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RANDOM ERROR MULTIPOLES IN COSINE MAGNETS

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Abstract

The analytical results for the random error multipoles generated by random errors in the current block positions are summarized and compared with the results of exact computer calculations. Bending magnets and quadrupoles are considered. Results are given for magnets having a 4-in. aperture and for a random error in the current block positions of 0.002 in.

Random errors in the position of the current blocks of a cosine magnet will produce random multipoles in the median plane field. An error in the position of a current block will produce an error field in the median plane which has both a vertical component, ΔB_z , and a radial component ΔB_r . Each of these components of the error field can be expanded in multipoles in the median plane according to

$$\Delta B = \sum_{l=0}^{\infty} \Delta B_l r^l \quad (1)$$

It was found by the writer in previous papers^{1,2} that a simple formula expresses the rms ΔB_l present due to a random rms error ϵ in either the radial or azimuthal positions of the current blocks. For the case of a dipole

$$\frac{\Delta B_l}{B_0} = \sqrt{\frac{2}{N_b}} \frac{l+1}{R^{l+1}} \epsilon \quad (2)$$

where B_0 is the dipole magnetic field, N_b is the total number of current blocks in the magnet, and R is the average radius of the current blocks. The cosine current distribution is assumed to be approximated by current blocks placed at equal intervals azimuthally and each carrying a current proportional to the cosine of the azimuthal angle. The derivation assumes the blocks have zero thickness in the radial direction.

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1. G. Parzen, BNL Accelerator Dept. Informal Report AADD-187 (1972).
 2. M. Month and G. Parzen, Particle Accelerators 2, 227 (1971).

The accuracy of the theoretical result for the random multipoles as given by Eq. (2) may be estimated by comparing this result with the computer results found by Dahl³ using the MAGFLD program⁴ for a magnet which has 6 blocks per quadrant which are 0.75 in. thick, whose inner radius is 2 in., and for a coil position error $\epsilon = 0.002$ in. The analytical results are given in Table I and a comparison of the analytical and computer results are given in Fig. 1. The error in the analytical results is about 30%, depending on the multipole one examines, and this error may be primarily due to the nonzero thickness of coils.

TABLE I. The random multipoles generated by a random error in the current block positions of 0.002 in. for a dipole and a quadrupole each having an aperture diameter of 4 in., and whose current blocks are 0.75 in. thick radially.

k	Dipole $\Delta B_k/B_0/10^{-3}$	Quadrupole $\Delta B_k/B_1/10^{-3}$
0	0.243	0.816
1	0.205	0.688
2	0.129	0.434
3	0.0726	0.244
4	0.0382	0.128
5	0.0193	0.0648
6	0.0095	0.0318
7	0.0046	0.0153
8	0.0022	0.0073
9	0.0010	0.0034
10	0.0005	0.0016
11		0.0007
12		0.0003
13		0.0001

3. P.F. Dahl, BNL Accelerator Dept. Informal Report CRISP 73-2 (1973).

4. G.H. Morgan, BNL Accelerator Dept. Informal Report AADD-145 (1970).

Figure 1

Random Error Multipoles
Dipole - 4-in. Aperture

The result given for the error multipoles generated by coil position errors in a dipole can be generalized to quadrupoles, or any higher order multipole. If the unperturbed field in the magnet is assumed to be

$$B = B_m r^m, \quad (3)$$

then the error multipole fields due to a random error ϵ in the current block positions are given by

$$\Delta B_\ell / B_m = \sqrt{\frac{2(m+1)}{N_b}} \frac{\ell+1}{R^{\ell+1}} \cdot R^m \epsilon, \quad (4)$$

where $m = 0$ for a dipole, $m = 1$ for a quadrupole, and $m = 2$ for a sextupole. These results can be found by slightly modifying the equations given in Ref. 2.

The random error multipoles present in a quadrupole are given in Table I for a quadrupole which has 3 blocks per octant which are 0.75 in. thick, and which has an aperture of 4 in. The random error in the current block positions is 0.002 in. Error multipoles are introduced in both the vertical component of field and the radial component of the field in the median, and the error multipoles are approximately the same for both components of the field.