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A MODEL RELATING STRESS, STRAIN, STRAIN RATE
AND TEMPERATURE WITH AN EXPERIMENTAL DATA FIT
FOR HIGH-PURITY TANTALUM

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A MODEL RELATING STRESS, STRAIN, STRAIN RATE
AND TEMPERATURE WITH AN EXPERIMENTAL DATA FIT
FOR HIGH-PURITY TANTALUM

ABSTRACT

Using the model developed as a result of a strain energy to failure investigation and experimental results from three separate sources for high-purity tantalum, a behavior model fit has been accomplished. Tantalum has strengthening "peaks" that have been attributed to interstitial oxygen, nitrogen and hydrogen. These cause deviation from the "normal" straight line between θ and temperature in the model. However, behavior is consistent. Increasing strain rate above 10^{-2} /sec produces a sudden anomalous shift toward ductile behavior. Though the shift is sudden, the effect holds throughout the higher strain rates. Tentative stress-temperature relationships have been established by the model for the temperature range from the cryogenic to 2000°K . These, however, only apply for strain rates $<10^{-2}$ /sec. The model also seems to be valid for the higher strain rates, but needs verification to extend the ranges beyond room temperature. Not enough data were available to establish the strain variation with temperature, but a very adequate room-temperature strain model was established. This strain model was not affected by the anomalous stress behavior with strain rate, indicating that no structural (phase) change is associated with the stress behavior.

GENERAL FORM OF THE MODEL

A model was constructed to describe the strain rate behavior of several materials ranging from brittle to very ductile.¹ The brittle material increased in both stress and strain as strain rate increased; the strain energy under the curves increased substantially. In general the more ductile materials showed a more constant energy, with decreasing strain offsetting increasing stress. This led to the postulation of a model consisting of an infinite number of stress-strain curves of different strain rates stacked in order of strain rate to form a solid. The axes of the solid then would be stress (σ), strain (ϵ) and strain rate ($\dot{\epsilon}$).

Definition of the limits of such a solid led to the reasoning that stress must be bounded by a maximum — perhaps the theoretical strength of the material — and a minimum based on change of state — melting or softening point — close to zero stress. A maximum strain could be envisioned as corresponding to this zero stress point with minimum strain elsewhere in the model. Stress would increase in the direction of increasing strain rate. The strain to fit the situations of both brittle and ductile materials behavior must decrease and increase with increasing stress. Therefore, two maximum strains must exist with one minimum strain.

A linear transition between maximum and minimum did not seem to fit the case, particularly for the strain axis. As an approximation, a sine function appeared reasonable.

Construction of the model is shown in Fig. 1 in orthogonal projection and in Figs. 2 and 3 in perspective views. The stress function is a 90- to 270-deg cut of a sine wave where

$$\sigma = \sigma_{\max} [1 - \sin (\theta + 270)].$$

At right angles to the stress plane, another sine wave is displaced 90 deg so that strain is the function

$$e = e_{\max} (1 - \sin \theta) + e_{\min}.$$

The minimum strain is found where θ is 90 deg and the stress is $\sigma_{\max}$:

$$e_{\min} = \frac{\sigma_{\max}}{E},$$

where E is the elastic modulus. Strain rate is then an inverse function of θ in the model. Temperature is a direct function of θ . It could be speculated that the limits of the temperature scale are absolute zero and the softening point of the material. The limits of the strain-rate scale could be dislocation velocity at the highest rate, and approach zero at the other end.

The procedure to construct a model for a given material is shown in Appendix A. Basically, it is an iterative process involving two points (of stress and strain) at different strain rates. Additional points serve to verify the model parameters of $\sigma_{\max}$, $e_{\max}$ and $e_{\min}$. Models were constructed¹ for gold, thorium and titanium. Strain rates varied from 10^{-6} /sec to 10^3 /sec in this experiment. The results indicated conformation to the model within 3 or 4 parts per hundred.

The models established in Ref. 1 were based on a single temperature. The model fit established for the uranium - 7.5 niobium - 2.5 zirconium alloy² covered five orders of magnitude in strain rate and a temperature range of 23° to 500° C. The data conformed well to the model, suggesting a high confidence level for interpolation and even extrapolation.

SPECIFIC FIT OF THE TANTALUM DATA TO THE MODEL

The tantalum data were obtained from three sources:^{1,3,4} (1) the tantalum (5B) data from Ref. 1 which covers five orders of magnitude in strain rate but is limited to room temperature; (2) the stress-strain data from Figs. 7 and 8 in Ref. 3 which covered seven orders of magnitude and temperatures from 78°K to room temperature; and (3) from

Fig. 1. Orthogonal projection of the model solid, with governing equations.

Fig. 2. Perspective view of the model, with four differently shaped stress-strain curves shown as dotted lines at 1, 2, 3 and 4.

Fig. 3. Perspective view of the model, illustrating the change of elastic modulus.

Ref. 4 covering both cryogenic and high temperature ranges at slow strain rates for two different heat treatments of electron-beam-melted material. These three sources all used high-purity material, and so appear reasonably comparable.

Data from Ref. 1 were used to establish the $\sigma_{\max}$, $e_{\max}$ and $e_{\min}$. The procedure for this is shown in Appendixes A and B which also describe subsequent steps in the analysis. Room temperature (25°C) data from Ref. 3 were then checked using the parameters determined from Ref. 1 and found to be in good agreement (Table 1). The temperature data from both Refs. 3 and 4 were then used to plot the θ -versus-temperature plot of Fig. 4. Agreement between the data of these two references in the cryogenic region for the recrystallized material is very good. The assumption was then made that the unknown "slow rate" of Ref. 4 must be comparable to the 10^{-4} /sec rate of Ref. 3. Tabular data from the three sources are given in Table 1, and this in turn is the source of the plots of Figs. 4 and 5.

Table 1. Data tabulation.

Data from Ref. 1 (recrystallized material)

$\sigma_{\max} = 350$ ksi	} see Appendix A for calculations
$e_{\max} = 0.595$ in./in.	
$e_{\min} = 0.013$ in./in.	

$\dot{e}$ (in./in./sec)	σ (ksi)	θ (deg)	Calculated e	Measured e (1/2 in. g.l.) (1/8 in. diam)	T (°K)
5×10^{-2}	62	145.37	—	0.270	298
5×10^{-2}	63	145.09	0.268	0.276	298
5×10^{-2}	64	144.80	0.265	0.264	298
2200	84	139.46	0.221	0.213	298
2550	89	138.22	0.212	0.220	298

Data from Ref. 3 (recrystallized material)

1×10^{-4}	35	154.16	0.349	—	298
8×10^{-2}	40	152.34	0.332	—	298
170	70	143.13	0.251	0.254	298
4900	93	137.25	0.204	0.212	298
1×10^{-4}	35	154.16			298
	41	151.99			202
	66	144.24			144
	71	142.86			115
	97	136.29			78

Fig. 4. Temperature versus θ for tantalum.

Fig. 5. Strain rate versus θ for recrystallized tantalum.

Table 1 (contd).

$\dot{\epsilon}$ (in./in./sec)	σ (ksi)	θ (deg)	Calculated ϵ	Measured ϵ (1/2 in. g.l.) (1/8 in. diam)	T (°K)
<u>Data from Ref. 4, p. 217, Fig. 112 (wrought stress-relieved)</u>					
	205	114.47			18
	153	124.25			78
	80	140.48			198
	52	148.37			298
<u>Data from Ref. 4, p. 218, Fig. 113 (recrystallized material)</u>					
	130	128.95			18
	105	134.43			78
	53	148.06			198
	36	153.79			298
<u>Data from Ref. 4, p. 207, Table 64 (wrought stress-relieved)</u>					
	60.5	145.81			298
	48.3	149.54			423
	47.0	149.96			518
	49.7	149.09			583
	54.7	147.53			638
	45.7	150.39			763
<u>Data from Ref. 4, p. 207, Table 64 (recrystallized material)</u>					
	29.4	156.35			298
	27.9	156.97			503
	31.1	155.66			598
	30.1	156.06			663
	28.1	156.88			698
	27.0	157.35			763
<u>Data from Ref. 4, p. 213, Fig. 108</u>					
	14.8	163.28			1450
	10	166.27			1590
	5.2	170.11			1750
	3.5	171.89			1940

SOME OBSERVATIONS

The temperature-versus- θ plots show shape and location conformity between the wrought stress-relieved material and the fully recrystallized material. Furthermore, the

Ref. 3 data fits well with the Ref. 4 cryogenic data. The Ref. 4 data are self-consistent; that is, the cryogenic, medium- and high-temperature range data are from separate investigators, yet they fit the model plot reasonably well. As noted before, all the materials were apparently high-purity, and had consistent metallurgical treatment.

Tantalum apparently is sensitive to what in Ref. 4 is called interstitial alloys. This sensitivity results in strengthening "peaks", as shown in Fig. 6. The "peak" about 600°K is caused by oxygen, nitrogen or carbon as an interstitial.⁴ The "peak" in the cryogenic region is thought to be the result of decreasing hydrogen solubility;⁴ the temperature-versus- θ plot of unaffected materials appears to be a straight line between absolute zero and the softening point² of the material. It seems reasonable to attribute these deviations from straight-line behavior to these interactions plus high-temperature chemical reactions. These "peaks" could be looked upon as shifts of the normal behavior slope.

Figure 5 contains the surprise of the tantalum model. Plotting strain rate against θ for the 25°C data yields an abrupt displacement at about 6×10^{-2} /sec. It is not caused by two separate data sets because the intermingled data from the two sources define the sudden offset. As strain rate increases, this sudden jump toward ductile behavior does not appear in the room-temperature strain model, suggesting that there is no structural (phase) change corresponding to the anomalous stress jump (see Ref. 2). It has been suggested that this jump is caused by the interstitial atoms losing their strengthening effect. Whatever the mechanism, the anomalous stress jump looks real.

There was not enough available strain data to establish the strain-model behavior with temperature. However, the room-temperature strain model looks consistent and reasonable.

SOME TENTATIVE CONCLUSIONS

The model fits the available data reasonably well, even though tantalum is sensitive to quite a few strengthening interstitial materials. The model does accentuate the anomalous stress-strain-rate behavior. It is consistent with known behavior in that the strain model shows no corresponding strain-strain-rate effect, indicating no structural (phase) change.

Based upon available data for the two material treatments and for strain rates of 6×10^{-2} /sec or slower, the curves can be extrapolated from room temperature in either direction to cryogenic or high-temperature regimes (Fig. 5).

No supporting data are known in the higher than 6×10^{-2} /sec strain rate at temperatures other than 25°C. An interesting question arises regarding tantalum behavior in this high-strain-rate region. If the higher strain rates negate the strengthening effect of the interstitials, then does the material temperature-versus- θ plot revert to the "normal" behavior slope of Fig. 6? If so, then the anomalous ductile behavior of tantalum at cryogenic temperatures could be explained, at least in terms of the model.

Fig. 6. Effects of interstitial oxygen, nitrogen, carbon and hydrogen on the behavior of tantalum. (Source data from Ref. 4.)

USE OF THE MODEL

To use the model to interpolate or extrapolate these data requires only choosing a strain rate and a temperature, and finding a θ from Fig. 5. This θ then will produce a stress from the equation

$$\sigma = \sigma_{\max} [1 - \sin (\theta + 270)].$$

The tantalum stress model is limited in the strain-rate regime above 6×10^{-2} /sec except at 25°C. Behavior with temperature is not known at the present time in this range.

The $e_{\max}$ is determined from an $e_{\max}$ versus temperature curve, and the corresponding strain is found from the equation

$$e = e_{\max} (1 - \sin \theta) + e_{\min}.$$

In the case of tantalum, only room-temperature strain data are available, and the model is presently limited in this respect.

ACKNOWLEDGMENTS

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REFERENCES

1. D. E. Lord, A Proposed Model Relating Stress, Strain, Strain Rate and Strain Energy, Lawrence Livermore Laboratory, Rept. UCID-16253 (1973).
2. D. E. Lord, A Model Relating Stress, Strain, Strain Rate and Temperature with an Experimental Data Fit for a Uranium - 7.5 Niobium - 2.5 Zirconium Alloy, Lawrence Livermore Laboratory, Rept. UCID-16254 (1973).
3. K. G. Hoge and A. K. Mukherjee, The Temperature and Strain Rate Dependence of the Flow Stress of Tantalum, Lawrence Livermore Laboratory, Rept. UCRL-74477 (1973).
4. F. F. Schmidt, Tantalum and Tantalum Alloys, DMIC Report 133, OTS PB 151091, July 25, 1960.

APPENDIX A. EQUATIONS DESCRIBING THE MODEL

$$\sigma = \sigma_{\max} [1 - \sin (\theta + 270^\circ)], \quad (1)$$

$$e = e_{\max} (1 - \sin \theta) + e_{\min}, \quad (2)$$

$$e_{\min} = \frac{\sigma_{\max}}{E}. \quad (3)$$

Putting (1) into the form

$$\begin{aligned} \theta &= \arcsin \left(1 - \frac{\sigma}{\sigma_{\max}} \right) - 270^\circ \\ \text{or} \quad \theta &= \arcsin \left(1 - \frac{\sigma}{\sigma_{\max}} \right) + 90^\circ \end{aligned} \quad (4)$$

and (2) into the form:

$$e_{\max} = \frac{e - e_{\min}}{1 - \sin \theta} \quad (5)$$

Evaluate (4) for values of $\frac{\sigma}{\sigma_{\max}}$ using a known σ and e (σ is in ksi and e in in./in.).
For tantalum (Table 1),

$$\frac{\sigma}{\sigma_{\max}} = \frac{62}{350} \quad \text{Point 1:}$$

$$\begin{aligned} \theta &= \arcsin \left(1 - \frac{62}{350} \right) + 90^\circ \\ &= 55.37^\circ + 90^\circ \\ &= 145.37^\circ \end{aligned} \quad (4)$$

$$e_{\min} = \frac{350 \times 10^3}{27 \times 10^6} = 0.013 \quad (3)$$

$$e_{\max} = \frac{0.270 - 0.013}{1 - \sin 145.37^\circ} = 0.595 \quad (5)$$

$$\frac{\sigma}{\sigma_{\max}} = \frac{89}{350} \quad \text{Point 5:}$$

$$\begin{aligned} \theta &= \arcsin \left(1 - \frac{89}{350} \right) + 90^\circ \\ &= 48.22^\circ + 90^\circ \\ &= 138.22^\circ \end{aligned} \quad (4)$$

$$e = (0.595) (1 - \sin 138.22^\circ) + 0.013$$

$$= 0.212$$

$$\text{Measured } e = 0.220$$

(2)

$$\frac{\sigma}{\sigma_{\max}} = \frac{64}{350} \quad \text{Point 3:}$$

$$\theta = \arcsin \left(1 - \frac{64}{350} \right) + 90^\circ$$

$$= 54.80^\circ + 90^\circ$$

$$= 144.80$$

(4)

$$e = (0.595) (1 - \sin 144.80^\circ) + 0.013$$

$$= 0.265$$

$$\text{Measured } e = 0.264$$

(2)

All other points were done in like manner.

APPENDIX B. PROCEDURES FOR ESTABLISHING AND USING THE MODEL

1. Establish $\sigma_{\max}$, $e_{\max}$ and $e_{\min}$ for a single temperature. Room-temperature data are the most consistent and make the best base. Appendix A covers this procedure.
2. Find $e_{\max}$ for each temperature group using the $\sigma_{\max}$ and $e_{\min}$ found in step 1.
3. Plot θ versus temperature for each strain-rate group. Extrapolation of these lines should intersect the 180° θ in the vicinity of the softening point of the material.
4. Plot θ versus strain rate (strain rate on multicycle log scale) for each group of temperatures. Use the room-temperature data to establish the line. Project a family of parallel lines based on the relation of θ and temperature found in step 3.
5. Plot $e_{\max}$ versus temperature. A continuous slope indicates single-phase material. Abrupt changes in slope indicate phase changes.

Once the model is established, interpolation and extrapolation can be done by choosing an $\dot{\epsilon}$ and a temperature. Then find the θ corresponding to these from the curve. Stress is then found from

$$\sigma = \sigma_{\max} [1 - \sin (\theta + 270)].$$

Then from the $e_{\max}$ versus temperature curve, the proper $e_{\max}$ is obtained and the strain is found from

$$e = e_{\max} (1 - \sin \theta) + e_{\min}.$$

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