + B \sum_{i=1}^N w_i t_i = \sum_{i=1}^N w_i z_i \quad (2)$$

and

$$A' \sum_{i=1}^N w_i t_i + B \sum_{i=1}^N w_i t_i^2 = \sum_{i=1}^N w_i t_i z_i. \quad (3)$$

Now, it is well known that an estimate of the variance of a fitted parameter is given by the corresponding diagonal element of the inverse of the matrix formed by the coefficients of the parameters from the left-hand sides of the above equations. Thus

$$\sigma^2(B) = \frac{\sum_{i=1}^N w_i}{\sum_{i=1}^N w_i \sum_{i=1}^N w_i t_i^2 - \left(\sum_{i=1}^N w_i t_i \right)^2} \quad (4)$$

for the variance of B . (Note that the parameter A is usually of no interest in such experiments.)

The weights w_i are given by the formula $1/[\sigma_i(z)]^2$, where $\sigma_i(z)$ is the variance of z_i . From the definition of z_i , we see that $\sigma_i(z) = \sigma_i(y)/y$, where $\sigma_i(y)$ is the variance of y_i . If it is assumed that this latter variance arises purely from counting statistics, we may write

$$\sigma_i(y) = \sqrt{y_i} \quad (5)$$

so that

$$w_i = y_i. \quad (6)$$

The expected value of y_i is

$$\langle y_i \rangle = \int_{t_{i-1}}^{t_i} C e^{Bt} dt = \frac{C}{B} (1 - e^{-Bt_i/N}) e^{Bt_i}, \quad (7)$$

where C is the instantaneous counting rate at the beginning of the run. From this, values for w_i may be derived and inserted into Eq. (4). This gives

$$\sigma^2(B) = \frac{B^3 \sum_{i=1}^N e^{B_i T/N}}{C(1 - e^{-BT/N}) \left[\sum_{i=1}^N e^{B_i T/N} \sum_{i=1}^N \left(\frac{B_i T}{N} \right)^2 e^{B_i T/N} - \left(\sum_{i=1}^N \frac{B_i T}{N} e^{B_i T/N} \right)^2 \right]} \quad (8)$$

C_0 is defined as the initial count rate in counts per period $C_0 \equiv C/B$ and $T_0 \equiv BT$ as the total run time in periods. Note that C_0 and T_0 take the sign of B , being negative for a descending exponential. The above expression in dimensionless form is obtained as

$$\frac{\sigma^2(B)}{B^2} = \frac{\sum_{i=1}^N e^{iT_0/N}}{C_0(1 - e^{-T_0/N}) \left[\sum_{i=1}^N e^{iT_0/N} \sum_{i=1}^N \left(\frac{iT_0}{N} \right)^2 e^{iT_0/N} - \left(\sum_{i=1}^N \frac{iT_0}{N} e^{iT_0/N} \right)^2 \right]}$$

An alternative method (the Gauss-Seidel method) that is commonly used for least-squares fitting of functions that are nonlinear in the fitted parameters involves expansion of the function in a Taylor's series in the parameters, truncating to first order, and solving for the increments of the parameters that minimize the

resulting least-squares sum. This process is iterated until the least-squares sum is minimized to the desired accuracy. If the foregoing derivation is carried out on the basis of this alternate method, a result identical to Eq. (9) is obtained. This suggests that the conclusions obtained here are independent of the method of minimizing the least-squares sum.

From the properties of geometric progressions, it is noted that

$$\sum_{i=1}^N e^{iT_0/N} = \frac{e^{N\gamma} - 1}{1 - e^{-\gamma}} \equiv S_0, \quad \text{where } \gamma \equiv \frac{T_0}{N}, \quad (10)$$

and further, that

$$\sum_{i=1}^N i e^{iT_0/N} = \frac{dS_0}{d\gamma} = S_0 \left(\frac{N}{1 - e^{-N\gamma}} - \frac{1}{e^\gamma - 1} \right) \quad (11)$$

and

$$\sum_{i=1}^N i^2 e^{iT_0/N} = \frac{d^2 S_0}{d\gamma^2} = S_0 \left(\frac{N^2}{1 - e^{-N\gamma}} + \frac{e^\gamma + 1}{(e^\gamma - 1)^2} - \frac{2N}{(1 - e^{-N\gamma})(e^\gamma - 1)} \right). \quad (12)$$

Then

$$\frac{\sigma^2(B)}{B^2} = \frac{S_0}{C_0 \gamma^2 (1 - e^{-\gamma})} \left[S_0 \frac{d^2 S_0}{d\gamma^2} - \left(\frac{dS_0}{d\gamma} \right)^2 \right]^{-1} \quad (13)$$

or

$$\frac{\sigma^2(B)}{B^2} = \frac{1}{C_0 \gamma^2} \left[\frac{e^{N\gamma} - 1}{(1 - e^{-\gamma})(e^\gamma - 1)} - \frac{N^2}{1 - e^{-N\gamma}} \right]^{-1}. \quad (14)$$

C_T is defined as

$$C_T = C_0(e^{T_0} - 1) \quad (15)$$

which is the total number of counts recorded during the run. Note that C_T is positive regardless of the sign

of B . Then

$$\frac{\sigma^2(B)}{B^2} = \frac{1}{C_T \gamma^2} \left[\frac{1}{(1 - e^{-\gamma})(e^\gamma - 1)} - \frac{N^2}{(1 - e^{-N\gamma})(e^{N\gamma} - 1)} \right]^{-1} \quad (16)$$

or

$$\frac{\sigma^2(B)}{B^2} = \frac{2}{C_T} \left[\frac{\left(\frac{T_0}{N} \right)^2}{\cosh \left(\frac{T_0}{N} \right) - 1} - \frac{T_0^2}{\cosh T_0 - 1} \right]^{-1}. \quad (17)$$

Since this is an even function of T_0 , the result will be

the same for ascending or descending exponentials with the same C_T , T_0 , and N . Also, note that the denominator goes to zero for $N = 1$, which is a reflection of the well known fact that one cannot estimate two parameters with one piece of data. If $N \rightarrow \infty$

FIG. IV-10-1. Fractional Standard Deviation of Result as a Function of Normalized Total Measuring Time for an Infinite Number of Intervals. ANL Neg. No. 112-5181.

FIG. IV-10-2. Ratio of Standard Deviation with Finite Number of Intervals to that with an Infinite Number. ANL Neg. No. 112-5183.

FIG. IV-10-3. Ratio of Standard Deviation with Finite Readout Time to that with Zero Readout Time for Ascending Exponentials. ANL Neg. No. 112-5182.

this equation reduces to

$$\frac{\sigma^2(B)}{B^2} = \frac{2}{C_T} \left(2 - \frac{T_0^2}{\cosh T_0 - 1} \right)^{-1}.$$

Since the quantity $[\cosh (T_0/N) - 1]$ increases more rapidly than $(T_0/N)^2$ as T_0/N increases, Eq. (18) gives the minimum values of the relative variance for any values of T_0 and C_T .

Results from this equation are plotted in Fig. IV-10-1. It can be seen that the error decreases with increasing $|T_0|$, as is reasonable, since observation for a longer time would be expected to yield more reliable information about the time behavior of the count rate. However, the degree of improvement is small for observation times greater than a few periods.

Figure IV-10-2 shows the ratio of the error for a finite N to the error for infinite N and the same $|T_0|$. Here it can be seen that the improvement in precision

FIG. IV-10-4. Ratio of Standard Deviation with Finite Readout Time to that with Zero Readout Time for Descending Exponentials. ANL Neg. No. 112-5184.

for more than a few intervals is not significant for the values of $|T_0|$ most likely to be used. For larger values of $|T_0|$, an increase in N causes a greater difference. Thus, it appears that the value of N used in many such measurements is larger than need be.

In the foregoing, it has been assumed that counting goes on during the entire run, and the time required to read out the count at the end of each interval has been neglected. This assumption is valid if the scaler is read in the parallel mode into a computer or a buffered magnetic-tape unit, or if a multichannel analyzer is used as a multiscaler. Here the readout requires only a few microseconds. However, other types of readout do take a significant amount of time, during which data cannot be taken. This loss of data affects the precision.

Since the readout time is a property of the equipment and is independent of the length of the counting interval, the total amount of time devoted to readout will increase as the number of intervals is increased. Such an increase will cause a greater fraction of the available data to be lost, and thus will give rise to an impairment of precision which in some cases, could counterbalance the small improvement from Fig. IV-10-2.

If Δ is the readout time in seconds, Eq. (7) becomes

$$\langle y_i \rangle = \int_{t_{i-1}}^{t_i - \Delta} C e^{Bt} dt = \frac{C}{B} e^{Bt_i} (e^{-B\Delta} - e^{-BT/N}). \quad (19)$$

Thus the factor $[1 - e^{-BT/N}]$ in the denominator of Eq. (8) must be replaced by $[e^{-B\Delta} - e^{-BT/N}]$. However, the total observed count, C'_T , now becomes

$$\begin{aligned} C'_T &= \sum_{i=1}^N C_0 e^{iT_0/N} (e^{-\delta} - e^{-T_0/N}) \\ &= C_0 (e^{-\delta} - e^{-T_0/N}) \frac{e^{T_0} - 1}{1 - e^{-T_0/N}}, \end{aligned} \quad (20)$$

where $\delta \equiv B\Delta$ is the readout time in periods (being negative for a descending exponential). Thus, the results quoted previously are still correct if C_T is replaced by C'_T .

To determine the effect of finite δ on the precision, the ratio C_T/C'_T is given by

$$\frac{C}{C'_T} = \frac{\sigma^2(B)|_{\delta}}{\sigma^2(B)|_{\delta=0}} = \frac{1 - e^{-T_0/N}}{e^{-\delta} - e^{-T_0/N}}. \quad (21)$$

Note that the symmetry between ascending and descending exponentials is now gone, because the fraction of counts lost during the readout time, which comes at the end of a counting interval, will obviously depend upon whether the counting rate was increasing or decreasing during the interval. The asymmetry is thus more marked for longer intervals.

This function is plotted on Figs. IV-10-3 and IV-10-4 for ascending and descending exponentials, respectively. Here the abscissa is the fraction of the total interval which is occupied by readout, and the ordinate is the ratio of the standard deviation with the given readout time to the standard deviation with zero readout time. It is apparent that the precision can be significantly impaired if the counting intervals are made small enough that the readout time becomes appreciable.

In some cases it is desired to have a larger number of intervals in order to provide for examination of residuals and statistical tests of goodness of fit. This is useful whenever departures from pure exponential behavior are considered possible; e.g., for reactor

periods, where the transient terms might not be negligible. If the readout time is significant, then a compromise must be made between the number of intervals needed for the proper application of the statistical tests and the impairment of precision that can be tolerated.

IV-11. Obtaining Space-Dependent Zero-Power Transfer Functions Using Statics Techniques

C. E. COHN

Recently considerable interest has arisen in the space-dependent aspects of the kinetic behavior of zero-power reactors. This behavior may be described by a space-dependent transfer function which gives the reactor's response, when observed at a given point, to a sinusoidal disturbance introduced at another point. Such transfer functions are currently obtained either by modal methods, in which the response to a disturbance is analyzed in terms of flux eigenfunctions; or by nodal methods, in which the reactor is treated as a number of intercoupled regions. Some work has been done on an alternative method by which such transfer functions may be calculated using variations of algorithms commonly used for obtaining static flux distributions.

We may begin by writing the one-group diffusion equation

$$D(\mathbf{r})\nabla^2\phi(\mathbf{r}, t) - \Sigma_{rem}(\mathbf{r})\phi(\mathbf{r}, t) + S_p(\mathbf{r}, t) + S_D(\mathbf{r}, t) + S_0(\mathbf{r}, t) = \frac{1}{v} \frac{\partial\phi(\mathbf{r}, t)}{\partial t}. \quad (1)$$

Here $\phi(\mathbf{r}, t)$ is the space- and time-dependent flux. D is the diffusion coefficient, Σ_{rem} is the removal cross section, and v is the neutron velocity. $S_p(\mathbf{r}, t)$ is a source term representing the prompt neutrons arising from fission, and is given by the equation

$$S_p(\mathbf{r}, t) = (1 - \beta)\nu\Sigma_f(\mathbf{r})\phi(\mathbf{r}, t), \quad (2)$$

where β is the delayed neutron fraction, ν is the mean number of neutrons per fission, and Σ_f is the fission cross section. $S_D(\mathbf{r}, t)$ is another source term representing the delayed neutron contribution, and is given by

$$S_D(\mathbf{r}, t) = \sum_i \lambda_i C_i(\mathbf{r}, t), \quad (3)$$

where λ_i are decay constants and the precursor densities

REFERENCES

1. D. J. Behrens, *Some Notes on the Nature of Experiments, on the Statistics of Counting, and on the Fitting of Exponential Decay Curves to the Results of Counting Experiments*, AERE T/R 629 (January 1951).
2. R. G. Cornell, *A New Estimation Procedure for Linear Combinations of Exponentials*, ORNL-2120 (September 1956).

$C_i(\mathbf{r}, t)$ are given by

$$\frac{\partial C_i(\mathbf{r}, t)}{\partial t} = \beta_i \nu \Sigma_f(\mathbf{r}) \phi(\mathbf{r}, t) - \lambda_i C_i(\mathbf{r}, t). \quad (4)$$

All source terms are in units of neutrons per unit volume per unit time.

Finally, $S_0(\mathbf{r}, t)$ is a source term which represents the external disturbance. Small changes in the configuration of the reactor (i.e., reactivity changes) may be treated as external sources wherever the assumption of linearity is valid.¹ This formulation may then be considered as a complete description of the linear zero-power space-dependent kinetics of the reactor.

All of the time-dependent quantities in the above equations (except S_0) may be resolved into a steady part, dependent on position only, plus a fluctuating part dependent on position and time. The steady parts obey the above equations with all time derivatives and S_0 equal to zero. When the appropriate subtractions are made, it is found that Eqs. (1) through (4) are satisfied by the fluctuating parts alone. [If the mean flux is not steady (e.g., if the reactivity is varying exponentially) a similar decomposition can still be made.]

We may now restrict ourselves to the consideration of sinusoidal disturbances, and write

$$\phi(\mathbf{r}, t) = \phi(\mathbf{r})e^{j\omega t} \quad (5)$$

where $\phi(\mathbf{r})$ is a complex amplitude dependent upon position and ω is the angular frequency. The amplitudes Σ_P , Σ_D , Σ_0 and C_i , also position-dependent, may be similarly defined.

Substituting into Eq. (4), we obtain

$$C_i = \frac{\beta_i \nu \Sigma_f \phi}{j\omega + \lambda_i} \quad (6)$$

and, from Eq. (3),

$$S_D = \nu \Sigma_f \phi \sum_i \frac{\lambda_i \beta_i}{j\omega + \lambda_i} \quad (7)$$

while Eq. (2) yields

$$\underline{S}_p = (1 - \beta)\nu\Sigma_f\phi \quad (8)$$

and Eq. (1) gives

$$D\nabla^2\phi - \left(\Sigma_r + \frac{j\omega}{v}\right)\phi + \underline{S}_p + \underline{S}_d + \underline{S}_0 = 0. \quad (9)$$

Determination of ϕ for a given $\underline{S}_0$ constitutes a solution to the problem.

Extension of this treatment to a multigroup model poses no difficulties. An equation similar to Eq. (9) is required for each group, with the appropriate intergroup coupling terms included. In addition, the contributions of the various sources to the various groups must be weighted according to the energy distributions of the source neutrons. The quantity $\nu\Sigma_f\phi$ appearing in Eqs. (7) and (8) must now be summed over all groups. Extension to higher order approximations to the transport equations is straightforward. For example, the telegrapher's equation, having a second time derivative, requires a term proportional to $\omega^2\phi$ in Eq. (9).

To obtain a solution, a value of ω and the spatial and energy distribution of $\underline{S}_0$ are specified. The neutron balance equations for the statics calculation are then modified as follows and applied to the various amplitudes. $\underline{S}_0$ is inserted. The prompt and delayed fission source terms are obtained by multiplying the usual fission source term by the constants

$$(1 - \beta) \quad \text{and} \quad \sum_i \frac{\lambda_i \beta_i}{j\omega + \lambda_i},$$

respectively. The quantity $j\omega/v$ is added to each removal cross section. The resulting inhomogeneous equations are then solved for ϕ by the usual methods. The magnitude and phase of ϕ then indicate the magnitude and phase of the reactor's response to the given sinusoidal source of amplitude $\underline{S}_0$ at the frequency ω . For a complete description of the space-dependent kinetics of the reactor, the calculation would have to be repeated for various values of ω and various source locations and energies.

Since the various amplitudes are complex, while the usual procedures for solving the multigroup neutron balance equations are designed for real quantities, the above formulation must be elaborated for practical computation. We write

$$\phi = \phi_R + j\phi_I$$

where ϕ_R and ϕ_I are real numbers giving, respectively, the in-phase and quadrature components of ϕ . The other amplitudes are treated similarly. (Since $\underline{S}_0$ represents the input to the system, its phase is arbitrary, so that it can be assumed to be real. However, an imaginary component is allowed in this argument in order to

maintain the generality of the formulation by providing for an input source whose phase is nonuniform over its extent.) Equations (7) through (9) then split into two equations each, and become, respectively

$$S_{DR} = \nu\Sigma_f \left(\phi_R \sum_i \frac{\lambda_i^2 \beta_i}{\omega^2 + \lambda_i^2} + \phi_I \sum_i \frac{\omega \lambda_i \beta_i}{\omega^2 + \lambda_i^2} \right) \quad (10)$$

$$S_{DI} = \nu\Sigma_f \left(-\phi_R \sum_i \frac{\omega \lambda_i \beta_i}{\omega^2 + \lambda_i^2} + \phi_I \sum_i \frac{\lambda_i^2 \beta_i}{\omega^2 + \lambda_i^2} \right) \quad (11)$$

$$S_{PR} = (1 - \beta)\nu\Sigma_f \phi_R \quad (12)$$

$$S_{PI} = (1 - \beta)\nu\Sigma_f \phi_I \quad (13)$$

$$D\nabla^2\phi_R - \Sigma_r\phi_R + \frac{\omega}{v}\phi_I + S_{DR} + S_{PR} + S_{OR} = 0 \quad (14)$$

$$D\nabla^2\phi_I - \Sigma_r\phi_I - \frac{\omega}{v}\phi_R + S_{DI} + S_{PI} + S_{OI} = 0. \quad (15)$$

Thus there are two coupled neutron balance equations for each group. The usual methods of handling down-scattering could handle the coupling in one direction, but the coupling in the other direction would then have to be treated as an upscattering. Furthermore, each source is split into two sources, so that even if the approximation were made that the prompt and delayed neutrons have the same energy spectrum, there would be two fission sources. Since existing statics codes cannot handle two fission sources, a new code for this purpose was written.

(Such a code has been written by R. N. Macdonald* and R. J. Johnson.*)

This was done in conjunction with the thesis research of Mr. Macdonald and Mr. Johnson for which the author is a consultant. The code is written in ALGOL for the Burroughs B-5000 computer. It is a one-dimensional, two-group cylindrical code, and has given good results in analyzing some space-dependent pile oscillator experiments on the NORA² critical. Work is also being done for applying this scheme to the CRAM code³ for two-dimensional calculations.)

The results of these calculations must be interpreted in terms of experimentally observable quantities. For a pile oscillator, this is quite straightforward. Here, the source $\underline{S}_0$ is made to represent the varying absorption of the oscillator, and is placed in the spatial regions and energy groups in which the oscillator operates. The calculated ϕ may then be directly compared with the observed response to the oscillator at various detector locations. The treatment of a pulsed or modulated source is similar.

Since the reactor noise observed at a given location is excited by random fluctuations occurring throughout the core, it is necessary to divide the reactor into a

* School of Nuclear Engineering, Georgia Institute of Technology.

number of regions in space and energy, and do the above calculation for each region. The extent of each region must be sufficiently small that the response of the reactor to a point source anywhere within the region will be uniform to the required degree of accuracy. For each region, the spectral density of an equivalent noise source¹ may be calculated to represent the random excitation in that region. If $\phi(\mathbf{r}_d, \mathbf{r})$ is the amplitude of the response observed at location $\mathbf{r}_d$ to a sinusoidal source of amplitude 1 neutron/sec distributed throughout the region centered at $\mathbf{r}$, and if $S^2(\mathbf{r})\Delta V$ is the spectral density of the equivalent noise source, with ΔV the volume of the region, then the spectral density of the reactor noise at $\mathbf{r}_d$ is given by

$$\sum |\phi(\mathbf{r}_d, \mathbf{r})|^2 S^2(\mathbf{r}) \Delta V,$$

where the summation is taken over all regions. Since phase does not enter here, the extent of each region is limited by uniformity in magnitude of response only.

If there are two detectors, located at $\mathbf{r}_{d1}$ and $\mathbf{r}_{d2}$, the cross-power spectrum between the fluctuations observed at the two locations is given by

$$\sum \phi^*(\mathbf{r}_{d1}, \mathbf{r}) \phi(\mathbf{r}_{d2}, \mathbf{r}) S^2(\mathbf{r}) \Delta V$$

with the summation again taken over all regions. Here

the response throughout each region must be uniform in magnitude and phase.

If the regions in the above sums are made very small, in the limit, passing from the sum to the integral, the individual equivalent noise sources are excited by an ever smaller number of random neutronic reactions per unit time. However, the formulas used to calculate such sources assume a large number of such reactions per unit time. It is not clear to the author at present how this apparent contradiction might be resolved or how it might affect practical calculations.

It is evident that the complete description of the space-dependent kinetics of a given reactor would use much computer time, since many calculations are required. However, some time can be saved by using the results from one calculation as the initial guess for another calculation with an adjacent frequency or source location, thus speeding the convergence.

REFERENCES

1. C. E. Cohn, *A Simplified Theory of Pile Noise*, Nucl. Sci. Eng. **7**, No. 5, 472 (1960).
2. P. T. Hansson and L. R. Foulke, *Investigations in Spatial Reactor Kinetics*, KR-43 (April 1963).
3. A. Hassitt, *A Computer Program to Solve the Multigroup Diffusion Equations*, TRG Report 229(R) (March 1962).

IV-12. Direct Measurement of Effective Capture-to-Fission Ratio at Low Flux

W. C. REDMAN and M. M. BRETSCHER*

The present and anticipated critical assembly programs at Argonne emphasize the need for a direct, accurate method for determining the effective capture-to-fission ratio, $\bar{\alpha}$, in low flux systems. Recently, the initial conversion ratio in undermoderated systems has been investigated indirectly in the Hi-C program. Critical experiments on a plutonium-fueled thermal reactor assembly are planned for EBWR. Because of the variation of alpha with effective neutron energy among such systems, it is desirable that the variation of alpha with neutron spectrum, or moderator to fuel ratio, be established experimentally. Furthermore, the ZPR-6 facility is being used for a long-term program of basic studies on the properties of fast reactor assemblies. Since breeding is one of the major motivations for the entire civilian fast reactor program, a need exists for a convenient means to establish $\bar{\alpha}$ in fast critical assemblies. This is the objective of a current investigation of the

feasibility of a pile oscillator method for the direct determination of capture-to-fission ratio in zero power systems.

METHOD

The reactivity associated with the insertion of a fissile sample in a critical reactor results from the combined effect of production of fission neutrons and the loss of reactor neutrons by both the fission and capture processes. A two-group perturbation theory treatment yields the following expression for this reactivity change:

$$\rho_v = \frac{R_f}{N_v} [\bar{v}W_f - (1 + \bar{\alpha})W_a], \quad (1)$$

where

$$N_v = \bar{v} \int_{\text{core}} \phi_1^\dagger(\mathbf{r}) \Sigma_{fc} \phi_2(\mathbf{r}) d\mathbf{r},$$

$$R_f = \text{fission rate in sample} = \int_{\text{sample}} \Sigma_{fs} \phi_2(\mathbf{r}) d\mathbf{r},$$

$$W_f = \int_{\text{sample}} \phi_1^\dagger(\mathbf{r}) \Sigma_{fs} \phi_2(\mathbf{r}) d\mathbf{r} / \int_{\text{sample}} \Sigma_{fs} \phi_2(\mathbf{r}) d\mathbf{r},$$

* Summer Resident Research Associate, Valparaiso University, Valparaiso, Indiana.

and

$$W_a = \int_{\text{sample}} \phi_2^\dagger(\mathbf{r}) \Sigma_{as} \phi_2(\mathbf{r}) d\mathbf{r} / \int_{\text{sample}} \Sigma_{as} \phi_2(\mathbf{r}) d\mathbf{r}.$$

W_f and W_a are the average values of the fast and slow flux adjoints, $\phi_1^\dagger(\mathbf{r})$ and $\phi_2^\dagger(\mathbf{r})$, and physically correspond to the weighting function for the importance of the fission neutrons produced within the sample and those absorbed within the sample, respectively. Σ_{fc} is the macroscopic fission cross section of the core while Σ_{fs} and Σ_{as} are the fission and absorption cross sections of the sample. $\bar{\nu}$ is the average number of neutrons released per fission in the sample while $\phi_2(\mathbf{r})$ corresponds to the slow flux.

This equation assumes that the sample is placed within a void in the reactor (otherwise Σ_{fs} and Σ_{as} are not the macroscopic cross sections of the sample), that the sample is located in a region where flux gradients are essentially zero, and that reactivity changes due to elastic and inelastic scattering effects within the sample are negligible.

It is necessary, through supplemental experiments, to establish the values for W_f and W_a . Use of a Cf-252 spontaneous fission source provides a convenient way to determine the fission neutron importance, from observation of the apparent reactivity ρ_{cf} associated with the insertion of a sample of known neutron strength S . From such a measurement

$$W_f = \frac{\rho_{cf} N_{cf}}{S}. \quad (2)$$

Here the quantity N_{cf} is related to the fission rate existing within the reactor core during measurements with the Cf-252 source.

In a similar way, the importance for absorbed neutrons is established using a material which responds in the same way to the reactor neutrons as does the fissile sample. For the initial measurements in an under-moderated assembly of moderately enriched uranium and light water, manganese has been used as a nuclear absorption equivalent for U-235. From reactivity measurements with a manganese sample having an absorption rate R_{Mn} ,

$$W_a = -\frac{\rho_{Mn} N_{Mn}}{R_{Mn}}. \quad (3)$$

Inserting the values for the worths of fission and reactor neutrons in Eq. (1) yields the following expression:

$$\rho_U = \frac{R_f}{N_U} \left[\bar{\nu} \frac{\rho_{cf} N_{cf}}{S} + (1 + \bar{\alpha}) \frac{\rho_{Mn} N_{Mn}}{R_{Mn}} \right]. \quad (4)$$

In the experimental procedure, the samples of interest are oscillated between a position at the center of the reactor and one almost completely removed from the re-

actor environment, and the resulting periodic variation in power level is observed. For this condition, the reactivity is proportional to the amplitude of the power level oscillation, ΔV , divided by the power level, V . Since each term in Eq. (4) contains reactivity, the quantity $\Delta V/V$ may be substituted for ρ , yielding the following expression:

$$(\Delta V/V)_U = \frac{R_f}{N_U} \left[\frac{\bar{\nu} N_{cf}}{S} \left(\frac{\Delta V}{V} \right)_{cf} + (1 + \bar{\alpha}) \frac{N_{Mn}}{R_{Mn}} \left(\frac{\Delta V}{V} \right)_{Mn} \right]. \quad (5)$$

In the experimental procedure followed here, all quantities are either determined at the same power level or normalized to the same level. Then $N_U = N_{cf} = N_{Mn}$ and so Eq. (5) may be written in terms of the measured quantities and available data as follows:

$$(\Delta V/V)_U = R_f \left[\bar{\nu} \left(\frac{\Delta V}{V} \right)_{cf} + \frac{1 + \bar{\alpha}}{R_{Mn}} \left(\frac{\Delta V}{V} \right)_{Mn} \right]. \quad (6)$$

Solving this expression for the desired quantity yields

$$1 + \bar{\alpha} = -\frac{R_{Mn}}{(\Delta V/V)_{Mn}} \left[\bar{\nu} \left(\frac{\Delta V}{V} \right)_{cf} - \frac{(\Delta V/V)_U}{R_f} \right]. \quad (7)$$

The experimental procedure involves oscillation of samples of U-235, Cf-252, and manganese; plus irradiation and subsequent absolute counting for samples of manganese and U-235. In addition, it is necessary that the neutron source be calibrated. These experimental results, combined with a knowledge of the number of neutrons per fission of U-235, yield a value of the quantity $1 + \bar{\alpha}$. Since $\bar{\alpha}$ for systems of particular interest is appreciably less than unity, extreme accuracy is required for each of the experimental determinations on which the derived value of $1 + \bar{\alpha}$ is based, if even moderate accuracy is to be obtained in the resulting value for $\bar{\alpha}$.

MEASUREMENTS

The ZPR-7 critical facility is being used for the verification of this proposed pile oscillator method for the determination of alpha. The initial tests have used a core geometry studied earlier as a part of the high conversion critical assembly program.¹ The core was composed of a cylindrical array of 948 of the aluminium-clad Hi-C fuel pins. These were arranged in a square lattice having a 1.24-cm pitch. The fuel was 3.04 w/o enriched UO_2 in the form of 0.94-cm diameter pellets stacked to a height of 122 cm. The core was moderated and reflected by light water. The arrangement of fuel pins was uniform except at the center of the core where five lattice positions in a cross arrangement were left

vacant to provide space for the insertion of the voided aluminum tube in which samples contained in 1.6-cm o.d. aluminum cans were oscillated.

The sample changer and pneumatically driven oscillator mechanism have been described previously.² The components used to record the variation in reactor power level resulting from oscillation of the samples are described elsewhere in this report (see Paper IV-13). The output of a Vibrating Reed Electrometer with a calibrated feedback circuit to buck out a portion of the signal from a neutron-sensitive ion chamber is recorded on a strip chart recorder and punched out every twentieth of a cycle on data cards. A computer program, RP-328X, is used for processing the data, yielding the Fourier coefficients of the sine and cosine terms of a third-order fit to the oscillatory signal. The amplitude of the first-order term relative to the d.c. level is used as a measure of the reactivity of the sample.

The oscillator measurements comprise the bulk of the experimental data. In addition, absolute determination of manganese absorption rate by coincidence counting, determination of fission rate by radiochemical procedures involving Mo-99, and a manganese bath calibration of the Cf-252 source are significant related measurements. Each of these is discussed in turn.

$\Delta V/V$

Sample sizes were chosen to give comparable reactivity effects, subject to the availability of materials and maintaining similar geometrical conditions. The main samples were foils of 93% U-235, 90% Mn-10% Cu, and approximately 1 μ g of Cf-252 in a teflon container. In addition, samples of copper, aluminum and teflon, and an empty can were used, in order to determine the net signals from the materials of principal interest.

Since the reactivity associated with the manganese sample is common to both terms in Eq. (7), it was chosen as a reference standard for all of the measurements. Three distinct sets of measurements were made, for each of the oscillated samples. All measurements were performed under identical experimental conditions insofar as could be determined from the daily calibration of the oscillatory signal circuitry and the power level channels on the ZPR-7 facility. The reactor power level was $3\frac{1}{2}$ W. This produced a thermal flux of approximately 6×10^6 n/(cm²)(sec) at the central core position occupied by the oscillated sample during the inserted part of its 15-sec cycle time.

An identical procedure was followed in each of the three distinct sets of measurements. Five sets of the reactor response to about twelve 15-sec cycles were obtained on punched cards for each of the samples oscillated. The manganese-copper reference sample was re-

measured frequently. The actual sequence of sample measurements was as follows: Cf-252 (three or more power levels), manganese-copper, teflon, copper, sample can, manganese-copper, aluminum, U-235, and manganese-copper.

The Cf-252 source is unique among these samples in that it contains an apparent reactivity effect which is independent of reactor power level (spontaneous neutron emission) in addition to the absorption effects which are directly proportional to the power. The amount of californium present in the source is less than a microgram and hence would not be expected to contribute an observable reactivity effect due to absorption of the sample. Nonetheless, the effect due to the teflon and aluminum sample cans, nylon sample can clip and magnesium insertion rod would, at significant power levels, dominate and even mask the effect due to the neutron emission. Consequently, the Cf-252 sample was oscillated at powers as low as 1/100 that used for the other measurements, in order to emphasize the effect due to introduction and removal of spontaneously emitted neutrons. A typical set of Cf-252 data is shown in Fig. IV-12-1. A least-squares fit of the amplitude of response as a function of power level provides a separation of the two contributions to the signal.

ABSOLUTE MANGANESE ABSORPTION RATE

Three separate irradiations of manganese-copper foils were made. In each run, three similar $2.5 \times 12.7 \times 0.005$ cm samples were placed at symmetrical positions in the turret of the sample changer. The first was the manganese-copper sample used as the reference material in the oscillator experiments. Five sets of oscillations were made with this sample. The sample at the second location was then inserted into the reactor for a 30-min irradiation. The third sample provided an indication of the amount of activity obtained by the irradi-

FIG. IV-12-1. Influence of Power Level on Reactor Response to Oscillated Cf-252 Spontaneous Fission Neutron Source. ANL Neg. No. 112-5331.

ated sample during its residence time in the turret. All three irradiations were performed under quite similar conditions. The current level read on the oscillation detector channel, and confirmed by the control instrumentation was used to duplicate the power level in the succeeding runs.

Following each irradiation, five 1.3-cm diameter foils were punched at 2.5-cm intervals along the lengths of both the irradiated and background samples. These foils were first gamma counted, in order to provide data from which to derive the required background correction. The five foils from each of the irradiated samples were then counted in the beta-gamma coincidence arrangement used in connection with the manganese bath procedure for source calibration.³ This yields an absolute disintegration rate for the foils.

From the gamma counting, an amount varying from 1 to 2.4 % of the core foil activity was attributed to background. The variation with position is consistent with the location of the sample changer in a flux gradient just above the 10-cm thick top water reflector. In the beta-gamma coincidence counting, the values within each set of foils, after correction for activity introduced during location in the turret, were in good agreement and show no evidence of a flux gradient at the central reactor location. One foil from each set was counted on the following day, some eight to nine half lives later. They had, of course, appreciably less activity and hence a greater uncertainty of statistical origin. Yet they were not in significant disagreement with the assumption that only manganese activity is detected in the beta-gamma coincidence counting. This agreement, combined with the known cross sections, half lives and decay schemes for the copper isotopes argues against any contribution of copper activity to the measured counting rates.

ABSOLUTE U-235 FISSION RATE

A procedure very similar to that used for the manganese absorption rate determination was followed for the determination of U-235 fission rate. Nearly identical samples of U-235, $12.7 \times 0.25 \times 0.013$ cm, were placed at three symmetrical locations within the turret of the sample changer. In addition, the reference manganese-copper sample was loaded adjacent to one of the U-235 foils. This U-235 foil and the standard manganese-copper sample were each oscillated to obtain five sets of values to interrelate their response. The second U-235 sample was irradiated for 30 min and the third was intended to provide background information. Three separate irradiations were made for the determination of the U-235 fission rate.

The total number of fissions experienced by each of the irradiated U-235 foils was determined by a radio-

TABLE IV-12-I. DATA FOR CALCULATION OF $(1 + \alpha)$

Quantity	Units	No. of Values	Average	Standard Error	%
$(\Delta V/V)_{Mn}$	(g Mn) ⁻¹	3	-1.987×10^{-3}	± 0.008	0.40
$(\Delta V/V)_U$	(g U-235) ⁻¹	3	4.543×10^{-3}	± 0.042	0.92
$(\Delta V/V)_{Cf}$	—	5	0.5573×10^{-3}	± 0.0037	0.67
R_{Mn}	absorptions/(sec) (g Mn	3	1.721×10^6	± 0.016	0.93
R_f	fissions/(sec) (g U-235	3	9.958×10^6	± 0.036	0.36
S	neutrons/sec	8	7.323×10^6	± 0.029	0.40
ν	neutrons/fission	5	2.442	± 0.006	0.25

chemical procedure⁴ involving the extraction and subsequent counting of Mo-99. Because of the complexity of the fission product decay, it proved more practical to derive the required background information from the measurements made in connection with the manganese irradiations. From those data, the magnitude of the flux within the sample changer relative to that at the position of irradiation within the core was determined. The background amounted to only 3 % of the total activity. Similarly, the low U-238 content, its very small fission cross section, and the predominance of low-energy neutrons rule out a detectable contribution from U-238 fissions.

STRENGTH OF THE Cf-252 SOURCE

The californium source used in the oscillator experiments was calibrated by the manganese bath technique (see Paper IV-20) on four separate occasions over a two-month interval. Samples of the manganous sulfate solution were withdrawn twice on each of these four occasions after an exposure sufficiently long that a saturated activity had been achieved. Usually each sample was counted twice. The source strength was determined from the absolute disintegration rate based on gamma-gamma coincidence counting combined with appropriate correction factors for the aliquot of solution counted, efficiency dependence of the gamma-gamma system, attenuation of gamma radiation in the manganese solution, and hydrogen absorption. A value of 2.646 yr for the total (alpha decay and spontaneous fission) half life of Cf-252 was used to correct these determinations to the date of the principal oscillator measurements.

RESULTS

Table IV-12-I summarizes the values obtained from the measurements described above, for the experimental

quantities on which the pile oscillator method for the determination of effective capture-to-fission ratio is based. The values for R_{Mn} , R_f , and $(\Delta V/V)_{cf}$ have been normalized to a common level, namely, $V = 1000$ mV. The indicated standard deviations of the mean reflect only the reproducibility of the measurements and do not allow for any additional uncertainty due to possible systematic errors. At the start of these experiments it was realized that the errors in the individual measurements had to be held in the vicinity of 1% if a useful value for alpha was to be obtained. This has been accomplished.

The accepted⁵ value of $\bar{\nu}$ for thermal fission of U-235 is included in the table. In calculating $(1 + \bar{\alpha})$, it must be reduced by 0.006 because 37.6% of the delayed neutrons are emitted while the oscillating U-235 sample is in the "out" position. Substituting this quantity and the other values of Table IV-12-I into Eq. (7) yields a value of 1.211 ± 0.018 for $(1 + \bar{\alpha})$. The quantity $\bar{\alpha}$, rather than $1 + \bar{\alpha}$, is of principal interest, so the uncertainty is magnified when unity is subtracted from this value. Using the more common probable error, equal to 0.6745 times the standard deviation, a value of $\alpha = 0.211 \pm 0.012$ results.

An approximate value for the effective capture-to-fission ratio can be deduced for comparison with the result obtained from the pile oscillator procedure. The only significant difference between the core used for the oscillator experiment and Hi-C core No. 8 was the replacement of the central five unit cells by moderator and void. Measurements¹ with 0.82-cm diameter, 0.005-cm thick manganese-copper foils in 0.051-cm thick cadmium and aluminum covers placed in the water moderator between fuel pins in core No. 8 gave a manganese-cadmium ratio of 3.37 ± 0.02 . The same measurement at the center of the oscillator tube gave a value of 5.15, indicating a softer spectrum as would be expected because of increased moderator to fuel ratio in the vicinity of the oscillator tube.

A U-235 fission cadmium ratio $[(CdR)_f]$ of 6.83 has been measured with 0.013-cm thick foils placed in the oscillator tube used in this experiment. THERMOS and GAM-1 calculations⁶ for Hi-C core No. 8 gave values of 0.1785 for α_{th} and 0.4667 for $\alpha_{epi\ cd}$. Combining these with the measured U-235 fission cadmium ratio in the oscillator tube yields the following semi-empirical value for the effective capture-to-fission ratio for U-235 in the oscillator tube:

$$\begin{aligned}\bar{\alpha} &= \alpha_{th} \frac{(CdR)_f - 1}{(CdR)_f} + \frac{\alpha_{epi\ cd}}{(CdR)_f} \\ &= 0.221.\end{aligned}$$

This value is slightly higher than the value derived

from the oscillator experiment, but agrees within the probable error ascribed to the oscillator value. The value for $\alpha_{epi\ cd}$ is not very sensitive to the degree of moderation and it is in reasonably good agreement with an average value of 0.464 obtained with a 0.051-cm cadmium thickness.⁷ A somewhat lower value of α_{th} , closer to the 2200 m/sec value of 0.175,⁽⁵⁾ can be justified. The manganese capture cadmium ratio and hence U-235 fission cadmium ratio within the oscillator tube is higher than that associated with Hi-C core No. 8. In addition, the calculations apply to a 0.94-cm diameter Hi-C fuel pellet, whereas less flux hardening exists for the 0.013-cm U-235 foils used in these measurements. Such a reduction improves the agreement.

COMMENTS

The development of a procedure for the direct measurement of capture-to-fission ratio in low flux systems is a significant addition to the techniques available for zero power reactor experimentation. Yet it would be premature to claim that this procedure is now available for application routinely without additional testing and further investigation. It is appropriate to enumerate several limitations, experimental difficulties and known, suspected, or possible systematic errors which warrant additional consideration.

The determination of the worth or importance of reactor and fission neutrons has involved the use of substitute materials for U-235. The cadmium ratio of 5.15 for manganese, compared with a U-235 absorption cadmium ratio of 5.68, indicates the extent to which manganese matches the response of U-235 to reactor neutrons. This U-235 absorption cadmium ratio is derived from the measured $(CdR)_f$ and calculated values for α_{th} and $\alpha_{epi\ cd}$. The average fission neutron energy for U-235 is 1.94 ± 0.05 MeV and for the spontaneous fission of Cf-252 is 2.2 ± 0.1 MeV. For both manganese and Cf-252, their use as substitutes is warranted, for lack of a better material. A calculated correction to both W_a and W_f , as measured with these substitute materials, appears to be the only feasible approach.

Certain difficulties exist in the experiment as performed to date. The Cf-252 source and the foils used for the cadmium ratio determinations were located near the bottom of the sample cans, while the other samples were distributed along the length at the wall of the can. If any of the effects produced are position-sensitive, then the various measurements were not performed under equivalent conditions.

Another limitation involves the possibility of effects due to inelastic scattering and moderation. This has been investigated to date only in a cursory fashion. For example, a 33.6-g sample of bismuth produced a signal barely perceptible above the noise level, giving a $\Delta V/V$

of $\sim 9 \times 10^{-6}$ /g, and so further such measurements were temporarily discontinued.

Finally, because of self-shielding effects, the derived value of alpha is strictly applicable only to the 0.013-cm thick U-235 foil for which measurements were made.

REFERENCES

1. A. R. Boynton, Q. L. Baird, J. M. Christenson, K. E. Plumlee, W. C. Redman, W. R. Robinson and G. S. Stanford, *High Conversion Critical Experiment*, Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010 (January 1965), pp. 33-43.
2. E. F. Groh, A. F. Engfer and R. A. Mattson, *A Pile Oscillator with Sample-Changing Features*, Nucleonics **17**, No. 2, 90 (1959).
3. A. De Volpi, K. G. A. Porges and R. N. Larsen, *Mn-56 Coincidence Counting Facility*, ANL-6760 (August 1963).
4. R. J. Armani, *Radiochemical Measurement Techniques in ZPR-6 and 9*, Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010, (January 1965), p. 228.
5. R. Sher and J. Felberbaum, *Least Squares Analysis of the 2200 m/s Parameters of U-233, U-235 and Pu-239*, BNL-918 (March 1965).
6. E. Pennington, Argonne National Laboratory (Private Communication).
7. L. J. Esch, et al., *A Measurement of Alpha for U-235 in the Epithermal Region*, Trans. Am. Nucl. Soc. **7**, No. 1, 78 (1964).

IV-13. Oscillation Measurement Procedure in Critical Experiments

K. E. PLUMLEE and C. E. COHN

Accuracy of determination of reactor response during sample oscillation is lessened by statistical fluctuations of the neutron flux, which occur randomly and independently of the response to be measured. Ordinarily there are several factors which may be adjusted to increase the magnitude of the desired response relative to random effects. If a restriction applies to one or more factors in a given experiment, greater use must be made of those remaining available. An example is found in the oscillation of a Cf-252 spontaneous fission source, as described in Paper IV-12. In contrast to typical experiments in which sample size could be increased as needed, the supply of Cf-252 was severely limited. A second limitation was that the spontaneous fission rate was independent of reactor flux whereas most samples react proportionally with flux level.

Much of the random variability resulting from statistical fluctuations may be cancelled out by averaging repetitive or sustained measurements. Semi-automatic equipment has been assembled which repetitively records reactor response during sustained sample oscillation. Periodic data entries are made on IBM data cards, and alternate sample insertion or withdrawal is automatically initiated after each tenth datum point. The recording equipment includes a conventional boron-lined ion current chamber, a linear amplifier, a calibrated bucking circuit, a chart recorder, and an encoder which facilitates readout by a card punch. A sample changer¹ permits manual selection and optional manual or automatic step insertion and removal of samples. Semi-automatic operation of the system is

accomplished by a control chassis. Operator intervention is necessary to start the cyclic measurements so that data card records synchronize with sample position. A lapse of data taking is necessary while the next sample is being selected, and until the reactor has been leveled with the new sample cycling in and out.

Analysis of the data by a computer provides fitted Fourier coefficients which are representative of the reactor response to each sample. It was assumed that the flux intensity record is a function $f(t)$ satisfying the restrictions stated in standard text books² for representation by a Fourier series wherein the following equations apply:

$$f(t) = \frac{b_0}{2} + \sum_{k=1}^{\infty} (a_k \sin kt + b_k \cos kt),$$

$$b_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) dt,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos nt dt,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin nt dt.$$

Simpson's Rule³ was applied in performing the quadrature necessary to approximate the integrals of the expressions for a_n , b_0 , and b_n :

$$\int_{x_0}^{x_0+nh} Y dx \approx \frac{h}{3} \sum_{i=1}^M C_i Y_i,$$

where

$$C = 1, 4, 2, 4, 2, \dots, 2, 4, 1.$$

TABLE IV-13-I. COMPUTED RESULTS

ACTOR LOADING 284 RUN 1 SAMPLE 1 DATA SET 1 NUMBER OF CYCLES—10

ORDER	COEF OF SINE TERM	COEF OF COSINE TERM	AMPLITUDE	PHASE ANGLE (DEGREES)
1	0.04250 +/- 0.00272	-0.23530 +/- 0.00248	0.23911 +/- 0.00248	-10.239 +/- 0.650
2	0.02911 +/- 0.00159	-0.01797 +/- 0.00180	0.03421 +/- 0.00165	-58.311 +/- 2.925
3	-0.04482 +/- 0.00203	-0.00287 +/- 0.00164	0.04491 +/- 0.00203	86.340 +/- 2.089

STEADY COMPONENT 0.50008 MEAN DEVIATION 0.00040 RMS DEVIATION 0.02998

FIG. IV-13-1. Fitted and Actual Data for an Oscillation (All Points Plotted from Computer Output). ANL Neg. No. 112-5725.

The solutions for b_0 , b_n , and a_n were as follows.

$$b_0 = \frac{1}{3(m-1)} [Y_1 + 4Y_2 + 2Y_3 + 4Y_4 + \cdots + Y_m]$$

where b_0 is summed over an odd number (m) of readings recorded during a measurement with a given sample and period,

$$b_n \approx \frac{1}{\text{No. cycles}} \sum_1^{\text{No. cycles}} \frac{1}{30} \sum_{i=1}^{21} C_i Y_i \cos \left(ni \frac{\pi}{10} \right),$$

$$a_n \approx \frac{1}{\text{No. cycles}} \sum_1^{\text{No. cycles}} \frac{1}{30} \sum_{i=1}^{21} C_i Y_i \sin \left(ni \frac{\pi}{10} \right).$$

The computer analysis yields computed values for b_0 , b_1 , b_2 , and b_3 and a_1 , a_2 , and a_3 , where the a 's and b 's are the Fourier coefficients defined previously. As may be seen by inspection of Fig. IV-13-1, reasonably close agreement was obtained between the fitted expression (*) and typical data points (X) which were plotted for purpose of comparison, using these terms. The fitted values resulted from a 201-data point input record (which may include any multiple of 20 data

points plus one extra). The mean deviations of the individual summations by cycles from the average are also given, yielding estimates of the standard deviations of a_n and b_n . The values computed are as follows for a_n (b_n follows the same pattern).

RMS dev of $a_n = \sigma_{an}$

$$= \frac{1}{\text{cycles}} \left[\sum_{i=1}^{\text{cycles}} (a_{ni} - \bar{a}_n)^2 \right]^{1/2},$$

$$= \left[\frac{1}{\text{cycles}} \sum_{i=1}^{\text{cycles}} a_{ni}^2 - \left(\frac{1}{\text{cycles}} \sum_{j=1}^{\text{cycles}} a_{nj} \right)^2 \right]^{1/2}.$$

The amplitudes and phase angles are also computed for the various Fourier components.

$$\text{Amplitude}_n = [b_n^2 + a_n^2]^{1/2},$$

$$\text{RMS fractional deviation of amplitude}_n = \frac{[b_n^2 \sigma_{bn}^2 + a_n^2 \sigma_{an}^2]^{1/2}}{[b_n^2 + a_n^2]^{1/2}}$$

$$\text{Phase}_n = 57.2958 [\text{Arc tan } (b_n/a_n)]$$

$$\text{RMS deviation in phase angle} \approx \frac{[b_n^2 \sigma_{an}^2 + a_n^2 \sigma_{bn}^2]^{1/2}}{[b_n^2 + a_n^2]^{1/2}}$$

The terms indicated above are listed in Table IV-13-I, as is the root-mean-square (RMS) deviation of data points from the fitted curve.

Consistent results have been obtained in comparing various standard samples on the basis of this technique, indicating that the analysis is adequate for accurate comparisons of sample worth. Although the computer program could easily be extended to obtain further terms of the a_n and b_n coefficients, this does not appear to be necessary.

REFERENCES

1. E. F. Groh, A. F. Engfer and R. A. Mattson, *A Pile Oscillator with Sample-Changing Features*, Nucleonics **17**, No. 2, 90 (February 1959).
2. I. S. Sokolnikoff and E. S. Sokolnikoff, *Higher Mathematics for Engineers and Physicists*, (McGraw-Hill Book Co., Inc., N.Y., 1941), 2nd Ed., p. 67.
3. J. B. Scarborough, *Numerical mathematical Analysis*, (Johns Hopkins Press, 1950), 2nd Ed., p. 132.

IV-14. Modification of the TREAT Integral Sodium Loops

L. E. ROBINSON, R. T. PURVIANCE and F. L. WILLIS

INTRODUCTION

An advanced phase of the Fast Reactor Safety experimental fuel meltdown program requires that clusters of fast reactor fuel elements be exposed in the presence of coolant to destruction during power excursions of the Transient Reactor Test Facility (TREAT). To realize this, a small integral sodium loop was designed and constructed to provide a high temperature flowing coolant within the reactor environment, in realistic simulation of fuel elements in an operating power reactor core. Prior to initial experiments with prototype EBR-II fuel pins, a number of modifications of these integral sodium loops were necessary to improve techniques of remote handling, pressure instrumentation, and the general operational characteristics of the loop assembly.

DESCRIPTION OF THE ORIGINAL LOOP DESIGN

The TREAT integral sodium loop is a package unit in which fast reactor fuel pins can be tested to destruction in stagnant or flowing sodium coolant. The loop contains a sodium reserve and flow system, an electromagnetic pump, power transformer, sodium flowmeter, sodium heaters, expansion chamber, fuel test section, pressure and temperature instrumentation, together with power and control wiring. The loop fits into a standard TREAT dummy fuel element, in a space $10 \times 10 \times 297$ cm. (Fig. IV-14-1).

The sodium circulation system is of the counterflow type, and consists of a cylindrical test section connected at the bottom to two return tubes which lead to a common central feed tube attached to the test section inlet (see Fig. IV-14-2). An electromagnetic pump in the feed tube forces the molten sodium upward through the test section fuel holder, thence down through the annulus between fuel holder and outer wall of the test section, through the return tubes and into the common feed tube containing the EM pump.

Power for the *ac* induction pump is supplied by a 2 kV-A, 60 cycle, single phase transformer which is suspended from the pump magnet on the feed tube at the bottom of the loop. Clamshell-type heaters enclosing the test section provide the heat to liquify the sodium within the test section and downcomer annulus. A cal-rod heater is shaped and bonded to the contours of the harp end of the loop to maintain the sodium in the return tubes liquid. Thermocouples surface-mounted on

the outer walls of the loop near the heaters provide temperature signals for the heater controls. The original loop design temperature of 500°C has been unattainable because of the 300°C temperature limitation of the power transformer.

An expansion chamber above the test section provides sufficient void volume to permit the thermal expansion of the loop sodium resulting from a 500°C temperature change. The void above the free surface of the molten sodium is filled with an inert gas blanket, and is relief-ported through a 450 psi rupture disk to an evacuated safety tank which is concentric with the expansion chamber of the loop.

Strain-gage bridge pressure transducers are mounted on pressure taps located above and below the loop test section to furnish inlet and outlet pressure data. An electromagnetic flowmeter, using a 1200-gauss permanent magnet, is located just above the EM pump, to measure the sodium flow rate through the feed tube at the fuel holder inlet. Transducer leads and thermocouples, along with power and control wiring, are brought out through the flange at the top of the loop in a flexible trunk cable to a multi-contact amphenol connector in the gripper cap of the dummy TREAT fuel can.

The fuel holder is cylindrical in shape and open at both ends, allowing sodium to flow past the fuel pins. It is designed to contain a cluster of seven EBR-II-type fuel pins. The fuel holder is suspended from the top flange of the loop by a tubular extension having an O-ring seal at the top. This test section assembly, shown in Fig. IV-14-3, is installed in the loop through the opening at the top of the top of the expansion chamber. Compressing an O-ring seal with the top flange closes the loop.

LOOP MODIFICATIONS

Loop Length Adjustment

To provide additional space above the loop closure flange for the power, control and instrumentation wiring, and for the cooling lines for pressure transducers the loop required shortening. As designed, the assembled and jacketed loop provided a cable storage space between gripper cap and closure flange of 9.5 cm with the top of the loop recessed approximately 12.5 cm from the top of the dummy TREAT fuel can when the gripper cap was removed. The restricted axial space

FIG. IV-14-1. Original Integral Loop, with TREAT Fuel and Test Fuel Holder Assembly. ANL Neg. No. 112-688.

did not allow convenient storage of the massive trunk cable of electrical wiring, and rendered the removal and insertion of the test fuel holder into the recessed loop entrance prohibitively difficult. The required shortening was accomplished by reducing the length of the loop throat above the expansion chamber.

FIG. IV-14-2. Integral Sodium TREAT Loop. ANL Neg. No. 112-5731 Rev. 1.

FIG. IV-14-3. Test Fuel Holder and Extension Assembly of Modified Loop. *ANL Neg. No. 112-4763A.*

The loop throat was cut off approximately 12 cm below the original flange and a newly designed flange of double depth was welded into place. The TREAT dummy element was cut into sections. One section houses the integral loop proper, from the base pedestal

FIG. IV-14-4. Modified Integral Sodium Loop. *ANL Neg. No. 112-3106.*

FIG. IV-14-4

the lower half of the double flange at the loop throat try. The remaining section, about 22 cm in length, was split symmetrically lengthwise into two parts, to enclose the storage space between the upper half of the

FIG. IV-14-5. Completed Modification of Integral Sodium Loop Top Section. ANL Neg. No. 112-3116.

throat flange and the base of the gripper cap (Fig. IV-14-4).

The gripper cap, of cast aluminum, was ported on all four corners, for the installation of quick-disconnect valves in the pressure transducer gas coolant inlet and exhaust lines, and in the vent line for the inert atmosphere within the TREAT dummy element external to the sodium loop. Four flexible metal tubes were connected to the quick-disconnect valves and silver-soldered to the cooling and vent lines passing through the loop throat flange. Sufficient clearance was allowed above the valves to permit the ready use of the TREAT facility fuel gripper tool in handling the loop experiments.

With the convenient removal of quick-disconnect valves, gripper cap and both halves of the TREAT dummy element upper section by remote means, the throat flange of the loop vessel can be exposed for the easy removal of the test fuel holder. The flexible coolant tubing and braid-shielded electrical cable can be arranged in positions which allow use of the remote-handling equipment without interference. The completed modification of the assembly top section is shown in Fig. IV-14-5.

PRESSURE INSTRUMENTATION

The pressure sensors selected for the integral sodium loops are bridge-connected unbonded strain-gage transducers, which have been used with considerable success in power reactors. The high-temperature transducer is 1.27 cm in diameter, 1.90 cm long, with a 0.156 cm mounting flange; it is diaphragm actuated, 0–150 psig in range, flush-mounted and has a rated maximum operational temperature of 600°F (316°C).

Originally the pressure transducers were enclosed in heavy metal heat-sink housings, and flange-mounted to extended U-shaped tubular pressure taps, with the transducers in direct contact with the sodium contained in the system. A leak-tight seal was achieved by the compression of a Belleville-coned square-sectioned nickel O-ring between transducer and tap flanges. The clamping stresses required to obtain the necessary deformation of flange metal to achieve a sodium-tight seal severely distorted the housing of the transducer, and effectively created a load-sensitive device. This usually resulted in a spurious electrical unbalance of the strain-sensitive bridge of the transducer beyond the capability of readout nulling circuits, under conditions of zero gage pressure.

During start-up of the integral loop, the cold-bend plugging of the extended-U pressure taps with solid sodium metal in conjunction with liquification of the sodium in contact with the transducer diaphragm, resulted in extreme overpressures which destroyed the

delicate sensing element. With subsequent thermal cycling of the loops, the sodium in contact with the transducer diaphragm froze within the pressure tap and continued to contract as the metal cooled to room temperature, resulting in permanent distortion of the thin diaphragm. Finally, the high-rated operational temperature of the transducer is valid only under isothermal gage conditions, for the device cannot function properly with a thermal gradient between diaphragm and transducer housing. Mounting the transducer with its diaphragm in direct contact with the molten sodium heat source resulted in high spurious unbalance of the sensor bridge.

These conditions led to the design, development and construction of the NaK-filled stand-off pressure transducer assembly. The transducer is isolated from the loop sodium by a low-spring-rate welded volume-compensating metal bellows and a 0.635 cm o.d. stainless steel connecting tube filled with Na-77K eutectic, which has a freezing point of -19°C (Fig. IV-14-6). Sodium inside the loop makes contact with the closed end of the bellows and pressures are transmitted to the transducer by the NaK column in the connecting tube.

The unit is fabricated by shielded-arc welding throughout with the transducer housing modified to

permit welding into a rigid mounting (see Paper IV-16). Voidless filling of the NaK stand-off tube is achieved by pressurization of the evacuated tube, under zero differential condition. The transducer diaphragm is maintained at near-ambient temperature by removal of conducted heat from the NaK in contact with the diaphragm, through a brazed-on extended-surface copper radiator enclosing the transducer mount and the adjacent portion of the stand-off tube.

The transducer with its copper radiator, is contained in a cylindrical cooling housing and is cooled by flowing gas directed over the transducer in vortex flow. The gas is supplied through the quick-disconnect valves in the gripper cap of the dummy TREAT fuel can through $\frac{1}{4}$ in. o.d. stainless steel tubing to the top of the cooling housings and exhausted through $\frac{5}{16}$ in. o.d. tubing at the housing hose.

The coolant lines were shaped to go from the cooling housings, along the loop, through the throat flange, to the valves in the gripper cap. Thin metal bands snugly hold the lines to the loop.

Isolating the transducer diaphragm from the sodium eliminated damage to the transducer. The cooling provisions allow the transducer to be operated isothermally at near ambient temperatures while the sodium in the

FIG. IV-14-6. Pressure Transducer NaK Standoff Assembly. ANL Neg. No. 112-5566-A.

op is far above a safe operating temperature for the transducer.

Due to the size and configuration of the NAK-filled stand-off pressure transducer assembly and the space limitations of the loop itself, the pressure tap locations had to be shifted to gain space for mounting the units. The upper pressure tap was raised 2.54 cm and the lower tap was lowered 1.27 cm and rotated 135 degrees.

High temperature asbestos-insulated armored 2-conductor lead wire to the transducer bridge circuit was installed through modified quick-disconnect connectors welded to the transducer cooling housing. A stainless steel overbraid-chromel-alumel thermocouple to indicate transducer temperatures enters the cooling housing in the same manner. The armored excitation and overbraid thermocouple wire replaced the stainless steel sheathed, MgO-insulated leads which had been extremely difficult to install.

The connectors provide a rigid means of fastening the instrumentation wiring to the transducer located inside the cooling housing. The special wire provides necessary flexibility, electrical shielding and protection against abrasion and shock of handling.

HEVI-DUTY HEATERS

First loop experiments in the TREAT reactor showed a need for higher capacities. As a result, the clamshell Hevi-Duty heaters around the loop test section were replaced with the same type heaters, but having approximately 100 watts more capacity.

Additional thermal insulation was added around the heaters, using high purity quartz wool. The outer wrap of glass cloth was replaced with high-temperature boron-free silica glass cloth. The modification provided more heat, improved heat retention, and better nuclear properties.

CAL-ROD TYPE HEATER

During insertion and removal of the loop from the TREAT dummy fuel element, a mechanical clearance problem existed. The cal-rod type heater that forms around the loop sodium return tubes, galled against the inside corners of the dummy element. The heater was bound to the loop by a close wrap of 0.020 in. diameter stainless steel wire. To correct this condition, the heater was removed and re-formed with a tighter radius. The heater was then fastened to the loop with thin (0.005 in. thick $\times$ 0.250 in. wide) stainless steel banding ribbon, spot welded in place.

The tighter radius on the heater along with the inner banding ribbon, eliminated the clearance problem. Also, the banding ribbon provided a quicker means of fastening the heater to the loop, and the heat transfer characteristics were improved.

SODIUM LEAK DETECTOR

As an addition to the safety features of the loop, a grid-type sodium leak detector was added to the loop below the sodium return tubes and above the pump power transformer. If a rupture in the loop should occur, electrical power to the loop would be disrupted automatically.

The power shutdown is accomplished by parallel wiring of the flowmeter output leads with the grid detector. Thus, falling sodium would strike the grid, shorting out the flowmeter output. With loss of flowmeter signal, a microswitch linked to the flowmeter recorder opens, and the loop power controller is dropped out. It should be noted that in the case of no-flow experiments, the detector circuit has no function. However, with the EM pump off, sodium would not be forcibly expelled in the event of a leak.

The detector itself consists of two wire grids, electrically insulated from each other. The $\frac{1}{16}$ in. diameter wire of the grids are placed and bonded with ceramic cement in closely spaced milled grooves in a transite plate. The assembly is baked dry and surface ground until the grids are flat and polished. The gap between wires of the grids is $\frac{1}{16}$ in.

INERT BLANKET GAS

It is necessary to supply the TREAT dummy fuel element with an inert gas blanket to prevent sodium fire in case of loop failure. The gas blanket is achieved by admitting helium or argon gas into the TREAT dummy element at the throat flange through the vent valve located in the gripper cap. The inert gas then exhausts from the loop housing ena through a $\frac{1}{32}$ in. diameter hole opening into the lower portion of the transducer coolant exhaust line. It is necessary to meter the inert gas flow and regulate the input pressure until flow is established. This pressure must be set to exceed slightly the pressure of the air in the coolant exhaust line.

If helium is used as the transducer coolant gas, the blanket gas would then enter the loop can through the small hole in the transducer coolant exhaust line and leave the dummy element through the vent line.

PREVENTING SODIUM FLOW BY-PASS

The bottom or inlet end of the TREAT sodium loop fuel holder fits loosely into a bored socket in the EM pump discharge line. Allowable machining tolerances on the mating pieces permit as much as 0.015 in. diametral clearance in the socketed joint. This could result in an appreciable sodium leak at that point, causing reduced flow through the fuel holder. Moreover, any flow-measuring device would include this leak-off along with that sodium which actually flowed through

the fuel element holder, rendering useless some of the most vital information to be gained from the loop experiments.

The existing design lent itself to the incorporation of a proprietary metallic face-type seal. This seal was installed in the bottom of the machined socket, where it rests against the shoulder. The bottom of the fuel holder has a threaded end fitting which can be screwed in or out, changing the over-all length of the fuel holder and extensions. By this means, during assembly of the loop, the fuel holder length is adjusted so that axial force is exerted against the seal upon tightening the flange. This sealing insures that all sodium flow reported by the transducers actually passes through the fuel element holder and that full EM pump capacity can be utilized effectively.

DRAIN-FILL LINE

Due to problems encountered in filling and draining the loop of sodium through its top, another sodium exit was fitted to the loop.

Prior to adding the new drain tap, sodium had to be put into the loop through the small valve mounted in the test section upper seal. This valve, being inside the loop, had to be heated to a temperature suitable for the transfer of sodium and was difficult to reach. Also, to empty the loop of sodium, it was necessary to invert the loop so the molten sodium could drain out, a cumbersome operation at best and almost impossible when remote operations were necessary.

With the fitting of the new drain-fill line at the base of the loop, it was possible to fill the loop with sodium by pressurization through the easily accessible line. Emptying the loop became a matter of heating the sodium and simply draining it into a container.

The drain-fill line consisted of an elbow tap which was welded to the bottom of the sodium return tube, and a connecting line of $\frac{1}{4}$ in. o.d. stainless steel tubing. The tubing is framed along the transformer to terminate in the base pedestal of the loop. The line is heated by a tape heater, spiral-wrapped around the steel tubing.

SODIUM LEVEL INDICATION

The practice of filling the integral loop from the bottom, following the addition of the heated drain-fill line, makes it practical to include a shorting electrode sodium level indicator. This would make it possible to add supplementary sodium following an experiment in order to bring the level to the proper point in the bottom of the expansion chamber.

With the molten sodium forced into the loop from the bottom, instead of through the top sealing flange, an electrode could be inserted into the expansion chamber from the top of the loop. A $\frac{1}{8}$ in. o.d. stainless steel sheathed, MgO-insulated thermocouple was prepared by baring the insulated leads at the end about $\frac{1}{4}$ in., and carefully shaping them to wide separation. The electrode was inserted through the seal plug in the loop top, and rigidly banded to the outside of the fuel holder extension, with the bare wire $\frac{1}{8}$ in. below and the end of the steel sheathing $\frac{1}{8}$ in. above the required sodium level. The two circuits, between the thermocouple wires and between either wire and ground, provides a simple but effective "High-Low" level indication.

CONCLUSION

The modified integral sodium loops have been used in two series of meltdown experiments in TREAT, and have performed satisfactorily under the conditions of initial temperature, flow rate and reactor power level specified. Auxiliary flow experiments on the test section of the loop proved the electromagnetic flowmeter to be more than 50 % deficient in output, and the EM pump to be capable of from 4 to 5 m/sec flow velocity through the loop fuel holder, instead of the 2.5 m/sec indicated by the flowmeter.

The present limitation of 260°C as initial sodium temperature can be removed and the integral loop used at its original design temperature of 500°C, by the replacement of the present epoxy-insulated pump power transformer with a new high temperature transformer which has been designed and tested. This modification is being considered for the future utilization of the integral TREAT sodium loops.

IV-15. Loading and Remote Handling of Integral TREAT Sodium Loops

F. L. WILLIS, R. J. SCHILTZ* and L. E. ROBINSON

The first three transient experiments have been performed with the modified TREAT integral sodium loops (see Paper IV-14) using prototype EBR-II fuel

* Reactor Engineering Division.

pins which have not been previously irradiated. There are no particular personnel hazards associated with the handling of such elements, even after their transient exposure.

The construction of the loops has been described in 1. The experimental fuel element is incorporated into a removable test section of the loop which consists of a fuel element holder, an extension tube, and an upper sealing flange. The specimen(s) under test is installed in a hexagonal cluster of seven in the fuel element holder. All testing thus far has been performed using one central fuel element surrounded by six dummy elements of identical dimensions.

However, future integral loop experiments will involve the installation of fuel elements which have been pre-irradiated. Furthermore, melting and cladding failure occurs in the TREAT tests, resulting in considerable fission product release. Because of these radioactivity hazards, provisions have been made for performing assembly and disassembly of the loops using remote techniques. Equipment for so doing has been installed in a 20-foot deep hole, bored for this purpose in the floor of a cave facility in Bldg. 330. The installation consists principally of a fixture for holding and positioning the loop while assembly or disassembly operations are performed. The positioning fixture can be hydraulically raised, lowered, and rotated to expose any portion of the loop to the cave window. The equipment can be retracted fully into the cave floor, and a flush-fitting shielding plug set into place over it.

The remote handling mechanism was tested before installation in the Bldg. 330 cave. Figure IV-15-1 shows the positioner during this testing with a loop in place ready for disassembly. The manipulators and the restricted view through the cave window were mocked up.

With the loop cold, and before addition of the atmospheric enclosure, the top section of the loop is disassembled, exposing—but not opening—the top flange of the test section. Figure IV-15-2 was taken just after this preliminary operation, and shows the lower section of the plastic atmospheric enclosure in place. Some of the gas and electrical connectors protruding from the top of the loop are shown, along with remote disassembly tools placed on the table top. The Lucite enclosure used to provide an inert gas atmosphere over the entire operation and the associated devices for handling the test section which appear in the photograph are not part of the permanent installation. Before proceeding further, it is necessary to establish an inert gas atmosphere over the operation. Fastening the upper plastic section in place completes the gas-tight enclosure. The interior is then purged with dried nitrogen gas until the sampling system shows an oxygen content of less than 2%. Not seen in Fig. IV-15-2 is an electrical connector which passes from the loop through a sealed opening in the plastic enclosure. A lead from the loop control console can be attached to this connector, supplying power to the loop heaters to melt the

contained sodium and permit withdrawal of the test section.

Within the enclosure is a gripper device which can be clamped onto the top of the test section. When in place, this gripper is raised with the crane hook to withdraw the test section from the loop and lowered into a shipping container stored inside the enclosure. By means of the gripper tool, sealing plugs are next installed on the loop and test section container, trapping the nitrogen atmosphere within them. Figure IV-15-3 shows the test section being withdrawn from the loop, while in Fig. IV-15-4, it is being inserted into its auxiliary storage container. Also in Fig. IV-15-4, the two seal plugs are shown stored in pockets provided inside the atmosphere enclosure, where they are accessible to the gripper tool.

Since the gripper must travel vertically some 50 in. in order to transfer the test section, a flexible gas-tight

FIG. IV-15-1. Remote Positioner Projecting from Guide Tube, with an Integral Loop and Auxiliary Storage Container in Place. ANL Neg. No. 112-4132.

FIG. IV-15-2. Lower Section of Atmospheric Enclosure in Place on Remote Positioner, with Gripper Cap, Dummy Can Clamshell and Upper Flange Removed from Loop. *ANL Neg. No. 112-4131.*

FIG. IV-15-3. Remote Positioner Retracted into Guide Tube, with Upper Section of Atmospheric Enclosure in Place, and Test Section Withdrawn from Loop by Gripper Tool. ANL Neg. No. 112-4129.

FIG. IV-15-4. Gripper Tool Lowering Test Section Into Auxiliary Storage Container, Inside of Atmospheric Enclosure. ANL Neg. No. 112-4126.

FIG. IV-15-5. Full Length View of Atmospheric Enclosure in Place On the Remote Positioner, with the Gripper Tool and its Plastic Bellows Seal. ANL Neg. No. 112-4128.

was required at the enclosure junction. Accordingly, an extension bellows was designed and fabricated out of vinyl plastic sheet. The partially extended bellows can be seen in Fig. IV-15-5. The spring-tensioned chains serve to fasten the removable Lucite enclosure to the top flange of the positioner mechanism, with adequate compression of the neoprene O-ring seals.

The atmospheric system is supplied from a nitrogen tank and the gas flow is metered into the enclosure to provide a continuous purge. The gas outlet line is

directed through an AEC standard filter before being vented, preventing escape of radioactive particulates into the cave interior. The cave ventilating system is capable of disposing of possible gaseous fission products.

REFERENCES

1. E. S. Sowa, C. R. Breden and L. W. Fromm, Jr., *Equipment used on the Performance of Experiments in Nuclear Reactors at the Argonne National Laboratory*, Nuc. Sci. Eng. **15**, 415 (1963).

IV-16. Pressure Transducers for TREAT Sodium Loops¹

L. E. ROBINSON and R. T. PURVIANCE

INTRODUCTION

Advanced phases of the fuel meltdown program include exposure of fuel elements to power transients in high-temperature flowing sodium loops placed in TREAT. The very short rise-time characteristics of pressure waves generated in the liquid-metal coolant necessitated development of a specialized pressure transduction system. Successful adaptation of commercially available strain-bridge pressure transducers resulted in a sensitive and durable sensor which permits in-pile measurements of steep-fronted pressure pulses in high-temperature sodium-filled systems.

DESIGN CRITERIA

A comprehensive listing of minimal criteria established for the design of adequate in-pile transient pressure instrumentation of the TREAT sodium loops is as follows:

1. Resistance to transient high-level neutron and gamma nuclear radiation fields.
2. Compatibility with sodium at elevated temperatures for prolonged periods.
3. Capability of reliable operation at elevated temperatures for prolonged periods.
4. Capability of response to signal frequencies up to 1000 cps and pulse-widths as narrow as 1 millisecond.
5. High sensitivity to sodium pressure waves, in terms of output/psi input.
6. Linearity of pressure/unit-output over the full range of the sensor.
7. Miniaturization to permit close coupling with the sodium system within the space limitations of the dummy element container of the Integral Loop.
8. Remote readout capability to permit signal

transmission to the recording mechanisms in a distant control room.

9. Zero temperature drift, to accommodate the extreme temperature changes realized during thermal cycling of the loops up to 500°C.

10. Zero mounting stress on the transducer to eliminate spurious force-stress signals during experiments.

11. Zero temperature gradient effect on operation under extreme temperature gradient changes realized during a test.

12. Durability in thermal cycling, with the freezing-contracting effects of sodium upon the sensor eliminated.

13. Low noise, with the sensor and its excitation and signal leads adequately protected by shielding from spurious electrical pick-up from AC-powered loop components and reactor accessories.

14. System containment integrity of the same order as that of the loop and zero sodium leakage over the full range of possible loop operational conditions.

15. Mounting convenience to permit easy removal, replacement, and maintenance.

A solution satisfying the above design criteria was realized by an adaptation of commercially available transducers in wide use on conventional high-temperature systems.

The strain-gage bridge-connected pressure transducer was chosen as the sensor, with a final choice being made between three competing and operationally equivalent devices. The transducer selected had been used in high radiation fields at steady state with good success, and has a maximum operational temperature rating of 600°F. It is about 1.5 cm in diameter by about 2 cm

long. Each is equipped with an integral mounting flange having a cross-section about 1.5 mm square. These gages have natural frequencies of 17–20 Kc and a frequency response which is essentially flat to 3000 cps. In the as-received condition, they exhibit no detectable departure from linearity over the full range. These gages are excited by a constant 5 V dc-voltage applied to the Wheatstone bridge composed of 2 active-2 inactive unbonded strain-wires supported on a star-spring mounting. An alternate method of excitation by ac-carrier wave, using a demodulator-to-analogue dc output amplifier, tends to clarify the output of the transducer through the elimination of spurious electrical noise. With either method of excitation, the full range output of the transducer is 21.5 to 24.5 millivolts of bridge unbalance. This is more than adequate for amplifier transmission to a distant recorder, and can be applied directly to millivolt indicators.

DESIGN FEATURES

The selected transducer was adapted to the sodium system by incorporation into an all-welded construction of Type 304 stainless steel, the material of the loops. Although such a transducer had never been successfully installed by welding, and the manufacturer stated that such treatment would render the transducer completely inoperative, a technique for the installation of the CEC transducers by welding was developed. It has been used successfully on more than two dozen transducers.

The all-welded assembly has provision for isolation of the stainless steel diaphragm of the transducer from the molten sodium of the loop by a pressure-transmitting column of NaK. The NaK, in turn is separated from the loop sodium by a multi-convolution welded metal bellows with a very low spring-rate. This serves as a thermal expansion volume compensator for the NaK column. It is fabricated of Type 321 stainless steel with a leaf thickness of 0.005 cm, a diaphragm closure of 0.025 cm thick placed at one end and a machined weld fitting at the other end.

The finished NaK-filled assembly is flange-mounted to a short-stemmed pressure tap on the loop. The high-temperature seal against leakage of sodium is achieved through use of a solid Type 302 stainless steel O-ring, in deformative compression between the mounting flanges. The O-ring is of rectangular cross-section, with exploded circle curvature of adjusted radii at the four corners, and is precisely fitted into 15 to 30 deg trapezoidal grooves. Flange compression is obtained with carbon steel bolts.

Protection of the transducer from the 500°C design temperature of the loop is accomplished by enclosing the transducer with an extended-surface copper radiator, within a thin-walled jacket through which cooling gas is circulated. This arrangement also eliminates spurious signals from external temperature gradients.

Excitation and signal leads are brought inside the cooling enclosure to the transducer connections through modified miniature tubing fittings, which provide a tight seal against excessive leakage of the coolant gas. All leads are paired (input and output) for shielding against cross-talk, and stray field electrical pick-up.

Electrical check-out tests and pressure recalibrations were made on the completed pressure transducer stand-off assemblies, with changes in sensitivity, linearity and zero shift noted. These tests revealed no detectable change in linearity, an average of 2.5 % shift in electrical zero reference, and an average of 3.9 % loss in full range sensitivity.

Strenuous thermal-cycling tests were performed on two prototype pressure transducer stand-off assemblies. These detectors were subjected to full range pressures while attached to a simulated sodium loop operated over a temperature range from ambient to 500°C. Operation was found to be satisfactory, with no detectable changes in linearity and sensitivity, and with no physical change or malfunction of the assemblies.

REFERENCES

1. L. E. Robinson and R. T. Purviance, *Pressure Transducers for TREAT Sodium Loops*, ANL-7107, (to be published).

IV-17. Remote Encapsulation of Pre-Irradiated Fuel Elements for Exposure in TREAT

F. L. WILLIS, R. J. SCHILTZ* and J. H. MONAWECK*

INTRODUCTION

As part of the Fast Reactor Safety program, reactor fuel elements have been given transient exposures in TREAT, up to and beyond meltdown, in order to study

their failure characteristics and the mechanisms of post-failure fuel movement. Early testing was confined to unirradiated specimens, but extension of the studies required the use of specimens which had been previously irradiated to various burnup levels.

* Reactor Engineering Division.

Initially, experiments were performed on sample pins retained, under an inert gas atmosphere, in stainless steel capsules¹. Encapsulation of the unirradiated specimens presented no particular problems. The pre-irradiated elements, however, were highly radioactive and their encapsulation in the conventional bench-top manner was not possible.

Therefore, equipment and techniques for in-cave remote assembly were developed, and the existing capsule for unirradiated elements was adapted to the requirements of remote assembly.

DEVELOPMENT

CAPSULES FOR UNIRRADIATED FUEL

The capsule used for testing of unirradiated fuel in TREAT has been described.¹ In essence, it consists of a stainless steel tubular can containing graphite liners which support the fuel pin in place and retain possible meltdown products. After assembly, each can is evacuated and back filled with helium gas at about 1 psia pressure before final sealing. Thermocouples are attached to the fuel pin cladding and their leads brought out through an electrical connector at the top of the capsule. Figure IV-17-1 shows the capsule components in addition to a view of a completed assembly.

CHANGES IN ASSEMBLY TECHNIQUE

Since the capsules for unirradiated specimens were hand assembled, most of the work was performed rather conveniently—including such delicate operations as welding thermocouple wires to the fuel pin and making soldered wire joints at the electrical connector. Performing the assembly in a shielded cave using only the comparatively awkward manipulator hands, however, would have been virtually impossible. Extensive changes in assembly technique were necessary, which led to the design of some specialized equipment and certain changes in the capsule components.

The basic practice adopted was that of doing the maximum portion of assembling and checking before beginning remote operations. A capsule subassembly was built which incorporated as much as possible of the thermocouple wiring, sealing devices, and support for the graphite pieces. This subassembly was clamped vertically in a support fixture so that the other components could be placed upon it one at a time during in-cave assembly operations.

A fixture was required to support and position the various pieces during this assembly. The support fixture used was a motor-driven chuck mounted, with axis vertical, on a cart which carried a small table, together with certain items of electrical gear and also fixtures for supporting the thermocouple wires before being welded to the sample. The subassembly was

clamped in the chuck and the other components placed upon it in the assembly sequence. Figure IV-17-2 shows the subassembly in the fixture during welding of the thermocouple wires. Note that the capsule was put together in an inverted position.

WELDING THERMOCOUPLES

To avoid the complication of handling delicate 28 gauge thermocouple wires with manipulator hands, the wires were positioned for welding before entering the cave. This was accomplished by fitting extension rods to the fixture which held the wires in place after they had been adjusted to a dummy fuel pin. A spring loaded electrode was developed which clamped the wire under proper pressure to the fuel pin during resistance welding of the wire to the pin cladding. The supporting rubber bands and excess wire, seen in Fig. IV-17-2 were cut away after welding.

WELDING CAPSULE BODY

A welding gun, using an inert gas-shielded electrode and mounted on the fixture, is used to integrate the two sections of the capsule. The capsule sections, still in the motor-driven chuck, are revolved past the electrode arc to form the weld seam joining them.

The electrodes for resistance welding of the thermocouples and arc welding the capsule body were each supplied with current by power sources located outside the cave.

LEAK CHECKING, GAS FILLING, SEALING

Another problem in remote assembly was presented by the specification calling for shipping the capsules with a contained atmosphere of about 1 psia of helium gas. This required the capsule weld to be impervious to helium gas and some means to be devised for proving it so. These final operations of gas filling, leak checking and sealing were combined under one development effort.

Development work led to incorporation into the capsule body of a small stainless steel tube, Fig. IV-17-3, which served as a capsule evacuation tube, leak detection outlet, and final sealing device. A separate testing system was developed which utilized a helium leak detector with a vacuum pump and appropriate valving and gauging manifold (see Fig. IV-17-4). When connected by vacuum hose and a slip-on fitting to the capsule evacuation tube, this gas system performed the following functions:

1. Proved the gas-tight integrity of the capsule body weld.
2. Established the specified helium atmosphere within the capsule.

FIG. IV-17-1. Capsule Components and Completed Assembly. ANL Neg. No. 112-1912-A.

3. Sealed the capsule evacuation tube while maintaining the above atmosphere.
4. Made a final leak check on the evacuation tube seal.

The above operations were accomplished in this manner:

The testing apparatus was connected to the evacuation tube and valved into the helium detector. The

1. EBR II FUEL ELEMENT
2. THERMOCOUPLE WIRE POSITIONED FOR WELDING
3. THERMOCOUPLE WIRE POSITIONER
4. RUBBER WIRE TENSIONER
5. WELDING ELECTRODE CLAMPED OVER WIRE
6. HINGED FUEL ELEMENT SUPPORT
7. CAPSULE SUBASSEMBLY CLAMPED IN CHUCK
8. PRE-POSITIONED GUN FOR WELDING CAPSULE BODY
9. GRAPHITE LINER
10. CAPSULE CAN

FIG. IV-17-2. Assembly Fixture with Partially Completed Capsule. *ANL Neg. No. 112-1906A.*

capsule was then revolved with the chuck while helium gas was blown through the previously described welding gun onto the newly completed weld. Any porosity at the weld would permit helium to pass through to the helium detector, indicating a leak.

The evacuation tube had an outer diameter of 0.32 cm and an inside diameter of 0.19 cm, in which a composite plug was held captive by suitable crimping (See Fig. IV-17-3). The plug diameter was 0.16 cm, leaving a 0.014 cm annulus through which gas could readily move. The sealing plug was especially made for this operation and consisted of a stainless steel lower section and a silver solder upper section. A groove

machined in the plug was charged with flux prior to insertion in the tube.

After the 1 psia helium atmosphere was established in the capsule, about 100 amperes of electrical current were passed from an electrode through the evacuation tube, causing it to heat and fuse the silver solder into the annulus surrounding the plug, thus effecting a seal.

A problem in leak checking the fused plug seal was posed by the fact that a massive leak at the plug would permit all the contained helium in the capsule to be rapidly pumped out before the leak detector could be brought into action. It is obvious that this would make accurate leak checking impossible. This uncertainty was

FIG. IV-17-3. Capsule Evacuation Tube and Sealing Device. ANL Neg. No. 112-1908A.

avoided by exposing to the seal, through the manifold, a small tank of helium at atmospheric pressure. Any large leak would then allow helium to flow from the tank into the capsule, which was at sub-atmospheric

pressure. Since the small tank had a volume approximately equal to that of the capsule, any transfer of gas would show up as a loss of pressure on the small tank gauge.

FIG. IV-17-4. Schematic Layout of Evacuation and Leak Detection System. ANL Neg. No. 112-5495 Rev. 2.

When no leak was revealed by this technique for finding large leaks, the helium leak detector was valved to the capsule. Since it was then certain that a helium atmosphere was trapped within the capsule, a minute leak at the seal would result in some of the contained helium escaping into the leak detector upon further testing.

With completion of leak testing the capsules were

ready for shipment to TREAT for performance of the transient experiment.

REFERENCES

1. C. E. Dickerman, E. S. Sowa, D. Okrent, J. Monaweck and L. B. Miller, *Studies of Fast Reactor Behavior Under Transient Heating to Failure. I. Initial Experiments on Metallic Samples in the Absence of Coolant*, ANL-6334, 19 (1961).

IV-18. Fast Neutron Transient Detection Hodoscope For Reactor Fuel Meltdown Studies

A. DE VOLPI and C. H. FREESE*

As part of the nuclear reactor safety program, fission fuel elements are tested at the Transient Reactor Test Facility (TREAT) under conditions leading to intentional meltdown. The reactor is programmed into a fast-rising period resulting in a high-flux transient lasting tens to hundreds of milliseconds. This power level is sufficient to cause test assemblies located at the outer periphery of the core to melt or rupture. Although in the past high speed color photographs have been obtained

for dry fuel pins, placement of a fuel assembly in a liquid sodium environment precludes visible spectrum observation.

As a natural adjunct in instrumentation for the test reactor TREAT, a hodoscope sensitive to penetrating fast fission neutrons has been designed. Discrete portions of an imaginary 16×20 matrix (drawn on a 2-in. horizontal by 20-in. vertical area centered on the fuel plane) are scanned by individual fast neutron detector channels (See Figs. IV-18-1 and IV-18-2). This

* Electronics Division.

FIG. IV-18-1. TREAT 334-Channel Fast Neutron Hodoscope. ANL Neg. No. 112-5610.

FIG. IV-18-2. Electronic Processing and Photographic Readout Assembly. ANL Neg. No. 112-5609.

FIG. IV-18-3. Electronic Processing Printed Circuit Card Schematic. ANL Neg. No. 143-1883.

FIG. IV-18-4. Diagram of Detection, Processing, and Display System for Fast Neutron Scanner. ANL Neg. No. 143-1882.

hodoscope, located outside the reactor core, 13 ft. from the test pin, is able to record the movement of fuel with a 0.15-in. horizontal resolution.

There are three basic elements of the hodoscope system: the collimator, the detection and electronic processing, and the readout system.

The collimator consists chiefly of thirty tapered steel plates assembled alongside each other. The taper provides the necessary horizontal divergence. By machining alternate grooves at angles down the length

of each plate, vertical divergence was introduced. These design features permit the placement of sufficient detectors on 1 in. centers at the rear of the collimator matrix while focal points at the fuel pin are separated by only 0.15 in.

Instrumentation was optimized with regard to cost. The "Hornyak button" scintillation and photo-multiplier high voltage are carried by a single coaxial cable. Each neutron signal is translated to a digital pulse in a circuit card (Fig. IV-18-3), which contains

low impedance pulse current input, feedback-stabilized amplifier, tunnel-diode discriminator, and shaping and driving stages, as illustrated in Fig. IV-18-4.

Ultimately 320 channels with possible instantaneous count rates of 10^6 pulses/sec in each channel would approach a total information rate of 10^7 counts/sec. For a detailed history of the event, it is necessary to identify each count with two parameters: location and time. Since the time span of an experiment is of the order of seconds, i.e., from the initiation of the neutron burst to the meltdown of the fuel element, a time resolution of the order of a millisecond is desirable. Existing multi-channel two-parameter analyzers cannot handle this duty factor and at the present time, digitized computer processing would be unnecessarily expensive.

The hodoscope display now being used is relatively inexpensive and fully able to accommodate the experi-

mental requirements. Each of the 320 channels has a digital output which gates into conduction a 6977 fluorescent vacuum tube. The standardized light flash, on for several microseconds, has sufficient intensity to provide an image well enough above fog on Kodak Royal-X Pan or 2475 Recording Film to be registered as a single event. A shutterless fast-framing camera with 98 % open time views the entire array of 320 lamps. With a picture rate of 1000/sec, each 16-mm frame then contains the accumulated number of flashes in each of 320 potential positions for 1 msec. A detector channel with a high count rate may yield several superimposed exposures on one location, while a detector channel with a low count rate may have none on a particular frame. Development conditions and framing rate are optimized to achieve some linearity in density.

IV-19. Remotely-Controlled Camera Stage for Hot-Cell Work

C. AUGUST

Meltdown experiments in TREAT with pre-irradiated fuel specimens necessitated the development of a variety of hot-cell equipment for the remote assembly of test capsules designed for exposure to reactor transients, disassembly of the capsules after exposure, and the post-transient examination of meltdown residues. Most of the required operations could be performed satisfactorily utilizing the handling and positioning capability offered by mechanical cave manipulators directly, or with appropriate add-on power assists and tooling refinements. However, during preliminary tests, it became obvious that the fragile, often finely-divided, and position-sensitive residues from meltdown experiments could withstand little "handling" without introducing further uncertainties in the data obtained from visual examination.

During post-mortem examinations of the fuel specimens, detailed photographic records of the fuel residues are obtained through use of a wall-penetrant hot-cell camera, similar to that described in Ref. 1. This device is a fixed-plate, 5×7 , limited-field view camera which yields photographs of $1\frac{1}{2}$ or $3\times$ image magnification. Focus is achieved with an elevating specimen stage, which is cantilever mounted on the inner wall of the cave. The stage is cable-driven vertically by a small capstan outside the cave. Photographing the 46 cm long typical fuel specimen previously required frequent repositioning of the sample with the cave manipulators

to allow consecutive viewing of its length with the limited-field camera. This resulted in continual disturbance of the fuel residue.

An auxiliary table to be used in conjunction with the vertical focusing stage, with the capability of remotely-controlled linear scanning motion in the horizontal plane along the X and Y axes, was required to alleviate the positioning and fragility difficulty and to facilitate obtaining coherent composite images. Design criteria were based upon the load capacity of the focusing stage; the need for smooth vibrationless motion controlled from outside the cave; and the maximum area occupied by the fuel sample, holder and residue under examination. The scanning table was to be as low as possible, have a maximum completed weight of 16 kg, a minimum capability of 56 cm longitudinal and 10 cm transverse travel, and a table load capacity over the full travel of 2.25 kg. The device was to be 110-volt ac powered, remotely-controlled, self-contained and semi-portable. In addition, since the table was intended for use within a hot cell, it was to be as free of contamination and entrapment of foreign material as possible, with maximum convenience of remote replacement of major components.

A traverse speed of 1 in./sec was selected as an optimum that could be adequately controlled remotely while providing a full traverse in minimum time. Standard Bodine motors equipped with all-metal slip-

latches and driving roller chains through sprockets were chosen as table drives because of their proven in-cave capabilities. Self-enclosing "V"-ways with lightly preloaded cam support roller bearings in treble mountings were used for the linear transfer members, to minimize platform drag while retaining maximum load capacity with least play and chatter.

Type 6061 aluminum was employed as structural material throughout the assembly. Longitudinal and transverse "V"-ways were brake-formed of the 0.3175 cm-thick stock used extensively. The longitudinal "V"-ways were supported on commercial 7.62 cm aluminum channels mounted on a 0.476 cm-thick base plate, resulting in a structure which was very rigid despite its low weight. The total weight of the assembled unit was 17.7 kg, exceeding the design value by $\approx 10\%$. This required only a slight adjustment of the focusing stage counterweights to bring the unbalanced load within the working range of the capstan.

Bat-type, momentary contact toggle switches located on an out-of-cave control box allow reversing scan direction with maximum response and execution of very slight incremental movements. Electrical egress

from the moving platform which houses the motor drives was obtained through a replaceable self-supporting Coiled Kord, tensioned to an electrical dulkhead connector fixed to the base structure of the table. This point was coupled with the operator control box by a common intercom cable inserted through a cave instrumentation port.

The length of unshielded roller chain, fixed as an "open" rack or ladder track, (readily subject to contamination), and the unitized motor-drives, (accessible after removal of the tray-top and the floating scatter shield), are remotely replaceable with the cave manipulators.

The unit performs the desired transversing motions very smoothly, without discernible coasting or over-travel, and gives sufficient control to closely position samples even when viewed under the higher ($3\times$) camera magnification.

REFERENCES

1. W. B. Doe, *Low Power Stereo Camera*, Proc. Fifth Hot Laboratory and Equipment Conference, Philadelphia, 1957 (Pergamon Press, New York, 1957) pp. 157-159.

IV-20. Improvements in Absolute Neutron Detection Techniques

A. DE VOLPI and K. G. PORGES

Absolute detection of high energy neutrons (such as fission sources) is strongly associated with the "manganese bath" system. In the course of a number of such calibrations, particularly in connection with the absolute determination of $\bar{\nu}$, the average neutron yield in fission of U-235 and Cf-252,⁽¹⁾ a number of improvements in various aspects of manganese bath assaying have been made. The changes relate not only to the bath itself, but also to the associated radioactivity counting and data processing. As indicated below, the work is partly described in several publications and more recent reports in the process of publication.

Much attention has been paid to precision, i.e. reproducibility of data over long periods of time (years). This appears to have been achieved within the limits of the existing design. For example, RMS spread, in the counting of two-liter samples of Mn-56 activity when compared with a 4π beta-gamma coincidence system, now is in the range of 0.4% over a period of the last half year, and has been about 0.67% for over two years.

These results have been obtained in a two stage counting facility² which permits accurate counting of weakly

activated solutions of $\text{Mn}^{56}\text{SO}_4$ drawn from an integrating neutron detector of the "manganese bath" type. Two-liter aqueous solutions are counted directly in a gamma-gamma coincidence arrangement which features relatively high efficiency and long-term stability. An electronic delay, mixing, splitting, and repeating technique is used to allow full utilization of radiation detected in both NaI(Tl) crystals while only two single channel analyzers are required. This system is calibrated on an absolute scale through another unit which counts small aliquots of a strongly activated sample mixed with liquid scintillator in a 4π beta-gamma coincidence mode. Signals from two high-gain, low-noise photomultiplier tubes viewing the same scintillation cell are put into coincidence to reduce thermionic photocathode noise below background level for room-temperature operation. Automatic punched-card readout with subsequent computer analysis is available. Otherwise incompatible requirements of convenience, stability, efficiency and accuracy are satisfied by using this two-stage process of direct counting backed up by calibration. The gamma-gamma

counter has twice the efficiency of ordinary gamma-gamma coincidence systems. Single-channel efficiencies for Mn-56 are 4% at 1.9 cps background and 9% at 1.5 cps background; coincidence efficiency is 0.2%. While intended primarily for precision detection of Mn-56, the system is readily adaptable to other radioisotope counting requirements.

The question of accuracy is a more difficult problem, since that requires an absolute standard of comparison. When dealing with uncertainties in the order of 1 or 2%, even the basic standards are subject to question. In the case of neutron source strength, there are available non-decaying sources which have been calibrated at several laboratories around the world. The NBS-II radium-gamma-beryllium source was loaned to us for nearly a year, during which time we compared the published values with those obtained under various conditions of operation. A further test of our equipment and procedures was a direct comparison of 2.5-h Mn-56 assays with the Natural Bureau of Standards through air shipments to them of samples from solutions counted at Argonne on the same day.

Indicative of the supporting work required in an over-all determination of neutron source strength is the following measurement: Since the manganese concentration enters directly into source strength calculations, a simple and accurate method of obtaining the MnSO_4 content is required. Determining solution density at a standard temperature fulfills such objectives. A polynomial representation of the concentration as a function of the solution density at 30°C has been obtained along with a graphical display which compares experimental data.³

In order to accurately and expeditiously process the large quantities of data produced in the various manganese bath activations and the supplemental tests required for stability and accuracy checks, the computer program COINC⁴ has been developed. The code may be used to reduce most types of coincidence counting data, including 4π beta-gamma and gamma-gamma. Input variables include particulars of identification, run time, clock times, total counts in the single and coincidence channels, and normalizing sample volume; the fixed system parameters are deadtimes and coincidence gate times for each channel, background rates, decay constants, and associated standard errors. The code computes corrected single channel and coincidence rates, specific disintegration rates at "zero" reference time, efficiencies, error limits, and a weighted mean

when more than one case is run at one time. The program is used with either manual or automatic data transcription. This FORTRAN-II code has been adapted to the IBM-704 and DDP-24.

As an outgrowth of the search for improved counting techniques, it was found that up to 40% efficiency can be obtained in detecting Mn-56 activity by Cherenkov radiation generated in a 10-ml dilute aqueous solution of MnSO_4 (with background about 1.2 cps). Data for higher concentrations indicate that at 300 g/liter total counting efficiency is 32%. Comparisons have been made with standard liquid scintillator techniques, demonstrating that Cherenkov counting allows the assay of larger quantities of inorganic salts but at lower efficiencies. The direct counting of 2-mil manganese foils provides as much as 86% beta efficiency, when the foil is inserted between two pieces of plastic scintillator. All these methods of counting have been carried out in a 4π two-tube, room temperature counting system.⁵

There are some additional features of the manganese bath system which tend to expand its usefulness outside the area originally intended. A $\text{Ra}(\alpha,n)\text{Be}$ source is being calibrated for possible use at Argonne. An independent determination of the manganese-to-hydrogen $1/v$ cross-section ratio is being obtained.

A recent experiment to determine alpha for a thermal core using an oscillating technique made use of both the neutron source determination and precision radioactivity capabilities of the system: a Cf-252 source was counted directly in the bath (and compared with the NBS standard), and manganese foils were calibrated in the 4π beta-gamma coincidence system. Calibrations and independent cross checks on the 4π beta counting system of the Argonne Reactor Physics Radiochemical Laboratory have been carried out.

REFERENCES

1. A. De Volpi and K. G. Porges, *Fission Neutron Yield Experiment*, Reactor Physics Division Annual Report July 1, 1963 to June 30, 1964, ANL-7010 (1965) p. 13.
2. K. G. A. Porges, A. De Volpi and R. N. Larsen, *Electronic Design of an Absolute Counting System for Mn-56*, Nucl. Instr. Methods **29**, No. 1, 157-169 (1964).
3. A. De Volpi, R. J. Armani and K. G. Porges, *Determination of Manganese Sulphate Content for Neutron Source Measurements*, J. Nucl. Energy **19**, 597-599 (1965).
4. A. De Volpi, K. G. Porges and G. Jensen, *Computer Code for Reduction of Coincidence Counting Data*, Intern. J. Appl. Rad. Isotopes (to be published).
5. A. De Volpi and K. G. A. Porges, *Cherenkov Counting of Aqueous Solutions*, Intern. J. Appl. Rad. Isotopes **16**, 496-498 (1965).

IV-21. Mn-55 and Co-59 Resonance Detectors

R. A. KARAM

Resonance detectors Mn-55, Co-59, and Na-23 provide spectral indices in the neutron energy range 1–100 keV. Manganese has several resonances in the energy range 350 eV to about 100 keV. Cobalt has two relatively large resonances at 4.3 and 5.0 keV and several small resonances between 5 and 50 keV. Sodium has one large resonance at 2.85 keV. The difference in activation between the outer and inner foils of several stacked foils gives the ratio of resonance to non-resonance activation. These detectors complement the spectral index measurements obtained by fission ratios and thus provide information about the lower portion of the neutron energy spectrum. The results reported here are preliminary and the technique is still under development.

Manganese foils of 98% purity (1% Cu and 1% Ni) were prepared by the Metallurgy Division for these experiments. The first experiment was designed to determine whether self shielding could be seen in a fast spectrum as typified by ZPR-6 Assembly No. 3.⁽¹⁾ Four foils with thicknesses of 1, 5, 10 and 20 mils respectively were irradiated at the center of the core for about 150 watt hours. The relative specific activity of each foil was determined by counting gamma rays with the counting system described by K. Plumlee and M. Wiggins.² By counting gammas of 0.822 MeV or greater, the gammas from the activation of Cu and Ni were not counted; the need for correcting the results was thus eliminated. The results of the first experiment are shown in Fig. IV-21-1. These results showed that self shielding in Mn-55 does exist.

FIG. IV-21-1. Manganese Self-shielding in ZPR-6, Assembly No. 3. ANL Neg. No. 112-5462.

FIG. IV-21-2. Manganese Activation in a Stack of Foils (Left Half). ANL Neg. No. 112-5460.

FIG. IV-21-3. Manganese Activation in a Stack of Foils (Right Half). ANL Neg. No. 112-5461.

The second experiment with manganese consisted of stacking together 39 foils of $\frac{1}{2}$ in. diameter. This stack comprised a central section consisting of nineteen 10-mil foils and two outer layers, one on each side of the central section, consisting of ten 1-mil foils each. The stack was irradiated at the center of ZPR-6 Assembly No. 3, in the void space ($2 \times 2 \times 1$ in.) provided for the samples used for the central worth measurements. The stack of foils was irradiated for about 10 watt hours. After the irradiation the foils were reduced in size from $\frac{1}{2}$ in. to $\frac{3}{8}$ in. diameter so that the edge effect on the inner foils was eliminated. The foils were counted in the manner described above and the results are shown in Figs. IV-21-2 and IV-21-3. Figure IV-21-2 shows the results of half the foils that were facing the void space. Figure

IV-21-3 shows the results of the other half of the foils, located next to the core materials at the center. The two curves in Figs. IV-21-2 and IV-21-3 are symmetric except in the region where resonance activation is present. This indicates that there were more resonance neutrons in the void space than near the core materials.

The experiment with Co-59 foils (99.9% pure) also showed definite resonance self shielding. The activation of Co-59, however, requires higher power levels than needed for Mn-55 because of the long half-life of Co-60. Consequently, the activation of Co-59 at a power level of 10 watts for 1 hour did not produce good statistics. This experiment will be repeated at higher power levels.

Sodium in the form of carbonate can also be used in a similar manner to give a spectral index at about 3 keV.

A search for a suitable compound of sodium for this technique is underway.

The results reported here clearly show that resonance detectors can be used for spectral indices in the low-energy portion of the neutron spectrum. Knowledge of the resonance parameters associated with such detectors makes the comparison between the measurements and calculations possible.

REFERENCES

1. F. H. Helm, W. Y. Kato, G. W. Main, H. H. Meister and G. K. Rusch, *Critical Facility Measurements of Sodium-Void Coefficient in a Large Disc-Shaped Fast Reactor Core*, Trans. Am. Nucl. Soc. **8**, No. 1, 245 (1965).
2. K. E. Plumlee and M. T. Wiggins, *Automatic Foil Activity Counting Facility and Data Reduction Program*, ANL-6628 (1962).

IV-22. Design Improvements and Sensitivity Tests of the FERD System for EBR-II

K. G. PORGES

The Fuel Element Rupture Detector (FERD), now installed in EBR-II, has been described previously¹ in general outline. In this report a brief account is given of the accrued maintenance experience which led to certain improvements as well as of a recent test to which the FERD system and confirming charged wire detector were subjected.

DETECTOR AND AMPLIFIER UNITS

Except for occasional tube and transistor replacement, detection electronics as well as detectors have not failed since having been turned on. The installation now includes three different types of detectors, two of which may eventually be replaced with the type demonstrating the best endurance record. If none of the detectors fail during the next year, it will be more practical to choose the type which has the highest detection efficiency, namely, a high-pressure BF₃ counter.

INFORMATION PROCESSING SYSTEM

This unit, installed in the control room, writes input counts in four channels on magnetic tape which, after a slight delay, is read out upon demand into a fast count rate meter and oscillograph. A number of design changes in the tape deck were required to allow trouble-free operation for a reasonable period and to reduce mechanical tape flutter (which might mask a weak

signal). To effect longer operation, a number of tape guides were replaced with rollers turning on precision bearings, and a new type of self-lubricated tape was put into use; to reduce flutter, a mechanical analogue of an "RL" filter was incorporated in the unit which now appears to perform satisfactorily.

SIGNAL STRENGTH TEST

Recently a test of all EBR-II cladding failure detection systems was made.² This consisted of introducing a single bare fuel pin attached to control rod No. 2 and running the reactor at 10 MW; the pin was then cycled in and out of the reactor at approximately 8-min periods. Both the fission chamber and boron detector channels of the FERD system yielded a two-fold increase in count rate when the rod was inserted. The time dependence of this signal could not be conveniently checked in this test, since insertion of the source was necessarily slow. Gamma background, furthermore, was not as high as it would eventually become after prolonged operation of EBR-II at maximum design power; this background, however, appears to be relatively unimportant. A more serious background, the exact cause of which is now being investigated, appears to be due to neutrons other than delayed neutrons released by the bare fuel pin. A section of the record of a slow chart recorder used for this test is shown in Fig. IV-22-1.

Fig. IV-22-1. Signal Strength Test—Record of a Slow Chart Recorder. ANL Neg. No. 112-5381.

REFERENCES

1. K. G. Porges, *EBR-II Fuel Element Failure Detector*, Reactor Physics Division Annual Report July 1, 1963 to June 30, 1964, ANL-7010 (1965), p. 231.

2. R. R. Smith and C. B. Doe, *Cladding Failure Simulation Test on EBR-II*, ANL-7067 (to be published).

IV-23. Standardization of Radiochemical Methods for Measurement of Uranium Fission Rate and U-238 Neutron Capture Rate

R. J. ARMANI and D. M. SMITH

In the past there has been disagreement between calculated and measured values of the ratio of U-238 capture to U-235 fission in fast reactors. A program was instituted to determine if the cause of the disagreement was due to the measurement.

The measurement is made radiochemically using two uranium foils. One foil is highly enriched in U-235, and the other is normal uranium or slightly depleted in U-235. The number of fissions in each foil is determined by Mo-99 analysis. The number of U-238 neutron captures is determined in the normal or depleted foil by β -239 analysis. From these two analyses, the ratio of β/σ_f^{25} is calculated.¹

The two measurements depend upon absolute beta

counting techniques and the calibration of the counting system was a possible source of error. A systematic error was indicated, since the calculations were consistently 10 to 20 % higher than the measured values.

FISSION RATE DETERMINATION

A foil exchange was initiated among four laboratories as a starting point in checking the general ability to determine fission rates by Mo-99 analysis. Uranium foils were irradiated in the Argonne Fast Source Reactor (AFSR) and distributed to the various groups and a general agreement was obtained in the fission analysis to within $\pm 2\%$.⁽²⁾ Another set of foils was subsequently distributed and agreement was again within $\pm 2\%$.

Another method was used to check the fission rate analysis. A 93 % enriched U-235 foil, 0.025 cm thick, was taped to the outside surface of an absolute fission counter. U-235 of identical composition was electroplated on a 0.0125-cm stainless steel plate which served as the bottom of the fission counter. The uranium foil for radiochemical analysis was located on the opposite side of the stainless steel plate. The separation between the uranium foils was 0.0125 cm; thus it was assumed that the fission rate per gram in both foils was the same. The weight of the fission counter foil was known to be better than 1 %, so that the fission rate could be determined to approximately 1 %.

The counter-fission foil assembly was irradiated in a thermal neutron flux. In order to minimize flux depression in the foils, the entire counter-fission foil assembly was wrapped with 0.025 cm of 93 % enriched U-235. In turn this was covered with 0.050 cm of cadmium, so that the thermal flux was filtered out and therefore a fairly hard flux reached the uranium foils. The fission rate per gram was determined by the absolute fission counter. Radiochemical analysis of the fission foil yielded agreement with the fission counter data to within ± 2 %.

The results of the fission rate determinations compared to other laboratories and the absolute fission counter measurements indicated that there was no substantial error in the analysis. If any experimental error exists, it must lie in the U-238 neutron capture measurement.

U-238 CAPTURE RATE DETERMINATION

A foil exchange program was again considered as in the case of fission rate determinations. At that time, only Los Alamos was known to have had experience with this method. Unfortunately, Los Alamos was not able to engage in a foil interchange. Consequently, it was decided to recalibrate the beta counting equipment. The counting efficiency was determined for various distances from the sample to the end window proportional counter. At the same time, the counting sample preparation was re-evaluated.

The relative precision of the measurement was determined by irradiating a set of depleted uranium foils in a thermal neutron flux. The foils were placed on the periphery of an aluminum disc and rotated during irradiation, so that each foil received the same integrated exposure. The foils were analyzed radiochemically for Np-239 activity. The results showed a standard deviation of ± 3 %. It was anticipated that the precision should be better than ± 3 %. The most obvious source of inconsistency was poor counting sample preparation of the Np-239. A new preparation technique was adopted, in which the neptunium was electroplated out

TABLE IV-23-I. COMPARISON OF 4π -BETA COUNTING TO BETA-GAMMA COINCIDENCE COUNTING AND RESULTS OF 4π -BETA COUNTING OF TWO NUCLEAR CHICAGO STANDARD SOLUTIONS

Isotope Counted	4π - β Activity, (dis/sec)/ μ l	Compared Activity, (dis/sec)/ μ l	Source of Compared Activity	4π - β Activity Compared Activity
P-32	373.8	373	Nuclear Chicago standard solution	1.002
Co-60	12.62	12.4	Nuclear Chicago standard solution	1.018
Co-60	5.868	5.980	β - γ coincidence counting	0.981
Fe-59	103.4	103.9	β - γ coincidence counting	0.995
Mn-56	135.3	134.0	β - γ coincidence counting	1.010
Au-198	18.15	18.23	β - γ coincidence counting	0.996

TABLE IV-23-II. RESULTS OF THE THREE Np-239 STANDARDIZATIONS END WINDOW PROPORTIONAL BETA COUNTER II

Standardization No.	Efficiency, %
1	13.45 ± 0.17
2	13.30 ± 0.11
3	13.34 ± 0.12
Average	13.37 ± 0.08

of solution onto a platinum disc. The disc was flamed in a Meeker burner, resulting in a uniform and almost invisible sample. Foils were irradiated again on the rotating aluminum disc. The new standard deviation was found to be ± 1.1 %.

The end window proportional beta-counting system was restandardized for Np-239 activity determinations. The efficiency for samples deposited on 0.0127-cm thick platinum discs and counted at the top (first) shelf through an aluminum absorber with a density of 22.2 mg/cm² was determined to be $13.37\% \pm 0.60\%$. The efficiency was obtained by counting aliquots of a standardized Np-239 solution in the end window proportional beta counter.

A 4π -proportional beta counter was used to determine the activity of the Np-239 solution. The counting of radioactive standards and comparison of samples using the 4π -beta system and a beta-gamma counting system demonstrated that satisfactory results are obtained by 4π counting. The results of these comparisons are listed in Table IV-23-I.

Aliquots of the Np-239 solution were pipetted onto platinum discs with a drop of Ludox solution added to

TABLE IV-23-III. SHELF FACTORS FOR END WINDOW PROPORTIONAL BETA COUNTER II

Shelf	Shelf Factor
1	1.0000
2	3.132 ± 0.009
3	8.231 ± 0.045
4	17.54 ± 0.07
5	31.43 ± 0.26

TABLE IV-23-IV. $\bar{\sigma}_c^{238}/\bar{\sigma}_f^{235}$

ZPR-3 Assembly No.	Radiochemical Measurement (M)	Calculated (C)	M/C	Corrected Measured Value	Corrected M/C
5	0.087	0.102	0.85	0.107	1.04
6F	0.084	0.103	0.82	0.103	1.01
23	0.089	0.105	0.85	0.109	1.04
15	0.080	0.110	0.73	0.098	0.90
10	0.093	0.110	0.85	0.114	1.04
36	0.097	0.111	0.87	0.119	1.07
22	0.091	0.113	0.81	0.112	0.994
24	0.092	0.116	0.79	0.113	0.969
20	0.100	0.116	0.86	0.123	1.06
12	0.100	0.120	0.83	0.123	1.02
17	0.104	0.122	0.85	0.128	1.04
42	0.103	0.125	0.82	0.126	1.01
14	0.109	0.126	0.86	0.134	1.06
35	0.111	0.126	0.88	0.136	1.08
29	0.115	0.126	0.91	0.141	1.12
43	0.114	0.139	0.82	0.140	1.01
Average:			0.84		1.03

facilitate more uniform deposits. The samples were evaporated by use of an infra-red lamp rather than by use of a hot plate, also to facilitate more uniform deposits. Samples prepared by electroplating yielded counting rates and uniformity similar to the samples prepared by evaporation. This is explained by the fact that the Np-239 standard solution is carrier-free, while the Np-239 extracted from a foil used for measurement carries a small amount of residue with it. Thus, there is

a need for electroplating the Np-239 extracted from the foil used for measurement in order to get uniform, weightless samples.

Three separate standardizations were run with good agreement among the three efficiencies found. These results are listed in Table IV-23-II.

The ratios between the first shelf and the four other shelves in the end window proportional beta counter were determined by counting a Np-239 source at the five different shelf geometries. The inverses of these ratios, which are called shelf factors, are listed in Table IV-23-III.

An efficiency of 0.164 was previously used in contrast to the newly measured value of 0.1337 ± 0.006 . The recalibration was done under the same geometric conditions used when making the $\bar{\sigma}_c^{238}/\bar{\sigma}_f^{235}$ measurements. This indicates that previous values of this measurement should be multiplied by a factor of 1.227. Table IV-23-IV from W. Davey³ contains radiochemical data from a number of ZPR-3 assemblies. Comparison with calculations shows that the measured values are about 16% lower than the calculated values. Two additional columns have been added which contain the same data corrected by the factor of 1.227. The average ratio of measured-to-calculated value of $\bar{\sigma}_c^{238}/\bar{\sigma}_f^{235}$ was determined to be 1.03. This indicates that the disagreement between measurements and calculations was due to an error in the calibration of Np-239 counting equipment; this has been resolved.

REFERENCES

1. R. J. Armani, E. F. Bennett, M. W. Brenner, M. M. Bretscher, C. E. Cohn, R. J. Huber, S. G. Kaufmann and W. C. Redman, *Improved Techniques for Low Flux Measurement of Prompt Neutron Lifetime, Conversion Ratio, and Fast Spectra*, Proc. IAEA Symp. on Exponential and Critical Experiments, Amsterdam, September 2-6, 1963, Vol. 1, pp. 227-259.
2. R. J. Huber, Argonne National Laboratory (Private Communication).
3. W. A. Davey, *The Ratio of U-238 Capture and U-235 Fission Cross Sections in Fast Reactors*, ANL-7034 (to be published).

IV-24. Multichannel Analyzers

G. S. STANFORD

A survey of the state of the art of multichannel pulse analysis is presented in Ref. 1 and it is intended chiefly to bring the reader with little prior knowledge of the field. There is no attempt to treat any topic in depth. The general principles of analysis of pulse height and time

of flight are outlined, with consideration given both to single-parameter and multi-parameter instruments. Memory systems, analog-to-digital conversion, and readout devices are discussed. Some of the features and limitations of modern analyzers are noted, and typical

performance specifications are given. Although the main emphasis is on instrumentation, mention is made of some of the applications of single- and multiparameter analysis.

REFERENCES

1. G. S. Stanford, *An Introduction to Multichannel Analyzers*, Nucl. Instr. Methods **34**, 1 (1965) or ANL-6881 (April 1964)

IV-25. Incorporation of Non-Standard Input-Output Devices Into a FORTRAN System

C. E. COHN

The FORTRAN language provides statements for initiating the transfer of alphanumeric information from or to various standard input or output media (cards, tape, printer, etc.). Non-standard media (e.g., graph plotter, oscilloscope display) may be readily incorporated into the system, as will be described here.

Such a problem arose in conjunction with a digital incremental graph plotter used with our DDP-24 computer. Our objective was to provide a means by which FORTRAN programs could call for the writing of alphanumeric legends on the graph. Routines of this type for other computers¹ use a Hollerith argument in a CALL statement to specify the legend to be written. However, DDP-24 FORTRAN will not recognize or transmit Hollerith arguments, so an alternative method of specifying legends had to be found. The method chosen is actually superior in that it makes available the full range of FORTRAN conversion and format processing capabilities.

The output facilities in the DDP-24 FORTRAN system² were found to lend themselves very readily to this function. When an output operation is specified, the compiler generates a calling sequence to a setup subroutine. This in turn plants a reference to another subroutine for handling the output characters. (These two subroutines are normally assembled together and form a single entry in the library.) The variables in the output list are then converted according to the specifications of the applicable FORMAT statement. As each output character is generated by the conversion subroutines, it is transmitted in binary-coded form to the character output subroutine, which inscribes it on the desired output medium. (The entire operation is coordinated by an input-output executive routine that resides in upper memory. A "carriage return flag" indicates the end of a line. If the output medium requires the transmission of a complete line, the output subroutine stores the characters in a buffer area until a complete line is ready.)

The difference between various output media, then, appears only in the character output subroutines.

Therefore, one may incorporate a non-standard output medium into the system by the relatively simple expedient of writing a character output subroutine suited to that medium and inserting the subroutine into the FORTRAN library. This is the approach that we adopted by writing a subroutine that could accept a BCD code and draw the corresponding character on the graph.

In writing such a subroutine, one problem that must be considered is how to refer to the output operation in source programs. The compiler will recognize only certain prescribed output statement types, each one being associated with a specific entry name for the setup subroutine. To avoid having to alter the compiler, it is necessary to restrict oneself to this list selecting an output statement type that is not tied up for a standard use. Since our machine has no magnetic tape units, the WRITE OUTPUT TAPE statement is free and was selected for this purpose. The compiler provides for transmission of the tape number to the setup subroutine, but it is ignored by the present version. However, we have recommended to programmers that a tape number of zero be used to allow for possible future expansion of the machine. Since zero is an illegal tape number, systems with tape units could combine this subroutine with the normal tape output subroutine in such a way that a tape number of zero would give graph plotting and a legal tape number would give conventional tape output. Also, other nonstandard output media could be handled by branching on other illegal tape numbers.

The subroutine has been equipped with an additional entry (named SETGRAPH) by which it may be instructed as to the size of the drawn characters and the positioning of the legend on the graph. A CALL to this entry with the appropriate arguments is executed before the WRITE OUTPUT TAPE in order to transmit the needed information. On subsequent WRITE OUTPUT TAPE statements, the lines of output appear one under the other just as in conventional printed output, until another CALL SETGRAPH is executed.

In this FORTRAN system, input operations are performed in an analogous manner. Here a setup subroutine plants a reference to a character input subroutine. The latter subroutine is called by the conversion subroutines whenever they are ready to process another character. It retrieves the next character from the input device (or from an input buffer area where an entire line is read in at one time). Here, inclusion of a nonstandard input medium could also be done in a straightforward manner.

FORTRAN systems for other computers handle

input and output in a similar way. However, in many systems, a single reference to a character input or output subroutine transmits a complete line of characters to or from a buffer area. This difference would not inhibit application of the techniques discussed here.

REFERENCES

1. California Computer Products, Inc., *Reference Manual, SCOOP Programming Systems for Digital Incremental Plotters*, (1963).
2. Computer Control Company, Inc., *FORTRAN II Compiler*, 1, Document No. 71-190A (August 1964).

IV-26. Calibration of Small Fission Chambers by the Coincidence Method

K. G. PORGES and A. DE VOLPI

Coincidence counting of fissions from a foil located inside a fission chamber (or gas scintillator) with prompt fission neutrons through an external detector is a convenient means of calibrating such fission chambers. These chambers then may be used for absolute fission rate measurements, as well as for measurements of various parameters (neutron multiplication and distribution, fission cross section, etc.) of the fission process. Because the finite thickness of the fissionable deposit reduces the detection efficiency for tracks nearly in the plane of the foil, while the angular distribution of neutrons emitted by the fission fragment is symmetric with respect to the fission track, one would expect a correlation factor to appear in the coincidence count. This factor may be shown¹ to amount to

$$F(E_f, E_n, \hat{k} \cdot \hat{d}) = \sum_{j=\text{even}} A_j(E_f) B_j(E_n) G_j(M_d) P_j(\hat{k} \cdot \hat{d}) / (2j + 1) \quad (1)$$

where $\hat{k}$ is a unit vector normal to the foil, and $\hat{d}$ is a corresponding unit vector along the axis of the cylindrical neutron detection which subtends a solid angle $(1 - M_d)/2$ at the fission foil. A_j are coefficients of a Legendre polynomial expansion of the dependence of fission chamber detection efficiency at a channel bias E_f on the direction of the fission track with respect to the foil normal, and B_j are corresponding coefficients of the distribution of neutrons (detected at neutron channel bias E_n) with respect to the angle between fission track and neutron emission. The functions G_j are given by:

$$G_j M_d = \int_{M_d} P_j(\hat{n} \cdot \hat{d}) \frac{d(\hat{n} \cdot \hat{d})}{2} \quad (2)$$

In a coincidence calibration of a flat fission chamber, Eq. (1) must be evaluated through experimental measurements of neutron angular distribution and angular dependence of fission chamber detection efficiency. Alternatively, a neutron detector of 4π geometry may be employed causing the limits of the integrals G_j to become ± 1 , whence Eq. (1) reduces to

$$F = A_0 B_0 = e_n e_f \bar{p} [1 - (e_n/2\bar{p})(\bar{v}^2 - \nu)]. \quad (3)$$

Practical 4π neutron detectors are, however, relatively sensitive to gamma radiation, and require heavy shielding as well as fairly elaborate electronics.

Another approach, which was resorted to in the $\bar{p}$ experiment now in progress, is the development of the fission chamber in spherical form (or, more conveniently, in hemispherical form). This has the effect of another integration of Eq. (1), resulting in a coincidence efficiency

$$B_0 A_0 G_0 = e_n e_f \bar{p} (1 - M_d)/2. \quad (4)$$

A number of chambers of this kind, shown in Fig. IV-26-1, were fabricated with either stainless steel or aluminum electrodes, with wall thicknesses down to 0.015 in., diameters of about 1-in. and spacing of 2-3 mm. The curved part of the inner electrode was electroplated with U-235 or Cf-252; gas-flow operation as well as pressures up to 3 atm CH₄ resulted in flat and stable plateaux for count rates up to 10⁵/sec, using current pulse amplification.^{2, 3} Coincidence gates down to 20 nsec, whose width was largely dictated by neutron flight time, were used when anti-walk discriminators were inserted in both channels; the other channel in this case was a Hornyak button, a 58-AVP photomultiplier and a gamma discrimination circuit.⁴

FIG. IV-26-1. Hemispherical Fission Chamber. ANL Neg. No. 112-5378.

REFERENCES

1. K. G. Porges and A. De Volpi, *Note on the Calibration of Small Fission Detectors*, (submitted), Nucl. Instr. Methods.
2. C. J. Rush, *New Technique for Designing Fast-Rise Transistor Amplifiers*, Rev. Sci. Instr. **35**, 149 (February 1964).
3. A. De Volpi, K. G. Porges and C. J. Rush, *Subnanosecond*

RC Rise Times from Ionization Counters, Bull. Am. Phys. Soc. **9**, 46 (January 1964).

4. K. G. Porges, A. De Volpi and P. Polinski, *Fast Cancellation of Gamma Radiation Background Pulses in Hornyak Buttons with an Active Circuit*, Rev. Sci. Instr. **35**, 1602-1604 (November 1964).

IV-27. Solid-State Neutron Spectrometer

R. A. KARAM

Monochromatic neutrons interacting with hydrogen produce recoil protons with a uniform pulse-height distribution that extends from zero up to the energy of the neutrons. Polychromatic neutrons produce a superimposed sum of such uniform distributions. The derivative of the recoil proton spectrum with respect to energy multiplied by the energy gives a quantity proportional to $\phi(E)$, the flux per unit energy per unit area and time. This principle was applied for spectrum measurements by placing a hydrogenous film against the sensitive area of a surface barrier solid state detector. The solid state detector measures the spectrum of the recoil protons produced by the n - p reaction. The chief advantage of this method is that wall effects which are troublesome in proportional counters¹ for higher energy neutrons are minimized if not eliminated. Also the small size of such a device minimizes perturbations of the neutron spectrum in a reactor. The main disadvantage is that gamma rays produce high background.

FIG. IV-27-1. Schematic for Recoil-Protons Ballistics. ANL Neg. No. 112-5650.

In a beam geometry, where all the neutrons travel in one direction, the collection of protons approaches 4π geometry provided the irradiator is placed against the sensitive surface of the detector in such a way that the

isitive surface overlaps the irradiator. Also, the irradiator and detector should be perpendicular to the beam. In an isotropic flux situation, the collection of protons approaches 2π geometry. Consider Fig. IV-27-1. The angular distribution of protons in the laboratory coordinate system produced from neutrons striking the irradiator from direction A will be bounded by the hemisphere EG. Thus, all protons produced from neutrons travelling along A will be counted. Protons produced by neutrons coming from the opposite direction of A will not be counted because there is no detector to receive them on the other side. Therefore, only half of the protons produced by neutrons from both sides of direction A will be collected. Now consider the neutrons striking the irradiator from direction B. The angular distribution of the protons, in this case, will be bounded by the hemisphere DF. A fraction of the recoil protons will escape undetected through angle Ω' . The same fraction that is lost through Ω' , however, will be detected from neutrons incident along the C direction through angle Ω . Furthermore, the energy correspondence between the protons lost through Ω' and those detected through Ω is 1:1. Thus, it is seen again that only half of all the recoil protons will be detected. Analogously, it is seen that regardless of the direction of the incident neutrons only half of the recoil protons will be detected for the case of isotropic flux. It is assumed here that the solid state detector does not perturb the flux and that the fraction of neutrons incident on the edge of the hydrogenous film is small and can be ignored.

In case the flux is not isotropic, the above analysis will not be valid. The breakdown in the argument takes place because the fraction of recoil protons lost through Ω' and the fraction gained through Ω will not be equal. To circumvent this difficulty, one can use two solid state detectors with the irradiator placed between them and thus obtain 4π -geometry.

The experimental arrangement of irradiator and detector is shown in Fig. IV-27-2. The hydrogenous film was $500 \mu\text{g}/\text{cm}$ rubber hydrochloride. Two surface-barrier solid state detectors were used: one had a depletion depth of 750 microns at a bias voltage of 100 volts (this device was free of hydrogenous material) and the other had a depletion depth of 290 microns at 100 volts bias (this device was not free of hydrogenous material). The presence of hydrogenous material in the structural components of a detector is undesirable because the use of such material can contribute appreciably to the background. The detector with a depletion depth of 750 microns (henceforth designated as detector A) was used in ZPR-6 Assembly No. 2 and the other detector (detector B) was used in the beam produced by a converter plate at the JUGGERNAUT reactor.

FIG. IV-27-2. Arrangement of Detector and Irradiator. ANL Neg. No. 112-5617.

Detector A broke down prior to the measurement at the JUGGERNAUT. The arrangement of the converter plate and detector at the JUGGERNAUT is shown in Fig. IV-27-3.

Two runs were taken in each case: One run with the irradiator in place and one run without the irradiator. The difference between the two runs was considered as the recoil proton spectrum. The ratio of signal to background was about 15% at the lower energy end of the spectrum; this ratio was only 3-4% at the higher end. Figure IV-27-4 shows the neutron spectrum obtained at the center of ZPR-6 Assembly No. 2. The fission rate of U-238 was calculated from this spectrum and it was found to be 3.1×10^5 fissions/g/min. The measured fission rate of U-238 was 4.3×10^5 fissions/g/min.³ The calculated fission rate is based on the fraction of the spectrum shown in Fig. IV-27-4. Figure IV-27-5 shows the spectrum obtained from the converter plate at JUGGERNAUT. The calculated fission rate from this spectrum and the measured fission rate of U-238 were 1.31×10^4 fissions/g/sec and 1.15×10^4 fissions/g/sec, respectively.

The results reported here are preliminary in nature. It was anticipated that the gamma-ray background would not be appreciable above 100 keV; however, such was not the case. The pulse-height spectrum background caused by gammas seemed to be quite high—up to 500 keV. It is conceivable that two or more “knocked off” electrons by gamma rays combine to produce one pulse. For these types of phenomena, thinner solid state detectors would be more suitable. On the other hand, if a detector had a depletion depth of less than 300 microns, recoil protons above 6 MeV would pass through and the device would become a dE/dx detector. Consequently, the pulse height would not be proportional to energy in the same manner as in an E detector. Thus, the simple conversion from proton recoil pulse-height spectrum to neutron spectrum would become more complicated. The background spectrum

FIG. IV-27-3. Arrangement of Converter Plate and Detector. ANL Neg. No. 112-5624.

FIG. IV-27-4. Neutron Spectrum Taken At Center of Assembly No. 2, ZPR-6. ANL Neg. No. 112-5615.

above 500 keV is also higher than one might expect from fast neutron interactions with the detector materials. The signal-to-background ratio in detector A (750 microns depletion depth) was lower than that of detector B (290 microns) even though detector B has some hydrogen material in the form of epoxy in its

structural material. This seems to indicate that gamma rays are the major contributor to the background.

The resolution was considered to be 10 %. The results reported by G. Dearnaley and A. Whitehead² indicate that 10 % resolution is reasonable.

The neutron beam from the converter plate of the

FIG. IV-27-5. Neutron Spectrum From Converter Plate At Juggernaut. ANL Neg. No. 112-5616.

JUGGERNAUT was slightly divergent. The decrease in neutron intensity over a distance of 12 in. was a factor of 2. Thus, under such conditions, the analysis of geometric effects is not strictly valid. The shape of the spectrum from the converter plate is not that of a fission spectrum. It is not known whether the materials around the converter plate are responsible for the shift from the fission spectrum.

Power fluctuations from run to run are certainly a source of error which can appreciably affect the results. No special effort was made to insure the constancy of the power level between runs other than reproducing the same bucked current on the ionization chamber (channel No. 3) in ZPR-6. The precautions with respect to the reproducibility of power level between runs were less stringent with the JUGGERNAUT since there were no bucking provisions in the monitoring equipment.

Although the calculated fission rates in U-238, based on the measured spectra in the center of ZPR-6 Assembly No. 2 and from a converter plate in the JUGGERNAUT, are in fair agreement with the measured

fission rate, one must not overlook the preliminary nature of the results. The indication that the spectrum in ZPR-6 is softer than the spectrum calculated by a 16-group one-dimensional diffusion code (RE 122) is also in agreement with the findings of E. Bennett.³

It appears that this method of spectrometry would be suitable for fast neutrons having energies above a few hundred kilovolts. Optimum conditions for carrying out spectral measurements with this technique will be investigated in the future. The measurement of the spectrum of neutrons from a Pu-Be source seems to be the next logical choice because, with such a source, the gamma background is very low.

REFERENCES

1. E. F. Bennett, "Proportional Counter Proton-Recoil Spectrometer with Gamma Discrimination", *Rev. Sci. Instr.* **33**, 1153-1160 (1962).
2. G. Dearnaley and A. B. Whitehead, "The Semiconductor Surface Barrier for Nuclear Particle Detection", *Nucl. Instr. and Methods*, **12**, 205-226 (1961).
3. E. F. Bennett and J. Haugsnes, Argonne National Laboratory (Private Communication).

IV-28. Note on the Reduction of Rossi-Alpha Data

C. E. COHN

In Rossi-alpha experiments where the decay of the prompt-neutron chains may be adequately described by one exponential, the data are commonly fit to a function consisting of a decaying exponential term and a constant term. It will be shown here that such a function is not exact, but is a good approximation in most experimental situations.

For the common type of time analyzer which records a maximum of only one event per sweep, it is convenient to do some preliminary data reduction before attempting a fit. Starting with the raw channel counts, C_i , one obtains P_i , which is an estimate of the probability that a sweep proceeding as far as channel i will terminate with an event in channel i , by use of the formula

$$P_i = C_i / \left(N_0 + \sum_{j=i}^n C_j \right). \quad (1)$$

Here n is the total number of channels and N_0 is the number of sweeps during which no events at all are recorded. The denominator is thus the total number of sweeps not terminating before channel i .

Now, each sweep proceeding as far as channel i can be considered as one trial in a biased coin-tossing game in which a fraction P_i of the trials result in a "head"; i.e., the recording of an event in channel i and termination of the sweep. Therefore, the process is governed by binomial rather than Poisson statistics, and the value of P_i computed from the above formula has associated with it an estimated standard deviation σ_i given by

$$\sigma_i = \sqrt{C_i(1 - P_i)} / \left(N_0 + \sum_{j=i}^n C_j \right). \quad (2)$$

In actual practice, the P_i are very small so that the result is practically equivalent to the Poisson prediction.

The theory of the Rossi-alpha experiment,¹ developed using one-group point-reactor kinetics, yields the expression

$$p(t)\Delta t = (A + Be^{-\alpha t})\Delta t. \quad (3)$$

Here $p(t)\Delta t$ is the probability of observing an event in the small time interval from t to $t + \Delta t$, given that there has been an event at $t = 0$ and irrespective of occurrences at intermediate times. As usual, it is assumed that Δt is sufficiently small that the probability of more than one event in the interval is negligible.

Now, Eq. (3) is not in the form where it may be used

to describe the results of measurements done with the type of time analyzer described above. However, an expression in the proper form may be obtained through a derivation which is identical to that used to derive the Poisson distribution, except for the second term in the parentheses on the right-hand side of Eq. (3).

Let

$$P(0, \Delta t) = 1 - (A + Be^{-\alpha t})\Delta t, \quad (4)$$

which is the probability of observing no event in the time interval Δt beginning at t . If $P(0, t + \Delta t)$ is the probability of having no event in the time from $t = 0$ to $t + \Delta t$ following an event at $t = 0$, then

$$P(0, t + \Delta t) = P(0, t)P(0, \Delta t) \quad (5)$$

where $P(0, t)$ is the probability of no events up to time t following an event.

Substituting Eq. (4) into Eq. (5) and rearranging yields

$$\frac{P(0, t + \Delta t) - P(0, t)}{\Delta t} = -(A + Be^{-\alpha t})P(0, t) \quad (6)$$

and passing to the limit of small Δt , using the definition of the derivative, yields

$$\frac{dP(0, t)}{dt} = -(A + Be^{-\alpha t})P(0, t). \quad (7)$$

The solution of this differential equation is

$$P(0, t) = \exp \left[-At - \frac{B}{\alpha} (1 - e^{-\alpha t}) \right], \quad (8)$$

given that $P(0, t) = 1$ for $t = 0$.

Now consider a Rossi-alpha experiment with a total number of sweeps M . Also, consider channel i of the time analyzer, which begins at $t = T_i$ and has width Δt . The total number of sweeps not terminating before channel i is then

$$MP(0, T_i). \quad (9)$$

Now, the probability that a sweep will terminate in the small time interval from t to $t + \Delta t$ is the probability of no events from time zero to t , multiplied by the probability of an event during Δt . Hence, the total number of sweeps so terminating is given by

$$MP(0, t)p(t)\Delta t. \quad (10)$$

The channel count C_i then is given by expression (10) summed over all time intervals included in the channel width. In the limit of small Δt ,

$$\begin{aligned} \mathcal{C}_i &= M \int_{T_i}^{T_i+\Delta T} P(0, t) p(t) dt = M \int_{T_i}^{T_i+\Delta T} \\ &\cdot \exp \left[-At - \frac{B}{\alpha} (1 - e^{-\alpha t}) \right] (A + B e^{-\alpha t}) dt \\ &= M \left(\exp \left[-AT_i - \frac{B}{\alpha} (1 - e^{-\alpha T_i}) \right] \right. \\ &\quad \left. - \exp \left\{ -A(T_i + \Delta T) - \frac{B}{\alpha} [1 - e^{-\alpha(T_i + \Delta T)}] \right\} \right). \end{aligned} \quad (11)$$

Eq. (1) indicates that P_i is given by Eq. (11) divided by Eq. (9), whence

$$P_i = 1 - \exp \left[-A\Delta T - \frac{B}{\alpha} e^{-\alpha T_i} (1 - e^{-\alpha \Delta T}) \right]. \quad (12)$$

Since the common type of time analyzer uses a clock oscillator and scaler to count the channels, the widths of all channels are inherently identical. Therefore, the following quantities which are independent of channel number may be defined:

$$C_1 = e^{-A\Delta T} \quad (13)$$

and

$$C_2 = \frac{B}{\alpha} (1 - e^{-\alpha \Delta T}),$$

giving

$$P_i = 1 - C_1 \exp(-C_2 e^{-\alpha T_i}), \quad (14)$$

which is evidently quite different in form from the aforementioned function commonly used to fit such data. However, the first exponential in Eq. (14) may be expanded in a Taylor's series to yield

$$P_i = 1 - C_1 + C_1 C_2 e^{-\alpha T_i} - \frac{1}{2} C_1 C_2^2 e^{-2\alpha T_i} + \dots \quad (15)$$

Here the first three terms are equivalent to the commonly-used function, and subsequent terms are corrections. If $C_2 \ll 1$, as is normally the case, the correction terms are small and the commonly-used function is a good approximation. Therefore, most runs yield almost identical results when fitted either way.

The effect of various perturbations on the fitting process may now be explored. First, consider the effect of a finite fixed time delay T_0 between the initiating event and the actual commencement of the sweep. Here, the quantity T_i in Eqs. (11) and (12) must be replaced by the quantity $(T_0 + T_i)$. The effect of T_0 may then be fully accounted for by adjoining a factor $\exp(-\alpha T_0)$ to the expression for C_2 in Eq. (13), leaving the form of Eq. (14) unchanged.

The analysis becomes somewhat more complicated if this initial time delay is variable. Such a situation arises when the channel boundaries are defined by pulses from a continuously-running clock oscillator.

Thus, after an initiating event is recorded, the analyzer must wait for the next clock pulse to define the beginning of the first channel. Since there is no correlation between clock pulses and neutrons, the resulting time delay T' may take on any value between zero and T'_0 , which is the period of the clock oscillator. If this period is equal to the channel width, then $T'_0 = \Delta T$. If, on the other hand, the oscillator runs faster and is scaled down to count the channels, and if the scaling circuitry is reset after each sweep so that the first clock pulse after an event initiates the first channel, then T'_0 will be smaller than ΔT by the scaling ratio.

The probability that a sweep with initial time delay between T' and $T' + \Delta T'$ will terminate with an event in channel i is

$$P_i(T') = 1 - C_1 \exp[-C_2 e^{-\alpha(T_i + T')}] \frac{\Delta T'}{T'_0} \quad (16)$$

since any value of T' within its range is equally probable. Summing over all values of T' and passing to the limit of small $\Delta T'$ yields

$$P'_i = 1 - \frac{C_1}{T'_0} \int_0^{T'_0} \exp[-C_2 e^{-\alpha(T_i + T')}] dT', \quad (17)$$

which is the probability that a sweep will terminate with an event in channel i irrespective of the value of T' . The result of the above integration must be expressed as an infinite series,² so that

$$\begin{aligned} P'_i &= 1 - C_1 + C_1 C_2 e^{-\alpha T_i} \left(\frac{1 - e^{-\alpha T'_0}}{\alpha T'_0} \right) \\ &\quad - \frac{C_1 C_2^2}{2} e^{-2\alpha T_i} \left(\frac{1 - e^{-2\alpha T'_0}}{2\alpha T'_0} \right) + \dots \end{aligned} \quad (18)$$

Each term of this expression is equal to the corresponding term of Eq. (15) multiplied by a factor of the form

$$(1 - e^{-k\alpha T'_0})/k\alpha T'_0,$$

which is independent of T_i and approaches unity as $\alpha T'_0$ approaches zero. Hence, the time delay does not affect the fitting when $\alpha T'_0 \ll 1$, and in any case when the correction terms of Eq. (15) are negligible.

It has been suggested by R. Karam³ that the results of a Rossi-alpha measurement in a reflected reactor should be fitted with two exponential decay modes, in an expression of the form

$$p(t)\Delta t = (A + B_1 e^{-\alpha_1 t} + B_2 e^{-\alpha_2 t})\Delta t. \quad (19)$$

Through a derivation analogous to that developed here, it may be shown that the probability of a sweep terminating with an event in channel i may now be written as

$$P_i = 1 - C_1 \exp[-(C_{21} e^{-\alpha_1 T_i} + C_{22} e^{-\alpha_2 T_i})]. \quad (20)$$

Its form is thus analogous to that of Eq. (14) and the foregoing discussion is applicable.

REFERENCES

1. J. D. Orndoff, *Prompt Neutron Periods of Metal Critical Assemblies*, Nucl. Sci. Eng. **2**, 450 (1957).
2. H. B. Dwight, *Tables of Integrals and Other Mathematical Data*, (The Macmillan Company, New York, 1961), 4th Ed., p. 134, Formula 568.1. (Here $x = e^{-aT'}$ and $a = -C_2e^{-aT_i}$.)
3. R. A. Karam, *Measurement of Rossi-Alpha in Reflected Reactors*, Trans. Am. Nucl. Soc. **7**, No. 2, 283 (1964).

IV-29. Serial Klystron-Magnetic Bunching

A. B. SMITH

The 3.0 MeV pulsed accelerator employed in the time-of-flight work (see Paper IV-31) is equipped with a magnetic bunching system.¹ The unit has been operational for thousands of hours providing 1 nsec bursts with peak pulse intensities of $\lesssim 10$ mA. The system is the key to successful study of fast neutron inelastic scattering. Despite the high intensities available the measurements are tedious and thus an effort has been made to increase the pulsed neutron source intensity by a factor of about ten. Work towards this end is now well along. A Klystron bunching and chopping system has been ordered for the radio frequency ion course. The pulsed ion source will provide ~ 100 nsec proton bursts at ~ 1 mA current. These bursts are bunched in a Klystron cavity to an approximate 10 nsec pulse having a peak intensity of ~ 10 mA. The ion packets are then accelerated through¹ the potential established by the electrostatic generator (up to 3.0 MeV) and

focused at the existing magnetic bunching system. The latter again bunches by a factor of ten yielding ~ 1 nsec bursts with peak intensities of ~ 100 mA.

Portions of the above systems are now operational and the remaining portions are expected to be operational in the immediate future. Even allowing for a reasonable degradation of performance from design specifications, particularly during the initial "run-in" period, the available pulsed intensity will be very large compared with that obtained from the existing machine and with that obtained elsewhere. This greater intensity will appreciably improve the speed and quality of elastic and inelastic scattering studies.

REFERENCES

1. R. C. Mobley, *Proposed Method for Producing Short Intense Monoenergetic Ion Pulses*, Phys. Rev. **88**, 360 (1952).

IV-30. Multi-angle Time-of-Flight System

C. H. NELSON and A. B. SMITH

An increase in the experimental sensitivity available for time of flight work is the most desirable method of obtaining improved quality and quantity in experimental work (see Paper IV-29). Nearly as effective is an increase in detector sensitivity and system developments toward this end are now nearly complete. Previously, scattered neutron distributions were determined angle by angle using a single detector and usually ten different angular measurements were involved. Apparatus, now in the final stages of testing, will permit the determination simultaneously of scattered neutron

angular distributions at ten angles. The result will not only provide a much increased speed of data acquisition but also will relax the existing stringent source monitoring and accelerator stability requirements. The simultaneous acquisition of such information requires that ten three-parameter data systems be simultaneously operational. This requires reliability and information storage capacity available only with solid state components and on line computer units. All such requisite instrumentation is now operational. The basic concept of the ten angle system is indicated in Fig. IV-30-1.

FIG. IV-30-1. Schematic Diagram of 10-angle Time-Of-Flight System Including: a) Pulsed Beam, b) Ten Detectors, c) Computer System with Measured Spectra Obtained by Scattering 1200 keV Neutrons from Ag. Both Elastic and Inelastically Scattered Neutron Groups are Evident. ANL Neg. No. 112-5626.

IV-31. Accelerator Digital System

J. F. WHALEN, R. ROGE, J. W. MEADOWS and A. B. SMITH

COMPUTER INTERFACE SYSTEM

An expanded interfacing system has been added to the accelerator laboratory on-line computer complex. This system enables the computer to select exclusively, upon command, a variety of external functions. Each of the external devices is assigned an individual binary code for identification. Before the computer communicates with a given external device, the identifying code is transmitted from the computer to the interface where the device is properly identified. The interface unit then gates the computer input and output data channels to the appropriate device. This procedure makes all external devices available to the computer at all times and requires no manual switching. It also allows the computer to read 24 binary bits of information per nuclear event on the same data channel by successively reading two 12 bits groups. Provision for additional identification codes has been made in order to accommodate expansion to possible future external functions.

ON-LINE DISPLAY SYSTEM

A new display system for the on line computer has been designed and is under construction.¹ A small magnetic drum memory is used for the intermediate storage of information during the display operation. The computer can read or write at random at any one of 8192 locations on the drum. These locations are organized into 4 bands of 12 tracks each. Each band stores 2048

bits. When the computer is not communicating with the drum, the contents of any selected portion of the drum memory may be displayed on a cathode ray tube (CRT). When using this system, it is necessary for the computer only to occasionally up-date the information on the drum memory. The pattern is sustained on the CRT by continuously interrogating the drum memory. This is performed independently of computer operation and the computer is thus free for the more important duties of data processing. This is in contrast to the present display system in which the computer is constantly interrupted from data processing to provide a sustained pattern on the CRT. For experiments in which the display does not consist simply of core memory content but rather of some analytical form of the primary data, the computer need only calculate the analytical form once. This analytical form, once stored on the drum, can be continuously displayed without occupying valuable core memory space or diverting the computer from data processing duties.

The display system will incorporate a spot (i.e. a calibrating marker) which can be manually moved at will about the face of the CRT. The coordinates of the spot, visually available from dial readings, are automatically recorded by the computer, allowing the experimenter to inform the computer directly and quantitatively of areas of interest in the data pattern. A block diagram of the new display system is shown in Fig. IV-31-1.

FIG. IV-31-1. Computer Controlled Display System with Auxiliary Memory. ANL Neg. No. 112-4123.

FIG. IV-31-2. Inter-relation of Components in the Automated System. ANL Neg. No. 112-5321.

FIG. IV-31-3. Flow Diagram of Program for Total Cross Section Measurements. ANL Neg. No. 112-5324.

EXECUTIVE CONTROL BY THE ON-LINE
COMPUTER

Many nuclear experiments utilizing an accelerator and a nuclear detector system require constant monitoring and supervision by the experimenter. For many of these experiments, the actual task of the experimenter consists of repetitive operations and decisions based upon the immediate status of the experimental data. A system has been designed and constructed whereby the experimenter can delegate his authority to execute the repetitive tasks to the computer.² In the system mode of operation the detectors, the Van de Graaff accelerator, and the computer function in a closed loop with the executive control residing in the computer program. Figure IV-31-2 shows the logic of the system. A number of external control options have been made available to the computer including: a) control of the Van de Graaff energy, b) sample and target movements, c) digitalization, transmission and processing of raw

data, and d) permanent recording of the experimental information.

In controlling the Van de Graaff potential, the computer must first determine the required neutron energy currently being called for by the executive program. After making a Q -value calculation for the particular reaction from appropriate conversion constants, the computer produces a control voltage at the output terminals of a digital to analog converter. The control voltage determines the deflecting potential across the plates of an electrostatic analyzer. The beam pick-up slits at the exit of the electrostatic analyzer provide input signals to a balanced amplifier which controls the accelerating potential with corona loading (the potential can be set within 0.025%). Other control parameters of the Van de Graaff are "locked in" with analog subloops so that, excluding "start-up" and extreme excursions, the computer controls the accelerator. Sample movement, detector movement, and beam

control are presently produced by relay action at the discretion of the executive program in the computer. Three analog-to-digital connectors, under the control of the computer, provide access to the detector system for data accumulation. Having processed the digitalized data and calculated the associated results, the computer automatically provides a permanent record of the experimental information on the typewriter, paper tape, line printer or the plotter. The closed loop system has been successfully used in neutron total cross section measurements.² The program flow chart is shown in Fig. IV-31-3. The entire experiment has been conducted continuously and unattended, except for routine maintenance, for several days at a time.

Appreciable benefits have been derived from the automated system in that routine operations are carried out accurately and with speed. Human errors inherently associated with repetitive operations are eliminated and the experimenter's time may be more efficiently utilized in conceptual activities.

INTEGRATION OF ADDITIONAL COMPUTATIONAL HARDWARE

The recent purchase of an additional computer and an auxiliary memory bank required modification of the existing hardware and software for maximum utilization of the equipment. Additional executive duties throughout the accelerator laboratory are being delegated to this computer complex, especially in the areas of accelerator and target control. The two computers will

assume a "Master-Slave" configuration with the executive control being vested in one computer while the second processes the data. Both computers communicate with each other through the common auxiliary memory bank. Their respective roles may be reversed at any time.

The two computers can also be used simultaneously in an independent mode if the experiment permits it. The additional advantage of having a "back up" instrument under these conditions greatly reduces the costly possibility of having valuable accelerator and personnel time wasted during computer "down" time, whether for routine maintenance or component failure. Reprogramming for the new configuration has been started and switch-gear to implement the expansion is under test.

REFERENCES

1. J. F. Whalen, *A Computer Controlled Display System for Nuclear Experiments*, Proc. European-American Nuclear Data Committee Conference on the Automatic Acquisition and Reduction of Nuclear Data, Karlsruhe, Germany, July 13-16, 1964, ed. K. H. Beckurts, W. Gläser, and G. Kruger. Gesellschaft für Kernforschung, Karlsruhe, 1964, pp. 399-408.
2. W. F. Miller, J. W. Meadows and J. F. Whalen, *On-line Data Acquisition and Feedback to Accelerator Control: A Status Report*. Proc. European-American Nuclear Data Committee Conference on the Automatic Acquisition and Reduction of Nuclear Data, Karlsruhe, Germany, July 13-16, 1964, ed. K. H. Beckurts, W. Gläser, and G. Kruger. Gesellschaft für Kernforschung, Karlsruhe, 1964, pp. 222-232.

Section V

Miscellaneous

The first thirteen papers in this section report efforts to extend and improve the accuracy of general reactor physics. The greatest stress is placed on resonance absorption and scattering effects. Continued emphasis on computing work in reactor physics is manifested in the next eight papers which include numerical analysis and code development. Associated with this work is the compilation of basic data for use in analysis and, in this connection, two papers describing activities of the Reactor Physics Constants Center are presented.

Concepts involving mostly preliminary analytical work to define possible objectives and probable areas of difficulty are contained in five papers. Three of these papers are continuations of concept studies reported in the 1963-1964 Annual Report, ANL-7010.

The last seven papers are concerned largely with problems in heat transfer, fluid dynamics in reactor safety experiments, fuel element stresses, and material properties.

V-1. Resonance Parameter Analysis

E. M. PENNINGTON

INTRODUCTION

Two Fortran codes, RESCAT and RESFIT, were written for the CDC-3600 computer to aid in the study of purely scattering resonances. This work was done in connection with the transmission measurements of total cross section by the Applied Nuclear Physics Section, Reactor Physics Division. The RESCAT code computes cross sections as a function of energy using single level and/or multilevel formulae. RESFIT uses multilevel formulae in determining resonance parameters from least squares fitting of experimental energy-cross section data. Both codes consider natural line shapes; that is, Doppler broadening is neglected. Orbital angular momentum values of $\ell = 0, 1$ and 2 are permitted. RESFIT is the successor to a single level code described briefly in Ref. 1.

THE MULTILEVEL FORMULAE

The total cross section, which is also the elastic scattering cross section when reactions are ignored, is

$$\sigma = \frac{2\pi}{k^2} \sum_{\ell} \sum_J g_J [1 - \text{Re } U_{\ell J}(E)]. \quad (1)$$

Here $k = \sqrt{2m/\hbar^2}(A/1 + A)E_{lab}^{1/2}$ where A is the mass number of the target, m is the neutron mass, and E_{lab} is the neutron energy in the laboratory system. The statistical factor $g_J = (2J + 1)/[2(2I + 1)]$ where I is the angular momentum of the target nucleus and J is the total angular momentum. Re refers to the real part of $U_{\ell J}(E)$. The quantity $U_{\ell J}$ is related to the R -matrix according to

$$U_{\ell J} = \frac{I_{\ell} \frac{1 - L_{\ell}^* R_{\ell J}}{O_{\ell} \frac{1 - L_{\ell} R_{\ell J}}{L_{\ell} R_{\ell J}}}, \quad (2)$$

which is Eq. (1.14) of Section IV-1 of Ref. 2. For the case of elastic scattering only, the R -matrix, defined in Eqs. (13) and (15), is simply the R -function. In Eq. (2), I_{ℓ} and O_{ℓ} are the incoming and outgoing waves of orbital angular momentum ℓ which may be expressed as

$$I_{\ell} = G_{\ell} - iF_{\ell} \quad (3)$$

and

$$O_{\ell} = G_{\ell} + iF_{\ell} \quad (4)$$

where

$G_{\ell}(\rho) = -\rho n_{\ell}(\rho)$ and $F_{\ell}(\rho) = \rho j_{\ell}(\rho)$ with n_{ℓ} and j_{ℓ}

being the spherical Bessel functions³. The argument $\rho = ka$, where a is the nuclear radius; while the factor L_{ℓ} in Eq. (2) is

$$L_{\ell} = \rho \frac{O'_{\ell}}{O_{\ell}} = \rho \frac{(F_{\ell} F'_{\ell} + G_{\ell} G'_{\ell})}{F_{\ell}^2 + G_{\ell}^2} + i \left(\frac{\rho}{F_{\ell}^2 + G_{\ell}^2} \right) = S_{\ell} + iP_{\ell}, \quad (5)$$

where S_{ℓ} and P_{ℓ} are termed the shift and penetration factors respectively. As explained in Section IV of Ref. 2, S_{ℓ} may be replaced by $\hat{S}_{\ell J} = S_{\ell} - B_{\ell J}$, where $B_{\ell J}$ is a constant. Upon substituting for I_{ℓ} , O_{ℓ} , L_{ℓ} and L_{ℓ}^* in Eq. (2), we get

$$U_{\ell J} = e^{2i\delta_{\ell J}} = \exp \left\{ -2i \left[\phi_{\ell} + \tan^{-1} \left(\frac{1}{Y_{\ell J}} \right) \right] \right\}, \quad (6)$$

where

$$\phi_{\ell} = \tan^{-1} (F_{\ell}/G_{\ell}) \quad (7)$$

and

$$Y_{\ell J} = \frac{\hat{S}_{\ell J} R_{\ell J} - 1}{P_{\ell} R_{\ell J}}. \quad (8)$$

Then, upon substituting Eq. (6) into Eq. (1),

$$\sigma = \sigma_p + \sigma_{res} + \sigma_{int}, \quad (9)$$

where

$$\sigma_p = \frac{4\pi}{k^2} \sum_{\ell} \sum_J g_J \sin^2 \phi_{\ell} = \frac{4\pi}{k^2} \sum_{\ell} (2\ell + 1) \sin^2 \phi_{\ell}, \quad (10)$$

$$\sigma_{res} = \frac{4\pi}{k^2} \sum_{\ell} \sum_J g_J \left(\frac{\cos 2\phi_{\ell}}{1 + Y_{\ell J}^2} \right) \quad (11)$$

and

$$\sigma_{int} = \frac{4\pi}{k^2} \sum_{\ell} \sum_J g_J \left(\frac{Y_{\ell J} \sin 2\phi_{\ell}}{1 + Y_{\ell J}^2} \right). \quad (12)$$

The R -function is expressed as a sum over resonances as

$$R_{\ell J} = \sum_{\lambda} \left(\frac{\gamma_{\lambda \ell J}^2}{E_{\lambda \ell J} - E} \right) + R_{\ell J}^{res} \quad (13)$$

with the resonance widths, $\Gamma_{\lambda \ell J}(E)$, being given in terms of the $\gamma_{\lambda \ell J}$ by

$$\Gamma_{\lambda \ell J}(E) = 2P_{\ell}(E)\gamma_{\lambda \ell J}^2; \quad (14)$$

$\gamma_{\lambda \ell J}^2$ is defined implicitly in Eq. (14).

The energy E , resonance energies $E_{\lambda lJ}$ and widths $\Gamma_{\lambda lJ}(E)$ in Eqs. (13) and (14) must all be measured in either the center-of-mass system or in the laboratory system. In the laboratory system, the widths and energies are larger by a factor $(A + 1)/A$.

In Eq. (13) the R_{lJ}^{res} term is the contribution to the R -function from energy levels $E_{\lambda lJ}$ outside the energy range under investigation. We write R_{lJ}^{res} in the manner used in Ref. 4 as

$$R_{lJ}^{res} = \sum_{n=1}^4 A_{n lJ} (E - E_{1/2})^{n-1}, \quad (15)$$

where $E_{1/2}$ is some energy in the range under consideration, and we limit the infinite sum in Eq. (15) to four terms. Upon substituting for R_{lJ} in Eq. (8), we get

$$Y_{lJ} = \frac{1}{\sum_{\lambda} \frac{P_l(E)}{P_l(E_{\lambda lJ})} \frac{\Gamma_{\lambda lJ}(E_{\lambda lJ})}{2(E - E_{\lambda lJ})} - \sum_n A_{n lJ} P_l(E) (E - E_{1/2})^{n-1}} + \left(\frac{S_l(E) - B_{lJ}}{P_l(E)} \right). \quad (16)$$

The single level equivalent of Eq. (16) is

$$X_{lJ} = Y_{lJ}^{single} = \frac{P_l(E_{\lambda lJ})}{P_l(E)} \frac{2(E - E_{\lambda lJ})}{\Gamma_{\lambda lJ}(E_{\lambda lJ})} + \frac{S_l(E) - B_{lJ}}{P_l(E)}. \quad (17)$$

When $l = 0$, $S_l(E) = 0$ identically, so that B_{0J} may be taken as zero in either multilevel or single level calculations.

It should be noted that the decomposition of the total cross section into "potential," "resonance" and "interference" terms, σ_p , σ_{res} and α_{int} , as given by Eq. (10), (11) and (12), is not unique. For example, one can go back to Eq. (6) and separate δ_{lJ} into two terms in the manner described in Appendix A of Ref. 5; this is the multilevel generalization of Eq. (1.19) of Section IV-I of Ref. 2. With such a separation, the potential scattering depends upon R_{lJ}^{res} .

Differences between cross sections calculated according to single level and multilevel formulae are especially large when the resonances belonging to the same R -function are close together, and when ϕ_l is near $\pi/4$. Figure V-1-1 illustrates such a case. This figure shows single level and multilevel cross sections for two s-wave resonances, each having $\Gamma = 10$ keV at the resonance energy, and with resonance energies of 740 keV and 750 keV. The target nucleus has $I = 0$ so that $g_J = g_{1/2} = 1$. Values of A and a were chosen so that $\phi_0 = \rho = \pi/4$ at 750 keV. The single level cross sections were calculated according to

$$\sigma^s = \frac{4\pi}{k^2} \sin^2 \rho + \frac{4\pi}{k^2} \cos 2\rho \left(\frac{1}{1 + X_1^2} + \frac{1}{1 + X_2^2} \right) + \frac{4\pi}{k^2} \sin 2\rho \left(\frac{X_1}{1 + X_1^2} + \frac{X_2}{1 + X_2^2} \right), \quad (18)$$

with

$$X_i = \frac{\rho_i}{\rho} \frac{2}{\Gamma_i(E_i)} (E - E_i),$$

while the multilevel cross sections were computed from

$$\sigma^M = \frac{4\pi}{k^2} \sin^2 \rho + \frac{4\pi}{k^2} \frac{\cos 2\rho}{1 + Y^2} + \frac{4\pi}{k^2} \sin 2\rho \left(\frac{Y}{1 + Y^2} \right), \quad (19)$$

with

$$Y = X_1 X_2 / (X_1 + X_2).$$

The multilevel cross sections in Fig. V-1-1 have minima of zero on the low energy sides of each resonance

and maxima of $4\pi/k^2$ on the high energy sides. Cross sections calculated with the single level formula actually become negative over a certain range of energy on the low energy side of the resonance having the lower value E_i , and greater than $4\pi/k^2$ over an energy range on the high energy side of the resonance having the larger value E_i . The fact that ρ is near $\pi/4$ over the range of the resonances helps make the interference effects large since the terms in Eqs. (18) and (19) involving $\sin 2\rho$ dominate those involving $\cos 2\rho$. Resonances with higher ℓ values in this energy range

FIG. V-1-1. Single Level and Multilevel Cross Sections for Two s-Wave Scattering Resonances. ANL Neg. No. 112-5621.

ould be involved with smaller values of ϕ_i and hence could not interfere with each other so drastically.

DESCRIPTION OF RESCAT AND RESFIT

The RESCAT code calculates cross sections according to the theory outlined above for up to 20 resonances and 205 energies. Single level or multilevel calculations or both are optional. The output for all problems includes σ_p as a function of energy, and the individual contributions from each ℓ value. If single level calculations are made, the corresponding resonance, interference and total cross sections are printed as functions of energy. Also σ_{res} and σ_{int} for each resonance are output. Likewise, for multilevel calculations, the resonance, interference, and total cross sections are output at each energy, followed by σ_{res} and σ_{int} corresponding to each R -function.

The RESFIT code reads as input up to 300 pairs of experimental energies and total cross sections, parameters for up to 5 "known" resonances, guesses for parameters for up to 5 "unknown" resonances, values of the $A_{n\ell J}$ for up to 5 "known" R -functions, and initial guesses of $A_{n\ell J}$ for up to 5 "unknown" R -functions. Total cross sections are calculated at each experimental energy, and the method of least squares is used to yield new parameters. The cross sections are then recalculated with the new parameters, and the use of the least squares procedure is repeated until convergence criteria are satisfied, or until the maximum number of iterations allowed by the input is reached.

In the least squares fitting procedure, it is desired to minimize

$$\sum_j d_j^2 = \sum_j (\sigma_E^j - \sigma_T^j)^2, \quad (20)$$

where σ_E^j and σ_T^j are the experimental and calculated total cross sections for the j th experimental energy. Equation (20) may be written as

$$\sum_j d_j^2 = \sum_j \left(\sigma_E^j - \sigma_{T_0}^j - \sum_{k=1}^M \frac{\partial \sigma_{T_0}^j}{\partial x_k} \delta x_k \right)^2, \quad (21)$$

where $\sigma_{T_0}^j$ is the calculated total cross section at some stage of the iterations and x_k stands for one of the M unknown parameters $\Gamma_{\lambda\ell J}(E_{\lambda\ell J})$, $E_{\lambda\ell J}$ or $A_{n\ell J}$. By setting the derivatives of $\sum_j d_j^2$ with respect to each of the δx_k equal to zero, we obtain a matrix equation for the δx_k . Solution of this equation yields new parameters equal to $x_k + \delta x_k$ with which to start the next iteration. The matrix equation is

$$\beta \delta x = \alpha, \quad (22)$$

ere

$$\beta_{km} = \sum_j \frac{\partial \sigma_{T_0}^j}{\partial x_k} \frac{\partial \sigma_{T_0}^j}{\partial x_m}, \quad (k, m = 1, 2, \dots, M) \quad (23)$$

and

$$\alpha_k = \sum_j \frac{\partial \sigma_{T_0}^j}{\partial x_k} (\sigma_E^j - \sigma_{T_0}^j), \quad (k = 1, 2, \dots, M). \quad (24)$$

Equation (22) is solved by use of the matrix inversion subroutine MATINV.⁶ Error estimates are provided for the final values of the parameters, unless convergence has not been achieved. It may be shown⁷ that the RMS error in parameter x_r is

$$(\delta x_r)_{\text{RMS}} = \sqrt{\beta_{rr}^{-1}} \sqrt{\frac{\sum_j d_j^2}{N - M}}, \quad (25)$$

where N is the number of energy cross section pairs in the input.

After each iteration in RESFIT, $\sum_j d_j^2$, the current values of the unknown parameters, and the vectors and matrices α , δx , β and β^{-1} are printed. The final cross section output for each energy consists of σ_p and its components for $\ell = 0, 1, 2$, σ_{res} , σ_{int} , σ_T , σ_E , and the σ_{res} and σ_{int} components corresponding to each R -function.

Test problems were run in which the "experimental" cross sections were actually calculated cross sections, and incorrect initial "guesses" were input for the unknown parameters. Convergence to the correct values for the parameters was achieved fairly rapidly.

The success of RESFIT when applied to actual experimental data has been rather limited so far, although parameters for a few resonances have been obtained. The energy range under investigation is roughly 300 to 1500 keV. Thus many p - and d -wave as well as s -wave resonances are present. Both p - and d -wave resonances are almost symmetric in this energy range and so are difficult to distinguish. Several values of J and hence g_J are usually allowed, with the number of possibilities increasing with I and ℓ . The s -wave resonances are quite unsymmetric, and have tails which provide a background cross section over a wide energy range. At low energies the cross section at a resonance peak is much greater than the potential scattering cross section, but this is no longer true in the energy range considered here. The average ratio of resonance width to level spacing is larger at higher energies than at lower energies, so that the resonances are not as well separated. For the reasons just outlined, the use of RESFIT in the energy range of interest is difficult and requires well considered input.

RESOLUTION CONSIDERATIONS

A study was made to determine whether neglect of the finite resolution in transmission experiments causes appreciable errors in resonance parameters derived from fitting experimental cross sections. A CDC 160-A

code was written to calculate apparent "experimental" cross sections according to

$$\sigma_{app}(E) = -\frac{1}{n} \ln T(E) \quad (26)$$

with

$$T(E) = \int_{-\infty}^{\infty} e^{-n\sigma(E')} R(E' - E) dE' \quad (27)$$

and

$n \equiv$ atom number density times sample thickness.

Here $\sigma_{app}(E)$ is the apparent cross section at energy E , $T(E)$ is the transmission, $\sigma(E)$ is the actual cross section, and $R(E' - E)$ is the resolution function which was taken to be a Gaussian with width ΔE at half-height. Of course, if $R(E' - E)$ were equal to $\delta(E' - E)$, the apparent cross section would be the true cross section.

The cross section $\sigma(E)$ in Eq. (27) was calculated as that from a single s -wave resonance, or from a single p -wave resonance including s -wave potential scattering. The apparent cross section output was punched by the 160-A and used as input to the single level predecessor of RESFIT. Calculations were done for both s - and p -wave resonances at 750 keV for several resonance widths Γ and resolution function widths ΔE . Values of A and a were chosen to make $\phi_0 = 45$ deg at 750 keV. The corresponding ϕ_1 is 6.85 deg.

It was found that $\delta\Gamma/\Gamma$ is roughly proportional to $(\Delta E/\Gamma)^2$ for small $\Delta E/\Gamma$, and that values for s -wave resonances were about twice as large as for p -wave resonances because of the greater asymmetry of the s -wave resonances. The errors in resonance energy, δE_i , are quite small, and are larger for the s -wave resonances because of the asymmetry. The values of $\sum_j d_j^2$ increase rapidly with increasing $\Delta E/\Gamma$. This is because the calculated height of a resonance will be essentially correct even though the finite transmission may depress the apparent height considerably when ΔE is not small in comparison with Γ . Thus the least squares fit code increases Γ somewhat to fit the apparent cross

TABLE V-1-I. RESOLUTION EFFECTS

ℓ	Γ , keV	ΔE , keV	$\delta\Gamma$, keV	δE_i , keV	$\sum_j d_j^2$, b^2
0	10.0	1.0	0.0261	0.005	0.00173
0	10.0	2.0	0.1005	0.020	0.02569
0	10.0	3.0	0.2131	0.042	0.11649
0	10.0	4.0	0.3510	0.071	0.32174
1	10.0	1.0	0.0132	0.001	0.00191
1	10.0	2.0	0.0507	0.006	0.02584
1	10.0	3.0	0.1061	0.012	0.12991
1	10.0	4.0	0.1712	0.020	0.35982

sections better in the wings of the resonance, but cannot improve the fit much near the peak. Hence $\sum_j d_j^2$ increases rapidly with increasing $\Delta E/\Gamma$, but $\delta\Gamma$ increases much more slowly. Table V-1-I shows the results for the cases with $\Gamma = 10$ keV.

The results in Table V-1-I indicate that resolution effects do not present a serious problem. By noting the similarities between the Gaussian resolution function and the Doppler broadening function, it may be demonstrated that Doppler broadening effects are quite negligible for resonances in the energy range of interest.

REFERENCES

1. E. Pennington, *Resonance Parameter Analysis*, Reactor Physics Division Annual Report July 1, 1963 to June 30, 1964, ANL-7010 (January 1965) p. 246.
2. A. M. Lane and R. G. Thomas, *R-Matrix Theory of Nuclear Reactions*, Rev. Mod. Phys. **30**, No. 2, Part I, 257-353 (April 1958).
3. L. L. Schiff, *Quantum Mechanics*, (McGraw-Hill Book Co., Inc., New York, 1949) p. 77-78.
4. F. W. K. Firk, J. E. Lynn and M. C. Moxon, *Analysis and Interpretation of the Neutron Cross Section of Vanadium Below 25 keV*, Proc. Phys. Soc. **82**, 477-487 (October 1963).
5. C. D. Bowman, E. G. Bilpuch and H. W. Newson, *s- and p-Wave Neutron Spectroscopy, Part VIII*, Annals of Physics **17**, 319-378 (March 1962).
6. B. S. Garbow, *ANF 402, Matrix Inversion with Accompanying Solution of Linear Equations (Fortran II)*, ANL Internal Memo (February 23, 1959).
7. F. B. Hildebrand, *Introduction to Numerical Analysis*, (McGraw-Hill Book Co., Inc., New York, 1956), Chapter 7.

V-2. Resonance Integrals in Slab Lattices Using Various Escape Probabilities

E. M. PENNINGTON

Resonance integrals were calculated for U-238 and U-238 O₂ slabs 0.5 cm thick separated by H₂O moderator of thicknesses 0.2, 0.5 and 1.0 cm. The calculations were done in order to compare the accuracies of four expressions for the escape probability P_0 , namely:

$$\text{Exact: } P_0^E = \frac{1}{x} \int_0^1 \frac{\mu(1 - e^{-x/\mu})(1 - e^{-y/\mu}) d\mu}{(1 - e^{-(x+y)/\mu})} \quad (1)$$

$$\text{Nordheim: } P_0^N = \frac{P_0^I(x)}{1 + \left(\frac{2C}{1-C} \right) x P_0^I(x)} \quad (2)$$

TABLE V-2-I. RESONANCE INTEGRALS (BARNS) AND PERCENT ERRORS
(Reproduced from Ref. 3)

Fuel	b, cm	I^E	I^N	I^G	I^W
UO ₂	0.2	11.007 (0.00)	10.878 (-1.17)	11.134 (+1.15)	10.688 (-2.90)
UO ₂	0.5	13.865 (0.00)	13.727 (-1.00)	13.915 (+0.36)	13.268 (-4.31)
UO ₂	1.0	15.577 (0.00)	15.500 (-0.49)	15.581 (+0.03)	14.820 (-4.86)
U	0.2	6.439 (0.00)	6.292 (-2.28)	6.523 (+1.30)	6.165 (-4.26)
U	0.5	8.557 (0.00)	8.423 (-1.57)	8.576 (+0.22)	8.106 (-5.27)
U	1.0	9.764 (0.00)	9.694 (-0.72)	9.758 (-0.06)	9.223 (-5.54)

$$\text{GAM-I: } P_0^G = P_0^I \left(\frac{x}{1-C} \right) \quad (3)$$

and

$$\text{WIGNER: } P_0^W = \frac{1}{1 + \frac{2x}{1-C}} \quad (4)$$

In Eq. (1) through (4), $x = \Sigma_1 a$ and $y = \Sigma_2 b$ where Σ_1 and Σ_2 are cross sections of fuel and water slabs and a and b are the corresponding thicknesses. The quantity $P_0^I(x) = (1/x) [\frac{1}{2} - E_3(x)]$ and is the escape probability for an isotropic spacially flat neutron source in an isolated slab, while the Dancoff factor is $C = 2E_3(y)$.

The calculations were carried out for the ten resonances of U-238 having the largest infinite dilution resonance integrals using either the NR or NRIA approximations¹ depending on the practical width. Six of the ten resonances, including those contributing over 90% of the total resonance integral, were calculated with the NRIA approximation. Using the notation of Ref. 1, the expressions for the resonance integrals are

$$\text{NR: } I = \frac{\sigma_p \Gamma_\gamma}{E_i} J(\xi, \beta) + \frac{\sigma_0 \Gamma_\gamma}{E_i} L(Z, \xi, \beta), \quad (5)$$

$$\text{NRIA(UO}_2\text{): } I = \frac{\sigma_m \Gamma}{E_i} J(\xi, \beta') + \frac{\sigma_0 \Gamma_\gamma}{E_i} L(Z', \xi, \beta'), \quad (6)$$

$$\text{NRIA(U): } I = \frac{\sigma_0 \Gamma_\gamma}{E_i} L'(Z'', \xi), \quad (7)$$

are

$$J(\xi, \beta) = \int_0^\infty \frac{\psi}{\psi + \beta} dX, \quad (8)$$

$$L(Z, \xi, \beta) = \int_0^\infty \frac{\psi^2}{\psi + \beta} P_0 \left[Z \left(\frac{\psi}{\beta} + 1 \right) \right] dX, \quad (9)$$

$$L'(Z'', \xi) = \int_0^\infty \psi P_0(Z'' \psi) dX, \quad (10)$$

and

$$\psi(\xi, X) = \frac{\xi}{2\sqrt{\pi}} \int_{-\infty}^\infty \frac{\exp[-\xi^2(X-Y)^2/4] dY}{1+Y^2}. \quad (11)$$

The functions, J , L and L' were evaluated on the CDC-3600 computer using 16-point Gaussian integration repeated over suitable intervals of X , depending on the parameters, from $X = 0$ to $X = 5000$. An asymptotic contribution was then added to yield the complete function. The Doppler broadening function $\psi(\xi, X)$ was calculated using the O'Shea-Thacher subroutine.² The E_3 functions appearing in the various P_0

TABLE V-2-II. VOLUME AND SURFACE CONTRIBUTIONS TO RESONANCE INTEGRALS (BARNS) AND PERCENT ERRORS

Fuel	b, cm	$I(V)$	$I^E(S)$	$I^M(S)$	$I^G(S)$	$I^W(S)$
UO ₂	0.2	5.832 (0.00)	5.175 (-2.49)	5.046 (+2.45)	5.302 (+2.45)	4.856 (-6.16)
UO ₂	0.5	5.832 (0.00)	8.033 (-1.72)	7.895 (+0.62)	8.083 (+0.62)	7.436 (-7.43)
UO ₂	1.0	5.832 (0.00)	9.745 (-0.79)	9.668 (+0.04)	9.749 (+0.04)	8.988 (-7.77)
U	0.2	0.328 (0.00)	6.111 (-2.41)	5.964 (+1.37)	6.195 (+1.37)	5.837 (-4.48)
U	0.5	0.328 (0.00)	8.229 (-1.63)	8.095 (+0.23)	8.248 (+0.23)	7.778 (-5.48)
U	1.0	0.328 (0.00)	9.436 (-0.74)	9.366 (-0.06)	9.430 (-0.06)	8.895 (-5.73)

FIG. V-2-1. Escape Probabilities for Various Approximations. ANL Neg. No. 112-5607.

were evaluated by series expansion for arguments less than unity and by an accurate rational approximation for arguments greater than one. For most values of x , $P_0^E(x)$ was computed by 16-point Gaussian integration repeated an adequate number of times; but for $x < 0.01$ the expansion

$$P_0^E(x) = \frac{1}{x} \left(\frac{1}{2} - E_3(x) \right) - x \int_0^1 \frac{e^{-(x+y)/\mu} d\mu}{\mu(1 - e^{-(x+y)/\mu})} - \frac{x^3}{12} \sum_{n=1}^{\infty} \frac{[1 + n(x+y)]e^{-n(x+y)}}{[n(x+y)]^2} \quad (12)$$

was used to avoid cancellation of leading terms.

Table V-2-I presents the resonance integrals and the corresponding percent errors for all four expressions for P_0 and both types of fuel. Table V-2-II gives the volume and surface contributions to the resonance integrals separately, with the volume contribution being that involving J and the surface contribution that involving L or L' and therefore P_0 . Thus Table V-2-II shows the effects of using the various P_0 more explicitly. Note that all approximations except that of Wigner yield the correct P_0 and hence the correct resonance

integral for large y . It is seen from the tables that the GAM-I approximation results agree best with exact values, with the Nordheim and Wigner approximations following in that order. The escape probability $P_0^G(x)$ is too large for large x and too small for small x , so that errors produced by using P_0^G have a tendency to cancel.

On the other hand, $P_0^N(x)$ is too small for all x , although it is more accurate than P_0^G over much of the range. The behavior of P_0^E , P_0^G and P_0^N is illustrated on Fig. V-2-1. The Wigner approximation gives values of P_0 which are much too small for large x , so that the corresponding resonance integrals are considerably underestimated.

REFERENCES

1. F. T. Adler, G. W. Hinman and L. W. Nordheim, *The Quantitative Evaluation of Resonance Integrals*, Proc. 1958 Geneva Conference 16, Paper No. 1988, pp. 155-171.
2. D. M. O'Shea and H. C. Thacher, Jr., *Computation of Resonance Line Shape Functions*, Trans. Am. Nucl. Soc. 6, No. 1, 36-37 (June 1963).
3. E. M. Pennington, *Resonance Integrals in Slab Lattices Using Various Escape Probabilities*, Trans. Am. Nucl. Soc. 8, No. 1, 288-289 (June 1965).

V-3. Tables of Resonance Integral Functions, L' (Z' , ξ)

E. M. PENNINGTON

The function $L'(Z', \xi)$, which is defined by

$$L'(Z', \xi) = \int_0^{\infty} \psi(\xi, x) P_0(Z' \psi) dx \quad (1)$$

occurs in the calculation of resonance integrals for absorbers without any admixed moderator in the NRIA approximation.¹ In Eq. (1) $\psi(\xi, x)$ is the Doppler broadened line shape function, while $P_0(Z' \psi)$ is the escape probability from the absorber element for a uniform and isotropic neutron source.

In converting the GAM-I code from the IBM-704 to the CDC-3600, it was discovered² that the L' tables on the old GAM-I tape were inaccurate, especially for small j where $Z' = 2^{-j} \times 10^4$. It was therefore decided to recalculate the values of L' for slab, cylindrical and spherical geometries, and to extend the tables somewhat. A Fortran program written for the CDC-3600 was used to compute values of L' for $\xi = 0.02, (0.01), 0.05, (0.05), 0.5$ and ∞ , and for $j = -2, (0.5), 14$.

The integral in Eq. (1) was evaluated by 16-point Gaussian integration repeated an adequate number of

times from $x = 0$ to 5000 with a small analytical contribution being added for $x > 5000$. In calculating ψ the O'Shea-Thacher subroutine³ was used, while the escape probabilities for the three geometries were computed from the usual formulae (e.g. Ref. 4) with various series, asymptotic and rational approximations involved. Ranges of integration were determined by using the natural line shape for ψ , the Wigner approximation for P_0 , and comparing numerical integration results with analytical results. It was found that doubling the number of ranges determined in this manner resulted in the same integrals to five or six figures for each original range, except the one starting at $x = 0$ for which the difference was sometimes a few tenths of a per cent. This is caused by the fact that the natural line shape may differ greatly from the broadened shape for small x . Thus a scheme was used to subdivide the first range. The program to calculate L' is not especially fast, but is quite accurate. The calculational methods are presented in more detail, along with the FORTRAN listings and the tables in Ref. 5.

REFERENCES

1. F. T. Adler, G. W. Hinman and L. W. Nordheim, *The Quantitative Evaluation of Resonance Integrals*, Proc. 1958 Geneva Conference, **16**, Paper No. 1988.
2. G. C. Jensen, Applied Mathematics Division, Argonne National Laboratory (Private Communication).
3. D. M. O'Shea and H. C. Thacher, Jr., *Computation of Reso-*

- nance Line Shape Functions*, Trans. Am. Nucl. Soc. **6**, No. 1, 36-37 (June 1963).
4. K. M. Case, F. de Hoffman and G. Placzek, *Introduction to the Theory of Neutron Diffusion*, Los Alamos Scientific Laboratory, Los Alamos, New Mexico (June 1953); also, Superintendent of Documents, U. S. Government Printing Office, Washington D. C.
5. E. M. Pennington, *Tables of Resonance Integral Functions*, $L'(Z', \xi)$, ANL-7024 (to be published).

V-4. Equivalence between Homogeneous and Heterogeneous Resonance Integrals in Cylindrical Geometry

H. H. HUMMEL

While the equivalence relation between homogeneous and heterogeneous resonance integrals¹ has proven very useful, it seemed desirable to investigate some of the assumptions made to be sure that the over-all procedure is sufficiently accurate for routine multigroup cross section evaluation.

A key assumption of the theory is that the flux in each region can be assumed spatially flat. The accuracy of this assumption is not investigated here. Another important assumption is that the probability, $P_0(E)$, that a uniformly distributed source of neutrons of energy E will escape from an isolated rod without suffering a collision is given by the rational approximation

$$P_0(E) = \frac{1}{1 + \frac{\Sigma_0(E)}{\Sigma_{e0}}}, \quad (1)$$

where $\Sigma_0(E)$ is the macroscopic total cross section of the rod, and

$$\Sigma_{e0} = \frac{S_0}{4V_0} = \frac{1}{2R_0}, \quad (2)$$

where S , V , and R are the surface, volume, and radius respectively of the rod. Subscripts 0 and 1 refer to fuel and moderator regions respectively. The rational approximation is not extremely accurate, and an improvement suggested by M. Levine² is to replace Eq. (1) by

$$P_0(E) = \frac{1}{1 + \frac{\Sigma_0(E)}{\Sigma_{e0} a}}, \quad (3)$$

where " a " is a constant, assumed to be the same for any resonance.

If one regards " a " as a function of E , as both " a " and will be in the following, clearly the rational approximation can be made exact, but it is then no longer use-

ful for determining an equivalence relationship. It was suggested by J. Otter³ that a mean value $\bar{a}$ be defined for each resonance by, in effect, integrating over the resonance by breaking it up into intervals, in each of which Eq. (2) is applied with appropriate constant values of parameters. However, it was found by Otter that Levine's assumption with a constant value of $\bar{a}$ of 1.35 for all resonances gave quite good results.

For a lattice, in which Dancoff interaction must be taken into account, if an equivalence relation is to be obtained, the escape probability for the rod in the lattice, P_0^* , must again be cast in the rational form

$$P_0^* = \frac{1}{1 + \frac{\Sigma_0}{\Sigma_{e0}^*}}. \quad (4)$$

An approximate relationship between P_0 and P_0^* ,

$$\frac{1}{P_0^*} = \frac{1}{P_0} + \frac{\Sigma_0 C}{\Sigma_{e0}(1 - C)} \quad (5)$$

has been developed^{4, 5, 6} using the assumption that paths in successive fuel pin cells are uncorrelated. When, as suggested by Levine,² the correction factor " a " is introduced for the lattice case also, one obtains from Eqs. (3), (4), and (5)

$$\Sigma_{e0}^* = \Sigma_{e0}(1 - C) \frac{a}{1 + (a - 1)C}. \quad (6)$$

The factor " a " is still that for P_0 and appears here because of the relation [Eq. (5)] between P_0 and P_0^* .

The equivalent σ_p , the effective reactor scattering cross section in barns per absorber atom to be used in a homogeneous resonance integral calculation, is then

$$\sigma_p = \sigma_{p0} + \frac{\Sigma_{e0}^*}{N_0}, \quad (7)$$

where σ_{p0} is the fuel rod scattering cross section per absorber atom in barns and N_0 is the concentration of absorber atoms in the fuel in atoms/cc $\times 10^{-24}$.

The accuracy of Eq. (6) was checked with the aid of a series of calculations kindly performed for the writer by E. Pennington, using his collision probability code.⁷ This code makes the flat flux approximation and uses an isotropic cell boundary condition. The calculations were carried out for monoenergetic neutrons for a 0.25 in. pin with steel clad and sodium coolant typical of a fast power breeder. Various values of the relevant cross sections were assumed to cover the range of escape probabilities of interest. Equation (6) was found to be quite accurate for "a" defined by the correct isolated rod escape probability and with C obtained from the calculation with Pennington's code. It was found also that clad and coolant regions could be homogenized in the calculation of C with good accuracy, a result previously obtained by E. Pennington (see Paper II-15) for water lattices. The discrepancy in $\gamma = 1 - C$ noted in this case between the two and three region cells did not exceed 1%.

An approximate method of calculating the Dancoff factor proposed by G. Bell⁵ is

$$\gamma_B = 1 - C_B = \frac{1}{1 + \frac{\Sigma_{cl}}{\Sigma_1}}, \quad (8)$$

where Σ_{cl} in this case is

$$\Sigma_{cl} = \frac{S_0}{4V_1} = \frac{R_0}{2(R_1^2 - R_0^2)} \quad (9)$$

and R_1 is the outer radius of the moderator (or in this case, homogenized clad and coolant). It has been found by D. Leslie, J. Hill, and A. Jonsson⁸ that γ does indeed depend to a good approximation only on Σ_1/Σ_{cl} , although Eq. (8) gives values about 12% too low at large values of Σ_1/Σ_{cl} , in the range 2 to 3. Calculations of γ were carried out in Ref. 9 with a collision probability cell calculation with isotropic return quite similar to E. Pennington's for a square cell with $V_1/V_0 = 1$. Calculations were also carried out in Ref. 9 for this same cell with a Monte Carlo calculation, with the result that the

collision probability results were about 3% too high at large Σ_1/Σ_{cl} . Values of γ calculated by E. Pennington (see Paper II-15) for a series of different water moderated lattices, as well as those discussed in the present paper, all fall very close, within 1 or 2% in most cases, to the curve of γ versus Σ_1/Σ_{cl} for the escape probability calculation of Ref. 9. If γ really does depend only on Σ_1/Σ_{cl} , then it seems one would do just as well by making an accurate determination of this universal curve and using an empirical fit to it. Whether this is really so perhaps requires further exploration. In any event, it is suggested that for the time being a modified Bell approximation be obtained by using a fit to the Monte Carlo curve of Ref. 8 for γ versus Σ_1/Σ_{cl} . A simple fit, which is 2% low at $\Sigma_1/\Sigma_{cl} = 3.0$ and closer at smaller values, is

$$\gamma = \gamma_B + \gamma_B^4(1 - \gamma_B). \quad (9)$$

In using Eq. (6), one is still faced with the problem of what mean value $\bar{a}$ of "a" to use. It would be possible to make $\bar{a}$ a function of C . This may not be worthwhile, however, since for C nearly unity the factor on the far right side of Eq. (6) is nearly unity regardless of $\bar{a}$; while for small C , the isolated rod case is approached for which $\bar{a} = 1.35$ seems satisfactory. The problem of defining $\bar{a}$ will be investigated by comparison of resonance integral calculations with more exact calculations.

REFERENCES

1. J. Chernick and R. Vernon, *Some Refinements in the Calculation of Resonance Integrals*, Nucl. Sci. Eng. **4**, 649 (1958).
2. M. M. Levine, *Resonance Integral Calculations for U-238 Lattices*, Nucl. Sci. Eng. **16**, 271 (1963).
3. J. M. Otter, *Escape Probability Approximations in Lumped Absorbers*, NAA-SR-9744 (1964).
4. W. Rothenstein, *Collision Probabilities and Resonance Integrals for Lattices*, Nucl. Sci. Eng. **7**, 162 (1960).
5. G. I. Bell, *A Simple Treatment for Effective Resonance Integrals in Dense Lattices*, Nucl. Sci. Eng. **5**, 138 (1959).
6. L. W. Nordheim, *The Theory of Resonance Absorption*, Proc. Symposium Appl. Math. **XI**, 58 (1961).
7. E. Pennington, *Collision Probabilities in Cylindrical Lattices*, Nucl. Sci. Eng. **19**, 215 (1964).
8. D. C. Leslie, J. G. Hill, and A. Jonsson, *Improvements to the Theory of Resonance Escape in Heterogeneous Fuel*, Nucl. Sci. Eng. **22**, 78 (1965).

-5. Calculation of Resonance Absorption Using Non-Asymptotic Neutron Sources in Two Region Cells

P. H. KIER

The standard methods of calculating resonance absorption of neutrons in lattices are based on the narrow resonance approximation in the moderator and the flat flux approximation in the fuel. These approximations are justified when considering absorption by fertile nuclides and when the moderator is hydrogenous, but they will introduce errors when considering fissile nuclides or mixtures of resonance absorbers with overlapping resonances.

In recent years, the problem of developing methods of calculating resonance absorption that include the effects of non-recovery of the neutron flux between resonances has received considerable attention.^{1, 2, 3} The method developed here treats a two-region circularized cell for above-thermal energies. Use of the isotropic return boundary condition and a fine energy mesh makes it well suited for treating small cells and Doppler effects.

An energy range of interest is divided into intervals, or groups, of equal lethargy width. Because the resonance cross section is a rapidly varying function of energy, the neutron flux distribution in the cell will be also. Consequently, to describe the flux adequately, groups must be extremely narrow. We shall specify therefore, that the width of a group is much less than the maximum lethargy gain acquired by a neutron in a collision with the resonant absorber. Use of such narrow groups allows us to assume that neutrons will not have more than one collision within a group; scattering collisions will cause neutrons to be scattered to groups of lower energy. Because a neutron will not have multiple collisions within a group, it is appropriate to develop a method based on first flight collision probabilities to calculate reaction rates for a group.

Since we shall consider only above-thermal energies, materials normally used as moderators have constant scattering cross sections and small absorption cross sections. The total cross section of the moderator will be taken as constant when considering escape probabilities. If it is also assumed that the flux entering or leaving the rod is isotropic and that the neutron source for a group has the form:

$$S(r) = \sum_n S_n F_n(r) \quad (1)$$

where S_n are group dependent constants and $F_n(r)$ are functions of the radial coordinate, we need compute the escape probability only once for the component of the

source in the moderator distributed as $F_n(r)$. The probability of neutrons slowing down in the moderator into a given group being absorbed upon directly entering the rod is then the product of the escape probability from the moderator and the collision probability in the rod for an isotropic surface flux.

The assumption that neutrons enter the rod isotropically is sound because of the geometric properties of rods. It is made with success by A. Amouyal and P. Benoist⁴ in their treatment of thermal utilization and, from the work of G. Stuart and R. Woodruff,⁵ the escape probability from a rod for an incident surface flux is only weakly dependent upon the angular distribution of the incident flux.

Three other assumptions are basic to the method:

- (1) Scattering is isotropic in the laboratory system of coordinates
- (2) Neutrons leaving the cell are returned isotropically
- (3) The neutron flux is asymptotic above the energy range of the calculation.

The first of these assumptions appears to be necessary if the method is to be tractable, the second assumption has been found to be a suitable boundary condition for small circularized cells⁶ and the third assumption is reasonable because the energy ranges of interest are far below the energies at which neutrons are born.

The neutron source distribution for a group is taken as a three term polynomial in each region:

$$S_i(r) = \sum_{n=0}^2 S_{in} \left(\frac{r}{a}\right)^{2n}, \quad (2)$$

$$i = 0, \text{ rod}; i = 1, \text{ moderator}$$

where a is the radius of the rod and S_{in} are group dependent coefficients. With the source shape given by Eq. (2), the collision rate in the rod is given by:

$$R_{col} = S_s(1 - p_s) + \sum_{n=0}^2 (1 - p_n) S'_{on}, \quad (3)$$

where S_s , the rate with which neutrons impinge upon the surface of the rod, is:

$$S_s = \sum_{n=0}^2 P_{an} S'_{1n} + T_{ba} \sum_{n=0}^2 (P_{bn} S'_{1n} + p_n T_{ab} S'_{on} + P_{an} p_s T_{ab} S'_{1n})$$

$$\begin{aligned}
& \sum_{j=0}^{\infty} (T_{bb} + T_{ba}T_{ab}p_s)^j \quad (4) \\
& = \sum_{n=0}^2 \left\{ P_{an}S'_{1n} \right. \\
& \quad \left. + \frac{T_{ba}[p_n T_{ab}S'_{on} + (P_{bn} + P_{an}p_s T_{ab})S'_{1n}]}{1 - T_{bb} - T_{ba}T_{ab}p_s} \right\}
\end{aligned}$$

where

b $\equiv$ outer radius of the moderator

P_{zn} $\equiv$ first flight probability of escape through the surface of the moderator at x for neutrons scattered into the group in the moderator and distributed as $(r/a)^{2n}$

T_{zy} $\equiv$ probability that neutrons entering the moderator isotropically through its surface at x escape through the surface at y without suffering an intervening collision

P_n $\equiv$ first flight escape probability from rod for neutrons scattered into the group in the rod and distributed as $(r/a)^{2n}$

P_s $\equiv$ probability that neutrons impinging upon the surface of the rod escape isotropically from the rod without suffering an intervening collision

and where

$$S'_{on} = \frac{\pi a^2}{n+1} S_{on}; S'_{1n} = \frac{\pi a^2}{n+1} [(b/a)^{2n+2} - 1] S_{1n}. \quad (5)$$

The first summation in Eq. (4) represents neutrons from the slowing down source in the moderator which impinge upon the rod's surface directly after escaping from the moderator; the second summation represents neutrons that impinge upon the rod's surface after leaving the cell and returning isotropically one or more times. The reaction rate for process z in material j is simply:

$$R_{zj} = R_{col}(\Sigma_{zj}/\Sigma_{rod}), \quad (6)$$

where

Σ_{zj} $\equiv$ macroscopic cross section for process z

Σ_{rod} $\equiv$ total macroscopic cross section.

In this model, the P_{zn} and T_{zy} are independent of group because the total cross section of the moderator is assumed to be constant. K. Cady and M. Clark⁷ have derived exact expressions for p_s and p_n , which are group dependent. For the neutron source to have the form given in Eq. (2), the flux must have the same form:

$$\begin{aligned}
\phi_i(r) &= \sum_{n=0}^2 \phi_{in} \left(\frac{r}{a}\right)^{2n}, \quad i = 0, \text{rod}; i = 1, \text{moderator} \quad (7)
\end{aligned}$$

where ϕ_{in} are group dependent coefficients. By taking

the spatial moments of the flux that results from a particular source distribution, Cady and Clark have been able to obtain ϕ_{on} in the form:

$$\phi_{on} = c_n S_s + \sum_{j=0}^2 c_{nj} S_{oj}, \quad (8)$$

where:

c_{nj} $\equiv$ contribution to flux coefficient ϕ_{on} from source neutrons distributed as $(\frac{r}{a})^{2j}$,

c_n $\equiv$ contribution to flux coefficient ϕ_{on} from isotropic surface flux.

The c_n and c_{nj} are group dependent since they depend upon the radius of the rod in mean free paths.

Thus far it has been shown how the flux distribution and the reaction rates in the rod for a group are obtained from the slowing down source distribution and the cross sections for materials in the rod. We must still obtain the flux distribution in the moderator, i.e., the ϕ_{1n} . At this point, there is enough information known for a group to determine the ϕ_{1n} by requiring the flux in the moderator to satisfy certain conditions. The following three conditions are used to obtain three simultaneous equations involving the ϕ_{1n} :

1. Equality of the fluxes at the interface:

$$\phi_1(a) = \phi_0(a) \quad (9a)$$

2. Neutron conservation; since each collision removes a neutron from the group, the collision rate in the group must equal the rate at which neutrons are scattered into the group or:

$$\begin{aligned}
\Sigma_{n,od} \int_{mod} \phi_1(r) dV &= \int_{mod} S_1(r) dV \\
&+ \int_{rod} S_0(r) dV - R_{col}, \quad (9b)
\end{aligned}$$

where

Σ_{mod} $\equiv$ macroscopic cross sections of the moderator

V $\equiv$ volume

3. The derivative of $\phi_1(r)$ vanishes at the surface of the cell:

$$\frac{d}{dr} \phi_1(b) = 0. \quad (9c)$$

Thus far it has been assumed that the neutron source distribution for the group is known. Since an asymptotic source is not assumed, the source is dependent upon the flux in those groups from which neutrons can be scattered into the group in question. A source component for the k th group is:

$$S_k^{in} = \sum_{j=1}^J \sum_{l=1}^{L_j} P_{lj} \Sigma_{s_{k-l,j}} \phi_{k-l}^{in}; \quad (i = 0, 1) \quad (10)$$

where:

P_{lj} $\equiv$ probability that a neutron is scattered

down ℓ groups in a scattering collision with material j

$\Sigma_{s_{k-\ell},j}$ $\equiv$ macroscopic scattering cross section of material j in group $k - \ell$

L_j $\equiv$ maximum number of groups a neutron can be downscattered in a single collision with an atom of material j

$\phi_{k-\ell}^{in}$ $\equiv$ flux component in group $k - \ell$.

Equation (10) would be an extremely time consuming expression to use because a neutron can be scattered through so many groups. Under the assumption of isotropic scattering P_{tj} has the form:

$$P_{tj} = \frac{1}{1 - \alpha_j} e^{-(t-1)\Delta u} (1 - e^{-\Delta u}) \Delta u, \quad (11)$$

where Δu is the lethargy width of a group and $(1 - \alpha_j)$ is the maximum fractional energy loss. From Eq. (11), it is seen that $P_t = e^{-\Delta u} P_{t-1}$, which allows Eq. (10) to be reduced to the much simpler form:

$$S_k^{in} = \sum_{j=1}^J P_{1j} \Sigma_{s_{k-1},j} \phi_{k-1}^{in} + e^{-\Delta u} \{ S_{k-1}^{in} - \sum_{j=1}^J P_{L_j} \Sigma_{s_{k-L_j-1},j} \phi_{k-L_j-1}^{in} \}. \quad (12)$$

It is impossible to store all of the fluxes necessary for exact evaluation of the last term of Eq. (12) for light materials, such as hydrogen or deuterium. Therefore for materials that can scatter neutrons through more groups than the number for which information is stored, we use the asymptotic flux for ϕ_{k-L-1} .

To summarize the treatment of a fine group, Eq. (12) is used to obtain the source distribution for a group. Then the cross sections, escape probabilities, and flux-source coefficients are obtained for the rod. Eqs. (3), (4) and (6) are then used to obtain the reaction rates; the flux distribution in the rod is obtained from Eq. (8) and the flux distribution in the moderator is obtained by solving Eqs. (9a), (9b) and (9c) for the ϕ_{1n} . We have now obtained the quantities of interest for the group and can repeat this procedure for the next group. It is assumed that the flux is asymptotic above the energy range being considered. With this assumption, we can work down in energy, group by group, without iteration, until the entire energy range of interest is covered. The reaction rates can be accumulated over specified energy intervals and then used to compute either resonance integrals or effective cross sections. The absorption probability is simply the ratio of the absorption rate to the rate at which neutrons flow down past the maximum energy of the calculation.

This method has been used to write a program in FORTRAN for the CDC-3600 called RIFF RAFF.⁸

TABLE V-5-I. RESONANCE ESCAPE PROBABILITIES FOR URANIUM ROD AT 300°K IN WATER MODERATED CELLS FOR THE ENERGY RANGE 14 TO 3 eV

Rod diameter, in.	V_{mod}/V_{rod}	p (Monte Carlo)	p (RIFF RAFF)
0.25	1	0.8740 ± 0.0019	0.8725
0.25	4	0.9543 ± 0.0012	0.9551
0.60	1	0.8974 ± 0.0017	0.8966
0.60	4	0.9711 ± 0.0009	0.9703

RIFF RAFF allows for ten materials in each region. There can be four resonance absorbers in the rod, each having sixty resonances. The non-resonance materials can have $1/v$ absorption cross sections with the restriction that the macroscopic absorption cross section of the moderator must be small compared with its scattering cross section. The running time for a problem with 20 resonances and 4000 fine groups is about 2 minutes.

The method has been modified to treat resonance absorption in homogeneous mixtures. Equation (12) is used to obtain the rate at which neutrons are scattered into a group. Because a neutron can have only one collision in a group, the collision rate is equal to the source and the flux is the collision rate divided by the total cross section for the group. The homogeneous version of the method has been used to write a program in FORTRAN for the CDC-3600 called MISH MASH.

Because of the assumptions of isotropic scattering and isotropic return, a severe test of the accuracy of the method is to consider absorption by a strong resonance in a small water moderated cell. The resonance escape probability for the energy interval 14 eV to 3 eV was computed by W. Rothenstein⁹ using a Monte Carlo program for small water moderated cells containing a uranium rod. RIFF RAFF was used to compute the resonance escape probability for these cells and in all four cases the results were within the statistical error of the Monte Carlo calculations. The results are summarized in Table V-5-I.

REFERENCES

1. F. E. Driggers, *A Method of Calculating Neutron Absorptions and Flux Spectra at Epithermal Energies*, CRRP-1197 (1964).
2. F. T. Adler and E. E. Lewis, *Comparison of Exact and Approximate Treatments of Resonance Effects in Cylindrical Lattices*, Trans. Am. Nucl. Soc. **7**, No. 1, 29 (1964).
3. C. A. Stevens and G. D. Joanou, *Investigation of the Flux-Recovery Assumption in Resonance Computations*, Trans. Am. Nucl. Soc. **8**, No. 1, 287 (1965).
4. A. Amouyal and P. Benoist, *New Method at the Determination of the Thermal Utilization Factor of a Cell*, CEA-571 (1956).
5. G. W. Stuart and R. W. Woodruff, *Method of Successive Generations*, Nucl. Sci. Eng. **3**, 339-373 (1958).

6. R. Sims, et al., *Failure of the Cell Cylindricalization Approximation in Closely Packed Uranium/Heavy Water Lattices*, Trans. Am. Nucl. Soc. **7**, No. 1, 9 (1964).
7. K. B. Cady and M. Clark, *Neutron Transport in Cylindrical Rods*, Nucl. Sci. Eng. **18**, 491-507 (1964).
8. P. H. Kier, *RIFF-RAFF, A Program for Computation of Resonance Integrals in a Two-Region Cell* ANL-7033 (1965).
9. W. Rothenstein, *Some Monte Carlo and Analytic Results for Resonance Capture in Lattices*, Nucl. Sci. Eng. **8**, 122-127 (1960).

V-6. Moderator Scattering and Flux Depression Near Resonances

P. F. GAST

In the approximations customarily used in computing resonance integrals, the first order solution for the neutron flux fails to show any reduction on the low energy side of the resonance.^{1, 2} This is due, in part, to neglect of any flux depression in the moderator, thus providing a channel by which the neutrons, most of which reach lower energies by moderator scattering, can bypass the resonance. When the moderator scattering is treated in the same approximation as heavy element scattering, the second order iterate for the flux is seen to have a form corresponding to the first-collision term in a Placzek solution. Repeated applications of the scattering operator would then be expected to lead to

the asymptotic solution. An approximate solution may then be obtained by replacing the first collision term by the asymptotic one, which then shows the expected reduction on the low energy side of the resonance.³

REFERENCES

1. R. Goldstein and E. R. Cohen, *Theory of Resonance Absorption of Neutrons*, Nucl. Sci. Eng., **13**, 132-140 (1962).
2. R. Goldstein, *Spectral Distribution of Neutrons in the Vicinity of a Resonance*, Trans. Am. Nucl. Soc., **7**, 1, p. 28 (1964), and Nucl. Sci. Eng., **19**, 359-362 (1964).
3. P. F. Gast, *Moderator Scattering and Flux Depression Near Resonances*, Nucl. Sci. Eng., **21**, 121 (1965).

V-7. A Simple Estimate of the Effects of Resonance Interference¹

C. N. KELBER

The accurate computation of capture in resonances shows that when resonances occur close together there may be a sizable effect on the capture rate because of flux perturbations.^{2, 3} While an accurate computation is a formidable problem, there are some conditions which a) occur reasonably often, and b) admit a simple approximate answer.

Such estimates are significant because accurate computations² are time consuming and require a large computing machine. Studies such as those cited in Refs. 2 and 3 show that flux perturbations often have a small but detectable influence on foil activations. In such a case, a reasonably accurate hand calculation is useful.

Suppose there are two resonances, labeled I and II, close together. Further assume that by reason of nuclide abundance or level parameters, resonance I dominates the shape of the flux near the resonant

energy. Then a simple estimate of the capture rate in resonance II may be obtained by integrating the capture cross section from resonance II times the flux determined by the presence of resonance I in the absence of resonance II.

The procedure is to express the capture cross section in resonance II in terms of the line shape of resonance I and a modifying function. We use unbroadened line shapes and expand σ_{a2} in the difference $x_1^2 - x_2^2$, where $x_i = 2(E - E_{r_i})/\Gamma_i$; E_{r_i} is the resonance energy and Γ_i is the total width.

For the flux we use the intermediate representation of R. Goldstein and E. Cohen⁴ and introduce their parameter β_λ given by

$$\beta_\lambda^2 = 1 + \frac{\sigma_0}{s + \lambda\sigma_p} \frac{\Gamma_\gamma + \lambda\Gamma_n}{\Gamma}$$

where

the resonance parameters are those of the dominant resonance (I)

σ_0 = cross section of resonance I

s = effective moderator scattering per atom of type I

σ_p = total potential scattering per atom of type I

λ = the intermediate representation parameter.

Then by contour integration of*

$$RI_2 = \frac{\Gamma_1}{2E_{r1}} \int_{-\infty}^{\infty} \psi_1 \sigma_{a2} dx_1,$$

where RI_2 is the resonance integral for the second resonance and ψ_1 is given approximately⁴ by

$$\psi_1 \approx (1 + x_1^2)/(\beta_\lambda + x_1^2)$$

and

$$\sigma_{a2} = \frac{\Gamma_{\gamma 2}}{\Gamma_2} \frac{\sigma_{0,2}}{1 + x_2^2},$$

we find

$$RI_2 = I_{0,2} \frac{E_2}{E_1} \left[1 + \frac{(1 - \beta_\lambda^2)\Gamma_1}{\beta_\lambda} \frac{\Gamma_2 + \beta_\lambda \Gamma_1}{(\Gamma_2 + \beta_\lambda \Gamma_1)^2 + 4(E_2 - E_1)^2} \right]$$

where $I_{0,2}$ is the infinite-dilution resonance integral.

The factor E_2/E_1 comes from neglecting the $1/E$ dependence of ψ_1 . In fact, if β_λ is set equal to unity

* Suggested by E. Pennington, ANL. The contour lies in the upper half plane.

(no resonance flux depression), then $RI_2 = I_{0,2}$, not $I_{0,2} E_2/E_1$ as would be obtained above. Therefore the ratio E_2/E_1 is omitted, yielding

$$RI_2 \approx I_{0,2} \left[1 + \frac{(1 - \beta_\lambda^2)\Gamma_1}{\beta_\lambda} \frac{\Gamma_2 + \beta_\lambda \Gamma_1}{(\Gamma_2 + \beta_\lambda \Gamma_1)^2 + 4(E_2 - E_1)^2} \right].$$

Consider as an example a mixture of H:U-238 = 1 and estimate the depression in the response of a gold foil from the interference between the 6.68 eV resonance of U-238 and the 4.91 eV resonance in gold.

For resonance I (U-238):

$$E_1 = 6.68 \text{ eV}$$

$$\sigma_0 = 2.192 \times 10^4 \text{ barns}$$

$$\Gamma_1 = 0.0264 \text{ eV}$$

$$\sigma_s = 20 \text{ barns}$$

$$\lambda = 0$$

$$\beta_\lambda^2 = 954.$$

For resonance II (gold):

$$E_2 = 4.91 \text{ eV}$$

$$\Gamma_2 = 0.1406 \text{ eV}.$$

By substitution we find $RI_2/I_{0,2} = 0.94$. A precise calculation¹ gives a value of 0.92 for this ratio.

REFERENCES

1. C. N. Kelber, *A Simple Estimate of the Effects of Resonance Interference*, Nucl. Sci. Eng. **22**, No. 1, 120-121 (1965).
2. C. N. Kelber, *Fluxes and Reaction Rates in the Presence of Interfering Resonances*, Trans. Am. Nucl. Soc. **6**, 2, 273 (1963).
3. W. K. Foel, R. A. Grimesey and S. Tong, *A Monte Carlo Study of Resonance Absorption in Gold and Indium Lumps*, Trans. Am. Nucl. Soc. **6**, 2, 272 (1963).
4. R. Goldstein and E. R. Cohen, *Theory of Resonance Absorption of Neutrons*, Nucl. Sci. Eng., **13**, 132-140 (1962).

V-8. Improved Rational Escape Probability in Lumped Absorbers¹

C. N. KELBER

When a resonance absorber is present as a lump of material the problem of determining the flux is complicated by the need to determine the probability that a neutron born at energy E in the lump will not escape the lump without a collision. This probability of non-escape, denoted by P , is difficult to compute and is even more difficult to integrate in determining the flux. For these reasons considerable effort has been expended on approximations to P which are easy to use in computation. The most familiar of these is the Wigner Rational Approximation (WRA):

$$P = \tau/(1 + \tau)$$

where

τ = the average optical path length.

It is well known² that under appropriate weak assumptions the resonance integral for a lumped absorber may be represented by that of an equivalent homogeneous system provided the escape probability from the lump is approximated by WRA. Unfortunately WRA is a serious underestimate of the escape probability over a wide range of optical thicknesses and a reasonable approach is to conserve the equivalence relation but attempt to improve the accuracy by fitting an additional material-dependent parameter to

the WRA. Such attempts have been made, for example, by M. Levine³ and G. Bell.⁴

A typical method, for example, is to change the optical path length, $\tau = \Sigma \ell$, to $\Sigma \ell / A$, where A is an empirical constant. Here Σ is the effective total macroscopic cross section, and ℓ is the mean chord length. This can be viewed as a change in the mean chord length, substituting ℓ / A for ℓ ; but ℓ typically appears in the definition of Σ , so care must be taken when following this procedure that the proper definition of Σ is made. If, in fact, ℓ / A is substituted for ℓ in Σ , we find the following expression for Σ in a frequently used approximation⁵

$$\Sigma = \frac{\Sigma_0}{1 + x^2} + \lambda \Sigma_p + \frac{A}{\ell}, \quad (1)$$

where

$$x = \frac{2}{\Gamma} (E - E_r)$$

E = neutron energy

E_r = resonance energy

Γ = total neutron width

Σ_p = macroscopic potential scattering cross section

Σ_0 = peak macroscopic resonant total cross section in the lump

λ = Cohen and Goldsteins' intermediate representation parameter ($0 \leq \lambda \leq 1$)⁽⁵⁾

However, A need not be the same constant for all resonances. More recently, J. Otter⁶ has attempted a procedure to determine how A should depend on the resonance parameters: Σ_0 , Γ , E_r , Σ_p , λ , ℓ .

Briefly the method is to define a function $A(\tau)$ implicitly by the relationship

$$P(\tau) = \frac{\tau/A}{1 + \tau/A}, \quad (2)$$

where $P(\tau)$ is the exact non-escape probability. Replacing the function $A(\tau)$ by 1 is the Wigner Rational Approximation.

If, for each resonance, $A(\tau)$ is replaced by some weighted average value appropriate to that resonance, the equivalence relationship for that resonance is preserved. Denoting the weighted average value of $A(\tau)$ by $\bar{A}$ we then find for each resonance a parameter $\bar{A}$ which gives an approximate non-escape probability, P_{app} ,

$$P_{app} = \frac{\tau/\bar{A}}{1 + \tau/\bar{A}}. \quad (3)$$

The problem resides in choosing a properly weighted average. In particular we seek a procedure which will preserve the value of the resonance integral. Recalling Eq. (1) we see that if $\bar{A}$ replaces the constant resonance independent value A in Eq. (1), the effective cross section is dependent on $\bar{A}$. Since $\bar{A}$ is not known *a priori* an iterative procedure will have to be used. The

TABLE V-8-I. ANGULAR WEIGHT POINTS
(RADIAN)S VERSUS n

(Note: $\theta_0 = 0$, $\theta_n = \pi/2$ for all n)

$\theta_i \backslash n$	2	3	4	5	6
$i = 1$	0.415855	0.268133	0.198953	0.158397	0.131657
2		0.586311	0.415855	0.325403	0.268133
3			0.687625	0.514226	0.415855
4				0.757418	0.586311
5					0.809580
$\theta_i \backslash n$	7	8	9	10	
1	0.112675	0.0984923	0.0874890	0.0787017	
2	0.228325	0.198953	0.176349	0.158397	
3	0.350617	0.303689	0.268133	0.240185	
4	0.485098	0.415855	0.364826	0.325403	
5	0.642326	0.540522	0.469298	0.415855	
6	0.850647	0.687625	0.586311	0.514226	
7		0.884169	0.725332	0.625048	
8			0.912273	0.757418	
9				0.936320	

scheme used follows Otter's closely. Otter has found satisfactory numerical fits to the relationship for $A(\tau)$ implied by Eq. (3). Make an initial guess at $\bar{A}$ (usually $\bar{A} = 1$ is a good choice) and compute the resonance integral using the exact function $A(\tau)$ given by Eq. (2), but with Σ determined by the guessed value of $\bar{A}$. Now compute the resonance integral using the approximate form given by Eq. (3) and determine a value of $\bar{A}$ from that form which makes the two values of the resonance integral equal. In general $\bar{A}$ thus determined will not equal the $\bar{A}$ used in finding Σ for use with Eq. (2). Hence, iterate the process using the most recently determined value of $\bar{A}$ to get Σ for use with Eq. (2), etc. It is shown in Ref. (1) that this process is equivalent to getting an underestimate of $\bar{A}$ by

$$\bar{A} \geq \frac{2}{\pi} \int_{-\pi/2}^{\pi/2} A[\ell \Sigma(\theta)] \cos^2 \theta d\theta, \quad (4)$$

where $\theta = \tan^{-1}[x(1 + \Sigma_0/\Sigma_m)^{1/2}]$

$$\Sigma_m = \lambda \Sigma_p + \bar{A}/\ell.$$

The most recent value of $\bar{A}$ is used in the right hand side of Eq. (4) to get a new estimate and the process iterated.

The integral for $\bar{A}$, Eq. (4), may be represented by a sum which, using the trapezoidal rule over n points, gives

$$\bar{A} = \left[\frac{1}{2} (A_0 + A_1) + \sum_{i=1}^{n-1} A_i \right] \frac{1}{n}$$

where $A_i = A[\ell \Sigma(\theta_i)]$. The θ_i are defined by

$$\int_{\theta_{i-1}}^{\theta_i} \cos^2 \theta d\theta = \frac{1}{n} \frac{\pi}{4},$$

where $\theta_0 = 0$.

The values of θ_i corresponding to various choices of are given in Table V-8-I.

An equation which overestimates $\bar{A}$ is given in Ref. (1).

REFERENCES

1. C. N. Kelber, *Improved Rational Escape Probability in Lumped Absorbers*, Nucl. Sci. Eng. **22**, 244-247 (June, 1965).
2. J. Chernik and R. Vernon, *Some Refinements in the Calculation of Resonance Integrals*, Nucl. Sci. Eng. **4**, 649-672 (1958).
3. M. M. Levine, *Resonance Integral Calculations for U-238 Lattices*, Nucl. Sci. Eng., **16**, 271-279 (1963).
4. G. I. Bell, *Theory of Effective Cross Sections*, LA-2327, (1959).
5. R. Goldstein and E. R. Cohen, *Theory of Resonance Absorption of Neutrons*, Nucl. Sci. Eng. **13**, 132-140 (1962).
6. J. M. Otter, *Escape Probability Approximations in Lumped Absorbers*, NAA-SR-9744 (August 1964).

V-9. Effects of Chemical Binding on the Doppler Broadened Cross Section of Uranium in a UO_2 Lattice*

C. R. ADKINS

Recent measurements of the Doppler effect have been performed for a temperature range (usually 500-1000°K for UO_2) such that the crystalline binding effects are presumed to be small. Also, recent calculations have utilized cross sections of resonances which were broadened by integrating over a Maxwellian velocity distribution for a system of weakly interacting particles, such as a gas. Corrections are then applied by defining an effective temperature, usually in terms of the Debye model or the Born-Von Karmon model of a crystal which attempts to account for the binding effects of the nuclei of the fuel. This technique has been known to give good results in the case where the binding of the fuel nuclide is weak relative to the energy imparted to it by the incoming neutron or in the case where the recoil energy of a particle in the lattice is known to be much greater than the maximum binding energy of the particle. The importance of the Doppler effect provides the impetus for studying this technique in detail, particularly for the case where the large recoil approximation is not necessarily justified. Even if the assumption of large recoil is valid, additional study is needed in the case of critical assemblies where the temperatures are not high, to correct the gas model results according to the phonon distribution of the crystalline medium.

HISTORICAL DEVELOPMENT

The concepts of Doppler broadening of nuclear quantum levels has been known for many years. The theory of the Doppler broadened line shape for nuclei in a gas was discussed by H. Bethe and G. Placzek.¹ The work of W. Lamb² considered a crystalline model of purely absorbing nuclei. Lamb,² and subsequently P. Egelstaff,³

showed that the results of the crystalline model reduced to those obtained by the gas model in the large recoil limit defined as

$$\Gamma + \Delta \gg 2h\omega_{\max} \quad (1)$$

and

$$\Delta = 2 \left(\frac{m}{M} E_0 \bar{E} \right)^{1/2}, \quad (2)$$

where

- Γ = total width of the quantum level
- Δ = Doppler width
- $h\omega$ = maximum binding energy of the crystal
- m = neutron mass
- M = nuclear mass
- E_0 = energy of the incident particle
- $\bar{E}$ = average energy per thermal mode of oscillation in the crystal.

It should be noted that the large recoil approximation need not necessarily specify a crystal in which the particles are loosely bound. This approximation applies to the case where the energy imparted to a nuclide, through collision recoil or the recoil associated with the decay from an excited state, is large in comparison with the maximum binding energy of that nuclide in the crystal.

The work of Bethe and Placzek¹, using the gas model, indicated that the Doppler broadened line shape retains the Gaussian shape. In contrast, Lamb's² work indicated that, for the case where the large recoil approximation does not hold, the Doppler broadened line shape becomes antisymmetrical and exhibits a fine structure due to the quantized modes of lattice vibration. Egelstaff³ corrected the gas model by using an effective temperature $T_{\text{eff}} = E/k$ and compared the corrected gas model with Lamb's results. The agreement

* Report of progress in work for Ph.D. thesis, Carnegie Institute of Technology, Pittsburgh, Pennsylvania.

was much better, but differences in self-shielding were still prevalent. Egelstaff³ and other investigators contended that, for most real reactor materials at realistic temperatures, the large recoil approximation as defined by Eq. (1) is valid, and that the gas model, modified by the use of an effective temperature usually gives good results.

INTRODUCTION

We wished to examine the case of uranium dioxide (UO_2) for two reasons: experimental data for the Doppler coefficient is available from the studies in critical assemblies; and UO_2 is assumed to be an ionic crystal which is expected to exhibit strong crystalline binding in relation to other types of crystals, and to have a reasonably high phonon frequency "cutoff" temperature in relation to the physical temperature in the critical assemblies. If the physical temperature is less than the "cutoff" temperature, it is reasonable to expect a deviation from the gas model calculation of the Doppler broadened line shape. Consequently, one might expect a change in the resonance integral as calculated by the gas model due to the change in the energy self-shielding of the line shape. From experimentally measured values of the specific heat, the Debye temperature (i.e., the phonon frequency "cutoff" temperature for an elastic continuum) has been measured for UO_2 , but the measurements exhibit a large degree of disagreement. The measured values lie in a range from 300–870°K,⁽⁴⁾ and one cannot decide upon a "best" value. Recent work at Chalk River⁵ places the Debye temperature at about 630°K for the physical temperature near room temperature (300°K) and above.

The Chalk River experiment consisted of studying the coherent one-phonon scattering of slow neutrons from a single crystal specimen of UO_2 in order to determine the dispersion curves in the high symmetry directions in the Brillouin zone. A shell model including ionic polarizabilities was fitted to the experimentally obtained dispersion curves to determine the phonon frequency distribution and subsequently the Debye temperature. The Debye temperature obtained in this manner is quite insensitive to the model used, since it is obtained from the integral over the phonon distribution.

One interesting conclusion is apparent from this experiment: the crystalline binding in UO_2 is not necessarily weak, which may change the Doppler coefficient as calculated by the gas model. The change is noted primarily because the effective temperature T_{eff} is greater than the physical temperature, and secondly because, depending upon the resonance chosen, the conditions of the large recoil approximation may be violated. Also, the Chalk River experiment showed a considerable concentration of phonons at low frequencies (acoustical modes) which invalidates the use of the Debye model

for calculations involving UO_2 . Hence, the Born-Von Karman model must be used for a more representative picture of UO_2 .

To study the crystalline behavior, the problem can be divided into two parts: development of cross sections (both scattering and absorption) based on a crystalline model, and the determination of the density of states distribution function (number of phonons per unit frequency interval) to be used in the calculation of the cross sections. Also, the average energy per mode of thermal vibration $\bar{E}$ and specific heat C_b can be obtained from this distribution, eliminating the need for a Debye temperature and the use of a Debye model.

DEVELOPMENT OF CROSS SECTIONS

The cross sections of interest, namely absorption, potential scattering, interference scattering, and resonance scattering cross sections, were developed from the reaction matrices for a crystalline material. The scattering cross sections were, at first, developed in the form of scattering kernels and then integrated over the solid angle and final energies to give cross sections as a function of initial neutron energy. The absorption cross section is essentially that given by Lamb,² although it was developed by a different method.

The three types of scattering cross sections are quite general, in that they describe a general crystal of the AB_n type without resorting to any specialized models—such as the Einstein model. Each includes an average over isotopes and total spin, where the spin average gives rise to an incoherent part in addition to the expected coherent part. The scattering cross sections are quite complicated. The most complex is the resonance scattering cross section which involves a triple integral, one for each of three times which may be interpreted as the collision time and the compound nucleus lifetime for two different nuclei. The interference scattering cross section involves a double integral, one over collision time and the other over a quantity which may be interpreted as the compound nucleus lifetime. The potential scattering cross section has a single integral over collision time. The resonance and interference scattering cross sections were studied in the short compound nucleus lifetime limit, or in another sense, the large level width approximation; that is,

$$\Gamma \gg \hbar\omega_{\text{max}}. \quad (3)$$

Since a short compound nucleus lifetime implies a large recoil energy, this approximation is equivalent to the large recoil approximation given by Eq. (1); a power series expansion may be used for all the various times and the sines may be truncated at terms of second order. We also drop the coherent scattering terms in all of the scattering cross sections, knowing that, at high energies,

these coherent terms will be damped out. Using these approximations we find:

1) The resonance and interference scattering cross sections reduce to the form given by the gas model plus many temperature-dependent correction terms which vanish for high incident neutron energy, and

2) The potential scattering cross section reduces to a constant plus temperature-dependent correction terms which vanish at high incident neutron energies.

In the large level width approximation, therefore, the crystalline form of the scattering cross sections reduces to the form given by the gas model. However, at low incident neutron energies (perhaps for a thermal resonance), the gas model results are *not* reproduced and the corrections terms must be incorporated. Also, if the large level width approximation does not hold, one must evaluate numerically the integrals in the cross sections and, in this case, look for even greater deviations from the gas model. It is expected that one could pick a material (like UO_2), and a resonance of that material (like one of the resonances in U), such that one could find significant differences between the crystalline and gas models, both in the shape of the scattering resonance and in the calculation of the resonance integral.

As in the work by Lamb,² the absorption cross section involves a single integral over collision time which reduces to the form given by the gas model in the short collision time limit [which is also the large recoil approximation given by Eq. (1)].

PHONON FREQUENCY DISTRIBUTION^{6,7}

All derived cross sections for a crystalline substance depend upon the determination of the phonon frequency distribution before they can be evaluated. The phonon frequency distribution is a histogram of the number of lattice vibrational quanta (phonons) per unit frequency interval. The method used was developed specifically from the case of UO_2 , although it is applicable to any face centered cubic crystal of the AB_n type (e.g. uranium carbide).

Uranium dioxide has the calcium fluoride structure; that is, it consists of three equal interpenetrating face centered cubic lattices consisting of O_I (oxygen belonging to the lattice of the first type), O_{II} (oxygen of the lattice of the second type), and U. If the uranium ion is located at the origin, O_I is located at $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4})a$ and O_{II} is located at $(-\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4})a$, where a is the side length of one of the face centered cubes. There are three atoms per primitive unit cell and, consequently, there will be nine branches of the dispersion relation in any direction. The dynamical matrix is 9×9 , and will give nine characteristic frequencies for each value of the wave propagation vector. Since the three interpenetrating lattices have the same lattice constant (a), the

reciprocal space is the same for all three lattice types. We incorporate the Born periodic boundary condition in the form of the Brillouin zone which takes the form of a truncated octahedron for the UO_2 crystal. The number of discrete points existing in the Brillouin zone is equal to the number of face centered cubic unit cells in the crystal, which approaches a continuum. We employ the root sampling method to the irreducible element of the Brillouin zone (that is, the smallest segment of the zone, $\frac{1}{48}$ of the total volume, which will reproduce the full zone under symmetry operations), covering this segment with a map of equally spaced points, choosing as many points as can be tolerated for a reasonable computer solution. Each point receives a weight corresponding to the number of times it occurs in the full zone when the symmetry operations are applied to the irreducible element. The dynamical matrix is diagonalized for each point in the irreducible element of the Brillouin zone, the result being the determination of a large number of frequency eigenvalues which may be plotted as a histogram.

The dynamical matrix representing the three equations of motion for each of the three nuclei in the primitive unit cell is derived in the harmonic approximation by considering a simple harmonic oscillator potential to exist between adjoining pairs of nuclei. The harmonic approximation assumes that all normal modes of vibration are independent, and neglects the contribution of anharmonic effects such as temperature effects, crystal dislocations, and surface effects, all of which cause interference between the normal modes. Using the theory of stresses on an elastic continuum (dispersionless media) in the form of the equations of equilibrium, compatibility, and Hooke's law, one may find the matrix equation describing the motion of an elastic solid in terms of the elastic constants, which are functions of Young's modulus, the shear modulus, and Poisson's ratio. This matrix may be compared with the dynamical matrix of UO_2 , when the latter is restricted to the acoustical branches for sufficiently long waves, to evaluate the interatomic force constants in terms of the elastic constants. Additional information, such as infrared absorption data may also be used to evaluate additional interatomic force constants. The specification of the interatomic force constants, and the values of the wave vectors (in terms of a map of the irreducible element of the Brillouin zone) completely establishes the matrix elements of the dynamical matrix and allows the determination of the frequency eigenvalues.

The histogram of the phonon frequency distribution for the UO_2 crystal is then plotted by defining a number of boxes along the abscissa (each box representing a frequency interval) and adding up the weights for each frequency lying within a particular box.

RESULTS

In order to calculate a phonon frequency distribution, a computer solution was obtained to map, point by point, the irreducible element of the Brillouin zone and provide the weights for each point. Other programs have been written to calculate the dynamical matrix elements, and to diagonalize the matrix to determine the frequency eigenvalues. We are currently considering 2805 points in the irreducible element of the Brillouin zone, which gives 25,245 eigenfrequencies. The final program forms a histogram of the frequencies using 200 boxes along the abscissa and plots the results using the Cathode Ray Tube and associated on-line plotting system for the CDC-3600. At the time of this writing, the program is in the final stages of debugging, and no complete phonon distributions have been obtained. However, once the phonon frequency distribution has been obtained for many different models of the crystal—both for single crystal and polycrystalline cases—the second problem lies in a computer program for the numerical solution of all the various cross sections. Once these cross sections are known, a comparison between the

crystalline model and the gas model for a uranium dioxide medium can be made.

REFERENCES

1. H. A. Bethe and G. Placzek, *Resonance Effects in Nuclear Processes*, Phys. Rev. **51**, 450 (1937).
2. W. E. Lamb, *Capture of Neutrons by Atoms in a Crystal*, Phys. Rev. **55**, 190 (1939).
3. P. A. Egelstaff, *The Doppler Broadening of Slow Neutron Resonances*, AERE-M1035 (1962).
4. J. Belle, Ed., *Uranium Dioxide, Properties and Nuclear Applications*, U. S. Atomic Energy Commission (1961).
5. G. Dolling, R. A. Cowley and A. D. B. Woods, *The Crystal Dynamics of Uranium Dioxide*, Chalk River Nuclear Laboratories, Chalk River, Ontario, Canada (unpublished).
6. J. DE LAUNAY, *Specific Heats and Lattice Vibrations*, Solid State Phys., F. Seitz and D. Turnbull, Eds. (Academic Press, New York, 1956), Vol. 2, p. 219.
7. A. A. Maraduden, E. W. Montroll and G. H. Weiss, *Theory of Lattice Dynamics in the Harmonic Approximation*, F. Seitz and D. Turnbull, Eds. (Academic Press, New York, 1963), Supplement 3.
8. R. K. Osborn and S. Yip, *The Foundations of Neutron Transport Theory*, ANS Monograph (1964).

V-10. Spatial Dependence of Prompt-Neutron Chains¹

R. A. KARAM

It was reported^{2, 3} that the prompt-neutron chains in reactors with reflectors constructed of low atomic weight material die out with modes besides the asymptotic mode. In this paper, the results of a study of the spatial dependence of the decay rates of the asymptotic mode and other modes (approximated by a single mode) are reported for Assemblies Nos. 4B and 6 of Zero Power Reactor-9 (ZPR-9)⁽⁴⁾.

The data were taken with two fission chambers feeding a 400-channel time analyzer. The channel width used in all of the measurements was 0.2 μ sec. The spatial dependence was studied by placing one chamber (A) at the center of the core and varying the position of the other chamber (B) in the radial direction. Another set of experiments consisted of placing Chamber B in the aluminum reflector at the core-reflector interface and moving Chamber A in increments from the center of the core toward Chamber B. Chamber A triggered the time sweep. If, during this sweep, an event took place in Chamber B, the sweep stopped, the pulse was stored in the appropriate channel and the time analyzer was reset. If an event did not take place in

Chamber B, the sweep continued until the end of the cycle, at which time the analyzer was reset. The triggering pulse was monitored on a scaler and a sweep that ended with no pulse stored anywhere was also monitored on a separate scaler. At the end of each run, the data was corrected according to the relation

$$P_j = C_j / \left(\sum_{i=1}^N C_i + S_N \right),$$

where

P_j $\equiv$ true number of pulses in channel j

C_j $\equiv$ observed number of pulses in channel j

C_i $\equiv$ observed number of pulses in channel i

N $\equiv$ total number of channels

S_N $\equiv$ number of sweeps that end with no data stored in any channel.

This correction of the data gives the number of counts per channel opening. The corrected data were fitted to the function

$$N(t) = BG + A_1 e^{a_1 t} + A_2 e^{a_2 t}$$

by the variable metric method for minimization.⁵ The

TABLE V-10-I. SPATIAL DEPENDENCE OF ROSSI-ALPHA DATA FITTED TO TWO EXPONENTIALS AND A BACKGROUND

Run/Assembly	Detector ^a Position	BG $\times 10^4$	($A_1 \times 10^4$) ^b	$\alpha_1 \times 10^{-6}$	($A_2 \times 10^{-4}$) ^c	$\alpha_2 \times 10^{-4}$	A_2/A_1
1/4B	A(23-23) B(23-24)	2.08 ± 0.01	1.26 ± 0.34	0.94 ± 0.34	4.10 ± 0.08	7.00 ± 0.16	3.25
2/4B	A(23-23) B(23-25)	1.83 ± 0.01	8.56 ± 8.40	3.98 ± 3.00	3.90 ± 0.04	7.31 ± 0.24	0.46
3/4B	A(23-23) B(23-27)	1.12 ± 0.01	1.74 ± 1.40	1.79 ± 1.02	2.36 ± 0.10	7.17 ± 0.14	1.36
4/4B	A(23-23) B(23-28) ^d	0.92 ± 0.01	1.60 ± 0.70	1.64 ± 0.51	1.62 ± 0.03	6.96 ± 0.16	1.01
5/4B	A(23-25) B(23-28) ^d	1.19 ± 0.01	0.45 ± 0.20	0.98 ± 0.50	1.58 ± 0.03	7.10 ± 0.20	3.50
6/4B	A(23-27) B(23-28) ^d	3.45 ± 0.01	0.42 ± 0.20	0.71 ± 0.50	1.56 ± 0.07	6.75 ± 0.42	3.68
1/6	A(23-23) B(23-24)	3.02 ± 0.01	1.70 ± 0.10	2.86 ± 1.10	1.60 ± 0.02	4.67 ± 0.31	0.94
2/6	A(23-23) B(23-30) ^d	1.33 ± 0.07	3.33 ± 0.11	2.60 ± 0.41	0.57 ± 0.01	4.10 ± 0.30	0.17

^a The letters A and B refer to detectors A and B; the numbers in parentheses are position indices; the first number refers to a row and the second number refers to a column. Position 23-23 is the center of the core; an increase of unity in the position index is equivalent to 5.5 cm.

^b A_1 is the coefficient of the exponential associated with α_1 .

^c A_2 is the coefficient of the exponential associated with α_2 or Rossi-alpha.

^d Indicates detector is in reflector at core-reflector interface.

FIG. V-10-1. Decay Rates of Prompt-Neutron Chains, Assembly No. 6, Run No. 2. ANL Neg. No. 112-4808.

symbol BG designates the background or uncorrelated pairs of events. Table V-10-I summarizes the results and Fig. V-10-1 shows a sample of the data.

The following remarks can be made about the results shown in Table V-10-I:

(a) α_2 is spatially independent

(b) α_1 and A_1 cannot be determined accurately at the present time because there are only a few channels in which the fast decay mode is seen

(c) Within the accuracy with which α_1 is determined, α_1 also seems to be spatially independent

(d) The coefficient A_1 appears to increase relative to the coefficient A_2 as the distance between the detectors increases. An exception to this observation is indicated by the results from Run 2, Assembly No. 4B

(e) In order to make the calculated and observed decay rates of the prompt-neutron chains compatible (see Paper III-23), the observed decay constants were averaged in the same manner as the decay constants of the delayed neutron groups were averaged, namely

$$\alpha_{av} = (A_1 + A_2) / \left(\frac{A_1}{\alpha_1} + \frac{A_2}{\alpha_2} \right).$$

α_{av} was found to be $8.9 \times 10^4 \text{ sec}^{-1}$, $8.9 \times 10^4 \text{ sec}^{-1}$ and $8.4 \times 10^4 \text{ sec}^{-1}$, respectively, for runs 1/4B, 5/4B and 6/4B. The calculated α for this assembly was $9.4 \times 10^4 \text{ sec}^{-1}$. The discrepancy between measurement and calculation is less than 10%. This averaging procedure with the other runs in Table V-10-I resulted in larger discrepancies between calculated and measured values. This could be due to the inaccuracy of α_1 and A_1 .

REFERENCES

1. R. A. Karam, *Spatial Dependence of the Decay Rates of Prompt-Neutron Chains in Reflected Reactors*, Trans. Am. Nucl. Soc. **8**, No. 1, 224 (1965).
2. R. A. Karam, *Measurement of Rossi-Alpha in Reflected Reactors*, Trans. Am. Nucl. Soc. **7**, No. 2, 283 (1964).
3. R. C. Doerner, K. K. Almenas, R. A. Karam, W. Y. Kato, W. G. Knapp and W. B. Loewenstein, *Physics Measure-*

ment in *Tungsten-Based Aluminum Reflected Reactors*, ANL-7010, (1964), pp. 116-123.

4. W. G. Knapp, R. C. Doerner, K. K. Almenas, C. E. Cohn and R. A. Karam, *Physics Measurements of Modified*

Tungsten-Based Aluminum-Reflected Fast Reactors, Trans. Am. Nucl. Soc. 8, No. 1, 241 (1965).

5. W. C. Davidon, *Variable Metric Method for Minimization*, ANL-5990 (Rev.) (1954).

V-11. On the Synthesis of Space Dependent Transfer Functions

C. K. SANATHANAN

INTRODUCTION

A procedure has been developed to synthesize the space dependent transfer function of a linear dynamic system using frequency response data. Use of the term, "transfer function" is generally associated with the space independent or the point model treatment of a dynamic system. The advantages of using the transfer function of a system for transient studies, synthesis, and control are well known.

The point model of a system becomes unrealistic when the system is described by partial differential equations in which the time and the space variables are not separable. The space modes of such a system exhibit different transient behavior. Therefore, when a certain forcing function—say sinusoidal—is applied to the system, the response would vary with the position at which it is measured. Figure V-11-1 illustrates such a situation for a forcing function of a certain shape $g(x)$ in space and sinusoidal in time. In Fig. V-11-1 the curves (1), (2) and (3) refer to the responses at three different locations, say x_1 , x_2 and x_3 respectively, where $x_1 < x_2 < x_3$. Note that the amplitude and the phase delay of the response increase with x . Suppose this observation is made for a band of frequencies of the forcing function at many different positions. The problem is to obtain an analytic expression for the system transfer function.

The need for an analytic expression arises particularly when the system under consideration is a part of a more complex system whose transient behavior and stability must be investigated. Consider for example a nuclear reactor in which the coolant is in a two-phase condition. Here, one is interested in the space dependent transfer function of the density of the coolant along the heated channels corresponding to a certain change in the reactor power. This transfer function may be determined experimentally by out-of-pile measurements. The nuclear parameters such as the diffusion coefficient, absorption cross section, removal cross section, etc., depend upon the coolant density. Consequently, any change in the coolant density results in a corresponding

reactivity feedback which in turn affects the transient behavior and stability of the reactor power and coolant flow.

THE TRANSFER FUNCTION

It is found that the space dependent transfer function of a system may be expressed as a series,^{1, 2, 3} in which each term is a product of a function of s , the Laplace transform variable representing time and a function of the space variable, x . Consequently, each term of the series gives the transient behavior of the corresponding space mode. The rapidity of convergence of the series would depend upon the choice of the space modes. One usually uses his qualitative understanding of the system behavior in selecting a set of space modes and, in many situations, some numerical experimentation is very useful. Sines and cosines, Legendre polynomials, Tchebyscheff polynomials, etc. are typical examples of representations of the space modes:

$$T(s, x) = T^0(s)P_0(x) + T^1(s)P_1(x) + T^2(s)P_2(x) + \dots \quad (1)$$

where P_0 , P_1 , etc., are the space modes, and T^0 , T^1 , etc., are the transfer functions for the time dependent part of the corresponding modes. Define $T^i(s)$ as

$$T^i(s) \equiv \frac{N^i(s)}{D^i(s)} = \frac{p_0^i + p_1^i s + p_2^i s^2 + \dots}{q_0^i + q_1^i s + q_2^i s^2 + \dots}, \quad (2)$$

where p_j^i and q_j^i are the unknown coefficients of a polynomial; they are to be determined from the frequency response data.

From the previous studies^{1, 2, 3} on the dynamic analysis of the type of systems not amenable to separability in space and time it is found that the natural frequencies of oscillation for each mode are the same.* In other words, the poles of the transfer functions $T^i(s)$ are identical for all i . The residues of these poles vary as a function of space, and hence one observes the

* This is an important consequence of representing the solution of a partial differential equation as a series.

variation of the amplitude and the phase delay of the transfer function along the x -direction. Therefore, when one synthesizes the transfer function at any space point, say x_k , as a ratio of two polynomials given by,

$$T_k(s, x_k) = \frac{A_k(s, x_k)}{B_k(s, x_k)}, \quad (3)$$

using the method given in Ref. 4, one may find that the coefficients of the denominator polynomials $B_k(s, x_k)$ do not vary appreciably with x_k , but those of the numerator polynomial $A_k(s, x_k)$ do. The variation with x_k of each of the coefficients of the numerator polynomial may be expressed as a series of any desired set of space modes. The coefficients of the series may be determined by minimizing the mean square error in their variation with x_k . Having obtained the spatial modal expansion of the coefficients of s^j of the numerator, one may, by rearranging, write the transfer function in the form

$$\begin{aligned} T(s, x) &= \frac{N^0(s)}{D(s)} P_0(x) + \frac{N^1(s)}{D(s)} P_1(x) + \dots \\ &= T^0(s) P_0(x) + T^1(s) P_1(x) \\ &\quad + T^2(s) P_2(x) + \dots \end{aligned} \quad (4)$$

Note that the denominator polynomial $D(s)$ is common to all modes.

THE METHOD OF SYNTHESIS

First of all, the transfer function $T_k(s, x_k)$ at the point x_k is synthesized as a ratio of two polynomials in s using the frequency response data (amplitude and phase delay for a band of frequencies) and employing the method given in Ref. 4:

$$T_k(s, x_k) = \frac{A_k(s)}{B_k(s)} = \frac{a'_{k0} + a'_{k1}s + a'_{k2}s^2 + \dots}{b_{k0} + b_{k1}s + b_{k2}s^2 + \dots} \quad (5)$$

In this method the coefficients of the polynomials $A_k(s)$ and $B_k(s)$ are determined by minimizing the mean square error in the amplitude of the transfer function over all the experimental frequencies.

This method is chosen because it is applicable to both minimum and nonminimum phase transfer functions, and the criterion used in the synthesis procedure is quite realistic (see Refs. 5-10 for other methods).

Even though one does not expect any variation for the denominator coefficients b_{k0}, b_{k1} , etc., the experimental and round-off errors introduce a slight variation with x .

Two methods are suggested to obtain the common denominator $B(s)$ of $T_k(s, x_k)$ for each k .

(1) One may take the average of each of the co-

FIG. V-11-1. Illustration of Space Dependent Response. ANL Neg. No. 112-5338.

efficients b_{kj} of s^j for all k :

$$b_j = \frac{1}{n} \sum_{k=1}^n b_{kj}, \quad (6)$$

where n is the number of space points. The common denominator is expressed as

$$B(s) = b_0 + b_1s + b_2s^2 + \dots \quad (7)$$

(2) One may factor each of the polynomials $B_k(s)$ for all k to obtain the pole locations. These pole locations are then averaged for all k and the common denominator polynomial is constructed from the averaged pole locations.

It is found that the second method is considerably more tedious and is not worth the trouble, except when the variations of the coefficients are large, say 1% or more of their average values.

After obtaining the common denominator, it is necessary to reconstruct the numerator polynomials from the frequency response data for all x_k 's, using the known common denominator, $B(s)$. This is done as follows. Define a new function of s and x

$$R(s, x) = T(s, x)B(s). \quad (8)$$

Frequency response data, similar to $T(s, x_k)$ is now obtained for $R(s, x_k)$, since $R(s, x_k)$ is known for all k 's. Let

$$R(s, x_k) \approx a_{k0} + a_{k1}s + a_{k2}s^2 + \dots \quad (9)$$

The coefficients a_{k0} , a_{k1} , etc., may be determined by employing the minimum mean square error in the amplitude of $R(s, x_k)$ as the criterion. Note that a_{k0} , a_{k1} , etc. may be slightly different from a'_{k0} , a'_{k1} , etc.

Therefore, one may write:

$$T_k(s, x_k) \approx \frac{a_{k0} + a_{k1}s + a_{k2}s^2 + \dots}{b_0 + b_1s + b_2s^2 + \dots} \quad (10)$$

In what follows, the x -variation of each of the coefficients a_{k0} , a_{k1} , etc., of the numerator polynomial of $T_k(s, x_k)$ is considered. Note that the coefficients a_{k0} , a_{k1} , etc., are real functions of x . The functional dependencies of the coefficients are explicitly stated by writing

$$a_{kj} = a_j(x_k). \quad (11)$$

Consider any coefficient $a_j(x)$. Its x -variation can be expressed by writing

$$a_j(x) \approx \lambda_{j0}P_0(x) + \lambda_{j1}P_1(x) + \lambda_{j2}P_2(x) + \dots \quad (12)$$

Let there be $N + 1$ terms in the right-hand member. In Eq. (1), the left-hand side is known for a set of k . Therefore, by minimizing the mean square error, one may determine the λ_{jt} for any particular choice of a set of space modes $P_0(x)$, etc. There are some powerful digital computer programs for the determination of these coefficients if the space modes are chosen as sines and cosines,¹¹ Legendre polynomials,¹² etc. If the set considered is the set of eigenfunctions of a certain differential equation in x (chosen on some physical grounds), some digital computer programming may become necessary to determine the λ_{jt} . The procedure however, is much the same as the standard procedures referred to before, except that there is now a different set of space modes, namely the eigenfunctions $P_j(x)$. Consider

$$T(s, x) = \frac{a_0(x) + a_1(x)s + a_2(x)s^2 + \dots}{b_0 + b_1s + b_2s^2 + \dots} \quad (13)$$

Let M be the degree of the numerator and denominator polynomials in Eq. (13). Substituting expressions for $a_0(x)$, $a_1(x)$, etc., from Eq. (12)

$$T(s, x) = \frac{\sum_{j=0}^M \sum_{t=0}^N \lambda_{jt} P_t(x) s^j}{b_0 + b_1s + b_2s^2 + \dots} \quad (14)$$

By rearranging,

$$T(s, x) = \frac{\sum_{t=0}^N \sum_{j=0}^M \lambda_{jt} s^j P_t(x)}{b_0 + b_1s + b_2s^2 + \dots} \quad (15)$$

Define

$$\sum_{j=0}^M \lambda_{jt} s^j \equiv N^t(s), \quad (16)$$

$$b_0 + b_1s + b_2s^2 + \dots \equiv D(s), \quad (17)$$

$$\frac{N^t(s)}{D(s)} \equiv T^t(s). \quad (18)$$

Substituting Eqs. (16)–(18) into Eq. (15),

$$\begin{aligned} T(s, x) &= \frac{N^0(s)}{D(s)} P_0(x) + \frac{N^1(s)}{D(s)} P_1(x) \\ &\quad + \frac{N^2(s)}{D(s)} P_2(x) + \dots \quad (19) \\ &= T^0(s)P_0(x) + T^1(s)P_1(x) \\ &\quad + T^2(s)P_2(x) + \dots \end{aligned}$$

This completes the synthesis procedure.

The following is a sample problem. Consider a frequency response function $T(s, x)$ from which the required frequency response data (amplitude and phase) are obtained for a set of 20 frequencies at 10 different values for x :

$$T(s, x) = \frac{0.2 - 0.1s}{1 + s + s^2} P_0(x) + P_1(x) \frac{0.1 - 0.2s}{1 + s + s^2}.$$

The transfer function synthesized is

$$\begin{aligned} T(s, x) &= \frac{0.2001 - 0.09980s + 0.0003s^2}{1 + 1.0002s + 0.999s^2} P_0(x) \\ &\quad + P_1(x) \frac{0.1003 - 0.2002s + 0.0005s^2}{1 + 1.0002s + 0.999s^2}. \end{aligned}$$

Note that the transfer function of each of the space modes P_0 and P_1 has zeros on the right-half plane, and hence nonminimum phase. The computing time on the Control Data 3600 computer was about one second for this example. Running time for a transfer function with 4 space modes each, having transfer functions with 4 poles and 4 zeros is found to average approximately 48 seconds.

REFERENCES

1. E. R. Quandt, *Analysis and Measurement of Flow Oscillations*, Chem. Eng. Progress Symposium Series, J. Heat Transfer, **57**, No. 32.
2. H. L. Garabedian and C. B. Leffert, *A Time-Dependent Analysis of Spacial Flux Distributions*, Nucl. Sci. Eng., **6**, 26 (1959).

1. C. K. Sanathanan, *Dynamic Analysis of Coolant Circulation in Boiling Water Nuclear Reactors*, ANL-6847 (1964).
4. C. K. Sanathanan and J. Koerner, *Transfer Function Synthesis as a Ratio of Two Complex Polynomials*, IEEE Transactions on Automatic Control, Vol. AC-8, No. 1 (1963).
5. E. C. Levy, *Complex Curve Fitting*, IRE Trans. On Automatic Control, Vol. AC-4, 37-44 (May 1959).
6. A. A. Kardashov and L. V. Karnyushin, *Determining System Parameters from Experimental Frequency Characteristics*, Automation and Remote Control **19**, 327 (1958).
7. V. M. Rushchinskii, *Determining an Approximate Expression for a Transfer Function of a Control Object from its Experimental Frequency Characteristics*, (in Russian) from a collection, "Production Automation", Acad. Sci., U.S.S.R. Press, Moscow, U.S.S.R., 2nd Ed. (1958).
8. G. I. Monastyrshin, *Processing Experimental Frequency Characteristics* Automation and Remote Control **21**, 294-300 (March 1960).
9. R. C. Nicholson and A. K. Orick, *A Least Squares Approach to Rational Functions*, IBM Corp., New York, N. Y., IBM-704 SHARE Distribution No. 859 (March 9, 1960).
10. C. F. Chen and W. Lipinski, *A Method for Determining EBWR Nuclear Reactor Transfer Functions from Frequency Response Data*, IEEE Region III Energy Conversion Meeting, Clearwater, Florida, May, 1964.
11. A. Raman, *Trigonometric Integrals*, Program Library AN-DI-08, Applied Mathematics Division, Argonne National Laboratory (FORTRAN-IBM-704), (November 1960).
12. G. J. Duffy, *LEGENDREVILLE—Least Square Legendre Polynomial Fit to Get the Best Set of Coefficients*, Program Library 1180/RE256, Applied Mathematics Division, Argonne National Laboratory, (June 1961).

V-12. The Neutron Flux Distribution in a Subcritical Multiplying Medium with a Constant External Source

D. H. SHAFTMAN

INTRODUCTION

The purposes of this contribution are two-fold. First, it summarizes the mathematical development of the method that has been used in solving a specific reactor problem for the Argonne Advanced Research Reactor (see Paper II-7). This problem concerns the placement of a neutron detector for reactor startup, when there is a background thermal neutron flux from photo-neutrons produced in the beryllium reflector after reactor shutdown. Second, although the basic method has been applied by various authors to the solution of a wide variety of reactor physics problems, it is hoped that additional insight will be provided by the coherent and dual solutions developed in this paper.

The setting for this development is multigroup diffusion theory, but this is not an essential restriction.

THE ONE-SHOT METHOD

Consider the N -group diffusion equations for a subcritical, multiplying medium with a time-independent external source, $S_j^e(\mathbf{r})$, in neutron energy group j and at vector position $\mathbf{r}$. The asymptotic solution, which will be designated as $\{\phi_j(\mathbf{r})\}_{j=1}^N$, satisfies the equations,

$$0 = \nabla \cdot D_j \nabla \phi_j - \Sigma_{rem,j} \phi_j + \sum_{k=1}^N \Sigma_{source}(k \rightarrow j) \phi_k + S_j^e, \quad 1 \leq j \leq N. \quad (1)$$

In Eq. (1), $\Sigma_{rem,j} \phi_j$ includes absorption as well as neutron slowing out of group j . The term,

$\Sigma_{source}(k \rightarrow j) \phi_k$ includes all events (fission, slowing down, etcetera) in group k which introduce neutrons into group j .

Let $\phi_j \equiv \sum_{m=0}^{\infty} \psi_{m,j}$, and substitute this series representation in Eq. (1) to get

$$0 = \nabla \cdot D_j \nabla \sum_{m=0}^{\infty} \psi_{m,j} - \sum_{m=0}^{\infty} \Sigma_{rem,j} \psi_{m,j} + \sum_{m=0}^{\infty} \sum_{k=1}^N \Sigma_{source}(k \rightarrow j) \psi_{m,k} + S_j^e, \quad 1 \leq j \leq N. \quad (2)$$

For $k_{eff} < 1$, it is well known that the representation for ϕ_j has convergence properties which justify the various multiplications and rearrangements of terms involved in going from Eq. (1) to Eq. (2). Indeed, the leakage operator also may be applied term by term in Eq. (2). The resulting equation may be solved from the recursion relationship:

$$0 = \nabla \cdot D_j \nabla \psi_{0,j} - \Sigma_{rem,j} \psi_{0,j} + \sum_{k=1}^N \Sigma_{source}^*(k \rightarrow j) \psi_{0,k} + S_j^e, \quad 1 \leq j \leq N, \quad (m=0)$$

$$0 = \nabla \cdot D_j \nabla \psi_{m,j} - \Sigma_{rem,j} \psi_{m,j} + \sum_{k=1}^N \Sigma_{source}^*(k \rightarrow j) \psi_{m,k} + \chi_j \sum_{k=1}^N (\nu \Sigma_f)_k \psi_{(m-1),k}, \quad 1 \leq j \leq N; m \geq 1. \quad (3)$$

In Eq. (3), $\{\chi_j\}_{j=1}^N$ is the fission-spectrum vector, assumed independent of the incident-neutron energy. This assumption, also, is not an essential restriction to the applicability of the method. The term $\Sigma_{\text{source}}^*(k \rightarrow j) \equiv \Sigma_{\text{source}}(k \rightarrow j) - \chi_j(\nu \Sigma_f)_k$.

Thus, Eqs. (3) yield the flux component $\psi_{m,j}$ as the time-asymptotic flux, in group j , in a non-multiplying medium with a time-independent "external" source, $S_{m,j}$, where

$$S_{m,j} \equiv \begin{cases} S_j^e, & \text{for } m = 0 \\ \chi_j \sum_{k=1}^N (\nu \Sigma_f)_k \psi_{(m-1),k}, & \text{for } m \geq 1 \end{cases}$$

The source $S_{m,j}$ is defined by finite induction in terms of the solution, $\{\psi_{(m-1),j}\}_{j=1}^N$, of the preceding problem in the sequence.

In the parlance of the REX¹ multigroup diffusion theory program for neutron flux calculations in one-dimensional geometries, each such problem is a "one-shot problem". Thus, if the initial external source distribution, S_j^e , and the reactor medium are radially or axially symmetric, and if S_j^e has the shape of the fundamental-mode distribution in the other directions, the m th component, $\{\psi_{m,j}\}_{j=1}^N$, may be solved with this one-shot option of REX. (It is implicitly assumed that otherwise REX is suitable for the problem, and vice versa.) One must then sum the components to obtain the total flux.

For some applications, it is of interest to obtain these successive-generation solutions, $\{\psi_{m,j}\}_{j=1}^N$, rather than simply to solve for the sum of these components. If, however, it is the sum which is sought, it is essential to be able to estimate the remainder, $\sum_{m=M+1}^{\infty} \psi_{m,j}$, after $M+1$ one-shot problems have been solved. To do so, assume that M is sufficiently large that the distributions $\psi_{M,j}$ have the "same" spatial shape as $\psi_{(M-n),j}$; $\psi_{(M-n+1),j}$; ...; $\psi_{(M-1),j}$, for some specified integer, n . What is meant here is that the shapes agree to within a preassigned error allowance, over a preassigned portion of the reactor. Presumably, then, for $m \geq M$, the $\psi_{m,j}$ also have the same shape as the $\psi_{M,j}$. It is assumed that the practical computing-machine problems of numerical solution, round-off, etcetera, have not become obnoxious.

As associated information, $\hat{k}_{effM}$ is obtained as the ratio of the total output source to the total input source for the $(M+1)$ th problem. Approximately, $\hat{k}_{effM}$ is the effective multiplication factor for the M th source distribution. Then $\hat{k}_{effm} \simeq \hat{k}_{effM}$ for all $m \geq M$. Consequently, $\psi_{(M+1),j} \simeq \hat{k}_{effM} \psi_{M,j}$, and, in general, $\psi_{m,j} \simeq \hat{k}_{effM}^{m-M} \psi_{M,j}$ for $m \geq M$. Therefore the remainder is estimated as

$$\sum_{m=M+1}^{\infty} \psi_{m,j} \simeq \frac{\hat{k}_{effM}}{1 - \hat{k}_{effM}} \psi_{M,j}$$

and the flux is given approximately as

$$\phi_j \simeq \sum_{m=0}^{M-1} \psi_{m,j} + \frac{1}{1 - \hat{k}_{effM}} \psi_{M,j}. \quad (4)$$

In particular, at any point r , the ratio of the multiplied group- j flux to the unmultiplied group- j flux is $\phi_j(r)/\psi_{0,j}(r)$.

THE DIRECT METHOD

An alternative method, for solving for the flux in a subcritical multiplying medium with a constant external source, also involves a sequence of one-shot problems. For a given problem in this sequence, the input source is the sum of the initial, external source, $\{S_j^e\}_{j=1}^N$, and the output source of the preceding one-shot problem. This is equivalent to solving:

$$0 = \nabla \cdot D_j \nabla \tilde{\psi}_{0,j} - \Sigma_{remj} \tilde{\psi}_{0,j} + \sum_{k=1}^N \Sigma_{\text{source}}^*(k \rightarrow j) \tilde{\psi}_{0,k} + S_j^e, \quad 1 \leq j \leq N;$$

and, for $m \geq 1$:

$$\begin{aligned} 0 &= \nabla \cdot D_j \nabla \tilde{\psi}_{m,j} - \Sigma_{remj} \tilde{\psi}_{m,j} \\ &+ \sum_{k=1}^N \Sigma_{\text{source}}^*(k \rightarrow j) \tilde{\psi}_{m,k} \\ &+ [S_j^e + \chi_j \sum_{k=1}^N (\nu \Sigma_f)_k \tilde{\psi}_{(m-1),k}]. \end{aligned} \quad (3')$$

The relationship between the $\tilde{\psi}_{m,j}$ and the terms $\psi_{n,j}$ discussed earlier is that $\tilde{\psi}_{m,j} = \sum_{n=0}^m \psi_{n,j}$. The proof is trivial; it is based upon the linearity of Eqs. (3) and (3') plus an application of the method of finite mathematical induction.

Therefore, $\phi_j = \lim_{m \rightarrow \infty} \tilde{\psi}_{m,j}$, $1 \leq j \leq N$.

If the sequence of one-shot problems is terminated after $M+1$ steps, yielding $\tilde{\psi}_{M,j}$ there remains the problem of estimating the remainder—which might be much larger than $\tilde{\psi}_{M,j}$ if $\hat{k}_{eff}$ is very close to unity! To do this, one determines $\hat{k}_{effM}$ and the asymptotic shape of the incremental flux components—say $\tilde{\psi}_{M,j} - \tilde{\psi}_{(M-1),j}$.

A reasonable definition for $\hat{k}_{effM}$ is:

$$\hat{k}_{effM} \equiv \frac{\int_{\text{reactor}} \sum_{k=1}^N (\nu \Sigma_f)_k [\tilde{\psi}_{M,k} - \tilde{\psi}_{(M-1),k}] dV}{\int_{\text{reactor}} \sum_{k=1}^N (\nu \Sigma_f)_k [\tilde{\psi}_{(M-1),k} - \tilde{\psi}_{(M-2),k}] dV}. \quad (4')$$

Then $\hat{k}_{effM} = \hat{k}_{effM}$, and, for $1 \leq j \leq N$,

$$\phi_j \simeq \tilde{\psi}_{(M-1),j} + \frac{1}{1 - \hat{k}_{effM}} [\tilde{\psi}_{M,j} - \tilde{\psi}_{(M-1),j}].$$

they should be, Eqs. (4') are the same as Equation (1).

EXAMPLE (SEE PAPER II-7)

The first one-shot method has been applied to determine the unmultiplied thermal (group-16) flux in the outer radial reflector of the shut-down AARR. Figure 2 of Paper II-7 shows how rapidly the shape of successive-generation fluxes approaches the asymptotic shape, even though the external (photo-neutron) source yields an unmultiplied neutron flux whose shape is very much different. Except for normalization, the compo-

nent fluxes in that figure have the relationship $\psi_m \equiv \psi_{m,16}$, for all m . For $m \geq 1$, all ψ_m were normalized to a fixed total fission neutron source rate from the core.

One must specify convergence criteria quite carefully, in applying this method to machine programs. When $\hat{k}_{effM}$ is very close to unity, a small deviation of $\hat{k}_{effM}$ from $k_{eff}^{asymptotic}$ may result in a large error in the estimate of the flux remainder, where the factor $1/(1 - \hat{k}_{effM})$ appears.

REFERENCES

1. *Reactor Physics Constants*, ANL-5800, 2nd Edition (1963), pp. 761-762 (Note: RP-122 is referred to also as REX.)

V-13. The Americium-Alpha-Lithium Neutron Spectrum

E. F. BENNETT

The Am-241- α -Li-7 reaction is interesting primarily because of the softer spectrum of emitted neutrons compared with those from reactions involving beryllium although the yield is lower by about a factor of 30.⁽¹⁾ Measurements have been made on the Po- α -Li neutron spectrum by D. Barton,² and a survey paper by W. Hess³ contains a theoretical calculation of this spectrum. Americium alphas are somewhat more energetic (180 keV) than are those from polonium.

The Am- α -Li neutron spectrum is well suited for study using the 4π -recoil technique.⁴ The reaction proceeds entirely through the Li-7 isotope; the threshold for Li-6 (α, n) is well above the energy of emitted alpha particles. The Q value for the reaction $\text{Li-7} + \text{He-4} \rightarrow \text{B-10} + n$ is -2.78 MeV and the ground state is the only level energetically possible for the residual B-10 nucleus. The maximum neutron energy is about 1.4 MeV.

The source contained 0.43 g of AmO₂ mixed with 5.0 g of natural lithium; total neutron emission was about 1.0×10^5 per second. The oxide powder and lithium metal were encapsulated in stainless steel; the temperature was raised above the melting point of lithium, and the source was shaken to obtain mixing. A secondary stainless steel shell was used to contain the source capsule. Total mass of source and container was 85 g and the external dimensions were 3.2 cm in diameter and 3.5 cm long.

Spectrum measurements were made using two different recoil-proton counters. The first was 5.0 cm in diameter with an effective length of 15 cm and was filled with CH₄ to a total pressure of $2\frac{1}{2}$ atm, including 10 cm

Hg of N₂ for calibration. Recoil-proton spectra between 75 keV and 1400 keV were recorded with this detector. Measurements in the energy interval between 20 keV and 110 keV were made using another probe having a 2.5-cm diameter and a 6.5-cm effective length and filled to a pressure of 3 atm of H₂ (with 20 cm Hg of CH₄ for quenching and 10 cm Hg of N₂ for calibration). The methane-filled counter had a wall thickness of 0.8 mm while the hydrogen-filled counter wall was 1.2 mm thick. The methane counter was enclosed in a concentric cylindrical lead shield 3.2 mm thick and covering the effective length, while the hydrogen counter was inserted within a $5 \times 5 \times 14$ cm lead block.

The lead used on both counters was sufficient to provide additional gamma shielding from the source without seriously distorting the shape of the neutron spectrum. In spite of this shielding, a copious flux of gamma rays was registered by both detectors over most of the region of their use. Some of this background may have been associated with scattering and absorption of source neutrons in the structural environment of the detectors but the major portion was from radioactive material in the source itself.

The source was placed at the side of the counters, as near as possible to the active counting volume, and was centered upon the active counting length. No measurements were taken with source and detector aligned axially. The weak source intensity precluded such an orientation although wall-effect distortion at the higher energies would have been lessened. Source and detector were suspended well away from floor and walls and it was established (by observing counting rate change

FIG. V-13-1. Neutron Spectrum From the Am- α -Li Reaction. ANL Neg. No. 112-4604.

versus source-detector separation) that no significant room return existed. Structural material, other than that associated with the counter and preamplifier, was of aluminum and was kept to a minimum.

Recoil-proton spectra were recorded for successive voltage increments corresponding to a factor of about two in energy. Spectra for the methane and hydrogen detectors overlapped between about 70 keV and 110 keV and were normalized over this region. Counts were accumulated for a total of about six days.

No correction for a variation of W , the average energy loss per ionized atom, with energy was applied to the methane results. The range of 1 MeV protons in the 5-cm diameter methane counter is 1.0 cm and a significant correction for distortion of the measured recoil-proton spectrum by wall and end effects above 1 MeV is required, although no correction was made to the data presented here.

Figure V-13-1 contains the neutron spectrum derived from this experiment (resolution is approximately 20%) together with the earlier results of Barton² using a diffusion cloud chamber and also the spectrum calculated by Hess.³ The experiments are in rough agreement and differ from the theoretical prediction. Reasons for this discrepancy have been offered by Hess. Since americium does not alloy with lithium, a mechanical mixture is

required and if the diameter of AmO₂ particles is comparable to the alpha particle range, then fewer energetic alphas penetrate to the lithium where (α, n) reactions occur. The neutron spectrum was calculated under the assumption of uniform mixing of alpha emitter and lithium reaction material. In addition to the mixing problem, degradation of the neutron energy also occurs by elastic scattering with lithium in the source and, to a much lesser extent, by inelastic scattering with the structural environment and lead shielding around the counter.

There are additional features of the measurements that should be noted. The spectrum rises gradually as energy drops, goes through a rather sudden increase at about 250 keV after having started to level off, and then falls rapidly as energy decreases below 50 keV. Statistics in the spectrum are within $\pm 3\%$ above 250 keV but worsen as energy decreases. The neutron spectrum is proportional to the slope of the recoil-proton distribution whereas the error in the spectrum (which is the error in the slope) is related to the square root of the total counts in the recoil-proton distribution. The recoil-proton distribution at low energies contains a large number of events arising from neutrons of relatively high energy. Because of the systematics of n-p scattering, energetic neutrons give rise to a flat recoil-proton distribution at lower energies. Consequently, the slope at low energy will be zero in response to fast neutrons but the statistical error in the slope will be larger than would have been the case if neutrons at higher energies were absent. Difficulty will generally be experienced in obtaining good precision at low energies where the spectrum is falling to zero rapidly as for the Am- α -Li result.

REFERENCES

1. E. F. Bennett, *Fast Neutron Spectroscopy by 4 π -Recoil Proportional Counting*, Nucl. Sci. Eng. (to be published).
2. D. M. Barton, *Measurement of the Neutron Spectrum from a Po-Li-7 Neutron Source*, LA-1609 (July 1953).
3. W. N. Hess, *Neutrons from (α, n) Sources*, Ann. Phys. **6**, 115 (1959).
4. E. F. Bennett, *A Study of the 1/E Slowing-Down Neutron Spectrum Using 4 π -Recoil Proportional Counters*, ANL-6897 (September 1964).

V-14. Reactor Computation Development

B. J. TOPPEL

In order to satisfy more effectively the needs of the Reactor Physics Division in the area of development of computation methods and codes suitable for high speed computers, a Reactor Computations Development Group has been formed within the Division. This group is primarily concerned with the major codes involved in the Division's work and, in particular, it is involved in the development of an automated and consistent reactor computation system. Such a system includes, for example, a cross section preparation code, reactivity codes, a depletion code, etc. The Group is currently concerned with implementing the portion of the system which generates multigroup cross sections. The code concerned is designated as MC².⁽¹⁾

The MC² program calculates multigroup constants from a basic evaluated nuclear data file (ENDF). Given the user's input specifications, the program forms group cross sections for a large number of fine groups. These cross sections are then averaged over either a pre-specified or a calculated weighting spectrum to produce the coarse group constants to be used in the neutronics portion of the computation system.

MC² is not yet complete although the logical framework for the CDC-3600 version of the program is essentially complete. A description of the present status of MC² on the CDC-3600, as well as plans for the next generation computer version is given in another paper in this report (see Paper V-16).

The codes making use of the multigroup cross sections comprise the neutronics computation part of the system. These codes provide solutions to various approximations of the Boltzmann equation, and will permit isotopic depletion studies, cost analysis studies, and various auxiliary calculations, such as perturbation calculations, neutron inventory calculations, and group contracting procedures to be performed.

Neutronics computations can supply input to, as well as receive output from the multigroup cross section computation portion of the overall system. Thus for example, the effects of spectral changes due to isotopic depletion during a burnup calculation can be made to modify the cross sections used in calculating the depletion, so that an iterative solution to the problem is effected. This automated interaction between the neutronics computations and MC² is intended for implementation on the next generation computer.

Various portions of the neutronics computations can be linked to expedite problem solution or economize on

FIG. V-14-1. Pictorial Representation of the Reactor Computation System. ANL Neg. No. 112-5140.

computing time. Examples of such interaction are: iteration between slab and cylindrical geometry in one dimension; use of one-dimensional results as the starting point for a two-dimensional calculation; few space point and/or few energy group results as the starting point for a more complex problem either in one- or two-dimensional problems.

The solutions to the Boltzmann equation are provided by either multigroup diffusion theory, or various of the multigroup transport theory solutions. The latter are the Sn, integral transport, or the Pn or Bn approximations. A long range addition to the system will be a Monte Carlo solution. Both one- and two-dimensional solutions can be obtained, and in the latter case, in RZ, ZR, Rθ, and XY geometries.

The standard Argonne input is utilized, both with regard to problem specifications and cross section specifications.

Depletion studies are accomplished in a manner similar to the procedures used in the CYCLE codes. However, new burnup codes will allow for radioactive decay in the depletion chains and will provide for more elaborate burnup chains than does the present CYCLE. The burnup calculations will be used either together with an Sn or diffusion theory neutronics calculation.

Attention will be given to allowing detailed modes of

fuel management, particularly in the two-dimensional case. For such a problem, interactions of axial and radial blanket management with core fuel management result in a large number of possible modes of operation.

A pictorial representation of the computation system is shown in Fig. V-14-1. The intended flexibility of the system permits bypassing various phases of the overall

capability of the system and obtaining output at any of the stages along the way.

REFERENCES

1. D. M. O'Shea, *Multigroup Constants Code MC²*, Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010 (January 1965) p. 270.

V-15. The Argonne Revised THERMOS Code

B. J. TOPPEL

The THERMOS code, developed by H. Honeck,^{1, 2} computes the scalar thermal neutron spectrum as a function of position in a lattice by solving the integral transport equation with isotropic scattering. One dimensional slab or cylindrical geometry can be used.

The original version of the code was intended for use on the IBM 704/709/7090 machines.

Greater speed and the two bank memory capacity of the CDC-3600 computer at Argonne prompted a re-programming effort in order to relax some of the original restrictions on dimensions of variables, and to provide a much more elaborate output edit. In connection with the latter effort, a modification of the associated cross section library preparation code was required.

An important objective of the revision was the development of a program which would produce directly all thermal group cross sections to be used with a multi-group cross section set. The revised code produces flux-averaged values of Σ_a , Σ_f , $\nu\Sigma_f$, and Σ_s , and D for the cell composition; for the isotopic constituents it supplies values for Σ_a , Σ_f , $\nu\Sigma_f$, Σ_s , and Σ_{tr} .

The usefulness of THERMOS has also been greatly increased by modifications which permit an automated sequence in which one problem generates a correct energy-dependent spatially-averaged set of cross sections for use in the next problem. This "pseudo-material" generating feature facilitates the solution of geometrically complex systems.

In generating pseudo-cross sections, the code computes the cross sections averaged over regions NRI to NRF using

$$\bar{\Sigma}_x(j) = \sum_{i=NRI}^{NRF} \Sigma_x(i, j) \frac{V_i \bar{\phi}_i(j)}{V \bar{\phi}(j)}$$

where x is absorption, fission, etc., j is the energy group, i is a region index ranging over region numbers NRI

TABLE V-15-I. COMPARISON OF PROBLEM DIMENSIONS FOR ORIGINAL AND ARGONNE-REVISED THERMOS CODES

Variable	Original Code	Revised Code
Number of space points	20	20
Number of speed groups	30	50
Number of isotopes used in the cell mixture	10	25
Number of isotopes used in the edit	10	25
Number of mixtures	5	5

to NRF , $\Sigma_x(i, j)$ is the region and group dependent cross section, V_i is the volume of region i , V is the sum of the V_i , $\bar{\phi}_i(j)$ is the region i average flux in group j , and $\bar{\phi}(j)$ is the average group j flux over the regions concerned. In the case of the diffusion constant, Σ_x is replaced by $1/[3\Sigma_{tr}(i, j)]$ where $\Sigma_{tr} = \Sigma_a + (1 - \bar{\mu})\Sigma_s$. The code allows the user to enter V_i values different from those used in computing the $\bar{\phi}_i(j)$, if desired.

The pseudo-materials are both written on a temporary tape and punched out on cards. The pseudo-materials may be used either in automated sequential problems, or at some later time employing the cards.

The original THERMOS methods of solution² have not been altered. However, considerable revision has been made in the subroutines concerned with calculation of the slab and cylindrical transport kernels so that the code now correctly allows for the presence of void regions in the system. The original Argonne IBM-704 THERMOS code could not cope with such void regions.

Table V-15-I compares limitations of various problem parameters for the original and Argonne revised THERMOS code.

Using the larger memory capacity of the CDC-3600, the cylinder and slab geometry options are now input

tions of the code. The original version consisted of two separate codes, one for each geometry. Having both geometries in a single code greatly increases the flexibility of the system, particularly when utilizing the pseudo-material option.

A detailed description of the input and output of the code, together with code listings is given in ANL-7023.⁽³⁾

V-16. Multigroup Constants Code MC²

D. M. O'SHEA, B. J. TOPPEL and A. L. RAGO*

Development of the multigroup constants program, MC², has been continued since the last annual report.¹ Particular emphasis has been given to methods of calculation over the resonance region.

With the increased amount of core storage available on the CDC-3600 it has been possible to expedite the calculation of the Doppler broadened line shape functions. The ψ and χ functions are now obtained by interpolating on a table of the real and imaginary parts of the complex probability integral, $W(Z)$.² The following bivariate interpolation scheme is used whenever the real or imaginary arguments of the complex probability integral are less than six:

$$\begin{aligned} W(x_0 + ph, y_0 + qk) &= \frac{q(q-1)}{2} W_{0,-1} + \frac{p(p-1)}{2} W_{-1,0} \\ &+ (1 + pq - p^2 - q^2) W_{0,0} + \frac{p(p-2q+1)}{2} W_{1,0} \\ &+ \frac{q(q-2p+1)}{2} W_{0,1} + pq W_{1,1} \end{aligned}$$

where

$$h = x_{i+1} - x_i, k = y_{j+1} - y_j$$

and

$$W_{i,j} = W(x_{i+0}, y_{j+0})$$

are the tabulated values.

For arguments of the complex probability integral greater than six and less than twelve, the following rational approximation is used:

$$W(Z) = iZ \left(\frac{0.5124242}{Z^2 - 0.2752551} + \frac{0.05176536}{Z^2 - 2.724745} \right),$$

* Applied Mathematics Division.

REFERENCES

1. H. C. Honeck, *The Distribution of Thermal Neutrons in Space and Energy in Reactor Lattices Part 1: Theory*, Nucl. Sci. Eng. **8**, 193 (1960).
2. H. C. Honeck, *A Thermalization Transport Theory Code for Reactor Lattice Calculations*, BNL-5826 (September, 1961).
3. B. J. Toppel and I. Baksys, *The Argonne Revised THERMOS Code*, ANL-7023 (March, 1965).

where

$$Z = x + iy.$$

For arguments of $W(Z)$ greater than twelve the first two terms of the continued fraction expansion are used. This approach permits one to evaluate the ψ and χ functions in a length of time equivalent to 35 machine language multiplications at the expense of about 8000 words of machine storage for the table. The interpolation is accurate to about four figures.

Cross sections in the unresolved resonance region are computed by taking averages over Porter-Thomas distributions of the neutron and fission widths. The program calculates both s - and p -wave neutrons and it includes a summation over spin states in each case. The program also permits energy variation of the fission width in the unresolved region. A flux correction factor is used consistent with the treatment in the resolved resonance region.

In the case of fission, for example, for a particular angular momentum and spin state,

$$\sigma_f(E^*) = \frac{\int_0^\infty \int_0^\infty P_n(x) P_m(y) y \Gamma_f J(\xi, \beta) dx dy}{\int_0^\infty \int_0^\infty P_n(x) P_m(y) \int_{D/2}^{D/2} \frac{1}{\psi + \beta} dE dx dy} \quad (1)$$

where P_n and P_m are Porter-Thomas distributions of n and m degrees of freedom, respectively. ψ is the Doppler broadened line shape discussed earlier, $J(\xi, \beta)$ is Dresner's J function, given by

$$J(\xi, \beta) = \int_0^\infty \frac{\psi(\xi, x)}{\psi(\xi, x) + \beta} dx$$

and σ_p is the potential scattering (barns) per heavy atom. Γ_f is the average fission width and D is the

average spacing between levels. E^* is a preassigned energy at which a resonance is assumed.

An improved method of calculating the function $J(\xi, \beta)$ and the flux correction factor

$$\int_{-D/2}^{D/2} \frac{1}{\psi + \beta} dE$$

has been developed to avoid excessive computation times in the unresolved region. It is evident from the convergence profile map of the function $W(Z)$ that only two or three terms of the continued fraction² are required to achieve five figure accuracy whenever either the real or imaginary parts of $W(Z)$ is greater than six. It has been possible to evaluate these integrals to five significant figures using two 16-point Gaussian quadratures. Abscissa for the quadratures are chosen such that for a particular set of resonance data, the upper limit of the first quadrature becomes $ReZ = 6$. The remaining contribution to the integral is obtained from the second quadrature, plus an asymptotic part in the case of the function $J(\xi, \beta)$.

A slightly modified version of the Bell approximation³ has been employed to provide a heterogeneity correction in both the resolved and unresolved resonance calculations. (Subscript one refers to fuel and two to moderator.) An equivalent σ_p is calculated as

$$\sigma_p = \sigma_{p0} + \sigma_m + \frac{\Sigma_{e,1}^*}{N_0}$$

where

$$\Sigma_{e,1}^* = \Sigma_{e,1}(1 - C) \left(\frac{a}{1 + (a - 1)C} \right)$$

C = Dancoff factor

σ_{p0} = potential scattering

N_0 = number of heavy density atoms.

For example, for cylinders

$$\Sigma_{e,1} = \frac{S_1}{4V_1} = \frac{1}{2R_1},$$

$$C = 1 - \alpha;$$

$$\alpha = \alpha_B + \alpha_B^4(1 - \alpha_B),$$

$$\alpha_B = \frac{1}{1 + \frac{\Sigma_{e,1}}{\Sigma_{e,2}}};$$

$$\Sigma_{e,2} = \frac{S_1}{4V_2} = \frac{R_1}{2(R_2^2 - R_1^2)}.$$

Here R_1 and R_2 are the radii of the pin and a homogenized clad coolant region, respectively. A value of 1.35 for a is presently used. The Bell approximation has been modified to give slightly higher values of α at higher optical chord length. Energy dependence of the scattering cross sections is also considered in these calculations.

In the section of the program which calculates inelastic scattering cross sections, the transition between the energy range of resolved levels and the unresolved region, has been made more gradual. The larger low lying levels are now extended well into the region where many levels are yet unresolved. The unresolved levels are accounted for by the statistical model. The statistical model is normalized to the difference between the total inelastic curve and the sum of extended resolved levels.

An option for having a fission spectrum represented by a function of the type $\alpha\sqrt{E} \exp(-E/T)$ has been added.

REFERENCES

1. D. M. O'Shea, *Multigroup Constants Code MC²*, Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010, (January 1965) p. 270.
2. D. M. O'Shea and H. C. Thacher, Jr., *Computation of Resonance Line Shape Functions*, Trans. Am. Nucl. Soc. **6**, 36 (June 1963).
3. G. I. Bell, *A Simple Treatment for Effective Resonance Absorption Cross Sections in Dense Lattices*, Nucl. Sci. Eng., **5**, 138 (1959).

V-17. Aids for Maintaining and Checking an Evaluated Nuclear Data File

D. M. O'SHEA, J. J. ZENOR and P. L. WALKER

An evaluated nuclear data file (ENDF) is a list on magnetic tape of nuclear cross sections ordered by a large number of parameters such as reaction type, incident particle energy, target nucleus, etc. In general this data is prepared by averaging one or more sets of experimental data which may also be listed on magnetic tape.

Basic experimental data is characterized by details of the experiment such as when and where performed, by whom, by what method, etc. Frequently the data is presented in the technical literature in ways inappropriate for direct inclusion in a data file, from which cross section evaluations can be made.

FIG. V-17-1. Example of a Typical Mechanically Plotted Graph of Basic Data in the Data File. (Mechanical plot was traced for publication.) ANL Neg. No. 112-5649.

FIG. V-17-2. Example of a Typical Display, Produced by the CALCOMP Plotter, of a Cross Section in the Evaluated Cross Section Library. (Mechanical plot was traced for publication.) ANL Neg. No. 112-5648.

When experimental data have to be transcribed, reduced, or otherwise manipulated, as in the generation and maintenance of an ENDF, errors can occur. Moreover, the procedures used for the compilation and reduction of basic data and the transcription of evaluated data, may inadvertently introduce large errors which may go undetected into the main stream of reactor physics calculations. A family of service routines has been prepared, therefore, which enables one to compare quickly new basic data with data already in the data library and to ascertain that evaluated data on tape have been correctly transcribed. These checks can be made by using data on the tape to produce a replica of the original data. If the replica is faithful there is assurance that the transcription is correct. (Of course there may be analytical errors; these have not been considered here.)

Since past data reports and most current ones in the technical literature rely on graphical presentation, the chief service routine now in use provides a graph of the data in the basic data file, and the curve in the ENDF prepared by the evaluator in the same style and size as the original graphical presentations. A quick scan by

the human eye is all that is needed to detect most errors since the graphs can be overlayed. Scales can be linear or logarithmic and the numerical value of a given interval is set by the user.

The graphs are prepared using a Calcomp plotter for which input has been prepared by the service routines. Two examples are shown here. Figure V-17-1 is a plot of experimental data in the file on the fission cross section of Pu-240. The major differences between new and old data can be isolated quickly for evaluation. Figure V-17-2 is a plot of the evaluated data file for Pu-240 fission. This is the set of data to be used in calculating multigroup constants.

The examples given do not illustrate the problems which arise in evaluating partial cross sections which are computed from nuclear parameters. Frequently the only check is on the total cross section or the total absorption. In such a case checking evaluated partial cross sections requires a full display of the parameters used. Service routines can easily retrieve the data, but it is not easy to make the origin of discrepancies obvious in these cases.

V-18. MACH 1, A One-Dimensional Diffusion Theory Calculation System

D. A. MENELEY, L. C. KVITEK and D. M. O'SHEA

INTRODUCTION AND GENERAL SPECIFICATIONS

The MACH 1 diffusion program is the result of an effort to increase the scope and usefulness of routine one-dimensional diffusion calculations. To increase overall efficiency, problem setup and computation time were also reduced. The program embodies very little original material. It is primarily an amalgamation of several separate programs into a single unit.

The code has been written for a Control Data Corporation 3600 machine with 64 K core storage and at least six tape drives. Much of the coding is compatible only with the 3600 FORTRAN language. Fixed storage allocation has been used throughout; this has resulted in the following problem size limitations: 20 regions, 150 mesh points, 30 energy groups, and 15 downscatter groups. In addition, hydrogen scattering may be used allowing full downscatter for one type of hydrogen in each region. Microscopic cross sections for up to 20 nuclides may be used for any one problem; up to 20 homogenized materials may be specified.

The main components of MACH 1 are a modified version of AIM-6,⁽¹⁾ the DEL² perturbation code, 1188/RE⁽³⁾ (beta effective and prompt neutron lifetime), RP 130⁽⁴⁾ (inhour curve), a group collapsing routine and a reaction summary. Cross sections may be obtained from a binary library tape or with the problem data at run time. When a series of problems is being run only those data which change from problem to problem need be specified for any problem following the first. Problem skipping is available if one or more problems of a series do not run on the first trial. This approach significantly reduces manual card manipulations.

The program has been in successful operation since May, 1965. Setup time has been reduced significantly, particularly for problems requiring the ancillary subroutines. While there are several undesirable limitations to the system, it is felt that it provides experience for construction of a more comprehensive integrated computation network. Detailed program descriptions and data specifications are given in Ref. 6.

MODIFICATIONS AND ADDITIONS TO THE AIM-6 DIFFUSION SOLUTION

The procedure for solution of the multigroup equations is the same as that in AIM-6 with the following exceptions. First, a correction term has been added to the regional fission source integral in spherical geometry,

when the trapezoidal rule leads to large errors near the origin. The fission source extrapolation during inner iterations has been changed to a damped linear extrapolation of the resultant source from two previous iterations. Running time has thus been reduced appreciably. The composition search option has been generalized to allow any combination of nuclides to be treated as fuel and/or diluent.

A constant neutron source distributed in space and energy may be included. Up to fifteen different source energy spectra may be specified by region. The spatial source distribution is specified for each mesh point.

ANCILLARY SUBROUTINES

PERTURBATION

This routine provides first order perturbation calculations for nuclide or material cross sections, transverse buckling changes, or any combination thereof. Any section of the reactor may be perturbed in any one "pass"; any number of "passes" may be performed for one problem. The transverse limits of integration may be less than or equal to the transverse extrapolated dimension. Available output information includes point by point, group by group flux gradients and perturbation components, the group sums of these quantities, interval and regional integrals of reactivity changes, and the total change in reactivity.

REACTION SUMMARIES

The summaries are in general given for regions, although some group dependent and some pointwise information is available. The results include core conversion and breeding ratios, energy group flux integrals, group dependent effective bucklings in the direction of calculation, group dependent transverse leakages, regional source integrals and peak and average fission densities. Also given are the regional capture, fission, and source integrals for each nuclide. Total fission densities are normally printed for each mesh point.

The reaction summary may be normalized to any desired total reactor power. In this case, the absolute fluxes are also given.

BETA EFFECTIVE, LIFETIME, AND INHOUR EQUATION

The effective delayed neutron fraction and prompt neutron lifetime may be calculated using the standard perturbation method. Relative worths of prompt and delayed fission neutrons for each nuclide are also given.

layed group information for each nuclide is incorporated in the cross section library tape.

The resulting effective delayed fractions for each family of each nuclide may be used to calculate the inhours per percent reactivity and the inhour curve for the reactor.

GROUP COLLAPSING

Macroscopic cross sections for each reactor region may be collapsed to any fewer group structure. Flux integrals in each region are used to weight the initial group cross sections. Output may be obtained in the following formats: standard Argonne, MACH 1, or modified Los Alamos.

REFERENCES

1. H. P. Flatt and D. C. Baller, *The AIM-6 Code*, NAA Program Description (1961).
2. L. C. Kvitek, *Del, A Perturbation Code Written for the IBM-704*, ANL-7052 (1965).
3. L. C. Kvitek, *Multigroup Calculations of Effective Delayed Neutron Fractions, Prompt Neutron Lifetime, and Related Kinetics Parameters. IBM-704 Program 1188/RE*, ANL-6511 (March 1962).
4. D. A. Meneley, Argonne National Laboratory (Private Communication).
5. M. Butler and J. M. Cook, *Univac Programs for the Solution of One-Dimensional Multigroup Reactor Equations*, ANL-5437 (October 1956).
6. D. A. Meneley and L. C. Kvitek, Argonne National Laboratory (Private Communication).

V-19. Numerical Calculation of Laplace Transforms

R. A. MEWALDT* and H. C. THACHER, JR.

The Laplace transform, defined by

$$f(s) \equiv L\{F(t)\} \equiv \int_0^{\infty} e^{-st} F(t) dt, \quad (1)$$

and its inversion are well established tools for the analysis of linear systems.^{1, 2} In reactor physics, they can be applied in the analysis of the slowing-down^{2, 3} of neutrons as well as in the areas of reactor kinetics and heat transfer. Unfortunately, there are many functions for which the transform and its inversion cannot be performed analytically, so that methods are needed for estimating numerical values of $f(s)$ for several values of s from numerical values of $F(t)$ at a set of values of t , and vice versa. This article reports a numerical test of a well-known method for accomplishing this task, and on an analytical study which explains, and partially remedies, its defects.

NUMERICAL TESTS OF SALZER'S FORMULAS

The most promising existing method of estimating Laplace transforms from equally spaced numerical data appeared to be one due to H. E. Salzer.⁴ In this method, the function $F(t)$ to be transformed is approximated by an interpolation polynomial of degree $n - 1$, where n is the number of data points. The transform of the function resulting is easily evaluated from the elementary transform $L\{t^n\} = n!/s^{n+1}$. Inverse transforms may be estimated in similar fashion⁵ by fitting numerical values of $f(s)$ at points $s = 1, 2, \dots, m$ an interpolation polynomial in $1/s$, with constant n zero.

Fortran programs were written to test these methods. Although the program for estimating $L\{F(t)\}$ checked Salzer's example, $J_0(x)$, and gave about 3-decimal accuracy for $L\{J_0\}$ for $s = 1(1)10$, it became apparent that serious round off errors were being incurred because of the subtraction of large quantities of the same sign. Estimates of the inverse transform were also disappointing.

ANALYSIS OF SALZER'S FORMULAS

More detailed consideration of Salzer's approach revealed at least two areas requiring further analysis. Although the use of sufficient precision in the calculations will control the numerical instability which caused difficulty in the experimental calculations, this remedy is expensive, and more stable formulas would be advantageous. In the second place, the lack of an adequate estimate of the truncation error incurred by replacing $F(t)$ by an interpolating polynomial of finite degree is a serious deficiency. Such an estimate is required not only to appraise the validity of the results, but also to select an adequate scaling of the independent variable in carrying out the calculation.

It should be emphasized that the estimation of Laplace transforms from a few function values is inherently dangerous. The degree of uncertainty is emphasized by the following simple example: Consider the set of functions for which $F(2j) = 0, j = 0, 1, 2, 3, 4, 5$, while $F(2j + 1) = 1, j = 0, 1, 2, 3, 4$. A formula for estimating the Laplace transform of functions based on these points will give the same estimate for all functions in the set. A few typical members of this

* Student, Lawrence University, Appleton, Wisconsin.

class of functions, with their analytical transforms, and values of the transform are shown in Table V-19-I. These functions are all continuous, and have bounded first derivatives. Yet the Laplace transforms at $s = 1$ differ by a factor of 9. Moreover, Salzer's 11-point formula, which gives the result in the last column, is seriously in error for functions with relatively small first derivatives.

The most general linear formula for estimating Laplace transforms based on equally spaced function values including the origin may be written as:

$$f(s) = \frac{1}{s} \sum_{k=0}^n \alpha_k(s, h) F(k, h) + R_n(s, h; F) \quad (2)$$

where the coefficients α_k are independent of the function F , and $R_n(s, h; F)$ is the truncation error. The formulas of most interest will be exact whenever $F(t)$ is a polynomial of degree m or less, with $m \leq n$. (Salzer's formulas are obtained by taking $m = n$.) This requirement imposes a set of constraints,

$$\sum_{k=0}^n \alpha_k(s, h) k^j = \frac{j!}{(sh)^j}; \quad (j = 0, 1, 2, \dots, m), \quad (3)$$

upon the coefficients $\alpha_k(s, h)$. To simplify the subsequent relations, we have chosen coefficients $\alpha_k(s, h)$ which differ from Salzer's A_k by a factor s .

For fixed s and h , $R_n(s, h; F)$ which we will frequently abbreviate to $R_n F$, is a linear functional, i.e. a linear rule which associates a real or complex scalar with each function in the set under consideration. The following discussion draws rather heavily on the concepts of modern functional analysis. The reader who is interested in more background in this area will find helpful introductions in A. Taylor,⁶ and in L. Collatz.^{7, 8}

By an extension of the Peano-Sard analysis,⁹ we can prove the following theorem:

THEOREM. Let $F(t)$ and its derivatives of all orders through $\nu - 1$ be continuous, and let $F^{(\nu-1)}(t)$ be absolutely continuous in $0 \leq t < \infty$. Let the Laplace transform of $|F^{(\nu)}(t)|$ exist for a given value of $s > 0$. Then for any formula of the form Eq. (2), and for any finite non-negative β

$$R_n(s, h; F) = \sum_{j=0}^{\nu-1} \frac{F^{(j)}(\beta)}{j!} R_n[s, h; (t - \beta)^j] + \int_0^\infty F^{(\nu)}(\theta) K_\nu(s, h; \theta) d\theta \quad (4)$$

with

$$K_\nu(s, h; \theta) = \begin{cases} \frac{e^{-s\theta}}{s^\nu} - \frac{1}{s(\nu-1)!} \sum_{k=[\theta/h]+1}^n \alpha_k(s, h) (kh - \theta)^{\nu-1}, & (\theta > \beta) \\ \frac{1}{s^\nu} e^{-s\theta} - \sum_{k=0}^{\nu-1} \frac{(-s\theta)^k}{k!} + \frac{1}{s(\nu-1)!} \sum_{k=0}^{[\theta/h]} \alpha_k(s, h) (kh - \theta)^{\nu-1} & (\beta > \theta) \end{cases} \quad (5)$$

where $[\theta/h]$ is the largest integer which does not exceed θ/h .

The expression Eq. (4) is more flexible than is

TABLE V-19-I. LAPLACE TRANSFORMS OF SOME SIMPLE FUNCTIONS WITH VALUES 0 AT $t = 0, 2, 4, 6, 8, 10$ AND 1 AT $t = 1, 3, 5, 7, 9$

	$F(t)$			
	$\frac{(1 - \cos \pi t)}{2}$	$ \sin(\pi t/2) $	$ x - 2 \text{ entier}(\frac{t+1}{2}) ^n$	$P_{10}(t)^b$
	$f(s)$			
	$\frac{(1 - \frac{s^2}{s^2 + \pi^2})}{2s}$	$\frac{2 \coth s}{\pi^2 + 4s^2}$	$\frac{\tanh(s/2)}{s^2}$	$\frac{1}{s} \tilde{P}_{10}(1/s)$
$f(1.0)$	0.4540	0.5948	0.4621	4.0702
$f(2.0)$	0.1779	0.2519	0.1904	4.2884
$f(3.0)$	0.0872	0.1377	0.1006	3.1612
$f(4.0)$	0.0477	0.0851	0.0603	2.3977
$f(5.0)$	0.0283	0.0572	0.0395	1.8682
$f(6.0)$	0.0179	0.0408	0.0276	1.4907
$f(7.0)$	0.0120	0.0305	0.0203	1.2143
$f(8.0)$	0.0084	0.0236	0.0156	1.0067
$f(9.0)$	0.0066	0.0188	0.0123	0.8474
$f(10.0)$	0.0045	0.0153	0.0100	0.7226

^a This function represents a sawtooth with slope +1 on the intervals $2k < t < 2k + 1$ and -1 on the intervals $2k + 1 < t < 2k + 2$.

^b This is the 10th degree interpolation polynomial based on the specified values. Salzer's formula transforms this function exactly.

needed for most applications, and holds, for example, even if the formula is not exact for polynomials of degree $\nu - 1$. If we restrict ourselves to formulas which obey Eq. (3) for $m \geq \nu - 1$, it can be shown that the two forms of Eq. (5) are equivalent, and that the summation in Eq. (4) vanishes for all β , so that the error is simply:

$$R_n F = \int_0^\infty F^{(\nu)}(\theta) K_\nu(s, h; \theta) d\theta. \quad (6)$$

Since the integer ν is subject only to the requirements that $F^{(\nu)}(t)$ must exist, be integrable, and that it not exceed $m + 1$, the theorem may be made the basis for several different analyses. Although the estimates will often be sharper for larger values of ν , they apply to a smaller class of functions, and require the consideration of high-order derivatives which are frequently clumsy

to manipulate. It is not always advantageous therefore to select ν as large as possible. Furthermore, the kernel

action $K_\nu(s, h; \theta)$ is more complicated to transform into a conveniently applicable form.

The behavior of $K_\nu(s, h; \theta)$, and the information available about $F^{(\nu)}(\theta)$ will often vary considerably over the range $0 \leq \theta < \infty$. It is therefore convenient to subdivide this range into a set of intervals I_i ($i = 1, 2, \dots, \omega$), where θ is in I_i if $\delta_{i-1} \leq \theta < \delta_i$, and where $\delta_0 = 0, \delta_\omega = \infty$. With this subdivision,

$$R_n F = \sum_{i=1}^{\omega} \int_{\delta_{i-1}}^{\delta_i} F^{(\nu)}(\theta) K_\nu(s, h; \theta) d\theta \equiv \sum_{i=1}^{\omega} \rho_i F^{(\nu)}. \quad (7)$$

The task of computing the remainder has thus been reduced to evaluating the ω linear functionals $\rho_i F^{(\nu)}$. In most cases, this task is as difficult as evaluating the original Laplace transform, and far less worthwhile. It is possible under many circumstances, however, to find a realistic upper bound on the magnitude of the $\rho_i F^{(\nu)}$ which will allow us to appraise the possible error incurred by neglecting R_n . For this, we must have a bound on some measure of the magnitude of $F^{(\nu)}(\theta)$ on the various intervals. Although almost any norm could be used, the sup, or Chebyshev norm

$$\|F^{(\nu)}(\theta)\|_i = \text{ess. sup.}_{\theta \in I_i} |F^{(\nu)}(\theta)| \quad (8)$$

i.e., the smallest number M such that $|F^{(\nu)}(\theta)| < M$ for almost all θ in I_i , is among the most readily available, and we will use it as an example.

To establish a bound on the remainder, we take advantage of the fact that the functions $F^{(\nu)}(\theta)$ with the norm Eq. (8) form a normed linear vector space. Hence, the conjugate space of all linear functionals, $\rho F^{(\nu)}$, over this space is also a normed linear vector space, with norm

$$\|\rho\| = \text{ess. sup.}_{\|F^{(\nu)}\| \leq 1} |\rho F^{(\nu)}|. \quad (9)$$

If $\|F^{(\nu)}\| < \infty$, then $\|(F^{(\nu)}/\|F^{(\nu)}\|)\| = 1$ and hence Eq. (9) implies

$$\left| \rho \frac{F^{(\nu)}}{\|F^{(\nu)}\|} \right| = \frac{1}{\|F^{(\nu)}\|} |\rho F^{(\nu)}| \leq \|\rho\| \quad (10)$$

or

$$\rho F^{(\nu)} < \|F^{(\nu)}\| \cdot \|\rho\|. \quad (11)$$

Thus, if we can find the norms of the functionals ρ_i in Eq. (7) we can bound the error by

$$|R_n F| \leq \sum_{i=1}^{\omega} \|\rho_i\| \cdot \|F^{(\nu)}\|_i \quad (12)$$

It will be possible to find a function satisfying the specified conditions for which the equality in Eq. (12) is true.

By considering the function

$$F^{(\nu)}(\theta) = \begin{cases} +1, & K(s, h; \theta) > 0 \\ 0, & K(s, h; \theta) = 0 \\ -1, & K(s, h; \theta) < 0 \end{cases} \quad (13)$$

it is apparent that the norm on ρ_i corresponding to the Chebyshev norm on $F^{(\nu)}$ is

$$\|\rho_i\| = \int_{\delta_{i-1}}^{\delta_i} |K_\nu(s, h; \theta)| d\theta. \quad (14)$$

In some cases, particularly for the last semi-infinite interval I_ω , it may turn out that the Chebyshev norm of $F^{(\nu)}$ is unacceptably large. Under these circumstances it is frequently possible to introduce a suitable positive weighting function, $w(\theta)$, and to bound the weighted norm,

$$\|F^{(\nu)}\|_\omega^{(w)} = \|w F^{(\nu)}\|_\omega. \quad (15)$$

The corresponding norm on ρ_ω is then

$$\|\rho_\omega\|_\omega^{(w)} = \int_{\delta_{\omega-1}}^{\infty} \left| \frac{K_\nu(s, h; \theta)}{w(\theta)} \right| d\theta \quad (16)$$

and as before

$$|\rho_\omega F^{(\nu)}| \leq \|\rho_\omega\|_\omega^{(w)} \cdot \|F^{(\nu)}\|_\omega^{(w)}.$$

SAMPLING RANGE ERROR

We now have a general scheme for bounding the error in applying any formula to any function $F(t)$ for which we have appropriate norms. For any given formula and given values of s and h , we choose a ν and compute the kernel function, $K_\nu(s, h; \theta)$, by Eq. (5), evaluating the $\|\rho_i\|$ by Eq. (14). Then for any function $F(t)$, the error bound is given by Eq. (12). The kernel function ordinarily depends upon the formula being used. However, for $\theta > nh$, the last sample point, even this dependence vanishes. Thus, it is convenient to let the interval I_ω extend from the last sample point to ∞ , so that the corresponding error term becomes

$$\rho_\omega F^{(\nu)} = \int_{nh}^{\infty} F^{(\nu)}(\theta) e^{-s\theta} s^{-\nu} d\theta. \quad (17)$$

Because of the infinite interval, even so simple a function as $t^{\nu+1}$ has an unbounded norm on I_ω , and we must resort to a weighted norm to obtain a usable bound. A convenient and relatively powerful weight is the exponential $e^{-\gamma\theta}$, with $0 \leq \gamma \leq s$. With this weight,

$$\|\rho_\omega\|_\omega^{(w)} = \frac{s^{-\nu}}{s - \gamma} \exp[-nh(s - \gamma)] \quad (18)$$

and

$$|\rho_\omega F^{(\nu)}| \leq \frac{s^{-\nu}}{s - \gamma} \exp[-nh(s - \gamma)] \text{ess. sup.}_{nh \leq \theta < \infty} |e^{-\gamma\theta} F^{(\nu)}(\theta)|. \quad (19)$$

For large enough nh , the magnitude of $\|\rho_\omega\|^{(w)}$ is dominated by the exponential factor, while $\|F^{(\nu)}\|_\omega^{(w)}$ is a nonincreasing function of nh . Thus, we can, and should, take nh large enough to insure that $|\rho_\omega F^{(\nu)}|$ is adequately small for the smallest s to be considered.

We illustrate the process by considering $F(t) = t^{\nu+1}/(\nu+1)!$ for which $F^{(\nu)}(t) = t$. For any $\gamma > 0$, $F^{(\nu)}(\theta)e^{-\gamma\theta}$ attains its maximum value over the whole range at $\theta = 1/\gamma$, and this maximum is $1/\gamma e$. Thus,

$$\|F^{(\nu)}\|_\omega^{(w)} \leq 1/\gamma e \quad (20)$$

where the inequality holds only if $1/\gamma < nh$. The contribution to the error due to the finite sampling range is bounded by

$$|\rho_\omega F^{(\nu)}| \leq \frac{e^{-nh}}{s^{\nu+2}} H(\gamma/s) \quad (21)$$

where

$$H(y) = e^{nhsy-1}/y(1-y). \quad (22)$$

The sharpest estimate will be obtained by selecting γ to minimize $H(\gamma/s)$. The appropriate value is

$$\gamma = \frac{nhs + 2}{2nh} [1 - \sqrt{1 - 4nhs(nhs + 2)^{-2}}]. \quad (23)$$

For this value of γ it can be shown that

$$H < (nhs + 2)^3/4nhs. \quad (24)$$

It can also be shown that for this function the error due to the finite sampling range is at least as large as $nhse^{-nhs}/s^{\nu+2}$.

Although a detailed analysis, such as the one outlined, is required for absolute certainty, the dominance of the exponential factor in Eq. (19) suggests that except for functions which grow almost exponentially, it will be sufficient to select a sampling range such that e^{-nhs} is one or two orders of magnitude smaller than the desired relative accuracy.

FORMULA-DEPENDENT ERROR

For $0 \leq \theta \leq nh$, the kernel function, and hence the error, depends upon the coefficients in the particular formula being used. It is clear from Eq. (5) that the $(\nu-1)$ st derivative of $K_\nu(s, h; \theta)$ is discontinuous at the points $\theta = kh$. It is therefore advantageous to allow the boundaries of the elementary intervals I_i to fall at these singular points, and our analysis hence forward will assume that this choice has been made.

In the i th interval, $(i-1)h \leq \theta < ih$, the kernel may be written in the form:

$$K_\nu(s, h; \theta) = \frac{e^{-s\theta}}{s^\nu} - \frac{1}{s} \sum_{j=0}^{\nu-1} \frac{(-\theta)^{\nu-1-j}}{j!(\nu-1-j)!} S_i^{(j)}, \quad (25)$$

where the parameters

$$S_i^{(j)} \equiv \begin{cases} h^j \sum_{k=i}^n \alpha_k(s, h) k^j, & (i \leq n) \\ 0, & (i > n) \end{cases} \quad (26)$$

are independent of θ , but depend upon the formula.

ZERO-ORDER ANALYSIS

For $\nu = 1$, Eq. (7) applies to all formulas which transform constants exactly, and to all continuous functions with piecewise continuous first derivatives. The error kernel, which simplifies to

$$K_1(s, h; \theta) = \frac{e^{-s\theta}}{s} - \frac{1}{s} S_i^{(0)}, \quad (27)$$

is monotone decreasing except at the points $\theta = ih$, where it has a jump of $-\alpha_i$. The first part of $K_1(1, 1; \theta)$ for Salzer's 11-point formula is shown in Fig. V-19-I.

Because of the monotonicity, $K_1(s, h; \theta)$ can change sign at most once within each elementary interval. This sign change will occur at $\theta = -\ln S_i^{(0)}/s$. Thus, letting

$$d_i = \begin{cases} i-1, & e^{-sh(i-1)} < S_i^{(0)} \\ -\frac{1}{sh} \ln S_i^{(0)}, & e^{-sh i} < S_i^{(0)} \leq e^{-sh(i-1)} \\ i, & S_i^{(0)} \leq e^{-sh i} \end{cases} \quad (28)$$

employing Eq. (14) we may express the norms of the elementary error functionals as

$$\begin{aligned} \|\rho_i\| &= \int_{(i-1)h}^{ih} |K_1(s, h; \theta)| d\theta \\ &= \int_{(i-1)h}^{d_i h} K_1(s, h; \theta) d\theta - \int_{d_i h}^{ih} K_1(s, h; \theta) d\theta \end{aligned} \quad (29)$$

so that

$$\begin{aligned} \|\rho_i\| &= \frac{1}{s^2} [e^{-sh i} (e^{sh} - 2e^{sh(i-d_i)} + 1) \\ &\quad + sh S_i^{(0)} (2i - 2d_i - 1)]. \end{aligned} \quad (30)$$

Since the $S_i^{(0)}$ are easily calculated from the α_k , Eqs. (12), (28) and (30) give us a straightforward method of computing an explicit upper bound on the truncation error for any formula. Moreover, this bound is actually attained for the saw tooth function with derivative $+1$ when $K_1(s, h; \theta) \geq 0$ and -1 when $K_1(s, h; \theta) < 0$. The values of $\|\rho_i\|$ for Salzer's 11-point formula are shown in Table V-19-II for $h = 1$, and several values of s . The sum, in the last row, is a bound on the error for a function with first derivative bounded by ± 1 . On comparing with Table V-19-I, where the last column is Salzer's estimate, it is apparent that the estimate is not overconservative.

FIG. V-19-1. Error Kernel $K_1(1,1;\theta)$ For Salzer's 11-Point Formula. ANL Neg. No. 112-5341.TABLE V-19-II. NORMS OF ERROR FUNCTIONAL FOR SALZER'S 11-POINT FORMULA
(Zero-Order Analysis, $h = 1.0$)

$i \backslash s$	$\ \rho_i(s)\ $									
	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
1	0.1969	0.1535	0.1542	0.1377	0.1222	0.1094	0.0989	0.0902	0.0828	0.0765
2	0.3688	0.5393	0.4348	0.3472	0.2832	0.2363	0.2011	0.1741	0.1529	0.1360
3	0.6371	0.3061	0.1937	0.1296	0.0874	0.0585	0.0382	0.0237	0.0132	0.0054
4	0.8819	1.1020	0.8595	0.6747	0.5414	0.4441	0.3716	0.3163	0.2731	0.2388
5	0.8353	0.6975	0.4875	0.3560	0.2662	0.2028	0.1569	0.1228	0.0971	0.0773
6	0.5274	1.0023	0.7861	0.6198	0.4994	0.4113	0.3453	0.2947	0.2552	0.2237
7	0.1975	0.1657	0.0896	0.0517	0.0279	0.0120	0.0011	0.0066	0.0119	0.0158
8	0.0285	0.4021	0.3363	0.2751	0.2289	0.1942	0.1678	0.1471	0.1306	0.1172
9	0.0067	0.2168	0.1973	0.1684	0.1450	0.1268	0.1125	0.1011	0.0918	0.0840
10	0.0026	0.2533	0.2247	0.1894	0.1615	0.1401	0.1234	0.1102	0.0995	0.0906
Σ	3.6827	4.8386	3.7637	2.9496	2.3631	1.9355	1.6168	1.3868	1.2081	1.0653

BEST ZERO-ORDER FORMULAS

If all we know about $F(t)$ is a set of equally spaced numerical values, and a bound on the first derivative, the best possible formula will be the one for which the $\|\rho_i\|$ are minimized. It is clear from a study of Fig. V-19-1 and Table V-19-II that the error kernels for Salzer's formulas are far from the best. The availability of a simple expression for the error functional enables us to obtain such optimum formulas in analytic form.

A brief consideration of Eq. (30) shows that $d\|\rho_i\|/dS_i^{(0)}$ is always negative for $S_i^{(0)} < e^{-sh_i}$, and

positive for $S_i^{(0)} > e^{-sh(i-1)}$. Hence, we need only consider the case where the kernel changes sign in the interval. Eliminating d_i from Eq. (30) by Eq. (28),

$$\|\rho_i\| = \frac{1}{s^2} \{ (e^{-sh_i}(e^{sh} + 1) + S_i^{(0)}[(2i-1)sh - 2 + 2\ln S_i^{(0)}]) \} \quad (31)$$

and

$$\frac{d\|\rho_i\|}{dS_i^{(0)}} = \frac{1}{s^2} [(2i-1)sh + 2\ln S_i^{(0)}]. \quad (32)$$

The value of $S_i^{(0)}$ which minimizes $\|\rho_i\|$ and the mini-

mum $\|\rho_i\|$ are thus

$$S_i^{(0)*} = e^{sh/2} e^{-sh}, \|\rho_i^*\| = \left(\frac{e^{sh/2} - 1}{s} \right)^2 e^{-sh}. \quad (33)$$

By the definition Eq. (26), the corresponding optimal coefficients are

$$\alpha_k^* = \begin{cases} S_k^{(0)*} - S_{k+1}^{(0)*} \\ = e^{-ksh} (e^{sh/2} - e^{-sh/2}), & (0 < k < n) \\ e^{sh/2} e^{-nsh} & (k = n). \end{cases} \quad (34)$$

The first coefficient, α_0^* , may now be evaluated from the condition Eq. (3), with $j = 0$:

$$\alpha_0^* = 1 - (e^{sh/2} - e^{-sh/2}) \sum_{k=1}^{n-1} (e^{-sh}) - e^{sh/2} e^{-nsh} = 1 - e^{-sh/2}. \quad (35)$$

Collecting all these results, we find for our best zero-order formula,

$$f(s) = \frac{1}{s} \left[(1 - e^{-sh/2}) F(0) + (e^{sh/2} - e^{-sh/2}) \sum_{k=1}^{n-1} e^{-ksh} F(kh) + e^{sh/2} e^{-nsh} F(nh) \right] + R_n(s, h; F) \quad (36)$$

where

$$|R_n(s, h; F)| < \left(\frac{e^{sh/2} - 1}{s} \right)^2 \sum_{k=1}^n e^{-ksh} \cdot \left(\text{ess. sup.}_{(k-1)h \leq t < kh} |F'(t)| \right) + \frac{e^{-(s-\gamma)nh}}{s(s-\gamma)} \left(\text{ess. sup.}_{nh \leq t} |e^{-\gamma t} F'(t)| \right). \quad (37)$$

The factor $[(e^{sh/2} - 1)/s]^2$ is at least as large as $(h/2)^2$, and thus limits the maximum allowable step size, while the factor $e^{-(s-\gamma)nh}$ sets a minimum bound on nhs , and thus on the number of points to be used.

For computational purposes, Eq. (36) has several advantages. In the first place, all coefficients are positive, so that the cancellation errors which caused serious losses of accuracy with Salzer's formulas will not be severe. In the second place, the coefficients are readily computable, and, except for the last one, are independent of the number of points. It is thus possible, under reasonable assumptions on the rate of growth of $F'(t)$, to simply continue computing terms in Eq. (36) until the contribution of the last term can be neglected.

Table V-19-III gives comparative results of applying Salzer's 11-point formula and Eq. (36) with $n = 11$ and $h = 1$ to a function which is known to be zero for

TABLE V-19-III. COMPARISON OF 11-POINT BEST ZERO ORDER FORMULA WITH SALZER'S

11-POINT FORMULA				
$F(t) = \begin{cases} 0 & t = 0, 2, 4, 6, 8, 10 \\ 1 & t = 1, 3, 5, 7, 9 \end{cases}$ $h = 1.0$				
s	Eq. (36)		Salzer's	
	$f(s)$	$\sum \ \rho_i\ $	$f(s)$	$\sum \ \rho_i\ $
1.0	0.4434	0.2449	4.0702	3.6827
2.0	0.1620	0.1155	4.2884	4.8386
3.0	0.0708	0.0706	3.1612	3.7637
4.0	0.0332	0.0476	2.3977	2.9496
5.0	0.0163	0.0339	1.8682	2.3631
6.0	0.0083	0.0251	1.4907	1.9355
7.0	0.0043	0.0192	1.2143	1.6168
8.0	0.0023	0.0151	1.0067	1.3868
9.0	0.0012	0.0120	0.8474	1.2081
10.0	0.0007	0.0098	0.7226	1.0653

$t = 0, 2, 4, 6, 8, 10$ and 1 for $t = 1, 3, 5, 7, 9$. It will be recalled that, as shown in Table V-19-I, the transforms of even relatively simple functions which take on these values can vary by factors of ten or more. Observing that the magnitudes of the derivatives of $0.5(1 - \cos \pi t)$ and $|\sin \pi t/2|$ are bounded by $\pi/2$, that the derivative of the sawtooth is ± 1 , while the derivative of the interpolating polynomial may be as large as 118, it is clear that our error bounds are satisfied, although disappointingly large.

HIGHER ORDER ANALYSIS

The disappointingly large error bounds on even the best zero-order formulas may be attributed, in part, to the very large class of functions for which they are valid: all continuous functions with piecewise continuous first derivatives. Some improvement in precision should be attained by restricting the class of functions, e.g., by increasing the value of ν . If we use $\nu = 2$, we need no longer consider functions with discontinuous first derivatives, or functions for which the second derivative is not continuous almost everywhere.

For $\nu = 2$, the error kernel on the i th interval

$$K_2(s, h; \theta) = \left(\frac{e^{-s\theta}}{s} + \theta S_j^{(0)} - S_j^{(1)} \right) / s. \quad (38)$$

This function is no longer monotone decreasing, but is concave upward, so that it may have as many as 2 zeros in each interval. Moreover, these zeros can no longer be expressed in terms of elementary functions. Determination of the error norms for a given formula thus requires the repeated numerical solution of the equation $K_2(s, h; \theta) = 0$. Although this is practical for a given formula, it is considerably more laborious.

Determination of best formulas is even more difficult, especially if the additional practical requirement that the coefficients be readily computable is considered. It is not too difficult, however, to find formulas which are appreciably better than Salzer's, and which do not require extensive computation.

CONCLUSIONS

The conclusions from this analysis are gloomy: it is unlikely that any linear formula based on equally-spaced function values will give precise and reliable estimates of the Laplace transform with a reasonable number of points. It is possible that selecting the data points less uniformly, so that the function is sampled more intensively where the exponential weight is large, would be more effective. One way of selecting the data points would be in geometric, rather than arithmetic progression:

$$t_k = t_0(1 + h)^k \quad (k = 0, 1, \dots, n) \quad (39)$$

Numerical studies reported by H. Nordén¹⁰ indicate some success along these lines, but a detailed analysis of the error does not appear to have been made.

V-20. Evaluation of Mathematical Functions

H. C. THACHER, JR.

In almost all calculations, a significant fraction of the effort is devoted to the evaluation of mathematical functions. It is therefore appropriate to pay some attention to the development and evaluation of methods for this task. During the past year, the following topics in this area have been studied: (1) The use of linear fractional transformations of the independent variable to improve the efficiency of approximations; (2) Finite difference interpolation in the complex plane; (3) Analytic solutions of transcendental equations by extended series reversion.

INDEPENDENT VARIABLE TRANSFORMATIONS IN APPROXIMATION

Most subroutines for computing functions are built around an approximate representation of the function as a polynomial or ratio of polynomials. As pointed out by C. Clenshaw,¹ truncated Fourier-Chebyshev series have many advantages over other forms for this purpose. Their computation requires relatively little effort, and a single set of coefficients provides a wide range of accuracies. When it converges rapidly, the

REFERENCES

1. R. V. Churchill, *Modern Operational Mathematics in Engineering*, (McGraw-Hill Book Co., Inc., New York, 1944).
2. I. N. Sneddon, *Fourier Transforms*, (McGraw-Hill Book Co., Inc., New York, 1951).
3. N. Corngold, *Slowing Down of Neutrons in Infinite Homogeneous Media*, Proc. Phys. Soc. (London) A70, 793 (1957).
4. H. E. Salzer, *Tables of Coefficients for the Numerical Calculation of Laplace Transforms*, (U.S. Department of Commerce, National Bureau of Standards, U.S. Government Printing Office, Washington, 1953) Applied Mathematics Series 30.
5. H. E. Salzer, *Tables for the Numerical Calculation of Inverse Laplace Transforms*, J. Math. and Phys. **37**, 89 (1958).
6. A. E. Taylor, *Introduction to Functional Analysis*, John Wiley and Sons, Inc., New York, 1958).
7. L. Collatz, *Functional Analysis as an Aid in Numerical Mathematics*, Proc. IFIP Congress 65 (Spartan Books, Inc., Washington, D.C., 1965), Vol. I, p. 145.
8. L. Collatz, *Funktionalanalysis und numerische Mathematik*, (Springer Verlag, Berlin, 1964).
9. A. Sard, *Linear Approximation*, (American Mathematical Society, Providence, R.I., 1963), Mathematical Surveys, No. 9, p. 14ff.
10. H. V. Nordén, *Numerical Inversion of the Laplace Transform*, Acta Accad. Aboensis, Math. Phys. **22**, No. 8, 1 (1961).

truncated series is very close to the best possible polynomial approximation. Unfortunately, functions with singularities on or near the interval of approximation are poorly represented by polynomials. This is reflected by slow convergence of the Chebyshev series, which not only necessitates the inclusion of many terms to attain the required accuracy, but also means that the truncated series differs significantly from the best approximation of the same degree. For such functions, approximations by ratios of polynomials, such as those constructed by C. Hastings,² are usually far more effective. Hastings' collection of approximations also includes notable examples of the benefits to be realized by judicious transformations of the dependent and independent variables. In most cases, these transformations appear to have been suggested by special analytic properties of the individual function. Transformation of the dependent variable to remove common types of singularity is now generally practiced where appropriate, but transformations of the independent variable appear to have been somewhat neglected.

The investigations described here were undertaken

with two aims: to attempt to combine the convenience of the Chebyshev series representation with the efficiency of the rational forms, and to explore the effects on best approximations to be expected from a family of simple transformations of the independent variable.

The expansions studied were of the form

$$F(x) = \sum_{k=0}^{\infty} c_k T_k^*(t) \quad (A \leq x \leq B) \quad (1)$$

where $T_k^*(t)$ is the shifted Chebyshev polynomial of degree k ,

$$t = \frac{\alpha + \beta x^n}{1 + \gamma x^n}, \quad (2)$$

n is either 1 or 2, depending upon the symmetry of $F(x)$, and $F(x)$ is either the function to be approximated, or a suitable auxiliary function. The parameters in the transformed independent variable were selected subject to the conditions $t(0) = A$, $t(1) = B$, so that but one parameter, usually β , was free to affect the convergence. Range and auxiliary function $F(x)$ were ordinarily those chosen by Clenshaw.¹

The coefficients c_k were found by rearranging the formal expansion of $F(x)$ as a power series in t . For satisfactory values of β , the power series usually converged rapidly enough so that extrapolation techniques³ were unnecessary. This method was selected because of its simplicity and flexibility compared to Clenshaw's recurrence method. The independent variable change frequently raises the order of the difference equation for the c_k prohibitively, and in one case which was treated by both methods led to a difference equation which had both rapidly increasing and rapidly decreasing extraneous solutions.

The formal power series coefficients were generated by changing the independent variable in a differential equation for $F(x)$, and applying the method of undetermined coefficients. This led to a recurrence which was evaluated numerically for each choice of the variable parameter.

For each function, a preliminary numerical survey was carried out using a FORTRAN program on the CDC 160A computer using 27-bit floating point arithmetic. After a suitable set of values of the parameters was found, the same program was run on the IBM 1620 computer, using 28-decimal floating point arithmetic. The loss of significance was estimated by comparing the two results, and also by comparing the sums and alternating sums with the true values of the functions, when known to sufficient accuracy. In most of the examples, the loss of significance with the final parameter set did not exceed 4 decimal places.

The economies resulting from the independent

TABLE V-20-I. NUMBER OF TERMS IN FOURIER-Chebyshev EXPANSION FOR SPECIFIED ACCURACY
[Note: $t = \beta x^n / (1 + \gamma x^n)$]

Function	$\frac{\arctan x}{x}$		$\frac{\operatorname{erf}(x)}{x}$		$\frac{\text{Dawson's Integral}}{x}$		$E_1(x) + \ln(x)$	
n	2		2		2		1	
Range	$-1 \leq x \leq 1$		$-4 \leq x \leq 4$		$-5 \leq x \leq 5$		$0 \leq x \leq 4$	
β	1.000 ^a	1.437	0.0625	0.0865	0.040 ^b	0.0613	0.2500	0.2915
10 Decimals	13	9	20	15	26	18	13	10
15 Decimals	19	14	26	20	34	23	17	13
20 Decimals	25	18	32	24	—	27	21	17
24 Decimals	—	22	36	27	—	31	24	19

^a Values from C. W. Clenshaw.¹

^b Values from D. G. Hummer.⁴

variable transformation for four different functions are summarized in Table V-20-I. These functions were chosen because their expansions in series of Chebyshev polynomials in x converge slowly. This is shown by the first column under each function, in which β has been chosen so that $\gamma = 0$, and $t = \beta x$. In all cases, a saving of 20–30% in the number of terms in the series is achieved by the choice of β given in the second column. With the same number of terms, the errors are reduced by factors of from 10^5 to 10^8 . Even this simple transformation thus has practical value when precisions of 10 decimals or more are required.

Figure V-20-1 illustrates the dependence of representative coefficients in the expansion on the parameter β for a typical function, $E_1(x) + \ln(x)$. The low order coefficients, as typified by c_5 , decrease monotonely, but slowly, as β increases. As the order of the subscript increases, the sharpness of the decline becomes more apparent, and alternations in sign of the coefficient appear. Unfortunately, the zeros of successive c_k do not coincide, so that a saving of at most one term can be attained by selecting β to coincide with one of the zeros. It is, however, possible to select a value of β which is uniformly good for a fairly wide range of numbers of terms. The second column under each function in Table V-20-I represents such a compromise value.

In all cases, the convergence of the series was rapid enough for the error of the truncated series to be very close to that for the best polynomial approximation with the same independent variable. Thus, we may expect the error of the best polynomial in t of degree

FIG. V-20-1. Dependence on β of Chebyshev Coefficients for $E_1(X) - \ln(X)$.

$$E_1(X) + \ln x = \sum' c_k T_k^* \left(\frac{\beta x}{1 + (\beta - 0.25)x} \right)$$

$c_k < D_0$ where line is dashed. ANL Neg. No. 112-4355 Rev.

$k - 1$ to have $k + 1$ extreme values in the neighborhood of a zero of c_k , where $|c_k| \ll |c_{k+1}|$, instead of the k extremes which would normally be expected. More significantly, the error of best polynomial approximations of degree n in t will not be a convex function of β , but will have local minima in the neighborhood of each of the zeros of c_{n+1} . Thus, methods like the Remes algorithm which depend upon continuously reducing the error cannot be expected to converge to the optimum choice. This is not too serious when the independent variable transformation contains but one parameter, so that an exhaustive search is feasible. For more flexible transformations, however, better techniques for locating global minima are badly needed.

FINITE DIFFERENCE INTERPOLATION OF ANALYTIC FUNCTIONS

The increasing memory capacity of modern large computers is again making the use of stored tables an attractive means of evaluating functions which are required often enough so that speed is of primary importance. This is so even for analytic functions of a

complex variable. The need for speed, moreover, makes it desirable to include as much of the interpolation process as possible in the table; i.e., to provide the table with "interpolation aids" such as reduced derivatives or differences, so that the coefficients which must be computed for each evaluation require a minimum number of arithmetic operations. Because of the typographical difficulties of representing differences in a bivariate table, most attention has hitherto been devoted to Lagrangian formulas,^{5, 6, 7} although P. Woodward and A. Woodward⁸ give interpolation formulas based on the values of the function, and the central differences $(\delta_{xx}^2 - \delta_{yy}^2)/2$ and $\delta_{xx}\delta_{yy}^2$ at the four corners of the square surrounding the point at which the interpolation is to be performed, and P. Woodward⁹ has tabulated the coefficients for this distribution of data. It appears, however, that the amount of computation required in interpolating for this arrangement is considerably greater than for the formula presented below which uses only function values and differences at the nearest base point.

Let the function to be interpolated be denoted by

$w(z) = w(x + iy) = u(x, y) + iv(x, y)$, and let it be tabulated on a rectangular grid of points $z_{m,n} = z_0 + mh + ink$, ($m, n = \dots, -1, 0, 1, \dots$). Let the point at which the interpolation is to be performed be $\bar{z} = z_0 + ph + iqk$, with $|p| \leq \frac{1}{2}$, $|q| \leq \frac{1}{2}$, and let $f(z)$ be analytic in a circle about z_0 with radius greater than the larger of h and k . Then the Taylor's series

$$w(z) = \sum_{j=0}^{\infty} \frac{w_0^{(j)}}{j!} (z - z_0)^j \quad (3)$$

converges for $\bar{z}$, and for the five points $z_{0,0} = z_0$, $z_{0,1}$, $z_{0,-1}$, $z_{1,0}$, and $z_{-1,0}$. In particular

$$w(z_{1,0}) = \sum_{j=0}^{\infty} \frac{w_0^{(j)}}{j!} h^j, \quad w(z_{-1,0}) = \sum_{j=0}^{\infty} \frac{w_0^{(j)}}{j!} (-h)^j, \quad (4)$$

$$w(z_{0,1}) = \sum_{j=0}^{\infty} \frac{w_0^{(j)}}{j!} (ik)^j, \quad w(z_{0,-1}) = \sum_{j=0}^{\infty} \frac{w_0^{(j)}}{j!} (-ik)^j. \quad (5)$$

Subtracting these series we find

$$w(z_{1,0}) - w(z_{-1,0}) = 2 \sum_{j=0}^{\infty} \frac{w_0^{(2j+1)}}{(2j+1)!} h^{2j+1}, \quad (6)$$

$$\begin{aligned} w(z_{0,1}) - w(z_{0,-1}) &= 2 \sum_{j=0}^{\infty} \frac{w_0^{(2j+1)}}{(2j+1)!} (ik)^{2j+1} \\ &= 2i \sum_{j=0}^{\infty} \frac{w_0^{(2j+1)}}{(2j+1)!} (-1)^j k^{2j+1} \end{aligned} \quad (7)$$

while adding,

$$w(z_{1,0}) + w(z_{-1,0}) = 2 \sum_{j=0}^{\infty} \frac{w_0^{(2j)}}{(2j)!} h^{2j}, \quad (8)$$

$$w(z_{0,1}) + w(z_{0,-1}) = 2 \sum_{j=0}^{\infty} \frac{w_0^{(2j)}}{(2j)!} (-1)^j k^{2j}. \quad (9)$$

In Eqs. (6) and (7), the differences are merely twice the mean first differences in the x and y directions at z_0 , while by subtracting $2w_0$ from Eqs. (8) and (9), we obtain the central second differences. Thus,

$$\mu \delta_x w_0 = \sum_{j=0}^{\infty} \frac{w_0^{(2j+1)}}{(2j+1)!} h^{2j+1}, \quad (10)$$

$$-i\mu \delta_y w_0 = \sum_{j=0}^{\infty} \frac{w_0^{(2j+1)}}{(2j+1)!} (-1)^j k^{2j+1},$$

$$\frac{\delta_{xx}^2 w_0}{2} = \sum_{j=0}^{\infty} \frac{w_0^{(2j+2)}}{(2j+2)!} h^{2j+2}, \quad (11)$$

$$-\frac{\delta_{yy}^2 w_0}{2} = \sum_{j=0}^{\infty} \frac{w_0^{(2j+2)}}{(2j+2)!} (-1)^j k^{2j+2}.$$

Multiplying the left member of Eq. (10) by k^3 , and the right member by h^3 , and adding

$$\begin{aligned} k^3 \mu \delta_x w_0 - i h^3 \mu \delta_y w_0 &= \frac{w_0^{(1)}}{1!} (k^3 h + h^3 k) \\ &+ \sum_{j=2}^{\infty} \frac{w_0^{(2j+1)}}{(2j+1)!} [k^3 h^{2j+1} + (-1)^j h^3 k^{2j+1}], \end{aligned} \quad (12)$$

while multiplying the left and right members of Eq. (10) by k and h , respectively, and subtracting,

$$\begin{aligned} k \mu \delta_x w_0 + i h \mu \delta_y w_0 &= \frac{w_0^{(3)}}{3!} (k h^3 + h k^3) \\ &+ \sum_{j=2}^{\infty} \frac{w_0^{(2j+1)}}{(2j+1)!} [k h^{2j+1} - (-1)^j h k^{2j+1}]. \end{aligned} \quad (13)$$

Similar operations on Eq. (11) give

$$\begin{aligned} \frac{k^4 \delta_{xx}^2 w_0 - h^4 \delta_{yy}^2 w_0}{2} &= \frac{w_0^{(2)}}{2!} (k^4 h^2 + h^4 k^2) \\ &+ \sum_{j=2}^{\infty} \frac{w_0^{(2j+2)}}{(2j+2)!} [k^4 h^{2j+2} + (-1)^j h^4 k^{2j+2}] \end{aligned} \quad (14)$$

and

$$\begin{aligned} \frac{k^2 \delta_{xx}^2 w_0 + h^2 \delta_{yy}^2 w_0}{2} &= \frac{w_0^{(4)}}{4!} (k^2 h^4 + h^2 k^4) \\ &+ \sum_{j=2}^{\infty} \frac{w_0^{(2j+2)}}{(2j+2)!} [k^2 h^{2j+2} - (-1)^j h^2 k^{2j+2}]. \end{aligned} \quad (15)$$

Using Eqs. (12), (13), (14), and (15) to eliminate the first four derivatives from Eq. (3)

$$\begin{aligned} w(z_0 + ph + iqk) &= w_0 + \frac{k^3 \mu \delta_x w_0 - i h^3 \mu \delta_y w_0}{h k (h^2 + k^2)} (ph + iqk) \\ &+ \frac{k^4 \delta_{xx}^2 w_0 - h^4 \delta_{yy}^2 w_0}{2 h^2 k^2 (h^2 + k^2)} (ph + iqk)^2 \\ &+ \frac{k \mu \delta_x w_0 + i h \mu \delta_y w_0}{h k (h^2 + k^2)} (ph + iqk)^3 \\ &+ \frac{k^2 \delta_{xx}^2 w_0 + h^2 \delta_{yy}^2 w_0}{2 h^2 k^2 (h^2 + k^2)} (ph + iqk)^4 \\ &- \frac{1}{(h^2 + k^2)} \sum_{j=2}^{\infty} \left[\frac{w_0^{(2j+1)}}{(2j+1)!} (ph + iqk) \right. \\ &\quad \left. + \frac{w_0^{(2j+2)}}{(2j+2)!} (ph + iqk)^2 \right] \\ &\cdot \{ [k^2 h^{2j} + (-1)^j h^2 k^{2j} - (h + k)(ph + iqk)^{2j}] \\ &\quad + [h^{2j} - (-1)^j k^{2j}] (ph + iqk)^2 \} \end{aligned} \quad (16)$$

The remainder, which we have expressed as a summation, is of the order of the fifth power of the larger of h and k .

For a square grid of base points, with $k = h$, the formula simplifies considerably:

$$\begin{aligned} w(z_0 + ph + iqh) &= w_0 \\ &+ \frac{\mu \delta_x w_0 - i \mu \delta_y w_0}{2} (p + iq) \\ &+ \frac{\delta_{xx}^2 w_0 - \delta_{yy}^2 w_0}{4} (p + iq)^2 \end{aligned} \quad (17)$$

$$+ \frac{\mu\delta_x w_0 + i\mu\delta_y w_0}{2} (p + iq)^3 \\ + \frac{\delta_{xx}^2 w_0 + \delta_{yy}^2 w_0}{4} (p + iq)^4 + O(h^5).$$

In terms of the real and imaginary parts of w , this becomes:

$$w(\bar{z}) = (u_0 + iv_0) \\ + \left\{ \frac{(\mu\delta_x u_0 + \mu\delta_y v_0) + i(\mu\delta_x v_0 - \mu\delta_y u_0)}{2} \right\} [p + iq] \\ + \left\{ \frac{(\delta_{xx}^2 u_0 - \delta_{yy}^2 u_0) + i(\delta_{xx}^2 v_0 - \delta_{yy}^2 v_0)}{4} \right\} \\ \cdot [(p^2 - q^2) + i(2pq)] \\ + \left\{ \frac{(\mu\delta_x u_0 - \mu\delta_y v_0) + i(\mu\delta_x v_0 + \mu\delta_y u_0)}{2} \right\} \\ \cdot [p(p^2 - 3q^2) + iq(3p^2 - q^2)] \\ + \left\{ \frac{(\delta_{xx}^2 u_0 + \delta_{yy}^2 u_0) + i(\delta_{xx}^2 v_0 + \delta_{yy}^2 v_0)}{4} \right\} \\ \cdot [(p^4 - 6p^2q^2 + q^4 + i4pq(p^2 - q^2))] + O(h^5). \quad (18)$$

In practice, the real and imaginary parts of the quantities in braces, which we can abbreviate to $t_j + i\theta_j$, will be stored in the table. Then $u(\bar{z})$ and $v(\bar{z})$ may be evaluated by

$$u(\bar{z}) = u_0 + \sum_{j=1}^n (t_j c_j - \theta_j \gamma_j) + O(h^{n+1}) \quad (19) \\ (n \leq 4),$$

$$v(\bar{z}) = v_0 + \sum_{j=1}^n (t_j \gamma_j + \theta_j c_j) + O(h^{n+1}) \quad (20) \\ (n \leq 4).$$

The number of operations required to evaluate the interpolation coefficients [the quantities in square brackets in Eq. (18)], may be minimized by applying the following identities:

$$c_2 = p^2 - q^2 = (p + q)(p - q) \quad (21)$$

$$\gamma_2 = 2pq = pq + pq \quad (22)$$

$$c_3 = p(p^2 - 3q^2) = p(p^2 - q^2) - 2pq^2 \\ = pc_2 - q\gamma_2 \quad (23)$$

$$\gamma_3 = q(3p^2 - q^2) = q(p^2 - q^2) + 2p^2q \\ = qc_2 + p\gamma_2 \quad (24)$$

$$c_4 = p^4 - 6p^2q^2 + q^4 = (p^2 - q^2)^2 - 4p^2q^2 \\ = c_2^2 - \gamma_2^2 = (c_2 + \gamma_2)(c_2 - \gamma_2) \quad (25)$$

$$= 4pq(p^2 - q^2) = 2c_2\gamma_2 = c_2\gamma_2 + c_2\gamma_2. \quad (26)$$

In Eqs. (22) and (26), the next to last form should be

TABLE V-20-II. OPERATION COUNTS FOR ANALYTIC INTERPOLATION

Order	Coefficient Evaluation			Interpolation		Cumulative Totals					
						Real or imaginary part			Both		
n	A	M	S	A	M	A	M	S	A	M	S
1	0	0	0	2	2	2	2	0	4	4	0
2	2	2	1	2	2	6	6	1	10	10	1
3	2	4	0	2	2	10	12	1	16	18	1
4	2	2	1	2	2	14	16	2	22	24	2

A = Number of additions and subtractions.

M = Number of multiplications.

S = Number of shifts.

used on a binary computer if multiplication by 2 is performed by incrementing the exponent (or shifting in fixed point). Otherwise the additive form is probably faster.

The number of operations required to perform interpolations of various orders given p and q is shown in Table V-20-II.

For comparison, a six-point interpolation formula on the points z_0 , $z_0 \pm h$, $z_0 \pm ih$, and $z_0 + h + ih$, for which the error is $O(h^3)$, requires 5 additions and 8 multiplications for each component, even if all appropriate function combinations of Eq. (6) are precomputed and stored at each base point. Operation counts for the Woodward formula, which uses data at four points, are even less favorable.

For practical error estimation, it is worth remarking that since both $w(z)$ and the interpolation polynomial are analytic, the error is also, and thus attains its maximum modulus on the boundary of any region. In particular, the maximum error in interpolation for either u or v in the square $|p| \leq \frac{1}{2}$, $|q| \leq \frac{1}{2}$ is less than the maximum modulus on the boundary. Testing of a routine based on this formula can therefore be restricted to points $z = z_0 + x + i(jh/2)$ or $z = z_0 + (jh/2) + iy$, where z_0 is one of the base points, and j is ± 1 .

SOLUTION OF TRANSCENDENTAL EQUATIONS BY SERIES REVERSION

The solution of transcendental equations is a frequent task in computation. In particular, the construction of good first-order formulas for Laplace transforms requires the solution of the equation

$$Y^2 \coth Y = \frac{1}{2} X^2 \coth X \quad (27)$$

for several values of X for each value of the transform which is to be evaluated. Such equations may, of course, be solved by iteration for each value of X , provided an adequate initial approximation to the root is available.

Often, however, an analytical approximation to the root is desirable either to furnish starting values for the iteration, or for direct application when only limited accuracy is required.

Let the equation

$$F(Y) = G(X) \quad (28)$$

have a known root, Y_0 for $X = X_0$. Let $F(Y)$ be analytic in the neighborhood of Y_0 , and let $G(X)$ be analytic in the neighborhood of X_0 . Further, suppose that $F'(Y_0) = F'_0 \neq 0$. Then the variable transformations $y = Y - Y_0$, $x = X - X_0$ allow us to write Eq. (28) in the form

$$\begin{aligned} f(y) &\equiv \frac{F(y + Y_0) - F(Y_0)}{F'_0} = g(x) \\ &\equiv \frac{G(x + X_0) - G(X_0)}{F'_0}, \end{aligned} \quad (29)$$

where $f(y)$ and $g(x)$ have the convergent power series representations

$$f(y) = \sum_{j=1}^{\infty} a_j y^j, \quad g(x) = \sum_{i=1}^{\infty} b_i x^i, \quad (30)$$

with $a_1 = 1$. In this note we will develop an algorithm for finding the coefficients in the power series for $y(x)$ where y is the root of Eq. (29) from the coefficients in the power series for $f(y)$ and $g(x)$. For $g(x) = x$, the algorithm reduces to the familiar one for reversion of power series.

For $j = 1, 2, \dots$, let y be a root of Eq. (29) and let

$$y^j = \sum_{i=j}^{\infty} c_{i,j} x^i \quad (31)$$

so that the desired coefficients are the $c_{i,1}$. The existence and continuity of y are assured in a neighborhood of 0 by the implicit function theorem.

Since $y^{j+1} = yy^j$, we may take the product

$$\begin{aligned} y^{j+1} &= \left(\sum_{i=1}^{\infty} c_{i,1} x^i \right) \left(\sum_{i=j}^{\infty} c_{i,j} x^i \right) \\ &= \sum_{i=j+1}^{\infty} \left(\sum_{k=1}^{i-j} c_{k,1} c_{i-k,j} \right) x^i \end{aligned} \quad (32)$$

and the $c_{i,j}$ obey the recurrence, for $i > j \geq 1$,

$$c_{i,j+1} = \sum_{k=1}^{i-j} c_{k,1} c_{i-k,j}. \quad (33)$$

Substituting Eq. (31) into Eq. (30); and rearranging

$$\begin{aligned} \sum_{j=1}^{\infty} a_j y^j &= \sum_{j=1}^{\infty} \sum_{i=j}^{\infty} a_j c_{i,j} x^i \\ &= \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} a_j c_{i,j} x^i = \sum_{i=1}^{\infty} b_i x^i, \end{aligned} \quad (34)$$

and so

$$c_{i,1} = b_i - \sum_{j=2}^i a_j c_{i,j}. \quad (35)$$

The algorithm for computing the $c_{i,j}$ is thus as follows: Set $c_{i,1} = b_i$. For $i = 2, 3, \dots$, compute $c_{i,j+1}$ for $j = i-1, i-2, \dots, 1$, by Eq. (33), and then compute $c_{i,1}$ by Eq. (35). The $c_{i,1}$ are the desired coefficients.

The resulting power series for $y(x)$ can be used in a variety of ways. In addition to direct application, it may be transformed to a series in Chebyshev polynomials, which may be truncated to give an approximation with close to the minimum possible maximum error over a specified range. Other possibilities include conversion to Padé approximants, and application of the various devices for accelerating convergence.

As an example, we may consider the equation

$$Y^Y = X, \quad (36)$$

with the known root $X_0 = Y_0 = 1$. Taking logarithms and making the transformations $Y = 1 + y$, $X = 1 + x$ leads to

$$\begin{aligned} (1+y) \ln(1+y) &= y + \sum_{j=2}^{\infty} (-1)^j \left(\frac{1}{j-1} - \frac{1}{j} \right) y^j \\ &= \ln(1+x) = \sum_{j=1}^{\infty} (-1)^{j-1} \frac{1}{j} x^j. \end{aligned} \quad (37)$$

Thus,

$$\begin{aligned} a_1 &= 1, \quad a_j = (-1)^j / j(j-1), \quad (j \geq 2); \\ b_j &= -(-1)^j / j, \quad (j \geq 1). \end{aligned} \quad (38)$$

Applying the algorithm to these coefficients leads to the $c_{i,j}$ array of which the first few elements are as follows.

i	j						
	1	2	3	4	5	6	7
1	1						
2	-1	1					
3	$\frac{3}{2}$	-2	1				
4	$-\frac{17}{6}$	4	-3	1			
5	$\frac{37}{6}$	$-\frac{26}{3}$	$\frac{15}{2}$	-4	1		
6	$-\frac{1759}{120}$	$\frac{81}{4}$	$-\frac{37}{2}$	12	-5	1	
7	$\frac{13279}{360}$	$-\frac{1003}{20}$	$\frac{187}{4}$	$-\frac{100}{3}$	$\frac{35}{2}$	-6	1

As might be expected from the fact that $Y(X)$ has a branch point at $X = e^{-1/3} = 0.6922 \dots$, this power series has a small radius of convergence. The corresponding continued fraction, however, obtained by the

qd algorithm, converges quite satisfactorily. Even for $X = 4$, the error of the sixth convergent is less than 2.4%, and the error of the 15th about 0.0015%.

REFERENCES

1. C. W. Clenshaw, *Chebyshev Series for Mathematical Functions*, National Physical Laboratory Mathematical Tables, Vol. 5, London HMSO, 1962.
2. C. Hastings, *Approximations for Digital Computers*, (Princeton University Press, Princeton, N. J., 1955).
3. H. C. Thacher, Jr., *Conversion of a Power to a Series of Chebyshev Polynomials*, Commun. ACM 7, 181 (1964).
4. D. G. Hummer, *Expansion of Dawson's Function in a Series of Chebyshev Polynomials*, Math. Computation 18, 317 (1964).
5. A. N. Lowan and H. E. Salzer, *Coefficients for Interpolation Within a Square Grid in the Complex Plane*, J. Math. Phys. 23, 155 (1944).
6. H. E. Salzer, *Coefficients for Complex Quartic, Quintic, and Sextic Interpolation Within a Square Grid*, J. Math. Phys. 27, 136 (1948).
7. H. E. Salzer, *Formulas for Complex Cartesian Interpolation of Higher Degree*, J. Math. Phys. 28, 200 (1949).
8. P. M. Woodward and A. M. Woodward, *Four-Figure Tables of the Airy Function in the Complex Plane*, Phil. Mag. 37, (7), 236 (1946).
9. P. M. Woodward, *Tables of Interpolation Coefficients for Use in the Complex Plane*, Phil. Mag. 39 (7), 594 (1948).

V-21. Stochastic Techniques for Reactor Analysis*

L. MILLER

The Doppler coefficient of reactivity is a major prompt control mechanism in the large fast reactors currently being studied. This interest has motivated efforts to develop more exact theoretical models and analytic methods for studying the effect. Many of these studies have been directed toward reducing the inherent errors when a heterogeneous system is treated as an "equivalent" homogeneous system. This approach has been reasonably successful. However, a more exact method of analysis is required to provide a basis of comparison for all approximation methods. To this end, some stochastic techniques applicable to reactor analysis have been developed and studied.

AN ABSOLUTELY RANDOM NUMBER GENERATOR

Basic to all stochastic techniques is a method for generating random numbers satisfying some known probability distribution. Until now no satisfactory method for generating truly random numbers has been known, so it has been customary to use numbers from a pseudo-random chain generated by the multiplicative or mixed congruential method. A method for obtaining random numbers by sampling arbitrary memory locations in a digital computer has been developed recently. Previous attempts to obtain random numbers by sampling memory locations failed because of bias in the results. A new technique for eliminating this bias has been developed and used as the basis for an absolutely random number generator. In its simplest form the technique is as follows:

Identical bit positions of successive memory locations are sampled. If the two sampled bits are (0, 0) or (1, 1) the result is discarded and another sample is taken. If the result is (0, 1) or (1, 0) then 0 or 1, respectively, is taken to be the first bit of our random number. The remaining bits are selected similarly. This procedure completely eliminates any bias due to an excess of "0" or "1" bits in the memory, since the probability of obtaining (0, 1) is exactly equal to the probability of obtaining (1, 0). Serial and moments tests of this procedure have produced excellent results.

MULTIPLE-WEIGHTING TECHNIQUES

The simplest use of stochastic theory is to evaluate an integral, e.g.

$$I = \int_a^b f(X)g(X) dX. \quad (1)$$

This can be evaluated by choosing N values of X_i from the probability distribution

$$P(X) = \begin{cases} f(X) & a < X < b \\ 0 & X < a \\ 0 & X > b \end{cases} \quad (2)$$

and evaluating $g(X_i)$ for each value of X_i . The approximate value of the integral is then

$$I = \frac{1}{N} \sum_{i=1}^N g(X_i). \quad (3)$$

If

$$f(\bar{X}) = \phi(\bar{X}) \Sigma_i(\bar{X}) \quad (4)$$

* Report of progress in work for Ph.D. thesis.

and

$$g(\bar{X}) = \Sigma_a^z(\bar{X})/\Sigma_t(\bar{X}), \quad (5)$$

where

$\phi(\bar{X})$ = scalar flux

$\Sigma_t(\bar{X})$ = total macroscopic cross section

$\Sigma_a^z(\bar{X})$ = macroscopic absorption cross section of isotope z

$\bar{X}$ = a point in 6-dimensional phase space

then I is the neutron absorption rate of isotope z . The points, $\bar{X}_i$, may be chosen from the probability distribution, $P(\bar{X})$, by following individual neutrons through phase space and taking the successive collision points to be the points $\bar{X}_i$. The value of I is then given by Eq. (3).

If the flux is due to a specified source, $S_1(\bar{X})$, it is possible to choose starting coordinates, $\bar{X}_0$, from a simple function $\tilde{S}(\bar{X})$, and obtain the correct value of the integral by correspondingly modifying $g(\bar{X})$. The necessary modification of $g(\bar{X})$ is most easily established by considering the contribution to the sum in Eq. (3) from the points $\bar{X}_{in}$ generated in the n th neutron history. Let $\bar{X}_{on}$ denote the starting coordinates for the n th neutron. The probability of the coordinates $\bar{X}_{on}$ being chosen, when the function $\tilde{S}(\bar{X})$ is used, is a factor of $\tilde{S}(\bar{X}_{on})/S_1(\bar{X}_{on})$ greater (or less) than the probability of the coordinates $\bar{X}_{on}$ being chosen when the function $S_1(\bar{X})$ is used. Accordingly, the probability of the points $\bar{X}_{in}$ being chosen is increased (or decreased) by the factor $\tilde{S}(\bar{X}_{on})/S_1(\bar{X}_{on})$ when the starting coordinates are selected from the distribution $\tilde{S}(\bar{X})$ instead of $S_1(\bar{X})$. The probability that $g(\bar{X}_{in})$ will contribute to the sum in Eq. (3) is multiplied by the same factor. However, to obtain the correct value of the integral, the probable contribution of $g(\bar{X}_{in})$ must remain constant. So, the value of $g(\bar{X}_{in})$ must be divided by $\tilde{S}(\bar{X}_{on})/S_1(\bar{X}_{on})$. The value of the integral is therefore given by

$$I = \frac{1}{N} \sum_{i=1}^N g(\bar{X}_i) S_1(\bar{X}_{on}) / \tilde{S}(\bar{X}_{on}) \quad (6)$$

Formulating the theory of stochastic evaluation in this way, we immediately observe that we can use one set of neutron trajectories with starting coordinates chosen from a simple distribution, $\tilde{S}(\bar{X})$, to obtain results for any number of specified sources, $S_i(\bar{X})$. We consider $S_i(\bar{X})/\tilde{S}(\bar{X})$ as a weighting function and refer to this as a multiple weighting technique.

The technique was applied to obtain the probability of escape from a finite rod, of neutrons incident on the rod with various incident angular distributions.

COMPUTATION OF TEMPERATURE DERIVATIVES

The absorption rate in each isotope of a system at a given temperature is easily obtained by stochastic techniques. The change in absorption rates as the temperature increases, can be obtained by taking the difference between absorption rates computed at two temperatures. However, this method is quite inefficient. An increase in reactor temperature of several hundred degrees will typically produce an increase in absorption rate of about one percent. If the change in absorption rates is desired to two significant digits, the absorption rates must be obtained to four significant digits. The time required to obtain four significant digits is a factor of 10,000 greater than the time required to obtain two significant digits, since the statistical error is approximately proportional to the inverse square root of the number of sample points.

In order to efficiently compute Doppler reactivity coefficients, it is desirable to have a code for directly computing the temperature derivative of the absorption rates. The integral to be evaluated is:

$$\bar{I} = \frac{\partial}{\partial T} \int \phi(\bar{X}) \Sigma_a(\bar{X}) d\bar{X}. \quad (7)$$

This may be given as:

$$\bar{I} = \int f(\bar{X}) \bar{g}(\bar{X}) d\bar{X}, \quad (8)$$

where

$$\bar{g}(\bar{X}) = \frac{\partial \Sigma_a / \partial T}{\Sigma_t} + \frac{1}{\phi} \frac{\partial \phi}{\partial T} \frac{\Sigma_a}{\Sigma_t} \quad (9)$$

and

$$f(\bar{X}) = \phi(\bar{X}) \Sigma_t(\bar{X})$$

as before, so that we can obtain the points $(\bar{X}_i)$ from the same trajectories used to compute the absorption rates.

The first term in the right hand member of Eq. (9) is easily evaluated at each point $\bar{X}_i$. The second term can be interpreted as the change in absorption rate due to perturbation of the neutron flux, $\phi(\bar{X})$. The fractional change in $\phi(\bar{X}_i)$ per degree is partly due to increased removal at the collision points preceding $(\bar{X}_i)$. This contribution can be evaluated by computing the ratio $(\partial \Sigma_t / \partial T) / \Sigma_t$ at each collision point preceding $(\bar{X}_i)$. Also $\phi(\bar{X}_i)$ is increased by the fraction $(\partial \Sigma_s / \partial T) / \Sigma_s$ at each collision point preceding $\bar{X}_i$. So, $(\partial \phi / \partial T) / \phi(\bar{X}_i)$ can be evaluated as the sum, over all collision points, $\bar{X}_p$, preceding $\bar{X}_i$, of

$$\frac{1}{\Sigma_s(\bar{X}_p)} \frac{\partial \Sigma_s(\bar{X}_p)}{\partial T} - \frac{1}{\Sigma_t(\bar{X}_p)} \frac{\partial \Sigma_t(\bar{X}_p)}{\partial T}. \quad (10)$$

The evaluation of the temperature derivative by this method has been programmed in CDC 3600 FORTRAN. A simple problem, for which the escape probability at two temperatures had been computed by another method, was selected as a test case. The problem geometry was a two region cell with a fuel and a moderator region. The energy range considered contained twenty-five resonances. In order to compare

the temperature derivative computed by this method with the earlier results, the assumption that the Doppler coefficient is proportional to the inverse of the temperature was used. The results differed by less than 2%, which is within the statistical error of the new calculation and the expected error in the earlier results.

V-22. Further Evaluation of a Rotating Power Generating System

A. J. ULRICH, R. O. BRITTAN, R. G. POST,* B. J. HENDERSON† and J. C. CARTER

INTRODUCTION

A proposed concept of a rotating electrical power generation system¹ has undergone further examination and evaluation. The system is of interest because it can operate in a zero-gravity field, using centrifugal forces to circulate the working fluid against the generator load and hydraulic pressure drops. It seems useful to review the description of the system as presented in Ref. 1.

One variation calls for rotating a natural-circulation, closed-flow circuit consisting of a nuclear reactor heat source in a peripheral position and connected by a riser and a downcomer to a separator-condenser heat sink near the axis (see Fig. V 22-1). The downcomer passes through the magnetic field of an MHD generator. The working fluid is a conducting metal having a mixed liquid-vapor phase in the riser and a liquid phase in the downcomer. The centrifugal force field causes a greater pressure gradient in the dense fluid of the downcomer than in the two-phase fluid of the riser. The net pressure difference circulates the fluid against the hydraulic pressure drops and impedance load of the MHD generator.

The cycle begins as heat is added by the nuclear reactor to the liquid metal before it enters the riser. The pressure diminishes as the liquid flows toward the axis of rotation, so that at some point in the riser the coolant drops below saturation pressure, and a lower-density, two-phase flow region is created. The two-phase fluid enters the separator-condenser wherein the vapor rejects heat to the sink and condenses. Upon leaving the condenser, the liquid enters the downcomer or high-density leg. Pressure builds up due to the centrifugal force field and increases as the periphery is

approached. The fluid has maximum mechanical energy as it enters the magnetic field of the generator near the periphery. After conversion of the excess fluid mechanical energy to electrical energy, the fluid returns along the periphery to the reactor heat source.

FIG. V-22-1. Diagram of a Rotating Power-Generating System. ANL Neg. No. 112-5620.

EVALUATION WORK

It has been noted¹ that an evaluation from a purely analytical standpoint would be unduly complex, so that a combined experimental-analytical investigation was recommended. The goal is that of describing the behavior and performance of the system in terms of the principal parameters: heat source strength, angular velocity, radial position, flow impedance, system pressures, and temperatures. It will thus be possible to determine the feasibility of the concept and to suggest and optimize practical designs incorporating the basic principles.

EXPERIMENTAL INVESTIGATION

The experimental phase called for construction and operation of a rotating heat transfer loop in which the simulation of the reactor heat source is effected by an

* University of Arizona, Tucson, Arizona.

† University of Georgia, Athens, Georgia.

FIG. V-22-2. Rotating Loop Schematic Diagram Reproduced from Reference 3. *ANL Neg. No. 112-5464*.

electric heater, the generator load by a variable impedance valve, and the metal working fluid by water. This "proof-of-principal" test is being conducted at the University of Arizona to develop the phenomenology, to provide a check on the analyses, and to establish stability characteristics.

The experimental investigation has been divided into two parts: the development of an electrolytic flow meter² and the construction of a rotating flow loop.³ Neither of these has been completed.

The electrolytic flow meter is being developed to provide an instrument which measures flow in a way which is independent of the gravitational field and which may be mounted on the rotating flow loop. Static calibration of the present version of the flow meter indicates that it is usable in the range of about 300 to 2000 g/min $\pm$ 8 g/min mass flow rate. In the downcomer of the rotating system this would amount to a range of about 5.4 to 30 cm/sec linear velocity. The meter measures flow by injecting an ionic marker in the fluid and detecting the marker at a position downstream. The average velocity during the measurement period may be determined from the time delay, which is inversely proportional to velocity and varies from 6.1 sec at the lower velocity limit to 1.1 sec at the higher limit. This time delay also determines the degree of dynamic equilibrium necessary for a measurement.

The rotating flow apparatus has been assembled but not completely tested. Two identical test loops are used, (see Fig. V-22-2), which counterbalance each other. The flow parameters are to be surveyed, with the mass flow determined by the heat balance method. The flow will then be compared to that in a fixed loop. The fixed loop

operates in the normal gravitational field of one g. Thus the length of riser and downcomer are greatly increased to provide the same driving pressure drop that is present in the rotating loop.

After these studies are completed the electrolytic flow meter and variable impedance valve in the high density leg are to be installed for more detailed measurements.

ANALYTICAL INVESTIGATIONS

PERFORMANCE OF A SYSTEM WHICH MAXIMIZES USE OF GRAVITATIONAL POTENTIAL ENERGY

The first example of a rotating system which comes to mind is one in which use of the gravitational potential is deliberately maximized. This is done by adjusting flow area in every part of the system, such that the velocity of the fluid remains constant even in the presence of change of state or expansion or contraction of the gas phase. By this artifice heat energy is changed to gravitational potential energy in the riser, gravitational potential energy changed to pressure energy in the downcomer, and pressure energy changed to electricity in the MHD generator. Then kinetic energy does not enter into the energy conversion process at all and the consequences of the deliberate maximum use of gravitational potential energy can be assessed.

In the course of this analysis, a number of things become apparent. Because flow velocities can be made low, friction losses in the system can be greatly reduced. On the other hand, the immediate result of allowing no increase in kinetic energy during the expansion in the riser is either an excessive increase in flow area as the two-phase fluid expands and moves radially inward, or limited expansion and acceptance of poor performance because of low exit quality. The conclusion is that the use of gravitational potential energy to the complete exclusion of kinetic energy as an intermediate form in the energy conversion process is not desirable.

SIMPLE FLOW CONSERVATION EQUATIONS FOR A ROTATING SYSTEM

Because of the need for inclusion of kinetic energy, the flow equations which are commonly used in the fixed frame of reference were transformed for use in a rotating frame of reference. Conservation of mass flow in the rotating system may be written as

$$\bar{\rho} (\mathbf{v} - \boldsymbol{\omega} \times \mathbf{r}) \cdot \mathbf{a} = \dot{M},$$

where $\bar{\rho}$ is the average density of the two phase fluid, $\mathbf{v}$ is the vector velocity in the fixed frame, $\boldsymbol{\omega}$ is the rotational angular velocity, $\mathbf{r}$ is the radial vector, $\mathbf{a}$ is a vector whose direction is perpendicular to and whose magnitude is equal to the flow cross sectional area, and

$\dot{M}$ is the mass flow rate. The conservation of linear momentum for steady state flow of the fluid, where the viscosity is zero and the flow in the rotating system is assumed to be irrotational, is given by

$$\frac{1}{2} \nabla [v_r^2 + (v_\theta + \omega r)^2] + 2\omega \times (v_r \hat{\mathbf{r}} + v_\theta \hat{\boldsymbol{\theta}}) + 2\omega \times (\omega \times \mathbf{r}) + \frac{1}{\rho} \nabla P = \frac{\mathbf{F}}{\rho},$$

where v_r is the speed and $\hat{\mathbf{r}}$ is the unit vector in the radial direction, and v_θ is the speed and $\hat{\boldsymbol{\theta}}$ is the unit vector in the θ direction, all in the rotating system. The pressure is P , $\mathbf{F}$ is any force acting on the body of the fluid other than the centrifugal force. The conservation of angular momentum leads to no new equation. It is redundant with the θ component of the linear momentum equation. The conservation of energy becomes

$$\frac{1}{2} [v_r^2 + (v_\theta + \omega r)^2] - \omega^2 r^2 + H + W = \frac{1}{2} [v_{r_1}^2 + (v_{\theta_1} + \omega r_1)^2] - \omega^2 r_1^2 + H_1,$$

where H is the enthalpy, W is the work done by the fluid in passing from position $\mathbf{r}$ to position $\mathbf{r}_1$ in the rotating frame of reference.

A BASIS FOR COMPARISON OF ROTATING AND NON-ROTATING SYSTEMS

With the flow kinetic energy now included, the rotating systems begin to look more like the non-rotating systems which have been investigated by many others. In fact, the conservation equations given above, which are applicable to the riser and downcomer, are the same as equations for a nozzle and diffuser, respectively, except that the gravitational potential energy change is

no longer negligible and v_θ is now augmented by the velocity imparted by the rotation of the system itself. With this analogy established, we may say that any system into which heat is added at a high temperature and pressure, and rejected at a low temperature and pressure, and which has a nozzle and a diffuser, or a pump, has its rotating analog.

The thermodynamic efficiency to be expected from the rotating system is essentially that of the fixed system. Any differences in performance of the two systems can be traced to differences in performance of particular components. These differences have not been assessed, but some examples which favor the rotating system are: separator—perhaps more efficient because of the centrifugal force field; condenser—more efficient because of the centrifugal force field; and simplified pumping because change of state is used directly to provide pumping power with no electrical power generation involved. Penalties which must be borne by any rotating system are the provision of a mechanism to start the system spinning, the difficulty involved in spinning a shield or providing a larger fixed shield, and the added mass due to added strength needed in a centrifugal force field.

REFERENCES

1. A. J. Ulrich, J. C. Carter, R. O. Brittan and R. G. Post, *Evaluation of a Rotating Power Generating System*, Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010 (1965), p. 256.
2. M. E. Satchell, *A Flow Meter to Measure the Mass Flow Rate of an Ionic Fluid*, Master's Thesis, Nuclear Engineering Department, University of Arizona (May 1965).
3. L. E. Stockett, *Measurement of Convective Fluid Flow in Centrifugal Fields*, Master's Thesis, Nuclear Engineering Department, University of Arizona (May 1965).

V-23. A Parametric Design Study of a Reactor System to Produce High Temperature Gaseous Lithium Hydride

M. B. RODIN and J. C. CARTER

Feasibility studies are being made on a reactor system using fission heat and neutron irradiation to directly heat a fluid to very high temperatures. Such a system might have special applications to space propulsion. For this case lithium hydride (LiH) was selected for the working fluid or propellant because of its physical and nuclear properties. The LiH propellant would first be preheated outside of the reactor with fission energy contained in the reactor coolant and then

heated to its final temperature in the reactor by interaction with the neutrons. As the gas reaches the desired temperature, it would be discharged to a nozzle and expanded to provide thrust in space.

Natural lithium is composed of 7.4 % Li-6 and 92.6 % Li-7. The heating in the reactor results from the interaction of Li-6 atoms and neutrons as shown in Eq. (1).

The irradiation products, helium-4 (He-4) and tritium (H-3) have 4.8 MeV of kinetic energy which is converted into thermal energy upon colliding with the residual atoms of Li and H. Since the heating occurs within the gas proper, it is possible to generate temperatures which are many factors greater than the temperature of the containing walls, which may of course require cooling. The concept provides a direct means of achieving gas temperatures in the range of 5,000–10,000°K which are normally obtained by using electrical methods.

On a weight basis, the (Li-6, n) reaction releases as much energy as the (U-235, n) fission reaction. Lithium-6 also has a very high cross section, about 900 barns, for thermal neutrons. A high cross section is required to achieve a high interaction rate.

The specific impulse (I_{sp}) obtained from a propellant is proportional to the square root of the ratio of the absolute temperature to its molecular weight. At reactor discharge conditions, it is expected that the LiH will have decomposed, so that the resultant I_{sp} will be the average of the two components, Li and H. Although Li is not as effective as hydrogen at a given temperature, it is relatively light and the combination of lithium and hydrogen does provide an acceptably effective propellant.

Preliminary neutron and heat balance calculations were made for two reactor configurations. In one system, the LiH was irradiated in the center of the reactor. Geometrically the core consisted of the irradiation chamber surrounded by an annular beryllium reflector, a duplex annular core having a thin inner ring fueled with U-235 O₂, and a thicker outer ring fueled with PuO₂. An annular nickel reflector around the core completed the assembly. In the second system, the LiH is irradiated around the periphery of the reactor. Geometrically, the cylindrical duplex core, comprised of adjacent regions of PuO₂ and U-235 O₂, is surrounded by an annular beryllium reflector and an annular irradiation chamber.

In both systems, liquid Li-7 was used as a coolant

and Nb-1% Zr alloy for fuel cladding and structure. Since for low energy neutrons U-235 has a lower value of capture-to-fission ratio than Pu-239, it was placed between the reflector and Pu-239 fuel to improve the ratio of excess neutrons absorbed in Li-6 per fission.

An analysis of the neutron balance for these particular reactor configurations indicates that in the second system, 1.2 neutrons per fission of fuel atoms would be absorbed in Li-6 and about one half of this value in the first system. The use of lithium enriched in Li-6 permits the use of low gas pressure in the irradiation chamber. Since the cost of lithium enriched in Li-6 is relatively low, the once-through usage is economically practicable.

Approximate heat balances based upon calculated physical properties of LiH were made for the two reactor configurations. The analysis indicated that in the second system about one half of the heat generated in the gas will be lost to the walls by conduction and convection; in the first system substantially less heat will be lost using the centrally located chamber since significantly less wall surface is exposed to the high temperature gas. It was assumed in this study that the gas is relatively opaque to radiant energy. Impingement losses by the irradiation products on the chamber walls were calculated and appeared to be small.

A preliminary weight analysis was made, and like all nuclear systems, the total mass is heavily dependent upon the shielding requirements. Another significant factor affecting weight is the coolant temperature. Although the temperature of the fission heat must be high enough for use in driving the reactor ancillary equipment, preheating the propellant and possibly for generating electrical power for other systems in the space vehicle, a large fraction of it must be dissipated by radiation. Since radiant energy emission rates vary inversely as the fourth power of the absolute temperature, significant savings in radiator weights can be achieved by using the highest possible coolant temperatures.

V-24. Efficiency of Direct Conversion of Nuclear Energy to Coherent Laser Light

J. A. DESHONG, JR.

INTRODUCTION

Interest in nuclear pumping of lasers stems largely from possible applications in space vehicles or remote navigation beacons where compact long-lived power

is necessary, e.g., a single direct-fission-fragment-pumped tube could be incorporated in the nuclear stage of a space vehicle to provide for optical communication, optical radar, etc., without loading the

small electrical supplies on such a vehicle with electrical pump requirements. Larger multiple units incorporated in the nuclear stage could be used for such things as welding and fabrication of metal parts, space ship repair while in space, etc.

Nuclear pumping experiments carried out elsewhere have been directed toward solid state pumping, because the maximum power output of such devices is large compared with present gas lasers. The lack of success with such solid state devices has stemmed mainly from nuclear radiation damage effects.

The gas laser on the other hand does not suffer from radiation damage, except that care must be taken to insure that mirrors and other components are resistant to nuclear radiation damage and that they are placed in as low-level nuclear fields as possible. An intensive survey of the gas laser literature was made at Argonne National Laboratory. This survey led to the conclusion that the helium-neon laser is the most suitable for nuclear pumping.¹ Consequently, the helium-neon laser was used for analytic purposes in the work reported herein, which is a theoretical study of nuclear pumping efficiency.²

INFLUENCE OF TUBE RADIUS AND GAS PRESSURE ON EFFICIENCY

Early experimental laser work indicated that electrically pumped continuous-wave gas lasers have maximum output when operating conditions satisfy the expression:³

$$p \times d = 3.5 \text{ torr-mm}$$

where

p = helium pressure in torr

d = tube diameter in mm

The ratio of helium to neon was 5/1, and the expression was found to hold for a tube diameter range of 1 to 15 mm. The existence of such a relation between pressure and diameter is assumed to be due in part to the need for optimizing the coupling of the electrical energy to the gas, and in part to the need for emptying the lower levels by diffusion to the container walls.

The electrical coupling problem which requires low pressures for efficient operation is not present in the case of direct nuclear pumping. Therefore, operation at much higher pressures should be possible, particularly since higher pressure enhances the coupling of the heavy fragment energy to the gas. Moreover, recent work has demonstrated pulsed-gas-laser peak outputs up to 80 W at pressures from 60 to 240 torr of helium in tubes of 2 to 4.45 cm i.d. and 150 cm length.^{4, 5, 6}

In the following work the conditions required to provide maximum nuclear energy coupling to the gas

are investigated. Expressions relating pressure and tube radius are derived subject only to the constraint of a maximum pressure. Pressure broadening of the fluorescent line starts around 100 torr with consequent increase in the oscillation threshold. The pressure limit of other experiments indicates that pressure broadening becomes dominant at about 240 torr; thus 300 torr was selected as the maximum usable pressure.

The radius of the single tube unit is the main variable. If for the moment the necessity for emptying the Ne_{1s} level by diffusion to the walls is neglected, the effects of increased radius on coupling and output can be considered. The neutron flux required to produce threshold density is independent of the tube radius (r), because the energy (ΔE) deposited in the gas increases with r and the heavy fragment source layer area increases with r , but the tube volume also increases as r^2 , just balancing off the former effects. The power output increases as the volume or r^2 ; the power input increases with r . Therefore, the efficiency should increase directly with the radius. However, the rate of energy deposition per unit path length (dE/dx) decreases as the fragment energy (E) decreases. This reduces the efficiency as the radius is increased, and there is the possibility of an optimum radius for maximum efficiency. The maximum useful tube diameter corresponds to the heavy fragment range, which is inversely proportional to gas pressure.

HIGH-PRESSURE LARGE-DIAMETER (HPLD) LASER PUMPED WITH U-235 FISSION FRAGMENTS

The fission fragment energy loss per cm of travel in general decreases as residual range decreases, because the fragment picks up electrons as it slows down.⁷ This effect can be approximated by a straight line relation between residual range and rate of energy loss. Conditions prevailing as the fragment enters the gas are designated by the subscript I and the conditions as it leaves are designated O . The average path length of the fragment through the gas from the point of entry to the point of exit is $2r$ (from Appendix B of Ref. 2). The average energy (ΔE) deposited in the gas by a fragment may now be calculated.

$$\begin{aligned} \Delta E &= \int_{R_I}^{R_O} (dE/dx) dR = \int_{R_I}^{R_O} -KR dR \\ &= \frac{K}{2} (R_I^2 - R_O^2); \end{aligned} \quad (1)$$

$$\text{but} \quad R_O = R_I - 2r; \quad (2)$$

$$\text{then} \quad \Delta E = 2Kr(R_I - r).$$

Also, since

$$(dE/dx)_I = Bp, \text{ where } p = \text{pressure in torr} \quad (3)$$

$$R_I = (dE/dx)_I / K = Bp/K,$$

and

$$\Delta E = 2r(Bp - Kr). \quad (4)$$

The constants B and K are calculated as follows from data of Appendix B of Ref. 2, for $E_I = 45.05$ MeV:

$$\left(\frac{dE}{dx}\right)_I = \frac{10.8(\text{MeV/cm-atm}) \times p(\text{torr})}{760(\text{torr/atm})}$$

$$= \frac{10.8p}{760} \text{ MeV/cm}$$

whence

$$B = 10.8/760 = 1.421 \times 10^{-2}.$$

It can be seen from Eq. (1) that

$$E_I = KR_I^2/2 = (Bp)^2/2K = (1.421)^2 \times 10^{-4} p^2/2K,$$

whence

$$K = 2.241 \times 10^{-6} p^2.$$

Substituting these values of B and K into Eq. (4),

$$\Delta E = rp(2.842 \times 10^{-2} - 4.482 \times 10^{-6} rp). \quad (5)$$

However, rp has a maximum limit corresponding to $\Delta E \leq E_I = 45.05$ MeV. Therefore, from Eq. (5)

$$rp_{\text{limit}} = 3170 \text{ torr-cm.}$$

Maximum efficiency is obtained when ΔE is maximum, that is, for the value of rp for which

$$\frac{d(\Delta E)}{d(rp)} = 2.842 \times 10^{-2} - 2 \times 4.482 \times 10^{-6} rp = 0, \quad (6)$$

$$rp_{\text{max. efficiency}} = \frac{2.842 \times 10^{-2}}{2 \times 4.482 \times 10^{-6}}$$

$$= 3170 \text{ torr-cm.}$$

Therefore, the maximum efficiency point corresponds with the limit condition above.

All that remains is to select the pressure, which will determine r for maximum efficiency and permit completion of the calculations. The pressure should be set as high as possible since the threshold flux is inversely proportional to pressure. The fact that pressure broadening of the fluorescence line begins to set in at about 100 torr must be considered, but experiments have been performed successfully with pressures up to 250 torr. Therefore, a pressure of 300 torr and a laser tube diameter of 20 cm will be used.

The following are data taken from Appendix B of Ref. 2.

$$p = 10 \text{ torr}$$

$$\text{tube diam.} = 1.125 \text{ cm}$$

$$\Delta E = 0.16 \text{ MeV/fragment}$$

$$\Phi_{TH} = 2.04 \times 10^{13} \text{ neutrons cm}^{-2} \text{ sec}^{-1}$$

$$(P_{IN})_{TH} = 4.2 \text{ kW thermal}$$

$$(P_{OUT})_{TH} = 1 \text{ mW}$$

$$\text{Efficiency} = P_{OUT}/P_{IN} = 2.4 \times 10^{-7}$$

where TH designates "lasing" threshold. Using these data the *HPLD U-235* laser may now be calculated as follows:

$$p = 300 \text{ torr}$$

$$\text{tube diam.} = 20 \text{ cm}$$

$$\Delta E = 45 \text{ MeV/fragment}$$

$$\Phi_{TH} = \frac{0.16}{45} \times \frac{20}{1.125} \times 2.04 \times 10^{13}$$

$$= 1.29 \times 10^{12}$$

$$(P_{IN})_{TH} = \frac{1.29 \times 10^{12}}{2.04 \times 10^{13}} \times \frac{20}{1.125} \times 4.2$$

$$= 4.7 \text{ kW thermal}$$

$$P_{(OUT)_{TH}} = 1 \text{ mW} \times \left(\frac{20}{1.125}\right)^2 = 316 \text{ mW.}$$

The efficiency has increased over that of the small-diameter low-pressure tube by a factor of $(316/4.7) \div (1/4.2) = 282$ times. The *HPLD U-235* laser efficiency is $0.316/4700 = 6.7 \times 10^{-5}$ at threshold. This value is about 25 times that for the electrically pumped case estimated to be 2.4×10^{-6} .⁽²⁾ The large diameter of 20 cm would probably require multiple-pass operation if wall collisions are the only means of emptying the lower level. The *HPLD B-10* laser is more favorable in this respect as will be seen in the next section.

HPLD LASER PUMPED WITH B-10 FRAGMENTS

The B-10 heavy fragment energy loss per cm of travel is essentially constant and independent of residual range except at energies which are low relative to 0.9 MeV, the energy of the fragment as it enters the gas. The average energy (ΔE) deposited in the gas by a fragment then is a straight-forward calculation.

$$\Delta E = \int_{R_I}^{R_O} (dE/dx)_I dR$$

$$= \int_{R_I}^{R_O} -K dR = K(R_I - R_O);$$

since

$$\begin{aligned} R_o &= R_I - 2r, \\ \Delta E &= 2Kr \end{aligned} \quad (7)$$

and

$$K = (dE/dx)_I = Bp. \quad (8)$$

Substituting Eq. (8) into Eq. (7) yields

$$\Delta E = 2B rp. \quad (9)$$

The constant B is obtained as follows from data of Appendix C of Ref. 2 for $E_I = 0.9$ MeV:

$$\left(\frac{dE}{dx}\right)_I = 4.4 \times 10^{-3} (\text{MeV}/10 \text{ torr-cm}) \times p(\text{torr})$$

whence

$$B = 4.4 \times 10^{-4} \text{ MeV/torr-cm.}$$

By substitution into Eq. (9),

$$\Delta E = 8.8 \times 10^{-4} rp. \quad (10)$$

However, rp has a maximum limit since $\Delta E \leq E_I = 0.9$ MeV. From Eq. (10),

$$rp_{\text{limit}} = \frac{0.9}{8.8 \times 10^{-4}} = 1020 \text{ torr-cm.}$$

In the case of B-10, the efficiency continues to increase with increasing radius up to the above limit.

Again it is desirable to set the pressure as high as possible for the same reasons given in the preceding section for the U-235 laser. Therefore, a pressure of 300 torr and a laser tube diameter of 5 cm will be used. From Appendix C of Ref. 2:

$$\begin{aligned} p &= 10 \text{ torr} \\ \text{tube diam.} &= 1.125 \text{ cm} \\ \Delta E &= 5.0 \times 10^{-3} \text{ MeV/fragment} \\ \Phi_{TH} &= 5.5 \times 10^{13} \text{ neutrons cm}^{-2} \text{ sec}^{-1} \\ (P_{IN})_{TH} &= 1.62 \text{ kW thermal} \\ (P_{OUT})_{TH} &= 1 \text{ mW} \\ \text{Efficiency} &= P_{OUT}/P_{IN} = 6.2 \times 10^{-7}. \end{aligned}$$

Then the HPLD B-10 laser may be calculated as follows:

$$\begin{aligned} p &= 300 \text{ torr} \\ \text{tube diam.} &= 5 \text{ cm} \\ \Delta E &= 0.9 \text{ MeV/fragment} \\ \Phi_{TH} &= \frac{5.0 \times 10^{-3}}{0.9} \times \frac{5}{1.125} \times 5.5 \times 10^{13} \\ &= 1.35 \times 10^{12} \end{aligned}$$

$$\begin{aligned} (P_{IN})_{TH} &= \frac{1.35 \times 10^{12}}{5.5 \times 10^{13}} \times \frac{5}{1.125} \times 1.62 = 179 \text{ W} \\ (P_{OUT})_{TH} &= 1 \text{ mW} \times \left(\frac{5}{1.125}\right)^2 = 19.8 \text{ mW}. \end{aligned}$$

The efficiency has increased over that of the small-diameter low-pressure tube by a factor of $(19.8/0.179) \div (1/1.62) = 179$ times. The HPLD B-10 laser efficiency is $(19.8 \times 10^{-3})/179 = 1.1 \times 10^{-4}$ at threshold. This value is about 46 times that for the electrically pumped case estimated earlier to be 2.4×10^{-6} . The 5 cm diameter is comparable to the 4.5 cm diameter used in pulse experiments at 240 torr reported in the literature; therefore, successful operation of the HPLD B-10 laser would seem optimistic if not assured.⁶

CONCLUSIONS

Large - diameter, high - pressure, direct - nuclear - pumped gas lasers show theoretical efficiencies two orders of magnitude larger than those obtained previously for small-diameter tubes. Lasers filled with helium at a pressure of 300 torr and made with large-diameter tubes optimized for that pressure require a flux of about 10^{12} neutrons $\text{cm}^{-2} \text{ sec}^{-1}$ to reach the lasing threshold. The power output of such types is about two orders of magnitude higher at threshold than the output of the small-diameter types, at least under pulse conditions.

Boron-10 used as a pumping medium has the following characteristics: (1) thermal efficiency is good; (2) the optimum diameter of 5 cm required for maximum efficiency at a pressure of 300 torr corresponds to the diameter of high-pressure lasers whose feasibility has already been demonstrated with electrical pumping; (3) residual radioactivity is low; (4) neutron absorption is high.

Uranium-235 used as a pumping medium exhibits the following properties: (1) thermal efficiency is fair; (2) the optimum diameter of 20 cm required for maximum efficiency at a pressure of 300 torr may require folding the light path into an annular shape near the walls to increase the rate at which the lower neon level is emptied. The feasibility of the folded light path has been demonstrated with electrically pumped lasers; (3) residual radioactivity is high; (4) neutron multiplication properties are available.

Thus, each of the two materials has advantages and disadvantages which must be considered for the particular use the designer has in mind. For example, the B-10 laser would act as a poison element in a reactor and could be considered as a control rod or could be used to attain longer reactor core life by depletion of the B-10. The U-235 laser, on the other hand, would permit consideration of a reactor using

such lasers as fuel elements in a large multiple array to achieve high laser beam power.

REFERENCES

1. L. O. Herwig, *Concepts for Direct Conversion of Stored Nuclear Energy to Laser Beam Power*, Trans. Am. Nucl. Soc. **7**, No. 1, 131 (June 1964).
2. J. A. DeShong, Jr., *Optimum Design of High-pressure, Large-diameter, Direct-nuclear-pumped Gas Lasers*, ANL-7030 (1965).
3. E. I. Gordon and A. D. White, *Similarity Laws for the Effect*

- of Pressure and Discharge Diameter on Gain of He-Ne Lasers, Appl. Phys. Letters **3**, 199 (December 1, 1963).
4. L. E. S. Mathias and N. H. Rock, *A Helium Neon Laser Amplifier*, Appl. Opt. **4**, 133 (January 1965).
5. H. A. H. Boot, D. M. Clunie and R. S. A. Thorn, *Pulsed Laser Operation in a High-pressure Helium Neon Mixture*, Nature **198**, 773 (May 25, 1963).
6. D. M. Clunie and N. H. Rock, *Optical Gain in Neon and Helium/Neon Pulsed Discharges*, Phys. Letters **13**, 213 (December 1, 1964).
7. J. Sharpe, *Nuclear Radiation Detectors*, (John Wiley and Sons, New York, Revised 1964), p. 29.

V-25. The Amplification of Coherent Gamma Photons

J. C. CARTER

INTRODUCTION

An exploration into the technology pertaining to the amplification of coherent gamma photons by means of a nuclear reactor has been initiated. The objective is that of determining the possibility of coupling directly or indirectly the phenomena of nuclear reactions to the phenomena of amplification by the stimulated emission of gamma photons.

The amplification of coherent beams of photons is a relatively new and rapidly developing technology as indicated by the development of masers and lasers. Extension of research into the gamma band of the spectrum presents challenging problems and hopefully commensurate rewards. For example, the penetrating power and divergence characteristics of gamma rays make them superior to longer wave length radiations for interplanetary communication.

Herein the term "maser" in the general sense applies to the entire spectrum of electromagnetic radiations, and specifically to the microwave band. The term "laser" applies to the light band, and "graser" to the gamma band.

The use of nuclear reactors as the source of energy for maser action is without precedent. The case for the nuclear lasers is presented by J. DeShong¹ (see Paper V-24). The work on nuclear grasers is still in the exploratory phase.

If fission energy and maser action in any band can be coupled, it will result in one of the more direct means of converting the energy of an atom to a form which can fulfill many functions. The current uses of coherent beams of electromagnetic waves are stated in nearly every paper and article on masers and lasers² (see references in Ref. 1). The unique function which

a nuclear graser can fulfill is that of the beam in space communication systems and in the precision navigation of space craft. A nuclear reactor with the exception of the sun and the stars is the only long-term source of energy which is ideally suited to a space environment. Also, grasers theoretically have insignificant amounts of divergence over distances associated with interplanetary space, if the experience with laser beams directed to the moon² is an indication.

The theory of the absorption and stimulated emission of photons is universally accepted and also substantiated by the design and operation of masers in the microwave and light wave bands. Stimulated emission is theoretically possible in any quantized system from any excited state to any lower state to which it can make a spontaneous radioactive transition.³ Therefore, the possibility of stimulated emission in the gamma band analogous to that phenomenon in the laser and maser bands exists. However, the accomplishment of it will be difficult since there will have to be multiplication during a single passage of photon avalanches initiated by spontaneous emission. This is so because gamma rays cannot be reflected. Also, only a fraction of the resonant cross section can be effective because of line-broadening with the nuclear environment.

While existing theories and basic principles of both nuclear reactors and masers are general enough to apply equally well over the whole frequency spectrum the physical problems associated with the integration of the theories, with the inversion of energy levels and with the stimulated emission in the gamma band, are quite different. Integration of the two theories into one comprehensive theory for nuclear graser systems is

ing explored concurrently with the physics of coherent beams of gamma photons.

The physical system of a nuclear graser is extremely complex and involves the interaction of physical phenomena of many disciplines. It is proposed to try to identify and study each of these phenomena individually and then to study their interaction within the framework of the two nuclear graser systems resulting from the ideas for integrating nuclear theory with the as yet vague graser theory. The two ideas are nuclear pumping, a direct means, and the use of radionuclides, an indirect means. These and the basic physics of some of the component phenomena of the nuclear graser systems are being studied. The studies have progressed to the point where considerations of the schemes and of some of the associated problems seem worthy of note. In addition, an initial outline suggesting future programmatic effort is presented.

NUCLEAR PUMPING

Nuclear pumping is a process in which nuclear reaction products transfer energy directly into a maser medium with the effect of producing an inverted population of excited states.

The passage of energetic nuclear particles through material is characterized by energy loss as a consequence of elastic and inelastic collisions with the electrons and nuclei in the medium giving rise to ionized and excited atoms, free electrons, and atomic recoils. The free electrons usually receive considerable energy (above that required for ionization) during an interaction with a heavy particle. This energy is dissipated in the maser medium by electron-atom interactions to form further ionized and excited atoms. For nuclear particles stopped in a gas medium, about one-half of the energy is lost to the gas atoms through secondary electron interactions. A sizable fraction of the energy lost by the particles can appear as atomic excitation which is distributed relatively uniformly throughout a medium. In some circumstances, an important contribution to the density of excited states is obtained through ionized atoms, created in the path of the particle, which recombine with electrons or negative ions to produce excited atoms. Also, the atomic recoils and nuclear reaction products will excite and ionize atoms in a medium. Thus energy losses by nuclear particles in matter provide a mechanism for producing atomic excitation.

The inversion of atomic levels of the maser medium with the energy of nuclear fission provides a constant and uniquely suitable, though inefficient, means of producing coherent beams of photons. The exciting energy may be provided by neutrons, gamma rays, fission products, and possibly in other forms existing

as a result of fission. The absorption of some of the energy of fission and the resulting isomeric states is an extremely complicated phenomenon because of nuclear interactions and competitions occurring simultaneously during the transfer of some of the energy.⁴

It has been demonstrated that energy can be absorbed at a rate at least equal to the rate of emission, otherwise lasers would not function. Whether this is so when the primary source of energy is a nuclear reactor has yet to be demonstrated. Theoretical calculations by investigators indicate that there is a good chance that it can be.^{1, 5, 6}

G. Baldwin, et al.⁷ states that, "If the medium should show the Borrmann effect⁸ in which photoelectric absorption is greatly reduced for radiation propagation in directions satisfying the Bragg condition and if as well resonance in emission and nuclear absorption could then occur at distinct energies, an extremely favorable case would be realized. However unlikely, its possibility should be investigated."

Nuclear pumping of a graser presents more theoretical difficulties than nuclear pumping of a laser. It is not certain that the stimulated emission of gamma photons can be accomplished in any reasonable arrangement of physical components and because the levels to which atoms must be excited by nuclear pumping are higher than those associated with a nuclear laser. The problem is currently being studied by several groups.^{7, 9}

If a nuclear reactor is used as the means of inverting the energy levels of the material, it will have to be able to put at least one quantum of energy into the isomer. Due to the direction and velocity of waves in the gamma region of the frequency spectrum, there can be no reflectors. The gamma is a once-through wave. There can be no stimulation of an infinite medium. However, amplification greater than two can be accomplished by having a maser whose coincident physical axis is quite long.

A limiting condition for the growth of a photon avalanche has been determined^{7, 9} and may be stated thus:

$$\frac{N}{N_0} \frac{\lambda^2}{2\pi\sigma_0} \frac{g_2}{g_1} \frac{\Gamma_r}{\Gamma} > 1,$$

where

$\sigma_0 \equiv$ the average atomic absorption cross section (photoelectric and Compton effect)

$h \equiv$ Planck's constant

$N \equiv$ effective excited state concentration

$N_0 \equiv$ total atom concentration

$\lambda \equiv$ wave length of the transient energy average

g_2 and $g_1 \equiv$ the respective statistical weights of upper and lower levels

Γ_r and $\Gamma \equiv$ the partial and total width of the gamma ray line.

Note that since $\lambda^2/2\pi\sigma_0$ is of the order of 10^3 , concentration and line width are the determining factors.

THE USE OF RADIONUCLIDES

An indirect means of coupling a nuclear reactor to a maser medium is one in which a material is irradiated within a nuclear reactor and then transferred to a maser medium.

This is a process in which energy is concentrated in the form of long-lived, artificially induced isomers over a relatively long period and released over a very short period. This achieves a concentration of energy which may not be possible in a direct coupled system.^{6, 10}

The active material is placed in an excited state by particle bombardment of a target isotope followed by the separation of the transmuted from the parent material. In the case of thermal neutron bombardment such as occurs in a nuclear reactor, a Szilard-Chalmers separation¹⁰ is made in which the target isotope is manufactured in a chemical component and the transmuted isomer is separated by nuclear recoil.

Due to the fact that the material has to be irradiated, separated, concentrated and placed in the maser medium, isomers with short lifetimes are not suitable. Also, in many isomeric transitions, internal conversion competes with radioactive decay. Therefore, isomers with large internal conversion coefficients are not desirable. Isomers should not be chemically inert and should have appreciable yield. About thirty isomers have been found which are potentially suitable.⁷

THE POSSIBILITY OF AMPLIFICATION

Gamma photons are electric and magnetic multiple radiations of various modes associated with the differences in angular momentum and parity between the two states involved in the transition.

Masers are devices for receiving energy in the form of low density incoherent photons and delivering it as high density coherent beams of photons. The total amount of power involved is quite small, but the power density is extremely high. Obviously more power must be absorbed than is delivered through induced emission because the spontaneous emission is unavailable.

The absorption and emission probabilities and the amount of power produced in the desired form are considered here.

In the atoms of a maser material there are N_m atoms in state m and N_n atoms in state n (ground state) with energies E_m and E_n . The frequency range ν_{mn} is then

$$\nu_{m,n} = \frac{E_m - E_n}{h},$$

where

$h \equiv$ Planck's constant.

Let u_ν equal the energy density of radiation per unit frequency range. Then the number of $n \rightarrow m$ transitions in time dt is

$$N_m B_{nm} u_\nu dt.$$

The energy absorbed is

$$E_{abs} = N_n B_{nm} u_\nu h \nu_{mn} dt.$$

The transition $m \rightarrow n$ is

$$n_m (A_{mn} dt + B_{mn} u_\nu dt).$$

A_{mn} , B_{mn} and B_{nm} are the Einstein coefficients for spontaneous emission, induced emission and absorption, respectively. They are probabilities that energy levels will change as a result of the absorption or emission of photons. It is desirable to have $A_{mn} \ll B_{mn}$.

The emitted energy from the maser is

$$E_{em} = N_m (A_{mn} dt + B_{mn} u_\nu dt) h \nu_{mn}.$$

Since energy emitted, E_{em} , must be greater than energy absorbed, (E_{em} and E_{abs}), and since $B_{nm} = (A_{mn} + B_{mn})$, N_m must be greater than N_n in order to maintain an excess of energy in the state m during dt .

The instantaneous power is

$$\frac{dE}{dt} = (N_m - N_n) B_{mn} u_\nu h \nu_{mn}.$$

The problem now becomes a matter of assigning numbers to the probabilities A and B .

The above is an ideal case. Actually there are many factors affecting the process which have to be taken into account before the final equations are formulated. These are the natural width of the emission spectrum, Doppler broadening, collision broadening, relaxation times and the saturation of energy levels.

EMISSION PROBABILITIES

It is assumed that each nucleus behaves as an independent system, interacting with radiation by emitting or absorbing photons, and that there exists an assemblage of two-level atomic systems in thermal equilibrium with a container at temperature T . In this case the ratio of populations is

$$\frac{N_n}{N_m} = \exp [-(E_n - E_m)/KT] = \exp (h \nu_{mn}/KT),$$

where

$h \equiv$ Planck's constant

$K \equiv$ Boltzmann's constant

$c \equiv$ velocity of light.

The radiation energy density u_ν is given by

$$u_\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{\exp(h\nu/KT) - 1} \quad (\text{Planck's Radiation Law}).$$

Although the radiation intensity increases without limit as the temperature increases, the population ratio approaches unity. A. Einstein¹¹ reasoned that radiation must therefore be equally capable of inducing downward as well as upward transitions between states. Denoting the spontaneous emission probability per atom per second by A_{mn} , the population balance equation is

$$\frac{dN_n}{dt} = -A_{mn}N_n + B_{mn}(N_m - M_n)u_\nu.$$

Consistency of these equations demands that

$$B_{mn} = \frac{c^3}{8\pi h\nu^3} A_{mn} \quad (\text{Bohr frequency condition})$$

so that the net excess rate of downward transition is

$$-\frac{dN_n}{dt} = B_{mn} \left[N_n \left(n + \frac{8\pi\nu^2}{c^3} \right) h\nu - N_m n_p h\nu \right],$$

where

$$n_p = \text{photons/cm}^3.$$

A_{mn} is evaluated from the expression

$$A_{mn} = \frac{64\pi^4 \nu_{mn}^3}{3h} \cdot \frac{|\bar{D}|^2}{dn},$$

where

$\bar{D} \equiv$ dipole matrix element of transition.

The photon flux ϕ in a masing medium may be analogous to neutron flux in which case the cross section for indirect emission is

$$\sigma = \frac{d\phi}{\phi N_n dx}.$$

NATURAL WIDTHS OF THE EMISSION SPECTRUM

The energy levels m, n of nuclear states are never narrow enough so that the electromagnetic radiations occur with a frequency ν_{mn} . There is a spread $\Delta\nu$ with a shape factor s . The maximum energy occurs at ν_{mn} and decreases either side of ν_{mn} . The half width is the distance from ν_{mn} to the frequency where the energy is that at ν_{mn} . The whole or natural width is twice the half width.

Every spectrum line has a natural breadth deter-

mined by the widths of the energy levels between which transitions occur to produce the radiation,

$$\Delta E \Delta t \simeq h/2\pi.$$

The average time spent by a nucleus in an excited state is directly related to the spontaneous emission probability. The probability A_{mn} that a system will change its state in time dt is $A_{mn} dt$. Therefore, the probability of spontaneous emission for the transition from state m to state n , ($m \rightarrow n$), is $1/\tau$. τ is the mean lifetime of the excited state m .

$$\Delta E \simeq \frac{h}{2\pi\Delta t} \simeq \frac{hA_{mn}}{2\pi},$$

$$\Delta E = h\Delta\nu,$$

$$\Delta\nu = \frac{\Delta E}{h} = \frac{A_{mn}}{2\pi}.$$

The intensity of radiation is the number of photons flowing through unit area per unit time,

$$I_{mn} = N_m h c \nu A_{mn}.$$

The intensity of a spectral line in emission, $I(\nu)$, is maximum at ν_{mn} . For $(\nu_{mn} - \nu) = \Delta\nu = A_{mn}/2\pi$, the intensity is half of the maximum intensity. The shape factor s is¹²

$$s(\nu, \nu_{mn}) = \frac{A_{mn}}{2\pi^2} \frac{1}{(\nu - \nu_{mn})^2 + A_{mn}^2/4\pi^2}$$

and

$$I(\nu) = I_{mn} s$$

RESONANT CROSS SECTIONS AND LINE BREADTH

The cross sections for resonance interactions can be derived in several ways. R. Babcock, et al.⁹ express the cross section for stimulated emission as

$$\sigma(E) = \frac{f\lambda^2}{2\pi(1+\alpha)} \frac{\Gamma}{\Gamma^1} \frac{\tau_1}{\tau_1 + T_2} \frac{(\Gamma^1/2)^2}{(E - E_0)^2 + (\Gamma^1/2)^2}$$

and the cross section for resonant absorption as

$$\sigma_a(E) = \frac{2I_2 + 1}{2I_1 + 1} \frac{f\lambda^2}{2\pi(1+\alpha)} \frac{\Gamma}{\Gamma^1} \cdot \frac{\tau_1}{\tau_1 + T_2} \frac{(\Gamma/2)^2}{(E - E_0)^2 + (T^1/2)^2},$$

where

$f \equiv$ resonant fraction* (Debye-Waller factor)

$\lambda \equiv$ wave length of the transient energy

$\alpha \equiv$ ratio of number of electrons emitted to number of photons emitted

$\Gamma \equiv$ natural width

* (At low ambient temperature $\log f$ varies as $(h\nu)^2/\theta$).

- $\Gamma^1 \equiv$ resonance width
 $I_{1,2} \equiv$ nuclear spin
 $E \equiv$ energy of photon $= h\nu$
 $E_0 \equiv$ energy at resonance peak
 $\tau \equiv$ mean lifetime
 $h \equiv$ Planck's constant
 $\theta \equiv$ Debye temperature
 $\nu \equiv$ frequency.

The accurate determination of these cross sections has been and is continuing to be made by theoretical calculations and some substantiating experimental work.

ISOMERIC STATES

An investigation of isomeric nuclear states is being made to determine which nuclei provide the best stimulated emission effect. The ratio of stimulated photon emission rate to spontaneous photon emission rate $\Delta R/R$ is given by⁹

$$\frac{\Delta R}{R} = \left[\frac{f^2 \lambda^2 N_m}{8\pi k(1 + \alpha)} \right] \frac{\tau_\Gamma}{\tau_m},$$

where

- $\alpha \equiv$ conversion electron/photon
 $\tau_\Gamma \equiv h/\Gamma$
 $\tau_m \equiv$ lifetime of state m .

A very large area of research is associated with the determination of the physical characteristics of nuclear radiation phenomena. The principle characteristics to be determined for likely isomers are:

1. The half lives of the energy states
2. Isomeric transition energy
3. Internal conversion coefficients
4. Energy level statistical weights
5. Nuclear spins
6. The stimulated emission cross sections
7. Non-resonant absorption cross sections
8. Recoil energy
9. Cross sections for capture in various energy levels.

This basic research can be conducted without association with any system.

PROPERTIES OF ENERGY LEVELS

Any system of particles occupying a limited volume of space and in a state of dynamic equilibrium must, according to the principles of quantum mechanics, have a total energy equal to one of a discrete set of values (the energy levels of the system). The determination of the energy level properties result from the interaction of phenomenologically and theoretical techniques. They are complementary. A large amount of work has been done and more will be done as a result of the stimulation provided by the laser development.¹³

The investigation of excitation of bound and virtual states, radioactive transition, and nuclear spectra, is so large in area that it will be necessary to restrict research efforts to the regions believed to have most significance in nuclear-maser systems.

MÖSSBAUER EFFECT

Gamma radiation loses a relatively large amount of its energy when there is a change in energy level of a nucleus due to the emission or absorption of gamma photons. This loss of energy goes into recoil motion of the emitting or absorbing nucleus.

The frequency of the gamma photon is changed as a result of the Doppler shift and the emission and absorption lines are displayed. The widths of the lines are broadened over what they would be if there were no recoil. Mössbauer discovered that there are recoilless emission and resonance absorption of gamma photons under some circumstances. When this can be accomplished, the energy loss is negligible. It is possible that the Mössbauer effect may be very important in graser action; therefore, the effect is being investigated with this possibility in mind.

The Mössbauer effect is present in certain types of crystals. It is, therefore, necessary to determine which crystals demonstrate this effect and the influence of the effect on the performance. The Mössbauer cross section is found to be⁷

$$\sigma_M = \frac{\lambda^2}{2\pi} \frac{w\beta f}{1 + \alpha} \frac{\tau_1}{\tau_1 + \tau_2} \frac{1}{1 + 4(E - E_{21})/\Gamma^2},$$

where

- $\lambda \equiv$ wave length of the transient energy
 $E \equiv$ energy of the photon
 $E_{21} \equiv$ excitation energy required to go to state 2 from state 1
 $\alpha \equiv$ ratio of spontaneously emitted electrons to photons. (Internal conversion coefficient)
 $\tau \equiv$ relaxation time
 $f \equiv$ Debye-Waller factor
 $w \equiv$ statistical weight factor
 $\beta \equiv$ branching ratio
 $\Gamma \equiv$ natural width.

An excited nucleus in the crystal lattice of a solid can emit a gamma ray with no momentum or kinetic energy transfer to the nucleus. This is so because atoms of a solid have strong binding energy in relation to the recoil energy and the recoil is not free. The recoil momentum is effectively accepted by a massive crystal as a whole rather than by the single nucleus. The recoilless emission is not a certain phenomenon. There exists a probability <1 that it will occur. In order to obtain an explicit estimate of the probability that the lattice remains in its initial state after emission of the

gamma ray, the assumption is made that the forces between atoms are harmonic. The expression for the probability as derived by H. Lipkin¹⁵ is

$$P(n_i n_s) \equiv P(n_s n_s) \\ = \exp \sum_s \{ -(2n_s + 1)[(\hbar k)^2 / 2M\hbar\omega_s] a_s^2 \}$$

where

- ω_s $\equiv$ angular frequency
- n_s $\equiv$ excitation state of the nucleus
- n_i $\equiv$ initial state of the nucleus
- a_s $\equiv$ a dimensionless coefficient
- $\hbar$ $\equiv$ Planck's constant/ 2π
- k $\equiv$ propagation number
- M $\equiv$ mass of nucleus.

Since the crystal effectively takes up the recoil momentum so that practically all of the excitation energy goes into the emitted gamma ray in these cases. The energy of the gamma rays is (for most practical purposes) equal to E_0 , the excitation energy. The spread in the energy of the gamma rays is due almost entirely to the natural line width arising from the finite lifetime of the excited state. This amounts to 4.7×10^{-6} eV for the 129 keV level in Ir-191 and to 4.57×10^{-9} eV for the 1.4 keV level in Fe-57.

If the gamma rays are passed through an absorber, which contains the same type of nuclei in the ground state as gave rise to the gamma rays which were emitted without recoil, then it is possible to have recoilless absorption of the gamma rays. The recoilless absorption line is also very sharp. Its width is approximately equal to the natural line width. This is much sharper than the thermal Doppler widths necessary for absorption by free atoms; namely, 4.57×10^{-9} eV compared with 0.02 eV for Fe-57 at 300°K. If the recoilless emission line and the recoilless absorption lines overlap significantly, then there is a strong probability that those gamma rays which were emitted without recoil will be absorbed without recoil when they pass through the absorber. If the source (or the absorber) is moved, then the energy of the recoilless emission line (or recoilless absorption line) is shifted due to the Doppler effect, and if the velocity of the source (or absorber) is high enough, the recoilless emission and absorption lines will no longer overlap significantly and the absorption of the gamma rays from the source decreases since Mössbauer found that for the 129 keV line in Ir-191 it was only necessary to move the source at a speed of 1.5 cm/sec to shift the recoilless emission line by its natural line width. This may be contrasted with the enormous speeds required to make up the recoil energy, in the case of gamma ray emission by free nuclei with recoil. The Mössbauer-effect absorption cross section depends critically upon the amount of

overlap of the recoilless emission and recoilless absorption lines. It can be seen that small shifts in the energy (or frequency) of either the recoilless emission or absorption lines changes the probability for the Mössbauer effect substantially and produces measurable effects.

SUGGESTIONS FOR THEORETICAL AND EXPERIMENTAL WORK

A study of the work of investigators^{7, 9} of maser action between nuclear states indicates that no existing nuclear reactor at a steady state can produce enough energy in a form which can excite the atoms of a known maser material to levels necessary to produce coherent gamma emission; however, coherent gamma emission can be achieved in crystals of highly concentrated radioactive isotopes. The crystals are obtained by irradiation followed by radiochemical separation (Szilard-Chalmers Process) and then by the growth of pure crystals of the isotope. The half-life of these isotopes will have to be long enough to permit the separation and growth of the crystals and still remain above the metastable energy level. This practical aspect needs evaluation.

The idea of nuclear masers implies a complex and difficult research and development project with a large reward for attaining the primary objective (a coherent beam of gamma photons) and with the likelihood of a large fall-out of technological contributions to many disciplines.

More mathematical analysis and experimental work on isolatable entities (such as experimentation with the Mössbauer effect) should be done before the complex and expensive proof-of-principal experiment for a system is initiated.

A list of suggested preliminary theoretical and experimental work follows:

Theoretical Work

1. The first and most important work is considered to be that of making a mathematical analysis of coherent emission mechanisms. It must be shown theoretically that one or more mechanisms exist which can produce the coherent emission of gamma photons. The analytical efforts to date have been naive; however, they could become very involved because of the complexity of the phenomena.
2. Anomalous transmission in crystals and lattice dynamics require investigation for this specific case.
3. The problem of growing perfect radioactive crystals should be the subject of thorough analytical work because a trial and error effort is almost certain to be expensive in every respect.
4. The field of spectroscopic research should be carefully checked for revisions and new data.

Experimental Work

It is advisable to be conservative in experimentation until the theory is developed. Some experiments are suggested which will be of value regardless of what turn the nuclear maser investigation takes, as

1. Mössbauer measurements with isomers having lifetimes one or more orders of magnitude greater than those presently employed in Mössbauer research.

2. Experiments with photon-maser techniques and Mössbauer isomers which attempt to change the recoilless fraction and to separate displaced times by artificially changing the spectrum of lattice vibrations.

3. Study of the Kössel lines in crystals containing gamma ray sources.

4. Attempt to grow crystals of radioactive materials and study their properties as a function of electromagnetic radiation rate.

The list of proposed research efforts are purposely kept small until a generally accepted mathematical analysis indicates that a nuclear maser is both physically possible and has a value commensurate with the estimated effort to develop it. Such analyses are being made by the advanced technology departments of several United States and Russian organizations. To date, no theory or evaluation is generally accepted.

The nuclear maser can have applications where extremely high energy density of photons with very short wave lengths are needed. Such applications are in weapon systems, communication systems, and the medical sciences.

Some of the unique features of nuclear masers are:

1. Nuclear excitation involves far greater energy per atom than chemical, other processes, e.g., thermal, etc.

2. Energy can be transformed initially at low concentration, stored at high concentration, and later transformed to extremely high density.

3. The long-lived isomer used in the device ensures a reasonably constant energy release rate.

REFERENCES

1. J. A. DeShong, *Optimum Design of High Pressure Large Diameter Direct Nuclear Pumped Gas Lasers*, ANL-7030, (1965).
2. C. A. Hogg and L. G. Sucsy, *Masers and Lasers*, (Maser/Laser Associates, Cambridge, Massachusetts, 1962).
3. R. H. Fowler, *Statistical Mechanics*, (Cambridge University Press, London, 1955) 2nd Edition.
4. R. D. Evans, *The Atomic Nucleus*, (McGraw-Hill Book Company, New York, 1955), p. 729, Fig. 3.1.
5. R. A. Shatoo, *Optical Masers*, N63-21469, (April 15, 1963).
6. L. O. Herwig, *Nuclear Pumping of Gas Laser Systems*, United Aircraft Corporation Research Laboratory, E. Hartford, Conn., (June 1, 1964).
7. G. C. Baldwin, et al., *On the Possibility of Maser Action Between Nuclear States*, Advanced Technology Laboratories—General Electric, 1 Rusl Road, Schenectady, New York, (May 1963).
8. G. Borrmann, *Trends in Atomic Physics*, (Interscience Publishers, Inc., New York, 1959) p. 262.
9. R. V. Babcock, et al., *Stimulated Emission of Gamma Radiation*, WEPL-63-927-115-R1, (December 1963).
10. R. A. Sharp and R. A. Schmitt, *Studies of the Szilard-Chalmers Process*, GA-910, (April 15, 1959).
11. A. Einstein, *Zur Quantentheorie der Strahlung*, *Phys. Zeitscher* **18**, 121 (1917).
12. G. Troup, *Masers*, (London Methuen and Co., Ltd., 1959).
13. G. Hersberg, *Atomic Spectra and Atomic Structure*, (Dover Publication, New York, 1944) p. 23.
14. H. Frauenfelder, *The Mössbauer Effect*, (W. A. Benjamin, Inc., New York, 1962).
15. H. J. Lipkin, *Some Simple Features of the Mössbauer Effect*, *Annals of Physics* **9**, 2 (1960) pp. 332-339.

V-26. Thermionic Energy Conversion with a Plasma Thermocouple

H. K. RICHARDS

NOMENCLATURE

E $\equiv$ volt/cm
 I $\equiv$ total current
 J $\equiv$ current density
 J_i $\equiv$ ion current density in plasma
 J_{i0} $\equiv$ forward ion current density at emitter
 K $\equiv$ Boltzmann's constant
 m $\equiv$ particle mass
 n^+ $\equiv$ density of particles moving from emitter to collector

n^- $\equiv$ density of particles moving from collector to emitter
 n $\equiv$ density in plasma
 n_{i0} $\equiv$ ion density produced at emitter
 n_0 $\equiv$ density of particles in storage space
 n_T $\equiv$ total density of ions and atoms between emitter and collector
 T_e $\equiv$ emitter temperature, °K
 T_c $\equiv$ collector temperature, °K
 T_0 $\equiv$ temperature of storage room, °K

u $\equiv$ dc velocity of particles
 J $\equiv$ time and space dependent velocity
 V $\equiv$ volt
 V_i $\equiv$ ionization voltage

Subscripts:

a — refers to atoms
 e — refers to electrons
 i — refers to ions
 α $\equiv$ fraction of atoms ionized
 β $\equiv$ propagation constant
 ϵ $\equiv$ electron charge
 ω $\equiv$ wave oscillation frequency, radians/sec.
 Φ_e $\equiv$ emitter work function
 Φ_c $\equiv$ collector work function
 ψ $\equiv$ voltage difference between sheath and emitter
 ψ_0 $\equiv$ voltage difference between plasma and emitter
 Other symbols will be shown at the proper place in the text.

ION OSCILLATIONS

BASIC CONSIDERATIONS

Experimental results for dc and rf power output from a cell with a Ta emitter and a Mo collector containing Cs, K and Na vapors are described in Ref. 1. However, theoretical treatment for the rf output due to ion oscillations is still very incomplete. A semi-empirical theory is developed here and compared with experimental results. The calculations are based on the treatment of J. Pierce,^{2, 3} P. Sturrock,⁴ B. Fried and R. Gould,⁵ and I. Bernstein and R. Keelsund,⁶ on instabilities in plasmas. The electron, ion and atomic densities considered are usually less than 10^{14} particles/cc. At an emitter-collector distance less than 3 mm the mean free path is so large that collisions may be neglected. The conditions for excitation of ion oscillations depend on the plasma ion and electron densities. Since these quantities require a knowledge of the sheath potential, this potential must first be derived.

EVALUATION OF SHEATH POTENTIAL

Plasma sheath calculations have been made by P. Auer⁷ and others.^{8,9} In this paper the calculations assume special conditions for the cell:

1. No collisions between particles in the emitter-collector space.

2. The emitter produces electrons at a temperature T_e according to Richardson's equation, and ions at T_e as given by the Saha-Langmuir equation.¹⁰

3. More ions than electrons are produced at the emitter (ion-rich plasma). Ions with a kinetic velocity equal to or exceeding the positive sheath potential enter the plasma and the balance are reflected back to

the emitter. If the motive of the collector and emitter are identical, the ions from the plasma are discharged at the collector and returned to the emitter as neutral atoms at T_e .

4. All electrons enter the collector.

The total density of ions and atoms in the emitter-collector space is given by¹¹

$$n_T = n_a^+ + n_a^- + n_i, \quad (1a)$$

where the density of atoms and ions moving from emitter to collector is

$$n_a^+ + n_i = \frac{n_0}{2} \sqrt{\frac{T_0}{T_e}} \quad (1b)$$

and the density of the atoms (no ions) moving from collector to emitter is

$$n_a^- = \frac{n_0}{2} \sqrt{\frac{T_0}{T_e}} \quad (1c)$$

so that

$$n_T = \frac{n_0 \sqrt{T_0}}{2} \left[\frac{1}{\sqrt{T_e}} + \frac{1}{\sqrt{T_c}} \right]. \quad (1d)$$

At the hot emitter the arriving atoms are partially ionized according to the Saha-Langmuir equation. These ions leave the emitter together with the neutral atoms. The fraction of atoms ionized is

$$\alpha = \frac{J_{i0}}{J_{i0} + J_a^+} = \frac{n_{i0}^+}{n_{i0}^+ + n_a^+} = \left\{ 1 + 2 \exp \left[-\frac{\epsilon}{KT_e} (\Phi_e - V_i) \right] \right\}^{-1} \quad (2a)$$

or

$$\frac{J_{i0}}{J_a^+} = \frac{n_{i0}^+}{n_a^+} = \frac{\exp \left[+\frac{\epsilon}{KT_e} (\Phi_e - V_i) \right]}{2}; \quad (2b)$$

and

$$n_a^+ = 2n_{i0}^+ \exp \left[-\frac{\epsilon}{KT_e} (\Phi_e - V_i) \right] \quad (2c)$$

or

$$n_a^+ = 2n_i^+ \left[\exp \frac{\epsilon \psi_0}{KT_e} \right] \left\{ \exp \left[-\frac{\epsilon}{KT_e} (\Phi_e - V_i) \right] \right\}. \quad (2d)$$

Since more ions than electrons are produced, a positive sheath will be formed at emitter and collector. The sheath will permit all electrons to pass and saturation current will flow for equal motive at collector and emitter.

First consider the emitter conditions:

The ion current density J_{i0} leaves the emitter. Due to the sheath potential difference, ψ_0 , (see Fig. V-26-1) the ion current density entering the plasma is

$$J_i = J_{i0} \exp (-\epsilon \psi_0 / KT_e). \quad (3a)$$

FIG. V-26-1. Emitter and Collector Plasma Sheaths. ANL Neg. No. 112-5698.

Then

$$n_i = n_{i0} \exp(-e\psi_0/KT_e) \quad (3b)$$

will be the ion density within the plasma, assuming a Maxwellian distribution at the emitter.

The fraction, $J_{i0}[1 - \exp(-e\psi_0/KT_e)]$ will be reflected back to the emitter. The densities will be given by

$$n_{i0} = J_{i0}\sqrt{\pi m_i/2KT_e} \quad \text{and} \quad n_i = J_i\sqrt{\pi m_i/2KT_e}.$$

The neutrality of the plasma requires $n_e = n_i$. Since the electrons are accelerated by the sheath potential, the average of the electron velocity is given by

$$u_e = \sqrt{\frac{KT_e}{m_e} + \frac{2e\psi_0}{m_e}}. \quad (3c)$$

The ion and electron densities are

$$J_e \left(\frac{KT_e}{m_e} + \frac{2e\psi_0}{m_e} \right)^{-1/2} = n_e = n_i, \quad (3d)$$

where J_e , the saturation current density, is given by Richardson's equation.

From Eqs. (2d) and (3d) it follows that

$$\begin{aligned} n_a^+ + n_i &= \frac{n_0}{2} \sqrt{\frac{T_0}{T_e}} \\ &= J_e \left[1 + 2 \left(\exp \frac{e\psi_0}{KT_e} \right) \cdot \left\{ \exp \left[-\frac{e}{KT_e} (\Phi_e - V_i) \right] \right\} \right] \\ &\quad \cdot \left[\frac{KT_e}{m_e} + \frac{2e\psi_0}{m_e} \right]^{-1/2}. \end{aligned} \quad (4)$$

Solving for $\exp e\psi_0/KT_e$,

$$\exp \frac{e\psi_0}{KT_e} = \frac{1}{2} \left[\exp \frac{e(\Phi_e - V_i)}{KT_e} \right] \cdot \left[\frac{n_0}{2J_e} \sqrt{\frac{T_0}{T_e}} \sqrt{\frac{KT_e}{m_e} + \frac{2e\psi_0}{m_e}} - 1 \right] \quad (5a)$$

or

$$\begin{aligned} e\psi_0 &= e(\Phi_e - V_i) \\ &+ KT_e \ln \frac{1}{2} \left[\frac{n_0}{2J_e} \sqrt{\frac{T_0}{T_e}} \sqrt{\frac{KT_e}{m_e} + \frac{2e\psi_0}{m_e}} - 1 \right]. \end{aligned} \quad (5b)$$

This equation is correct only for $J_e =$ saturation current density, and for excess ions. Since

$$\frac{n_0}{2} \sqrt{\frac{T_0}{T_e}} = n_i + n_a^+ \quad \text{and} \quad \sqrt{\frac{KT_e}{m_e} + \frac{2e\psi_0}{m_e}} = u_e$$

it is necessary that

$$(n_i + n_a^+)u_e > J_e \quad \text{for} \quad n_e = n_i$$

and the value of ψ_0 must be positive since otherwise the plasma would be electron rich.

ψ_0 can be found by iteration. n_0 is determined from the cesium pressure in the storage space or cesium trap. T_e is measured by an optical pyrometer. The current density J_e is measured in amperes or milliamperes. V_i and ψ_0 are known ionization voltage and work func-

FIG. V-26-2. Ion Current as a Function of Sheath Potential. ANL Neg. No. 112-5699.

FIG. V-26-3. Condition for Plasma Stability and Instability. ANL Neg. No. 112-5700. Rev.

tion, respectively. For any given alkaline metal the ψ_0 can be calculated¹¹ and plotted.

In Fig. V-26-2, ψ_0 is given for cesium plasma as a function of current density with the cesium vapor density n_0 as parameters from $n_0 = 10^{11}$ to 10^{14} .

EXCITATION OF ION OSCILLATIONS DUE TO PLASMA INSTABILITIES

Pierce^{2, 3} has derived a general expression describing space charge waves using the Boltzmann, continuity, and space charge equations and assuming that the time and space variables are small perturbations.

Analogously, the wave equations for ions and electrons can be obtained. Since the mean free paths of the particles are large compared with the emitter-collector gap, the gradient of the pressure can be neglected and the equations (using the notations of this paper) appear as follows for a neutral plasma. [Time and space variables are proportional to $\exp j(\omega t - \beta x)$.]

Boltzmann:

$$m_i n_0 \omega v_i - m_i n_0 u_i \beta v_i = n_0 \epsilon \beta V,$$

$$m_e n_0 \omega V_e - m_e n_0 u_e \beta v_e = -n_0 \epsilon \beta V.$$

continuity:

$$n_i - n_0 \beta v_i - u_i \beta n_i = 0,$$

$$n_e - n_0 \beta v_e - u_e \beta n_e = 0.$$

space charge:

$$\beta^2 V = 4\pi \epsilon [n_i - n_e].$$

From these, for

$$\omega_e^2 = 4\pi \epsilon^2 n_0 / m_e \quad \text{and} \quad \omega_i^2 = 4\pi \epsilon^2 n_0 / m_i,$$

the describing equation for the oscillations is found to be

$$1 = \frac{\omega_e^2}{(\omega - \beta u_e)^2} + \frac{\omega_i^2}{(\omega - \beta u_i)^2} \quad \text{since } \omega_i^2 \ll \omega_e^2 \quad (6a)$$

because

$$\frac{\omega_i^2}{\omega_e^2} = \frac{m_e}{m_i}, \quad (6b)$$

so that

$$\frac{\omega_i^2}{\omega_e^2} = \begin{cases} 1/246000 & \text{for Cs} \\ 1/72400 & \text{for K} \\ 1/42500 & \text{for Na.} \end{cases}$$

Neglecting second order terms, Eq. (6a) can be written

$$\begin{aligned} 0 = \omega^4 - 2\omega^3 \beta \omega_e \\ - \omega^2 (\omega_e^2 - \beta^2 u_e) + 2\omega \beta u_i (\omega_e^2 - \beta^2 u_e^2) \\ - \beta^2 [u_i (\omega_e^2 - \beta^2 u_e^2) + \omega_i^2 u_e^2]. \end{aligned} \quad (7a)$$

There are four solutions, but only those for ions are of interest here. In this case $\omega \ll \beta u_e$, for $\beta < 32$ or $x_0 > 0.5$ mm. Computer calculations show that the first two terms in Eq. (7a) may be disregarded if $|\omega_e^2 - \beta^2 u_e|$ is not very small. Hence, Eq. (7a) can be replaced by the approximation

$$0 = \omega^2 - 2\omega \beta u_i + \beta^2 u_i^2 + \frac{\omega_i^2 u_e^2}{\omega_e^2 - \beta^2 u_e^2} \quad (7b)$$

from which

$$\omega = \beta u_i \pm \frac{\beta u_e \omega_i}{\sqrt{\beta^2 u_e^2 - \omega_e^2}} \quad \text{for } \frac{\beta^2 u_e^2}{\omega_e^2} > 1 \quad (7c)$$

and

$$\omega = \beta u_i \pm j\beta \frac{\omega_i}{\omega_e} u_e \left(1 - \frac{\beta^2 u_e^2}{\omega_e^2}\right)^{-1/2} \quad \text{for } \frac{\beta^2 u_e^2}{\omega_e^2} < 1. \quad (7d)$$

See Fig. V-26-3 for condition of plasma stability as a function of J and $\beta^2 u_e^2 / \omega_e^2$ as parameter.

No excitation due to plasma instabilities occurs in Eq. (7c), but in Eq. (7d) ω and β are complex quantities and can be written

$$\omega = \omega_r + j\alpha; \quad \beta = \beta_1 + j\beta_2 \quad (7e)$$

The time dependent solution for the variable quantities is therefore proportional to

$$e^{j\omega t} = e^{j\omega_r t} e^{\pm \alpha t},$$

where $\omega_r = 2\pi f$ is the real portion of ω ; the imaginary portion, $j\alpha$, determines the excitation or decay of the oscillations due to instabilities.¹² Note that Eq. (7b) is valid only for small amplitudes. Otherwise the assumptions used for the derivation would no longer be correct and nonlinear equations will have to be substituted. In

addition, damping due to recombination, collision and scattering out of phase will reduce the excitation.

Replacing

$$u_e \text{ by } \sqrt{\frac{KT_e}{m_e} + \frac{2\epsilon\psi}{m_e}} \quad \text{and} \quad u_i \text{ by } \sqrt{\frac{KT_e}{m_i}}$$

in Eq. (7d) yields

$$\omega = \omega_r + j\alpha = \beta \sqrt{\frac{KT_e}{m_i}} \pm \frac{j\beta \sqrt{\frac{2\epsilon\psi + KT_e}{m_e}} \cdot \frac{\omega_i}{\omega_e}}{\sqrt{1 - \frac{\beta^2}{\omega_e^2} \left[\frac{2\epsilon\psi + KT_e}{m_e} \right]}} \quad (8a)$$

Then, using ψ_0 for ψ and the relations in (6b) and (7e), the real and imaginary parts of ω are found to be

$$\left. \begin{aligned} \omega_r &= \beta_1 \sqrt{\frac{KT_e}{m_i}} \pm \beta_2 \sqrt{\frac{2\epsilon\psi_0 + KT_e}{m_i}} \\ &\quad \pm \frac{[3\beta_1^2\beta_2 - \beta_2^3] m_i}{2\omega_e^2 m_e} \left(\sqrt{\frac{2\epsilon\psi - KT_e}{m_i}} \right)^3 \\ \text{and} \\ j\alpha &= j \left[\beta_2 \frac{\sqrt{KT_e}}{m_i} \pm \beta_1 \sqrt{\frac{2\epsilon\psi + KT_e}{m_i}} \right. \\ &\quad \left. \pm \frac{[\beta_1^3 - 3\beta_1\beta_2^2] m_i}{2\omega_e^2 m_e} \left(\sqrt{\frac{2\epsilon\psi + KT_e}{m_i}} \right)^3 \right] \end{aligned} \right\} \quad (8b)$$

β_1 can be determined by the boundary condition as

$$\beta_1 = \frac{n\pi}{x_0} \quad \text{or} \quad \frac{n\pi}{2x_0}, \quad n = 1, 2, \dots$$

x_0 = emitter-collector distance, and is one quarter wave length when the emitter is grounded and the collector is open.

β_2 is not given by boundary conditions.

However, an estimate can be made, using an experimental value of ω_r . Omitting the last term for ω_r in Eq. (8b), β_2 follows

$$\beta_2 = \frac{\omega_r - \beta_1 \sqrt{\frac{KT_e}{m_i}}}{\sqrt{\frac{2\epsilon\psi + KT_e}{m_i}}} \quad (9)$$

The space-time solution is now proportional to

$$e^{j\omega_r t} e^{-\alpha t} e^{-j\beta_1 x} e^{\beta_2 x} = (\cos \omega_r t)(\cos \beta_1 x) e^{-\alpha t} e^{\beta_2 x}. \quad (10)$$

Here the term $e^{-\alpha t}$ shows the time dependent increase if α is negative, or decrease if α is positive.

From experimental data it was found that β_2 varies greatly; α will be two valued, as is the term,

$$\pm \beta_1 \sqrt{(2\epsilon\psi + KT_e)/m_i}$$

in Eq. (8b).

The space dependent term, $e^{\beta_2 x}$, will show an increase of amplitude with emitter-collector distance for a positive β_2 , provided that β_2 is not itself proportional to $1/x_0$, as is β_1 .

EXPERIMENTAL CONFIRMATION OF THEORY

The experiments¹¹ were conducted with the emitter grounded and the collector connected by a capacitance to an rms voltmeter and oscilloscope. A choke excluded the rf current from ground. As a result the collector was open for rf and the emitter-collector distance, x_0 , represented a quarter wave length for the ion oscillations.

The frequencies used for calculation were measured at $-2V$ at the collector. This position is as close as possible to zero motive between emitter and collector and the assumptions made previously should be approximately correct.

Table V-26-I shows in Columns 1 to 3 the measured values of emitter temperature, cesium concentration in the storage space and current density, respectively. Column 4 gives the emitter-collector distance in mm and β_1 in cm^{-1} . Column 5 represents the energy of the electron during acceleration in the plasma sheath. Column 6 gives the values for $\beta_1^2 u_e^2$ and ω_e^2 , the ratio of which determines theoretically from Eqs. (7c) and (7d) whether or not the instabilities will excite plasma oscillations. Column 7 shows the measured ω_r , Columns 8 and 9 the measured rms voltages at zero and -2 volts. In Column 10 the β_2 are calculated from ω_r and Eq. (9). Column 11 shows the increase of amplitude with emitter-collector distance.

As is seen, (e.g., cases 1-3) when

$$\frac{\beta_1^2 u_e^2}{\omega_e^2} < 1$$

oscillations are excited. The rms voltages decrease with decreasing emitter-collector distance, in qualitative agreement with the $e^{\beta_2 x_0}$ in Column 11. This observation is also confirmed in all measurements and shows complete agreement between experiment and theory. The conditions for excitation are usually fulfilled, but in some cases oscillations start at a lower value of ω_e^2 than expected, e.g., Table V-26-I (Test No. 6), while in Test Nos. 20, 21, 24, and 25 the theory agrees with the experimental results.

A comparison of the β_2 's indicates that the values decrease with increasing cesium densities, do not vary much with emitter-collector distance (Test No. 1-12) and are usually close for moderate current and emitter temperature variations. However, it should be emphasized that the frequencies vary in many experiments irregularly. Frequencies may double or triple quite suddenly. This is not surprising, since $\beta_1 = n\pi/2x_0$ and "n" might assume any number.

TABLE V-26-I. EXPERIMENTAL DATA FOR COMPARISON WITH THEORY

Test No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	T_e , °K	n_{Cs} , ^a atoms per cc $\times 10^{-12}$	J_e , mA/cm ²	x_0 , mm	β_1 , ^b cm ⁻¹	$\frac{KT_e}{2} +$ $e\psi$, ^b eV	$\beta_1^2 u_e^2$, ^b sec ⁻² $\times$ 10^{-17}	ω_e^2 , ^b sec ⁻² $\times$ 10^{-17}	$\frac{\beta_1^2 u_e^2}{\omega_e^2}$	ω_r at -2V, $\times 10^{-6}$	rms Voltage at zero V	rms Voltage at -2V	β_2 , ^b cm ⁻¹	$e^{\beta_2 x_0}$
1	2103	0.3	13	2	7.85	0.85	1.79	41.7	0.043	1.38	0.6	0.42	10.5	8.16
2	1979	0.35	10	1	15.7	0.95	7.6	36	0.21	2.07	0.3	0.29	12.9	3.7
3	1990	0.4	13	0.5	31.4	1.05	35.4	42	0.84	2.36	—	—	10.25	0.68
4	2010	3	3.8	3.5	4.5	1.55	1.06	10.25	0.10	1.19	0.28	0.13	7.15	10.3
5	2000	3	3.6	2.3	6.83	1.55	2.47	9.75	0.25	1.38	0.2	0.1	7.7	5.9
6	2000	3	3.2	1.0	15.7	1.55	13.1	8.14	1.61	1.57	0.09	—	6.95	1.9
7	2071	11	7	2	7.85	1.75	3.62	18.3	0.20	1.38	0.4	0.2	7.05	4.1
8	2071	11	7	1	15.7	1.75	14.45	18.3	0.79	1.44	0.1	0.08	5.68	1.76
9	2071	11	7	0.5	31.4	1.75	58.9	18.3	3.20	1.82	0.15	0.1	4.45	1.24
10	2373	0.75	160	2	7.85	0.75	2.0	60.8	0.033	1.195	2	0.8	9.68	6.45
11	2350	1.1	230	1	15.7	0.70	6.04	91.8	0.066	2.01	1.4	0.62	14.5	3.80
12	1813	0.25	0.54	2.1	7.48	1.3	2.65	1.51	1.75	—	—	—	—	—
13	1973	0.24	6.6	2.1	7.48	1.1	2.2	21	0.10	—	—	—	—	—
14	2073	0.24	16.5	2.1	7.48	0.9	1.82	57.7	0.03	1.38	0.8	0.25	10.1	8.3
15	2163	0.24	71.0	2.1	7.48	0.65	1.36	290	0.005	0.753	1.0	0.18	5.4	2.9
16	2029	1.7	5.0	2	7.85	1.4	2.95	14.3	0.21	1.76	0.28	0.1	10.55	8.2
17	2029	1.7	5.0	1	15.8	1.4	11.8	14.3	0.83	2.19	0.1	0.06	11.6	3.2
18	2029	1.7	5.0	0.5	31.6	1.4	47.3	14.3	3.30	2.5	0.1	0.06	9.9	1.64
19	1773	11	0.27	1.9	8.26	2.4	5.7	0.58	9.80	—	—	—	—	—
20	1853	11	1.8	1.9	8.26	1.9	4.3	4.66	0.92	1.02	—	—	4.6	2.4
21	1992	11	6.0	1.9	8.26	1.6	3.84	16.2	0.24	1.07	0.67	0.07	5.2	2.7
22	2176	11	37	1.9	8.26	1.5	3.54	103	0.034	1.21	1.4	0.3	5.5	2.75
23	1805	120	0.45	1.5	10.48	2.5	9.3	0.956	9.70	—	—	—	—	—
24	1930	120	3.7	1.5	10.48	2.3	8.36	8.40	0.99	1.00	0.3	0.05	3.52	1.69
25	2076	160	10.76	1.5	10.48	2.2	8.5	14.6	0.58	1.00	0.7	0.25	3.50	1.69

^a n_{Cs} is calculated from actual temperature and pressure data.

^b $\beta_1 = \pi/2X_0; e\psi$ from Fig. 2; $u_e^2 = (KT_e + 2e\psi)/m_e$; $\omega_e^2 = 4\pi e^2 n_0/m_e$; $\beta_2 = (\omega_r - \beta_1 \sqrt{KT_e/m_i})/\sqrt{(KT_e + 2e\psi)/m_i}$.

As was reported before,¹¹ the frequencies increase with a more positive voltage at the collector. This effect can now be explained qualitatively. If the motive at the collector becomes more positive than the motive of the emitter, undischarged ions are reflected from the collector sheath and, upon returning to the emitter, increase the emitter sheath and ψ_0 . Indeed Eq. (8b) shows that $|\omega_r|$ is given by

$$|\omega_r| = \left| \beta_1 \sqrt{\frac{KT_e}{m_i}} \pm \beta_2 \sqrt{\frac{2e\psi_0 + KT_e}{m_i}} \right| \pm \text{higher terms}$$

should increase therefore with ψ_0 . Note that in the experiments $e\psi_0$ is always $\gg KT_e/2$, and β_1 and β_2 are of the same order of magnitude.

RECENT EXPERIMENTAL RESULTS

COMPARISON OF DC AND RF POWER OUTPUT

In previous experiments the *rf* power output was measured with an *rms* voltmeter across an inductance-free resistor. Since the *rf* output is usually not sinusoidal but may consist of pulses, the *rms* instruments do not give correct readings. In recent experiments light bulbs were substituted and the output was measured photometrically by comparison with dc heated lamps. The results indicated that the *rf* power output is about 20 % higher than assumed previously. Some results are given in Table V-26-II. The output was higher at an emitter-collector distance of 2 mm than at 1 mm separation in agreement with the behavior of $e^{\beta_1 x}$, as noted earlier. The power was measured at zero V dc at the collector or a slightly negative value.

TABLE V-26-II. POWER OUTPUT FROM THE CESIUM DIODE

Emitter Temperature, °K	Cs Pressure, mm Hg $\times 10^{-3}$	Cs Atom Density, atoms/cc $\times 10^{-13}$	Emitter-Collector Distance, mm	dc Power, mW	rf Power, mW	rf/dc Power Ratio, %
2082	3.6	6.3	2	50	6	12
2206	3.6	6.3	2	200	35	17.5
2240	3.6	6.3	1	300	10	3.3
2279	1.5	3.2	2	460	115	25
2270	1.5	3.2	1	500	23	4.6
2394	2.5	5.5	2	1000	190	19
2354	2.5	5.5	1	930	?	?
2284	8	20	2	310	44	14
2268	8	20	1	500	25	5
2380	8	20	2	900	95	10.5

TABLE V-26-III. PEAK-TO-PEAK AND AMPLITUDE VOLTAGES OF SHORT PULSES FROM A CESIUM DIODE AT A CESIUM PRESSURE OF 3.6×10^{-3} mm Hg

Emitter Temperature, °K	Emitter-Collector Distance, mm	DC Voltage at Collector					
		0		-1		-2	
		P-P-V	AMPL-V	P-P-V	AMPL-V	P-P-V	AMPL-V
2082	2	7.5	—	5	—	4	—
2206	2	17.5	15	12	10	—	—
2240	1	11.0	8	10	7	—	—

RF VOLTAGE PULSES

The rf output is frequently produced in the form of single pulses separated by several microseconds. Some experimental data showing this phenomenon are presented in Table V-26-III. The voltages were measured from peak to peak (P-P-V) or the large amplitudes (AMPL-V) were counted from a zero line. The applied dc voltages were zero, -1 V and -2 V. P-P-V up to 17 V have been observed.

CONCLUSIONS

The theoretical treatment was based on an approximation for small amplitude oscillations and the assumptions outlined in the section, "Evaluation of Sheath Potential". The condition for oscillations due to instabilities was compared with experimental results and a fair, but not complete agreement, was observed. The experimentally observed increase of rf amplitude and power with increasing emitter-collector distance from 0.5 to 2 mm is in full agreement with the theory. The theory explains qualitatively the measured increase in frequency with increasing positive collector potential.

REFERENCES

1. H. K. Richards, *Thermionic Energy Conversion with a Plasma Thermocouple*, Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010 (1965) p. 251.
2. J. Pierce, *Possible Fluctuation in Electron Streams Due to Ions*, J. Applied Physics **19**, 23 (1948).
3. J. Pierce, *Increasing Space Charge Waves*, J. Applied Physics **20**, 1060 (1949).
4. P. A. Sturrock, *Excitation of Plasma Oscillations*, Phys. Rev. **117**, 1426 (1960).
5. B. D. Fried and R. W. Gould, *Longitudinal Ion Oscillations in a Hot Plasma*, Phys. of Fluids **4**, 139 (1961).
6. I. B. Bernstein and R. M. Keelsund, *Ion Wave Instabilities*, Phys. of Fluids **3**, 937 (1960).
7. P. Auer, *Potential Distributions in a Low Pressure Thermionic Converter*, J. Applied Physics **31**, 2096 (1960).
8. A. L. Eichenbaum and K. G. Hernquist, *Space Charge Instabilities in Synthesized Plasma*, J. Applied Physics **32**, 16 (1961).
9. C. Birdsall and W. Bridges, *Space Charge Instabilities in Electron Diodes and Plasma Converters*, J. Applied Physics **32**, 2611 (1961).
10. I. Langmuir and K. Kingdon, *Contact Potential Measurements with Observed Films*, Phys. Rev. **34**, 129 (1929).
11. H. K. Richards, *DC and High Frequency Voltage and Power Output and Interaction in Cesium, Potassium and Sodium Thermionic Converters*, TID-7687, pp. 165-179, Appendix I, (February 1964).
12. J. G. Linhart, *Plasma Physics*, (North Holland Pub. Co., Amsterdam, 1960) pp. 133-137.

V-27. Reactor Physics Constants Center—Information Storage and Retrieval

L. TEMPLIN, H. THACHER and (Mrs.) P. L. WALKER

As an aid to the preparation of Reactor Physics Constants Center publications, the investigation and development of a key word storage and retrieval system for reactor physics information was begun during the

past year. Attention has been given to the compilation of a thesaurus of key words and to the mechanics of storing and retrieving desired information.

Selection of key words for the thesaurus has been

ed principally upon reviews of numerous pertinent books and of approximately nine hundred technical reports. The frequency of occurrence of the key words selected to date has been tabulated with respect to the titles of the technical reports. Preparation of the thesaurus continues.

A set of programs designed to allow for experimentation with the key word method of indexing was written for the CDC-160A computer. Speed and machine efficiency were sacrificed in the programs in favor of convenience of use and flexibility since the objective was not a production system but an experimental tool. The programs were written in FORTRAN.

The system consists of three programs: one to make a key word dictionary tape, another to make or add to a library tape, and a third to search for desired references. The programs are used in that order when making and checking out a new set of key words and references.

Only the search routine and previously made tapes are needed for finding references in a system already used.

The dictionary maker prepares, or adds to, a list of key words, and produces a magnetic tape. The list may be any desired length and is stored on a specified number of 'pages' of 128 or less words. The words may be up to 16 characters in length.

The library maker prepares, or adds to, a list of references with associated key words, and produces a magnetic tape. Each entry consists of up to 80 characters of identification such as: author, title, name of journal, page, etc. There may be up to 32 key words of 16 characters each associated with each entry. The program checks to see that each of the key words is in the dictionary.

The search routine takes a set of key words and searches the library for references which refer to all the requested key words. It then prints out a list of these references.

V-28. Reactor Physics Constants Center—Resonance Integral and Age Compilation

P. J. PERSIANI and A. E. MCARTHY

The Reactor Physics Constants Center published Newsletter No. 11,⁽¹⁾ the third in a series of Newsletters dealing with neutron resonance integral and age data for assorted isotopes, elements, and mixtures of materials. The first two such publications were Newsletters Nos. 1 and 10. Newsletter No. 11 includes data accumulated from the literature, by private communication and by calculations done at Argonne National Laboratory since the publication of Newsletter No. 10 in April, 1963. It is intended that Newsletter Nos. 1, 10 and 11 be used together as one volume.

The resonance integrals for Cu-63 and Cu-65 were computed in the single-level Breit-Wigner approximation, using the ANL-RP/266 code.² The resonance parameters were taken from BNL-325, 2nd Ed., Supplement 1. Since no radiation widths were listed, the average widths, Γ_γ , of 5.5 and 10.7 eV were used for Cu-63 and Cu-65 respectively. These widths were calculated such that the computed capture cross sections for the 0.025 eV neutrons agreed with the experimental values.

In the case of Cu-63 the resolved levels contributed 0.0 barns and the unresolved levels contributed 0.62 barns to the integral. The discrepancy between the

computed and experimental values (see Newsletters Nos. 1, 10 and 11) indicate that Cu-63 may be a non-1/v capturing isotope.

The disagreement between calculated and experimental values (see Newsletters, Nos. 1, 10 and 11) for the resonance integral of Cu-65 suggests that this may also be a non-1/v absorber. The low energy dependence of the capture cross section of the two isotopes should be determined experimentally.

The reported experimental value of 377 ± 34 b in Ref. 3 for Dy-164 agrees with the computed value of 382.16 b.⁴ The computed resonance integral for this isotope involved using a radiative width of 76 mV for the bound level and a width of 115 mV for the positive levels. This choice of widths allowed very good agreement between experimental and calculated capture cross sections in the low energy region.

The discrepancies between the analytical and experimental values for the other dysprosium isotopes remain unresolved.

The experimental value of 2595 b for the 54 minute In-115 isotope reported in Ref. 5 is consistent with the value of 2500 b (1.3 eV cut-off) for the 54 min component of the 54 min and 14 sec isotope reported in

Ref. 6. Included in Ref. 6 is the measured total value (54 min and 14 sec activity) of 3480 ± 120 b which is in agreement with the value of 3300 ± 850 b (Ref. 2) obtained by the absorption technique. The computed value of 3250 b (Ref. 2) is in good agreement with these experimental values.

The resonance integral of 82.7 ± 1.8 b reported in Ref. 7 for Th-232 is now in good agreement with the experimental value of 85 b reported in Ref. 8 and Newsletter No. 1. However, the computed value of 94 b (Refs. 2 and 4) still lies between the above values and the measured value of 110 b reported in Ref. 9 and Newsletter No. 1.

REFERENCES

1. A. E. McCarthy and P. J. Persiani, *Neutron Resonance Integral and Age Data*, Argonne National Laboratory, Reactor Physics Constants Center Newsletter No. 11 (January 1965).
2. P. J. Persiani, J. J. Kaganove and A. E. McCarthy, *Neutron Resonance Integral and Age Data*, Argonne National Laboratory, Reactor Physics Constants Center Newsletter No. 10 (April 1963).
3. J. J. Scoville, E. Fast and J. W. Rogers, *Burnup Cross Sections and Resonance Integrals of Dysprosium*, Trans. Am. Nucl. Soc. **7**, No. 1, 85 (June 1964).
4. P. J. Persiani, J. J. Kaganove and A. E. McCarthy, *The Contribution of Negative Energy Levels to Resonance Integrals*, Trans. Am. Nucl. Soc. **6**, No. 1, 39-40 (June 1963).
5. H. L. Brown, Jr., T. J. Connolly and W. K. Foel, *Effective Resonance Integrals and Interference Between Resonances of Indium-115, Gold, and Rhenium-185*, Trans. Am. Nucl. Soc. **5**, No. 2, 375-376 (November 1962).
6. K. H. Beckurts, et al., *Thermal Activation Cross Sections and Resonance Integrals of In-115*, Nucl. Sci. Eng. **17**, No. 3, 329-335 (November 1963).
7. M. Brose, *The Infinite Dilute Resonance Integral of Thorium*, Nucl. Sci. Eng. **19**, No. 2, 244-245 (June 1964).
8. F. J. Johnston, J. Halperin and R. W. Stoughton, *The Thermal Neutron Cross Section of Th-232 and the Resonance Integrals of Th-232, Th-233 and Co-59*, J. Nucl. Energy **11**, No. 2-4, 95-100 (1960).
9. R. B. Tattersall, et al., *Pile Oscillator Measurements of Resonance Absorption Integrals*, AERE-R/2887 (August 1959).

V-29. Vaporization of Sodium in Reactor Coolant Channels

D. R. MacFARLANE

NOMENCLATURE

t	$\equiv$ time
z	$\equiv$ axial distance along channel
α_i	$\equiv$ fraction of channel area occupied by i th component
ρ_i	$\equiv$ density of i th component
G_i	$\equiv$ mass flow rate of i th component
G_t	$\equiv \sum_{i=1}^2 \alpha_i G_i \equiv$ total mass flow rate
H_i	$\equiv$ enthalpy of i th component
x_i	$\equiv$ quality of i th component
P	$\equiv$ pressure
g	$\equiv$ gravitational constant
$\frac{\partial r}{\partial z}$	$\equiv$ frictional pressure gradient
γ	$\equiv$ ratio of channel perimeter to cross sectional area
$\phi(z, t)$	$\equiv$ heat flux
x_{11}	$\equiv$ Martinelli flow parameter
β	$\equiv$ density ratio, liquid/vapor
μ	$\equiv$ viscosity
f	$\equiv$ friction factor
T_f	$\equiv$ fuel temperature
$Q(z, t)$	$\equiv$ fuel heat generation rate
C_f	$\equiv$ fuel heat capacity

ρ_f	$\equiv$ fuel density
k_f	$\equiv$ fuel thermal conductivity
r	$\equiv$ fuel rod radius
$h(G, H)$	$\equiv$ convection heat transfer coefficient
$T_w(z, t)$	$\equiv$ channel wall temperature
$T_c(z, t)$	$\equiv$ coolant temperature
D_H	$\equiv$ channel hydraulic diameter
a	$\equiv$ exponent in friction factor correlation
$\bar{G}$	$\equiv$ average channel flow rate
	$= \frac{1}{L} \int_0^L G(z, t) dz$
L	$\equiv$ length of coolant channel
L_p	$\equiv$ length of liquid leg between pump and reactor
A_r	$\equiv$ reactor flow area/coolant pipe cross section
v'	$= \frac{1}{G_t^2} \sum_{i=1}^2 \alpha_i \frac{G_i^2}{\rho_i}$
λ	$\equiv$ latent heat of vaporization
T_S	$\equiv$ local saturation temperature

INTRODUCTION

The work described herein is part of a continuing program to obtain a better understanding of coolant vaporization effects in large, fast power reactors. It has

been pointed out that the accurate description of a boiling sodium accident depends on more information about the possible movement of sodium in such an accident.¹ This study was undertaken in an effort to help provide that information. Although all the work performed so far is analytic in nature, it is likely that the results will be useful in laying guidelines for future planned transient boiling experiments with sodium. While the problem formulation is quite general, the work is oriented toward the large (1000 MWe) fast breeder reactor having uranium oxide or uranium carbide fuel rods.

FORMULATION OF PROBLEM

As a starting point in the analysis, it is assumed that the reactor has no flux or power gradients in the radial direction (infinite slab) and has identical coolant channels and rod-type fuel elements. The heat transfer coolant flow analysis is thus simplified to the description of a single fuel element and its associated coolant. Two basic types of transients are considered:

(1) mild over-power transients with reactivity inputs which would be characteristic of a control rod drive malfunction, and

(2) coastdown of the coolant flow resulting from pump failure. In both of these situations, it is assumed that the control system shutdown fails to operate. These are highly credible occurrences in the operation of a power reactor and therefore are interesting in a fast breeder reactor safety analysis. Basically, the method of approach consists of seeking solutions for the coupled hydrodynamic conservation equations and the fuel heat conduction equation. Reactivity feedback via the reactor kinetic equations has been omitted in the initial formulation of the problem. The transient equations for conservation of mass, momentum, and energy for a two-phase fluid flowing in a channel are: continuity,

$$\frac{\partial}{\partial t} \left(\sum_{i=1}^2 \alpha_i \rho_i \right) + \frac{\partial}{\partial z} \left(\sum_{i=1}^2 \alpha_i G_i \right) = 0; \quad (1)$$

momentum,

$$\frac{\partial}{\partial t} \left(\sum_{i=1}^2 \alpha_i G_i \right) + \frac{\partial}{\partial z} \left(\sum_{i=1}^2 \alpha_i \frac{G_i^2}{\rho_i} \right) + \frac{\partial p}{\partial z} + g \sum_{i=1}^2 \alpha_i \rho_i + \frac{\partial \tau}{\partial z} = 0; \quad (2)$$

energy,

$$\frac{\partial}{\partial t} \left(\sum_{i=1}^2 \alpha_i \rho_i H_i \right) + \frac{\partial}{\partial z} \left(G_t \sum_{i=1}^2 x_i H_i \right) = \gamma \phi(z, t). \quad (3)$$

The symbols are defined at the beginning of this paper). These equations, derived by J. Meyer,² assume

no heat conduction in the fluid in the direction of flow and they neglect pressure and temperature gradients transverse to the flow. Also, the terms in the energy equation, giving the effect of system pressure on energy content, are neglected since they are negligible in the systems considered here.

In the absence of better data, the correlation of R. Martinelli and R. Lockhart³ is used as the basis for the equation of state in the two-phase fluid. Thus, the vapor void fraction is given by

$$\alpha = 1.0 - \left(1.0 + \frac{21.0}{x_{tt}} + \frac{1.0}{x_{tt}^2} \right)^{-1/2}, \quad (4)$$

where

$$x_{tt} = \left(\frac{1-x}{x} \right)^{0.9} \left(\frac{1}{\beta} \right)^{0.5} \left(\frac{\mu_t}{\mu_g} \right)^{0.1}. \quad (5)$$

Likewise, the Martinelli correlation is used to calculate the friction factor for two-phase flow in Eq. (2). The friction factor correlation is given by

$$\frac{f_{tp}}{f_{sp}} = \phi_t^2 = 1.0 + \frac{20.0}{x_{tt}} + \frac{1.23}{x_{tt}^2}. \quad (6)$$

The appropriate equation for transient conduction in the fuel is

$$\nabla^2 T_f + \frac{Q(z, t)}{k_f} = \frac{C_f \rho_f}{k_f} \frac{\partial T_f}{\partial t}, \quad (7)$$

which is approximated by

$$\frac{\partial^2 T_f}{\partial \ell^2} + \frac{1}{r} \frac{\partial T_f}{\partial r} + \frac{Q(z, t)}{k_f} = \frac{C_f \rho_f}{k_f} \frac{\partial T_f}{\partial t}, \quad (8)$$

since the axial conduction in the rods can be neglected relative to the radial conduction. The heat flux at the channel wall as a function of position and time is given in the usual way by an equation of the form

$$\phi(z, t) = h(G, H)[T_w(z, t) - T_c(z, t)], \quad (9)$$

where the heat transfer coefficient, $h(G, H)$, is given by an established correlation for the subcooled region and by a modified correlation in a two-phase region. An additional thermal resistance between the fuel and the coolant to allow for the cladding and cladding-to-fuel contact resistance is included as a constant value.

The boundary conditions required to complete the problem formulation are:

- (1) temperatures at time zero in the coolant and fuel,
- (2) coolant temperature at channel inlet as a function of time,
- (3) fuel heat generation as a function of time,
- (4) channel outlet pressure and pump outlet pressure as a function of time.

With reference to the available pump pressure boundary

condition, the model for the reactor used here assumes that all coolant channels are behaving identically. For this reason the momentum of the fluid leg in the piping between the pump outlet and the reactor inlet must be considered in the overall momentum balance. As long as flow cannot bypass the core, the inertia of this fluid will have significant effects on flow changes during transients. Ideally, the characteristics of the complete coolant loop should be brought into the picture in attempting to solve a given problem. Since this unduly complicates the problem at this stage, a compromise has been adopted wherein only several of the important coolant loop parameters are considered.

The various approximations which can be made to the above set of equations to derive practical methods of solution are given in an excellent review article by J. Meyer.⁴ These various approximations have been applied in the ART⁵ and CHIC⁶ series of programs developed for transient analysis of water-moderated reactors. In essence, the practical methods all use the energy equation Eq. (3) in the form which neglects the pressure contribution. This is a reasonable assumption for all reactor coolant systems. Also, the momentum equation Eq. (2) is used in a spacewise integrated (momentum integral model) form. Since the codes are directed toward the analysis of water-moderated reactors under tens of atmospheres of pressure, the additional assumption is made that the coolant properties may be determined at some fixed reference pressure, p^* , which is constant during the transient. In describing a boiling liquid metal at near atmospheric pressures, however, this assumption of a constant reference pressure is no longer valid. Therefore, the method of solution under development for sodium systems, while retaining the essential features of momentum integral model approach, does consider the effect of local pressures on coolant properties. The effect of vapor-liquid slip is also considered in the present analysis since it is implied in the Martinelli-Lockhart void fraction correlation Eq. (4).

STEADY STATE SOLUTIONS

The initial phase of the work consisted of obtaining solutions to the steady state forms of the hydrodynamics equations Eqs. (1) (2) and (3). The purpose of this work was to obtain some feeling for the ranges of the parameters involved in boiling sodium systems and also to calculate initial conditions for the transient problems. For steady state conditions the continuity equation is

$$\frac{\partial G_t}{\partial z} = 0, \quad (10)$$

which states the constancy of mass flow rate with position. The momentum and energy equations can be in-

tegrated directly to obtain the steady state solutions. In the analysis, three regimes are treated; subcooled liquid, two-phase, and superheated vapor. Accordingly, the integrated forms of the momentum and energy equation for these three segments of the coolant channel are written as:

For $T_c < TS$ ($x = 0.0$)

energy,

$$H_t(z_1) - H_t(0) = \frac{\gamma}{G_t C_t} \int_0^{z_1} \phi(z') dz'; \quad (11)$$

momentum,

$$P \Big|_0^{z_1} = -\frac{G_t^2}{2gD_H} \int_0^{z_1} \frac{f_{sp}(G, H)}{\rho_t(H)} dz' - \int_0^{z_1} \rho_t(H) dz' - \frac{G_t^2}{\rho_t(H)g} \Big|_0^{z_1}. \quad (12)$$

For $T_c = TS$ ($0.0 \leq x \leq 1.0$)

energy,

$$x(z_2) = \frac{1}{\lambda(z_2)} \cdot \left\{ \frac{\gamma}{G_t} \int_{z_1}^{z_2} \phi(z') dz' - [H_t(z_2) - H_t(z_1)] \right\}; \quad (13)$$

momentum,

$$P \Big|_{z_1}^{z_2} = -\frac{1}{g} \sum_{i=1}^2 \alpha_i \frac{G_t^2}{\rho_i} \Big|_{z_1}^{z_2} - \int_{z_1}^{z_2} \sum_{i=1}^2 \alpha_i \rho_i dz' - \frac{G_t^2}{2gD_H \rho_t} \int_{z_1}^{z_2} f_{sp}(G, H) (1-x)^{2-\alpha} \phi_t^2 dz'. \quad (14)$$

For $T_c > TS$ ($x > 1.0$)

energy,

$$H_g(z_3) - H_g(z_2) = \frac{\gamma}{G_t C_g} \int_{z_2}^{z_3} \phi(z') dz'; \quad (15)$$

momentum,

$$P \Big|_{z_2}^{z_3} = -\frac{G_t^2}{2gD_H} \int_{z_2}^{z_3} \frac{f_{sp}(G, H, P)}{\rho_g(H, P)} dz' - \frac{G_t^2}{g\rho_g(H, P)} \Big|_{z_2}^{z_3}. \quad (16)$$

An ALGOL program was written to solve this set of equations. The method used was an iterative technique wherein the energy equation was first used to calculate an enthalpy profile for the channel. This result was then used to calculate pressure and temperature dependent coolant properties which were used in the momentum equation to calculate a new pressure profile. Iteration was continued between the momentum and energy equation until two successively calculated pressure profiles agreed within a specified convergence criterion. In the calculation, a switch was made from the subcooled single phase equations to the two-phase equations whenever the coolant temperature

FIG. V-29-1. Quality and Vapor Void Fraction Profiles for Channel in Steady Boiling with Various Flow Rates. ANL Neg. No. 112-5644.

FIG. V-29-2. Quality and Vapor Void Fraction Profiles for a Channel in Steady Boiling with Various Flow Rates. ANL Neg. No. 112-5645.

FIG. V-29-3. Pressure Profiles for Steady Boiling at Various Flow Rates. ANL Neg. No. 112-5642.

needed the local saturation temperature. Likewise, a tech was made from the two-phase equations to the reheated vapor equations whenever the quality exceeded 1.0. In this manner the point where boiling

FIG. V-29-4. Pressure Profiles for Steady Boiling at Various Flow Rates. ANL Neg. No. 112-5641.

FIG. V-29-5. Pressure Profiles for Steady Boiling at Various Flow Rates. ANL Neg. No. 112-5643.

starts in the channel shifted from iteration to iteration until convergence was obtained.

Some results of the steady state calculations for a reactor with 0.635 cm diam rods and 50 v/o coolant are presented in Figs. V-29-1 through V-29-6. In all of these problems the heat flux distribution was assumed to be a chopped cosine and the maximum heat flux was an input parameter. Also specified for each problem was coolant flow rate, inlet enthalpy, and channel exit pressure. The Martinelli-Lockhart correlations given in Eqs. (4) and (6) for vapor void fraction and two-phase friction factor, respectively, were used to predict those quantities. The fuel rod temperature calculation was not included as part of the steady state study since this does not contribute particularly to the understanding of the steady flow behavior. In Figs. V-29-1 and V-29-2

*Denotes points where superheating of vapor starts

FIG. V-29-6. Total Channel Pressure Drop Versus Flow Rate For Various Maximum Channel Heat Fluxes. ANL Neg. No. 112-5640.

the quality and void fraction profiles are given for two different peak heat fluxes at various flow rates. The channel exit pressure for all data presented in the figures is 2.0 atm. The salient feature of these results is the abrupt increase of the coolant void fraction when boiling starts. This agrees qualitatively with the results obtained by various experimenters studying boiling of liquid metals, although the exact numerical values are, of course, subject to question. The channels for which results are presented have blanket sections at the end with markedly reduced heat generation rates. This accounts for the flattening of the quality profiles at the end of the channel. The slight increase in vapor quality at the end of the channel occurred because there was actually some flashing of liquid due to the steep pressure gradient there.

Pressure profiles for successively decreasing flow rates in a channel having a maximum heat flux of 100 calories/cm²-sec at three different exit pressures are shown in Figs. V-29-3, V-29-4, and V-29-5. Of primary interest in this series of figures is the appearance of discontinuities in the 2.0 atm curves (Fig. V-29-5.) This phenomenon has not been studied in detail, but some rather cursory comparisons of the results in which curves with the discontinuity were obtained with theoretical estimates by H. Fauske⁷ for critical flow rates with sodium have shown a rough agreement. The implication

is that the curves showing the discontinuity are describing a physically impossible situation wherein critical flow has been exceeded. Figure V-29-6 shows the effect of channel heat flux on the total channel pressure drop as a function of flow rate. Pressure drop versus flow rate curves are shown for maximum channel heat fluxes of 50, 100 and 200 calories/cm²-sec. As shown by the curves, the pressure drop at first decreases with decreasing flow rate until boiling starts, whereupon the pressure drop increases abruptly.

TRANSIENT SOLUTIONS

Successful solution of the transient hydrodynamics equations has not been completed as yet; however, the method of approach and some preliminary results are briefly summarized here. The scheme of solution utilizes Eqs. (1) and (3) and the momentum equation in integral form, which is

$$\frac{\partial G}{\partial t} = \frac{1}{L_p A_r + L} \left[\Delta p - G^2 v' \right]_{L_p}^{L_p+L} \int_{L_p}^{L_p+L} \left(\bar{p} + \frac{f|G|G}{2gD_H \rho} \right) dz \quad (17)$$

In the above form the momentum equation is integrated from the pump exit to the reactor channel outlet. In doing this, the assumption is made that only the acceleration pressure drop is important in the pipe leading from the pump to the reactor. Thus the parameters L_p and A_r in the equation represent the length of the liquid leg between pump and reactor and the ratio between reactor flow area and the coolant pipe cross sectional area, respectively. In this way the inertial effects of the liquid flowing into the reactor vessel are included.

Briefly, the iterative scheme used in the ALGOL program for the transient case works as follows. After calculation of initial conditions for the problem, the momentum integral equation is used to calculate the average coolant flow rate at the next succeeding point in time. This average flow rate is then used in the energy difference equation to calculate the coolant enthalpy profile at the succeeding time. A switch from the single-phase to the two-phase form of the energy equation is controlled by comparing the coolant temperature with the saturation temperature at each point. Once boiling starts, an iterative solution is required in the two-phase region. Since the mass flow rate is no longer constant with position in the two-phase region, it is necessary to iterate between the energy and continuity equations in the boiling section of the channel. Once a convergence criterion has been satisfied the calculation proceeds to the next time step by calculating a new average coolant flow rate via the momentum integral equation. The two forcing functions which may be used for the transient problems are:

- (1) an exponentially decreasing available pump outlet pressure, and
- (2) an exponentially increasing fuel element heat generation rate, corresponding to a constant reactor period.

The calculation of the fuel rod temperatures at each time step is straightforward. The technique and difference equations employed are essentially the same as those used in the ARGUS code.⁸

The real problems with the transient calculation at present occur with the two-phase hydrodynamics equations. Successful calculations have been performed in the two-phase region with up to 20% of the channel boiling. Results of these calculations show oscillations in the outlet flow even though the inlet flow is decreasing monotonically. At this point it is not clear if the oscillations are real or if they are the result of instabilities in the numerical solution. Because of the method of integrating across the point where boiling starts it is necessary that the boiling point advance about one space node per time step for the differencing approximation to be valid. One source of difficulty until now has been that this condition is not necessarily met with a constant time step. Therefore, the program is being modified so that the size of the time step is under program control once boiling has started. The space steps remain constant.

It appears that it will be necessary to restrict the solutions to cases where the maximum coolant temperature occurs at the end of the channel at the inception of boiling, at least initially. This means that the effect of a lower heat generation blanket zone at the end of the core will not be treated in detail as had been planned.

Thus far, in the successful calculations in the two-phase region, a constant coolant saturation temperature has been used, even though the other coolant properties are pressure dependent. Since the effect of local pressure on coolant saturation temperature is important and determines where boiling starts in the model, it is necessary that local pressure be taken into account. This is the last refinement planned for the code before using it to calculate the behavior of a series of representative reactor systems.

REFERENCES

1. H. H. Hummel, et al, *Sodium Void Effect*, Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010 (1965) p. 86.
2. J. E. Meyer, *Conservation Laws in One Dimensional Hydrodynamics*, Bettis Technical Review, WAPD-BT-20, 61 (September 1960).
3. R. W. Lockhart and R. C. Martinelli, *Proposed Correlation of Data for Isothermal Two-Phase, Two-Component Flow in Pipes*, Chem. Eng. Progress **45**, No. 1, 39 (January 1949).
4. J. E. Meyer, *Hydrodynamic Models for the Treatment of Reactor Thermal Transients*, Nucl. Sci. Eng. **10**, 269 (July 1961).
5. J. E. Meyer, et al, *ART—A Program for the Treatment of Reactor Thermal Transients on the IBM-704*, WAPD-TM-156, (November 1959).
6. G. Birkhoff and T. F. Kimes, *CHIC Programs for Thermal Transients*, WAPD-TM-245, (February 1962).
7. H. K. Fauske, *Two Phase Critical Flow with Application to Liquid-Metal Systems (Mercury, Cesium, Rubidium, Potassium, Sodium and Lithium)*, ANL-6779 (1963).
8. D. F. Schoeberle, J. Heestand and L. B. Miller, *A Method of Calculating Transient Temperatures in a Multiregion, Axisymmetric, Cylindrical Configuration, The ARGUS Program, 1089/RE248, written in FORTRAN II*, ANL-6654 (1963).

V-30. Parametric Studies Using a Coolant Expulsion Model

C. DICKERMAN, W. STEPHANY and A. WRIGHT*

INTRODUCTION

Knowledge of the effects of pressure generated by vaporizing coolant is of importance to the Fast Reactor Safety Program for two principal applications:

- (1) planning and analyzing transient meltdown experiments performed with sodium loops in the Transient Reactor Test Facility (TREAT); and
- (2) performing accident analyses for fast reactor

cores in cases in which significant reactivity changes might result from coolant expulsion or from motion of fuel under the influence of coolant motion and pressure.

Because of the lack of experimental data on transient boiling of liquid metals, and the absence of an adequate theoretical description for such systems (or, indeed even for the comparatively well-investigated case of water), a theoretical parameter study has been performed using representations in which detailed heat transfer phenomena do not enter explicitly. Emphasis was

* Student, Northwestern University, Evanston, Illinois.

placed on the study of the effects of physical parameters, such as coolant channel cross sectional area, channel length, mass loading, and power. In principle, at least, the model is sufficiently general that heat transfer data could be combined with it, when those data become available. This study represents an extension of the earlier, detailed calculations by R. Brittan.¹

BASIC MODEL

The basic assumption used is that the actual physical coolant channel can be replaced, for purposes of calculation, by an equivalent channel of the same effective cross section, inside of which all of the liquid coolant is combined into a "piston", behind which the vaporized coolant exerts a pressure. In addition, it is also assumed that:

- (1) energy is distributed adiabatically and isothermally in the heated mass whose temperature determines the vapor pressure of the coolant,
- (2) vaporization occurs instantaneously,
- (3) the only force acting on the piston is that resulting from the vapor pressure of the coolant,[†]
- (4) the heated portion of the system is always at saturation conditions and in equilibrium,
- (5) the channel cross section remains constant, and
- (6) the difference in specific heat of liquid and vapor has a negligible effect on the problem.

The basic equations are the following:

The volume of vapor is defined by

$$V(t) = G(P) \frac{m_f(t)}{P(t)}, \quad (1)$$

where m_f is the flashed off (vaporized) mass, P is the vapor pressure, t is time, and G is derived from material properties. In addition, V is also defined by application of Newton's Second Law of Motion, as

$$V(t) = A^2 \int_0^t \int_0^{t'} \frac{P(t'')}{m_L(t'')} dt'' dt', \quad (1a)$$

where A is the cross sectional area, and m_L is the total liquid mass to be accelerated. That is, the volume equals "piston" motion times cross sectional area, where piston motion is obtained by double time-integration of acceleration.

The power available for vaporization, Q_2 , is

$$Q_2(t) = Q_1(t) - \frac{m_n}{D(P)} \frac{dP(t)}{dt}, \quad (2)$$

where Q_1 is the total power, m_n is the heated mass, and D is the derivative of pressure with respect to enthalpy. Finally, the flashed off mass is determined by

[†] This is relaxed in the computer code used for part of the parameter study.

$$m_f(t) = b \int_0^t Q_2(t') dt', \quad (3)$$

where b is the reciprocal of the heat of vaporization.

At the beginning of a calculation using this model, input of energy causes the temperature of the heated mass to increase, producing an increase in pressure. [Because of the unique pressure-temperature relationship along the saturation curve, it is not necessary for temperature to enter the calculations explicitly.] Then, as the pressure causes the liquid piston to move, coolant is vaporized to fill the "void" behind the piston. Thus, as the calculation continues, more and more energy goes into vaporization until eventually $Q_2 = Q_1$ and a pressure maximum is reached.

This basic model was developed by G. Rosenberg, S. Mehta, and R. Ray and it was used to estimate maximum pressures for the condition $Q_2 = Q_1$, in the case of extreme experimental conditions in the large TREAT sodium loop.² Their analytical solution of the equations was developed further by C. Sayles,³ (assuming constant parameters and properties) in order to obtain pressure as a function of time. This formulation was coded for computer solution using a differential equation subroutine, developed separately,⁴ for solution of the key equations.¹ Mathematical difficulties were encountered when this basic method was used to solve problems with the analytical equations reformulated for variable parameters and non-constant properties. As a result, a more direct solution of Eqs. (1), (1a), (2) and (3) was prepared, using the finite-difference method described by R. Richtmyer.⁵ In this final form, the basic equations are

$$\begin{aligned} \dot{Z} \left(T + \frac{\Delta t}{2} \right) &= \dot{Z} \left(t - \frac{\Delta t}{2} \right) \\ &+ \frac{A[P(t) - P(0)]}{m_L(t)} \Delta t - g \Delta t \\ &- \gamma \left(t - \frac{\Delta t}{2} \right) \left[\dot{Z} \left(t - \frac{\Delta t}{2} \right) \right]^2 \Delta t, \end{aligned} \quad (4)$$

where $\dot{Z}$ is the velocity of the liquid piston, g is the acceleration due to gravity, and γ is a friction term. If $\dot{Z} < 0$, then $\dot{Z}$ is set equal to zero. Distance is calculated directly by time integration of $\dot{Z}$. Enthalpy of the coolant, H , is calculated from

$$H(t + \Delta t) = \left\{ \int_0^{t+\Delta t} Q_1 dt' - \frac{AZ(t + \Delta t)P(t)}{G(P)b} \right\} \frac{1}{m_n} + H[P(0)] \quad (5)$$

and H and P are related using tabulated physical property values. After iterating between P and H , m_L is calculated from

$$m_L(t + \Delta t) = m_L(0) - \frac{AZ(t + \Delta t)P(t + \Delta t)}{G(P)}. \quad (6)$$

FIG. V-30-1. (a) Comparison of Triangular q Function and "Fitted" Square Wave Pulse. (b) Comparison of ΔP Calculated Using the Two Power Functions. ANL Neg. No. 112-4671 Rev. 1.

Although convergence difficulties were found in certain problems run with exponentially increasing power, they occurred for values of Z well in excess of heated channel lengths for physically-interesting systems.

This study of transient pressure pulses was carried out along two parallel paths:

(1) a general investigation using a simplified analytic model in which vaporization effects are treated in the spirit of a first order perturbation, based on Eq. (2) with $Q_2 = 0$.

(2) a more restricted set of calculations using the digital code based on Eqs. (4), (5) and (6) with, however, g and γ set equal to zero in order to make more nearly transparent the effects of parameters.

Calculations were run, using basic material properties, for a variety of cases—both constant and variable parameters and properties—using the digital code. Results were described and correlated using the simplified analytic model.

The basic development of the approximate analytic model is as follows:

Pressure rise, ΔP is given by integrating Eq. (2), with $h_2 = 0$,

$$\Delta P(t) = \int_0^t qD \, dt', \quad (7)$$

where q is specific power. Then acceleration is

$$\ddot{Z}(t) = \int_0^t \frac{AqD}{m_L(0)} \, dt'; \quad (8)$$

velocity is given by

$$\dot{Z}(t) = \int_0^t \int_0^{t'} \frac{AqD}{m_L(0)} \, dt'' \, dt', \quad (9)$$

and distance follows, as

$$Z(t) = \int_0^t \int_0^{t'} \int_0^{t''} \frac{AqD}{m_L(0)} \, dt''' \, dt'' \, dt'. \quad (10)$$

These equations can be readily solved for t as a func-

TABLE V-30-I. COMPARISON OF PARAMETRIC DEPENDENCE FOR THE CONSTANT PARAMETER CASE AND ONE VARIABLE D CASE

Quantity	Constant D Case	Variable D Case
$\Delta P(t)$	t q_1	$t^{1.5}$ $q_1^{1.5}$
$\dot{Z}(t)$	t^2 q_1 A $m_L(0)^{-1}$	$t^{2.5}$ $q_1^{1.5}$ A $m_L(0)^{-1}$
$Z(t)$	t^3 q_1 A $m_L(0)^{-1}$	$t^{3.5}$ $q_1^{1.5}$ A $m_L(0)^{-1}$
$t(Z)$	$q_1^{0.33}$ $A^{-0.33}$ $m_L(0)^{0.33}$ $Z^{0.33}$	$q_1^{-0.43}$ $A^{-0.29}$ $m_L(0)^{0.29}$ $Z^{0.29}$
$\Delta P(Z)$	$q_1^{0.67}$ $A^{-0.33}$ $m_L(0)^{0.33}$ $Z^{0.33}$	$q_1^{0.86}$ $A^{-0.43}$ $m_L(0)^{0.43}$ $Z^{0.43}$
$\dot{Z}(Z)$	$q_1^{0.33}$ $A^{0.33}$ $m_L(0)^{-0.33}$ $Z^{0.67}$	$q_1^{0.43}$ $A^{0.29}$ $m_L(0)^{-0.29}$ $Z^{0.71}$

TABLE V-30-II. COMPUTER PROBLEM SUMMARY SODIUM PROPERTIES, CONSTANT POWER

Problem No.	P_0 , atm	A , cm ²	$\frac{m_n q_1}{\times 10^{13}}$ ergs/sec	$m_L(0)$, g	m_n , g
12	2.7	12.187	3.0	25,000	310
13	2.7	12.187	3.0	10,000	310
14	2.7	12.187	10.0	10,000	310
15	2.7	12.187	10.0	1,000	310
16	1.0	12.187	3.0	10,000	310
17	2.7	1.58	0.3	100	31
18	2.7	9.678	3.0	13,950	317
19	2.7	9.678	10.0	13,950	317

FIG. V-30-2. Pressure and Expulsion Distance as Functions of Time for Reference Problem. ANL Neg. No. 112-5446.

tion of Z if constant values of q and D are used. Exploratory calculations made for power pulses and reasonably-varying material properties showed that "suitably-averaged" values of q and D exist for typical cases of interest for reactor channels, indicating that the approximate analytical model based on constant parameters could be used for the correlation of experiments (see Paper III-19). This point is shown in Fig. V-30-1,

for the case of a $q(t)$ function that is an isosceles triangle, ranging from $-t_0$ to t_0 , with peak at time zero. Figure V-30-1a contains two power pulses: one, the given triangular pulse, and the other, a square wave pulse selected to give an approximate fit to the ΔP curve for the triangular q function. Figure V-30-1b compares ΔP relationships calculated from the two power pulses using Eq. (7). Although this is a relatively

TABLE V-30-III. SCALED PRESSURE RISES FOR CONSTANT POWER, SODIUM PROPERTY CALCULATIONS, 30 CM EXPULSION

Problem No.	Computed Pressure Rise, $\times 10^6$ dyne/cm ²	Calculated Pressure Rise From Constant Case, $\times 10^6$ dyne/cm ²	Calculated Pressure Rise From Scaling Laws, Table V-30-I, Variable Case, $\times 10^6$ dyne/cm ²	Error, %	
				Constant case	Variable case
12	56.9	—	—	—	—
13	36.9	41.8	38.4	13.3	4.1
14	114.9	92.7	108.1	-19.3	-5.9
15	39.0	43.3	40.2	11	3.1
16	34.2	41.8	38.4	22.2	12.2
17	18.1	17.9	12.8	-0.1	-29.3
18	18.0	49.8	48.2	3.8	0.4
19	133.7	110.6	135.5	-17.3	1.3

extreme case, the square wave pulse gives a reasonable fit over a range in ΔP values of almost an order of magnitude. Since Eqs. (7), (8), (9) and (10) have the same functional dependence upon the product (qD) as on q , this example also can be used to indicate the possibility of averaging D .

RESULTS

Scaling relationships were derived using the approximate analytical model for constant q , and both the case of constant D and the case of a variable D providing a fit to sodium properties for $P(0)$ equal to one atm, with $P(t) \lesssim 20$ atm. Parametric dependence for both cases are given in Table V-30-I. The table shows that the general behavior for this particular variable case is similar to that for the constant case. In general, experimentally, the two cases would be nearly indistinguishable.

A summary of computer problems using sodium properties and constant power is given in Table V-30-II. The reference problem (No. 12) is based on typical experimental loop dimensions; it has a heated mass of 310 grams, total coolant mass of 25 kg, heated channel of 30 cm, and total power of 3 MW. Results of the computer calculation of pressure and distance of expulsion are shown in Fig. V-30-2. Pressure is approximately 59 atm when the coolant leaves the heated section. The initial portion of the pressure curve is concave upward; this results from the non-linear relationship between vapor pressure and saturated liquid enthalpy. The latter (concave-downward) part of the curve results from appreciable energy going into vaporization. At the time of expulsion, about 0.6 gram of sodium has been vaporized. At the time of maximum pressure (160 m sec) the pressure is 82 atm, and the coolant slug has been expelled 152.7 cm, well beyond the physical length of the heated channel.

Although not readily apparent from the figure, the distance of expulsion is still approximately given by the scaling relationships of Table V-30-I: the calculated Z

values are proportional to $t^{3.49}$, whereas the constant case predicts proportionality to t^3 , and the variable case of Table V-30-I predicts $t^{3.5}$.

One further point should be made here, with respect to real systems. The coolant channel cross sectional area in the heated section may be appreciably different from that of the channel beyond the heated section. If, for example, the area in the heated length is 40 % of that beyond, then the equivalent piston of problem No. 12 will have traveled only about 40 % or 30 cm (or 12 cm) when the heated section is voided of liquid. Thus the actual peak pressure would be reduced (see Fig. V-30-2) to about 41 atm. This change in effective peak pressure may also be estimated using the scaling relationships of Table V-30-I, which predict ΔP to be proportional to $Z^{0.33}$ or $Z^{0.43}$ for the constant and one variable cases, respectively. (These two cases yield 43 atm and 40 atm, respectively.)

Pressure calculated for $Z = 30$ cm, using the approximate analytical scaling relationships of Table V-30-I and the reference problem, are shown in Table V-30-III, for the problems described in Table V-30-II. Also given in Table V-30-III are the pressures calculated by the computer code for an expulsion distance of 30 cm. Somewhat better agreement is indicated for the variable property case than for the scaling laws of the constant property case. The degree of agreement shown for the constant case and problem 17 is fortuitous, since 30 cm expulsion is reached just after the time of maximum pressure and the perturbation method of the approximate analytical model is really outside its proper range of application. This problem is included to show an extreme example of the use of the scaling relationships.

Although these studies have not been checked by extensive experimental investigations, even for water, they do appear to explain some of the trends of in-pile experiments performed using water by Wentz, Wright and Zivi (see Paper III-19) and some low-power check runs by J. Thurman⁶ made with water to provide a crude test of the basic model.

REFERENCES

1. R. O. Brittan, *Coolant Pressures During Transients*, Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010 (1965) p. 165.
2. G. Rosenberg, S. Mehta and R. Ray, Argonne National Laboratory (Private Communication).
3. C. Sayles, Argonne National Laboratory (Private Communication).
4. P. M. Lukehart, *Algorithm 218*, Kutta Merson, Comm. A.C.M. 6, No. 12, 737 (1963).
5. R. D. Richtmyer, *Difference Methods for Initial-Value Problems*, (Interscience Publishers, Inc., New York, 1957).
6. J. Thurman, Argonne National Laboratory (Private Communication).

V-31. The Effects of the Thermal Characteristics of UO_2 Upon the Dynamics of High Performance Fast Reactors

C. K. SANATHANAN, J. C. CARTER and R. O. BRITTAN

INTRODUCTION

The limitations on thermal performance of reactor fuels, both statically and dynamically, are governed by the thermal diffusivity of the fuel material. Estimates of the peak central temperatures are necessary to determine limiting power densities for normal operation, and physical behavior in abnormal circumstances. In addition, complete temperature distributions are necessary to determine coupling between heat generation and cooling, both statically and dynamically.

It has been possible to meet this problem in the past because fuel materials have had high thermal conductivity, and because fuels have had operating temperatures limited by low melting points. When the conductivity is high, its variation with temperature low, and thermal gradients small, the heat conduction equation can be greatly simplified. Errors in assuming "constant" values for the thermal diffusivity are insignificant and can be ignored.

Now, however, UO_2 fuels have been proposed for large fast reactors.^{1,2} In calculating temperature distributions for these fuels, large errors will ensue if simplified methods are employed. UO_2 has very low thermal conductivity which decreases with temperature to a minimum, then increases sharply. It is desirable to operate a reactor fueled with UO_2 with centerline temperatures approaching the melting point in order to realize maximum power densities. The large center-to-edge temperature ratios result in conductivities on both sides of the minimum, with thermal radiation strongly influencing heat flow.

In addition, UO_2 fuels are subjected to transient heating experiments in TREAT which may result in rapid temperature rises to well above the melting point. For these tests it is necessary to predict the temperature so that the burst intensity can be properly planned. This is true especially if the damage threshold is approached

cautiously in a series of shots. The unreliability of temperature measurements in the UO_2 make extrapolations to the next test in a series useless.

Because of these design and test requirements, a study has been made in which new calculational methods for determining transient temperature distributions in UO_2 fuel elements have been devised and determination of transient temperature and heat flow behavior undertaken.

SIGNIFICANCE OF VARIABLE CONDUCTIVITY

The effect of neglecting variation of thermal conductivity and its gradient can be shown by analysis of the usual steady state heat conduction equation for a flat plate:

$$\frac{d}{dx} \left[K \frac{dT}{dx} \right] + q = 0, \quad (1)$$

with boundary conditions

$$dT/dx|_{x=0} = 0 \quad (2)$$

$$-K dT/dx|_{x=l} = ql, \quad (3)$$

where T is temperature, x is the space coordinate, q is volumetric heat generation rate, l is the plate half thickness, and K is thermal conductivity. Equation (1) may be rewritten as

$$\frac{d^2T}{dx^2} + \frac{dK}{dT} \left(\frac{dT}{dx} \right)^2 + q = 0. \quad (4)$$

When dealing with materials in which variation of K with T is insignificant, and the thermal gradient (dT/dx) is small, the quantity in parentheses in Eq. (4) is neglected.³⁻⁷ However, this simplification would lead to significant error if the material were UO_2 .

From reported measurements,⁸⁻²⁶ the thermal conductivity of UO_2 is small at low temperatures and decreases significantly as temperature increases. Then,

as thermal radiation becomes significant in transferring heat, the conductivity begins to increase sharply. The term expressing conductivity as a function of temperature in the range below that where radiation is significant can be taken as

$$K = A/(B + T), \quad (5)$$

where A and B are constants and have been assigned the values $A = 42$ and $B = 400$ on the basis of experimental observation.¹⁶

Using Eq. (5), Eq. (4) has been solved for $T(x)$ exactly and with the parenthetic term neglected. The comparative results are displayed in Fig. V-31-1 for $q = 1000$ watts/cc and a coolant temperature of 500°K . The center plane temperature yielded by the simplified solution is seen to be 50 % higher than that given by the exact solution.

This is a simple steady state problem for illustrative purposes, since temperatures reached are in the range where thermal radiation is significant. With the radiation component of K increasing nearly as the cube of the temperature the thermal gradient of K takes on greater significance in the central region of the plate. However, the example chosen dramatically illustrates the errors which may be introduced by neglecting the temperature and conductivity gradients in calculating temperature distributions in low conductivity materials like UO_2 . For example, this feature makes the ARGUS code⁴ in its present form unsuitable for use in calculation of temperature distributions in UO_2 elements (especially those to be tested in TREAT).

THERMAL DIFFUSIVITY OF UO_2

Before going into the study of transient effects, some discussion of the particular make-up of thermal dif-

FIG. V-31-1. Comparison of the Approximate with the Exact Temperature Distribution in a UO_2 Plate. ANL Neg. No. 112-5764.

FIG. V-31-2. Conductivity of UO_2 Versus Temperature. ANL Neg. No. 112-5765.

fusivity of UO_2 is in order. The diffusivity is defined as the ratio of thermal conductivity to heat capacity (which in turn is the product of specific heat and density). Diffusivity is the key parameter signifying the space-time dependent behavior of heat flow and temperature. In many materials it varies little with temperature and is frequently considered constant in making calculations. However in a low conductivity material like UO_2 it is dominated by $K(T)$ and varies significantly.

With respect to $K(T)$, the predominant mechanisms of heat transfer in ceramics are generally recognized to be phonon conduction and photon conduction (internal radiation).

At low temperatures, phonon conduction is dominant; and its effect, in most cases, diminishes with temperature. On the other hand, photon conduction becomes increasingly significant at higher temperatures. In materials such as UO_2 , photon conduction is so significant at higher temperatures (say above 1800°K) that it causes the overall conductivity of these materials to increase with temperature. This has been observed by several investigators.^{14, 24} The phonon conduction component of the thermal conductivity is usually expressed as in Eq. (5),^{16, 25} that is

$$K_{\text{phon}} = A/(B + T). \quad (5a)$$

The photon conduction component is considered to be²⁶

$$K_{\text{phot}} = (16n^2\sigma/3\beta_r)T^3, \quad (6)$$

where

n = index of refraction

σ = Stefan-Boltzmann constant

β_r = inverse mean free path of radiation.

It has been found²⁶ that β_r is essentially proportional to T in crystalline UO_2 . The temperature dependence of β_r is much more complex in non-crystalline UO_2 , and is not fully understood. The total effective thermal conductivity is evidently the sum of Eqs. (5a) and (6), that is

$$K(T) = A/(B + T) + (16n^2\sigma/3\beta_r)T^3. \quad (7)$$

The values of A and B , and the coefficient of T^3 may be obtained from experimental data. However, to facilitate the calculations which follow it was found more convenient to express $K(T)$ as a third degree power series:

$$K = k_0 + k_1T + k_2T^2 + k_3T^3. \quad (8)$$

This allows determination of the coefficients ($k_0, k_1, \dots$) either from Eq. (7) or directly from an experimental plot of K versus T by a curve-fitting procedure²⁸ which leads to errors of less than $\frac{1}{2}\%$. Use of the series expression has distinct computational advantages over the form of Eq. (7) and facilitates extension of the computational methods to other ceramic materials.

After a careful study of the recent publications on the conductivity of UO_2 (both experimental and theoretical)^{3,8, 10, 14, 18-26} it is found that the variation of conductivity with temperature can be given a certain range of values shown by curves K_2 , K_3 and K_4 in Fig. V-31-2. In this figure, curve K_1 is obtained by assuming the unrealistic condition of no radiative transfer in the medium and hence represents the value used earlier in the simple steady state problem as given by Eq. (5) using values of A and B from Ref. 16. That is,

$$K = 42/(400 + T). \quad (9)$$

The other component of the diffusivity (heat capacity) also varies with temperature.³ Although variation is not pronounced, the variation is taken into account in determining the transient behavior of the temperature. Depending on the porosity, irradiation level and stoichiometry, the volumetric heat capacity may increase or decrease with temperature. It appears that this variation is linear and within $\pm 8\%$ in the temperature range 500–3000°K. Hence it will be necessary to express the product of specific heat, $c(T)$, and density, $\rho(T)$, as

$$c(T)\rho(T) = g_0 + g_1T, \quad (10)$$

where

$$g_0 = 3.31 \text{ watt-sec/cc}$$

$$g_1 = \pm 1.27 \times 10^{-4} \text{ watt-sec/cc/}^\circ\text{K}.$$

These elements of the thermal diffusivity have a pro-

found effect on the transient heat transfer processes within the UO_2 medium. It is this effect that is examined in the study to follow.

TRANSIENT THERMAL CHARACTERISTICS OF UO_2

A critical evaluation of the transient performance of a reactor system involves the simultaneous solution of a complex system of differential equations describing heat transfer in the fuel element and the coolant, and the neutron kinetics in the reactor.

This particular study is directed towards analyzing the transient thermal characteristics of UO_2 as the first step in the evaluation without reference to any specific reactor system. Later it is planned to extend the studies to a system represented by a UO_2 -fueled, high performance fast reactor for the purpose of understanding the influence of these thermal characteristics on the dynamics of the system. At present, indications are that the future system study will employ an analog-digital hybrid computer. For this portion of the study, the analog computer is best suited because of the parametric nature of the study and the time dependence. A brief account of this initial phase is presented here.

Although the fuel elements in the fast reactor of interest will probably be cylindrical, flat plate geometry is used here with no cladding to simplify the space dependence. In addition, the temperature of the coolant and the heat transfer coefficient at the fuel-coolant interface are assumed to be constant. (As the next step in the study, these constraints will be removed and the computational scheme will be extended to include cylindrical geometry.)

Under these constraints, the time dependent conservation of energy equation in one dimensional space is well known to be

$$\frac{\partial}{\partial x} \left[K(T) \frac{\partial T}{\partial x} \right] + g = c(T)\rho(T) \frac{\partial T}{\partial t}. \quad (11)$$

The boundary conditions chosen are

$$\partial T / \partial x \big|_{x=0} = 0 \quad (12)$$

$$-K(T) \partial T / \partial x \big|_{x=l} = h(T_l - T_c), \quad (13)$$

where x and t are the independent space and time coordinates, respectively. l is the plate half-thickness, g is the volumetric heat generation rate; h is the heat transfer coefficient at the fuel-coolant interface, T_l is the fuel boundary temperature, and T_c is the coolant temperature. $K(T)$ is given by Eq. (8), and $c(T)\rho(T)$ is given by Eq. (10).

It is assumed that the material is isotropic and that the volumetric heat generation rate is uniform, so that the temperature distribution is taken to be symmetrical with respect to the plate midplane.

FIG. V-31-3. Sectionalization of Space for Analog Computation. ANL Neg. No. 112-5758.

The half thickness is divided into N sections of width h as shown in Fig. V-31-3. The temperature distribution between centers of any two consecutive sections is assumed to be linear. This enables one to integrate Eq. (11) within each of these sections, thereby leaving only $T_0, T_1, T_2, \dots, T_N, T_{N+1}$ as the unknown functions of time. T_0 and T_{N+1} are outside the boundaries. This method of solution of the heat conduction equation is found to require the minimum number of sections. Several other differential equation formulations were investigated before arriving at this conclusion.

Now consider any section, i , and integrate Eq. (11) in this section:

$$K_{i,i+1} \frac{(T_{i+1} - T_i)}{h} - K_{i-1,i} \frac{(T_i - T_{i-1})}{h} + qh = \frac{\partial}{\partial t} \int_{(i-1)h}^{ih} c_p T dx, \quad (14)$$

where $K_{i,i+1}$ is the conductivity at the boundary of section i and $i+1$, and is evaluated at $T = (T_i + T_{i+1})/2$.

To show the influence of variable conductivity alone, c_p is assumed constant. The added influence of variable heat capacity is studied separately. Hence two sets of equations must be developed.

If c_p is not a function of T , the right-hand side of Eq. (14) may be expressed as

$$\left. \begin{aligned} c(T) \rho(T) \frac{\partial}{\partial t} \int_{(i-1)h}^{ih} T dx &= c_p \frac{\partial}{\partial t} \int_{(i-1)h}^{ih} T dx \\ &= c_p \frac{\partial}{\partial t} \left[\int_0^{h/2} \left(T_i + \frac{T_{i-1} - T_i}{h} x \right) dx \right. \\ &\quad \left. + \int_0^{h/2} \left(T_i - \frac{T_i - T_{i+1}}{h} x \right) dx \right]. \end{aligned} \right\} \quad (15)$$

Due to the symmetry about the center plane

$$T_0 = T_1 \quad (1st \text{ boundary condition}). \quad (16)$$

The outer boundary temperature is,

$$T_l = \frac{T_N + T_{N+1}}{2}.$$

Applying the second boundary condition

$$-K_{N,N+1} \left(\frac{T_{N+1} - T_N}{h} \right) = h(T_l - T_c). \quad (17)$$

Thus, one may write N equations of the form of Eq. (14), for $i = 1, 2, \dots, N$. The boundary conditions give two more equations. These $N+2$ equations are used to solve for the $N+2$ unknown temperatures $T_0, T_1, \dots, T_{N+1}$.

However, if c_p is assumed to vary linearly with respect to T as given by Eq. (10), the right-hand side of Eq. (14) may be integrated to yield

$$\left. \begin{aligned} \int_{(i-1)h}^{ih} c(T) \rho(T) \frac{\partial T}{\partial t} dx &= \int_{(i-1)h}^{ih} (g_0 + g_1 T) \frac{\partial T}{\partial t} dx \\ &= hg_0 \frac{\partial}{\partial t} \left[\frac{1}{8} T_{i-1} + \frac{3}{4} T_i + \frac{1}{8} T_{i+1} \right] \\ &\quad + \frac{1}{6} hg_1 \frac{\partial}{\partial t} \left[\frac{1}{4} T_{i-1}^2 + \frac{11}{2} T_i^2 + \frac{1}{4} T_{i+1}^2 \right]. \end{aligned} \right\} \quad (18)$$

It is noted that the above method of solution of Eq. (11) has the distinct advantage that there are no memory requirements in the analog computer to deal

FIG. V-31-4. The Effect of Radiative Transfer Upon Temperature Distribution in a Plate of UO_2 . ANL Neg. No. 112-5762.

FIG. V-31-5. The Effect of Radiative Transfer Upon Temperature Gradient. ANL Neg. No. 112-5796.

with the nonlinearity of the equation and the boundary conditions. Further, the method is equally applicable to cylindrical geometry, and is found to require the minimum number of spatial subdivisions to achieve a certain desired accuracy in the solution.

Since an exact analytic solution of Eq. (11) is available for the steady state case,²⁹ it was compared with the analog solution for various numbers of subdivisions,

FIG. V-31-6. Distribution of Temperature Resulting From a Step Input of Heat, $K(T) = K_1(T)$. ANL Neg. No. 112-5763.

N . It was found that $N = 5$ is adequate for this study, since the error for this case is less than $\frac{1}{2}\%$. For this example, $K(T)$ is taken from curve K_4 , Fig. V-31-2, $q = 1000$ watts/cc, $l = 0.25$ cm, $T_c = 500^\circ\text{K}$, $h = 5.67$ watts/cm²/°K and c_p is held constant and given by Eq. (10) with $g_1 = 0$. It is noted that the computed curve touches the theoretical curve in each subsection and that the small error is always ≥ 0 .

The following significant results of the present study include the effect of internal radiation upon the steady state and transient distribution of the temperature, its gradient within the medium, and the effect of c_p being a function of temperature.

FIG. V-31-7. Distribution of Temperature Resulting From a Step Input of Heat, $K(T) = K_4(T)$. ANL Neg. No. 112-5759.

TABLE V-31-I. TIME CONSTANTS (sec)

	$T_c = 500^\circ\text{K}$	$T_c = 1000^\circ\text{K}$
$K_1(T)$	3.40	4.80
$K_2(T)$	3.25	4.00
$K_3(T)$	2.95	3.45
$K_4(T)$	2.60	2.85

FIG. V-31-8. Frequency Response of the Center Plane Temperatures. ANL Neg. No. 112-5760.

FIG. V-31-9. The Effect of $c \times \rho$ Upon the Response. ANL Neg. No. 112-5761.

The steady state distribution of temperature for each of the conductivity versus temperature curves of Fig. V-31-2 is shown in Fig. V-31-4. This figure clearly shows that photon conduction has a significant influence upon the steady state temperature distribution.

Radiative transfer also has a sizable effect upon the temperature gradient in the UO_2 plate (which incidentally determines the thermal stress in the material). This is illustrated in Fig. V-31-5. The effect of increasing the radiative transfer upon the transient response is illustrated in Figs. V-31-6 and V-31-7 considering the time dependent temperature distribution resulting from a step increase in the internal heat generation rate

of 1000 watts/cc for $T_c = 1000^\circ\text{K}$. Each figure is normalized to the corresponding steady state center plane temperature rise. $K_1(T)$ and $K_4(T)$ from Fig. V-31-2 are used. The relative speed with which the temperature distributions attain their steady states may be observed from these figures.

The familiar "time constant" used in connection with linear systems may also be chosen as a measure to indicate the speed of response. This "time constant" is defined as the time taken by the response to reach $[1 - (1/e)]$ of its final value for a step increase of the forcing function. The "time constants" associated with the center plane temperature responses corresponding to the various conductivity curves of Fig. V-31-2 for coolant temperature of 500 and 1000°K respectively are obtained for a step increase in the volumetric heat generation rate of 1000 W/cc (Table V-31-I).

The effect of internal radiation on the transient behavior may also be illustrated by considering the frequency response. This consists of observing the amplitude and phase delay of oscillations of the center plane temperature for a variation of the volumetric heat generation rate q , of the form

$$q = q_0 + A \sin \omega t, \quad (19)$$

where q_0 is a certain steady value.

The amplitudes and phase responses of the center plane temperatures are given in Fig. V-31-8 for a coolant temperature of 1000°K . It is apparent from this figure that the internal radiation effects are more pronounced at higher temperatures and for lower frequencies.

It is also clear that the attenuation of the amplitude of the response is quite rapid with increasing frequency. This means that high frequency fluctuations in the heat generation rate encountered in reactor operations have little disturbing effect upon the UO_2 fuel temperature for frequencies of practical interest,* even though its thermal conductivity (and consequently its ability to diffuse heat) is considerably smaller than the metallic fuels.

A linear variation of $c\rho$ with respect to T is considered next. $c\rho$ is chosen as in Eq. (10), and the integration is carried out using Eq. (18). q is taken as 1000 watts/cc (applied as a step function), $T_c = 1000^\circ\text{K}$, and the conductivity is given by curve K_4 from Fig. V-31-2. Figure V-31-9 gives the response of the center plane temperature for the $+$ and $-$ values of g_1 . Note that the "time constant" is 3.1 sec for the positive value of g and 2.3 sec for the negative value of g_1 .

In conclusion it is stressed that internal radiation has a significant effect upon both the steady state and transient thermal response of UO_2 which in turn affects

* The "cut off" frequency for UO_2 is about 15 times less than that for metallic uranium.

the reactor design using this material. The recent experimental determinations of thermal conductivity indicate that there is a substantial contribution from internal radiation. However, at present there is disagreement in the quantitative functional relationship of conductivity with temperature. The range of experimental functional relationships shown in Fig. V-31-2 for UO_2 covers a wide variation of its properties, such as oxygen content, porosity and structure. An experimental determination of the variation of conductivity with temperature corresponding to the exact form of UO_2 used in any design is, therefore, essential to a realistic evaluation of the physical behavior of that specific reactor design.

REFERENCES

1. Chicago Operations Office, Reactor Engineering Division, USAEC, *An Evaluation of Four Design Studies of a 1000 MWe Ceramic Fueled Fast Breeder Reactor*, COO-279, (December 1, 1964).
2. W. R. Simmons, P. J. Persiani and T. R. Bump, *Fast Reactor Test Facility (FARET) and Initial Experimental Program*, ANS National Topical Meeting on Fast Reactor Technology, Detroit, Michigan, April 26-28, 1965.
3. W. D. Kingery, A. M. Ross and J. Belle, *Uranium Dioxide: Properties and Nuclear Applications*, (J. Belle, ed.), USAEC, Washington, D.C., (1961), chapter titled *Thermal Conductivity*.
4. D. F. Schoeberle, J. Heestand and L. B. Miller, *A Method of Calculating Transient Temperatures in a Multiregion, Axisymmetric, Cylindrical Configuration. The ARGUS Program, 10891/RE248, Written in Fortran II, ANL-6654* (1963).
5. J. Heestand, D. F. Schoeberle and L. B. Miller, *The Calculation of Transient Temperature Distributions in a Solid Cylindrical Pin, Cooled on the Surface*, ANL-6237 (1960).
6. P. Greebler, D. B. Sherer and N. H. Walton, *FORE—A Computational Program for the Analysis of Fast Reactor Excursions*, GEAP-4090, (1962).
7. R. J. Wagner, *Heat-1, A One-Dimensional Time Dependent or Steady State Heat Conduction Code for the IBM-650*, IDO-16867, (1963).
8. W. D. Kingery, et al., *Thermal Conductivity, X, Data for Several Pure Oxide Materials Corrected to Zero Porosity*, J. Amer. Ceram. Soc. **37**, 108 (1954).
9. J. C. Jamieson and A. W. Lawson, *High Temperature Heat Conductivity of Some Metal Oxides*, J. Appl. Phys. **29**, 1313 (1958).
10. A. D. Feith, *Thermal Conductivity of UO_2 by a Radial Heat Flow Method*, TID-21668, General Electric Co., Cincinnati.
11. J. L. Bates, *Thermal Conductivity of UO_2 Improves at High Temperatures*, Nucleonics **19**, No. 6, 83 (1961).
12. W. D. Kingery and M. C. McQuarrie, *Thermal Conductivity: 1, Concepts of Measurement and Factors Affecting Thermal Conductivity of Ceramic Materials*, J. Amer. Ceram. Soc. **37**, No. 2, 67 (1954).
13. M. F. Lyons, et al., *UO_2 Pellet Thermal Conductivity from Irradiations with Central Melting*, Trans. Am. Nucl. Soc. **7**, No. 1, 106 (1964).
14. A. D. Feith, *The Thermal Conductivity of UO_2 up to 2500°C*, J. Nuclear Materials **16**, 231 (1965).
15. J. C. Hedge and I. B. Fieldhouse, *Measurement of Thermal Conductivity of Uranium Oxide*, AECU-3381, (September 20, 1956).
16. R. Scott, *Thermal Conductivity of UO_2* , AERE-M/R-2526, (March 1958).
17. T. J. Pashos, et al., *Irradiation Behavior of Ceramic Fuels*, Proc. 1964 Geneva Conference, Paper No. A-CONF. 28/P/240.
18. Power Reactor Technology **6**, No. 3, 12 (1963).
19. J. E. May, et al., *Observations on the Thermal Conductivity of Uranium Dioxide*, Trans. Am. Nucl. Soc. **5**, No. 2, 473 (1962).
20. J. L. Daniel, J. Matolich, Jr. and H. W. Deem, *Thermal Conductivity of UO_2* , HW-69945, (September 1962).
21. D. W. Lee and W. D. Kingery, *Radiation Energy Transfer and Thermal Conductivity of Ceramic Oxides*, J. Amer. Ceram. Soc. **43**, No. 11, 594 (1960).
22. J. Lambert Bates, *Thermal Conductivity of UO_2 Improves at High Temperatures*, Nucleonics **19**, No. 6, 83 (1961).
23. D. H. Whitmore, *Excitation Processes in Ceramics and Anomalous Increase in Thermal Conductivity at Elevated Temperatures*, J. Appl. Phys. **31**, 1109 (1960).
24. R. Viskanta and R. J. Grosh, *Heat Transfer in Thermal Radiation Absorbing and Scattering Media*, International Developments in Heat Transfer, Part IV, p. 820, ASME, New York, (1961).
25. R. D. Reiswig, *Thermal Conductivity of UO_2 to 2100°C*, J. Am. Ceram. Soc. **44**, No. 1, 48 (1961).
26. R. Viskanta, *Influence of Internal Thermal Radiation on Heat Transfer in UO_2 Fuel Elements*, Nucl. Sci. Eng. **21**, 13 (1965).
27. S. Chandrasekhar, *Radiative Transfer*, (Dover Publications, Inc., New York, 1960).
28. B. Garbow, *A General Program for Least Square Polynomial Fit (FORTRAN II)*, ANME 209, ANL Code Center (1961).
29. H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*, (Clarendon Press, Oxford, 1959), p. 89.

V-32. Fuel Element Thermal Displacements and Stresses for Reactor Feedback and Structural Integrity Analyses

C. K. YOUNGDAHL

In studying stability and control problems of nuclear reactors, it is important to know the effects of temporal and spatial variations of the core temperature distribution on the elastic displacements of individual fuel pins. A knowledge of the corresponding stress distributions in the fuel elements is of value in investigating the structural integrity of the reactor core.

For purposes of elastic analysis fuel pins may be idealized as solid circular cylinders, tubes, or rods with concentric cladding; the corresponding elasticity problems are most conveniently formulated in cylindrical coordinates. A new set of four stress functions, related to the standard Papkovitch-Neuber stress functions has been devised for treating three-dimensional thermoelasticity problems in cylindrical coordinates. The principal feature of these stress functions is that the governing differential equations and boundary conditions expressed in terms of the stress functions can be transformed using standard integral transformations; this is not the case if the Papkovitch-Neuber functions are employed.

In many problems the complete set of four stress functions need not be used. If the body forces vanish only three are required. For torsionless axially-symmetric problems a complete solution is given by two

stress functions while one stress function will suffice for the case of pure torsion.

To illustrate the use of the stress functions and the associated integral transform technique, example problems for two elementary geometries were solved. The elasticity solution for these two regions was obtained for arbitrary temperature, body force, and surface traction distributions, the only restriction being that all loadings were to possess a Fourier series in the angular coordinate. In particular, it was not assumed that the loadings were axially-symmetric. The first problem, which required a transformation with respect to the axial coordinate, was that of an infinitely long circular rod, loaded symmetrically about the midplane; the second problem, requiring a transformation with respect to the radial coordinate, was that of the elastic half-space.

The techniques developed for solving the two example problems will be employed to treat the more difficult but more physically-realistic thermoelasticity problems for finite rods, tubes, and rods with cladding. The application of the systematic transform method to these very complicated problems will make it possible to solve them in a routine, if laborious, fashion.

V-33. A Graphical Method for Determining the Radial Distribution of Thermal Stress in Fuel Rods Possessing Temperature-Dependent Material Properties

C. K. YOUNGDAHL

During a reactor transient the thermal stresses in a fuel rod caused by severe temperature gradients may be large enough to cause fracture. Some reactor fuel materials have elastic properties which are strongly temperature dependent in the temperature ranges encountered in reactor core environments, and this dependence should be taken into account in calculating stress levels. Because experimental measurements of the variations of these properties with temperature are usually given in graphical form, a graphical technique was devised for determining the radial variation of the axial stress in a

cylindrical fuel rod having a large temperature variation in the radial direction.

It is assumed that the temperature is a given function of the rod radius, that the rod is unrestrained, and that initially plane cross sections of the fuel rod remain plane after the heating takes place. By elementary strength of materials analysis it can be shown that

$$\sigma(r) = E[T(r)]\{e_A - e_T[T(r)]\}, \quad (1)$$

with

$$e_A = \frac{2\pi}{AE_A} \int_0^R rE[T(r)]e_T[T(r)] dr, \quad (2)$$

$$E_A = \frac{2\pi}{A} \int_0^R rE[T(r)] dr, \quad (3)$$

$$A = \pi R^2. \quad (4)$$

In the above, R is the outer radius of the rod, A is its cross sectional area, T is the temperature, σ is the normal stress in the axial direction, E is the Young's modulus of the material, E_A is the average Young's

modulus for the cross section, $e_T(T)$ is the thermal strain in a test specimen uniformly heated to a temperature T , and e_A is the average strain of the cross section. Given experimentally determined curves for E and e_T as functions of temperature and assuming or calculating a temperature profile $T(r)$, the integrands appearing in Eqs. (2), (3) can be plotted as functions of r . The integration in Eqs. (2), (3) can be performed by measuring the area under these curves, resulting in values for E_A and e_A . The right-hand side of Eq. (1) is then completely known so that $\sigma(r)$ can be plotted.

V-34. Properties of Materials Near Their Thermodynamic Critical Point

D. MILLER

The behavior of substances near their thermodynamic critical point is of considerable interest to reactor safety as well as to the understanding of the fundamental properties of matter. Because of the large compressibilities in the region of the critical point, the details of the behavior of density versus energy or temperature in this region will strongly determine the characteristics of the pressure versus energy or temperature for reactor materials in this region. In particular, in the case where the material completely fills the available volume, the pressure increases quite rapidly with energy and the large adiabatic compressibility must be considered in calculating shock wave propagation.

In a discussion of the shape of the pressure-volume (pv) curve in the vicinity of the critical point, J. Rowlinson¹ showed that at any temperature below but near the critical the difference between the critical density, ρ_c , and the vapor density, ρ_V , or the liquid density, ρ_L , is a linear function of the difference between the critical temperature, T_c , and the temperature T on logarithmic scales. This can be expressed mathematically as

$$(\rho_L - \rho_c) = a(T_c - T)^{k_1}$$

and

$$(\rho_c - \rho_V) = b(T_c - T)^{k_2}.$$

These expressions can be put in reduced coordinates by dividing by the appropriate critical property. The subscript R indicates this reduced property.

$$(\rho_{LR} - 1) = A(1 - T_R)^{\beta_L}$$

$$(1 - \rho_{VR}) = B(1 - T_R)^{\beta_V}.$$

These relations are valid only near the critical point

and are strongly similar to the "ideal-fluid" equations of E. Guggenheim,² who gave the values $\frac{1}{3}$ to β_L and β_V and found a good fit for the inert gases and several common fluids.

Recent studies by J. Essam and M. Fisher³ and M. Fisher⁴ using the lattice model of a fluid—which reproduces the two-phase behavior—has indicated that β is probably $\frac{5}{16}$. A. Michels, B. Blaisse and C. Michels⁵ found that their data for CO_2 could be fitted with greater accuracy if $\beta = 0.357$. M. Weinberger and W. Schneider,⁶ who took special precautions for the effects of gravity in their measurements on xenon, found $\beta = 0.345 \pm 0.015$ for $(1 - T_R)$ as low as 3×10^{-5} and $(\rho_{LR} - 1) = 0.08$.

The lattice gas model also indicates that

$$\rho_{LR} - \rho_{VR} = C(1 - T_R)^{\beta}$$

and for the pressure,

$$1 - p_R = D(1 - T_R)^{\delta}.$$

In order to resolve the discrepancy between theoretical and experimental values of the exponents, data for a wide variety of fluids were plotted and constants fitted to the linear portion. The fluids studied included a quantum fluid, typical organic and polar materials, metals and a fused salt. The results are shown in Table V-34-I. Blanks in this table indicate that data was not available or had not been correlated. A sample plot of experimental information for Hg is shown in Fig. V-34-1.

These preliminary results did not use least squares or statistical techniques for rejection of experimental information, but are presented to give progress in the

TABLE V-34-I. CONSTANTS FOR PROPERTIES NEAR THE CRITICAL POINT

Material	T_c , °K	ρ_c , g/cm ³	P_c , atm	Z_c	A	β_L	B	β_V	C	β	D	δ
Hg	1733	4.70	1587	0.477	2.15	0.410	1.42	0.323	—	—	3.91	1.005
Rb	2093	0.356	155.7	0.217	3.22	0.536	1.71	0.379	4.73	0.466	—	—
Cs	2043	0.430	134.6	0.248	3.42	0.541	1.25	0.257	4.41	0.414	—	—
BiCl ₃	1178	1.210	118.1	0.318	2.24	0.356	1.76	0.333	—	—	—	—
SO ₂	430.6	0.518	75.245	0.263	2.77	0.427	1.96	0.380	4.60	0.398	4.63	0.954
H ₂ O	647.3	0.3145	225.65	0.243	2.44	0.346	2.04	0.346	4.16	0.374	6.42	0.969
CO ₂	304.2	0.468	72.83	0.274	2.35	0.383	1.71	0.325	3.30	0.337	5.50	0.958
N ₂ O	309.594	0.4525	71.596	0.274	2.21	0.372	2.21	0.372	4.48	0.374	1.17	0.982
NH ₃	405.46	0.2362	111.5	0.242	2.42	0.373	1.80	0.320	4.00	0.336	4.58	0.917
Xe	289.75	1.105	57.9	0.289	1.90	0.358	2.15	0.374	3.67	0.354	—	—
He-4	5.1994	0.06948	2.260	0.305	0.637	0.414	0.637	0.414	—	—	—	—
Freon-12	384.7	0.555	39.6	0.274	2.30	0.380	2.80	0.445	—	—	—	—
Perfluoropropane	345.06	0.628	2.6115	0.276	2.58	0.421	1.45	0.262	3.88	0.341	3.98	0.852
Methylchloride	416.26	0.3630	65.919	0.268	2.39	0.389	1.52	0.292	3.73	0.334	5.54	0.995
Tertiary butyl alcohol	509.06	0.258	41.76	0.287	3.36	0.490	1.68	0.308	5.13	0.428	5.18	0.888
1,1-difluoro ethylene	302.86	0.414	45.51	0.283	2.19	0.363	1.52	0.288	—	—	5.05	0.955
Average					2.41	0.410	1.73	0.339	4.19	0.379	4.60	0.943

FIG. V-34-1. Reduced Property Plot for Mercury. *ANL Neg No. 112-5676*.

study. Except for water and helium, the exponents are generally different for vapor and liquid and the reduced liquid density function generally has a greater slope. The coefficients A and B are approximately equal and have values from about 1 to 3. The effect of small changes in the critical properties on these constants will be examined.

The exponents range from about $\frac{1}{4}$ to $\frac{1}{2}$ and are not ratios of integers. An important conclusion of this

study is that the density or volume cannot be expanded in a Taylor's series about the critical point since the exponents are not equal to one divided by an even integer.

Consideration of the exponents for the reduced density difference and the pressure function shows that the density difference does not have a constant exponent but that the value is approximately $\frac{1}{3}$. The exponent for the pressure is approximately 1.0 but usually slightly smaller.

In general the reduced liquid density function was linear for $T_R > 0.6$, the reduced vapor density function for $T_R > 0.7$, the reduced density difference for $T_R > 0.8$ and the reduced pressure function for $T_R > 0.9$.

REFERENCES

1. J. S. Rowlinson, *Liquids and Liquid Mixtures*, (Butterworth's Scientific Publications, Ltd., London, 1959) Ch. 5.
2. E. A. Guggenheim, *The Principle of Corresponding States*, *J. Chem. Phys.* **13**, 253 (1945).
3. J. W. Essam and M. E. Fisher, *Padé Approximant Studies of the Lattice Gas and Ising Ferromagnet Below the Critical Point*, *J. Chem. Phys.* **38**, 802 (1963).
4. M. E. Fisher, *Correlation Functions and the Critical Region of Simple Fluids*, *J. Math. Phys.* **5**, 944 (1964).
5. A. Michels, B. Blaisse and C. Michels, *The Isotherms of CO₂ in the Neighborhood of the Critical Point and Round the Coexistence Line*, *Proc. Roy. Soc. London* **A160**, 358 (1937).
6. M. A. Weinberger and W. E. Schneider, *On the Liquid-Vapor Coexistence Curve of Xenon in the Region of the Critical Temperature*, *Can. J. Chem.* **30**, 442 (1952).

V-35. Thermophysical Properties of Reactor Materials at High Temperatures

D. MILLER and R. D. JOHNSON*

The physical properties of reactor materials at high temperatures are an important link between the reactor kinetics equations and the hydrodynamic behavior in the analysis of a fast reactor accident. Determination of the energy appearing as kinetic energy provides insight into the potential damage which can occur during a proposed nuclear excursion.¹

Energy released in hypothetical nuclear accidents can lead to temperatures and pressures beyond those possible with static experiments and present technology. Resort must therefore be to theory and correlation for the estimation of these physical properties at extreme conditions.

Several analyses for the estimation of the properties of uranium^{2,3} have relied on the application of the well known law of corresponding states⁴ which has found useful application to substances whose thermodynamic critical states are not very far removed from room temperature.

This principle can be shown to be inapplicable by considering several fluids for which experimental pressure-density data exist to the critical point. O. Hougen, et al.⁴ showed that a third parameter, the critical compressibility Z_c , was necessary to correlate the properties for the ordinary chemical substances. The normal range of this variable is from 0.24 to 0.30. The critical properties measured for several substances—mercury, a heavy metal; bismuth chloride, a fused salt; acetonitrile, a polar organic material; and deuterium, a quantum fluid are given in Table V-35-I.

Empirical rules for estimating critical properties cannot be relied upon. One such rule states that the critical temperature is 1.5 times the boiling temperature. For mercury, bismuth chloride and cesium these temperature ratios are 2.75, 1.66 and 2.0 respectively. Similar rules for the volumetric behavior are equally unreliable.

Although the law of corresponding states appears inapplicable, it should provide a useful guide until more satisfactory estimation procedures can be tested.

As a temporary expedient to explore the temperature, pressure and density ranges of interest, the density-temperature relation for the saturated liquid-vapor region has been estimated up to the critical point by the "rectilinear law" method.⁵ This method assumes that one half the sum of the liquid and vapor densities is linear up to the critical point. This empirical law has

been substantiated for quantum fluids, such as hydrogen and helium; polar liquids such as ammonia and hydrogen fluoride; a fused salt, BiCl_3 ; and metals such as mercury⁵, cesium, rubidium and sodium⁶ and many ordinary fluids.

Using data for the density of the liquid, as a function of temperature along the saturation curve, and estimates of the vapor density using a vapor pressure curve and its extrapolation, the coexistence curves can be estimated. The technique is illustrated in Fig. V-35-1 for uranium. The estimated constants for various other reactor materials are given in Table V-35-II. Care must be taken in the use of these estimates since they require extrapolation of experimental data far beyond the region of measurement. This is particularly true for vapor pressures which are generally determined below one mm of Hg pressure.

An additional avenue being explored is the analysis of explosive shock measurements and fitting of the results to a Grüneisen equation of state.⁷ It appears that the linear relation between shock velocity and particle velocity obtained from the experiments can be estimated from low pressure static compressibility and sonic velocity measurements and from the Grüneisen coefficient evaluated by using thermal expansion and heat capacity data. This relation can be substituted in the Rankine-Hugoniot equations⁸ and an equation of state estimated for the shocked condition.

If the relation for the best fit between the shock velocity, U_s , and the shock particle velocity, U_p , is given by

$$U_s = C + SU_p \quad (1)$$

then, in the limit of zero particle velocity, the shock velocity must be the sonic velocity (and = C) which is available from sonic test data. Also, at zero pressure the Grüneisen ratio, γ , is given by

$$\gamma_0 = \frac{V\alpha}{\kappa C_v} = 2S - 1, \quad (2)$$

where

V = specific volume

α = volumetric thermal expansion coefficient

κ = isothermal compressibility

C_v = heat capacity at constant volume.

The quantity κ can also be derived from sonic velocity measurements and the others are generally available.

If the constants C and S in Eq. (1) can be obtained

* Student, Northwestern University, Evanston, Illinois.

TABLE V-35-I. MEASURED CRITICAL PROPERTIES

	T_c , °K	ρ_c , g/cm ³	P_c , atm	$Z_c = \frac{P_c}{RT_c} \frac{M}{\rho_c}$
Mercury	1733 ± 50	4.70 ± 0.20	1587 ± 50	0.477
BiCl ₃	1178 ± 5	1.210 ± 0.006	118.1	0.318
Acetonitrile	547.9	0.237	47.7	0.19
Deuterium	38.3	0.0151	16.3	1.37

then the pressure, p , after the shock is related to the density, ρ , by

$$p = \rho_0 C^2 \left(1 - \frac{\rho_0}{\rho}\right) / \left[1 - S \left(1 - \frac{\rho_0}{\rho}\right)\right]^2$$

and to the energy by

$$E = E_{0^\circ K} + \frac{C^2}{2} \left(1 - \rho_0/\rho\right)^2 / \left[1 - S(1 - \rho_0/\rho)\right]^2$$

$$= E_{0^\circ K} + \frac{p}{2\rho_0} \left(1 - \frac{\rho_0}{\rho}\right).$$

The temperature can be calculated using corrections for electronic heat capacity. This theoretical approach would enable an approximate equation of state for uranium or plutonium compounds of potential reactor interest to be obtained from simple, inexpensive and safe measurements.

TABLE V-35-II. ESTIMATED CRITICAL PROPERTIES OF REACTOR MATERIALS

Material	Critical Temperature, T_c °K	Critical Density, ρ_c g/cm ³	Critical Pressure, P_c atm	Critical Compressibility, $Z_c = \frac{P_c}{RT_c} \frac{M}{\rho_c}$
Uranium	12,500 ± 700	3.22 ± 0.36	6860 ± 1500	0.493 ± 0.15
Plutonium	10,000 ± 500	1.414 ± 0.380	1800 ± 350	0.371 ± 0.020
UO ₂	8970	1.507	1120	0.27 ^a
Iron	7000	1.22	3060	0.245
Sodium	2780	0.17	440	0.285
Beryllium	8600	0.435	14000	0.411

^a Assumed.

The effects of ionization and dissociation on the high temperature properties are being studied. As an upper limit for extrapolation purposes the equation of state at extremely high pressures and temperatures is being studied using Thomas Fermi-Dirac theory suitably corrected for correlation, exchange and finite temperature.

REFERENCES

1. D. Okrent, J. M. Cook, D. Satkus, R. B. Lazarus and M. W. Wells, *AX-I, A Computing Program for Coupled Neu-*

FIG. V-35-1. Density-Temperature Relation for Uranium Estimated by Rectilinear Law Method. ANL Neg. No. 112-5635.

tronics-Hydrodynamics Calculations on the IBM-704, ANL-5977 (1959).

2. R. H. Brout, *Equation of State and Heat Content of Uranium*, APDA-118 (February 1957).
3. H. Elrod, *Estimation of the Critical Temperature and Pressure of Uranium*, NDA 14-122 (February 1956).
4. O. A. Hougen, K. M. Watson and R. A. Ragatz, *Chemical Process Principles*, (Thermodynamics, John Wiley, New York 1959) 2nd Ed., Vol. 2.

5. A. V. Grosse, *The Temperature Range of Liquid Metals and an Estimate of their Critical Constants*, J. Inorganic and Nuclear Chemistry **22**, 23 (1961).
6. I. Dillon, *Critical Temperatures and Densities of Alkali Metals*, ANL-7025 (1965).
7. G. E. Duvall and G. R. Fowles, *Shock Waves*, High Pressure Physics and Chemistry, Ed. by R. S. Bradley (Academic Press, New York 1963) Vol. II, p. 209.
8. M. H. Rice, R. G. McQueen and J. M. Walsh, *Compression of Solids by Strong Shock Waves*, Solid State Physics, Ed. by F. Seitz and D. Turnbull, (Academic Press, New York 1958) Vol. VI.

Section VI

Publications

(April 1, 1964 to June 30, 1965)

Open Literature

- Armani, R. J., Bennett, E. F., Brenner, M. W., Bretscher, M. M., Cohn, C. E., Huber, R. J., Kaufmann, S. G. and Redman, W. C., *Improved Techniques for Low Flux Measurement of Prompt Neutron Lifetime, Conversion Ratio and Fast Spectra*, Proc. IAEA Symp. on Exponential and Critical Experiments, Amsterdam, September 2-6, 1963. Intern. Atomic Energy Agency, Vienna, 1964. Vol. 1, pp. 227-259.
- Chernick, J., Auerback, T. and Avery, R., *Reactor Physics and Performance*, Nucl. News **7**, 20-23 (October 1964).
- Cohn, C. E., *Applicability of Simple Reactor Kinetics to the Interpretation of Reactor Noise Experiments*, Trans. Symp. on Noise Analysis in Nuclear Systems, University of Florida, Gainesville, November 4-6, 1963. J. Nucl. Energy: Reactor Sci. Technol. **18**, 31-32 (1964).
- Cohn, C. E., *Errors in Noise Measurements Due to the Finite Amplitude Range of the Measuring Instrument*, Rev. Sci. Instr. **35**, 701-703 (June 1964).
- Cohn, C. E., *Note on the Simulation of Higher-Order Linear Systems with Single Operational Amplifiers*, Proc. IEEE **52**, 874 (July 1964). Letter
- Cohn, C. E., *Linear Detectors for Noise Measurements*, Rev. Sci. Instr. **35**, 638-639 (May 1964). Note
- DeShong, J. A., Jr., Daniels, R. E., Hildebrand, R. H. and Meyer, P., *Spark Chamber and Magnet System for Photographing Cosmic Ray Tracks at Balloon Altitudes*, Rev. Sci. Instr. **35**, 1035-1039 (August 1964).
- DeVilliers, J. A., Engelbrecht, C. A., Vonach, W. G. and Smith, A. B., *Neutron Scattering from Au, Hg and Tl*, Z. Physik **183**(4), 323-340 (1965).
- Dickerman, C. E. and Stephany, W. P., *Correlation of In-Pile Pressure Measurements*, Nucl. Struct. Eng. J. **1**(3), 342 (1965).
- Gast, P. F., *Moderator Scattering and Flux Depression Near Resonances*, Nucl. Sci. Eng. **21**, 191 (January 1965). Letter
- Gold, R., *Limitations in the Orthonormal Expansion of Neutron Spectra with Activation Measurements*, Nucl. Sci. Eng. **20**, 493-512 (December 1964).
- Groh, E. F. and Cohn, C. E., *A Simple Compact Rod Drive Using a Stepping Motor*, Nucl. Sci. Eng. **20**, 290-297 (November 1964).
- Hwang, R. N., *Doppler Effect Calculations with Interference Corrections*, Nucl. Sci. Eng. **21**, 523-525 (April 1965).
- Kelber, C. N., *A Simple Estimate of the Effects of Resonance Interference*, Nucl. Sci. Eng. **22**, 120-121 (May 1965). Letter
- Kelber, C. N., *Improved Rational Escape Probability in Lumped Absorbers*, Nucl. Sci. Eng. **22**, 244-247 (June 1965).
- pinski, W. C., Harrer, J. M. and Ramp, R. L., *Control and Instrumentation*, Reactor Handbook. Vol. IV, Engineering, ed., Stuart McLain and J. H. Martens. Interscience Publishers, New York, 1964. Ch. 8, pp. 346-400.
- Loewenstein, W. B., *The Control of Fast Reactors: Current Methods and Future Prospects*, Proc. IAEA Symp. on Physics and Material Problems of Reactor Control Rods, Vienna, November 11-15, 1963. Intern. Atomic Energy Agency, Vienna, 1964. pp. 633-665.
- McCarthy, A. E. and Persiani, P. J., *Neutron Resonance Integral and Age Data*, Reactor Physics Constants Center Newsletter No. 11.
- McVean, R. L., Davey, W. G. and Fischer, G. J., *The Role of the Fast Critical Facility in the Power Reactor Program*, Proc. IAEA Symp. on Exponential and Critical Experiments, Amsterdam, September 2-6, 1963. Intern. Atom Energy Agency, Vienna, 1964. Vol. 1, pp. 85-124.
- Meadows, J. W., Whalen, J. F. and Smith, A. B., *Extensive Experience with an On-Line Computer in a Nuclear Laboratory*, Proc. European-American Nuclear Data Committee Conf. on the Automatic Acquisition and Reduction of Nuclear Data, Karlsruhe, Germany, July 13-16, 1964, ed. K. H. Beckurts, W. Glaser, and G. Kruger. Gesellschaft fur Kernforschung, Karlsruhe, 1964, pp. 132-142.
- Miller, D., *Maximum Flow Rate of a Single Component, Two-Phase Mixture (Discussion of Paper by F. J. Moody)*, J. Heat Transfer **87**, 141 (February 1965).
- Miller, W. F., Meadows, J. W., Whalen, J. F. and Smith, A. B., *On-Line Data Acquisition and Feedback to Accelerator Control: A Status Report*, Proc. European-American Nuclear Data Committee Conf. on the Automatic Acquisition and Reduction of Nuclear Data, Karlsruhe, Germany, July 13-16, 1964, ed. K. H. Beckurts, W. Glaser, and G. Kruger. Gesellschaft fur Kernforschung, Karlsruhe, 1964, pp. 222-232.
- Moldauer, P. A., *Statistical Theory of Nuclear Collision Cross Sections*, Phys. Rev. **135**, B642-B659 (August 10, 1964).
- Moldauer, P. A., *Statistical Theory of Nuclear Collision Cross Sections. II. Distributions of the Poles and Residues of the Collision Matrix*, Phys. Rev. **136**, B947-B952 (November 23, 1964).
- Moldauer, P. A., *Average Compound Nucleus Cross Sections*, Rev. Mod. Phys. **36**, 1079-1084 (October 1964).
- Moldauer, P. A., *Theory of Average Cross Sections and Fluctuations*, Comptes Rendus du Congres International de Physique Nucleaire, Paris, July 2-8, 1964, ed. P. Gugenberger. Centre National de la Recherche Scientifique, Paris, 1964. Vol. II, pp. 681-682.
- Moldauer, P. A., *On the Surface Absorption of Low Energy Nucleons by Nuclei*, Comptes Rendus du Congres International de Physique Nucleaire, Paris, July 2-8, 1964, ed. P. Gugenberger. Centre National de la Recherche Scientifique, Paris, 1964. Vol. II, pp. 851-852.
- Pennington, E. M., *Collision Probabilities in Cylindrical Lattices*, Nucl. Sci. Eng. **19**, 215-220 (June 1964).
- Porges, K. G. and Bjorkland, J. A., *Gain Stabilization of Ana-*

- logue Amplifiers in the Nanosecond Regime, Proc. Intern. Symp. on Nuclear Electronics, Paris, November 25-27, 1963. European Nuclear Energy Agency, 1964. pp. 811-818.
- Porges, K. G., Devolpi, A. and Larsen, R. N., *Electronic Design of an Absolute Counting System for Mn-56*, Nucl. Instr. and Methods **29**(1), 157-168 (September 1964).
- Porges, K. G., Devolpi, A. and Polinski, P. W., *Fast Cancellation of Gamma Radiation Background Pulses in Hornyak Buttons with an Active Circuit*, Rev. Sci. Instr. **35**, 1602 (November 1964). Note
- Redman, W. C., Plumlee, K. E. and Baird, Q. L., *Power Reactor Design at Zero Power*, Proc. IAEA Symp. on Exponential and Critical Experiments, Amsterdam, September 2-6, 1963. Intern. Atomic Energy Agency, Vienna, 1964. Vol. 1, pp. 3-27.
- Robinson, L. E., Dickerman, C. E., Carlander, R., August, C. and Mueller, C. J., *Behavior of Uranium Sulfide Fast-Reactor-Type Fuel Specimens under Transient Heating in TREAT*, Nuclear Appl. **1**(2), 168-175 (April 1965).
- Simmons, W. R., Persiani, P. J. and Bump, T. R., *Fast Reactor Test Facility (FARET) Initial Experimental Program*, Proc. American Nuclear Society National Topical Meeting on Fast Reactors, Detroit, April 26-28, 1965. ANS-100, pp. 273-274.
- Smith, A. B., Engelbrecht, C. A. and Reitmann, D., *Elastic and Inelastic Scattering of Fast Neutrons from Co, Cu and Zn*, Phys. Rev. **135**, B76-B82 (July 13, 1964).
- Smith, A. B. and Vonach, W. G., *A Compendium of Fast Neutron Scattering Studies*, Comptes Rendus du Congres International de Physique Nucleaire, Paris, July 2-8, 1964, ed. P. Eugenberger. Centre National de la Recherche Scientifique, Paris, 1964. Vol. II, pp. 772-774.
- Spinrad, B. I., *Desegregating the Atom*, Nucl. News **8**, 3 (June 1965).
- Spinrad, B. I., *Ramifications of Nuclear Energy*, Bull. Atom. Sci. **21**, 21-24 (May 1965).
- Stanford, G. S., *An Introduction to Multichannel Analyzers*, Nucl. Instr. Methods **34**, 1-9 (1965).
- Thacher, H. C. Jr., *Certification of Algorithms 191 and 192—Hypergeometric and Confluent Hypergeometric* (A. P. Relph, Commun. Assoc. Computing Mach., July 1963) Commun. Assoc. Computing Mach. **7**, 244 (April 1964).
- Thacher, H. C. Jr., *Remark on Algorithm 60—Romberg Integration* (F. L. Bauer, Commun. ACM **4**, 255, June 1961), Commun. Assoc. Computing Mach. **7**, 420-421 (July 1964).
- Thacher, H. C. Jr., *Certification of Algorithm 128—Summation of Fourier Series*, (M. Wells, Commun. ACM **5**, October 1962), Commun. Assoc. Computing Mach. **7**, 421 (July 1964).
- Thacher, H. C. Jr., *Algorithm 181. Complementary Error Function. Large X*, Commun. Assoc. Computing Mach. **7**, 702 (December 1964).
- Vonach, W. G., Smith, A. B. and Moldauer, P. A., *Shell Model Effects on Neutron Optical Model Absorption*, Phys. Letters **11**, 4 (1964).
- Walter, C. M. and Dickerman, C. E., *TREAT Study of the Penetration of Mollen Uranium and U/5 Wt. % Fs Alloy through Type 304 Stainless Steel*, Nucl. Sci. Eng. **18**, 518-524 (April 1964).
- Whalen, J. F., *A Computer Controlled Display System for Nuclear Experiments*, Proc. European-American Nuclear Data Committee Conf. on the Automatic Acquisition and Reduction of Nuclear Data, Karlsruhe, Germany, July 13-16, 1964, ed. K. H. Beckurts, W. Glaser and G. Kruger. Gesellschaft fur Kernforschung, Karlsruhe, 1964, pp. 399-408.
- Whalen, J. F., Meadows, J. W. and Larsen, R. N., *On-Line Operation of a Digital Computer in Nuclear Physics Experiments*, Rev. Sci. Instr. **35**, 682-690 (June 1964).

Reports

- Bennett, E. F., *A Study of the 1/E Slowing-Down Neutron Spectrum Using 4 π -Recoil Proportional Counters*, ANL-6897 (September 1964).
- Carter, J. C., Sparks, D. W. and Tessier, J. H., *The Steady State and Transient Kinetics of EBR-I, Mark-III*, ANL-6851 (March 1964).
- Carter, J. C., Sparks, D. W. and Tessier, J. H., *The Period Effect in Reactor Dynamics*, ANL-6852 (April 1964).
- Carter, J. C., Sparks, D. W. and Tessier, J. H., *Period Effects in Reactor Dynamics*, Reactor Kinetics and Control. Proc. Symp. held at the University of Arizona, March 25-27, 1963, ed. L. E. Weaver. Atomic Energy Commission Symp. Ser. No. 2, Oak Ridge, TID-7662 (April 1964) pp. 53-90.
- Carter, J. C., Sparks, D. W. and Tessier, J. H., *The Steady State and Transient Kinetics of EBR-I, Mark-III*, Reactor Kinetics Control. Proc. Symp. held at the University of Arizona, March 25-27, 1963, ed. L. E. Weaver. Atomic Energy Commission Symp. Ser. No. 2, Oak Ridge, TID-7662 (April 1964) pp. 405-418.
- Cohn, C. E., *Applicability of Simple Reactor Kinetics to the Interpretation of Reactor Noise Experiments*, Noise Analysis in Nuclear Systems. Proc. Symp. on Noise Analysis in Nuclear Systems, University of Florida, November 4-6, 1963. TID-7679 (June 1964) pp. 307-320.
- Dickerman, C. E., *Kinetics of the Transient Reactor Test Facility*, Reactor Kinetics and Control. Proc. Symp. held at the University of Arizona, March 25-27, 1963, ed. L. E. Weaver. Atomic Energy Commission Symp. Ser. No. 2, Oak Ridge, TID-7662 (April 1964) pp. 419-433.
- Dickerman, C. E., Robinson, L. E., Sowa, E. S. and Skladzien, S. B., *Studies of Fast Reactor Fuel Element Behavior under Transient Heating to Failure. II. In-Pile Experiments on UO₂ Samples in the Absence of Coolant*, ANL-6845 (January 1965).
- Gold, R., *An Iterative Unfolding Method for Response Matrices*, ANL-6984 (December 1964).
- Gopinath, D. V. and Dickerman, C. E., *Calculations of Coherence of Failure for Hypothetical Meltdown Accidents in an EBR-II-Like Reactor*, ANL-6844 (March 1964).
- Iskenderian, H. P., *Use of Depleted Uranium in Thermal Reactors with Slightly Enriched Fuel to Achieve High Neutron Economy and High Burnup*, ANL-6843 (February 1964).
- Kato, W. Y., Fischer, G. J. and Dates, L. R., *Safety Analysis Report, Argonne Fast Critical Facility (ZPR-VI)*, ANL-6271 (December 1963).
- Kirn, F. S. and Loewenstein, W. B., *EBR-II Wet Critical Experiments*, ANL-6864 (October 1964).
- MacFarlane, D. R. and Madell, J. T., *Preliminary Physics and Heat Transfer Design Calculations for a Rocket Fuel Test Reactor (RFTR)*, ANL-6828 (August 1964).
- Marchertas, A. H., Fistedis, S. H. and Brittan, R. O., *Analytical and Experimental Investigation of a Nuclear Reactor Support Structure*, ANL-6781 (March 1965).
- Meneghetti, D., *Introductory Fast Reactor Physics Analysis*, ANL-6809 (December 1963).
- Moldauer, P. A., Engelbrecht, C. A. and Duffy, G. J., *NEARREX, a Computer Code for Nuclear Reaction Calculations*, ANL-6978 (December 1964).
- Moon, D. P., *Consistent Heavy-Atom Cross-Section Evaluation*

- for Plutonium Recycle Calculations, ANL-6972 (December 1964).
- Pennington, E. M., *Cylindrical Lattice Collision Probability Codes, B692/RP*, ANL-6836 (February 1964).
- Pennington, E. M. and Boettinger, W. L., *Foil Activation Programs, B512/RP*, ANL-6822 (January 1964).
- Persiani, P. J., Watanabe, A., Wolff, U., Grifoni, S. A. and Warman, R. A., *Some Considerations on the Meltdown Problem for FARET*, ANL-6935 (September 1964).
- Plumlee, K. E., *Flux Trap Experiments in D₂O-Moderated Thoria-Urania Cores*, ANL-6915 (October 1964).
- Rago, A. L. and Hummel, H. H., *ELMOE, An IBM-704 Program Treating Elastic Scattering Resonances in Fast Reactors*, ANL-6805 (January 1964).
- Reactor Physics Division Annual Report, July 1, 1963 to June 30, 1964, ANL-7010.
- Richards, H. K., *Thermionic Energy Conversion for DC and High Frequency Power Output*, AMU-ANL Conference on Direct Energy Conversion, November 4-5, 1963. ANL-6802 (December 1963) pp. 152-160.
- Sanathanan, C. K., *Dynamic Analysis of Coolant Circulation in Boiling Water Nuclear Reactors*, ANL-6847 (April 1964).
- Schoeberle, D. F., Heestand, J. M. and Miller, L. B., *A Method of Calculating Transient Temperatures in a Multiregion, Axisymmetric, Cylindrical Configuration. The Argus Program, 1089/RE248, Written in Fortran II*, ANL-6654 (November 1963).
- Smith, A. B., ed., *Reports to the AEC Nuclear Cross Sections Advisory Group Meeting at E. I. du Pont de Nemours and Co., Savannah River Laboratory, January 28-30, 1964. WASH-1046.*
- Spinrad, B. I., *Why Study Direct Conversion?* AMU-ANL Conference on Direct Energy Conversion, November 4-5, 1963. ANL-6802 (December 1963) pp. 3-14.
- Spinrad, B. I., *Symposium Introduction*, A Symposium on High Temperature Conversion—Heat to Electricity, Tucson, Arizona, February 19-21, 1964. TID-7687, pp. 1-9.
- Stanford, G. S., *An Introduction to Multichannel Analyzers*, ANL-6881 (April 1964).
- Abstracts*
- Almenas, K. K., *Study of Reflector-Based Control of Fast Nuclear Rocket Reactors*, Trans. Am. Nucl. Soc. **8**(1), 165-166 (June 1965).
- Almenas, K. K., Knapp, W. G. and Doerner, R. C., *Properties of Aluminum Reflectors of Fast Critical Assemblies*, Trans. Am. Nucl. Soc. **7**(2), 304 (November 1964).
- Avery, R., Greebler, P., Hummel, H., Hwang, R. N., Meneghetti, D., Moldauer, P. A., Nims, J. B. and Smith, A. B., *Physics of Fast Reactors*, Atoms for Peace Conference, 1964. Third United Nations Intern. Conf. on the Peaceful Uses of Atomic Energy, Geneva, August 31-September 9, 1964. Report by the U. S. Delegation, U. S. Atomic Energy Commission, Washington, D. C., pp. 107-108.
- Baird, Q. L., Boynton, A. R., Robinson, W. R. and Christenson, J. M., *Initial Conversion Ratio and Epicadmium to Subcadmium Uranium-238 Capture Rate Measurements in Very Undermoderated Slightly Enriched Oxide Cores*, Trans. Am. Nucl. Soc. **7**(1), 80 (June 1964).
- annett, E. F., *Fast Neutron Spectroscopy by 4π Recoil Proportional Counting*, Trans. Am. Nucl. Soc. **7**(2), 369 (November 1964).
- Boynton, A. R., *Experimental Thermal-Disadvantage Factors in Some Very Undermoderated 3%-Enriched, Water-Uranium Oxide Lattices*, Trans. Am. Nucl. Soc. **8**(1), 249 (June 1965).
- Buccino, S. G., Gemmell, D. S., Lee, L. L., Schiffer, J. P. and Smith, A. B., *Si-28 (d,n)P-29 and Mg-24 (d,n)Al-25 Reactions*, Bull. Am. Phys. Soc. **10**, 511 (April 1965).
- Buccino, S. G. and Smith, A. B., *Be-9 (d,n)B-10 Reaction*, Bull. Am. Phys. Soc. **9**, 628 (October 1964).
- Butler, D. K., Hummel, H. H., Loewenstein, W. B. and Meneghetti, D., *Use of Beryllium Zones to Enhance Safety of Fast Breeder Reactors*, Trans. Am. Nucl. Soc. **8**(1), 239-240 (June 1965).
- Chien, Ji-Peng and Smith, A. B., *Fast-Neutron Cross Sections of Na and Al*, Bull. Am. Phys. Soc. **10**, 567 (June 1965).
- Cox, S. A., *Elastic Neutron Scattering from Intermediate-Weight Elements*, Bull. Am. Phys. Soc. **10**, 576 (June 1965).
- Dickerman, C. E. and Robinson, L. E., *Photographic Experiments on Meltdown of Irradiated Metallic Uranium Fuel Pins*, Trans. Am. Nucl. Soc. **7**(1), 137 (June 1964).
- Dickerman, C. E., Robinson, L. E. and Stewart, R. R., *Fast-Reactor Safety Experiments on EBR-II Type Thorium-Uranium Fuel*, Trans. Am. Nucl. Soc. **8**(1), 309 (June 1965).
- Doerner, R. C., *Precision Reactivity Measurement Facility at Argonne National Laboratory*, Trans. Am. Nucl. Soc. **8**(1), 261-262 (June 1965).
- Doerner, R. C., Knapp, W. G. and Almenas, K. K., *Physics Measurements in Tungsten-Based Aluminum Reflected Fast Reactors*, Trans. Am. Nucl. Soc. **7**(2), 236 (November 1964).
- Fischer, G. J., Meneley, D. A. and Groh, E. F., *Doppler-Effect Measurements in Plutonium-Fueled Fast Assemblies*, Trans. Am. Nucl. Soc. **7**(2), 252 (November 1964).
- Gast, P. F., *Evaluation of Breeders from a Conservation Standpoint*, Trans. Am. Nucl. Soc. **7**(2), 530 (November 1964).
- Gemmell, D. S., Lee, L. L., Jr., Schiffer, J. P. and Smith, A. B., *Study of (d,n) Reactions on Fe-54 and Ni-58*, Bull. Am. Phys. Soc. **9**, 676 (October 1964).
- Gold, R., *Calculation of Spectral Distortion Due to Pile-Up Effect*, Rev. Sci. Instr. **36**, 784-794 (June 1965).
- Gopinath, D. V., Dickerman, C. E. and Bryant, L. T., *Calculations on Coherence of Failure in Hypothetical Meltdown Accidents in an EBR-II-Like Reactor*, Trans. Am. Nucl. Soc. **7**(1), 138 (June 1964).
- Grifoni, S. A., Nozawa, M. and Kato, W. Y., *Sodium-Void Coefficient Measurements in a Large Carbide Fast Assembly*, Trans. Am. Nucl. Soc. **7**(2), 250 (November 1964).
- Helm, F. H., Kato, W. Y., Main, G. W., Meister, H. H. and Rusch, G. K., *Critical Facility Measurements of the Sodium-Void Coefficient in a Large Disc-Shaped Fast-Reactor Core*, Trans. Am. Nucl. Soc. **8**(1), 245-246 (June 1965).
- Iskenderian, H. P., *Use of Depleted Uranium in Thermal Reactors with Slightly Enriched Fuel to Achieve High Neutron Economy and Burnup*, Trans. Am. Nucl. Soc. **7**(1), 197-198 (June 1964).
- Karam, R. A., *Measurement of β_{eff} and Power Calibration in Fast Reactors*, Trans. Am. Nucl. Soc. **7**(2), 248 (November 1964).
- Karam, R. A., *Measurement of Rossi-Alpha in Reflected Reactors*, Trans. Am. Nucl. Soc. **7**(2), 283 (November 1964).
- Karam, R. A., *Spatial Dependence of the Decay Rates of Prompt-Neutron Chains in Reflected Reactors*, Trans. Am. Nucl. Soc. **8**(1), 224-225 (June 1965).
- Kier, P. H., *A Method of Calculating Mutual Shielding Between Resonances of Different Nuclides*, Trans. Am. Nucl. Soc. **8**(1), 286-287 (June 1965).

- Kirn, F. S. and Loewenstein, W. B., *EBR-II Wet Critical Experimental Results*, Trans. Am. Nucl. Soc. 7(1), 172 (June 1964).
- Knapp, W. G., Doerner, R. C., Almenas, K. K., Cohn, C. E. and Karam, R. A., *Physics Measurements of Modified Tungsten-Based Aluminum-Reflected Fast Reactors*, Trans. Am. Nucl. Soc. 8(1), 241 (June 1965).
- Link, L. E., Fischer, G. J. and Zebroski, E. L., *Fuel Cycle Economics of Fast Reactors*, Atoms for Peace Conference, 1964. Third United Nations Intern. Conf. on the Peaceful Uses of Atomic Energy, Geneva, August 31–September 9, 1964. Report by the U. S. Delegation, U. S. Atomic Energy Commission, Washington, D. C., pp. 95–96.
- Loewenstein, W. B., *Fast-Reactor Reactivity Coefficients; The Effect of Excess Reactivity*, Trans. Am. Nucl. Soc. 7(2), 249 (November 1964).
- Loewenstein, W. B., Almenas, K. K., Doerner, R. C. and Knapp, W. G., *Recent Developments in the Physics Design of Some Nuclear Rockets*, AIAA Bull. 2, 249 (May 1965).
- MacFarlane, D. R. and Madell, J. T., *Investigation of a Rocket Fuel Test Reactor (RFTTR)*, Trans. Am. Nucl. Soc. 7(1), 68–69 (June 1964).
- Main, G. W., Helm, F. H. and Meister, H. H., *Effective Fission-Ratio Measurements in a Series of Fast Reactors*, Trans. Am. Nucl. Soc. 8(1), 242–243 (June 1965).
- McCarthy, W. J., Dickerman, C. E., Klickman, A. E. and Nicholson, R. B., *Fast Reactor Safety Research*, Atoms for Peace Conference, 1964. Third United Nations Intern. Conf. on the Peaceful Uses of Atomic Energy, Geneva, August 31–September 9, 1964. Report by the U. S. Delegation, U. S. Atomic Energy Commission, Washington, D. C., p. 127–128.
- Meadows, J. W., Whalen, J. F. and Smith, A. B., *Thick Target Neutron Yields for 6-BeV Protons*, Bull. Am. Phys. Soc. 9, 460 (April 1964).
- Meneghetti, D., *Evaluation of Group Sections for Light Reflectors of Fast Reactors*, Trans. Am. Nucl. Soc. 8(1), 247–248 (June 1965).
- Meneghetti, D. and Ishikawa, H., *Dimensions of Plutonium Cores by Extrapolation of Perturbed Uranium Cores*, Trans. Am. Nucl. Soc. 7(1), 37 (June 1964).
- Meneghetti, D. and White, J. R., *Calculations of ZPR-III Fast Assemblies Using a Twenty-Six Group Cross Section Set*, Trans. Am. Nucl. Soc. 7(2), 237 (November 1964).
- Moldauer, P. A., *Statistical Theory of Nuclear Cross Sections*, Bull. Am. Phys. Soc. 9, 505 (April 1964).
- Nelson, P. A., Butler, D. K., Chasanov, M. G. and Meneghetti, D., *Fuel Properties and Nuclear Performance of Fast Reactors Fueled with Molten Chlorides*, Trans. Am. Nucl. Soc. 8(1), 153–154 (June 1965).
- Okrent, D., Cohen, K. B. and Loewenstein, W. B., *Some Nuclear and Safety Considerations in the Design of Large Fast Power Reactors*, Atoms for Peace Conference, 1964. Third United Nations Intern. Conf. on the Peaceful Uses of Atomic Energy, Geneva, August 31–September 9, 1964. Report by the U. S. Delegation, U. S. Atomic Energy Commission, Washington, D. C., p. 112.
- O'Shea, D. M., Hummel, H. H., Loewenstein, W. B. and Okrent, D., *Twenty-Six Group Cross Sections*, Trans. Am. Nucl. Soc. 7(2), 242 (November 1964).
- Pennington, E. M., *Resonance Integrals in Slab Lattices Using Various Escape Probabilities*, Trans. Am. Nucl. Soc. 8(1), 288–289 (June 1965).
- Pennington, E. M. and Plumlee, K. E., *Internal Thermal-Column Experiments with ThO_2/UO_2 -Fueled, D_2O -Moderated and -Reflected Cores*, Trans. Am. Nucl. Soc. 7(1), 74 (June 1964).
- Persiani, P. J., Watanabe, A., Wolff, U., Grifoni, S. A. and Warman, R. A., *Preliminary Considerations on the Safety Analysis of FARET*, Trans. Am. Nucl. Soc. 7(2), 501 (November 1964).
- Plumlee, K. E., *Theory and Methods of Correction for Resolution Losses of Counting Equipment*, Trans. Am. Nucl. Soc. 7(2), 341 (November 1964).
- Robinson, L. E., Dickerman, C. E. and August, C., *Behavior of Uranium Sulfide Fast-Reactor-Type Fuel Specimens under Transient Heating in TREAT*, Trans. Am. Nucl. Soc. 7(1), 105–106 (June 1964).
- Rusch, G. K., Bohme, E. R., Dates, L. R., Grifoni, S. A., Kato, W. Y., Main, G. W., Meister, H. H., Nozawa, M. and Stover, R. L., *Physics Measurements on a 600-Liter Fast Neutron Carbide Core*, Trans. Am. Nucl. Soc. 7(2), 236 (November 1964).
- Rusch, G. K. and Helm, F. H., *The Influence of Heterogeneity on the Measured Sodium-Void Coefficient in a 950-Liter Pancake Fast Core*, Trans. Am. Nucl. Soc. 8(1), 246–247 (June 1965).
- Sanathanan, C. K., *Further Development in Dynamic Analysis of Coolant Circulation in Boiling Water Nuclear Reactors*, Trans. Am. Nucl. Soc. 7(2), 509 (November 1964).
- Sanathanan, C. K., Carter, J. C. and Miraldi, F., *Dynamic Analysis of Coolant Circulation in Boiling-Water Nuclear Reactors*, Trans. Am. Nucl. Soc. 7(1), 150 (June 1964).
- Smith, A. B. and Guenther, P. T., *Some Elastic Neutron Angular Distributions*, Trans. Am. Nucl. Soc. 7(2), 267 (November 1964).
- Spinrad, B. I., *Speculations on Future Ramifications of Nuclear Energy*, Trans. Am. Nucl. Soc. 7(2), 527 (November 1964).
- Stanford, G. S. and Plumlee, K. E., *Linear Relationship Between Cadmium Ratio and Reactor Core Composition*, Trans. Am. Nucl. Soc. 8(1), 270–271 (June 1965).
- Thalgott, F. W., Best, G. H., Carpenter, S. G., Davey, W. G., Kato, W. Y., Long, J. K. and Morewitz, H. A., *Fast Critical Experiments and Their Analysis*, Atoms for Peace Conference, 1964. Third United Nations Intern. Conf. on the Peaceful Uses of Atomic Energy, Geneva, August 31–September 9, 1964. Report by the U. S. Delegation, U. S. Atomic Energy Commission, Washington, D. C., pp. 110–111.
- Till, C. E., *Doppler Broadening of Resonances by Sample Motion*, Trans. Am. Nucl. Soc. 7(2), 273 (November 1964).
- Ulrich, A. J., *Evaluation of a Thermionic Diode for Space Power Using a Liquid Metal Electron Collector*, Trans. Am. Nucl. Soc. 7(2), 377 (November 1964).
- Vonach, W. G., Engelbrecht, C. A. and Smith, A. B., *Elastic and Inelastic Scattering of Fast Neutrons from Tl, Hg and Au*, Bull. Am. Phys. Soc. 9, 461 (April 1964).
- Vonach, W. G. and Smith, A. B., *Scattering of Fast Neutrons from Ag, In and Cd*, Bull. Am. Phys. Soc. 10, 498 (April 1965).
- Whalen, J. F., Rogue, R. R. and Smith, A. B., *Fast-Neutron Total Cross Sections with an Automated Facility*, Bull. Am. Phys. Soc. 9, 651 (October 1964).