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Comments on a Dual Multiparticle Theory
with Nonlinear Trajectories*

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Abstract

Some features of the nonlinear trajectories of a dual multiparticle theory are examined and the development of the theory is discussed.

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Within the past year and a half a dual multiparticle theory with nonlinear trajectories has been developed¹⁻⁷ in a manner which closely parallels the development of the generalized Veneziano model⁸⁻¹². The theory includes the Veneziano model as a limiting case. The following work has been done since the four-point function was originally proposed²: (1) explicit construction of N-point tree graphs¹, (2) verification that Veneziano tree graphs are obtained in the limit of linear trajectories,^{1,5} (3) investigation of conjectured rules for loop diagrams⁴ analogous to the rules of Kikkawa, Sakita and Virasoro⁹, (4) factorization⁶ of the tree graphs analogous to the work of Fubini and Veneziano¹⁰, (5) reformulation of the tree graphs in terms of an operator formalism⁷ analogous to that of Fubini, Gordon and Veneziano,¹¹ (6) discovery of Ward-like identities,⁶ and (7) application² of the Adler self-consistency condition¹³ analogous to the applications of Lovelace¹⁴ and Ademollo, Veneziano and Weinberg¹⁵. Three of these advances (4-6) are quite new.

In this note we will first answer some questions which have been raised concerning the four-point Born term, B_4 . We will then review the rules for constructing the N-point Born term B_N , emphasizing the simple, symmetric way in which duality is incorporated and, finally, comment on the present state of the theory.

The model arose in an attempt to find the most general meromorphic, dual Born term and to thereby avoid specialization to the Veneziano model. We have not proved that this is the most general possible model. However, no more general meromorphic, dual Born terms have yet been proposed.¹⁶ The four-point function of the model is given by

$$B_4(s, t) = G(\sigma, \tau) / G(\sigma) G(\tau) \quad (1)$$

where

$$G(\sigma) = \prod_{\ell=0}^{\infty} (1 - \sigma q^{\ell}), \quad (2)$$

q is a parameter, $0 < q < 1$, $\sigma = as + b$ and $\tau = at + b$. The poles of B_4 occur at $\sigma, \tau = q^{-j}$, $j = 0, 1, 2, \dots$ with polynomial residues of order j and the trajectory function is

$$\alpha(t) = -\frac{\ln \tau}{\ln q}. \quad (3)$$

As $|s| \rightarrow \infty$ for fixed t , B_4 has the Regge behavior $B_4(s, t) \sim (-as)^{\alpha(t)}$.

If the coefficients a and b are chosen so that the q dependence of τ near $q = 1$ is of the form

$$\tau = 1 + (1-q) \tau'(t) + (1-q)^2 \tau''(t) + \dots$$

then

$$\lim_{q \rightarrow 1} \alpha(t) = \tau' = \alpha_{\text{Veneziano}}. \quad (4)$$

Thus, we can have trajectories which are linear or experimentally indistinguishable from linear trajectories.

We now consider two questions which have been raised concerning the logarithmic trajectory functions:

(a) Trajectories of the form (3) have a left hand cut. In general, trajectories are not complex below threshold¹⁷. The exception to the rule occurs when two or more trajectories intersect. This is in fact what happens in our model. If we denote the integer-spaced daughter¹⁸ trajectory functions of the model by $\alpha_n(t) = \alpha(t) - n$, we see that $1/\alpha_n = 0$ for all n at $\tau = at + b = 0$ and $t = \infty$. These are just the branch points of α_n . Thus, the branch points occur when the leading trajectory and all of the daughters collide at the point at

infinity ($1/j = 0$) in the j -plane. In the linear limit, Eq. (4), the branch point at $t = -b/a$ moves off towards $t = -\infty$ where it annihilates the other branch point, leaving a linear trajectory. Of course the analyticity properties of α had to be physically acceptable since the resultant B_4 does possess the necessary analytic properties to be a physically allowable Born term.

(b) The real part of $\alpha(t)$ rises as $t \rightarrow -\infty$. Since as $|s| \rightarrow \infty$, B_4 is Regge behaved for any finite t , this rise of α on the left leads to a unitarity violation for t sufficiently large and negative. Thus, as $|s| \rightarrow \infty$ for fixed $z = \cos \theta_s$, we have^{1,2}

$$|B_4(s, t)| \sim \exp [-(\ln q)^{-1} (\ln a|s|) \ln |as(1-z)/2|] \quad (5)$$

which gets large because $\ln q$ is negative. This means that $B_4(s, t)$ cannot be a satisfactory approximation to the complete unitarized amplitude in the fixed angle region. However, since fixed z with $s \rightarrow \infty$ involves large momentum transfer, there is no physical reason to expect that the Born term should be a good approximation in this region. In a unitarized theory with B_4 as its Born term, higher order diagrams should be important in the large momentum transfer region and should change the effective $\alpha(t)$ for large negative t .

Since Eq. (5) holds for z near -1, $B_4(s, t)$ also increases as $s \rightarrow \infty$ for fixed u . This bad behavior was mentioned in the original paper² on B_4 and was elaborated upon by Capra¹⁹. Since the origin of the bad fixed u behavior is the same as that of the bad fixed z behavior, any higher order effect which damps out one will damp out the other. Physically we expect higher order corrections to the fixed u behavior of $B_4(s, t)$ to be important because B_4 can be written as a sum

of either s- or t-channel resonance contributions while u-channel exchange contributions are not contained in B_4 . Thus $B_4(s,t)$ only describes peripheral processes for fixed t or fixed s and is not expected to give a good representation of the full scattering amplitude at fixed u and large s. A completely crossing symmetric Born term, $B_4(s,t) + B_4(s,u) + B_4(t,u)$ has u-channel exchange contributions only in $B_4(s,u) + B_4(t,u)$. Only these terms can be expected to give a phenomenologically acceptable representation of the scattering amplitude for $|s| \rightarrow \infty$ with u fixed. It is a special property of the Veneziano limit that $B_4(s,t)$ also remains bounded in this limit. However, even in the Veneziano case, trouble can occur for fixed u when satellites or lower lying trajectories are introduced²⁰.

We thus see that $B_4(s,t)$ is a physically acceptable Born term since it violates unitarity only in that region where one expects higher order corrections to be important. There remains the problem of constructing a consistent unitary dual theory with B_4 as a Born term. Consequently, we have embarked on a program which is analogous to that advocated for the Veneziano model by Kikkawa, Sakita and Virasoro⁹. This is of course a large undertaking which is only at its early stages of development. Some reason for optimism can be found in the fact that the development of the nonlinear theory has so closely paralleled the development of the generalized Veneziano model⁸⁻¹².

The essential first step in the above mentioned program is the construction of the N-point Born term B_N with spin zero external lines. We will now review the reasoning which led to its construction. For this purpose we first note that the infinite product representation (1)

of $B_4(s, t)$ can be expanded¹ in the following double power series in σ and τ

$$B_4(s, t) = \frac{G(\sigma\tau)}{G(\sigma)G(\tau)} = \sum_{n,m=0}^{\infty} \frac{\sigma^n}{f_n} q^{nm} \frac{\tau^m}{f_m} \quad (6)$$

where $f_n = (1 - q) \cdot \dots \cdot (1 - q^n)$ for $n > 1$ and $f_0 = 1$. Eq. (6)

converges for $|\sigma| < 1$, $|\tau| < 1$. If we set $\tau = 0$ in Eq. (6), we obtain the expansion

$$\frac{1}{G(\sigma)} = \sum_{n=0}^{\infty} \frac{\sigma^n}{f_n} \quad (7)$$

Without the q^{nm} factor in Eq. (6), the n and m sums would have decoupled and by Eq. (7) we would have obtained $[G(\sigma)G(\tau)]^{-1}$ which is just the product of the denominator factors of Eq. (1). The residues of the poles in σ would then have had poles in τ , in contrast with Eq. (6) where the residues of the poles in σ are polynomials in τ . Thus, the coupling factor q^{nm} in Eq. (6) prevents simultaneous poles in σ and τ but allows poles in either σ or τ alone. We thus call q^{nm} a simultaneous pole killer or "duality factor". With the above guide we can write down almost immediately the generalization of Eq. (6) to the N -point function B_N corresponding to a given set of Feynman diagrams such as all the planar tree graphs.

The positions of poles in B_N are determined as follows: if p_i is the momentum of the particle associated with an internal line L_i of one of the diagrams of the given set, then B_N must possess poles in the variable p_i^2 at values determined from

$$\sigma_i \equiv a_i p_i^2 + b_i = q^{-j}, \quad j = 0, 1, 2, \dots, \quad (8)$$

where a_i and b_i are constants and q is the same parameter that

appears in B_4 . This guarantees that the mass spectrum associated with B_N is consistent with that deduced from B_4 . We first construct a function with all the above poles by forming the product

$$B_N^{\text{sing}} = \prod_i 1/\pi G(\sigma_i) \quad (9)$$

where $G(\sigma)$ is defined by Eq. (2). The product (9) is taken over all lines L_i which carry distinct momentum p_i . B_N^{sing} is not a satisfactory candidate for B_N because it contains simultaneous poles in all the variables p_i^2 . If we insert the expansion (7) for $1/G$ into Eq. (9) we obtain a multiple power series in the variables σ_i

$$B_N^{\text{sing}} = \sum_{n_1, \dots, n_r=0}^{\infty} \frac{\sigma_1^{n_1}}{f_{n_1}} \frac{\sigma_2^{n_2}}{f_{n_2}} \dots \frac{\sigma_r^{n_r}}{f_{n_r}} \quad (10)$$

We want to construct a function B_N which has no simultaneous poles in any pair of dual variables p_i^2 and p_j^2 (p_i^2 and p_j^2 are dual variables if there is no Feynman diagram in the given set of tree graphs which contains both lines L_i and L_j). Using the analogy with Eq. (6) for B_4 we can construct such a function B_N by the following simple rule:
Rule: For each pair of dual variables p_i^2 and p_j^2 introduce a "pole killing" factor $q^{n_i n_j}$ under the multiple sum in Eq. (10).

Using Eqs. (6) and (7), it is easy to verify¹ that insertion of these $q^{n_i n_j}$ factors into Eq. (10) gives a B_N which has the pole and residue structure of the totality of Feynman diagrams of the given set.

In order for the resulting dual Born term, B_N , to form the basis for construction of a dual multiparticle theory, B_N must possess the basic factorization property.¹⁰ The proof of factorization for B_N has recently been carried out by Yu, Baker and Coon.⁶ Although their

work on factorization is in principle analogous to that of Fubini and Veneziano¹⁰ on the Veneziano N-point function, it is in detail completely different as are the results. The degeneracy of the pole in B_N at $\sigma_i = q^{-j}$ is found⁶ to be $\sim 6^j$. The number 6 enters through the use of $O(4,2)$ scalar products in the factorization. Since the poles are spaced exponentially, the degeneracy grows only as some power of the mass. Thus, even though the degeneracy in the nonlinear theory is greater than the degeneracy of the Veneziano model, the exponential mass spectrum compensates for the increased degeneracy so that the resulting asymptotic density of states is less than in the Veneziano model.

From the factorized amplitude we obtain the N-point Born term with two external lines having arbitrary spin. Using this we have obtained expressions for the N-point planar loop diagrams. However, we have not yet been able to evaluate the integrands in the general case, because we have not yet found a convenient technique to facilitate evaluation of the traces which define these integrands.

As in the Veneziano model, the structure of the theory is more clearly understood by introducing an operator formalism. This formalism⁷ differs fundamentally from that used in the linear theory.¹¹ In our case, the totality of spin states is generated by a single set of noncommuting creation operators²¹ acting on a vacuum state. The commutation relations of these operators with their adjoints differ from the usual harmonic oscillator commutation relations. Because the analysis of this operator structure has not been fully carried out, the development of the nonlinear theory has not yet proceeded as far as the development of the Veneziano model. When

more detailed properties of the nonlinear theory are understood, we will have a better idea of whether the nonlinear B_N can form the basis for a complete dual theory, if indeed such a theory can be constructed at all. At present we are unaware of any physical or aesthetic reason for preferring either the linear or the nonlinear theory.

REFERENCES AND FOOTNOTES

1. M. Baker and D. D. Coon, Phys. Rev. 2, 2349 (1970).
2. D. D. Coon, Phys. Letters 29B, 669 (1969).
3. D. D. Coon, Phys. Rev. 186, 1422 (1969).
4. M. Baker and D. D. Coon, Phys. Rev. (to be published).
5. M. Baker and D. D. Coon, to be published.
6. S. Yu, M. Baker and D. D. Coon, to be published.
7. M. Baker, D. D. Coon and S. Yu, to be published.
8. G. Veneziano, Nuovo Cimento 57A, 190 (1968).
9. Kikkawa, Sakita and Virasoro, Phys. Rev. D1, 3258 (1970).
10. S. Fubini and G. Veneziano, Nuovo Cimento 64A, 811 (1969).
11. S. Fubini, D. Gordon and G. Veneziano, Phys. Letters 29B, 697 (1969); Y. Nambu, International Conference on Symmetries and Quark Models, Wayne State University (1969, unpublished).
12. V. Alessandrini, D. Amati, M. LeBellac and D. Olive, CERN preprint TH. 1160 and references contained therein.
13. S. Adler, Phys. Rev. 137, B1022 (1965).
14. C. Lovelace, Phys. Letters 28B, 264 (1968).
15. M. Ademollo, G. Veneziano and S. Weinberg, Phys. Rev. Letters 22, 83 (1969).

16. Trajectories of the form of those in Refs. 1-7 have also been found by E. Cremmer and J. Nuyts, Nucl. Phys. B26, 151 (1971) and J. Golden, preprint, Dublin Institute for Advanced Studies.
17. W. R. Frazer, Proceedings of the International School of Physics "Enrico Fermi," (Academic Press, New York, 1968) p. 94: A. O. Barut and D. E. Zwanziger, Phys. Rev. 127, 974 (1962): N. F. Bali, Shu-Yuan Chu, R. N. Haymaker and Chung-I Tan, 161, 1450 (1967).
18. Integer-spaced daughters are obviously present since the polynomial residues of B_4 are not Legendre polynomials.
19. F. Capra, Phys. Letters 30B, 53 (1969).
20. As $|s| \rightarrow \infty$ for fixed u , a Veneziano term of the form $V_4(s, t) = \frac{\Gamma(n-\alpha(s)) \Gamma(m-\alpha(t))}{\Gamma(l-\alpha(s)-\alpha(t))}$ has the behavior $V_4(s, t) \sim s^{u-4M^2-2\alpha(0)-1+n+m} / \sin \pi \alpha(s)$. Thus, for large s in the neighborhood of the positive, real axis and u fixed, $V_4(s, t)$ behaves as a power of s which becomes appreciable if $\alpha(0)$ is too small or n and m are too large. For example, with negligible external masses ($M=0$), if we set $u = 0$, $n = m = 1$ and $\alpha(0) = 0$, $V_4 \sim s$ as $s \rightarrow \infty$ near the real axis.
21. This is in contrast with the infinite set of commuting creation operators of Ref. 11.