233
7-5-73

LA-5230-T

THESIS

Ar. 209

33436

A Study of the (π^+, p) and (π^+, pp)
Reactions on Selected Light Nuclei

los alamos
scientific laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87544

9/8/8

UNITED STATES
ATOMIC ENERGY COMMISSION
CONTRACT W-7405-ENG. 36

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

This report is a printed version of a thesis submitted as partial fulfillment of the requirement for an advanced academic degree and as such represents the independent work of the author. It has not been edited by the Technical Information staff.

Printed in the United States of America. Available from
National Technical Information Service
U. S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22151

Price: Printed Copy ~~\$3.00~~; Microfiche \$0.95

97.60

LA-5230-T

Thesis

UC-34

ISSUED: May 1973

A Study of the (π^+, p) and (π^+, pp) Reactions on Selected Light Nuclei

by

Edward D. Arthur

This report is derived from a dissertation submitted to the Physics Department of the Graduate School of the University of Virginia in partial fulfillment of requirements for the Degree of Doctor of Philosophy.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

UNCLASSIFIED

TABLE OF CONTENTS

Abstract	x
Introduction	1
I. THEORY OF PION ABSORPTION	5
Introduction	5
Pion Absorption - Detailed Theory	8
The Pole Approximation	19
Pion Absorption Followed by Single Nucleon Emission	23
II. EXPERIMENTAL DETAILS	30
Introduction	30
Nodoscope and Scintillator Trigger Counters	37
Targets	38
NaI Counters	40
Spark Chambers	44
Electronics - Event Logic	46
Electronics - NaI and dE/dX Pulse Heights	52
Data Acquisition System	54
Online Analysis During the Experiment	58
Calibration Procedures During the Experiment	61
III. DATA REDUCTION AND ANALYSIS	63
Introduction	63
Offline Data Analysis	63

III.	Corrections to the NaI Pulse Height	64
	Overall Calibrations	67
	Geometrical Acceptance and Corrections to the (π^+, pp) Data	69
	Efficiency - General Remarks	70
	Determination of the Variables for the Efficiency Function $A(\bar{X})$	72
	Geometrical Efficiency Calculations	74
	Determination of the Monte Carlo Input Beam . . .	75
	Regions of Integration - Ellipse Definition . . .	78
	Corrections Used in the Determination of Cross Sections	86
	Determination of Errors	87
IV.	RESULTS - $A(\pi^+, pp)A-2$ REACTION	90
	Introduction	90
	Pole Model Analysis of the (π^+, pp) Process . . .	91
	Deuterium Results	95
	The ${}^6\text{Li}$ Data	97
	Ground State Results	102
	Results - ${}^4\text{He}$ Excited States	114
	${}^{16}\text{O}$ Results - Missing Mass Spectra	124
	${}^{16}\text{O}$ Results - 2 MeV to 10 MeV Missing Mass . . .	140
	${}^{14}\text{N}$ Results - Missing Mass Distributions	150
	${}^{14}\text{N}$ Peak 1 - Results	160
	${}^{14}\text{N}$ Peak 2 - Results	171

V.	RESULTS - $A(\pi^+, p)A-1$ REACTION	176
	Introduction	176
	Results - ^9Be , ^{12}C , ^{16}O	177
	Results - ^6Li	187
VI.	SUMMARY AND CONCLUSIONS	192
APPENDIX A.	BEAM TRANSPORT MATRICES	197
APPENDIX B.	ELECTRONICS	199
APPENDIX C.	TAPE AND STORAGE FORMAT	201
APPENDIX D.	BERKELEY EXPERIMENT PARAMETER LIST	203
APPENDIX E.	ORTHONORMAL . POLYNOMIALS	207
APPENDIX F.	EFFICIENCY CORRECTIONS AND ERRORS	208
APPENDIX G.	THE PWIA DIFFERENTIAL CROSS SECTION EXPRESSION	210
APPENDIX H.	CLUSTER MOTION WAVE FUNCTIONS	219
ACKNOWLEDGMENTS		223
REFERENCES		224

FIGURES

1.1	Rescattering Terms, πN Hamiltonian	12
1.2	Pole Model Diagram for the (π^+, pp) Reaction	20
2.1	Experimental Geometry (Overview)	31
2.2	Experimental Geometry, (π^+, p) Data.	34
2.3	Separation of Target Elements	39
2.4	Details, NaI Detector	42
2.5	Spark Chamber Efficiency as a Function of Trigger Delay	47
2.6	Event Logic (Overview)	48
2.7	Event Logic, (π^+, pp) Events	49
2.8	(π^+, p) Event and Event Register Logic	50
2.9	Light Pulser Event Logic	53
2.10	Linear Electronics	55
2.11	Nidbus Data Control Structure	56
2.12	ONLINE Flow Chart	60
3.1	Comparison of Experimental and Monte Carlo Beam Phase Space	77
3.2	Integration Regions, Θ_B and Θ_{AB} Space, ${}^6\text{Li}$ Data	80
3.3	Integration Regions, Θ_B and Θ_{AB} Space, ${}^{16}\text{O}$ Data	81
3.4	Integration Regions, Θ_B and Θ_{AB} Space, ${}^{14}\text{N}$ Data	82
3.5	Integration Ellipse Definition Parameters	83
4.1	Excitation Energy, ${}^6\text{Li}$ (π^+, pp) ${}^4\text{He}$ Events, for All Opening Angles and for Deuteron Angles	98
4.2	Excitation Energy, ${}^6\text{Li}$ Events, Off Deuteron Opening Angles	99

4.3	Excitation Energy, ${}^6\text{Li}$ Events, for Various Regions of Recoil Momentum	103
4.4	Recoil Momentum, ${}^6\text{Li}$ Events, for Various Regions of Excitation Energy	104
4.5	Experimental Recoil Momentum Distributions, ${}^6\text{Li}(\pi^+, pp){}^4\text{He}$ (Ground State)	107
4.6	Relative Momentum Distributions, ${}^6\text{Li}(\pi^+, pp){}^4\text{He}$ (Ground State)	108
4.7	Treiman-Yang Distributions, ${}^6\text{Li}(\pi^+, pp){}^4\text{He}$ (Ground State), On Deuteron Angles	109
4.8	Treiman-Yang Distributions, ${}^6\text{Li}(\pi^+, pp){}^4\text{He}$ (Ground State), Off Deuteron Angles	110
4.9	Center of Mass Angle Dependence, ${}^6\text{Li}$ Ground State Events. .	115
4.10	Opening Angle Distribution, ${}^6\text{Li}$ Ground State Events	116
4.11	${}^6\text{Li}(\pi^+, pp){}^4\text{He}$ (Excited States) Excitation Energy Spectra for Various Regions of Recoil Momentum	118
4.12	${}^6\text{Li}$ Excited States Excitation Energy Spectra for Various Opening Angle Regions	119
4.13	Recoil Momentum Distributions, ${}^6\text{Li}$ Excited States	120
4.14	Relative Momentum Distributions, ${}^6\text{Li}$ Excited States	121
4.15	Treiman-Yang Distributions, ${}^6\text{Li}$ Excited States, On Deuteron Angles	122
4.16	Treiman-Yang Distributions, ${}^6\text{Li}$ Excited States, Off Deuteron Angles	123
4.17	Opening Angle Distribution, ${}^6\text{Li}$ Excited States	125
4.18	Center of Mass Angle Dependence, ${}^6\text{Li}$ Excited States	126
4.19-a	Excitation Energy, ${}^{16}\text{O}$ Events, with $T = 0$ and $T = 1$ Theoretical Curves of Kopaleishvili	127
4.19-b	Excitation Energy, ${}^{16}\text{O}$ Events, with Combined Theoretical Predictions (Kopaleishvili) for $T = 0$ and $T = 1$ States	129

4.19-c	Excitation Energy, ^{16}O Events, with Fractional Parentage Curves (Cohen-Kurath)	132
4.19-d	Excitation Energy, ^{16}O Events, with Summed Contribution from Curves in Fig. 4.19-c	133
4.20	Excitation Energy, ^{16}O Events, for Various Opening Angle Regions	136
4.21	Excitation Energy, ^{16}O , for Different Recoil Momentum Values	137
4.22	Fit of $ \phi(p) ^2$ to $(\Delta\sigma/\text{PS})$ Distribution, ^{16}O Events (-2 to 10 MeV)	139
4.23	Recoil Momentum Distributions, ^{16}O Events	142
4.24	Relative Momentum Distributions, ^{16}O Events	143
4.25	Treiman-Yang Distributions, ^{16}O Events, Deuteron Angles	145
4.26	Treiman-Yang Distributions, ^{16}O Events, Off Deuteron Angles	146
4.27	Opening Angle Distributions, ^{16}O Events	147
4.28	Center of Mass Angle Distributions, ^{16}O Events	151
4.29-a	Excitation Energy, ^{14}N Events, with $T = 0$ and $T = 1$ Calculated Curves (Kopaleishvili)	152
4.29-b	Excitation Energy, ^{14}N Events with Sum of Curves Appearing in Fig. 4.29-a	155
4.29-c	Excitation Energy, ^{14}N Events, with Fractional Parentage Curves (Cohen-Kurath)	156
4.29-d	Excitation Energy, ^{14}N Events, with Sum of Curves from Fig. 4.29-c	157
4.30	Excitation Energy, ^{14}N Events, for Various Regions of Recoil Momentum	161
4.31	Excitation Energy, ^{14}N Events, for Different Opening Angle Values	162
4.32	Fit of $ \phi(p) ^2$ to $(\Delta\sigma/\text{PS})$ Distribution, ^{14}N Events (0 to 20 MeV)	164

4.33	Recoil Momentum Distribution, ^{14}N Events (Peak 1)	165
4.34	Relative Momentum Distribution, ^{14}N Events (Peak 1)	166
4.35	Treiman-Yang Distributions, ^{14}N Events (Peak 1), On Deuteron Angles	168
4.36	Treiman-Yang Distributions, ^{14}N Events (Peak 1), Off Deuteron Angles	169
4.37	Opening Angle Distributions, ^{14}N Events (Peak 1)	170
4.38	Relative and Recoil Momentum Distributions. ^{14}N Events (Peak 2)	172
4.39	Treiman-Yang Distributions, ^{14}N Events (Peak 2)	173
4.40	Opening Angle Distribution, ^{14}N Events (Peak 2)	174
5.1	^8Be Excitation Energy Spectrum	178
5.2	^{11}C Excitation Energy Spectrum	179
5.3	^{15}O Excitation Energy Spectrum	180
5.4	Angular Distribution, $^{12}\text{C}(\pi^+, p)^{11}\text{C}$ Events	182
5.5	^{12}C Experimental Results and Theoretical Curves.	185
5.6	^{16}O Experimental Point and Theoretical Curves	186
5.7	^5Li Excitation Energy Spectrum, $\theta = 53^\circ$	189
5.8	Angular Distribution, $^6\text{Li}(\pi^+, p)^5\text{Li}$ Events	190
G.1	Impulse Approximation, (π^+, pp) Reaction	211

TABLES

1	Targets and Angles Covered for (π^+ , pp) Events	3
2	Integration Regions, ^6Li Data	84
3	Integration Regions, ^{16}O and ^{14}N Data	85
4	Free Deuteron Cross Section Results	.96
4-a	Ratio of Ground State ^4He Events to Excited State Events . .	.101
5	Experimental and PM Cross Sections, ^6Li Data	.112
6-a	Level Excitation Predictions of Kopaleishvili for the $^{16}\text{O}(\pi^+, \text{pp})^{14}\text{N}$ Reaction	130
6-b	Fractional Parentage Calculations of Cohen-Kurath for Pickup A = 16(0,0)	134
7	Experimental and PWIA Cross Sections, ^{16}O Data	148
8-a	Level Excitation Predictions of Kopaleishvili for the $^{14}\text{N}(\pi^+, \text{pp})^{12}\text{C}$ Reaction	.154
8-b	Fractional Parentage Calculations of Cohen-Kurath for Pickup A = 14(1,0)	.158
9	Cross Sections - (π^+ , p) Data	.183

A STUDY OF THE (π^+, p) AND (π^+, pp)

REACTIONS ON SELECTED LIGHT NUCLEI

by

Edward D. Arthur

ABSTRACT

The capture in flight of positive pions on the p-shell nuclei ${}^6\text{Li}$, ${}^{14}\text{N}$, and ${}^{16}\text{O}$ with the subsequent ejection of two fast protons has been studied. The momentum of the incoming pion was determined through the use of a magnetic spectrometer. The measurement of the energy and angle of the two emitted protons, along with the knowledge of the pion momentum, allowed a complete determination of the final state to be made. Experimental distributions in opening angle, recoil momentum, relative momentum, and Treiman-Yang angle for low-lying states of the residual nuclei were compared with the predictions of the pole model and the plane wave impulse approximation based on the assumption of absorption of the pion on a quasi deuteron. Agreement was found between the experimental distributions and the predictions of these models. The center-of-mass angle dependence of the cross section for events resulting from capture on ${}^6\text{Li}$ and ${}^{16}\text{O}$ was found to follow the $A(0.23 + \cos^2\theta_{\text{cm}})$ dependence of the cross section for capture on a free deuteron. Excitation energy spectra from capture on ${}^{14}\text{N}$ and ${}^{16}\text{O}$ did not agree well with existing theoretical calculations.

The rarer process of the capture of positive pions leading to the emission of one proton was also measured. The nuclei studied were ${}^6\text{Li}$, ${}^9\text{Be}$, ${}^{12}\text{C}$, and ${}^{16}\text{O}$. Angular distributions were obtained for ${}^{12}\text{C}$ and ${}^6\text{Li}$. For ${}^9\text{Be}$ and ${}^{16}\text{O}$ only upper limits were obtained for the cross section since there was little contribution to the excitation spectrum of the residual nucleus above that from background. The results obtained for ${}^{12}\text{C}$ were found to be lower than previous experiments and existing theories. The dependence of the ${}^6\text{Li}$ cross section upon laboratory angle was found to decrease rapidly for angles less than 60° and then to become fairly constant in the measured range up to 90° .

INTRODUCTION

The purpose of this short introduction is to provide an overview of the experiment. This is necessary because of the fact that two experiments were actually done and a variety of targets were investigated. It is intended that this introduction will describe the main reasons for doing these experiments and will indicate what information it was hoped would be obtained from them. First the discussion will center on the (π^+ ,pp) portion of the experiment, and then a short description of the (π^+ ,p) experiment will be presented.

Previous (π^+ ,pp)^{1,2} experiments have investigated absorption in flight on a variety of nuclei. Most of the results from these experiments have been discussed in terms of a simple model based on the absorption of a π^+ on a quasi-deuteron through a direct mechanism. One experiment² has made a thorough analysis of their events from the reaction ${}^6\text{Li} (\pi^+, \text{pp}) {}^4\text{He}$ in terms of this "pole" approximation. The analysis was made for events taken with an opening angle (160°) which corresponded to the kinematics of the free deuteron reaction $\pi^+ + d \rightarrow p + p$. Satisfactory agreement was obtained between experiment and pole model predictions over a range of incident pion energies.

We sought through our experiment to extend the region over which the pole approximation could be tested. We also sought to investigate the reaction on several p-shell nuclei with better resolution than had been obtained previously. The angles covered and

the targets used in the (π^+ ,pp) measurements are presented in Table 1. For the lithium target, runs were made at three values of opening angles ($63^\circ - 98^\circ$, $80^\circ - 80^\circ$, $42^\circ - 123^\circ$) which corresponded to the opening angle for the free deuteron process but which had different center of mass angles. By covering a range in the center of mass angle, we hoped to measure the dependence of the (π^+ ,pp) cross section on this angle. We also took data for opening angles which were off the deuteron angle in an effort to see any breakdown in the agreement between the experimental and pole model results. The same procedure was followed for the oxygen data. With the nitrogen target we were not able to take data at several opening angle settings because of the size of the target.

We have presented the results of our (π^+ ,pp) data in a form which eliminates any bias due to geometrical effects. Results from previous (π^+ ,pp) experiments did not do this but instead presented distributions which were biased by the acceptance of their apparatus. The unbiased presentation of our data presents a form which is readily comparable with any theory.

We have compared our data whenever possible with existing theoretical predictions for the (π^+ ,pp) process. Since the number of (π^+ ,pp) theoretical calculations is small compared with calculations performed for two nucleon emission following the absorption of a stopped π^- meson, we have made our main comparison with the theoretical predictions of the pole model and the plane wave impulse approximation.

TABLE 1

TARGETS AND ANGLES COVERED FOR (π^+ , pp) EVENTS

<u>${}^6\text{Li}$ Target</u>		<u>D_2O Target</u>	
<u>θ_1</u>	<u>θ_2</u>	<u>θ_1</u>	<u>θ_2</u>
38°	98°	25°	113°
53	98	37.5	113
63	98	50	113
78	98	65	113
80	80	80	113
42	123	61	100

<u>N_2 Target</u>	
<u>θ_1</u>	<u>θ_2</u>
57	105

Our measurement of the (π^+ ,p) process sought to improve upon the results of the one previous (π^+ ,p) experiment³ by investigating this process on a number of nuclei with improved energy resolution. Most of the (π^+ ,p) data was taken at an angle of 25° . An exception to this was the case of ${}^6\text{Li}$ for which data were taken at angles of 42° , 53° , 63° , 78° , and 80° .

In the following chapter a general description of the theory of pion absorption on one and two nucleons will be presented along with a short description of the pole approximation. Next the experimental setup and the methods used in the data analysis will be described. Finally the results of the (π^+ ,pp) and (π^+ ,p) data will be presented.

CHAPTER 1

THEORY OF PION ABSORPTION

Introduction

Since the early 1960's it has been recognized that the use of pion interactions⁴⁻⁶ offers a unique tool for nuclear structure studies. The pion has two striking features which make it different from other nuclear probes. The first feature is the fact that a pion is a boson and can thus be absorbed in nuclei if energy and momentum balance requirements are met. This property simplifies the kinematical analysis of products in the final state. Secondly, the pion has isobaric spin one which allows for three charge states of the pion; π^+ , π^0 , and π^- . Charge exchange reactions are possible with pions, making more types of investigations of reactions possible. For example, during a reaction a neutron can be changed into a proton making it more easily detectable than in its initial state.

The study of pion absorption in nuclei allows other unique possibilities for the study of nuclear structure. When a pion is absorbed by a nucleus it brings in a large amount of energy (≥ 140 MeV) and relatively little momentum. If the pion is absorbed on one nucleon then all of the available energy is given to the nucleon resulting in a momentum greater than 500 MeV/c. For momentum conservation to be satisfied the initial momentum of the nucleon must also have been greater than 500 MeV/c. This is relatively unlikely, however, since it is difficult to find a nucleon having a momentum greater than the Fermi momentum, 250 MeV/c. Thus pion absorption by

one nucleon is strongly suppressed and the dominant mode for absorption is considered to be via two closely correlated nucleons. If the energy of the incoming pion is shared equally by two nucleons then each would have an energy of approximately 70 MeV and thus a momentum of 350 MeV/c. The two nucleons are emitted approximately back to back with a relative momentum $\frac{1}{2} (\hat{p}_1 - \hat{p}_2)$. Therefore the two nucleons are very close together ($\leq .6$ fermis) at the moment of capture, and pion absorption provides a tool for probing short range pair correlations in the initial nucleus.

The absorption of a pion with the subsequent ejection of two fast nucleons leads to a way of probing two hole excited states of the nucleus in an analagous way to the probing of one hole excited states by (p,2p) reactions.⁷ The use of pion absorption to study such two hole excitations is much easier than the use of other reactions, for instance (p,3p) reactions, since one has only to determine the kinematical parameters for two particles in the final state. Such two hole states manifest themselves as structure in the summed energy of the two emitted nucleons or in the excitation energy spectrum of the residual nucleus.

The study of the angular correlations of the two emitted nucleons can provide information concerning the momentum distribution of the correlated pair relative to the rest of the nucleus. Since the outgoing nucleons obtain their high relative momentum components from interaction with the nucleon-nucleon hardcore it may be possible to obtain information concerning the short range part of nucleon-

nucleon potential. The absorption of pions with the emission of heavy particles would provide information concerning the presence of nucleon clusters within the nucleus.

The concept of pion absorption by two correlated nucleons was first advanced by Breuckner⁸ in 1951. Breuckner assumed that pion absorption involved two nucleons with the rest of the nucleus acting as a spectator and also that absorption was inverse of pion production in the collision of two nucleons. In his theory, pion capture on complex nuclei could be related to pion capture on the deuteron by the following expression

$$\frac{1}{Z} \sigma(\pi + A \rightarrow \text{star}) = \Gamma \sigma(\pi + d \rightarrow 2N)$$

in which Γ allowed for the effect of the remaining $A-2$ nucleons on the capturing pair. The two nucleon capture mode has been verified by numerous experiments,⁹⁻¹⁶ mainly through observation of the angular distribution of the outgoing nucleon pair.

Recently experiments¹⁷⁻¹⁹ have been performed in which evidence has been obtained for modes of absorption other than the two nucleon mode. These experiments have measured the yields and in one case¹⁹ the angular distribution of the heavier particles emitted. The existence of sizeable fractions of deuterons and tritons lead to the concept of pion absorption on more complicated nucleon structures such as alpha clusters within the nucleus.

Most of the theoretical calculations describing absorption on light nuclei have been done within the framework of the two nucleon model. Thus a more detailed picture will be given in the following section.

Pion Absorption - Detailed Theory

The information which pion absorption can yield about nuclear structure is masked somewhat by an incomplete knowledge of various aspects of the absorptive and nuclear processes involved. For example, in order to obtain information concerning initial state correlations one must know to what extent final state interactions between the outgoing nucleons as well as nuclear distortions (nucleon-nucleus scattering) mask the initial state correlations. The difficulties involved in the theoretical analysis of pion absorption center around the following four main areas:

- (1) an accurate description of the wave function of the incoming pion.
- (2) a reliable description of the $\pi - N$ interaction in the relative kinematical region.
- (3) the introduction of initial state correlations.
- (4) the description of the effects of scattering between the two outgoing nucleons and the interaction between the outgoing nucleons and the residual nucleus.

For the case of the absorption of a stopped π^- meson from an atomic S or P orbital, the initial state wave function of the pion is fairly well known. The problem is more difficult for the absorption of a $\pi^\pm$ in flight. Often the form for the incident π meson is taken as an undistorted plane wave.²⁰

To describe the interaction Hamiltonian most authors assume the following form from the pion-nucleon interaction

$$H \propto G \bar{\Psi} \gamma_5 \tau \phi \Psi \quad ^4$$

This reduces to the following when written nonrelativistically and in a Gallilean invariant form²¹

$$(1-1) \quad H = \frac{f}{m} \sum_{i=1}^A [\sigma \cdot \nabla_{\pi} \tau \cdot \phi - \frac{m}{M} \tau \cdot \phi \sigma \cdot \nabla_N]_i$$

Here f is the pion nucleon coupling constant, m and M are the mass of the pion and nucleon respectively, ϕ is the pion field, σ and τ are the Pauli spin and isospin matrices. The sum extends over all nucleons. However, since the absorption is assumed to take place only on 2 closely correlated nucleons with the remainder of the nucleus acting as spectator, the sum is usually taken over $i = 1, 2$. The first term in the expression for H corresponds to a P wave absorption, the second to an S wave absorption. Some authors^{20,22} point out that for a pion captured in flight with kinetic energy greater than 20 MeV, only the first term involving P wave absorption is important, and therefore drop the second term describing S wave absorption.

Other authors^{23,24} have maintained that it is extremely difficult to calculate the matrix element for the nuclear capture process using the basic π -N interaction Hamiltonian. Instead they use a phenomenological approach assuming a zero range approximation for the interaction. The form for the matrix element is described using four independent amplitudes. The values for the amplitudes are then determined empirically from pion production in nucleon-nucleon collisions.

Using the phenomenological approach Eckstein²³ was able to calculate with quite good agreement to experimental results, the ratio of the triton mode to all capture modes for π^- absorption on He^4 . However this approach has been criticized for several reasons. First, the form chosen for the interaction is not a sum of simple one body operators but is more complicated. Since any correlations between the nucleons in the initial state are incorporated only in the value for the independent coefficients, it is difficult to extract information concerning initial correlations using this method.

In a series of theoretical calculations for pion absorption and production Koltun and Reitan²⁵⁻²⁷ have used the following form for the π nucleon interaction Hamiltonian

$$H = H_0 + H_1 + H_2 .$$

Here H_0 is the nonrelativistic Gallilean invariant Hamiltonian described previously. The terms labeled H_1 and H_2 are derived from the direct and charge exchange interactions which have been used to explain low energy S wave scattering of pions by nucleons. The forms for H_1 and H_2 are

$$(1-2) \quad \begin{aligned} H_1 &\propto (\lambda_1 / \mu) \phi^2(x) \\ H_2 &\propto (\lambda_2 / \mu^2) \tau \cdot \phi(x) \times \pi \end{aligned}$$

where $\phi(x)$ is the pion field, $\pi(x)$ is the conjugate pion field operator, μ is the mass of the pion, and λ_1 and λ_2 are coefficients which are fixed phenomenologically from S wave phase shifts

for pion nucleon scattering. The pion is produced (absorbed) in the lowest approximation (Fig. 1-1) by H_0 with a correction resulting from the pion being produced (absorbed) at one nucleon by H_0 and then rescattered at the second nucleon by H_1 or H_2 . The virtual meson produced may propagate backward or forward in time. In their calculation of the cross section for the reaction $\pi^+ + d \rightarrow p + p$, Koltun and Reitan²⁵ found a large contribution resulting from the charge exchange S wave interaction. The inclusion of the rescattering term in the $\pi - N$ interaction had a noticeable effect in their calculation of the process ${}^6\text{Li}(\pi^+, 2n){}^4\text{He}$,²⁶ in that the absorption rate is smaller when scattering is not included. Also the shape of the distribution of the momentum of the neutron pair depends on whether the pion scattering term is included or not.

In the calculation of the transition rate for absorption on two nucleons one employs the following expression²¹ which is a result of first order perturbation theory

$$(1-3) \quad W = \bar{\sum} \sum \rho(E) |\langle f | H | i \rangle|^2$$

where $\bar{\sum}$ is the average over initial states and $\sum$ indicates the sum over final states. The term $\rho(E)$ is the density of final states; i and f refer to the initial and final states.

The method for calculating the transition rate using the expression for W usually proceeds as follows. The initial state is usually taken to be a wave function formed from the product of an antisymmetrized wave function for the two absorbing nucleons in an initial state α_i times a wave function for a system of $A-2$ nucleons

Figure 1.1 Rescattering Term, πN Hamiltonian

given by some nuclear model. If one neglects the exchange or interaction of the outgoing nucleons and the residual nucleus, the final state wave function is the product of the wave function of the emitted pair and the wave function for the residual nucleus.

In most calculations the initial and final state wave functions are factored into their relative and center of mass parts.^{26,28,29} For the initial 2N wave function the separation becomes

$$\varphi_i(\vec{r}_1, \vec{r}_2) = \psi(\vec{r}) \phi_i(\vec{R})$$

where $\vec{r} = \vec{r}_1 - \vec{r}_2$ and $\vec{R} = \frac{1}{2}(\vec{r}_1 + \vec{r}_2)$. Similarly the final state 2N wave function is separated into relative and center of mass parts and has the form (neglecting interaction of the outgoing nucleons with the residual nucleus)

$$\varphi_f(\vec{r}_1, \vec{r}_2) = \psi_f(\vec{r}) e^{i\vec{p} \cdot \vec{R}}$$

where $\vec{p} = \frac{1}{2}(\vec{p}_1 + \vec{p}_2)$. The interaction Hamiltonian H is taken between two states, one of which occurs when the pion is in a state α and the other which represents a vacuum state. This operator, denoted by H_{12} , is then transformed into a form employing relative and center of mass co-ordinates.

In order to include initial state correlations the wave function of the absorbing pair of nucleons has to be modified. The form chosen is usually a multiplicative factor which contains a one parameter function describing the effects of the nuclear hard core on the model wave function. An expression of this type has been formulated by Dabrowski³⁰

where

$$(1-4) \quad f(r) = 1 - \chi(r)$$

$$\chi(r) = \begin{cases} 1 & r < r_c \\ \exp[-B(r/r_c)^2 - 1] & r > r_c \end{cases}$$

Here B is the correlation parameter and r_c is the radius of the repulsive core of the NN interaction.

Koltun and Reitan have taken the initial state correlations into account using the method of Dawson, Talmi, and Walecka.³¹ In this method the relative motion of the pair is modified from that given by the harmonic oscillator shell model through integration of a Schrodinger equation for each state of relative motion with a combination of an harmonic oscillator and a phenomenological (Hamada-Johnston) potential. This method not only introduces the desired short range correlation between the pair motion of the absorbing nucleons but also introduces orbital angular momentum admixture through the tensor potential. The authors found that both types of effects were important in their treatment of $p+p \rightarrow d+\pi^+$

The approximation for the final state wave function of the two emitted nucleons usually takes one of three following forms. The first is to assume a plane wave form in which no effect due to the interaction between the outgoing nucleons is included. This approach was used by Breuckner⁸ in his calculation of π^- capture of complex nuclei. This method has also been used by Divakaran in his calculation of π^- absorption modes on ${}^3\text{He}$.²⁴ The use of

plane waves has been criticized as being a poor approximation for the description of the two nucleon final state. A better approximation includes the final state interaction between the two outgoing nucleons through the use of an asymptotic form derived for NN scattering. Thus the expression $\psi(\vec{r})$ becomes at all distances

$$\psi_f(r) \propto j_l(kr) - \tan \delta_l m_l(kr) \quad 20,21,28$$

The phase shifts used in this expression are taken from experimental NN scattering data.

A third and even better form is the exact solution of the NN scattering wave equation through the use of a realistic nucleon-nucleon potential (Hamada-Johnston). This approach has been used by Kopaleishvili²² in his study of $(\pi^\pm, NN)$ processes on light nuclei, by Koltun and Reitan in their study of π^- absorption of ${}^6\text{Li}$ and ${}^4\text{He}$, and by Eisenberg³² in a study of π^- absorption on ${}^{12}\text{C}$ and ${}^{16}\text{O}$. The shape of certain distributions and the value for transition rates can be greatly affected by the form of the final state interaction which is chosen. For example, calculations of π^- absorption on ${}^{16}\text{O}$ performed by Eisenberg, first using an asymptotic approximation for the final state interaction and then an exact solution gave transition rates which were much higher in the latter case. Also the shape of the cross section for equal energy sharing between the two emitted nucleons differed greatly in the two treatments.

Most theoretical approaches to two nucleon emission from pion absorption neglect the interaction of the outgoing nucleons with the residual nucleus. This interaction is also commonly known as nuclear

distortions. Kaushal and Waghmare³³ have recently included both the interaction between the outgoing nucleons and the effects of the nuclear distortion in a Hartree-Fock calculation of π^- absorption on ^{12}C . Their approximation for the effects of nuclear distortion consists of the following. Since the energy of each outgoing nucleon is large, the wave function of the outgoing pair is written as

$$u_{k_j}^{(-)}(r_j) = e^{ik_j r_j - iS_j} \quad j=1,2$$

in the high energy approximation. The quantity S_j is given by the relation

$$S_j = \frac{1}{v_j} \int_0^\infty V(r_j + \hat{k}_j) dr_j$$

where v_j is the nuclear velocity. The integration is taken along the unit vector k_j for r_j from 0 to $+\infty$. The nuclear interaction V is assumed to be an optical model potential.

Assuming that the optical potential has a strength $-(U_0 + iW_0)$ then several effects arise as a result of nuclear distortion. First, both total momentum K and relative momentum k are multiplied by the factor

$$\left[(1 + U_0/k_i v_i) - iW_0/k_i v_i \right]$$

the real part of which causes a shift in the magnitude of the momenta. The imaginary part of this expression describes absorption. An extra factor of the form

$$\exp \left[-2W_0/v_i (R_1 + R_2) \right]$$

occurs in the absorption rate. Here $R_j = (R_0 - \frac{2}{3}r_j^2)^{\frac{1}{2}}$ where R_0 is the square well radius.

Applying these methods to the calculation of the (π^-, NN) process on ^{12}C , Kaushal and Waghmare found that the effects resulting from nuclear distortion are more important than those from final state nucleon-nucleon scattering.

There are several quantities and distributions which provide a test for the validity of the assumptions used in various treatments of pion absorption. The first quantity is the total absorption rate which is extremely sensitive to the assumptions made in the various models, particularly with respect to the wave function in relative coordinates. The calculation of the relative probability of excitation of various levels of the residual nucleus also provides a test of the applicability of a nuclear model to the process in question. The shape of distributions such as the opening angle distribution is dependent on the assumptions made concerning the effects of final state nucleon-nucleon scattering and nuclear distortions. As an example Koltun and Reitan found differing shapes in the total momentum distribution in their calculation of ${}^6\text{Li}(\pi^-, nn){}^4\text{He}$ depending whether or not S wave rescattering was included in their interaction Hamiltonian.

A final distribution of interest, especially when comparing data to a simple pole model approximation, is the Treiman-Yang angular distribution.³⁴ For the reaction

$$A(\pi^+, 2N)B$$

the Treiman-Yang angle is defined in the pion rest frame as the angle

between the plane defined by the momenta of the two ejected nucleons and the plane defined by the momenta of the target nucleus A and the recoil nucleus B. The Treiman-Yang angle distribution has been shown by Shapiro³⁵ to be related to the angular momentum of A and B and to the "pole" or transferred particle in the reaction. Because the "pole" approximation has been used to compare the results of previous (π^+ ,pp) experiments it will be briefly described at this point.

The Pole Approximation

Shapiro³⁶ has suggested that the use of Feynman graphs offers some advantages in the description of nuclear reactions at high energies (≥ 100 MeV). The use of Feynman graphs allows one to avoid explicit use of absorption operators and instead use the measured amplitude of a real process.

The pole model description for the reaction $A(\pi^+, 2p)B$ is shown in Fig. 1-2. Here no interaction of the outgoing nucleons with each other or with the residual nucleus is assumed since this would lead to more complicated Feynman diagrams.

The square modulus of the matrix element for the reaction $A + B \rightarrow B + y + z$ (denoted by $|M|^2$) can be related to the corresponding element $|M'|^2$ for the elementary process $i + x \rightarrow y + z$ (in this case the reaction $\pi^+ + d \rightarrow p + p$). If the spin of the virtual particle is zero or if the particle i and the system B are in a relative S state with respect to each other then the expression for $|M|^2$ takes the form

$$(1-6) \quad |M|^2 \propto \frac{\Theta^2 |F(p_R)|^2}{(p_R^2 + \alpha^2)^2} |M'|^2$$

In this expression $|F(p_R)|$ is the form factor for the motion of the virtual particle, i.e. its center of mass momentum wave function multiplied by the factor $(p^2_{\text{recoil}} + \alpha^2)^2$ where α is determined from the binding energy of the virtual particle in the nucleus A . The term Θ^2 describes the probability that the absorbing nucleus can be

Figure 1.2 Pole Model Diagram for the (π^+, pp) Reaction

described as a given state of B and the virtual particle i. For the case where absorption on a quasi-deuteron is assumed then $|M'|^2$ assumes a form proportional to $\lambda_1(\tilde{E}) + \lambda_2(\tilde{E}) \cos^2 \tilde{\Theta}$ where $\tilde{E}$ and $\tilde{\Theta}$ are defined in the center of mass of the $\pi^+ + d \rightarrow p + p$ reaction.

The applicability of the pole model is limited to regions of small momentum transfers,³⁵ i.e.

$$0 \leq q^2 < 2m_{Bi} \mathcal{E}$$

where $\mathcal{E}$ is the binding energy of virtual particle i in the nucleus A and m_{Bi} is the reduced mass of B and i. At large q^2 it becomes necessary to take into account graphs containing singularities far removed from the physical region. In this case a simple pole approximation breaks down.

The Treiman-Yang angle distribution was mentioned previously as being a possible test for the validity of the pole approximation to a particular reaction. If the diagram shown previously correctly describes the process then the cross section as a function of Treiman-Yang angle should be isotropic if the virtual particle has spin 0 or if the virtual particle and the residual nucleus are in a relative S state, or if the channel spin $S_{np} + S_R$ is zero. Thus the isotropy of the Treiman-Yang distribution should give an indication of how well the $A(\pi^+, pp)B$ process is described by the pole approximation.

Finally the performance of the pole model can be determined through the use of the quantity Θ^2 (defined as the effective number of deuteron clusters in the target nucleus). The quantity Θ^2 is

determined experimentally by taking the ratio of $\sigma_{\text{EXP}} / \sigma_{\text{POLE MODEL}}$.
It gives an indication of the agreement between the measured cross
section and the cross section determined from the pole model.

Pion Absorption Followed by Single Nucleon Emission

It was noted previously that pion absorption followed by the emission of a single nucleon is a relatively rare process since energy and momentum conservation require the absorbing nucleon to have momentum components considerably higher than the Fermi momentum. Although the process occurs only a fraction of the time (on the order of a few percent) compared with absorption followed by two nucleon emission, it has several unique features which make it attractive both theoretically and experimentally. Experimentally (π, N) reactions allow one to probe single hole states in an analagous manner to $(p, 2p)$ or $(e, e'p)$ reactions. However this process has the advantage of having only one particle in the final state. Positive pion absorption followed by the emission of a proton is more easily studied experimentally than absorption of a bound pion, since for a bound pion a neutron must be detected in the final state.

Theoretically the study of the (π, N) process can provide a test of the perturbation theory usually employed in calculations of pion absorption reactions. In the theoretical treatment of single nucleon emission following pion absorption complications usually present in two nucleon emission such as initial state correlations and final state interactions between the two outgoing nucleons are absent.

Using the results of first order time dependent perturbation theory, the cross section for the process $A(\pi^+, p)A-1$ is given by³⁸

$$(1-7) \quad \sigma(\theta, \varphi) = \frac{M}{(2\pi)^2} E_\pi \frac{k}{q} \sum |K_f| H |i\rangle|^2$$

where M is the mass of the nucleon, q and k are the momentum of the pion and nucleon, and $E_\pi = \sqrt{\mu^2 + q^2}$. Theoretical calculations for this process usually assume an interaction Hamiltonian H which is the non relativistic Gallilean invariant pseudoscalar-pseudoscalar interaction given previously in expression 1-1. The simplest diagram for the (π^+, p) reaction is given below.

Here the pion which is on the mass shell interacts with a neutron which is off the mass shell through the interaction H .

The simplest type of calculation approximates the initial pion and outgoing nucleon through the use of undistorted plane waves. Eisenberg and Letourneux³⁸ have performed a calculation based on this approximation for the reaction (π^+, p) on ^{16}O and ^{12}C . Harmonic oscillator wave functions were used to represent the neutron hole state of the residual nucleus.

Several difficulties arise in the calculation of the $A(\pi^+, p)A-1$ process. The first difficulty involves the uncertainty in the knowledge of the momentum distribution of the neutron in the nucleus, especially in the case of very high momentum components. Other difficulties involve the approximations used in the calculation. The use of undistorted plane waves for the incoming pion and outgoing proton wave functions is a poor approximation since it neglects initial and final state interactions. The use of harmonic oscillator wave functions to describe the neutron hole state introduces a great sensitivity in the value of the calculated cross section as a function of the harmonic oscillator parameters.

In the perturbation expression for the cross section

$$\sigma \propto \sum_j |\langle f | H | i \rangle|^2$$

there arise terms formed from the product of the neutron wave function and the exponent $i(\hat{q} - \hat{k}) \cdot \hat{r}$ which emphasize the very high momentum components of the bound state wave function. The use of an harmonic oscillator wave function offers a poor fit to the detailed shape of the neutron wave function. Also in the expression for $\sigma(\theta, \varphi)$ there arise coefficients made up of terms which nearly cancel. The use of plane waves in the calculation makes the problem very sensitive to these delicate cancellations and very large fluctuations occur in the calculated cross section for small differences in proton momentum.

Kaushal and Waghmare³⁹ have also used the first order time dependent perturbation approach with undistorted plane waves for the pion and proton to calculate the cross section for (π^+, p) on ^{12}C

and ¹⁶O. Their calculation differs somewhat from that of Eisenberg and Letourneux in that they assume the nucleons in the initial nucleus are in Hartree Fock (HF) orbits. To use HF wave functions they expand these wave functions in terms of an harmonic oscillator basis

$$|n'ljm\rangle_{\text{HF}} = \sum_n C_{n'n}^{(l)} |nljm\rangle$$

where $C_{n'n}^{(l)}$ are mixing coefficients obtained from a self consistent calculation. These coefficients also introduce correlations between the nucleons in the nucleus which have the effect of increasing the calculated cross section over the value obtained when correlations are not present. Their calculated result is somewhat higher than the experimental data point of Whitten et al.³ When corrections due to final state nucleon nucleus scattering are included the value of the calculated cross section is lowered and agrees better with experiment. However the results of Kaushal are also sensitive to the values of the oscillator well parameter used in the calculation. Jones and Eisenberg⁴⁰ have redone the calculation of the (π^+ ,p) process on ¹²C employing the same first order time dependent perturbation theory approach but using more realistic wave functions for the incident pion and outgoing proton. For the outgoing proton, optical model wave functions were used. The wave function for the initial state neutron was generated using a Woods-Saxon well. To generate the incoming pion wave function two optical models were used--the Kisslinger and the Kroll-Kisslinger model. The wave equation for the pion is given in both models by

$$(-\nabla^2 + \mu^2) \phi(\vec{r}) = \{ (E_\pi - V_c)^2 + A b_0 q_0^2 \rho(\vec{r}) - \vec{\nabla} \cdot [A b_1 \rho(\vec{r}) (1 + \frac{1}{3} \xi A b_1 \rho(\vec{r}))^{-1}] \vec{\nabla} \} \phi(\vec{r})$$

where μ , E_π and q_0 are the mass, total energy, and incident energy of the pion. V_c is the coulomb potential for the target nucleus A; $\rho(\vec{r})$ is the nuclear density. The quantities b_0 and b_1 are constants related to pion nucleon scattering. The quantity ξ is equal to 0 in the ordinary Kisslinger model. However in the Kroll-Kisslinger model ξ is greater than 0. Thus in the Kroll-Kisslinger equation the fact that ξ is greater than 0 produces in the potential a non-linear dependence on the density of scattering centers.

The use of optical model wave functions and neutron wave functions generated by a Woods-Saxon well helped reduce some of the extreme fluctuations in the cross section that had arisen when plane wave and harmonic oscillator wave functions were used. However their calculated cross section was a factor of one-hundred greater than the then existing data point.

Even with the improvements in the techniques of Jones and Eisenberg, there are still several uncertainties present which have yet to be resolved. The first pertains to the use of an optical potential generated from elastic scattering to describe scattering which takes place off the mass shell. Secondly, since the distances probed by the reaction are quite small (~ 0.4 fermi), then short range correlation effects may be present which cannot be described using an optical potential.

In an approach due originally to Ruderman,⁴¹ Ingram et al.⁴² have made a calculation of single pion production in bound nuclei. This is of course the inverse of pion absorption followed by single nucleon emission and the cross sections calculated for production can be related to the absorption cross section through detailed balance. Ruderman's approach assumed that all π^+ production originates in the two nucleon reaction $pp \rightarrow d\pi^+$ and therefore the process $pA \rightarrow (A+1)\pi^+$ can be related to this reaction.

Using this method Ingram et al obtained the following expression for the reaction $pA \rightarrow (A+1)\pi^+$

$$\left(\frac{d\sigma}{d\Omega}\right)_{pA \rightarrow (A+1)\pi^+} = ((2\pi+1)^2/B_{pA}) \rho_A g_A |\langle \pi, (A+1) | T | pA \rangle|^2$$

The quantities B_{pA} , ρ_A , and g_A are the relative velocity of the incoming particles, the density of the states factor, and the average over initial and sum over the final spin states. The $(A+1)$ operator T is approximated by a sum over all the protons in A of the two nucleon operator T' given by the expression

$$\left(\frac{d\sigma}{d\Omega}\right)_{pp \rightarrow d\pi^+} = \frac{(2\pi)^4}{B_{pp}} \rho_i g_i |\langle nd | T' | pp \rangle|^2$$

After some manipulation the expression for $(d\sigma/d\Omega)_{pA}$ is expressed in terms of the elementary differential cross section $(d\sigma/d\Omega)_{pp}$ as in the following expression

$$(d\sigma/d\Omega)_{pA} \propto |F(\Delta)|^2 (d\sigma/d\Omega)_{pp}$$

where $\Delta = (A-1)/A \vec{k} - \frac{A-1}{A+1} \vec{k}_\pi$ and $F(\Delta)$ is a form factor.

The cross section calculated using the single nucleon approximation is proportional to the term $\exp(-\Delta^2 / \alpha^2)$ when harmonic oscillator wave functions are used. The fact that the Ruderman cross section is proportional to $\exp(-\Delta^2 / 3 \alpha^2)$ reflects the division of the momentum transfer between three nucleons, two of which are in the residual and the other in the target nucleus.

If the two nucleon description of the $\pi^+ A \rightarrow (A-1)p$ is a more appropriate description than the single nucleon description, then the πN reaction loses some of its attractiveness for studying one hole states. In such a case the distinction between (π, N) and (π, NN) reactions becomes blurred.

CHAPTER 2

EXPERIMENTAL DETAILS

Introduction

This experiment was performed at the 184 inch cyclotron, Lawrence Radiation Laboratory, Berkeley, California. Positive pions were produced using the external proton beam of the cyclotron. Particles of momentum 159 MeV/c were swept by bending magnets into our experimental apparatus.

The experimental setup is shown in Fig. 2-1. After having been deflected by the first bending magnet, the beam was slightly focussed by a set of quadrupoles and then was deflected by a second C magnet onto our scattering target. A four by sixteen element hodoscope and a spark chamber box placed after the quadrupole allowed the determination of the angle and position of the beam particles prior to entering the second bending magnet. Proton contamination in the beam was ranged out by using a thin sheet of polyethelene. The exit angle and position of the emerging beam particles were determined by a spark chamber box which followed the bending magnet. A 4" x 4" x $\frac{1}{16}$ " counter defined the target region. Sets of spark chamber planes on each arm of a six foot diameter scattering platform served to measure the position and angle of the outgoing proton. Two $\frac{1}{16}$ inch dE/dX counters mounted on each arm were used for particle identification. The energy of the protons was measured using thick NaI detectors. The scattering platform, spark chamber boxes, NaI detector mounts,

Figure 2.1 Experimental Geometry (Overview)

and the spectrometer magnet are shown in the picture on the following page. A $C_1-C_2-C_3-C_4-C_5-C_6$ coincidence indicated that a (π^+ , pp) event had occurred within the target region.

In this experimental setup two other triggers indicated that a (π^+ , p_A) or (π^+ , p_B) event had occurred. Only for the investigation of the (π^+ , p) process on ${}^6\text{Li}$ were the NaI crystals thick enough to stop the emitted protons. It was necessary to employ a slightly different experimental setup for our (π^+ , p) measurements taken on ${}^9\text{Be}$, ${}^{12}\text{C}$, and ${}^{16}\text{O}$. This setup is shown in Fig. 2-2. A thick copper absorber was placed in front of the forward NaI crystal in order that the 200 MeV protons could be stopped in the crystal. Two extra dE/dX counters were added to provide better particle identification. With either setup an event coincidence triggered the spark chambers as well as the data acquisition system which read out and then transmitted to an on-line computer the measured experimental parameters. The computer was used for online data analysis as well as to store the acquired data on magnetic tape.

After the experiment was completed the raw data were analyzed and good events were written on summary tapes for further analysis and histogramming. A more complete description of the experimental details will be given in the following sections.

Beam

The external proton beam from the 184 inch cyclotron was used to produce the π^+ beam needed in the experiment. Protons coming out of the cyclotron "snout" were focussed on a production target by two

Figure 2.2 Experimental Geometry, (π^+, p) Data

sets of quadrupole magnets. The production target consisted of a 10 centimeter thick, 8 kilogram block of copper. Secondary particles produced by the protons striking the production target, and which came off at an angle of 30° went down a channel where those having momentum of approximately 160 MeV/c were swept by a Berkeley C magnet into our experimental area. This beam then passed through a set of two quadrupoles for further focussing. The proton contamination in the beam was ranged out by using a $\frac{1}{8}$ inch thick piece of polyethelene after the quadrupoles and immediately before the first set of hodoscope elements. Helium bags inserted after the production target and extending the length of the beam channel up to a point before the scattering platform served to reduce multiple scattering in the beam.

The energy spread in the pion beam was approximately 20 MeV. The determination of the momentum of each pion occurred in the following way. A second Berkeley C magnet, outfitted with shims on the pole faces to provide a more uniform magnetic field over a greater area, served as spectrometer. The position and angle of the incident particle were measured using three sets of X and Y spark chamber planes positioned before the bending magnet. A hodoscope and chamber plane tilted at 45° were installed in front of this box in an effort to make track reconstruction more accurate when multiple tracks occurred. Likewise, the exit angle and position were determined by three sets of X and Y planes positioned after the magnet. A field map of the bending magnet was made and was used to obtain first and

second order transport matrices through the magnet. A more detailed description of the method used to obtain the transport matrix elements is given in Appendix A.

The flux of particles from the beam and from background was extremely high at $S_1 - S_4$ of Fig. 2-1, and multiple tracks occurred a large fraction of the time. Although the chambers could handle up to four tracks at one time, they were still flooded and track reconstruction was difficult. The problem was remedied by installing a hodoscope with elements approximately one centimeter wide which gave the horizontal position of the particle before entering the magnet. From the position and angle of the particle measured after the magnet and the position before the magnet, the pion momentum was calculated using transport matrices which traced the particle backwards through the magnet. The resolution on the measurement of the energy of the beam particles was approximately 1 MeV.

Periodic checks were made at various points throughout the experiment to determine the percentage of pions in the beam. The majority of the measurements used the time of flight of the beam particles between counters 1 and 2 although in the early stages of setup some dE/dX measurements were made. The composition of the beam was determined to be

π^+	62 %
μ^+	9.6 %
e^+	27.8 %

Hodoscope and Scintillator Trigger Counters

The hodoscope consisted of four horizontal and sixteen vertical bins which were used to help determine the x and y position of an incident beam particle. Four "fingers" of $1.5" \times 6" \times \frac{1}{16}$ NE 102* plastic scintillator comprised the horizontal elements of the hodoscope. Each finger was viewed through a curved light pipe by an RCA 8575 phototube.** The design of the sixteen vertical elements was more complex. Six counters each consisting of $6" \times \frac{1}{16}"$ pieces of scintillator but with varying horizontal dimensions were used to form the binary register hodoscope shown below.

Once again each of the six counters was viewed by an RCA 8575 phototube. The photomultiplier tubes were equipped with voltage divider

* Nuclear Enterprises, Ltd., San Carlos, Calif.

** Radio Corporation of America, Harrison, N. J.

chains designed at Los Alamos and consisting of $\frac{1}{2}$ watt, 1% precision metal film resistors. The target counter consisted of a 4" x 4" x $\frac{1}{16}$ " sheet of NE 102 scintillator. The two dE/dX counters used on each arm were made of a 4" diameter, $\frac{1}{16}$ " thick disk of NE 102 viewed through a curved light pipe. All detectors used in the experiment were wrapped with a sheet of .00025" aluminized mylar covered by one layer of black photographic tape. Each detector was protected from magnetic field effects by two layers of magnetic shielding. The inner shield consisted of a layer of .060" of Netic-Conetic metal extending approximately two inches beyond the photocathode. The outer magnetic shields consisted of $\frac{1}{4}$ inch thick, three inch diameter iron pipes.

Targets

During the course of the experiment the following targets were used-- D_2O , ^{14}N , ^{12}C , 9Be , and 6Li . All of these targets were used for both (π^+ ,pp) and (π^+ ,p) runs with the exception of 9Be which was used only for (π^+ ,p) data and ^{14}N which was used for (π^+ ,pp) data. All targets consisted of three elements, separated by a distance of $1\frac{1}{8}$ inches center to center. The target in which the event originated was determined by the location of the intersection of both proton tracks and the pion track (for the π^+ ,pp case). Then a test was used to determine the actual target number. The resolution on such a vertex reconstruction was .3-.4 centimeters and therefore each of the three targets could easily be resolved as evidenced in the dot plot of the target distribution for π^+ ,pp events on 6Li shown on the following page. The use of three target elements allowed

Figure 2.3 Separation of Target Elements

us to increase our event rate while minimizing the loss in overall energy resolution. A short description of each target appears below.

D_2O : Three sheets of $.28g/cm^2$ thick jelled heavy water. The jellied solution was encapsulated between two $.0005''$ thick mylar sheets and was mounted on a plexiglass frame. The purity of the heavy water target was 91%.

^{14}N : Three target elements each $.12g/cm^2$ thick. In each of the three elements liquid nitrogen flowed between two sheets of $.001''$ thick mylar.

6Li : Three sheets of $.337g/cm^2$, 99% pure 6Li isotope enclosed in an inert atmosphere of 4He to prevent oxidation.

^{12}C : Three lucite sheets, each $.381g/cm^2$ thick.

9Be : Three sheets of pure 9Be isotope each of $.231 g/cm^2$ thickness.

NaI Counters

The detectors used to measure the energy of the emitted protons consisted of two five inch diameter by four inch thick NaI crystals.* The crystals were selected for their uniformity of pulse height response across the crystal to gamma rays from ^{137}Cs . A five inch diameter by one inch thick light pipe transmitted the light from the crystal to a RCA 4525 phototube. The light pipe also housed a Ferranti Gallium Phosphide light emitting diode.** An overall view

* Harshaw Chemical Company, Cleveland, Ohio

** Ferranti Electric, Inc., Long Island, N. Y.

of the detector is shown in Fig. 2-4. The NaI crystals, the phototube, the voltage divider, and the preamplifier were housed in a temperature stabilized environment to prevent gain drifts resulting from temperature fluctuations. A Borg-Warner^{*} heat pump attached to a three inch thick threaded copper block acted as a heat sink and thereby cooled the environment housing the components listed above. A layer of one inch thick polyurethane foam provided the necessary insulation. A thermistor located in the back of the assembly led to a control unit which turned the heat pumps on or off as needed to maintain an even temperature. The temperature in the detector assemblies was kept at approximately 13°C; the accuracy of the controllers was .5°C.

The voltage divider network used with the phototubes was specially designed for the experiment. The photocathode was at ground potential and the voltage on the anode was approximately +1800 volts. Precision metal film resistors were used in the construction of the resistor string to provide good temperature stability. In order to minimize pulse height shifts resulting from count rate variations a second high voltage supply was connected to the first four dynodes. Zener diodes for stabilization were tried in the earlier stages of design but were found to be too noisy and were discarded. The linear signal was taken from the third dynode and fed into an Ortec 109 charge sensitive preamplifier mounted inside the detector housing. The

* Borg-Warner Thermoelectrics, Des Plaines, Illinois.

Figure 2.4 Details, NaI Detector

linear signal was taken from one of the early dynodes in the resistor string to minimize gain changes due to temperature instabilities in the high voltage power supplies and also to insure good linearity.

The Ferranti XP 50 light emitting diode housed in the detector was used during the experiment for various tests and also to monitor any gain fluctuations in the phototube or electronics network. A light output approximately equivalent to a 150 MeV proton was obtained when a current of about 300 nanoamps at 12 volts was applied across the diode. The diode was temperature compensated so that temperature changes of the diode could be offset by the appropriate diode voltage change. The temperature coefficient of the diode was $-.7\%/^{\circ}\text{C}$. The method used to obtain precision pulses for triggering the diode was to drive a transistor far into the region of saturation. The temperature compensating network monitored the change in resistance of a thermistor placed close to the diode in the detector assembly. Any change in resistance was translated into a voltage change which was then fed to the sensing network of the precision power supply used to trigger the light pulser. Tests prior to the experiment gave a temperature variation in light output of about $+.08\%/^{\circ}\text{C}$. During the experiment some of the effectiveness of the light pulser was negated by undetected and uncorrected voltage changes in the circuit. However the light pulser information was used for various checks and tests during the experiments and also in the preliminary analysis of data after the completion of the experiment.

Magnetic shielding for the phototube was provided by two sets

of magnetic shields which extended five inches in front of the photocathode. The outer shield consisted of .060" thick nickel iron which reduced a magnetic field down to the point where the second shield of .060" thick mu metal could become effective. The magnetic shielding was tested during the experiment by varying the position of the detector with relation to the bending magnet used to determine the momentum of the pions and noting the position of the light pulser peak on a pulse height analyzer. Magnetic field effects would have produced a change in this pulse height but none were detected.

Spark Chambers

The spark chambers used in the experiment were of the magnetostrictive readout variety. The chambers were housed in RF tight aluminum boxes. Each box contained three X and three Y spark chamber planes. All connections--gas, high voltage, clearing field, signal input and output, and monitoring were made via feed-throughs which were grounded to the box. All of the boxes were machined to an accuracy of a few thousands of an inch and during the experiment the positions of the spark chamber boxes were known to a likewise accuracy. The chambers themselves were attached to aluminum mounting arms and their position on these arms was measured precisely before the experiment. This arrangement allowed us to determine the position of the chamber fiducials very accurately while permitting easy interchange or replacement of individual chamber units.

The chambers were manufactured by Science Accessories Corporation.* Each of the spark chamber boxes mounted on the scattering platform housed two copper wire chambers and one stainless steel wire chamber. The stainless steel chambers were placed on the target side of each of the boxes in an effort to decrease the effect of multiple scattering on our angular resolution. The copper wire chambers contained .005 inch diameter copper wire at a spacing of 50 wires per inch. The stainless steel chambers contained .0025 inch steel wire at the same spacing. The active area of the chambers was six inches by six inches.

An event coincidence dropped a voltage of 6000 to 6500 volts across the chambers and a spark occurred at the position of an ionization track from a charged particle. The spark set up a current in a neighboring wire. The current in the wire had associated with it a magnetic field which caused a change in permeability of a special wire "wand". A longitudinal wave was then set up in the wire. Finally a coil at the end of the wand measured the change in flux and this signal was sent to a SAC 1101 high impedance preamplifier. Fiducial and spark pulses were transformed by a comparator from a tripolar pulse into a standard logic level pulse. The first fiducial then turned on a 25 MHz clock. Four scalers were attached to each wand which were in turn turned off by the arrival of a spark pulse. Thus each plane could handle up to four sparks.

* Science Accessories Corporation, Southport, Conn.

A clearing field of 60 volts was applied to the chambers to sweep out ionized particles in order to prepare the chamber for the next set of tracks. A gas mixture of 90% neon and 10% helium flowed through all chambers at a rate of approximately 15 cubic feet per hour. The spark chamber gas was not vented but instead was purified and recycled through the system. The multiple track efficiency for each group of 3X and 3Y planes ranged from 97 to 99 percent. The spark chambers had a spatial resolution of .6 - .7 millimeters FWHM as determined from cosmic ray measurements.

Electronics - Event Logic

The trigger electronics were designed so that a coincidence from a (π^+ ,pp) or (π^+ ,p) event could go to our control room and the resulting trigger signal could go back to the spark chambers with as small a delay as possible. The efficiency of the spark chambers remained high for approximately 300 nanoseconds after an event coincidence was registered. A measurement of spark chamber efficiency versus trigger delay is shown in Fig. 2-5. The time delay between the occurrence of an event and a trigger from our event logic to the spark chambers was about 200 nanoseconds. Thus no duplicate set of fast electronics was needed inside the experimental cave.

Figure 2-6 shows an overall view of the fast logic system. More details of the logic system are shown in figures 2-7 and 2-8. The signal for an event coincidence was obtained as follows. The signals from hodoscope counters C_1-1 through C_1-4 were fed into a

2

Figure 2.5 Spark Chamber Efficiency as a Function of Trigger Delay

Figure 2.6 Event Logic (Overview)

Figure 2.7 Event Logic, (π^+, pp) Events

Figure 2.8 (π^+, p) Event and Event Register Logic

a LRS four fold discriminator and then were summed using an EGG DC mixer to produce a composite signal. A complete description of each NIM module appears in Appendix B. Similarly, hodoscope counters C_1-5 through C_1-10 were summed to produce a second composite signal. The signal from the target trigger counter, after being fed into a discriminator, was used to form a coincidence with the composite signals from the hodoscope counters. Such a coincidence indicated that a beam particle had passed through the bending magnet and had entered the target region. Signals from the trigger counters on each arm were fed into a coincidence unit to indicate whether a suitable trigger had occurred on arms A and/or B. For a (π, pp) event a coincidence between the " π ", A, and B signals was required. The logic for a (π^+, p_A) or (π^+, p_B) event was slightly different. Once again a " π " signal was required and a coincidence between the trigger counters on arm A or those on arm B was required. In addition the anode signal from the appropriate NaI had to be above a threshold corresponding to proton energies of 100 - 200 MeV.

With each (π^+, pp) or (π^+, p) event a coincidence was also formed between the event signal and the signal from each of the hodoscope elements. The resulting signal from this coincidence was then fed into the event register, which consisted of a series of latches that were set when triggered. The contents of the event register were subsequently decoded to determine which hodoscope element had fired.

A third type of trigger to the system in addition to the (π^+, pp)

and (π^+ , p) triggers was a light pulser trigger. The logic diagram for a light pulser event is shown in Fig. 2-9. A signal was produced by pulse generator #1 which was set to pulse approximately six times per minute. The signal from pulse generator #1 went into a discriminator for shaping and then into the event register where a flag was set which would signal the occurrence of a light pulser event. The output from the event register triggered an EGG trigger unit which provided a pulse to the NIDBUS system through an OR logic gate. The EGG trigger unit produced two other pulses, the first of which was used to eventually pulse the precision pulser to the light diode after being shaped and amplified. The second pulse was regenerated and delayed, and served to gate open the 8000 channel ADC's during a light pulser event.

Electronics--NaI and dE/dX Pulse Heights

Linear signals from the NaI detectors went from an Ortec 109 preamp into an Ortec 440 amplifier, where after being shaped and amplified, they were sent to a Northern Scientific 8000 Channel ADC. The anode signals from each of the trigger counters on arms A and B were split, with one of the signals going into a discriminator to eventually be used in the event coincidence, while the other signal went into an EGG linear gate for pulse shaping and then into an EGG 128 Channel ADC. These trigger counter pulse heights were later to be used for dE/dX information. A coincidence formed by any of the three previously mentioned event triggers was set into a fan out and was used to trigger a pulse generator which gated open the two 8000

Figure 2.9 Light Pulser Event Logic

Channel ADC's so that the pulse height could be recorded. Similarly, a signal from the event OR circuit strobed information into each of the six EGG ADC's. The linear electronics is shown in Fig. 2-10.

Data Acquisition System

The data acquisition system for the experiment consisted of an LRL Berkeley NIDBUS data control unit coupled with an interface designed and built at Los Alamos Scientific Laboratory to channel data into the computer. The computer used was XDS Sigma 2,^{*} with a memory of 32,000 16 bit words, a disk with a storage capacity of 300,000 words, and a magnetic tape unit for reading and writing data tapes of density 556 bits per inch. A simplified schematic of the NIDBUS data system and the interface is shown in Fig. 2-11.

A command from the computer was needed initially to reset the program controls (1 and 2) of the NIDBUS master control unit. A subsequent event coincidence then triggered the master control unit and also sent an interrupt to the computer. When the master control unit was triggered the contents of the two high resolution ADC's, the six 128 Channel ADC's, and the SAC Midas scalers (spark chamber coordinate scalers) were read via program 1. Twenty LeCroy scalers were read out via program 2. These scalers contained information such as the number of pions, the total number of triggers, external proton current readings, and the numbers of (π^+ ,pp) and (π^+ ,p)

* Xerox Data Systems, El Segundo, Calif.

Figure 2.10 Linear Electronics

Figure 2.11 NIDBUS Data Control Structure

triggers. A real event coincidence also generated a .3 → .4 second inhibit which prevented another event from being accepted until the spark chamber trigger network could recharge completely. The contents of the data string were then sent to the interface and then into buffers inside the computer. After the data string was transmitted to the computer the NIDBUS control unit was reset, thus making it ready to receive another event.

In case of a light pulser trigger a flag was set inside the interface and only the contents of the high resolution ADC's were sampled. Light pulser pulse heights were stored in a special buffer inside the computer. With light pulser events the spark chambers were not pulsed and there was no need to generate a long inhibit gate as was the case with true events.

The interface and computer also performed operations besides the control of data taking. A clock inside the computer activated the scan of a digital voltmeter^{*} at regular 3 second intervals. The digital voltmeter was a 2's complement, 12 bit device capable of sampling voltages on 64 channels with a choice of limits from 0 - 10 volts, 0 - 1 volt, and 0 - 100 millivolts. Limits for each of the channels could be loaded into the computer via cards or typed in on the control teletype. If the voltage strayed outside the present limits, a warning was typed on the teletype at which time the problem

* Fifth Dimension, Inc., Princeton, N.J.

could be corrected or the limits changed. The contents of the DVM were stored in a special buffer within the computer. The clock interrupt also activated a scope refresher routine at 32 millisecond intervals at which time a buffer was examined and the data contained in it was transmitted to the storage scope.

When the buffer containing the data in a compressed form (i.e. - spark chamber scaler overflows eliminated) became full, the contents of the event, scaler, DVM, header, and light pulser buffers were written on tape. This usually occurred every seven to nine events. The tape storage format is given in Appendix C. The header buffer contained information pertinent to the identification of the run such as run number, target, date, angles and so forth.

Online Analysis During the Experiment

The Sigma 2 computer had two operating configurations consisting of foreground and background modes. The foreground mode allowed uninterrupted data taking through the use of the NIDBUS data acquisition system. The background mode allowed one to run fortran programs such as those needed for online analysis or equipment testing. The foreground mode always operated at the highest priority and the amount of time the foreground was in use determined the speed of execution of any background program.

The ONLINE⁴³ analysis program was the principal program used during the experiment for data analysis. The main function of this online analysis was to check to make sure that all facets and components of the experiment worked properly. From the knowledge of the spark

chamber tracks and the measured proton energies ONLINE could calculate all kinematical parameters of interest. The total number of parameters calculated was 50, any one of which could be displayed through the use of 18 histograms or printed out at selected intervals on the teletype. A routine was available to display the X and Y co-ordinates of a set of tracks in the four spark chamber modules. Scaler information was printed out regularly and the DVM voltages and light pulser pulse heights could be sampled. Finally at the end of a run all histograms could be printed on the teletype along with the efficiency of the 26 spark chamber planes for (π^+ ,pp) and (π^+ ,p) events.

A simplified flow chart of the program is shown in Fig. 2-12. The program was written in segments which were stored on disk until called by a short main calling routine which resided in the central memory. Segment 1 served to initialize the program. Parameters and constants needed for the calculations were read in and histograms could be defined. Segments 2, 3, and 4 formed the analysis loop. In these segments the event buffer was read and the spark chamber scaler counts were translated into physical co-ordinates. Possible tracks were then formed from the spark locations in each plane. The tracks in the module were then fitted to an event occurring in the target. A χ^2 value for each combination of tracks was calculated and the combination giving the lowest χ^2 for a straight line fit was taken as the correct one. The track intersections at the targets were determined and the overall χ^2 was computed for the event. The pion momentum was calculated using the measured pion trajectory through the second

Figure 2.12 ONLINE Flow Chart

bending magnet. Proton energies were determined from the NaI pulse heights and the pion and proton momentum, energies, and angles were used for the kinematical analysis. Depending on the type of event, (π^+, pp) or (π^+, p), parameters were then calculated for three body or two body final states. Quantities of interest that were calculated were the missing mass or excitation energy of the residual nucleus, the opening angle between the two protons (π^+, pp), the angle between the pion and the proton, the recoil momentum of the residual nucleus and the relative momentum of the two emitted protons for (π^+, pp) events. Histograms were also incremented in this segment.

Segment 5 contained service routines used to display histograms, check the contents of the DVM and scaler, and print out the histograms.

Calibration Procedures During the Experiment

The calibration of the NaI detectors presented some difficulties during the experiment. Attempts to obtain a calibration through the extrapolation of pulse heights determined from known gamma ray energies up to the energy of the protons measured in the experiment proved completely unreliable. Efforts to calibrate the detectors by placing them in a reduced intensity pion beam were unsuccessful also. The calibration procedure which finally proved reliable was the use of the $\pi^+ + d \rightarrow p + p$ reaction. In this reaction the measurement of the vector quantities $\vec{p}$, $\vec{p}_1$, $\vec{p}_2$ provided nine determined (measured) kinematical quantities. Six quantities are needed to define the final state. However, the conservation of energy and momentum between initial and final states provided four constraints which meant that

the reaction had been overdetermined. The knowledge of only two quantities made it possible to calculate from simple two body kinematics the remaining four needed to describe the final state. The momentum of the incoming pion was well known and was used along with the angle of the proton in either arm to calculate the energies of the protons occurring in the reaction. The calculated energies of the protons were then corrected for energy loss occurring in the targets and in the material on each spark chamber arm. This corrected energy was then compared with the measured proton energy determined from the pulse height and some proportionality constant. The true proportionality constant was then determined by the constraint that the distribution of $T^{\text{cal}} - T^{\text{measured}}$ had to be centered about 0. During this procedure the intercept at zero pulse height was held constant. Determination of both the calibration slope and intercept by means of a straight line fit to π^+d data taken at different arm angles will be described in a later section. The fact that for the π^+d reaction $\vec{p}_R = 0$ implies that the quantities $\vec{p}_\pi$, $\vec{p}_1$, and $\vec{p}_2$ are completely coplanar. This constraint provided a check on the alignment of the equipment.

CHAPTER 3

DATA REDUCTION AND ANALYSIS

Introduction

Before meaningful distributions could be obtained from the experimental data, a thorough and complete analysis had to be done. Some analysis had been done on line during the operation of the experiment, but this was done mainly as a check to see how well the experiment was operating. A more detailed analysis had to be performed off line for the following reasons:

(1) to determine a set of consistent calibrations for the NaI detectors.

(2) to correct for geometrical and other effects so that the data could be presented in an unbiased form.

(3) to determine what cuts would have to be made on the data before it was presented in final form.

Offline Data Analysis

The computer code for offline track reconstruction and data analysis was similar to the ONLINE program described in Chapter 2. The program differed in the fact that there were some modifications made for use on the CDC 6600 computer. Also the program was changed slightly to calculate more parameters of interest, a list of which appears in Appendix D.

In order to obtain histograms and dot plots of experimental parameters it was necessary to rerun the offline program for each set of histograms. This proved to be a long and costly process since for each use of the program the particle tracks had to be reconstructed. It was only necessary to reconstruct the tracks once, and thus the program was modified to write summary tapes for analyzed (π^+ ,pp) or (π^+ ,p) events which contained the 400 calculated parameters. These summary tapes were then read by another program which could recompute the proton energies and all of the necessary kinematical quantities as well as produce histograms and dot plots.

Corrections to the NaI Pulse Height

In order to obtain the best possible resolution from the NaI detectors several corrections to the pulse height were tried. Pulse height variations could come primarily from temperature drifts, count rate fluctuations, and non-uniformities across the face of the NaI crystal. In order to test the effects of count rate variations on the pulse height, measurements were made for various beam intensities using the peak of the light pulser distribution. The peak position was found to change 5 per cent for a factor of 10 change in beam. The beam rate fluctuations were examined for each run and no fluctuations were found that could produce a noticeable shift in the pulse height. Any run to run changes were taken into account by adjusting the calibration slope for that particular run.

Throughout any particular run the temperature fluctuations were found to be small, usually less than $.5^{\circ}\text{C}$. Towards the end of

the experiment the temperature of the NaI detectors was reset about 4°C lower than normal when the equipment was restarted. This difference in temperature provided a basis for the determination of the combined temperature coefficient for the phototube and crystal, from which the results could be applied to correct the data. To determine the exact value of the coefficient, events from the deuteron data were used. Using the theoretical pulse heights determined from deuteron kinematics a quantity δ was defined by

$$\delta = (PH^{CAL} - PH^{COR}) / PH^{CAL}$$

The term PH^{COR} represented the pulse height corrected for temperature effects and drifts determined by the light pulser and was related to the measured pulse heights by

$$PH^{COR} = (A_1 + A_2 \times \text{temperature} + A_3 \times LP) \times PH^{MEASURED}$$

A χ^2 minimization procedure was used to obtain the value for A_1 , A_2 , and A_3 which could be used to correct the data. It was later determined that the light pulser variations merely reflected the drifts due to temperature and so a correction was made in terms of temperature alone.

The final type of correction to the pulse height arose from non-uniformities across the face of the NaI crystal. Before the experiment a map had been made of the pulse height response along the crystal using 35 MeV protons from the $d(^3\text{He}, p)^4\text{He}$ reaction and 662 keV gamma rays from ^{137}Cs . At that time some non-uniformities

were found, mainly at the outer edges of the crystal.

The position corrections were obtained in the following manner. The pulse heights, which had been corrected for temperature, were multiplied by a factor which took into account the variations as a function of x and y across the crystal, i.e.

$$PH = PH^{\text{CORRECTED}} \times (1 - f(x, y))$$

where the function $f(x, y)$ was expressed in terms of a set of two dimensional orthonormal polynomials. The form of $f(x, y)$ was

$$f(x, y) = \sum_{l=0}^{\infty} \sum_{M=0}^{\infty} a_{lM} \varphi_M^l(x, y)$$

where a_{lM} were the coefficients to be determined and $\varphi_M^l(x, y)$ were a set of polynomials orthonormal on a unit circle. The polynomials employed went up to order 3 and are given in Appendix E. The coefficients a_{lM} were determined by a χ^2 minimization of the expression

$$\delta = (PH^{\text{CORRECTED}} \times (1 - f(x, y)) - PH^{\text{CAL}}) / PH^{\text{CAL}}$$

where once again PH^{CAL} was the pulse height calculated from deuteron kinematics. The advantage of using orthonormal polynomials for the expansion of $f(x, y)$ was that the estimates for the coefficients could be obtained in an infinite series in a simple way without having to solve a complicated series of linear equations. The corrections were applied to pulse heights and were found to decrease the σ for the missing mass peak by 5% - 15%. The greatest change occurred when

events were lumped toward one side of the crystal as was the case with data taken at angles off the nominal settings for deuteron kinematics.

Overall Calibrations

Two methods for determining a set of calibration constants were used in the data analysis. For the D_2O data the process was fairly simple since the overdetermined deuteron kinematics could be used to extract a calibration. During the experiment the bulk of the deuterium data was taken at two arm angle settings, $50^\circ - 113^\circ$ and $100^\circ - 61^\circ$. From deuteron kinematics, and for a pion of energy of 70 MeV, the proton energies for arms 1 and 2 are centered at 122.3 MeV and 86.4 MeV for the first angle settings and 92.9 MeV and 116.3 MeV for the second settings. With the knowledge of pulse heights for two energy points it was possible to make a straight line fit to obtain the calibration slope and intercept based on data from all the deuterium runs. Actually the position of the deuteron break-up peak shifted slightly between runs as a result of slight voltage changes, changes in beam intensity and the like. Therefore, the overall intercept determined by the method described above was held fixed, while the slope was calculated from the constraint that the distribution $T = T^{cal} - T^{meas}$ had to be centered at 0. This is the same calibration method as described in Chapter 2.

The lithium data presented more of a problem in terms of extracting a calibration. For some lithium runs there were nearby D_2O runs which could be used to aid in the calibration. However,

there was a portion of the data which could not easily be compared to D_2O data because of the absence of nearby D_2O runs. Finally it was decided to determine the slopes by keeping the energy intercepts for the two detectors constant and minimizing the width of the missing mass distribution for 6Li events going to the 4He ground state, while constraining the average value of missing mass to be 0. The missing mass can be approximated by the following expression

$$\Delta M \sim T_{\pi} + M_{\pi} - T_1 - T_2 - |p_R| + \text{ENERGY LOSS IN ARMS}$$

or written in terms of calibration constants

$$\Delta M \sim T_{\pi} + M_{\pi} - S_1 \cdot PH1 - S_2 \cdot PH2 - |p_R| + \text{ENERGY LOSS} + \text{INTERCEPT}$$

where $S1$ and $S2$ are calibration slopes. Thus by a χ^2 minimization of this expression, values for $S1$ and $S2$ could be obtained when the combined intercept of the two detectors was held fixed.

This method of determining calibration constants was still lacking in that even though $\overline{\Delta M}$ was constrained to lie at 0, there was no constraint on the proton energy in each arm. Thus it was possible for the proton energy in arm A to be too high while the energy in arm B was too low, with a net cancellation of these effects in the missing mass determination. The parameter most sensitive to an incorrect proton energy was the Treiman Yang angle. The sensitivity of the Treiman Yang angle was such that a one per cent change in proton energy could produce a four per cent change in the asymmetry of the distribution where the asymmetry was defined as

$$\{ N_{\text{events}} (\Phi_{\text{TY}} \leq 90^\circ) - N_{\text{events}} (90^\circ < \Phi_{\text{TY}} \leq 180^\circ) \} / N_{\text{TOTAL}}$$

Thus a further constraint was needed on the energies of the individual protons. This was accomplished by the determination of the average energies for on-deuteron angle kinematics through the use of a pole model calculation to be described later. In this method the quantity

$$(\Delta M_i^2 / \sigma_{\Delta M}^2 + (\text{Int}^{(1)} - \text{Int}^{(1)}(\text{limit})) / \sigma_{\text{INT}^{(1)}}^2 + (\text{Int}^{(2)} - \text{Int}^{(2)}(\text{limit})) / \sigma_{\text{INT}^{(2)}}^2)$$

was minimized subject to the constraints that the average proton energies were those calculated from the pole model. Here the quantities $\text{Int}(\text{limit})$ were obtained from the overall fits to the deuterium data as well as was the value for σ^2_{int} . The two right hand terms in the above expression helped constrain the intercept value and prevented the intercepts from attaining ridiculous values. The success of this method of calibration was evident in that the calibration constants obtained were consistent with the values obtained from nearby deuteron runs.

The nitrogen runs were sandwiched between deuterium runs and the calibrations used for them were those determined from the deuterium data.

Geometrical Acceptance and Corrections to the (π^+ ,pp) Data⁴⁴

In order to present experimental data in a form which is readily comparable with theory, the experimental events have to be

corrected for the geometrical efficiency of the apparatus. Otherwise a particular theory has to be integrated over the special geometry of an experiment. Such was the case with previous (π^+ ,pp) experiments in which the data was not presented with corrections for geometrical effects.^{1,2}

Thus the efficiency function was obtained for our experimental geometry and was used to correct the experimental distributions for (π^+ ,pp) events such as opening angle, recoil momentum and Treiman Yang angle. This process will be described in the following sections.

Efficiency - General Remarks

In an experiment one measures a distribution Q made up of a group of values $\vec{x}$. This can be related to the true or source distribution by the expression

$$Q(\vec{x}) = \int A(\vec{x}, \vec{y}) T(\vec{y}) d\vec{y}$$

where $Q(\vec{x})$ is the distribution measured in the experiment having values $\vec{x}$; $T(\vec{y})$ is the source or parent distribution having values $\vec{y}$; and $A(\vec{x}, \vec{y})$ is the probability that a true event with values $\vec{y}$ is observed by the apparatus to have values $\vec{x}$. If measurement errors are neglected then generally $\vec{x}$ and $\vec{y}$ should be the same and $A(\vec{x}, \vec{y})$ becomes

$$A(\vec{x}, \vec{y}) = \delta(\vec{x} - \vec{y}) A(\vec{y})$$

Then the experimentally observed events $Q(x)$ is related to the parent distribution by

$$Q(\bar{x}) d\bar{x} = A(\bar{x}) T(\bar{x}) d\bar{x}$$

If one produces a known distribution $T(\bar{x})$ (for example by generating a uniform distribution through a Monte Carlo method), then the efficiency function can be obtained by dividing $T(\bar{x})$ by the final distribution as seen by the apparatus $Q(\bar{x})$ for each $\bar{x}$ in R . If the efficiency function $A(\bar{x})$ has been obtained in this manner the corrected "physics" data can be obtained by dividing the measured data by the efficiency function $A(x)$ for that value of x . Or, in other words, if the efficiency $A(\bar{x})$ is known, then an estimator for the integral of the parent distribution over the region R (i.e.

$$\lambda = \int_R T(\bar{x}) d\bar{x}) \text{ is given by}$$

$$\hat{\lambda} = \frac{1}{N_{\pi}} \sum_{\bar{x} \in R} \frac{1}{A(\bar{x})}$$

where $\hat{\lambda}$ is the estimator for λ . Thus to obtain unbiased distributions each experimental event is weighted by $1/A(\bar{x})$.

For the geometry employed in this experiment an exact calculation of the efficiency would have probably been impractical. Therefore, a Monte Carlo calculation of the efficiency was made. In this calculation a distribution of events was generated across the targets and rays were projected in a cone defined by the detectors.

From the number of Monte Carlo "successes" one could determine the solid angle of the apparatus as well as the efficiency function $A(\vec{x})$ for the variables $\vec{x}$. The program for the Monte Carlo generation of events will be described in a later section. Once the efficiency function $A(\vec{x})$ had been obtained, the (π^+, pp) events recorded on the summary tapes were histogrammed with each histogram being incremented by $1/A(x)$ for each event that fell within the limits of the histogram instead of the value 1 which would have been the case if no efficiency corrections were attempted.

For the (π^+, p) data analysis the primary quantity of interest was the differential cross section $(d\sigma/d\Omega)$ and the Monte Carlo program was used to calculate the detector solid angle. There were no individual experimental distributions where efficiency corrections were made for each event. Therefore the material described in the following sections will pertain to the (π^+, pp) data corrections.

Determination of the Variables for the Efficiency Function $A(\vec{x})$

The efficiency function $A(x)$ for (π^+, pp) events can be expressed in terms of any set of variables $\vec{x}$ as long as the physics of the reaction can be expressed in terms of them. Ten quantities are needed to specify the three body final state, but four are given by conservation equations. A set of six variables $\vec{x}$ can be specified by the quantities

$$\Theta_1, \Theta_2, \varphi_2, T_1, T_2, T_\pi$$

as measured in the laboratory system. Here Θ_1 and Θ_2 are the polar angles of the two emitted protons with respect to the incident pion; φ_2 is the azimuthal angle of the second proton or more exactly

$$\cos(\varphi_2) = (\vec{\pi} \times \vec{p}_1) \cdot (\vec{p}_2 \times \vec{\pi})$$

The quantities T_1 , T_2 , and T_π are the kinetic energies of the two protons and the pion. Other quantities such as the opening angle, recoil momentum, or various center of mass quantities could define the variables $\vec{x}$. However the set listed above has the advantage of natural decoupling. The energy of the pion is a function of the beam system only. The angular dependence is a function of geometry only and the energy acceptances are determined by the respective thresholds and the known dependence for proton interactions in NaI crystals. With the choice of these 5 quantities to represent the variables $\vec{x}$, the efficiency function $\Lambda(\vec{x})$ becomes

$$\begin{aligned} A(\Theta_1, \Theta_2, \varphi_2, T_1, T_2, T_\pi) = & A_1(\Theta_1, \Theta_2, \varphi_2) \\ & \times A_2(T_1) \times A_3(T_2) \times A_4(T_\pi) \end{aligned}$$

The term $A_4(T_\pi)$ can be ignored since for most cases only averages over T_π are considered. Thus the total efficiency function is

$$A = A_1 \times A_2 \times A_3$$

With this definition of A the following method was used to correct the events for acceptance. The acceptance function $A_1(\Theta_1, \Theta_2, \varphi_2)$ was generated from the Monte Carlo code and a three dimensional histogram of A was punched out. This efficiency histogram was then used by a histogramming routine. The functions $A_2(T_1)$ and $A_3(T_2)$ were calculated from the data of Measday et. al.^{45,46} for the percent of interaction for a proton of kinetic energy T in NaI. The total efficiency A was then used to give corrected histograms of experimental results.

Geometrical Efficiency Calculations

The computer code GEOLUG⁴⁷ was used to make the Monte Carlo calculations which determined the geometrical efficiency of the apparatus. The code operated as follows. A gaussian beam was generated across the face of the front spark chamber module using input parameters so that the generated beam phase space would correspond as closely as possible to the experimental beam. From its position on the spark chamber module a beam track was projected through a trigger counter which defined the target region. If the projected position on the trigger counter was outside the horizontal and vertical limits of the trigger counter, then a new pion would be generated. Next the target number was selected randomly and the position of the beam particle on the target was determined. At this point the particle was an accepted "try" and a counter was advanced. If the particle did not lie within the target area then a new event was generated. For

each event occurring on the targets, two outgoing particles were generated isotropically. For computing efficiency a cone was defined for each detector and events were generated into this cone. Thus each "try" was weighted by the fraction $\Delta\Omega$ of the solid angle subtended by the cone. The position of the outgoing particle was determined at the face of the detector and if both particles lay within the radius of the respective detectors then the event was accepted and fifty parameters describing the event were calculated. Otherwise a new particle was generated and the process began again. Interesting parameters could be displayed in up to eighteen histograms and a three dimensional efficiency histogram could be obtained.

Determination of the Monte Carlo Input Beam

The results obtained from the Monte Carlo code were fairly sensitive to the type of beam generated and to the distribution of events across each of the three targets. The attempt was made to reproduce the true experimental beam phase space parameters as closely as possible. Unfortunately during the experiment inadequate direct measurements were made on the incident pion beam. To determine the experimental beam characteristics we had to rely on beam phase space distributions obtained from either (π^+, p) or (π^+, pp) triggers. Events from either of these two triggers could produce biased phase space distributions. For example, events from (π^+, pp) triggers were dominated mainly by free deuteron kinematics, and for on-deuteron angle runs the distribution of events across the targets was fairly symmetric. However, at arm angle settings off the nominal

deuteron kinematics, the accepted events would be distributed so that opening angles corresponding to a free deuteron opening angle ($\sim 160^\circ$) would be preferred. This "physics" effect led to asymmetric distributions of events across the targets. The distribution of (π^+ ,p) events was found to drop rapidly at forward angles and then level off at backward angles. The (π^+ ,p) events taken at angles greater than 80° produced the least amount of bias in the beam distributions. Therefore the X and Y centroids and sigmas, and the corresponding quantities for the X and Y angles of the beam particles were obtained from an average over backward angle (π^+ ,p) runs. The centroids obtained from these distributions were checked with the corresponding values obtained from (π^+ ,pp) events taken on deuteron kinematics. The distribution for beam phase space parameters for data and Monte Carlo simulation is shown in figure 3-1. The data are for (π^+ ,p) events taken at $\Theta = 98^\circ$.

The agreement between Monte Carlo and experimental beam phase space was good except for regions on the extremities of the beam distribution. The uncertainty at the edges of the beam affected the distribution of Monte Carlo events on the targets and hence the number of efficiency counts in poor efficiency regions could fluctuate greatly for small changes in the beam distribution. This led to the definition of integration regions where the apparatus efficiency would be high enough to be relatively insensitive to uncertainties in the Monte Carlo beam. The next section will describe these integration regions and the error in efficiency associated with

Figure 3.1 Comparison of Experimental and Monte Carlo
Beam Phase Space

uncertainties in the beam.

Regions of Integration - Ellipse Definition

The integration ellipses defined in this section have a two-fold purpose.

(1) In order for an accurate comparison of a theoretical model to experimental data, a precise region of integration over the variables $\vec{x}$ must be explicitly defined.

(2) The integration regions define for each angular setting a region of high efficiency such that the sensitivity of the corrected data to errors in the Monte Carlo parameters is minimized.

The laboratory variables used to define the integration region were:

- (a) Θ_B - the angle between the direction of the pion and the second proton.
- (b) Θ_{AB} - the opening angle between protons 1 and 2.
- (c) ψ_2 - the azimuthal angle previously defined.
- (d) a branch number.

The first three quantities are straightforward - the fourth requires some explanation. If one describes a (π^+ , pp) event in terms of Θ_A , Θ_B , and ψ_2 then the event is unambiguously defined. However, if the event is described in terms of Θ_{AB} , Θ_B , and ψ_2 , then for each value of Θ_B a cone defined by the value of Θ_{AB} is generated which gives rise to some ambiguity in the value for Θ_A . The branch number is then defined by the sum of Θ_A^{proj} and Θ_B where Θ_A^{proj} is the value of Θ_A projected on the π - Θ_B

plane. The following conventions are defined for the branch number:

$$\Theta_A^{\text{proj}} + \Theta_B \leq 180^\circ \quad \text{branch number} = 1$$

$$\Theta_A^{\text{proj}} + \Theta_B > 180^\circ \quad \text{branch number} = 2$$

The integration region in Θ_B versus Θ_{AB} space was most easily defined by an ellipse. This ellipse was constructed such that a cut would be made on regions where the efficiency was less than 10 - 15 percent of the maximum efficiency value. The angular spread in φ_2 was almost identical for each set of arm angles taken with the same setting for Θ_B . Therefore a constant region for φ_2 was defined for each setting. Actually the geometrical efficiency of the apparatus depends on four angles, the angles listed here and the azimuthal angle of the first proton, φ_1 . Since the function $A(\vec{x})$ has been defined in terms of only a particular set of angles needed to describe the physics of the reaction, we therefore integrate over 2π in the first azimuthal angle φ_1 when presenting cross section results.

The integration ellipses in Θ_B versus Θ_{AB} space are shown in figures 3-2, 3-3, 3-4, and 3-5 for the arm settings at which (π^+ , pp) events were taken for ${}^6\text{Li}$, ${}^{16}\text{O}$, and ${}^{14}\text{N}$. Tables 2 and 3 give the pertinent parameters which define each ellipse as well as the limits for φ_2 and the branch number.

Figure 3.2 Integration Regions, θ_B and θ_{AB} Space,
 ${}^6\text{Li}$ Data

Figure 3.3 Integration Regions, θ_B and θ_{AB} Space, ^{16}O Data

Figure 3.4 Integration Regions, θ_B and θ_{AB} Space
 ^{14}N Data

CENX: Center of ellipse on θ_B axis
CENY: Center of ellipse on θ_{AB} axis
b, a : Major and minor axis of ellipse
p : Angle of rotation
 θ_{AB}^{LL} : Lower limit of θ_{AB} covered
 θ_{AB}^{UL} : Upper limit of θ_{AB} covered

Figure 3.5 Integration Ellipse Definition Parameters

Table 2

INTEGRATION REGION PARAMETERS - ${}^6\text{Li}$ DATA

<u>Arm Settings</u>		<u>Cenx</u>	<u>Ceny</u>	<u>a</u>	<u>b</u>	<u>ρ</u>	<u>Θ_{AB}^{LL}</u>	<u>Θ_{AB}^{UL}</u>	<u>Region</u>	<u>ϕ_2</u>
<u>A</u>	<u>B</u>									
38°	98°	98°	136°	3.93	26.48	60°	110°	158°	1	-10° to 10°
53°	98°	98°	151°	3.93	26.48	60°	130°	170°	1	-10° to 10°
63°	98°	98°	161°	3.93	26.48	60°	140°	180°	1	-10° to 10°
78°	98°	98°	176°	4.95	26.48	60°	158°	180°	1	-10° to 10°
80°	80°	78.5°	156°	4.1	24.7	60°	143°	176°	1	-10° to 10°
42°	123°	121°	160°	3.83	26.1	61°	143°	176°	1	-10° to 10°

Table 3
INTEGRATION REGIONS ^{16}O AND ^{14}N DATA

^{16}O :	Arm Settings		Cenx	Ceny	<u>a</u>	<u>b</u>	<u>ρ</u>	Θ_{AB}^{LL}	Θ_{AB}^{UL}	Region	ϕ_2
	<u>A</u>	<u>B</u>									
	25°	113°	113°	138°	3.93	26.1	60°	118°	156°	1	-12.5° to 12.5°
	37.5°	113°	113°	151°	3.93	26.48	60°	130°	168°	1	-12.5° to 12.5°
	50°	113°	111°	160°	3.86	23.87	60°	140°	176°	1	-12.5° to 12.5°
	65°	113°	106°	165°	3.93	17.1	60°	154°	180°	1	-12.5° to 12.5°
	80°	113°	106°	175°	4.1	10.2	60°	164°	180°	1	-12.5° to 12.5°
	100°	61°	59.5°	158°	4.24	23.85	61°	138°	178°	1	-12.5° to 12.5°
^{14}N :	57°	105°	105°	162°	4.24°	24.4	60°	140°	178°	1	-10.0° to 10.0°

Even with the ellipse definitions described here there was still some uncertainty in the acceptance as a function of the input parameters chosen for the Monte Carlo beam. This uncertainty was determined by two methods. Opening angle distributions were made for Monte Carlo efficiency events with sets of beam parameters which differed from the parameters nominally chosen for the "correct" beam. These sets of parameters reflected the uncertainty in the Monte Carlo beam since they represented some of the extreme beam values found during an examination of the experimental beam parameters. These opening angle distributions for efficiency events were then compared with the efficiency distribution obtained when the "correct" beam was used. The difference in the two distributions gave an indication of the uncertainty in efficiency due to the uncertainty in the Monte Carlo beam input parameters. The average uncertainty in efficiency was found to be 5 to 6 percent with the present ellipse cuts. A second and somewhat similar method involved the efficiency correction of selected experimental data with efficiency functions generated with Monte Carlo beam parameters which differed from the accepted values. Experimental data corrected with efficiency distributions generated with the accepted beam parameters were compared with this data. The difference in the two sets of corrected experimental data averaged approximately 4 to 5 percent.

Corrections Used in the Determination of Cross Sections

To obtain the true number of events N^{cor} to be used in the cross section, the number of detected events N^{det} had to be divided

by a correction factor. For (π^+ , pp) events

$$N^{\text{COR}} = N^{\text{DET}} / (N_{\pi} \cdot N_{\text{EFF}}^{\text{A}} \cdot N_{\text{EFF}}^{\text{B}} \cdot N_{\text{EFF}}^{\text{VERTEX}} \cdot N_{\text{EFF}}^{\text{D}} \cdot N_{\text{INT}}^{\text{A}} \cdot N_{\text{INT}}^{\text{B}})$$

where

- (1) N_{π} is the per cent of beam particles which are pions;
- (2) $N_{\text{EFF}}^{\text{A}}, N_{\text{EFF}}^{\text{B}}, N_{\text{EFF}}^{\text{D}}$ are the X and Y multiple track efficiency for spark chamber modules A, B, and D;
- (3) $N_{\text{EFF}}^{\text{VERTEX}}$ is the combined efficiency for module C and the efficiency for (π^+ , pp) event vertex reconstruction;
- (4) $N_{\text{INT}}^{\text{A}}, N_{\text{INT}}^{\text{B}}$ are the per cent of protons seen in the NaI crystals after interactions.

For (π^+ , p) events the number of corrected events is given by a similar expression except that in (2) only $N_{\text{EFF}}^{\text{A}}, N_{\text{INT}}^{\text{A}}$ or $N_{\text{EFF}}^{\text{B}},$

$N_{\text{INT}}^{\text{B}}$ appears depending on whether the (π^+ , p) event occurred in arm A or B. Also the $N_{\text{EFF}}^{\text{VERTEX}}$ is the efficiency for C plus the efficiency for reconstruction of a (π^+ , p) vertex. The values of the correction factors used for (π^+ , pp) and (π^+ , p) events are given in Appendix F.

Determination of Errors

The (π^+ , pp) and (π^+ , p) cross sections which are quoted in the following chapters have associated with them errors arising from several sources.

Statistical errors make up the first class of errors. For the (π^+ , p) data the statistical error is determined by the square

root of the number of events. For the (π^+ ,pp) data the determination of the statistical error is complicated by the fact that an event by event correction was made for the efficiency of the apparatus. The statistical error in this case also depends on the number of events generated in the Monte Carlo efficiency distribution as well as the number of experimental events.

The second class of errors in the determination of the (π^+ ,pp) and (π^+ ,p) cross sections arises from uncertainties in the values of various efficiencies used to obtain the true number of events, N^{cor} , described in the previous section. The contributions to this type of error are the following:

(1) Uncertainty in the percentage of pions in the beam.

(2) Uncertainty in the determination of the overall spark chamber efficiency, especially with respect to the determination of the vertex reconstruction efficiency.

(3) Errors in the determination of the percent of proton interactions in the absorber and NaI crystals. The percent interaction was taken from the theoretical calculations of Measday, et al,⁴⁵ and the experimental measurements of Palmieri and Wolfe.⁴⁶ The error was estimated on the basis of the difference in values quoted by the two sources. The total error resulting from various efficiencies was taken as the sum of the squares of these errors listed here. The numerical values of these errors appear in Appendix F along with the values for the correction factors.

For the (π^+ ,p) data there occurs one other source of

which must be included in the presentation of cross section results. This error is related to the uncertainty in the energy extrapolation from our calibration point up to the region for energetic protons from the (π^+ ,p) process. This error in energy extrapolation is a result of the error on the determination of the calibration constant and the energy intercept at zero pulse height. Errors also arise from the nonlinearity of the response of the NaI detectors to high energy protons. The total extrapolation error was estimated to be approximately 2 MeV.

CHAPTER 4

RESULTS - $A(\pi^+, pp)A-2$ REACTION

Introduction

This experiment was designed in part to test the two nucleon absorption mechanism and in particular the applicability of the quasi-deuteron pole model to this type of reaction. Previous experiments tested the pole model in a narrow region around angular settings that corresponded to opening angles for the free deuteron process. This experiment, through its greater angular coverage and better energy resolution, extended the region of interest over which the pole model would be tested in order to see at what point it began to break down. In this experiment the influence of the pole mechanism on the (π^+, pp) process leads to distributions which make the reaction process readily identifiable. The most important of these characteristics are listed below.

(1) Dominance of the pole mechanism leads to small values of recoil momentum

(2) A reaction process in which the target nucleus can be described by a pole diagram in terms of a quasi-deuteron and an A-2 system in an S state of relative motion yields an isotropic distribution in Treiman-Yang angle (Chapter 1).

(3) Events dominated by the pole mechanism occur mainly in a narrow band about the opening angle corresponding to absorption on a free deuteron.

(4) From the pole model behavior the cross section for events measured on "deuteron" angles should exhibit the same dependence on the center of mass angle as the free deuteron cross section i.e. an $A + \cos^2 \Theta_{CM}$ dependence.

The data will be presented then to identify the effects listed above and to point out deviations from them.

Pole Model Analysis of the (π^+, pp) Process

The main aspects of the pole model have been described in Chapter 1. Only the detailed expressions used to generate pole model distributions which were compared with the data will be described here. The expression for the differential cross section given by the pole model is^{48,2}

$$(4-1) \quad d\sigma = \Theta^2 \frac{\alpha}{\pi^2} \frac{W^2}{m_{np} p_\pi} \frac{|F(p_R)|^2}{(p_R^2 + \alpha^2)^2} \frac{p'_\pi}{p'} \left(\frac{d\sigma}{d\Omega} \right)_{np} \frac{p_1 p_2 dE_1 d\Omega_1 d\Omega_2}{|1 - \vec{v}_2 \cdot \vec{v}_R / v_2^2|}$$

This expression was derived by the CERN group for their (π^+, pp) analysis and is used here for comparison of the data obtained in the two experiments. The quantities which appear in the cross section expression are described below.

The quantity α is related to the binding energy of the deuteron in the target nucleus through the expression

$$(4-1-2) \quad \alpha = \sqrt{2ME_B} / \hbar$$

Here E_B is the binding energy and M is the reduced mass defined by

$$\frac{1}{M} = \frac{1}{m_{np}} + \frac{1}{M_R}$$

(2) $F(p_R)$ is the form factor which describes the momentum distribution of the quasi-deuteron in the target nucleus. The normalization for $F(p_R)$ is taken such that for S wave motion of the C.M. in a δ function potential, $F(p_R) = 1$. With this normalization

$|F(p_R)|^2$ is given by

$$|F(p_R)|^2 = (\pi^2/\alpha)(\alpha^2 + p_R^2)^2 |\varphi(p_R)|^2$$

where $\varphi(p_R)$ is the Fourier transform of the wave function representing the motion of the clusters

$$\varphi(\vec{p}_R) = (1/2\pi)^{3/2} \int d^3\vec{r} \psi(\vec{r}) e^{i\vec{p}\cdot\vec{r}}$$

(3) The quantities p'_π , p' , W^2 , and $(\frac{d\sigma}{d\Omega})_{np}$ are defined in the center of mass of the two outgoing protons; W^2 is the square of the center of mass energy; p'_π and p' are the center of mass momentum of the pion and proton; $(\frac{d\sigma}{d\Omega})_{np}$ is the center of mass cross section for the reaction $\pi^+ + d \rightarrow p + p$.

(4) The quantity $\left| \frac{p_1 p_2}{1 - \vec{v}_1 \cdot \vec{v}_2 / v_1^2} \right|$ is a phase space term.

(5) The term Θ^2 is defined as the probability that a target nucleus can be described in terms of a quasi-deuteron and an A-2 residual system. The quantity is a measure of the "effective" number of deuterons present in the target nucleus.

For the analysis of the oxygen and nitrogen data we applied the results of Balashov et al.^{60,61} who developed a generalized theory for the quasi elastic knockout of deuterons assuming a plane wave impulse approximation. Using shell model harmonic oscillator wave

functions, they calculated the fractional parentage coefficients and reduced widths θ^2 for the virtual particle breakup of an initial nucleus (A) which corresponds to a state (E, T, J) of a residual (A-2) nucleus. Here the quasi-deuteron is composed of two p-shell nucleons and can have values of angular momentum $l = 0$ or $l = 2$ with respect to the center of motion of the (A-2) system.

The differential cross section in the plane wave impulse approximation (PWIA) is given below⁵⁷

$$(4-2) \quad d\sigma = |\varphi(p_R)|^2 W^2 \frac{p_\pi'}{p'} \left(\frac{d\sigma}{d\Omega} \right)_{np} \\ \times \frac{E_3 p_1 p_2^2}{p_0 E_{21}} \frac{d\Omega_1 d\Omega_2 dE_1}{(p_2 E_3 + E_2 (p_2 - p_0 \cos \Theta_2 + p_1 \cos (\Theta_1 + \Theta_2)))}$$

The quantities W^2 , p_π^l , p' , and $(d\sigma/d\Omega)_{np}$ appear in Expression 4-1 and have been defined previously. The quantity E_3 is the recoil energy and E_{21} is equal to $\sqrt{p_R^2 + m_d^2}$ where p_R is the reduced momentum and m_d is the reduced mass of the deuteron. Once again $\varphi(p_R)$ is the Fourier transform of the relative motion of the clusters. The remaining quantities p_0 , p_1 , p_2 refer to the momentum of the pion and the two outgoing protons. This expression reduces to the form for the Charpak differential cross section (4-1) when the normalization given previously for $F(p_R)$ is used, and when the effective number of deuterons is defined as the following

$$\theta^2 = \int d^3\tilde{p} \varphi^2(p_R)$$

The derivation of the differential cross section expression (4-2) based on a method given by Jain et al.⁵⁷ appears in Appendix G. Also presented in Appendix G is a proof of the equivalency of the pole model and PWIA form for the differential cross section.

The presentation of the (π^+, pp) data will be mainly through the use of five kinematical variables. The first of these is missing mass, ΔM , defined as

$$(4-3) \quad \Delta M = [(E_\pi + M_A - E_1 - E_2)^2 - (\vec{p}_\pi - \vec{p}_1 - \vec{p}_2)^2]^{\frac{1}{2}} - M_{A-2}$$

where M_A and M_{A-2} are the masses of the target and residual nuclei. Missing mass is also known as the excitation energy of the residual nucleus.

The quantities $\vec{p}_{\text{recoil}}$ and $\vec{p}_{\text{relative}}$ are defined as

$$(4-4-a) \quad \vec{p}_{\text{recoil}} = \vec{p}_\pi - (\vec{p}_1 + \vec{p}_2)$$

and

$$(4-4-b) \quad \vec{p}_{\text{relative}} = \vec{p}_1 - \vec{p}_2$$

The recoil momentum distribution is especially interesting. According to the impulse approximation, it should lead to information concerning the momentum distribution of the absorbing pair, since in the laboratory $\vec{p}_{np} = -\vec{p}_{\text{recoil}}$.

The Treiman-Yang angle has been defined previously. Distributions in the Treiman-Yang angle will be presented because of its importance in determining the role of the pole mechanism in the reaction. The final quantity is the opening angle, or the angle between the two emitted protons, which is interesting from a general theoretical point

of view. It is also of importance to the pole model since for events dominated by this mechanism, the opening angle distribution should follow the distribution for the free deuteron process.

Deuterium Results

The overconstrained two body reaction of π^+ absorption on a free deuteron was used to check the performance of the experimental system. Results from this reaction were also used to test the assumptions made in the Monte Carlo codes which would be used to generate pole model cross sections and distributions. The differential cross section in the center of mass system for the process $\pi^+ + d \rightarrow p + p$ can be parameterized in terms of the well known $A + \cos^2 \theta_{CM}$ dependence where θ_{CM} is the center of mass angle. The differential cross section is then given by

$$(4-5) \quad \left(\frac{d\sigma}{d\Omega} \right)_{CM} = \sigma_{TOT} / 2\pi (A + \cos^2 \theta_{CM}) / (A + 1/3)$$

where the values for σ_{TOT} and A were obtained from the results of Richard-Serre et al.⁴⁹ and are the following

$$\sigma_{TOT} = 7.3 \pm 0.7 \text{ mb}$$

$$A = 0.23 \pm .04.$$

This expression was integrated over our experimental geometry and the values obtained were compared with the values obtained in the experiment. The results appear in Table 4.

The first two sets of θ_A and θ_B angles in the table correspond to opening angles of $\sim 160^\circ$ which is the opening angle for deuteron events. The two sets of angles correspond to different center of mass

TABLE 4

FREE DEUTERON CROSS SECTION RESULTS

<u>θ_A ($^\circ$)</u>	<u>θ_B ($^\circ$)</u>	<u>Measured Value (μb)</u>	<u>Calculated Value (μb)</u>
50	113	$2.94 \pm .37$	$2.83 \pm .2$
100	61	$1.98 \pm .28$	$2.0 \pm .15$
65	113	$1.09 \pm .19$	$0.9 \pm .1$
37.5	113	$1.95 \pm .3$	$1.8 \pm .13$
25	113	$.23 \pm .07$	$.12 \pm .02$

angles. The agreement between measured and calculated cross sections is good. The next three sets of angles correspond to opening angle values which are less or greater than the nominal "deuteron" angle of $\sim 160^\circ$. Deuteron events with opening angles of 160° could still be detected at these values of θ_A and θ_B although their distribution across the targets would be asymmetric. The asymmetry provided a test for the beam input parameters and overall assumptions used in the Monte Carlo code. The agreement between data and Monte Carlo cross sections is reasonable even for arm settings the opening angles of which are as much as $\pm 15^\circ$ from the deuteron angle of 160° . At the 25° - 113° setting there is a factor of two difference between the cross sections. This is not surprising since the events occur in a region which is very sensitive to the tails of the beam distribution and for which the geometrical efficiency is extremely poor.

The ${}^6\text{Li}$ Data

Missing mass distributions for the ${}^6\text{Li} (\pi^+, pp) {}^4\text{He}$ reactions are shown in Figs. 4-1, 4-2. The distributions are for data taken with $\theta_B = 98^\circ$. The distributions include only the events which lie within the integration ellipses defined in Chapter 3 and within the limits of the azimuthal angle defined there. The uppermost portion of Fig. 4-1 presents data summed over all opening angles covered in the experiment. The next distribution shows data which lie within an opening angle on or near the opening angle for the free deuteron process. The two remaining distributions in Fig. 4-2 present missing

Figure 4.1 Excitation Energy, ${}^6\text{Li}(\pi^+, pp){}^4\text{He}$ Events,
for All Opening Angles and for Deuteron Angles

Figure 4.2 Excitation Energy, ${}^6\text{Li}$ Events,
Off Deuteron Opening Angles

mass distributions for events the opening angles of which lie to either side of the free deuteron angle. The missing mass distributions are made up of two regions of interest. The first peak corresponds to the capture of the π^+ on the two weakly bound p-shell nucleons in ${}^6\text{Li}$ leading to the ground state of ${}^4\text{He}$. The higher excited states evident in peak 2 involve a more complicated explanation and will be discussed later. The missing mass resolution for the ground state peak is 4.5 MeV FWHM. With this resolution we were unable to separate any individual levels which may lie in the second peak. However various cuts do provide suggestions of nuclear structure.

This type of spectrum has been observed in the results of previous (π^+ ,pp)^{1,2} and (π^- ,nn)^{10,14} experiments and in (p,pd)^{50,51} reactions performed at low and medium energies.

For opening angles on or near the so called free deuteron angle most events correspond to absorption leaving ${}^4\text{He}$ in its ground state. In the cluster model this can be described as the absorption of a π^+ on a deuteron-like structure which is in some state of relative motion with an alpha particle. For opening angles which lie to either side of the free deuteron opening angle the ground state peak is suppressed and more contribution to the missing mass spectra comes from the excited region $18 \leq \Delta M \leq 40$ MeV. The ratio of ground state events ($-5 \leq \Delta M \leq 8$ MeV) to excited states ($18 \leq \Delta M \leq 40$ MeV) as a function of opening angle appears in Table 4-a.

Golovanova has studied the (π^+ ,pp) reaction on ${}^6\text{Li}$ assuming a cluster model representation for lithium. He interprets the excitation

TABLE 4-a

RATIO OF GROUND STATE ^4He EVENTS TO EXCITED STATE EVENTS

<u>Opening Angle Range (Degrees)</u>	<u>Ratio</u>
130 to 180	0.67
155 to 165	1.22
130 to 155	0.26
165 to 180	0.41

energy spectra in terms of absorption on a quasi deuteron (g.s. peak), absorption on a quasi alpha (peak two, $\Delta M > 25$ MeV), and absorption on two nucleons, one of which lies within the quasi alpha, the other in the quasi deuteron. Golovanova obtains a ratio for the relative excitation due to these three processes of 1/1.5/0.2. Our results integrated over all our opening angles do show qualitative agreement with his calculations.

Figure 4-3 shows the missing mass spectra for low and high recoil momentum cuts. One criterion for the applicability of the pole model is that the recoil momentum is low. When low recoil momentum cuts are made most of the contribution from the higher excited states is eliminated while the contribution from ground state events is hardly affected. The fact that the ground state peak is associated mainly with low recoil momentum events is illustrated in Fig. 4-4. Here recoil momentum is plotted for various regions of missing mass. Events in the ground peak have momentum which is less than 100 MeV/c for the most part.

Ground State Results

In this section we will present distributions obtained for events which occur in the region of missing mass, -5 to 8 MeV. Since these events are characterized by low recoil momentum we have used a quasi free mechanism to describe the reaction leading to the ${}^4\text{He}$ ground state. In order to apply the pole model cross section given in expression 4-1 we have made the following assumptions.

- (1) ${}^6\text{Li}$ can be described in terms of a deuteron and an alpha

Figure 4.3 Excitation Energy, ${}^6\text{Li}$ Events, for Various Regions of Recoil Momentum

Figure 4.4 Recoil Momentum, ${}^6\text{Li}$ Events, for Various Regions of Excitation Energy

particle bound together in an s-state of relative motion.

(2) In this approximation initial state interactions between the incoming pion and the target nucleus can be neglected. Likewise final states interactions between the two outgoing protons are not included beyond the effects included in the cross section for free absorption on a deuteron. In addition the wave functions have not been antisymmetrized.

(3) The free π^+ ,d cross section in the center of mass was obtained as function of energy and angle using a parameterization by Charpak² based on the results of Rose⁵³ and Measday et al⁴⁸ i.e.

$$(4-6) \quad \left. \frac{d\sigma}{d\Omega} \right)_{\pi^+d} = \frac{1}{2\pi} \frac{2}{3} \left(\frac{p'}{m_{\pi}c} \right)^2 \left(\frac{0.24}{\eta} + 0.74\eta \right) \\ \times \left(\frac{A + \cos^2 \Theta_{cm}}{A + 1/3} \right) \quad \text{mb/sr}.$$

In this expression $\eta = \frac{p'_{\pi}}{m_{\pi}c}$, p'_{π} and p' are the center of mass momentum of the pion and proton; Θ_{cm} is the center of mass angle. The parameter A was chosen to be 0.23.

To compute the form factor $F(p_R)$ we used the Hulthen wave function

$$(4-7) \quad \psi(r) = N_H (e^{-\alpha r} - e^{-\lambda r})/r$$

to describe the motion of the clusters. The quantity α is given by expression 4-1-a and is equal to 60.5 MeV/c for the binding energy of a deuteron in ${}^6\text{Li}$ of 1.47 MeV. The parameter λ was determined by a fit to give best agreement between theory and experiment. The value

chosen was $\lambda = 180$ MeV/c although the pole model results were relatively insensitive to λ in the range 120 to 250 MeV/c. The calculated cross section obtained from expression 4-1 using $\lambda = 120$ MeV/c and $\lambda = 180$ MeV/c differed by six percent.

The pole model cross section was integrated over our experimental geometry by the Monte Carlo method. Distributions in recoil momentum, Treiman-Yang angle, and ϕ_{TY} , were obtained and are compared in Figs. 4-5, 4-6, 4-7, 4-8 with the data for various opening angle cuts. Once again the region of integration was defined using the ellipses described in Chapter 3. Both the pole model and experimental data have been corrected for geometrical efficiency and for double counting due to overlap in the integration ellipses. Pole model events were normalized to the number of experimental events in a region around the free deuteron opening angle ($155^\circ < \theta_{AB} \leq 165^\circ$). The pole model would be expected to fit the shapes of the distributions best in the opening angle region corresponding to absorption on a free deuteron. This normalization was then used for the pole model events occurring off the free deuteron angle. This type of comparison serves to point out regions where the pole model and data distributions may disagree. Within our statistics the shape of the pole model curves agree quite well with the data distributions even for opening angles far off the free deuteron opening angle.

The Treiman-Yang distributions for events occurring on or around the free deuteron opening angle should be isotropic since the pole model predicts an even distribution in the cross section as a

Figure 4.5 Experimental Recoil Momentum Distributions,
 ${}^6\text{Li} (\pi^+, pp) {}^4\text{He}$ (Ground State)

Figure 4.6 Relative Momentum Distributions, ${}^6\text{Li}(\pi^+, pp){}^4\text{He}$ (Ground State)

Figure 4.7 Treiman-Yang Distributions, ${}^6\text{Li}(\pi^+, pp){}^4\text{He}$ (Ground State), On Deuteron Angles

Figure 4.8 Treiman-Yang Distributions, ${}^6\text{Li}(\pi^+, pp){}^4\text{He}$
(Ground State), Off Deuteron Angles

function of ϕ_{TY} when the absorbing cluster and the residual nucleus are in a relative s-state.^{35,36} Our distributions are biased toward high values of ϕ_{TY} . This is a result of the geometrical limitations of our apparatus which prevented our apparatus from accepting large values of p_R in certain directions. When a recoil momentum cut of 100 MeV/c or less is made then the distributions become more isotropic. This is shown in Fig. 4-7 for data obtained for $155^\circ < \theta_{AB} \leq 165^\circ$ with and without recoil momentum cuts.

Another test of the pole model occurs in the comparison of the cross section σ_{cal} calculated using expression 4-1 with the cross section σ_{exp} measured in our experiment. The results are given in Table 5 and have been integrated over the ellipses defined in Chapter 3. The values for σ_{cal} and σ_{exp} obtained by integrating over our entire acceptance for off deuteron angle runs are subject to errors resulting from events occurring in low efficiency regions and also resulting from the sensitivity of the Monte Carlo distributions to the input beam phase space. The values obtained as a result of integration over regions of good efficiency are much less sensitive to errors arising from these two causes. From the ratio of $\sigma_{exp} / \sigma_{cal}$ one can deduce the value for θ^2 or the "effective number of deuterons" in the ${}^6\text{Li}$ nucleus. These values for θ^2 also appear in Table 5 for the integration regions described above. The values for θ^2 obtained for the on-deuteron angle runs at $63^\circ - 98^\circ$, $42^\circ - 123^\circ$, and $80^\circ - 80^\circ$ are consistent with the value of 1.55 obtained by Charpak et al.² The calculated values of θ^2 again illustrate that the region of applicability

TABLE 5

EXPERIMENTAL AND POLE MODEL CROSS SECTIONS ${}^6\text{Li}$ DATA

θ_A	θ_B	Exp. Cross Sec. (μb) (-5 to 8 MeV)	PM Cross Sec. (μb)	θ^2
63	98	143 ± 18	$118 \pm 5. \text{ ub}$	$1.21 \pm .15$
53	98	135 ± 19	114.8 ± 6	$1.18 \pm .17$
38	98	10.3 ± 3.1	8.6 ± 1.1	$1.193 \pm .36$
78	98	$67. \pm 13.4$	59.5 ± 4.2	$1.13 \pm .23$
42	123	309.7 ± 43	221.75 ± 13.3	$1.4 \pm .2$
80	80	102.3 ± 16.3	72.7 ± 4.4	$1.39 \pm .22$

of the pole model extends even off the opening angle for free deuteron events.

The values for the effective number of deuterons obtained here are different from those obtained from $(p, pd)^{54,55}$ and $(p, p\alpha)^{56,57}$ reactions at lower energies. Values of θ^2 obtained from these reactions are of the order of .3 although a value of .8 was obtained for a ${}^6\text{Li}$ $(p, pd){}^4\text{He}$ reaction at 600 MeV.⁵⁰ Also Trouel and Bierter⁵⁸ have found a value of 1.44 in their analysis of elastic scattering of alpha particles on ${}^6\text{Li}$. The value of θ^2 seems, therefore, to depend on the nuclear reaction and it is difficult to compare absorption and quasi elastic knockout reactions since different momentum components are probed in the two types of reactions. Charpak et al.² have also explained the greater than unity difference between the quasi free cross section and the elementary process in the following manner. According to Breuckner⁸ the probability for the reaction $\pi + 2N$ to occur is a function of $|\psi(\xi)|^2$ where $\psi(\xi)$ is the wave function of the relative motion of the two nucleons and ξ is some separation distance at which absorption takes place. In the case of a bound deuteron the separation distance is closer to ξ than for a free deuteron, and hence the probability for absorption is greater.

Another test of the pole model is to check experimentally the assumption that the pole model cross section should behave much the same as the free cross section - i.e. that it should exhibit a dependence upon the square of the cosine of the center of mass angle. In this experiment we were able to extend our coverage to other center

of mass angles to check this prediction. The results are shown in Fig. 4-9 for events with low recoil momentum and on the deuteron angle, for events off the deuteron angle, and for all events. The dependence of the events upon the center of mass angle can be fit by a function of the form $A(.23 + \cos^2 \theta_{CM})$ which is exactly the dependence observed for the free cross section.

Finally Fig. 4-10 presents a distribution in opening angle for ground state events along with a curve based on the prediction of the pole model, normalized as previously described. Although our geometrical efficiency allowed us to detect events at opening angles from 110° to 180° , most of the events occurred in a region centered at the free deuteron opening angle of 161° with a FWHM of 12° . The fit of the pole model is quite good, and it appears that the pole model interpretation is quite reasonable even for events occurring off the deuteron angle.

Results - ^4He Excited States

The events corresponding to ^4He excited states peak at approximately 30 MeV. This region of the missing mass spectrum can be interpreted, using the shell model, as corresponding to the case where at least one nucleon is removed from the 1s shell. Also at these excitation energies contributions from absorption leading to the following ^4He breakup reactions can occur.

<u>Reaction</u>	<u>Threshold (MeV)</u>
$\pi^+ + {}^6\text{Li} \rightarrow t + p$	19.8
$\rightarrow {}^3\text{He} + n$	20.6
$\rightarrow d + d$	23.843
$\rightarrow n + p + d$	26.1
$\rightarrow 2n + 2p$	28.3

Figure 4.9 Center of Mass Angle Dependence, ${}^6\text{Li}$ Ground State Events

Figure 4.10 Opening Angle Distribution, ${}^6\text{Li}$
Ground State Events

The Charpak group² found that the structure of the ${}^4\text{He}$ excited states could not be explained using only four body phase space curves.

Figures 4-11, 4-12 illustrate the details of this missing mass region for high and low recoil momentum cuts and for opening angle cuts. Excitation in the region 20 to 23 MeV is weak, indicating that the $T = 0$ states at 20.2 (0^+), 21.4(0^-), and 22.4(2^-) MeV are not strongly excited. The peaking at 30 MeV occurs in the region where the predicted $T = 1$ states for ${}^4\text{He}$ would lie.

Another explanation for the existence of this peak has been advanced by several authors. Their explanation is that this peak arises from π^+ absorption on an alpha particle. It has already been mentioned that Golovanova⁵² has interpreted the excitation energy spectrum for ${}^6\text{Li}$ in terms of absorption on a deuteron and absorption on nucleons within an alpha particle. Kolybasov³⁷ has calculated distributions for opening angle, recoil momentum, and summed proton energy spectra for a pole mechanism where the absorbing cluster is an α particle. He approximated the matrix element for free absorption on an α particle as being equal to a constant. His curves obtained for an α particle model are wider than those for a deuteron pole mechanism but are less than the phase space for the ${}^6\text{Li}(\pi^+, pp){}^4\text{He}$ reaction.

Charpak and the CERN group investigated the reaction ${}^4\text{He}(\pi^+, pp){}^2\text{He}$ and found the distributions corresponding to the second peak resembled qualitatively the distributions obtained for the free process on ${}^4\text{He}$.

Figures 4-13, 4-14, 4-15, 4-16 illustrate unbiased distributions for recoil momentum, relative momentum, and ϕ_{TY} integrated over an

Figure 4.11 ${}^6\text{Li}(\pi^+, pp) {}^4\text{He}$ (Excited States) Excitation Energy Spectra for Various Regions of Recoil Momentum

Figure 4.12 ${}^6\text{Li}$ Excited States Excitation Energy Spectra for Various Opening Angle Regions

Figure 4.13 Recoil Momentum Distributions, ${}^6\text{Li}$ Excited States

Figure 4.14 Relative Momentum Distributions, ${}^6\text{Li}$ Excited States

Figure 4.15 Treiman-Yang Distributions, ${}^6\text{Li}$ Excited States, On Deuteron Angles

Figure 4.16 Treiman-Yang Distributions, ${}^6\text{Li}$ Excited States, Off Deuteron Angles

opening range from 110° to 180° . The integration is carried over the ellipses previously defined. No attempt has been made to compare the data with a theoretical model because of the complexities in the interpretation of events in the second peak. There are some features, however, which show qualitative agreement with the quasi deuteron pole model. The recoil momentum distribution for events with opening angles in the range $155^\circ \leq \theta_{AB} \leq 165^\circ$ is peaked at approximately 75 MeV/c while the recoil momentum distributions for the two other angle cuts ($\theta_{AB} < 155^\circ$, $\theta_{AB} \geq 165^\circ$) have their peaks at approximately 150 MeV/c. The Treiman-Yang angle distributions for the three opening angle cuts differ in that the events for $110^\circ \leq \theta_{AB} \leq 155^\circ$ and $165^\circ < \theta_{AB} \leq 180^\circ$ are slightly more asymmetric than the distributions for events which lie in the region $155^\circ < \theta_{AB} \leq 165^\circ$. If a low recoil momentum cut is made for events occurring in this region then the Treiman-Yang distribution becomes fairly isotropic (see Fig. 4-15). The opening angle distribution for events in the second peak is shown in Fig. 4-17. The distribution is much wider than for the ground state events but is still peaked at an opening angle of 161° to 162° . Finally Fig. 4-18 shows a distribution of events from peak two as a function of the square of the cosine of the center of mass angle. Here the cross section for these events also follows the $.23 + \cos^2 \theta_{CM}$ for the free deuteron process as did the ground state events.

^{16}O Results - Missing Mass Spectra

The missing mass spectrum for π^+ absorption on ^{16}O leading to residual states in ^{14}N is shown in Fig. 4-19-a. The main feature of

Figure 4.17 Opening Angle Distribution, ${}^6\text{Li}$ Excited States

Figure 4.18 Center of Mass Angle Dependence, ${}^6\text{Li}$ Excited States

Figure 4.19-a Excitation Energy, ^{16}O Events, with $T=0$ and $T=1$ Theoretical Curves of Kopaleishvili

the spectrum is the strong peaking in the region of 0 to 10 MeV missing mass. The excitation of the ground state appears small. Little excitation is noted for the missing mass region greater than 20 MeV.

Also shown in the figure are the theoretical predictions of Kopaleishvili⁵⁹ for the probability of excitation of individual levels in the residual nucleus folded with our resolution function. The calculations were made for a pion energy of 100 MeV and assumed absorption only by two p-shell nucleons. Scattering of the two outgoing nucleons from the residual nucleus was neglected but interactions between the ejected nucleons were included. The calculations have presumably been made by integrating over all opening angle and recoil momentum values. Curve one represents the contribution from $T = 0$ states while the contribution of $T = 1$ state appears in curve two. The normalization is such that the area under curve one and the data distribution are equal in the energy range from -2 to 20 MeV. The curve representing the sum of the $T = 0$ and $T = 1$ states contribution appears in Fig. 4-19-b. The area under the theoretical curve has again been normalized to the data in the region of -2 to 20 MeV. The spin-isospin numbers describing the states, the energy of the state as calculated by Kopaleishvili, the actual energy of the level in ^{14}N , and the probability for excitation of the state (in arbitrary units) appear in Table 6-a.

Our results show some disagreement with these predictions. It is difficult to arrive at definite conclusions because of our energy resolution and the fact that we have obtained events only for a portion of the total integration space. However, it does appear as though

Figure 4.19-b Excitation Energy, ^{16}O Events, with Combined Theoretical Predictions (Kopaleishvili) for $T = 0$ and $T = 1$ States

TABLE 6-a

LEVEL EXCITATION PREDICTIONS OF KOPALEISHVILI FOR
 THE $^{16}\text{O}(\pi^+, pp)^{14}\text{N}$ REACTION

<u>(J, T)</u>	<u>E(Kopaleishvili)</u>	<u>E(Exp)</u>	<u>Calculated Strength (Arbitrary Units)</u>
1,0	0	0	3.8
0,1	2	2.313	1.6
1,0	4	3.95	5.
2,0	5→6	5.106	3.5
2,1	9	9.17	3.8
1,1	12	12?	2.5
3,0	12?	12.7	.8
0,1	15	15?	1.4
2,1	18	18?	3.75
1,0	20	20	1.6

most of the contribution to our missing mass spectrum arises from excitation of the $J = 2^-$, 1^- , 3^- ($T = 0$) states at 5.1, 5.7, and 5.8 MeV rather than the 1^+ ($T = 0$) state at 3.95 MeV.

Our excitation spectrum can also be compared with the fractional parentage coefficients obtained by Cohen and Kurath⁶³ for two nucleon transfer in the $1p$ shell. In their calculations the wave function for N $1p$ nucleons is related to the wave function for $(N - 2)$ $1p$ nucleons when two nucleons are removed. The two transferred nucleons are coupled to angular momentum J and isospin T . The transition strengths are calculated for the case when the orbital angular momentum of the transferred nucleons is 0 with respect to the $N-2$ system, and for the case when the two nucleons are in a D state with respect to the residual system. Figure 4-19-c presents their calculation for pickup with $A = 16$ ($J = 0$, $T = 0$). The contributions from $T = 0$ and $T = 1$ states have been separated as well as have the contribution of $l = 0$ and $l = 2$ transitions for each value of T . Table 6-b lists the spin and isospin number of the level, its energy, and the values of the coefficients for $l = 0$ and $l = 2$ transfer. Figure 4-19-d presents the summed contribution from the states listed in Table 6-b.

Our results are in poor agreement with these calculated strengths. Cohen and Kurath predict a large excitation of the ground state through a $l = 2$ transfer whereas we see little evidence for such excitation. An extremely large contribution for the $J = 2$, $T = 1$ state at 9.2 MeV is expected on the basis of their calculated strength. Once again we see little excitation in this region.

Figure 4.19-c Excitation Energy, ^{16}O Events, with Fractional Parentage Curves (Cohen-Kurath)

Figure 4.19-d Excitation Energy, ^{16}O Events, with Summed Contribution from Curves in Fig. 4.19-c

TABLE 6-b

FRACTIONAL PARENTAGE CALCULATIONS OF COHEN - KURATH

FOR PICKUP A = 16(0,0)

<u>(J, T)</u>	<u>Energy (Exp)</u>	<u>Strength (L = 0)</u>	<u>Strength (L = 2)</u>
1,0	0. MeV	.016	2.704
1,0	3.94	2.675	.081
1,0	6.2	0.308	.214
0,1	2.31	2.212	
0,1	8.62	0.788	
2,1	9.2		12.721
2,1	10.2		2.277

Figures 4-20, 4-21 present missing mass distributions for regions of opening angle on and around the free deuteron angle as well as off this angle. Also shown are missing mass distributions for high and low recoil momentum cuts. Here, as in the case for all distributions relating to (π^+ , pp) data (unless otherwise noted), the data have been integrated over the region previously defined in Chapter 3. Low recoil momentum cuts enhance the peaking in the region 0 to 10 MeV while most of the contribution from higher values of missing mass is eliminated. When high recoil momentum cuts are made more contribution from the missing mass region of 15 to 30 MeV occurs. Opening angle cuts produce the same effect, with cuts on the free deuteron angle resulting in the dominance of the peak between 0 to 10 MeV, and off deuteron angle cuts leading to greater contributions for the higher regions of the missing mass spectrum.

We have used the PWIA form for the differential cross section given in expression 4-2 to obtain theoretical curves to compare with the oxygen experimental data. The expression $|\phi(p_R)|^2$ was calculated using the shell model harmonic oscillator wave functions given by Balashov to describe the relative motion of the clusters. The possible values for the quantum numbers n and l are $n = 2, l = 0$ and $n = 1, l = 2$. The values for the square of the Fourier transform then become ⁶²

$$\begin{aligned}
 l=0 \quad |\phi_0(p_R)|^2 &= \frac{1}{4\pi} \frac{6}{\sqrt{\pi}} \frac{1}{p_0^3} \left(1 - \frac{2}{3} \left(\frac{p_R}{p_0}\right)^2\right)^2 \exp\left(-\left(\frac{p_R}{p_0}\right)^2\right) \\
 (4-8) \\
 l=2 \quad |\phi_2(p_R)|^2 &= \frac{1}{4\pi} \frac{16}{15\sqrt{\pi}} \frac{1}{p_0^3} \left(\frac{p_R}{p_0}\right)^4 \exp\left(-\left(\frac{p_R}{p_0}\right)^2\right)
 \end{aligned}$$

Figure 4.20 Excitation Energy, ^{16}O Events, for Various Opening Angle Regions

Figure 4.21 Excitation Energy, ^{16}O , for Different Recoil Momentum Values

The normalization is taken such that

$$\int |\phi_l(p_R)|^2 d^3p = 1 \quad \text{for } l=0,2$$

The square of the Fourier transform is actually some combination of contributions from S and D state deuterons and the expression for

$$|\phi(p_R)|^2 \quad \text{becomes}$$

$$(4-9) \quad |\phi(p_R)|^2 = \alpha_0 |\phi_0(p_R)|^2 + \alpha_2 |\phi_2(p_R)|^2$$

where α_0 and α_2 give the relative probability for contributions from $l = 0$ and $l = 2$ deuterons. The quantity p_0 is related to the harmonic oscillator frequency

$$p_0 = (\hbar \mu \omega)^{1/2}$$

with μ being the reduced mass of the deuteron in the nucleus. We have determined the values for p_0 , α_0 , and α_2 in the following manner. From expression 4-2 it can be seen that the quantity ($\Delta\sigma$ /PHASE SPACE) is proportional to the square of the Fourier transform. To best approximate the shape of the $\Delta\sigma$ /PS distribution different values of p_0 were tried and for each value of p_0 a least squares fit was made to determine α_0 and α_2 . The distribution of ($\Delta\sigma$ /PHASE SPACE) for oxygen events is presented in Fig. 4-22 along with the calculated values of $\phi(p_R)$ which gave the best fit to the data in the range of recoil momentum 0 to 250 MeV/c. The values obtained for α_0 and α_2 were 66% and 34%; p_0 was determined to be 180 MeV/c.

The quantity p_0 is related to the harmonic oscillator length parameter r_0 by the expression $p_0 r_0 = \hbar$ where r_0 is

Figure 4.22 Fit of $|\phi(p)|^2$ to $(\Delta\sigma/PS)$ Distribution, $^{16}_0$ Events (-2 to 10 MeV)

defined by

$$r_0 = \left(\frac{\hbar}{\mu\omega} \right)^{\frac{1}{2}}$$

Our value of $p_0 = 180$ MeV/c results in a value of r_0 equal to 1.1 fm.

The value of r_0 can be related to the r.m.s. radius of a nucleus through the following expression for the case of shell model harmonic oscillator wave functions ⁶⁴

$$(4-10) \quad \langle r^2 \rangle_{SM} = \frac{Z-2}{Z} r_0^2 + \frac{3}{2} r_0^2.$$

With this expression we obtain a value of 1.6 fm. for the r.m.s. radius of ^{16}O . This is smaller than the value of 2.65 fm. ($r_0 = 1.76$ fm.) obtained from electron scattering measurements. ⁶⁴

This disagreement in the value which we obtained for r_0 and the value obtained from electron scattering data indicates a breakdown of the PWIA. Attempts to fit the distribution of Fig. 4-22 with a value of p_0 corresponding to a larger value of r_0 resulted in a theoretical curve which was too narrow to fit the experimental data in the momentum range involved.

^{16}O Results: -2 MeV to 10 MeV Missing Mass

In this section we present distributions for the events occurring in the region of excitation energy -2 to 10 MeV. These limits encompass the peak which is the main feature of the missing mass spectrum for $^{16}\text{O}(\pi^+, pp) ^{14}\text{N}$ events. All data have been corrected for geometrical efficiencies and have been integrated over the ellipse regions defined previously. Unless otherwise noted the data are from runs taken with

$\theta_B = 113^\circ$. Since the events associated with this region of missing mass are mainly of low recoil momentum, we have calculated theoretical curves using the plane wave impulse approximation described in the previous sections.

Figure 4-23 presents recoil momentum distributions for three ranges of opening angles corresponding to the free deuteron opening angle and to values to either side of the deuteron angle. Figure 4-24 shows relative momentum distributions for the same values of opening angles. With the experimental distributions curves corresponding to PWIA calculations are given. As with the lithium data the curves have been normalized to the experimental data in the region around the deuteron angle ($158^\circ < \theta_{AB} \leq 168^\circ$). For opening angles around the free deuteron angle, the values of recoil momentum are low, with the peak centered at approximately 70 MeV/c. For opening angle regions off the free deuteron angle the recoil momentum distribution is wider and is shifted towards higher values of recoil momentum. For off deuteron angle data one might expect that some of the width in the recoil momentum distribution arises from the scattering of one or both of the emitted protons off the residual nucleus which would redistribute the values of the recoil momentum. If this is the the case then some of the information concerning the momentum distribution of the absorbing pair in the target nucleus is lost. The PWIA predictions agree qualitatively with the experimental recoil momentum distributions. Agreement is also obtained between experimental and calculated distributions in relative momentum. The relative momentum peaks at 830 MeV/c for on deuteron

Figure 4.23 Recoil Momentum Distributions, ^{16}O Events

Figure 4.24 Relative Momentum Distributions, ^{16}O Events

angles. Off the deuteron angle the distribution is wider and is shifted towards lower values for events in the opening range 110° to 158° . The shift is in the opposite direction for events with $168^\circ < \theta_{AB} \leq 180^\circ$. The calculated curves reproduce these general characteristics.

Figure 4-25 presents Treiman-Yang angle distributions for events on deuteron angle with and without recoil momentum cuts. The distribution becomes isotropic for low values of recoil momentum. Distributions for the other two values of opening angle appear in Fig. 4-26. The distributions are somewhat more asymmetric but this behavior is also reflected by the PWIA curves.

The opening angle distribution for oxygen events is given in Fig. 4-27. The peak occurs at a value of $162^\circ - 163^\circ$ which is the value of the opening angle for absorption on a free deuteron. The opening angle peak (FWHM = 22°) is wider than the distribution obtained for ${}^6\text{Li}$ ground state events. There is agreement between the experimental distribution and the curve obtained from the PWIA based on shell model harmonic oscillator wave functions although the calculated curve is slightly wider than the data. The fact that the opening angle peaks at the value corresponding to the free process again confirms the role of the quasi-deuteron mechanism when applied to the oxygen data.

We have obtained the value for the effective number of deuterons, θ^2 , in ${}^{16}\text{O}$ from the ratio of the experimentally determined cross section to that obtained from the PWIA. Our results give a value approximately 10 for θ^2 . The results obtained for θ^2 for each arm setting is given in Table 7. Our value for the effective number of deuterons is in

Figure 4 25 Treiman-Yang Distributions, ^{16}O Events, Deuteron Angles

Figure 4.26 Treiman-Yang Distributions, ^{16}O Events,
Off Deuteron Angles

Figure 4.27 Opening Angle Distributions, ^{16}O Events

TABLE 7

EXPERIMENTAL AND PWIA CROSS SECTIONS ^{16}O DATA

θ_A	θ_B	Exp Cross Sec (μb) (-2 to 10 MeV)	PWIA Cross Sec (μb)	θ^2
50	113	279 ± 42	30.3 ± 1.5	9.2 ± 1.56
100	61	231.2 ± 46	21.88 ± 1.1	10.5 ± 2
25	113	158.9 ± 52	14.2 ± 1.2	11.2 ± 3.6
65	113	148 ± 49	17.38 ± 1.5	8.7 ± 2.8
37.5	113	323 ± 78	29.64 ± 2.1	10.89 ± 2.6

qualitative agreement with the results obtained by Balashov et al.^{60,61} for the reaction $^{16}\text{O}(p, pd)^{14}\text{N}$. His value for Θ^2 is 10.3 if one sums over the 0 MeV (1^+), 3.9 MeV (1^+), 5.1 MeV (2^-) levels and includes both $l = 0$ and $l = 2$ transfers. We were not able to separate individual states and have therefore assumed that the relative contributions of the $l = 0$ and $l = 2$ terms to $|\phi(p_R)|^2$ was the same for all levels excited in the missing mass region of -2 to 10 MeV. Our value for Θ^2 is also in agreement with the results of Beregi et al.⁶⁵ who calculated the effective number of deuterons through the use of the oscillator shell model and considered the cluster constructed only from p-shell nucleons. They obtained a Θ^2 of 16.5 assuming a deuteron with an internal state $J = 1^+$, $T = 0$. Our value obtained for Θ^2 is somewhat different than the qualitative estimates made by Charpak et al.² for the effective number of deuterons in ^{16}O . From phase space restrictions they assumed that their measured cross section would involve only the transfer of a $l = 0$ deuteron. They compared the ratio of the cross section for events with the missing mass in the range of 0 to 20 MeV with the cross section for π^+ absorption on a free deuteron. They obtained a value of 2 for Θ^2 which agreed with the results of Balashov if only $l = 0$ transfers were included. Our fit to the $(\Delta\sigma/PS)$ distribution showed that for our phase space a sizable contribution arose from the transfer of a $l = 2$ deuteron. If the $l = 2$ term was not included we obtained unrealistic fits to this distribution over the range in recoil momentum of 0 to 250 MeV/c.

For oxygen (π^+ , pp) events we went to one other value of the center of mass angle. The range in the square of the cosine of the center of mass angle is not as great as for the lithium data but some information can still be obtained concerning the dependence of the cross section on $\cos^2 \theta_{CM}$. The dependence is shown in Fig. 4-28 for various regions of recoil momentum and opening angle. The results generally follow the center of mass angle dependence of the free deuteron cross section. An exception may be events which have high recoil momentum and which lie off the deuteron angle. The cross section is less strongly dependent on $\cos^2 \theta_{CM}$ than the deuteron cross section. Since our statistics are not good in this region, and since our range in center of mass angle is not great, definite conclusions are difficult to reach.

^{14}N Results - Missing Mass Distributions

Figure 4-29-a presents the missing mass distribution for the reaction $^{14}\text{N}(\pi^+, pp)^{12}\text{C}$ integrated over opening angles of 140° to 178° .

The main features of the spectrum are

- (1) little or no excitation of the ground and first excited states;
- (2) strong excitation of a group of states around 12 MeV;
- (3) excitation of a group of states centered at approximately 30 MeV.

Kopaleishvili⁵⁹ has also made a calculation of the probability of excitation of individual levels in the residual ^{12}C nucleus resulting from π^+ absorption on ^{14}N . The assumptions made in the calculations

Figure 4.28 Center of Mass Angle Distributions, ^{16}O Events

Figure 4.29-a Excitation Energy, ^{14}N Events, with $T = 0$ and $T = 1$ Calculated Curves (Kopaleishvili)

have been described in the section dealing with ^{16}O results. Listed in Table 8-a are the spin and isospin numbers of the states, the excitation energy calculated by Kopaleishvili, the excitation energy of the level as it appears in the level structure of ^{12}C , and the excitation strength of each level (in arbitrary units).

The curves appearing in the figure give the predictions of Kopaleishvili for the excitation of $T = 0$ and $T = 1$ states folded with our resolution function of 4.5 MeV. The areas under the theoretical curve for $T = 0$ states and the experimental distribution have been normalized in the region -2 to 20 MeV. Figure 4-29-b presents the curve which is the combination of contributions from $T = 0$ and $T = 1$ states. Our experimental results disagree with these predictions in that we do not see any evidence of excitation of the 2^+ state at 4.44 MeV. Our excitation spectrum indicates that the $T = 0$ states at $10.3(1^-)$, $11.8(2^-)$, and $12.7(1^+)$ MeV are excited strongly rather than the $4^+(T=0)$ state at 14.08 MeV as predicted by Kopaleishvili.

The results of the fractional parentage calculations of Cohen and Kurath⁶³ for pickup in the target $A = 14(J = 1, T = 0)$ appear in Fig. 4-29-c. As with the oxygen data we have separated the contribution for $T = 0$ and $T = 1$ states. For each value of T the contributions for $\ell = 0$ and $\ell = 2$ transfer have also been separated. The curve appearing in Figure 4-29-d is the composite curve obtained by including the contribution from both $T = 0$ and $T = 1$ states, as well as the contribution from $\ell = 0$ and $\ell = 2$ transfers. The spin and isospin numbers describing the state, its energy, and the calculated strengths for $\ell = 0$ and $\ell = 2$ transfers appears in Table 8-b.

TABLE 8-a

LEVEL EXCITATION STRENGTHS OF KOPALEISHVILI

FOR THE $^{14}\text{N}(\pi^+, pp)^{12}\text{C}$ REACTION

<u>(J, T)</u>	<u>E (Theor.)</u>	<u>E (Exp)</u>	<u>Calculated Strength</u>
1,0	0 MeV	0 MeV	0.5
2,0	4	4.44	1.0
4,0	12	14.08	4.
0,0	13	13?	0.6
1,1	15	15.11	0.5
1,0	~ 16.	16.	0.5
2,1	~ 16.	16.6	5.0
2,0	20.	19.4	0.4

Figure 4.29-b Excitation Energy, ^{14}N Events with Sum of Curves
Appearing in Fig. 4.29-a

Figure 4.29-c Excitation Energy, ^{14}N Events, with Fractional Parentage Curves (Cohen-Kurath)

Figure 4.29-d Excitation Energy, ^{14}N Events, with Sum of Curves from Fig. 4.29-c

TABLE 8-b

FRACTIONAL PARENTAGE CALCULATIONS OF
COHEN - KURATH FOR PICKUP A = 14(1, 0)

<u>(J,T)</u>	<u>Energy (Exp)</u>	<u>Strength (L = 0)</u>	<u>Strength (L = 2)</u>
0,0	0 MeV	.015	.355
2,0	4.44	.652	.151
0,0	7.65	.016	.333
1,0	10.8	.157	.003
1,0	12.7	.280	.036
2,0	13.35	.063	.022
2,0	19.4	.107	.099
1,1	15.2	.033	2.416
2,1	16.6		2.453
1,1	19.2	1.383	.002
2,1	19.6		.324
3,1	22.5?		1.808
1,1	23.6	.339	.002
2,1	23.9?		.965

Our results are in poor agreement with these predictions. A strong excitation of the ground state through the transfer of a $\ell = 2$ deuteron and a strong excitation of the $J = 2, T = 0$ state at 4.4 MeV through a $\ell = 0$ transfer are predicted. We see no evidence for the excitation of these states. The predictions for the excitation of $T = 1$ states also fail to fit our spectrum. According to their calculated strengths, the strongest contribution to the spectrum should come from the excitation of the $J = 1, T = 1$, state at 15.2 MeV and the $J = 2, T = 1$ state at 16.6 MeV. We see relatively little excitation in the region where these states should occur.

The shell model configuration of ^{14}N is $(1s_{1/2})^4(1p_{3/2})^8(1p_{1/2})^2$. In the cluster model ^{14}N can be described as being composed of the ground state of ^{12}C and a quasi-deuteron composed of the two $p_{1/2}$ shell nucleons. The absence of capture leading to ground state events and the predominance of higher excited state events leads to the conclusion that the π^+ is absorbed not on two $p_{1/2}$ shell nucleons but instead that absorption takes place on two of the nucleons in the $p_{3/2}$ shell.

The second peak in the spectrum could possibly arise as the result of capture on a $1s$ shell nucleon correlated with a $1p$ shell nucleon. The peak can also be explained in terms of capture in the ^{14}N nucleus ($S = 1, T = 0$) on a singlet deuteron ($S = 0, T = 1$) leading to states of $S = 1, T = 1$ in the recoil nucleus ^{12}C . For ^{12}C these states lie much higher in excitation energy than $S = 1, T = 0$ states resulting from capture on a triplet deuteron ($S = 1, T = 0$).

Figures 4-30 and 4-31 present missing mass distributions with high and low recoil momentum cuts, and with opening angle cuts in the region of the free deuteron opening angle and off the deuteron angle. For low momentum cuts the peak at 12 MeV (peak 1) predominates while for higher values of recoil momentum more contribution arises from the second peak at 30 MeV. Opening angle cuts on the free deuteron angle enhance peak 1 and lead to a decrease in the contribution to the missing mass spectrum arising from the second peak. Off the deuteron angle the contribution from peak 2 becomes stronger. To point out further differences in the composition of events in peak 1 and peak 2 we will present in the following sections various distributions for events in the missing mass region 0 to 20 MeV and 20 to 40 MeV.

^{14}N Peak 1 - Results

Since the events in the first missing mass peak can be associated with recoil momentum values which are mostly less than 100 MeV/c, we have applied the PWIA to calculate theoretical distributions in recoil momentum, relative momentum, and Treiman- Yang angle. A comparison with the data is hampered somewhat by the poor statistics that we obtained resulting from low count rates. Also because the liquid nitrogen target was bulky we were not able to make extended runs at different values of opening angle. However our geometrical efficiency at the arm setting of $\theta_A = 57^\circ$ and $\theta_B = 105^\circ$ was good enough to allow us to see events in the region of 140° to 178° . As was the case with the oxygen data we obtained the value for p_0 , and the values for α_0 and

Figure 4.30 Excitation Energy, ^{14}N Events, for Various Regions of Recoil Momentum

Figure 4.31 Excitation Energy, ^{14}N Events, for Different Opening Angle Values

α_2 from a fit of the form factor to the $(\Delta\sigma/\text{PHASE SPACE})$ versus recoil momentum for the ^{14}N data. Our phase space acceptance was good for recoil momentum up to more than 250 MeV/c. So therefore, the fit was made in the region of recoil momentum 0 to 250 MeV/c. The values obtained for p_0 , α_0 , and α_2 are

$$p_0 = 180 \text{ MeV/c}$$

$$\alpha_0 = .63$$

$$\alpha_2 = .37$$

Figure 4-32 presents the distribution of $(\Delta\sigma/\text{PHASE SPACE})$ along with the curve obtained using the above values for p_0 , α_0 , and α_2 in the expression for the form factor squared

$$|\phi(p_R)|^2 = \alpha_0 |\phi_0(p_R)|^2 + \alpha_2 |\phi_2(p_R)|^2$$

Using the value of p_0 obtained from the fit to our data, we obtain a value of the harmonic oscillator length parameter r_0 , equal to 1.1 fm. This leads to an r.m.s. radius for nitrogen of 1.6 fm. This is considerably smaller than the 2.48 fm. obtained from electron scattering.⁶⁴ As was the case with the oxygen data, this indicates some breakdown in the applicability of the PWIA since the values obtained for r_0 are not completely realistic. Attempts to fit the $(\Delta\sigma/\text{PS})$ distribution with a p_0 corresponding to a more reasonable value of r_0 led to a theoretical curve which was too narrow to fit the data over this range of recoil momentum

The experimental distributions and calculated curves for recoil and relative momentum appear in Figs. 4-33, 4-34 for various regions of opening angles. The theoretical curves have been normalized to the

Figure 4.32 Fit of $|\phi(p)|^2$ to $(\Delta\sigma/\text{PS})$ Distribution, ^{14}N
Events (0 to 20 MeV)

Figure 4.33 Recoil Momentum Distribution, ^{14}N Events (Peak 1)

Figure 4.34 Relative Momentum Distribution, ^{14}N Events (Peak 1)

data distributions in the region $157 \leq \theta_{AB} \leq 167$. Within the statistics the data distributions and the calculated curves agree in shape and magnitude even for off deuteron angles. The Treiman-Yang distributions as a function of opening angle cuts are shown in Fig. 4-35 and Fig. 4-36. For the data on and around the free deuteron opening angle a distribution for low values of recoil momentum ($p_R \leq 100$ MeV/c) has been made. The distribution is isotropic as predicted when absorption occurs on a cluster which is in a relative s state with respect to the rest of the nucleus.

Finally a distribution in opening angle is shown in Fig. 4-37. The distribution peaks around the free deuteron opening angle of 162° . Its width is comparable to the width of the opening angle distribution for ^{16}O events occurring in the region $-2 \leq \Delta M \leq 10$ MeV.

We have obtained a value for the effective number of deuterons in ^{14}N equal to 5.6 ± 0.8 . This value for θ^2 is higher than the value obtained by Balashov^{60,61} for the (p,pd) reaction on ^{14}N . If one sums the results of Balashov over the $J = 0$, $T = 0$ states at 0.0 and 10.1 MeV, the $J = 1$, $T = 0$ states at 12.73 and 19.5 MeV, and the $J = 2$, $T = 0$ states at 4.43, 16.1, and 18.4 MeV, then the value for θ^2 is .7940. Our excitation energy spectrum differs from the results of Balashov in that he predicts strong excitation of the 2^+ state at 4.4 MeV whereas we see almost no excitation. If one does not include this level in the sum, then his value for θ^2 becomes .498. One difference between the calculated θ^2 of Balashov and the value which we obtain is the fact that he included only the transfer of a $l = 0$ deuteron.

Figure 4.35 Treiman-Yang Distributions, ^{14}N Events
(Peak 1) On Deuteron Angles

Figure 4.36 Treiman-Yang Distributions, ^{14}N Events
(Peak 1) Off Deuteron Angles

Figure 4.37 Opening Angle Distributions, ^{14}N Events
(Peak 1)

Our ($\Delta\sigma$ /PHASE SPACE) distribution was fit with a contribution from $l = 0$ and $l = 2$ terms. The reason Balashov does not include a $l = 2$ term is not clear. For instance Cohen and Kurath, in a similar fractional parentage calculation, have included the $l = 2$ term. If the $l = 0$ and $l = 2$ coefficients of Cohen and Kurath for pickup in the ^{14}N nucleus are summed over the levels for 0 to 20 MeV excitation energy, then the contribution for $l = 0$ terms is about 57% while the $l = 2$ contribution is 43%.

^{14}N Peak 2 - Results

Unbiased distributions in recoil and relative momentum for peak 2 events are shown in Fig. 4-38. The distributions have been integrated over the range of 140° to 178° in opening angle and over missing mass values from 20 MeV to 40 MeV. The recoil momentum distribution is peaked at higher values than was the case for the recoil momentum distribution for peak 1 events. There is a large contribution from events with recoil momentum greater than 150 MeV/c. Some of the events at high values of recoil momentum may arise from the scattering of one or both of the protons off the recoiling nucleus thereby redistributing the recoil momentum distribution.

Figure 4-39 presents Treiman-Yang angle distributions with and without recoil momentum cuts. For low recoil momentum values the distribution is fairly isotropic.

The opening angle distribution for peak 2 events is shown in Fig. 4-40. The opening angle distribution shows some peaking at the

Figure 4.38 Relative and Recoil Momentum Distributions
 ^{14}N Events (Peak 2)

Figure 4.39 Treiman-Yang Distributions,
 ^{14}N Events (Peak 2)

Figure 4.40 Opening Angle Distributions, ^{14}N Events (Peak 2)

free deuteron opening angle of 162° . The opening angle distribution is wide with a large contribution arising from events with opening angles in the region 170° to 180° .

CHAPTER 5

RESULTS: $A(\pi^+, p)A-1$ REACTION

Introduction

This chapter describes the results of our measurement of cross sections for the (π^+, p) reaction on ${}^9\text{Be}$, ${}^{12}\text{C}$, and ${}^{16}\text{O}$. For these nuclei the data were taken at a laboratory angle of 25° with the special arrangement described in chapter 2 - that is, with the addition of absorber to reduce the energy of the emitted proton (~ 200 MeV) so that it could be stopped in our detector, and with the addition of two extra counters for better dE/dX separation of protons and charged particle background. Also described are the results from the ${}^6\text{Li}(\pi^+, p){}^5\text{Li}$ reaction. The protons from this process had a maximum energy of 178 MeV. This energy occurred for emission at a laboratory angle of 30° , which was the most forward angle covered by our apparatus. These protons lost about 2.7 MeV in passing through the material in front of the NaI counter, and entered the crystal with an energy of approximately 175 MeV. The range of a 175 MeV proton in NaI is 9.4 cm and our detectors were 10.16 cm thick which corresponds to the range of a 184 MeV proton. The effective thickness of the crystal was even greater for protons which did not enter it perpendicularly and which did not scatter out its sides. Care was taken in the data analysis of the data to include only those protons whose projected position at the rear of the detector was within the limits of the crystal. The situation improved as we went to larger angles,

since the proton energy decreases for larger laboratory angles. Our crystals were therefore thick enough to stop the most energetic proton from the ${}^6\text{Li}(\pi^+, p){}^5\text{Li}$ process, and we were able to take (π^+, p) data simultaneously with data from the ${}^6\text{Li}(\pi^+, pp){}^4\text{He}$ reaction.

Results - ${}^9\text{Be}$, ${}^{12}\text{C}$, ${}^{16}\text{O}$

Figures 5-1, 5-2, and 5-3 show the missing mass spectra obtained as a result of the following (π^+, p) reactions:

Also shown with these missing mass distributions are the thresholds for various background reactions which have three and four particles in the final state. The curves which appear in the distributions represent the phase space of these reactions, adjusted so as to obtain the best fit with the data. The data have been corrected for inefficiencies resulting from event vertex reconstruction (12%) and from proton losses in the absorber and NaI (24%). The (π^+, p) missing mass resolution was 3 MeV FWHM. The errors quoted are statistical. Systematic errors are small and no attempt has been made to include them. Events which appear at values of negative missing mass are background probably due to scattered electrons and scattered pions which converted to π^0 . In the latter case the decay γ would be detected by the NaI detector. For the oxygen and beryllium data

Figure 5.1 ^8Be Excitation Energy Spectrum

Figure 5.2 ${}^{11}\text{C}$ Excitation Energy Spectrum

Figure 5.3 ^{15}O Excitation Energy Spectrum

there is little or no contribution to the missing mass spectrum in the region 0 to 18 MeV other than that associated with pion and electron background. Only for the $^{12}\text{C}(\pi^+,p)^{11}\text{C}$ process do we see any contribution to the missing mass in excess of background. Our missing mass resolution did not permit us to separate individual levels in the residual ^{11}C nucleus.

Our geometrical acceptance for an arm setting of 25° , at which these (π^+,p) events were taken, allowed us to cover a range in πp angles from 15° to 40° with good efficiency. For our ^{12}C data we had enough (π^+,p) events to divide them into two regions of πp angle, the first of which was centered at 20° and extended from 15° to 25° , and the second which extended from 26° to 38° and was centered at 32° . We present in Table 9 the cross sections obtained from the ^{12}C data within these two angular regions. Also presented in the table are the results obtained for the (π^+,p) process on ^9Be and ^{16}O . Since for these two nuclei the data do not rise substantially above the background levels, the results presented are considered to be upper limits.

Our ^{12}C results are compared with two previous measurements of the $^{12}\text{C}(\pi^+,p)^{11}\text{C}$ process in figure 5-4. These measurements were performed by Witten et al.³ at laboratory angles of 0° and 11° with a resolution of 12 MeV. They measured cross sections of $430 \pm 110 \mu\text{b/sr}$ (0°) and $640 \pm 130 \mu\text{b/sr}$ (11°) for capture on a $1p_{3/2}$ nucleon leading to the ground and low lying states of ^{11}C . It appears that our cross section results are lower than those obtained from these experiments.

Figure 5.4 Angular Distribution, $^{12}\text{C}(\pi^+, p)^{11}\text{C}$ Events

TABLE 9

CROSS SECTIONS $-(\pi^+, p)$ REACTION

<u>Nucleus</u>	<u>Θ(Deg)</u>	<u>ΔM Interval of Integration</u>	<u>Cross Section ($\mu\text{b/sr}$)</u>
^9Be	25	0 - 16	35 ± 11
^{12}C	20	0 - 6	78 ± 30
^{12}C	20	0 - 8	110 ± 39
^{12}C	32	0 - 6	41 ± 18
^{12}C	32	0 - 8	58 ± 23
^{16}O	25	0 - 8	11 ± 6

Our results are more consistent with a recent measurement of positive production by 185 MeV protons on ^{12}C leading to states in ^{13}C .⁶⁶ In this experiment emission leading to the ground state was measured to be about 200 nb/sr for a laboratory angle of 25° . By detailed balance the cross section to the ground state for the reaction $^{13}\text{C}(\pi^+, p)^{12}\text{C}$ would be 4 - 5 $\mu\text{b/sr}$. Discounting the difference of one unit in isotopic spin and assuming that each state for ^{11}C in the interval 0 to 8 MeV is equally excited, then the cross section for $^{12}\text{C}(\pi^+, p)^{11}\text{C}$ would be about 25 $\mu\text{b/sr}$.

Figure 5-5 illustrates the prediction of various calculations for capture on a $p_{3/2}$ nucleon. Curve 1 is obtained from the results of LeTourneux and Eisenberg³⁸ who calculated the capture on one nucleon assuming no distortion of the incoming pion and outgoing wave functions. Curve 2 is due to Jones and Eisenberg⁴⁰ and includes distortions of both the pion and proton. Curve 3 represents the calculations of Kaushal and Waghmare³⁹ using a Hartree Fock model of the nucleus and assuming plane waves for the pion and outgoing proton. The only curve which obtains a result close to our values is the curve of Eisenberg and Letourneux. However, this calculation suffers from an extreme sensitivity of the cross section to the harmonic oscillator parameters used to represent the neutron-hole state of the residual nucleus.

Kaushal-Waghmare and Eisenberg-Letourneux have also calculated the cross section for the $^{16}\text{O}(\pi^+, p)^{15}\text{O}$ reaction using the method described above. Figure 5-6 shows their calculated curves for capture

Figure 5.5 ^{12}C Experimental Results and Theoretical Curves

Figure 5.6 ^{16}O Experimental Point and Theoretical Curves

on a $p_{1/2}$ or $p_{3/2}$ shell nucleon along with the upper limit which we obtain for the cross section measured at 25° .

Results - ${}^6\text{Li}$

The (π^+, p) data described in this section were obtained during our (π^+, pp) runs on ${}^6\text{Li}$. We were therefore able to measure the differential cross section for laboratory angles in the range of 30° to 93° .

For the (π^+, p) process on ${}^6\text{Li}$ the energy of the protons ranges in value from 165 to 178 MeV for laboratory angles in the interval 94° to 30° . Our NaI detectors were able to stop these protons without the need for an additional absorber as was the case with the (π^+, p) measurements described previously. The NaI calibrations used in the analysis of this data were obtained from the (π^+, pp) process leading to the ground state of ${}^4\text{He}$. The method of extracting these calibrations has been described in Chapter 3. For forward arm settings the energy of the protons used in the calibrations were approximately 120 MeV, while for the backward angles the protons were of energy 80 MeV. For the forward arm we were able to extrapolate up to the (π^+, p) proton energy of 162 to 175 MeV with a ± 2 MeV uncertainty. Since the calibration of the backward arm was obtained with lower energy protons, and the response of the backward NaI was not linear up to energies of 160 MeV, we were not able to use reliably the data taken at arm angles of 98° and 123° . Here the uncertainty in the energy extrapolation was $\pm 5-6$ MeV.

For these measurements of the ${}^6\text{Li}(\pi^+, p){}^5\text{Li}$ reaction we did not have the two extra dE/dx counters which were inserted for the (π^+, p) data taken at 25° . Our particle identification was not as good for this case but we were still able to separate π^+ 's and electrons from protons. A missing mass spectrum made with a dE/dx cut for π^+ 's and electrons is essentially flat in the missing mass region of -10 to 50 MeV.

Figure 5-7 presents a missing mass distribution for events taken at a laboratory angle of 53° . Noticeable in the distribution is a peak centered at 5 MeV with a FWHM of 6 MeV which corresponds to excitation of the 4.5 MeV level in ${}^5\text{Li}$. This level has a width of 4 to 5 MeV. Our missing mass resolution for protons of these energies is about 3 MeV. Little evidence is seen for capture leading to the ground state of ${}^5\text{Li}$.

Shown with the missing mass spectrum are the thresholds for background reactions involving π^+ capture leading to the emission of a proton. The residual nucleus ${}^5\text{Li}$ decays into ${}^4\text{He}$ and a proton at an energy of -1.98 MeV on this missing mass scale. Protons from π^+ capture on ${}^6\text{Li}$ with the ejection of two energetic protons, and which lead directly to states in the residual ${}^4\text{He}$ nucleus, result in high values of ${}^5\text{Li}$ missing mass.

Figure 5-8 presents the measured cross section as a function of laboratory angle for events arising from capture on a $p_{1/2}$ nucleon. The cross sections have been determined by summing over events in the range of -5 to 15 MeV missing mass. No attempt has

Figure 5.7 ${}^5\text{Li}$ Excitation Energy Spectrum, $\theta = 53^\circ$

Figure 5.8 Angular Distribution, $^6\text{Li}(\pi^+, p)^5\text{Li}$ Events

been made to correct for background due to pions and electrons which were not discriminated against by our dE/dX counters and which may appear in our missing mass spectrum. The data have been corrected for spark chamber efficiency and vertex reconstruction efficiency (88%) and for losses in the NaI detector resulting from inelastic processes (28%). The errors on the data points include both statistical errors and the error resulting from the uncertainty in the energy calibration at these proton energies.

The cross section in this range of laboratory angle (30° to 94°) decreases as one goes to larger angles. For $\Theta = 36^\circ$ we measured a cross section of $65 \mu\text{b/sr}$ while at $\Theta = 87^\circ$ the value for the cross section was $\sim 19 \mu\text{b/sr}$.

There are some indications that the cross section values level off for backward angles. As mentioned before our data taken for arm settings greater than 80° suffered from large errors due to calibration and pulse height response. This limits the conclusions that one can draw concerning the behavior of the cross section for angles greater than 90° .

CHAPTER 6

SUMMARY AND CONCLUSIONS

We have examined the (π^+ ,pp) process on a variety of nuclei for a variety of opening angles. Our data have been presented in an unbiased form suitable for comparison with any available theory. In particular we have compared our experimental data with the predictions of the pole model or plane wave impulse approximation, both on and off the deuteron angle.

For the process ${}^6\text{Li}(\pi^+,pp){}^4\text{He}$ (ground state) we have found quite good agreement between the data and pole model predictions when a Hulthen wave function is used to describe the motion of the deuteron with respect to the alpha cluster. This agreement was consistent for all distributions which we examined both on the deuteron angle and off. The cross section for ground state events followed the $(A + \cos^2 \Theta_{cm})$ dependence predicted by the pole model even for events with relatively high recoil momentum, and which were off the deuteron angle. The value obtained for the effective number of deuterons, Θ^2 , was consistent with the value obtained by a previous (π^+ ,pp) experiment. The agreement between ground state events and the pole model is not too surprising since the ${}^6\text{Li}$ nucleus can be described very well in terms of some state of relative motion between a quasi-deuteron and a quasi-alpha cluster.

The interpretation of events resulting from absorption leading to excited states of ${}^4\text{He}$ is complicated due to our inability to distinguish individual final states. These events have been interpreted

in terms of absorption on a quasi alpha particle. The peak may also be due to effects resulting from the interactions of one or both of the emitted protons with the recoiling alpha core. Data in this peak have been shown, through the opening angle distribution and the center of mass angle dependence, to exhibit general characteristics which suggest that these events are also influenced by the quasi deuteron absorption mechanism.

The events arising from π^+ absorption on ^{16}O and ^{14}N have been analyzed in terms of the plane wave impulse approximation with shell model harmonic oscillator wave functions as a basis for the form factor. Qualitative agreement between experimental and PWIA distributions was obtained, although the agreement was not as good as was the case in the pole model analysis of ^6Li ground state events. The opening angle distribution given by the PWIA was found to be wider than the data distribution. With the use of harmonic oscillator wave functions, the PWIA predicted more high momentum components than we saw experimentally. The agreement between the PWIA and the experimental data may be improved if a different wave function is used to obtain the form factor. There are indications that oxygen events having high recoil momentum and occurring off the deuteron angle may exhibit some deviations from the center of mass behavior of the free deuteron cross section. Deviations from the predictions of the plane wave impulse approximation may be expected to occur since the quasi-deuteron is bound inside the oxygen nucleus with an energy of approximately 22 MeV. The quasi-deuteron is rather far off the mass shell, and the assumption that the cross section for

absorption on a quasi-deuteron exhibits the same features as the cross section for absorption on a free deuteron is probably not valid.

The comparison of the PWIA predictions and $^{14}\text{N}(\pi^+, pp)^{12}\text{C}$ events occurring with a ^{12}C excitation energy of around 12 MeV (peak 1) showed general agreement. The comparison was hampered somewhat by poor statistics and the fact that our coverage in opening angle was not as great as was the case with ^6Li and ^{16}O . We were also not able to test the center of mass behavior of peak 1 events.

Our calculated values for the effective number of deuterons in ^{16}O agreed with the results of Balashov when compared in the region of excitation energy 0 to 10 MeV. Agreement was obtained even though some very general assumptions were made concerning the probability of the transfer of a $l = 0$ or $l = 2$ deuteron. For ^{14}N our values of Θ^2 disagreed sharply with the predictions of Balashov. Some of the disagreement may result from the fact that we had a contribution arising from the transfer of a $l = 2$ deuteron whereas Balashov calculated a $l = 0$ transfer even though both types were possible.

Our excitation spectra obtained for absorption on ^{16}O and ^{14}N showed little excitation of the ground state for both nuclei. Our results were in disagreement with the level strengths predicted by Kopaleishvili, and with the fractional parentage calculations of Cohen and Kurath.

To summarize our (π^+, pp) results one can say for the case of ^6Li ground state events the pole approximation applies quite well even relatively far off the deuteron angle. Further experiments will

need increased event rates in order to investigate regions in opening angle further from the deuteron angle. Increased energy resolution is needed to better understand the events in the region of the excited states of ^4He . The oxygen and nitrogen data show general agreement with the predictions of the plane wave impulse approximation. There are some regions where deviations from the PWIA are indicated. Definite conclusions concerning these regions are hampered due to poor statistics. Once again a requirement for future experiments is a higher event rate so that data can be gathered with better statistics in regions of low cross sections. A better comparison with theory could be made if improved energy resolution allowed one to separate individual levels.

Our results for the (π^+, p) reaction on ^9Be , ^{12}C , and ^{16}O show that the experimental cross sections are lower than most theoretical predictions. The results vary widely from nucleus to nucleus with a measurable cross section obtained for ^{12}C , whereas virtually no excitation was seen for ^9Be and ^{16}O . The (π^+, p) data suggest that this process provides a sensitive test of the assumptions used to describe the incoming pion and the outgoing proton. Nuclear structure effects also play an important role in the values obtained for the cross sections.

The lithium (π^+, p) data show definite excitation of the $p_{1/2}$ unbound state at 5 MeV with perhaps some excitation of the ground state. Angular distributions indicate that the cross section levels off as one goes to backward angles. At the present time no theoretical calculation exists for the $^6\text{Li}(\pi^+, p)^5\text{Li}$ reaction.

For future (π^+ ,p) experiments, the requirements are much the same as was mentioned for (π^+ ,pp) experiments. Increased beam intensity will be needed to measure low cross sections and to obtain angular distributions. Sufficient energy resolution will be needed to distinguish individual final states for a variety of nuclei.

APPENDIX A

BEAM TRANSPORT MATRICES

The first and second order beam transport matrices were calculated through the use of the program SOTRM,⁶⁷ which for a given magnetic field, integrates along a specified set of rays to determine the transformation elements. A field map of the pion spectrometer served as input to the program. The program then calculated the matrix elements in the following manner.

For a specified magnetic field one can choose a reference particle which has momentum $\vec{p}_0$ and which experiences a field $\vec{B}_0$. This particle defines a reference curve s_0 , and by integrating the equations of motion for the reference particle

$$\frac{d}{dt} \vec{p}_0 = \frac{q_0}{m} \vec{p}_0 \times \vec{B}_0$$

and

$$\frac{d}{dt} \vec{r}_0 = \frac{\vec{p}_0}{m}$$

its coordinates can be found in a global reference frame. The motion of nearby particles is governed by similar equations. By transforming the equations of motion for a nearby particle so that its trajectory is described in terms of the reference orbit, then its coordinates can also be found.

The coordinates of the reference orbit and those of a nearby particle are then used to obtain the transformation elements. The orbit of a particle depends upon its initial conditions. By using a

suitable coordinate system and a reference orbit, the transformation solution can be expanded in terms of the initial conditions. The initial condition vector is specified by the five quantities

$$\vec{t} \equiv \{ \delta x, \delta \Theta, \delta y, \delta \psi, \delta P/p_0 \}$$

which are defined with respect to the reference orbit. The coordinates of the reference orbit are used as the origin of the system. The solution vector is expanded in terms of the initial vector through the use of a Taylor series. The first and second partial derivatives are calculated and are used to obtain the first and second order matrix elements. For example the first partial derivative can be obtained by defining the initial vector such that all components are zero except for the j^{th} component

$$t^{(j)} \equiv \{ 0, 0, a_j, 0, 0 \}$$

If $y(t^{(j)})$ is the value of the solution for a particular initial vector $t^{(j)}$, then the partial derivative F_{ij} can be obtained by tracing a reference ray and five adjacent rays. The second order terms are found in a similar but more complicated manner. Here the tracing of a greater number of rays is needed to obtain the diagonal and off-diagonal matrix elements.

APPENDIX B

ELECTRONICS

Listed below are the name, number, and manufacturer of the modules used in the logic system described in Chapter 3.

<u>Model</u>	<u>Description</u>	<u>Manufacturer</u>
AN102	DC Mixer	E. G. and G.*
T140	Quad Discriminator (Zero Cross)	E. G. and G.
C102	Dual Coincidence	E. G. and G.
C104	Coincidence (Four Fold)	E. G. and G.
F108	Dual Fanout	E. G. and G.
AD128	128 Channel ADC	E. G. and G.
TR104	Dual Trigger	E. G. and G.
TH200	TAC	E. G. and G.
LG102	Linear Gate and Stretcher	E. G. and G.
520	100 MHz Scaler	LRS**
153	Binary to BCD Converter	LRS
321B	Quad Discriminator	LRS
156	Scaler Digital Display	LRS

*E. G. and G., Inc., Salem, Mass.

**Lecroy Research Systems Corp., West Nyack, N.Y.

<u>Model</u>	<u>Description</u>	<u>Manufacturer</u>
1148	Spark Chamber Scaler	SAC*
1308	Readout Indicator	SAC
1110	Test Generator	SAC
1109	Spark Chamber Preamp	SAC
7050	Nidbus Master Control	LBL**
7920	Word Adder	LBL
8590	Bit Scrambler	LBL
8410	Teletype Control	LBL

* Science Accessories Corporation, Southport, Conn.

** Lawrence Berkeley Laboratory, Berkeley, Calif.

APPENDIX C

TAPE & STORAGE FORMAT

			Pointer	
			Mailbox	
Start Tape Record	10 words	Flags	AB	NR5Z
	10	Preliminary Info.	AC	BUFF
	740	Event Buffer	A9	SCBF
	60	Scaler Buffer	AD	HDSF
	100	Header Buffer	AE	DW13
	80	DVM Buffer	AF	LPBF
End Long Record	30	L.P. Info. & Hisc.	80	SVFL
	40	Overflo Reg.	81	

EVENT BUFFER
(as written on tape)

APPENDIX D

BERKELEY EXPERIMENT PARAMETER LIST

1	$(x_1 - x_{ave})/o$	0=no missing tracks	51	kx	$\vec{k} = \vec{p}_A + \vec{p}_B - \vec{p}_\pi$
2		1=A missing	52	ky	
3		2=B	53	kz	
4		3=C	54	k	
5		4=D	55	$\Delta E = (E_A + E_B + E_N) - (E_\pi + M_T)$	
6	NTYPE	5=2 or more missing	56	cos θ_{pp}	
7		1=no missing sparks	57	θ_A	
8	NSPARK	2=1 or more missing sparks	58	θ_B	
9	KIRACK	1=only 1 track in each chamber	59	sin α	
10		2=more than 1 track in any ch.	60	sin α^2	
11	CHISQUARE(JP=1,8)		61	$\cos^{-1}(\cos \theta_{pp})$	
12		straight line fit for each plane	62	pa - pb	
13			63	PHA	
14			64	PHB	
15			65	1 for entry into Cerb	
16			66	IEVENT	
17	X^2 (vertex)		67	ΔM	
18			68	LEVENT(1) (event register)	
19	X^2 (overall fit)		69	IFIRE	
20	Ta		70	cos $\theta_{\pi A}$	
21	TB		71	cos $\theta_{\pi B}$	
22	T π		72	Ta - Ta (deut.)	
23	ΔM		73	Tb - Tb (deut.)	
24	cos θ_{pp}	(π^+ , 2p)	74		
25	K		75	JTILT(1)	
26	Xave	target	76	JTILT(2)	
27	Yave		77	NB(1,25)	
28	X	mod. C	78	NB(2,25)	
29	X'		79	NB(1,26)	
30	Y	mod. D	80	NB(2,26)	
31	Y'		81	LEVENT(4)a = ADC - A1	
32	X		82	LEVENT(4)b = ADC - A2	
33	X'		83	LEVENT(5)a = ADC - B1	
34	Y		84	LEVENT(5)b = ADC - B2	
35	Y'		85	LEVENT(6)a = TOF	
36	DST (hodoscope par.)		86	LEVENT(6)b	
37	IKODD		87	$\theta_A + \theta_B$	
38	Vert. hodo. element		88	$\Delta p/p$ (pion)	
39	Horiz. hodo. element		89	RUN NUMBER	
40	KIRACK(JP=1,8)		90	EVENT COUNT	
41			91		
42			92	$\cos^{-1}(\cos \theta_{np})$	
43		number of tracks in each plane	93	ϕ_{TY} (Treiman-Yang angle)	
44			94		
45			95	# of L.P.'s	
46	ΔM		96	ELAPSED TIME (minutes)	
47			97	LIGHT PULSER A (average)	
48	cos θ_{np}	(π^+ , p)	98	LIGHT PULSER B (average)	
49	Q		99	CHIX (vertex)	
50			100	CHIX (vertex)	

101 $\sqrt{(ADC-A1)(ADC-A2)}$
 102 $\sqrt{(ADC-B1)(ADC-B2)}$
 103 Ta (deuteron)
 104 Tb (deuteron)
 105 ΔE (deuteron)
 106
 107
 108 (Ta + Tb) (deuteron)
 109 (Ta + Tb)
 110
 111
 112
 113
 114
 115
 116 SCALERS 1-12 (UNCLEARED)
 117 CUMULATIVE TOTALS
 118
 119
 120
 121
 122
 123
 124
 125
 126
 127
 128
 129
 130 SCALERS 1-12 (UNCLEARED)
 131 INCREMENTS
 132
 133
 134
 135
 136
 137
 138 SCALERS 13-20 (CLEARED)
 139 INCREMENTS
 140
 141
 142
 143
 144
 145
 146 SCALERS 13-20 (CLEARED)
 147 CUMULATIVE TOTALS
 148
 149
 150
 151 PION NUMBER (CUMULATIVE)
 152 PION NUMBER (INCREMENT)
 153
 154
 155
 156

157
 158
 159
 160
 161
 162 CHISQ-P (1+2)
 163 CHISQ-P (3+4)
 164 CHISQ-P (1+2+3+4)
 165 CHISQ-PI(1+2)
 166 CHISQ-PI(3+4)
 167 CHISQ-PI(1+2+3+4)
 168
 169
 170
 171 1 IF CHISQ-P < CHISQ-PI (Using 4 Counters)
 172
 173
 174 CHISQ-P (individual counters)
 175
 176
 177 CHISQ-PI (individual counters)
 178
 179
 180
 181
 182
 183
 184
 185
 186
 187
 188
 189
 190
 191
 192
 193
 194
 195
 196
 197
 198
 199
 200
 201 EVENT REGISTER +5.2V
 202 INTERFACE +5 V
 203 INTERFACE +8
 204 INTERFACE -8
 205 SCOPE DAC -35
 206 SCOPE +35
 207 NIDCUSS RACK +24
 208 NIDCUSS RACK -24
 209 NIDCUSS RACK +12
 210 NIDCUSS RACK -12
 211 NIDCUSS RACK +6
 212 SAC SC SCALERS +6

CHI-Squared
 for Proton
 Hypothesis
 CHI-Squared for
 Pion Hypothesis

DVI
 CHANNELS
 0-24

213	NIM RACK	+24		269		
214	NIM RACK	-24		270		
215	NIM RACK	+12		271		
216	NIM RACK	-12		272		
217	CLEARING FLD.-SPARE MOD			273		
218	"			274		
219	"			275		
220	"	E1		276		
221	"	D3		277		
222	"	D2		278		
223	"	D1		279		
224	"	C3		280		
225	"	C2		281	Xa	
226	CLEARING FIELD	C1		282	Ya	TRAJECTORY EXTRAPOLATION CRYSTAL A (REAR),
227	"	B3		283	Ra	
228	"	B2		284	Ca	
229	"	B1		285	Xb	
230	"	A3		286	Yb	TRAJECTORY EXTRAPOLATION CRYSTAL B (REAR)
231	"	A2		287	Rb	
232	"	A1		288	Cb	
233	S.C. HIGH VOLTAGE	A		289		
234	"	B		290	$\left\{ \begin{array}{l} P_A + P_B \\ P_A - P_B \end{array} \right\}$	Center of mass quantities; sum vector, difference vector, and included angle
235	"	C		291	$\cos c$	
236	"	D		292	$\left\{ \begin{array}{l} P_A + P_B \\ P_A - P_B \end{array} \right\}$	Same quantities in lab system
237	"	E		293	$\cos c$	
238	SHUNT-HERMES			294		
239	SHUNT-PAN			295		
240	SHUNT-SELLONA			296		
241	TEMP-NaI A			297		
242	"	B		298		
243				299	EVENT REGISTER-BIT 0 (MOST SIG.)	
244				300	"	1
245				301	"	2
246				302	"	3
247				303	"	4
248				304	"	5
249	HV1-NaI A			305	"	6
250	"	B		306	"	7
251	HV2-NaI A			307	"	8
252	"	B		308	"	9
253	VOLTAGE-LIGHT PULSER A			309	"	10
254	"	B		310	"	11
255				311	"	12
256				312	"	13
257				313	"	14
258				314	"	15
259				315	X ² (lowest)	
260						
261						
262						
263						
264						
265						
266						
267						
268						

316 σ_1 }
 317 σ_2 }
 318 σ_3 }
 319 σ_4 } MULTIPLE SCATTERING
 320 σ_5 } ERRORS
 321 σ_6 }
 322 σ_7 }
 323 σ_8 }
 324 X_a } π, P_a INTERCEPT
 325 Z_a }
 326 X_b } π, P_b INTERCEPT
 327 Z_b }
 328 X } AVERAGE INTERCEPTS
 329 Z }
 330 $\sqrt{Ed}^2$ for $(\pi, 2p)$ INTERCEPT
 331 X } ROTATED
 332 Z }
 333 Δx } INCREMENT BETWEEN BEST
 334 ΔZ } TARGET ORIGIN AND LINE
 INTERCEPT
 335 $\sqrt{(\Delta x)^2 + (\Delta Z)^2}$
 336 } $X(NT=1,3)$ Target θ
 337 }
 338 }
 339 } $Y(NT=1,3)$
 340 }
 341 }
 342 } $X^2(NT=1,3)$
 343 }
 344 }
 345 } $Y^2(NT=1,3)$
 346 }
 347 }
 348 } $X^2(NT=1,3)$
 349 }
 350 }
 351 } X^2 RANK ORDERED, MIN. TO MAX.
 352 }
 353 }
 354 } Y^2 RANK ORDERED, MIN. TO MAX.
 355 }
 356 }
 357 }
 358 }
 359 TRACK(1,NTRACK,JP)
 360 POSITION IN CHAMBERS
 361
 362
 363
 364
 365 TRACK(2,NTRACK,JP)
 366 ANGLES IN CHAMBERS
 367
 368

369 Ta-Tb all events tried as $(\pi, 2p)$
 370 $F(1)$ }
 371 $F(2)$ } Added Range
 372 $F(3)$ }
 373 TPACOR } Calculated energies corrected
 374 TPBCOR } for absorber in arcs
 375 PHACAL } Calculated Pulse
 376 PHBCAL } Heights
 377 δ^A } $\delta = \frac{PH - PHCAL}{PH}$
 378 δ^B }
 379 PHACOR } Corrected Pulse Heights
 380 PHBCOR }
 381
 382 $\cos \theta_1$ }
 383 $\cos \theta_2$ }
 384 $W = E_1^2 + E_2^2$ }
 385 P' } Defined in C.N. of two
 386 P'_π } Protons
 387 θ_1 }
 388 θ_2 }
 389 $f_1^2 (\times 10^{-9})$ } $f_1 = \frac{v^2 P'_\pi}{P'_\pi P'_2} P_1 P_2$
 390 f_1^2 }
 391 P_R^2 } $f_T = f_1 / (P_R^2 + x^2)^2$
 392 }
 393 θ_A } $x = 60 \text{ MeV/c}$
 394 θ_B }
 395 $(\cos \theta_1)^2$ } Polar Angles in Modules A,B
 396
 397
 398
 399 δ^A_{COR} } $\delta_{COR} = \frac{PH_{COR} - PH_{CAL}}{PH_{COR}}$
 400 δ^B_{COR} }

APPENDIX E

ORTHONORMAL POLYNOMIALS

Listed below are the sets of polynomials (up to order 3) orthonormal on a unit circle, which were used to obtain the pulse height maps of the NaI crystals described in Chapter 3.

<u>n</u>	<u>m</u>	<u>Φ_{nm}</u>
0	0	1
1	0	2X
1	1	2Y
2	0	$\sqrt{2} (3X^2 + Y^2 - 1)$
2	1	$2 \sqrt{6} XY$
2	2	$\sqrt{2}(X^2 + 3Y^2 - 1)$
3	0	$4/\sqrt{5} (5X^3 + 3XY^2 - 3X)$
3	1	$4 (3X^2Y + Y^3 - Y)$
3	2	$4 (X^3 + 3XY^2 - X)$
3	3	$4/\sqrt{5} (3X^2Y + 5Y^3 - 3Y)$

APPENDIX F

EFFICIENCY CORRECTIONS AND ERRORS

Presented in the following tables are the correction factors needed to obtain the true number of events, N^{cor} , used to determine (π^+, p) and (π^+, pp) cross section. Also shown is the error associated with each correction factor.

CORRECTIONS - (π^+, pp) DATA

<u>Symbol</u>	<u>Description</u>	<u>Value</u>	<u>Error</u>
N_{π}	Fraction of π 's in beam	.62	.05
N_{eff}^A	Efficiency, spark chamber box A	.98	.02
N_{eff}^B	Efficiency, box B	.98	.02
$N_{\text{eff}}^{\text{C vertex}}$	Combined efficiency for box C and (π^+, pp) event vertex reconstruction	.86	.05
N_{int}^A	Fraction of protons seen after interaction in arm A absorber, NaI - A	.88	.04
N_{int}^B	Same as above for arm B	.90	.04

In the following table we present correction factors for (π^+, p) events. Since all (π^+, p) data presented here were taken with arm A, only efficiencies will be given for that arm. Also the values for N_{π} and N_{eff}^A have been given previously and will be omitted here.

CORRECTIONS - (π^+ , p) DATA

<u>Symbol</u>	<u>Description</u>	<u>Value</u>	<u>Error</u>
C vertex Neff	Combined efficiency for box C and for (π^+ , p) vertex reconstruction	.88	.05
N ^A _{int}	Fraction of protons seen after interactions in the absorber and NaI on arm A	.76* .72**	.04

* For (π^+ , p) data taken with copper absorber

** For ${}^6\text{Li}$ (π^+ , p) ${}^5\text{Li}$ data

APPENDIX G

THE PWIA DIFFERENTIAL CROSS SECTION EXPRESSION

In this Appendix we present a derivation of the plane wave impulse approximation expression of the differential cross section $\frac{d^3\sigma}{d\Omega, d\Omega_2, dE_2}$ for the (π^+, pp) process based largely on a similar derivation by Jain et al.⁵⁷ for the quasi elastic knockout of clusters. We will not present all of the steps in the derivation but will refer to the results obtained by Jain. It is our intention to show how the cross section obtained by Jain can be generalized to the case of the (π^+, pp) reaction. Finally we will present a reduction of the PWIA cross section expression into the pole model form used by the CERN group² to illustrate the equivalent nature of these two cross section expressions.

The co-ordinate system that will be used in the derivation is shown in Fig. G-1. The initial wave function of the target nucleus A is taken to be

$$\psi_A = \psi_b \psi_B \psi(\vec{r}_b - \vec{r}_B)$$

where ψ_b is the wave function of the cluster, b in its ground state (a quasi deuteron in the case of the (π^+, pp) reaction), ψ_B is the ground state wave function of the residual A-2 system, and $\psi(\vec{r}_b - \vec{r}_B)$ is the wave function of the relative motion of the clusters. This expression corresponds to the first term in the cluster model expansion of the ground state wave function for the

THREE BODY SYSTEM INITIAL FINALTWO BODY SYSTEM INITIAL FINALFigure G.1 Impulse Approximation, (π^+, pp) Reaction

nucleus A. In Fig. G-1 the initial state consists of an incident pion (1) with momentum $\vec{p}_0$, and a quasi deuteron (2) in some state of relative motion with a residual A-2 system (3). The final state consists of the particles 1' and 2' (the protons) produced by the absorption of the pion, and the recoil nucleus (3). The c.m. co-ordinate of the target nucleus is represented by

$$(1) \quad \vec{r}_A = \frac{(m_2 \vec{r}_2 + m_3 \vec{r}_3)}{(m_2 + m_3)}$$

and the relative co-ordinate between the two clusters is given by

$$(2) \quad \vec{s} = (\vec{r}_2 - \vec{r}_3)$$

In the target nucleus A clusters 2 and 3 have momenta $\vec{q}_2$ and $\vec{q}_3$.

The wave functions of the unbound particles are represented by momentum eigenfunctions which are plane waves

$$(3) \quad \langle \vec{r} | \vec{p} \rangle = \left(\frac{1}{2\pi} \right)^{3/2} \exp(i \vec{p} \cdot \vec{r}).$$

The initial state wave function is given by

$$|i\rangle = |p_0, A\rangle$$

where $|A\rangle$ is the intrinsic state of the target. The final state is

$$|f\rangle = |\vec{p}_1, \vec{p}_2, \vec{p}_3\rangle$$

and the transformation matrix T for the reaction becomes in the laboratory system

$$(4) \quad T_{fi} = \langle f | T | i \rangle = \langle \vec{p}_1, \vec{p}_2, \vec{p}_3 | T^{3B} | \vec{p}_0, A \rangle$$

where T^{3B} is the three body transition operator. The use of the impulse approximation implies that particle 1 interacts only with

particle 2, and that particle 3 is a spectator to the reaction. The three body operator T^{3B} is replaced by the two body operator T^{2B} which describes the interaction of particles 1 and 2. In this approximation initial state interactions of particles 1 and 3 as well as final state interactions between particles 1' or 2' with 3 are neglected. Interaction and exchange effects are included through the use of T^{2B} . The transition matrix element becomes using the impulse approximation

$$(5) \quad T_{fi} = \langle \vec{p}_1', \vec{p}_2', \vec{p}_3 | T^{2B} | \vec{p}_0, A \rangle$$

The fact that the target nucleus is at rest in the laboratory system implies that the momentum of cluster 2 is equal and opposite to that of cluster 3. Furthermore, the transition matrix element can be written as⁵⁷

$$(6) \quad T_{fi} = \langle \vec{p}_1', \vec{p}_2' | T^{2B} | \vec{p}_0, -\vec{p}_3 \rangle \phi(-\vec{p}_3) \\ = T_{fi}^{2B} \phi(-\vec{p}_3)$$

where $\phi(-\vec{p}_3)$ is the Fourier transform of the motion of the clusters and $T_{fi}^{2B} = \langle \vec{p}_1', \vec{p}_2' | T^{2B} | \vec{p}_0, -\vec{p}_3 \rangle$. The momentum of cluster 2 is equal and opposite to the momentum of the residual nucleus $\vec{p}_3$, and thus this type of reaction can provide information about the momentum distribution of clusters (2) in the target.

A general expression for the cross section is given by⁶⁸

$$(7) \quad d\sigma_{fi} = \frac{4\pi^2}{V_{rel}} d^3p_1 d^3p_2 d^3p_3 \frac{\delta^4(p_f - p_i)}{(2S_1+1)(2S_2+1)} \sum_{S_i} \sum_{S_f} |t_{fi}^{3B}|^2$$

where t_{fi} is related to the scattering matrix S by

$$(8) \quad \langle f | S | i \rangle = \delta_{fi} - i S^*(P_f - P_i) t_{fi}$$

Since we have used the impulse approximation t_{fi}^{3B} is replaced by t_{fi}^{2B} for the two body system

$$t_{fi}^{3B} = \phi(-\vec{p}_3) t_{fi}^{2B}$$

The spins of the colliding particles are those of particle 1 and the cluster 2. In order to relate t_{fi}^{2B} to the center of mass cross section, it is advantageous to define the Lorentz invariant amplitude $M^{2B}(\vec{p}_f, \vec{p}_i)$

$$(9) \quad |M^{2B}(\vec{p}_f, \vec{p}_i)|^2 = |\sqrt{E_1 E_2} t_{fi}^{2B} \sqrt{E_0 E_{2i}}|^2$$

where E_1 and E_2 are the energies of the two particles 1' and 2', E_0 is the energy of the incident pion, and E_{2i} is the initial energy of the absorbing cluster 2

$$E_{2i}^2 = m_2^2 + p_3^2$$

The expression for $d\sigma$ becomes

$$(10) \quad d\sigma_{fi} = \frac{(2\pi)^2 d^3 p_1 d^3 p_2 d^3 p_3}{v_{rel} E_0 E_{2i} E_1 E_2} \frac{1}{(2S_1+1)} \frac{1}{(2S_2+1)} S^*(P_f - P_i) \\ \times |\phi(-\vec{p}_3)|^2 \sum_{S_i} \sum_{S_f} |M_{fi}^{2B}|^2$$

The two body Lorentz invariant matrix element is related to the cross section in the center of mass by⁶⁸

$$(11) \quad \frac{d\sigma}{d\Omega} = \left(\frac{2\pi}{W}\right)^2 \left(\frac{p_f'}{p_i'}\right) \frac{1}{(2S_1+1)} \frac{1}{(2S_2+1)} \sum_{S_f} \sum_{S_i} |M_{fi}^{2p}|^2$$

where W is the total center of mass energy, and p_i' and p_f' are initial and final c.m. momenta. For the case of the free process, $\pi^+ + d \rightarrow p + p$, $p_f' = p_p'$ and $p_i' = p_\pi'$. Substituting this result into expression (10) gives

$$(12) \quad d\sigma_{fi} = \frac{d^3 p_1' \cdot d^3 p_2' \cdot d^3 p_3}{V_{rel} E_0 E_{2i} E_1 E_2} \delta^4(p_f - p_i) |\phi(-\vec{p}_3)|^2 W^2 \\ \times \left(\frac{p_\pi'}{p_p'}\right) \left(\frac{d\sigma}{d\Omega}\right)_{np}$$

The relative velocity is equal to p_0/E_0 and

$$\delta^4(p_f - p_i) = \delta^3(\vec{p}_0 - \vec{p}_3 - \vec{p}_1 - \vec{p}_2) \delta(E_f - E_i).$$

Substituting this into (12) gives

$$(13) \quad d\sigma_{fi} = \frac{p_1^2 \cdot dp_1 \cdot d\Omega_1 \cdot p_2^2 \cdot dp_2 \cdot d\Omega_2 \cdot d^3 \vec{p}_3}{p_0 E_{2i} E_1 E_2} \delta(\vec{p}_0 - \vec{p}_3 - \vec{p}_1 - \vec{p}_2) \\ \times \delta(E_f - E_i) |\phi(-\vec{p}_3)|^2 W^2 \frac{p_\pi'}{p_p'} \left(\frac{d\sigma}{d\Omega}\right)_{np}.$$

Since particle 3 is unobserved we integrate over $\vec{p}_3$ and obtain

$$(14) \quad d\sigma_{fi} = p_1' \cdot p_2' \cdot \delta(E_f - E_i) |\phi(-p_3)|^2 W^2 \frac{p_\pi'}{p_p'} \left(\frac{d\sigma}{d\Omega}\right)_{np} \\ \times \frac{dp_2 \cdot dE_1 \cdot d\Omega_1 \cdot d\Omega_2}{p_0 E_{2i} E_2'}$$

We are interested in the differential cross section projected onto the E_1' axis. Thus integrating over dp_2' using the relationship

$$\frac{d}{dp_2'} = \frac{dp_2'}{dE_f} dE_f$$

where

$$E_f = E_1' + \sqrt{m_2^2 + p_2'^2} + \sqrt{m_3^2 + (\vec{p}_0 - \vec{p}_1' - \vec{p}_2')^2}$$

$$E_i = E_0 + m_A$$

and the energy δ function, we obtain

$$(15) \quad \frac{d^3\sigma}{dE_1' d\Omega_1' d\Omega_2'} = \frac{p_1' p_2'^2 E_3}{p_0 E_{2i}} \frac{1}{(p_2' E_3 + E_2 \cdot (p_2' - p_0 \cos \Theta_2 + p_1' \cos(\Theta_1' - \Theta_2)))} \\ \times |\phi(-p_2)|^2 W^2 \frac{p_\pi'}{p_1'} \left(\frac{d\sigma}{d\Omega} \right)_{np}$$

which is the PWIA result for the differential cross section given in expression (4-2).

It now remains to relate expression (15) to the pole model cross section expression where indices 1 and 2 now refer to the outgoing protons.

$$(16) \quad \frac{d^3\sigma}{dE_1 d\Omega_1 d\Omega_2} = \Theta^2 \frac{\alpha}{\pi^2} \frac{W^2}{(p_R^2 + \alpha^2)^2} \frac{|F(p_R)|^2}{m_{np} p_\pi} \frac{p_\pi'}{p_1'} \left(\frac{d\sigma}{d\Omega} \right)_{np} \frac{p_1 p_2}{|1 - \vec{v}_2 \cdot \vec{v}_R| \frac{v_2^2}{v_1^2}}$$

In expression (15) the term

$$(17) \quad E_3 p_1 p_2' / (p_2 E_3 + E_2 (p_2 - p_0 \cos \Theta_2 + p_1 \cos(\Theta_1 - \Theta_2)))$$

can be related to the phase space term

$$\vec{p}_1 \cdot \vec{p}_2 / |1 - \vec{v}_2 \cdot \vec{v}_R / v_2^2|$$

appearing in the pole model expression. When the following relationships are used

$$(18) \quad \cos(\Theta_1 - \Theta_2) = \frac{\vec{p}_1 \cdot \vec{p}_2}{p_1 p_2}$$

$$\cos \Theta_2 = \frac{\vec{p}_2 \cdot \vec{p}_0}{p_2 p_0}$$

then

$$(19) \quad (p_2 - p_0 \cos \Theta_2 + p_1 \cos(\Theta_1 - \Theta_2)) = p_2 \left(1 - \frac{\vec{p}_2 \cdot \vec{p}_0}{p_2^2} (\vec{p}_0 \cdot \vec{p}_1) \right) \\ = \vec{p}_2 (\vec{p}_0 \cdot \vec{p}_1) / p_2^2$$

since $\vec{p}_0 = \vec{p}_1 + \vec{p}_2 + \vec{p}_3$. Expression (19) becomes

$$E_3 p_1 p_2^2 \left(\frac{1}{p_2 E_3 - \frac{(\vec{p}_2 \cdot \vec{p}_0) p_2 E_2}{p_2^2}} \right)$$

and after a few algebraic steps this is seen to be equal to

$$\frac{p_1 p_2}{|1 - \vec{v}_2 \cdot \vec{v}_R / v_2^2|}$$

The term $E_{2i} = \sqrt{m_2^2 + \vec{p}_2^2}$ is approximately equal to m_2^* (the reduced mass of the deuteron) since m_2^* is much larger than p_2^2 .

With these substitutions, expression (15) becomes

$$\frac{d^3\sigma}{d\Omega_i d\Omega_f dE_i} = \frac{1}{p_0 m_{np}} |\phi(p_R)|^2 W^2 \frac{p'_\pi}{p'_r} \left(\frac{d\sigma}{d\Omega} \right)_{np} \frac{p_1 p_2}{|1 - \frac{\vec{v}_2 \cdot \vec{v}_R}{v_2^2}|}$$

If the following quantities are set equal to each other

$$|\phi(p_R)|^2 = \Theta^2 \frac{\alpha}{\pi^2} \frac{|F(p_R)|^2}{(\alpha^2 + p_R^2)^2}$$

then the PWIA and pole model expressions for the differential cross section are equal. This is seen to be the case since Θ^2 appearing in expression (16) is just a normalization factor. Secondly, the expression given in Chapter 4 for the form factor $|F(p_R)|^2$ multiplied by $\frac{\infty}{\pi^2} \left(\frac{1}{p_R^2 + \kappa^2} \right)^2$ is just the Fourier transform of the Hulthen wave function

$$\phi(p_R) = \frac{1}{(2\pi)^{3/2}} \int d^3\vec{r} \psi_H(\vec{r}) e^{i\vec{p} \cdot \vec{r}}$$

(see Appendix H). Thus the PWIA and pole model cross section expressions are equivalent.

APPENDIX H

CLUSTER MOTION WAVE FUNCTIONS

We present in this Appendix the wave functions used to represent the motion of the clusters in the pole model and PWIA analysis of (π^+, pp) events described in Chapter 4. We begin with the wave equation for motion in a δ function potential to illustrate the normalization for the form factor, $F(p_R)$, used in expression (4-1). The solution for motion in a δ function potential is⁶⁹

$$\psi(r) = \sqrt{\frac{\alpha}{2\pi}} e^{-\alpha r}/r$$

where the normalization is such that

$$\int |\psi(\vec{r})|^2 d\vec{r} = 1$$

The Fourier transform, $\phi(p)$, of this wave function is determined from the relationship

$$\phi(p) = \frac{1}{(2\pi)^{3/2}} \int e^{i\vec{p}\cdot\vec{r}} \psi(\vec{r}) d\vec{r}$$

which leads to

$$\phi(p) = \frac{\sqrt{\alpha}}{\pi} \frac{1}{(\alpha^2 + p^2)}$$

Substituting this into the expression for $F(p_R)$ defined in Chapter 4 gives $F(p_R) = 1$ for the c.m. motion in a δ function potential.

The Hulthen wave function used to describe the motion of the clusters in the analysis of the lithium data is given by

$$\psi(r) = \sqrt{\frac{\alpha\lambda(\alpha+\lambda)}{2\pi}} \frac{1}{(\lambda-\alpha)} \frac{1}{r} [e^{-\alpha r} - e^{-\lambda r}]$$

with the Fourier transform of this wave function equal to

$$\phi(p) = \frac{\sqrt{\alpha\lambda(\alpha+\lambda)}}{\pi} \frac{(\lambda+\alpha)}{(\alpha^2+p^2)(\lambda^2+p^2)}$$

The expression for $|F(p_R)|^2$ then becomes

$$|F(p_R)|^2 = \frac{\lambda(\alpha+\lambda)^3}{(\lambda^2+p_R^2)^2}$$

For the oxygen and nitrogen data we used shell model harmonic oscillator wave functions to describe the motion of the clusters.

The general form for the harmonic oscillator wave function is⁷⁰

$$\psi(r) = N_{nl} \exp\left(-\frac{r^2}{2r_0^2}\right) r^l v_{nl}(r) Y_m^l(\theta, \varphi)$$

where r_0 is the harmonic oscillator length parameter

$$r_0 = \left(\frac{\hbar}{m\omega}\right)^{\frac{1}{2}}$$

N_{nl} is a normalization constant,

$$N_{nl}^2 = \frac{2^{\ell-n+3} (2\ell+2n-1)!!}{\sqrt{\pi} (n-1)! [(2\ell+1)!!]^2} \left(\frac{1}{r_0^3}\right)^\ell$$

and $U_{nl}(r)$ is the associated Laguerre polynomial

$$\begin{aligned} U_{nl} &= L_{n+\ell-\frac{1}{2}}^{\ell+\frac{1}{2}}(r/r_0) \\ &= \sum_{k=0}^{n-1} (-1)^k 2^k \binom{n-1}{k} \frac{(2\ell+1)!!}{(2\ell+2k+1)!!} \left(\frac{r^2}{r_0^2}\right)^k \end{aligned}$$

The square of the Fourier transform for the harmonic oscillator wave function is given by⁶²

$$\begin{aligned} |\phi(p)|^2 &= N_{nl} \left(\exp(-p/p_0)^2\right) \left(p/p_0\right)^{2\ell} |Y_\ell^m(p)|^2 \\ &\times \left| \sum_{k=0}^{n-1} (-1)^k 2^k \binom{n}{k} \frac{(2\ell+1)!!}{(2\ell+2k+1)!!} \left(p/p_0\right)^{2k} \right|^2 \end{aligned}$$

where

$$N_{nl} = \frac{2^{\ell-n+3} (2\ell+2n-1)!!}{\sqrt{\pi} (n-1)! [(2\ell+1)!!]^2 p_0^3}$$

The harmonic oscillator quantum numbers used in the oxygen and nitrogen data analysis were $n=2, l=0$, $n=1, l=2$. For the case in which $n=2, l=0$ the above expression gives

$$|\phi(\rho)|^2 = \frac{6}{\sqrt{\pi}} \frac{1}{4\pi\rho_0^3} \left(1 - \frac{2}{3}\left(\frac{\rho}{\rho_0}\right)^2\right)^2 \exp\left(-\frac{\rho}{\rho_0}\right)^2$$

For $n=1$ and $l=2$

$$|\phi(\rho)|^2 = \frac{1}{\pi} \frac{4}{15\sqrt{\pi}} \frac{1}{\rho_0^3} \left(\frac{\rho}{\rho_0}\right)^4 \exp\left(-\frac{\rho}{\rho_0}\right)^2$$

where the average has been taken over the values $-2 \leq m \leq 2$.

ACKNOWLEDGMENTS

The experimental investigations presented here were made in collaboration with members of the Medium Energy Physics Division of the Los Alamos Scientific Laboratory, as well as with participants from other universities. In particular, the author would like to express his gratitude to his thesis advisor, Dr. Stanley Sobottka of the University of Virginia, and to his site advisor, Dr. Robert Macek of group MP-6, LASL, for their guidance and support during all phases of the experiment. Special thanks are due to Dr. Robert Burman, MP-7, for his help and discussions during the experiment, and to Dr. Arch Thiessen, MP-4, for his invaluable assistance in the preparation of experimental equipment. The work of Dr. William Shlaer, who wrote the machine language computer programs used in the experiment, is acknowledged. Thanks are due to Dr. James Amato, formerly of MP-7, and Jean Oostens, formerly of MP-4, for their contribution to data collection and analysis. The work of the remaining participants in this experiment, Drs. Peter Barnes and W. C. Lam of Carnegie Mellon University, Drs. Peter Fessenden and Wayne Swenson of Oregon State University, and Drs. David Axen and Martin Salomon of the University of British Columbia is gratefully acknowledged.

The author would like to thank Bob Rajala, MP-1, and Charlie Dalton, MP-4, for their technical assistance and expertise contributed during the preparation and execution of the experiment. Also appreciated was the hospitality of the staff of the 184-Inch Cyclotron, Lawrence Berkeley Laboratory, which was shown to the LASL group during their stay at LBL. Finally, the financial support given to the author by Associated Western Universities, Inc., during the period of this experiment is gratefully acknowledged.

REFERENCES

1. R. L. Burman and M. E. Nordberg, Jr., Phys. Rev. Lett. 2(1968) 229.
2. J. Favier, T. Bressani, G. Charpak, L. Massonnet, W. E. Meyerhof and Č. Zupaničič, Nucl. Phys. A169 (1971) 540.
3. T. R. Witten, M. Blecher, and K. Gotow, Phys. Rev. 174 (1968) 1166; High Energy Physics and Nuclear Structure (Plenum Press, N.Y. 1970), ed., S. Devons, p. 374 ff.
4. T. Ericson, Phys. Lett. 2 (1962) 278.
5. T. Ericson, in Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, 1962.
6. M. Jean, in Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, 1962.
7. M. Jean, Nuovo Cimento Supplement 4 (1964) 400.
8. K. A. Brueckner, Phys. Rev. 84 (1951) 258.
9. S. Ozaki, R. Weinstein, G. Glass, E. Loh, L. Neimala and A. Wattenberg, Phys. Rev. Letters 4 (1960) 533.
10. H. Davies, H. Muirhead, and J. N. Wouds, Nucl. Phys. 78 (1966) 663.
11. M. E. Nordberg, Jr., K. F. Kinsey, and R. L. Burman, Phys. Rev. 165 (1968) 1096.
12. D. Chesire and S. E. Sobottka, Phys. Lett. 30B (1970) 244.
13. A. O. Vaisenberg, E. D. Kolgalova, and M. V. Rabin, Soviet Physics JETP 20 (1965) 854.
14. F. Calligaris, C. Cernigoi, I. Gabrielli, and F. Pellegrini, Nuc. Phys. A126 (1969) 209.
15. V. F. Kosmach, A. A. Kotov, and V. I. Ostroumov, Soviet Journal of Nuclear Physics 8 (1969) 18.
16. J. Afanas'ev and V. I. Ostroumov, Soviet Journal of Nuclear Physics 1 (1965) 463.
17. K. O. H. Ziock, C. Cernigoi, and G. Gorini, Phys. Lett. 33B (1970) 471.

18. P. J. Castleberry, L. Coulson, R. C. Minehart, and K. O. H. Ziock, Phys. Lett. 34B (1971) 57.
19. D. M. Lee, Thesis, University of Virginia (1971).
20. T. I. Kopaleishvili, Soviet Journal of Nuclear Physics 4 (1967) 382.
21. J. M. Eisenberg and J. LeTourneux, Nuc. Phys. B3 (1967) 47.
22. T. I. Kopaleishvili, Nuc. Phys. B1 (1967) 335.
23. S. G. Eckstein, Phys. Rev. 129 (1963) 413.
24. P. P. Divakaran, Phys. Rev. 139B (1965) 387.
25. D. S. Koltun and A. Reitan, Phys. Rev. 141 (1966) 1413.
26. D. S. Koltun and A. Reitan, Phys. Rev. 155 (1967) 1139.
27. D. S. Koltun and A. Reitan, Nuc. Phys. B4 (1968) 629.
28. R. Guy, J. M. Eisenberg, and J. Letourneux, Nuc. Phys. A112 (1968) 689.
29. R. I. Jibuti and T. I. Kopaleishvili, Nuc. Phys. 55 (1964) 337.
30. J. Dabrowski, Proc. Phys. Soc. 71 (1958) 658.
31. J. F. Dawson, I. Talmi, and J. D. Walecka, Ann. Phys. 18 (1962) 339.
32. W. Elsaesser and J. M. Eisenberg, Nuc. Phys. A144 (1970) 44.
33. R. S. Kaushal and Y. R. Waghmare, Nuc. Phys. A144 (1970) 449.
34. S. B. Treiman and C. N. Yang, Phys. Rev. Lett. 8 (1962) 140.
35. I. S. Slapiro, in Proceedings of the International School of Physics "Enrico Fermi," Course 38 (1967).
36. I. S. Shapiro, Soviet Physics Uspekhi 10 (1968) 515.
37. V. M. Kolybasov and T. A. Lomonosova, Soviet Journal of Nuclear Physics 11 (1970) 325.
38. J. Letourneux and J. M. Eisenberg, Nuc. Phys. 87 (1966) 331.
39. R. S. Kaushal and Y. R. Waghmare, Phys. Lett. 31B (1970) 637.

40. W. B. Jones and J. M. Eisenberg, Nuc. Phys. A154 (1970) 49.
41. M. Ruderman, Phys. Rev. 87 (1952) 383.
42. C. H. Q. Ingram, N. W. Tanner, and J. J. Domingo, Nuc. Phys. B31 (1971) 331.
43. Analysis program written by R. L. Burman with additions and modifications by R. J. Macek, W. C. Lam, J. Amato, and I. Arthur.
44. R. J. Macek, Private Communication.
45. D. F. Measday and C. Richard-Serre, Nuc. Instr. Meth. 76 (1969) 45.
46. J. N. Palmieri and J. Wolfe, Nuc. Instr. Meth. 76 (1969) 55.
47. Geometrical Efficiency Code written by S. E. Sobottka with modifications and additions by W. C. Lam, R. J. Macek, and E. Arthur.
48. E. Ferrari and F. Selleri, Nuovo Cimento Supplement 24 (1962) 453.
49. C. Richard-Serre, W. Hirt, D. F. Measday, E. G. Michaelis, M.J.M. Saltmarsh, and P. Skarek, Nuc. Phys. B20 (1970) 413.
50. J. C. Alder, W. Pollhopf, W. Kossler, C. F. Perdrisat, W. K. Roberts, P. Kitching, G. A. Moss, W. C. Olsen, and J. R. Priest, Phys. Rev. C6 (1972) 18.
51. I. A. Mackenzie, S. K. Mark, and Tseh Y. Li, Nuc. Phys. A178 (1971) 225.
52. N. F. Golovanova and N. S. Zelenskaya, Soviet Journal Nuclear Physics 8 (1969) 158.
53. C. M. Rose, Phys. Rev. 154 (1967) 1305.
54. R. B. Liebert, Thesis, The University of Rochester (1971)
55. C. L. Ruhla, M. Riou, J. P. Garron, J. C. Jacmart, and L. Massonnet, Phys. Lett. 2 (1962) 44.
56. C. L. Ruhla, M. Riou, M. Gusakow, J. C. Jacmart, M. Liu, and L. Valentin, Phys. Lett. 6 (1963) 282.
57. M. Jain, P. G. Roos, H. G. Pugh, and H. D. Holmgren, Nucl. Phys. A153 (1970) 99.
58. P. Troel and W. Bierter, Phys. Lett. 29B (1969) 21.

59. T. I. Kopaleishvili, I. Z. Machabeli, G. Sh. Gogsadze, and N. B. Krupennikova, *Soviet Journal of Nuclear Physics* 7 (1968) 198.
60. V. V. Balashov, A. N. Boyarkina, and I. Rotter, *Nucl. Phys.* 59, (1964) 417.
61. V. V. Balashov and A. N. Boyarkina, *Izv. Akad. Nauk. USSR (ser. fiz)* 28 (1964) 277.
62. J. R. Pizzi, Thesis, University of Lyon (1970).
63. S. Cohen and D. Kurath, *Nucl. Phys.* A141 (1970)
64. L.R.B. Elton, Nuclear Sizes, Oxford University Press, London, 1961.
65. P. Beregi, N. S. Zelenskaja, V. N. Neudatchin, and Yu. F. Smirnov, *Nuc. Phys.* 66 (1965) 513.
66. S. Dahlgren, P. Grafström, B. Hölstad, and A. Asberg, Preprint, Uppsala University (1971).
67. A program to generate first and second order transport matrices, E. R. Close, UCRL-19823.
68. K. Nishijima, Fundamental Particles, W. A. Benjamin, Inc., New York, 1964.
69. L. I. Schiff, Quantum Mechanics, McGraw-Hill, New York, 1955.
70. R. R. Roy and B. P. Nigam, Nuclear Physics, John Wiley & Sons, Inc., New York, 1967.

☆ U. S. GOVERNMENT PRINTING OFFICE: 1973 — 784-278/17